

Macroeconomic Implications of Learning and Financial Frictions in Interdependent Economies

Eleni Iliopoulos* Erica Perego† Thepthida Sopraseuth‡§

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Abstract

This paper investigates the rationale behind international policy coordination in a two-country model with financial frictions where economic agents are not fully rational. Under adaptive learning, private agents form over-optimistic as well as over-pessimistic expectations on the state of the economy, due to their partial knowledge of the world they are living in. We show that the interaction between learning and financial frictions further amplifies the macroeconomic effects of the financial accelerator. In addition, as private agents form expectations using domestic variables only, they tend to under-estimate the international propagation of shocks. This leads to more asymmetric fluctuations and lower business cycle synchronization. Under learning, the model generates volatile and persistent departure from uncovered interest rate parity, as well as data-consistent estimates in UIP regressions. We then assess the opportunity of monetary policy coordination under learning. We show that learning entails a greater need for both output and inflation stabilization, thereby requiring a more aggressive action from central banks. However, even under coordinated policies, agents cannot learn the true dynamics of their globalized economy because of their partial knowledge. Therefore, the joint monetary policy fails to fully anchor expectations at the international level.

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*U. of Evry and Cepremap. eleni.iliopoulos@univ-evry.fr

†Cepremap. erica.perego@cepreamap.org

‡U. of Cergy Pontoise (THEMA). thepthida.sopraseuth@u-cergy.fr

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1 Introduction

We build a two-country model with imperfect risk sharing and financial frictions à la Bernanke et al. (1999). Moreover, we assume that agents learn gradually about the structural parameters of their economy. To our knowledge, our work is the first one studying stabilization policies in a two-country model with financial frictions and adaptive learning. In doing so, we address several important issues.

Anchoring expectations is at the heart of policy makers' concerns. Mario Draghi recently stressed that "*it has been essential that the ECB has acted - and is continuing to act - to bring inflation back towards 2% and ensure the firm underpinning of inflation expectations.*" (November 2014, Frankfurt, Frankfurt European Banking Congress) and the need for "*credibility of monetary policy in anchoring inflation expectations*" (February 2016, speech at SUERF conference).¹ These statements illustrate the key role of expectations formation for policy makers. By investigating macroeconomic dynamics with financial frictions in a globalized world, we revisit the notion of anchoring expectations. Indeed, due to financial frictions, in addition to inflation expectations, mis-perceptions about future financial conditions, asset prices and return on investment affects current macroeconomic dynamics. Using a 2-country model, we will also assess the ability of central banks to anchor the other country's expectations. In particular, in our setting, central banks will face heterogeneous beliefs across countries.

Our analysis suggests that learning constitutes a source of amplification and propagation. Macroeconomic variables display amplified dynamics under learning with respect to rational expectations. Mis-perception about future asset prices and returns on capital results in current changes in capital demand, investment and net worth, thereby amplifying the effects of the financial accelerator. Under learning, the strong interaction between financial frictions and forecast errors generates larger responses of the economy to exogenous shocks, whatever the nature of the shock. The originality of our work also lies in exploring the consequence of learning in an international setting. Under rational expectations, as well as under learning, one of the main drivers of the international propagation of shocks lies in the uncovered interest rate parity condition (UIP). This arbitrage condition is a force that tends to align domestic and foreign nominal interest rates, which is a key determinant of the cost of loans. With convergent interest rates, credit conditions tend to be similar across countries. This tends to generate positive co-movements of output across countries, as in Faia (2007a) under

¹Past statements in the same vein by predecessors include: "*It is absolutely essential to ensure that inflation expectations remain firmly anchored in line with price stability over the medium term*" (Trichet, as president of ECB in 2009, University of Munich). Analogously, "*Many of the most interesting issues in contemporary monetary theory require an analytical framework that involves learning by private agents and possibly the central bank as well*" (Ben Bernanke, as chairman of the US Federal reserve in 2007, NBER Summer Institute). Finally, Peter Praet, chief economist at ECB, recently stated that "*It's key for a central bank to keep inflation expectations anchored, especially in a period of slack in the economy, and we have some signals that these inflation expectations are still fragile. [...] A possible de-anchoring of inflation expectations together with a lot of slack is a dangerous cocktail.*" (Interview for Bloomberg, November 2015).

rational expectations. Under learning, the international propagation of shocks is more asymmetric. Since agents learn only about their local state variables (Home information bias), private agents in the other country do not fully track the UIP, thereby missing a crucial propagation mechanism. This leads to more asymmetric international spillovers. As a result, in our view, the output co-movement, that 2-country models usually fail to generate (Obstfeld & Rogoff (2001)), is still a puzzle.

Finally, under learning, UIP does not hold, which is consistent with the data. Indeed, under learning, due to the Home information bias, agents misperceive the response of the interest rate to fluctuations coming from abroad. This leads to departure from UIP. The model under learning displays persistent and volatile deviations from UIP. In addition, using simulated data, a regression of realized exchange rate changes on interest rate differentials produce coefficient estimates well below one, which is consistent with the data². In contrast, under rational expectations, the model fails to generate persistent and volatile deviation from UIP. UIP regression produces a coefficient of one, which confirms that the model under rational expectation fails to generate departure from UIP.

We then explore macroeconomic dynamics if central banks agree to coordinate their monetary policies to the purpose of stabilizing fluctuations in both economies. We compare the non-coordination scenario to the coordination case. The benchmark calibration is considered as the non-coordination scenario as the Fed and the ECB have displayed little coordination over the past decades. The estimated Taylor rules reflect this lack of international coordination. The coordination scenario is based on the minimization of a standard loss function à la Orphanides & Williams (2008), so as to obtain optimal simple rules (OSR). We find that under rational expectations, central banks use fluctuations in inflation to dampen the effects of the financial accelerator through the Fisher effect. For instance, in the case of a domestic technological shock, the domestic central bank lets inflation fall more under optimal simple rules than under the benchmark calibration. This tends to increase the real value of debt, which dampens the effect of the financial accelerator so that output volatility is dampened. This offsets in turn the increase in inflation volatility. Similar effects are observed in the foreign country. Under learning, while the Home central bank succeeds in achieving output stabilization in response to the productivity domestic shock, the Foreign central banker cannot properly anchor private agents' expectations. Indeed, as Foreign agents learn about Foreign variables only, private agents' expectations in the Foreign country deviate from the "true economy". Therefore, they do not fully understand the international spillover of monetary policies and eventually expect inflation to fall more. As a result, Foreign agents expect improved future credit conditions, that stimulate in turn the demand for capital and amplify the effects of the financial accelerator. The Foreign central banks' failure to anchor Foreign expectations leads to difficulties in stabilizing output.

Finally, notice that, unlike Orphanides & Williams (2007), anchoring inflation expectations (via a greater weight on inflation into the Taylor rule or the loss function) is not sufficient to stabilize the economy. In fact, because of financial frictions, agents expectations' on future financial variables

²See for instance Engel (2016) for a survey on the UIP puzzle.

such as asset prices and capital return do determine credit behaviours and can trigger the financial accelerator. Not surprisingly, OSR under learning requires that the central banker gives also a specific and greater role to output stabilization.

Our work builds a bridge among the literatures on international macroeconomics, financial imperfections and adaptive learning, respectively. To our knowledge, our paper is the first one to combine the 3 elements. We argue that they are all relevant in studying the stabilizing policies in a globalized world with financial frictions. Rychalovska et al. (2016) develop a closed-economy setting with financial frictions à la Bernanke et al. (1999) and adaptive learning. They show how this framework captures well the financial distress of the recent years. Indeed, there is a strong interaction between learning and financial frictions due to mis-perception of asset prices and expected returns on investment. In our paper we recover the strong interaction between learning and financial frictions. Pintus & Suda (2013) uses a model with financial frictions à la Kiyotaki & Moore (1997) with learning to illustrate the interaction between learning and financial frictions. We extend their results to an international setting to explore the propagation of financial shocks under learning. Faia (2007*b*) and Kolasa & Lombardo (2014) analyze optimal policies in interdependent economies with financial frictions. However, their papers are developed under the assumption of rational expectations and cannot account for the implications of agents learning gradually about fundamentals. In addition, in their models, UIP holds, which is counterfactual. Our model generates departure from UIP. Orphanides and Williams (2007, 2008) and Gaspar et al. (2011) study monetary policies under learning. These works emphasize the role of central banks in disciplining inflation expectations. However, they consider a standard New Keynesian closed-economy framework where financial markets are perfect. Our work complements theirs by adding both the international dimension and financial frictions. Chen & Kulhanavitt (2008) examine monetary-policy rules under learning in a 2-country DSGE model with price rigidities. Milani & Park (2015) develop a small open economy DSGE model with learning. Both papers discard financial frictions. Our paper fills this gap. Finally, in international macroeconomics, the impact of expectational errors on exchange rate dynamics and UIP has been explored by several papers (Lewis (1989), Gourinchas & Tornell (2004), Ilut (2012), among others). However, to our knowledge, only a few (cited above, including ours) develop learning in a full-fledged general equilibrium model.

The paper is organized as follows. We describe the model in Section 2 before presenting the modelling of learning (Section 3). We compare the macroeconomic implications of learning on Impulse Response Functions (hereafter IRFs, in Section 4) and policy design (Section 5). Section 6 concludes.

2 A two-country model with financial frictions

In this paper we study two interconnected economies characterized both by imperfect international risk sharing and domestic financial frictions due to a costly-state verification problem à la Bernanke et al. (1999).³ Our benchmark model aims thus at tracking the amplification mechanisms associated to the financial accelerator and the international transmission of shocks.

Each country is populated by representative households whose members receive both revenues arising from labor work in wholesale firms and profits coming from their retail activity. Households have access to international markets where they can invest in international bonds (or get indebted); they can also lend their savings to domestic (foreign) banks. As in Bernanke et al. (1999), each economy is also populated by entrepreneurs, who produce capital and decide over investment and labor inputs so as to produce wholesale goods. Capital production is affected by capital adjustment costs. To finance their production activity entrepreneurs have access to loans from domestic (foreign) banks. However, as their activity is affected both by aggregate and idiosyncratic risk, the bank cannot observe their profits. In every period, a share of existing entrepreneurs defaults and exits from the market. The bank faces thus a costly-state-verification problem: in case the entrepreneur declares default, the bank needs to engage in a (costly) monitoring activity. Therefore, the interest rate on loans paid by entrepreneurs is greater than the one at which deposits are remunerated. There is thus a spread between lending and borrowing rates, which is carried by entrepreneurs as a risk premium. Once all production uncertainty is solved, retailers aggregate wholesale goods and sell (export) the final good to domestic (foreign) consumers. Retailers are monopolistic competitors, and their activity is affected by price rigidities à la Rotemberg. Having said that, as rigidities hit the retailing activity only, the exchange-rate pass through between countries is perfect.

In what follows, we will focus on the main features of our benchmark model. Notice that starred variables refer to the foreign country. For simplicity, we will denote by H the domestic country and by F the foreign one. Our calibration will be based on US and Euro Area data, two large economies with floating exchange rate. For a detailed description of the model and the list of equilibrium equations, see Appendix A.

2.1 Households

Households in country H maximize their flow of expected utilities subject to a budget constraint. Their resources come from labor activity in wholesale firms and profits arising from the retailing activity. Households consume a basket of both domestically and foreign produced goods (with a bias in favour of domestic goods), lend funds to (perfectly competitive) banks and invest in international imperfect markets. The first order conditions of their problem (in real terms of the domestic good) read as:

³See Faia (2007a) and Faia (2007b) among others.

$$U'_{Nt} + U'_{ct} \frac{W_t}{P_t} = 0 \quad (1)$$

$$U'_{ct} = \beta E_t \left[\frac{R_t}{\pi_{t+1}} U'_{ct+1} \right] \quad (2)$$

$$U'_{ct} = \beta E_t \left[R_t^F U'_{ct+1} \frac{e_{t+1}}{\pi_{t+1} e_t} \right] \quad (3)$$

where U'_N is the marginal disutility of labor effort, U'_c is the marginal utility of consumption and $\frac{W_t}{P_t}$ are real wages. Equation (23) is indeed the optimality condition associated to labor effort. The domestic nominal interest rate is denoted as R , agents discount rate is denoted by β and π is domestic CPI inflation so that equation (24) is the standard Euler equation associated to domestic deposits. Finally, as e represents the nominal interest rate and R^F is the rate on international bonds, b^* , equation (25) is the optimality equation associated to international bonds.

Due to a risk premium associated to debt accumulation, there is a spread between the return on international securities received (paid) by domestic agents and the one paid (received) by foreign ones. In particular, following Schmitt-Grohe & Uribe (2003), the spread is a function of the (real) value of the country's net foreign asset position so that the interest rate on international bonds is defined as:

$$R_t^F = R_t^* + p(-b_t^*) \quad (4)$$

where $p(-b_t^*)$ is a country-specific interest rate premium and R^* is the foreign nominal interest rate.

Foreign households face the same optimization problem as domestic households except for the fact that international bonds are denominated in their own currency.

By combining agents' Euler equations we obtain the following uncovered interest parity condition:

$$U'_{ct} = \beta E_t \left[(R_t^* + p(-b_t^*)) U'_{ct+1} \frac{e_{t+1}}{\pi_{t+1} e_t} \right]$$

and thus:

$$U'_{ct} = \beta E_t \left[\left(\frac{U_{ct}^*}{\beta E_t \left[\frac{U_{ct+1}^*}{\pi_{t+1}^*} \right]} + p(-b_t^*) \right) U'_{ct+1} \frac{e_{t+1}}{\pi_{t+1} e_t} \right] \quad (5)$$

so that marginal utilities across countries are equalized up to a spread for the country risk. Notice finally that terms of trade are the ratio of the price of domestic goods over the price of foreign goods, $tot_t = \frac{P_{Ht}}{e_t P_{Ft}^*} = \frac{f_t}{e_t^* f_t^*}$, where $f_t \equiv \frac{P_{Ht}}{P_t^c} = f_{t-1} \frac{\pi_{Ht}}{\pi_t^c}$ and $f_t^* \equiv \frac{P_{Ft}^*}{P_t^{*c}} = f_{t-1}^* \frac{\pi_{Ft}^*}{\pi_t^{c*}}$.

2.2 Entrepreneurs

We now focus on domestic entrepreneurs (the problem of F entrepreneurs is symmetric). As in Bernanke et al. (1999), entrepreneurs are risk neutral and choose the optimal level of both capital and labor inputs to be used for wholesale production. Once idiosyncratic uncertainty is solved, wholesale output is:

$$Y_t = A_t F(K_{t-1}, N_t)$$

where K denotes capital, N is labor and A denotes exogenous total factor productivity:

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^{as}$$

Capital evolves as:

$$K_t = (1 - \delta)K_{t-1} + I_t$$

where δ is the depreciation rate and I is investment. The optimality condition with respect to labor is:

$$f_t \frac{Y_{N,t}}{X_t} = \frac{W_t}{P_t}$$

where $Y_{N,t}$ denotes the first derivative of output w.r.t. labor and X_t the gross markup of retail goods over wholesale goods (i.e. $\frac{1}{X_t} = \frac{P^w}{P_H}$ where, in turn, P^w is the wholesale output price and P_H is the price of the domestic production). The optimal investment decision verifies:

$$Q_t = \left[1 + \Phi' \left(\frac{I_t}{K_{t-1}} \right) K_{t-1} \right]$$

where Q_t is the (real) price of capital and it is different from one around the steady-state because of capital adjustment costs. The mean return from holding one unit of capital is thus:

$$R_t^k = \frac{\pi_t}{Q_{t-1}} \left[\frac{Y_{K,t-1}}{X_t} f_t + \Phi' \left(\frac{I_t}{K_{t-1}} \right) \frac{I_t}{K_{t-1}} - \Phi \left(\frac{I_t}{K_{t-1}} \right) + Q_t(1 - \delta) \right] \quad (6)$$

where the first term in the brackets represents the domestic-currency yields of one unit of capital, $\frac{Y_{K,t-1}}{X_t} f_t$; the second one is the reduction in adjustment costs, $\Phi' \left(\frac{I_t}{K_{t-1}} \right) \frac{1}{K_{t-1}} - \Phi \left(\frac{I_t}{K_{t-1}} \right)$; and the third term represents the returns from selling that unit of non-depreciated capital, $Q_t(1 - \delta)$.

2.3 Loan contract and wealth accumulation

In each period t , a continuum of entrepreneurs (indexed by j) needs to finance the purchase of new capital K_t^j that will be used for production in period $t + 1$. The entrepreneur engages in a financial contract *before* the realization of the idiosyncratic shock, ω^j . Indeed, at the moment in which the contract is signed, both the bank and the entrepreneur do not know the rate of return of capital, $\omega^j R^k$.

In every period, each entrepreneur owns end-of-period internal funds for an amount nw_t^j (in real terms of the consumption good). As in Bernanke et al. (1999), we assume that the required funds for investment exceed internal funds, and thus:

$$l_t^j = Q_t K_t^j - nw_t^j \geq 0 \quad (7)$$

where l_t^j denotes the loans needed by entrepreneur j to finance investment projects. Default occurs when the return from the investment $\omega_{t+1}^j R_{t+1}^k Q_t K_t^j$ is lower than the amount that needs to be repaid $R_t^L l_t^j$, i.e.,

$$\omega_{t+1}^j \leq \tilde{\omega}_{t+1}^j \equiv \frac{R_t^L l_t^j}{R_{t+1}^k Q_t K_t^j}$$

where $\tilde{\omega}$ is the threshold level for the productivity idiosyncratic shock below which entrepreneurs default. We denote by R^L the borrowing rate paid by entrepreneurs. We recall that, as Bernanke et al. (1999), the borrowing rate R^L is an endogenous result of the optimal debt contract proposed by banks to entrepreneurs.⁴ Indeed, the bank knows that the entrepreneur has an incentive to declare default so as not to pay back its debt. As shocks are specific to each entrepreneur, j , each time s/he declares default, the bank needs to engage in a monitoring activity. As in Bernanke et al. (1999), we suppose that in each period only a fraction of entrepreneurs survives while the other fraction defaults and goes out from the market. Moreover, following Christiano et al. (2011) and Kolasa & Lombardo (2014), we suppose that the survival rate of entrepreneurs follows the exogenous process (also called wealth shock):

$$\log \varsigma_t = \rho_\varsigma \log \varsigma_{t-1} + \varepsilon_t^\varsigma$$

This shock specifically hits the survival rate of entrepreneurs, and thus, the share of wealth that is accumulated in the economy. Indeed, when more entrepreneurs are alive, more wealth is accumulated.

It is possible to rewrite banks' net capital output share as a function of the threshold default level, ω^j :

⁴For all details on the optimal contract problem, see Iliopoulos et al. (2016)

$$\Gamma(\tilde{\omega}_{t+1}^j) = \int_{\underline{\omega}}^{\tilde{\omega}_{t+1}^j} \omega_{t+1}^j f(\omega) d\omega + \tilde{\omega}_{t+1}^j \int_{\tilde{\omega}_t^j}^{\bar{\omega}} f(\omega) d\omega$$

and the implied monitoring cost share:

$$\mu G(\tilde{\omega}_{t+1}^j) \equiv \mu \int_{\underline{\omega}}^{\tilde{\omega}_{t+1}^j} \omega_{t+1}^j f(\omega) d\omega$$

so that the optimal contract results from maximizing banks' expected real profits:

$$E_t \left\{ \left[1 - \Gamma(\tilde{\omega}_{t+1}^j) \right] R_{t+1}^k Q_t K_t^j \right\}$$

under the bank participation constraint

$$\left[\Gamma(\tilde{\omega}_{t+1}^j) - \mu G(\tilde{\omega}_{t+1}^j) \right] R_{t+1}^k Q_t K_t^j = R_t \left(Q_t K_t^j - n w_t^j \right) \quad (8)$$

which implies zero profits.

Because i) only a share of entrepreneurs remains alive in every period and ii) both the cut-off value and the external finance premium are linear with respect to the capital-wealth ratio, aggregation across entrepreneurs is possible. By aggregating wealth, the optimality condition resulting from the bank optimal program can be rewritten as:

$$E_t \frac{R_{t+1}^k}{R_t} = E_t \frac{1}{\frac{[1 - \Gamma(\tilde{\omega}_{t+1})][\Gamma'(\tilde{\omega}_{t+1}) - \mu G'(\tilde{\omega}_{t+1})]}{\Gamma'(\tilde{\omega}_{t+1})} + [\Gamma(\tilde{\omega}_{t+1}) - \mu G(\tilde{\omega}_{t+1})]} \quad (9)$$

or

$$E_t \frac{R_{t+1}^k}{R_t} = \rho(\tilde{\omega}_{t+1}) \quad (10)$$

where $\rho'(\tilde{\omega}) \succeq 0$. $\rho(\tilde{\omega})$ is the external finance premium. The ratio $E_t \frac{R_{t+1}^k}{R_t}$ captures the cost of finance, which reflects in turn the existence of monitoring costs. Using equation (36), we get

$$[\Gamma(\tilde{\omega}_{t+1}) - \mu G(\tilde{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} \frac{Q_t K_t}{n w_t} = \left(\frac{Q_t K_t}{n w_t} - 1 \right) \quad (11)$$

With equations (39) and (40), equation (41) defines a relationship between the external finance premium (EFP) and the leverage ratio $\frac{Q_t K_t}{n w_t^j}$. Surviving entrepreneurs accumulate wealth. We assume that the wealth belonging to defaulting entrepreneurs is instead consumed by existing ones. Thus, the consumption level of surviving entrepreneurs is:

$$C_t^e = (1 - \varsigma_t) [1 - \Gamma(\tilde{\omega}_t)] \frac{R_t^k Q_{t-1}}{\pi_t} K_{t-1}$$

Aggregate wealth can be written as

$$nw_t = \varsigma_t R_t^k \frac{Q_{t-1}}{\pi_t} K_{t-1} - \frac{\varsigma_t}{\pi_t} \left[R_{t-1} + \frac{\mu G(\tilde{\omega}_t) R_t^k Q_{t-1} K_{t-1}}{(Q_{t-1} K_{t-1} - nw_{t-1})} \right] (Q_{t-1} K_{t-1} - nw_{t-1}) \quad (12)$$

with $\frac{\mu G(\tilde{\omega}_t) R_t^k Q_{t-1} K_{t-1}}{(Q_{t-1} K_{t-1} - nw_{t-1})}$ the risk premium factor.

2.4 Final good production

As in Bernanke et al. (1999), retailers aggregate wholesale goods to the purpose of producing final ones. They operate in a monopolistic competition framework and price setting is affected by nominal rigidities à la Rotemberg. Retailers' optimization problem entails the following Phillips curve:

$$(\pi_{Ht} - 1) \pi_{Ht} = Y_t \frac{v}{\omega_P} \left[\frac{1}{X_t} - \frac{(v-1)}{v} \right] + \beta E_t \left[\frac{U'_{ct+1}}{U'_{ct}} (\pi_{Ht+1} - 1) \frac{f_{t+1}}{f_t} \pi_{Ht+1} \right] \quad (13)$$

where π_H denotes producer price inflation in country H, v denotes the elasticity of substitutions between domestic varieties, and ω_P is the Rotemberg parameter of price rigidity.

Analogously, country F retailers' problem entails the following Phillips curve:

$$(\pi_{Ft}^* - 1) \pi_{Ft}^* = Y_t^* \frac{v}{\omega_P} \left[\frac{(1-v)}{v} + \frac{1}{X_t^*} \right] + \beta \omega_P E_t \left[\frac{U'^*_{ct+1}}{U'^*_{ct}} (\pi_{Ft+1}^* - 1) \frac{f_{t+1}^*}{f_t^*} \pi_{Ft+1}^* \right]$$

where π_F^* denotes producer price inflation in country F .

2.5 Monetary policy

We suppose that in each country the monetary policy follows empirical Taylor rules. Therefore, the monetary rule in country H is:

$$R_t = (R_{t-1})^\chi \left(\bar{R}^n \left(\frac{\pi_t}{\bar{\pi}} \right)^{b_\pi} \left(\frac{y_t}{\bar{y}} \right)^{b_y} \right)^{1-\chi} mp_t \quad (14)$$

In country F ,

$$R_t^{*n} = (R_{t-1}^{*n})^{\chi^*} \left(\bar{R}^{*n} \left(\frac{\pi_t^*}{\bar{\pi}^*} \right)^{b_\pi^*} \left(\frac{y_t^*}{\bar{y}^*} \right)^{b_y^*} \right)^{1-\chi} mp_t^* \quad (15)$$

with a mp_t and mp_t^* temporary monetary policy shocks, such that:

$$\begin{aligned}\log mp_t &= \rho_{mp} \log mp_{t-1} + \varepsilon_t^{mp} \\ \log mp_t^* &= \rho_{mp} \log mp_{t-1}^* + \varepsilon_t^{mp}\end{aligned}$$

2.6 Calibration

Each period corresponds to one quarter. The calibration of this model is mostly based on the works of Christiano et al. (2014) (hereafter, CMR) and Kolasa & Lombardo (2014) (hereafter, KL). We assume that the Home country is the Euro area and the Foreign country refers to the US. Table 1 summarizes the calibration.

Preferences: We let the instantaneous utility function be $U_t = \frac{C_t^{1-\sigma}}{1-\sigma} + \Psi \log(1 - N_t)$. The intertemporal elasticity of substitution for consumption is set in both countries equal to 2, consistently with the literature. The disutility of labor parameter is set in both countries equal to 2.6 so as to insure that labor is normalized at 1/3 at the steady state. The discount factor is in both country equal to 1/1.01147, consistently with an annual interest rate as in CMR. The share of foreign goods into the domestic basket, γ , is equal to 0.4, as in KL and the elasticity of substitution between foreign vs domestic goods is 1.5 as in Faia (2007b). The elasticity of substitution among varieties v is set equal to 6 as in CMR among others.

Production: The wholesale production function is a Cobb-Douglas, $Y_t = a_t K_t^\alpha N_t^{1-\alpha}$ where α is set to 0.36 and the capital depreciation rate is 0.025 as in CMR among others. The capital adjustment costs parameter Φ is set to 5.2 in both countries as in CMR. The Rotemberg parameters are calculated both for the EU and the US as in Monacelli (2009) starting from CMR estimates of the Calvo parameters in the EU and the US (around 0.7 and 0.6, respectively).

Financial parameters: Given our limited number of financial shocks, the monitoring cost parameter, μ is set to the same level in both countries. We let $\mu = 0.21$ in both countries, based on CMR. The interest rate premium parameter, $\zeta = 0.000742$ as in Schmitt-Grohe & Uribe (2003). We set the share of surviving entrepreneurs $\varsigma = 0.978$, consistently with Christiano, Rostagno & Motto (2010).

Monetary policy: The weight on inflation and output in Taylor rules are set as in KL. We let $b_\pi = 2$ and $b_y = 0.15$. In the Foreign country, the same parameters are fixed consistently with the estimates of CMR on US data. We let $b_\pi^* = 2.6$ and $b_y^* = 0.36$. We let $\chi = 0.6$ in both countries, which is a little lower than what is currently used in the above mentioned literature but is consistent with the interval considered for estimation in Ascari et al. (2011).

Shocks : All shocks are log-normal AR(1). The persistence parameter of the productivity shock is set in both countries to 0.85, while the standard deviation of ε_t^{as} is set to 0.024 as in KL. At the steady state $a = 1$ in both countries. The persistence of the riskiness shock is set to 0.97, as in CMR and its standard deviation to 0.04, consistently with KL and CMR. We follow KL to set the persistence of the wealth shock (mortality shock) to 0.5 and its standard deviation to 0.012. Finally, the standard deviation of the monetary shock is set to 0.001, as in KL.

Table 1: Calibration

β	discount factor	0.9887	CMR (2014)
σ_c	elasticity of intertemporal substitution	2	KL (2014)
γ	share of foreign goods in domestic basket	0.4	KL (2014)
η	elasticity of substitution between home and foreign goods	1.5	Faia (2007b)
ν	elasticity of substitution between varieties	6	CMR (2014)
α	production function	0.36	CMR (2014)
δ	capital depreciation	0.25	CMR (2014)
ϕ	capital adjustment costs	5.2	CMR (2014)
ω_p	Rotemberg parameter	41.75	CMR (2014), Monacelli (2009)
ω_{p^*}	Rotemberg parameter	30	CMR (2014), Monacelli (2009)
μ	monitoring cost	0.21	CMR (2014)
ζ	int.rate premium	0.000742	SGU (2003)
b^*	s.s. NFA	0	SGU (2003)
ζ	share of surviving entrepreneurs	0.978	KL (2014)
χ	weight on lagged int. rate into TR	0.6	ACR (2011)
χ^*	weight on lagged int. rate into TR	0.6	ACR (2011)
b_π	weight on inflation into TR	2	KL (2014)
b_{π^*}	weight on inflation into TR	2.6	KL (2014), CMR (2014)
b_y	weight on output gap into TR	0.15	KL (2014)
b_{y^*}	weight on output gap into TR	0.36	KL (2014), CMR (2014)
ρ_a	persistence techno shock	0.85	KL (2014)
σ_a	sd techno shock	0.024	KL (2014)
ρ_σ	persistence risk shock	0.97	KL (2014), CMR (2014)
σ_σ	sd risk shock	0.04	KL (2014), CMR (2014)
ρ_ζ	persistence wealth shock	0.5	KL (2014)
σ_ζ	sd wealth shock	0.012	KL (2014)
σ_{mp}	sd monetary shock	0.001	KL (2014)
Ψ	disutility of labor	2.6	Labor normalized at 1/3 at ss

CMR(2014) refer to Christiano et al. (2014), KL(2014) to Kolasa & Lombardo (2014), SGU (2003) to Schmitt-Grohe & Uribe (2003), ACR (2011) to Ascari et al. (2011).

3 Learning

3.1 Adaptive learning

Under rational expectations, all agents in both countries know about the structure of all shocks as well as the state variables in both countries. As a result, when agents forecast an economic

variable, whether Home or Foreign, they use all worldwide available information. As expectations are rational, the perceived law of motion, used for forecasting purposes, is the actual law of motion observed in the economy. In our paper, private agents engage in adaptative learning. As mentioned by Sims (1980), there are many ways of modeling non-rational behavior. In this paper, we adopt the standard view put forward by Evans & Honkapohja (2001). This provides a useful starting point to compare our work to the literature. The model is approximated at order 1, as in Evans & Honkapohja (2001). The reduced form is then

$$k_t = a_1 E_t k_{t+1} + a_2 k_{t-1} + b_1 z_t + b_2 z_{t-1} \quad (16)$$

$$z_t = \rho z_{t-1} + \varepsilon_t \quad (17)$$

with z_t the vector of shocks and k_t a vector of all endogenous variables in the model. Notice that agents know all shocks in the economy, as in Evans & Honkapohja (2001). In addition, only 1-step ahead forecasts matter for the current economic decisions.

Private agents have beliefs on the evolution of macroeconomic variables in the economy, based on their Perceived Law of Motion (PLM):

$$k_t = \phi_{k,t-2} x_{t-1} + \phi_{z,t-2} z_{t-1} \quad (18)$$

Private agents think that endogenous variables k_t are a function of a set of observed variables x_{t-1} and shocks z_{t-1} . Private agents use the PLM to forecast economic variables

$$E_t k_{t+1} = \phi_{k,t-1} k_t + \phi_{z,t-1} z_t \quad (19)$$

The actual evolution of macroeconomic variables in the economy is obtained by replacing the expected value from equation (19) into the reduced form (equation (16)). It is thus necessary to define:

- i*). The set of observed variables x included in the PLM (equation (18)).
- ii*). The methodology used to obtain time-varying coefficients ϕ in the PLM.
- iii*). The methodology defined in *ii*). will rely on a recursive loop. Define the initialization of ϕ .

Many choices can be made at this stage. Obviously, any of these choices affect the macroeconomic behavior of the economy. We describe below the rationale behind each of our choices.

3.2 Modelling choices

i). **Information set used for adaptative learning: Home information bias** Under rational expectations, all agents from both countries observe all economic variables in the world. They

use this wide information set in their PLM and forecasting model. In contrast, under learning, agents have an imperfect knowledge of their economic environment. They then use a reduced information set when forming their expectations ⁵. The literature on learning shows that the dynamics of macroeconomic variables are affected by the choice of whether private agents use a smaller forecasting model. When learning is based on the same set of state variables as under rational expectations, amplification and propagation mechanisms are small in models where only one-period-ahead forecasts matter for current decisions (Eusepi & Preston (2011)). The literature points out that small forecasting models improve the model’s fit to the data (Slobodyan & Wouters (2012a), Ormeno & Molnar (2015), Hommes et al. (2015)). We then depart from rational expectations by assuming also that private agents do not consider the full information set. In particular, we assume Home information bias: in each country, private agents base their PLM on local state variables only. This assumption is consistent with empirical evidence based on SPF and ECB SPF data. This evidence suggests that changes in US inflation expectations are mainly driven by US data and, symmetrically, changes in EA inflation expectations are mainly driven by EA data ⁶. Moreover:

- Endogenous state variables in each country are: net worth nw , capital k , nominal interest rate R , price of capital q .
- Home private agents, whether households or entrepreneurs, use Home endogenous state variables only in their perceived law of motion, together with international bonds b^* and terms of trade.
- Symmetrically, Foreign private agents, whether households or entrepreneurs, use Foreign variables only in their perceived law of motion, together with international bonds b^* and terms of trade.

ii). Learning method. In this work, we assume that agents update their belief using a stochastic-gradient constant-gain (using the semantics in Carceles-Poveda & Giannitsarou (2007)).

$$\phi_t = \phi_{t-1} + gain \times x_{t-1} (k_t - x'_{t-1} \phi_{t-1}) \quad (20)$$

Private agents form adaptative expectations: after observing the state of the economy, they correct their previous estimate of ϕ using their forecast error. Under adaptive learning, individuals behave much like econometricians, using new observations on macroeconomic conditions to update their

⁵Unlike Orphanides & Williams (2007), central banks have access to all information. We discard any learning behavior from central banks and leave this point for future research.

⁶This claim is based on the following empirical exercise. We use SPF Survey data of inflation expectations in the US and the EA. We also use observed data from US FRED Economic database and AWM model for the Euro Area. We then regress changes in US expected inflation on past changes in US inflation and GDP, as well as on Euro Area data on a sample spanning 1970Q1-2016Q1. Changes in US expectations are mainly driven by US data. EA variables appear non significant or insignificant with a much lower estimated coefficient than the ones for US data. We perform the symmetric exercise for Euro Area data on a sample spanning 1999Q1-2016Q1: analogously, changes in EA expectations are mainly driven by EA data. See Appendix B.3.1 for a description of the data.

estimates of key economic relationships. Private agents do not cease to update coefficients in their PLM, based on their forecast errors. They engage in real-time perpetual learning.⁷

In the literature, private-agent updating parameters *gain* lie between 0.01 and 0.05 (Orphanides & Williams (2005), W.A. & Evans (2006), Milani (2007), Slobodyan & Wouters (2012b)). $1/\textit{gain}$ can be interpreted as an indication of how many past observations agents take into account to form their expectations. With $\textit{gain} = 0.03$ ($\textit{gain} = 0.01$, respectively), private-agents approximately use 33 quarters or 8 years of data (25 years of data or 100 quarters, respectively). We choose $\textit{gain} = 0.03$ as a benchmark value, which is close to the the values chosen in the papers on monetary policy under learning ($\textit{gain} = 0.02$ in Orphanides & Williams (2008), $\textit{gain} = 0.03$ in Gaspar et al. (2011)). In addition, this value also lie in the range estimated by Milani (2007) using Bayesian techniques and Orphanides & Williams (2005) who exploit data on expectations from the Survey of Professional Forecasters.

iii). Initialization of the learning loop Equation (20) is based on a recursive specification, which requires initialization (ϕ_0). Initialization is a crucial step as errors on initial perception take a long time to die out (Carceles-Poveda & Giannitsarou (2007), Slobodyan & Wouters (2012b)). In our model, when PLM are initialized below the values of rational expectations ($\phi_{learning}^0 < \phi_{RE}^0$), macroeconomic dynamics tend to remain below the ones observed under rational expectations. The reverse is also true.⁸ In order to pin down initial beliefs, we draw them in a normal distribution.⁹ Moreover, we choose the mode of the distribution, so that the model under learning replicates the output response to a technological shock, as identified in a structural VAR on US data (Christiano, Trabandt & Walentin (2010), Gali & Rabanal (2004)). We choose this empirical evidence as a benchmark because of the abundant and consensual empirical literature on the impact of productivity shocks on output. Also, when looking at variance decomposition, output fluctuations are mainly driven by local technological shocks. This also follows the spirit of Slobodyan & Wouters (2012b) who optimize initial beliefs to maximize the in-sample fit of the model with learning.

Appendix B describes the learning algorithm. We make sure that the resulting business cycle properties of forecast errors are consistent with the ones computed on expectation survey data (US and ECB Survey of Profesional Forecasters). In particular, the model under learning correctly predicts the persistence of 1-quarter ahead forecast errors on inflation, persistence and cyclicity

⁷We do not use learning based on recursive least square, or learning that use the inverse of the variance-covariance matrix of x in the *gain* parameter. Indeed, this introduces a technical difficulty as the variance-covariance matrix must be of full rank, which is not the case if some variables of x are not linearly independent. This is, for instance, the case for nominal interest rates, terms of trade and external debt, through the UIP. This restricts the information set available to private agents in each country. Circumventing this difficulty is left for future research.

⁸This phenomenon was also remarked by Carceles-Poveda & Giannitsarou (2007).

⁹In practice, initial beliefs cannot be too far away from rational expectations. Beliefs can actually explode if initialized too far away from the rational expectations. Projection facility can discipline exploding beliefs. However, the results would then be driven by exploding beliefs. We check that, given our initial beliefs, the actual law of motion is stable. We check that this is also the case along the simulations. The projection facility disciplines exploding beliefs, but is rarely triggered.

of nominal interest rate (See Appendix B.3). Under rational expectations, these stylized facts on forecasts errors cannot be matched: all forecast error display no persistence at all (by definition of rational expectations) and no cyclicalities.

4 Inspecting the economic mechanisms

4.1 Impulse Response Functions

Financial frictions in interdependent economies under learning. In this section, we use an impulse response analysis (IRFs) to investigate transmission mechanism. Figures here below display the response to one-standard-deviation shock under rational expectations (RE). Under adaptive learning (AL), IRFs are obtained through simulations: repeated random draws of innovation ϵ from the calibrated distribution. Indeed, as can be seen from the learning algorithm in Appendix B, IRFs depend on the state of the economy (the value of the shock) and the state of beliefs in the perceived law of motion at the time of the shock. We summarize the outcomes by displaying in IRFs under learning the 80% range of simulated IRFs as well as its median value. We checked that the median value of the shock under learning is the same as under rational expectations. Under rational expectations, in a model approximated at order 1, IRFs do not depend on the state of the economy. The model is very rich. IRFs illustrate

- a).* The effects of financial frictions
- b).* in interdependent economies with neo-keynesian features
- c).* both under rational expectations and learning

In what follows, we analyse the transmission mechanisms of our economy in response to the aggregate stochastic processes specific to our model. *a).* was investigated by Bernanke et al. (1999), and more recently by Christiano et al. (2011) in a small open economy setting. Our IRFs for the local economy, after a local shock, are similar to the ones displayed in Christiano et al. (2011). *b).* relates to the international transmission of exogenous shocks in interdependent economies with financial frictions, which was analyzed by Kolasa & Lombardo (2014), Faia (2007*a*) and Faia (2007*b*) among others. The IRFs are consistent with theirs. Our contribution lies in *c).* As a result, in our comments below, we will quickly summarize the economic mechanisms under rational expectations in open-economy (*a).* and *b).*) before stressing the different macroeconomic dynamics under learning. In addition, for the sake of brevity, we describe below the effects of Home shocks, as the effects of Foreign shocks are nearly symmetric.¹⁰

¹⁰In the benchmark calibration, countries are asymmetric with respect to their price rigidity and Taylor rules

Lessons from the analysis of IRFs The lessons from the analysis of IRFs can be summarized as follows:

- Local propagation of domestic shock :
 - Under rational expectations, as well as under learning, for all shocks, the large weight on price stability in the Taylor rule makes the nominal interest rate responsive to all shocks. This endogenous monetary policy response affects the cost of loans, which tends to amplify the initial effects of the shocks.
 - Under learning, the economy is more responsive to shocks. Macroeconomic variables display amplified dynamics under learning with respect to rational expectations, as in Rychalovska et al. (2016). Mis-perceptions about future asset prices and returns on capital results in current changes in capital demand, investment and net worth, thereby amplifying the effects of the financial accelerator. The strong interaction between financial frictions and forecast errors generates larger responses of the economy under learning to exogenous shocks, whatever the nature of the shock.
- International propagation of domestic shock :
 - Under rational expectations, as well as under learning, one of the main drivers of the international propagation of shocks lies in the uncovered interest rate parity (UIP). This arbitrage condition is a force that tends to align domestic and foreign nominal interest rates, which is a key determinant of the cost of loans. With convergent interest rates, credit conditions tend to be similar across countries, which tends to generate positive co-movement of output across countries, as in Faia (2007a).
 - Under learning, the international propagation of shocks is more asymmetric. Since agents only take into account their local state variables in their forecasting model, private agents in the other country do not properly perceive the UIP, thereby missing a crucial propagation mechanism. This leads to more asymmetric international spillover.
- The behavior of nominal interest rate plays a key role in the macroeconomic dynamics in each economy. The volatility and speed of adjustment are affected by the responsiveness of monetary policy to changes in output and prices. Coordination of monetary policy is then a relevant issue to investigate.
- the interaction between the frictions in the economy and learning is key in understanding the impact of learning on macroeconomic dynamics. We check that, in absence monitoring costs $\mu = 0$ and under flexible prices, IRFs under AL are not significantly different from the ones as under RE, for the domestic country after a domestic shock.

4.2 Home Positive technological shock

Figures 1-5 display the impulse response functions following a positive Home technological shock.

4.2.1 Rational expectations

Effects in the Home country. The productivity shock has a positive impact on output (Fig. 1, (a)), capital accumulation (Fig. 2, (b)) and investment. Moreover, as expected, it entails a downward pressure on domestic prices and inflation (Fig. 2, (f)) that dampens in turn domestic interest rates through the Taylor rule (Fig. 3, (a)). As expected, the (real) price of capital jumps up (Fig. 2, (a)) and entrepreneurs accumulate more net wealth (Fig. 2, (c)). The financial accelerator is at work.

Terms of trade also deteriorate (Fig. 3, (c)) while the nominal exchange rate appreciates. The reaction of (nominal) spreads and the external finance premium (hereafter EFP) depends on the degree of price stickiness. If prices are flexible, the interaction of relative price dynamics trigger a decrease in spreads. This further stimulates investment. If prices are sticky, as in our calibration, nominal spreads increase, consistently with Christiano et al. (2011).

Thanks to the increased productivity, the price of wholesale production decreases so that retailers's marginal costs decrease and their profits increase. As households receive profits, they consume more (Fig. 2, (d)) and work less. They also invest in international bonds (Fig. 3, (b)), so that the net external position improves together with the trade balance.

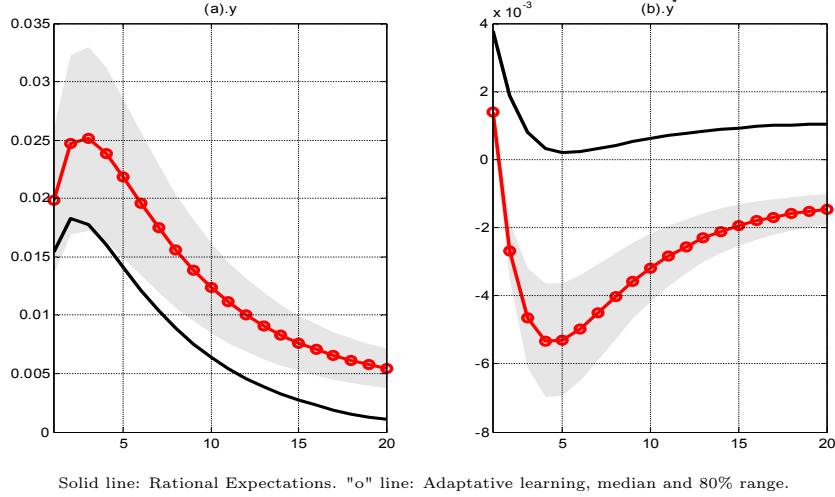
International spillovers. The shock is transmitted through the interest parity condition. Because of a lower international rate and the dynamics of the net external position, the interest rate abroad decreases (Fig. 3, (d)), i.e.:

$$E_t [R_t] = E_t \left[R_t^F \frac{e_{t+1}}{e_t} \right]$$

where $R_t^F = R_t^* + p(-b_t^*)$. A lower interest rate also stimulates current Foreign consumption (Fig. 4, (d)) so that Foreign production abroad becomes more expensive. However, as imported goods from Home are cheaper (Foreign terms of trade improve), the aggregate inflation index in the Foreign country decreases (Fig. 3,(e)). Finally, because of the lower external finance premium, entrepreneurs abroad invest and produce more (Fig. 4, (b)), further stimulating Foreign output (Fig. 1, (b)). This can be clearly seen by substituting the interest rate of each country into the UIP by the definition of the EPF and the return of capital, in the spirit of Kolasa & Lombardo (2014). Indeed, the UIP transmits all disturbances hitting the interest rate of one country to foreign capital markets. Under RE, international business cycle co-move positively. This is consistent with the findings in Faia (2007a) who stress that the financial spillover effect solves the comovement puzzle in international

macroeconomics.

Figure 1: Output responses to a positive Home technological shock



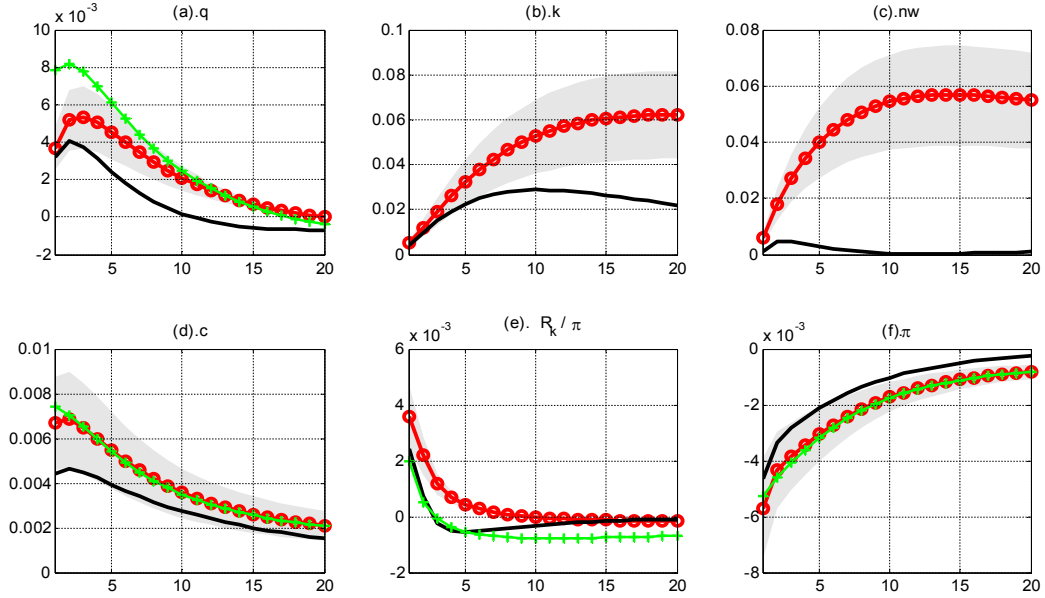
4.2.2 Adaptive learning

Learning affects the macroeconomic dynamics through expectation terms in equations determining agents' intertemporal decisions. The forecasts of future inflation, consumption, entrepreneurs' productivity threshold, asset prices and real return on investment affect current behaviors: household's savings, entrepreneurs' investment and lending decisions, as well as price dynamics.

We report on IRFs the expected value $E_t[x_{t+1}]$ of forward variables that affect current behavior ($x = q, \frac{R_k}{\pi}, c, \pi, \tilde{\omega}$). We interpret an overly pessimistic or optimistic mis-perception of variable x whenever the expected value (the forecast $E_t[x_{t+1}]$ using the *Perceived* Law of Motion) significantly departs from the realized value under learning (*Actual* Law of Motion). Moreover, for the sake of clarity, we report in a separate Figure (Figure 5) the forecasts under RE and under AL, to illustrate the gap in expectations under both scenarios.

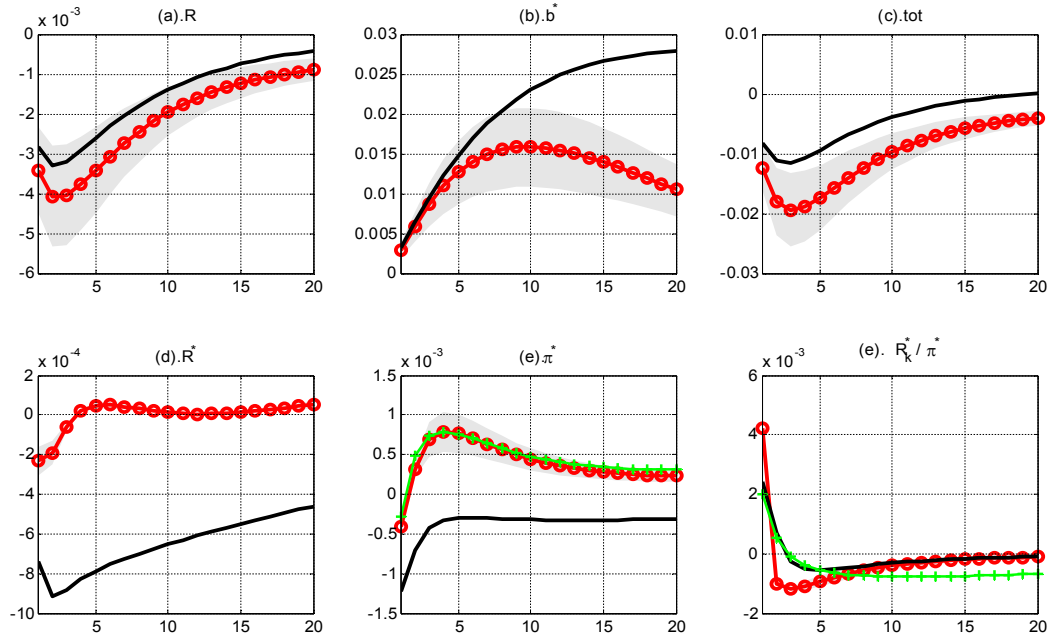
Effects in the Home country. Following the Home productivity shock, the price of capital increases as in the standard Bernanke et al. (1999) model. However, as Home agents' initial beliefs are over-optimistic about the effects of the technological shock, the initial increase in the price of capital is stronger than under RE (Fig. 2, (a)). This triggers a more pronounced dynamics of the financial accelerator mechanism through net worth and investment (Fig. 2, (b), (c)). This results in a stronger increase in output and, at the same time, a stronger decrease in inflation (2, (f)) driven by a greater expansion of production (Fig. 1, (a)). Households increase their consumption more than before (Fif. 2, (d)) thanks to greater profits. As inflation drops more than under RE (Fig. 2, (f)), the monetary reaction is even more accomodative (Fig. 3, (a)). By lowering the cost of loans, the expansionary monetary policy also fuels the expansion of credit, investment and output.

Figure 2: IRFs to a positive Home technological shock



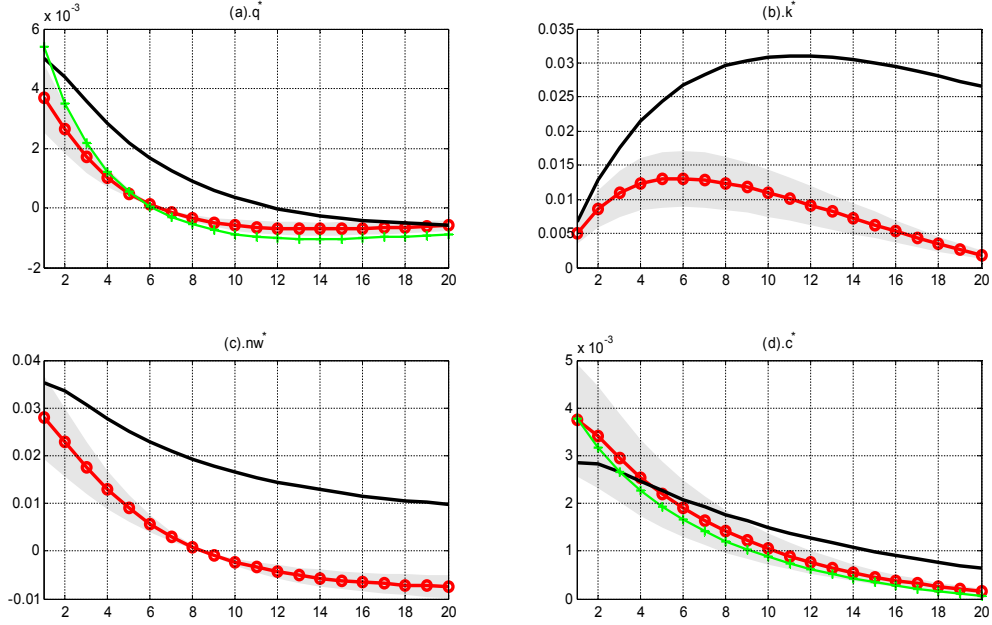
Solid line: Rational Expectations. "o" line: Adaptive learning, median and 80% range. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of variable x based on PLM, median value. Only for forward-looking variables.

Figure 3: IRFs to a positive Home technological shock



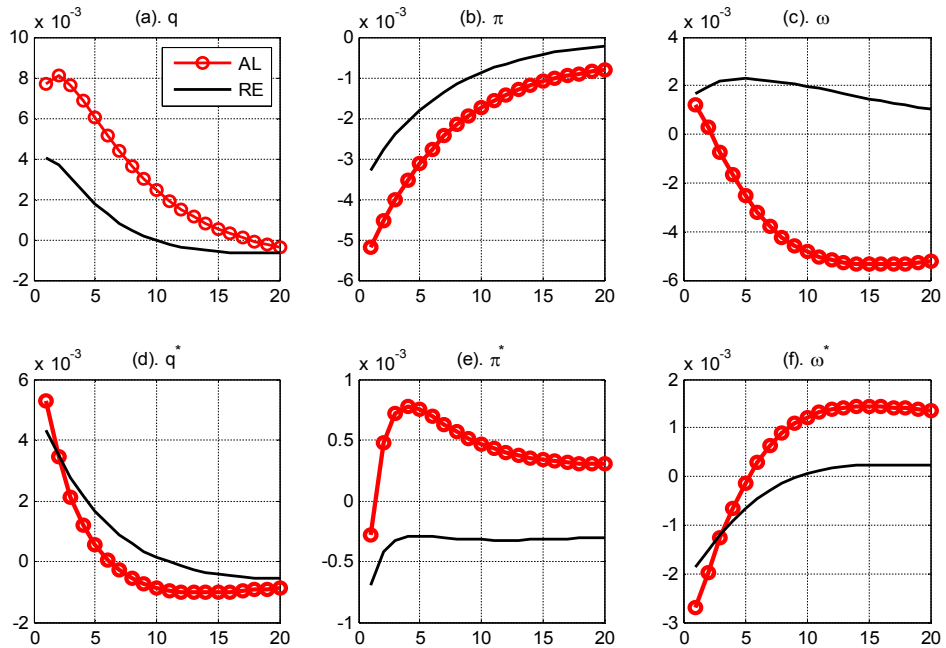
Solid line: Rational Expectations. "o" line: Adaptive learning, median and 80% range. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of variable x based on PLM, median value. Only for forward-looking variables.

Figure 4: IRFs to a positive Home technological shock



Solid line: Rational Expectations. "o" line: Adaptive learning, median and 80% range. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of variable x based on PLM, median value. Only for forward-looking variables.

Figure 5: IRFs to a positive Home technological shock. 1-quarter ahead forecast $E_t[x_{t+1}]$ forecast of variable x based on PLM.



Solid line: Rational Expectations. "o" line: Adaptive learning, median value.

International spillovers. In response to the shock, because of greater returns, H agents' investment arbitrage favour physical capital rather than international bonds (Fig. 4, (d)). Differently from the RE case in the previous section, foreign agents do not internalize the positive spillover stemming from the UIP and cannot track its effects on Foreign financial markets. Indeed, under learning, the Foreign nominal interest rate does not fall as much as its Home counterpart Fig. 3, (d)). Therefore, the fall of the cost of loans is smaller than under rational expectations. The consequence is a smaller increase in net worth, capital (Fig. 4, (b), (c)) and output in the Foreign country (Fig.1, (b)).¹¹

For the sake of brevity, the IRFs of the other shocks are displayed in Appendix C.

5 Monetary policy under learning in a globalized world

5.1 Optimal Simple Rules

The above analysis of transmission mechanisms has shown the importance of interest rate dynamics for the propagation of shocks both throughout each separate financial market and among countries. Given the asymmetric response of each country's economy to shocks, monetary cooperation may introduce a stabilization mechanism through the role of expectations. In what follows we reassess the role of monetary policy coordination (in the spirit of Obstfeld & Rogoff (2002)) in a world characterized by financial frictions and adaptive learning. To this purpose, we compare the macroeconomic outcomes when countries coordinate to maximize a joint criterion versus a case in which the monetary policy of each country follows an historical independent Taylor rule. Because of adaptive learning à la Evans & Honkapohja (2001), the model is approximated at order 1. Therefore, it is not possible to develop an analysis based on standard welfare measures (Schmitt-Grohe & Uribe (2004)).¹² Following the literature on monetary policies with adaptive learning (see Orphanides & Williams (2008) and Gaspar et al. (2011)), our welfare criterium will rely on minimizing a loss function.

No coordination. We consider the benchmark calibration where the coefficients of the Taylor rule in each country is based on values estimated on US and Euro Area data. Chairs of central banks across the Atlantic Ocean do not meet before choosing their respective monetary policies.

¹¹The UIP is a key element to track international spillovers. In order to illustrate its importance, we perform an experiment in which private agents take into account the other country's nominal interest rate in their perceived law of motion. The information set used in the agents' forecasting model is expanded to include the other country's nominal interest rate. The IRFs under learning, in the Foreign country, get closer to the IRFs under rational expectations. IRFs are reported in Appendix D. One might suggest that this is trivial as the information set gets closer to the one used under RE. Let us stress that, under rational expectations, each agent information set includes all state variables in all countries. Even in the case of expanded information set under learning, with only the addition of the other country's interest rate in the PLM, the gap between the information sets under learning and under RE remains large. Moreover, the inclusion of a different state variable does not entail as close IRFs as it is the case for the interest rate

¹²Indeed, as shown by Kim & Kim (2003), this could give rise to spurious results: rigorous welfare analysis shall be based on models approximated at order 2, at the very least.

We then suppose that there is currently little coordination in monetary policies in the US and the EA. The estimated coefficients of the Taylor rule reflect this lack of coordination.

Coordination. In the spirit of the literature on international coordination, joint-policy making refers to policy design in both countries under the maximization of a joint criterion. We follow the literature on learning and minimize the following standard loss function:

$$\mathcal{L} = \text{var}(y) + \text{var}(y^*) + \text{var}(\pi) + \text{var}(\pi^*) \quad (21)$$

where $\text{var}(\cdot)$ denotes the unconditional variance. The monetary policy instruments are the nominal interest rates in both countries. Central banks coordinate to stabilize both economies, i.e. minimize a loss \mathcal{L} equal to the sum of unconditional variances of output and CPI inflation. In doing so, we extend to a 2-country setting the papers on monetary policy under learning such as Orphanides & Williams (2008) and Gaspar et al. (2011). As in Orphanides & Williams (2008), the loss function puts equal weights on output and inflation¹³. Based on a standard New Keynesian model, they conclude that learning requires the policy maker to be more conservative (i.e., give more weight to inflation into the loss function), with respect to the rational expectations case. Indeed, as inflation is the forward-looking variable of their model, it is important to anchor agents' expectations on it. In our work, financial frictions interact with learning. As showed in the above analysis, the interplay of these features significantly amplifies the dynamics of the model. More in particular, financial frictions introduce additional forward looking variables triggering the financial accelerator (so as to affect output). By giving equal weight to both inflation and output (as in the benchmark calibration of Orphanides & Williams (2008)), we focus our attention on both the mechanisms associated to i) the New Keynesian features and ii) the financial imperfections of our model.

In addition, as countries are of equal size, the loss function displays equal weights on each country's variables. Notice in particular that the loss function includes CPI inflation of each economy. Therefore, policy makers indirectly coordinate on terms of trade. We remind the reader that terms of trade do belong to the learning information set of both domestic and foreign private agents, respectively. Therefore, even central banks under coordination do not *per se* enlarge the information set of agents in both countries, they maximize a common objective including common variables. This pins down terms of trade. Coordinated terms-of-trade dynamics affect in turn the information set of domestic and foreign agents, respectively, together with their expectations. In this sense, coordination can be interpreted as a mean to share information. In practice, joint-policy making refers to searching for the coefficients of both Taylor rules (3 coefficients in each country), that minimize \mathcal{L} . The optimization algorithm allows for asymmetric Taylor rules because countries are not fully symmetric (different degrees of price stickiness).

¹³With a model approximated at order 1, it is not possible to derive the relative weights on output and inflation stabilization from the structural parameters, as Woodford (2003).

5.2 Results

Table 2 displays the quantitative results. Column (1) displays the numbers for the benchmark calibration, interpreted as the "non-coordination" case. Columns (2) and (3) display the optimized Taylor coefficients under RE (column (2)) and under AL (column (3)). First, we remark that the loss under learning is always greater than under rational expectations. This is consistent with the fact that under learning macroeconomic dynamics are more volatile than under rational expectations. Monetary policy is not able to reduce the volatility gap between AL and RE.

Table 2: OSR results

	No coordination (1)	OSR RE (2)	OSR AL (3)
Parameters in Taylor rule			
χ persistence	0.60	0.74	0.59
χ^* persistence	0.60	0.75	0.58
b_π	2.00	1.89	1.98
b_{π^*}	2.60	1.90	1.97
b_y	0.15	0.77	1.33
b_{y^*}	0.36	0.71	1.55
Weights in Taylor rule			
$(1 - \chi)b_\pi$	0.80	0.49	0.81
$(1 - \chi)b_y$	0.06	0.20	0.54
$(1 - \chi^*)b_{\pi^*}$	1.04	0.48	0.82
$(1 - \chi^*)b_{y^*}$	0.14	0.18	0.65
Loss function			
\mathcal{L}	0.0017 (RE)	0.0013	
	0.0030 (AL)		0.0021

5.2.1 Understanding OSR under rational expectations.

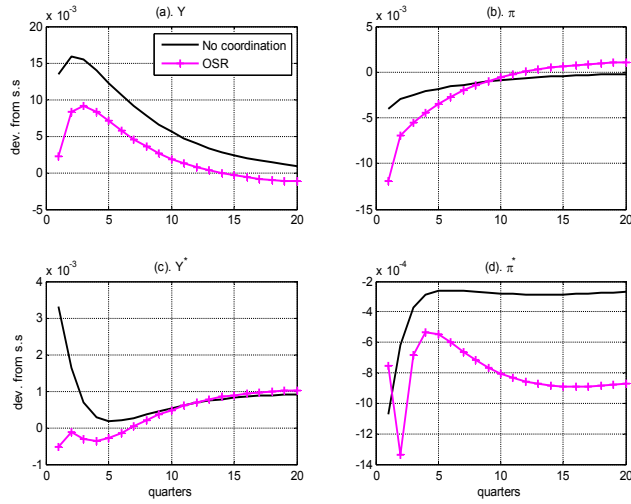
Let us first have a look at columns (1) and (2) under RE. The OSR procedure leads to nearly symmetric coefficients across countries, suggesting that the calibrated difference in price rigidities is not large enough to justify asymmetric monetary policies. Secondly, under OSR, when central bankers are eager to stabilize output and price volatility, the response of interest rates becomes more persistent and the weight on output increases. Pure inflation targeting is not consistent with the OSR outcomes, as weight on output gaps in the Taylor Rules are different from zero. This is not surprising. Indeed, as we showed in the analysis of transmission mechanisms, financial frictions introduce a trade-off between pure price stabilization and the stabilization of the EFP and capital (see also Kolasa & Lombardo (2014)).

We now focus on transmission mechanisms in the cooperative (OSR) case as opposed to the non-cooperative case. Figure 6 displays the IRFs after a Home positive technological shock.¹⁴ As

¹⁴Indeed, the productivity shock allows to resume the main mechanisms making OSR different from the non-

expected, domestic output under OSR is less volatile: the Home central banker lets domestic CPI inflation go down further so as to increase the external financial premium at Home and dampen capital fluctuations. Inflation falls more so that the real value of debt goes up, which tends to dampen the effects of the financial accelerator (Fisher effect). Therefore, Home central bank achieves better stabilization of domestic output but lets CPI inflation be more volatile. This result illustrates the presence of an inflation-output variability tradeoff.

Figure 6: IRFs to an increase in Home technological shock under RE, no coordination case versus OSR

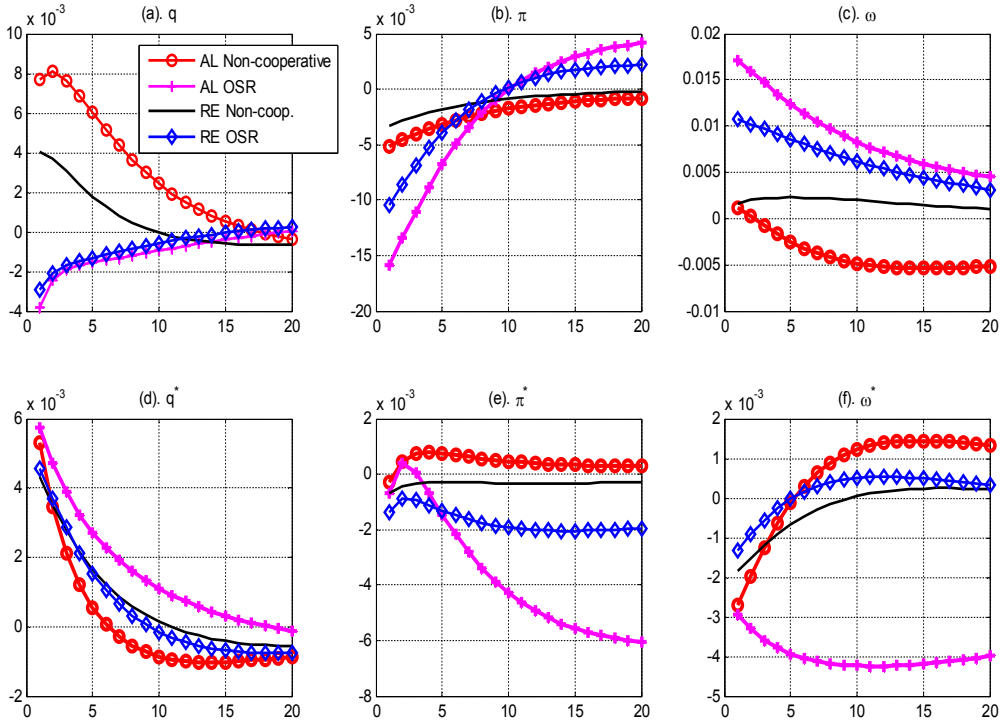


Analogously, the Foreign central banker allows Foreign inflation to be more negative and during more periods (Foreign inflation fluctuates more than in the benchmark calibration), which dampens the financial accelerator in the Foreign economy. The financial spillover effect generating a positive international co-movement in the benchmark calibration is dampened by OSR. Simulated moments in the Foreign country show that the fall in Foreign output volatility is the main driver of stabilization in country F.

Figure 7 reports private agents' expectations, based on a 1-period ahead projection of their perceived law of motions. Under rational expectations, forecasts are based on a PLM with fixed coefficients (it is not subject to sample variations). In contrast, under learning, forecasts depend on the realization of shocks. For the sake of clarity, we report only the mean value obtained with the simulated data under AL. Figure 7 illustrates how central banks succeed in anchoring private agents' expectations. In particular, under OSR with RE, domestic private agents' forecasts are consistent with tighter credit conditions (Figure 7, panels (a), (b), (c), lower price of capital, greater deflation, higher productivity threshold). In the Foreign country, expectations under OSR are driven by the stronger deflation, which dampens in turn the financial accelerator.

cooperative case

Figure 7: IRFs to an increase in Home technological shock, no coordination case versus OSR. 1-quarter ahead forecast of variable $x : E_t[x_{t+1}]$



Solid line: Rational Expectations. Non-cooperative case. Diamond: Rational Expectations. OSR. "o" line: Adaptive learning, median. Non-cooperative case. "+" line: Adaptive learning, OSR, median value.

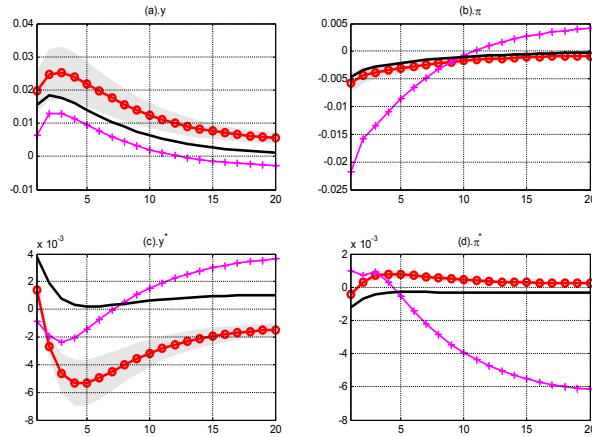
5.2.2 Understanding OSR under adaptive learning.

Columns (2) and (3) in Table 2 suggest that, under AL, central banks are more responsive to fluctuations in inflation and output (Taylor rules are characterized by larger weights on inflation and output and lower persistence on interest rates). We examine the IRFs under AL after a positive Home technological shock (Figure 8, one-standard deviation shock). The analysis of panels (a) and (b) in Figures 8 and 6 suggest that, the Home central bank pursues the same strategy as under RE: inflation is allowed to go down further so as to dampen the magnitude of the financial accelerator. This stabilizes output fluctuations. Simulated moments confirm that, under OSR with learning, fluctuations in output are minimized. This stabilizing effect is stronger than the increase in inflation volatility.

Under learning, in the foreign country (panels (c) and (d) in Figure 8) output response is dampened under OSR, with respect to the non-cooperative case. Foreign CPI inflation initially slightly increases before being negative for a longer time under OSR than under the benchmark calibration. As in the Home country, the Foreign central banker accepts larger inflation fluctuations to achieve better output stabilization. The Foreign central bank tries to implement the same strategy as the

one described under RE: she lets inflation fall for a long time, which tends to increase the real value of debt, thereby offsetting the positive financial spillover effect. However, the Foreign central bank's strategy is not as successful, as under RE, in stabilizing output.

Figure 8: IRFs to an increase in Home technological shock under adaptive learning, no coordination case versus OSR



Solid lin : Rational Expectations. Non-cooperative case. "o" line: Adaptive learning, median and 80% range. Non-cooperative case. "+" line: Adaptive learning, OSR. median value.

Figure 7 provides an intuition for the reason behind the difficulties faced by the central banker in stabilizing the Foreign economy. In the Home country (panels (a), (b), (c)), domestic private agents understand the Home central bank's policy under OSR. Indeed, their expectations (based on their perceived law of motion) are in line with the central bank's policy (i.e. OSR under RE). They expect low asset prices, lower inflation and a large threshold for productivity. This is consistent with the central bank's objective to stabilize output, by dampening the favorable effects of the financial accelerator. Therefore, even though there is still a (small) gap with expectations under RE, private agents' expectations in the Home country are well anchored by the domestic monetary policy.

This is not the case in the Foreign country. In fact under OSR, Foreign agents expect a prolonged episode of reduced inflation (Figure 7, (e)), which reduces their expected net wealth, through the Fisher effect. This limits their ability to borrow from the bank and is in line with the Foreign central bank's objective of dampening the financial accelerator. However, for the reasons discussed above, they miss the international transmission of monetary policies through the UIP. Therefore, they expect higher collateral prices q^* and a smaller productivity threshold $\tilde{\omega}^*$ (Figure 7, (d), (f)). This makes Foreign private-agents willing to invest more: the demand for capital increases, the general equilibrium price of capital goes up fuelling in turn the financial accelerator (this stimulates the response of Foreign output in subsequent quarters, see Figure 8, (d)). This macroeconomic behavior is at odds with the Foreign central bank's objective to dampen the financial accelerator. The Foreign central bank thus fails to properly anchor private agents' expectations. This can be

seen on Figure 7, panels (d), (e), (f): in the Foreign country, under OSR with AL, expectations significantly differ from the RE case.

6 Conclusions

Our analysis show that learning constitutes a powerful propagation mechanism when countries are interconnected and characterized by financial imperfections. Indeed, as in Rychalovska et al. (2016), learning interacts with the financial accelerator, further amplifying it. Moreover, because of imperfect knowledge of fundamentals, private agents cannot track the international transmission of shocks. Therefore, the uncertainty about interest-rate dynamics affects developments on financial markets and is transmitted to output. This is at the roots of an asymmetric transmission of shocks. We thus assess the opportunity of monetary policy coordination under learning. We show that learning entails a greater need for both output and inflation stabilization. However, even under coordinated policies, agents cannot learn the true dynamics of their globalized economy, because of their partial knowledge. Therefore, the joint monetary policy is not able to anchor expectations in response to shocks hitting Foreign countries.

Our analysis considers adaptive learning on the fundametalns of the economy (the coefficients of the model), as in Evans & Honkapohja (2001). Further research should account for partial knowledge of the exogenous processes. Analogously, the opportunity of monetary coordination should be evaluated with a better focus on data.

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Appendix

A Model

The model is two-country version of Bernanke et al. (1999) (as in Faia (2007*b*), Kolasa & Lombardo (2014) and Christiano et al. (2011), among others). Each country H and F, respectively, is populated by households, entrepreneurs and retailers. Households consume (both domestic and foreign produced) goods, work, lend funds to domestic (foreign) banks and receive profits from retailers. They also have access to international markets, where they can buy international bonds (or get indebted). As in Bernanke et al. (1999), entrepreneurs decide over labor and capital inputs to the purpose of producing wholesale goods in a perfect competition framework. Installing capital entails adjustment costs. Entrepreneurs are subject to aggregate and idiosyncratic shocks. To finance their activity, they have access to bank loans. However, banks do not observe idiosyncratic shocks so that their relationship is affected by agency problems. Because of the costly state verification problem, borrowers need to pay a risk premium. This entails a spread between the rate paid by entrepreneurs and the one at which deposits are remunerated.

Wholesale goods are then purchased by retailers and distributed both in the domestic country (H) and in the foreign one (F). Retailers operate in a monopolistic competition framework and prices of domestic (foreign) goods are affected by nominal rigidities à la Rotemberg in each country. The exchange rate pass through is perfect and price rigidities enter only at the domestic (foreign) level. The retailing activity is operated by households.

A.1 Households

A.1.1 Domestic households

Household in country H maximizes the following flow of expected utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

where β is the discount rate, C_t denotes aggregate consumption and N_t labor effort. The utility function $U(C_t, N_t)$ verifies the standard properties, $U'_c > 0, U''_c < 0, U'_N < 0$. The aggregate consumption basket is a Dixit-Stiglitz aggregator including both domestically produced goods and foreign ones, i.e.:

$$C = \left[(1 - \gamma) C_H^{\frac{\eta-1}{\eta}} + \gamma C_{HF}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

and thus, the CES-related CPI price index is:

$$P = \left[(1 - \gamma) P_H^{1-\eta} + \gamma P_F^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

where P_H is the price of domestically-produced goods and P_F the one of foreign ones (in domestic currency). Also, $(1 - \gamma)$ represents the degree of home bias and $\eta > 0$ the elasticity of substitution between domestic and foreign goods.

Agents' budget constraint can be written in real terms of domestic goods as¹⁵:

$$C_t + d_t + b_t^* \leq R_{t-1} \frac{d_{t-1}}{\pi_t} + R_{t-1}^F \frac{b_{t-1}^*}{\pi_t} \frac{e_t}{e_{t-1}} + \frac{W_t}{P_t} N_t + \frac{\Pi_t}{P_t} \quad (22)$$

where d are households' deposits in the bank, R is the deposit rate, e is the nominal exchange rate, R_{t-1}^F is the return received (paid) on foreign-denominated international bonds (debt) b_{t-1}^* . We denote by e the nominal exchange rate (ie, the price of the foreign currency) and π_t is CPI inflation. Given that $\frac{W}{P}$ are real wages and $\frac{\Pi}{P}$ real profits deriving from the retailing activity, the first order conditions of agents' problem read as:

$$U'_{N_t} + U'_{c_t} \frac{W_t}{P_t} = 0 \quad (23)$$

$$U'_{c_t} = \beta E_t \left[\frac{R_t}{\pi_{t+1}} U'_{c_{t+1}} \right] \quad (24)$$

$$U'_{c_t} = \beta E_t \left[R_t^F U'_{c_{t+1}} \frac{e_{t+1}}{\pi_{t+1} e_t} \right] \quad (25)$$

where equation (23) is the optimality condition associated to labor effort and equation (24) is the standard Euler equation associated to domestic deposits. Equation (25) is the one associated to international bonds.

Due to imperfect capital mobility and/or in order to capture the existence of risk associated to debt accumulation, there is a spread between the return on international securities received (paid) by domestic agents and the one paid (received) by foreign ones. Following Schmitt-Grohe & Uribe (2003), this spread is a function of the (real) value of the country's net foreign asset position, i.e.:

$$R_t^F = R_t^* + p(-b_t^*) \quad (26)$$

¹⁵The budget constraint in nominal terms writes as:

$$P_t C_t + D_t + B_t^* e_t \leq R_{t-1} D_{t-1} + R_{t-1}^F B_{t-1}^* e_t + W_t N_t + \Pi_t$$

where P are domestic prices and all capital letters are written in nominal terms. Therefore, international bonds in real terms of domestic consumption can be written as $b_t^* = e_t B_t^* / P_t$.

where p is a country-specific interest rate premium such that

$$p(-b_t^*) = -\zeta \left(e^{b_t^*} - 1 \right) \quad (27)$$

with $\zeta > 0$.

A.1.2 Foreign households

Foreign households face a symmetric optimization problem as domestic households except for the fact that international bonds are denominated in their own currency. For simplicity, we mark by an asterisk all variables referring to the foreign country. Foreign households thus maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^*, N_t^*)$$

subject to the budget constraint:

$$C_t^* + d_t^* - b_t^* \leq R_{t-1}^* \frac{d_{t-1}^*}{\pi_t^*} - R_{t-1}^* \frac{b_{t-1}^*}{\pi_t^*} + \frac{W_t^*}{P_t^*} N_t^* + \frac{\Pi_t^*}{P_t^*} \quad (28)$$

where d^* and b^* denote foreign deposits and the international bond, respectively. As for domestic agents, the aggregate consumption basket is a Dixit-Stiglitz aggregator including both domestically produced goods and foreign ones, i.e.:

$$C^* = \left[\gamma C_H^{*\frac{\eta-1}{\eta}} + (1-\gamma) C_F^{*\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

and thus, the CES-related CPI price index is:

$$P^* = \left[\gamma P_H^{*1-\eta} + (1-\gamma) P_F^{*1-\eta} \right]^{\frac{1}{1-\eta}}$$

The first order conditions read:

$$U_{ct}^{*'} \frac{W_t^*}{P_{ct}^*} = -U_{Nt}^{*'} \quad (29)$$

$$U_{ct}^{*'} = \beta R_t^* E_t \left[\frac{U_{ct+1}^{*'}}{\pi_{t+1}^*} \right] \quad (30)$$

$$U_{ct}^{*'} = \beta R_t^* E_t \left[\frac{U_{ct+1}^{*'}}{\pi_{t+1}^*} \right] \quad (31)$$

Where equations (29), (30) and (31) are the foreign counterpart of equations (23), (24) and (25),

respectively. The returns on the deposits and on the international securities in the Foreign country are clearly equalized by an arbitrage condition.

A.1.3 UIP

By combining equations Euler equations (31) with (25) and (26) we obtain the following uncovered interest parity condition:

$$U'_{ct} = \beta E_t \left[(R_t^* + p(-b_t^*)) U'_{ct+1} \frac{e_{t+1}}{\pi_{t+1} e_t} \right]$$

and thus:

$$U'_{ct} = \beta E_t \left[\left(\frac{U_{ct}^{*'}}{\beta E_t \left[\frac{U_{ct+1}^{*'}}{\pi_{t+1}^*} \right]} + p(-b_t^*) \right) U'_{ct+1} \frac{e_{t+1}}{\pi_{t+1} e_t} \right] \quad (32)$$

so that marginal utilities across countries are equalized up to a spread for the country risk.

A.2 Entrepreneurs

We now focus on domestic entrepreneurs (the problem of F entrepreneurs is perfectly symmetric). We stick to Bernanke et al. (1999) and assume that entrepreneurs decide over investment decisions. They choose indeed the optimal level of capital and labor to be used for current production. Notice that only capital accumulated in previous periods can be used for production and that capital accumulation is subject to capital adjustment cost, $\Phi \left(\frac{I_t}{K_{t-1}} \right) K_{t-1}$. Capital adjustment costs are such that they disappear at the steady-state, $\Phi \left(\frac{I_t}{K_{t-1}} \right) = 0$ and $\Phi' \left(\frac{I_t}{K_{t-1}} \right) > 0, \Phi'' \left(\frac{I_t}{K_{t-1}} \right) < 0$. Entrepreneurs are wholesale producers and once idiosyncratic uncertainty is solved, wholesale output is:

$$Y_t = A_t F(K_{t-1}, N_t)$$

where A is total factor productivity and it is defined by the following exogenous stochastic process:

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^{as}$$

They are risk neutral and maximize the following stream of utilities:

$$E_0 \sum_{t=0}^{\infty} (\varsigma \beta)^t C_t^e \quad \text{with } \varsigma \beta \leq \beta$$

Once all uncertainty is solved, entrepreneurs' resources (in real terms of domestic consumption) come

from loans from banks, l_t , and wholesale output, $f_t \frac{F(K_{t-1}, N_t)}{X_t}$. We denote by X_t the gross markup of retail goods over wholesale goods (ie the ratio between the wholesale output price, P^w and the price of the domestic production, P_H is equal to $\frac{1}{X}$, so that $\frac{1}{X} = \frac{P^w}{P_H}$) and f_t the ratio between the domestic producer price, P_H and the domestic consumption price, P , ie, $f_t = \frac{P_H}{P}$. Entrepreneurs pay the wage bill, $\frac{W}{P}N$, and the costs associated to capital accumulation, $I_t + \Phi\left(\frac{I_t}{K_{t-1}}\right)K_{t-1}$. Capital evolves as:

$$K_t = (1 - \delta)K_{t-1} + I_t$$

In each period, entrepreneurs need to pay the interests on their loans, $R_{t-1}^L \frac{l_{t-1}}{\pi_t}$. However, because of idiosyncratic uncertainty, banks are subject to a costly-state-verification problem. There is thus a spread between the borrowing and the lending rate. The lending rate is the result of an optimal contract (see the following).

The first order conditions with respect to labor and investment, respectively, read as:

$$f_t \frac{Y_{N,t}}{X_t} = \frac{W_t}{P_t}$$

$$Q_t = \left[1 + \Phi' \left(\frac{I_t}{K_{t-1}} \right) K_{t-1} \right]$$

where Q_t is the (real) price of capital. The mean return from holding one unit of capital is:

$$R_t^k = \frac{\pi_t}{Q_{t-1}} \left[\frac{Y_{K,t-1}}{X_t} f_t + \Phi' \left(\frac{I_t}{K_{t-1}} \right) \frac{I_t}{K_{t-1}} - \Phi \left(\frac{I_t}{K_{t-1}} \right) + Q_t(1 - \delta) \right] \quad (33)$$

and can be decomposed into: i) the domestic-currency yields of one unit of capital, $\frac{Y_{K,t-1}}{X_t} f_t$; ii) the reduction in adjustment costs, $\Phi' \left(\frac{I_t}{K_{t-1}} \right) \frac{1}{K_{t-1}} - \Phi \left(\frac{I_t}{K_{t-1}} \right)$; iii) the returns in selling that unit of non-depreciated capital, $Q_t(1 - \delta)$.

A.3 Loan contract

During period t , a continuum of entrepreneurs (indexed by j) needs to finance the purchase of new capital K_t^j that will be used for production in period $t+1$. Each entrepreneur engages in a financial contract before the realization of the idiosyncratic shock, ω^j . Once the shock is realized, the return of capital is thus $\omega^j R^k$.

Before entering the loan contract, each entrepreneur owns end-of-period internal funds for an amount nw_t^j (in real terms of the consumption good) and seeks to finance the purchase of new capital $Q_t K_t^j$. As in ?, we assume that the required funds for investment exceed internal funds, and thus:

$$l_t^j = Q_t K_t^j - nw_t^j \geq 0 \quad (34)$$

Default occurs when the return from the investment $\omega_{t+1}^j R_{t+1}^k Q_t K_t^j$ happens to be below the amount that needs to be repaid $R_t^L l_t^j$. The entrepreneur defaults thus if

$$\omega_{t+1}^j \leq \tilde{\omega}_{t+1}^j \equiv \frac{R_t^L l_t^j}{R_{t+1}^k Q_t K_t^j}$$

where $\tilde{\omega}$ is the threshold level for the productivity idiosyncratic shock. We follow Christiano et al. (2014) and introduce a shock increasing the idiosyncratic risk. Risk increases when the standard deviation of the threshold level for the idiosyncratic shock, $\tilde{\omega}$, goes up, because the dispersion of entrepreneurs' outcome goes up as well. The uncertainty shock hitting σ , the standard deviation of the idiosyncratic shock, is thus:

$$\log \sigma_t = \rho_\sigma \log \sigma_{t-1} + \varepsilon_t^\sigma$$

A.3.1 Optimal debt contract

The contract is signed before the realization of uncertainty. Let $\Gamma(\tilde{\omega}_t^j)$ denote the fraction of net capital output received by the lender where

$$\Gamma(\tilde{\omega}_{t+1}^j) = \int_{\underline{\omega}}^{\tilde{\omega}_{t+1}^j} \omega_{t+1}^j f(\omega) d\omega + \tilde{\omega}_{t+1}^j \int_{\tilde{\omega}_t^j}^{\bar{\omega}} f(\omega) d\omega$$

As stressed by ?, the bank does not observe idiosyncratic shocks and entrepreneurs could declare default to the purpose of not repaying back their debt. The bank needs thus to engage in a costly monitoring activity. As this latter is operated when the entrepreneur declares default, monitoring costs are:

$$\mu G(\tilde{\omega}_{t+1}^j) \equiv \mu \int_{\underline{\omega}}^{\tilde{\omega}_{t+1}^j} \omega_{t+1}^j f(\omega) d\omega$$

and the net share received by the lender is thus $\Gamma(\tilde{\omega}_{t+1}^j) - \mu G(\tilde{\omega}_{t+1}^j)$.

The arbitrage condition for the bank implies to make zero profit, and thus:

$$\left[\Gamma(\tilde{\omega}_{t+1}^j) - \mu G(\tilde{\omega}_{t+1}^j) \right] R_{t+1}^k Q_t K_t^j = R_t l_t \quad (35)$$

Using (34) and (35), we obtain the following participation constraint:

$$\left[\Gamma(\tilde{\omega}_{t+1}^j) - \mu G(\tilde{\omega}_{t+1}^j) \right] R_{t+1}^k Q_t K_t^j = R_t \left(Q_t K_t^j - n w_t^j \right) \quad (36)$$

where equation (36) is written in real terms of the consumption basket.

The optimal contract is a pair $(\tilde{\omega}_{t+1}^j, K_t^j)$ maximizing entrepreneurs' expected real profits

$$E_t \left\{ \left[1 - \Gamma \left(\tilde{\omega}_{t+1}^j \right) \right] R_{t+1}^k Q_t K_t^j \right\}$$

subject to (36). Let χ_t denote the Lagrange multiplier associated with (36).

The problem's optimality condition with respect to $\tilde{\omega}_{t+1}^j$ reads:

$$\Gamma' \left(\tilde{\omega}_{t+1}^j \right) = \chi_t \left[\Gamma' \left(\tilde{\omega}_{t+1}^j \right) - \mu G' \left(\tilde{\omega}_{t+1}^j \right) \right] \quad (37)$$

the one with respect to K_t^j is:

$$\left[1 - \Gamma \left(\tilde{\omega}_{t+1}^j \right) \right] R_{t+1}^k Q_t + \chi_t \left[\Gamma \left(\tilde{\omega}_{t+1}^j \right) - \mu G \left(\tilde{\omega}_{t+1}^j \right) \right] R_{t+1}^k Q_t - \chi_t R_t Q_t = 0$$

and can be rewritten as

$$\frac{R_{t+1}^k}{R_t} \left(\left[1 - \Gamma \left(\tilde{\omega}_{t+1}^j \right) \right] + \chi_t \left[\Gamma \left(\tilde{\omega}_{t+1}^j \right) - \mu G \left(\tilde{\omega}_{t+1}^j \right) \right] \right) = \chi_t \quad (38)$$

Using (37) and (38), we get the external finance premium:

$$E_t \frac{R_{t+1}^k}{R_t} = E_t \frac{1}{\frac{\left[1 - \Gamma \left(\tilde{\omega}_{t+1}^j \right) \right] \left[\Gamma' \left(\tilde{\omega}_{t+1}^j \right) - \mu G' \left(\tilde{\omega}_{t+1}^j \right) \right]}{\Gamma' \left(\tilde{\omega}_{t+1}^j \right)} + \left[\Gamma \left(\tilde{\omega}_{t+1}^j \right) - \mu G \left(\tilde{\omega}_{t+1}^j \right) \right]} \quad (39)$$

and

$$E_t \frac{R_{t+1}^k}{R_t} = \rho \left(\tilde{\omega}_{t+1}^j \right) \quad (40)$$

with $\rho' \left(\tilde{\omega}_t^j \right) \geq 0, \rho \left(\tilde{\omega}_{t+1}^j \right)$ is dubbed as the external finance premium. The ratio $E_t \frac{R_{t+1}^k}{R_t}$ captures the cost of finance, which reflects in turn the existence of monitoring costs.

By using equation (36) and aggregating, we get

$$\left[\Gamma \left(\tilde{\omega}_{t+1} \right) - \mu G \left(\tilde{\omega}_{t+1} \right) \right] \frac{R_{t+1}^k}{R_t} \frac{Q_t K_t}{n w_t} = \left(\frac{Q_t K_t}{n w_t} - 1 \right) \quad (41)$$

With equations (39) and (40), equation (41) defines a relationship between the external finance

premium (EFP) and the leverage ratio $\frac{Q_t K_t}{nw_t^j}$. Indeed, all firms face the same EFP.

$$\begin{aligned} \frac{R_{t+1}^k}{R_t} &= \frac{\left(\frac{Q_t K_t}{nw_t} - 1\right)}{\frac{Q_t K_t}{nw_t} [\Gamma(\tilde{\omega}_{t+1}) - \mu G(\tilde{\omega}_{t+1})]} \\ &= \frac{1}{[\Gamma(\tilde{\omega}_{t+1}) - \mu G(\tilde{\omega}_{t+1})]} - \frac{1}{\frac{Q_t K_t}{nw_t} [\Gamma(\tilde{\omega}_{t+1}) - \mu G(\tilde{\omega}_{t+1})]} \end{aligned}$$

Notice that the larger the leverage ratio, the greater the EFP.

A.3.2 Net worth accumulation

Surviving entrepreneurs accumulate wealth. As in ? the wealth belonging to defaulting entrepreneurs is consumed by existing ones. Thus, aggregate net worth at the end of period t is:

$$nw_t = \varsigma_t [1 - \Gamma(\tilde{\omega}_t)] \frac{R_t^k Q_{t-1}}{\pi_t} K_{t-1} \quad (42)$$

where ς is the share of surviving entrepreneurs. Following Christiano et al. (2011) and Kolasa & Lombardo (2014), we suppose that the survival rate of entrepreneurs follows the exogenous process (also called wealth shock):

$$\log \varsigma_t = \rho_\varsigma \log \varsigma_{t-1} + \varepsilon_t^\varsigma$$

This shocks specifically hit the survival rate of entrepreneurs, and thus, the share of wealth that is accumulated in the economy. Indeed, when more entrepreneurs are alive, more wealth is accumulated.

Entrepreneurs' consumption is thus:

$$C_t^e = (1 - \varsigma_t) [1 - \Gamma(\tilde{\omega}_t)] \frac{R_t^k Q_{t-1}}{\pi_t} K_{t-1}$$

In what follows we will assume that the share of surviving entrepreneurs follows an exogenous stochastic process. Lagging (36) we obtain:

$$\Gamma(\tilde{\omega}_t) R_t^k Q_{t-1} K_{t-1}^j = \mu G(\tilde{\omega}_t) R_t^k Q_{t-1} K_{t-1}^j + R_{t-1} (Q_{t-1} K_{t-1} - nw_{t-1})$$

so that aggregate wealth, equation (42), can be rewritten as

$$nw_t = \varsigma_t R_t^k \frac{Q_{t-1}}{\pi_t} K_{t-1} - \frac{\varsigma_t}{\pi_t} \left[R_{t-1} + \frac{\mu G(\tilde{\omega}_t) R_t^k Q_{t-1} K_{t-1}}{(Q_{t-1} K_{t-1} - nw_{t-1})} \right] (Q_{t-1} K_{t-1} - nw_{t-1}) \quad (43)$$

with $\frac{\mu G(\tilde{\omega}_t) R_t^k Q_{t-1} K_{t-1}}{(Q_{t-1} K_{t-1} - nw_{t-1})}$ the risk premium factor, that depends on R_t^k .

A.4 Production

We follow Bernanke et al. (1999) by introducing monopolistic competition at a retailer level.¹⁶ Retailers aggregate domestic (foreign) goods in each country and distribute them both in the domestic country and abroad.

Their activity is subject to price adjustment costs à la Rotemberg. Notice that in country H (F) price inertia is at a domestic (foreign) retailer level. Therefore, the exchange rate pass through is complete among countries. Retailers buy wholesale goods and transform them in final-retailed goods, that can be consumed domestically or exported.

As in Bernanke et al. (1999), let $Y_t(i)$ be the quantity of output sold by retailer i in terms of wholesale goods. $P_t(i)$ is the nominal price of the final good. Total final consumption goods are aggregated à la Dixit-Stiglitz into the following basket of individual retail goods:

$$Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{v-1}{v}} di \right)^{\frac{v}{v-1}} \quad (44)$$

where $v > 1$ is the elasticity of substitutions among varieties. The corresponding price index is:

$$P_t = \left(\int_0^1 P_t(i)^{1-v} di \right)^{\frac{1}{1-v}}$$

and the demand curve facing each retailer is thus:

$$Y_t^d(i) = \left[\frac{P_t(i)}{P_t} \right]^{-v} Y_t \quad (45)$$

Each monopolistic firm chooses the sequence $\{P_t(i)\}_{t=0}^{\infty}$ to maximize the stream of nominal profits,

$$E_t \left\{ \sum_{t=0}^{\infty} \Lambda_{t,t+1} \Pi_t(i) \right\}$$

where

$$\Pi_t(i) = Y_t(i) [P_t(i) - P_t^w] - \frac{\omega_P}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 P_t(i)$$

and, from the household problem:

$$\Lambda_{t,t+1} = \frac{\beta E_t [U'_{ct+1}]}{U'_{ct} \pi_{t+1}} = \frac{1}{R_t}$$

¹⁶Wholesale producers are indeed the above-analyzed entrepreneurs.

The maximization of profits is subject to the demand:

$$Y_t(i) \succeq \left(\frac{P_t(i)}{P_t} \right)^{-v} Y_t \quad (46)$$

Retailers' optimization problem entail the following Phillips curve:

$$(\pi_{Ht} - 1) \pi_{Ht} = Y_t \frac{v}{\omega_P} \left[\frac{1}{X_t} - \frac{(v-1)}{v} \right] + \beta E_t \frac{U'_{ct+1}}{U'_{ct}} (\pi_{Ht+1} - 1) \frac{f_{t+1}}{f_t} \pi_{Ht+1} \quad (47)$$

Analogously, country F retailers' problem entail the following Phillips curve:

$$\omega_P (\pi_{Ft}^* - 1) \pi_{Ft}^* = X_{Ft}^* v \left[\frac{(1-v)}{v} + \frac{1}{X_t^*} \right] + \beta \omega_P E_t \frac{U'^*_{ct+1}}{U'^*_{ct}} (\pi_{Ft+1}^* - 1) \frac{f_{t+1}^*}{f_t^*} \pi_{Ft+1}^*$$

Finally, terms of trade are the ratio of the domestic goods over the price of foreign prices, $tot_t = \frac{P_{Ht}}{e_t P_{Ft}^*} = \frac{f_t}{e_t^* f_t^*}$, where $f_t \equiv \frac{P_{Ht}}{P_t^c} = f_{t-1} \frac{\pi_{Ht}}{\pi_t^c}$ and $f_t^* \equiv \frac{P_{Ft}^*}{P_t^{*c}} = f_{t-1}^* \frac{\pi_{Ft}^*}{\pi_t^{*c}}$. Notice that because of home bias, the law of one price holds for the domestic and foreign basket of goods, separately. Indeed $P_{Ht} = e_t P_{Ht}^*$ and $P_{Ft} = e_t P_{Ft}^*$ but the real exchange rate generally differs from one, $P_t \neq e_t P_t^*$.

A.5 Monetary policy and market equilibria

To close the model, we suppose that in each country the monetary policy follows a standard Taylor rule targeting both the output gap and CPI inflation. Therefore, in country H the monetary-policy rule is:

$$R_t = (R_{t-1})^\chi \left(\bar{R}^n \left(\frac{\pi_t}{\bar{\pi}} \right)^{b_\pi} \left(\frac{Y_t}{y} \right)^{b_y} \right)^{1-\chi} mp_t \quad (48)$$

In country F,

$$R_t^{*n} = (R_{t-1}^{*n})^{\chi^*} \left(\bar{R}^{*n} \left(\frac{\pi_t^*}{\bar{\pi}^*} \right)^{b_\pi^*} \left(\frac{Y_t^*}{y^*} \right)^{b_y^*} \right)^{1-\chi} mp_t^* \quad (49)$$

with a mp_t and mp_t^* temporary monetary policy shocks, such that:

$$\log mp_t = \rho_{mp} \log mp_{t-1} + \varepsilon_t^{mp}$$

and

$$\log mp_t^* = \rho_{mp} \log mp_{t-1}^* + \varepsilon_t^{mp}$$

Using output (44) and recalling that the law of one price holds ($P_{Ht} = e_t P_{Ht}^*$), the aggregate demand for domestic output is:

$$X_{Ht} = (1 - \gamma) \left[\frac{P_{Ht}}{P_t^c} \right]^{-\eta} X_t^c + \left[\frac{P_{Ht}}{e_t P_t^{*c}} \right]^{-\eta} \gamma^* X_t^{c*} + \frac{\omega_P}{2} (\pi_{Ht} - 1)^2$$

that can be rewritten as:

$$X_{Ht} = (1 - \gamma) [f_t]^{-\eta} X_t^c + [tot_t f_t^*]^{-\eta} \gamma^* X_t^{c*} + \frac{\omega_P}{2} (\pi_{Ht} - 1)^2$$

where X_t^c is aggregate domestic demand:

$$X_t^c = C_t^c + I_t + C_t^e + \mu G(\tilde{\omega}_t) R_t^k \frac{Q_{t-1}}{\pi_t} K_{t-1} + \Phi \left(\frac{I_t}{K_{t-1}} \right) K_{t-1}$$

Analogously, in country F the aggregate demand for foreign goods is:

$$X_{Ft}^* = (1 - \gamma^*) [f_t^*]^{-\eta} X_t^{*c} + \left[\frac{f_t}{tot_t} \right]^{-\eta} \gamma X_t^c + \frac{\omega_P}{2} (\pi_{Ft}^* - 1)^2$$

where X_t^{*c} is aggregate foreign demand:

$$X_t^{*c} = C_t^{*c} + I_t^* + C_t^{*e} + \mu^* M(\tilde{\omega}_t^*) R_t^{*k} \frac{Q_{t-1}^*}{\pi_t^*} K_{t-1}^* + \Phi \left(\frac{I_t^*}{K_{t-1}^*} \right) K_{t-1}^*$$

As the activity of banks do not cross borders, the demand for loans has to be equal to the real supply of loans for both countries:

$$\begin{aligned} d_t &= l_t \\ d_t^* &= l_t^* \end{aligned}$$

The world net supply of bonds is zero. Finally, the current account equation is:

$$b_t^* - \frac{b_{t-1}^*}{\pi_t} \frac{e_t}{e_{t-1}} = (R_{t-1}^F - 1) \frac{b_{t-1}^*}{\pi_t} \frac{e_t}{e_{t-1}} + f_t Y_t - \left(C_t^c + I_t + C_t^e + \mu G(\tilde{\omega}_t) R_t^k \frac{Q_{t-1}}{\pi_t} K_{t-1} \right) \quad (50)$$

B Learning algorithm

B.1 Learning loop

1. Draw a vector of innovations ϵ_t
2. Start with x_0, ϕ_0 (Initialization)

3. ALM :

$$\begin{pmatrix} k_t \\ z_t \end{pmatrix} = \begin{pmatrix} T_k(\phi_{t-1}) & T_z(\phi_{t-1}) \\ 0 & \rho \end{pmatrix} \begin{pmatrix} k_{t-1} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} V(\phi_{t-1}) \\ 1 \end{pmatrix} \epsilon_t$$

$$z_{t-1} = \rho z_t + \epsilon_t$$

4. Using the realized observation k_t, z_t , the agents obtain a new estimate of ϕ

$$\phi_t = \phi_{t-1} + gain \quad x_{t-1} (k_t - x'_{t-1} \phi_{t-1})$$

then back to step 3 until the end of the sample.

Fixed-gain learning does not necessarily converge towards RE (Evans & Honkapohja (2001)). In addition, learning may cause the dynamic system to explode. Following Marcat & Sargent (1989), as it is a standard procedure in the learning literature, we invoke a "projection facility" in the simulations whenever updated beliefs lead to explosive dynamics in the actual law of motion. Projection facility can be interpreted as the idea that private agents reject updates that lead to unlikely economic dynamics. They tend to favor stable models. When the projection facility is invoked, the update of the belief is rejected. Beliefs are back to the initial value, plus a ridge factor of 0.00001. Projection facility is rarely encountered (less than 1% of the simulated data, with $gain = 0.03$). Results are not driven by explosive beliefs.

B.2 Evolutions of PLM for 1 simulation

In this section, we illustrate the learning process by looking at the update of coefficients ϕ in private agents' perceived law of motion. Let us consider the case of consumption.

Under rational expectations, consumption policy rule is a function of all state variables: i.e. all shocks (whether Home or Foreign), and all state variables in the economy (net worth, capital asset price, nominal interest rate, in Home and Foreign countries, along with terms of trade and external debt.) The coefficients in the perceived law of motion are constant. Figures 9 and 10 display coefficients of PLM under RE. They all appear constant.

Under learning,

- The PLM of Home consumption c is a function of all shocks in the economy (as in Evans & Honkapohja (2001)) and domestic state variables only (nw, k, q, R), along with terms of trade and net foreign asset position b^* : $c_t = \phi_{t-2} x_{t-1}$ with $x = \{a, a^*, \dots, nw, k, q, R, b^*, tot\}$. Time-variation in coefficients ϕ is illustrated in Figure 9 displayed for 1 simulation.
- The PLM of Foreign consumption c^* is a function of all shocks in the economy (as in Evans & Honkapohja (2001)) and Foreign state variables only (nw^*, k^*, q^*, R^*), along with terms of

Figure 9: Coefficients ϕ in PLM of $c. 1$ simulation

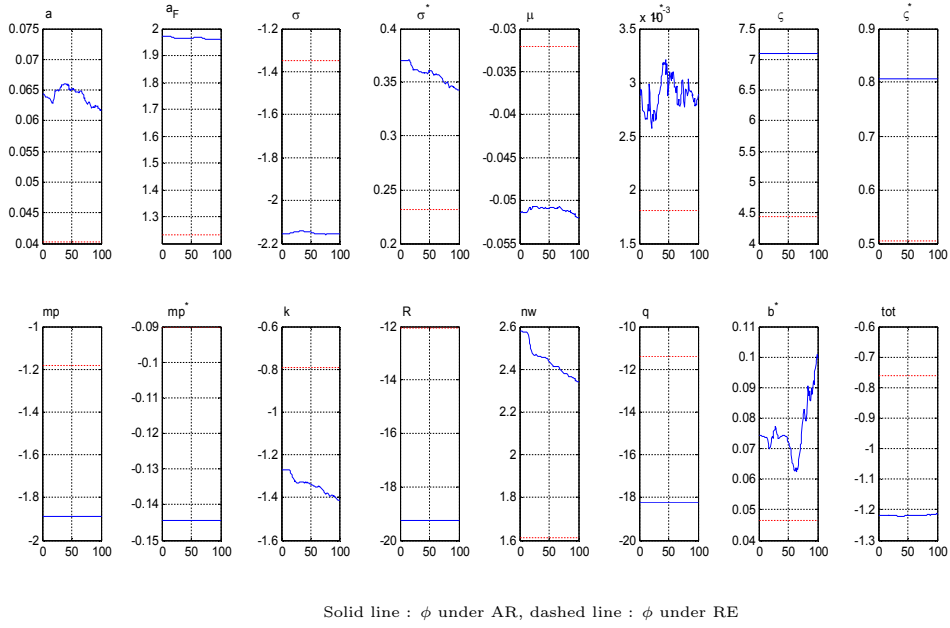
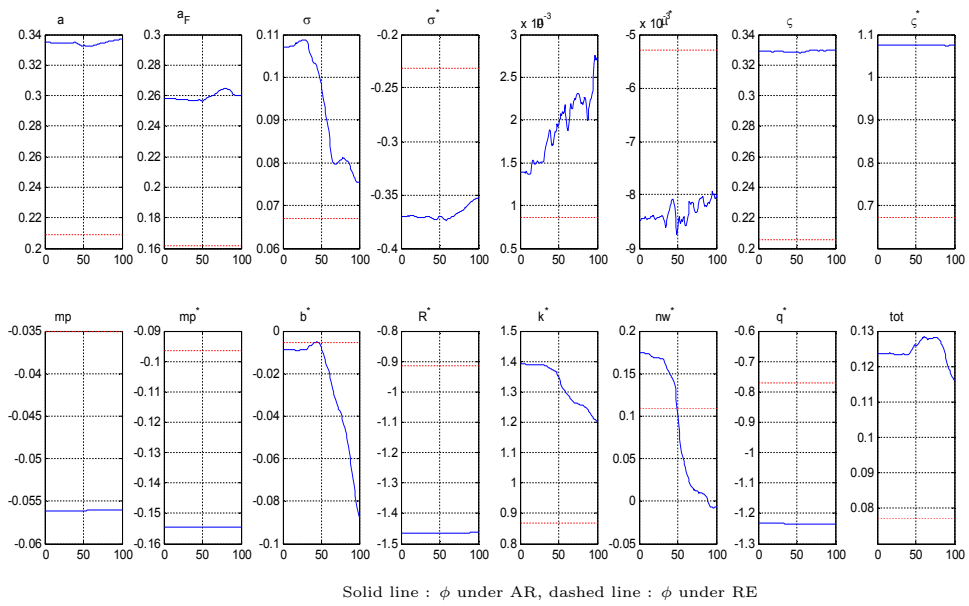


Figure 10: Coefficients ϕ in PLM of $c. 1$ simulation



trade and net foreign asset position b^* : $c_t^* = \phi_{t-2} x_{t-1}^*$ with $x = \{a, a^*, \dots, nw^*, k^*, q^*, R^*, b^*, tot\}$. Time-variation in coefficients ϕ is illustrated in Figure 10 displayed for 1 simulation.

Notice in Figures 9 and 10 that ϕ keep being updated. This is due to the constant gain in the learning loop.

B.3 Business cycle statistics of forecasts errors

B.3.1 Data:

Survey Data of Inflation Expectations :

- US SPF : 1968Q4-2106Q3. From the Federal Reserve Bank of Philadelphia. Forecasts for the quarterly growth of the GDP price index. Seasonally adjusted index. Mean forecasts (rather than median forecasts) because Euro Area data only report mean forecasts. In order to maximize the size of the sample, we consider GDP price rather than expectations on CPI (available only after 1981Q1). 1-quarter ahead forecasts only because this is the only forecasting horizon that is common with the euro area survey. The US SPF survey also provides forecasts for interest rates (nominal or real, short-term). We also use them in Appendix B.3.2.
- ECB SPF : 1999Q1-2016Q3. Quarterly data on Inflation forecasts. Mean point estimates (all survey rounds) are considered. Inflation is defined as year on year percentage change of the Harmonised Index of Consumer Prices published by Eurostat. For the purpose of comparing US and EA data, we focus 1-Quarter ahead forecasts. This forecasting horizon is common across surveys.

Macroeconomic data:

- US FRED Economic Database : GDP price deflator, real GDP. Seasonally adjusted. 1968Q4-2106Q3 that is the same sample as US SPF.
- Euro Area AWM model : Euro Area macroeconomic time series from the Area Wide Model (AWM) dataset. Quarterly time series from 1970Q1 to 2014Q4. Harmonised Index of Consumer Prices. Real GDP. Seasonally adjusted.

B.3.2 Fluctuations in forecast errors : Model versus Data

We report in Table 3 the business cycle statistics of 1-quarter ahead forecast errors (defined as the forecast minus the realized value) found in the SPF data and in the learning model. We do not report business cycle statistics for the model under rational expectations as all statistics are

nearly zero. Under rational expectations, the model cannot match fluctuations in forecast errors. In columns (1) and (2), Table 3 displays the autocorrelation of one-period-ahead forecast errors, found in US (column (1)) and ECB (column (2)) data. In the learning model, we report the same statistics found on simulated data in the Foreign (US) country and Home (EA) countries. The forecast of short-term nominal interest rate is available in US data only. In the data, forecast errors are positively autocorrelated. This systematic behavior is completely at odds with rational expectations.

Table 3: Fluctuations in forecast errors : Model versus Data

	Persistence		Cyclicality	
	US SPF (1)	ECB SPF (2)	US SPF (3)	ECB SPF (4)
Inflation				
Data	0.54	0.21	-0.31	0.016
Model	0.15	0.19	0.18	0.19
Nominal interest rate				
Data	0.27		-0.17	
Model	0.24		-0.14	

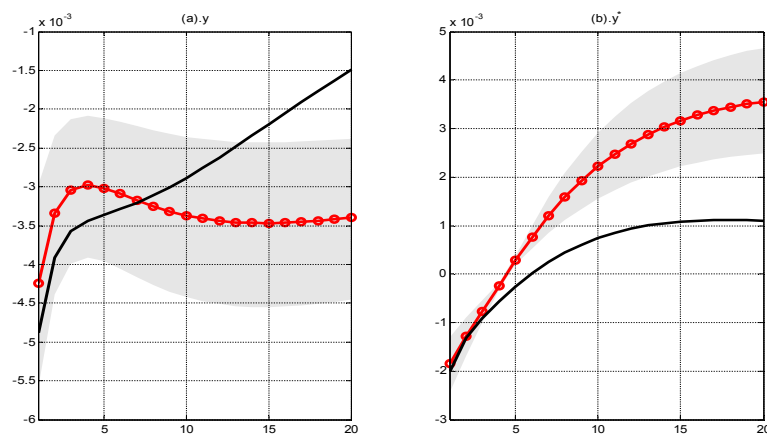
C IRFs for other shocks

C.1 Risk shock (Home increase of idiosyncratic risk)

IRFs are displayed in Figures 11-14.

C.1.1 Rational expectations

Figure 11: Output responses to a Home increase in idiosyncratic risk

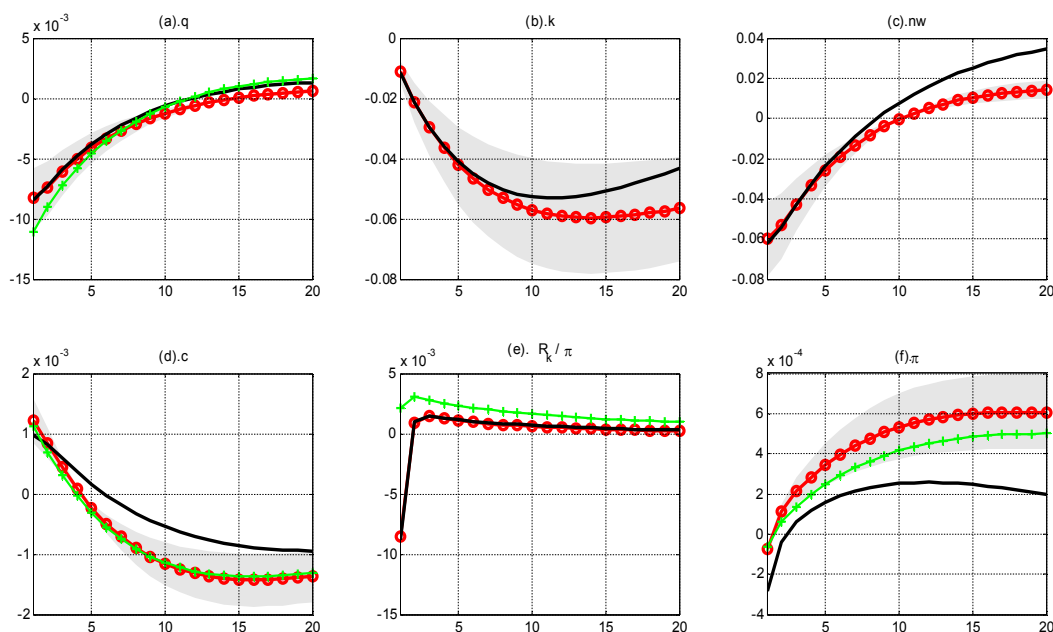


Solid line: Rational Expectations. "o" line: Adaptive learning, median and 80% range.

Effects in the Home country. As in Christiano et al. (2014), when risk is high, spreads (and EFP) are large and credit to entrepreneurs is low. With fewer financial resources, investment falls. The equilibrium quantity of capital decreases (Fig. 12, (b)). With this decline in the purchase of goods, output and employment drop (Fig. 11, (a)).

As in Bernanke et al. (1999), the net worth of entrepreneurs falls (Fig. 12, (c)). This occurs because their capital incomes fall with the decline in economic activity, and because they suffer capital losses (the price of capital drops, Fig. 12, (a)). The overall decline in economic activity results in a short lived drop in the marginal cost of production and thus, in inflation (Fig. 12, (e)). Households' consumption slightly increases (Fig. 12, (d)) because of greater profits. Terms of trade slightly deteriorate (Fig. 13, (c)) and then improve while the exchange rate depreciates. The (relative) drop in consumption triggers a trade (and current account) surplus, together with the accumulation of external assets (Fig. 13, (b)). As emphasized by Christiano et al. (2014), the risk shock reproduces a countercyclical credit spread and procyclical investment, consumption, employment, inflation, stock market, and credit. This is consistent with the data.

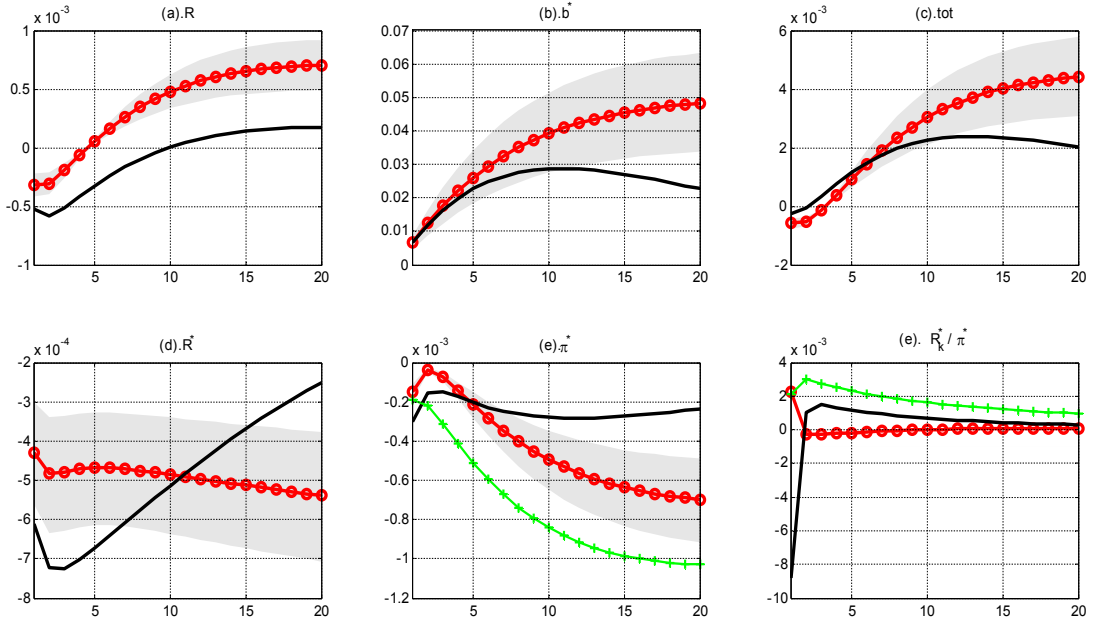
Figure 12: IRFs to a Home increase in idiosyncratic risk



Solid line: Rational Expectations. "o" line: Adaptive learning, median and 80% range. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of variable x based on PLM, median value. Only for forward-looking variables.

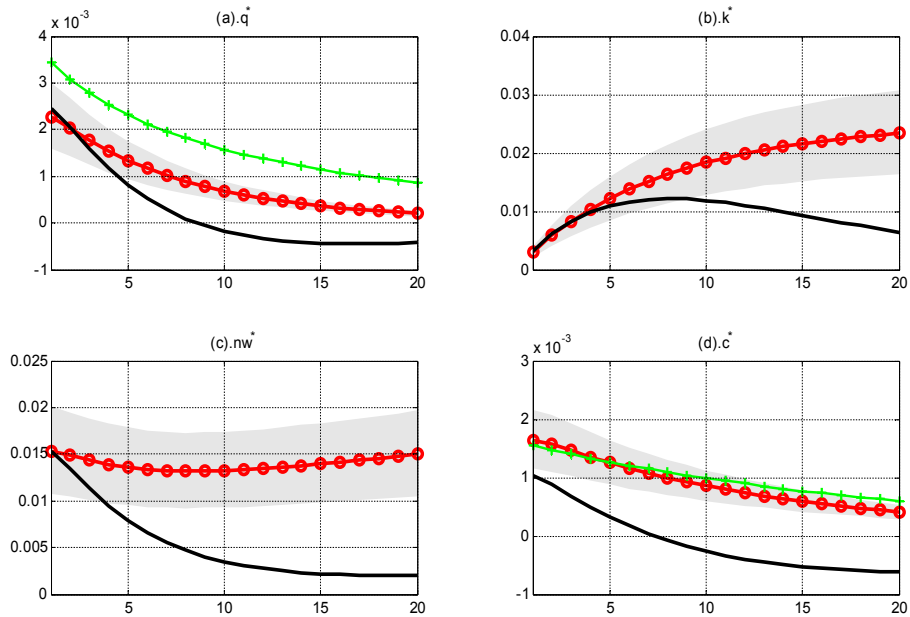
International spillovers. Lower interest rates are transmitted abroad (Fig. 13, (d)) so as to stimulate entrepreneurs' investment, capital gains and the accumulation of wealth (Fig. 14). Households consume more (Fig. 14, (d)) and work less (positive income effect for the household). Notice however that, because of lower labor effort, output decreases on impact before picking up (Fig. 11, (b)).

Figure 13: IRFs to a Home increase in idiosyncratic risk



Solid line: Rational Expectations. "o" line: Adaptive learning, median and 80% range. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of variable x based on PLM, median value. Only for forward-looking variables.

Figure 14: IRFs to a Home increase in idiosyncratic risk



Solid line: Rational Expectations. "o" line: Adaptive learning, median and 80% range. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of variable x based on PLM, median value. Only for forward-looking variables.

C.1.2 Adaptive learning

Effects in the Home country. Under learning, the effect of the uncertainty shock is amplified by more pessimistic expectations on the price of capital (Fig. 12, (a)). With pessimistic views on the future price of collateral, credit conditions deteriorate. As the price of collateral falls more, so does net worth, the expected return on capital, hence capital demand (Fig. 12, (b), (c)) and output (Fig. 11, (a)). The financial accelerator amplifies the initial adverse effects of the shocks. As production remains low for a longer time, inflation increases further (Fig. 12, (f)). Therefore, the Taylor rule entails greater policy rates than under RE (Fig. 13, (a)). Also, agents substitute current consumption with investment in the international bond (Fig. 13, (b)).

International spillovers. In the foreign country, private agents do not properly take into account the impact of the Home policy rate on their Foreign nominal interest rate, (as for the productivity shock analyzed above). As a result, the Foreign nominal interest rate remains accommodative for a longer time (Fig. 13, (d)). The effect of this more expansionary monetary policy is to stimulate the activity in the foreign country in terms of capital, net worth, consumption (Fig. 14) and output (Fig. 11, (b)).

C.2 Home negative entrepreneurs' survival shock

When more entrepreneurs default, less wealth is accumulated. This is why a negative survival shock can be interpreted as an exogenous fall in entrepreneurs' net worth. IRFs are displayed in Figures 15-18.

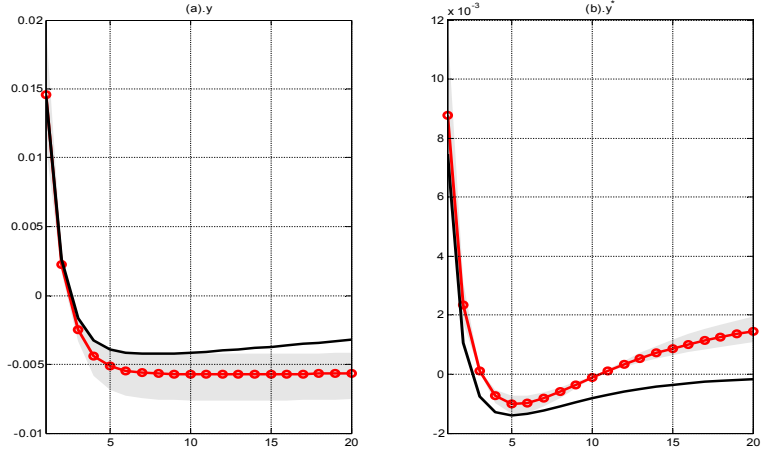
C.2.1 Rational expectations

Effects in the Home country. The shock causes financial distress at Home.¹⁷ There are less entrepreneurs alive. Therefore, net wealth goes down (Fig. 16, (c)). As a consequence, external funds are more expensive (both the spread and the EFP increase). Investment goes down, together with capital and its price (Fig. 16,(a), (b)). The financial accelerator makes the financial distress persistent because the price of collateral falls (Fig. 15, (a)). This depresses further entrepreneurs' net wealth (Fig. 16, (c)) and worsens their access to cheap lending. Consumption falls (Fig. 16, (d)) as well as investment. Aggregate demand goes down.

The fall in output (Fig. 15, (a)) outpaces the decrease in aggregate demand, which causes inflation to go up (Fig. 16, (f)) together with the deficit of both the trade balance and the current account. Because of the Taylor rule, the upward pressure on prices calls for monetary tightening (Fig. 17, (a)).

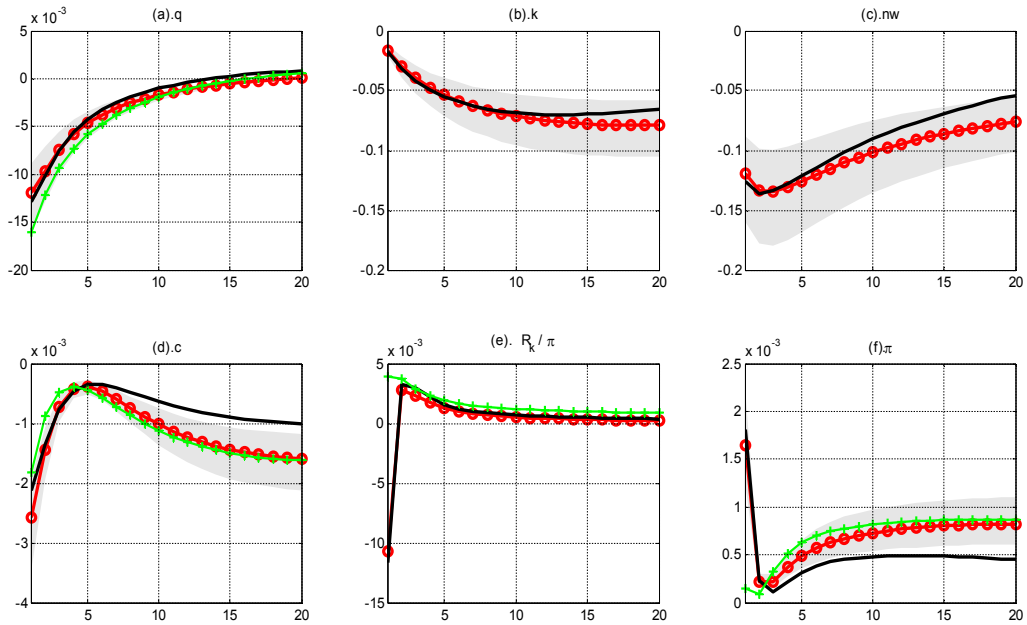
¹⁷The initial increase in output is due to the negative wealth effect on household's labor supply.

Figure 15: Output responses to a Home negative entrepreneurs' survival shock



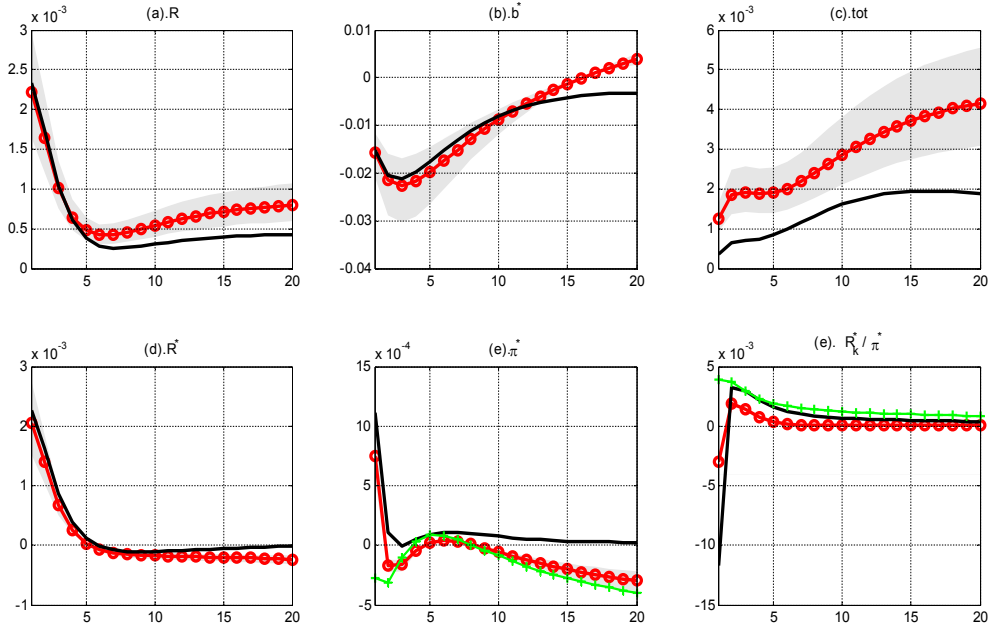
Solid line: Rational Expectations. "o" line: Adaptive learning, median and 80% range.

Figure 16: IRFs to a Home negative entrepreneurs' survival shock



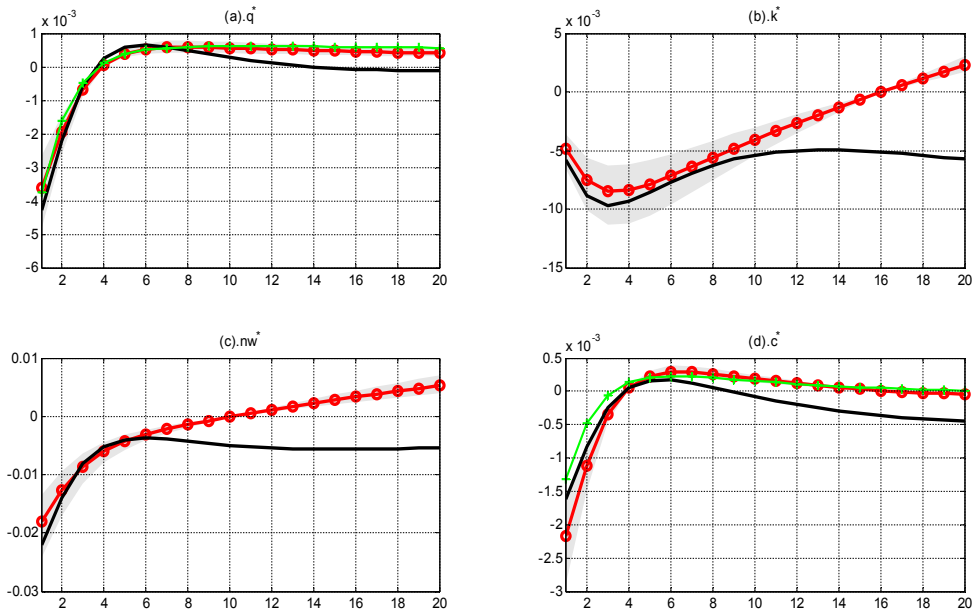
Solid line: Rational Expectations. "o" line: Adaptive learning, median and 80% range. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of variable x based on PLM, median value. Only for forward-looking variables.

Figure 17: IRFs to a Home negative entrepreneurs' survival shock



Solid line: Rational Expectations. "o" line: Adaptive learning, median and 80% range. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of variable x based on PLM, median value. Only for forward-looking variables.

Figure 18: IRFs to a Home negative entrepreneurs' survival shock



Solid line: Rational Expectations. "o" line: Adaptive learning, median and 80% range. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of variable x based on PLM, median value. Only for forward-looking variables.

Terms of trade improve (Fig. 17, (c)) and the exchange rate depreciates. The net foreign position worsens (Fig. 17, (b)). Because of the uncovered interest parity condition, foreign interest rates go up as well (Fig. 15, (d)).

International spillovers. Because of interest rates dynamics (through the UIP), entrepreneurs abroad see their EFP increase. As the external finance premium and spreads go up, investment fall together with capital and its price (Fig. 18, (a), (b)). The shock thus affects Foreign firms as a negative supply shock and Foreign prices increase (Fig. 17, (e)).

C.2.2 Adaptive learning

Effects in the Home country. Under learning, the Home agents are more pessimistic about the impact of the shock: they expect a lower price of capital (Fig. 16, (a)). As a consequence net worth and capital fall more than under RE (Fig. 16, (b),(c)). The financial accelerator is then at work with persistent pessimistic expectations on the price of collateral.

Also output falls more than in the RE case (Fig. 15, (a)), stronger inflation increases more (Fig. 16, (f)). Thus, the monetary tightening is stronger (Fig. 17, (a)).

International spillovers. In the foreign country, agents do not correctly track the spillovers deriving from the Home-country shock and underestimate the impact on their own nominal interest rate. As a result, they are more optimistic about expected capital return (Fig. 17, (f)) and the price of capital (Fig. 18, (a)). This implies a greater response of capital, net worth (Fig. 18, (b), (c)) and output (Fig. 15, (b)) under learning than under RE. As production is greater, inflation is lower (Fig. 17, (e)), and the monetary policy is more accomodative (Fig. 17, (d)).

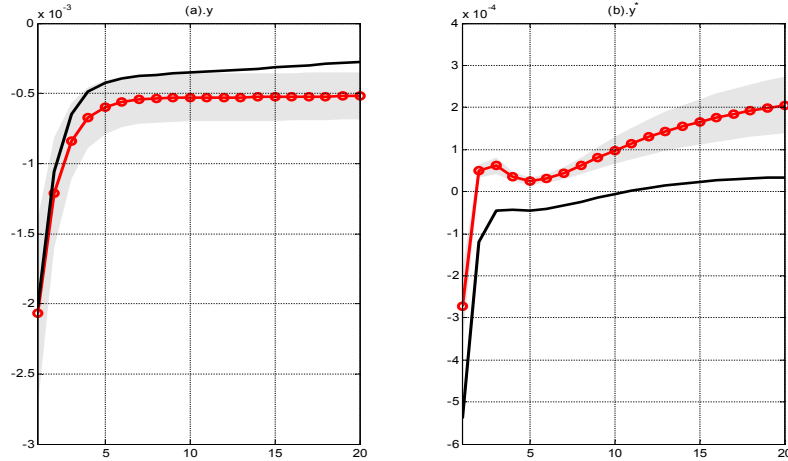
C.3 Monetary tightening: increase in Home nominal interest rate

IRFs are displayed in Figures 19-22.

C.3.1 Rational expectations

In presence of (significant) price rigidities, a monetary shock triggers an increase in the domestic interest rate (Fig. 20, (b)). This dampens domestic demand both through households consumption (Fig. 20, (d)) and lower entrepreneurs' investment (Fig. 20, (b)). The shock also dampens prices (Fig. 20, (f)) as aggregate demand falls. The lower demand for capital results in a decrease in the price of collateral (Fig. 20, (a)). Net wealth goes down (Fig. 20, (c)) due to the joint effect of the fall in the price of capital and the Fisher effect (Fig. 20, (f)), which increases the real burden of the debt. Spreads and the EFP increase because of the higher risk of default.

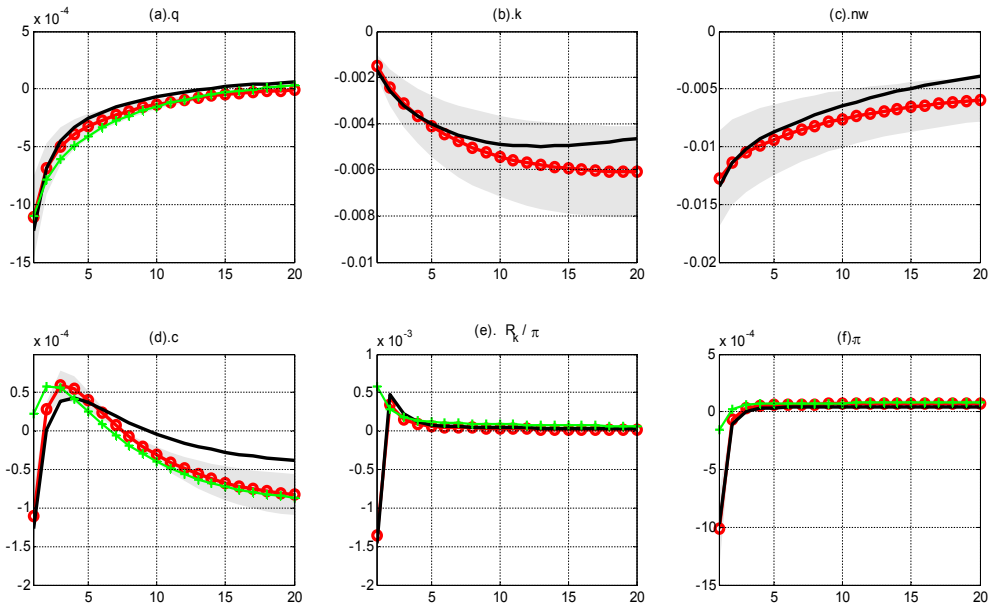
Figure 19: Output responses to an increase in Home nominal interest rate



Solid line: Rational Expectations. "o" line: Adaptive learning, median and 80% range.

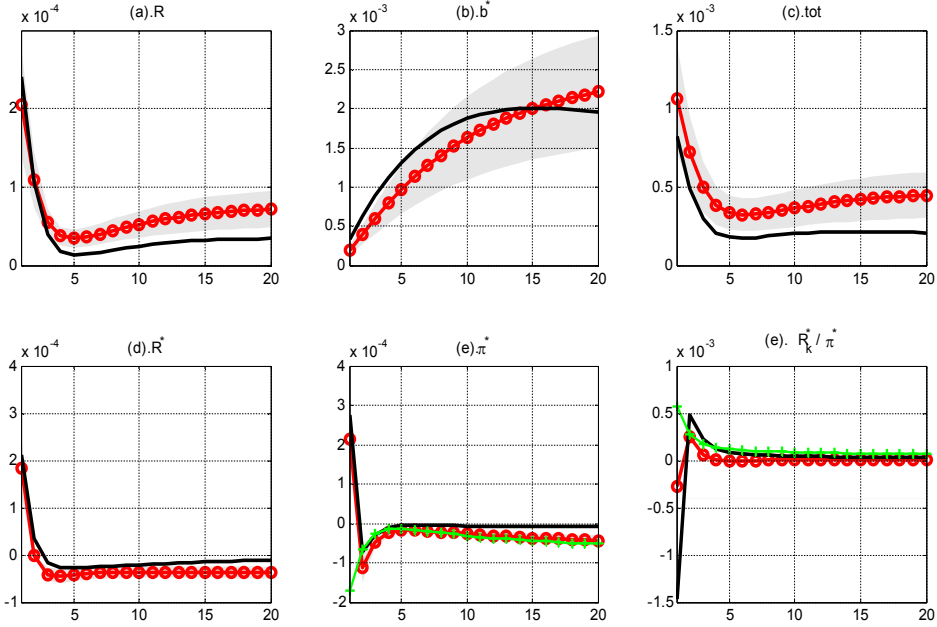
The contraction in domestic demand generates a trade surplus, i.e. international asset holdings by Home households increases. In response to the monetary tightening, the exchange rate appreciates (Fig. 21, (b)), consistently with the results in Kollmann (2001) and Christiano et al. (2011). Thus, terms of trade improve (Fig. 21, (c)). The increase in interest rates is transmitted abroad with analogous recessionary effects through UIP (Fig. 21, (d)). Monetary tightening abroad increases the cost of loans, which spreads the Home recession to the Foreign country.

Figure 20: IRFs to an increase in Home nominal interest rate



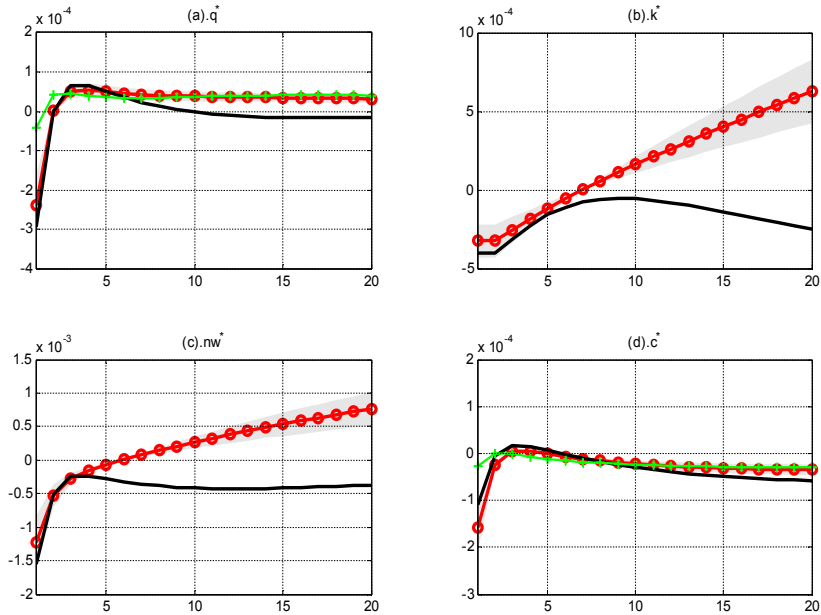
Solid line: Rational Expectations. "o" line: Adaptive learning, median and 80% range. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of variable x based on PLM. median value. Only for forward-looking variables.

Figure 21: IRFs to an increase in Home nominal interest rate



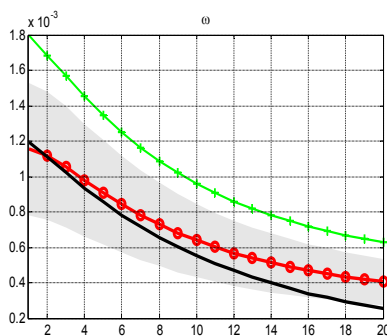
Solid line: Rational Expectations. "o" line: Adaptive learning, median and 80% range. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of variable x based on PLM, median value. Only for forward-looking variables.

Figure 22: IRFs to an increase in Home nominal interest rate



Solid line: Rational Expectations. "o" line: Adaptive learning, median and 80% range. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of variable x based on PLM, median value. Only for forward-looking variables.

Figure 23: IRFs to an increase in Home nominal interest rate



Solid line: Rational Expectations. "o" line: Adaptive learning, median and 80% range. "+" line: Adaptive learning, $E_t[x_{t+1}]$ forecast of variable x based on PLM, median value. Only for forward-looking variables.

C.3.2 Adaptive learning

Under learning, agents expect that a monetary policy tightening in the Home country has a more negative impact on the default threshold (Fig.23).¹⁸ Notice that the threshold level of productivity is the endogenous outcome of an optimal contract pinning down it together with the borrowing rate. The greater the risk of default, the larger the premium that borrowers need to pay. Thus, misperceptions about the default threshold are linked to expectations on borrowing rates. The expectation of a stronger increase in the default's threshold is a signal to the entrepreneur that the chances of bankruptcy go up. The higher risk of default increases the external finance premium. Entrepreneurs hold a pessimistic view on future credit conditions, which amplify the recessionary impact of monetary tightening. The economy under learning is characterized by lower price of capital, investment, net worth (Fig.20). Expecting stronger inflation (Fig.20, (f)) pushes inflation a touch higher than in the RE case. This triggers a tighter policy rate (Fig.21, (a)). Therefore, consumption drops longer (Fig.20, (d)) together with output (Fig.19,(a)). As the drop in demand outpaces the drop in output, the economy experiences a trade surplus and an increase of foreign assets (Fig.21, (b)).

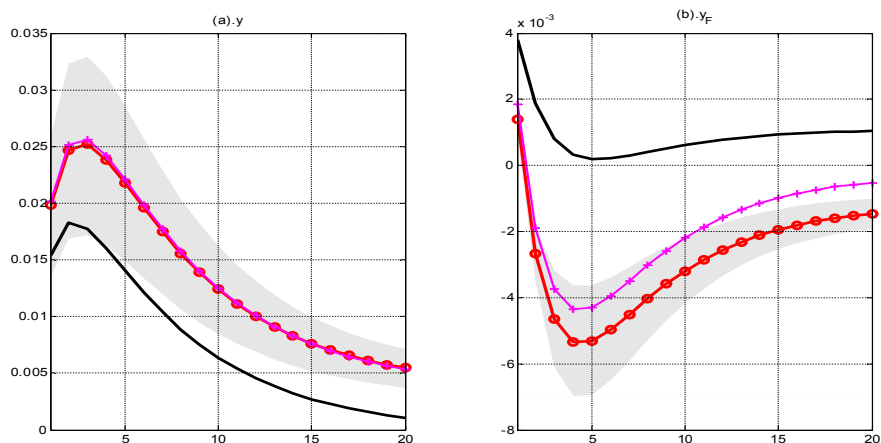
In the foreign country agents expect a less tight response of monetary policy (Fig.21, (d)) and this has a more positive effect on capital accumulation, net worth, consumption and output (Fig.22).

¹⁸In the previous section, for the sake of brevity, we did not display expectations on the default threshold $\tilde{\omega}$ because the mis-perceptions on this variable were behaving like other expectations on asset price or real return on capital.

D IRFs after a Home technological shock with an information set that includes the other country's nominal interest rate

In the domestic country, the macroeconomic effects under learning with or without information sharing are barely different (Figures 24 and 25). This illustrates that the inclusion of the other country's nominal interest rate in Home agents' information set does not matter. It matters only to the extent that agents are concerned with international spillover of shocks occurring in the trading partner. In the foreign country, the negative international transmission of the Home country shock is dampened when Foreign private agents include the Home nominal interest rate in their information set. The macroeconomic dynamics under learning move closer to the one observed under rational expectations, with reduced volatility of Foreign macroeconomic variables when Foreign agents take into account the Home nominal interest rate in their perceived law of motion.

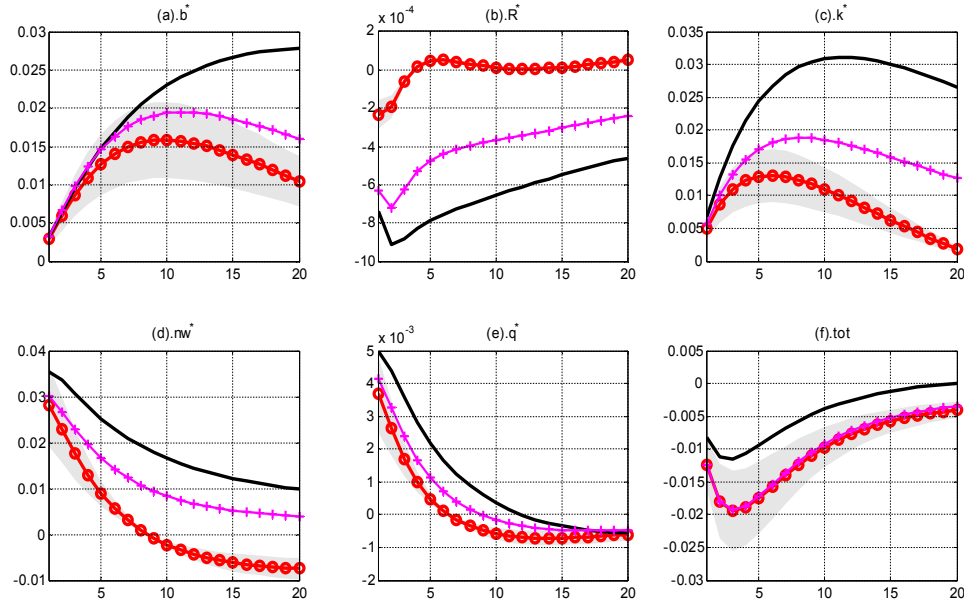
Figure 24: IRFs to a positive Home technological shock



Solid line : Rational Expectations. "o" line : Adaptive learning, median and 80% range. "+" line : Adaptive learning, information set includes nominal interest rate in the other country. Median.

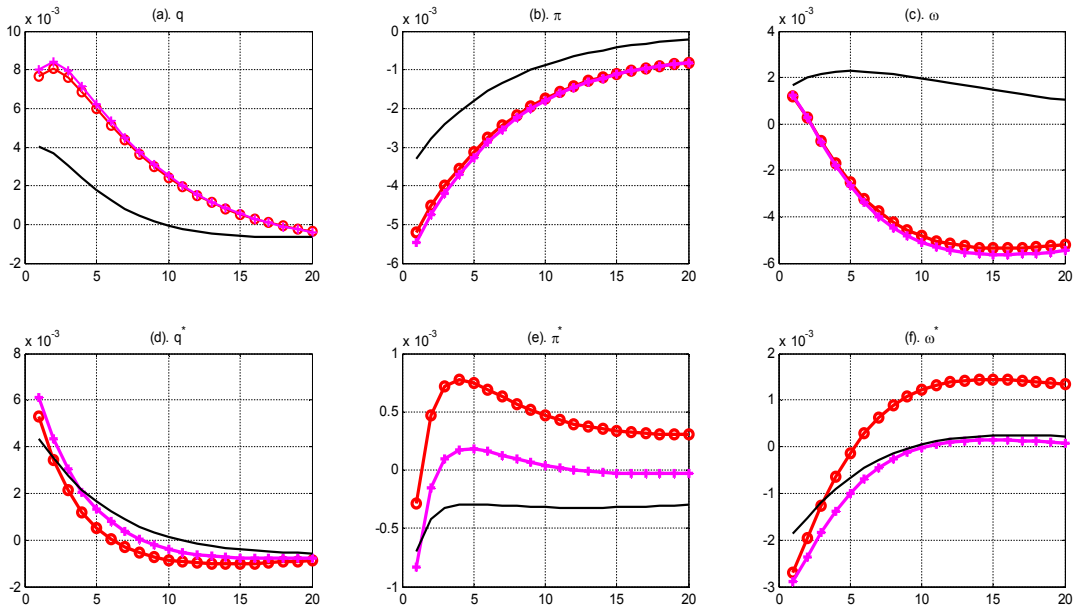
After a domestic technology shock the knowledge of the other country's monetary policy changes the behaviour of foreign agents. As they now observe a decrease in the home policy rate, they understand the positive spillovers to the foreign country and their demand for goods increases, driving foreign inflation to increase less. The foreign policy rate, reacting to inflation, decreases more and this has a more expansionary effect on all the other variables in the foreign economy. Foreign expectations are more in line with the ones obtained under rational expectations (Figure 26, panels (d), (e), (f)).

Figure 25: IRFs to a positive Home technological shock



Solid line : Rational Expectations. "o" line : Adaptive learning, median and 80% range. "+" line : Adaptive learning, information set includes nominal interest rate in the other country. Median.

Figure 26: IRFs to a positive Home technological shock: 1-step ahead forecast $E_t[x_{t+1}]$



Solid line : Rational Expectations. "o" line : Adaptive learning, median and 80% range. "+" line : Adaptive learning, information set includes nominal interest rate in the other country. Median.