Learning, house prices and macro-financial linkages

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Abstract: To account for the linkages between the credit market, the housing market and the real sector, which were strikingly evidenced by the subprime financial crisis in the US, we develop a small-scale DSGE model in which agents form misspecified beliefs about future house prices but otherwise behave rationally given those beliefs. In the model with learning, both standard productivity shocks and financial shocks (loan-to-value ratio shocks and lenders' preference shocks) prove able to generate endogenously persistent booms in house prices. Because real-estate assets serve as collateral for household and entrepreneurial debt in the context of financial frictions, long-lasting excess volatility in house prices generated by the learning mechanism in response to shocks in turn affects the financial sector and thus propagates to the real sector. This powerful amplification and propagation mechanism significantly improves the ability of the model to simultaneously replicate the volatility in house prices and in business cycle variables beginning in the mid 1980s in the US in response to small shocks along with additional features of the dynamics of house prices, and can account for some aspects of the macro-financial linkages observed during the subprime financial crisis.

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1 Introduction

The US subprime financial crisis, which started in the mortgage credit market in 2007 following a sudden decrease in house prices, and finally propagated to the real sector, has revealed the strong linkages between the credit market, the housing sector and the real sector. Such macro-financial linkages have been accentuated since the early 2000s with the fast development of mortgage debt contracts, in which house prices determined how much agents could borrow. Therefore, it seems crucial to explain the dynamics of house prices to understand credit and business cycles over the recent period. However, as for other assets, such as stocks, patterns of excess volatility in house prices relative to fundamentals (such as rents or interest rates) have been apparent (see for instance Gelain and Lansing (2014) and Granziera and Kozicki (2015)).

As stated by Piazzesi and Schneider (2016), "a major outstanding puzzle is the volatility of house prices – including but not only over the recent boom-bust episode. Rational expectations models to date cannot account for house price volatility – they inevitably run into "volatility puzzles" for housing much like for other assets. Postulating latent "housing preference shocks" helps understand how models work when prices move a lot, but is ultimately not a satisfactory foundation for policy analysis. Moreover, from model calculations as well as survey evidence, we now know that details of expectation formation by households – and possibly lenders and developers – play a key role" (p. 5).

To simultaneously explain several puzzling features of the dynamics of house prices and the volatility of standard macroeconomic and financial variables over time in the US since the mid-1980s and also more specifically in the run-up to the Great Recession, this paper thus presents a stylized small-scale DSGE model with capital adjustment costs in the spirit of Iacoviello (2005), in which impatient households and entrepreneurs can borrow from patient households against a fraction of their net worth, i.e., the expected value of their real-estate holdings.

The specificity of the propagation mechanism in the present model is that it relies on subjective expectations and Bayesian learning about future house prices. Indeed, following a rich set of recent literature dealing with learning on house prices, we assume that agents do not understand how house prices form through the housing market clearing equation. Agents instead believe that house price growth equals the sum of two components: a persistent component and a normally distributed transitory component. This perceived law of motion is motivated by the empirical behavior of house prices. Indeed, house prices display episodes of persistent increase followed by episodes of persistent decrease. We assume that agents cannot separately observe the two components; therefore, they optimally learn over time the unknown persistent time-varying component based on past data. Such a learning mechanism seems very intuitive: when observing an increase in house prices, agents do not know whether this increase will last or whether this increase is only transitory. They thus try to evaluate the persistence of the increase based on their past experience. Adam et al. (2012), Kuang (2014), Adam et al. (2016a) and Adam et al. (2016b) show that such a specification for the perceived law of motion of asset prices yields beliefs that are both nearly rational (in the sense that they are close in distribution to the model's outcomes) and successful in jointly explaining asset price data and survey data.

In addition, as emphasized by Iacoviello (2015), productivity shocks, which are traditional drivers of business cycles in most DSGE models, are unlikely to fully account for the Great Moderation dynamics and for the recent financial crisis in the US. It seems that since the mid-1980s, and even more since the 2000s, business cycles have been mainly financial in the US. Building on this approach, we introduce two additional shocks: loan-to-value ratio shocks and lenders' time preference shocks.

A positive loan-to-value ratio shock and a negative lenders' discount factor shock can both provide easier and less costly access to the mortgage credit market. Indeed, a positive shock to the loan-to-value ratio directly relaxes the tightness of the borrowing constraint. Following the shock, impatient households and entrepreneurs can borrow a higher fraction of their net worth. As for the negative lenders' discount factor shock, it mimics a general context of suddenly higher willingness to lend, independently of borrowers' situations or decisions, and it generates a decrease in the endogenous mortgage rate. Thus, many papers note the role of the decrease in the mortgage interest rates at the beginning of the 2000s in the US in driving the steep increase in house prices observed until 2006 (see for instance Adam et al. (2012)).

In addition, in order to introduce the smallest degree of freedom into the model and the smallest deviation from the rational expectations assumption, along with only two new state variables (that characterize agents' beliefs and house prices), we follow Winkler (2016) in assuming that expectations of all variables are rational conditional on house price expectations. We thus adapt Winkler's approximation method for solving the learning model – which is close to a rational expectations solving method. Such an approach avoids the limitations of the widespread parameterized expectations method, which consists in substituting expectations terms by parameterized forecast functions directly into first order conditions, whereas in most cases, variables in expectations terms do depend on the own decisions of agents. Our approach yields the following results.

First, the learning process generates a strong feedback mechanism between house prices and beliefs. House price expectations are self-reinforcing: when house prices are expected to grow faster, housing demand increases and thus house prices indeed grow faster (due to the fixity of housing supply), which in turn makes expectations more optimistic and fuels the increase in house prices. Therefore, the model with learning is able to generate endogenously persistent booms-and-busts in real-estate prices and thus strong volatility of house prices relative to output in response to small macroeconomic or financial shocks. In addition, by comparison with the rational expectations version of the model, the learning model replicates the positive sign of the autocorrelation in house price growth observed in US data over the 1985-2015 period.

Second, the model with learning features an amplified financial accelerator mechanism. Indeed, the variation in house price expectations obtained under learning triggers strong changes in the tightness of collateralized borrowing constraints. In addition, under learning, house prices and beliefs are slow-moving state variables, which further propagates the amplified effect. Therefore, small shocks are sufficient to simultaneously generate strong volatility in house prices, credit and most macroeconomic variables as observed over the 1985-2015 period in the US, and responses of variables to shocks are hump-shaped. We show that such a strong volatility arises due to the combination of learning with credit market frictions, rather than learning alone.

Thus, the results show that even if the small-scale DSGE model relies only on a small number of frictions (standard real frictions – capital adjustment costs –, financial frictions – credit market frictions related to asymmetry of information between lenders and borrowers – and informational frictions – imperfect knowledge of the house price law of motion[1]), this is sufficient for the model with learning to explain puzzling features of house price dynamics while simultaneously accounting for macro-financial linkages in the US since the mid-1980s. Relying on a small quantity of new ingredients relative to a simple and standard borrower-saver baseline model with collateralized borrowing constraints has the advantage of making it easier to disentangle the separate and joint effects of each ingredient and to assess how they can help explain the existence of simultaneous cycles in credit, housing and macroeconomic variables.

The remainder of the paper is organized as follows. Section 2 presents the related literature. Section 3 describes the baseline model with collateralized borrowing constraints, financial frictions and capital adjustment costs. Section 4 explains the formation of beliefs

[1] In addition, the housing supply is fixed, which can be interpreted as an additional rigidity in the housing market.

about future house prices based on a specification that is standard in the recent Bayesian learning literature and describes the equilibrium under learning. Section 5 displays the simulated results obtained in the learning model, compares them to the rational expectations model and discusses how they can help explain features of the joint dynamics of real variables, house prices and credit since the mid-1980s in the US. Finally, Section 6 concludes.

2 Related literature

This paper is related to two strands of the literature that have for the most part remained separate. The first strand relates to the relaxation of the rational expectations assumption in standard asset pricing models, whereas the second strand relates to the role of the housing market in the business cycle.

The first strand aims at modelling expectations that are more consistent with the results of survey data and at better replicating house price dynamics. Relying on survey data about house price expectations, Case et al. (2012) and Ling et al. (2015) identify patterns that contradict the rational expectations assumption. This is hardly surprising given that economists themselves seem to make systematic errors in forecasting future real-estate prices in a context of strong uncertainty about the future evolution of the housing market. Empirical evidence thus provides strong support for relying on learning about house prices for explaining excess volatility. Thus, Gelain and Lansing (2014) and Granziera and Kozicki (2015) explain house price volatility by introducing simple but not microfounded extrapolative models of house price expectations.

By contrast, we directly follow Adam et al. (2012), Kuang (2014), Adam et al. (2016a) and Adam et al. (2016b) in specifying the perceived law of motion of asset prices and we optimally derive the beliefs' law of motion, relying on optimal Bayesian updating.

Even if we follow Adam et al. (2012), Kuang (2014), Adam et al. (2016a), Adam et al. (2016b) and Caines (2016) regarding the specification of beliefs, the latter model exchange economies in which consumption and output streams are exogenous. Therefore, these studies cannot account for the impact of the dynamics of house prices on the business cycle. An exception is the recent paper by Winkler (2016) mentioned above. In contrast to this paper, we focus on house prices rather than on stock prices and our model features financial shocks and household debt to better explain the recent period.

The literature on the role of the housing market in the business cycle is thus the second strand of literature this paper relates to.[2] In particular, several papers investigate the linkages between the asset market, the credit market and the real sector in production economies with financial frictions, where consumption and production are endogenous, featuring a well-known financial accelerator mechanism (Kiyotaki and Moore (1997), Bernanke et al. (1999)).

Most papers featuring a financial accelerator mechanism have focused on the production sector, but recent papers focusing on real-estate assets rather than on other assets have also modeled financial accelerator dynamics related to household borrowing, which seems consistent with the role of household debt in the recent financial crisis (Aoki et al. (2004), Iacoviello (2005), Iacoviello (2015)).

However, in most of the papers that feature housing assets as collateral, at least part of the dynamics of house prices is driven by exogenous changes directly related to the housing sector. The most common approach consists in introducing housing price shocks, housing demand shocks, or housing technology shocks (Iacoviello (2005), Darracq-Paries and Notarpietro (2008), Iacoviello and Neri (2010)). Such ingredients are not very helpful in understanding asset price dynamics, as the latter thus remain largely exogenous,

[2]See Davis and Van Nieuwerburgh (2014) for an extensive literature review on the several aspects of the relation between housing, finance and the macroeconomy which go far beyond those investigated in the present paper.

and they usually require imposing high volatility in exogenous innovations. Other elements of explanations resort to monetary policy shocks or financial conditions shocks (Aoki et al., 2004) or to non-time separable preferences (Jaccard, 2012). In all cases, this set of explanations, based on standard rational expectations specifications, is difficult to reconcile with distinct measures of house price expectations because the latter reveal the existence of systematic forecast errors. Even in papers modelling adaptive learning about financial variables such as a leverage shock process, house price shocks are introduced to replicate the boom-bust pattern observed in the 2000s (e.g. Pintus and Suda (2016)).[3]

By contrast, we only introduce discount factor shocks in the lending sector and loan-to-value ratio shocks (in addition to the more standard productivity shock), such that the dynamics of the housing market are initially driven by shocks related to the credit market and not directly by shocks related to the housing market. The response of house prices to exogenous shocks is thus more endogenous, less close to the shock, and more consistent with patterns observed during the last boom-and-bust episode in the housing market. Indeed, the steep increase in house prices that started in 2001 in the US arised as a consequence of relaxed financial conditions and the fast development of mortgage credit (e.g. Mian and Sufi (2009), Demyanyk and Van Hemert (2011), Dell'Arricia et al. (2012)).[4] Introducing learning about asset prices in production economies with a housing market, in accord with the successful results obtained in exchange economies, then amplifies the response to exogenous shocks, which is much smaller under rational expectations.

In the next section, we turn to the description of the baseline model.

[3]Similarly to Pintus and Suda (2016), several recent papers have investigated the role of adaptive learning about macroeconomic and credit variables in business cycle models, including Eusepi and Preston (2011), Milani and Rajbhandari (2012) and Milani (2014). By contrast, we model optimal Bayesian learning about asset prices and our main focus is on replicating the volatility in asset prices without introducing house price shocks as a shortcut.

[4]The 2011 U.S. Financial Crisis Inquiry Commission Final Report on the Causes of the Financial and Economic Crisis in the United States presents a similar view on the chain of events that triggered the crisis: collapsing mortgage-lending standards fueled credit and housing demand, and thus fueled a housing boom, which in turn fueled credit. When house prices fell, the mortgage-credit sector then collapsed.

3 The baseline model

The baseline model is close to the extended model presented in Iacoviello (2005), except that we focus only on real frictions. The model features a discrete-time, infinite horizon economy with three sectors: lenders in the form of patient households and borrowers in the form of both entrepreneurs and impatient households. The housing stock in the economy is exogenous and normalized to 1. All variables are expressed in units of a single consumption good, which also serves as an investment good.

3.1 Lenders

Following Iacoviello (2005), we assume that a set of households displays a high discount factor relative to other households. Those households weight more future periods in their intertemporal utility function; they are more patient and thus more willing to postpone consumption and save money through lending. Their preferences take the standard following form:

$$\max E_0 \sum_{t=0}^{\infty} \beta_P^t d_t [\ln(C_{t,P}) + j \ln(H_{t,P}) + \psi \ln(1 - N_{t,P})]. \tag{1}$$

Patient households thus value consumption $C_{t,P}$, housing services provided by real-estate holdings $H_{t,P}$[5] and leisure hours equal to $1 - N_{t,P}$, where $N_{t,P}$ are working hours. Patient households discount future periods with the discount factor β_P, j is the weight allocated to housing services in the utility function and ψ is the weight allocated to leisure. d_t is a time preference shock that follows an autoregressive process in the form of:

$$\ln(d_t) = \rho_d \ln(d_{t-1}) + \varepsilon_{d,t}, \tag{2}$$

[5]Throughout the paper, we use interchangeably the terms 'houses', 'real-estate holdings' and 'real-estate (or housing) assets'.

where $\rho_d < 1$ and $\varepsilon_{d,t}$ follows a normal distribution with mean zero and variance σ_d. The interpretation of such a discount factor shock is that time preferences are time varying and patient households can suddenly display more or less preference for current consumption, housing services and leisure. We introduce this shock to mimic a context of higher willingness to lend, independently of borrowers' net worth. An exogenous increase in the discount factor of lenders notably decreases the mortgage interest rate.

The intertemporal flow of funds constraint of patient households writes as follows:

$$C_{t,P} + q_t H_{t,P} + B_t = w_t N_{t,P} + R_{t-1} B_{t-1} + q_t H_{t-1,P}, \tag{3}$$

where q_t is the price of houses, B_t is the debt held by patient households, R_t is the (gross) interest rate on debt and w_t is the wage. Housing assets are traded in each period.

The inter-temporal first-order conditions with respect to housing, debt and hours worked are standard, except that the preference shock is included:

$$d_t \frac{1}{C_{t,P}} q_t = \beta_P E_t \left[\frac{1}{C_{t+1,P}} q_{t+1} d_{t+1} \right] + j d_t \frac{1}{H_{t,P}}. \tag{4}$$

$$\frac{d_t}{C_{t,P}} = \beta_P E_t \left[\frac{d_{t+1}}{C_{t+1,P}} R_t \right]. \tag{5}$$

$$\frac{w_t}{C_{t,P}} = \frac{\psi}{1 - N_{t,P}}. \tag{6}$$

3.2 Entrepreneurs

Entrepreneurs own the capital stock and own the firm and maximize the intertemporal utility of consumption streams:

$$\max E_0 \sum_{t=0}^{\infty} \beta_F^t [\ln(C_{t,F})], \tag{7}$$

subject to the following flow of funds constraint:

$$C_{t,F} + q_t H_{t,F} + R_{t-1} B_{t-1,F} + w_t N_t + I_t = Y_t + B_{t,F} + q_t H_{t-1,F}, \tag{8}$$

where β_F is the entrepreneurs' discount factor, $C_{t,F}$ is consumption, $H_{t,F}$ represents real-estate holdings, $B_{t,F}$ is debt, N_t is labor demand, I_t is investment and Y_t is output.[6] The production function is a typical Cobb-Douglas production function, with three factors of production: labor, capital and housing. Both capital and housing become productive only after one period:

$$Y_t = A_t K_{t-1}^{\alpha} H_{t-1}^{v} N_t^{1-\alpha-v}. \tag{9}$$

A_t follows a standard $AR(1)$ process in the log:

$$\ln(A_t) = \rho_a \ln(A_{t-1}) + \varepsilon_{a,t}, \tag{10}$$

where $\varepsilon_{a,t}$ follows a normal distribution with mean zero and variance σ_a. Adjusting capital too fast is costly (notably because installing new machines implies temporarily not running some of the existing machines). Therefore, the capital accumulation equation takes the standard following form under capital adjustment costs (Hayashi, 1982):

$$K_t = I_t + (1-\delta) K_{t-1} + \frac{\phi}{2} \left(\frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1}, \tag{11}$$

where K_t is the predetermined capital stock, δ is the capital depreciation rate and ϕ is a parameter governing the size of the capital adjustment cost.

Entrepreneurs can borrow a limited amount of debt and face a collateralized borrow-

[6] Entrepreneurs' consumption being residual income after investment, labor costs, housing and interest payments have been made, the decision problem of entrepreneurs is equivalent to maximizing a concave function of discounted dividends.

ing constraint in which housing assets play the role of pledgeable assets:

$$B_{t,F} \leq m_F a_{m,t} E_t \left[q_{t+1} \frac{H_{t,F}}{R_t} \right], \tag{12}$$

where the term $E_t[q_{t+1}\frac{H_{t,F}}{R_t}]$ represents expected future asset value, m_F is the loan-to-value ratio and $a_{m,t}$ is a loan-to-value ratio shock that follows the law of motion below:

$$\ln(a_{m,t}) = \rho_{a_m} \ln(a_{m,t-1}) + \varepsilon_{a_m,t}, \tag{13}$$

where $\varepsilon_{a_m,t}$ follows a normal distribution with mean zero and variance σ_{a_m}. We assume that, due to asymmetry of information between lenders and borrowers, the lender can recover only some fraction of the pledgeable assets in case of default, implying that $m_F < 1$. Even though the entrepreneurs' borrowing constraint is not directly microfounded in the present model, it is both standard (e.g. Kiyotaki and Moore (1997), Iacoviello (2005) and Iacoviello (2015) to mention just a few) and intuitive. Indeed, first, the borrowing constraint (12) implies that the borrowing capacity of agents depends on the expected future value of their assets, because assets can be seized and sold by the lender in case of default. Second, the borrowing constraint says that transaction costs arising when the lender seizes borrowers' assets reduce the final recovery value.

The first-order conditions for firms with respect to labor, debt and real-estate assets write:

$$w_t = \frac{(1-\alpha-v)Y_t}{N_t}, \tag{14}$$

$$\frac{1}{C_{t,F}} = \beta_F E_t \left[\frac{1}{C_{t+1,F}} \right] R_t + \mu_{H,t}, \tag{15}$$

and

$$\frac{q_t}{C_{t,F}} = \beta_F E_t \left[\frac{1}{C_{t+1,F}} \left(q_{t+1} + \frac{vY_{t+1}}{H_{t,F}} \right) \right] + \mu_{H,t} m_H a_{m,t} E_t \left[\frac{q_{t+1}}{R_t} \right], \tag{16}$$

where $\mu_{H,t} \geq 0$ is the Lagrange multiplier associated with the borrowing constraint. The complementary slackness condition writes:

$$\mu_{H,t} \left[B_{t,F} - m_F a_{m,t} E_t \left[q_{t+1} \frac{H_{t,F}}{R_t} \right] \right] = 0. \tag{17}$$

The first-order condition with respect to labor is standard, except that the share of labor in the production function depends not only on the share of capital but also on the share of real-estate assets in the production function.

The Lagrange multiplier $\mu_{H,t}$ associated with the borrowing constraint appears in the previous two equations, which shows that financial frictions act as an inter-temporal wedge in the first-order conditions by comparison to standard first-order conditions. Housing price expectations, and not only housing prices, matter for determining the entrepreneurs' demand for housing. This feature is reinforced by the fact that the Lagrange multiplier in the borrowing constraint enters the first-order condition with respect to another asset (housing) because buying more of this asset today relaxes the borrowing constraint in the future. As a consequence, learning will prove able to generate dynamics that are distinct from rational expectations ones even for similar states of the economy.

Note also that in the non-stochastic steady state, the Lagrange multiplier associated with the borrowing constraint of impatient households μ_I is equal to $(\frac{\beta_P - \beta_F}{\beta_F})\frac{1}{C_F}$. Therefore, the discount factor of lenders must be strictly higher than the discount factor of borrowers to ensure that the Lagrange multiplier associated with the borrowing constraint is strictly positive and thus that the borrowing constraint is binding. Therefore, $\beta_F < \beta_P$ is a necessary condition for ensuring that the borrowing constraint is binding in a neighborhood of the steady state.

The combination of the first-order conditions with respect to capital and to investment yields:

$$\frac{1}{C_{t,F}\left(1-\phi(\frac{I_t}{K_{t-1}}-\delta)\right)}=$$

$$\beta_F\frac{1}{C_{t+1,F}}\left(\frac{\alpha Y_{t+1}}{K_t}+\frac{1}{1-\phi(\frac{I_{t+1}}{K_t}-\delta)}(1-\delta-\frac{\phi}{2}\left(\frac{I_{t+1}}{K_t}-\delta\right)^2+\phi(\frac{I_{t+1}}{K_t}-\delta)\frac{I_{t+1}}{K_t})\right). \tag{18}$$

When the capital adjustment cost parameter ϕ is null, this equation reduces to the standard first-order condition with respect to capital.

3.3 Impatient households

The preferences of impatient households are similar to those of patient households except that their time preference rate β_I differs ($\beta_I < \beta_P$). This assumption, which is standard in a borrower-saver model, makes impatient households willing to borrow rather than lend, as evidenced below for the non-stochastic steady state equilibrium. The maximization program of impatient households is thus the following:

$$\max E_0\sum_{t=0}^{\infty}\beta_I^t[\ln(C_{t,I})+j\ln(H_{t,I})+\psi\ln(1-N_{t,I})] \tag{19}$$

s.t.

$$C_{t,I}+R_{t-1}B_{t-1,I}+q_tH_{t,I}=w_tN_{t,I}+q_tH_{t-1,I}+B_{t,I} \tag{20}$$

$$B_{t,I}\leq m_IE_t\left[q_{t+1}\frac{H_{t,I}}{R_t}\right](1+a_{m,t}). \tag{21}$$

All variables indexed by I for impatient households are equivalent to similar variables indexed by P for patient households that were presented above. $m_I < 1$ is the loan-to-value ratio of impatient households.

Impatient households face a borrowing constraint similar to that of entrepreneurs, subject to the same loan-to-value ratio shock, which can thus be interpreted as a general financial liberalization shock when it is positive. Indeed, when a positive loan-to-value ratio shock hits, a lower expected value of real-estate assets is required to guarantee the same amount of borrowing; the borrowing limit mechanically increases.

The first-order conditions with respect to housing, labor supply and debt write:

$$\frac{q_t}{C_{t,I}} = \beta_I E_t\left[\frac{q_{t+1}}{C_{t+1,I}}\right] + j\frac{1}{H_{t,I}} + \mu_{I,t} m_I a_{m,t} E_t\left[\frac{q_{t+1}}{R_t}\right], \tag{22}$$

$$\frac{w_t}{C_{t,I}} = \frac{\psi}{1-N_{t,I}}, \tag{23}$$

$$\frac{1}{C_{t,I}} = \beta_I E_t\left[\frac{1}{C_{t+1,I}}R_t\right] + \mu_{I,t}, \tag{24}$$

where $\mu_{I,t} \geq 0$ is the Lagrange multiplier associated with the borrowing constraint of the impatient households. The complementary slackness condition writes:

$$\mu_{I,t}\left[B_{t,I} - m_I a_{m,t} E_t\left[q_{t+1}\frac{H_{t,I}}{R_t}\right]\right] = 0. \tag{25}$$

In the first-order condition with respect to housing, the last term is what differentiates the impatient households' first-order condition from that of patient households. Indeed, for borrowers, buying real-estate assets also presents the advantage of relaxing the borrowing constraint.

Similar to the entrepreneurs case, the Lagrange multiplier associated with the borrowing constraint of impatient households μ_I is equal to $(\frac{\beta_P-\beta_I}{\beta_I})\frac{1}{C_I}$ in the deterministic steady state, requiring $\beta_I < \beta_P$ for the borrowing constraint to bind in the steady state.

3.4 Market clearing

Finally, the model is closed by adding market clearing conditions, and standard transversality conditions are imposed. The model features four markets: a goods market, credit market, labor market and housing market.The market clearing condition on the goods market is:

$$Y_t = I_t + C_{t,P} + C_{t,F} + C_{t,I}. \tag{26}$$

Bonds are assumed to be in zero-net supply:

$$B_t = B_{t,F} + B_{t,I}. \tag{27}$$

The equilibrium condition on the labor market is:

$$N_t = N_{t,P} + N_{t,I}. \tag{28}$$

Finally, the market clearing condition on the housing market is:

$$H_{P,t} + H_{F,t} + H_{I,t} = 1. \tag{29}$$

In the rational expectations case, the model is solved relying on standard perturbation methods. Appendix A provides a summary of the model's equations under the assumption that the borrowing constraint of both entrepreneurs and impatient households is binding and thus that the associated Lagrange multipliers $\mu_{F,t}$ and $\mu_{I,t}$ are strictly positive. When numerically solving the model both under rational expectations and under learning, we verify that this assumption holds true in all simulations, as in Iacoviello (2005) and Iacoviello (2015).

We now describe the model under subjective expectations, when agents no longer form rational expectations about the law of motion of house prices, while still holding model-consistent beliefs for all other variables. We then compare the results of the rational expectations model and of the subjective expectations model and discuss their relative relevance for understanding the recent joint dynamics of housing prices, credit and macroeconomic variables.

4 The learning model

4.1 Optimal Bayesian learning

Following a recent trend in the literature on learning regarding asset prices (Adam et al. (2012), Kuang (2014), Adam et al. (2016a), Adam et al. (2016b), Winkler (2016)), we now assume that agents in the economy do not understand the endogenous process through which house prices form. The actual equilibrium price results from the equalization of the demand for housing of the three sectors in the model to the exogenous supply of housing. However, market participants have imperfect knowledge of the market process and do not properly understand how house prices form. Instead, agents observe house prices realizations and try to determine whether the observed evolution is permanent or temporary. They thus try to evaluate the persistence of a given variation in house prices based on their past experience. Price determination is indeed a difficult task for atomistic agents as it implies perfect knowledge of the mapping between state variables and prices and thus perfect knowledge about other agents' knowledge and subsequent optimal decisions. Instead of taking into account the housing market clearing condition, atomistic market participants believe that logged house prices follow an exogenous process which takes the following form:

$$\ln(q_t) - \ln(q_{t-1}) = \ln(\mu_t) + \ln(\eta_t), \tag{30}$$

where η_t is a temporary disturbance and where the time-varying persistent component μ_t follows the process:

$$\ln(\mu_t) = \ln(\mu_{t-1}) + \ln(\nu_t), \tag{31}$$

where ν_t is an additional disturbance. This specification for the perceived exogenous process driving house prices is consistent with the empirical behavior of house prices. Indeed, in the data, episodes of persistent increase in house prices are followed by episodes of persistent decrease in house prices. In addition, Adam et al. (2012), Kuang (2014), Adam et al. (2016a), Adam et al. (2016b) and Winkler (2016) show that the optimal beliefs derived from this perceived law of motion present features that are consistent with survey data.

Agents perceive the innovations η_t and ν_t to be normally distributed according to the following joint distribution:

$$\begin{pmatrix} \ln(\eta_t) \\ \ln(\nu_t) \end{pmatrix} \sim N\left(\begin{pmatrix} -\frac{\sigma_\eta^2}{2} \\ -\frac{\sigma_\nu^2}{2} \end{pmatrix}, \begin{pmatrix} \sigma_\eta^2 & 0 \\ 0 & \sigma_\nu^2 \end{pmatrix} \right), \tag{32}$$

where σ_η and σ_ν are small.

Note that all sectors in the economy share common beliefs on the house price process. Agents perfectly observe house price realizations q_t, but they are not able to separately observe the persistent component and the transitory component of what they believe to be the exogenous process driving house price dynamics. Therefore, they face an optimal filtering problem and come up with the best statistical estimate $\ln(\widehat{\mu}_t)$ of the persistent component $\ln(\mu_t)$ in each period t. Due to normality of residuals and the linearity of the process, optimal Bayesian filtering amounts to standard Kalman filtering in the set-up. Again following the related literature, we assume that the prior distribution of beliefs is a normal distribution with mean parameter $\ln(\widehat{\mu}_0)$ and dispersion parameter σ_0. Because we take the deterministic steady state as a starting point in our simulations below, as is usual in DSGE models analyses, we set the prior mean and dispersion parameters at their steady state values.[7] The prior mean belief about house price growth is thus set at $\ln(\widehat{\mu_0}) = 0$, and prior uncertainty σ_0^2 is set at its Kalman filter steady state value σ^2:[8]

$$\sigma^2 = \frac{-\sigma_\nu^2 + \sqrt{(\sigma_\nu^2)^2 + 4\sigma_\nu^2\sigma_\eta^2}}{2}. \tag{33}$$

Agents' subjective probability measure P is specified jointly by equations 30, 31 and 32, by prior beliefs and by knowledge of the productivity, loan-to-value ratio and lenders' discount factor random processes.

The posterior distribution of beliefs in time t following some history up to period t, ω_t, is $\ln(\mu_t|\omega_t) \sim N(\ln(\widehat{\mu_t}), \sigma^2)$, where $\ln(\widehat{\mu_t})$ is given by the following optimal updating rule:

$$\ln(\widehat{\mu_t}) = \ln(\widehat{\mu_{t-1}}) - \frac{\sigma_\nu^2}{2} + g\left[\ln(q_t) - \ln(q_{t-1}) + \frac{\sigma_\eta^2 + \sigma_\nu^2}{2} - \ln(\widehat{\mu_{t-1}})\right]. \tag{34}$$

This unique recursive equation – in which g is the Kalman filter gain, which optimal expression is $\frac{\sigma^2}{\sigma^2+\sigma_\nu^2}$ – fully characterizes agents' beliefs about house price growth, which are summarized in each period t by the state variable μ_t. The Kalman filter gain governs the size of the updating in the direction of the last forecast error. Logically, the Kalman filter gain increases in the signal-to-noise ratio $\frac{\sigma_\nu^2}{\sigma_\eta^2}$ because a higher noise-to-signal ratio means that changes in house prices are driven to a higher extent by changes in the persistent component μ_t relative to changes in the transitory noise η_t, and thus the last forecast error is more informative for predicting future house prices. Agents believe that house prices in period t are such that:

$$\ln(q_t) = \ln(q_{t-1}) + \ln(\widehat{\mu_{t-1}}) - \frac{\sigma_\eta^2 + \sigma_\nu^2}{2} + z_{1t}, \tag{35}$$

[7]Note that in the steady state, agents assume both $\bar{\eta}$ and $\bar{\nu}$ to be equal to zero.

[8]This implies that posterior uncertainty will remain at its steady state value following new house price realizations because it is already starting at its maximal value.

where z_{1t} is seen by agents as an exogenous forecast error, normally distributed with mean 0 and variance σ_z, whereas it is actually endogenous and is equal to the difference between the expected growth rate of house prices and the actual growth rate formed endogenously on the housing market (up to a negligible constant).

Given the perceived law of motion for the growth rate of house prices, agents form optimal beliefs and make optimal decisions. Therefore, agents are 'internally rational' despite not holding rational expectations on the future dynamics of house prices.[9] We now define more specifically internal rationality in the context of our model.

4.2 Internally rational expectations equilibrium

Definition 1: Internal rationality for each sector in each period t

- Patient households are internally rational if they choose $(C_{t,P}, H_{t,P}, N_{t,P}, B_t)$ to maximize the expected utility (1) subject to the budget constraint (3), for the given subjective probability measure P.

- Entrepreneurs are internally rational if they choose $(C_{t,F}, H_{t,F}, B_{t,F}, N_t, K_t, I_t)$ to maximize the expected utility (7) subject to the budget constraint (8), the production function (9), the capital accumulation equation (11) and the complementary slackness condition (17), for the given subjective probability measure P.

- Impatient households are internally rational if they choose $(C_{t,I}, H_{t,I}, N_{t,I}, B_{t,I})$ to maximize the expected utility (19) subject to the budget constraint (20) and to the complementary slackness condition (25), for the given subjective probability measure P.

[9]The concept of internal rationality was defined by Adam and Marcet (2011): "[i]nternal rationality requires that agents make fully optimal decisions given a well-defined system of subjective probability beliefs about payoff relevant variables that are beyond their control or "external", including prices." By contrast, "[e]xternal rationality postulates that agents' subjective probability belief equals the objective probability density of external variables as they emerge in equilibrium" (p. 1225).

We can thus define the internally rational expectations equilibrium of our model.

Definition 2: The internally rational expectations equilibrium

The internally rational expectations equilibrium is defined by:

- The subjective probability measure P over the space Ω of all possible realizations of variables which are external to agents' decisions.[10]
- A sequence of contingent choices $\{C_{t,P}, C_{t,F}, C_{t,I}, H_{t,P}, H_{t,F}, H_{t,I}, B_t, B_{t,F}, B_{t,I}, N_t, N_{t,P}, N_{t,I}, K_t, I_t\} : \Omega^t \to \mathbb{R}_+^{14}$ such that the internal rationality of each agent defined above is satisfied.
- A sequence of equilibrium prices $\{q_t, R_t, w_t\}_{t=0}^{\infty}$ where $(q_t, R_t, w_t) : \Omega^t \to \mathbb{R}_+^3$ such that markets clear in each period t and all realizations in Ω are almost surely in P.

We now describe briefly how the internally rational expectations equilibrium is solved for.

4.3 Solving the model under imperfect market knowledge

For solving the model under subjective expectations, we resort to lagged beliefs updating, which aims at avoiding the simultaneous determination of beliefs and house prices. Indeed, according to equation 34, the mean belief about house price growth $\ln(\widehat{\mu}_t)$ in period t depends on current house prices q_t. At the same time, house prices in period t depend on the expectations of future house price growth and thus on the current mean belief $\ln(\widehat{\mu}_t)$. To avoid this issue due to self-referential learning, we assume lagged beliefs updating, which means that agents only rely on lagged information when updating their beliefs.

[10]Note that under learning the space of realizations of variables external to agents' decision includes the realizations of house prices, whereas such realizations provide information redundant with that provided by exogenous fundamental realizations under rational expectations because agents understand the mapping of fundamentals into house prices.

This assumption is made in all the papers that rely on the same specification of beliefs as ours and is also standard in the general self-referential learning literature. It consists in rewriting the beliefs updating equation rule 34 as:

$$\ln(\widehat{\mu_t}) = \ln(\widehat{\mu_{t-1}}) - \frac{\sigma_\nu^2}{2} + g\left[\ln(q_{t-1}) - \ln(q_{t-2}) + \frac{\sigma_\eta^2 + \sigma_\nu^2}{2} - \ln(\widehat{\mu_{t-2}})\right]. \tag{36}$$

The slightly modified updating rule means that in period t, agents update their mean belief in the direction of the forecast error of the previous period rather than of the current period. Consequently, the mean belief $\ln(\widehat{\mu_t})$ is now predetermined at time t, and equilibrium house prices are determined in time t by the housing market clearing condition. Lagged beliefs updating thus ensures that the equilibrium is unique. Adam et al. (2016a) provide microfoundations for this updating rule with delayed information. Following Winkler (2016), we treat the lagged forecast error as an exogenous disturbance z_{2t} in the belief system of agents in period t to justify why agents can see their forecast error in period t but do not adjust their beliefs in its direction until period $t+1$.

Under imperfect market knowledge, all expected future realizations are conditional on the subjective probability measure P. The system of equations characterizing the policy function under P includes the first-order conditions (4-6), (14-16), (18 and (22-24), the flow of fund constraints (3), (8), and (20), the production function (9), the capital accumulation equation (11), the complementary slackness conditions (17-25), market clearing conditions (27) and (28), random processes (2), (10) and (13) and the beliefs' updating equation (36). The market clearing condition on the housing market is not included in the system under the subjective probability measure P because agents do not understand how house prices form.

Solving this subjective system of equations yields the subjective policy functions, obtained under the probability measure P. However, subjective policy functions do not characterize the actual equilibrium house prices, which, despite being seen as exogenous, arise endogenously in the model through the market clearing condition.

Therefore, to solve the model under imperfect market knowledge, we rely on two steps, following the method proposed by Winkler (2016). First, we numerically solve for the coefficients of the approximate subjective policy function h of the system of equations described above in the neighborhood of the deterministic steady state, relying on standard perturbation methods. Second, we solve for the approximate actual policy function, i.e., the objective policy function g, by deriving actual endogenous house prices from the subjective policy function h, relying on chain rules derivation. Therefore, we obtain the derivatives of the Taylor expansion of the actual policy function g in the neighborhood of the deterministic steady state and we thus obtain a numerical approximation for g, which fully characterizes the numerical solution to the learning model. The method is explained with more details in Appendix B and closely follows that presented in Winkler (2016) (see in particular Appendix C in Winkler's paper).

5 Results: asset price volatility and macro-financial linkages

After solving for the model both under rational expectations and under imperfect market knowledge as explained above, we show to what extent the introduction of learning about house prices affects the housing market dynamics and the transmission of aggregate disturbances in the baseline model.

We first detail the calibration strategy. The time interval in the model is a quarter.

5.1 Calibration strategy

Static parameters are calibrated to target steady state values of variables or in reference to the literature, and dynamic parameters are either calibrated to target second-order moments, either derived from the data, or based on the literature.[11] All the data used in the calibration process and for assessing the results are detailed in Appendix C.

The first set of parameters is static parameters $(\beta_P, \beta_I, \beta_F, \alpha, \psi, m_1, m_2, \delta, j)$, which affect the steady state. The discount factor of patient households β_P is set at 0.9934 such that the steady state mortgage rate $\bar{R}$ equals the mean of the average 1-year adjustable mortgage rate in the US over the period 1985-2015. The discount factors of impatient households and firms (β_I, β_F) are set at 0.94, following Iacoviello (2015). Such a rather low value ensures that the borrowing constraints are easily binding in the simulations. The weight on leisure in the household utility function is set at $\psi = 2$, the weight on housing in the household utility function is set at $j = 0.075$ and the share of housing in the consumption goods production function is set at $v = 0.05$. The first former parameter value implies that households allocate around one-third of their active time to work and that the Frisch elasticity of labor supply is around 2, which is consistent with values used in the macroeconomic literature (see Peterman (2016)) and with the calibration in Iacoviello (2015). The last two parameter values imply a steady state entrepreneurial share of housing of 24%. From the US data over the 1985-2015 period regarding the average labor share to output, and given that we set the housing share to $v = 0.05$, we get $\alpha = 0.31$ for the share of capital in the production function. The capital depreciation rate δ is set at the standard value of 0.025, corresponding to a 10% annual depreciation.

The loan-to-value ratios for entrepreneurs m_F and for impatient households m_I are set at 0.5, which is consistent with the value estimated in Iacoviello (2005) and Kuang (2014). This value implies a significant degree of financial frictions, which plays a crucial role in the model mechanism, as will be shown below, and is such that the borrowing constraints are always binding throughout the simulations.

[11] Future work will complement the analysis by separately estimating dynamic parameters of the model through the simulated method of moments for the rational expectations model and the subjective expectations model.

In what regards the dynamic parameters of the model, the persistence parameter of the productivity shock ρ_a is estimated from the US data over the period 1985-2015, during which the linearly detrended Solow residual displays strong persistence, with $\rho_a = 0.9765$. For the persistence parameters of the lenders' preference shock and of the loan-to-value ratio shock, we rely on the literature and set $\rho_d = 0.830$ and $\rho_{a_m} = 0.850$ (see Primiceri et al. (2006) and Iacoviello (2015)).

The only parameter related to learning is the Kalman filter gain g.[12] As an initial guess for the parameter g, we set it at the low value 0.0056, following Winkler (2016). Even if our model is distinct from Winkler's model, this value for g generates consistent shapes for the impulse response functions. In addition, although this value is relatively lower than standard values usually found in the literature on learning in exchange economies, the learning model is able to simultaneously replicate the volatility in asset prices and in output, as shown below. Such a result suggests that in production economies with learning, a low degree of updating in the direction of recent forecast errors (and thus a low noise-to-signal ratio) is sufficient to generate a strong amplification mechanism relative to rational expectations models, even though beliefs display only small variations.[13] We provide a sensitivity analysis to the value of the Kalman gain g in subsection 5.4.

Finally, we set the values of $(\phi, \sigma_a, \sigma_d, \sigma_{a_m})$ such that a set of second-order moments (output, house prices, debt and investment) is equal to that in the data. In particular, the capital adjustment cost parameter ϕ mainly affects the volatility of investment, whereas the standard deviation of the productivity shock σ_a mainly affects the volatility of output, the standard deviation of the lenders' preference shock σ_d mainly affects the volatility of

[12] Indeed, in practice, the perceived variances σ_η and σ_ν, which determine the perceived variance of the forecast errors, do not affect the results as they are chosen to be very small, suggesting that even a low degree of uncertainty gives room for a strong impact of the learning mechanism.

[13] Actually, the powerful propagation mechanism triggered by learning in production economies in response to shocks can generate unstable dynamics or converging but too persistent oscillatory impulse response functions to shocks, which imposes a trade-off between the advantage of generating strong responses following very small shocks and the disadvantage of generating too persistent or unstable responses to shocks.

house prices and the standard deviation of the loan-to-value ratio shock σ_{a_m} mainly affects the volatility of debt.

The calibrated capital adjustment cost ϕ is high, showing that the learning mechanism generates strong amplification of shocks. The calibrated variance of the productivity shock σ_a is slightly higher than the one obtained when estimating total factor productivity in the data, but it allows for a better replication of output volatility and it is close to the value estimated in Iacoviello (2015).

The calibrated variances of the two additional shocks are higher than that of the productivity shock, consistent with the idea that financial shocks became stronger – notably relative to productivity shocks – in the run-up to the Great Recession. The shock variances are relatively small, which shows that the model proves able to generate enough volatility for a set of small shocks.[14] Finally, in order to compare the results of the learning model and of the rational expectations model, we recalibrate the dynamic parameters $(\phi, \sigma_a, \sigma_d, \sigma_{a_m})$ to improve the fit with the four targeted moments under rational expectations. However, the replication of the second-order moments of output, investment and debt under rational expectations is obtained at the expense of the replication of the volatility in house prices. Indeed, matching the volatility in house prices would require imposing high shock variances that would lead the rational expectations model to overpredict the volatility in other variables. Therefore, we set $(\phi, \sigma_a, \sigma_d, \sigma_{a_m})$ such that output, investment and debt volatility are replicated under rational expectations and such that house price volatility is the highest possible given this constraint.

Table 1 gathers the values of all parameters used in the model simulations.

[14] As a comparison, over a roughly similar period, Iacoviello (2015) estimates the volatility of the housing demand shock that he implements in his model in addition to many other shocks at 0.0346, which is even higher than the sum of the volatility of the three shocks in our model. Our model proves anyway able to fully replicate the dynamics of house prices while providing a more endogenous explanation of these dynamics. This reveals the powerful amplification mechanism generated under learning.

Parameter	Calibrated value (Learning)	Calibrated value (Rational expectations)
β_P	0.9934	0.9934
β_I	0.94	0.94
β_F	0.94	0.94
ψ	2	2
j	0.075	0.075
v	0.05	0.05
α	0.31	0.31
m_F	0.5	0.5
m_H	0.5	0.5
δ	0.025	0.025
ϕ	11.3	11.8
ρ_a	0.9765	0.9765
ρ_d	0.830	0.830
ρ_{a_m}	0.850	0.850
σ_a	0.00684	0.00725
σ_d	0.00906	0.0118
σ_{a_m}	0.00859	0.02
g	0.0056	NA

Table 1: Calibration

Under rational expectations, stronger shock variances are required in comparison to the learning case. Such a result reveals the strong amplification in the responses to shocks generated by the learning mechanism.

5.2 Asset price moments and business cycle moments

We now look at standard business cycle moments and asset price moments in order to assess whether the model with learning can help reconcile the two sets of moments over the last 30 years. Table 2 compares the volatility of house prices and of standard business cycle variables in the data with that in the learning model. Table 2 presents both the moments that were directly targeted in the calibration strategy (output, house prices, investment and debt volatility) and moments that were not directy targeted. The latter include asset price moments – the autocorrelation of house prices, the autocorrelation of house price growth and the contemporaneous correlation of house prices with output –, along with additional business cycle moments – consumption, hours worked and labor productivity. Table 2 also reports the moments obtained in the rational expectations model for values of dynamic parameters identical to those in the learning model ("RE Calibration 1") and for values of dynamic parameters specifically calibrated for this version of the model ("RE Calibration 2"). The empirical quarterly data and the model-generated data are both logged (except for the mortgage rate and the house price growth rate) and hp-filtered with parameter 1600. The model-generated consumption is the sum of patient household consumption, impatient household consumption and entrepreneur consumption.

	US Data Q1 1985-Q4 2015	Learning	RE Calibration 1	RE Calibration 2
σ_{Y_t}	0.011	0.011	0.010	0.011
$\frac{\sigma_{I_t}}{\sigma_{Y_t}}$	3.34	3.34	2.66	3.34
$\frac{\sigma_{B_t}}{\sigma_{Y_t}}$	3.42	3.42	2.66	3.42
$\frac{\sigma_{q_t}}{\sigma_{Y_t}}$	2.04	2.04	1.03	1.53
$\frac{\sigma_{C_t}}{\sigma_{Y_t}}$	0.73	0.82	0.86	0.84
$\frac{\sigma_{N_t}}{\sigma_{Y_t}}$	1.51	0.38	0.21	0.40
$\frac{\sigma_{\frac{Y_t}{N_t}}}{\sigma_{Y_t}}$	0.79	0.82	0.91	0.90
$\rho(q_{t-1}, q_t)$	0.92	0.86	0.73	0.72
$\rho(\ln \frac{q_t}{q_{t-1}}, \ln \frac{q_{t-1}}{q_{t-2}})$	0.20	0.40	-0.02	-0.03
$\rho(q_t, y_t)$	0.53	0.62	0.85	0.72

Table 2: Business cycles and house prices moments

Regarding the business cycle moments that were not targeted in the calibration strategy, they are roughly similar for the learning model and the rational expectations model with the second calibration. Both models generate slightly too much volatility for both consumption (due notably to high capital adjustment costs) and labor productivity relative to the data, even if the learning model overpredicts the latter to a lesser extent. Unsurprisingly, the model has more difficulties in replicating the volatility of hours, both under learning and under rational expectations. This is a well-known issue in standard basic real business cycle models due to the simplicity of labor market and labor supply decisions.[15] For small values of shock variances identical to those calibrated for the learning model, the rational expectations model performs poorly: it is unable to replicate the high volatility in house prices, debt and investment.

Interestingly, the learning model is able to replicate additional features of the house price dynamics that were not targeted in the calibration strategy. First, the learning model well predicts the strong autocorrelation in house prices $\rho(q_{t-1}, q_t)$, whereas the rational expectations model under-predicts this autocorrelation. Second, the model with learning predicts a positive autocorrelation in house price growth $\rho(\ln \frac{q_t}{q_{t-1}}, \ln \frac{q_{t-1}}{q_{t-2}})$, whereas the model with rational expectations predicts a negative autocorrelation. Third, the learning model replicates the procyclicality of house prices $\rho(q_t, y_t)$, whereas the rational expectations model over-predicts the positive correlation with output to a larger extent relative to the learning model.

All in all, it appears that the learning model is able to simultaneously replicate the high volatility in house prices relative to output and most of the volatility in standard business cycle variables over the period 1985-2015 in addition to stylized facts regarding the behavior of house prices, which already indicates a surprisingly good success given the simplicity of the model.

Our quantitative results suggest that learning about house prices offers a powerful and intuitive mechanism for explaining excess volatility in house prices in a simple and standard production economy without resorting to too many additional assumptions and ingredients, in a context where the empirical validity of the rational expectations assumption is called into question.[16] To better understand the amplification mechanism at play

[15]Several assumptions were made in the literature to overcome this limitation inherent to standard real business cycle theory, such as indivisible labor or the search model of the labor market. The main focus of the paper is not labor market dynamics, but future work will aim at completing the labor market side of the baseline model in order to better explain the volatility of hours.

[16]As Glaeser and Nathanson (2015) state regarding their model of learning on house prices in an exchange economy set-up: "Many other forms of irrationality may exist, and it may be possible to discover a rational model that can reconcile all the facts. Yet it is remarkable that this relatively modest deviation from rationality predicts outcomes so much closer to reality than the standard rational model."

in the model, we now compare the impulse response functions following the three shocks under learning and under rational expectations.

5.3 Impulse response functions analysis

The impulse response functions are log-deviations from the steady state (except for the mortgage rate, which is in percentage point deviations) in response to a one-standard-deviation positive productivity shock, a one-standard-deviation positive loan-to-value shock and a one-standard-deviation negative lenders' preference shock, both under rational expectations and under learning.[17] We assess the impact of a negative lenders' preference shock because this shock decreases the preference for the current period in the lenders' utility function and thus increases the willingness of patient households to lend to other sectors of the economy. Therefore, the mortgage rate decreases, which corresponds to what was observed in the recent housing boom-and-bust episode.

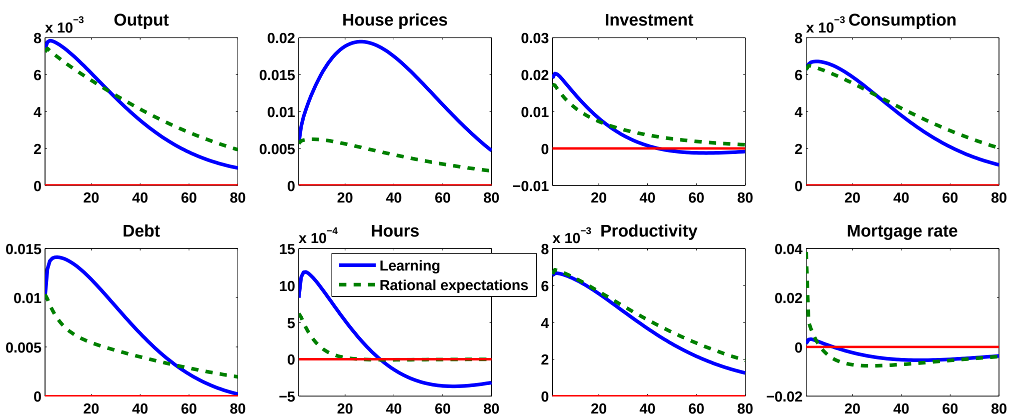

Figure 1: Impulse response functions following a positive productivity shock

First, regarding responses to positive productivity shocks (Figure 1), contemporaneous responses to the shock all display similar signs under both rational expectations and learn-

[17]To compare impulse response functions under subjective and rational expectations following similar size of innovations, the standard deviations of shocks that we retain are those calibrated for the learning model.

ing (which are in line with standard results in business cycles theory). However, house prices, debt and hours respond much more strongly to a productivity shock under learning and display clear hump-shaped responses. Output, investment, consumption, labor productivity and the mortgage rate also display hump-shaped response functions in the learning case in comparison to the rational expectations case. If output and consumption increase first more strongly in response to the shock under learning, labor productivity and the mortgage rate increase less. The results are in line with Winkler (2016), who also finds hump-shaped responses to productivity shocks under learning.

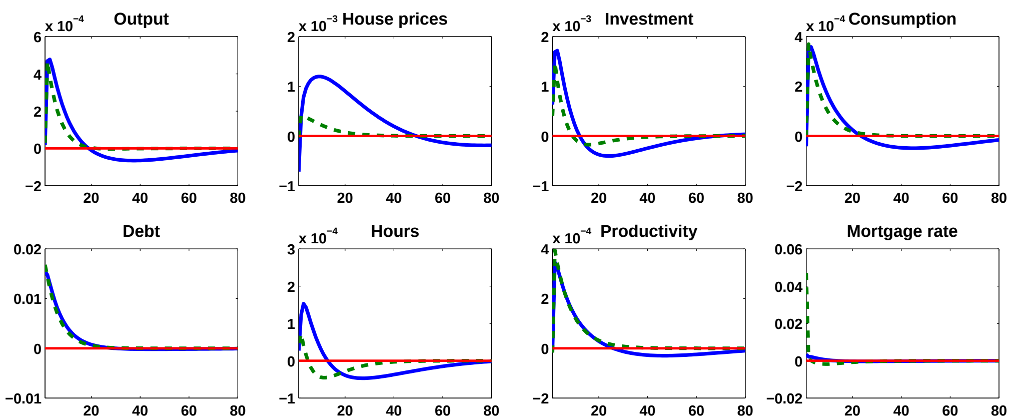

Figure 2: Impulse response functions following a positive loan-to-value ratio shock

Positive shocks to the loan-to-value ratio (Figure 2) increase the borrowing capacity of borrowers in the context of binding borrowing constraints. Borrowers thus increase their debt (and the mortgage rate increases). Equilibrium debt then gradually decreases as the shock lacks persistence. First, housing demand decreases, due mainly to patient households being able to lend more. They thus save relatively less through housing investment. This effect can also occur through the collateral motive: impatient households and firms can borrow the same amount of debt with less collateral. They thus need less real-estate assets to guarantee their debt. The decrease in total housing demand then triggers a decrease in house prices, because the housing supply is constrained. However, house prices then increase, as the borrowing capacity gradually decreases. This increase in house prices is exacerbated under learning due to a specific internal propagation mechanism (which is explained below). Thus, learning generates endogenously persistent booms in house prices in response to a loan-to-value ratio shock. As for total consumption, it first decreases, driven by a decrease in the consumption of patient households – who can save more by lending more following the loan-to-value ratio shock – and by a decrease in the consumption of entrepreneurs – who invest more in capital following the shock.

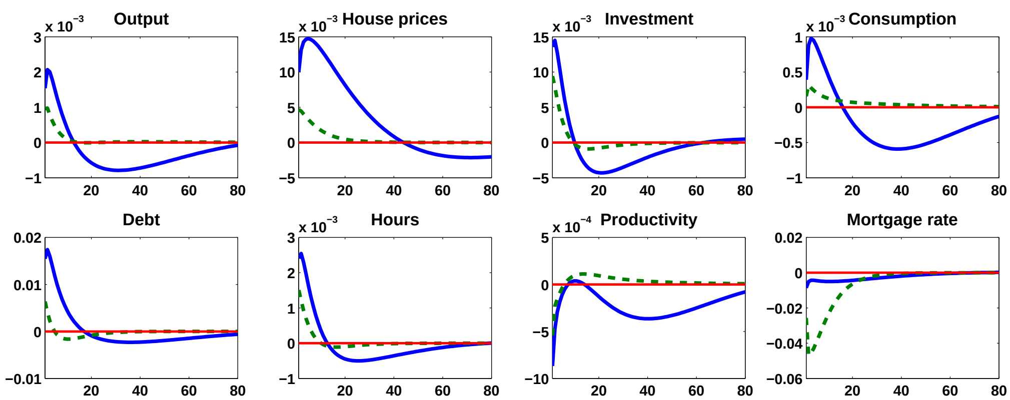

Figure 3: Impulse response functions following a negative lenders' preference shock

A negative lenders' preference shock, which implies that patient households value the current period less and are thus more willing to save money through lending, triggers a decrease in the mortgage rate (Figure 3). This is in accord with what was observed in 2001 in the US prior to the housing market boom. The shock triggers a strong increase in all variables, except in labor productivity. Impatient households and firms can borrow at a lower rate and can thus invest more in housing and capital following the shock, as well as consume more. Learning generates strong responses in house prices, investment, output, debt and consumption in response to the shock.

Therefore, we see that learning generates stronger responses to shocks, that are, in most cases, hump-shaped. The effect of productivity shocks and financial shocks is thus amplified and propagated by learning.

To understand the mechanism at play, we now present the joint dynamics of the expected house price growth rate and of the actual growth rate in response to a one-standard-deviation negative lenders' preference shock (Figure 4). The mechanism is similar for the other shocks.

We observe that house prices grow in response to the shock. The realized growth rate $\ln(P_t) - \ln(P_{t-1})$ (blue curve, left-hand scale) is higher than the expected growth rate $\ln(\widehat{\mu_{t-1}})$ (red curve, left-hand scale), implying positive forecast errors (which are roughly measured as the difference between the two curves). According to equation (36), when (lagged) forecast errors are positive, beliefs are updated upwards. This contributes to a rise in the demand for housing and thus to an increase in house prices (green curve, right-hand scale) because the housing supply is fixed. This in turn fuels the increase in the expected growth rate. As the forecast error (that is, the distance between the two series) decreases, at some point, the expected growth rate becomes equal to the actual growth rate. This implies that forecast errors become null (at the point where the blue curve and the red curve intersect). In this case, the expected growth rate remains constant in the next period, and demand for housing grows more slowly. The realized growth rate of house prices thus decreases below the expected value. Therefore, the expected growth rate decreases in the next period. This decrease also causes the actual growth rate of house prices to decrease and to become negative, meaning that house prices start to decrease.

As the expected growth rate is now higher than the actual growth rate, i.e., forecast errors are negative, this generates the reverse feedback mechanism. A similar mechanism is at work until house prices finally go back to the steady state, under stable dynamics. Cyclical variations in forecast errors thus lead to cyclical variations in beliefs, house prices and other variables, as forecast errors are state variables, and so are the mean belief and house prices, which co-move with forecast errors. This mechanism, specific to learning,

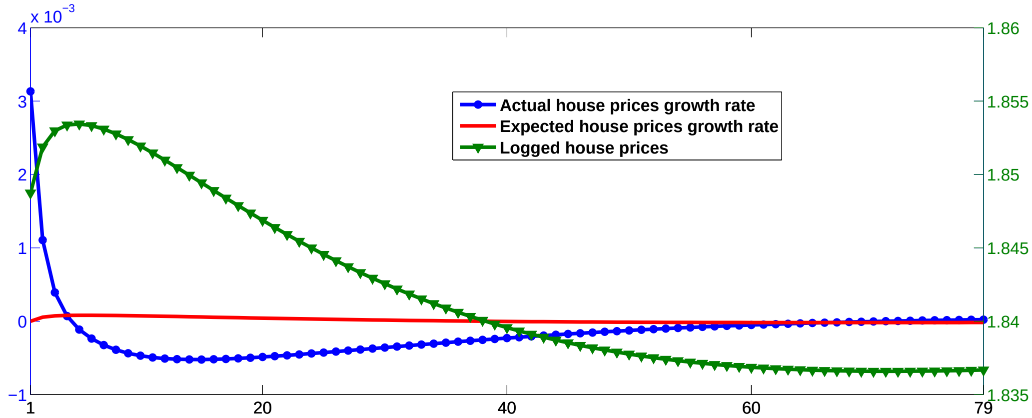

Figure 4: House prices and the dynamics of beliefs

then allows for the replication of episodes of sustained increases in house prices followed by a steep persistent decrease in response to only one-quarter-lasting shocks. Learning indeed generates self-reinforcing dynamics and endogenous reversals (at the time when the expected growth rate is equal to the actual growth rate).

The simple model thus features several macro-financial linkages, which are exacerbated under learning. First, the model generates transmission of the macroeconomic shock to the credit market. Second, the model generates transmission of the financial shocks to the real sector. Learning on house prices reinforces those macro-financial linkages in response to both macroeconomic and financial shocks. Indeed, the learning mechanism induces a feedback loop between beliefs and house prices. In response to fundamental shocks, house prices shift, which affects both the beliefs and the other macroeconomic and financial variables in the general equilibrium setting. Shifts in beliefs and in other variables in turn affect house prices, and the transmission mechanism from house prices to other variables starts again, generating strong and persistent propagation of fundamental shocks until all variables converge to their steady state value.

Even if default is not accounted for in the baseline model, which prevents modelling the dynamics of financial losses and their consequences on the credit supply and the real sector, the model with learning is, however, able to qualitatively replicate the joint dynamics of macro-financial variables similar to those observed during the recent financial crisis. Thus, the model with learning is able to generate a credit and housing boom as the result of financial shocks. The increase in house prices contributes to fuel the simultaneous increase in credit due to collateralized borrowing constraints and thus to fuel the increase in investment, consumption and output. However, at some point, the increase in credit and macroeconomic variables reverts, as the effects of the transitory initial shock decline and as house prices grow more slowly. The reversal is further propagated when the expected house price growth rate catches up to the actual growth rate, which triggers an endogenous bust in house prices.

Finally, we now assess the sensitivity of output and asset price volatility to key parameters of the model, namely the size of the financial frictions governed by the parameters m_F and m_I and the parameter governing the learning process g in order to assess the respective impact of learning and of financial frictions in driving the dynamics of the model in response to shocks.

5.4 Sensitivity analysis to key parameters: the joint role of learning and financial frictions

We have shown how learning affects the joint dynamics of business cycle variables and house prices relative to the rational expectations set-up in a model with financial frictions. These financial frictions play a crucial role in propagating the dynamics enhanced by the learning mechanism. This is important to see that it is not learning alone that generates the strong volatility in house prices in the model but the combination of learning and financial frictions.

To show this, we provide a sensitivity analysis to the value of the parameters governing the extent of the financial frictions, that is, the parameters m_F (in the borrowing constraint of entrepreneurs) and m_I (in the borrowing constraint of impatient households), which represent loan-to-value ratios. For simplicity, we keep m_F and m_I equal in the sensitivity analysis, and we assess how their value affects the volatility of both house prices and output, with all other parameters value, notably the learning parameter g, held constant. Second, we study the sensitivity of the volatility of house prices and of output to the learning parameter g (Figure 5).[18]

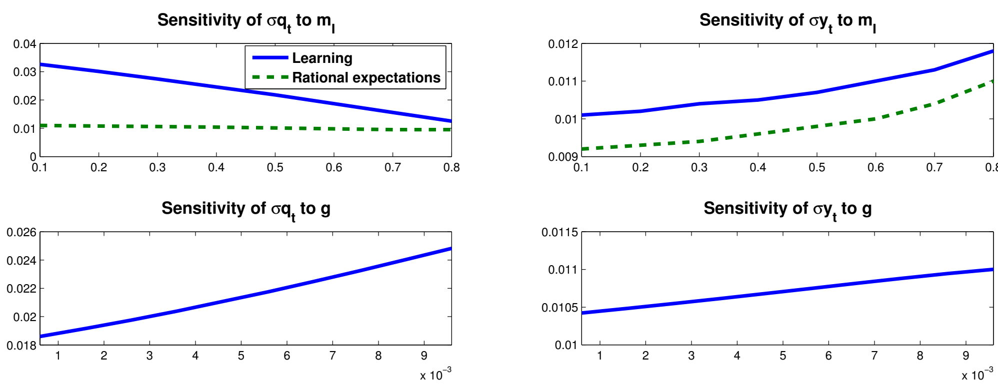

Figure 5: Sensitivity of the volatility of house prices and output to financial frictions and learning parameters

The top left-hand-side panel reveals that the volatility of house prices decreases when the degree of financial frictions (which decreases in m_F) decreases. This result is obtained both in the learning and in the rational expectations case.

However, high financial frictions (i.e., low m_F and m_I) only generate high volatility of house prices in the learning model (the green dotted curve remaining almost constant).

Interestingly, when the degree of financial frictions is low, house price volatility in the learning case is much closer to that in the rational expectations case, showing the crucial role of financial frictions in making the learning mechanism able to replicate the volatility observed in the data.

Such a result can be rationalized as follows. With high financial frictions, when pro-

[18] We restrict the sensitivity analysis to parameter values for which borrowing constraints are always binding throughout the simulations given the calibrated variance of shocks.

ductivity suddenly increases, when the loan-to-value ratio suddenly increases, or when the lenders' preference for the present period suddenly decreases, the amount of credit in the economy does not increase as much as it would with lower financial frictions, and the demand for housing tends to increase more. Learning then generates self-reinforcing expectations about future house prices that propagate the increase in house prices.

As for the volatility of output (top right-hand-side panel), on the reverse, it increases when financial frictions decrease because entrepreneurs can increase their debt more and thus produce relatively more following the distinct shocks. In addition, the sensitivity of output volatility to the degree of financial friction displays much less difference between the two models. This result shows that lower financial frictions increase the volatility of output, whereas they decrease the volatility of house prices, implying a trade-off between the two. Learning can help reconcile both aspects by amplifying the dynamics of asset prices.

The two bottom panels show how the volatility of house prices and of output both increase when the Kalman filter gain g increases, reflecting a stronger reaction of beliefs (and thus of all other variables because beliefs are a state variable) to new house price realizations. Indeed, a higher g means that agents believe that changes in the time-varying drift μ_t play a stronger role in explaining variations in house prices relative to changes in the transitory noise η_t, which are unpredictable. Agents thus update more strongly their beliefs in the direction of the last forecast error in response to changes in house prices. Indeed, the last forecast error is then more informative for predicting future variations in house prices.

In addition, as the persistence parameter of the loan-to-value ratio shock process ρ_a estimated in the most recent period (since the early 2000s) on household data in the US is significantly higher (see Pintus and Suda (2016)) than the value retained in earlier literature, we display the volatility of output, house prices, debt and investment as functions of this parameter. However, it appears that in our model the volatility of output, house prices and investment is quite insensitive to the persistence parameter of the leverage shock process (Figure 6).

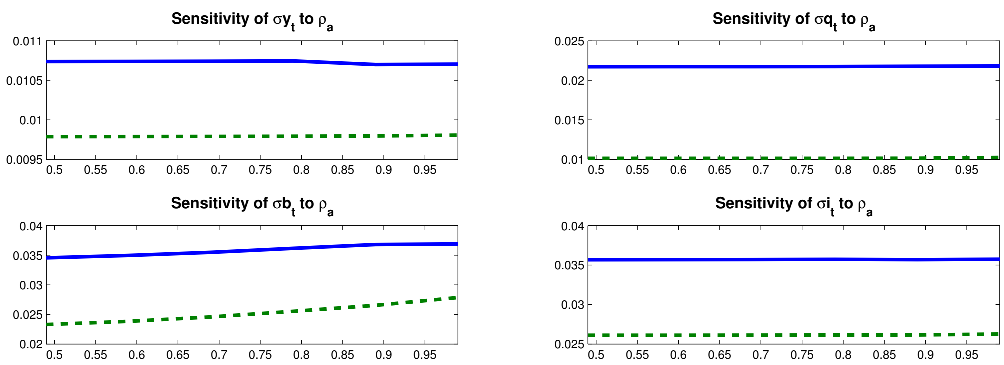

Figure 6: Sensitivity of key variables to the persistence of the loan-to-value ratio shock process

Finally, our results on the joint role of learning and financial frictions to explain asset prices volatility are in line with Winkler (2016) in the context of a distinct set-up directed at explaining stock prices instead of house prices, with the same learning mechanism but different financial frictions and with Kuang (2014) who obtains a similar result in a pure exchange economy. Our own results thus tend to further support the idea that learning in combination with financial frictions significantly amplifies and propagates the impact of both macroeconomic and financial shocks on house prices.

6 Conclusion

The present paper proposes an interpretation of the recent macro-financial linkages involving feedback transmission channels between the credit market, the housing market and the real sector, based on imperfect market knowledge, and thus on Bayesian learning regarding house prices. We incorporate the learning process into an otherwise standard borrower-saver model with a production sector, collateralized borrowing constraints and financial frictions. We show how learning generates strong amplification and propagation of small aggregate disturbances in combination with credit frictions. We show that the learning mechanism enables the replication of persistent boom episodes in house prices followed by endogenous busts in response to shocks affecting the credit market, in accord with what was observed before and during the recent subprime financial crisis. The set-up helps simultaneously replicate several features of the dynamics of house prices and most standard business cycle moments of the last 30 years.

This paper thus presents a parsimonious and stylized model that helps better account for the recent joint dynamics of house prices, credit and macroeconomic variables, and paves the way for many potential extensions in which house price excess volatility would be better accounted for, based on an explanation with strong intuitive appeal and consistent with the rejection of the rational expectations hypothesis evidenced by survey data. The model can thus be seen as a starting point for many future avenues of research, that would focus on deriving optimal policies in a context of significant volatility in asset prices, with strong consequences on other macroeconomic and financial variables. In particular, the set-up could be extended by introducing nominal frictions and investigating optimal monetary policy.

The model could also be extended by allowing for default – and therefore by modelling lenders' expectations of default and thus endogenous risk premia – to better account for additional features of the recent subprime financial crisis. In a model allowing for default and a house price appreciation-dependent mortgage rate, the learning mechanism would prove able to generate increased mortgage rates in response to the endogenous reversal in house prices. This would then generate an endogenous increase in the default rate, triggering losses in the financial sector, a credit crunch and recession. The learning mechanism thus seems very promising for explaining the recent financial crisis without resorting to somewhat less explanatory exogenous shocks.

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A The rational expectations model

A.1 The first-order conditions

$$d_t \frac{1}{C_{t,P}} q_t = \beta_P E_t \left[\frac{1}{C_{t+1,P}} q_{t+1} d_{t+1} \right] + j d_t \frac{1}{H_{t,P}}.$$

$$\frac{d_t}{C_{t,P}} = \beta_P E_t \left[\frac{d_{t+1}}{C_{t+1,P}} R_t \right].$$

$$\frac{w_t}{C_{t,P}} = \frac{\psi}{1 - N_{t,P}}.$$

$$w_t = \frac{(1 - \alpha - v) Y_t}{N_t}.$$

$$\frac{1}{C_{t,F}} = \beta_F E_t \left[\frac{1}{C_{t+1,F}} \right] R_t + \mu_{H,t}.$$

$$\frac{q_t}{C_{t,F}} = \beta_F E_t \left[\frac{1}{C_{t+1,F}} \left(q_{t+1} + \frac{v Y_{t+1}}{H_{t,F}} \right) \right] + \mu_{H,t} m_H a_{m,t} E_t \left[\frac{q_{t+1}}{R_t} \right].$$

$$\frac{1}{C_{t,F} \left(1 - \phi(\frac{I_t}{K_{t-1}} - \delta) \right)} =$$

$$\beta_F \frac{1}{C_{t+1,F}} \left(\frac{\alpha Y_{t+1}}{K_t} + \frac{1}{1 - \phi(\frac{I_{t+1}}{K_t} - \delta)} (1 - \delta - \frac{\phi}{2} \left(\frac{I_{t+1}}{K_t} - \delta \right)^2 + \phi(\frac{I_{t+1}}{K_t} - \delta) \frac{I_{t+1}}{K_t}) \right).$$

$$B_{t,F} = m_H a_{m,t} E_t \left[\frac{q_{t+1}}{R_t} \right] H_{t,F}.$$

$$\frac{q_t}{C_{t,I}} = \beta_I E_t \left[\frac{q_{t+1}}{C_{t+1,I}} \right] + j \frac{1}{H_{t,I}} + \mu_{I,t} m_I a_{m,t} E_t \left[\frac{q_{t+1}}{R_t} \right].$$

$$\frac{w_t}{C_{t,I}} = \frac{\psi}{1 - N_{t,I}}.$$

$$\frac{1}{C_{t,I}} = \beta_I E_t \left[\frac{1}{C_{t+1,I}} R_t \right] + \mu_{I,t}.$$

$$B_{t,I} = m_I a_{m,t} E_t \left[\frac{q_{t+1}}{R_t} \right] H_{t,I}.$$

A.2 The flow of fund constraints

$$C_{t,P} + q_t H_{t,P} + B_t = w_t N_{t,P} + R_{t-1} B_{t-1} + q_t H_{t-1,P}.$$

$$C_{t,F} + q_t H_{t,F} + R_{t-1} B_{t-1,F} + w_t N_t + I_t = Y_t + B_{t,F} + q_t H_{t-1,F}.$$

$$C_{t,I} + R_{t-1} B_{t-1,I} + q_t H_{t,I} = w_t N_{t,I} + q_t H_{t-1,I} + B_{t,I}.$$

A.3 The production function and the capital accumulation equation

$$Y_t = A_t K_{t-1}^{\alpha} H_{t-1}^{v} N_t^{1-\alpha-v}.$$

$$K_t = I_t + (1-\delta) K_{t-1} + \frac{\phi}{2} \left(\frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1}.$$

A.4 The shock processes

$$\ln(d_t) = \rho_d \ln(d_{t-1}) + \varepsilon_{d,t}.$$

$$\ln(A_t) = \rho_a \ln(A_{t-1}) + \varepsilon_{a,t}.$$

$$\ln(a_{m,t}) = \rho_{a_m} \ln(a_{m,t-1}) + \varepsilon_{a_m,t}.$$

A.5 The market clearing equations

$$Y_t = I_t + C_{t,P} + C_{t,F} + C_{t,I}.$$

$$B_t = B_{t,F} + B_{t,I}.$$

$$N_t = N_{t,P} + N_{t,I}.$$

$$H_{P,t} + H_{F,t} + H_{I,t} = 1.$$

B The approximation method

Appendix B presents a summary of the original method presented in Winkler (2016) applied to the specific case of our model.

A general way to describe the learning model is to write it as the combination of the two following systems of equations under the subjective probability measure P:

$$E_t^P[f(y_{t+1}, y_t, x_t, u_t, z_t)] = 0, \tag{37}$$

$$E_t^P[\phi(y_{t+1}, y_t, x_t, u_t, z_t)] = 0, \tag{38}$$

where y_t is a vector of endogenous variables in period t, y_{t+1} is a vector of these variables in the next period, x_t is a vector of state variables such that $x_t = Cy_{t-1}$, where C is a matrix including 0 and 1 values, and y_{t-1} is a vector of the endogenous variables in the previous period. u_t is a vector of stochastic disturbances with σ the vector of variances of these exogenous disturbances. z_t is a vector of variables perceived by agents as iid exogenous disturbances that affect the house price process and the belief updating process, with zero mean and variance σ_z. They are assumed to be uncorrelated with u_t. They can be interpreted as forecast errors regarding house prices and are assumed to be null in the steady state. E_t^P is the expectations operator under the subjective probability measure P.

As discussed in Section 3, the elements of z_t are actually not exogenous. They are determined endogenously by market clearing on the housing market, and thus by a second set of equilibrium conditions, given by the system of equations (38), which is unknown to agents.

In the specific case of our model, the second set of equilibrium conditions takes the form of two equations:

$$H_{I,t} + H_{F,t} + H_{P,t} - 1 = 0, \tag{39}$$

and

$$z_{2t} - z_{1,t-1} = 0. \tag{40}$$

Solving the model under the subjective probability measure P first amounts to finding the function h, that we call the subjective policy function, such that all endogenous variables are expressed as functions of state variables x_t and of the two types of exogenous disturbances u_t and z_t only (and of the shock variances): $h(x_t, u_t, z_t, \sigma)$. We obtain a numerical approximation for the subjective policy function by implementing standard perturbation methods for numerically solving the system of equations characterizing the policy function under P that includes the first-order conditions (4-6), (14-16), (18 and (22-24), the flow of fund constraints (3), (8), and (20), the production function (9), the capital accumulation equation (11), the complementary slackness conditions (17-25), market clearing conditions (27) and (28), random processes (2), (10) and (13) and the beliefs' updating equation (36). This step yields the coefficient of the Taylor expansion of h around the deterministic steady state. This is the first step of the approximation method.

Secondly, we aim at finding the values of z_t such that the market clearing condition on the housing market is actually satisfied, that is, such that $H_{I,t} + H_{F_t} + H_{P,t} = 1$. The elements of z_t can thus be written as a function r of state variables x_t, stochastic disturbances u_t and shock variances σ: $z_t = r(x_t, u_t, \sigma)$. We thus need to approximate the function r

(which, at this point, is assumed to exist and to be unique, which is verified ex-post in the case of our model) in order to approximate the objective policy function g, which is such that, in equilibrium, $y_t = g(x_t, u_t, \sigma)$. By applying chain rules for derivation, g can be approximated in the neighborhood of the non-stochastic steady state at the first order as follows:

$$y_t = g(x_t, u_t, \sigma) = h(x_t, u_t, r(x_t, u_t, \sigma), \sigma)$$
$$\simeq g(\bar{x}, 0, 0) + (h_{x_t} + h_{z_t} r_{x_t})(x_t - \bar{x}) + (h_{u_t} + h_{z_t} r_{u_t})u_t + (h_\sigma + h_{z_t} r_\sigma)\sigma,$$

where $\bar{x}$ is the vector of steady state values of the state variables.

To derive the objective policy function g, we still need to find the approximate values of r_{x_t}, r_{u_t} and r_σ in the steady state. These derivatives can be otained by total differentiation of the second set of equilibrium conditions (38) at the deterministic steady state. Indeed, the second set of equilibrium conditions (38) can be rewritten as:

$$0 = \Phi(x_t, u_t, \sigma) = E_t^P \left[\phi(y_{t+1}, y_t, x_t, u_t, z_t)\right] =$$
$$E_t^P \left[\phi\left[\underbrace{h\Big(Ch(x_t, u_t, r(x_t, u_t, \sigma), \sigma), u_{t+1}, z_{t+1}, \sigma\Big)}_{y_{t+1}}, \underbrace{h\Big(x_t, u_t, r(x_t, u_t, \sigma), \sigma\Big)}_{y_t}, x_t, u_t, \underbrace{r\Big(x_t, u_t, \sigma\Big)}_{z_t}\right]\right].$$

Total differentiation at the steady state makes it possible to obtain the derivatives of r, which are, in our case, as follows (due to the fact that y_{t+1} and u_t do not appear in the second set of equilibrium conditions (39-40) and thus $\phi_{y_{t+1}} = 0$ and $\phi_{u_t} = 0$):

$$\frac{d\Phi}{dx}(\bar{x}, 0, 0) = \phi_{y_t} h_{x_t} + \phi_{x_t} + (\phi_{y_t} h_{z_t} + \phi_{z_t}) r_{x_t} = 0$$
$$\Leftrightarrow r_{x_t} = -(\phi_{y_t} h_{x_t} + \phi_{x_t})(\phi_{y_t} h_{z_t} + \phi_{z_t})^{-1}.$$

$$\frac{d\Phi}{du}(\bar{x},0,0) = \phi_{y_t}h_{u_t} + (\phi_{y_t}h_{z_t} + \phi_{z_t})r_{u_t} = 0$$
$$\Leftrightarrow r_{u_t} = -\phi_{y_t}h_{u_t}(\phi_{y_t}h_{z_t} + \phi_{z_t})^{-1}.$$

$$\frac{d\Phi}{d\sigma}(\bar{x},0,0) = \phi_{y_t}h_{\sigma} + (\phi_{y_t}h_{z_t} + \phi_{z_t})r_{\sigma} = 0$$
$$\Leftrightarrow r_{\sigma} = -\phi_{y_t}h_{\sigma}(\phi_{y_t}h_{z_t} + \phi_{z_t})^{-1} = 0,$$

because $h_\sigma = 0$.

Because the matrix $\phi_{y_t}h_{z_t} + \phi_{z_t}$ is invertible in our model, the function r exists and is unique.

For higher order approximations, the method is similar, even though the calculations are trickier, notably because they imply deriving Kronecker products of large matrices (see Winkler (2016) for more details).

C Data series

All data series are extracted from the Federal Reserve Bank of St Louis database (FRED), mainly from the US Flow of Funds Statistics, and are expressed in real terms using the GDP Implicit Price Deflator. Series used for deriving aggregate entrepreneur loans and household loans are similar to those used in Iacoviello (2015).

Variable	**Series**
Output	Gross Domestic Product
House prices	All-transactions House Prices Index
Investment	Private Non-Residential Fixed Investment + Durable Consumption
Consumption	Personal Consumption Expenditures, Nondurable Goods
Debt	Sum of Entrepreneur Loans and of Household Loans
Hours	Hours of All Persons, Nonfarm Business Sector
Capital	Annual Capital Stock at Constant National Prices
Mortgage rate	1-year adjustable rate (average in the US)