A Quantitative Analysis of the Retail Market for Illicit Drugs

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Abstract

We develop a theoretical framework to study illicit drugs markets, and we estimate it using data on purchases of crack cocaine. Buyers are searching for high-quality drugs, but they can determine drugs' quality (i.e., their *purity*) only after consuming them. Hence, sellers can rip-off first-time buyers, or can offer higher-quality drugs to induce buyers to purchase again from them. In equilibrium, a distribution of qualities persists. The estimated model implies that sellers' moral hazard reduces the average and increases the dispersion of drug purity, thereby affecting drug consumption. Moreover, the estimated model implies that increasing penalties may increase the purity and the affordability of drugs traded, because it increases sellers' relative profitability of targeting loyal buyers versus first-time buyers.

PRELIMINARY AND INCOMPLETE

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1   Introduction

How do markets for illicit commodities, such as narcotics, differ from regular markets? What would happen to the consumption and prices of narcotics if their trade were legalized? How do changes in the intensity of enforcement affect them?

We seek to understand these issues by building and estimating a model that focuses on pervasive sellers' moral hazard as the distinguishing characteristic of the market for illicit drugs (i.e., the "cutting" of drugs).[1] We quantify the effects of sellers' moral hazard on drugs' pure-gram prices and drugs' consumption, possibly providing some insights on how market outcomes would differ if this market were legal. The presence of moral hazard leads to counter-intuitive effects of policing, as well.

We model a market in which buyers with heterogeneous willingness to pay for drugs search for sellers with heterogeneous costs of supplying drugs. Following the key insight of Galenianos, Pacula, and Persico (2012), buyers cannot observe drug purity before consuming it—i.e., illicit drugs are *experience* goods; this is one key way in which the model captures an illegal market, in which quality is non-contractible and no institutions can enforce quality standards.[2] Buyers' inability to ascertain quality creates a trade-off for sellers. On one hand, they can offer zero-purity drugs to first-time buyers, thereby maximizing instantaneous profits. On the other hand, they can offer higher-quality drugs that induce buyers to purchase again from them, thereby increasing their customer base. In equilibrium, a distribution of quality levels persist: high-cost sellers choose to cheat and rip-offs their (first-time) buyers, whereas low-cost sellers offer positive purity levels, with the lowest-cost sellers offering the purest drugs.

In our quantitative analysis, we estimate the model combining two distinct datasets that provide key pieces of information on the crack cocaine market: 1) the distribution of drug qualities offered in the market; 2) how frequently buyers purchase drugs; and 3) whether buyers purchased drugs from their regular sellers. Overall, the model fits the data well.

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[1]For instance, we document in Section 3 that a significant proportion of drug purchases—more than 10 percent— involve zero purity level, i.e., are complete rip-offs. It is hard to find a legal market with comparable levels of outright fraud. Moreover, the cutting of drugs happens at the retail level, since wholesale operations (i.e., transportation) are more efficient if drugs are pure, and the testing of drug purity is a common practice in large wholesale transactions.

[2]We abstract from *why* the market is illegal, presumably because of externalities that market participants impose on non-market participants. Rather, we focus on buyer-seller relationships *given* that the market is illegal. See Fryer, Heaton, Levitt, and Murphy (2013) for an analysis of some of the externalities due to crack cocaine.

Moreover, the estimates imply that sellers' profits are extremely skewed, with very few (low-cost) sellers reaping substantial profits, whereas most sellers earn less than the minimum wage, in agreement with the descriptive evidence reported by Levitt and Venkatesh (2000).

We use our parameterized model to perform counterfactual analyses. Specifically, the model allows us to quantify the role of sellers' moral hazard due to buyers' inability to verify the quality of drugs. This type of buyers' imperfect information is one key characteristic of illegal markets because, in a legal market, the quality of the product is more-easily verifiable and contractible. Our counterfactual analysis quantifies the effect of this information friction on market outcomes, possibly providing some insights on how outcomes would differ if the market were legal, with buyers having better information about product quality before trading. This counterfactual analysis reveals that zero-purity drugs disappear from the market. Moreover, sellers have to increase quality to induce their first-time occasional buyers to purchase and, possibly, to become their loyal customers and, thus, the average purity increases by approximately 13 percent and the standard deviation of purity decreases by approximately 75 percent. Hence, a larger fraction of buyers is matched to a regular seller, thereby affecting buyers' drug consumption.

We further use our model to study the role of differential penalties on buyers and on sellers. In the past 30 years the U.S. have markedly increased the enforcement and severity of drug laws—i.e., the so-called "war on drugs." One important outcome of this policy is that the number of people arrested for drug-related offenses has tripled in the past 30 years, whereas the number of arrests for non-drug related offenses has barely changed over the same period. Interestingly, during the same period, drugs have become dramatically *cheaper* and *purer*. We find that increasing enforcement on buyers and/or sellers leads to an increase in the average quality offered in the market, thereby making drugs more affordable. The reason is that higher penalties decrease the number of sellers, thus making it more difficult for buyers to meet them. In a market with moral hazard, this lower meeting rate decreases sellers' incentives to make quick profits by selling zero-purity drugs. Instead, sellers increase the qualities of drugs to attract loyal-buyers. More generally, the counterfactual analyses highlight that long-term relationships are more valuable in a market with less frequent search. Thus, to the extent that an increase in police enforcement reduces the intensity of search in the market, it helps strengthen the long-term relationships that help overcome the inherent moral hazard problem in an illegal market and, therefore, leads to greater average quality. Hence, our analysis suggests that increasing penalties may have contributed to the observed increased affordability and purity of retail drugs in the U.S. Of course, the market for drugs

has changed in many ways over time (among others, through economies of scale in the transportation of drugs to the U.S.); nonetheless, we find interesting that our model is consistent with these time-series differences.

The paper proceeds as follows. Section 2 reviews the literature. Section 3 introduces the data. Section 4 presents the theoretical model. Section 5 presents our quantitative analysis: we estimate the model, illustrate its main implications, and perform counterfactual analyses. Section 6 concludes. The appendices present the details of the solution of the model with observable quality, and of the calculation of the density of buyers' preferences in the ADAM dataset, respectively.

2 Related Literature

The paper contributes to several strands of the literature. The first one is the literature on illegal markets. Most previous papers on illicit drugs markets focus on the role of consumers' rationality and addiction in the demand for illicit drugs (Stigler and Becker, 1977; Schelling, 1984; Becker and Murphy, 1988; Grossman and Chaloupka, 1998). Most theoretical models of the market structure build on the traditional economic assumptions of perfect information and/or a centralized market (a notable exception is the discussion of information issues in drug markets in Reuter and Caulkins, 2004); Bushway and Reuter (2008) presents a review of this literature. This framework abstracts from two defining features of illicit markets that we instead focus on: non-contractibility and search frictions. Hence, it cannot address how penalties affect meeting rates and the distribution of purity in the market.

This paper is also related to the empirical literature on the structural estimation of search models. Most dynamic applications focus on labor markets; Eckstein and Van den Berg (2007) provide an insightful survey of this literature, and the closest empirical paper within the labor-market context is Bontemps, Robin, and Van den Berg (1999). Almost all empirical applications to product markets use static models: recent contributions include Hortaçsu and Syverson (2004), Hong and Shum (2006), Wildenbeest (2011), and Allen, Clark, and Houde (2014). Instead, we focus on the role of long-term buyer-seller relationships in a product market, and innovate on all previous empirical contributions by seeking to understand the quantitative effects of an additional friction—i.e., the imperfect observability of product quality at the time of the transaction—and of its interaction with penalties that affect

buyer-seller meetings.[3]

In doing so, the paper also contributes to the literature that studies firms' quality decisions when quality is not observable or not contractible. Most of these papers examine the markets for legal commodities. Hence, search and matching frictions play a smaller role than in the market for crack cocaine, in which turnover of both buyers and sellers are very important. Among theoretical contributions, Gale and Rosenthal (1994) presents a model in which buyers have to pay a cost before finding a high-quality seller. Many empirical contributions have analyzed the role of quality certification, and consumers' and suppliers' responses to it; for a thorough survey, see Dranove and Jin (2010). Empirical analyses of moral hazard behavior have mainly focused on the behavior of intermediaries; see, among others, Iizuka (2007, 2012) for the case of prescription drugs. Hubbard (1998) and Schneider (2012) present direct evidence on sellers' moral hazard in the case of vehicle inspections and repairs, respectively. This paper innovates on previous descriptive work by estimating an equilibrium model that allows a quantification of the role of the non-observability of quality on market outcomes.

3   Data

We combine two distinct datasets. The first is an extensive database on drug purchases. The second is a survey that collects information about drug use among those committing crimes. We now describe each dataset in more detail.

STRIDE—The System to Retrieve Information from Drug Evidence (STRIDE) is a database of drug exhibits sent to Drug Enforcement Administration (DEA) laboratories for analysis. Exhibits in the database are from the DEA, other federal agencies, and local law enforcement agencies. The data contain records of acquisitions of illegal drugs by undercover agents and informants of the DEA. Economic analyses of markets for illegal drugs have widely used STRIDE, although it is not a representative sample of drugs available in the United States.[4]

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[3]Gavazza (2013) also estimates a dynamic search model, with a focus on asset markets.

[4]Horowitz (2001) questions the reliability of the STRIDE data set, noting that the time series of drug prices in Washington, D.C. differ depending on which agency collected the data (DEA or other law enforcement agency). However, Arkes, Pacula, Paddock, Caulkins, and Reuter (2008) show that the inconsistencies identified by Horowitz (2001) largely disappear simply by controlling for the size of the transaction (above or below 5 grams) when combined with other data cleaning issues that Horowitz (2001) raise. Mindful of this finding, we are careful to restrict our analysis to the relatively narrow sample of transactions whose value is

The entire dataset has a total of approximately 915,000 observations for the period 1982-2007 for a number of different drugs and acquisition methods. We focus on crack cocaine and keep the observations acquired through purchases (i.e., we drop seizures) and clean the data of missing values and other unreliable observations, as Arkes, Pacula, Paddock, Caulkins, and Reuter (2008) suggests. We use the STRIDE data to present trends for our entire sample period, but our quantitative analysis of Section 5 uses data for the years 2001-2003 because of the time limitations of our other data source, as described below. Moreover, since the focus of our model is on retail transactions, we include in our estimation sample only purchases with a value of less than $200 in real 1983 dollars. We further drop purchases with a value of less than $25, as most of these purchases (representing seven percent of our sample of transactions) resemble "introductory offers" that the model will not consider; our empirical results are very similar if we include these observations, as well.

ADAM—The Arrestee Drug Abuse Monitoring (ADAM) data set is a quarterly survey of persons arrested or booked on local and state charges within the past 48 hours in various ADAM metropolitan areas in the United States.[5] The survey asks about the use, importance and role of drugs and alcohol. The arrestees participate in the survey voluntarily under full confidentiality.[6] In addition to interviewing arrestees, urine samples are requested and analyzed for validation of self-reported drug use. Since 2000, the survey includes a drug market procurement module, and collects information on the arrestee's most recent drugs purchase for all arrestees who report having used drugs in the previous 30 days. Information collected includes number of drug purchases in the past 30 days, and whether they last purchased from their regular dealer. We have data from the 2001-2003 surveys.

3.1 Data Description

Table 1 provides summary statistics of the main variables used in the quantitative analysis. Panel A refers to the STRIDE Dataset, and Panel B to the ADAM dataset.

Panel A reports some interesting patterns. While the transactions display some hetero-

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below 200 constant 2003 dollars. Also, Arkes, Pacula, Paddock, Caulkins, and Reuter (2008) show that the price series for different drugs obtained from STRIDE predict, in a Granger sense, the number of drug-related admissions to emergency rooms (DAWN data set). Overall, we feel that Arkes, Pacula, Paddock, Caulkins, and Reuter (2008) make a compelling case for the usefulness of the STRIDE dataset when used carefully, i.e., without aggregating across transactions of vastly different sizes.

[5]The number of these areas changes from years to year based on the availability of the data. From 2001 to 2003, it has been 33, 36 and 39, respectively.

[6]Dave (2008) reports that only about ten percent of the arrestees reject the interview request.

TABLE 1: Summary Statistics

PANEL A: STRIDE (N=2,138)	Mean	Std. Dev.	Min	Max
Price (2003 dollars)	101.279	44.770	25.241	200.000
Weight (Grams)	1.178	1.095	0.037	9.500
Potency (%)	58.074	26.538	0.000	98.000
Pure Quantity	0.689	0.688	0.000	4.422
Pure Grams per $100	2.159	1.697	0.000	9.980
PANEL B: ADAM (N=14,713)				
Obtained Drug in Last 30 Days (%)	0.740	0.438	0.000	1.000
Purchased from Regular Dealer (%)	0.525	0.499	0.000	1.000
Number of Purchases in Last 30 Days, Matched	16.331	11.124	1.000	30.000
Number of Purchases in Last 30 Days, Unmatched	11.548	10.419	1.000	30.000

Notes—This table provides summary statistics of the variables used in the empirical analysis. Panel A presents summary statistics of the variables obtained from the STRIDE dataset; Panel B presents summary statistics of the variables obtained from the ADAM dataset. Drug prices have been deflated using the GDP Implicit Price Deflator, with 2003 as the base year.

geneity in their dollar values, the coefficient of variation of PURE QUANTITY (the product of WEIGHT and POTENCY) is substantially larger. We take the ratio of PURE QUANTITY and PRICE to construct the variable PURE GRAMS PER $100; figure 1 displays its empirical distribution, which displays substantial variation, with 12.6 percent of the observations having a value of zero—i.e., complete ripoffs.

Panel B reports that 74 percent of all arrestees purchased crack cocaine in the past 30 days. Of those who purchased it, the average number of PURCHASES IN PAST 30 DAYS equals 12.98 (thus, the unconditional average of PURCHASES IN PAST 30 DAYS is 9.6). Of those who purchased crack cocaine, 52.5 percent report consuming from their regular source. Interestingly, individuals purchasing from their regular dealers report an average of 16.3 PURCHASES IN PAST 30 DAYS, whereas individuals purchasing either from an occasional source or from a new source have an average of 11.5 PURCHASES IN PAST 30 DAYS. The model will interpret this difference as different consumption rates between buyers who are currently matched to a seller and buyers who are currently not matched, taking into account that buyers will choose to match with a seller depending on their preferences for drugs.

Overall, these two datasets provide a rich description of the retail crack-cocaine market
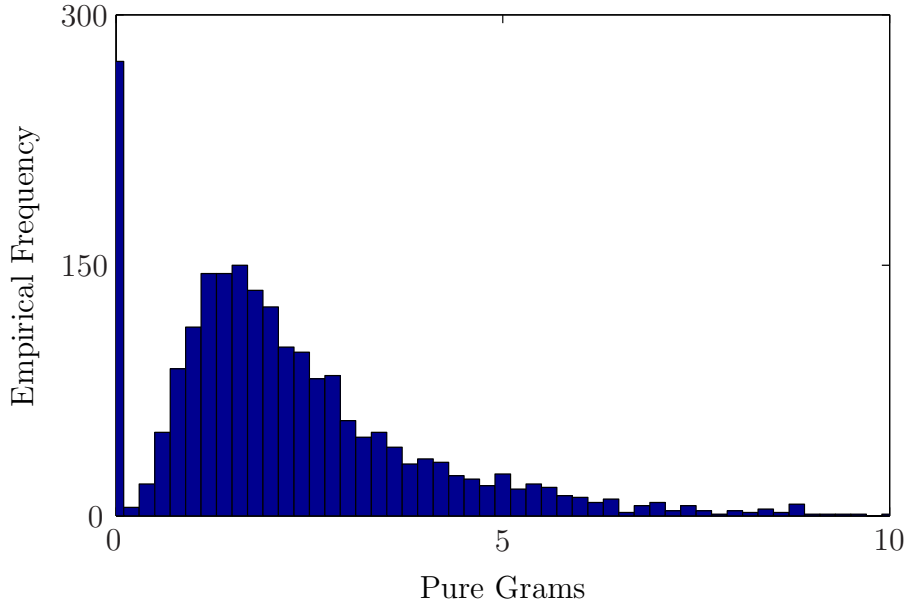
FIG. 1: Histogram of pure grams of crack cocaine per $100.

and are well-suited to investigating the importance of search frictions and of imperfect observability, and the role of buyer-seller relationships. Specifically, our model interprets the dispersion of PURE GRAMS PER $100 as departure of the law-of-one-price originating from both search frictions and imperfect observability. Moreover, the ADAM dataset is useful to measure the frequency of buyer-seller long-term relationships and buyers' consumption rates. Since ADAM likely oversamples drug users, we will explicitly account for sample selection in our quantitative analysis.[7]

With all their advantages, however, the datasets pose some challenges. For example, both datasets are cross-sectional, which implies that we do not observe buyers' and sellers' behavior over time. Nonetheless, in our view, the main limitation is that we do not observe individual sellers transacting with several buyers. This limitation implies that a model in which sellers discriminate between different buyers, while theoretically feasible, would be difficult to identify with the available data. Therefore, the model will (successfully) match the data by focusing on heterogeneity between sellers, rather than heterogeneity within sellers.

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[7]For example, 2003 National Survey on Drug Use and Health reports that the fraction of non-institutionalized individuals aged 12 and older who use drugs is substantially lower than those who report using drugs in ADAM.

4 The Model

Time runs continuously, the horizon is infinite and the future is discounted at rate r.

There is a continuum of potential buyers of measure $\overline{B}$ who are heterogeneous with respect to their preferences for consuming drugs. A buyer's marginal utility of consuming drugs is denoted by z and is distributed according to a continuous, connected and log-concave distribution $\overline{M}(\cdot)$ with support $[0, \overline{z}]$.[8] Each buyer decides whether to participate the market. If he does not participate, his payoff is zero. If he participates, he pays entry cost K_B, which represents the possibility of arrest,[9] and he trades with sellers. The measure of buyers who participate in the market is denoted by B and the distribution of their types is denoted by $M(\cdot)$. Buyers maximize their expected discounted utility.

There is a continuum of potential sellers of measure $\overline{S}$ who are ex ante identical. A seller decides whether to pay entry cost K_S and participate in the market. If he participates, he draws the cost of providing drugs c from distribution $D(\cdot)$ which is continuous and connected with support $(0, \infty)$.[10] The measure of sellers who participate in the market is denoted by S. Sellers maximize their discounted steady state profits.

Buyers and sellers that participate in the market meet and trade with each other. At any point in time, a participating buyer is either matched with a seller (his "regular" seller) or he is unmatched. There are two types of meetings: "new" meetings, where a buyer and a seller meet for the first time, and "repeat" meetings, where a buyer meets his regular seller. At a meeting a transaction takes place and is followed by the transition between the matched and unmatched state.

In a transaction the buyer pays a fixed price p and receives quality q. The quality q is chosen by the seller at cost cq. At the time of the transaction, the buyer and seller both observe p but the quality q fetched by p cannot be determined by the buyer. After the transaction, the buyer consumes and the quality of the purchase is perfectly revealed. The instantaneous utility that a type-z buyer receives from consuming quality q is equal to zq.

The main assumption on sellers' behavior is that, once they decide on the quality level that they offer, they commit to their decision forever. That is, a seller supplies the same quality at all times and, as a result, the buyer knows the quality that he will receive from a

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[8]The heterogeneity in z could arise because of differences in innate preferences, or because of differences in addictions Becker and Murphy (1988), although we do not model the path to addiction.

[9]This cost can also be formulated as a flow cost without affecting the results.

[10]The model's information structure (described below) makes it trivially optimal for a seller to stay in the market once he has paid the entry cost.

particular seller once he has sampled from him.

After transacting with a new seller, the buyer decides whether to *match* with that seller. Specifically, if the buyer is unmatched, he chooses whether to match with the seller or to remain unmatched; if the buyer is already matched with a different seller, he chooses whether to match with new seller, thereby severing the earlier match, or to return to his previous regular seller. Therefore, matches may be endogenously dissolved when a matched buyer participates in a new meeting and chooses to switch to the new seller. Additionally, at exogenous rate δ a match is dissolved and the buyer becomes unmatched.

The flow of new meetings is determined by a meeting function $m(B, S)$. The meeting function has constant returns to scale, is increasing and concave in both arguments, and satisfies $m(0, S) = m(B, 0) = 0$ and the Inada conditions. Denote the buyer-seller ratio by $\theta = \frac{B}{S}$ and let the rate at which a buyer meets with a new seller by $\alpha_B(\theta)$ and the rate at which a seller meets with a new buyer by $\alpha_S(\theta)$. We have:

$$
\begin{aligned}
\alpha_B(\theta) &= \frac{m(B, S)}{B}, \\
\alpha_S(\theta) &= \frac{m(B, S)}{S}.
\end{aligned}
$$

Our assumptions imply that $\alpha_B(\cdot)$ is strictly decreasing and $\alpha_S(\cdot)$ is strictly increasing in the buyer-seller ratio θ.

The flow of repeat meetings is equal to γ, which is the rate that a matched buyer contacts his regular seller. Therefore, matched and unmatched buyers meet new sellers at the same rate $\alpha_B(\theta)$ and matched buyers additionally meet their regular sellers at rate γ.

A potential buyer decides whether to participate in the market. Denote the value of an unmatched buyer of type z who participates in the market by $\overline{V}_z$. A buyer of type z compares the costs and benefits of entering the market and he participates if and only if

$$
r\overline{V}_z \geq K_B
$$

A participating buyer chooses the reservation quality for becoming matched with a new seller, as a function of his type and whether he is currently matched or unmatched. Let R_z denote the reservation quality of an unmatched buyer of type z and let $H(\cdot)$ denote the distribution of unmatched buyers' reservation qualities. The reservation quality of a matched buyer is, trivially, the quality that he receives from his regular seller.

A potential seller decides whether to participate in the market and, if so, he chooses the quality level q that maximizes his steady state profits conditional on his cost c. Steady state profits have two components: the margin per transaction $(p - cq)$ and the steady state flow of transactions $t(q)$. Steady state profits equal:

$$\pi_c(q) = (p - cq)t(q).$$

Let $q^*(c)$ denote the quality that a seller of type c offers, and let $F(\cdot)$ denote the resulting distribution of qualities offered in the market.

Thus, sellers' free entry condition implies that sellers' expected profits equal their entry cost:

$$\int_0^\infty \pi_c(q^*(c))dD(c) = K_S.$$

Definition 1 *An equilibrium is the actions of buyers $M(\cdot), H(\cdot), B$ and the actions of sellers $q^*(c), F(\cdot), S$ such that all agents optimize.*

4.1 The Buyers

We derive the buyers' optimal action, taking as given the distribution of offered qualities $F(\cdot)$ and the number of participating sellers S.[11]

Consider an individual buyer of type z who takes the behavior of sellers and other buyers as given. When unmatched, the buyer meets a new seller at rate $\alpha_B(\theta)$, pays price p, receives quality q which is a random draw from $F(\cdot)$ and matches with the seller if the quality level exceeds his reservation, R_z. His value is given by:

$$r\overline{V}_z \;=\; \alpha_B(\theta)\Big(z\int_0^{\overline{q}} xdF(x) + \int_{R_z}^{\overline{q}}(V_z(x) - \overline{V}_z)dF(x) - p\Big). \tag{1}$$

When matched with a seller who offers quality q, the buyer meets his regular seller at rate γ and meets a new seller at rate $\alpha_B(\theta)$. If the new seller's quality is above q, then the buyer matches with the new seller and leaves his current seller. His value is:

$$rV_z(q) \;=\; \gamma(zq - p) + \alpha_B(\theta)\Big(z\int_0^{\overline{q}} xdF(x) + \int_q^{\overline{q}}(V_z(x) - V_z(q))dF(x) - p\Big) + \delta(\overline{V}_z - V_z(q)). \tag{2}$$

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[11]Conditional on $F(\cdot)$, the optimal quality choice $q^*(c)$ of each seller type does not affect buyers' payoffs.

Buyers decide whether to participate in the market and, if so, their reservation quality for becoming matched. The Proposition characterizes that decision.

Proposition 2 *Given $F(\cdot)$ and θ:*

1. *The value of participating in the market for a type-z buyer is:*

$$r\overline{V}_z = \alpha_B(\theta)\left(z\int_0^{\overline{q}} x dF(x) + z\int_{\frac{p}{z}}^{\overline{q}} \frac{\gamma(1-F(x))}{r+\delta+\alpha_B(\theta)(1-F(x))}dx - p\right). \tag{3}$$

2. *A type-z buyer participates in the market if and only if $z \geq \hat{z}(F,\theta)$, where possibly $\hat{z}(F,\theta) > \overline{z}$.*

3. *The optimal reservation quality is*

$$R_z = \frac{p}{z}. \tag{4}$$

Proof. See Appendix A. ∎

Notice that the reservation quality is decreasing in buyers' marginal utility. For a given q, a buyer's utility from consuming is increasing in z and therefore so is his willingness of matching with the sellers who offers q. Furthermore, R_z does not depend on the distribution of offered qualities, $F(\cdot)$, because the arrival rate of new sellers is the same when matched and unmatched.[12]

We now aggregate the decisions of all buyers, thereby endogenizing the measure of participating buyers and the buyer-seller ratio θ. Conditional on the quality distribution $F(\cdot)$ and the number of participating sellers S, we have the following characterization of the market.

Proposition 3 *Given $F(\cdot)$ and S:*

1. *If $p \geq \overline{z}\int_0^{\overline{q}} x dF(x)$, then there is no buyer entry: $B = 0$.*

2. *If $p < \overline{z}\int_0^{\overline{q}} x dF(x)$, then there is a unique buyer type $z^* \leq \overline{z}$ such that a buyer participates in the market if and only if $z \geq z^*$.*

3. *The marginal buyer type is given by the solution to:*

$$\alpha_B\left(\frac{\overline{B}(1-\overline{M}(z^*))}{S}\right)\left(z^*\int_0^{\overline{q}} x dF(x) + z^*\int_{p/z^*}^{\overline{q}} \frac{\gamma(1-F(x))}{r+\delta+\alpha_B\left(\frac{\overline{B}(1-\overline{M}(z^*))}{S}\right)(1-F(x))}dx - p\right) = K_B \tag{5}$$

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[12]See Galenianos, Pacula, and Persico (2012) for a different modeling assumption where R_z does depend on the full distribution.

Proof. See Appendix A. ∎

The distribution of buyer types and reservation qualities can now be described. Let $z(R)$ denote the buyer type whose reservation quality is equal to R. Rearranging equation (4) we have:

$$z(R) = \frac{p}{R}.$$

Furthermore, note that $R_{z(R)} = R$ and $z \leq z(R) \Leftrightarrow R_z \geq R$. Given z^*, the equilibrium distribution of reservation qualities mirrors the distribution of marginal utilities.

The corollary summarizes the results.

Corollary 4 *The marginal type z^* completely characterizes buyers' behavior.*

1. *The measure of buyers in the market is given by:*

$$B = \overline{B}(1 - M(z^*))$$

2. *The distribution of buyer types in the market is given by:*

$$M(z) = \begin{cases} 0 & \text{if } z \leq z^* \\ \frac{\overline{M}(z) - \overline{M}(z^*)}{1 - \overline{M}(z^*)} & \text{if } z \geq z^* \end{cases}$$

3. *The distribution of reservation qualities in the market retains the log-concavity of $\overline{M}(\cdot)$ and is given by:*

$$H(R) = \begin{cases} 0 & \text{if } R \leq \underline{R} \\ \frac{1 - \overline{M}\left(\frac{p}{R}\right)}{1 - \overline{M}(z^*)} & \text{if } R \in [\underline{R}, \overline{R}] \\ 1 & \text{if } R \geq \overline{R} \end{cases}$$

where $\underline{R} = R_{\overline{z}} = \frac{p}{\overline{z}}$ and $\overline{R} = R_{z^*} = \frac{p}{z^*}$.

This completes the characterization of buyers' behavior.

4.2 The Sellers

This Section characterizes the optimal decisions of the sellers, taking as given the marginal buyer type z^*, which determines the measure of buyers who participate B and the (log-concave) distribution of reservation qualities $H(\cdot)$.[13]

We proceed in three steps. First, we characterize the profits of an individual seller, taking as given the actions of buyers and other sellers. Second, we derive the optimal action $q^*(c)$, and resulting quality distribution $F(\cdot)$, for a given buyer-seller ratio, θ. Finally, we determine the measure of sellers who participate in the market, S.

A seller's transactions come from two sources: new buyers, who purchase from that seller for the first time, and repeat buyers, who purchased from that seller in the past and decided to match with him. The flow of transactions is $t(q) = t_N + t_R(q)$ where t_N represents sales to new buyers and $t_R(q)$ represents sales to the seller's repeat (or, regular) buyers. Steady state profits are therefore equal to:

$$\pi_c(q) = \big(t_N + t_R(q)\big)\big(p - cq\big)$$

The flow of new transactions is equal to the rate that a seller is contacted by new buyers, which does not depend on the quality offered:

$$t_N = \alpha_S(\theta) = \theta\alpha_B(\theta)$$

The flow of repeat transactions to a seller who offers quality q depends on the number of regular buyers, denoted by $l(q)$, and the rate at which these buyers contact their regular seller, γ:

$$t_R(q) = \gamma l(q)$$

Unlike new transactions, the flow of repeat transactions depends on the quality level offered by the seller. A seller who offers quality q gains regular customers when he is sampled by unmatched buyers whose reservation is below q and when he is sampled by matched buyers whose regular seller offers less than q. Similarly, he loses his regular customers when they sample a seller offering quality higher than q and also when the match is exogenously disrupted, at rate δ.

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[13]Conditional on the distribution of reservation qualities, the distribution of buyer types does not affect sellers' payoffs.

Before characterizing a seller's profits in more detail, we derive some necessary conditions on the quality distribution which will prove useful below.

Lemma 5 *In equilibrium, the quality distribution $F(\cdot)$ is continuous on $[0, \bar{q}]$ and has support on a subset of $\{0\} \cup [\underline{q}, \bar{q}]$ for some $\underline{q} \in [\underline{R}, \overline{R}]$.*

Proof. See Appendix A. ∎

Taking θ and $F(\cdot)$ (satisfying Lemma 5) as given, the following Proposition characterizes the steady state profits of a type-c seller.

Proposition 6 *The steady state profits of a type-c seller who offers quality q are:*

$$
\pi_c(q) = \begin{cases} \alpha_B(\theta)\theta\left(1 + \dfrac{\gamma\delta H(q)}{\left(\delta + \alpha_B(\theta)(1-F(q))\right)^2}\right)(p - cq) & \text{if } q \geq \underline{R}, \\ \alpha_B(\theta)\theta p & \text{if } q < \underline{R}. \end{cases}
$$

Proof. See Appendix A. ∎

The following Proposition characterizes the optimal decision $q^*(c)$ of sellers, taking as given the buyer-seller ratio, θ and buyers' reservation distribution, $H(\cdot)$.

Proposition 7 *Given $H(\cdot)$ and θ, we have the following:*

1. *There is a unique seller type $\bar{c}$ which is determined by the solution to:*

$$
p = \left(p - \bar{c}\underline{q}(\bar{c})\right)\left(1 + \frac{\gamma\delta H(\underline{q}(\bar{c}))}{(\delta + \alpha_B(\theta)D(\bar{c}))^2}\right),
$$

   where $\underline{q}(c)$ is the solution to:

$$
-c\left(1 + \frac{\gamma\delta H(\underline{q})}{(\delta + \alpha_B(\theta)D(c))^2}\right) + (p - c\underline{q})\frac{\gamma\delta H'(\underline{q})}{(\delta + \alpha_B(\theta)D(c))^2} = 0.
$$

2. *Sellers with $c > \bar{c}$ offer zero quality: $q^*(c) = 0$.*

3. *Sellers with $c \leq \bar{c}$ offer positive quality $q^*(c)$ which is strictly decreasing in c and is determined by the solution to the differential equation*

$$
\dot{q}^*(c) = -\frac{2\gamma\delta(\frac{p}{c} - q^*(c))H(q^*(c))\alpha_B(\theta)D'(c)}{(\delta + \alpha_B(\theta)D(c))\left((\delta + \alpha_B(\theta)D(c))^2 + \gamma\delta H(q^*(c)) - \gamma\delta(\frac{p}{c} - q^*(c))H'(q^*(c))\right)}
\tag{6}
$$

14

with initial condition $q^*(\overline{c}) = \underline{q}(\overline{c})$.

Proof. See Appendix A. ∎

Corollary 8 *The quality distribution is given by:*

$$F(q) = 1 - D\big(q^{*-1}(q)\big).$$

Having fully characterized $F(\cdot)$ for given θ, we now determine the number of sellers S who participate in the market.

Proposition 9 *Given $H(\cdot)$ and B there is a unique S such that $\Pi = K_S$.*

Proof. See Appendix A. ∎

This completes the characterization of sellers' behavior.

4.3 Equilibrium

The equilibrium is a fixed point on the marginal buyer type. Given z^*, the measure of participating buyers B and the distribution of their reservations $H(\cdot)$ are uniquely determined (Corollary 4). This, in turn, determines the measure of sellers who enter the market S (Proposition 9) and the quality distribution $F(\cdot)$ (Corollary 8). Finally, $F(\cdot)$ and S determine the marginal buyer type (Proposition 3, equation (5)). The marginal type is defined on a closed and bounded set $[0, \overline{z}]$ and Proposition 10 follows.

Proposition 10 *An equilibrium exists.*

5 Quantitative Analysis

The model does not admit an analytic solution for all endogenous outcomes. Hence, we choose the parameters that best match moments of the data with the corresponding moments computed from the model's numerical solution. We then study the quantitative implications of the model evaluated at the estimated parameters.

5.1 Parametric Assumptions

We estimate the model using the data described in Section 3, assuming that they are generated from the model's steady state. We set the unit of time to be one month.

Unfortunately, the data lack some detailed information to identify all parameters. Therefore, we fix some values. Specifically, the discount rate r is traditionally difficult to identify, and we set it to $r = .01$. Moreover, since we use the normalized the variable PURE GRAMS PER $100, we set the price to be equal to $p = \$100$. Furthermore, we set sellers' monthly opportunity cost K_S to be $\$2,000$, which is broadly in line with drug-dealers' average earnings reported by Levitt and Venkatesh (2000).

We further make parametric assumptions about the distributions of buyers' and of sellers' heterogeneity. Specifically, we assume that the distribution $M(\cdot)$ of buyers' taste for drugs z is lognormal with unknown parameters μ_z and σ_z. This implies that the distribution $H(\cdot)$ of reservation qualities $R = \frac{p}{z}$ is also lognormal with parameters $\mu_R = \log p - \mu_z$ and σ_z.

Moreover, if quality $q(c)$ is a strictly monotonic (and, thus, invertible) function of cost c, we can estimate directly the distribution $D(c)$ from the empirical distribution of q: $D(c) = 1 - F(q)$. However, we need to specify a parametric distribution for $D(c)$ because several sellers with different costs c choose $q(c) = 0$. Thus, we assume that the distribution of the inverse of sellers' costs $1/c$ follow a Pareto distribution with lower bound $\frac{1}{c_M}$ and shape parameter $\xi \geq 1$. This implies that the distribution of costs c is:

$$D(c) = \left(\frac{c}{c_M}\right)^{\xi}, \quad c \in [0, c_M]. \tag{7}$$

The shape parameter ξ captures the dispersion of costs. If $\xi = 1$, the cost distribution is uniform on $[0, c_M]$. As ξ increases, the relative number of high-cost sellers increases, and the cost distribution is more concentrated at these higher cost levels. As ξ goes to infinity, the distribution becomes degenerate at c_M.

We further assume that drug qualities q are measured with error. More specifically, we assume that the reported qualities $\hat{q}$ and the "true" qualities q are related as:

$$\hat{q} = q\epsilon,$$

where ϵ is a measurement error. We assume that ϵ has a lognormal distribution and restrict its mean to be equal equal to 1, which implies that the parameters μ_ϵ and σ_ϵ of the lognormal distribution satisfy $\mu_\epsilon = -.5\sigma_\epsilon^2$. The assumption of measurement error on wages is quite

common in the literature that structurally estimates search models of the labor market. In our application, it is plausible as well, and it could also account for some unobserved seller behavior that the model does not consider (i.e., price discrimination), thereby allowing us to fit the quality distribution better. In particular, as Lemma 5 highlights, the model implies a gap in the quality distribution between the complete rip-offs $q^* = 0$ and the minimum positive quality $\underline{q}$. Figure 1 shows that the empirical distribution displays this qualitative feature, and the measurement ϵ allows it to more precisely match its magnitude.

Finally, we explicitly model the selection into the ADAM sample. Specifically, we assume that a buyer of type z is in ADAM if $\log(z) + \eta \geq 0$, where η is a random variable independent of z, distributed according to a normal distribution with mean μ_η and standard deviation σ_η. Hence, this selection equation features that buyers with higher preferences for drugs (and, thus, greater drug consumption) are more likely to be in the ADAM dataset. Appendix C reports the details of the derivation of the density of drug users' preferences z in ADAM; we will use this density to compute simulated moments that we match to their empirical counterparts.

5.2   Estimation and Identification

We estimate the vector of parameters $\psi = \left\{\alpha, \gamma, \delta, K_B, \mu_R, \sigma_R, c_M, \xi, \sigma_\epsilon, \mu_\eta, \sigma_\eta\right\}$ using a minimum-distance estimator that matches key moments of the data with the corresponding moments of the model. More precisely, for any value of these parameters, we solve the model of Section 4 to find its equilibrium: the mass B of active buyers and their distribution of reservation qualities $H(\cdot)$, and the mass S of active sellers and their distribution $F(\cdot)$ of offered qualities. We then calculate two sets of moments, one that we match to a set of moments computed from the STRIDE dataset, and one that we match to a set of moments computed from the ADAM dataset.

The first set $\mathbf{m}_1\left(\psi\right)$ is composed by these moments of the quality distribution:

1. The fraction of rip-offs $q = \hat{q} = 0$.[14]

2. The mean of quality for $\hat{q} > 0$.

3. The standard deviation of quality for $\hat{q} > 0$.

4. The median of quality for $\hat{q} > 0$.

---

[14]Note that $q^* = 0$ if and only if $q = 0$.

5. The skewness of quality for $\hat{q} > 0$.

6. The kurtosis of quality for $\hat{q} > 0$.

Moreover, at each value of the parameters, we simulate buyers-sellers meetings and consumption patters (i.e., the α, δ and γ shocks), using the distributions of preferences z and buyer-seller matches that take into account the selection into ADAM (see Appendix C). We then compute the second set $\mathbf{m}_2(\psi)$ composed by these moments:

1. The fraction of users who purchased from their regular dealer, among those who purchased drugs in the last 30 days.

2. The average number of purchases of those who purchased drugs in the last 30 days and made their last purchase from their regular dealer (in the simulation, a purchase from a regular dealer is defined as a purchase from the same seller than the previous purchase).

3. The average number of purchases of those who purchased drugs in the last 30 days and did not made their last purchase from their regular dealer.

4. The standard deviation of the number of purchases of those who purchased drugs in the last 30 days and made their last purchase from their regular dealer.

5. The standard deviation of the number of purchases of those who purchased drugs in the last 30 days and did not made their last purchase from their regular dealer.

6. The fraction of individuals who purchased drugs in the last 30 days.

7. The incarceration rate. While the ADAM data do not report directly this number, aggregate statistics for the United States say that this number is approximately equal to one percent of the population, and we use this figure in our estimation.

The minimum-distance estimator chooses the parameter vector ψ that minimizes the criterion function

$$\left(\mathbf{m}(\psi) - m_S\right)' \Omega \left(\mathbf{m}(\psi) - m_S\right),$$

where $\mathbf{m}(\psi) = \begin{bmatrix} \mathbf{m}_1(\psi) \\ \mathbf{m}_2(\psi) \end{bmatrix}$ is the vector of stacked moments computed from the model evaluated at ψ, and m_S is the vector of corresponding sample moments. Ω is a symmetric, positive-definite weighting matrix. In practice, we use the inverse of the matrix $E(m_S' m_S)$.

TABLE 2: Parameter Estimates

α	1.1925 [,]	μ_z	3.7108 [,]
γ	16.4463 [,]	σ_z	0.1255 [,]
δ	0.7486 [,]	c_M	28.5324 [,]
K_B	110.6650 [,]	ξ	13.9526 [,]
σ_ϵ	0.4140 [,]	μ_η	-12.4632 [,]
σ_η	3.7624 [,]		

Notes—This table reports the estimates of the parameters. 95-percent confidence intervals in brackets are obtained by bootstrapping the data using 100 replications (to be computed).

Although the model is highly nonlinear, so that (almost) all parameters affect all outcomes, the identification of some parameters relies on some key moments in the data. Specifically, the moments of the quality distribution identify the parameters of the distribution D of sellers' heterogeneity, of the distribution of the measurement error, and contribute to the identification of the parameters of the distribution M of buyers' heterogeneity. The moments of buyers' consumptions identify the meeting rates α and γ, the destruction rate δ, and contribute to the identification of the parameters of the distribution M of buyers' heterogeneity. The fraction of individuals who purchased drugs in the last 30 days and the incarceration rate identify the parameters of the distribution of the unobservable η that contributes to the selection into the ADAM sample. From the distribution of buyers' heterogeneity, we can then recover buyers' cost K_B.

5.2.1   Estimates

Table 2 reports estimates of the parameters, along with 95-percent confidence intervals obtained by bootstrapping the data using 100 replications (to be computed).

The magnitude of the parameter α indicates that a buyer meets a new seller, on average, every $\frac{30}{\alpha} = 25$ days. The parameter γ indicates that a matched buyer purchases, on average, approximately 16 times every month. However, the buyer-seller match lasts, on average, only $\frac{30}{\delta} = 40$ days. Buyers' monthly cost K_B is quite low, approximately equal to \$110.

The parameters c_M and ξ of sellers' cost distribution imply that the range of sellers' cost is $[0, 28.53]$, but their average cost is 18.75, as $\xi = 13.95$ implies that most sellers have costs close to the upper bound c_M. Moreover, the estimates of the parameters of the distribution of buyers' heterogeneity imply that all buyers with taste $z \geq z^* = 25.4179$ are active in the market and, among those active, the average taste is approximately equal to 40 and the standard deviation is approximately equal to 10.

Finally, the variance of the measurement error is estimated to be quite small, indicating that the model without any error already captures the data quite well.

5.2.2 Model Fit

Before considering some broader implications of our results, we examine the fit of the estimated model. Table 3 presents a comparison between the empirical moments and the moments calculated from the model at preliminary parameters. Overall, the model matches the moments of the quality distribution quite well. The largest discrepancy is in the variance of the offered qualities, that is the model implies a dispersion of the offered qualities that is lower than that of the observed distribution. Nonetheless, the model captures well both the fraction of ripoffs and the higher-order moments of the quality distribution. The model matches the moments of the distribution of buyers' consumptions slightly less precisely than those of the quality distribution, but overall it captures quite well the difference in consumption rates between matched and unmatched buyers.

To further appreciate how the model compares to the quality data in a perhaps more-intuitive way, Figure 2 displays the histogram of the quality distribution obtained from a model simulation using the estimated parameters reported in Table 2. The comparison with the empirical distribution of Figure 1 corroborates that the model matches the qualitative and quantitative features of the distribution of drug quality quite well.

5.3 Model Implications

The estimated parameters reported in Table 2 imply that all sellers with costs $c \in (\bar{c}, c_M] = (28.28, 28.53]$ rip their buyers off by choosing $q = 0$. While this interval is small, the mass of sellers in it equals 12.73 percent of sellers, because the shape parameter ξ of the Pareto distribution is large, implying that the right tail of the cost distribution has a large mass. For sellers with costs $0 \leq c \leq \bar{c} = 28.28$, $q(c)$ is the solution to the differential equation (6): sellers' quality choices are strictly decreasing in their costs, as Lemma 7 says.

TABLE 3: Model Fit

	DATA	MODEL
FRACTION OF RIP-OFFS	0.1263	0.1163
AVERAGE PURE GRAMS PER $100, $\hat{q} > 0$	2.4708	3.1656
ST. DEV. PURE GRAMS PER $100, $\hat{q} > 0$	1.5895	1.4028
MEDIAN PURE GRAMS PER $100, $\hat{q} > 0$	2.0277	2.9172
SKEWNESS PURE GRAMS PER $100, $\hat{q} > 0$	1.5145	1.4383
KURTOSIS PURE GRAMS PER $100, $\hat{q} > 0$	5.6143	6.6745
FRACTION OBTAINED DRUG IN LAST 30 DAYS	0.7404	0.8167
FRACTION LAST PURCHASED FROM REGULAR DEALER	0.5248	0.6657
AVERAGE NUMBER OF PURCHASES, MATCHED BUYER	16.3315	13.9184
AVERAGE NUMBER OF PURCHASES, UNMATCHED BUYER	11.5484	9.1222
ST. DEV. NUMBER OF PURCHASES, MATCHED BUYER	11.1244	6.2765
ST. DEV. NUMBER OF PURCHASES, UNMATCHED BUYER	10.4188	5.7544
INCARCERATION RATE	0.0100	0.0100

Notes—This table reports the values of the empirical moments and of the simulated moments calculated at the estimated parameters reported in Table 2.

The estimated parameters imply that sellers' positive qualities lie in the interval $[2.63, 3.77]$. Hence, the model implies a non-trivial dispersion of drug qualities, and the measurement error helps to match the larger observed dispersion.

Sellers' quality choice $q(c)$ implies that sellers' markups $\frac{p-cq(c)}{p}$ are non-monotonic, with the lowest- and highest-cost sellers charging the highest ones (equal to 1, as either c or q equals 0) and a seller with cost $c = 25.52$ charging the lowest one; the average sellers' markup $\frac{\int_0^{c_M}(p-cq(c))dD(c)}{p}$ equals 16.5 percent. On average, sellers make approximately 130 transactions $t(q)$ per month, and the distribution of transactions $t(q)$ has a large range—the lowest-quality (i.e., higher-cost) sellers make approximately 50 monthly deals and the highest-quality sellers make approximately 300 monthly deals—and is skewed towards sellers with fewer transactions. Sellers' profits have a large range and are highly skewed as well: the lowest-quality' seller is earning approximately $1,300 per month, the highest-quality' seller is earning approximately $12,600 per month, and the average seller is earning $K_S = \$2,000$. The shape of the distribution of profits matches reasonably well the evidence reported by Levitt and Venkatesh (2000).
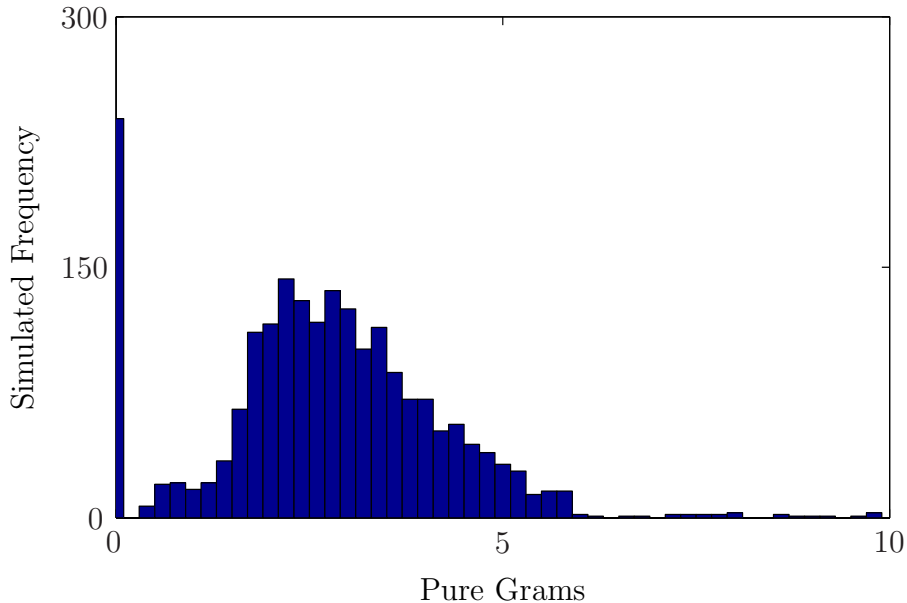
21

FIG. 2: Histogram of pure grams per \$100, simulated data.

Figure 3 compares the equilibrium distribution of qualities consumed by first-time (i.e., unmatched) buyers and the equilibrium distribution of qualities consumed by regular (i.e., matched) buyers. It displays the key features of the distribution of qualities characterized in Lemma 5, most notably the mass point at zero quality. Of course, no matched buyers consumes zero quality from his regular dealer. Moreover, as buyers move up over time in the offered quality distribution by switching to sellers that offer higher-quality drugs, they are more likely to be matched to higher-quality sellers. Hence, the cumulative $G(q)$ first-order stochastically dominates the cumulative $F(q)$. Matched buyers consume drugs that have an average quality of $\int_{\underline{q}}^{\bar{q}} q g(q)\, dq = 3.31$, whereas unmatched buyers consume drugs that have an average quality of $\int_{0}^{\bar{q}} q f(q)\, dq = 2.81$, indicating that buyers' switching behavior and buyer-seller relationships have a substantial effect on the qualities that regular buyers are consuming relative to the distribution of qualities that first-time buyers are consuming.

5.4 Counterfactual Analyses

In this Section, we use our model to understand the quantitative effects of two key features of illegal markets: 1) imperfect observability and, thus, sellers' moral hazard; and 2) penalties on market participants.
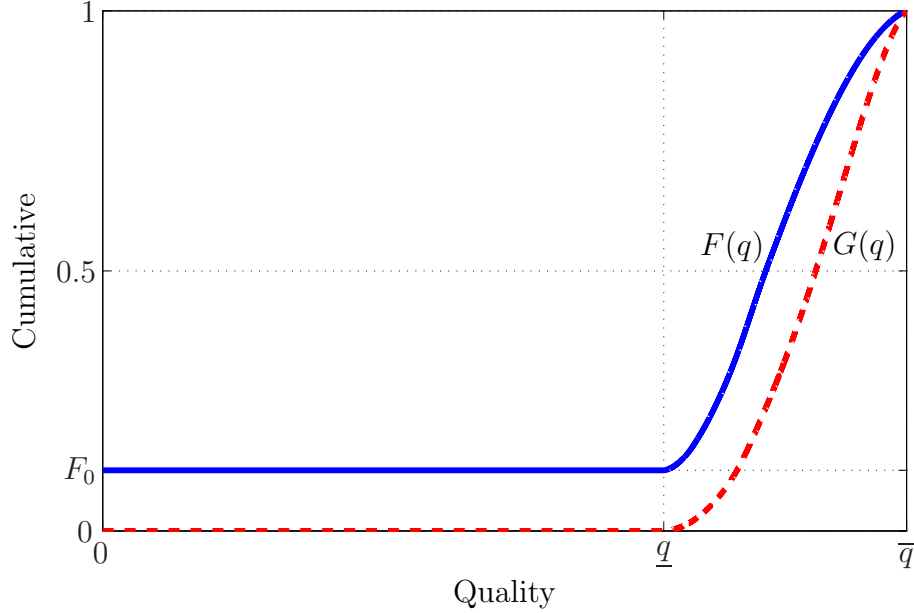
FIG. 3: The figure displays the cumulative distribution functions of quality $F(q)$ (solid line) and $G(q)$ (dashed line). Based on the parameter estimates reported in Table 2.

5.4.1   The Role of Sellers' Moral Hazard

In order to understand how sellers' moral hazard affects market outcomes, we modify the model of Section to allow buyers to observe drug purity before purchasing it. Appendix B reports the full derivation of the equilibrium. We highlight here how the observability of q modifies buyers' values, sellers' profits and, thus, the equilibrium distribution of quality q.

When buyers observe q before transacting, they purchase only if the seller offers a sufficiently high q, so that their flow payoff $zq - p$ is non-negative. Thus, their value functions are:

$$r\bar{V}_z = \alpha_B(\theta) \int_0^{\bar{q}} \max\left\{z\tilde{q} - p + \max\left\{V_z(\tilde{q}) - \bar{V}_z, 0\right\}, 0\right\} dF(\tilde{q}),$$

$$rV_z(q) = \gamma(zq - p) + \alpha_B(\theta) \int_0^{\bar{q}} \max\left\{z\tilde{q} - p + \max\left\{V_z(\tilde{q}) - V_z(q), 0\right\}, 0\right\} dF(\tilde{q}) + \delta(\bar{V}_z - V_z(q)).$$

Buyers' purchase decisions affect sellers' profits, as well. Specifically, the rate at which an individual seller offering quality q transacts with a new buyer depends on the meeting rate $\alpha_S(\theta)$ *and* on the probability $H(q)$ that the seller's quality q is above the buyer's reservation value (this probability does not arise when drug quality is observable, as every meeting results in a sale):

$$t_N(q) = \alpha_S(\theta)H(q) = \alpha_B(\theta)\theta H(q).$$

23

Since loyal buyers know the quality that the seller is offering, equation (8) still characterizes the rate $t_L(q)$ at which an individual seller offering quality q transacts with loyal buyers. Therefore, sellers' steady state profits are:

$$\pi_c(q) = \alpha_B(\theta)\theta H(q)(p - cq)\left(1 + \frac{\gamma\delta}{(\delta + \alpha_B(\theta)(1 - F(q)))^2}\right), \qquad q \geq \underline{q}.$$

Hence, sellers' optimal quality choice is:

$$\dot{q}^*(c) = \frac{2(p - cq^*(c))\gamma\delta\alpha_B(\theta)D'(c)}{\left(\frac{H'(q^*(c))}{H(q^*(c))}(p - c\hat{q}(c)) - c\right)(\delta + \alpha_B(\theta)D(c))\left((\delta + \alpha_B(\theta)D(c))^2 + \gamma\delta\right)}$$

with initial condition $\underline{q}(c_M)$, determined by the profit-maximization of the highest-cost seller c_M:

$$\max_q \pi(c_M) = \max_q \alpha_B(\theta)\theta H(\underline{q})(p - c_M\underline{q})\left(1 + \frac{\gamma\delta}{(\delta + \alpha_B(\theta))^2}\right).$$

Thus, the equilibrium distribution of qualities is:

$$F(q) = 1 - D(q^{*-1}(q)).$$

We compute the resulting equilibria for two alternative cases: 1) a partial-equilibrium case in which buyers make optimal purchase decisions and sellers choose the optimal quality to offer, but buyers' and sellers' masses and types are unchanged relative to the benchmark case; and 2) a general-equilibrium case in which, in addition to the partial-equilibrium optimizations, buyers and sellers also make optimal entry decisions—i.e, a buyer's entry threshold z^{**} satisfy $r\bar{V}_z = K_B$ the and the mass of sellers S^{**} satisfies $\int \pi(q(c))dD(c) = K_S$. We believe that the partial-equilibrium case is useful to focus exclusively on the effects of sellers' moral hazard due the imperfect observability of drugs' purity.

Table 4 reports the quantitative values of market outcomes for the counterfactuals of observable drug purity for the partial-equilibrium case and the general-equilibrium case. Overall, market outcomes differ substantially when buyers observe drug purity and when they do not. Moreover, the quantitative effects are very similar in the partial- and general-equilibrium cases.

Specifically, if drug purity is observable, sellers cannot rip-off buyers and, thus, zero-purity drugs disappear from the market. Similarly, sellers have to increase quality to induce their first-time occasional buyers to purchase and, possibly, to become their loyal customers.

24

TABLE 4: Observable Quality

	BASELINE	OBSERVABLE q, PARTIAL EQ.	OBSERVABLE q, GENERAL EQ.
FRACTION OF RIP-OFFS	0.1163	0	0
AVERAGE PURE GRAMS PER $100	2.8239	3.2054	3.2016
ST. DEV. PURE GRAMS PER $100	1.0330	0.2564	0.2533
ACTIVE BUYERS, IN MILLIONS	12.5000	12.5000	12.7511
ACTIVE SELLERS, IN MILLIONS	1.1460	1.1460	1.0262
FRACTION OF MATCHED BUYERS	0.5806	0.6144	0.5949
AVERAGE NUMBER OF PURCHASES	10.6198	11.1134	10.6790
AVERAGE PURE GRAMS CONSUMED	34.6709	37.0466	35.5435

Notes—This table reports market outcomes in the counterfactual cases in which buyers can observe drugs' purity before purchasing.

Hence, the average purity increases by approximately 13 percent and the standard deviation of purity decreases by more than 75 percent. Therefore, a larger fraction of buyers is matched to a regular seller, thereby increasing buyers' purchases and consumption by approximately five and seven percent, respectively

The general equilibrium case highlights three additional effects relative to the partial-equilibrium case. When buyers can observe drug quality, their average purity increases, thereby attracting a larger number of active buyers relative to the baseline case. However, since it is expensive to supply high-quality drugs, sellers' profits decrease relative to the baseline case. Hence, in equilibrium, fewer sellers enter the market. In turn, it becomes more difficult for buyers to meet new sellers, thereby affecting their purchases and their consumption. Hence, in the general-equilibrium case, the extensive margins of consumption increase relative to the baseline case, but less than in the partial-equilibrium case. Nonetheless, because of the increase in the number of buyers as well, aggregate drug consumption increases by approximately 2.5 percent relative to the baseline case.

The results reported in Table 4 could provide some insights on how outcomes would differ if drugs markets were legal. Buyers' imperfect information is one key way in which our model captures an illegal market, because, in legal markets for similar commodities (i.e., tea, coffee, cigarettes), buyers are better (although perhaps not fully) informed about the

TABLE 5: The Effect of Penalties

	BASELINE	LOWER K_S	LOWER K_B
FRACTION OF RIP-OFFS	0.1163	0.1667	0.1346
AVERAGE PURE GRAMS PER $100	2.8239	2.6597	2.7588
ST. DEV. PURE GRAMS PER $100	1.0330	1.2177	1.1099
ACTIVE BUYERS, IN MILLIONS	12.5000	12.5043	12.5485
ACTIVE SELLERS, IN MILLIONS	1.1460	1.3083	1.1710
FRACTION OF MATCHED BUYERS	0.5806	0.5826	0.5775
AVERAGE NUMBER OF PURCHASES	10.6198	10.7760	10.6812
AVERAGE PURE GRAMS CONSUMED	34.6709	35.0489	34.7359

Notes—This table reports market outcomes in the counterfactual cases in which buyers' cost K_B and sellers' cost K_S are 10-percent lower than in the baseline case, respectively.

quality of the product they are purchasing. While a full legalization counterfactual requires many additional assumptions (for example, on the destruction rate δ and on the efficiency of the matching process), our analysis illustrates that the average quality of drugs will increase if drugs markets were legal because of buyers' better information.

5.4.2   The Role of Penalties

The United States has witnessed a large increase of penalties on drugs' buyers and sellers in the last 30 years. Moreover, different countries have adopted quite different penalties to drug users. Most notably, several European countries have mild or no penalties on illicit drugs' buyers and strong penalties on drugs' sellers, whereas the United States enforce strict penalties on both buyers and sellers. Legal penalties on drug trade obviously affect sellers' costs K_S and buyers' costs K_B and, thus, in this Section, we use our model to understand how these costs K_S and K_B affect market outcomes.[15] More specifically, we perform two counterfactuals: in the first one, we decrease sellers' cost K_S by 10 percent (i.e., from $2,000 to $1,800) relative to the baseline model of Section 4 with ex-ante unobservable quality; in the second one, we decrease buyers' cost K_B by 10 percent relative to the baseline model.

Table 5 reports the quantitative values of market outcomes for the two counterfactuals

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[15]Different policing interventions affect the market through different parameters. For example, prison sentencing guidelines more likely affect buyers' and sellers' costs, whereas police patrolling more likely affects the destruction rate δ and the efficiency of the matching function and, thus, the meeting rate.

cases, displaying interesting results. Specifically, a 10-percent smaller sellers' cost K_S increases the equilibrium mass S of active sellers by approximately 14 percent. Hence, the meeting rate $\alpha(\theta)$ between buyers and sellers increases. This increase makes it easier for buyers to purchase drugs and, thus, tends to increase the number of active buyers in the market. Moreover, a higher meeting rate shifts sellers' relative profitability of targeting first-time buyers or loyal buyers. Specifically, a higher meeting rate increases sellers' incentives to make quick profits and to rip-off buyers by selling $q = 0$, thereby decreasing the qualities they offer. As a result, the fraction of rip-offs F_0 increases, and the average qualities consumed by first-time buyers decreases. Table 5 reports that these effects on the drug quality distribution are quantitatively large: the fraction of rip-offs increase by more than 40 percent, decreasing the average drug quality by almost six percent, and increasing the variance of drug quality by approximately 17 percent, relative to the baseline case.

This decrease in drug quality tends to decrease the number of active buyers in the market, whereas a higher meeting rate tends to increase it. As a result of these opposing forces, the equilibrium number of buyers increases by a very small amount, less than one percent. However, the higher meeting rate also implies that it is easier for buyers to switch to sellers that offer higher-quality drugs. Hence, the fraction of matched buyers increases, thereby increasing active buyers' average number of purchases and average quantity of pure cocaine consumed; however, the magnitude of this increase is small, approximately one percent.

Similarly, a 10-percent smaller buyers' cost K_B increases the equilibrium mass B of active buyers by a small, amount, approximately one percent. This increase in aggregate demand increases the number of sellers by approximately two percent, indicating that the elasticity of supply is larger than the elasticity of demand. As a result, the meeting rate $\alpha(\theta)$ between buyers and sellers increases. A higher meeting rate increases sellers' incentives to make quick profits and to rip-off buyers by selling $q = 0$, decreasing the offered qualities. Hence, the fraction of rip-offs F_0 increases, and the average qualities consumed by first-time buyers decreases. The last column of table 5 reports that these effects on the drug quality distribution are quantitatively sizable: the fraction of rip-offs increase by more than 15 percent, decreasing the average drug quality by almost three percent, and decreasing the variance of drug quality by approximately 7.5 percent, relative to the baseline case.

The equilibrium fraction of matched buyers decrease because the newly-entered buyers have a lower preference z and, thus, a higher reservation value R, and because offered drugs are of lower quality then those offered in the baseline case. These effects quantitatively dominate the effect of the opposite sign that a higher meeting rate makes it is easier for

buyers to switch to sellers that offer higher-quality drugs. Nonetheless, the average number of active buyers' purchases and their consumption are higher than in the baseline case, although the quantitative effects are negligible.

Overall, the results reported in Table 5 highlight that changes in penalties has larger effects on the distribution of drugs offered on the market than on buyers' purchase and consumption patterns, as buyers' switching between sellers allows them to adjust their consumption. Hence, these results suggest that it may be difficult to infer buyers' consumptions exclusively from samples of drug purchases. Moreover, the results further highlight the role of buyers' imperfect/incomplete information at the time of purchase. Specifically, in the product-market version of Burdett and Mortensen (1998), which features complete/perfect information about the quality received by first-time buyers, any change to market primitives that increases buyers' meeting rate $\alpha(\theta)$ unambiguously improves the average offered quality. In contrast, our analysis illustrates that, when buyers' information is imperfect/incomplete, changes, such as those of penalties, that increase the meeting rate give sellers' greater incentives to make quick profits by selling $q = 0$, thus decreasing the average offered quality. Similarly, other market interventions, such as police patrolling, could alter the efficiency of the matching process and, thus, the meeting rate, thereby affecting the quality distribution.

Moreover, the results indicate that increasing penalties may help strengthen the long-term relationships between buyers and sellers that help overcome illegal markets' informational problems. Hence, our analysis suggests that increasing penalties may have contributed to the observed increased purity of retail drugs in the U.S. Similarly, the UN office for Drugs and Crime reports that price-adjusted purity of drugs is lower in Europe than in the United States,[16] whereas penalties for market participants—buyers, in particular—are lower in Europe than in the United States. Of course, there are potentially many other differences among U.S. markets over time, and between the U.S. market and the European one. Nonetheless, we find interesting that our model is consistent with these across-market differences.

6 Conclusions

This paper develops a framework to understand illicit drug markets. We focus on two key characteristics of illegal markets: 1) the inability to verify/contract the quality of the good; and 2) penalties on market participants. We estimate the model using data on the

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[16]See `http://www.unodc.org/unodc/secured/wdr/Cocaine_Heroin_Prices.pdf`.

U.S. market for crack cocaine. The model fits the data well. Our counterfactual analysis implies that sellers' moral hazard reduces the average and increases the dispersion of drug purity, thereby reducing drug consumption. Moreover, the estimated model implies that increasing penalties may increase the purity and the affordability of drugs traded, because it increases sellers' relative profitability of targeting loyal buyers versus first-time buyers.

At the same time, the model has some limitations. In our view, perhaps the main limitation is that sellers commit to a single level of quality and do not discriminate between first-time and loyal buyers. As explained in Section 3, we do not directly observe this discrimination in the data, which implies that a model that allows this discrimination could be difficult to identify with the available data. Nonetheless, our theoretical framework delivers rich heterogeneity between buyers, and our empirical model successfully captures the matches the heterogeneity of the data.

APPENDICES

A Proofs

Proof of Proposition 2. The reservation quality for an unmatched buyer can be found by equating the value of remaining unmatched with the value of becoming matched at R_z:

$$\overline{V}_z = V_z(R_z),$$

which implies equation (4).

Using integration by parts, the value of being unmatched satisfies:

$$r\overline{V}_z = \alpha_B(\theta)\Big(z\int_0^{\overline{q}} q\,dF(q) + \int_{R_z}^{\overline{q}} V_z'(q)(1 - F(x))dx - p\Big).$$

Differentiating equation (2) with respect to q, we obtain

$$V_z'(q) = \frac{\gamma z}{r + \delta + \alpha_B(\theta)(1 - F(q))}.$$

Combining the previous two equations yields equation (3).

The value of participating in the market is negative for buyers who receive no utility from consuming and is strictly increasing in a buyer's marginal utility of consumption:

$$
\begin{aligned}
r\overline{V}_0 &= -\alpha_B(\theta)p < 0, \\
\frac{\partial r\overline{V}_z}{\partial z} &= \alpha_B(\theta)\Big(\int_0^{\overline{q}} x\,dF(x) + \int_{R_z}^{\overline{q}} \frac{\gamma(1 - F(x))}{r + \delta + \alpha_B(\theta)(1 - F(x))}dx + \frac{p}{z^2}\frac{\alpha_B(\theta)\gamma(1 - F(R_z))}{r + \delta + \alpha_B(\theta)(1 - F(R_z))}\Big) > 0.
\end{aligned}
$$

The value of participating can be made arbitrarily large by increasing z and therefore a buyer participates in the market only if his type is high enough. ∎

Proof of Proposition 3. Fix a type-z buyer and consider his value of participating in the market as a function of the buyer-seller ratio, θ. If there are very many buyers per seller, so that he never meets with a seller ($\lim_{\theta\to\infty} \alpha_B(\theta) = 0$), the buyer's value of participating is strictly below the entry cost:

$$\lim_{\theta\to\infty} r\overline{V}_z = 0 < K_B.$$

The value of participating in the market is strictly decreasing in θ, i.e. it is strictly increasing in the rate of meeting with sellers:

$$\frac{\partial r\overline{V}_z}{\partial \theta} = \alpha'_B(\theta)\left(z\int_0^{\overline{q}} xdF(x) - p\right) + \frac{z\alpha'_B(\theta)(r+\delta)}{\alpha_B(\theta)^2}\int_{p/z}^{\overline{q}} \frac{\gamma(1-F(x))}{\left(\frac{r+\delta}{\alpha_B(\theta)} + 1 - F(x)\right)^2}dx < 0.$$

If there are very few buyers per seller, that a buyer meets with a seller arbitrarily often $(\lim_{\theta\to 0}\alpha_B(\theta) = \infty)$, the value of participating is given by:

$$\lim_{\theta\to 0} r\overline{V}_z = \lim_{\theta\to 0}\alpha_B(\theta)\left(z\int_0^{\overline{q}} xdF(x) - p\right)$$

and the condition for participation is:

$$\lim_{\theta\to 0}\alpha_B(\theta)\left(z\int_0^{\overline{q}} xdF(x) - p\right) > K_B \Leftrightarrow z > \frac{p}{\hat{q}}.$$

Therefore, a type-z buyer never participates in the market regardless of θ if $z\int_0^{\overline{q}} xdF(x) \leq p$ and there is no buyer entry $(B = 0)$ when $\overline{z}\int_0^{\overline{q}} xdF(x) \leq p$, which proves part 1 of the Proposition.

When $\overline{z}\int_0^{\overline{q}} xdF(x) > p$, a type-$z$ buyer participates if $z\int_0^{\overline{q}} xdF(x) > p$ *and* if θ is low enough. Conversely, given θ, there is a $z(\theta)$, with $z(\theta)\int_0^{\overline{q}} xdF(x) > p$, such that

$$r\overline{V}_{z(\theta)} = K_B,$$

so that a buyer participates if and only if $z \geq z(\theta)$ and the measure of buyers in the market is:

$$B = \overline{B}(1 - \overline{M}(z(\theta))).$$

Given S, this leads to

$$\theta(z(\theta)) = \frac{\overline{B}(1 - \overline{M}(z(\theta)))}{S}.$$

We now show that, given S and $F(\cdot)$, there is a unique z^* such that $z^* = z(\theta(z^*))$. We show that as z^* increases, the participation value of the marginal type increases after taking

into account the effect on θ:

$$\frac{dr\overline{V}_{z^*}}{dz^*} = \frac{\partial r\overline{V}_{z^*}}{\partial z^*} + \frac{\partial r\overline{V}_{z^*}}{\partial \theta}\left(-\overline{BM}'(z^*)\right) > 0.$$

Therefore, there is a unique z^* such that the unmatched value of the marginal buyer is exactly equal to K_B and it is defined by equation (5).

This completes the proof of parts 2 and 3. ∎

Proof of Lemma 5. For $q \in [0, \underline{R})$ we have $t(q) = t_N$ which implies that $\pi_c(0) > \pi_c(q)$ for $q \in (0, \underline{R})$. Therefore either $q = 0$ or $q \geq \underline{q}$ for some $\underline{q} \geq \underline{R}$. If $\underline{q} > \overline{R}$ then $t(q) = t(\overline{R})$ for $q \in [\overline{R}, \underline{q}]$ which implies that $\pi_c(\overline{R}) > \pi_c(q)$ for $q \in (\overline{R}, \underline{q}]$. Therefore, $\underline{q} \leq \overline{R}$. The previous point proves that F is constant (and hence continuous) on $[0, \underline{q}]$. Standard arguments (as in Burdett and Mortensen, 1998) prove continuity on $[\underline{q}, \overline{q}]$. ∎

Proof of Proposition 6. Recalling that the lowest reservation quality among buyers is $\underline{R}$, any seller who offers $q < \underline{R}$ does not have any regular customers.

We determine the number of unmatched buyers and their type distribution using the fact that, in steady state, the flow of buyers into and out of the matched state must equal. Let $n(R)$ denote the number of buyers who are unmatched and whose type is less than R. The total number of unmatched buyers is $n(\overline{R}) \equiv \overline{n}$. An unmatched buyer of type R becomes matched after transacting with a seller who offers above-reservation quality which occurs at rate $\alpha_B(\theta)(1 - F(R))$. A matched buyer exits the matched state when his match is exogenously destroyed which occurs at rate δ. In steady state:

$$n'(R)\alpha_B(\theta)(1 - (F(R))) = \delta(BH'(R) - n'(R)) \Rightarrow n'(R) = \frac{\delta BH'(R)}{\delta + \alpha_B(\theta)(1 - F(R))}.$$

Hence, the mass $n(R)$ satisfies:

$$n(R) = \int_{\underline{R}}^{R} \frac{B\delta}{\delta + \alpha_B(\theta)(1 - F(x))} dH(x),$$

and, thus, the mass of matched buyers is:

$$\begin{aligned}
B - \overline{n} &= B\left(1 - \int_{\underline{R}}^{\overline{R}} \frac{\delta}{\delta + \alpha_B(\theta)(1 - F(x))} dH(x)\right) \\
&= \int_{\underline{R}}^{\overline{R}} \frac{B\alpha_B(\theta)(1 - F(x))}{\delta + \alpha_B(\theta)(1 - F(x))} dH(x).
\end{aligned}$$

32

Let $G(\cdot)$ denote the received quality distribution for matched buyers. The mass of matched buyers receiving quality up to q is given by $(B - \overline{n})G(q)$. An unmatched type-R buyer flows into this group if $R \le q$ and he samples a seller who offers quality less than q, which occurs at rate $\alpha_B(\theta)(F(q) - F(R))$. A buyer flows out of this group if the match is exogenously destroyed or if he samples a new seller whose quality if greater than q, which occurs at rate $\delta + \alpha_B(\theta)(1 - F(q))$. Equating these flows yields:

$$\alpha_B(\theta) \int_{\underline{R}}^{q} \big(F(q) - F(x)\big)dn(x) \;=\; (B - \overline{n})G(q)\big(\delta + \alpha_B(\theta)(1 - F(q))\big)$$

$$\Rightarrow (B - \overline{n})G(q) \;=\; \frac{\alpha_B(\theta)B\delta \int_{\underline{R}}^{q} \frac{F(q)-F(x)}{\delta+\alpha_B(\theta)(1-F(x))}dH(x)}{\delta + \alpha_B(\theta)(1 - F(q))}.$$

Thus, $G'(q)$ satisfies (we assume, and later verify, that F is differentiable):

$$(B - \overline{n})G'(q) = \frac{\alpha_B(\theta)B\delta F'(q)H(q)}{\big(\delta + \alpha_B(\theta)(1 - F(q))\big)^2},$$

The number of buyers who are matched with a seller offering q is $(B - \overline{n})G'(q)$ and the number of sellers offering quality q is $SF'(q)$. Therefore the number of matched buyers per seller at quality q is given by:

$$l(q) = \frac{(B - \overline{n})G'(q)}{SF'(q)}$$

which implies that the flow of transactions from regular buyers is:

$$t_R(q) = \frac{\gamma \alpha_B(\theta)\theta\delta H(q)}{\big(\delta + \alpha_B(\theta)(1 - F(q))\big)^2}. \tag{8}$$

Combining results completes the proof of Proposition 6. ∎

Proof of Proposition 7. The proof has three parts. Part 1 proves that $q^*(c)$ is increasing. Part 2 characterizes the marginal seller, $\overline{c}$, which determines $q^*(c)$ for $c > \overline{c}$. Part 3 characterizes the optimal quality choice of sellers with $c < \overline{c}$.

Part 1: Consider sellers 1 and 2 with $c_1 > c_2$ and denote their actions by q_1 and q_2. In equilibrium $q_2 > 0 \Rightarrow q_2 > q_1$. The proof is by contradiction. Suppose that $q_2 > 0$ and $q_2 \le q_1$. Therefore:

$$(p - c_1 q_1)t(q_1) \ge (p - c_1 q_2)t(q_2) \Rightarrow p(t(q_1) - t(q_2)) \ge c_1\big(t(q_1)q_1 - t(q_2)q_2\big).$$

33

Seller 2 chose quality q_2 over q_1. Therefore:

$$(p - c_2 q_2)t(q_2) \geq (p - c_2 q_1)t(q_1) \Rightarrow p(t(q_1) - t(q_2)) \leq c_2\big(t(q_1)q_1 - t(q_2)q_2\big),$$

which, combined with the assumption $c_1 > c_2$, yields the desired contradiction. Going through the same steps, it is easy to show that $q_2 = 0 \Rightarrow q_1 = 0$. One corollary is that $F(q^*(c)) = 1 - D(c)$.

Part 2: We assume that the seller who offers the lowest positive quality is of type c, find the optimal quality that he offers $\underline{q}(c)$ and show that it is suboptimal for sellers of different types to offer a lower (but positive) level of quality, if his profits are equal to the profits of offering zero quality. Then, we show that there is a unique type $\overline{c}$ such that the profits when offering the $\underline{q}(\overline{c})$ are equal to the profits of offering zero quality.

We showed above that all sellers with $c' < c$ offer positive quality and sellers with $c' > c$ offer lower quality than the type-c seller. Therefore, $F(0) = 1 - D(c)$. Denote the profits of the seller who offers the lowest positive quality by:

$$\underline{\pi}_c(\underline{q}) = \alpha_B(\theta)\theta(p - c\underline{q})\left(1 + \frac{\gamma\delta H(\underline{q})}{(\delta + \alpha_B(\theta)D(c))^2}\right)$$

Notice that the level of profits for this seller do not depend on the exact shape of $F(\cdot)$ over and above the mass at zero.

The optimal choice for a type-c seller who offers the lowest positive quality is denoted by $\underline{q}(c)$ and is determined as the root of

$$\underline{\pi}'_c(\underline{q}) = \alpha_B(\theta)\theta\left[-c\left(1 + \frac{\gamma\delta H(\underline{q})}{(\delta + \alpha_B(\theta)D(c))^2}\right) + (p - c\underline{q})\frac{\gamma\delta H'(\underline{q})}{(\delta + \alpha_B(\theta)D(c))^2}\right] \qquad (9)$$

where the the log-concavity of $H(\cdot)$ guarantee that the second order condition is negative:

$$\underline{\pi}''_c(\underline{q}) = \alpha_B(\theta)\theta\frac{-2c\gamma\delta H'(\underline{q}) + (p - c\underline{q})\gamma\delta H''(\underline{q})}{(\delta + \alpha_B(\theta)D(c))^2} < 0$$

Consider sellers of type $c' \neq c$. We showed that it is suboptimal for a seller with $c' < c$ to offer a quality level below that of a type-c seller. We now show that a seller with $c' > c$ will earn lower profits than the type-c seller if he offers his optimal positive quality. Differentiating

34

profits at the optimally chosen lowest positive quality with respect to the seller type we have:

$$\frac{\partial \pi_c(\underline{q}(c))}{\partial c} = \pi'_c(\underline{q}(c))\frac{d\underline{q}(c)}{dc} - \underline{q}(c)t(q) < 0$$

The first term is zero by the envelope condition and the second term is negative because higher costs reduce margins.

As a result, if the type-c seller is indifferent between 0 and $\underline{q}(c)$ we have:

$$\pi_{c'}(0) = \pi_c(0) = \underline{\pi}(\underline{q}(c)) > \underline{\pi}_{c'}(\underline{q}(c'))$$

and it is optimal for a seller with $c' > c$ to offer zero quality, if the type-c seller is the marginal type.

We now show that the marginal type exists and is unique. Using our assumptions on the support of $D(\cdot)$:

$$\lim_{c\to\infty} \underline{\pi}_c(\underline{q}(c)) = \lim_{c\to\infty}(p - c\underline{q}(c))t(\underline{q}(c)) \quad < \quad \lim_{c\to\infty}\pi_c(0)$$
$$\lim_{c\to 0} \underline{\pi}_c(\underline{q}(c)) = \lim_{c\to 0}pt(\underline{q}(c)) \quad > \quad \lim_{c\to\infty}\pi_c(0)$$

As the type of the marginal seller changes, his profits change as follows:

$$\frac{d\underline{\pi}_c(\underline{q}(c))}{dc} = \pi'_c(\underline{q}(c))\frac{d\underline{q}(c)}{dc} + \frac{\partial \underline{\pi}_c(\underline{q}(c))}{\partial c} + \frac{\partial \underline{\pi}_c(\underline{q}(c))}{\partial D(c)}D'(c) < 0.$$

The sum of the first and second terms are negative for the same reasons as above. The third term is negative because

$$\frac{\partial \underline{\pi}_c(\underline{q}(c))}{\partial D(c)} = -\alpha_B(\theta)\theta\frac{(p - c\underline{q}(c))\gamma\delta H(\underline{q}(c))2\alpha_B(\theta)}{(\delta + \alpha_B(\theta)D(c))^3} < 0.$$

Thus, there is a unique $\overline{c}$ such that the profits from offering $\underline{q}(\overline{c})$ are exactly equal to the profits from offering zero and no seller of type $c' \neq \overline{c}$ can do better by offering a lower positive quality level. Equating $\pi_{\overline{c}}(0)$ with $\underline{\pi}_{\overline{c}}(\underline{q}(\overline{c}))$ and going through the algebra yields the equation that determines $\overline{c}$:

$$p = (p - \overline{c}\underline{q}(\overline{c}))\left(1 + \frac{\gamma\delta H(\underline{q}(\overline{c}))}{(\delta + \alpha_B(\theta)D(\overline{c}))^2}\right)$$

where $\underline{q}(\overline{c})$ is defined by the root of equation (9).

Part 3: We now determine $q^*(c)$ for $c < \overline{c}$. We assume that an optimal schedule $q^*(c)$ exists and rewrite the profits of a type-c seller as if he decides which other type c' to imitate rather than which quality to offer. In other words, his profits from offering some quality q' are written in terms of imitating type c' who offers quality $q' = q^*(c')$. We have:

$$\pi_c(c') = \alpha_B(\theta)\theta(p - cq^*(c'))\left(1 + \frac{\gamma\delta H(q^*(c'))}{(\delta + \alpha_B(\theta)D(c'))^2}\right)$$

The advantage of formulating the choice in terms of c' rather than q' is that the term in the denominator depends on the exogenous type distribution $D(\cdot)$ rather than the endogenous quality distribution $F(\cdot)$. The quality distribution will be recovered once $q^*(c)$ is constructed.

Differentiate profits with respect to c'

$$\begin{aligned}\pi_c'(c') &= \alpha_B(\theta)\theta c\left(-q^{*\prime}(c')\left(1 + \frac{\gamma\delta H(q^*(c'))}{(\delta + \alpha_B(\theta)D(c'))^2}\right)\right.\\ &\left.+ \left(\frac{p}{c} - q^*(c')\right)\gamma\delta\frac{H'(q^*(c'))q^{*\prime}(c')(\delta + \alpha_B(\theta)D(c')) - H(q^*(c'))2\alpha_B(\theta)D'(c')}{(\delta + \alpha_B(\theta)D(c'))^3}\right)\end{aligned}$$

By construction, profits are maximized when $c' = c$ and we can therefore set the derivative to zero and rearrange to arrive at equation (6). This differential equation and the initial condition $q^*(\overline{c}) = \underline{q}(\overline{c})$ determine $q^*(c)$ for $c < \overline{c}$.

This completes the proof. ∎

Proof of Proposition 9. Steady state profits for every type of seller are increasing in θ:

$$\frac{d\pi_c(q^*(c))}{d\theta} = \frac{\partial\pi_c(q^*(c))}{\partial\theta} + \frac{\partial\pi_c(q^*(c))}{\partial q}\frac{dq^*(c)}{d\theta}.$$

The first term is clearly positive and the second terms is zero by the envelope theorem. Furthermore:

$$\begin{aligned}\lim_{\theta\to 0}\pi_c\big(q^*(c)\big) &= 0,\\ \lim_{\theta\to\infty}\pi_c\big(q^*(c)\big) &> K_S,\end{aligned}$$

Therefore, there is a unique θ such that sellers' expected profits equal K_S. Given B, this determines uniquely the measure of sellers S who participate in the market. ∎

B   Observable Drug Purity

In this Appendix, we characterize the market equilibrium when buyers can observe drug quality before purchasing it.

B.1   The Buyers

Proposition 11 *Given $F(\cdot)$ and S:*

1. *If $\frac{p}{q} \geq \bar{z}$ then there is no buyer entry: $B = 0$.*

2. *If $\frac{p}{q} < \bar{z}$ then there is a unique buyer type $z^* \leq \bar{z}$ such that all buyers with $z > z^*$ participate in the market and all buyers with $z \leq z^*$ do not.*

3. *The measure of buyers in the market is $B = \bar{B}(1 - \tilde{M}(z^*))$ and the distribution of their types in the market is given by*

$$M(z) = \begin{cases} 0 & \text{if } z \leq z^* \\ \frac{\tilde{M}(z) - \tilde{M}(z^*)}{1 - \tilde{M}(z^*)} & \text{if } z \geq z^* \end{cases}$$

4. *The marginal buyer type is given by the solution to:*

$$z^* \alpha_B(\theta) \int_{p/z^*}^{\bar{q}} \left(1 + \frac{\phi}{r + \delta + \alpha_B(\theta)(1 - F(q))}\right)(1 - F(q))dq = K_B \qquad (10)$$

5. *The reservation quality of a type-z buyer who participates in the market is $R_z = \frac{p}{z}$ and the distribution of reservation qualities in the market is*

$$H(R) = \begin{cases} 0 & \text{if } R \leq \underline{R} \\ \frac{1 - M(\frac{p}{R})}{1 - M(z^*)} & \text{if } R \in [\underline{R}, \overline{R}] \\ 1 & \text{if } R \geq \overline{R} \end{cases}$$

where $\underline{R} = R_z = \frac{p}{z}$ and $\overline{R} = R_{z^*} = \frac{p}{z^*}$.

A buyer observes the quality offered before purchasing. Therefore, at each state he has to choose whether to consume and whether to become matched with that seller. His value

37

functions are given by:

$$r\bar{V}_z \;=\; \alpha_B(\theta)\int_0^{\bar{q}}\Big(\max\big[z\tilde{q}-p+\max[V_z(\tilde{q})-\bar{V}_z,0],0\big]\Big)dF(\tilde{q})$$

$$rV_z(q) \;=\; \phi(zq-p)+\alpha_B(\theta)\int_0^{\bar{q}}\Big(\max\big[z\tilde{q}-p+\max[V_z(\tilde{q})-V_z(q),0],0\big]\Big)dF(\tilde{q})+\delta(\bar{V}_z-V_z(q))$$

The reservation quality for consumption is the same regardless of whether the buyer is matched or not and is denoted by $\hat{R}_z$. Comparing the static costs and benefits of consumption we have:

$$\hat{R}_z \;=\; \frac{p}{z}$$

When the buyer is matched with a seller who offers q, his reservation for matching with a new seller is q. When the buyer is unmatched, his reservation is denoted by R_z. Equating the two value functions delivers the reservation quality for becoming matched:

$$R_z = \hat{R}_z = \frac{p}{z}$$

We can rewrite the value functions as follows:

$$r\bar{V}_z \;=\; \alpha_B(\theta)\int_{\frac{p}{z}}^{\bar{q}}\big(z\tilde{q}-p+V_z(\tilde{q})-\bar{V}_z\big)dF(\tilde{q})$$

$$rV_z(q) \;=\; \phi(zq-p)+\alpha_B(\theta)\left(\int_{\frac{p}{z}}^{\bar{q}}\big(z\tilde{q}-p\big)dF(\tilde{q})+\int_q^{\bar{q}}\big(V_z(\tilde{q})-V_z(q)\big)dF(\tilde{q})\right)+\delta(\bar{V}_z-V_z(q))$$

An individual buyer takes as given the actions of sellers $\{F(\cdot),S\}$ and other buyers (B) and decides whether to participate in the market. The actions of other agents are summarized as $\{F(\cdot),\theta\}$. To examine the individual buyer's choice, we write his value of participating $\bar{V}_z$ in a more convenient way.

Using integration by parts the value of being unmatched can be written as:

$$\begin{aligned}
r\bar{V}_z &\;=\; \alpha_B(\theta)\left((zq-p+V_z(q)-\bar{V}_z)F(q)|_{R_z}^{\bar{q}}-\int_{R_z}^{\bar{q}}(z+V_z'(q))F(q)dq\right)\\
&\;=\; \alpha_B(\theta)\left(z\bar{q}-p+V_z(\bar{q})-\bar{V}_z-F(R_z)(zR_z-p+V_z(R_z)-\bar{V}_z)-\int_{R_z}^{\bar{q}}(z+V_z'(q))F(q)dq\right)\\
&\;=\; \alpha_B(\theta)\left(z(\bar{q}-R_z)+V_z(\bar{q})-V_z(R_z)-\int_{\frac{p}{z}}^{\bar{q}}(z+V_z'(q))F(q)dq\right)
\end{aligned}$$

where we used $R_z = \frac{p}{z}$ and $\bar{V}_z = V_z(R_z)$ in the last equality.

Using the fundamental theorem of calculus:

$$
\begin{aligned}
r\bar{V}_z &= \alpha_B(\theta)\left(\int_{\frac{p}{z}}^{\overline{q}}(z + V_z'(x))dx - \int_{\frac{p}{z}}^{\overline{q}}(z + V_z'(x))F(x)dx\right) \\
&= \alpha_B(\theta)\int_{\frac{p}{z}}^{\overline{q}}\left[z + V_z'(q)\right](1 - F(x))dx
\end{aligned}
$$

Differentiate the value of being matched with respect to q and rearrange to get:

$$
V_z'(q) = \frac{\phi z}{r + \delta + \alpha_B(\theta)(1 - F(q))}
$$

Combining the previous two equations:

$$
r\bar{V}_z = z\alpha_B(\theta)\int_{\frac{p}{z}}^{\overline{q}}\left(1 + \frac{\phi}{r + \delta + \alpha_B(\theta)(1 - F(x))}\right)(1 - F(x))dx
$$

We now determine whether a buyer of type z participates in the market. Notice that buyers with $z \leq \frac{p}{\overline{q}}$ have no benefit from participating in the market and never enter. For buyers with $z > \frac{p}{\overline{q}}$ we have:

$$
\begin{aligned}
\lim_{\theta \to 0} r\bar{V}_z &= \lim_{\theta \to 0} \alpha_B(\theta)\int_{\frac{p}{z}}^{\overline{q}}(1 - F(x))dx > 0 \\
\lim_{\theta \to 0} r\bar{V}_z &= 0
\end{aligned}
$$

Therefore, a buyer with $z > \frac{p}{\overline{q}}$ might enter if the arrival rate of new meetings is high enough and does not enter if $z \leq \frac{p}{\overline{q}}$ regardless of θ. As a corollary, if $\bar{z} \leq \frac{p}{\overline{q}}$ then no buyer enters. Furthermore, fixing a buyer's type and increasing θ, i.e. increasing the buyer-seller ratio, reduces that buyer's value of participating in the market:

$$
\frac{\partial r\bar{V}_z}{\partial \theta} = z\int_{\frac{p}{z}}^{\overline{q}}\left(\alpha_B'(\theta) + \frac{\alpha_B'(\theta)(r + \delta)}{\alpha_B(\theta)^2}\frac{\phi}{\left(\frac{r+\delta}{\alpha_B(\theta)} + 1 - F(x)\right)^2}\right)(1 - F(x))dx < 0.
$$

In the limit, if a buyer never meets with sellers, then he does not enter:

$$
\lim_{\theta \to \infty} r\bar{V}_z = 0 < K_B.
$$

39

Therefore, for each buyer of type z with $z > \frac{p}{\bar{q}}$ there is a unique $\theta(z)$ such that he participates if $\theta \le \theta(z)$ and stays out otherwise.

The value of participating in the market is, unsurprisingly, negative for buyers who receive no utility from consuming and is strictly increasing in a buyer's marginal utility of consumption:

$$
\begin{aligned}
r\bar{V}_0 &= -\alpha_B(\theta)p < 0 \\
\frac{\partial r\bar{V}_z}{\partial z} &= \alpha_B(\theta)\int_0^{\bar{q}} x\, dF(x) + \alpha_B(\theta)\int_{R_z}^{\bar{q}} \frac{\phi(1-F(x))}{r+\delta+\alpha_B(\theta)(1-F(x))}dx + \frac{p}{z^2}\frac{\alpha_B(\theta)\phi(1-F(R_z))}{r+\delta+\alpha_B(\theta)(1-F(R_z))} > 0
\end{aligned}
$$

Taking θ as given, there is a unique $z(\theta)$ such that a buyer participates if $z \ge z(\theta)$ and does not participate otherwise.

We now prove that z^* is unique, taking into account that the number of buyers depends on z^* according to $B = \bar{B}\left(1 - \tilde{M}(z^*)\right)$. First, note that when $z^* = 0$ we have $r\bar{V}_{z^*} < K_B$. Furthermore, when $z^* = \bar{z}$ we have $r\bar{V}_{\bar{z}} > K_B$, assuming of course that $\bar{z} > \frac{p}{\bar{q}}$, because otherwise no buyers enter.

To prove that the uniqueness of z^* we need to show that the value of the marginal type is increasing in his own type. The unmatched value of the marginal buyer depends on z^* as follows:
$$
\frac{dr\bar{V}_{z^*}}{dz^*} = \frac{\partial r\bar{V}_{z^*}}{\partial z^*} + \frac{\partial r\bar{V}_{z^*}}{\partial \theta}\left(-\bar{B}\tilde{M}'(z^*)\right) > 0
$$
Therefore, there is a unique z^* such that the unmatched value of the marginal buyer is exactly equal to K_B and it is defined by equation (5). This completes the proof of Proposition 11, parts 2, 3 and 4.

Finally, let $z(R)$ denote the buyer type whose reservation quality is equal to R. Rearranging equation (4) we have:
$$
z(R) = \frac{p}{R}
$$
Furthermore, note that $R_{z(R)} = R$ and $z \le z(R) \Leftrightarrow R_z \ge R$. Given z^*, the equilibrium distribution of reservation qualities mirrors the distribution of marginal utilities.

This completes the characterization of buyers' behavior.

B.2 The Sellers

We derive the sellers' profits and describe their actions, taking as given the measure of buyers who participate B and the distribution of reservation qualities $H(\cdot)$. The distribution of buyer types does not affect sellers over and above the distribution of reservation qualities.

A measure S of sellers participate in the market, which is determined through free entry. Each seller draws the marginal cost c of providing a unit of quality from some distribution $D(\cdot)$. The problem of a seller of type c is to choose a level of quality $\hat{q}(c)$ that maximizes his steady state profits. Steady state profits have two components: the margin per transaction and the steady state flow of transactions. The profit margin from each transaction is equal to $p - cq$. The flow of transactions is $t(q) = t_N(q) + t_L(q)$ where $t_N(q)$ refers to new buyers and $t_L(q)$ refers to loyal buyers. Steady state profits ar:

$$\pi_c(q) = (p - cq)\big(t_N(q) + t_L(q)\big).$$

We first derive some necessary conditions on the distribution of offered qualities.

Lemma 12 *In equilibrium, the quality distribution F:*

1. *has support on a subset of $[\underline{q}, \overline{q}]$,*

2. *$\underline{q} \in [\underline{R}, \overline{R}]$,*

3. *is continuous on $[0, \overline{q}]$.*

Proof. For $q \in [0, \underline{R})$ we have $t(q) = 0$ and therefore $q \geq \underline{q}$ for some $\underline{q} \geq \underline{R}$. If $\underline{q} > \overline{R}$ then $t(q) = t(\overline{R})$ for $q \in [\overline{R}, \underline{q}]$ which implies that $\pi_c(\overline{R}) > \pi_c(q)$ for $q \in (\overline{R}, \underline{q}]$. Therefore, $\underline{q} \leq \overline{R}$. The previous point proves that F is constant (and hence continuous) on $[0, \underline{q}]$. Standard arguments (as in Burdett-Mortensen) prove continuity on $[\underline{q}, \overline{q}]$. ∎

In the following sections we characterize the flow of transactions for any F that satisfies the previous Lemma and then we characterize the seller's optimal quality choice $\hat{q}(c)$.

B.2.1 Characterization of profits

We take $H(\cdot)$, $F(\cdot)$ and θ as given and calculate the steady state profits that a type-c seller would enjoy for any quality q. The main result is summarized in the next proposition.

Proposition 13 *The steady state profits of a seller of type c who offers quality q are:*

$$\pi_c(q) \quad = \quad \alpha_B(\theta)\theta H(q)(p - cq)\left(1 + \frac{\phi\delta}{\left(\delta + \alpha_B(\theta)(1 - F(q))\right)^2}\right), \qquad q \geq \underline{q}.$$

To determine profits, we need to first determine the flow of a seller's transactions as a function of the quality he offers. The rate at which an individual seller transacts with a new buyer equals the meeting rate times the probability the seller's quality is above the buyer's reservation:

$$t_N(q) \quad = \quad \alpha_S(\theta)H(q) = \theta\alpha_B(\theta)H(q)$$

The flow of transactions from loyal buyers is given by:

$$t_L(q) = \phi l(q)$$

where $l(q)$ is the steady steady number of loyal buyers of a seller offering q.

The number of loyal buyers per seller offering q is given by:

$$l(q) = \frac{(B - \bar{n})G'(q)}{SF'(q)}$$

where $\bar{n}$ is the number of unmatched buyers, $(B - \bar{n})G'(q)$ is the number of buyers who are matched with a seller offering q and $SF'(q)$ is the number of sellers offering quality q.

We determine the number of unmatched buyers and their type distribution. In steady state, the flow of buyers from the unmatched to the matched state must equal the flow out of the matched state and into the unmatched state. Let $n(R)$ denote the number of buyers who are unmatched and whose type is less than R. The total number of unmatched buyers is therefore given by $n(\overline{R}) \equiv \bar{n}$.

An unmatched buyer of type R becomes matched after transacting with a seller who offers above-reservation quality which occurs at rate $\alpha_B(\theta)(1 - F(R))$. A matched buyer exits the matched state when his match is exogenously destroyed which occurs at rate δ. As a result, in steady state the following holds:

$$n'(R)\alpha_B(\theta)(1 - (F(R))) = \delta(BH'(R) - n'(R)) \Rightarrow n'(R) = \frac{\delta BH'(R)}{\delta + \alpha_B(\theta)(1 - F(R))}$$

Alternatively, this can be written as:

$$n(R) = \int_{\underline{R}}^{R} \frac{B\delta}{\delta + \alpha_B(\theta)(1 - F(x))} dH(x)$$

Therefore, we have:

$$\begin{aligned}
\bar{n} &= \int_{\underline{R}}^{\overline{R}} \frac{B\delta}{\delta + \alpha_B(\theta)(1 - F(x))} dH(x) \\
B - \bar{n} &= B\left(1 - \int_{\underline{R}}^{\overline{R}} \frac{\delta}{\delta + \alpha_B(\theta)(1 - F(x))} dH(x)\right) \\
&= \int_{\underline{R}}^{\overline{R}} \frac{B\alpha_B(\theta)(1 - F(x))}{\delta + \alpha_B(\theta)(1 - F(x))} dH(x)
\end{aligned}$$

We now characterize $G(\cdot)$. The mass of matched buyers receiving quality up to q is given by $(B - \bar{n})G(q)$. An unmatched type-R buyer flows into this group if $R \leq q$ and he samples a seller who offers quality less than q, which occurs at rate $\alpha_B(\theta)(F(q) - F(R))$. A buyer flows out of this group if the match is exogenously destroyed or if he samples a new seller whose quality if greater than q, which occurs at rate $\delta + \alpha_B(\theta)(1 - F(q))$. Equating these flows yields

$$\begin{aligned}
\alpha_B(\theta) \int_{\underline{R}}^{q} \left(F(q) - F(R)\right) dn(R) &= (B - \bar{n})G(q)\left(\delta + \alpha_B(\theta)(1 - F(q))\right) \\
\Rightarrow (B - \bar{n})G(q) &= \frac{\alpha_B(\theta) \int_{\underline{R}}^{q} \left(F(q) - F(x)\right) dn(x)}{\delta + \alpha_B(\theta)(1 - F(q))} \\
&= \frac{\alpha_B(\theta) B\delta \int_{\underline{R}}^{q} \frac{F(q) - F(x)}{\delta + \alpha_B(\theta)(1 - F(x))} dH(x)}{\delta + \alpha_B(\theta)(1 - F(q))}
\end{aligned}$$

Some algebra leads to:

$$(B - \bar{n})G'(q) = \frac{\alpha_B(\theta) B\delta F'(q) H(q)}{\left(\delta + \alpha_B(\theta)(1 - F(q))\right)^2}$$

which implies that the flow of transactions from loyal buyers is:

$$t_L(q) = \frac{\phi \alpha_B(\theta)\theta\delta H(q)}{\left(\delta + \alpha_B(\theta)(1 - F(q))\right)^2}$$

Combining results completes the proof of the Proposition.

B.2.2 The sellers' optimal quality choice

We now characterize the distribution of offered qualities, $F(\cdot)$ and the number of sellers who enter the market taking as given the number of buyers B and the distribution of their reservation values $H(\cdot)$.

Lemma 14 *Consider sellers 1 and 2 and denote their actions by q_1 and q_2. We have $c_1 > c_2 \Rightarrow q_2 > q_1$.*

Proof. The proof is by contradiction. Suppose that $c_1 > c_2$ and $q_2 \leq q_1$. Recall that profits are given by $\pi_c(q) = (p - cq)t(q)$.

Seller 1 chose quality q_1 over q_2. Therefore:

$$
\begin{aligned}
(p - c_1 q_1)t(q_1) &\geq (p - c_1 q_2)t(q_2) \\
\Rightarrow p(t(q_1) - t(q_2)) &> c_1\big(t(q_1)q_1 - t(q_2)q_2\big) \\
\Rightarrow p(t(q_1) - t(q_2)) &> c_2\big(t(q_1)q_1 - t(q_2)q_2\big)
\end{aligned}
$$

where the strict inequality results from $c_1 > c_2$.

Seller 2 chose quality q_2 over q_1. Therefore:

$$(p - c_2 q_2)t(q_2) \geq (p - c_2 q_1)t(q_1) \Rightarrow p(t(q_1) - t(q_2)) \leq c_2(t(q_1)q_1 - t(q_2)q_2)$$

Therefore, $q_1 = q_2$ is the only possibility that satisfies the above inequalities.

Now consider any seller 3 with $c_1 > c_3 > c_2$. Such a seller exists because the support of $D(\cdot)$ is connected. If $q_1 = q_2 = \tilde{q}$ then the analysis above means that $q_3 = \tilde{q}$ as well. In that case, there is a mass of sellers offering $\tilde{q}$ equal to $D(c_1) - D(c_2)$ which is inconsistent with equilibrium (see Lemma 12). ∎

One corollary of the previous Lemma is that $F(\hat{q}(c)) = 1 - D(c)$.

We now determine sellers' optimal $\hat{q}(c)$.

Proposition 15 *Given $H(\cdot)$ and θ the optimal quality choice for sellers of type $c < c^*$ is given by the solution to the differential equation*

$$\hat{q}'(c) = \frac{2(p - c\hat{q}(c))\phi\delta\alpha_B(\theta)D'(c)}{\left[\frac{H'(\hat{q}(c))}{H(\hat{q}(c))}(p - c\hat{q}(c)) - c\right]\left(\delta + \alpha_B(\theta)D(c)\right)\left[(\delta + \alpha_B(\theta)D(c))^2 + \phi\delta\right]}$$

where $\underline{q}(c^*)$ is the initial condition, determined by the solution of

$$\max_q \pi\left(c^*\right) = \max \alpha_B(\theta)\theta H(\underline{q})(p - c^*\underline{q})\left(1 + \frac{\phi\delta}{\left(\delta + \alpha_B(\theta)\right)^2}\right).$$

The distribution of qualities is:

$$F(q) = 1 - D(\hat{q}^{-1}(q)).$$

To characterize the function of optimal quality offer $\hat{q}(c)$ we rewrite the profits of a type-c seller as if he decides which other type c' to imitate rather than which quality to offer. In other words, his profits from offering some quality q' are written in terms of imitating type c' who offers quality $q' = \hat{q}(c')$. We have:

$$\pi_c(c') = \alpha_B(\theta)\theta H(\hat{q}(c'))(p - c\hat{q}(c'))\left(1 + \frac{\phi\delta}{(\delta + \alpha_B(\theta)D(c'))^2}\right)$$

The advantage of formulating the choice in terms of c' rather than q' is that the term in the denominator depends on the exogenous type distribution $D(\cdot)$ rather than the endogenous quality distribution $F(\cdot)$. The quality distribution will be recovered once $\hat{q}(c)$ is constructed.

Differentiate profits with respect to c'

$$
\begin{aligned}
\pi_c'(\hat{c}; c) \;=\; & \alpha_B(\theta)\theta\Big(H'(\hat{q}(c'))\hat{q}'(c)(p - c\hat{q}(c))\big(1 + \frac{\phi\delta}{(\delta + \alpha_B(\theta)D\left(c'\right))^2}\big) \\
& -c\hat{q}'(c)H(\hat{q}(c))\big(1 + \frac{\phi\delta}{(\delta + \alpha_B(\theta)D\left(c'\right))^2}\big) \\
& -H(\hat{q})(p - c\hat{q}(c))\frac{2\phi\delta\alpha_B(\theta)D'(c')}{(\delta + \alpha_B(\theta)D\left(c'\right))^3}\Big)
\end{aligned}
$$

By construction, profits are maximized when $\hat{c} = c$ and we can therefore set the derivative to zero and rearrange to arrive at

$$\hat{q}'(c) = \frac{2(p - c\hat{q}(c))\phi\delta\alpha_B(\theta)D'(c)}{\left[\frac{H'(\hat{q}(c))}{H(\hat{q}(c))}(p - c\hat{q}(c)) - c\right]\left(\delta + \alpha_B(\theta)D(c)\right)\left[(\delta + \alpha_B(\theta)D(c))^2 + \phi\delta\right]}$$

which determines $\hat{q}(c)$.

Notice that all terms of $\hat{q}'(c)$ are always positive except for the first term of the denominator. Therefore the sign of $\hat{q}'(c)$ is the same at the sign of $T(c)$ where

$$T(c) = \frac{H'(\hat{q}(c))}{H(\hat{q}(c))}(p - c\hat{q}(c)) - c$$

Define $\underline{c}$ such that $\hat{q}(\underline{c}) = \overline{R}$ and notice that $c < \underline{c} \Rightarrow \hat{q}(c) > \overline{R} \Rightarrow H'(\hat{q}(c)) = 0$ which means that $T(c) < 0$. For $c > \underline{c}$, we have that $T(c) = 0$ for $q = \underline{q}$ where $\hat{q}(c^*) = \underline{q}$. Therefore, for $c \in (\underline{c}, c^*)$ we have $T(c) < 0$ and $\hat{q}'(c) < 0$.

Having fully characterized $F(\cdot)$, we turn to determining the number of sellers S who choose to enter the market.

Proposition 16 *Given $H(\cdot)$ and B there is a unique S such that $\Pi = K_S$.*

The key for this proposition is that profits for every type of seller are increasing in θ:

$$\frac{d\pi_c(q)}{d\theta} = \frac{\partial\pi_c(q)}{\partial\theta} + \frac{\partial\pi_c(q)}{\partial q}\frac{dq}{d\theta}$$

The first term is positive. The second terms is zero by the envelope theorem. Furthermore:

$$\lim_{\theta \to 0} \pi_c(q) = 0$$
$$\lim_{\theta \to \infty} \pi_c(q) > K_S$$

which proves the Proposition.

This completes the characterization of sellers' behavior.

C   Selection into ADAM

Since $\log(z)$ and η are normally distributed and independent of each other, the distribution of $\log(z) + \eta$ is normal with mean $\mu_z + \mu_\eta$ and variance $\sigma_z^2 + \sigma_\eta^2$. Thus, the proportion of buyers $P(A)$ that is in ADAM (i.e., the incarceration rate) is:

$$
\begin{aligned}
P(A) &= \Pr\left(\log(z) + \eta \geq 0\right) = 1 - \Phi\left(\frac{-\mu_z - \mu_\eta}{\sqrt{\sigma_z^2 + \sigma_\eta^2}}\right) \\
&= \Phi\left(\frac{\mu_z + \mu_\eta}{\sqrt{\sigma_z^2 + \sigma_\eta^2}}\right).
\end{aligned}
$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal. Moreover, a buyer enters the market if $\log(z) \geq \log(z^*)$. Therefore, a buyer is both in ADAM and in the market if $\log(z) \geq \log(z^*)$ and $\log(z) + \eta \geq 0$:

$$
P(M, A) = \int_{\log(z^*)}^{\infty} \Phi\left(\frac{\log(z) + \mu_\eta}{\sigma_\eta}\right) \frac{1}{\sigma_z} \phi\left(\frac{\log(z) - \mu_z}{\sigma_z}\right) d\log(z)
$$

where $\phi(\cdot)$ denotes the probability density function of a standard normal.

Thus, the fraction of drug users in ADAM is :

$$
\begin{aligned}
P(M|A) &= \frac{P(M, A)}{P(A)} \\
&= \frac{\int_{\log(z^*)}^{\infty} \Phi\left(\frac{\log(z) + \mu_\eta}{\sigma_\eta}\right) \frac{1}{\sigma_z} \phi\left(\frac{\log(z) - \mu_z}{\sigma_z}\right) d\log(z)}{\Phi\left(\frac{\mu_z + \mu_\eta}{\sqrt{\sigma_z^2 + \sigma_\eta^2}}\right)},
\end{aligned}
$$

and the density of the log of drug users' preferences z in ADAM satisfies:

$$
\frac{\frac{1}{\sigma_z} \phi\left(\frac{\log(z) - \mu_z}{\sigma_z}\right) \Phi\left(\frac{\log(z) + \mu_\eta}{\sigma_\eta}\right)}{\int_{\log(z^*)}^{\infty} \Phi\left(\frac{\log(z) + \mu_\eta}{\sigma_\eta}\right) \frac{1}{\sigma_z} \phi\left(\frac{\log(z) - \mu_z}{\sigma_z}\right) d\log(z)} \quad \text{for } z \geq z^* \tag{11}
$$

and zero otherwise. In practice, the density (11) is the density of buyers' preferences $\frac{1}{\sigma_z} \phi\left(\frac{\log(z) - \mu_z}{\sigma_z}\right)$ weighted by the probability $\Phi\left(\frac{\log(z) + \mu_\eta}{\sigma_\eta}\right)$ of being in ADAM.

References

ALLEN, J., R. CLARK, AND J.-F. HOUDE (2014): "Search frictions and market power in negotiated price markets," Discussion paper, National Bureau of Economic Research.

ARKES, J., R. L. PACULA, S. PADDOCK, J. CAULKINS, AND P. REUTER (2004): "Technical Report for the Price and Purity of Illicit Drugs Through 2003," *Office of National Drug Control Policy. November.*

ARKES, J., R. L. PACULA, S. M. PADDOCK, J. P. CAULKINS, AND P. REUTER (2008): "Why the DEA Stride data are still useful for understanding drug markets," Discussion paper, National Bureau of Economic Research.

BECKER, G., AND K. MURPHY (1988): "A Theory of Rational Addiction," *The Journal of Political Economy*, 96(4), 675–700.

BONTEMPS, C., J.-M. ROBIN, AND G. VAN DEN BERG (1999): "An empirical equilibrium job search model with search on the job and heterogeneous workers and firms," *International Economic Review*, 40(4), 1039–1074.

BURDETT, K., AND D. MORTENSEN (1998): "Wage differentials, employer size, and unemployment," *International Economic Review*, 39(2), 257–273.

BUSHWAY, S., AND P. REUTER (2008): "Economists' contribution to the study of crime and the criminal justice system," *Crime and Justice*, 37(1), 389–451.

DAVE, D. (2008): "Illicit drug use among arrestees, prices and policy," *Journal of Urban Economics*, 63(2), 694–714.

DAVIES, R. B. (2010): "Mandatory minimum sentencing, drug purity and overdose rates," *The Economic and Social Review*, 41(4), 429–457.

DRANOVE, D., AND G. JIN (2010): "Quality Disclosure and Certification: Theory and Practice," *Journal of Economic Literature*, 48(4), 935–963.

ECKSTEIN, Z., AND G. J. VAN DEN BERG (2007): "Empirical labor search: A survey," *Journal of Econometrics*, 136(2), 531–564.

FREEBORN, B. A. (2009): "Arrest avoidance: law enforcement and the price of cocaine," *Journal of Law and Economics*, 52(1), 19–40.

FRYER, R. G., P. S. HEATON, S. D. LEVITT, AND K. M. MURPHY (2013): "Measuring Crack Cocaine and its Impact," *Economic Inquiry*, 51(3), 1651–1681.

GALE, D., AND R. W. ROSENTHAL (1994): "Price and quality cycles for experience goods," *The RAND Journal of Economics*, 25(4), 590–607.

GALENIANOS, M., R. L. PACULA, AND N. PERSICO (2012): "A Search-theoretic Model of the Retail Market for Illicit Drugs," *Review of Economic Studies*, 79, 1239–1269.

GAVAZZA, A. (2013): "An Empirical Equilibrium Model of a Decentralized Asset Market," Discussion paper, London School of Economics.

GROSSMAN, M., AND F. J. CHALOUPKA (1998): "The demand for cocaine by young adults: a rational addiction approach," *Journal of Health Economics*, 17(4), 427–474.

HONG, H., AND M. SHUM (2006): "Using price distributions to estimate search costs," *The RAND Journal of Economics*, 37(2), 257–275.

HOROWITZ, J. L. (2001): "Should the DEA's STRIDE data be used for economic analyses of markets for illegal drugs?," *Journal of the American Statistical Association*, 96(456), 1254–1271.

HORTAÇSU, A., AND C. SYVERSON (2004): "Search costs, product differentiation, and welfare effects of entry: a case study of SP 500 index funds," *Quarterly Journal of Economics*, 119(4), 403–456.

HUBBARD, T. N. (1998): "An empirical examination of moral hazard in the vehicle inspection market," *The RAND Journal of Economics*, 29(2), 406–426.

IIZUKA, T. (2007): "Experts' agency problems: evidence from the prescription drug market in Japan," *The RAND Journal of Economics*, 38(3), 844–862.

——— (2012): "Physician agency and adoption of generic pharmaceuticals," *The American Economic Review*, 102(6), 2826–2858.

KUZIEMKO, I., AND S. LEVITT (2004): "An empirical analysis of imprisoning drug offenders," *Journal of Public Economics*, 88(9), 2043–2066.

LEVITT, S., AND S. VENKATESH (2000): "An economic analysis of a drug-selling gang's finances," *Quarterly Journal of Economics*, 115(3), 755–789.

REUTER, P., AND J. P. CAULKINS (2004): "Illegal 'lemons': price dispersion in cocaine and heroin markets," *Bulletin on Narcotics*, 56(1-2), 141–165.

SCHELLING, T. C. (1984): "Self-command in practice, in policy, and in a theory of rational choice," *The American Economic Review*, 74(2), 1–11.

SCHNEIDER, H. S. (2012): "Agency problems and reputation in expert services: Evidence from auto repair," *The Journal of Industrial Economics*, 60(3), 406–433.

STIGLER, G. J., AND G. S. BECKER (1977): "De Gustibus Non Est Disputandum," *The American Economic Review*, 67(2), 76–90.

WILDENBEEST, M. R. (2011): "An empirical model of search with vertically differentiated products," *The RAND Journal of Economics*, 42(4), 729–757.