Equity weights under various conceptions of fairness

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Introduction

- Weighted CBA:

$$\sum_i \beta_i wtp_i \gtrless 0$$

- Distributional weights in CBA require interpersonally comparable indexes and a social welfare function $W(u_1, ..., u_n)$, so that

$$\beta_i = \frac{\partial W}{\partial u_i} \frac{\partial u_i}{\partial m_i}$$

1. Interpersonal comparisons
2. Social welfare function
    1. without risk
    2. with risk

Interpersonal comparisons

- Interpersonal comparisons are value-laden: who is better off?
- Three approaches:
  1. Objective list (e.g., capabilities)
  2. Subjective utility (e.g., satisfaction or happiness)
  3. Compare indifference curves (e.g., money-metric utility: budget that gives indirect utility equal to current utility)
- 1 and 3 prove that "utility" indexes are not necessary
- Only 2 and 3 respect intrapersonal comparisons at a given moment (2 may have problems over time due to adaptation and self-anchoring)
- Only 3 can respect interpersonal comparisons done by individuals themselves (at least those having the same preferences). More generally, *a higher indifference curve corresponds to a better situation*.

Examples of approach 3

- Equivalence approach: Define a set of reference situations S_λ that are ranked in an obvious way, then compute

$$u_i\left(x_i\right) = u_i\left(S_\lambda\right)$$

  and use λ to compare individuals.
- Example: money-metric utility. S_λ is a budget (that can extend to non-market goods, by giving them a price)
- Adler's approach: Rely on scaled VNM utilities (0 and 1 at reference situations)
- This too only involves ordinal preferences (it avoids the scaling problems of subjective utility).
- **Fairness principles**:
  (a) respect preferences (incl. for interpersonal comparisons);
  (b) for some reference situations, preferences are not needed for the comparisons.

The three approaches summarized

$$\text{Subj.Utility} = f\,(\text{aspects; preferences over sure options; risk attitude; scaling})$$

	aspects	preferences	risk attitudes	scaling
objective	$\surd$			
subj. utility	$\surd$	$\surd$	$\surd$	$\surd$
equivalence	$\surd$	$\surd$	$(\surd)$	
scaled VNM	$\surd$	$\surd$	$\surd$	

"A higher indifference curve is better": equivalence satisfies it in absence of risk for indifference curves on sure options; in contrast, scaled VNM respects it only when the full indifference curve on risky prospects is higher.

A practical issue

$$\text{Subj.Utility} = f\left(\begin{array}{c}\text{observable aspects; unobservable aspects;}\\ \text{preferences; risk attitudes; scaling}\end{array}\right)$$

	obs. aspects	unobs. asp.	pref.	risk att.	scaling
objective	$\surd$				
subj. utility	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
equivalence	$\surd$		$\surd$	$(\surd)$	
scaled VNM	$\surd$		$\surd$	$\surd$	

What is more important? Eliminate scaling or include unobservable aspects?

Example (Fleurbaey Luchini Muller Schokkaert Health Econ 2013): CBA weights when u is money-metric utility ("equivalent income" with good health as the reference) and the only aspects observed are income and health.

In Decancq Fleurbaey Schokkaert (wp), it seems that the residual of the satisfaction equation reshuffles the distribution at the bottom a lot.

Social welfare

- In absence of risk, the problem of the social welfare function is simple: how much inequality aversion over the individual indexes?
- Under risk, choosing the social welfare function is complex: ex ante versus ex post, and issues within the latter
- This interferes with the choice of indexes

The Harsanyi trilemma

- It is impossible to combine 3 properties:
  1. Pareto (respecting individual risk attitudes)
  2. Expected social welfare (rationality in social evaluation)
  3. Inequality aversion over VNM utility functions
- Harsanyi (1955) shows that 1&2 imply that the SWF must be affine in individual VNM utilities
- This leaves three options:
  a) 1&2: (Weighted) utilitarianism (Harsanyi's choice)
  b) 1&3: Inequality-averse SWF on expected utilities (Diamond's choice)
  c) 2&3: Ex post inequality-averse SWF (Adler's choice)

The problems with utilitarianism and ex ante egalitarianism

- Utilitarianism: There is no reason why inequality aversion over income (a social ethical issue) should coincide with individual risk aversion (an individual, prudential issue)
- Ex ante egalitarianism: it violates not just independence and time consistency, but statewise dominance. Very irrational.
- One can make it rational by saying that even ex post, the only thing that matters is the initial expected utilities. This does not respect individual preferences (in interpersonal comparisons): the lucky is better off than the unlucky even if their expected utilities were equal.

The problem with ex post egalitarianism

- It does not respect risk attitudes; BUT...

1. It can respect some: Statewise dominance & Pareto when equality ex post is guaranteed $\rightarrow$ SW $=$ Expected EDE
   (EDE: "equally distributed equivalent":
   $W\left(u_1, ..., u_n\right) = W\left(EDE, ..., EDE\right)$)
   Proof:
   - In every state, replace the final allocation by its EDE
   - One obtains a prospect with full equality ex post
   - Apply Pareto: individual EU, and they are equal
   (What if equality is not defined in terms of VNM utilities? Later)
2. Pareto not compelling when people disagree ex post (spurious unanimity)
3. It can also take account of the ex post utility cost of ex ante risk (and replicate Diamond's preference for a fair lottery)

The tetralemma

- It is hard to combine:
  1. Pareto (even weakened to cases of ex post equality)
  2. Statewise dominance (weaker than Expected social welfare)
  3. Inequality-aversion (in a flexible way)
  4. Separability (Independence of the utilities of the indifferent riskfree)
- Compromises (Fleurbaey Zuber 2013): for instance,

$$\sum_{s \in S} \pi_s \prod_{i \in N} (\varepsilon U_i^s - 1)^{\frac{1}{n}}$$

  This is more inequality averse than $\frac{1}{1-\alpha} \sum_{i \in N} (U_i^s)^{1-\alpha}$, for a given $\alpha > 1$, if and only if U_i is confined to $\left(\frac{1}{\varepsilon}, \frac{1}{\varepsilon} \frac{\alpha}{\alpha - 1}\right)$.
- Note that it depends on n; Independence of the indifferent riskfree would bring back weighted utilitarianism.

When individual indexes are not VNM utilities

- Interpersonal comparisons ex post made with $v_i(x_{is})$, risk preferences in VNM $u_i(x_i)$.
- A new version of Harsanyi's theorem
- (Fleurbaey Zuber 2014) Pareto when there is no inequality ex post & Expected social welfare $\rightarrow$ SWF affine in the individual EU of the EDE
  Proof:
  - $EW(u_1, ...u_n) = H(Eu_1, ..., Eu_n)$ on the domain $U(EDE)$.
  - All functions EW, Eu_i are mixture preserving.
  - Therefore EW is affine in $(Eu_1, ..., Eu_n)$.
- Open question: how to choose the weights?

A particular solution?

- Pareto for allocations with ex post equality or no risk and Statewise dominance $\rightarrow EDE\left(CE\left(EDE\right)\right)$
  Proof (Fleurbaey Zuber 2014):
  - in each final allocations, replace with the EDE $v_i = v_j \; (\forall i,j)$
  - this is an egalitarian prospect, which is Pareto indifferent to $CE\left(EDE\right)$: i.e., for each individual replace the EDE prospect by its certainty-equivalent
  - one then has a riskless allocation (unequal when heterogeneous risk aversion) that can be evaluated by the EDE again.
- Problem with this: it satisfies statewise dominance but violates eventwise dominance.
- Exception to this violation: it is an Expected social welfare when there is an individual with the greatest risk aversion, and the EDE = min. Another argument for the maximin?

Example of violation of eventwise dominance

	Act f			Act g		
States	1	2	3	1	2	3
Individual 1	1	3	$3a-4$	0	b	$3a-3$
Individual 2	1	3	$3a-4$	0	b	$3a-3$
EDE	1	3	$3a-4$	0	b	$3a-3$
$ce_1(EDE)$		1			0	
$ce_2(EDE)$		a			$a-1+b/3$	
Conditional on $\{1,2\}$						
$ce_1(EDE)$		1			0	
$ce_2(EDE)$		2			$b/2$	

For a and b large enough and moderate inequality aversion, f is worse than g conditionally on $\{1,2\},\{3\}$; but better than g unconditionally.

Subjective probabilities and ambiguity

- Subjective probabilities:
  - spurious unanimity
  - rationality of the evaluator: use the evaluator's own beliefs.
- Ambiguity aversion?
  - violation of eventwise dominance
  - rejection of free information
- BUT a rational evaluator may take account of individual aversion to paternalism

What next?

- Identify the demands of rationality (EW? Why?)
- Incorporate ex ante fairness and ex ante perspective in the ex post evaluation
- The risk on population size
- Applications of the ex post approach

VSL

- Usual ex ante approach: WTP for fatality risk reduction
- Two problems: 1) ex ante (hard to apply to social decisions); 2) everyone dies (so far, few exceptions)
- Ex post approach: social WTP for life extension

$$W\left(U_1\left(c_1, l_1\right), \ldots, U_n\left(c_n, l_n\right)\right)$$

  where c consumption, l longevity (Cf Adler, Hammitt, Treich)
- Social WTP for Δl for a fraction of the population: depends on who benefits and who pays.
- Final formula similar to weighted version of classical VSL: fraction replaces probability, inequality aversion replaces risk aversion...

Insurance markets

- Driven by ex ante preferences
- Work well for replaceable losses
- Problematic for irreplaceable losses: may penalize the losses that reduce marginal utility

Unemployment insurance

(Gollier 1991, 1993)
VNM utility $u(c - m_i\ell)$, where m_i is i's reservation wage, and $\ell = 1$ when employed
In state s, consumption w_s for the workers, b_s for the unemployed.
Expected profits must satisfy: $\sum_s p_s(f_s(L_s) - w_s L_s - (1 - L_s)b_s) \geq \bar{v}$, where $(1 - L_s)b_s$ is perfect experience rating
Spot-market: $f_s'(L_s) = w_s - b_s$. A worker is employed if $w_s - m_i \geq b_s$, so that total employment is determined by

$$L_s = F(w_s - b_s),$$

where F is the CDF of m_i.

$$Eu_i = \sum_s p_s[u(\max\{w_s - m_i, b_s\})].$$

Unemployment insurance

Full ex ante Pareto efficiency in risk sharing would require making $\max\{w_s - m_i, b_s\}$ a constant for every worker i, which is impossible with a uniform allowance b_s. A constrained efficient allowance program $\{b_s\}$ must make the following expression a constant across states of nature:

$$\int_0^{w_s-b_s} \lambda(m)\, u'(w_s - m)\, dF(m) + \int_{w_s-b_s}^{+\infty} \lambda(m)\, u'(b_s)\, dF(m).$$

The consequence is that b_s will be countercyclical with respect to w_s.
Let SW be the CE(EDE):

$$ESWF = \sum_s p_s u \circ \varphi^{-1}\left(\int_0^{w_s-b_s} \varphi(w_s - m)\, dF(m) + \int_{w_s-b_s}^{+\infty} \varphi(b_s)\, dF(m)\right)$$

The constant expression is now:

$$\frac{u'(EDE_s)}{\varphi'(EDE_s)}\left(\int_0^{w_s-b_s} \varphi'(w_s - m)\, dF(m) + \int_{w_s-b_s}^{+\infty} \varphi'(b_s)\, dF(m)\right)$$

Now b_s may be cyclical (in particular if φ is more concave than u).

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