What is a SIFI?
On the Systemic Importance of Financial Institutions as determined by an Extended CAPM with Systemic Risk [1]

Jean-Charles Garibal [a, d], Patrick Kouontchou [b], Bertrand Maillet [a, c, 2]

[a] Variances - Paris, France.
[b] Univ. Lorraine (CEREFIGE) - Metz, France.
[c] EMLyon Business School (CEFRA) - Paris, France.
[d] Univ. Orleans (LEO/CNRS) - Orleans, France.

Very preliminary version – May 2017

Abstract

We propose herein to test an extension of the traditional Fama and French (1993) extended Capital Asset Pricing Model (three-factor CAPM), in which is added a factor represented by an Index of Systemic Risk Measures (ISRM), built thanks to a Sparse Principal Component Analysis (SPCA) of a large set of systemic risk measures, as recently proposed by Giglio et al. (2016). The empirical tests of the CAPM with Systemic risk (CAPMS) we run on the American market, show that the new systemic risk factor is highly significant when pricing assets. We lastly propose an original application of the CAPMS related to a new methodology for designating and ranking Systemically Important Financial Institutions (SIFI), based on ordered significant sensitivities to the ISRM: the more sensitive, the most important.

JEL Classification: C45, C53, C58, G01, G11.

Keywords: Systemic Risk, CAPM, SIFI

---

[1] We thank here Sylvain Benoit, Christophe Hurlin, Catherine Lubochinsky, Joël Petey, Roger Stein, Bertrand Tavin, Sessi Tokpavi and Anne-Gaël Vaubourg, for comments, positive feedback and encouragements. All authors thank the *GRI in Financial Services* (www.globalriskinstitute.org), as well as the Louis Bachelier Institute for supports. Extra materials linked to this article can be found at: www.systemic-risk-hub. **Preliminary version (*work in progress*) - do not quote, reference or distribute without an explicit agreement of the authors. The usual disclaimer applies.**

[2] EMLyon Business School (CEFRA) et Variances. Correspondance: Pr. Bertrand Maillet, EMLyon Business School - Paris Campus, 15 boulevard Diderot, F-75012 Paris, France. *Courriel* : bmaillet@em-lyon.com.

1. Introduction

The recent financial crisis of 2008 was characterized both by the speed of financial contagion and by its strong negative impact on the real sector - with the consequent contraction of economic activity in many developed countries. One of the major issues in financial economics as a result of these turbulent events was, first, to try to agree on one (or more) definition(s) of systemic risk, multifaceted by nature, when emphasizing one or such essential characteristic of financial institutions. Indeed, the intention was to highlight the different aspects of this risk: size of the financial institution in shock, leverage risk and extreme market liquidity shortage phenomenon of interconnections between actors and contagion shock, have been identified as key elements of a systemic crisis. Once these aspects were identified, the objective was to build relevant analysis tools for measuring the systemic risk. Many authors have proposed measures reflecting both the general state of the system to distinguish the main institutions contributing to the overall risk. The academic literature also accompanied the new imperative by banking regulatory authorities, proposing a number of systemic risk or metric measurements. Indeed, there are two types of measures: the individual measures that assess systemic risk institutions in isolation and those that are designed to measure the overall systemic risk. In the first category, we can find for instance the Conditional Value-at-Risk (CoVaR) of Adrian and Brunnermeier (2016), the Marginal Expected Shortfall (MES) of Acharya et al. (2013) and Brownlees and Engle (2017), and the SRISK by Acharya et al. (2012) and Brownlees and Engle (2017). In the second group of measures, we find, among the main, the Spillover Index (SI) of Diebold and Yilmaz (2009) and the Dynamic Causality Index (DCI) of Billio et al. (2012).

However, recent works showed that the definition of a "good" measure of systemic risk still remains unresolved, 1) because of the observed empirical redundancies in the various measures of systemic risk, 2) because of the model risk associated with their estimates and 3) due to the absence of an objective criterion to tell us about the relevance of the different approaches. Thus, recent articles have focused on the implied model risk in the implementation of these different metrics. Daníelsson et al. (2016) and Benoit et al. (2017a) show that a large majority of individual measures of systemic risk strongly depends upon extreme percentiles of returns, and therefore inheriting the risk of such uncertain quantities. Actually, model risk seems to be largely underestimated in practice (Boucher et al., 2014 and 2016), and it is heterogeneous in the different steps, leading to significant discrepancies in the rankings of systemic institutions (Benoit et al., 2017a; Nucera et al., 2016; Kouontchou et al., 2015). Consequently, measures of overall systemic risk constructed as a weighted sum of individual measures are also subject to model risk (see Moreno and Peña, 2013).

A solution that recently appeared in the literature on systemic risk to mitigate the model risk is to construct aggregate indexes from different metrics existing systemic risk. The objective is to obtain a risk index which diversifies the model risk. As part of the quantification of the overall systemic risk, this approach is retained by Giglio et al. (2016) who identify an aggregate index at a given date as the common signal extracted from time-series of various metrics of systemic risk, recovered through a Principal Component Analysis (PCA). In their empirical investigations, it appears that this index predicts periods of sharp slowdown in economic activity - which is ultimately the economic criterion that should be the most important. Regarding the individual measures, Nucera et al. (2016) adopt a similar technical approach and they infer a rating issued from other noisy and divergent rankings of competing measures. Beyond the diversification of the model risk, it should be noted that the aggregation also allows us to synthesize the different dimensions of systemic risk (size, leverage, interconnections, liquidity etc.).

Once the overall level of systemic risk is established, the objective of this article was consequently to provide a simple theoretical framework to illustrate several channels through which systemic risk could affect asset prices. This model is a useful first step in understanding how a number of recent empirical findings could be explained. Indeed, systemic risk intuitively seems to play a role in the assessment of asset prices, although this role is not easily identified due to the nonlinear relationship between systemic risk to asset prices. Also, the role of interconnections between institutions has been highlighted since the 2008-2009 crisis. We try to show in the following how systemic risk alters the equilibrium relationship of the Capital Asset Pricing Model (CAPM; see Sharpe, 1964), and the estimations of risk premia related to other factors. We thus show that a portfolio with a high systematic risk has also a significant systemic risk. More specifically, our methodology proceeds in three steps.

In the first step, we retain as a construction tool for the aggregate index: the "Sparse" Principal Component Analysis (SPCA). This method of dimension reduction, as opposed to the standard PCA used in Giglio et al. (2016) in the building of systemic risk indexes, allows us to select a predefined number of active components in the index. In this case, it has the advantage to retain, for the construction of the aggregated risk index, the measures that best explain some output targeted data observations.

The second step is devoted to the endogenization of the smoothing parameter that governs the scarcity of the SPCA. In our case, the optimal value of this parameter is obtained by retaining the aggregate index that Granger causes extreme variations in economic activity. The inference is here performed using the nonparametric test of causality in extreme risks of Hong et al. (2009). This approach has the advantage to test for a large number of time windows in the causal relation, with higher discounting the longest horizons.

The third step concerns the pricing of financial assets. Whether the systemic risk related to some extent to each asset, has an impact or not on prices is hereafter our first question. If so, the CAPM does not take into account the existence of a so-called "systemic risk premium". Indeed, the Capital Asset Pricing Model (CAPM), based on the definition of two parameters (see Sharpe, 1964; Lintner, 1965; Mossin, 1966), is the reference model for the valuation of financial assets and premiums related to factors of risk. However, recent experience of financial crises highlighted the importance systemic risk; in other words, the risk that the financial system as a whole collapses should enter into the global picture. This has led some economists to question the validity of such a model, merely because it does not explicitly take into account the existence of systemic risk. Intuitively, it seems that the existence of potential systemic shocks, particularly marked for financial institutions in the last crisis, is expected to have an impact on asset prices and should allow investors to differentiate securities. Holding a portfolio of systemic institutions should in fact lead to the revelation of a specific premium. It thus seems now important to consider studying the impact of systemic risk in terms of valuation of financial assets. According to our empirical analysis conducted in the US market, the systemic risk revealed to be a significant component of compensation on certain securities.

Furthermore, protecting the financial system is the main aim of some regulatory bodies. Detection and surveillance of the most important institutions, called Systemically Important Financial Institutions (SIFI), is one of their main objective. Our intuition is that sensitivities of financial institutions to a global systemic risk indicator, measured in a sound asset pricing model such as the proposed Capital Asset Pricing Model with Systemic risk (CAPMS), can help to better designate and control such important institutions, based on an objective valuation criterion. This leads to an original application of the CAPMS regarding the designation and ranking of SIFI, as a final added-value and by-product of our proposal for asset pricing with systemic risk.

The rest of this article is organized as follows. In the first part, we present the construction method of systemic risk index with an illustration on the US market. In the second part, we present the CAPM and its extension (Fama and French, 1993), and we present the Capital Asset Pricing Model with Systemic risk (CAPMS), as well as our main empirical results on the US Equity market. Finally, we elaborate a proposition for a methodology when designating and ranking SIFI, based on sensitivities of financial institutions. The last section ultimately concludes.

2. On an Index of Systemic Risk Measures (ISRM)

To integrate systemic risk as another factor, complementary to the systematic and specific risk, it is necessary to use a precise measurement of risk. In recent years, a strong literature has been developed on identification of Systemically Important Financial Institutions by quantitative measures to characterize the conditional link between different financial institutions and the market as a whole. However, given the many dimensions of systemic risk, these individual measures hardly detect systematically the potentially systemic institutions.

The use of factor analysis as information aggregation tool from a set of systemic risk measures is a new approach. Moreno and Peña (2013) use a Principal Component Analysis (PCA) on a set of companies for building a systemic risk index. Giglio et al. (2016) use principal component analysis to build a systemic risk index and test its predictive power of future shocks on macroeconomic variables using quantile regression. Nucera et al. (2016) run principal component analysis on a set of six systemic risk measures. However, their study differs from Giglio et al. (2016) as they apply a PCA on the ranking of 113 companies in the financial sector through a series of systemic risk measures and not on a set of companies over a period of time as in Giglio et al. (2016) in their study from a set of 19 measures of systemic risk. We hereafter summarize, complement and extend the work by Giglio et al. (2016) and Nucera et al. (2016), mainly considering the databases first used in Brownlees and Engle (2017) and, secondly, by Giglio et al. (2016).

2.1. About Systemic Risk Measures

The financial literature has proposed numerous quantitative measures that can be used to identify potentially systemic institutions. We can group them into several categories.

First, the individual systemic risk measures are defined from econometric models of specific risk to the institution. This is the Conditional Value-at-Risk (CoVaR) and Delta

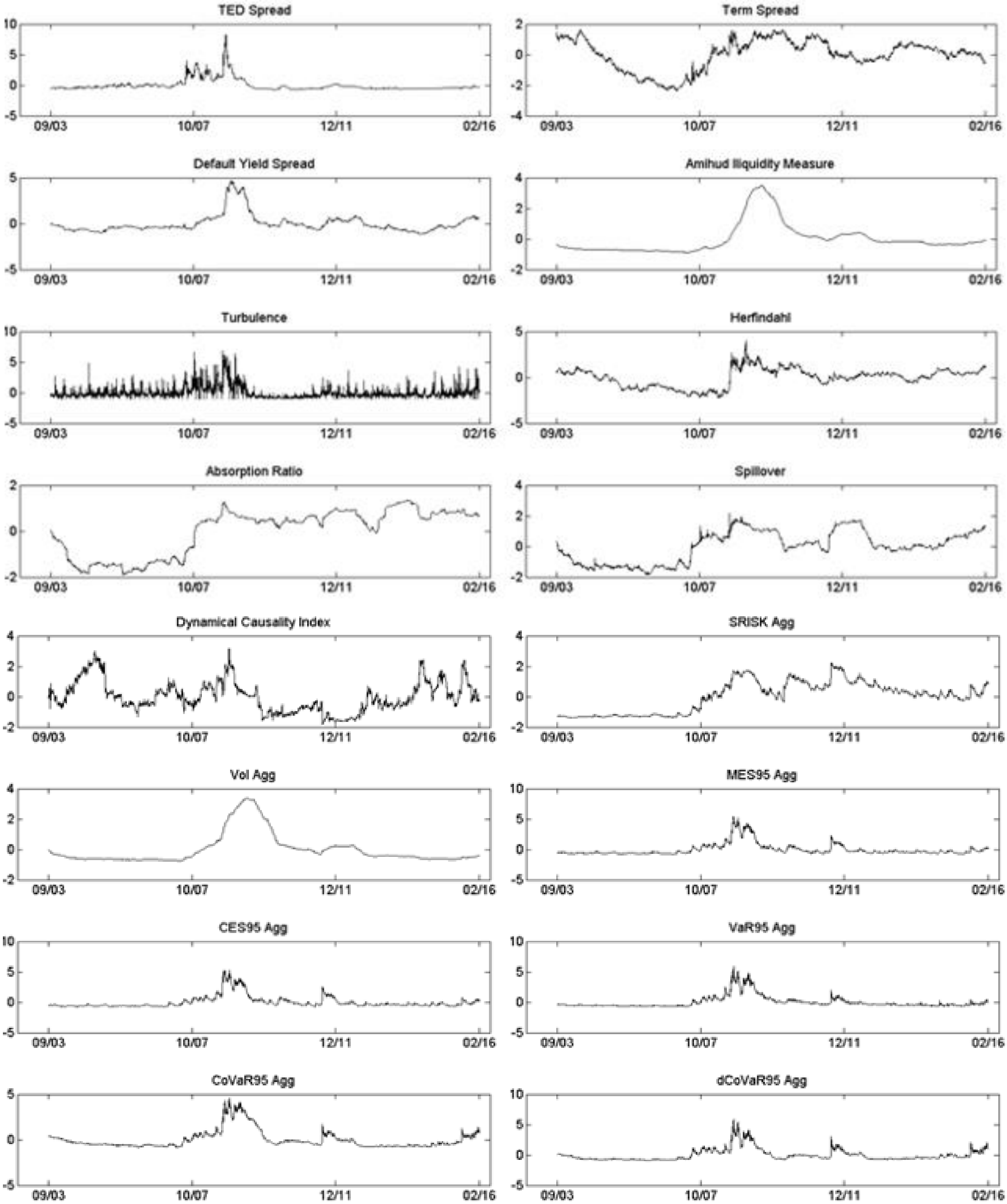
Conditional Value-at-Risk (ΔCoVaR) by Adrian and Brunnermeier (2016), and the Marginal Expected Shortfall (MES) of Acharya et al. (2013) and Brownlees and Engle (2017), and SRISK of Acharya et al. (2012) and Brownlees and Engle (2017), and Amihud Illiquidity Measure proposed by Amihud (2002).

Secondly, other measures focus specifically on an important aspect of systemic risk, i.e. the level of interconnection of financial institutions or the financial system concentration. In this category, we select the Spillover Index (SI) by Diebold and Yilmaz (2009), the Dynamic Causality Index (DCI) of Billio et al. (2012), the measurement of Turbulence by Kritzman and Li (2010), the Absorption Ratio of Kritzman et al. (2011), and the concentration Herfindahl-Hirschman Index. Thirdly, some macro-financial variables are generally used to complement the analysis, serving as leading indicators of economic activity (see Estrella and Trubin, 2006; Chen et al., 2009). We retain hereafter in our analysis: the Credit Default Yield Spread which measures the difference between the yield on corporate bonds rated BAA and the rated AAA by Moody's, as Chen et al. (2009) show that this variable is an aggregate measure of the risk of robust credit frictions (tax and liquidity) in the bond market; the TED Spread, which measures the difference between the LIBOR three-month rate and sovereign interest rates in three months: an increase of this variable is the sign that lenders expect an increase in credit risk in the interbank lending market; and finally the Term Spread measures the slope of the yield curve, and corresponds to the yield spread between 10-year Treasury bonds and three months money market maturities, since this variable may serve as a leading indicator of the economic activity (e.g. Estrella and Trubin, 2006).
We consider also the volatility (Vol) and the Value-at-Risk (VaR) aggregated across the system to take into account the evolution of its variability.

We illustrate the dynamics of these different systemic risk indicators in the following Figure 1 from daily data financial institutions from the US market over the period from the 09/03/2003 to the 02/26/2016. We see in this Figure a significant increase in all global systemic risk measures over the period 2007-2008 which is the period of the financial crisis. Similarly, although a common trend seems to emerge from the dynamics of the series, there are still some disparities between these measures. These differences may stem from the fact that systemic risk is multidimensional, each of the different metrics is modelling a specific dimension.

This result is confirmed by the analysis of correlations between different risk measures.

Figure 1: Dynamics of global systemic risk measures



Source: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016; authors' computation.
Note: These measures (without considering the macroeconomic variables) are estimated from rolling windows of one year. Here are presented the z-scores of these measures and are in the following order: TED Spread, Term Spread, Default Yield Spread, Amihud Illiquidity Measure, Turbulence, Herfindahl-Hirschman Index, Absorption Ratio, Spillover, Dynamical Causality Index, SRISK Agg, Vol Agg, MES95 Agg, CES95 Agg, VaR95 Agg, CoVaR95 Agg, ΔCoVaR95 Agg.

Indeed, from the matrix of the correlations presented in Table 1 below, we can note that all correlations are statistically significant at a nominal risk level of 5%, except for the correlations between the Amihud Illiquidity Measure and the TED Spread and between the Dynamical Causality Index and the Default Yield Spread for the Pearson correlations. The exception of the correlations for the Spearman correlations are between the TED Spread, the aggregate SRISK and the aggregate volatility, between the Turbulence Index and the aggregated SRISK, between the Herfindahl-Hirschman Index and the Dynamical Causality Index and between the aggregated CoVaR and the Dynamical Causality Index.

Table 1: Pearson and Spearman correlations of systemic measures

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}	M_{13}	M_{14}	M_{15}	M_{16}
M_1	1.00	-.11	.55	.00	.65	-.19	.06	.26	.21	.16	.28	.69	.69	.67	.57	.68
M_2	-.49	1.00	.18	.54	-.09	.68	.54	.45	-.01	.44	.47	.24	.24	.31	.33	.25
M_3	.41	.08	1.00	.52	.40	.39	.34	.61	.09	.56	.74	.88	.87	.87	.89	.85
M_4	-.29	.63	.54	1.00	-.05	.60	.45	.61	-.22	.56	.93	.49	.48	.57	.62	.43
M_5	.48	-.28	.17	-.20	1.00	-.11	.09	.23	.20	.15	.16	.52	.53	.52	.45	.56
M_6	-.39	.63	.29	.64	-.24	1.00	.41	.43	.03	.47	.51	.35	.33	.38	.43	.35
M_7	-.19	.33	.32	.64	.05	.30	1.00	.84	-.24	.83	.37	.36	.37	.37	.31	.42
M_8	.08	.32	.71	.76	.14	.40	.72	1.00	-.23	.84	.61	.57	.58	.59	.60	.66
M_9	.17	-.05	-.21	-.39	.25	.02	-.17	-.21	1.00	-.31	-.07	.13	.13	.11	.17	.14
M_{10}	-.01	.28	.61	.84	.02	.38	.67	.75	-.36	1.00	.53	.60	.61	.59	.50	.59
M_{11}	-.03	.61	.65	.88	-.05	.47	.48	.76	-.32	.74	1.00	.72	.70	.79	.82	.64
M_{12}	.26	.26	.66	.61	.23	.26	.49	.72	-.13	.74	.74	1.00	1.00	.97	.92	.94
M_{13}	.26	.25	.66	.60	.22	.25	.49	.73	-.11	.74	.72	.99	1.00	.97	.91	.94
M_{14}	.15	.44	.70	.75	.16	.36	.48	.79	-.23	.76	.89	.91	.89	1.00	.94	.93
M_{15}	.34	.38	.68	.46	.22	.35	.23	.67	.01	.40	.67	.71	.71	.77	1.00	.94
M_{16}	.25	.34	.67	.54	.27	.33	.55	.81	-.02	.62	.67	.81	.82	.82	.85	1.00

Source: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016; authors' computation.
Note: M_1 to M_{16} represent the 16 systemic risk measures and are in the following order: M_1 = *TED Spread,* M_2 = *Term Spread,* M_3 = *Default Yield Spread,* M_4 = *Amihud Illiquidity Measure,* M_5 = *Turbulence,* M_6 = *Herfindahl-Hirschman Index,* M_7 = *Absorption Ratio,* M_8 = *Spillover Index,* M_9 = *Dynamical Causality Index,* M_{10} = *SRISK,* M_{11} = *Vol,* M_{12} = *MES95,* M_{13} = *CES95,* M_{14} = *VaR95,* M_{15} = *CoVaR95,* and M_{16} = *ΔCoVaR95.* Above, the off-diagonal elements are the Pearson correlations, the Spearman correlations are below. Non-significant correlations at a nominal risk level of 5% are in grey.

The strongest correlations (above .90) are those related to global systemic risk measures corresponding to average data instant time-series of individual measures (CoVaR, ΔCoVaR, MES). It is therefore necessary to propose an indicator that integrates all dimensions of risk (aggregated model).

2.2. An Aggregated Index of Systemic Risk Measures

We present in this section the methodology for the construction of the aggregated index of overall systemic risk (see Kouontchou et al., 2015). First, we present the PCA sparse approach for dimension reduction, and secondly, we present the optimal choice of systemic risk index through the causality test in extreme risks of Hong et al. (2009) for selecting the most parsimonious index.

2.2.1. The Sparse PCA Method

The PCA is a decomposition of a data set on the basis of orthogonal functions which are determined from the data. These functions, which are linear combinations of the original variables, are supposed to reproduce a large extent of the existing variability in the data, and they correspond to the most important main axes or components. From a statistical point of view, if we consider a matrix M of dimension (T, p) of the initial normalized data (see Benoit et al., 2017b on the importance of this point in the context of detecting SIFI)), the first component (main axis) is denoted by a vector of dimension p as the solution of the following program:

$$\begin{aligned} &\max_{x \in \mathbb{R}^p} \{x' A x\} \\ &s.t. \|x\|_2 = 1, \end{aligned} \tag{1}$$

where $A = T^{-1}M'M$ is the covariance matrix of M of dimension (p, p), M' is its transpose, T the size of the sample and $\| x \|_2$ stand for the 2-norm of vector x.

The first component is obtained by minimizing the empirical variance of the projected data in an identification constraint associated with a specific norm. The projection data on this component makes it possible to obtain a factor noted F of dimension (T, p), with $F = Mx$ whose variance, called eigenvalue, is equal to $\lambda = T^{-1}F'F$, is the criterion in the optimization program (1). In the construction of aggregated systemic risk indices, the index is generally associated with the factor F (Moreno et Peña, 2013; Giglio et al., 2016).

The previous optimization program that provides the first component and the dominant factor has an equivalent representation in terms of linear regression (Zou et al., 2006). Indeed, it is shown that in the linear regression that reads:

$$F = M\beta + U, \tag{2}$$

where the dependent variable F (respectively, the explanatory variables M) is the dominant factor of the PCA (initial data matrix respectively) and U is the error term. The normalized value of the Ordinary Least Square (OLS) estimator of the parameter vector β is equal to the first component, that is:

$$x = \frac{\hat{\beta}}{\|\hat{\beta}\|_2}, \tag{3}$$

with $\|.\|_2$ the 2-norm.

Zou et al. (2006) propose to modify the linear regression represented by Eq. (2) in order to obtain the main sparse component from the expression (3). Indeed, if x^s is this component, it is equal to:

$$x^s = \frac{\hat{\beta}^s}{\|\hat{\beta}^s\|_2}, \tag{4}$$

where $\hat{\beta}^s$ is the solution of the constrained following regression (or penalized) below:

$$\begin{gathered} F = M\beta^s + U \\ s.t. \|\beta^s\|_1 = \sum_{j=1}^{p} |\beta_j^s| \leq \delta. \end{gathered} \tag{5}$$

The parameter $\delta \geq 0$ defines the upper limit of norm 1 of the parameter vector β^s. Regression (5) introduced by Tibshirani (1996) is known as the Least Absolute Shrinkage and Selection Operator (LASSO), and its primary goal is to make a variable selection. The limit behaviour of this regression can be summarized as follows. When δ tends to zero, the number of active elements (different from zero) in $\hat{\beta}^s$, and therefore in the "sparse" component x^s, also approaches zero - the degenerated limit case being when $\delta = 0$, where $\hat{\beta}^s$ and x^s correspond to the zero vector; in the opposite case when δ tends to infinity, the regression (5) is the unrestricted regression (2), and x^s is exactly equal to x, i.e. the main component of a conventional PCA and the number of active elements then takes its maximum value$p$.

This method applied in our systemic risk framework has the advantage of providing a main component that summarizes the variability in systemic risk indicators, using only a few of them. Beyond the parsimony that brings the SPCA, it should be mentioned that the variable selection is made using the usual trade-off between bias and variance. Indeed, under the usual conditions of exogeneity of the error term U in the regression (2), the estimator $\hat{\beta}$ is unbiased. The additional constraint in regression (5) helps to reduce the variance of the estimator by introducing bias.[3] Therefore the main factor from a SPCA has a more stable temporal dynamic. As highlighted above, this property is desirable since the implementation of regulatory policies should not be based on noisy and erratic metrics of systemic risk. Finally, note that the dominant factor of the SPCA is obtained by projecting the data matrix M on the sparse component x^s, with $F^s = Mx^s$.

Table 2: Variable decomposition of the sparse principal components

δ	1.000	1.309	1.402	1.711	1.834	1.933	2.017	2.022	2.050	2.125	2.139	2.199	2.249	2.304	2.327	2.388
Id_k	Id_1	Id_2	Id_3	Id_4	Id_5	Id_6	Id_7	Id_8	Id_9	Id_{10}	Id_{11}	Id_{12}	Id_{13}	Id_{14}	Id_{15}	Id_{16}
k	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$	$k=8$	$k=9$	$k=10$	$k=11$	$k=12$	$k=13$	$k=14$	$k=15$	$k=16$
M₁	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.01	.04
M₂	.00	.00	.00	.00	.00	.00	.02	.02	.03	.04	.05	.05	.06	.06	.06	.07
M₃	.00	.00	.00	.00	.00	.00	.00	.00	.01	.03	.04	.05	.05	.07	.07	.08
M₄	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.05
M₅	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.01	.03	.03	.04
M₆	.00	.39	.41	.49	.51	.53	.54	.54	.54	.55	.55	.55	.55	.56	.57	.57
M₇	.00	.00	.00	.17	.24	.30	.34	.34	.35	.38	.39	.40	.40	.40	.41	.41
M₈	1.00	.92	.91	.83	.79	.75	.73	.73	.72	.70	.70	.70	.70	.69	.69	.68
M₉	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.03	.04	.06	.06	.07
M₁₀	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.02	.03	.04	.05	.06
M₁₁	.00	.00	.00	.00	.00	.00	.00	.00	.00	.01	.01	.03	.04	.08	.10	.06
M₁₂	.00	.00	.00	.00	.00	.02	.06	.06	.06	.05	.04	.05	.05	.04	.04	.05
M₁₃	.00	.00	.00	.00	.00	.00	.00	.00	.01	.05	.06	.05	.06	.06	.05	.05
M₁₄	.00	.00	.08	.23	.24	.23	.20	.19	.19	.16	.15	.15	.13	.10	.08	.05
M₁₅	.00	.00	.00	.00	.05	.10	.13	.13	.14	.14	.14	.13	.12	.07	.06	.07
M₁₆	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.05	.06	.05

Source: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016; authors' computation.

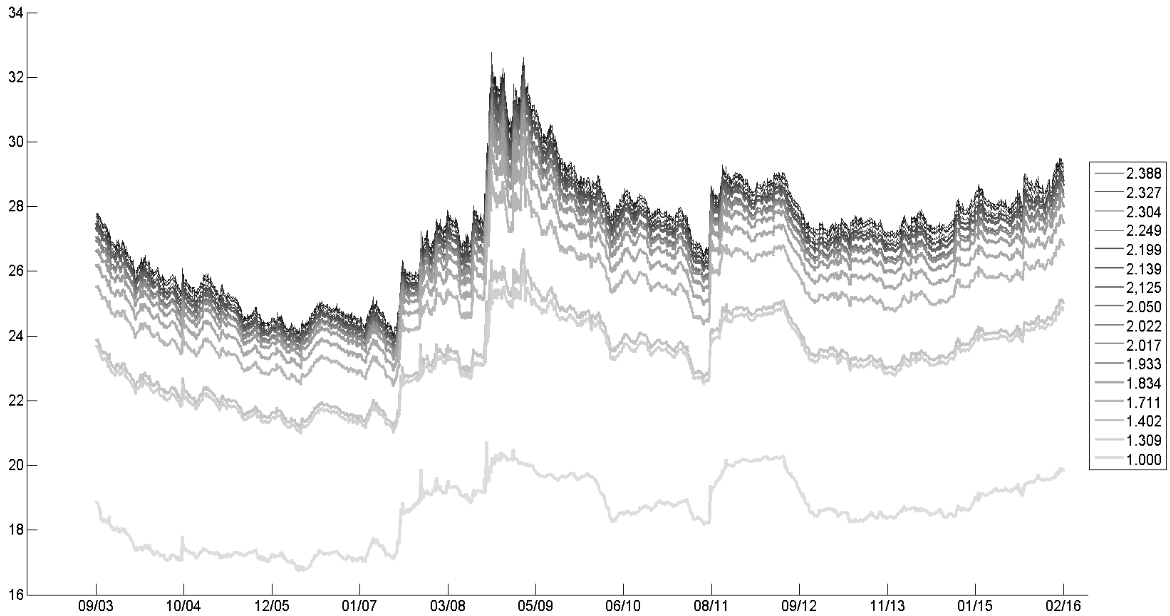
Note: M_1 to M_{16} represent the 16 systemic risk measures and are in the following order: M_1 = *TED Spread,* M_2 = *Term Spread,* M_3 = *Default Yield Spread,* M_4 = *Amihud Illiquidity Measure,* M_5 = *Turbulence,* M_6 = *Herfindahl-Hirschman Index,* M_7 = *Absorption Ratio,* M_8 = *Spillover Index,* M_9 = *Dynamical Causality Index,* M_{10} = *SRISK,* M_{11} = *Vol,* M_{12} = *MES95,* M_{13} = *CES95,* M_{14} = *VaR95,* M_{15} = *CoVaR95,* and M_{16} = *ΔCoVaR95.*

Table 2 shows the dominant principal component derived from the SPCA methodology for different values of the parameter δ. When $\delta = 1$, which corresponds to the strongest constraint in regression (5), the number of active global systemic risk measures in the dominant component is equal to $k = 1$, and corresponds to the M8 measure, namely the Spillover Index of Diebold and Yilmaz (2009). When δ increases, the constraint becomes lighter and other additional systemic risk measures enter into the dominant component. For illustration, when $\delta = 1.933$, six measures are active in the index, namely the concentration Herfindahl-Hirschman Index, the Absorption Ratio

3. Here we find a compromise between bias and variance in the so-called regression "RIDGE". Arbitrage nonetheless addresses the norm 1 and not the norm 2 of the "LASSO" regression.

of Kritzman et al. (2011), the Spillover Index of Diebold and Yilmaz (2009), the aggregated MES[4] by Acharya et al. (2010), the aggregated Value-at-Risk and the aggregated CoVaR of Adrian and Brunnermeier (2016). For the highest value, all measures are active in the dominant component, and it is the first component of a classic PCA. Note that the latest systemic risk measure to be active in component is the Amihud Illiquidity Measure.

Figure 2: Dynamics of the various SPCA indexes (as functions of δ)



Source: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016; authors' computation.
Note*:* Are presented the dynamics of all the aggregate indexes obtained from SPCA for a specific value of δ.

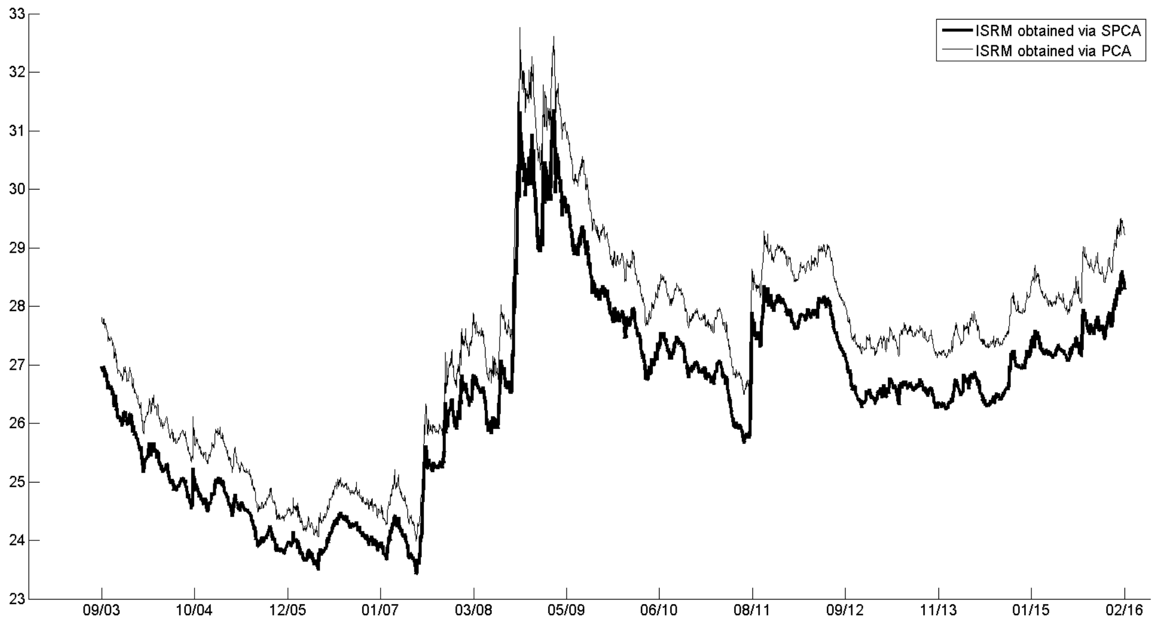
Figure 2 shows the dynamics of all 16 aggregate indexes of systemic risk obtained through the analysis of the main sparse components for a given value of the truncation parameter δ, varying from $\delta = 1.000$ to $\delta = 2.388$. For the first value, when $\delta = 1.000$, the aggregate index is nothing else than the Spillover Index of Diebold and Yilmaz (2009) and is the most stable. For the last value, where $\delta = 2.388$, the aggregate index corresponds to the aggregation of all systemic risk measures (the 16 measures are included in the analysis). The dynamics of the other indices are between these two limit case indexes. Indeed, the addition of any extra factor in the index

[4] For some authors, and since its definition relies on the effect on a financial institution of an extreme market movement, the MES should not be taken as a systemic risk measure... We see here that the information content of such a statistic is singular and different from the one in other measures.

increases its variability, which will be between the variability of the two indexes included in the limiting cases as displayed in the following Figure.

Figure 3 below, displays the dynamics of the SPCA and PCA cases of the 16 factors (or aggregate indexes) from the sparse components of Table 1. Indeed, although the dynamics of the two aggregate indices are very similar, the temporal variability is not equal. The most stable index obviously corresponds to the case with a variance equal to 1. This index is identical to the Spillover Index of Diebold and Yilmaz (2009). The most volatile aggregate index is obtained for $\delta = 2.388$ and is equal to the dominant factor of a conventional PCA, with an estimated variance of 3.195. Other unrepresented aggregated indexes show variances between these two values. We thus find, with this analysis, the primary objective of the SPCA, namely the temporal stabilization of factors, is to define our aggregate index. However, as already mentioned, this stabilization is achieved *via* a bias-variance arbitrage, and thus induces a decrease in the quality of the representation (explained variance).

Figure 3: Dynamics of the SPCA and PCA indexes



Source: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016; authors' computation.
Note: Are plotted the two limit cases of the set of indexes. The optimal choice is in bold and the index obtained by the PCA is in dashed line.

The following section is dedicated to the optimal choice of the aggregate index of systemic risk among the 16 competing indices which are all special combinations of our 16 systemic risk measures.

2.2.2. Optimal Choice within the Set of the Competing Sparse Principal Components

Our aim in this section is to assess to what extent the aggregate index can be considered a leading indicator of economic activity. This approach is the one used by Giglio et al. (2016) to measure the predictive power of their aggregate index extracted from a classical PCA. Indeed, these authors, *via* a quantile regression, test whether extreme variations in the industrial production is explained by the lagged value of the index of systemic risk - compared to a non-conditional specification excluding the index.

The method we adopt in this section is, however, different, in the sense that we assess to what extent the positive extreme movements of the aggregate index of systemic risk (when systemic risk is high) Granger-cause the negative extreme movements in the industrial production. As highlighted above, this approach is consistent with the intuition that only the extreme movements of the aggregate index can explain systemic events, inducing strong slowdowns in the future economic activity. We use for this purpose the causality test in distributions tails developed by Hong et al. (2009).

For a brief description of the test, let us note $y_{1,t} = \Delta P_t$ the monthly change in industrial production, and $Q_{1,t}(\alpha;\theta_1)$ the quantile at the order α of the distribution of $y_{1,t}$, with θ_1 a vector of parameters associated with the specification of the dynamic of $y_{1,t}$. Here we follow Giglio et al. (2016) by setting α to 20%. For monthly data, note here this is a reasonable choice since it allows to have samples with limited sizes and a significant number of observations in the left tail of the distribution y_t. Let $Hit_{1,t}(\alpha;\theta_1)$ the dummy variable defined as:

$$Hit_{1,t}(\alpha;\theta_1) = \begin{cases} 1 & if\ \ y_{1,t} \leq \ Q_{1,t}(\alpha;\theta_1) \\ 0 & otherwise \end{cases}. \tag{6}$$

This variable equals 1 when the change in industrial production is extreme and negative, corresponding to a severe contraction of economic activity. In the same manner, let us denote $y_{2,t} = -\Delta F_t^S$ the opposite of the monthly change in the aggregate index of systemic risk[5] obtained *via* the PCA « *sparse* » methodology, and $Hit_{2,t}(\alpha;\theta_2)$ the dummy variable defined as:

5. Monthly data for each aggregate index are obtained as averages of daily data of Figure 2. In total, we have 130 observations for competitor aggregate indices, and 130 monthly observations for industrial production.

$$Hit_{2,t}(\alpha;\theta_2) = \begin{cases} 1 & if \;\; y_{2,t} \leq \; Q_{2,t}(\alpha;\theta_2) \\ 0 & otherwise \end{cases}. \tag{7}$$

Note that this variable equals 1 when the change in aggregated systemic index is extreme and positive indicating a systemic event. The null hypothesis testing in Hong et al. (2009) is:

$$E\left[Hit_{1,t}(\alpha;\theta_1)|\Omega_{t-1}\right] = E\left[Hit_{1,t}(\alpha;\theta_1)|\Omega_{1,t-1}\right], \tag{8}$$

wherein the sets of information on the date *t*-1 are defined respectively by:

$$\begin{cases} \Omega_{t-1} = \left\{\left(y_{1,s}, y_{2,s}\right), s \leq t-1 \right\} \\ \Omega_{1,t-1} = \left\{y_{1,s}, s \leq t-1\right\}. \end{cases} \tag{9}$$

Under the null hypothesis, the positive extreme movements of the aggregate index of systemic risk have no predictive power on the negative extreme movements in industrial production. The test statistic proposed by the authors depends on a weighted sum of the estimated correlations between $Hit_{1,t}\left(\alpha;\hat{\theta}_1\right)$ and $Hit_{2,t}\left(\alpha;\hat{\theta}_2\right)$ where $\hat{\theta}_1$ and $\hat{\theta}_2$ are consistent estimators of $\theta_1$and θ_2. This weighted sum is defined by:

$$Z = T\sum_{j=1}^{T-1} \kappa^2(j/d)\hat{\rho}(j), \tag{10}$$

With the function $\kappa(.)$ of the type decreasing kernel[6], d the truncate parameter[7] and $\hat{\rho}(j)$ the cross-correlation of order *j* between $Hit_{1,t}\left(\alpha;\hat{\theta}_1\right)$ and $Hit_{2,t}\left(\alpha;\hat{\theta}_2\right)$ equals to:

$$\hat{\rho}(j) = \frac{\hat{\gamma}(j)}{\hat{s}_1\hat{s}_2}, \tag{11}$$

where $\hat{s}_1$ and $\hat{s}_2$ refer to the standard deviation of $Hit_{1,t}\left(\alpha;\hat{\theta}_1\right)$ and $Hit_{2,t}\left(\alpha;\hat{\theta}_2\right)$ respectively and $\hat{\gamma}(j)$ cross-covariance of order j defined by:

$$\hat{\gamma}(j)\begin{cases} T^{-1}\sum_{t=l+j}^{T-1}\left\{\left[Hit_{1,t}\left(\alpha;\hat{\theta}_1\right)-\hat{\pi}_1\right]\left[Hit_{2,t-j}\left(\alpha;\hat{\theta}_2\right)-\hat{\pi}_2\right]\right\} \; for \; 0 \leq j \leq T-1 \\ T^{-1}\sum_{t=l-j}^{T-1}\left\{\left[Hit_{1,t+j}\left(\alpha;\hat{\theta}_1\right)-\hat{\pi}_1\right]\left[Hit_{2,t}\left(\alpha;\hat{\theta}_2\right)-\hat{\pi}_2\right]\right\} \; for \; 1-T \leq j \leq 0, \end{cases} \tag{12}$$

6. We use the kernel function from Daniell that induces optimal properties for causality test. Cf. Hong et al. (2009) for further details.

7. When this parameter *d* increases, the value of the function that plays in the formula (10) as weighting is higher for low values of *j* lags.

with $\hat{\pi}_1$ and $\hat{\pi}_2$ the empirical means of $Hit_{1,t}(\alpha; \hat{\theta}_1)$ and $Hit_{2,t}(\alpha; \hat{\theta}_2)$ respectively. We therefore denote that the particularity of the Z statistic is the fact that all possible lags are considered, with a discount of the most distant lags. Also, in the current context of applying this test, the inclusion of a high number of lags, helps to capture the stronger or weaker inertia in the reaction of the economy to a systemic event. Under the null hypothesis of no causality in extreme movements, Hong et al. (2009) demonstrate that:

$$U = \frac{Z - C_T(d)}{[D_T(d)]^{1/2}}, \tag{13}$$

follows a standard normal distribution, with:

$$C_T(d) = \sum_{j=1}^{T-1} (1 - j/T)\kappa^2(j/d), \tag{14}$$

and:

$$D_T(d) = 2\sum_{j=1}^{T-1} (1 - j/T)(1 - (j+1)/T)\,\kappa^4(j/d). \tag{15}$$

The statistical U is therefore used for inference. The Monte Carlo simulations carried out by Hong et al. (2009) show that the test has good properties at a finite distance. It is important here to note that the minimum sample size considered by the authors in the simulations is $T = 500$, and the minimum quantile is 5% (approximately 25 observations in the tails of distributions). We have here with our monthly data of the changes in industrial production and changes in competitors aggregated indices only 129 observations. With a 20% quantile, this leaves us also 25 cases. It is close to the test application conditions, namely the existence of a relatively not too small number of data in the tails of distributions.

The results of causality tests for the different competing indices (denoted Id_1 to Id_{16}) are summarized in Table 3, for two values of truncation parameter d, ranging from $d = 15$ and $d = 20$. The null hypothesis of no causality from positive and extreme monthly variations of each aggregate index of systemic risk to the negative and extreme monthly variations in industrial production, is rejected in all configurations at a nominal 5% threshold. When closely reading this Table, the optimal index derived from the SPCA methodology is the aggregate index 14. Indeed, whatever the value of d, this index appears to be the most parsimonious: it is constructed from only 14 systemic risk measures, and is relatively stable over time, whist it has the highest predictive power (high test statistic) on severe contractions in the economic activity. We note here the analogy between our approach to identify the optimal aggregate index and the traditional model selection *criteria* (AIC, BIC).

The optimal aggregate index (that we are going to name the ISRM from now on) is thus, based on the results of Table 3, entirely determined by: the Default Yield Spread, the Term Spread, the Herfindahl-Hirschman Index, the Absorption Ratio by Kritzman et al. (2011), the Spillover Index by Diebold and Yilmaz (2009), the SRISK by Acharya et al. (2012) and Brownlees and Engle (2017), the Aggregated Vol, the MES of Acharya et al. (2010), the Component Expected Shortfall of Banulescu and Dumitrescu (2015), the Value-at-Risk and the CoVaR and the ΔCoVaR of Adrian and Brunnermeier (2016), the Dynamical Causality Index of Billio et al (2012) and the turbulence index of Kritzman and Li (2010).

The largest contributor to the aggregate index is the Spillover Index of Diebold and Yilmaz (2009) with a weight of .69; conversely, the one with the lowest impact is the Turbulence index of by Kritzman and Li (2010) with a weight of .03. Finally, three complementary dimensions of systemic risk are taken into account in our aggregate index: the liquidity (Amihud Illiquidity Measure), the contagion effect measure (the Spillover Index) and the concentration risk component (measured by the Herfindahl-Hirschman Index), in addition to the size of the institution and leverage effect encompassed in the SRISK.

Table 3: Causality tests in extreme movements

	SPCA															PCA
δ	1,000	1,309	1,402	1,711	1,834	1,933	2,017	2,022	2,050	2,125	2,139	2,199	2,249	2,304	2,327	2,388
Id_k	Id_1	Id_2	Id_3	Id_4	Id_5	Id_6	Id_7	Id_8	Id_9	Id_{10}	Id_{11}	Id_{12}	Id_{13}	Id_{14}	Id_{15}	Id_{16}
k	*k=1*	*k=2*	*k=3*	*k=4*	*k=5*	*k=6*	*k=7*	*k=8*	*k=9*	*k=10*	*k=11*	*k=12*	*k=13*	*k=14*	*k=15*	*k=16*
U(15)	4,77	1,61	1,61	5,54	4,08	4,08	4,08	4,08	4,08	5,07	5,07	6,90	6,90	**10,43**	7,60	7,60
U(20)	5,48	1,48	1,48	5,66	3,91	3,91	3,91	3,91	3,91	5,06	5,06	7,22	7,22	**11,04**	7,75	7,75

Source: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016; authors' computation.
Note: The Table shows the value of the $U(.)$ statistic of Hong et al. (2009) in Eq. (13) for inference on causality from monthly variations of each aggregate index to the monthly change in industrial production. Id_1 to Id_{16} correspond to the various aggregated indices of systemic risk. The threshold for significance at nominal risk level of 5% is 1.96.

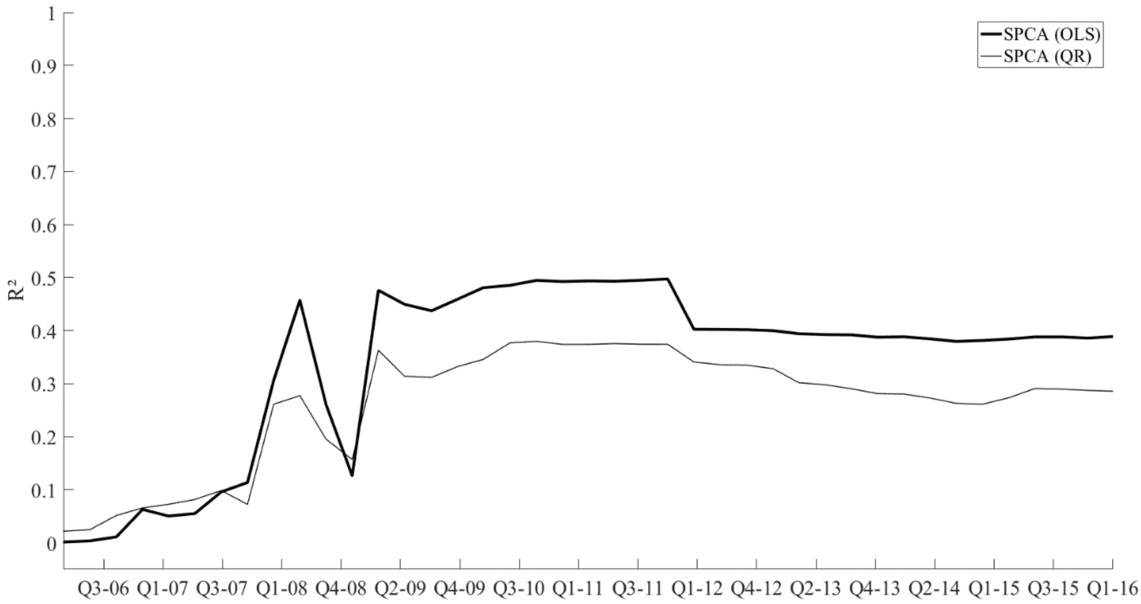
In the following, we continue our analysis on the relationship between the GDP and the ISRM to see if the ISRM can explain future variations of the GDP. We also conduct the same analysis between the market index and the ISRM using the indexes obtained by SPCA and by PCA.

We start by computing quarterly series of the ISRM, the GDP and the market index (the S&P500 here). With Ordinary Least Squares (OLS) and Quantile Regression (QR), we estimate the relationship between the GDP/market index and the systemic risk index (lagged) from the Q1-06 to the Q4-15 following the equation:

$$\dot{y}_{t+1} = \mu + \lambda I S\dot{R} M_t + \xi_t, \qquad (16)$$

where $\dot{y}_{t+1} = (y_{t+1} - y_t / y_t)$ is either the variation rate of the macroeconomic variable or later the variation rate of the market index, $IS\dot{R}M_t = (ISRM_t - ISRM_{t-1}/ISRM_{t-1})$ is the variation rate of the systemic risk index and ξ_t is the residual at time t.

Figure 4: R^2 dynamic for the GDP-ISRM relationship

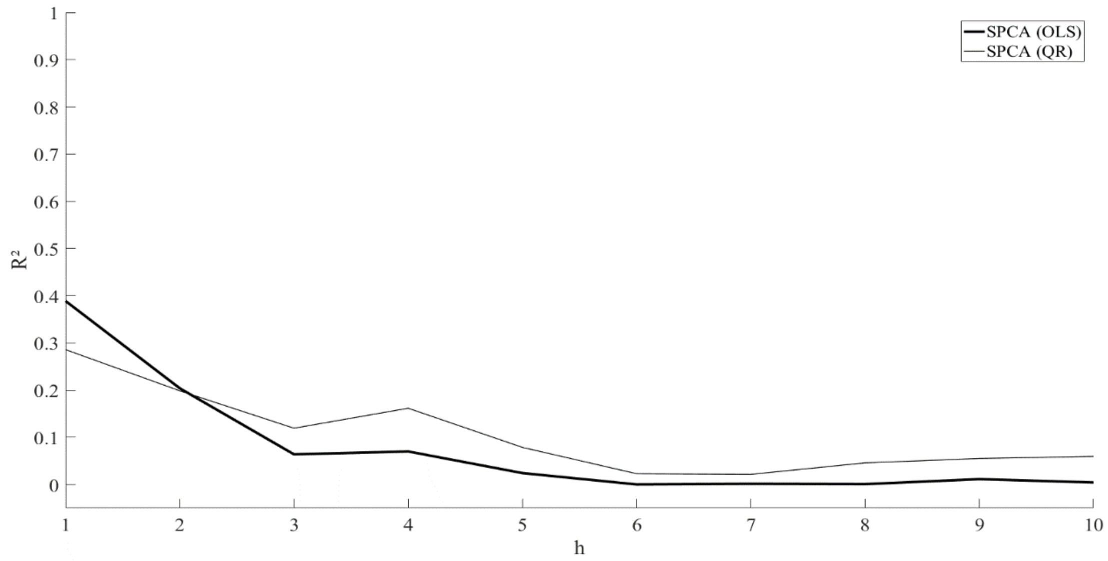


Source: Bloomberg, quarterly GDP series from the Q1-06 to the Q4-15; authors' computation.
Note: The Figure shows the dynamic of the R^2 for the relationship between GDP and systemic risk index using ordinary least squares (OLS) and Quantile Regression (QR) with a 20th percentile for a given t+1 horizon (dynamic evolution of the link according to t);

Figure 4 shows the dynamic of the R^2 for the relationship between the one ahead period GDP and systemic risk index (lagged) using Ordinary Least Squares (OLS) and Quantile Regression (QR) at the present period. The explicative power seems to increase during the financial crisis of 2008-2009. We can distinguish three change in regime periods along the entire sample. Here are plotted the two R^2 dynamics that represents in fact the two estimation methods (OLS) and (QR) for the index obtained by Sparse-Principal Component Analysis (SPCA).

We continue our analysis by looking for what is the better forecast horizon of the ISRM to predict changes in the GDP growth rate. Figure 5 represents the R^2 of the relationship between GDP and ISRM at the $t + h$ period ahead from 1 to 10 periods. Here, the R^2 seems to goes down slowly as the forecast horizon increases.

Figure 5: R² dynamic for the GDP-ISRM relationship with respect to *t+h* period ahead



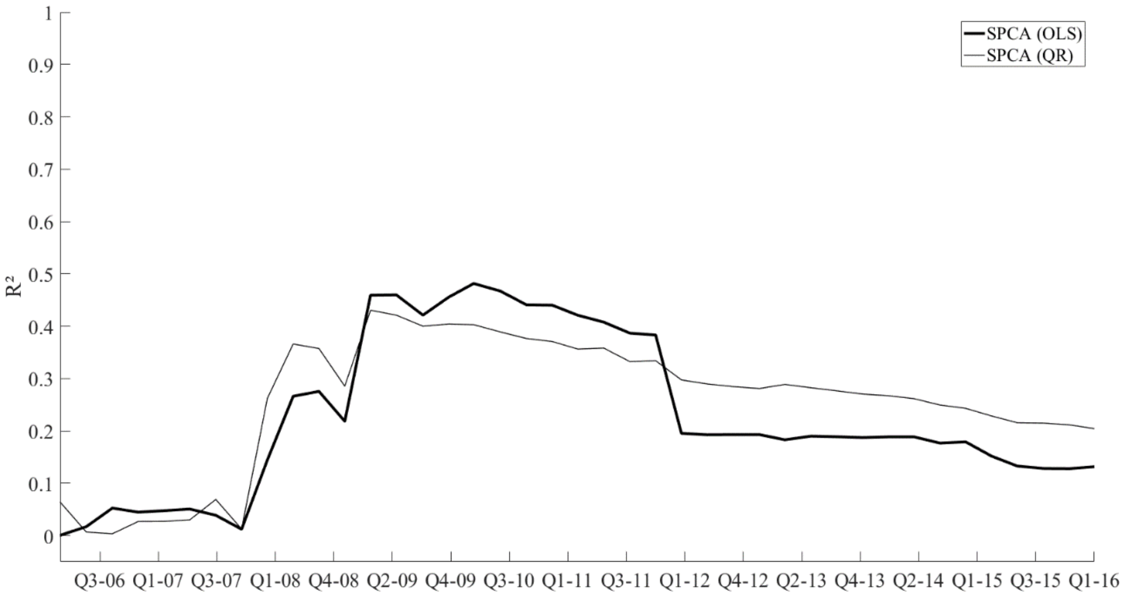
Source: Bloomberg, quarterly GDP series from the Q1-06 to the Q4-15; authors' computation.
Note: The Figure shows the dynamic of the R^2 for the relationship between GDP and systemic risk index using ordinary least squares (OLS) and Quantile Regression (QR) with a 20th percentile for a given t+h horizon (mean general relation according to h).

Now we do the same analysis for the relationship between the market index represented here by the S&P500 and the ISRM.

As previously viewed with the GDP, Figure 6 shows the dynamic of the R^2 for the relationship between the one ahead period market index and systemic risk index (lagged) using OLS and Quantile Regression at the present period. The explicative power seems to increase during the financial crisis of 2008-2009 again and there is also at least two change in regime periods along the entire sample.

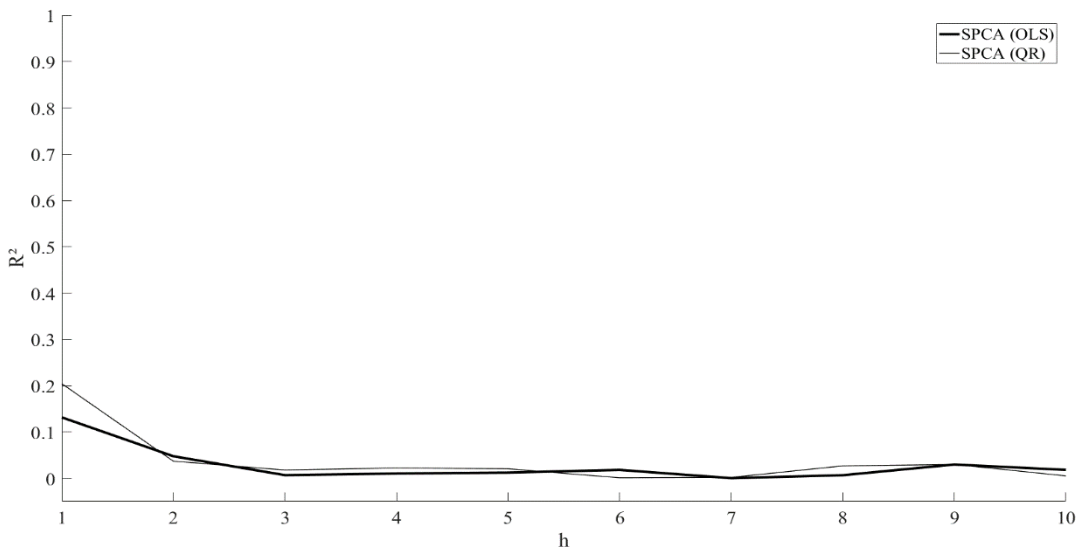
Then, we also continue the analysis by looking for again what is the better forecast horizon of the ISRM to predict changes in the market index. Figure 7 represents the R² of the relationship between the market index and ISRM at the $t + h$ period ahead from 1 to 10 periods. Here, the R^2 seems to be very low and stable along all the horizons.

Figure 6: R² dynamic for the S&P500-ISRM relationship



Source: Bloomberg, quarterly market index series from the Q1-06 to the Q4-15; authors' computation.
Note: The Figure shows the dynamic of the R^2 for the relationship between GDP and systemic risk index using ordinary least squares (OLS) and Quantile Regression (QR) with a 20th percentile for a given t+1 horizon (dynamic evolution of the link according to t);

Figure 7: R² dynamic for the S&P500-ISRM relationship with respect to *t*+*h* period ahead



Source: Bloomberg, quarterly market index series from the Q1-06 to the Q4-15; authors' computation.
Note: The Figure shows the dynamic of the R^2 for the relationship between GDP and systemic risk index using ordinary least squares (OLS) and Quantile Regression (QR) with a 20th percentile for a given t+h horizon (mean general relation according to h).

Giglio et al. (2016) use two estimators called Principal Component Quantile Regression (PCQR) and Partial Quantile Regression (PQR) to estimate the relationship between future realizations of macroeconomic variables and a latent factor based on systemic risk measures.

In the next section, we will use the ISRM as an additional factor in the traditional CAPM, but now, with Systemic risk (CAPMS).

3. On the CAPM with Systemic risk (CAPMS)

In the following, we briefly come back to the traditional Capital Asset Pricing Model (CAPM). Then we pursue with the presentation of its extended version by Fama and French (1993), who deal with the existence of some market anomalies. Finally, we introduce an extra factor linked to the global systemic risk in the financial system that we add into the classical Fama and French (1993) model. This supplementary factor comes from the Index of Systemic Risk Measures (ISRM) built in the previous section, and is so based on an optimal SPCA of several systemic measures.

3.1 From the canonical CAPM to the augmented CAPM with Systemic risk

In the equilibrium relationship proposed by Sharpe (1964), asset returns can be explained by the returns of the market portfolio such as:

$$r_i = r_f + \beta_i\left(r_M - r_f\right) + \varepsilon_i, \tag{17}$$

where r_i and r_M are respectively the returns of the asset i and the returns of the market portfolio, r_f is the risk-free rate supposed to be a constant and ε_i the residual term with a zero mean and β_i the sensitivity to the systematic risk as such:

$$\beta_i = \frac{Cov(r_i, r_M)}{\sigma^2(r_M)}. \tag{18}$$

This relationship allows us to determine, under certain conditions, the expected return of the asset i with respect to the systematic risk premium which reads:

$$E(r_i) = r_f + \beta_i\left[E(r_M - r_f)\right], \tag{19}$$

where $E(r_i)$ and $E(r_M)$ are respectively the expected return of the asset i and the expected return of the market.

This brings us directly to the version of the CAPM extended to the Fama-French (1993) three-factor model, which can be written as such:

$$E(r_i) = E(r_f) + \beta_i[E(r_M - r_f)] + \gamma_i[E(r_{SMB} - r_f)] + \theta_i[E(r_{HML} - r_f)], \qquad (20)$$

where r_{SMB} is the return of the Small Minus Big factor (denoted SMB) - related to the profitability gap between a portfolio composed of small cap assets and a portfolio composed of big cap assets, and where r_{HML} is the High Minus Low factor (denoted HML) - linked to the profitability gap between a (value) portfolio composed of assets with a high ratio of Book Equity out of Market Value, and a (growth) portfolio composed of assets that have a low ratio.

Thus, portfolios composed with low long-term returns (corresponding to "losers"), will tend to have a high ratio of Book Equity out of Market Value, whilst, conversely, portfolios with relatively high long-term returns (corresponding to "winners"), will tend to have a low ratio of Book Equity out of Market Value.

As a supplementary explanatory variable, we add thereafter the optimal systemic risk index mentioned in the previous section, as an additional factor in the Fama-French (1993) three-factor model. We thus obtain a 4-factor model (20) below, such as (with previous notations):

$$r_{i,t} = \alpha_i + \beta_i r_{M,t} + \theta_i r_{SMB,t} + \gamma_i r_{HML,t} + \varphi_i r_{ISRM,t} + \varepsilon_{i,t}, \qquad (21)$$

where the returns of ISRM at time *t* denoted $r_{ISRM,t}$ is the fourth factor.

Once we have written the asset valuation relations in their simplest form, insisting on linear relation between return and premiums, we now have to deal with other financial market peculiarities.

In particular, we first have to take into account in the estimation technique the potential inter-relations between asset returns. We use Seemingly Unrelated Regressions Estimation (SURE, Zellner, 1962), of the proposed model (see Kraus and Litzenberg, 1976; Barone-Adesi et al., 2003; Galagedera and Maharaj, 2008).

Indeed, let us consider a system of *N* simultaneous equations, of which the typical *i*-th equation defined in Eq. 20. In a more compact way, the latter vectors can be stacked into an *NT*-dimensional vector *r*, with a corresponding arrangement for the error terms, coefficient vectors and regressors, as such:

$$r = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix}, \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{bmatrix}, \ F = \begin{bmatrix} F_1 & 0 & \cdots & 0 \\ 0 & F_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & F_{K+1} \end{bmatrix},$$

with r the vector of returns of size $(NT \times 1)$ where $r_i = (r_{i,t=1}, \dots, r_{i,t=T})'$ for $i = [1, \dots, N]$, ε the vector of error terms of size $(NT \times 1)$ where $\varepsilon_i = (\varepsilon_{i,t=1}, \dots, \varepsilon_{i,t=T})'$ and F the $[NT \times (K+1)T]$ matrix of factors whose diagonal terms are defined as such: $F_1 = (1, 1 \dots, 1)'$, $F_2 = (r_{M,t=1}, \dots, r_{M,t=T})'$, $F_3 = (r_{SMB,t=1}, \dots, r_{SMB,t=T})'$ and $F_4 = (r_{HML,t=1}, \dots, r_{HML,t=T})'$.

This leads to the following equation:

$$r = F\Lambda + \varepsilon, \tag{22}$$

where $\Lambda = (\Lambda_1, \Lambda_2, \ldots, \Lambda_N)$ of size $[(K+1)T \times 1]$ with $\Lambda_i = (\alpha_i,\ \beta_i,\ \theta_i,\ \gamma_i,\ \varphi_i)'$ is the vector of loadings of size [$1 \times (K+1)$], which is a system that is identical to the one proposed by Zellner (1962).

Therefore, given that $\varepsilon_{i,t}$ is the error for the *i*-th asset in the *t*-th time period, the assumption of contemporaneous disturbance correlation, but not correlation over time, implies that the covariance matrix within this SURE system denoted $V(\varepsilon)$ is:

$$V(\varepsilon) = \Sigma \otimes I, \tag{23}$$

where:

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1T} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2T} \\ \vdots & \vdots & & \vdots \\ \sigma_{T1} & \sigma_{T2} & \cdots & \sigma_{TT} \end{bmatrix}, \tag{24}$$

with I the identity matrix of dimension N, Σ the covariance matrix of size $(T \times T)$ where $\sigma_{ii} = E(\varepsilon_i \varepsilon_i)$ is the variance of the residual of ε_i and $\sigma_{ij} = E(\varepsilon_i \varepsilon_j)$ for $i \neq j$ and $\otimes$ the Kronecker product notation indicating that each element of Σ is multiplied by an identity matrix. With the previous notations, the classical Ordinary Least Squares estimator for the vector Λ is:

$$\widehat{\Lambda}_{LS} = (F'F)^{-1}F'r, \tag{25}$$

and the Generalized Least Squares estimator (assuming that Σ is known) reads:

$$\widehat{\Lambda}_{GLS} = [F'(\Sigma \otimes I)^{-1}F]^{-1}F'(\Sigma \otimes I)^{-1}r. \tag{26}$$

This system finally corresponds to the CAPMS when Λ corresponds to the sensitivities of the three Fama-French plus the systemic risk factors, whilst the following empirical results are based on the system of equations 25 and 26 leading to single linear relation in 20.

Other extensions of the three-factor CAPM of Fama and French (1993) may be considered. Thus, Carhart (1997) proposes to add the momentum effect as an additional factor. This factor denoted Winners Minus Losers (WML in short) represents the Momentum effect, which is related to the profitability gap between yields on individual stocks that over-performed (corresponding to winners) and those that underperformed (corresponding to losers) in the market. We do not take into account yet this development and future research (in progress) will take this factor into account.

In the following are reported empirical tests of CAPMS in the US market.

3.2 Empirical Tests and Robustness Checks of the CAPMS on the Main American Financial Institutions

In this section, we conduct empirical tests and robustness checks of the CAPMS on the Main American Financial Institutions based on two samples.

First, we compare in the following the three-factor model of Fama and French (1993) (model I) and the very same model but extended to systemic risk (Model II), i.e., by adding our systemic risk index as a fourth factor, we evaluate differences in the values of the parameters and coefficients of determination adjusted for the CAPM three-factor Fama and French (1993) with or without taking into account the systemic risk index as an additional factor.

Secondly, we conduct some robustness checks in order to check the relationship supposing that systemic risk may contaminates the other factors.

Unlike the literature on testing the CAPM, which involves testing the equation of the risk premium, we seek to test the CAPMS, and in particular the significance of the systemic risk factor. To do this, we refer to the work of Black et al. (1972) and resume their method of classification of securities in the portfolios. We first start by creating 10 portfolios, which are constructed in two stages. Calculating β of each asset by least squares regression is performed first. Then assets are grouped into the 10 portfolios according to a sorting rule on β. At time t, the performance of each portfolio is the average returns of each of the securities that compose it.

For the empirical analysis in this section, we use the daily database from a panel of 95 financial institutions in the US market up to the 26th of February 2016. This database contains the market prices of different securities and different ratios of their market capitalizations in daily frequency during the period of September 2, 2003 to June 24, 2014 extracted from Bloomberg. We split our database in two samples[8]. The first sample is the same used by Brownlees and Engle (2017), based on daily data series from the 01/03/2005 to the 06/30/2010. The second is based on the sample of Giglio et al. (2016), on monthly data series from the 01/1926 to the 12/2011. For the sake of simplicity in comparing the results, our sample starts on the 09/03/2003 to the 12/30/2011 in a daily basis.

[8] The empirical test for the entire sample from the 09/03/2003 to the 02/26/2016 with robustness using decontaminated factors, is provided in appendix A.6.

3.2.1 Empirical Tests

In this subsection, we test empirically the CAPMS on the main American financial institutions. The following results are based on the equation system of Eq. (26) related to the linear relationship in Eq. (21).

Table 5: Fama and French (1993) three-factor model with or without a systemic risk factor on the main American financial institutions (1st Sample – Brownlees and Engle, 2017)

We estimate the parameters' values and their t-statistics for each portfolio according to the two models used: the Fama-French (1993) three-factor model (I), and the Fama-French (1993) three-factor model with systemic risk as a fourth factor (II) as in Eq. (21).

$$\begin{cases} r_{i,t} = \alpha_i^{(I)} + \beta_i^{(I)} r_{M,t} + \theta_i^{(I)} r_{SMB,t} + \gamma_i^{(I)} r_{HML,t} + \varepsilon_{i,t}^{(I)} & (I) \\ r_{i,t} = \alpha_i^{(II)} + \beta_i^{(II)} r_{M,t} + \theta_i^{(II)} r_{SMB,t} + \gamma_i^{(II)} r_{HML,t} + \varphi_i^{(II)} r_{ISRM,t} + \varepsilon_{i,t}^{(II)} & (II) \end{cases}$$

	Model I (without systemic risk)					Model II (with systemic risk)					
	$\alpha_i^{(I)}$	$\beta_i^{(I)}$	$\theta_i^{(I)}$	$\gamma_i^{(I)}$	$\overline{R^2}$	$\alpha_i^{(II)}$	$\beta_i^{(II)}$	$\theta_i^{(II)}$	$\gamma_i^{(II)}$	$\varphi_i^{(II)}$	$\overline{R^2}$
Group 1	**7.96**	**1.23**	-.05	**-1.14**	72.76%	**7.84**	**1.23**	-.05	**-1.14**	-.05	72.76%
	(2.14)	(28.16)	(.37)	(7.29)		(2.09)	(29.80)	(.41)	(7.44)	(.16)	
Group 2	**12.00**	**1.07**	-.03	-1.09	70.91%	**1.23**	**1.07**	-.03	**-1.08**	-.20	70.87%
	(3.02)	(17.79)	(.26)	(13.39)		(2.98)	(20.90)	(.39)	(13.54)	(.56)	
Group 3	**7.62**	**1.04**	-.07	**-1.05**	74.73%	**7.61**	**1.05**	-.06	**-1.05**	.08	74.76%
	(2.39)	(20.85)	(.90)	(13.95)		(2.36)	(23.35)	(1.01)	(14.48)	(.24)	
Group 4	**5.56**	**1.01**	-.13	**-.97**	75.23%	**5.84**	**1.00**	-.14	**-.96**	-.24	75.28%
	(2.66)	(29.49)	(1.83)	(10.48)		(2.79)	(27.51)	(1.88)	(10.68)	(1.92)	
Group 5	**8.86**	**.96**	.05	**-.68**	63.11%	**8.93**	**.96**	.04	**-.68**	-.06	63.11%
	(2.41)	(22.55)	(.46)	(9.21)		(2.40)	(22.68)	(.45)	(9.51)	(.26)	
Group 6	3.59	**.93**	-.13	**-.91**	70.47%	3.62	**.91**	-.15	**-.90**	-.35	70.45%
	(1.55)	(18.14)	(1.44)	(13.93)		(1.54)	(18.42)	(1.61)	(14.61)	(1.82)	
Group 7	**4.70**	**.90**	.19	**-.34**	61.05%	**4.78**	**.89**	.17	**-.33**	-.26	61.11%
	(2.00)	(17.16)	(1.50)	(3.20)		(1.99)	(16.58)	(1.44)	(3.14)	(1.47)	
Group 8	**3.66**	**.84**	.01	**-.80**	59.93%	3.57	**.81**	-.02	**-.78**	**-.71**	60.20%
	(.97)	(19.36)	(.10)	(7.75)		(.94)	(19.22)	(.23)	(8.10)	(3.18)	
Group 9	-2.39	**.79**	.07	**-.26**	55.16%	-2.69	**.77**	.04	**-.24**	**-.58**	55.90%
	(1.02)	(18.05)	(.62)	(2.78)		(1.11)	(18.78)	(.36)	(2.64)	(4.19)	
Group 10	7.98	.64	.05	-.07	39.71%	1.02	**.63**	.04	-.06	**-.24**	39.98%
	(.41)	(12.99)	(1.30)	(1.14)		(.53)	(12.88)	(1.07)	(1.02)	(2.31)	

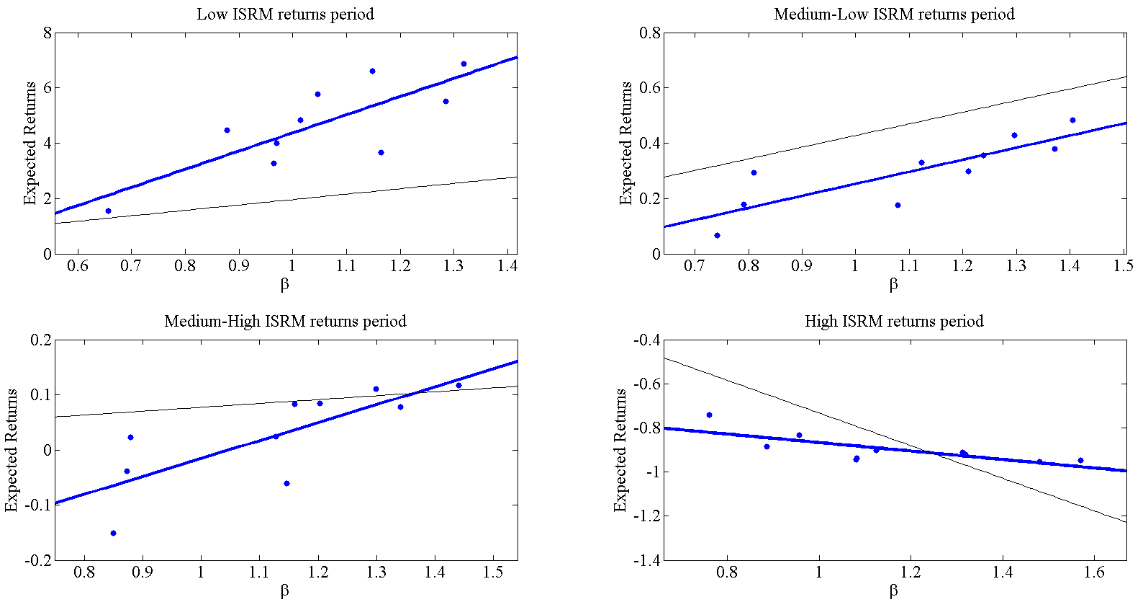
Source: Bloomberg, daily data from the 01/03/2005 to the 06/30/2010 (1st Sample – Brownlees and Engle, 2017) for a set of 95 financial institutions; authors' computation.
Note: The Table provides the estimated values of the parameters using the Zellner (1962) Seemingly Unrelated Regression Estimation (SURE) method *via* GLS, see Eq. (26) and their t-statistic (values in brackets) corrected by Newey-West (1987) for each model. The values of the parameter α are expressed in 10^{-4}. Bold parameters' values are significant at a 5% level.

Table 5 provides the estimates, for the first sample, of the three-factor model of Fama and French (1993) and the model extended to systemic risk (additional factor) while Table 6 does the same but for the second sample. In the two Tables, the two competitive models are compared for 10 portfolios (Groups of stocks).

Most of the portfolios have a significant systemic risk factor and the effect of this factor on the portfolios' returns is negative. In the next Table, we done the same exercise but on the second sample.

Figure 8 shows the evolution of the empirical relationship between expected returns and the level of the risk premium on the market factor, the β parameter in various systemic risk environments for the first sample, while Figure 9 does the same for the second sample.

Figure 8: Evolution of the empirical relationship between expected returns and the market risk factor in various systemic risk environments (1st Sample – Brownlees and Engle, 2017)



Source: Bloomberg, daily data from the 01/03/2005 to the 06/30/2010 (1st Sample – Brownlees and Engle, 2017) for a set of 95 financial institutions; authors' computation.
Note: We first create 10 portfolios based on rankings on the estimated betas relative to the market. The figure shows the relationship between expected returns and the market risk factor when systemic risk factor is low, medium or high. The Low period corresponds to a systemic risk factor $r_{ISRM,t} \in [-0.0227, -0.0012[$, the Medium-low period corresponds to $r_{ISRM,t} \in [-0.0012, -0.00015[$, the Medium-high period to $r_{ISRM,t} \in [-0.00015,\ 0.00089[$ and the High period corresponds to $r_{ISRM,t} \in [0.00089, 0{,}0413[$. The x-axis represents the level of the beta of the portfolios and the y-axis, the expected annualized returns. The thin line is the relationship predicted by the CAPM and the bold line is calculated by regression of the expected returns on the betas.

In Figure 8, the traditional unconditional relationship between risk and return is reversed when systemic risk is high, and thus becomes negative gradually negative as systemic risk increases. In a low systemic risk environment represented by the upper left quadrant, a portfolio with a high β will have higher expected returns in contrast to a portfolio with low β. In a high systemic risk environment represented by the lower right quadrant, a portfolio with a high β will have lower expected returns (even negative) in contrast to a portfolio with a low β.

On the first sample, which replicates the one used by Brownlees and Engle (2017), based on daily data series from the 01/03/2005 to the 06/30/2010, we find that systemic risk is significant for almost all the portfolios, and the relationship between expected returns and systematic risk is inverted when systemic risk increases. We now turn to do the same exercise on the second sample.

Table 6: Fama and French (1993) three-factor model with or without a systemic risk factor on the main American financial institutions (2nd Sample – Giglio et al., 2016)

We estimate the parameters' values and their t-statistics for each portfolio according to the two models used: the Fama-French (1993) three-factor model (I), and the Fama-French (1993) three-factor model with systemic risk as a fourth factor (II) as in Eq. (21).

$$\begin{cases} r_{i,t} = \alpha_i^{(I)} + \beta_i^{(I)} r_{M,t} + \theta_i^{(I)} r_{SMB,t} + \gamma_i^{(I)} r_{HML,t} + \varepsilon_{i,t}^{(I)} & (I) \\ r_{i,t} = \alpha_i^{(II)} + \beta_i^{(II)} r_{M,t} + \theta_i^{(II)} r_{SMB,t} + \gamma_i^{(II)} r_{HML,t} + \varphi_i^{(II)} r_{ISRM,t} + \varepsilon_{i,t}^{(II)} & (II) \end{cases}$$

	Model I (without systemic risk)					Model II (with systemic risk)					
	$\alpha_i^{(I)}$	$\beta_i^{(I)}$	$\theta_i^{(I)}$	$\gamma_i^{(I)}$	$\overline{R^2}$	$\alpha_i^{(II)}$	$\beta_i^{(II)}$	$\theta_i^{(II)}$	$\gamma_i^{(II)}$	$\varphi_i^{(II)}$	$\overline{R^2}$
Group 1	6.64	**1.18**	-.03	**-1.12**	71.69%	6.55	**1.18**	-.04	**-1.12**	-.01	71.69%
	(1.90)	(19.72)	(.24)	(7.35)		(1.85)	(19.00)	(.26)	(7.47)	(.02)	
Group 2	**11.30**	**1.05**	.03	**-1.03**	69.61%	**11.30**	**1.05**	.03	**-1.03**	.02	69.61%
	(3.10)	(18.63)	(.28)	(10.23)		(3.00)	(20.83)	(.31)	(10.21)	(.05)	
Group 3	5.88	**.99**	-.10	**-1.05**	73.49%	**6.50**	**1.00**	-.09	**-1.06**	.31	73.48%
	(1.93)	(17.63)	(1.42)	(13.40)		(2.11)	(17.88)	(1.31)	(13.61)	(1.20)	
Group 4	**5.42**	**.99**	-.13	**-.98**	73.98%	**5.79**	**.97**	-.15	**-.97**	**-.34**	74.11%
	(2.70)	(21.74)	(1.68)	(10.39)		(2.94)	(19.91)	(1.83)	(10.62)	(2.06)	
Group 5	5.61	**.96**	.07	**-.69**	61.93%	5.77	**.96**	.07	**-.69**	.07	61.93%
	(1.71)	(17.00)	(.62)	(8.99)		(1.73)	(16.86)	(.70)	(9.28)	(.30)	
Group 6	1.31	**.89**	-.16	**-.88**	69.18%	1.34	**.88**	-.16	**-.87**	-.01	69.19%
	(.50)	(16.93)	(1.75)	(10.93)		(.52)	(16.82)	(1.73)	(11.26)	(.06)	
Group 7	3.32	**.86**	.14	**-0.36**	59.44%	2.93	**.85**	.13	**-.35**	-.12	59.49%
	(1.41)	(12.28)	(1.07)	(3.08)		(1.23)	(12.15)	(1.03)	(3.04)	(.92)	
Group 8	7.31	**.81**	.01	**-.80**	59.04%	6.70	**.79**	-.02	**-.78**	**-.59**	59.36%
	(1.93)	(17.39)	(.09)	(7.45)		(1.80)	(17.41)	(.20)	(7.75)	(3.10)	
Group 9	1.84	**.77**	.08	**-0.25**	54.22%	2.12	**.75**	.05	**-.23**	**-.56**	55.03%
	(.81)	(14.83)	(.66)	(2.52)		(.90)	(15.23)	(.42)	(2.36)	(4.01)	
Group 10	2.95	**.63**	.02	-.08	42.11%	2.97	**.62**	.01	-.07	**-.20**	42.41%
	(1.29)	(12.65)	(.32)	(1.32)		(1.32)	(12.52)	(.11)	(1.21)	(1.97)	

Source: Bloomberg, daily data from the 09/03/2003 to the 12/30/2011 (2nd Sample – Giglio et al., 2016) for a set of 95 financial institutions; authors' computation.

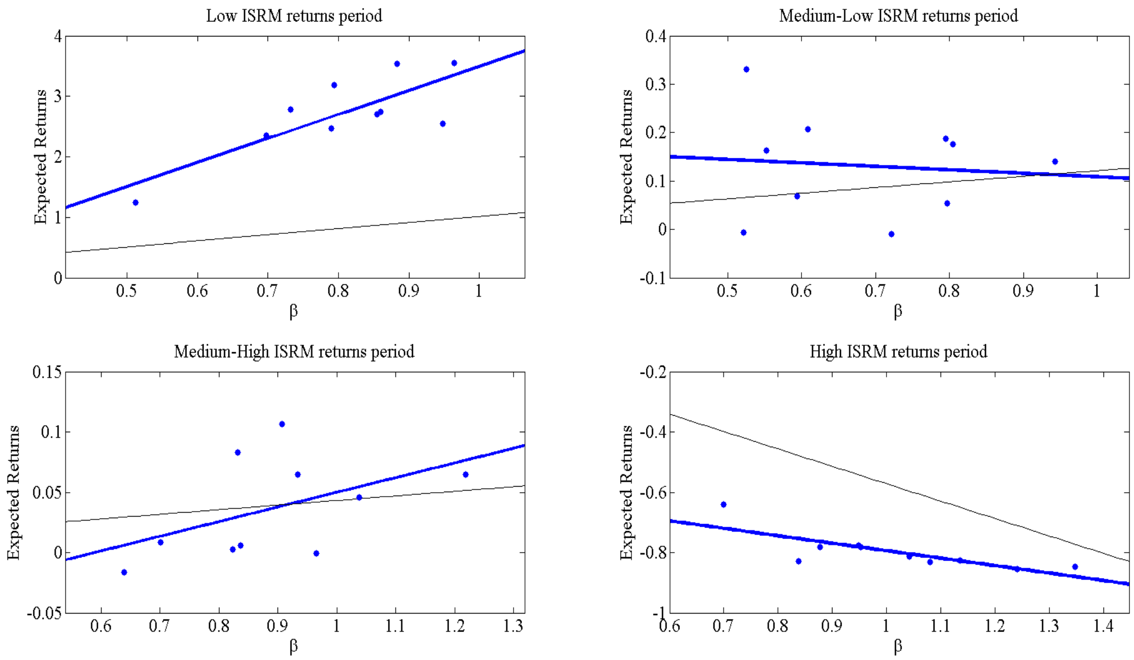
Note: The Table provides the estimated values of the parameters using the Zellner (1962) Seemingly Unrelated Regression Estimation (SURE) method *via* GLS, see Eq. (26) and their t-statistic (values in brackets) corrected by Newey-West (1987) for each model. The values of the parameter α are expressed in 10^{-4}. Bold parameters' values are significant at a 5% level.

As in Table 5, Table 6 provides the estimates of the three-factor model of Fama and French (1993) and the model extended to systemic risk (additional factor) for the second sample.

Again, most of the portfolios have a significant systemic risk factor. But there exist some portfolios in which systemic risk has no impact.

Like Figure 8, Figure 9 shows the evolution of the empirical relationship between expected returns and the level of the risk premium to the market factor, the β parameter in various systemic risk environments but for the second sample. Again, the relationship is reversed and becomes gradually negative as systemic risk increases.

Figure 9: Evolution of the empirical relationship between expected returns and the market risk factor in various systemic risk environments (2nd Sample – Giglio et al., 2016)



Source: Bloomberg, daily data from the 09/03/2003 to the 12/30/2011 (2nd Sample – Giglio et al., 2016) for a set of 95 financial institutions; authors' computation.
Note: We first create 10 portfolios based on rankings on the estimated betas relative to the market. The figure shows the relationship between expected returns and the market risk factor when systemic risk factor is low, medium or high. The Low period corresponds to a systemic risk factor $r_{ISRM,t} \in [-0.0227, -0.0012[$, the Medium-low period corresponds to $r_{ISRM,t} \in [-0.0012, -0.00015[$, the Medium-high period to $r_{ISRM,t} \in [-0.00015,\ 0.00089[$ and the High period corresponds to $r_{ISRM,t} \in [0.00089, 0{,}0413[$. The x-axis represents the level of the beta of the portfolios and the y-axis, the expected annualized returns. The thin line is the relationship predicted by the CAPM and the bold line is calculated by regression of the expected returns on the betas.

By doing the same exercise as on the first sample, we find the same results on the second sample based on the one used by Giglio et al. (2016) from the 09/03/2003 to the 12/30/2011 in a daily basis. In the following subsection, we propose to decontaminate each factor from the systemic risk factor, to see if systemic risk can be more significant.

3.2.2 Robustness checks

In this subsection, we wonder if is it possible that a part of the sensitivity to the systemic risk is absorbed by the other Fama-French factors.

To bring an answer to this question, we then regress the Fama-French factors on the systemic risk factor to obtain decontaminated or orthogonalized[9] Fama-French factors and compare the new model specifications with respect to the previous one on the two samples of Brownlees and Engle (2017) and Giglio et al. (2016).

To be more explicit, we compare in the next results, our benchmark: Model II (with raw factors) with two new specifications using decontaminated and orthogonalized factors which are respectively: Model III (with decontaminated factors) and Model IV (with orthogonalized factors). Their own specifications are based on factors' transformations and are defined in each Table with their parameter estimations and t-statistics for our 10 groups previously constructed.

In the following, we do these exercises on the first and second samples for the decontaminated factors. For the orthogonalized factors, we just present the results of the estimated relationship for each portfolio group with the explicative power and their t-statistic on the entire sample, i.e., from the 09/03/2003 to the 02/26/2016.

Table 7 compares the previous model specification in Model II (raw factors) with the new model specification in Model III (decontaminated factors). Decontaminated factors are obtained by avoiding the effect of the systemic risk factor on the others. Using decontaminated factors from the systemic risk factors, we obtain a better significant systemic risk factor (with as expected, an unchanged total explicative power of the model). Bold parameters' values are significant at a 5% level. We note that the φ coefficient related to systemic risk is significant for most portfolios. Specifically, the systemic risk should be considered in 9 out of 10 portfolios. The addition of systemic risk as an additional factor also changes slightly the estimated parameters of the canonical relationship CAPM extended to three-factor model of Fama and French (1993).

[9] For the sake of simplicity, we just present the results based on all the sample for the orthogonalized factors since, as expected, they are also the same for each sample, i.e., no parameter changes and the φ coefficient related to systemic risk is significant for most portfolios.

Table 7: Fama and French (1993) three-factor model with systemic risk factor (raw or decontaminated) on the main American financial institutions (1st Sample – Brownlees and Engle, 2017)

We estimate the parameters' values and their t-statistics for each portfolio according to the two models used: the Fama-French (1993) three-factor model with systemic risk as a fourth factor (II), and the Fama-French (1993) three-factor model with systemic risk as a fourth factor as in Eq. (21) but by decontaminating the other factors from the ISRM factor (III) following the equation:

$$r_{i,t} = \alpha_i^{(III)} + \beta_i^{(III)}\hat{\varepsilon}_{M,t} + \theta_i^{(III)}\hat{\varepsilon}_{SMB,t} + \gamma_i^{(III)}\hat{\varepsilon}_{HML,t} + \varphi_i^{(III)} r_{ISRM,t} + \varepsilon_{i,t}^{(III)} \quad (III)$$

with:

$$\begin{cases} \hat{\varepsilon}_{M,t} = r_{M,t} - \hat{b}_M r_{ISRM,t} \\ \hat{\varepsilon}_{SMB,t} = r_{SMB,t} - \hat{b}_{SMB} r_{ISRM,t} \\ \hat{\varepsilon}_{HML,t} = r_{HML,t} - \hat{b}_{HML} r_{ISRM,t}. \end{cases}$$

	Model II (with raw factors)						Model III (with decontaminated factors)					
	$\alpha_i^{(II)}$	$\beta_i^{(II)}$	$\theta_i^{(II)}$	$\gamma_i^{(II)}$	$\varphi_i^{(II)}$	$\overline{R^2}$	$\alpha_i^{(III)}$	$\beta_i^{(III)}$	$\theta_i^{(III)}$	$\gamma_i^{(III)}$	$\varphi_i^{(III)}$	$\overline{R^2}$
Group 1	**7.84**	**1.23**	-.05	**-1.14**	-.05	72.76%	**7.32**	**1.23**	-.05	**-1.14**	**-2.65**	72.76%
	(2.09)	(29.80)	(.41)	(7.44)	(.16)		(1.96)	(29.80)	(.41)	(7.44)	(5.57)	
Group 2	**1.23**	**1.07**	-.03	**-1.08**	-.20	70.87%	**11.18**	**1.07**	-.03	**-1.08**	**-2.57**	70.87%
	(2.98)	(20.90)	(.39)	(13.54)	(.56)		(2.87)	(20.90)	(.39)	(13.54)	(5.96)	
Group 3	**7.61**	**1.05**	-.06	**-1.05**	.08	74.76%	**7.16**	**1.05**	-.06	**-1.05**	**-2.21**	74.76%
	(2.36)	(23.35)	(1.01)	(14.48)	(.24)		(2.21)	(23.35)	(1.01)	(14.48)	(5.53)	
Group 4	**5.84**	**1.00**	-.14	**-.96**	-.24	75.28%	**5.44**	**1.00**	-.14	**-.96**	**-2.30**	75.28%
	(2.79)	(27.51)	(1.88)	(10.68)	(1.92)		(2.60)	(27.51)	(1.88)	(10.68)	(12.73)	
Group 5	**8.93**	**.96**	.04	**-.68**	-.06	63.11%	**8.56**	**.96**	.04	**-.68**	**-1.94**	63.11%
	(2.40)	(22.68)	(.45)	(9.51)	(.26)		(2.31)	(22.68)	(.45)	(9.51)	(5.88)	
Group 6	3.62	**.91**	-.15	**-.90**	-.35	70.45%	3.25	**.91**	-.15	**-.90**	**-2.22**	70.45%
	(1.54)	(18.42)	(1.61)	(14.61)	(1.82)		(1.37)	(18.42)	(1.61)	(14.61)	(7.78)	
Group 7	**4.78**	**.89**	.17	**-.33**	-.26	61.11%	4.48	**.89**	.17	**-.33**	**-1.80**	61.11%
	(1.99)	(16.58)	(1.44)	(3.14)	(1.47)		(1.87)	(16.58)	(1.44)	(3.14)	(7.35)	
Group 8	3.57	**.81**	-.02	**-.78**	**-.71**	60.20%	3.23	.81	-.02	**-.78**	**-2.47**	60.20%
	(.94)	(19.22)	(.23)	(8.10)	(3.18)		(.85)	(19.22)	(.23)	(8.10)	(7.16)	
Group 9	-2.69	**.77**	.04	**-.24**	**-.58**	55.90%	-2.92	**.77**	.04	**-.24**	**-1.74**	55.90%
	(1.11)	(18.78)	(.36)	(2.64)	(4.19)		(1.20)	(18.78)	(.36)	(2.64)	(11.26)	
Group 10	1.02	**.63**	.04	-.06	**-.24**	39.98%	.86	**.63**	.04	-.06	**-1.05**	39.98%
	(.53)	(12.88)	(1.07)	(1.02)	(2.31)		(.45)	(12.88)	(1.07)	(1.02)	(13.08)	

Source: Bloomberg, daily data from the 01/03/2005 to the 06/30/2010 (1st Sample – Brownlees and Engle, 2017) for a set of 95 financial institutions; authors' computation.

Note: The Table provides the estimated values of the parameters using the Zellner (1962) Seemingly Unrelated Regression Estimation (SURE) method *via* GLS, see Eq. (26) and their t-statistic (values in brackets) corrected by Newey-West (1987) for each model. The values of the parameter α are expressed in 10^{-4}. Bold parameters' values are significant at a 5% level.

Like on the first sample, results on the second sample are better than when compared to the raw model. We obtained a better significant systemic risk factor (with, once again, an unchanged total explicative power of the model for all portfolios).

Table 8: Fama and French (1993) three-factor model with systemic risk factor (raw or decontaminated) on the main American financial institutions (2nd Sample – Giglio et al., 2016)

We estimate the parameters' values and their t-statistics for each portfolio according to the two models used: the Fama-French (1993) three-factor model with systemic risk as a fourth factor (II), and the Fama-French (1993) three-factor model with systemic risk as a fourth factor as in Eq. (21) but by decontaminating the other factors from the ISRM factor (III) following the equation:

$$r_{i,t} = \alpha_i^{(III)} + \beta_i^{(III)} \hat{\varepsilon}_{M,t} + \theta_i^{(III)} \hat{\varepsilon}_{SMB,t} + \gamma_i^{(III)} \hat{\varepsilon}_{HML,t} + \varphi_i^{(III)} r_{ISRM,t} + \varepsilon_{i,t}^{(III)} \quad (III)$$

with:

$$\begin{cases} \hat{\varepsilon}_{M,t} = r_{M,t} - \hat{b}_M r_{ISRM,t} \\ \hat{\varepsilon}_{SMB,t} = r_{SMB,t} - \hat{b}_{SMB} r_{ISRM,t} \\ \hat{\varepsilon}_{HML,t} = r_{HML,t} - \hat{b}_{HML} r_{ISRM,t}. \end{cases}$$

	Model II (with raw factors)						Model III (with decontaminated factors)					
	$\alpha_i^{(II)}$	$\beta_i^{(II)}$	$\theta_i^{(II)}$	$\gamma_i^{(II)}$	$\varphi_i^{(II)}$	$\overline{R^2}$	$\alpha_i^{(III)}$	$\beta_i^{(III)}$	$\theta_i^{(III)}$	$\gamma_i^{(III)}$	$\varphi_i^{(III)}$	$\overline{R^2}$
Group 1	6.55	**1.18**	-.04	**-1.12**	-.01	71.69%	6.08	**1.18**	-.04	**-1.12**	**-2.38**	71.69%
	(1.85)	(19.00)	-(.26)	-(7.47)	-(.02)		(1.73)	(19.00)	-(.26)	-(7.47)	-(5.83)	
Group 2	**11.3**	**1.05**	.03	**-1.03**	.02	69.61%	**10.91**	**1.05**	.03	**-1.03**	**-2.20**	69.61%
	(3.00)	(20.83)	(.31)	-(10.21)	(.05)		(2.89)	(20.83)	(.31)	-(10.21)	-(5.84)	
Group 3	**6.50**	**1.00**	-.09	**-1.06**	.31	73.48%	**6.09**	**1.00**	-.09	**-1.06**	**-1.77**	73.48%
	(2.11)	(17.88)	-(1.31)	-(13.61)	(1.20)		(1.98)	(17.88)	-(1.31)	-(13.61)	-(5.73)	
Group 4	**5.79**	**.97**	-.15	**-.97**	-.34	74.11%	**5.42**	**.97**	-.15	**-.97**	**-2.24**	74.11%
	(2.94)	(19.91)	-(1.83)	-(10.62)	-(2.06)		(2.75)	(19.91)	-(1.83)	-(10.62)	-(10.99)	
Group 5	5.77	**.96**	.07	**-.69**	.07	61.93%	5.41	**.96**	.07	**-.69**	**-1.73**	61.93%
	(1.73)	(16.86)	(.70)	-(9.28)	(.30)		(1.63)	(16.86)	(.70)	-(9.28)	-(5.53)	
Group 6	1.34	**.88**	-.16	**-.87**	-.01	69.19%	1.01	**.88**	-.16	**-.87**	**-1.70**	69.19%
	(.52)	(16.82)	-(1.73)	-(11.26)	-(.06)		(.39)	(16.82)	-(1.73)	-(11.26)	-(7.59)	
Group 7	2.93	**.85**	.13	**-.35**	-.12	59.49%	2.65	**.85**	.13	**-.35**	**-1.51**	59.49%
	(1.23)	(12.15)	(1.03)	-(3.04)	-(.92)		(1.12)	(12.15)	(1.03)	-(3.04)	-(7.26)	
Group 8	6.70	**.79**	-.02	**-.78**	**-.59**	59.36%	6.37	**.79**	-.02	**-.78**	**-2.21**	59.36%
	(1.80)	(17.41)	-(.20)	-(7.75)	-(3.10)		(1.71)	(17.41)	-(.20)	-(7.75)	-(7.42)	
Group 9	2.12	**0.75**	.05	**-.23**	**-.56**	55.03%	1.91	**.75**	.05	**-.23**	**-1.63**	55.03%
	(.90)	(15.23)	(.42)	-(2.36)	-(4.01)		(.81)	(15.23)	(.42)	-(2.36)	-(10.55)	
Group 10	2.97	**.62**	.01	-.07	**-.20**	42.41%	2.83	**.62**	.01	-.07	**-0.92**	42.41%
	(1.32)	(12.52)	(.11)	-(1.21)	-(1.97)		(1.26)	(12.52)	(.11)	-(1.21)	-(10.65)	

Source: Bloomberg, daily data from the 09/03/2003 to the 12/30/2011 (2nd Sample – Giglio et al., 2016) for a set of 95 financial institutions; authors' computation.

Note: The Table provides the estimated values of the parameters using the Zellner (1962) Seemingly Unrelated Regression Estimation (SURE) method *via* GLS, see Eq. (26) and their t-statistic (values in brackets) corrected by Newey-West (1987) for each model. The values of the parameter α are expressed in 10^{-4}. Bold parameters' values are significant at a 5% level.

In Table 9, we use orthogonalized factors (rather than decontaminated factors from the ISRM factor) on all the sample. Rather than just avoid the effect of the systemic risk factor on the other factors, we now turn to avoid the effect of each factor on the others.

Table 9: Fama and French (1993) three-factor model with systemic risk factor (raw or orthogonalized) on the main American financial institutions (All Sample)

We estimate the parameters' values and their t-statistics for each portfolio according to the two models used: the Fama-French (1993) three-factor model with systemic risk as a fourth factor (II), and the Fama-French (1993) three-factor model with systemic risk as a fourth factor as in Eq. (21) but with orthogonalized factors from the ISRM factor (IV) following the equation:

$$r_{i,t} = \alpha_i^{(IV)} + \beta_i^{(IV)} \hat{\varepsilon}_{M,t} + \theta_i^{(IV)} \hat{\varepsilon}_{SMB,t}^{**} + \gamma_i^{(IV)} \hat{\varepsilon}_{HML,t}^{***} + \varphi_i^{(IV)} r_{ISRM,t} + \varepsilon_{i,t}^{(IV)} \quad (IV)$$

with:

$$\begin{cases} \hat{\varepsilon}_{M,t} = r_{M,t} - \hat{b}_M r_{ISRM,t} \\ \hat{\varepsilon}_{SMB,t}^{**} = r_{SMB,t} - \left(\hat{b}_{SMB}^{*} r_{M,t} + \hat{b}_{SMB}^{**} r_{ISRM,t}\right) \\ \hat{\varepsilon}_{HML,t}^{***} = r_{HML,t} - \left(\hat{b}_{HML}^{*} r_{M,t} + \hat{b}_{HML}^{**} r_{SMB,t} + \hat{b}_{HML}^{***} r_{ISRM,t}\right). \end{cases}$$

	Model II (with raw factors)						Model IV (with orthogonalized factors)					
	$\alpha_i^{(II)}$	$\beta_i^{(II)}$	$\theta_i^{(II)}$	$\gamma_i^{(II)}$	$\varphi_i^{(II)}$	$\overline{R^2}$	$\alpha_i^{(IV)}$	$\beta_i^{(IV)}$	$\theta_i^{(IV)}$	$\gamma_i^{(IV)}$	$\varphi_i^{(IV)}$	$\overline{R^2}$
Group 1	**5.83**	**1.12**	-.06	**-1.15**	-.12	70.12%	**5.53**	**1.49**	**.28**	**-1.15**	**-2.31**	70.12%
	(2.27)	(14.02)	(.86)	(8.02)	(.45)		(2.16)	(17.23)	(2.84)	(8.02)	(6.88)	
Group 2	**5.77**	**1.01**	.00	**-1.06**	-.18	68.67%	5.49	**1.35**	**.31**	**-1.06**	**-2.22**	68.67%
	(1.99)	(18.68)	(.02)	(12.92)	(.81)		(1.89)	(21.36)	(5.15)	(12.92)	(8.18)	
Group 3	3.93	**.95**	-.04	**-1.05**	.12	72.10%	3.67	**1.29**	**.27**	**-1.05**	**-1.83**	72.10%
	(1.73)	(14.57)	(.81)	(14.80)	(.54)		(1.62)	(18.15)	(5.10)	(14.80)	(7.71)	
Group 4	. 82	**.94**	-.07	**-.94**	-.30	71.71%	.58	**1.24**	**.20**	**-.94**	**-2.09**	71.71%
	(.43)	(16.55)	(1.24)	(11.90)	(1.92)		(.30)	(20.22)	(3.11)	(11.90)	(12.02)	
Group 5	**6.42**	**.88**	.03	**-.71**	-.05	60.41%	3.40	**1.14**	**.15**	**-.86**	**-1.81**	66.61%
	(2.95)	(14.15)	(.54)	(11.32)	(.26)		(1.63)	(23.12)	(2.47)	(14.52)	(9.75)	
Group 6	3.62	**.86**	-.11	**-.86**	-.20	66.61%	**6.19**	**1.11**	**.24**	**-.71**	**-1.66**	60.41%
	(1.74)	(17.82)	(1.57)	(14.52)	(1.34)		(2.86)	(18.54)	(3.97)	(11.32)	(7.40)	
Group 7	**3.69**	**.78**	.10	**-.41**	**-.32**	57.12%	3.09	**1.00**	**.21**	**-.73**	**-1.97**	57.07%
	(2.33)	(9.87)	(1.21)	(4.34)	(2.06)		(1.09)	(18.86)	(3.58)	(8.34)	(8.09)	
Group 8	3.30	**.76**	-.01	**-.73**	**-.50**	57.07%	**3.49**	**.91**	**.21**	**-.41**	**-1.57**	57.12%
	(1.16)	(17.71)	(.10)	(8.34)	(2.90)		(2.20)	(12.02)	(3.36)	(4.34)	(8.73)	
Group 9	-1.51	**.71**	.00	**-.29**	**-.53**	52.54%	-1.67	**.80**	.08	**-.29**	**-1.51**	52.54%
	(.87)	(14.10)	(.05)	(3.92)	(3.89)		(.96)	(15.64)	(1.67)	(3.92)	(10.08)	
Group 10	3.20	**.61**	.04	-.09	**-.26**	32.58%	3.06	**.64**	.06	-.09	**-.98**	32.58%
	(.97)	(12.99)	(1.12)	(1.39)	(2.91)		(.93)	(13.82)	(1.66)	(1.39)	(13.79)	

Source: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016 (All Sample) for a set of 95 financial institutions; authors' computation.
Note: The Table provides the estimated values of the parameters using the Zellner (1962) Seemingly Unrelated Regression Estimation (SURE) method *via* GLS, see Eq. (26) and their t-statistic (values in brackets) corrected by Newey-West (1987) for each model. The values of the parameter α are expressed in 10^{-4}. Bold parameters' values are significant at a 5% level.

Table 9 strengthens our previous results, since, as expected, the effect of systemic risk, now isolated, is still very significant[10]. Another interesting result is that the SMB factor becomes significant when using orthogonalized factors rather than raw or decontaminated factors from the ISRM factor, meaning that the SMB factor has some information content as well as the HML factor.

[10] Compared to the model (II), expressions of the *SMB* and *HML* factors differ in Models (III) and (IV).

By testing the CAPMS and doing some robustness checks, we find the same results on the two samples and on the all sample. Systemic risk is significant for most portfolios. However, the relationship between expected returns and systematic risk tends to be inverted when systemic risk is high. In the following section, we propose an original application of the CAPMS, that is, designating and ranking SIFI.

4. An application of the CAPMS: designating and ranking SIFI

We hereafter propose an original application of the CAPMS, attempting to measure the systemically importance of financial institutions by the sensitivities to the global systemic risk factor.

4.1. What is a SIFI?

The systemic importance of an institution should be explained by the potential threat to the economy that the collapse of this specific identified institution would generate. Large banks, insurance companies, clearing houses, finance companies and investment funds are natural examples of potential candidates to be considered as Systemically Important Financial Institutions (SIFI). However, in order to determine which institutions can actually be designated as SIFI, it is necessary to define what are the key features of systemic risk and how to quantify it.

Until recently the principle of the too-big-to-fail was prevalent. A large institution was considered systemic by this simple fact. But too-interconnected-to-fail, leverage, market exposures, conditional states of the market, size and complexity of the institution have to be taken into account to judge if an institution is a SIFI or not. These various aspects are crucial and this is the reason why several measures have been recently developed in the academic literature. Indeed, Bisias et al. (2012), already report 31 measures of systemic risk in which their analysis focuses on critical themes in systemic risk measurement and management, whilst the literature is still growing with the number of systemic risk measures. This growth is extensive because of the different sources of public or private data now available. In this perspective, Benoit et al. (2017a) review the extensive literature on systemic risk and connect it to the current regulatory debate and identify a gap between two main approaches. The first one studies different sources of systemic risk in isolation, uses confidential data, and inspires targeted but complex regulatory tools. The second approach uses market data to produce global measures which are not directly connected to any particular theory, but could support a more efficient regulation. They argue that bridging this gap will require encompassing theoretical models and improved data disclosure. Building systemic risk measures that can fill this gap must include a comprehensive view of systemic risk in order to be used as a baseline for financial and monetary policies to maintain a stable financial system. De Bandt et al. (2000), argue that a comprehensive view of systemic risk has to integrate bank failure contagion with financial markets spillover effects and payment and settlement risks. Until recently, these measures were implemented separately, even though systemic risk is a clearly multidimensional phenomenon. In Giglio et al. (2016), several measures are used together in order to build a set of systemic risk indexes with the use of a reduction dimension technique, namely a classical PCA.

More generally, there are two main traditional approaches for identifying a SIFI: indicator methods and modelling methods. The first approach relies on factors related to an entity's contribution to systemic risk, and on an *ad hoc* score that is used to decide whether it is a SIFI. For example, measures of an entity's size and reliance on short-term debt may be weighted and combined to produce a systemic risk score. If the entity scores above a certain threshold, then it is identified as a SIFI. Important questions include how such *criteria* should be selected, measured and weighted for different types of financial institutions. The second approach attempts to mathematically model the interconnections among firms. The models are calibrated using available data and simulated future crisis scenarios to estimate the size of potential losses and Spillover effects between institutions. Those estimates are then used to determine an institution's contribution to systemic losses and its status as a SIFI.

At present, regulatory bodies typically employ indicator approaches to identify SIFI. They are considered to be more straightforward and simple to administer (in the sense of operational costs and delivering a flexible and efficient regulation). While a full modelling approach is not yet considered practical at this time, combining the techniques, or incorporating a model's output as a factor in an indicator method, may help to bridge the gap (see also Benoit et al., 2017b).

When we compare our actual framework to other current approaches used to define SIFI, the first similarity is in the use of indicator and modelling methods to establish a score. Secondly, we base our approach in a set of systemic risk measures and use a reduction dimension technique in order to obtain a single output. Our approach, however, differs from the current ones since we propose to combine indicator and modelling methods and use them altogether. In this methodology, we use the Sparse-PCA to define a systemic risk factor, then the canonical Fama-French (1993) three-factor CAPM, extended to the previously defined systemic risk factor, for ultimately determine a ranking based on the impact of the different institutions on the global systemic risk. Thus, we incorporate a model's output as a factor in an indicator method.

4.2. Methodology for Designating SIFI according to the CAPMS

Our proposed methodology ultimately contains 5 steps. First, a panel of systemic risk measures is computed (Cf. Bisias et al., 2012, for the main ones), all focusing on the various aspects of the systemic risk that characterize financial institutions. Since this phenomenon is multidimensional *per* nature, we also use other measures such as credit spreads, VaR, book and market leverage, in order to take into account market conditions, size, leverage, interconnections, instability and credit conditions in the system. All measures use publicly available data. Of course, several adaptations will reside on the definition of appropriate systemic risk measures for specific types of financial institutions.

Secondly, we aggregate the information extracted from the panel of systemic risk measures defined at the first step, by using reduction dimension technique, namely a Sparse-PCA, where sparseness implies a smaller number of the signal samples that are significantly different from zero. This approach identifies the aspects of the systemic risk that represent the components leading to a potential market collapse over a given period. Moreover, this approach gives us the visibility on which of the relevant variables may cause a systemic event, and provide an assessment on these variables. Once we have identified the principal components from the model, we proceed to the construction of an aggregated Index of Systemic Risk Measures (ISRM) using the specific weighted sum of these components, connecting the final choice of the Sparse-PCA nuisance parameter (the smoothness parameter) and the variations of the future GDP (Cf. step 2 in appendix A.3). More precisely, we evaluate which of the potential indexes (amongst the various potential Sparse-PCA candidates) can be considered as an advanced indicator of economic activity. We use for this purpose the truncation parameter - which governs the smoothness and the sparseness of the components, that leads to the most parsimonious ISRM (*via* the statistical test of Hong et al., 2009, that introduces a

Granger causality-in-risk test - choosing what are the variations that best explain the extreme variations of the GDP in various horizons).

Thirdly, we add the parsimonious ISRM in the Fama-French (1993) three-factor model and we obtain a new risk premium level: the systemic risk premium level. That is its level and also its relevance through the corrected t-statistic that gives us an objective ranking *criterion*. The higher the systemic risk sensitivity, the more Systemically Important the Financial Institution.

Of course, distinctions between firms of national *versus* global systemic importance should be expected and can be obtained both 1) by selecting institutions (and market index) from a national, a global market or a specific economic sector, and 2) by building an index of systemic risk measures that are relevant for the related market. Note also here that reverse SIFI designation based on our approach is always feasible, since the methodology is dynamic, flexible and easy to understand. Moreover, all the data is publicly available and the methodology is easy to compute, just by updating the ISRM index, the return series and the GDP growth rate. The model and the methodology could thus be run on every quarter, year…, and rankings adapted to new market and financial institution conditions.

Ranking potential SIFI by their sensitivities to the systemic risk has, obviously, to take into account in some way the significance level of these sensitivities. But, the severity of the assessment should be adapted to market conditions and resources of regulators. Should be selected in our proposal a certain percentile (60% in our illustrative test) of the main SIFI ranked according to the significance of their sensitivities to the systemic risk. For example, if the confidence threshold is set to .1%[11], and the given percentile is 60% of the significant financial institutions, we find 32 SIFI in our preliminary studies. In this case, all Office of Financial Research (OFR) detected SIFI[12] are present in the sample of SIFI signalled by our proposal.

The main idea behind the methodology is that if regulator bodies can impose a decrease on systemic risk measures for the (main) SIFI - ranked according to their sensitivities to the systemic risk to which they contribute themselves, then, the Index of Systemic Risk Measures will be constrained, and thus the global systemic risk reduced. The number of defined SIFI, in our opinion, should be 1) large enough in order to avoid missing an important player; 2) large enough for being able to control the global risk

[11] There is no difference for a range of .1% to 1% significance levels while there is a little difference for 5% to 10% significance levels (see Table 12).

[12] With the important difference for HSBC (see Figure 10).

when imposing marginal restrictions on entity systemic risk measures; 3) time-varying in some rational ways in order to reduce the moral hazard strategic game of being considered as protected, since designated as a stamped-and-protected SIFI, and 4) not too large for being able to use efficiently the scarce resources of regulators.

4.3. Empirical Results: Designating and Ranking American Financial Institutions

In this part, we conduct our ranking methodology on the 12 selected SIFI according to the ranking published by the OFR in 2013. These SIFI are the 12 Bank Holding Companies (BHC) listed by the OFR in 2013 that have assets over $250 billion.

Table 10 presents the estimation results of the three-factor model of Fama-French (1993) extended to an additional systemic risk factor for the 12 Bank Holding Companies (BHCs) on two samples: from 09/03/2003 to 06/24/2014 and from 09/03/2003 to 02/26/2016. The results are particularly interesting. Indeed, the coefficient affected to the systemic risk factor is significant for all the banks on the first sample but this coefficient seems to become less significant in the time. This evolution can be related with the regulation settlement and the first publication of the SIFI ranking by the OFR.

Table 11 below provides the ranking of 12 BHCs ordered with respect to their systemic risk score of the OFR ranking established in 2013. The eight grey-shaded BHCs were Global Systemically Important Banks (-SIBs) as of 2013. We establish the ranking obtained from the systemic-risk extended Fama-French (1993) three-factor model on four periods: over the year 2013, over the period 2003-2013, over the period 2003-2014 and over the period 2003-2016.

In Table 12 below, the proposed methodology is used on a database of 60 financial institutions using both the Ordinary Least Squares (OLS) technique and the Zellner (1962) Seemingly Unrelated Regression Estimation (SURE) method, to estimate the Fama-French (1993) three-factor model extended to the systemic risk factor, in order to find whether they are SIFI or not. This Table represents the 60% of the most important financial institutions related to systemic risk. For example, in Panel A, we find 32 SIFI (if we use OLS) and 46 SIFI (if we use the SURE method) respectively at .1% significance level (extremely severe threshold), that correspond in fact to 27 and 19 SIFI of the percentile (60% here) of significant Financial Institutions. In other words, these percentiles represent 45% and 32% respectively of the total number of Financial Institutions.

Table 10: Estimates of the Fama-French (1993) three-factor model with an additional Systemic Risk Factor on the 12 OFR-selected SIFI

In the Table below are presented the estimation of the Fama-French (1993) three-factor model with systemic risk, following the equation:

$$r_{i,t} = \alpha_i^{(II)} + \beta_i^{(II)} r_{M,t} + \theta_i^{(II)} r_{SMB,t} + \gamma_i^{(II)} r_{HML,t} + \varphi_i^{(II)} r_{ISRM,t} + \varepsilon_{i,t}^{(II)} \quad (II).$$

where the factors are the market returns denoted $r_{M,t}$, the Small Minus Big portfolio returns denoted $r_{SMB,t}$, and the High Minus Low portfolio returns, denoted $r_{HML,t}$ and where the systemic risk factor denoted $r_{ISRM,t}$ is the additional fourth factor.

	From 09/03/2003 to 06/24/2014						From 09/03/2003 to 02/26/2016					
	$\alpha_i^{(II)}$	$\beta_i^{(II)}$	$\theta_i^{(II)}$	$\gamma_i^{(II)}$	$\varphi_i^{(II)}$	$\overline{R^2}$	$\alpha_i^{(II)}$	$\beta_i^{(II)}$	$\theta_i^{(II)}$	$\gamma_i^{(II)}$	$\varphi_i^{(II)}$	$\overline{R^2}$
JPM	1.62	**1.30**	**-.31**	**-.11**	**-2.76**	59.09%	3.30	**.71**	**-.07**	**-.84**	.07	52.23%
	(.33)	(61.15)	(6.68)	(2.19)	(27.45)		(.84)	(32.96)	(3.22)	(3.66)	(.76)	
C	**1.56**	**1.64**	**-.37**	.06	**-2.48**	45.36%	3.71	**.71**	**-.07**	**-.28**	-.23	44.02%
	(3.13)	(76.35)	(8.06)	(1.31)	(25.19)		(.83)	(21.55)	(2.23)	(7.27)	(1.83)	
BAC	-2.16	**1.64**	**-.11**	-.05	**-3.17**	47.79%	7.40	**.90**	**-.23**	**-1.27**	**.44**	58.66%
	(.44)	(79.06)	(2.38)	(1.05)	(32.68)		(1.88)	(41.44)	(1.19)	(46.14)	(4.92)	
WFC	1.34	**1.28**	**-.25**	**-.34**	**-2.98**	48.34%	4.30	**.57**	**-.08**	**-.11**	**-.25**	26.72%
	(.27)	(61.44)	(5.52)	(6.91)	(29.63)		(1.34)	(26.12)	(3.43)	(4.08)	(2.79)	
GS	3.62	**1.21**	**-.14**	.03	**-1.34**	54.87%	1.50	**.81**	**-.09**	**-.57**	**-.58**	47.01%
	(.73)	(55.31)	(3.06)	(.66)	(13.79)		(.38)	(37.89)	(4.22)	(2.95)	(6.40)	
MS	-.56	**1.88**	.08	**.18**	**-1.70**	52.54%	**1.20**	**.83**	**-.12**	**-1.76**	-.11	58.53%
	(1.13)	(88.34)	(1.76)	(3.70)	(17.30)		(2.58)	(39.34)	(5.13)	(64.45)	(1.21)	
USB	-.22	**1.09**	**-.23**	**-.31**	**-2.26**	51.42%	5.76	**.97**	**.04**	**-.93**	-.09	33.75%
	(.04)	(50.42)	(5.01)	(6.54)	(22.43)		(1.28)	(29.42)	(1.22)	(22.76)	(.71)	
PNC	-9.10	**1.14**	**-.29**	**-.16**	**-2.18**	45.40%	**2.70**	**.49**	**2.23**	**-1.30**	**-2.28**	17.47%
	(1.83)	(53.77)	(6.37)	(3.37)	(21.92)		(4.60)	(15.99)	(71.58)	(32.59)	(16.23)	
BK	.64	**1.35**	-.05	**-.35**	**-1.69**	58.45%	6.96	**.55**	**-.15**	**-2.19**	**-3.13**	34.60%
	(.13)	(61.94)	(1.14)	(7.31)	(16.95)		(1.55)	(17.31)	(4.27)	(56.16)	(22.76)	
HSBC	7.21	**1.00**	**-.12**	.01	**-.46**	59.33%	**9.78**	**1.13**	**.08**	**-.48**	**-.43**	55.71%
	(1.45)	(48.04)	(2.57)	(.23)	(4.52)		(2.18)	(33.83)	(2.19)	(11.46)	(3.51)	
STT	-3.58	**1.47**	**-.09**	**-.26**	**-.85**	47.74%	5.70	**.77**	**-.12**	**-1.68**	-.05	57.36%
	(.87)	(67.66)	(2.04)	(5.45)	(8.58)		(1.45)	(36.39)	(5.36)	(6.76)	(.56)	
COF	-2.73	**1.50**	**-.23**	.02	**-2.18**	49.16%	2.52	**1.17**	**.21**	**-.98**	**-.72**	33.50%
	(.67)	(67.59)	(5.08)	(.43)	(21.77)		(.56)	(38.65)	(6.73)	(24.32)	(5.43)	

Source: Bloomberg. Net Asset Values in USD; cylindrical sample of American financial institutions (Cf. Brownlees and Engle, 2017); authors' computations.

Note: This Table reports the estimates of the Fama-French (1993) three-factor model in which we add a fourth factor of systemic risk for the 12 BHCs listed by the OFR in 2013. We estimate the relation for each SIFI by using the Zellner (1962) SURE method *via* GLS, see Eq. (26) and their t-statistic (values in brackets) corrected by Newey-West (1987). There are two estimation periods. In the left part of the Table, the estimation period starts from the 09/032003 to the 06/24/2014. In the right part, it starts form 09/03/2003 to 23/02/2016. The α are expressed in 10^{-4} terms.

Figure 10 below provides the comparison between SIFI designated by the OFR ranking in 2013, and the choice of a percentile (60% here) of the SIFI ranked by their sensitivities to systemic risk given a significance level fixed here to .1%. In dark-grey are the percentile (60% here corresponds to 32 financial institutions) of SIFI detected by the three-factor model extended to systemic risk, and in white are the Financial Institutions (FI) not belonging to this percentile (i.e. not significant). The dark-grey-bars are the SIFI detected following our proposal, the light-grey-bars are the SIFI detected by the OFR and the solid-mid-grey-bars are the SIFI detected both by the three-factor model extended to systemic risk and the OFR. Panel A is based on the estimation period started from the 09/03/2003 to the 06/24/2014 and Panel B is based on the same starting date until 02/26/2016.

Table 11: Comparative rankings of SIFI

Panel A: Comparative rankings over three periods

BHCs	OFR ranking	Fama-French (1993) three-factor with ISRM (ranking)			
	2013	2013	2003-2013	2003-2014	2003-2016
JPM	1	4	4	3	3
C	2	1	3	4	7
BAC	3	7	1	1	11
MS	4	3	10	8	12
GS	5	2	8	10	5
WFC	6	8	2	2	1
BK	7	9	9	9	10
STT	8	11	11	11	6
HSBC	9	12	12	12	2
USB	10	10	6	5	8
PNC	11	6	7	6	4
COF	12	5	5	7	9

Panel B: Correlations with the 2013 OFR ranking

	01/2013-12/2013			2003-2013			2003-2014			2003-2016		
Correlations*	ρ	τ	γ	ρ	τ	γ	P	τ	γ	ρ	τ	γ
The Sample	.52	.36	.52	.38	.21	.38	.42	.33	.42	-.07	-.03	-.07

Source: OFR. This Table shows BHCs with assets over $250 billion. The eight grey-shaded BHCs were G-SIBs as of 2013; authors' computations.
Note: In Panel A, the column named "OFR 2013 ranking" represents the ranking of the OFR. The last column named "Fama-French (1993) three-factor with ISRM ranking" provides the ranking for 4 periods 2013 (01/2013-12/2013), then 2003-2013 and 2003-2014 and 2003-2016. In Panel B, the correlations for each ranking period based on the Fama-French (1993) three-factor with ISRM are presented. Panel B shows the corresponding correlation coefficients (*ρ: Spearman; τ: Kendall; γ: Pearson). Authors computations.

Table 12: Number of detected SIFI for a given significance level

Panel A: For the estimation period starts from the 09/03/2003 to the 06/24/2014

	SIFI at .1% significance level		SIFI at .5% significance level		SIFI at 1% significance level		SIFI at 5% significance level		SIFI at 10% significance level	
Estimation Method	OLS	SURE	OLS	SURE	OLS	SURE	OLS	SURE	OLS	SURE
Significant Financial Institutions	46	32	49	32	51	32	53	34	54	34
Percentile (60%) of significant Financial Institutions	27	19	29	19	30	19	31	20	32	20
In percent of the total number of Financial Institutions	45%	32%	48%	32%	50%	32%	52%	33%	53%	33%

Panel B: For the estimation period starts from the 09/03/2003 to the 02/26/2016

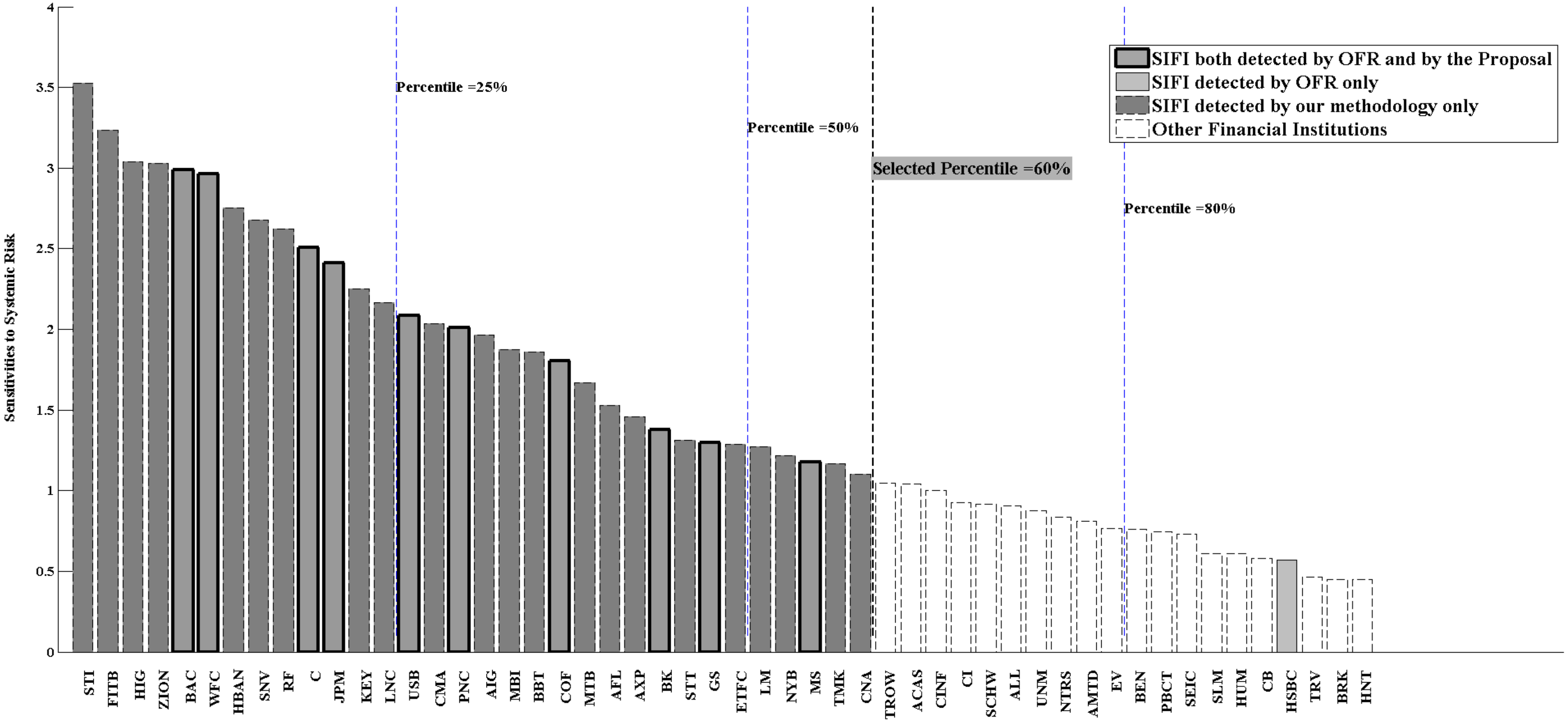
	SIFI at .1% significance level		SIFI at .5% significance level		SIFI at 1% significance level		SIFI at 5% significance level		SIFI at 10% significance level	
Estimation Method	OLS	SURE	OLS	SURE	OLS	SURE	OLS	SURE	OLS	SURE
Significant Financial Institutions	23	36	26	38	29	38	37	45	40	49
Percentile (60%) of significant Financial Institutions	13	21	15	22	17	22	22	27	24	29
In percent of the total number of Financial Institutions	22%	35%	25%	37%	28%	37%	37%	45%	40%	48%

Source: Bloomberg. Net Asset Values in USD; cylindrical sample of American Financial Institutions (Cf. Brownlees and Engle, 2017).

Note: This Table reports the number and the percentage of SIFI over a sample of 60 potential SIFI using the OLS and SURE methods. Each row is decomposed into five columns according to the significance level also decomposed into two main columns according to the method used. The first row gives the estimation method, the second gives the number of significant financial institutions, the third provides the retained number of these institutions according to a selected percentile (60% here) and the last row gives the percentage of retained financial institutions on the total number of financial institutions. Each Panel provides the detections respectively for a threshold of .5%, 1%, 5% and 10%. For Panel A, the estimation period starts from the 09/03/2003 to the 06/24/2014. For Panel B, the estimation period starts from the 09/03/2003 to the 02/26/2016. Authors' computations.

Figure 10: Comparison of SIFI detection

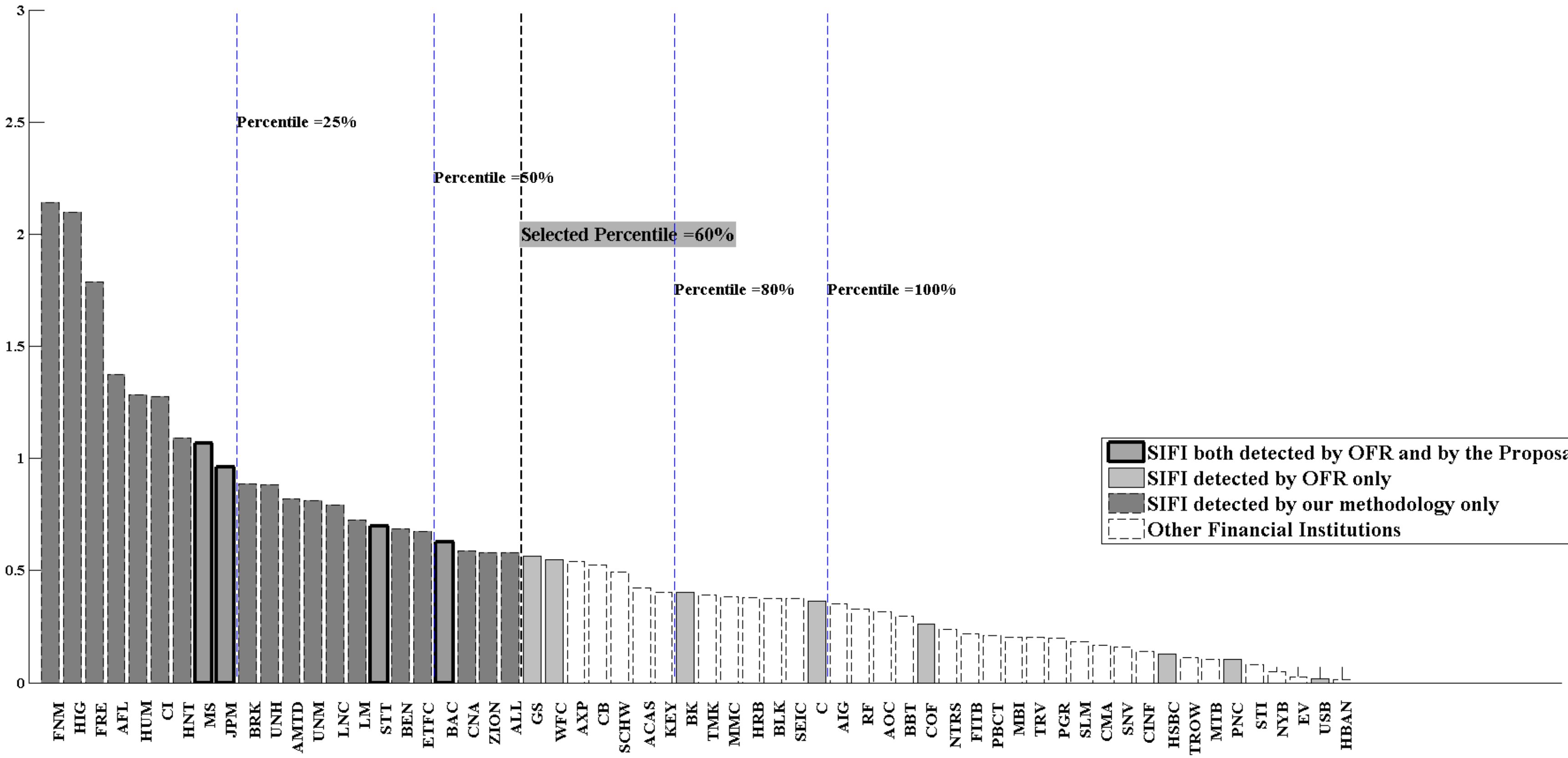
Panel A: Comparison on the period from the 09/03/2003 to the 06/24/2014



Source: Bloomberg. Net Asset Values in USD; cylindrical sample of American financial institutions (Cf. Brownlees and Engle, 2017).

Note: This Figure reports the ranked SIFI according to their sensitivities to systemic risk given a significance level of .1%. In dark-grey are the percentile (60% here) of SIFI detected by the three-factor model extended to systemic risk and in dashed white-bar are the other FI not belonging to the percentile. The grey-bars are the SIFI detected by a methodology. The estimation period starts from the 09/03/2003 to the 06/24/2014. The sensitivities are expressed in absolute values. Authors' computations.

Panel B: Comparison on the period from the 09/03/2003 to the 02/26/2016



Source: Bloomberg. Net Asset Values in USD; cylindrical sample of American financial institutions (Cf. Brownlees and Engle, 2017).

Note: This Figure reports the ranked SIFI according to their sensitivities to systemic risk given a significance level of .1%. In dark-grey are the percentile (60% here) of SIFI detected by the three-factor model extended to systemic risk and in dashed white-bar are the other FI not belonging to the percentile. The grey-bars are the SIFI detected by a methodology. The estimation period starts from the 09/03/2003 to the 02/26/2016. The sensitivities are expressed in absolute values. Authors' computations.

Conclusion

Following Giglio et al. (2016) and from a set of quantitative measures of systemic risk, we begin our study with the construction of a systemic risk index on the US equity market. The rationality of the exercise lies in the multiplicity of global systemic risk metrics introduced in the literature since the last global financial crisis, and existing differences between these metrics. These emerge as each metric evaluates a particular facet of systemic risk, but with a model risk attached to each measure. Our methodology is based on a Sparse Principal Component Analysis (SPCA), which, at the difference of a traditional PCA, selects a small number of systemic risk measures for the construction of some aggregate indices. Consequently, the resulting indices are more parsimonious and have, by construction, a more stable dynamic. We then analyse each of these indices and their components, and select the index that best predicts the extreme variations of the future GDP in the sense of the Hong et al. (2009) causality test in extreme movements. Then we proceed first by assessing the relationship between the optimal systemic risk index and the future real activity or the market index. Second, we compare our approach with the one used by Giglio et al. (2016). As expected, we also find a significant relationship between the optimal systemic risk index and the future real activity.

Then, we test a new model, called Capital Asset Pricing Model with Systemic risk (CAPMS), as the three-factor model of Fama and French (1993) to which we add our systemic risk index. Using the Black et al. (1972) portfolio sorting methodology, we obtain significant coefficients for most portfolios in the US market. This new illustration thus helps to better explain variations in asset returns, while essentially finding, however, the original relationship generated by the three-factor model of Fama and French (1993). Robustness tests using decontaminated factors from the additional systemic risk factor and orthogonalized ones strengthen our results.

Finally, we propose an original application of the CAPMS that could also be used for detection of SIFI, with the underlying idea of being able to estimate the sensitivities of various financial institutions to systemic risk. Thus, a ranking of these institutions, according to the most statistically significant parameters, might be based on the CAPMS: the importance of financial institutions would be determined by the level of their link to the global systemic risk. Our approach could contribute to the definition and detection of SIFI, within an explicit and rational framework based on a traditional pricing model. This is, in our opinion, of major interest for macro-prudential regulation and the stability of the financial system as a whole, especially if the developed model allows us to develop an Early Warning Signal based on forecasts of the various components of the systemic risk.

Beyond our approach, it could be interesting, first, to try to better complement the panel of systemic risk measures used in the construction of our systemic risk index, for extracting even more information. Second, to verify further the robustness of CAPMS, we should extend this study to other markets. Indeed, such studies would increase the number of assets that make up our portfolio, increasing the explanatory power of CAPMS through the significance of the estimated parameters. Indeed, the Office of Financial Research (OFR) and the regulators like the Financial Stability Board (FSB) with the Basel Committee on Banking Supervision (BCBS) update and rank the Systemically Important Financial Institutions (SIFI) in order to apply to them higher capital requirement. It appears that many SIFI are European financial institutions. In addition, other extensions are also possible such as, for instance, adding additional factors (Carhart, 1997). Finally, in the framework of the recent approach by Billio et al. (2015), it may be interesting to consider the evolution of connections between institutions to better reflect the micro-structural market changes and connections in terms of sector, credit and liquidity. One can also imagine a systemic impact coefficient that is no longer a scalar but a vector, each element being attached to a stock or portfolio. To conclude on the improvements, a novel approach will be to build systemic early warning systems on the basis of the recent improvements in early warning signals proposed by Candelon et al. (2014) and on a set of systemic risk measures to predict potential systemic crisis in the future.

References

Acharya, V., Engle, R., Richardson, M., 2012. Capital shortfall: a new approach to ranking, regulating systemic risks. American Economic Review 102(3), 59-64.

Acharya, V., Pedersen, L., 2005. Asset pricing with liquidity risk. Journal of Financial Economics 77, 375-410.

Acharya, V., Pedersen, L., Philippon, T., Richardson M., 2016. Measuring Systemic Risk. Review of Financial Studies 30(1), 2-47.

Adrian, T., Brunnermeier, M., 2016. CoVaR. American Economic Review 106(7), 1705-1741.

Amihud, Y., 2002. Illiquidity and stock returns: cross-section and time-series effect. Journal of Financial Markets 5(1), 31-56.

Amihud, Y., Mendelson, H., 1986. Asset pricing and the bid-ask spread. Journal of Financial Economics 17, 223-249.

D'Aspremont, A., El Ghaoui, L., Jordan, M., Lanckriet, G., 2007. A direct formulation for sparse PCA using semidefinite programming. Society for Industrial and Applied Mathematics Review 49(3), 434-448.

Banz, R., 1981. The relationship between return and market value of common stocks. Journal of Financial Economics 9, 3-18.

Banulescu, G., Dumitrescu, E., 2015. Which are the SIFIs? A Component Expected Shortfall CES approach to systemic risk. Journal of Banking and Finance 55, 575-588.

Barone-Adesi, G., Gagliardini, P., Urga, G., 2003. Testing asset pricing models with coskewness. Journal of Business and Economic Statistics 22(4), 474-485.

Benoit, S., Colletaz, G., Hurlin, C., Perignon, C., 2014. A theoretical and empirical comparison of systemic risk measures. HEC Paris Research Paper FIN-2014-1030, 47 pages.

Benoit, S., Hurlin, C., Perignon, C., 2015. Implied risk exposures. Review of Finance 19(6), 2183-2222.

Benoit, S., Colliard, J., Hurlin, C., Perignon, C., 2017a. Where the risks lie: a survey on systemic risk. Review of Finance 21(1), 109-152.

Benoit, S., Hurlin, C., Perignon, C., 2017b. Pitfalls in systemic-risk scoring. HEC Paris Research Paper FIN-2013-1005, 73 pages.

Billio, M., Getmansky, M., Lo, A., Pelizzon, L., 2012. Econometric measures of connectedness and systemic risk in the finance and insurance sectors. Journal of Financial Economics 104(3), 535-559.

Billio, M., Caporin, M., Panzica, R., Pelizzon, L., 2015. Network connectivity and systemic risk. University Ca' Foscari Venezia Working Paper, 43 pages.

Bisias, D., Flood, M., Lo, A., Valavanis, S., 2012. A survey of systemic risk analytics. Annual Review of Financial Economics 4, 255-296.

Black, F., Jensen, M., Scholes, M., 1972. The Capital Asset Pricing Model: some empirical tests. In: M. C. Jensen (Ed.), Studies in the Theory of Capital Markets.

Boucher, C., Kouontchou, P., Maillet, B., 2015. Du risque des mesures de risque systémique. Revue Economique 2(67), 263-278.

Boucher, C., Daníelsson, J., Kouontchou, P., Maillet, B., 2014. Risk models-at-risk. Journal of Banking and Finance 44, 72-92.

Brennan, M., Subrahmanyam, A., 1996. Market microstructure and asset pricing: on the compensation for illiquidity in stock returns. Journal of Financial Economics 41, 441-464.
Brownlees, T., Engle, R., 2017. SRISK: a conditional capital shortfall index for systemic risk measurement. Review of Financial Studies, 30(1), 48-79.
Brunnermeier, M., Cheridito, P., 2013. Measuring and allocating systemic risk. Princeton University Working Paper, 22 pages.
Canina L., Figlewski, S., 1993. The informational content of implied volatility. Review of Financial Studies 6(3), 659-681.
Candelon B., Dumitrescu E.-I., Hurlin C., 2014. Currency crisis early warning systems: Why they should be dynamic. International Journal of Forecasting 30, 1016-1029.
Carhart, M., 1997. On persistence in mutual fund performance. Journal of Finance 52, 57-82.
Chordia, T., Roll, R., Subrahmanyam, A., 2001. Market liquidity and trading activity. Journal of Finance 56, 501-530.
Daníelsson, J., James, K., Valenzuela, M., Zer, I., 2016. Model risk of risk models. Journal of Financial Stability, 23, 79-91.
De Bandt, O., Hartmann, P., 2000. Systemic risk: a survey. European Central Bank Working Paper 35, 79 pages.
Diebold, F., K. Yilmaz 2009. Measuring financial asset return and volatility spillover, with application to global equity markets. Econometric Journal 199, 158-171.
Eleswarapu, V., 1997. Cost of transacting and expected returns in the NASDAQ market. Journal of Finance 52, 2113-2127.
Easley, D., Hvidkjaer, S., O'Hara, M., 2002. Is information risk a determinant of asset returns? Journal of Finance 57, 2185-2221.
Engle, R., Siriwardane, E., 2016., Structural GARCH: the volatility-leverage connection. Harvard Business School Finance Working Paper 16-009, 62 pages.
Estrella, A., Trubin, M., 2006. The yield curve as a leading indicator: some practical issues. Current issues in Economics and Finance 12(5), 8 pages.
Fama, E., French, K., 1993. The cross-section of expected stock returns. Journal of Finance 47, 427-465.
Galagedera, D., Maharaj, E., 2008. Wavelet time-scales and conditional relationship between higher-order systematic co-moments and portfolio returns. Quantitative Finance 8(2), 201-215.
Giglio, S., Kelly, B., Pruitt, S., 2016. Systemic risk and the macroeconomy: an empirical evaluation. Journal of Financial Economics 119(3), 457-471.
Gilli, M., Roko, I., 2006. Using economic and financial information for stock selection. Swiss Finance Institute Research Paper 6, 23 pages.
Glosten, L., Jagannathan, R., Runkle, D., 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks. Journal of Finance 48(5), 1779-1801.
Gouriéroux, C., Monfort A., 2013. Allocating systemic risk in a regulatory perspective. International Journal of Applied and Theoretical Finance 16(7), 1-20.
Hong, Y., Liu, Y., Wang, S., 2009. Granger causality in risk and detection of Extreme risk spillover between financial markets. Journal of Econometrics 150(2), 271-287.

Hurlin, C., Kouontchou, P., Maillet, B., 2010. Un MEDAF à plusieurs moments réalisés. Brussels Economic Review 53, 457-480.
Koenker, R., Hallock, K., 2001. Quantile regression. Journal of Economic Perspectives 15, 143-156.
Kouontchou, P., Maillet, B., Modesto, A., Tokpavi, S., 2017. Quand l'union fait la force : un indice de risque systémique. Forthcoming in Revue Economique, 24 pages.
Kouontchou, P., Lendasse, A., Miche, Y., Maillet, B., 2013. Forecasting financial markets with classified tactical signals. European Symposium on Artificial Neural Networks ESANN-2013, 363-368.
Kraus, A., Litzenberger, R., 1976. Skewness preferences and the valuation of risk assets. Journal of Finance 31(4), 1085-1100.
Kritzman, M., Li, Y., 2010. Skulls, financial turbulence, and risk management. Financial Analysts Journal 66(5), 30-41.
Kritzman, M., Li, Y., Page, S., Rigobon, R., 2011. Principal components as a measure of systemic risk. Journal of Portfolio Management 37(4), 112-126.
Kyle, A., 1985. Continuous auctions and insider trading. Econometrica 53(6), 1315–1335.
Lintner, J., 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. Review of Economics and Statistics 47, 13-37.
Moreno, M., Peña, J., 2013. Systemic risk measures: the simpler the better? Journal of Banking and Finance 37, 1817-1831.
Mossin, J., 1966. Equilibrium in a capital asset market. Econometrica 34(4), 768-783.
Newey, W., West, K., 1987. A simple, positive semi-definite, heteroscedasticity and autocorrelation consistent covariance matrix. Econometrica 55(3), 703-08.
Nucera, F., Schwaab B., Koopman, S., Lucas A., 2016. Information in systemic risk rankings. Journal of Empirical Finance 38, 461–475.
Sharpe, W., 1964. Capital asset prices: a theory of market equilibrium under conditions of risk. Journal of Finance 19(3), 425-442.
Zellner, A., 1962. An efficient method for estimating Seemingly Unrelated Regressions, tests for aggregation bias. Journal of the American Statistical Association 57(298), 348-368.

Zou, H., Hastie, T., Tibshirani, R., 2006. Sparse principal component analysis. Journal of Computational and Graphical Statistics 15(2), 265-286.

Appendices

In these Appendices, we provide the systemic risk measures used in the building of the systemic risk index (Section A.1) on a set of American financial institutions (Section A.2) and we provide the sketch of the algorithm to finally rank the SIFI (Section A.3). We provide also some robustness checks using Independent Component Analysis (Section A.4) and complementary theoretical following Giglio et al. (2016) in the next appendix (Section A.5). We also provide the results of the estimated CAPMS based on the entire sample (Section A.6).

A.1. Systemic risk measures

In the building of the ISRM, we use the following systemic risk measures in order to capture the instability, the economic activity, the degree of interconnection and concentration and the liquidity conditions in the system (Cf. Giglio et al., 2016). These measures can be classified in four categories:

- Financial macroeconomic variables used as advanced indicators of the economic activity: Credit Default Yield Spread, TED Spread and Term Spread.
- Aggregated systemic risk measures among the financial institutions: the Conditional Value-at-Risk (CoVaR) and the Delta Conditional Value-at-Risk (ΔCoVaR) of Adrian and Brunnermeir (2016) and the Marginal Expected Shortfall (MES) of Acharya et al. (2013) and Brownlees and Engle (2017).
- Systemic risk measures used to take into account degree of interconnection and the concentration in the system: the Spillover Index of Diebold and Yilmaz (2009), the Dynamic Causality Index (DCI) of Billio et al. (2012), the Turbulence Measure of Kritzman and Li (2010), the Absorption Ratio (AR) of Kritzman et al. (2011) and the Herfindahl-Hirschman Index.
- We also use the *Amihud Illiquidity Measure* (*AIM*) of Amihud (2002) to take into account the liquidity conditions in the system.

A.2. American financial institutions database

From a database that contains NAV and characteristics of 95 (large) financial institutions (*e.g.*, Brownlees and Engle, 2017), we build two cylindrical databases of Net Asset Values in USD for 60 financial institutions from the 01/03/2005 to the 06/30/2010 for the first sample (same sample used by Brownlees and Engle, 2017) and from the 09/03/2003 to the 12/30/2011 also in a daily basis in order to replicate the sample used by Giglio et al. (2016).

Table 13. Tickers of Financial Institutions

Banques (30)		Assurance (32)		Courtiers (10)		Autres (23)	
BAC	Bank of America	ABK	Ambac Financial Group	AGE	A.G. Edwards	ACAS	American Capital
BBT	BB&T	AET	Aetna	BSC	Bear Stearns	AMP	Ameriprise Financial
BK	Bank of New York Mellon	AFL	Aflac	ETFC	E-Trade Financial	AMTD	TD Ameritrade
C	Citigroup	AIG	American International Group	GS	Goldman Sachs	AXP	American Express
CBH	Commerce Bancorp	AIZ	Assurant	LEH	Lehman Brothers	BEN	Franklin Resources
CMA	Comerica inc	ALL	Allstate Corp	MER	Merrill Lynch	BLK	Blackrock
HBAN	Huntington Bancshares	AOC	Aon Corp	MS	Morgan Stanley	BOT	CBOT Holdings
HCBK	Hudson City Bancorp	WRB	W.R. Berkley Corp	NMX	Nymex Holdings	CBG	C.B. Richard Ellis Group
HSBC	Hong Kong & Shanghai Banking Corporation	BRK	Berkshire Hathaway	SCHW	Schwab Charles	CBSS	Compass Bancshares
JPM	JP Morgan Chase	CB	Chubb Corp	TROW	T. Rowe Price	CIT	CIT Group
KEY	Keycorp	CFC	Countrywide Financial			CME	CME Group
MI	Marshall & Ilsley	CI	CIGNA Corp			COF	Capital One Financial
MTB	M & T Bank Corp	CINF	Cincinnati Financial Corp			EV	Eaton Vance
NCC	National City Corp	CNA	CNA Financial corp			FITB	Fifth Third Bancorp
NTRS	Northern Trust	CVH	Coventry Health Care			FNM	Fannie Mae
NYB	New York Community Bancorp	FNF	Fidelity National Financial			FRE	Freddie Mac
PBCT	Peoples United Financial	GNW	Genworth Financial			HRB	H&R Block
PNC	PNC Financial Services	HIG	Hartford Financial Group			ICE	Intercontinental Exchange
RF	Regions Financial	HNT	Health Net			JNS	Janus Capital
SNV	Synovus Financial	HUM	Humana			LM	Legg Mason
SOV	Sovereign Bancorp	LNC	Lincoln National			NYX	NYSE Euronext
STI	Suntrust Banks	MBI	MBIA			SEIC	SEI Investments Company
STT	State Street	MET	Metlife			SLM	SLM Corp
UB	Unionbancal Corp	MMC	Marsh & McLennan				
USB	US Bancorp	PFG	Principal Financial Group				
WB	Wachovia	PGR	Progressive				
WFC	Wells Fargo & Co	PRU	Prudential Financial				
WM	Washington Mutual	SAF	Safeco				
WU	Western Union	TMK	Torchmark				
ZION	Zion	TRV	Travelers				
		UNH	Unitedhealth Group				
		UNM	Unum Group				

Source: Bloomberg. Cylindrical sample of 60 American financial institutions (Cf. Brownlees and Engle, 2017).
Note: This Table reports the tickers of the institutions.

A.3. Methodological Details on SIFI Detection based on the CAPMS

The proposal methodology is defined in five steps. In step one, we build the time series of a set of systemic risk measures and use their z-score as the input for the building of the Index of Systemic Risk Measures (ISRM). In step two, we use the SPCA method to obtain the weights and the components of each measure in order to build the index using a specific sum of these components weighted by their weights. In step 3, conduct the statistical test of Hong et al. (2009) to obtain the most parsimonious ISRM that based on a Granger causality-in-risk, i.e., which parameters leads to changes in the components that best explain the extreme drawdowns of the GDP. In step 4, we add the ISRM as an additional factor in the Fama-French (1993) three-factor model and estimate parameters of the model using the Zellner (1962) Seemingly Unrelated Regression Estimation (SURE) that considers the inter-relations between securities. Finally, in step 5, choose a percentile (60% here) of the most sensitive financial institutions to systemic risk given a specific significance level (.1% in our test).

Table 14: Sketch of the algorithm for SIFI detection based on the CAPMS

Step 1: **Systemic Risk Measures**

Select and compute the time-series of the systemic risk measures and gather their z-score in a matrix that is the input to build the Index of Systemic Risk Measure (ISRM).

Step 2: **Reduction dimension techniques and ISRM**

Use Sparse-PCA to obtain the components and the weights of each measures and compute the specific weighted sum of them to obtain several ISRM. Among them, we have to choose the most parsimonious index in the next step.

Step 3: **ISRM and the Macroeconomy**

Choose the optimal Sparse-PCA is guided by the statistical test of Hong et al. (2009). Indeed, the truncation parameter which determines the smoothness and the sparseness of the components provides the most parsimonious ISRM *via* the statistical test of Hong et al. (2009), that introduces a new concept of Granger causality in risk i.e., which changes of the components explain the extreme drawdowns of the GDP.

Step 4: **Fama-French three-factor with ISRM**

Add the most parsimonious ISRM as an additional factor in the Fama-French (1993) three-factor model and estimate it using Newey-West corrected t-statistics of the Zellner (1962) Seemingly Unrelated Regression Estimation (SURE) method that takes into account the inter-relations between equities.

Step 5: **Final ranking of SIFI**

Conclude by choosing a percentile (60% here) of the most sensitive financial institutions to systemic risk from a specific significance level (.1% in our example). Then, finally, rank the SIFI with respect to their systemic risk sensitivities and name the sample of them as SIFI.

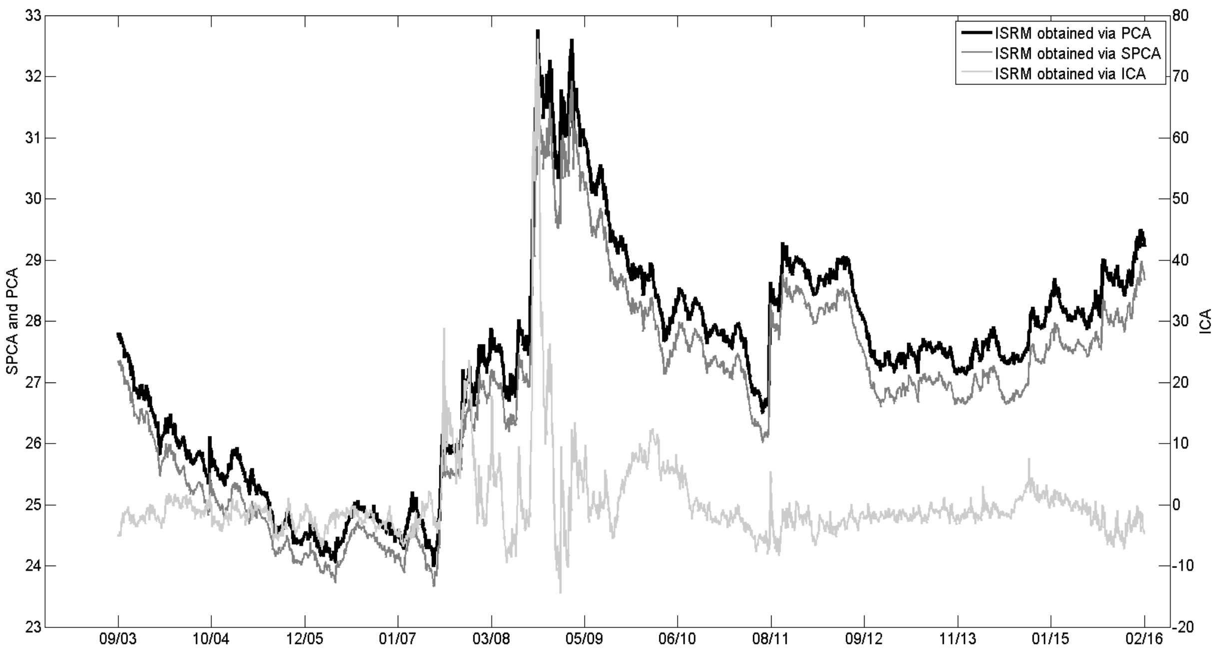
A.4 Robustness check with Independent Component Analysis

This appendix is devoted to the ISRM computed with the Independent Component Analysis and its use in the CAPMS.

Independent component analysis (ICA) implies equal weights (i.e. same importance) among the systemic risk measures in the building of the index. ICA methodology implies equal weights since components are independent in the sense of their all moments.

Figure 11 compares the ISRM indexes computed from PCA and the SPCA methodologies with the ISRM computed by ICA. ISRM computed by ICA is more stable with a low volatility than the two other competitive methodologies since equal weights are accorded to each retained systemic risk measure rescaled to have equal variances.

Figure 11: Three different methodologies to obtain ISRM



Source: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016; authors' computation.
Note: Are plotted three indexes following three different methodologies.

With this new index in hand, we estimate the Fama-French three-factor model where the systemic factor is computed from ICA and serves as an additional fourth factor.

Table 15 provides the estimation parameters for the CAPMS with systemic risk computed from SPCA (Model II) and the same model where systemic risk is computed from ICA (Model VIII).

Table 15: Estimates of the Fama-French (1993) three-factor model with an additional Systemic Risk Factor on the 12 OFR-selected SIFI

In the Table below are presented the estimation of the Fama-French (1993) three-factor model with systemic risk, following the equation (21):

$$r_{i,t} = \alpha_i^{(VIII)} + \beta_i^{(VIII)}\, r_{M,t} + \theta_i^{(VIII)} r_{SMB,t} + \gamma_i^{(VIII)}\, r_{HML,t} + \varphi_i^{(VIII)}\, r_{ISRM,t} + \varepsilon_{i,t}^{(VIII)} \qquad (VIII)$$

where the factors are the market returns denoted $r_{M,t}$, the Small Minus Big portfolio returns denoted $r_{SMB,t}$, and the High Minus Low portfolio returns, denoted $r_{HML,t}$ and where the systemic risk factor denoted $r_{ISRM,t}$ is the additional fourth factor. The ISRM index is compute following two methodologies: SPCA (Model II) and ICA (Model VIII).

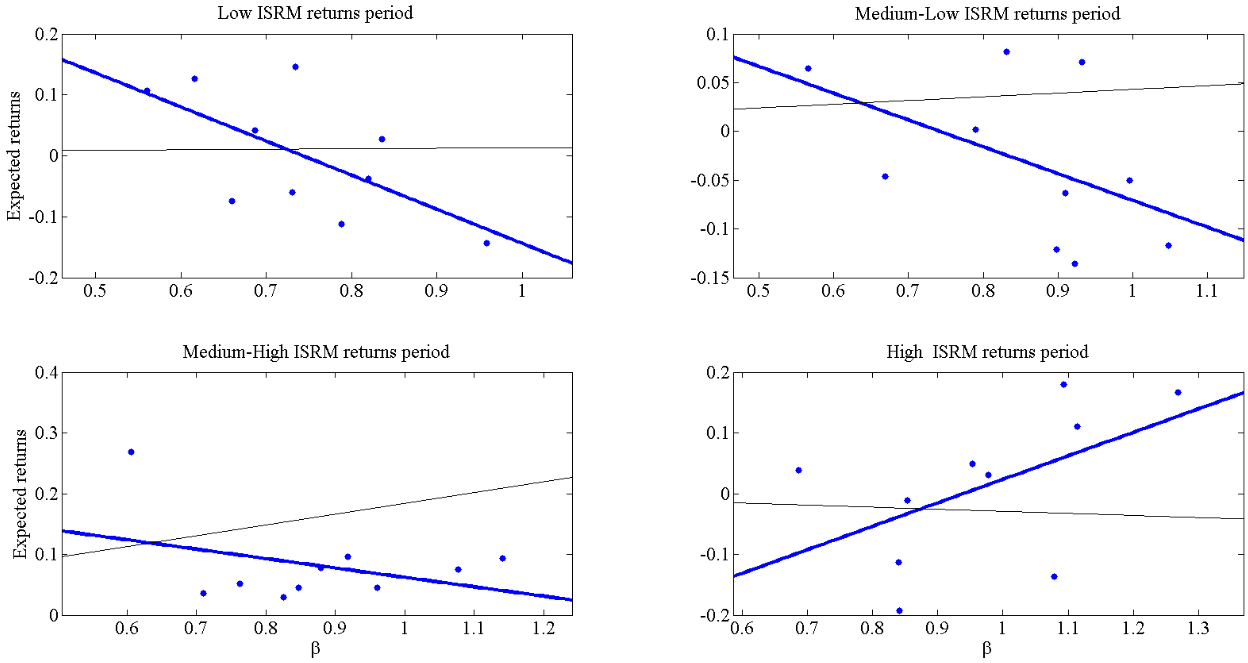
	Model II (with systemic risk by SPCA)						Model VIII (with systemic risk by ICA)					
	$\alpha_i^{(II)}$	$\beta_i^{(II)}$	$\theta_i^{(II)}$	$\gamma_i^{(II)}$	$\varphi_i^{(II)}$	$\overline{R^2}$	$\alpha_i^{(VIII)}$	$\beta_i^{(VIII)}$	$\theta_i^{(VIII)}$	$\gamma_i^{(VIII)}$	$\varphi_i^{(VIII)}$	$\overline{R^2}$
Group 1	**5.83**	**1.12**	-.06	**-1.15**	-.12	70.12%	**6.18**	**1.11**	-.04	**-1.11**	.02	70.18%
	(2.27)	(14.02)	(.86)	(8.02)	(.45)		(2.85)	(57.53)	(1.85)	(46.00)	(.71)	
Group 2	**5.77**	**1.01**	.00	**-1.06**	-.18	68.67%	**5.12**	**1.01**	-.03	**-1.07**	**-.07**	68.68%
	(1.99)	(18.68)	(.02)	(12.92)	(.81)		(2.36)	(52.37)	(1.27)	(44.26)	(2.52)	
Group 3	3.93	**.95**	-.04	**-1.05**	.12	72.10%	**5.47**	**.96**	**-.07**	**-1.01**	.03	72.19%
	(1.73)	(14.57)	(.81)	(14.80)	(.54)		(2.52)	(50.04)	(3.51)	(41.87)	(1.14)	
Group 4	. 82	**.94**	-.07	**-.94**	-.30	71.71%	2.45	**.95**	**-.08**	**-.93**	-.02	71.52%
	(.43)	(16.55)	(1.24)	(11.90)	(1.92)		(1.13)	(49.34)	(4.13)	(38.49)	(.73)	
Group 5	**6.42**	**.88**	.03	**-.71**	-.05	60.41%	**5.18**	**.88**	**.04**	**-.68**	.02	60.44%
	(2.95)	(14.15)	(.54)	(11.32)	(.26)		(2.39)	(45.90)	(1.96)	(28.27)	(.57)	
Group 6	3.62	**.86**	-.11	**-.86**	-.20	66.61%	2.99	**.85**	**-.10**	**-.84**	**-.07**	66.61%
	(1.74)	(17.82)	(1.57)	(14.52)	(1.34)		(1.38)	(44.41)	(5.16)	(34.94)	(2.51)	
Group 7	**3.69**	**.78**	.10	**-.41**	**-.32**	57.12%	3.93	**.80**	**.09**	**-.42**	-.02	56.90%
	(2.33)	(9.87)	(1.21)	(4.34)	(2.06)		(1.81)	(41.71)	(4.69)	(17.44)	(.72)	
Group 8	3.30	**.76**	-.01	**-.73**	**-.50**	57.07%	4.24	**.77**	**-.01**	**-.74**	**-.14**	56.90%
	(1.16)	(17.71)	(.10)	(8.34)	(2.90)		(1.95)	(40.25)	(.51)	(3.73)	(4.98)	
Group 9	-1.51	**.71**	.00	**-.29**	**-.53**	52.54%	.13	**.72**	.02	**-.31**	**-.09**	51.65%
	(.87)	(14.10)	(.05)	(3.92)	(3.89)		(.06)	(37.55)	(1.04)	-(12.73)	(3.18)	
Group 10	3.20	**.61**	.04	-.09	**-.26**	32.58%	**4.67**	**.61**	**.05**	**-.14**	-.02	32.46%
	(.97)	(12.99)	(1.12)	(1.39)	(2.91)		(2.15)	(31.81)	(2.72)	(5.90)	(.78)	

Source: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016 for a set of 95 financial institutions; authors' computation.

Note: The Table provides the estimated values of the parameters using the Zellner (1962) SURE method *via* GLS and their t-statistic (values in brackets) for each model. The values of the parameter α are expressed in 10^{-4}. Bold parameters' values are significant at a 5% level.

Figure 12 shows the evolution of the empirical relationship between expected returns and the level of the risk premium to the market factor, the β parameter in various systemic risk environments. The relationship becomes positive gradually as systemic risk increases. In a low systemic risk environment represented by the upper left quadrant, a portfolio with a high β will have less (even negative) expected returns in contrast to a portfolio with low β. In a high systemic risk environment represented by the lower right quadrant, a portfolio with a high β will have higher expected returns in contrast to a portfolio with low β.

Figure 12: Evolution of the empirical relationship between expected returns and the market risk factor in various systemic risk environments using a systemic risk factor computed by ICA



Source: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016 for a set of 95 financial institutions; authors' computation.
Note: We first create 10 portfolios based on rankings on the estimated betas relative to the market. The figure shows the relationship between expected returns and the market risk factor when systemic risk factor is low, medium or high. The Low period corresponds to a systemic risk factor $r_{ISRM,t} \in [-0.662, -0.0303[$, the Medium-low period corresponds to $r_{ISRM,t} \in [-0.3023, -0.0015[$, the Medium-high period to $r_{ISRM,t} \in [-0.0015, 0.027[$ and the High period corresponds to $r_{ISRM,t} \in [0.027, 1.804[$. The x-axis represents the level of the beta of the portfolios and the y-axis, the expected annualized returns. The thin line is the relationship predicted by the CAPM and the bold line is calculated by regression of the expected returns on the betas.

A.6 CAPMS estimation based on the whole estimation period

The Table 16 provides the estimates of the three-factor model of Fama and French (1993) and the model extended to systemic risk (additional factor).

Table 16 compares the previous model specification in Model II (raw factors) with the new model specification in Model III (decontaminated factors). Using decontaminated factors, we obtained a better significant systemic risk factor with an unchanged total explicative power of the model.

Table 16: Fama and French (1993) three-factor model with or without a systemic risk factor on the main American financial institutions

We estimate the parameters' values and their t-statistics for each portfolio according to the two models used: the Fama-French (1993) three-factor model (I), and the Fama-French (1993) three-factor model with systemic risk as a fourth factor (II) as in Eq. (22).

$$\begin{cases} r_{i,t} = \alpha_i^{(I)} + \beta_i^{(I)} r_{M,t} + \theta_i^{(I)} r_{SMB,t} + \gamma_i^{(I)} r_{HML,t} + \varepsilon_{i,t}^{(I)} & (I) \\ r_{i,t} = \alpha_i^{(II)} + \beta_i^{(II)} r_{M,t} + \theta_i^{(II)} r_{SMB,t} + \gamma_i^{(II)} r_{HML,t} + \varphi_i^{(II)} r_{ISRM,t} + \varepsilon_{i,t}^{(II)} & (II) \end{cases}$$

	Model I (without systemic risk)					Model II (with systemic risk)					
	$\alpha_i^{(I)}$	$\beta_i^{(I)}$	$\theta_i^{(I)}$	$\gamma_i^{(I)}$	$\overline{R^2}$	$\alpha_i^{(II)}$	$\beta_i^{(II)}$	$\theta_i^{(II)}$	$\gamma_i^{(II)}$	$\varphi_i^{(II)}$	$\overline{R^2}$
Group 1	**6.17**	**1.11**	-.04	**-1.11**	70.17%	**5.83**	**1.12**	-.06	**-1.15**	-.12	70.12%
	(2.84)	(57.48)	(1.91)	(45.97)		(2.27)	(14.02)	(.86)	(8.02)	(.45)	
Group 2	**5.16**	**1.01**	**-0.02**	**-1.07**	68.63%	**5.77**	**1.01**	.00	**-1.06**	-.18	68.67%
	(2.37)	(52.36)	(1.06)	(44.22)		(1.99)	(18.68)	(.02)	(12.92)	(.81)	
Group 3	**5.45**	**.96**	**-.07**	**-1.01**	72.18%	3.93	**.95**	-.04	**-1.05**	.12	72.10%
	(2.51)	(49.99)	(3.62)	(41.84)		(1.73)	(14.57)	(.81)	(14.80)	(.54)	
Group 4	2.46	**.95**	**-.08**	**-.93**	71.51%	. 82	**.94**	-.07	**-.94**	-.30	71.71%
	(1.13)	(49.31)	(4.08)	(38.46)		(.43)	(16.55)	(1.24)	(11.90)	(1.92)	
Group 5	**5.18**	**.88**	.04	**-.68**	60.44%	**6.42**	**.88**	.03	**-.71**	-.05	60.41%
	(2.38)	(45.86)	(1.91)	(28.24)		(2.95)	(14.15)	(.54)	(11.32)	(.26)	
Group 6	3.03	**.85**	**-.10**	**-.84**	66.54%	3.62	**.86**	-.11	**-.86**	-.20	66.61%
	(1.39)	(44.41)	(4.97)	(34.91)		(1.74)	(17.82)	(1.57)	(14.52)	(1.34)	
Group 7	3.94	**.80**	**.09**	**-.42**	56.89%	**3.69**	**.78**	.10	**-.41**	**-.32**	57.12%
	(1.81)	(41.70)	(4.76)	(17.43)		(2.33)	(9.87)	(1.21)	(4.34)	(2.06)	
Group 8	**4.31**	**.78**	.00	**-.74**	56.60%	3.30	**.76**	-.01	**-.73**	**-.50**	57.07%
	(1.98)	(40.29)	(.10)	(30.70)		(1.16)	(17.71)	(.10)	(8.34)	(2.90)	
Group 9	.17	**.72**	.03	**-.31**	51.44%	-1.51	**.71**	.00	**-.29**	**-.53**	52.54%
	(.08)	(37.57)	(1.31)	(12.72)		(.87)	(14.10)	(.05)	(3.92)	(3.89)	
Group 10	**4.68**	**.61**	**.06**	**-.14**	32.45%	3.20	**.61**	.04	-.09	**-.26**	32.58%
	(2.15)	(31.80)	(2.79)	(5.89)		(.97)	(12.99)	(1.12)	(1.39)	(2.91)	

Source: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016 for a set of 95 financial institutions; authors' computation.

Note: The Table provides the estimated values of the parameters using the Zellner (1962) SURE method *via* GLS. and their t-statistic (values in brackets) for each model. The values of the parameter α are expressed in 10^{-4}. Bold parameters' values are significant at a 5% level.

Most of the portfolios have a significant systemic risk factor. But is it possible that a part of the sensitivity to the systemic risk in absorbed in the other Fama-French factors? We then regress the Fama-French factors on the systemic risk factor in order to obtain decontaminated Fama-French factors and compare the new model specification with respect to the previous one.

Table 17: Fama and French (1993) three-factor model with or without a systemic risk factor on the main American financial institutions

We estimate the parameters' values and their t-statistics for each portfolio according to the two models used: the Fama-French (1993) three-factor model with systemic risk as a fourth factor (I), and the Fama-French (1993) three-factor model with systemic risk as a fourth factor as in Eq. (21) but by decontaminating the other factors from the ISRM factor (II) following the equation:

$$r_{i,t} = \alpha_i^{(III)} + \beta_i^{(III)} \hat{\varepsilon}_{M,t} + \theta_i^{(III)} \hat{\varepsilon}_{SMB,t} + \gamma_i^{(III)} \hat{\varepsilon}_{HML,t} + \varphi_i^{(III)} r_{ISRM,t} + \varepsilon_{i,t}^{(III)} \quad (III)$$

with:

$$\begin{cases} \hat{\varepsilon}_{M,t} = r_{M,t} - \hat{b}_M r_{ISRM,t} \\ \hat{\varepsilon}_{SMB,t} = r_{SMB,t} - \hat{b}_{SMB} r_{ISRM,t} \\ \hat{\varepsilon}_{HML,t} = r_{HML,t} - \hat{b}_{HML} r_{ISRM,t}. \end{cases}$$

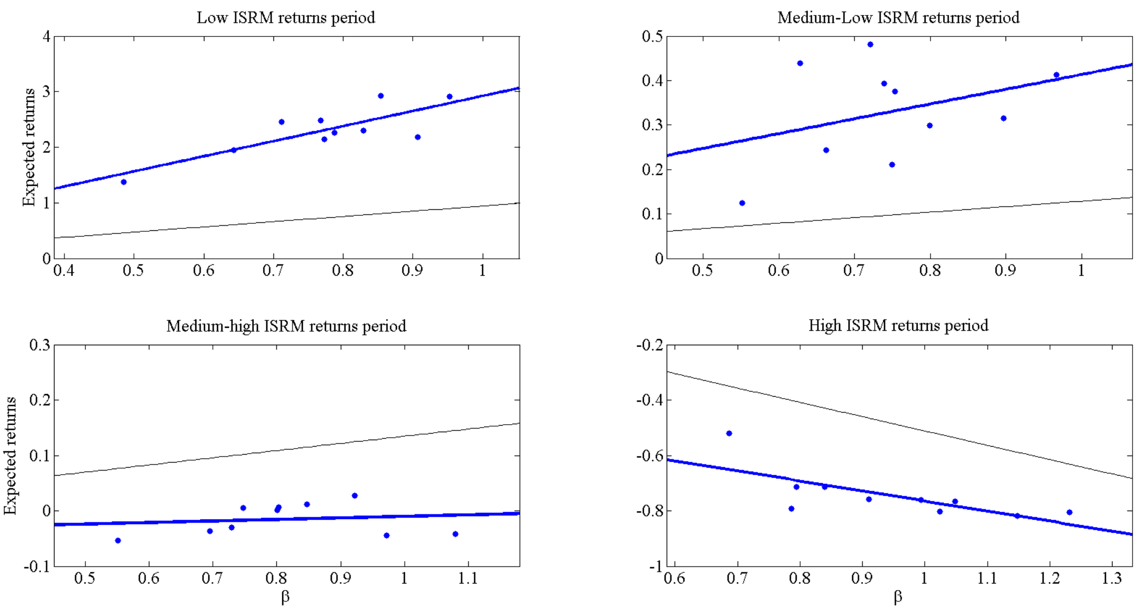
	Model II (with raw factors)						Model III (with decontaminated factors)					
	$\alpha_i^{(II)}$	$\beta_i^{(II)}$	$\theta_i^{(II)}$	$\gamma_i^{(II)}$	$\varphi_i^{(II)}$	$\overline{R^2}$	$\alpha_i^{(III)}$	$\beta_i^{(III)}$	$\theta_i^{(III)}$	$\gamma_i^{(III)}$	$\varphi_i^{(III)}$	$\overline{R^2}$
Group 1	**5.83**	**1.12**	-.06	**-1.15**	-.12	70.12%	**5.39**	**1.12**	-.06	**-1.15**	**-2.31**	70.12%
	(2.27)	(14.02)	(.86)	(8.02)	(.45)		(2.11)	(14.02)	-(.86)	-(8.02)	-(6.88)	
Group 2	**5.77**	**1.01**	.00	**-1.06**	-.18	68.67%	5.36	**1.01**	.00	**-1.06**	**-2.22**	68.67%
	(1.99)	(18.68)	(.02)	(12.92)	(.81)		(1.85)	(18.68)	-(.02)	-(12.92)	-(8.18)	
Group 3	3.93	**.95**	-.04	**-1.05**	.12	72.10%	3.55	**.95**	-.04	**-1.05**	**-1.83**	72.10%
	(1.73)	(14.57)	(.81)	(14.80)	(.54)		(1.56)	(14.57)	-(.81)	-(14.80)	-(7.71)	
Group 4	. 82	**.94**	-.07	**-.94**	-.30	71.71%	.47	**.94**	-.07	**-.94**	**-2.09**	71.71%
	(.43)	(16.55)	(1.24)	(11.90)	(1.92)		(.24)	(16.55)	-(1.24)	-(11.90)	-(12.02)	
Group 5	**6.42**	**.88**	.03	**-.71**	-.05	60.41%	**6.10**	**.88**	.03	**-.71**	**-1.66**	60.41%
	(2.95)	(14.15)	(.54)	(11.32)	(.26)		(2.82)	(14.15)	(.54)	-(11.32)	-(7.40)	
Group 6	3.62	**.86**	-.11	**-.86**	-.20	66.61%	3.30	**.86**	-.11	**-.86**	**-1.81**	66.61%
	(1.74)	(17.82)	(1.57)	(14.52)	(1.34)		(1.58)	(17.82)	-(1.57)	-(14.52)	-(9.75)	
Group 7	**3.69**	**.78**	.10	**-.41**	**-.32**	57.12%	**3.44**	**.78**	.10	**-.41**	**-1.57**	57.12%
	(2.33)	(9.87)	(1.21)	(4.34)	(2.06)		(2.18)	(9.87)	(1.21)	-(4.34)	-(8.73)	
Group 8	3.30	**.76**	-.01	**-.73**	**-.50**	57.07%	3.00	**.76**	-.01	**-.73**	**-1.97**	57.07%
	(1.16)	(17.71)	(.10)	(8.34)	(2.90)		(1.06)	(17.71)	-(.10)	-(8.34)	-(8.09)	
Group 9	-1.51	**.71**	.00	**-.29**	**-.53**	52.54%	-1.70	**.71**	.00	**-.29**	**-1.51**	52.54%
	(.87)	(14.10)	(.05)	(3.92)	(3.89)		-(.98)	(14.10)	-(.05)	-(3.92)	-(10.08)	
Group 10	3.20	**.61**	.04	-.09	**-.26**	32.58%	3.05	**0.61**	.04	-.09	**-.98**	32.58%
	(.97)	(12.99)	(1.12)	(1.39)	(2.91)		(.93)	(12.99)	(1.12)	-(1.39)	-(13.79)	

Source: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016 for a set of 95 financial institutions; authors' computation.
Note: The Table provides the estimated values of the parameters using the Zellner (1962) SURE method via GLS. and their t-statistic (values in brackets) for each model. The values of the parameter α are expressed in 10^{-4}. Bold parameters' values are significant at a 5% level.

Bold parameters' values are significant at a 5% level. We note that the φ coefficient related to systemic risk is significant for most portfolios. Specifically, the systemic risk should be considered in nine out of ten portfolios. The addition of systemic risk as an additional factor also changes slightly the estimated parameters of the canonical relationship CAPM extended to three-factor model of Fama and French (1993).

Figure 13 shows the evolution of the empirical relationship between expected returns and the level of the risk premium to the market factor, the β parameter in various systemic risk environments. The relationship is reversed and becomes negative gradually as systemic risk increases. In a low systemic risk environment represented by the upper left quadrant, a portfolio with a high β will have higher expected returns in contrast to a portfolio with low β. In a high systemic risk environment represented by the lower right quadrant, a portfolio with a high β will have lower expected returns (even negative) in contrast to a portfolio with low β.

Figure 13: Evolution of the empirical relationship between expected returns and the market risk factor in various systemic risk environments



Source: Bloomberg, daily data from the 09/03/2003 to the 02/26/2016 for a set of 95 financial institutions; authors' computation.

Note: We first create 10 portfolios based on rankings on the estimated betas relative to the market. The figure shows the relationship between expected returns and the market risk factor when systemic risk factor is low, medium or high. The Low period corresponds to a systemic risk factor $r_{ISRM,t} \in [-0.0227, -0.0012[$, the Medium-low period corresponds to $r_{ISRM,t} \in [-0.0012, -0.00015[$, the Medium-high period to $r_{ISRM,t} \in [-0.00015,\ 0.00089[$ and the High period corresponds to $r_{ISRM,t} \in [0.00089, 0{,}0413[$. The x-axis represents the level of the beta of the portfolios and the y-axis, the expected annualized returns. The thin line is the relationship predicted by the CAPM and the bold line is calculated by regression of the expected returns on the betas.