Risk Sharing with Excludable and Non-Excludable Common Goods

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Introduction

This project brings together two literatures:

1. **Environmental literature** on the co-operation over the common good's use and the associated tragedy of the commons.
2. **Risk sharing literature** with limited commitment (starting with Kehoe and Levine (1993) and Kocherlakota (1996)).
    - Models tested empirically with data from traditional villages engaging into risk sharing (e.g. Udry (1994), Ligon, Thomas and Worrall (2002)).

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Introduction

This project: investigate theoretically the relationship between risk sharing and co-operation over the common goods in presence of voluntary participation constraints.

Key ingredients:

1. Production economy with limited commitment (e.g. Kehoe and Perri (2002)).
2. Common good (à la Nordhaus and Boyer (2000) or Golosov, Hassler, Krusell and Tsyvinski (2014)).

Preview of results

- Risk sharing helps sustaining co-operation over the common good.
  - like transfers from richer to poorer countries, linking participation with trade agreements, punishments upon deviation etc.
- Unlike these, risk sharing improves efficiency of the production rights assignment.
- Internalizing the externalities increases the aggregate output and so improves risk sharing.
- Complementarity?
- Outside option important with limited commitment. Even more so in presence of externalities, many (>2) agents and associated incentives to free ride: analyze the role of excludability from the commons.

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Outline of the talk

1. Introduction
2. Model economy with two agents: the mechanism
   - Allocations without risk sharing
   - Voluntary co-operation with risk sharing
3. Model with many agents: the role of excludability
4. Decentralizing the constrained efficient allocation
5. Conclusion and policy discussion

Model economy with 2 agents

- Infinite horizon, discrete time, 2 risk averse agents with $U(c_{i,t}, n_{i,t}) = \log\left(c_{i,t} - n_{i,t}^{\phi}\right)$.
- No saving or borrowing.
- Ex-post heterogeneity due to i.i.d. markovian shocks to productivity $\theta_{i,t} \in \Theta = \left(\theta^1, ..., \theta^{N_\theta}\right)$, $0 < \theta^1 < ... < \theta^{N_\theta} < \infty$.
- Perfect observability.
- Production $y_{i,t} = (1 - D(P_t))\,\theta_{i,t} n_{i,t}$ (à la Golosov et al. (2014)).
- Status of the common good: $1 - D(P_t)$, with $P_t = P_{1,t} + P_{2,t}$.
- Assumptions: $D'(P_t) > 0$, $P'(n_{i,t}), P''(n_{i,t}) > 0 \Rightarrow \frac{\mathrm{d}(1-D(P_t))}{\mathrm{d}n_{i,t}} < 0$.
- Technical assumption: $\frac{\mathrm{d}^2(1-D(P_t))}{\mathrm{d}^2 n_{i,t}} \leq 0$.

Outline

Autarky

Autarkic value $V_{i,t}^{aut}(\theta^t)$ to each agent i given by solution to:

$$\max_{\{c_{i,t},n_{i,t}\}} \mathrm{E}\left[\sum_{s=t}^{\infty} \beta^{s-t}\left(\log\left(c_{i,s} - n_{i,s}^{\phi}\right)\right)|\theta^t\right]$$

subject to:

$$\begin{aligned} c_{i,s} &\leq (1 - D(P_s))\,\theta_{i,s} n_{i,s}\ \forall s \geq t \\ n_{i,s} &\in [0,1] \end{aligned}$$

FOC wrt n:

$$(1 - D(P_s))\,\theta_{i,s} + \frac{\mathrm{d}(1 - D(P_s))}{\mathrm{d}n_{i,s}}\theta_{i,s} n_{i,s} = \phi\, n_{i,s}^{\phi-1}$$

Tragedy of the commons - *private* vs *social* marginal costs:

$$\frac{\mathrm{d}(1 - D(P_s))}{\mathrm{d}n_{i,s}}\theta_{i,s} n_{i,s} \text{ vs } \frac{\mathrm{d}(1 - D(P_s))}{\mathrm{d}n_{i,s}}(\theta_{1,s} n_{1,s} + \theta_{2,s} n_{2,s})$$

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Voluntary co-operation over the CG

The centralized problem of co-operation among 2 agents:

$$\max_{\left\{c_{i,t},n_{i,t}\right\}_{t\geq 0}} \sum_{i=1}^{2}\sum_{t=0}^{\infty}\sum_{\theta^t}\lambda_i\beta^t\pi\left(\theta^t\right)\log\left(c_{i,t}-n_{i,t}^{\phi}\right)$$

subject to:

$$\left(\zeta_i\left(\theta^t\right)\right)\;\; c_{i,t}\leq\left(1-D\left(P_t\right)\right)\theta_{i,t}n_{i,t}\;\forall i,t$$

$$\left(\mu_i\left(\theta^t\right)\right)\;\; \sum_{s=t}^{\infty}\sum_{\theta^s}\beta^{s-t}\pi\left(\theta^s|\theta^t\right)\log\left(c_{i,s}-n_{i,s}^{\phi}\right)\geq V_{i,s}^{aut}\left(\theta^s\right)\;\forall i \textit{ and } s\geq t$$

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The centralized problem of co-operation among 2 agents:

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Voluntary co-operation over the CG (cont.)

Formulate recursive Lagrangian using Marcet and Marimon's (2017) method:

$$\inf_{\mu_i \geq 0} \sup_{\left\{c_{i,s}, n_{i,s}\right\}_{s \geq t, i \in \{1,\ldots,N\}}} \sum_{i=1}^{2} \sum_{t=0}^{\infty} \sum_{\theta^t} \beta^t \pi\left(\theta^t\right) \{ M_i\left(\theta^{t-1}\right) \log\left(c_{i,t} - n_{i,t}^{\phi}\right)$$
$$+\mu_i\left(\theta^t\right)\left[\log\left(c_{i,t} - n_{i,t}^{\phi}\right) - V_{i,t}^{aut}\left(\theta^t\right)\right] + \zeta_i\left(\theta^t\right)\left[\left(1 - D\left(P_t\right)\right)\theta_{i,t} n_{i,t} - c_{i,t}\right]\}$$

where $M_i\left(\theta^t\right) = M_i\left(\theta^{t-1}\right) + \mu_i\left(\theta^t\right)$ and $M_i\left(\theta^{-1}\right) = \lambda_i$. The FOCs read:

$$\begin{aligned}
\frac{c_{1,t} - n_{1,t}^{\phi}}{c_{2,t} - n_{2,t}^{\phi}} &= \frac{\frac{1}{\zeta_1}\left(M_1\left(\theta^{t-1}\right) + \mu_1\left(\theta^t\right)\right)}{\frac{1}{\zeta_2}\left(M_2\left(\theta^{t-1}\right) + \mu_2\left(\theta^t\right)\right)} \ \forall i,j \\
\phi n_{i,t}^{\phi-1} &= \left(1 - D\left(P_t\right)\right)\theta_{i,t} + \frac{\mathrm{d}\left(1 - D\left(P_t\right)\right)}{\mathrm{d}n_{i,t}} \sum_{j=1}^{2} \frac{\zeta_j}{\zeta_i} \theta_{j,t} n_{j,t} \ \forall i
\end{aligned}$$

Voluntary co-operation over the CG (cont.)

FOCs again:

$$
\begin{aligned}
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\end{aligned}
$$

Implications:

- Co-operation leads to some internalization of externalities and so increases the aggregate production.
- However, it may be unstable.
- PC binding ($\mu_i > 0$) when shock realization far apart.
- Participation can be induced via adjustments in the production rights.
- Thus, allocation is inefficient.

Outline

Constrained Efficient Allocation

The centralized problem of the constrained efficient contract among 2 agents:

$$\max_{\left\{c_{i,t},n_{i,t}\right\}_{t\geq 0}} \sum_{i=1}^{2}\sum_{t=0}^{\infty}\sum_{\theta^t}\lambda_i\beta^t\pi\left(\theta^t\right)\log\left(c_{i,t}-n_{i,t}^{\phi}\right)$$

subject to:

$$\left(\zeta\left(\theta^t\right)\right)\ \ c_{1,t}+c_{2,t}\leq\left(1-D\left(P_t\right)\right)\left(\theta_{1,t}n_{1,t}+\theta_{2,t}n_{2,t}\right)\ \forall i,t$$

$$\left(\mu_i\left(\theta^t\right)\right)\ \ \sum_{s=t}^{\infty}\sum_{\theta^s}\beta^{s-t}\pi\left(\theta^s|\theta^t\right)\log\left(c_{i,s}-n_{i,s}^{\phi}\right)\geq V_{i,s}^{aut}\left(\theta^s\right)\ \forall i \text{ and } s\geq t$$

Constrained Efficient Allocation (cont.)

FOCs read:

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\end{aligned}$$

vs FOCs without risk sharing:

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Implications:

- Co-operation leads to full internalization of externalities ($\Rightarrow$ aggregate production above the level from co-op. w/o risk sharing).
- Stability improved: unilateral deviation implies total autarky $\Rightarrow$ only the high productivity type's PC can be binding ($\mu_i > 0$).

Complementarity between risk sharing and common good Def :

- Risk sharing allows for sustainable co-operation over the common good's use.
- Solving the tragedy of the commons increases the total output and so improves risk sharing.

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Model with many agents: the role of excludability

- $\bar{N}$ risk averse agents with or without access to common good; $N \leq \bar{N}$ agents co-operating over common good (and possibly risk sharing).
- Denote by $\mathcal{N}$ ($\mathcal{N}^{nc} = \bar{\mathcal{N}} \backslash \mathcal{N}$) the set of co-operating (non co-op.) agents with cardinality $|\mathcal{N}| = N$ ($|\mathcal{N}^{nc}| = \bar{N} - N$).
- State of the common good:
  - if non-excludable: $(h(|\mathcal{N}|) - D(P_t(\mathcal{N}, \mathcal{N}^{nc})))$ for everyone;
  - if excludable: $(h(|\mathcal{N}|) - D(P_t(\mathcal{N}, \emptyset)))$ for co-operators,

    $(h(1) - D(P_t(\emptyset, \mathcal{N} + \mathcal{N}^{nc})))$ for non co-operators.
- Assume: The CG state function is increasing in number of co-op. agents, i.e. if $\mathcal{N} \subset \mathcal{N}'$ (and so $\mathcal{N}^{nc\prime} \subset \mathcal{N}^{nc}$), then:
  - under excludability:
    $[h(|\mathcal{N}'|) - D(P_t(\mathcal{N}', \emptyset))] > [h(|\mathcal{N}|) - D(P_t(\mathcal{N}, \emptyset))]$;
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Model with many agents: the role of excludability

- $\bar{N}$ risk averse agents with or without access to common good; $N \leq \bar{N}$ agents co-operating over common good (and possibly risk sharing).
- Denote by $\mathcal{N}$ ($\mathcal{N}^{nc} = \bar{\mathcal{N}} \backslash \mathcal{N}$) the set of co-operating (non co-op.) agents with cardinality $|\mathcal{N}| = N$ ($|\mathcal{N}^{nc}| = \bar{N} - N$).
- State of the common good:
  - if non-excludable: $(h(|\mathcal{N}|) - D(P_t(\mathcal{N}, \mathcal{N}^{nc})))$ for everyone;
  - if excludable: $(h(|\mathcal{N}|) - D(P_t(\mathcal{N}, \emptyset)))$ for co-operators,

    $(h(1) - D(P_t(\emptyset, \mathcal{N} + \mathcal{N}^{nc})))$ for non co-operators.
- Assume: The CG state function is increasing in number of co-op. agents, i.e. if $\mathcal{N} \subset \mathcal{N}'$ (and so $\mathcal{N}^{nc\prime} \subset \mathcal{N}^{nc}$), then:
  - under excludability:
    $[h(|\mathcal{N}'|) - D(P_t(\mathcal{N}', \emptyset))] > [h(|\mathcal{N}|) - D(P_t(\mathcal{N}, \emptyset))]$;
  - under non-excludability:
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Model with many agents: the role of excludability

As the set of co-operators $\mathscr{N}$ gets larger:

- Value of CG+RS co-operation goes up due to:
  - better consumption smoothing;
  - improvement in the state of the common good.
- Value of autarky:
  - if CG non-excl.: goes up due to improvement in the state of the CG;
  - if CG excl.: constant.

Implications:

- with excludability there is complementarity between risk sharing and co-operation over the common good;
- with non-excludability:
  - extent of risk sharing may be non-monotone in $|\mathscr{N}|$;
  - risk sharing and solving the common good problem may become substitute to each other;
  - existence of a non-autarkic solution among many agents not guaranteed (similar to Genicot and Ray (2003)).

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Cap-and-trade decentralization: definition

Definition: An eq-m with $\{B_i, \bar{P}\}$, for initial conditions $\{a_{i,0}, \tilde{a}_{i,0}\}$ has quantities $\{c_i, a_i, \tilde{a}_i, n_i\}$ and prices $\{q, \tilde{q}\}$ s.t.:
(i) for each $i \in \{1, ..., N\}$, $\{c_i, a_i, \tilde{a}_i, n_i\}$ solve:

$$\begin{aligned} V_{i,t}\left(a_{i,\theta^t}, \theta^t\right) = \max \log\left(c_{i,t} - n_{i,t}^{\phi}\right) & + & \beta \sum_{\theta' \in \Theta} \pi\left(\theta' | \theta_t\right) V_{i,t+1} \\ s.t.:\ (\psi) \quad a_{i,\theta'} & \geq & B_{i,t+1}\left(\theta^t, \theta'\right)\ \forall \theta' \in \Theta \\ (\phi) \quad \tilde{a}_{i,\theta_t} & \geq & P_{i,t} \\ c_{i,t} + \sum_{\theta'} q_t\left(\theta^t, \theta'\right) a_{i,\theta'} + \tilde{q}_t\left(\theta^t\right) \tilde{a}_{i,\theta_t} & \leq & y_{i,t} + a_{i,\theta^t} \end{aligned}$$

(ii) markets clear:

$$\begin{aligned} \sum_i c_{i,t} & = & \sum_i y_{i,t}\ \forall \theta^t \\ \sum_i a_{i,\theta'} & = & 0\ \forall \theta' \\ \sum_i \tilde{a}_{i,\theta^t} & = & \bar{P}\left(\theta_t\right)\ \forall \theta_t \end{aligned}$$

(ii) $\bar{P}\left(\theta_t\right)$ is set optimally to $\sum_{i=1}^{N} P_{i,t}\left(\theta_t\right)$ from the planner's problem.

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Cap-and-trade decentralization: implications

- Constraint $\tilde{a}_{i,\theta_t} \geq P_{i,t}$ will always hold with equality as by construction the pollution permits are perishable.
  - Why do firms save using permits? Missing markets? Firms' informational constrains?
- Decentralization gives micro-foundations for the proposal of the Market Stability Reserve:
  - Market clearing condition $\sum_i \tilde{a}_{i,\theta^t} = P(\theta_t)\ \forall \theta_t$ means that the total number of pollution permits should be varying together with the aggregate state of the economy - in line with the purpose of the MSR.

Conclusion and policy discussion

This project shows that:

1. Risk sharing facilitates co-operation over the common goods and ensures efficient assignment of production rights.
2. Excludability allows for mutual reinforcement of the two institutions.
3. Underlying common goods may have important implications for the extent of sustainable risk sharing.

Policy implications (e.g. if EU introduces E-UI or deposit insurance):

1. if possible, ensure (credible) exclusion from common goods (e.g. common trade or labor market).
2. If exclusion not possible (e.g. environment or peace), take into account these common goods when deciding about the extent of risk sharing.

Defining complementarity

Definition of complementarity (see e.g. Vives (2005)):

$$\sum_{i=1}^{N} u^{RS}(c_i) - \sum_{i=1}^{N} u^{\emptyset}(c_i) < \sum_{i=1}^{N} u^{RS+CG}(c_i) - \sum_{i=1}^{N} u^{CG}(c_i)$$

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