Financial Shocks and Job Flows[*]

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Abstract

The labor market recovery since the end of the Great Recession has been characterized by a marked decline in labor market turnover. In this paper, we provide evidence that the housing crisis and financial nature of the Great Recession account for this decline in job flows. We exploit MSA-level variation in job flows and housing prices to show that a decline in housing prices diminishes job creation and lagged job destruction. Moreover, we document differences across firm size and age categories, with middle-sized firms (20-99 employees) and new and young firms (firms less than 5 years of age) most sensitive to a decline in house prices. We propose a quantitative model of firm dynamics with collateral constraints, calibrating the model to match the distribution of employment by firm size and age. Financial shocks in our firm dynamics model depresses job creation and job destruction and replicates the empirical pattern of the sensitivity of job flows across firm age and size categories.

Keywords: Job flows, financial frictions.
JEL Classification: E44, J60

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1 Introduction

The labor market recovery since the trough of the Great Recession in 2009 has been characterized by a pronounced decline in labor market churn. As shown in Figure 1, job creation and job destruction during the recovery have fallen relative to their pre-recession averages, despite positive net job creation during each period.[1]

Figure 1: US job flows
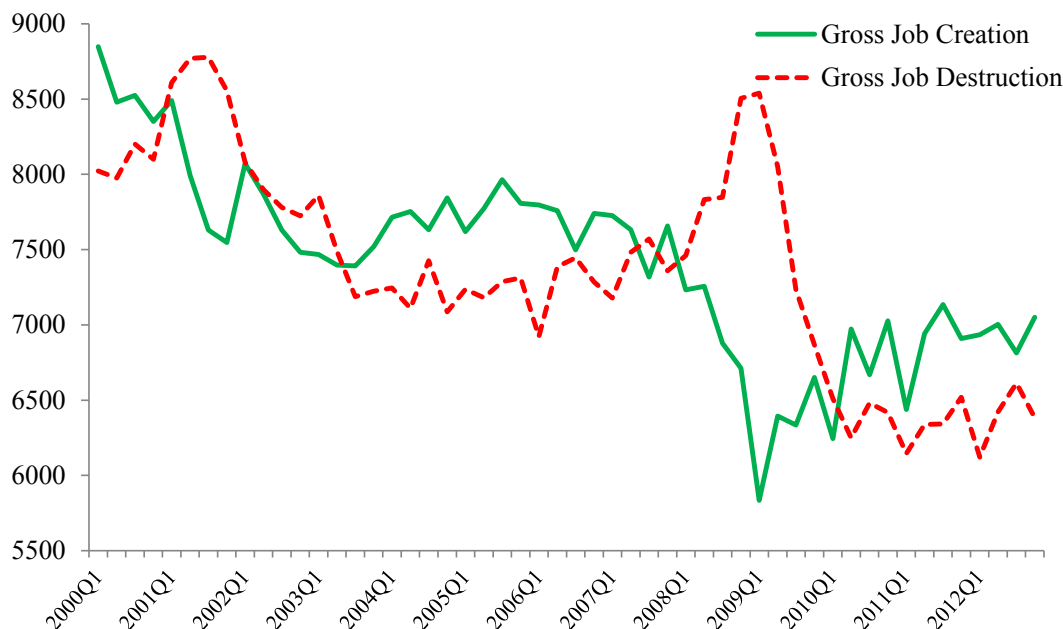
The figure shows aggregate US job flows for 2000Q1-2012Q4 from the Business Employment Dynamics.

An extensive literature has documented the importance of labor market turnover in the process of labor reallocation and productivity growth[2]. Moreover, higher job flow rates in the time series and cross section typically coincide with a healthier labor market, characterized by higher levels of employment growth and lower unemployment rates. The decline in job flows exhibited in the Great Recession may provide clues explaining the slow recovery in the labor market.

Recent work by Haltiwanger, Jarmin and Miranda (2012) demonstrates that new and young firms account for a disproportionate share of job creation and job destruction due to their "up and out" pattern of expansion. Using 2000-2006 averages from the Business Dynamics Statistics, new firms account for about 3% of employment but for nearly 17% of job creation. Similarly, young firms (defined as firms 5 years or younger) account for 12% of employment but 16% of job creation and 21% of job destruction respectively. Given the disproportionate contribution of new and young

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[1]Haltiwanger, Jarmin and Miranda (2011) also document the sharp fall in job creation in the Great Recession, and Haltiwanger, Jarmin and Miranda (2012) show an acceleration in the decline of employment and job creation among young firms.

[2]For an overview, see Haltiwanger (2012) and Davis and Haltiwanger (1999)

Figure 2: Change in job creation and house prices
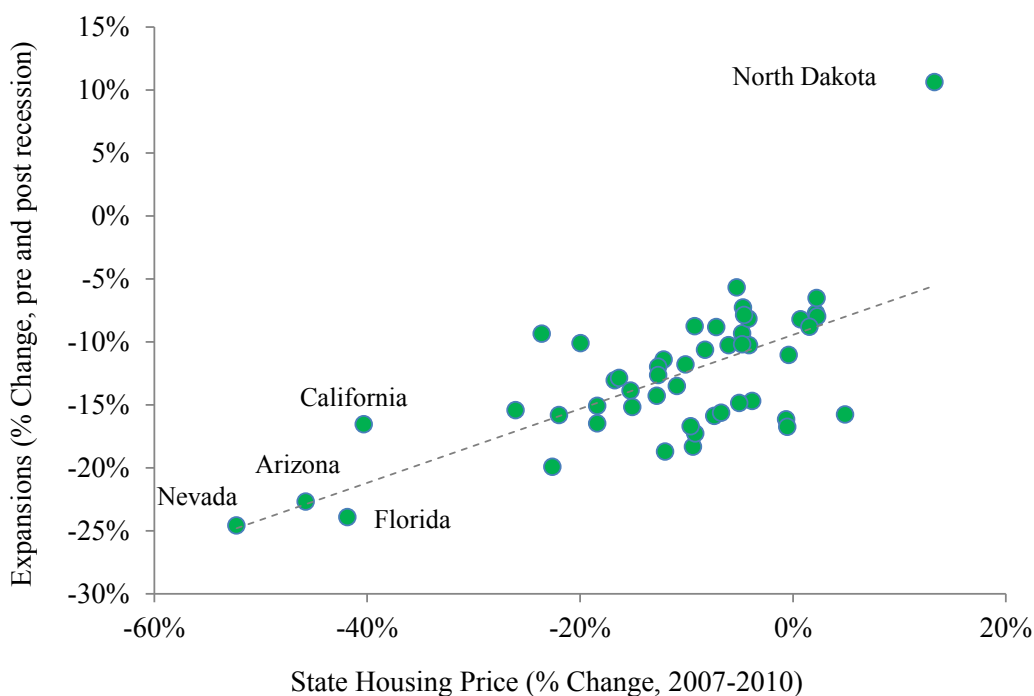
The figure shows the change in job creation and the change in house prices across US states. Change in job creation is measured as percent change in job creation at expanding establishments taking the post-recession (2010-2012) average relative to the prerecession average (2004-2006).

firms to labor market churn, some economists in the business press have speculated that the credit crisis may have had a disproportionate impact on young businesses. They argue that a decline in house prices may impair the formation of new firms and the expansion of existing firms by reducing the value of collateral and thereby restricting their access to external finance. Figure 2 provides some suggestive support for this hypothesis, showing a strong correlation between the decline in house prices at the state level and the decline in job creation from expanding establishments as measured in the Business Employment Dynamics.

In this paper, we argue in favor of this hypothesis - that the financial crisis and sharp decline in housing prices explain the large and persistent decline in job creation and job destruction. In particular, we argue that a collateral shock tightens the availability of credit for new firms and young expanding firms leading to a decline in aggregate job creation and job destruction. We investigate this hypothesis both empirically and theoretically.

In our empirical work, we provide direct evidence that a decline in collateral values diminishes job creation and lagged job destruction using MSA-level data from the Business Dynamics Statistics (BDS). We exploit MSA-level variation in job flows and housing prices to examine the effects of movements in MSA housing prices on job flows. To address issues of causality, we control for MSA

and time fixed effects and add direct controls for the business cycle. We also utilize an IV approach based on differences across MSA in their sensitivity to movements in aggregate US house prices. This land supply elasticity approach - used in the literature to examine the effect of collateral shocks on real variables - is applied here to examine the effect of housing prices on job flows.

To analyze the theoretical effect of a collateral shock on job flows and employment, we build a model of firms dynamics with financial frictions and decreasing returns to scale. Newly born firms and young firms accumulate assets and expand towards their efficient scale. Mature firms are financially unconstrained and are free to expand or contract subject to idiosyncratic shocks to firm productivity. Firms differ in productivity levels so that some businesses remain small without any binding financial constraint - not all small firms are financially constrained in our model. Our model is calibrated to target the average size and age distribution of employment in the Business Dynamics Statistics.

Our empirical results show that a shock to housing prices reduces job creation persistently and reduces job destruction with a lag. These results hold under both the OLS and IV specifications and are robust to alternative controls for the MSA business cycle. Moreover, we document differences across firm age and size categories in the sensitivity of job flows to housing price shocks. In particular, we find that job creation for middle-sized firms (firms with 20-99 employees) and new and young firms exhibit the strongest sensitivity to housing price declines. Similar patterns hold for job destruction with middle-size firms and young firms (less than 5 years old) exhibiting a lagged decline in job destruction in response to a decline in housing prices.

Using our firm dynamics model, we show that collateral shocks diminish job creation and job destruction over the transition. Moreover, the collateral shock replicates the empirical pattern of job flows sensitivity across firm size and age, generating firm size and age patterns that are qualitatively different from productivity shocks. Collateral shocks have relatively stronger effects on aggregate job creation while productivity shocks have a stronger effect on job destruction. The collateral shock diminishes job creation for larger firm size categories by reducing employment demand at credit-constrained firms and shifting the size distribution towards smaller firms. This effect is partially offset by the general equilibrium effect of a decline in wages which leads unconstrained firms to expand. As a result, job flows for middle-sized firms exhibit the greatest sensitivity to housing price shocks. Likewise, a tightening collateral constraint reduces job creation for new and young firms by increasing the required asset level to achieve a given level of employment. Collateral shocks diminish job destruction by shrinking the size of firms leading to lower job destruction from firm deaths. Our model highlights the various mechanisms culminating in lower job flows from a collateral tightening and supports the conclusion that the empirical pattern we document reflects financial shocks rather than productivity shocks.

1.1 Related Literature

Our paper is related to several strands of literature. Firstly, our empirical work is related to a literature documenting the real effect of housing price shocks. Recent papers by Gan (2007) and Chaney, Sraer and Thesmar (2012) examine the effect of collateral shocks on firm investment. Chaney, Sraer and Thesmar (2012) use firm-level financial data to show that a decline in the value of real estate for a firm's headquarters has a statistically significant effect of firm investment. Adelino, Schoar and Severino (2013) documents that small business starts and employment levels showed a strong sensitivity to increases in housing prices during the boom years from 2002-2007. Both papers use the land supply elasticity instruments proposed in Saiz (2010), and our IV strategy follows a similar approach. Our paper differs from this literature in its focus on both job creation and job destruction patterns instead of employment growth or unemployment rates.

Likewise, our work draws on and contributes to an empirical literature documenting differences in job flows across firm size and age categories. The influence of startups and young firms on job creation and job destruction is documented in Haltiwanger, Jarmin and Miranda (2010), but we establish facts about the sensitivity of job flows across firm size and age to housing price shocks. Our empirical work is closest to contemporaneous work by Fort et al. (2012) that examines the cyclical role of housing prices on employment and job flows. Our results are consistent with their results and differ primarily in the use of an instrument variables approach to establish the causal effect of housing price shocks on job flows by firm size and age. Additionally, we reconcile the empirical patterns we document with a firm dynamics model with financial frictions.

Finally, our work is related to an emerging literature on quantitative firm dynamics models. Our model comes closest to Khan and Thomas (2011) who study the effect of a credit shock in a model with collateral constraints and firm-specific capital. They find that credit shock recessions behave differently than productivity driven recessions, but they do not explore the implications for their mechanism on job flows. Also, the mechanism at work in our model is similar to the type of financial frictions emphasized in recent firm dynamics models by Siemer (2013) and Schott (2013). However, we focus on the job flows effect of financial shocks and identifying financial shocks relative to productivity shocks whereas these papers focus primarily on explaining jobless recoveries. Gomes (2001) and Cooley and Quadrini (2001) also build firm dynamics models with various financial frictions to fit facts on the firm age and firm size distribution and stylized facts about the financing of small versus large businesses.

The paper is organized as follows: Section 2 discusses our data and presents empirical results on the link between collateral values and job creation and job destruction. Section 3 presents a simple continuous time firm dynamics model and characterizes firm behavior. Section 4 discusses our benchmark discrete time firm dynamics model. Section 5 describes our calibration strategy and shows the quantitative effect of collateral shock and productivity shocks. Section 6 concludes.

2 Empirical Strategy and Results

2.1 Empirical Strategy

Any test of the hypothesis that an increase in financial frictions diminishes job flows must overcome several challenges of both measurement and causality. Our empirical strategy addresses these issues by using MSA-level variation in job flows and financial conditions to determine the causal effect of increased financial frictions on job flows.

The first issue we confront is finding suitable proxy for financial conditions at the MSA level. Since financial constraints are not typically observable, we use data on the growth rate of MSA house prices as a proxy for financial conditions. To the extent that lending to firms is secured by either the firm's real estate or the owner's real estate, movements in housing prices should directly affect the ability of a firm to obtain financing. For firms with access to corporate debt and equity markets, housing price movements may be a poor proxy for financial conditions. However, the vast majority of firms do not issue debt or equity securities, instead relying upon bank financing or other forms of collateralized finance. Fairlie and Krashinsky (2012) provide direct evidence for changes in housing equity on entrepreneurship using data from the Current Population Survey, while Schmalz, Sraer and Thesmar (2013) show in French data that higher house prices increase the probability of becoming an entrepreneur and, conditional on starting a business, increase the initial scale of the firm. Adelino, Schoar and Severino (2013) also document the importance of the collateral channel in the employment at small establishments.

In addition to finding a suitable proxy for financial frictions, the relative dearth of job flows data in the time series limits any analysis of the effect of financial frictions on job flows in the aggregate data. Instead, we exploit MSA-level variation in job flows and housing prices to improve the power of our estimates and increase useful variation from state and regional housing price booms.

The most significant challenge in establishing a causal affect of housing price movements on job flows is ruling out an aggregate demand channel that drives the correlation between job flows and housing prices. We address this concern in several ways. Firstly, we include state and time fixed effects to account for the business cycle and differences across states in job flows. Secondly, to control for MSA-specific demand shocks, we include controls for the business cycle. Our baseline regression takes the following form:

$$y_{it} = \alpha_i + \delta_t + \gamma(L)\,\Delta GSP_{it} + \beta(L)\,\Delta hp_{it} + \epsilon_{it}$$

where y_{it} is job creation or job destruction for MSA i at time t. ΔGSP_{it} represents the growth rate of the MSA-level business cycle variable, while Δhp_{it} is the growth rate of MSA housing prices. Our coefficient of interest is the sum of the coefficients $\beta(1)$ on MSA housing prices.

Alternatively, we also adopt an IV strategy following the methodology laid out in the empirical literature on the real effect of housing price shocks. In our IV estimates, we use both a Bartik

approach and the land supply elasticity approach, using elasticities computed in Saiz (2010). Under the Bartik approach, MSA-level house price growth is instrumented with US house price growth interacted with an MSA dummy. This IV strategy is similar to the methodology used in Nakamura and Steinsson (2011) in their study of government spending multipliers. The authors use movements in national government defense spending as an instrument for state-level government spending by exploiting differences in state sensitivity to government defense expenditures.

Our other IV approach interacts the MSA-level land supply elasticities computed in Saiz (2010) with national house prices. In both cases, the identifying assumption is that whatever causes movements in national house prices is uncorrelated with MSA-specific aggregate demand shocks. Our IV regression takes the following form:

$$y_{it} = \alpha_i + \delta_t + \beta\left(L\right)\Delta\widetilde{hp}_{it} + \epsilon_{it} \qquad \text{(2nd stage)}$$

$$\Delta hp_{it} = \alpha_i + \delta_t + \rho_i(L)\Delta hp_t + u_{it} \qquad \text{(1st stage)}$$

where $\Delta\widetilde{hp}_{it}$ is the fitted value for MSA house prices obtained from the first-stage regression of MSA house prices on national house prices interacted with an MSA dummy or with the Saiz land supply elasticity. As before, the coefficient of interest is the sum of coefficients $\beta(1)$ measuring the effect of housing prices on job flows.

To further examine the effect of housing prices on job flows, we decompose the effect of housing prices on job flows by firm size and firm age categories. As before, we utilize both OLS and IV specifications. Our OLS specification is a generalization of the MSA-level job flows regression:

$$y_{iht} = \alpha_i + \delta_t + \kappa_h + \gamma_h\left(L\right)\Delta GSP_{it} + \beta_h\left(L\right)\Delta hp_{it} + \epsilon_{iht}$$

where y_{iht} is job creation or job destruction for MSA i, in year t and category h. In addition to MSA and time fixed effects, we include category fixed effects. In these regressions, we allow both the MSA business cycle variable and MSA house prices to have differential effects on job flows across categories, and our coefficient of interest is $\beta_h(1)$ - the sum of coefficients of MSA house prices by category. The IV specification is analogous to the IV specification for aggregate job flows, where the instrument is now national house price growth interacted with a MSA-category dummy (Bartik approach) or the MSA land supply elasticity (Saiz approach):

$$y_{iht} = \alpha_i + \delta_t + \kappa_h + \beta_h\left(L\right)\Delta\widetilde{hp}_{it} + \epsilon_{iht} \qquad \text{(2nd stage)}$$

2.2   Data

We draw on several distinct data sources for measures of job flows, house prices, and MSA measures of the business cycle. Data on job flows comes from the Business Dynamics Statistics compiled by the US Census Bureau. The Business Dynamics Statistics is drawn from the Census Bureau's Longitudinal Database (LBD), a confidential database that tracks employment at the establishment

and firm level over time. The Business Dynamics Statistics report job creation and job destruction by firm age and size categories at the state and MSA level; prior to the development of BDS, these data were only available to researchers with access to confidential Census microdata. The job flows data in the BDS is drawn from Census Bureau's Business Register, which consists of the population of firms and establishments with employees covered by unemployment insurance or filing taxes with the Internal Revenue Service[3].

Specifically, we use data on gross job creation and job destruction at the MSA level from 1982-2011, where job creation measures the increase in employment at new firms or expanding firms and job destruction measures the decrease in employment at exiting firms or contracting firms. Firm level employment is recorded in March of each year and job flows are measured with respect to employment in the previous year. Our data set includes job flows from 366 MSAs resulting in a panel of 30 x 366 observations.

Our house price data comes from the Federal Housing Finance Agency's MSA level house price indices. We use the all-transactions indexes which provide a quarterly time series of housing prices from 1975 to present. These data are not seasonally adjusted, but we use year-over-year changes in the log of the house price index as our measure of MSA housing price growth. National housing prices are measured in the same way using the national house price index[4].

MSA-level business cycle measures come from the Bureau of Economic Analysis (BEA). Our baseline measure for the MSA business cycle is the growth rate of MSA personal income. We use measures of annual personal income and compute the growth rate as the change in the log of MSA personal income. Since job flows are measured as of March in a given year, we use the growth rate of MSA personal income in the previous year. For example, an observation of job creation for a given MSA in 2010 is matched with the growth rate of MSA personal income in 2009. Since housing prices are reported quarterly, no similar lag is required for house price growth. In addition to personal income, we also use real MSA gross product growth and employment growth as alternative proxies for the business cycle from BEA regional data.

2.3 Empirical Results

2.3.1 Aggregate Job Flows

Table 1 displays the coefficients of MSA housing price growth on job creation and job destruction at the MSA-level. MSA job creation and job destruction are converted to logs and detrended using a linear MSA-specific time trend. As Table 1 shows, both the OLS and IV specifications give statistically significant coefficients for MSA house prices on job creation on impact and with a lag. For job destruction, the impact effect of house prices is negative, but the second lagged coefficient

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[3]A more complete description of the BDS and access to job flows data is available at http:/ www.census.gov/cesdataproducts/bds/

[4]Housing price data may be downloaded from: http://www.fhfa.gov

Table 1: Effect of housing prices on aggregate job flows

	Job Creation			Job Destruction		
	OLS	IV (Bartik)	IV (Saiz)	OLS	IV (Bartik)	IV (Saiz)
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A						
Current housing price growth	0.36**	0.31**	0.59**	-0.32**	-0.22	-0.18
	(0.04)	(0.14)	(0.23)	(0.05)	(0.15)	(0.41)
1 year lagged housing price growth	0.18**	0.05	-0.62**	0.12**	-0.45**	-0.20
	(0.04)	(0.19)	(0.30)	(0.05)	(0.19)	(0.52)
2 years lagged housing price growth	-0.01	0.18**	0.45**	**0.27****	**0.62****	**0.49****
	(0.04)	(0.07)	(0.12)	(0.04)	(0.09)	(0.17)
Num. Obs.	8977	2565	2565	8977	2565	2565
First stage F-test		19.0	17.7		19.0	17.7
		18.8	12.8		18.8	12.8
Panel B						
Sum of coefficients	**0.52****	**0.54****	**0.41****	0.08	-0.05	0.11
	(0.05)	(0.08)	(0.10)	(0.05)	(0.08)	(0.12)

Panel A of the table presents coefficient estimates relating job flows to housing price growth at the MSA level. Panel B presents the sum of the coefficient estimates on current, 1 year and 2 years lagged housing price growth. Each column of the table reports results from a different regression. The dependent variable is MSA-level job creation in the first three columns and MSA-level job destruction in the last three columns. ** - coefficient estimate significant at the 5% level. Standard errors are in parentheses.

is positive implying that a decline in house prices reduces lagged job destruction. It is worth noting that since the sample ends in March 2011, our estimates for the effect of house prices on job flows are exploiting variation that does not fully include the weak recovery after the Great Recession[5].

Table 1 also computes the sum of the coefficients on housing prices. For job creation, the sum of the coefficients is positive and statistically significant indicating that housing price movements have a persistent effect on job creation. For job destruction, the sum of the coefficients under the baseline OLS and IV specifications is not statistically different from zero. However, excluding the impact effect, the sum of the lagged coefficients of housing prices on job destruction is positive and statistically significant across both the OLS and IV specifications. For the IV regressions, current and lagged house prices are instrumented with F-statistics above 10 under both the Bartik and the Saiz approach. Our OLS results are robust to using either real MSA gross product growth or MSA employment growth as cyclical controls and are robust to using first-differenced job flows instead of linearly detrended job flows. Additionally, are results continue to hold in state level data instead

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[5]Figure 1 uses a different data set, the Business Employment Dynamics, maintained by the Bureau of Labor Statistics that is available with a shorter delay and at quarterly frequencies.

Table 2: Effects on housing prices on job flows by firm size

	Job Creation			Job Destruction		
	OLS	IV (Bartik)	IV (Saiz)	OLS	IV (Bartik)	IV (Saiz)
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A						
1-19 employees	0.34**	0.20**	-0.19*	**-0.34****	**-0.46****	**-0.82****
	(0.04)	(0.06)	(0.12)	(0.04)	(0.07)	0.12
20-99 employees	**0.86****	**0.84****	**0.84****	0.15**	0.001	0.04
	(0.06)	(0.07)	(0.10)	(0.06)	(0.07)	(0.07)
100+ employees	0.49**	0.70**	0.69**	**0.83****	**0.93****	**1.42****
	(0.07)	(0.09)	(0.13)	(0.07)	(0.08)	(0.14)
Num. Obs.	26931	7695	7695	26931	7695	7695
Panel B						
H = (20-99 emp)	**0.52****	**0.64****	**1.03****	**0.48****	**0.46****	**0.78****
- (1-19 emp)	(0.05)	(0.05)	(0.12)	(0.04)	(0.06)	(0.11)

The table presents the effect of housing price growth at the MSA level on job flows by firms size categories (1-19, 20-99 and 100+ employees). Each column in the table reports results from a different regression. The dependent variable is job creation in the first three columns and job destruction in the last three columns. The numbers reported are the sum of the effects of current, 1 year and 2 years lagged changes in house price growth on job flows by firms size. Panel B reports the difference in the effect of housing price changes on job flows between medium (20-99 employees) and small firms (1-19 employees). ** - coefficient estimate significant at the 5% level, *- coefficient estimate significant at the 10%. Standard errors are in parentheses.

of MSA-level data.

2.3.2 Category-Specific Job Flows

We first consider job flows by firm size, and consider three categories: small firms (1-19 employees), medium-sized firms (20-99 employees), and large firms (100+ employees). Firm size assigns size categories based on an average of employment in the previous year and employment in the current year raising potential issues of reclassification bias (see Moscarini and Postel-Vinay (2012) for a discussion). However, initial firm size data is not available at the MSA level, and our results are unchanged in state level data using size categories based on initial firm size.

Table 2 displays the results from the category-specific regression of job creation and job destruction on housing prices. The table shows the sum of coefficients on MSA housing prices, $\beta_h(1)$ under the OLS and IV specifications. For job creation, middle-sized firms exhibit the highest sensitivity to housing prices, followed by large firms and small firms respectively. In the case of the IV specification, the coefficient of housing prices on job creation for small firms is negative meaning a decrease in house prices raises job creation at small firms. Job destruction for middle-sized firms display a positive coefficient on housing prices under all specifications, though the coefficients are not statistically significant under IV specifications. Job destruction falls for large firms, but the

Table 3: Effect of housing prices on job flow by firm age

	Job Creation			Job Destruction		
	OLS	IV (Bartik)	IV (Saiz)	OLS	IV (Bartik)	IV (Saiz)
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A						
Births	**1.05****	**0.80****	**1.11****			
	(0.07)	(0.09)	(0.14)			
Young Firms,	0.80**	0.87**	1.17**	**0.44****	**0.44****	**0.87****
1-5 years	(0.08)	(0.11)	(0.15)	(0.08)	(0.09)	(0.17)
Mature Firms,	-0.70**	-0.96**	-1.76**	-0.26**	-0.57**	-0.80**
6+ years	(0.06)	(0.10)	(0.20)	(0.06)	(0.12)	(0.18)
Num. Obs.	26931	7695	7695	17954	5130	5130
Panel B						
H = Births - Mature	**1.74****	**1.76****	**2.87****	**0.70****	**1.02****	**1.67****
or Young - Mature	(0.08)	(0.13)	(0.25)	(0.08)	(0.13)	(0.24)

The table presents the effect of housing price growth at the MSA level on job flows by firm age categories (births, 1-5 and 6+ years old). Each column in the table reports results from a different regression. The dependent variable is job creation in the first three columns and job destruction in the last three columns. The numbers reported are the sum of the effects of current, 1 year and 2 years lagged changes in house price growth on job flows by firms age. The first three columns of panel B reports the difference in the effect of housing price changes on job flows between new and mature firms. The last three columns of panel B present the difference in the effect of housing price between young and mature firms. ** - coefficient estimate significant at the 5% level, *- coefficient estimate significant at the 10%. Standard errors are in parentheses.

positive job destruction coefficient for large firms is heavily influenced by a positive impact coefficient. When restricted to lagged coefficients (not shown in Table 2), middle-sized firms display a positive and statistically significant coefficent on house prices with the coefficient for large firms falling closer to zero.

Table 2 also shows that the difference in coefficients between middle-sized firms and small firms is statistically significant across all specifications for both job creation and job destruction. In contrast, the difference for middle and large sized firms for job creation is generally not significant. As with the result for overall job flows, the general pattern of a positive coefficient of job flows on housing prices at middle-sized firms and stronger response relative to small-sized firms is robust to use of state-level data and alternative controls for the MSA business cycle.

We also consider job flows by firm age categories: new firms, young firms (1-5 years of age), and mature firms (6+ years of age). These firm age categories are same categories used in Haltiwanger, Jarmin and Miranda (2010). Table 3 shows that job creation at new firms exhibit the strongest response to housing prices followed closely by job creation at young firms. By contrast, job creation at mature firms increases with a decline in housing prices. This pattern is preserved across all specifications and the coefficients remain statistically significant. By definition, new firms have

Table 4: Effect of housing prices on firm entry and exit

	Firm Entry			Firm Exit for Firms Age 1			Firm Exit for Firms Age 2		
	OLS	IV (Bartik)	IV (Saiz)	OLS	IV (Bartik)	IV (Saiz)	OLS	IV (Bartik)	IV (Saiz)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel A									
Current housing price growth	0.25**	-0.03	-0.19	-0.36**	-1.20**	-1.11**	-0.82**	-1.45**	-0.92
	(0.03)	(0.12)	(0.30)	(0.06)	(0.16)	(0.39)	(0.10)	(0.28)	(0.70)
1 year lagged housing price gr.	0.30**	-0.24**	0.01	**0.45***	**0.73***	**0.59***	0.16*	-0.18	-0.72
	(0.03)	(0.11)	(0.42)	(0.06)	(0.13)	(0.29)	(0.09)	(0.34)	(0.94)
2 years lagged housing price gr.	0.31**	0.58**	0.27				**0.71***	**0.93***	**1.18***
	(0.03)	(0.08)	(0.27)				(0.09)	(0.15)	(0.59)
Num. Obs.	8977	2565	2565	9249	2586	2586	8951	2562	2562
Panel B									
Sum of coefficients	**0.86***	**0.32***	**0.10**	0.09*	-0.48**	-0.53**	0.05	-0.69**	-0.46
	(0.06)	(0.13)	(0.24)	(0.05)	(0.13)	(0.21)	(0.10)	(0.14)	(0.33)

Panel A of the table presents coefficient estimates relating firm entry and exit to housing price growth at the MSA level. Panel B presents the sum of the coefficient estimates on current, 1 year and 2 years lagged housing price growth. Each column of the table reports results from a different regression. The dependent variable is the number of new firms in the first three columns, the number of exiting 1 year old firms in columns (4)-(6) and the number of exiting 2 years old firms in columns (7)-(9). ** - coefficient estimate significant at the 5% level. Standard errors are in parentheses.

zero job destruction. Job destruction at young firms shows a positive and statistically significant coefficient on housing prices, while job destruction at mature firms moves inversely to housing prices. As the last row of Table 3 show that the difference in coefficients on job creation for new firms versus mature firms is statistically significant, as is the difference in coefficients on job destruction for young firms versus mature firms. As before, these patterns are preserved in state-level data and with the use of alternative MSA business cycle controls.

2.3.3 Firm Entry and Exit

In addition to documenting the effect of housing prices on job flows, we can also examine the behavior of firm entry and exit due to housing price shocks. Firm entry or exit on the MSA level is expressed in logs. The first column of Table 4 displays the effect of housing prices on firm entry, showing a significant effect of housing price movements on firm entry rates in the OLS specification while the coefficients dissipate under the IV specifications. The second column of Table 4 looks at firm death rates for firms 1 year of age. We find some evidence that housing price shocks initially lead to a higher death rate but subsequently reduce death rates in the next period. This is consistent with the effect of house price shocks on firm entry - if a negative house price shock reduces entry on impact, then it should lower firm exits for firms 1 year of age with a lag. Similarly, the last column of Table 4 provides evidence that house price shocks reduce firm exit with a 2 period lag consistent

with a strong effect of housing prices on firm entry. The sum of the coefficients for house prices on firm exits are either poorly estimated or inconclusive in sign providing no definitive evidence of an overall effect of housing price shocks on firm deaths.

3 Simple Model

In this section, we build a simple continuous time firm dynamics model to characterize the effect of a collateral shock on asset accumulation, employment, and job flows. This simple model illustrates the mechanisms at work that cause a collateral shock to diminish job creation and job destruction at young firms and middle-sized firms. The core framework is a real business cycle model. To this we add (i) a financial friction that limits the amount of borrowing, (ii) firm heterogeneity, (iii) and a decreasing returns production technology.

The economy consists of three types of agents: households, heterogeneous firms, and intermediaries. Each household consumes, supplies labor and trades in a market for capital. The household consists of measure n of workers. Workers supply labor to firms and return their wages to the household. Each firm hires workers from households and buys capital from intermediaries to produce. Intermediaries issue one-period real risk- free bonds and rent capital to firms. Every period σ firms die and transfer their assets to the household and σ new firms are born; these firms receive an initial transfer of assets from the households. There is a single consumption good in the economy that serves as the numeraire good. There are two types of assets in the economy: capital and the risk-free one period real bonds. Capital can be freely converted to the consumption good and back using a one-to-one technology. There is no aggregate uncertainty in the economy and the only idiosyncratic uncertainty is the risk of death for individual firms.

3.1 Households

Let $c(t)$ be consumption and $n(t)$ be labor supply. Then household preferences are given by the following expression

$$\int_0^\infty e^{-\rho t} U\left[c - v\left(n\right)\right] dt. \tag{1}$$

The household faces a budget constraint

$$\dot{a} = wn + ra + \Pi - c, \tag{2}$$

where r is the return on household assets a, ρ is the rate of time preference, Π is net payouts to the household from the ownership of firms, wn is household labor income. We assume preferences with no wealth effect on labor supply to simplify the analysis of equilibrium in the labor market.

The household takes its initial assets a_0 and the equilibrium behavior of prices as given. In addition, for the problem to be well-defined we add the natural debt limit constraint

$$a(t) \geq - \int_t^\infty [w(t)n(t) + \Pi(t)] \left[- \int_t^s r(s)ds \right] ds.$$

3.2 Firms

The economy is composed of a measure 1 of firms which produce homogeneous output. Firms behave competitively on the output, capital and labor markets. Each firm faces an exogenous rate of exit σ in which case the firm transfers its assets to the household and disappears forever. Every period σ firms exit and σ new firms are born with an initial endowment of assets a_0[6].

Each firm has productivity of $z\epsilon$, where z is common across firms while ϵ is a firm-specific productivity. Both values are constant over time for a given firm. We assume that $\epsilon \in \{\epsilon_L, \epsilon_H\}$ with $\epsilon_L < \epsilon_H$. We also assume that the probability of being born with a high firm-specific productivity is μ, i.e., $Pr(\epsilon = \epsilon_H) = \mu$.

We assume that the firms use $\Lambda_{t,t+\tau} = e^{-\rho\tau} U'(c_{t+\tau})/U'(c_t)$ as their discount factor between periods t and $t + \tau$. Each firm maximizes the present discounted value of its terminal wealth. Formally, each firm maximizes

$$\max_{\{n_{t+\tau}, k_{t+\tau}\}_{\tau=0}^\infty} \int_0^\infty e^{-\sigma\tau} \Lambda_{t,t+\tau} a_{t+\tau} d\tau, \tag{3}$$

where $a_{t+\tau}$ is wealth of the firm in period $t + \tau$, k is the amount of capital the firm decides to rent in period t. The firm faces two constraints. First, the wealth accumulation equation:

$$\dot{a} = \pi + ra, \tag{4}$$

where

$$\pi = z\epsilon \left(k^\alpha n^{1-\alpha} \right)^\phi - r_k k - wn, \tag{5}$$

where first term $z\epsilon \left(k^\alpha n^{1-\alpha} \right)^\phi$ represents a decreasing-returns-to-scale production function. The second and the third terms represents cost of capital and labor inputs respectively.

Second, the firm faces a financial constraint of the following form:

$$k \leq \chi a, \tag{6}$$

where χ denotes the borrowing capacity which is common across firms; $\chi = \infty$ corresponds to frictionless capital rental market and $\chi = 1$ to self-financing. This specification reflects the prediction of financial contracts in models with limited contract enforcement. See Evans and Jovanovic (1989) for an early use of this specification of the financial constraint[7].

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[6] a_0 will be chosen to match the average share of employment at new firms in the US.

[7] Buera and Shin (2011) argue that this type of financial constraint can be derived by assuming limited liability on the side of the firms and one period punishment for not honoring the promises.

3.3 Intermediaries

Households and firms have access to competitive intermediaries that receive deposits (issue risk-free one-period bonds) and rent out capital at rate r_k to firms. The zero profit condition of the competitive intermediaries implies:

$$r_k = r + \delta, \tag{7}$$

where δ is the depreciation rate of capital.

3.4 Competitive Equilibrium

A competitive equilibrium is paths for $\{c(t), a^H(t), n^H(t), a^F(t), n^F(t), k(t), w(t), r(t), r_k(t)\}_{t=0}^{\infty}$ such that

1. households solve (1) and (2) given initial level of assets $a^H(0)$ taking prices $\{w(t), r(t)\}_{t=0}^{\infty}$ as given;

2. firms solve (3) - (5) given initial level of assets $a^F(0)$ taking prices $\{w(t), r(t)\}_{t=0}^{\infty}$ as given;

3. markets clear:

   (a) the capital market clears:

$$\mu\sigma \int_0^{\infty} e^{-\sigma t} k(t, \epsilon_H) dt + (1-\mu)\sigma \int_0^{\infty} e^{-\sigma t} k(t, \epsilon_L) dt$$
$$= \mu\sigma \int_0^{\infty} e^{-\sigma t} a^F(t, \epsilon_H) dt + (1-\mu)\sigma \int_0^{\infty} e^{-\sigma t} a^F(t, \epsilon_L) dt + a^H(t) \text{ for all } t;$$

   (b) the labor market clears:

$$n^H(t) = \mu\sigma \int_0^{\infty} e^{-\sigma t} n(t, \epsilon_H) dt + (1-\mu)\sigma \int_0^{\infty} e^{-\sigma t} n(t, \epsilon_L) dt \text{ for all } t.$$

3.5 Characterization of the firm problem

In this section we characterize the steady state equilibrium of the economy in which prices are constant over time but firms enter and exit at rate σ.

Household optimality requires:

$$\dot{c} = -\frac{u'(c)}{u''(c)}(r - \rho), \tag{8}$$

$$w = v'(n). \tag{9}$$

The first equation is the standard continuous time Euler equation. The second line is the labor supply condition which equates the real wage with the marginal disutility of working. See Appendix

A for the derivation details. Equation (8) immediately implies that the real interest rate is equal to the discount factor in the steady state, i.e., $r = \rho$.

To describe the solution to the firm's problem, we specify the Hamiltonian:

$$\mathcal{H} = e^{-\sigma t} \Lambda_{0,t} a + \lambda_F \left[z\epsilon \left(k^\alpha n^{1-\alpha} \right)^\phi - r_k k - wn + ra \right] - \eta [k - \chi a]$$

The maximum principle implies:

$$\mathcal{H}_k = \lambda_F [z\epsilon\alpha\phi k^{\alpha\phi-1} n^{(1-\alpha)\phi} - r_k] - \eta = 0, \tag{10}$$

$$\mathcal{H}_n = \lambda_F [z\epsilon(1-\alpha)\phi k^{\alpha\phi} n^{(1-\alpha)\phi-1} - w] = 0, \tag{11}$$

$$\dot{\lambda}_F = - \left\{ e^{-\sigma t} \Lambda_{0,t} + \lambda_F r + \eta\chi \right\}, \tag{12}$$

$$k \leq \chi a, \ \eta \geq 0, \eta[k - \chi a] = 0. \tag{13}$$

It is convenient to break the solution into two cases.

Case 1 Assume that the collateral constraint binds. This implies that $\eta > 0$ and $k = \chi a$. The labor demand condition - equation (11) - requires:

$$n = \left[\frac{z\epsilon\phi(1-\alpha)}{w} \chi^{\alpha\phi} a^{\alpha\phi} \right]^{\frac{1}{1-\phi(1-\alpha)}}. \tag{14}$$

Substituting optimal employment and capital in output function we can rewrite the law of motion for assets as follows:

$$\dot{a} = Aa^\psi - Ba,$$

where $A = \left(z\epsilon\chi^{\alpha\phi}\right)^{\frac{1}{1-\phi(1-\alpha)}} \left[\frac{\phi(1-\alpha)}{w}\right]^{\frac{\phi(1-\alpha)}{1-\phi(1-\alpha)}} [1 - \phi(1-\alpha)], B = r_k\chi - r; \psi = \phi\alpha/(1 - \phi(1-\alpha)) < 1.$

Lemma 1. *The solution to the law of motion of a capital-constrained firm is*

$$a = \left\{ \frac{A}{B} - \left(\frac{A}{B} - a_0^{1-\psi} \right) e^{-B(1-\psi)t} \right\}^{1/(1-\psi)}, \tag{15}$$

where a_0 is the initial level of wealth.

Solution $a(t, \chi, \epsilon)$ is monotonic in t, can be convex or concave in t, non-monotonic in χ and increasing in ϵ (see Appendix A.3 for the details). Because assets and labor demand are related by equation (14) the same conclusion can be reached about properties of employment. Labor demand $n(t, \chi, \epsilon)$ is monotonic in t, may be convex or concave in t, is non-monotonic in χ, and increasing in ϵ.

Case 2 The collateral constraint does not bind. This implies $\eta = 0$ and $k < \chi a$. Optimality with respect to labor and capital implies:

$$\frac{\alpha}{1-\alpha} \frac{n}{k} = \frac{r_k}{w} \tag{16}$$

Thus, the optimal level of capital and labor can be expressed in terms of prices:

$$n^* = (z\epsilon\phi)^{1/(1-\phi)} \left(\frac{\alpha}{r_k}\right)^{\alpha\phi/(1-\phi)} \left(\frac{1-\alpha}{w}\right)^{(1-\alpha\phi)/(1-\phi)}$$

$$k^* = (z\epsilon\phi)^{1/(1-\phi)} \left(\frac{\alpha}{r_k}\right)^{[1-(1-\alpha)\phi]/(1-\phi)} \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)\phi/(1-\phi)}$$

Combining the two cases The optimal capital and labor choices are $k(t) = \min\left\{\overline{k}(t), k^*\right\}$ and $n(t) = \min\left\{\overline{n}(t), n^*\right\}$, where $\overline{k}(t), \overline{n}(t)$ are the constrained optimal choice of capital and labor.

3.6 Characterization of partial equilibrium effects

Figure 3 shows the firm-level employment dynamics for two firms with different levels of idiosyncratic productivities. The more productive firm cannot achieve its optimal level immediately and has to grow before it reaches its optimal employment level $n^*(\epsilon_H)$. In contrast, the low-productivity firm can immediately jump to its optimal level of employment $n^*(\epsilon_L)$.

Figure 3: Firm employment dynamics.
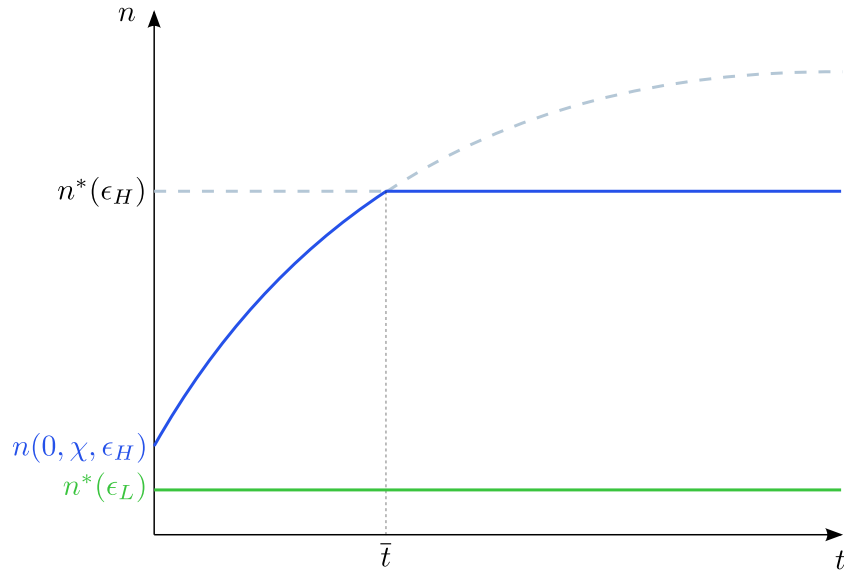
The assets of an unconstrained firm continue to grow according to $\dot{a} = \pi^* + (1+r)a$, where π^* is the profit level of an unconstrained firm. Figure 3 demonstrates the role of age and size in identifying the effect of a collateral shock. The collateral constraint is irrelevant for the low productivity firms; a tightening of the collateral constraint has no impact on employment for these firms. In contrast, for high-productivity firms which over time, the collateral constraint impacts their rate of growth while leaving the optimal level employment unchanged.

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Let's $\bar{t}$ denote the moment in time when a firm grows out of its financial constraint (assuming that the firm was financially constrained at the beginning of its life). This time $\bar{t}$ solves the equation $\bar{k}\left(\bar{t}\right) = k^*$.

Lemma 2. *Consider two financial constraint parameters χ_L, χ_H where $\chi_L < \chi_H$. Denote $T(\chi_L, \chi_H)$: $\bar{k}\left(T(\chi_L, \chi_H), \chi_L\right) = \bar{k}(T(\chi_L, \chi_H), \chi_H)$ - that is, $T(\chi_L, \chi_H)$ is the time at which the employment path of the credit constrained high-productivity firm crosses under χ_L and χ_H respectively. Assume that $\bar{t}(\chi_L) < T(\chi_L, \chi_H)$. Then it follows that:*

1. *$n(t, \chi_L) \leq n(t, \chi_H)$,*

2. *$\bar{t}(\chi_H) < \bar{t}(\chi_L)$.*

The results of the lemma are presented in Figure 4. The lemma shows that a collateral shock depresses employment at the high-productivity credit constrained firms and extends the time it takes for the firm to reach its optimal size. As a result, employment at the high- productivity firm is depressed at every age level after an adverse collateral shock. However, since the optimal unconstrained size of the firm is unchanged, job creation for any given firm is unchanged over its lifecycle conditional on surviving long enough to reach its optimal size.

Figure 4: Firm employment dynamics: comparative statics with respect to χ.
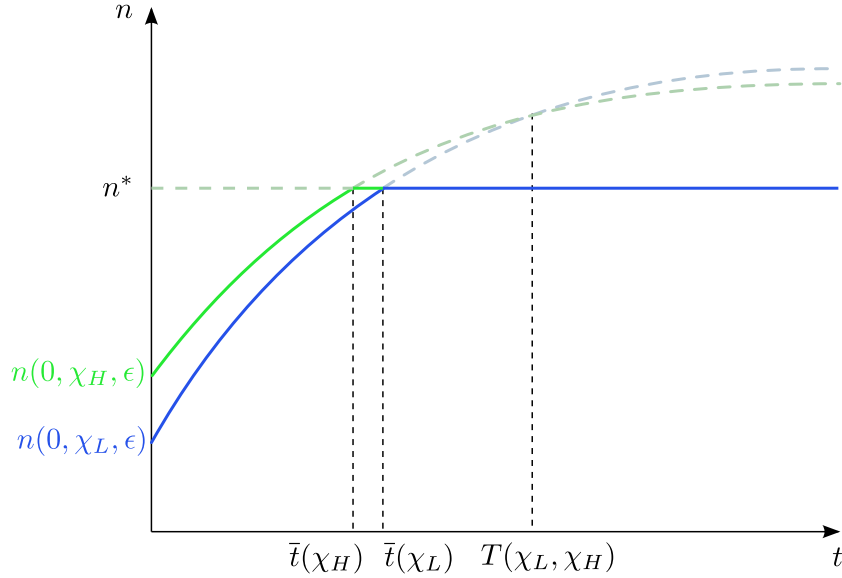
If the firm's optimal unconstrained size is left unchanged, how does a collateral shock depress job creation and job destruction? Given the constant hazard rate of exit σ, since it takes longer to reach optimal size, fewer firms survive, thereby lowering aggregate job creation. Since job destruction in this setting is attributable solely to firm exits, the decrease in job destruction is due to fewer firms surviving to any given level of employment. It can be shown that job destruction $JD = \sigma N$ where N is aggregate employment.

An aggregate productivity shock z has a qualitatively different effect on employment paths for firms than a collateral shock χ. A productivity shock depresses employment at all ages; while the optimal size of the firm is independent of the collateral constraint, productivity directly affects the optimal size. As such, a productivity shock will affect both high and low-productivity firms, and constrained and unconstrained firms. However, like Khan and Thomas (2011), a productivity shock will interact with the financial constraints to generate asymmetric effects on firm employment across age and size categories.

3.7 Characterization of general equilibrium effects

Summing across firms leads to the following expression for the aggregate labor demand in the economy

$$N^d = \mu \left[\sigma \int_0^{\bar{t}} n(t, \epsilon_H, w, r, r_k, \chi) e^{-\sigma t} dt + \sigma \int_{\bar{t}}^{\infty} n^*(\epsilon_H, w, r, r_k) e^{-\sigma t} dt \right]$$

$$+ (1 - \mu) \left[\sigma \int_0^{\bar{t}} n(t, \epsilon_L, w, r, r_k, \chi) e^{-\sigma t} dt + \sigma \int_{\bar{t}}^{\infty} n^*(\epsilon_L, w, r, r_k) e^{-\sigma t} dt \right]. \quad (17)$$

In a dynamic steady state equilibrium r and r_k are pinned down by the household preferences and the depreciation rate. Hence, any change in χ will have a direct effect on aggregate labor demand and an indirect effect through an equilibrium change in wages w.

Assumption 1. *Parameters of the model are such that in equilibria to be considered all firms with low idiosyncratic productivity are not credit constrained.*

This assumption implies that the aggregate demand for labor can be expressed as follows

$$N^d = \mu \left[\sigma \int_0^{\bar{t}} n(t, \epsilon_H, w, r, r_k, \chi) e^{-\sigma t} dt + \sigma \int_{\bar{t}}^{\infty} n^*(\epsilon_H, w, r, r_k) e^{-\sigma t} dt \right]$$

$$+ (1 - \mu) n^*(\epsilon_L, w, r, r_k) \quad (18)$$

Lemma 3.

- *(PE) Assume that w is constant and $\bar{t}(\chi_L) < T(\chi_L, \chi_H)$, then $N^d(w, \chi_L) < N^d(w, \chi_H)$.*

- *(GE + No wealth effect) Assume that $\bar{t}(\chi_L) < T(\chi_L, \chi_H)$, then $N(w(\chi_L), \chi_L) < N(w(\chi_H), \chi_H)$.*

Proof: See Appendix A.4 for a proof. ∎

As Lemma 3 shows, under certain mild conditions, a tightening in the collateral constraint reduces aggregate labor demand when wages are held constant. Employment at low-productivity firms is unchanged, while employment at the high-productivity firms falls for firms that are credit constrained and remains unconstrained for firms with sufficient assets. Since firm employment is

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weakly lower after the collateral shock, aggregate employment falls. Wage adjustment partially offsets the direct effects of the collateral shock. A lower wage raises optimal size of all firms thereby increasing employment at unconstrained firms. So long as the labor supply is upward-sloping, wage adjustment will not fully undo the direct effect of the collateral shock on aggregate employment.

4 Benchmark Model

In this section, we describe a dynamic stochastic general equilibrium model with endogenous and exogenous firm entry and exit. The agents, markets, assets and production technology are the same as in the simple model. We add uncertainty to aggregate productivity and the collateral constraint as well as uncertainty to firm-specific productivity. We will also assume that the exogenous exit rate depends on the firm age.

Time is discrete and is indexed by $t = 1, 2, \ldots$. The common component of productivity z_t follows a Markov process that takes values in the set $\mathbf{Z}$ with conditional distribution $H(z_{t+1}|z_t)$. The idiosyncratic component of productivity ϵ_t follows a Markov process that takes values in $\mathfrak{E}$ and has conditional distribution $G(\epsilon_{t+1}|\epsilon_t)$. Financial shock χ_t also follows a Markov process with values in $\mathcal{K}$ and conditional distribution $Q(\chi_{t+1}|\chi_t)$.

In addition to the aggregate states, the firm's problem will be characterized by the firm's initial level of assets, its current level of idiosyncratic productivity and its age. Thus, the aggregate state of the economy x_t consists of $\{z_t, \chi_t, a_t^H, \mu_t\}$, where μ_t is the distribution of firms over assets, idiosyncratic shocks and age, and a_t^H is the wealth of the representative household. Observe that if the households were heterogenous we would need to keep track of the households distribution over their state space. However, because households are identical they can be summarized by a single state variable a_t^H.

Denote $\mu' = \Gamma(z, \chi, a^H, \mu)$ as the law of motion for the firm distribution. Also denote $\Phi(x_{t+1}|x_t)$ as the conditional distribution of the aggregate state. This conditional distribution can be expressed as follows:

$$d\Phi\left[z', \chi', (a^H)', \mu'|z, \chi, a^H, \mu\right] = \mathbb{1}\left[\mu' = \Gamma(z, \chi, a^H, \mu)\right]\mathbb{1}\left[(a^H)' = f(z, \chi, a^H, \mu)\right]$$
$$\cdot dH(z'|z)dQ(\chi'|\chi), \tag{19}$$

where $f(\cdot)$ is firms policy function.

4.1 Households

The household problem can be summarized as follows:

$$V^H(a,x) = \max_{c,n,a'} \left\{ u\left[c - v(n)\right] + \beta \int V^H(a',x')d\Phi(x'|x) \right\},$$

$$\text{s.t.} \quad c + a' = wn + (1+r)a + \Pi.$$

Households choose consumption c, labor supply n, and next period assets a' subject to a standard budget constraint taking firm profits as given.

4.2 Firms

We assume there are two types of firms: incumbent firms and prospective entrants. All operating firms must pay fixed cost $c_F > 0$ every period. The incumbent firm solves the following problem:

$$V^F(\epsilon,a,\tau,x) = \max \left\{ 0, -c_F + \max_{k,n,a'} \int \left[\sigma_\tau \Lambda' a' + [1 - \sigma_\tau]V^F(\epsilon',a',\tau+1,x') \right] d\Phi(x'|x)dG(\epsilon'|\epsilon) \right\},$$

$$\text{s.t.} \quad a' = z\epsilon \left(k^\alpha n^{1-\alpha}\right)^\phi - r_k k - wn + (1+r)a,$$

$$k \leq \chi a.$$

Incumbent firms operate a decreasing returns to scale production technology subject to idiosyncratic and aggregate productivity shocks. Firms choose whether to exit or to stay and maximize a weighted average of the next period assets and their continuation value, where the weight σ_τ reflects the exogenous probability of exit of a τ years old firm. Firms choose capital k, next period assets a', and employment n subject to a standard accumulation equation for assets and the same constraint on renting capital described in the previous section.

There are two possibilities for entry. First, firms enter exogenously: each period a measure M_0 of firms enter without paying the cost of entry. Second, measure M of prospective firms can pay a fixed cost c_E to enter. The firms that enter in the current period start producing immediately. The decision to pay c_E occurs after the firms learn the current level of aggregate productivity and a signal about their current idiosyncratic productivity. The value of a potential entrant is:

$$V^E(q,a_0,x) = \int V^F(\epsilon,a,\tau=0,x)dH(\epsilon|q) \tag{20}$$

Signal q is drawn from unconditional distribution $Q(q)$. The distribution of idiosyncratic productivity conditional on the shock is $H(\epsilon|q)$. Firms that pay to enter will enter if and only if:

$$V^E(q,a_0,x) \geq c_E.$$

4.3 Intermediaries

Perfectly competitive intermediaries operate identically as described in the previous section. The zero-profit condition for intermediaries implies:

$$r_k = r + \delta$$

4.4 Recursive Equilibrium

A recursive equilibrium is a collection of functions $V^H(a,x), V^F(\epsilon, a, \tau, x), V^E(q, a, x), c(a, x), a'_H(a, x),$ $n(\epsilon, a, \tau, x), k(\epsilon, a, \tau, x), a'_F(\epsilon, a, \tau, x), w(x), r(x), r_k(x), \Gamma(x), \Lambda(x)$ such that:

1. households, firms, intermediaries optimize;

2. capital, labor and goods markets clear;

3. Γ: for all Borel $\mathcal{E} \times \mathcal{A} \in \mathbf{R}^+ \times \mathbf{R}^+$
   and $\tau \geq 1$

$$\mu'(\mathcal{E} \times \mathcal{A}, \tau) = (1 - \sigma_{\tau-1}) \int_{\epsilon' \in \mathcal{E}} \int_{(\epsilon, a) \in \mathcal{B}(x, \tau, \mathcal{A})} d\mu(\epsilon, a, \tau) dG(\epsilon'|\epsilon) \quad \text{(survived incumbents)}$$

   and $\tau = 0$

$$\mu'(\mathcal{E} \times \mathcal{A}, 0) = \mathbb{1}\left(a_0 \in \mathcal{A}\right) M_0 \int_{\epsilon \in \mathcal{E}_E(x)} dQ(q)dH(\epsilon|q) \quad \text{(exogenous entry + no exit)}$$

$$+ \mathbb{1}\left(a_0 \in \mathcal{A}\right) M \int_{\epsilon \in \mathcal{E}_E(x)} \int_{q \in \mathcal{Q}_E(x, a_0)} dQ(q)dH(\epsilon|q), \quad \text{(endogenous entry + not exit)}$$

   where

$$\mathcal{Q}_E(x, a_0) = \{q : V^E(q, a_0, x) \geq c_E\},$$
$$\mathcal{E}_E(x) = \{\epsilon : \epsilon \in \mathcal{E}, \ V^F(\epsilon, a_0, \tau = 0, x) \geq 0\},$$
$$\mathcal{B}(x, \tau, \mathcal{A}) = \{(\epsilon, a) : V^F(\epsilon, a, \tau, x) \geq 0 \text{ and } \pi(x, \epsilon, a) + (1 + r(x))a \in \mathcal{A}\},$$

given μ_0.

4.5 Stochastic Steady State without Endogenous Firms Entry and Exit

In this section, we assume that $c_E = \infty$ and $c_F = 0$, i.e., firms enter and exit exogenously. A stochastic steady state is an equilibrium in which there are no aggregate shocks, i.e., χ is constant and z is constant, and prices, distribution of firms and assets of households do not change over time.

Household optimality requires that in the steady state equilibrium the real interest rate equals the discount factor:

$$\frac{1}{\beta} = 1 + r, \tag{21}$$

and the wage equals marginal disutility of working:

$$w = v'(n) \tag{22}$$

Firm optimality with respect to capital and labor implies standard factor demand conditions:

$$z\epsilon_t \phi \alpha k_t^{\alpha\phi-1} n_t^{(1-\alpha)\phi} = r_k + \frac{\eta_t}{\lambda_t}, \tag{23}$$

$$z\epsilon_t \phi (1-\alpha) k_t^{\alpha\phi} n_t^{(1-\alpha)\phi-1} = w. \tag{24}$$

These conditions are the discrete time equivalent of the firm's policy functions described in the previous section.

5   Quantitative Predictions of the Model

To explore the quantitative implications of our model, we calibrate a version of our benchmark model without endogenous entry and exit. We also assume that firms' idiosyncratic productivity level does not change so that there are no transitory shocks to firm productivity. We examine the effect of collateral and aggregtae productivity shocks in our model on overall job flows and the distribution of job creation and job destruction across firm size and firm age categories along the transition path.

5.1   Calibration Strategy and Targets

Our calibration strategy chooses several common parameters from the literature. Given that our empirical evidence on job flows is observed in annual data, we use annual values for several common parameters. As shown in Table 5, the household's discount rate β and the capital share α are standard. The depreciation rate of capital δ is set to match the depreciation rate for structures. The parameter ϕ governing the degree of decreasing returns to scale is set at 0.95, comparable to values chosen in Cooley and Quadrini (2001) and Khan and Thomas (2011). We experiment with several different values for the Frisch elasticity ν to gauge the importance of labor supply response in our quantitative experiment. A Frisch elasticity of zero conforms to the case of a vertical labor supply curve, while an infinite Frisch elasticity conforms to the case of a horizontal labor supply curve. In the former case, wages adjust so that total employment is unaffected by the collateral shock. In the latter case, wages are unchanged so employment is demand determined. In effect, this case conforms to the partial equilibrium effect of the collateral shock or, equivalently, the effect

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Table 5: Calibration values

Aggregate Parameters		Value
Discount rate	β	0.96
Depreciation rate	δ	0.025
Capital share	α	0.3
Decreasing returns	φ	0.95
Frisch elasticity	ν	0,∞
Initial assets	a_0	8
Collateral constraint	χ	20

of a collateral shock with perfect real wage rigidity. In our preferred calibration, we choose a Frisch elasticity of $\nu = 5$ - at the higher end of calibrated Frisch elasticities in the macro literature.

It remains to choose an initial level of assets a_0, the collateral constraint parameter χ, firm exit rate σ, and a support and distribution of idiosyncratic productivity levels ϵ. We select the distribution of the idiosyncratic productivity levels to target the distribution of employment by mature firms in the data. In our model, firm that survive sufficiently long converge towards an optimal level of employment. We take averages of employment share by firm size categories for firms over 21 years of age in the Business Dynamics Statistics from 2000-2006, and we back out the implied level idiosyncratic productivity so that the optimal employment size of the firm is at the midpoint of the employment bin range. We choose the distribution of firms over idiosyncratic productivity levels to target the share of employment by firm size in the data. Table 6 shows the size bins used and the employment shares that our calibration targets. The last column shows the implied distribution of firms that matches the employment shares we are targeting, showing that most firms are small, low-productivity firms.

Table 6: Idiosyncratic shock calibration

Size Bins	Employment distribution in % (Data)	Firm distribution in % (Model)
[1;4]	2.5	44.0
[5;9]	3.6	22.5
[10;19]	5.1	15.3
[20;49]	8.5	9.8
[50;99]	7.1	4.1
[100;249]	9.8	2.5
[250;499]	7.0	0.8
[500;999]	6.4	0.4
[1000;2499]	8.6	0.2
>2500	41.4	0.5

Instead of choosing a single constant exit rate for firms, we choose time-dependent exit rates

Table 7: Exit rate calibration

Firm Age	Firm distribution in % (Data)	Firm distribution in % (Model)
0	10.2	10.2
1	7.7	7.7
2	6.6	6.6
3	5.8	5.8
4	5.2	5.2
5	4.7	4.7
6-10	18.0	17.9
11-15	12.6	12.6
16-20	8.8	9.7
21-25	6.2	6.4
>26	14.4	13.4

for the first five years before a constant exit rate for firms older than five years. We do this to capture the declining hazard of firm exit - with an endogenous firm extensive margin, selection effects would generate this declining hazard for young firms without the need exogenous differences in exit rates. In the absence of endogenous entry and exit, the firm's policy functions and the steady state are unaffected by the assumption of time-dependent exit rates. We choose entry and exit rates to match the empirical age distribution of firms using 2000-2006 averages from the BDS. Table 7 provides the age distribution of firms and the distribution implied by our calibration. By construction, the empirical distribution and model distribution match for firms age 0-5, but differs for older ages when a constant exit rate is assumed. The exit rate for firms older than age 5 is $\sigma = 0.069$ and implies a model age distribution that closely matches the empirical distribution.

The final parameters that we choose are the initial level of assets a_0 and the collateral constraint parameter χ. We jointly choose these parameters shown in Table 5 to best match the distribution of employment by firm age and size. The empirical and model distributions are shown in Table 8. Our calibration closely matches the age distribution of employment and does a reasonable job matching the size distribution of employment. Our calibration has a somewhat lower distribution of employment among new and younger middle-sized and larger firms and consequently too large employment share for small firms. The size distribution for new and young firms is determined in part by the initial level of assets a_0. Heterogeneity in initial asset levels would likely allow us to match the size distribution for new and young firms.

5.2 Effect of Collateral and Productivity Shocks

We consider the transition path for a permanent 20% tightening of the collateral constraint parameter from $\chi = 20$ to $\chi = 16$. This tightening conforms to the magnitude of the drop experienced

Table 8: Distribution of employment by firm size and age

	1-19 emps	20-99 emps	100+	Total
Panel A: data				
Births	1.5	0.8	0.6	**2.8**
1-5 years	5.7	3.4	3.0	**12.1**
6+ years	12.1	13.6	59.3	**85.0**
Total	**19.3**	**17.8**	**62.9**	
Panel B: model				
Births	2.9	0.2	0.0	**3.0**
1-5 years	6.1	6.1	1.6	**13.7**
6+ years	11.1	15.6	56.6	**83.2**
Total	**20.0**	**21.8**	**58.2**	

Panel A of the table presents the distribution of employment over firms size and age categories in % in Business Dynamics Statistics database over 2000-2006. Panel B shows the distribution of employment over firms size and age categories in % in steady state of the baseline calibration of our model.

in US housing prices during the Great Recession. The collateral shock is modeled as a permanent shock given the persistence of the drop in nominal US housing prices, with prices five years since the start of the recession still 15-20% below. The results we present are unchanged for shocks that last five years or longer and then gradually normalize.

The left panel of Figure 5 displays the transition paths for employment under a financial shock, while the bottom panel illustrates the same path for a productivity shock that generates a similar long-run decline in employment. We display both transition paths with an infinite Frisch elasticty and with a Frisch elasticity of $\nu = 5$. For simplicity, in both cases, the rental rate is held constant (i.e. small open economy assumption).

As the transition paths illustrate, both permanent financial and productivity shock generate large effects of employment on impact. The firm dynamics model generates little endogenous propagation in subsequent periods, but the financial shock does reduce employment in subsequent periods as effects filter through the age distribution of firms. Due to the tightened collateral constraint, large firms that exit are replaced by smaller firms reducing overall employment. With wage adjustment, this effect is somewhat offset.

Figure 6 displays the transition paths for gross job creation and job destruction under the financial and productivity shocks. The financial shock, shown in panels (a) and (b), reduces employment by sharply reducing job creation, while the productivity shock, shown in panels (c) and (d) reduces employment through a sharp effect on job destruction. The partial equilibrium effect of the financial shock on job creation in panel (a) is particularly stark with job creation dropping some 80% relative to steady state while job destruction increases only slightly. Wage adjustment in panel (b) reduces the large effect of a financial shock on job creation to a plausible magnitude but preserves the relatively larger effect of financial shocks relative to productivity shock on gross

Figure 5: Employment transition paths with permanent financial and productivity shocks
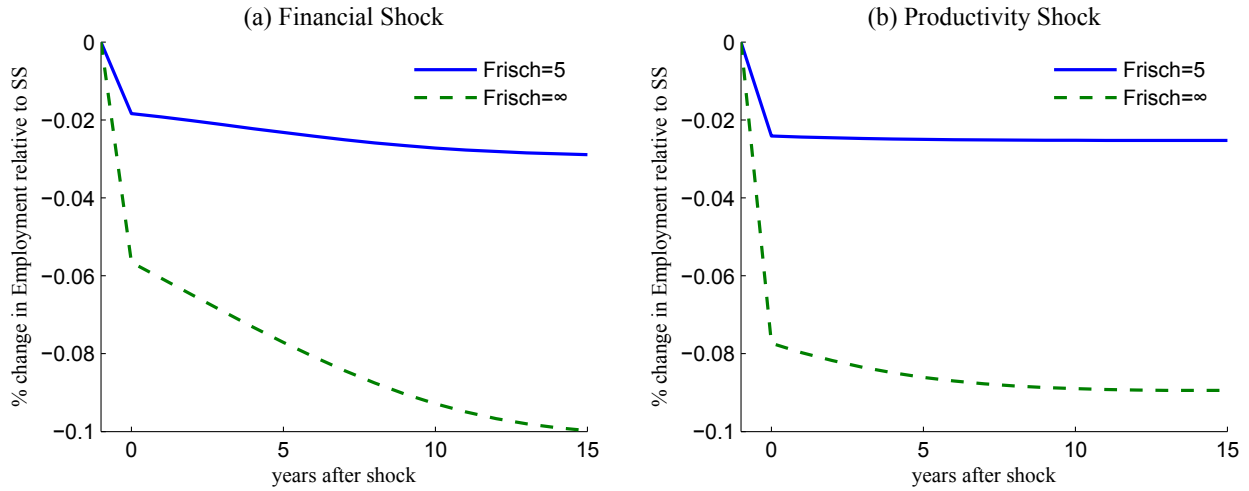
The figure displays the transition paths for employment under financial and productivity shocks. The size of productivity shock is chosen to deliver a similar decline in unemployment as under the financial shock for the case of Frisch elasticity of 5.
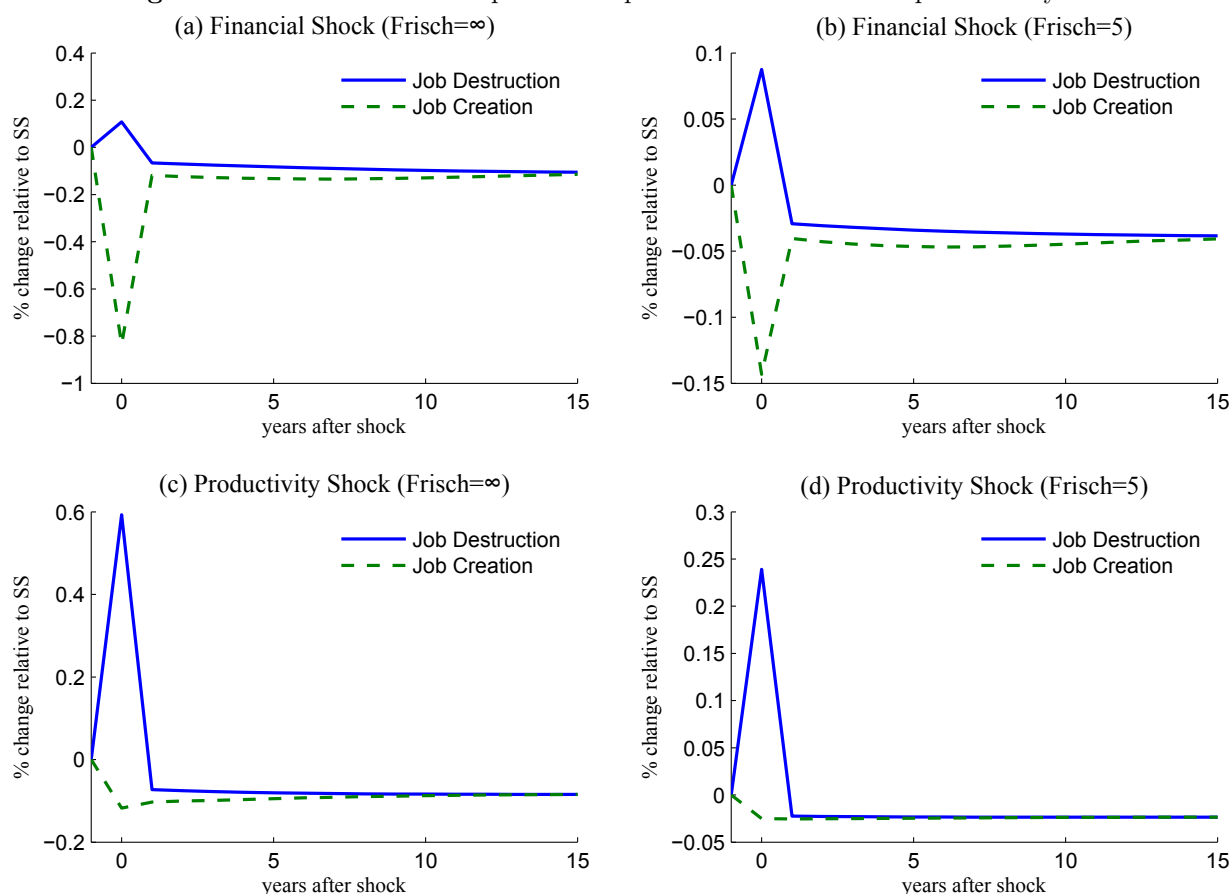
job creation.

A financial shock reduces employment by reducing job creation as both new firms are smaller and existing firms grow less in the next period. Job destruction also increases because a tighter financial constraint leads some growing firms to reduce employment in the next period. A productivity shock reduces employment by increasing job destruction since all unconstrained firms contract in size, while exerting a smaller effect on the path of employment growth of those firms still growing out of their financial constraint.

Interestingly, this pattern fits the response of job flows in the last two recessions as seen in Figure 1: the 2001 recession characterized by a relatively sharp response of job destruction, while the 2008 recessions characterized by a strong response of job creation. Indeed, in 2008, job destruction did not exceed the levels reached in 2001 in a much shallower recession. After the impact period, job creation from a financial shock falls below its steady state level and converges towards a lower level. Job destruction behaves similarly converging towards the lower level of job creation. While permanent productivity shocks also reduce job flows in the long-run, financial shocks result in a larger reduction in job flows; in the case of constant wages, job flows fall by 10% of steady state levels under a permanent financial shock while job flows fall by 8% of steady state levels - somewhat less - under a productivity shock.

Table 9 displays the effect of financial and productivity shocks on the distribution of job creation and job destruction by firm age and size categories. The table shows the average effect over the first three years after the shock with job flows expressed as percentages of the initial (steady state) level. The left-hand side displays the job flows effects of a permanent financial shock, while the right-hand side displays the job flows effects of a permanent productivity shock. The three columns in

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Figure 6: Job flows transition paths with permanent financial and productivity shocks

(a) Financial Shock (Frisch=∞)

(b) Financial Shock (Frisch=5)

(c) Productivity Shock (Frisch=∞)

(d) Productivity Shock (Frisch=5)

The figure displays the transition paths for gross job creation and job destruction under the financial and productivity shocks. The numbers plotted display changes relative to the initial (steady state) levels. For example, job creation declines by 80% on impact after the financial shock. The effects of the financial shock are shown in panels (a) and (b), while the productivity shock effects are shown in panels (c) and (d).

each panel conform to different values for the Frisch elasticity: first column is the case of a infinite Frisch elasticity (rigid wages), the second column is the case of a zero Frisch elasticity (vertical labor supply), and the highlighted column is our preferred specification.

As the first and eighth rows show, the collateral shock depresses job creation and job destruction both relative to productivity shocks and relative to steady state so long wages do not fall to maintain the same level of employment. The fall in job creation under a financial shock is particularly stark in the PE case with rigid wages with job creation falling 31%. Consistent with our aggregate job flows regressions, job destruction shows little response to a collateral shock over a three year period - an increase on impact is offset by subsequent declines in job destruction in the following periods. By contrast, job destruction rises well above steady state levels under a negative productivity shock.

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5.2.1 Age Effects

The effect of the collateral shock on job creation by age largely mirrors the empirical patterns we document. For firm age categories, younger firms exhibit the strongest response to a collateral shock followed by new firms and mature firms. In our empirical results, new and young firms exhibited a similar long-run decline in job creation in response to a decline in housing prices. Our model exhibits a more muted response of job creation at new firms. However, the absence of an active entry margin in our calibration likely accounts for this discrepancy. Given our results on firm entry and house prices, an active entry margin should increase the sensitivity of job creation among new firms. In the absence of wage adjustment, mature firms also exhibit a large decrease in job creation in response to a collateral shock. However, the general equilibrium response of wages offsets the effect of the financial shock on mature firms by lowering wages and raising employment at unconstrained firms. In our preferred specification, the drop in wages is sufficient to raise employment at mature firms above their steady state level, consistent with the negative sign on the coefficient estimated in our firm age regressions.

Table 9: Effect of shocks on job flows

		Permanent Financial Shock			Permanent Productivity Shock		
		Frisch = ∞	Frisch = 0	Frisch = 5	Frisch = ∞	Frisch = 0	Frisch = 5
Panel A: Job Creation							
	Aggregate	-0.31	0.02	-0.07	-0.11	0.01	-0.03
Age	Births	-0.12	-0.08	-0.09	-0.04	0.00	-0.01
	1-5 years	-0.49	-0.32	-0.37	-0.09	0.01	-0.02
	6+ years	-0.41	0.26	0.09	-0.18	0.01	-0.04
Size	1-19 emps	-0.05	0.13	0.07	-0.06	0.00	-0.02
	20-99 emps	-0.50	-0.11	-0.22	-0.11	0.01	-0.03
	100+ emps	-0.49	-0.02	-0.15	-0.14	0.01	-0.03
Panel B: Job Destruction							
	Aggregate	-0.01	0.02	0.01	0.20	0.01	0.07
Age	1-5 years	-0.05	-0.04	-0.04	0.04	0.00	0.02
	6+ years	0.01	0.04	0.03	0.25	0.01	0.09
Size	1-19 emps	0.05	0.09	0.08	0.18	0.00	0.06
	20-99 emps	-0.07	-0.05	-0.06	0.17	0.01	0.06
	100+ emps	0.00	0.02	0.01	0.22	0.01	0.08

The table displays the effect of permanent negative financial shock (a 20% decline in collateral constraint parameter) and permanent negative productivity shocks (the size of the shock produces the same long-run decline in aggregate employment level) on the distribution of job creation and job destruction by firm age and size categories. The table shows the average effect over the first three years after the shock with job flows expressed as changes relative to the initial (steady state) level. For example, the aggregate job creation declines by 31% on average over the first three years after the shock. The first three columns display the job flows effects of a permanent financial shock, while the last three columnds display the job flows effects of a permanent productivity shock. Each column conforms to different values for the Frisch elasticity: an infinite Frisch elasticity (rigid wages), zero Frisch elasticity (vertical labor supply), and the highlighted column is our preferred specification.

Our model also does a good job of matching the empirical patterns of housing prices on job destruction by firm age. Job destruction falls for young firms relative to mature firms under a financial shock, consistent with the empirical ordering we documented. Moreover, job destruction among mature firms increases 3% relative to initial levels, consistent with the negative coefficient for job destruction at mature firms in our firm age regressions. On impact, job destruction rises at both young and mature firms since tighter financial constraints lower capital and labor demand. After impact, destruction falls at young firms because they become smaller after the collateral shock. Therefore, the jobs destroyed by these firms when they exit also fall. In contrast, for mature firms, there are two competing effects in periods after impact: given exogenous exit rates, fewer firms survive to their optimal size reducing job destruction, however, as wages fall, optimal size increases leading to greater job destruction for unconstrained firms that exit.

In contrast, productivity shocks generate a more uniform effect across firm age categories. As the highlighted column shows, productivity shocks generate largely uniform declines in job creation across firm age categories. Mature firms rather than young firms exhibit the sharpest declines to a negative productivity shock. Additionally, a productivity shock raises job destruction at both young and mature firms, though the shock does have a larger relative effect on mature firms similar to a financial shock.

5.2.2 Size Effects

For firm size categories, our model matches the ordering of sensitivity across size categories with middle-sized firms experiencing the biggest decline in job creation relative followed by large firms and small firms respectively. In the case of small firms, a negative collateral shock generates an increase in job creation consistent with the sign of the coefficient on housing prices in our regressions. A collateral shock has a stronger effect on medium-sized and large firms because the collateral constraint is more likely to bind for high productivity, growing firms. Low productivity (small) firms need not accumulate sizable assets to achieve their optimal size, while high productivity firms must wait to accumulate capital to achieve optimal size. These higher productivity firms transit through the middle-sized employment category and are most sensitive to the effects of a tighter collateral constraint on their growth rates. With wage adjustment, the effect on large firms is partially offset by increased job creation among unconstrained firms - lower wages raise their optimal size and increases job creation at these high productivity unconstrained firms.

In terms of job destruction, we find declines in job destruction for middle-sized firms while an increase in job destruction for small firms and little effects for large firms. This ordering is consistent with our empirical evidence for small and middle-sized firms and consistent with the finding that collateral shocks raise job destruction for small firms. While our empirical estimates for large firms do not fit the model predictions, this anomoly is driven entirely by the impact

response of job destruction in our regressions. For large firms, job destruction decreases with a negative housing price shock in the MSA-level regressions. However, if we restrict our attention to the lagged response of job destruction, the empirical pattern across firm size categories is consistent with our model.

In the model, job destruction falls for middle-sized and larger firms because constrained firms become smaller after a collateral shock, while unconstrained firms do not become bigger. As a result, higher productivity firms spend a longer period of time as smaller firms. When these firms exit, this decreases job destruction among middle-sized firms but increases job destruction for small firms. In general equilibrium, lower wages offset this effect for the highest productivity firms mitigating the effect of the collateral shock on job destruction for large firms relative to middle-sized firms.

As with firm age categories, the response of job flows to a productivity shock is more uniform across size categories than the response to a collateral shock. Across all specifications, job creation falls most for large firms in response to a negative productivity as opposed to middle-sized firms. Similarly, job destruction at large firms is most sensitive to a negative productivity shock and job destruction increases across all size categories. These patterns and signs are at odds with the empirical patterns we document further supporting the view that our empirical strategy successfully identifies financial shocks as opposed to productivity shocks or other business cycle shocks.

6   Conclusion

The US housing crisis has raised concerns that depressed real estate values may inhibit firm formation and expansion, disrupting the process of innovation and labor market turnover essential for a healthy economy. An extensive literature has documented the importance of real estate collateral for new firms to obtain lending and for small businesses to obtain financing for expansion. Moreover, recent work also documents the disproportionate contribution of new and young firms to overall labor market turnover. Given these facts, it stands to reason that job flows may be particularly sensitive to a decline in collateral values.

In this paper, we provide support for this hypothesis illustrating the empirical and theoretical link between job flows and housing prices. Using MSA-level variation in job flows and housing prices, we show that both job creation and lagged job destruction decline in response to a fall in housing prices. We control for aggregate demand effects by introducing direct controls for the business cycle and by using a land supply elasticity approach common in the empirical literature on the real effects of collateral shocks. We also document size and age patterns in the sensitivity of job flows to housing prices, showing that job flows for new and young firms (0-5 years of age) are most sensitive to housing prices shocks as are job flows for medium-sized firms (20-99 employees).

We build a simple firm dynamics model with collateral constraints and examine the effect of a collateral shock on overall job flows and job flows by firm size and age category. We show

analytically in a simple version of our model that a collateral shock must reduce employment, job creation and job destruction, and demonstrate why a collateral shock should have stronger effects for young firms and medium-sized firms. We calibrate our benchmark model to match the distribution of employment by firm size and age observed in the data. Our calibrated model replicates the empirical pattern of job flow sensitivity to a collateral shock by firm size and age categories.

Future work will extend our numerical work to include transitory firm productivity shocks to match the overall level of job flows and better match the distribution of job flows across age and size categories. Given the importance of house prices on firm entry, we will also extend our quantitative work to include an endogenous entry margin.

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A Simple Model: Characterization

A.1 Household

The Hamiltonian for the household problem is a sum of instantaneous utility function and the right-hand side of the wealth evolution equation multiplied by the Lagrange multiplier λ_H

$$\mathcal{H} = e^{-\rho t}[u(c) - v(n)] + \lambda_H[wn + ra + \Pi - c]$$

Maximum principle necessarily implies

$$\mathcal{H}_n = -e^{-\rho t}v'(n) + \lambda_H w = 0$$
$$\mathcal{H}_c = -e^{-\rho t}u'(c) - \lambda_H = 0$$
$$\dot{\lambda}_H = -\lambda_H r$$

If we substitute out the Lagrange multiplier we get

$$\dot{c} = -\frac{u'(c)}{u''(c)}(r - \rho) \tag{A.1}$$

$$w = \frac{v'(n)}{u'(c)} \tag{A.2}$$

A.2 Proof of Lemma 1

To solve equation

$$\dot{a} = Aa^{\psi} - Ba,$$

introduce the following change of variables $y = \log a$. Hence,

$$\dot{y} = Ae^{(\psi-1)y} - B.$$

We can rewrite this equation as follows

$$\frac{dy}{Ae^{(\psi-1)y} - B} = dt.$$

Rearranging we get

$$\frac{1}{B(\psi - 1)}\left(\frac{d\left(Ae^{(\psi-1)y} - B\right)}{Ae^{(\psi-1)y} - B} - d[(\psi - 1)y]\right) = dt.$$

Integrating this equation leads to

$$\log\left[Ae^{(\psi-1)y} - B\right] - (\psi - 1)y = B(\psi - 1)t + const.$$

Transforming back to original variable

$$\log[A - Ba^{1-\psi}] = B(\psi - 1)t + const.$$

Since $a(t = 0) = a_0$ we have

$$\log\left[\frac{A - Ba^{1-\psi}}{A - Ba_0^{1-\psi}}\right] = B(\psi - 1)t.$$

This can be expressed as

$$a = \left\{\frac{A - (A - Ba_0^{1-\psi})e^{-B(1-\psi)t}}{B}\right\}^{1/(1-\psi)}.$$

A.3 Properties of $a(t, \chi, \epsilon)$

Monotonicity in t. By taking first order derivative of (15) with respect to time we can show that depending on the initial condition the following three cases are possible

$$
a'(t) \begin{cases} > 0 & \text{for all } t \text{ if } a_0 < \left(\frac{A}{B}\right)^{1/(1-\psi)} \\ = 0 & \text{for all } t \text{ if } a_0 = \left(\frac{A}{B}\right)^{1/(1-\psi)} \\ < 0 & \text{for all } t \text{ if } a_0 > \left(\frac{A}{B}\right)^{1/(1-\psi)}. \end{cases}
$$

Note that once we combine the two cases it we will not be optimal to borrow up to firms borrowing limit $k = \kappa a$ if $a_0 > (A/B)^{1/(1-\psi)}$. This implies that in equilibrium all the firms will never decrease its level of capital rentals.

Convexity in t. $\dot{a} = Aa^\psi - Ba$ implies $\ddot{a} = \left(\psi A a^{\psi-1} - B\right)\dot{a} = \left(\psi A a^{\psi-1} - B\right)\left(Aa^\psi - Ba\right)$. Hence,

$$
a''(t) \begin{cases} > 0 & \text{if } a < \left(\psi\frac{A}{B}\right)^{1/(1-\psi)} \\ < 0 & \text{if } a \in \left(\left(\psi\frac{A}{B}\right)^{1/(1-\psi)}, \left(\frac{A}{B}\right)^{1/(1-\psi)}\right) \\ > 0 & \text{if } a > \left(\frac{A}{B}\right)^{1/(1-\psi)}. \end{cases}
$$

Monotonicity in χ.

$$
\begin{aligned}
\frac{da^{1-\psi}}{d\chi} &= \frac{\partial a^{1-\psi}}{\partial A}\frac{dA}{d\chi} + \frac{\partial a^{1-\psi}}{\partial B}\frac{dB}{d\chi} \\
&= \frac{A}{B}\left(1 - e^{-B(1-\psi)t}\right)\underbrace{\left[\frac{A_\chi}{A} - \frac{B_\chi}{B}\right]}_{<0} + \underbrace{\left(\frac{A}{B} - a_0^{1-\psi}\right)}_{\lessgtr 0} e^{-B(1-\psi)t}(1-\psi)tB_\chi
\end{aligned}
$$

because

$$
\frac{A_\chi}{A} - \frac{B_\chi}{B} = \frac{\alpha\phi}{1-\phi(1-\alpha)}\frac{1}{\chi} - \frac{r_k}{r_k\chi - r} < 0.
$$

Hence,

- if $a_0 < \left(\frac{A}{B}\right)^{1/(1-\psi)}$ then there exists $T_a < \infty$ such that

$$
a_2(t, \chi) \begin{cases} > 0 & \text{if } t < T_a, \\ = 0 & \text{if } t = T_a, \\ < 0 & \text{if } t > T_a, \end{cases}
$$

- if $a_0 > \left(\frac{A}{B}\right)^{1/(1-\psi)}$ then $a_2(t, \chi) < 0$.

∎

A.4   Proof of Lemma 3

PE effect.

$$\int_0^{\bar{t}(\chi_L)} n(t, \epsilon_H, w, r, r_k, \chi_L) e^{-\sigma t} dt + \int_{\bar{t}(\chi_L)}^{\infty} n^*(\epsilon_H, w, r, r_k) e^{-\sigma t} dt$$

$$< \int_0^{\bar{t}(\chi_H)} n(t, \epsilon_H, w, r, r_k, \chi_L) e^{-\sigma t} dt + \int_{\bar{t}(\chi_H)}^{\infty} n^*(\epsilon_H, w, r, r_k) e^{-\sigma t} dt$$

$$< \int_0^{\bar{t}(\chi_H)} n(t, \epsilon_H, w, r, r_k, \chi_H) e^{-\sigma t} dt + \int_{\bar{t}(\chi_H)}^{\infty} n^*(\epsilon_H, w, r, r_k) e^{-\sigma t} dt$$

GE effect.

In the absence of wealth effect the equilibrium on labor market can be expressed as follows $N = N^d(w, \chi) = N^s(w)$. Taking full derivative we obtain

$$\frac{dN}{d\chi} = N_1^s(w) \frac{N_2^d(w, \chi)}{N_1^s(w) - N_1^d(w, \chi)} > 0,$$

where $N_1^s(w) > 0, N_2^d(w, \chi) > 0, N_1^d(w, \chi) < 0$ are the derivative of the corresponding functions. ∎

Job creation and job destruction.   Firm-level job creation is

$$jc(t) = \max\{\dot{n}(t), 0\} \begin{cases} = \dot{\bar{n}}(t) & \text{if } t < \bar{t} \\ = 0 & \text{if } t \geq \bar{t} \end{cases}$$

Note that for any firm that reaches its optimal size it is true that

$$n^*(t) = \bar{n}(0) + \int_0^{\bar{t}} jc(t) dt \tag{A.3}$$

The first term takes into account that at the moment of birth the firm increases its employment from 0 to $\bar{n}(0)$.