Sin licenses revisited*

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November 12, 2014

Abstract

We analyze personalized regulation in the form of sin licenses (O'Donoghue and Rabin 2003, 2005, 2007) to correct the distortion in the consumption of a harmful good when consumers suffer from varying degrees of self-control problems. We take into account demand uncertainty, which generates a trade-off between flexibility and the commitment provided by sin licenses. We also account for the possibility that consumers may trade the sin good in a secondary market, which partially erodes the commitment power of sin licenses. We show that if sophisticated consumers are allowed to choose any general, individualized pricing-scheme for sin goods, they will choose a system of sin licenses. Nevertheless, sin licenses do not implement the first best in our general setting. Under certain conditions, social welfare will increase if linear taxation of sin goods is *supplemented* by a system of voluntary sin licenses; but welfare might decrease if the linear tax was *replaced* by sin licenses.

Keywords: self-control problems, sin licenses, non-linear pricing, demand uncertainty, secondary markets

JEL: H21, H30, I18

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*Earlier versions of the paper have been circulated under the title "Personalised regulation of sinful consumption".

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1 Introduction

A large body of literature in behavioral economics suggests that consumers sometimes make mistakes. A prominent example is excessive consumption of harmful goods such as alcohol, tobacco and unhealthy food; such excessive consumption can be caused for example by self-control problems. We reconsider the use of so called sin licenses to regulate the consumption of harmful commodities, first suggested by O'Donoghue and Rabin (2003) and also discussed in O'Donoghue and Rabin (2005, 2007).

In most countries, alcohol and tobacco are subject to linear taxation i.e. the tax rate is the same for all individuals and units consumed. Accordingly, the previous literature on regulating harmful consumption (see for example O'Donoghue and Rabin (2003; 2006), Gruber and Köszegi (2004), and Haavio and Kotakorpi (2011)) has overwhelmingly concentrated on linear taxation (so called sin taxes).[1] When people have self-control problems, they may value sin taxes as a commitment device which allows them to lower consumption. However, when consumers are heterogenous, a linear scheme will not achieve the first-best outcome: a tax based on some measure of average self-control problems will distort the consumption of individuals without a self-control problem and will be too low for individuals with severe self-control problems. Indeed, many economists remain sceptical about using instruments such as sin taxes to combat problems associated with the lack of self-control.[2]

Recent literature has started to consider personalized regulation. O'Donoghue and Rabin (2005) consider a scheme of so called sin licenses, where consumers may purchase 1 cent licenses that permit them to buy one unit of the sin good in the future tax free, whereas purchases without the license are subject to a prohibitively high tax. O'Donoghue and Rabin (2005) conclude that sin licenses achieve the first-best outcome if individuals can forecast their future tastes accurately. Further, even though many mechanisms that are based on voluntary participation can in general only be expected to work for sophisticated individuals, sin licenses have the desirable property that at least in some stylized settings, they also work for naives: As a naive person is unaware of his self-control problem and therefore assumes that he will prefer the optimal level of consumption in the future, he would ex ante ask for the optimal amount of sin licenses.[3]

[1] See Kanbur, Pirttilä and Tuomala (2006) for an analysis of sin taxes within the broader context of non-welfarist optimal taxation and Cremer et al. (2010) for a related analysis of commodity taxation under habit formation and myopia.

[2] On this discussion, see e.g. Gregory Mankiw's column in the New York Times http://www.nytimes.com/2010/06/06/business/06view.html?_r=1. Further, the possibility of government failure may reduce the effectiveness and desirability of paternalistic policies in general - see for example Glaeser (2006) for a critical view on paternalism.

[3] The concepts of sophistication and naivete (complete unawareness of ones' self-control problem),

The optimality results concerning sin licenses partly rest on the assumption that the sin good cannot be traded between consumers (or there is e.g. perfect policing that will eliminate any secondary or black market activity). However, if the authorities attempted to implement this type of regulation, there would ex post be incentives to create a secondary market, where individuals with serious self-control problems buy the sin good from individuals without self-control problems. This is because, under a scheme that would implement the first-best, individuals with a low level of self-control problems would be subject to less stringent regulation than individuals with serious self-control problems. Further, when future tastes are uncertain, the use of commitment devices may involve interesting trade-offs between commitment and flexibility, and a scheme that imposes a strict ceiling on consumption (as in O'Donoghue and Rabin's original example), might have undesirable properties in such a setting.[4]

We analyze the welfare properties of personalized regulation of harmful consumption in a setting where (i) there is preference uncertainty, so that there is a trade-off between achieving commitment and maintaining flexibility to adjust consumption in the face of preference shocks; and (ii) we account for the possibility of trading the sin good in a secondary market. We consider a generalization of O'Donoghue and Rabin's system of sin licenses, where consumers decide in period $t-1$ on a quota of sin licenses to be used for consumption in period t. Sin licenses are given out for free. Purchases without a license incur a tax, whereas purchases with a license are subject to a reduced tax. The tax rates are constrained by the possibility of secondary market activity.

Our focus is on the following question: if sophisticated individuals were allowed to choose any non-linear personalized pricing scheme for the sin good, what type of a scheme would they choose? Perhaps surprisingly, we show that consumers will under

were discussed already by Strotz (1955-6) and Pollak (1968) and have been analyzed in numerous papers - see for example O'Donoghue and Rabin (1999) for an analysis of the implications of both sophistication and naivete, and O'Donoghue and Rabin (2001) for a model that introduces a formalization of the intermediate case of partial naivete.

[4]A related literature studies consumers' attempts to achieve self-control in the market (Heidhues and Kőszegi 2009, DellaVigna and Malmendier 2006, Kőszegi 2005). Market mechanisms have the advantage of being voluntary and personalized, but there are two caveats: (i) market mechanisms in general do not work for naives; and (ii) they may be ineffective in achieving commitment. For example, in a competitive market, a consumer may reach a contract with one firm to limit the supply of harmful commodities, but another firm will have an incentive to supply the commodity at marginal cost. E.g. Kőszegi (2005) has argued that market-based mechanisms for alleviating self-control problems are often ineffective. See also Gottlieb (2008) for a related analysis. Heidhues and Kőszegi (2010) and Eliaz and Spiegler (2006) analyze firms' incentives to take advantage of consumers' self-control problems. For a review of issues related to the use of commitment devices, see Bryan, Karlan and Nelson (2010). Sin licenses can be viewed as an attempt to combine the positive sides of market mechanisms (voluntary, personalized regulation) and government regulation (wider scope and better commitment, since public policy cannot be changed over night).

certain rather general conditions in fact choose a scheme of sin licenses. An important part of the intuition is that sin licenses provide a cheap way of achieving commitment to a given level of consumption. When considering the merits of personalized regulation, therefore, this result shows that we can concentrate on analyzing the welfare properties of sin licenses.

Further, we show that even though the sin licensing scheme is preferred by sophisticated consumers, it is not socially optimal. The intuition is that whereas individual consumers pay attention to the monetary costs of the regulating scheme, and would like to achieve commitment at the lowest possible cost (to themselves), these costs (at the individual level) are irrelevant from a social point of view, as any revenue collected by the regulating scheme can be redistributed back to consumers in a lump-sum fashion. Hence the objectives of the social planner and the consumer diverge, even when the consumer is sophisticated.

Nevertheless, since the socially optimal regulating mechanism is likely not implementable, sin licenses may have a place in the regulator's toolkit. The main policy conclusions from our analysis are two-fold. First, the currently existing systems with linear taxation should *not* be replaced by a pure system of sin licenses: this would reduce the welfare of naives and might also reduce the welfare of sophisticates. Second, however, it is possible that welfare might be improved by supplementing the current linear tax with a voluntary system of sin licenses. This would have no effects on naives, whereas there are cases where it would improve welfare for sophisticates. Hence linear taxation and sin licenses should be thought of as complements, not substitutes. However, it is interesting to note that even supplementing a linear tax with a voluntary system of sin licenses is not always beneficial: perhaps surprisingly, there are cases where voluntary self-regulation may be detrimental for social welfare.

Regarding the trade-off between commitment and flexibility, our paper is closely related to Amador, Werning and Angeletos (2006), who study a sophisticated individual's choice of how to regulate the level of savings, when the individual suffers from self-control problems.[5] Self-control problems may in general lead to too high current consumption and hence inadequate savings. Amador et al. show that in certain circumstances, sophisticated consumers would opt for a minimum-savings policy. Since a floor on savings is equivalent to a ceiling on current consumption, it is easy to see that their result is closely related to ours: in its simplest form, a scheme of sin licenses

[5] Another related paper is Galperti (2013), who considers the trade-off between commitment and flexibility in a setting where a monopolist and/or social planner attempts to provide mechanisms that optimally screen consumers with varying degrees of self-control problems.

simply implies an upper limit on sin goods consumption. Our analysis differs from that in Amador et al. in that we analyze the relationship between the individual's preferred policy and the social optimum, and show that in general they do not coincide. This finding also motivates our comparison between the merits of voluntary personalized regulation *vs.* mandatory uniform regulation. We also relate our findings to some key elements of the optimal taxation literature.

The rest of the paper proceeds as follows. The model is introduced in Section 2. The properties of sin licenses in the presence of preference uncertainty and secondary market trade are examined in Section 3. Section 4 turns to the case of general non-linear personalized pricing of sin goods and derives our main result: if consumers are allowed to choose any personalized non-linear pricing scheme for sin goods, they (under certain conditions) opt for a system of sin licenses. Section 5 considers the role of sin licenses in regulating harmful consumption as a substitute or complement to current linear taxation. Section 6 provides a further discussion of our results and Section 7 concludes.

2 The model

We consider a model where consumers have a quasi-hyperbolic discount function (Laibson 1997), using a set-up that is similar for example to O'Donoghue and Rabin (2003; 2006). In the model, consumers suffer from varying degrees of self-control problems. Life-time utility of an individual is given by

$$U_t = (u_t, ..., u_T) = u_t + \beta_i \sum_{s=t+1}^{T} \delta^{s-t} u_s, \tag{1}$$

where $\beta_i, \delta \in (0,1)$ and u_t is the periodic utility function. We assume that the quasi-hyperbolic discount factor β has a distribution function $F(\beta)$ with mean $E(\beta)$ and median β_{med}. Throughout the paper we consider the general case where β has the support $[\beta_L, \beta_H]$, with $0 \leq \beta_L < \beta_H \leq 1$. Quasi-hyperbolic discounting implies that preferences are time-inconsistent: discounting is heavier between today and tomorrow, than any two periods that are both in the future.

We assume that utility is quasilinear with respect to a composite good (z). Consumer utility is also affected by the consumption of another good (x), which is harmful in the sense that it yields positive utility in the short-run, but has some negative effects in the long-run. Specifically, we assume that periodic utility is given by

$$u_t\left(x_t, x_{t-1}, z_t\right) = \theta_{it} v(x_t; \gamma_i) - h\left(x_{t-1}\right) + z_t, \tag{2}$$

where $v' > 0, v'' < 0$ and the harm function[6] is characterized by $h' > 0$ and $h'' \geqslant 0$. We allow individuals to differ in their preferences for the sin good: this heterogeneity is captured by the parameter γ_i, where a higher value of γ_i is taken to imply a higher taste for the sin good ($v_{x\gamma} > 0$). θ_{it} is an individual-specific preference shock that is realized in period t.

We assume that there is no borrowing or lending. Given this assumption and our specification for the periodic utility function in (2), in each period t an agent whose objective is to maximize (1) chooses x_t so as to maximize $u(x_t) = \theta_{it} v(x_t; \gamma_i) - \beta_i \delta h\left(x_t; \gamma_i\right) + z_t$. Maximization is subject to a per-period budget constraint $px_t + z_t \leq B + S$. We assume that product markets are competitive and normalize the producer price to 1, and $p = 1 + \tau$ denotes the consumer price of good x, where τ is a possible per unit tax on good x. B is the consumer's income (taken to be exogenous) and S is a possible lump-sum subsidy received by the consumer from the government. Taxes and subsidies will be modelled in more detail in later sections. Given the above specification, the demand for good x satisfies[7]

$$\theta_i v'(x^*(\theta_i); \gamma_i) - \beta_i \delta h'\left(x^*(\theta_i)\right) = p. \tag{3}$$

However, the time-inconsistency in preferences implies that the consumer would like to change his behavior in the future: Maximizing (1) from the next period onwards would amount to maximizing $u^o(x) = \theta v\left(x\right) - \delta h\left(x\right) + z$ each period.[8] Therefore, when thinking about future decisions, the consumer would like to choose consumption levels that maximize $u^o(x)$.

In general the issue of how to conduct welfare analysis when consumers have time-inconsistent preferences is far from straight-forward, and this question has received

[6] As in O'Donoghue and Rabin (2006), we assume that the marginal benefits and marginal costs of consumption are independent of past consumption levels. In such a setting, it is not essential that the harm is modelled as occuring only in the period following consumption - h can be thought of as the discounted sum of harm occurring in all future periods. See Gruber and Köszegi (2004) for an analysis where past consumption affects current marginal utility.

[7] We have dropped the time index t, since with our specification consumption is constant accross periods.

[8] See equation (1) and think of a consumer in period t, making consumption decisions for period $t+1$ onwards.

considerable attention in the literature. In the current paper, we take the so-called "long-run criterion" as the appropriate welfare criterion - that is, we take the utility function $u^o(x)$ to be the one that is relevant for welfare evaluation. This has been a standard choice in the literature on sin taxes based on models of quasi-hyperbolic discounting (see for example Gruber and Köszegi (2004), O'Donoghue and Rabin (2003; 2006)). There are clear reasons that justify this choice of welfare criterion in the present setting: Firstly, we assume that regulation is implemented from the period after the policy decision is made. Therefore, consumers themselves in any given period agree that $u^o(x)$ is the relevant utility function from the point of view of making regulatory policy. Secondly, $u^o(x)$ is the utility function that applies to all periods except for the present one. Since we consider an infinite number of periods, the weight of any single period should be negligible as long as periods are sufficiently short.[9] This latter consideration applies irrespective of the timing of the model.[10]

Given the above assumptions, the optimal level of consumption satisfies

$$\theta_i v'(x^o(\theta_i); \gamma_i) - \delta h'(x^o(\theta_i)) = p. \tag{4}$$

Because of quasi-hyperbolic discounting ($\beta_i < 1$), the equilibrium level of consumption of the harmful good (x^*) is higher than the optimal level (x^o).

[9] See also Bernheim and Rangel (2007, p.14) for a similar argument.

[10] In situations involving time-inconsistent preferences, either the long-run criterion or the multiself Pareto criterion have typically been used for welfare analysis. The latter views the different preferences of the individual at different points in time in terms of different "selves", and applies the Pareto criterion in this multiself-setting. See e.g. O'Donoghue and Rabin (2006, p. 1829) and O'Donoghue and Rabin (2007, p. 220) for arguments supporting the long-run criterion and Bhattacharya and Lakdawalla (2004) for an anlysis using the Pareto criterion. Note that the optimal policy (derived according to the long-run criterion) in our setting is part of the set of multiself Pareto optimal policies, since deviating from this policy would make the current self worse off. It is also very close to being optimal for each future self as long as periods are sufficiently short. Bernheim and Rangel (2009) build a choice-based framework for conducting welfare analysis with behavioral individuals, and show that in general their framework does not lead to either of these commonly used welfare criteria. Nevertheless, their framework, with some refinements, gives a justification for the long-run criterion in settings such as ours (see Theorem 11 in their paper). On the other hand, the literature is not even unanimous on whether behavioral welfare analysis can be based (solely) on choices (see e.g. Köszegi and Rabin (2007) and Sugden (2004)).

3 Sin licenses with preference uncertainty and (potential) secondary market trade

3.1 Preference uncertainty

We consider a continuous-demand generalization of O'Donoghue and Rabin's system of sin licenses (O'Donoghue and Rabin 2005), where consumers decide in period $t-1$ on a quota of sin licenses (y) to be used for consumption in period t. Sin licenses are given out for free. Purchases without a license incur a per-unit tax τ_1, whereas purchases with a license are subject to a reduced tax τ_2. The simplest case is the one involving pure sin licenses (as in O'Donoghue and Rabin 2005), where $\tau_2 = 0$ so that purchases with a license are tax free whereas τ_1 is set at a level that prohibits consumption without a license. We analyze a modified system where the possibility of secondary market trade puts an upper bound on τ_1, and τ_2 may be above zero (but nevertheless $\tau_2 < \tau_1$). In the current subsection, we analyze sin licenses in the presence of preference uncertainty, and introduce the trade-off between flexibility and commitment, which is a crucial feature associated with a system of sin licenses - or any other commitment device - in the presence of demand uncertainty. The implications of secondary market trade are deferred to the next subsection.

Assume that the preference shock θ has a cumulative distribution function $F(\theta)$ over some support $\left[\underline{\theta}, \overline{\theta}\right]$. The period t preference shock is realized in that period. In the previous period, when the amount of sin licenses is chosen, the consumer only knows the distribution of θ.[11] Expected indirect utility (of the long-run self), given the amount of sin licenses demanded, y, is

$$V(y) = E_\theta \left[\theta v(x(\theta, \beta)) - \delta h(x(\theta, \beta)) - px(\theta, \beta)\right]$$

where

$$x(\theta, \beta) = \min\{y, x^*(\theta, \beta)\}$$

and $x^*(\theta, \beta)$ is given by (3). The actual consumption level $x(\theta, \beta)$ is therefore chosen by the short-run self, who knows the realization of the preference shock θ, but faces self-control problems (if $\beta < 1$). Through the system of sin licenses the long-run self can attempt to influence the choices of the short-run self.

[11] In order to simplify the notation, in what follows we leave out the parameter γ. This can be done without loss of generality, since at this stage we focus on the choices of an individual consumer, with a given γ. We also drop the indeces denoting individuals and time.

If the preference shock realization is small, $\theta \leq \theta_1(y;\beta)$, the realized consumption falls short of the consumer's quota of sin licenses $x(\theta,\beta) = x^*(\theta;\beta) < y$. On the other hand, if the shock realization is large $\theta > \theta_1(y;\beta)$, the consumer uses the entire quota of sin licenses $x(\theta,\beta) = y$. The critical value $\theta_1(y;\beta)$ is given by

$$\theta_1(y,\beta) = \frac{\beta\delta h'(y) + p}{v'(y)}. \tag{5}$$

Denote the optimal (rational) level of consumption, with preferences θ, by $x^o(\theta)$. We show in Appendix A1 that fully rational consumers, with $\beta = 1$, choose $\theta_1(y;\beta = 1) \geq \overline{\theta}$,or equivalently $y^*(\beta = 1) \geq x^o(\overline{\theta})$. For a rational consumer, there is no need to ex ante constrain the ex post choices, and in equilibrium $x(\theta,\beta = 1) = x^o(\theta)$ for all θ i.e. rational consumers consume optimally under a scheme of sin licenses. (Pure) sin licenses, unlike linear taxes, have the desirable property that they do not distort the decisions of fully rational individuals[12].

Consumers with self control problems ($\beta < 1$), on the other hand, choose $y^*(\beta) < x^o(\overline{\theta})$ and sin licenses do not in general implement the first-best for them. For consumers with self-control problems, sin licenses imply a trade-off between flexibility in the face of preference shocks vs. commitment in the face of self-control problems. On the one hand, the consumer would like to consume the (ex ante) optimal amount even when the preference shock is large. This argument favors flexibility, and hence acquiring a large quota of sin licenses: If high consumption is due to a high preference shock, it would be optimal to be able to accommodate this taste. On the other hand, however, high consumption may arise due to self-control problems, rather than preferences. This argument favors commitment, and acquiring a smaller amount of sin licenses.

Hence for low levels of the preference shock, $\theta < \theta_1$, realized consumption is at the laissez-faire level $x(\theta) = x^*(\theta,\beta;p)$. On the other hand, for $\theta > \theta_1$, $x(\theta)$ is constant at $x(\theta) = y$. Hence sin licenses imply excessive flexibility for low preference shock realizations, and excessive commitment (no flexibility in the face of preference shocks) for high preference shock realizations. The outcome and the associated trade-offs are characterized in more detail in the appendix.

3.2 (Potential) secondary market trade

We next turn to the implications of secondary market trade, while continuing also to take into account demand uncertainty. As in the previous subsection, each unit of

[12] See the discussion on asymmetric paternalism for example in Camerer et al. (2003).

sin licenses allows consumers to buy one unit of the sin good at price p. The price p includes a possible tax, so that $p = 1 + \tau_1$. In contrast to the previous section, we now assume that without sin licenses, the consumer can buy sin goods at the (per unit) price $q > p$. Depending on circumstances (e.g. the specifics of legislation and regulation) this trade can take place in either primary markets or in secondary (black) markets.

We assume that secondary market trade involves a transaction cost k per unit of goods traded. If the government allows consumers without sin licenses to buy the good at price $q \leq p+k$, this trade takes place in the primary market. The government then collects the price difference $q-p$ in the form of a sin tax τ_2; i.e. τ_2 is now set at a level that does not prohibit consumption without a sin license, but nevertheless $\tau_2 > \tau_1$ so that a license allows purchases at a reduced tax. If this form of primary market trade without a sin license (at price $q \leq p+k$) is not allowed, the transactions may take place in the secondary market at price $p + k$. In sum, the potential for secondary market trade caps the price difference relating to purchases with or without a license, so that $q - p \leq k$.

When secondary market trade is possible, the consumer's expected indirect utility, with a given amount of sin licenses y is

$$V(y) = E_\theta \left[\theta v(x(\theta, \beta)) - \delta h(x(\theta, \beta)) - px(\theta, \beta) - (q-p)x^s(\theta, \beta)\right]$$

where

$$x^s(\theta, \beta) = \max\{0, x(\theta, \beta) - y\}$$

is the amount of sin goods bought without a license at price q.

Realized consumption $x(\theta; \beta)$ is characterized by the following expressions

$$x(\theta;\beta) \begin{cases} x^*(\theta;\beta) & \text{if } \theta < \theta_1(y;\beta) \\ y & \text{if } \theta_1(y;\beta) < \theta < \theta_2(y;\beta) \\ x^{**}(\theta;\beta) & \text{if } \theta > \theta_2(y;\beta) \end{cases}$$

where $x^*(\theta, \beta)$ is given by (3), $\theta_1(y; \beta)$ is given by (5), $x^{**}(\theta; \beta)$ is characterized by

$$\theta v'(x^{**}(\theta;\beta)) - \beta\delta h'(x^{**}(\theta;\beta)) - q = 0 \tag{6}$$

and $\theta_2(y; \beta)$ is given by

$$\theta_2(y;\beta) = \frac{\beta\delta h'(y) + q}{v'(y)}. \tag{7}$$

Notice in particular, that the quota of sin licenses y now determines consumption only at the medium range of preference shock realizations $\theta_1(y;\beta) < \theta < \theta_2(y;\beta)$. As in the previous section, with small shocks ($\theta < \theta_1$), realized consumption falls short of the quota. On the other hand, now that consumption without a license is not prohibited, the consumer yields to the temptation of purchasing more of the good from the secondary market when the preference shock realization is high ($\theta > \theta_2$).

A key point to note is that the presence of (potential) secondary market trade alters the trade-off between flexibility and commitment associated with sin licenses: the possibility of secondary market trade partially erodes the commitment-power of sin licenses, as the consumer has the option of resorting to secondary market purchases.

The effects of the secondary market on the commitment properties of sin licenses depend on the costs of secondary market trade. If the costs are very high, no consumer is tempted by the secondary market ($\theta_2 = \bar{\theta}$). As the transaction cost declines, the threshold value θ_2 declines: consumers with lower consumption needs are tempted by the secondary market, and the region of preference shock realizations where sin licenses have some commitment power (the gap between θ_1 and θ_2) shrinks. (Recall that below θ_1, the quota of sin licenses is not binding and consumption is at the laissez-faire level.) When secondary markets are perfect ($k = 0$), $\theta_2 = \theta_1$ (see (5) and (7)), sin licenses have no commitment power and the outcome associated with a system of sin licenses corresponds to the laissez-faire outcome for all consumers. The outcome of the model with preference uncertainty and potential secondary market trade is illustrated in Figure 1.

The fact that sin licenses in some cases result in laissez-faire outcome raises some interesting questions: since linear taxation would in many cases be welfare improving compared to no regulation (O'Donoghue and Rabin 2006), it may be that a simple linear tax in fact results in a superior outcome compared to sin licenses, when secondary markets exist. It is therefore not clear a priori, whether more complicated non-linear schemes such as sin licenses improve welfare compared to simple linear taxation. We will revisit this question in Section (5), after considering regulating sin goods through general, non-linear pricing schemes.

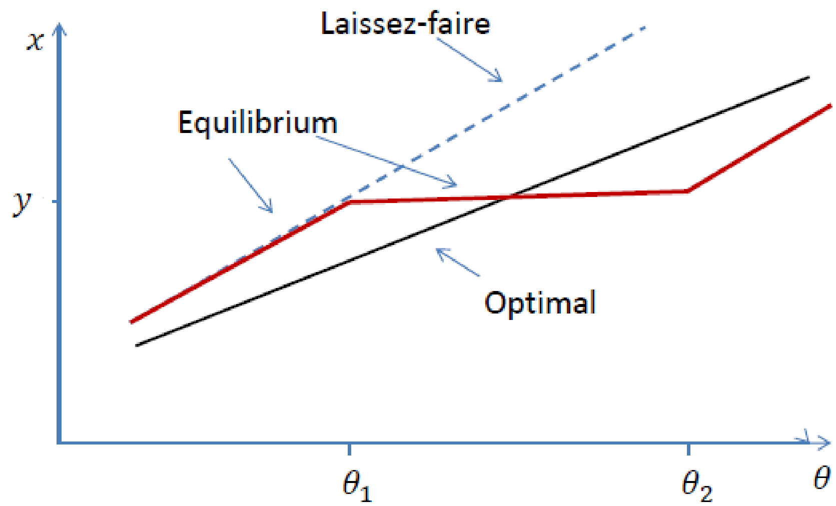

Figure 1: Equilibrium consumption under sin licenses with demand uncertainty and potential secondary market trade.

4 General non-linear pricing scheme: Mechanism design approach

In the previous section, the commitment scheme was constrained to be of a particular, simple type: a lower tax τ_1 up to a quota y of the sin good, and a higher tax τ_2 thereafter. This corresponds to a generalized version of the sin license scheme discussed in earlier literature. In the current section, we consider regulating sin goods via a completely general, personalized, non-linear pricing scheme. As in the case of sin licenses, the idea is that there is voluntary self-selection, i.e. the consumer himself chooses in period t the pricing scheme to be applied to his purchases of the sin good in period $t+1$. That is, the consumer self-selects what type of regulation should be applied to his purchases of the sin good in the future.

Assume that the long-run self chooses a general non-linear pricing scheme $T(x)$, where $T(x)$ is the total price for x units of the sin good. The scheme is chosen so as to maximize

$$E_\theta [V(\theta)] = \int_{\underline{\theta}}^{\overline{\theta}} V(\theta) f(\theta) d\theta \tag{8}$$

where[13]

$$V(\theta) = \theta v(x(\theta)) - \delta h(x(\theta)) - T(x(\theta)). \tag{9}$$

[13] Clearly, x also depends on the degree of self-control problems β, i.e. $x(\theta;\beta)$, but to simplify notation, we have left β out of the formulas.

Given the pricing scheme $T(x)$, the quantity of sin goods is chosen by the ex post self, who maximizes

$$\widehat{V}\left(\widehat{\theta}, \theta\right) = \theta v\left(x\left(\widehat{\theta}\right)\right) - \beta\delta h\left(x\left(\widehat{\theta}\right)\right) - T\left(x\left(\widehat{\theta}\right)\right) \tag{10}$$

where $x\left(\widehat{\theta}\right)$ is the consumption level intended for type $\widehat{\theta}$. We assume that the pricing scheme has to satisfy the following constraints:

$$T'(x) \geq p \tag{11}$$

where the price floor $p = 1 + \tau_1 \geq 1$ (the sin good is not subsidized, and it may be subject to a sin tax τ_1) and

$$T'(x) \leq q \tag{12}$$

where the price ceiling $q \leq p + k$ (at each point, the per unit price has to be no bigger than the secondary market price $p + k$, however the government can choose a lower ceiling). Further, assume that the revenues from the pricing scheme are redistributed back to consumers via uniform lump-sum subsidies.

We derive the conditions characterizing the consumers's optimal choice of $T(x)$ in Appendix A3. The main result of this analysis is summarized in the following Proposition:

Proposition 1 *Assume that consumers are sophisticated and the distribution of the preference shock θ is such that the hazard rate $\lambda(\theta) = \frac{f(\theta)}{1-F(\theta)}$ is non-decreasing. Then a system of sin licenses implements the consumers' preferred personalized non-linear pricing scheme.*

Proof. See Appendix A3. ■

The proposition shows that the system of sin licenses proposed in the literature has the interesting property that it would actually be the system that would be chosen by sophisticated consumers among all possible, completely general and individualized non-linear pricing schemes for sin goods. The solution is therefore again, as in the previous section, characterized by excessive flexibility (laissez-faire consumption) at low levels of the preference shock ($\theta < \theta_1$), and excessive commitment at higher level of the preference shock: the solution to the non-linear pricing problem is a bunching equilibrium where all individuals for whom $\theta \in [\theta_1, \theta_2]$ consume $x(\theta) = y$. On the other hand, the possibility of secondary market trade undermines the commitment power of sin licenses at high preference shock realizations, as consumers with $\theta > \theta_2$ are tempted by the secondary market.

The key to the intuition behind the result is to note that in choosing the regulating scheme, consumers have two objectives: they want to minimize the monetary costs of regulation, while at the same time reducing distortions in consumption. A scheme of pure sin licenses, where consumption of the sin good is subject to no tax up to the quota y, provides the cheapest possible means of achieving (at least some level of) commitment. The intuition for the bunching equilibrium is discussed in more detail in Section 6.

However, the scheme chosen by the consumer is not socially optimal. In contrast to the consumer's two objectives (minimizing costs as well as distortions), the social planner has only one objective, to minimize distortions. For the social planner, one consumer's cost is another consumer's gain, and hence the social planner does not care about the costs of the sin license scheme to any one consumer. Hence, the consumer's objective differs from that of the social planner. In fact, a social planner would never choose a system of sin licenses for any given consumer: We shown in the Appendix that the optimal solution would have the marginal price $T'\left(x^{o}\left(\theta\right)\right)$ increasing in θ for the entire range $\left[\underline{\theta}, \overline{\theta}\right]$. In the case of sin licenses, this is obviously not the case.

We further show in the Appendix that if the costs of the regulatory scheme for any given consumer were neutralized by personalized subsidies, the scheme chosen by the consumer would in fact coincide with the one that the social planner would choose. This further highlights the fact that it is the cost minimization motive that causes the consumer to choose sin licenses rather than the socially optimal scheme. Note that both cases - the one where the social planner chooses the regulatory mechanism for each consumer, and the one where personalized subsidies are used - should be thought of as hypothetical thought experiments: in order to implement either of these systems, we would need to assume that the social planner has information on each consumer's degree of self-control problems and the harm function associated with sin good consumption.[14]

[14] We show in the Appendix, however, that if the social planner were to choose the pricing scheme for the consumer, information on the distribution of θ i.e. on the type of uncertainty faced by the consumer would *not* be needed.

5 Role of sin licenses in regulating the consumption of harmful goods

Above, we have shown that sin licenses are in general not the socially optimal policy, and (if free to choose) the social planner would never choose a sin licensing scheme for any given consumer. Does this imply that sin licenses have no role to play in regulating harmful consumption? Based on our analysis, such a conclusion would be premature. A first point to note is that the first-best policy above is not implementable, as it would require information (e.g. on the extent of individuals' self-control problems) that the social planner is very unlikely to have. The sin licensing scheme, on the other hand, is based on self-selection, and is therefore part of a feasible policy package.

An interesting question relates to whether sin licenses are a superior regulating mechanism compared with linear taxation. Goods such as alcohol are currently in practice subject to linear taxation - would we do better by replacing linear taxation with a system of sin licenses? Interestingly, the answer to this question is in general ambiguous. As was explained above, the cost-minimization objective that the consumers have when choosing among different regulating mechanisms leads them to prefer a zero tax for low preference shocks. This leads to excessive (laissez-faire) consumption for low preference shocks, and raising τ_1 above zero would move consumption decisions closer to the social optimum. Mandatory linear taxes may therefore have some desirable commitment properties compared to voluntary sin licenses.

Even though we cannot draw clear-cut conclusions on whether the current system of linear sin taxes should be replaced with sin licenses, we can analyze the question whether welfare could be increased if we *supplement* a linear sin tax with a voluntary system of sin licenses. The following Proposition shows that linear sin taxes and sin licenses can be thought of as complements, in the sense that it is in general not optimal to rely on one type of regulation (linear tax vs. sin licenses) only, but social welfare (denoted by W) can be improved if elements of the other system are introduced to supplement a purely linear sin tax / a pure system of voluntary sin licenses. More specifically, we consider the welfare effects of introducing a marginal sin tax on top of a system of sin licenses, on the one hand; and the welfare effects of introducing marginal sin licenses on top of a linear sin tax, on the other. (By 'marginal sin licenses' we refer to a system where a minimal degree of personalized regulation is introduced on top of a linear tax, in the sense that compared to a situation where only a linear tax τ_1 is in place, the consumer chooses a level of consumption y above which a marginally higher tax rate $\tau_1 + d\tau$ is applied.)

Proposition 2 *(i) Supplementing a system of sin licenses with a (marginal) linear sin tax would improve welfare. (ii) Supplementing a linear sin tax τ_1 with a system of (marginal) sin licenses would improve welfare, if $\frac{\partial W}{\partial \tau_1} \geq 0$ (sufficient condition).*

Proof. See Appendix A4. ■

A combination of a linear tax and a voluntary scheme of sin licenses is potentially useful as it would impose some commitment in cases where a pure system of sin licenses would be excessively geared towards flexibility (at low levels of the preference shock); and on the other hand, it would allow individuals to opt for stricter regulation than in the case of a pure linear tax, in case they feel more commitment would be beneficial (at high levels of the preference shock). The intuition for the condition $\frac{\partial W}{\partial \tau_1} \geq 0$ is the following. In this case a small tax increase would improve welfare, but introducing a marginal sin license instead has the added benefit of allowing for self-selection: the tax increase is implemented only for those consumers for whom tighter regulation is beneficial. Interestingly, the above Proposition also implies that a marginal sin license would improve welfare when the linear sin tax is set optimally ($\frac{\partial W}{\partial \tau_1} = 0$).

The main message conveyed by Proposition 2 is that the two systems of regulation - linear taxation and sin licenses - can be thought of as being complements, not substitutes. A system combining both linear taxes and sin licenses would therefore be optimal. However, the Proposition concerns marginal reforms, and therefore does not tell us whether any given reform is welfare improving. The following Proposition provides a (sufficient) condition that allows one to assess ex-post whether introducing a general sin licensing scheme on top of a linear sin tax is welfare improving.

Proposition 3 *Suppose that a linear sin tax τ is replaced by a hybrid system of sin taxes and sin licenses. Each unit of sin licenses allows consumers to buy one unit of the sin good at price $p = 1 + \tau_1$ where $\tau_1 \leq \tau$. Without sin licenses, the consumer can buy sin goods at the (per unit) price $q = 1 + \tau_2$, where $\tau_2 \geq \tau$. This reform would improve welfare, if i) all consumers are sophisticated, ii) if the hazard rate $\lambda(\theta)$ is non-decreasing, and iii) if the reform does not lead to a reduction in tax revenue (sufficient conditions).*

Proof. See Appendix A5. ■

While Proposition 2 considered marginal sin licenses, Proposition 3 allows for any voluntary sin licensing scheme chosen by the consumers, implemented on top of a linear sin tax. The intuition for the Proposition is that if tax revenue does not decline after the introduction of sin licenses, we can infer that the consumer's choice of his license quota was not determined by the motivation to minimize costs (i.e. to escape the sin tax), but rather by the motivation to minimize distortions; the latter is the motivation that the social planner would agree with, and hence sin licenses are guaranteed to improve welfare. (Note that the result is *not* related to any concern for tax revenue per se. The sole objective for the social planner is to minimize distortions from sin tax consumption.[15]) It should be noted however that this is a sufficient condition, and sin licenses may improve welfare in other situations as well. Nevertheless, it is interesting to note that introducing a regulation mechanism that is completely voluntary is not guaranteed to improve welfare even when consumers are sophisticated.

6 Discussion

In the current section, we first briefly discuss the implications of naivete about self-control problems for our results. We then turn to relate our analysis to the study of non-linear income taxation, and further develop the intuition behind the bunching equilibrium obtained above. Finally, we present some (highly) suggestive historical evidence on the effects of one form of personalized regulation, namely the system of alcohol purchase permits that was in place in Finland and Sweden in the mid-20th century.

Naivete So far, we have assumed that consumers are sophisticated, i.e. they are fully aware of their degree of self-control problems. Let us next consider the case of naive individuals. (Definition / discussion of naivete to be added.) A naive individual believes that he will consume optimally in the future, and hence sees no reason for restricting his future consumption choices. He therefore chooses his quota of sin licenses in such a way as to obtain full flexibility to adjust to future preference shocks and always obtains the maximum possible quota of sin licenses. Hence the level of taxes relating to purchases without licenses (τ_2) is irrelevant for a naive person, and only the tax τ_1 matters. We thus obtain the result that for a naive individual, sin licenses are equivalent to a linear tax.

However, it should further be noted that if the choice of the linear tax was left for the naive person himself, he would opt for a zero tax. Clearly, in the case of naives, self-selection into regulation does not work, and the welfare of naives would be improved

[15] This is a reasonable objective. The received wisdom from the literature on optimal commodity taxation is that as a general rule, from the point of view of efficient revenue collection, commodity taxes should be uniform - see e.g. the Mirrlees Review. Hence the main (legitimate) reason for higher taxes on sin goods would be to elimiate distortions.

if a positive linear tax was imposed by the social planner.

This result is in stark contrast to the result obtained in the case when there is no demand uncertainty or secondary markets. In that case, sin licenses implement the first-best outcome for both sophisticates and naives. (O'Donoghue and Rabin 2005.) This result is however overhauled in the present, more general case: sin licenses bring no welfare improvement for naives, compared to simple linear regulation.

This also implies that we can generalize the above results relating to the policy implications of our analysis, to a society consisting of both sophisticated and naive individuals. First, the currently existing systems with linear taxation should not be replaced by a pure system of sin licenses: this would reduce the welfare of naives (moving from a situation with some beneficial commitment to none) and might also reduce the welfare of sophisticates, as argued above. Second, welfare could be improved by supplementing the current linear tax with a voluntary system of sin licenses. This would have no effects on naives, whereas it would improve welfare for sophisticates.

Intuition for non-linear personalized pricing and comparison to non-linear income taxation Consider increasing the (personalized) marginal price $T'(x(\theta))$ of the sin good at some consumption level $x(\theta)$. This change involves both benefits and costs to the consumer. First, starting from laissez-faire (with $T'(x(\theta)) = 1$), the benefit is that actual consumption $x(\theta)$ will decrease and move closer to the (ex ante) optimal level $x^o(\theta)$. Hence the internality wedge (i.e. the distortions caused by self-control problems) $\Psi(x(\theta)) = (1-\beta)\delta h'(x(\theta))$ will decrease. This beneficial effect tends to be large, if the initial internality wedge (at consumption level $x(\theta)$) is large. Furthermore notice the marginal price $T'(x(\theta))$ will affect the consumer's (ex post) choices when the preference shock is θ, and consumption is $x(\theta)$. Hence the beneficial effect of raising the marginal price $T'(x(\theta))$ tends to be large, if the density $f(\theta)$ is high.

The cost of raising the marginal price $T'(x(\theta))$, on the other hand, is that at all higher preference shocks $\theta' \geq \theta$, and higher consumption levels $x(\theta') \geq x(\theta)$, the consumer will have to pay an increased *total* price $T(x(\theta'))$ for the sin goods (s)he buys. The costs (i.e. the negative effects of raising the marginal price $T'(x(\theta))$) therefore tend to be high, if the probability $\Pr(\theta' \geq \theta) = 1 - F(\theta)$ is high.

To summarize, the benefits of raising the marginal price $T'(x(\theta))$ are proportional to $f(\theta)\Psi(x(\theta))$, while the costs are proportional to $1-F(\theta)$. The ratio of benefits and costs is $\lambda(\theta)\Psi(x(\theta))$, where $\lambda(\theta) = \frac{f(\theta)}{1-F(\theta)}$ is the hazard rate. If the *benefit-cost* ratio $\lambda(\theta)\Psi(x(\theta))$ is high, it is (ex ante) attractive for the consumer to raise the marginal price.

The situation at hand is a mirror image of a typical non-linear taxation problem: the basic logic is similar, but turned upside down. Here the consumer's objective is to have large (corrective) behavioral effects, combined with minimal payments. In the non-linear taxation problem, the objective is to extract large payments (i.e. tax revenue) with minimal (distorting) behavioral effects (on labor supply).

In the non-linear taxation problem, it is optimal to have a high marginal tax $T'(w(\theta))$ rate in a certain income bracket $w(\theta)$ (or at productivity level θ), if there are not that many people whose labor supply is affected by that marginal tax rate, i.e. if the density $\widehat{f}(\theta)$ is low. On the other, the distortions tend to be low, if the elasticity of labor supply (which we denote by $\widehat{\Psi}(\theta)$) at productivity level of θ is low. Moving to the benefit side, raising the marginal tax rate $T'(w(\theta))$ will increase the tax income collected from all income groups higher than $w(\theta)$. This benefit tends to be large, if the frequency mass $1-\widehat{F}(\theta)$ is large.

Putting the above arguments together, in the case of non-linear taxation, the *cost-benefit* ratio of increasing the marginal tax rate is $\widehat{\lambda}(\theta)\widehat{\Psi}(\theta)$, where $\widehat{\lambda}(\theta) = \frac{f(\widehat{\theta})}{1-F(\widehat{\theta})}$ is the hazard rate. Notice that here a high ratio $\lambda(\theta)\widehat{\Psi}(\theta)$ means high costs, relative to benefits, and speaks against a tax increase. In our situation of non-linear (personalized) pricing, we got a similar ratio $\lambda(\theta)\Psi(x(\theta))$, but with the opposite interpretation: in our application $\lambda(\theta)\Psi(x(\theta))$ is the *benefit-cost* ratio, and a high ratio means that the marginal price should be raised.

Understanding the bunching equilibrium: Why do consumers choose the system of sin licenses? Let us next consider the intuition behind our key result in more detail, paying particular attention to the bunching property of the equilibrium. Why does the consumer choose a mechanism that entails the same realized amount of consumption for different levels of the preference shock? In other words, why is it optimal for the consumer to choose a mechanism that is highly inflexible in the sense that it does not allow different consumption needs to be accommodated?

Intuitively, a major reason for consumers to choose the system of sin licenses is the low monetary cost of such a system (from the point of view of an individual consumer). Indeed, under the basic / pure vanilla system of sin licenses (with no potential secondary market trade undermining the system) the consumer pays the lowest possible price (the producer price) for his/her consumption. Recall that the benefit-cost ratio of increasing the marginal price is given by $\lambda(\theta)\Psi(x(\theta))$. Indeed the hazard rate $\lambda(\theta) = \frac{f(\theta)}{1-F(\theta)}$ appearing in this choice criterion reflects the concern for the monetary costs.

The root of the bunching phenomenon lies in the fact that the importance of the monetary cost minimization motive varies with the level of the preference shock, and the level of consumption. With our assumptions (increasing hazard rate $\lambda(\theta)$), the monetary cost minimization motive is strongest at the low end of the consumption and pricing schedule; intuitively, high marginal prices at the low end of the consumption / pricing schedule would increase the consumer's (total) monetary costs with a high probability. At the high end of the consumption / pricing schedule the incentives for monetary cost minimization are weaker; intuitively, a high marginal price at the high end of the schedule increases the consumer's (total) monetary costs with a relatively low probability, since the consumer has (very) high consumption needs (very high θ) only infrequently.

Hence, the cost minimization motive dominates at the low end of consumption, whereas at the high end of the consumption / pricing schedule, there is more room for the corrective motive. Note also that the internality wedge $\Psi(x(\theta))$ is increasing in consumption (given the convexity of $h(x(\theta))$), which also serves to make the corrective motive stronger at high levels of the preference shock. The problematic feature is that the rising relative importance of the corrective motive would call for a consumption profile that is partially *decreasing* in the preference shock θ. It is easy to understand that a scheme where realized consumption should depend negatively on the need to consume θ cannot be implemented. If the scheme stipulates that $x(\theta') < x(\theta)$ for $\theta' > \theta$, it is clear that ex post, the consumer with high consumption needs (preference shock θ') will pick the higher consumption level $x(\theta)$, rather than the lower consumption level $x(\theta')$, stipulated by the putative scheme.

The solution to this problem is bunching. If the decreasing consumption scheme cannot be implemented, the best alternative (for the ex ante self) is to implement a constant consumption scheme. Hence, in equilibrium consumption will be the same for all θ above a certain threshold. Such a solution can be implemented with sin licenses.

Evidence Real life evidence on the functioning of personalized regulation of harmful consumption is hard to find, as real tax schedules tend to be linear. An interesting piece of evidence - albeit being admittedly highly suggestive - comes from the experience of attempts to control alcohol purchases in Finland and Sweden mainly in the 1940s and 1950s. In both countries, official purchases of alcohol were only allowed upon showing a special identity card, and purchases were recorded on the card. (Below, we call these cards "alcohol purchase permits"; a more colloquial term often used is the "booze card"). Both systems had elements of attempting to implement a crude form of personalized regulation. In Finland, purchases were monitored more closely for those individuals suspected of being prone to alcohol misuse, and high consumption could make a person lose his entitlement to hold a permit. In Sweden, a person's quota depended on certain characteristics such as age, gender and family status, and the permit could be completely denied from individuals suspected of misuse. (Häikiö 2007.) An important caveat is, however, that the above systems differ from sin licenses in that they were not voluntary, as in some cases the choice of not receiving a permit was imposed from outside (in which case opting for *lighter* regulation was not possible). Thus, those systems cannot be thought of as an indication of what a genuine system of sin licenses - personalized, voluntary regulation - would look like.[16] Nevertheless, they may provide some evidence about the self-control properties of personalized regulation, as those systems offered individuals the possibility to opt for *stricter* regulation by choosing not to obtain a permit.

What can the experience with these systems tell us about the pros and cons of personalized regulation as a self-control device? First, during the latter part of the 1940s (when the Finnish system was in full operation) only 30-50% of individuals over the age 20 obtained the permit to purchase alcohol each year. (Häikiö 2007.) At the same time, the number of abstainers was relatively high: in 1946, 20 % of Finnish consumers were life-long abstainers (i.e. had never had any alcohol, a very strict definition of abstinence) and 49 % had not drunk any alcohol within the past month. (Sulkunen 1979.) This suggests that for some individuals, not obtaining a permit may indeed have functioned as an effective self-control device. On the other hand, the gap between the number of abstainers and those who held a permit suggests that alcohol was also consumed without a permit.[17]

Second, it is also interesting to note that widespread secondary markets posed significant challenges for these systems. For instance in Sweden, a large proportion of offences associated with the misuse of alcohol were committed by individuals who did not hold an permit to purchase alcohol in the first place. In Finland, a significant proportion of individuals whose permit was withdrawn for a fixed period never re-applied for it. It is clear however that these individuals had not stopped alcohol

[16] Naturally, practical implementation of the scheme would also be very different today - see e.g. Cowell (2008) for a discussion of the use of smart cards in taxation.

[17] It should be noted however, that to some extent this was legitimate: alcohol was also served in restaurants, where a personal permit was not required from customers. However, alcohol consumption in restaurants at the time was not very common, and for example in 1948, purchases with a permit accounted for 86 % of total official alcohol consumption in Finland (Häikiö 2007).

consumption but rather found unofficial means of obtaining alcohol more convenient than the official route (Immonen 1980).

To sum up, the above evidence is broadly consistent with our story: it appears that the system of personal licenses may have helped some individuals to limit their alcohol consumption. Further, the main problems with the system arose because of unofficial trade.

7 Conclusion

We have analyzed the regulation of harmful consumption when consumers have self-control problems, there is demand uncertainty, and consumers can trade the good in a secondary market. In particular, we have examined the idea that it may be useful to implement personalized regulation, where sophisticated consumers individually self-select in advance the type of regulation that should be applied to their consumption of harmful goods in the future. We have analyzed the central trade-offs associated with these types of commitment devices in the presence of demand uncertainty - namely the trade-off between commitment in the face of self-control problems, and flexibility in the face of demand uncertainty.

We show that if sophisticated consumers are allowed to select any general, non-linear personalized pricing scheme, they will in fact choose a relatively simple scheme of sin licenses, with a zero tax rate up to a given level of consumption y and a higher (constant) tax rate thereafter. We have also shown that even though the sin licensing scheme is preferred by sophisticated consumers, it is not socially optimal. Nevertheless, sin licenses may have a place in the regulator's toolkit. The main policy conclusions from our analysis are two-fold. First, the currently existing systems with linear taxation should *not* be replaced by a pure system of sin licenses: this would reduce the welfare of naives and might also reduce the welfare of sophisticates. Second, however, welfare could in some cases be improved by supplementing the current linear tax with a voluntary system of sin licenses. This would have no effects on naives, whereas it might improve welfare for sophisticates. Hence linear taxation and sin licenses should be thought of as complements, not substitutes.

Appendix

A1. Appendix to Section 3.1

Let us consider the demand for sin licenses under preference uncertainty. If the preference shock realization is small, $\theta \leq \theta_1(y;\beta)$, the realized consumption falls short of the amount mandated by sin licenses $x(\theta,\beta) = x^*(\theta;\beta) < y$. If the shock realization is large $\theta > \theta_1(y;\beta)$, the consumer uses the entire quota of sin licenses $x(\theta,\beta) = y$. The critical value $\theta_1(y;\beta)$ is given by

$$x^*(\theta_1,\beta) = y$$

or

$$\theta_1 v'(y) - \beta\delta h'(y) - p = 0$$

or

$$\theta_1(y,\beta) = \frac{\beta\delta h'(y) + p}{v'(y)} \tag{13}$$

(Notice: In the next section we are going to define a second critical value of the preference shock, $\theta_2(y,\beta)$, where $\theta_2(y,\beta) > \theta_1(y,\beta)$. Hence the notation.)

The consumer chooses the amount of sin licenses y so as to maximize

$$\begin{aligned} V(y) &= \int_{\underline{\theta}}^{\theta_1(y;\beta)} \left[\theta v(x^*(\theta;\beta)) - \delta h(x^*(\theta;\beta)) - px^*(\theta;\beta)\right] dF(\theta) \\ &\quad + \int_{\theta_1(y;\beta)}^{\overline{\theta}} \left[\theta v(y) - \delta h(y) - py\right] dF(\theta) \end{aligned}$$

The first-order condition is given by

$$\begin{aligned} & \left[\theta_1(y;\beta) v(x^*(\theta_0;\beta)) - \delta h(x^*(\theta_0;\beta)) - px^*(\theta_0;\beta)\right] dF(\theta_1) \frac{d\theta_1(y;\beta)}{dy} \\ & - \left[\theta_1(y;\beta) v(y) - \delta h(y) - py\right] dF(\theta_0) \frac{d\theta_1(y;\beta)}{dy} \\ & + (1 - F(\theta_1)) \left(E\left[\theta \mid \theta \geq \theta_1(y;\beta)\right] v'(y) - \delta h'(y) - p\right) \\ = \; & 0. \end{aligned}$$

Since $x^*(\theta_0;\beta) = y$, the terms on rows one and two clearly cancel out each other, and the first-order condition simplifies to the form

$$E\left[\theta \mid \theta \geq \theta_1(y;\beta)\right] v'(y) - \delta h'(y) - p = 0. \tag{14}$$

We still have to study the second-order condition, which can be expressed in the form

$$v''(y)\int_{\theta_1}^{\overline{\theta}} \theta f(\theta)\, d\theta - \delta h''(y)\int_{\theta_1}^{\overline{\theta}} f(\theta)\, d\theta - \left[\theta_1 v'(y) - \delta h'(y) - p\right] f(\theta_1)\frac{\partial \theta_1}{\partial y} \tag{15}$$

Since

$$\theta_1 v'(y) - \delta h'(y) - p = -(1-\beta)\,\delta h'(y)$$

and (by (13))

$$\frac{\partial \theta_1(y,\beta)}{\partial y} = \frac{v'(y)\,\beta\delta h''(y) - (\beta\delta h'(y) + p)\,v''(y)}{\left[v'(y)\right]^2} = \frac{\beta\delta h''(y)}{v'(y)} - \theta_1\frac{v''(y)}{v'(y)}$$

the second-order condition (15) can be rewritten as

$$\begin{aligned} & v''(y)\int_{\theta_1}^{\overline{\theta}} \theta f(\theta)\, d\theta - \delta h''(y)\int_{\theta_1}^{\overline{\theta}} f(\theta)\, d\theta \\ & -(1-\beta)\frac{\delta h'(y)}{v'(y)}\theta_1 f(\theta_1)\, v''(y) + (1-\beta)\frac{\delta h'(y)}{v'(y)} f(\theta_1)\,\beta\delta h''(y) \end{aligned}$$

Next, the first-order condition implies that (see also Appendix A3, equation (65) below)

$$(1-\beta)\frac{\delta h'(y)}{v'(y)} = \frac{\int_{\theta_1}^{\overline{\theta}} (1-F(\theta))\, d\theta}{(1-F(\theta_1))}$$

and the second-order condition can be further re-written as

$$\begin{aligned} & v''(y)\int_{\theta_1}^{\overline{\theta}} \left[\theta f(\theta) - (1-F(\theta))\frac{\theta_1 f(\theta_1)}{1-F(\theta_1)}\right] d\theta \\ & -\delta h''(y)\int_{\theta_1}^{\overline{\theta}} \left[f(\theta) - \beta(1-F(\theta))\frac{f(\theta_1)}{1-F(\theta_1)}\right] d\theta \\ = \; & v''(y)\int_{\theta_1}^{\overline{\theta}} \left\{\left[\theta\lambda(\theta) - \theta_1\lambda(\theta_1)\right](1-F(\theta))\right\} d\theta \\ & -\delta h''(y)\int_{\theta_1}^{\overline{\theta}} \left\{\left[\lambda(\theta) - \beta\lambda(\theta_1)\right](1-F(\theta))\right\} d\theta \end{aligned} \tag{16}$$

Now, from the expression (16) it is immediately clear that the second-order condition holds if $\lambda'(\theta) \geq 0$ (sufficient condition).

ii) Denote the optimal / rational level of consumption, with preferences θ, by $x^o(\theta)$. This level of consumption is given by

$$\theta v'(x^o(\theta)) - \delta h'(x^o(\theta)) - p = 0 \tag{17}$$

a) Let us show that fully rational consumers, with $\beta = 1$, choose

$$\theta_1(y; \beta = 1) \geq \overline{\theta},$$

or equivalently

$$y^*(\beta = 1) \geq x^o\left(\overline{\theta}\right).$$

By (13), one can clearly see that

$$E\left[\theta \mid \theta \geq \theta_1(y; \beta)\right] v'(y) \geq \theta_1(y; \beta) v'(y) = \beta\delta h'(y) + p$$

where strict inequality applies when $\theta_1(y; \beta) < \overline{\theta}$, and equality applies iff $\theta_1(y; \beta) = \overline{\theta}$. Then also

$$E\left[\theta \mid \theta \geq \theta_1(y; \beta)\right] v'(y) - \delta h'(y) - p \geq -(1-\beta)\delta h'(y) \text{ for all } y.$$

Now it is easy to see that for $\beta = 1$, the first-order condition (14) can only hold if $\theta_1(y; \beta) = \overline{\theta}$.

b) Next, consider consumers with self-control problems ($\beta < 1$). Assume that $\overline{\theta} < \infty$. To show that consumers with self control problems choose $y^*(\beta) < x^o\left(\overline{\theta}\right)$, assume by contrast that $y^*(\beta) = x^o\left(\overline{\theta}\right)$. From equation (17) we get $\delta h'\left(x^o\left(\overline{\theta}\right)\right) + p = \overline{\theta} v'\left(x^o\left(\overline{\theta}\right)\right)$. Using this result, the left-hand side of the first-order condition (14) then takes the form

$$\left(E\left[\theta \mid \theta \geq \theta_1\left(x^o\left(\overline{\theta}\right); \beta\right)\right] - \overline{\theta}\right) v\left(x^o\left(\overline{\theta}\right)\right) \tag{18}$$

By (13)

$$\theta_1\left(x^o\left(\overline{\theta}\right); \beta\right) = \frac{\beta\delta h'\left(x^o\left(\overline{\theta}\right); \beta\right) + p}{v'\left(x^o\left(\overline{\theta}\right)\right)} \tag{19}$$

Clearly $\theta_1\left(x^o\left(\overline{\theta}\right); \beta\right) < \overline{\theta}$: if $\theta_1\left(x^o\left(\overline{\theta}\right); \beta\right)$ were equal to $\overline{\theta}$, (19) would imply $\overline{\theta} v'\left(x^o\left(\overline{\theta}\right)\right) - \beta\delta h'\left(x^o\left(\overline{\theta}\right); \beta\right) + p = 0$; but this contradicts the first-order condition characterizing the optimal choice $x^o\left(\overline{\theta}\right)$ (look at equation (17) with $\theta = \overline{\theta}$). Finally, since

$\theta_1\left(x^o\left(\overline{\theta}\right);\beta\right) < \overline{\theta}$,

$$\begin{aligned} & E\left[\theta \mid \theta \geq \theta_1\left(x^o\left(\overline{\theta}\right);\beta\right)\right] v'\left(x^o\left(\overline{\theta}\right)\right) - \delta h'\left(x^o\left(\overline{\theta}\right)\right) - p \\ = \ & \left(E\left[\theta \mid \theta \geq \theta_1\left(x^o\left(\overline{\theta}\right);\beta\right)\right] - \overline{\theta}\right) v\left(x^o\left(\overline{\theta}\right)\right) < 0 \end{aligned}$$

Thus the optimal choice of $y^*(\beta)$ must be lower than $x^o\left(\overline{\theta}\right)$.

We can further characterize the outcome with sin licenses for consumers with self-control problems as follows. In the first-best optimum, consumption would be different for each level of the preference shock ($x(\theta) = x^o(\theta)$). Clearly, sin licenses are too blunt a tool to achieve this objective: under the license system, the consumer with self-control problems actually achieves the first-best only when the preference shock realization happens to be $\theta^0 = \theta^0(y)$, where $\theta^0(y)$ is given by $x\left(\theta^0\right) = y$. More generally, under the system of sin licenses, we have four possible outcomes for a given consumer, depending on the level of the preference shock realization:

1. $x(\theta) = x^*(\theta, \beta; p) > x^o(\theta)$, when $\theta \leq \theta_1(y, \beta)$: the quota of sin licenses is not binding, consumption equals the laissez-faire choice of the short-run self, and the license system provides no commitment.

2. $x^o(\theta) < x(\theta) = y < x^*(\theta, \beta; p)$, when $\theta_1(y, \beta) \leq \theta < \theta^0(y)$: realized consumption is higher than optimal, but the sin license system provides some commitment (since consumption is lower than short-fun laissez-faire)

3. $x(\theta^o) = y = x^o(\theta^o)$: realized consumption equals optimal consumption

4. $x(\theta) = y < x(\theta^o)$, when $\theta > \theta^0(y)$: there is commitment at the expense of flexibility, and realized consumption is lower than optimal consumption

Choosing the optimal quota y involves finding the right balance between these trade-offs. In particular, a tighter quota (a smaller y), improves consumer welfare over segment 2 (since actual consumption moves closer to optimal consumption), but lowers consumer welfare over segment 4 (since actual consumption moves further away from optimal consumption).

A2. Appendix to Section 3.2

Assume that without sin licenses, the consumer can buy sin goods at unit price q. The consumer then chooses the amount of sin licenses y so as to maximize

$$\begin{aligned} V(y) &= \int_{\underline{\theta}}^{\theta_1(y;\beta)} \left[\theta v\left(x^*(\theta;\beta)\right) - \delta h\left(x^*(\theta;\beta)\right) - px^*(\theta;\beta)\right] dF(\theta) \\ &\quad + \int_{\theta_1(y;\beta)}^{\theta_2(y;\beta)} \left[\theta v(y) - \delta h(y) - py\right] dF(\theta) \\ &\quad \int_{\theta_2(y;\beta)}^{\overline{\theta}} \left[\theta v\left(x^{**}(\theta;\beta)\right) - \delta h\left(x^{**}(\theta;\beta)\right) - py - q(x^{**}(\theta;\beta) - y)\right] dF(\theta). \end{aligned}$$

subject to (5) and (7). The first-order condition of this problem takes the form

$$\begin{aligned} &\left[F\left(\theta_2(y;\beta)\right) - F\left(\theta_1(y;\beta)\right)\right]\left[E\left[\theta \mid \theta_2(y;\beta) \geq \theta \geq \theta_1(y;\beta)\right] v'(y) - \delta h'(y) - p\right] \\ &\qquad + \left[1 - F\left(\theta_2(y;\beta)\right)\right](q - p) = 0. \end{aligned} \tag{20}$$

This form is obtained noting that $x^*(\theta_1;\beta) = x^{**}(\theta_2;\beta) = y$, and thus the terms involving $\frac{d\theta_1(y;\beta)}{dy}$ and $\frac{d\theta_2(y;\beta)}{dy}$ cancel out.

We still need to study the second-order condition, which takes the form

$$\begin{aligned} &\int_{\theta_1}^{\theta_2} \widetilde{\theta} f\left(\widetilde{\theta}\right) d\widetilde{\theta} v''(y) - \int_{\theta_1}^{\theta_2} f\left(\widetilde{\theta}\right) d\widetilde{\theta} \delta h''(y) + \left[\theta_2 v'(y) - \delta h'(y) - q\right] f(\theta_2) \frac{d\theta_2}{dy} \\ &- \left[\theta_1 v'(y) - \delta h'(y) - p\right] f(\theta_1) \frac{d\theta_1}{dy}. \end{aligned} \tag{21}$$

Next, notice that (by (5) and (7))

$$\theta_1 v'(y) - \delta h'(y) - p = \theta_2 v'(y) - \delta h'(y) - q = -(1-\beta)\delta h'(y)$$

Also notice that

$$\begin{aligned} \frac{\partial \theta_1(y,\beta)}{\partial y} &= \frac{v'(y)\beta\delta h''(y) - \left(\beta\delta h'(y) + p\right) v''(y)}{\left[v'(y)\right]^2} \\ &= \frac{\beta\delta h''(y)}{v'(y)} - \theta_1 \frac{v''(y)}{v'(y)} = \theta_1\left[\frac{h''(y)}{\delta h'(y) + p} - \frac{v''(y)}{v'(y)}\right] \end{aligned}$$

and

$$\begin{aligned}\frac{\partial \theta_2(y,\beta)}{\partial y} &= \frac{v'(y)\,\beta\delta h''(y) - (\beta\delta h'(y)+q)\,v''(y)}{[v'(y)]^2} \\ &= \frac{\beta\delta h''(y)}{v'(y)} - \theta_2\frac{v''(y)}{v'(y)} = \theta_2\left[\frac{\delta h''(y)}{\delta h'(y)+q} - \frac{v''(y)}{v'(y)}\right],\end{aligned}$$

Then the second-order condition (21) can be rewritten as

$$\begin{aligned}&\int_{\theta_1}^{\theta_2}\theta f(\theta)\,d\theta v''(y) - \int_{\theta_1}^{\theta_2} f(\theta)\,d\theta\delta h''(y) \\ &-(1-\beta)\,\delta h'(y)\left[f(\theta_2)\left(\frac{\beta\delta h''(y)}{v'(y)} - \theta_2\frac{v''(y)}{v'(y)}\right) - f(\theta_1)\left(\frac{\beta\delta h''(y)}{v'(y)} - \theta_1\frac{v''(y)}{v'(y)}\right)\right] \\ =\;& v''(y)\int_{\theta_1}^{\theta_2}\theta f(\theta)\,d\theta + v''(y)\left[\theta_2 f(\theta_2) - \theta_1 f(\theta_1)\right]\frac{(1-\beta)\,\delta h'(y)}{v'(y)} \qquad (22) \\ &-\delta h''(y)\int_{\theta_1}^{\theta_2} f(\theta)\,d\theta - \delta h''(y)\,\beta\left[f(\theta_2) - f(\theta_1)\right]\frac{(1-\beta)\,\delta h'(y)}{v'(y)}\end{aligned}$$

Next, the first-order condition (20) implies that (see also Appendix A3, equation (**??**) below)

$$\frac{(1-\beta)\,\delta h'(y)}{v'(y)} = \frac{\int_{\theta_1}^{\theta_2}(1-F(\theta))\,d\theta}{F(\theta_2(y;\beta)) - F(\theta_1(y;\beta))}$$

Hence, the second-order condition can be further rewritten as

$$\begin{aligned}& v''(y)\int_{\theta_1}^{\theta_2}\left[\theta f(\theta) + (1-F(\theta))\frac{\theta_2 f(\theta_2) - \theta_1 f(\theta_1)}{F(\theta_2) - F(\theta_1)}\right]d\theta \\ & -\delta h''(y)\int_{\theta_1}^{\theta_2}\left[f(\theta) + \beta(1-F(\theta))\frac{f(\theta_2) - f(\theta_1)}{F(\theta_2) - F(\theta_1)}\right]d\theta \\ =\;& v''(y)\int_{\theta_1}^{\theta_2}\left\{\left[\theta\lambda(\theta) + \frac{\theta_2 f(\theta_2) - \theta_1 f(\theta_1)}{F(\theta_2) - F(\theta_1)}\right](1-F(\theta))\right\}d\theta \\ & -\delta h''(y)\int_{\theta_1}^{\theta_2}\left\{\left[\lambda(\theta) + \beta\frac{f(\theta_2) - f(\theta_1)}{F(\theta_2) - F(\theta_1)}\right](1-F(\theta))\right\}d\theta\end{aligned}$$

or

$$\begin{aligned}& v''(y)\int_{\theta_1}^{\theta_2}\left\{\left[\theta\lambda(\theta) + \theta_2\lambda(\theta_2)\,\Psi(\theta_2) - \theta_1\lambda(\theta_1)\,\Psi(\theta_1)\right](1-F(\theta))\right\}d\theta \\ & -\delta h''(y)\int_{\theta_1}^{\theta_2}\left\{\left[\lambda(\theta) + \beta\lambda(\theta_2)\,\Psi(\theta_2) - \beta\lambda(\theta_1)\,\Psi(\theta_1)\right](1-F(\theta))\right\}d\theta \qquad (23)\end{aligned}$$

where

$$\Psi(\theta_j) = \frac{1 - F(\theta_j)}{F(\theta_2) - F(\theta_1)}, \quad j = 1, 2$$

From expression (23) one can see that the second-order condition hinges on the hazard rate $\lambda(\theta)$. However, unlike in the case with no (potential) secondary market trade (see expression 16) above), the second-order condition is not necessarily satisfied when $\lambda'(\theta) \geq 0$: the terms $\theta_1 \lambda(\theta_1) \Psi(\theta_1)$ and $\beta \lambda(\theta_1) \Psi(\theta_1)$ can potentially be so large (in absolute value) that they make the expression (23) positive. If the second-order condition is not satisfied, the consumer chooses a corner solution: either a large y that never binds ex post consumption choices, or small y that always binds ex post choices (so that the consumer's ex post choices will always depend on the higher price q).

Marginal sin licenses

Next, let us study a system of *marginal* sin licenses, such that $\tau_2 = \tau_1 + d\tau$ and $q = p + d\tau$, where $d\tau$ is (very) small. With marginal sin licenses, the first-order condition (20) takes the form

$$\theta_1 v'(y) - \delta h'(y) - p + \frac{1}{\lambda(\theta_1)} v'(y) = 0 \tag{24}$$

The critical value θ_1 is still given by (5), while the second critical value θ_2 is characterized by

$$d\theta_{21} \equiv \theta_2 - \theta_1 = \frac{d\tau}{v'(y)}$$

Combining this first-order condition with equation (5) yields

$$(1 - \beta)\, \delta h'(y) = \frac{1}{\lambda(\theta_1)} v'(y)$$

Next, with marginal sin licenses, the second-order condition (23) takes the form

$$v''(y) \left[\theta_1 \lambda(\theta_1) + 1 + \frac{f'(\theta_1)\, \theta_1}{f(\theta_1)} \right] (1 - F(\theta_1))\, d\theta_{21}$$
$$-\delta h''(y) \left\{ \left[\lambda(\theta_1) + \beta \frac{f'(\theta_1)}{f(\theta_1)} \right] (1 - F(\theta_1)) \right\} d\theta_{21}$$

The second-order condition holds if (sufficient condition)

$$\lambda(\theta_1) + \frac{f'(\theta_1)}{f(\theta_1)} \geq 0$$

Notice that

$$\lambda'(\theta_1) = \lambda(\theta_1)\left[\lambda(\theta_1) + \frac{f'(\theta_1)}{f(\theta_1)}\right]$$

Hence, the second-order condition holds if the hazard rate is non-decreasing, $\lambda'(\theta) \geq 0$.

A3. Appendix to Section 4 and Proof of Proposition 1

Let us study the mechanism design problem characterized by (8)-(12). From the ex post self's problem (10) we get the first-order incentive constraint

$$\frac{\partial \widehat{V}(\theta, \theta)}{\partial \widehat{\theta}} = 0 \tag{25}$$

or, using (10)

$$[\theta v'(x(\theta)) - \beta\delta h'(x(\theta)) - T'(x(\theta))]\frac{dx}{d\theta} = 0. \tag{26}$$

We also have the second-order incentive constraint

$$\frac{\partial^2 \widehat{V}(\theta, \theta)}{\partial \widehat{\theta}^2} \leq 0. \tag{27}$$

By totally differentiating (25) we get

$$\frac{\partial^2 \widehat{V}(\theta, \theta)}{\partial \widehat{\theta}^2} + \frac{\partial^2 \widehat{V}(\theta, \theta)}{\partial \widehat{\theta}\partial\theta} = 0 \Leftrightarrow \frac{\partial^2 \widehat{V}(\theta, \theta)}{\partial \widehat{\theta}^2} = -\frac{\partial^2 \widehat{V}(\theta, \theta)}{\partial \widehat{\theta}\partial\theta}. \tag{28}$$

Combining (27) and (28) shows that the second-order incentive constraint can also expressed as

$$\frac{\partial^2 \widehat{V}(\theta, \theta)}{\partial \widehat{\theta}\partial\theta} \geq 0 \Leftrightarrow v'(x(\theta))\frac{dx}{d\theta} \geq 0$$

and finally, since $v'(x(\theta)) > 0$, the second-order condition boils down to

$$\frac{dx(\theta)}{d\theta} \geq 0. \tag{29}$$

Quite simply, consumption must be non-decreasing in the short-run self's type, or realization of the preference shock, θ.

To sum up, the first-order incentive constraint is (26), and the second-order incentive constraint is (29).

We also assume that the unit price has to be between the price floor p and the price ceiling q, that is the pricing scheme has to satisfy the constraints (11) and (12).

To sum up, the ex ante self wants to maximize (8) subject to (26), (29), (11) and (12).

Solving the problem. As a first step, we aim to eliminate the pricing schedule $T(x(\theta))$. To do so, let us define

$$\widehat{w}(\theta) = \widehat{V}(\theta, \theta) \tag{30}$$

Then, the first-order incentive condition (26) implies that (this is just the envelope theorem)

$$\frac{d\widehat{w}(\theta)}{d\theta} = \frac{\partial \widehat{V}(\theta, \theta)}{\partial \theta} = v(x(\theta))$$

and

$$\widehat{w}(\theta) = \widehat{w}(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} v\left(x\left(\widetilde{\theta}\right)\right) d\widetilde{\theta}. \tag{31}$$

Then using (10), (30) and (31) gives

$$T(x(\theta)) = \theta v(x(\theta)) - \beta\delta h(x(\theta)) - \int_{\underline{\theta}}^{\theta} v\left(x\left(\widetilde{\theta}\right)\right) d\widetilde{\theta} - \widehat{w}(\underline{\theta}). \tag{32}$$

Alternatively, we could have derived equation (32) by integrating the first-order incentive constraint (26).

Benchmark case 1: The social planner chooses the pricing scheme for the consumers The social planner maximizes

$$W = E_i\left[E_{\theta_i}\left[V_i(\theta_i)\right]\right] = \int_i \int_{\underline{\theta}}^{\overline{\theta}} V_i(\theta_i) f_i(\theta_i) \, d\theta_i di \tag{33}$$

(where the inner expectation is over shock realizations θ_i of consumer i, and the outer expectation is over consumers i) subject to the incentive constraints (26) and (29), the pricing constraints (11) and (12), and the government budget constraint

$$E_i[S_i] = E_i E_{\theta_i}\left[T_i(\theta_i) - p x_i(\theta_i)\right]. \tag{34}$$

Plugging (9) and (34) into (33) shows that the government ends up maximizing

$$W = E_i\left[E_{\theta_i}\left[V_i(\theta_i)\right]\right] = E_i\left[E_{\theta_i}\left[\theta_i v(x(\theta_i)) - \delta h(x(\theta_i)) - p x_i\right]\right] \tag{35}$$

subject to (26) and (29), (11) and (12). Since (in this hypothetical thought experiment) the planner chooses a personalized pricing scheme / mechanism for each consumer, the planner's objective for an individual consumer i is

$$W_i = E_{\theta_i}\left[V_i\left(\theta_i\right)\right] = E_{\theta_i}\left[\theta_i v\left(x\left(\theta_i\right)\right) - \delta h\left(x\left(\theta_i\right)\right) - px_i\right]. \tag{36}$$

Essentially, the government does not care about the redistributive effects of the pricing schemes: one consumer's monetary loss is an other consumer's gain (through the system of subsidies). The planner just wants to implement (subject to constraints (26), (29), (11) and (12)) an allocation that is as close as possible to the (ex ante) first best for each consumer and for preference shock realization.

Benchmark case 2: The consumer chooses the pricing scheme, but there are no redistributive effects Let us consider a hypothetical benchmark case, where pricing schemes $T(x)$ chosen by different consumers have not effects on the expected (re)distribution of income. Hence we assume

$$S = S(T)) = E_\theta\left[T\left(x\left(\theta\right)\right)\right] - pE_\theta\left[x\left(\theta\right)\right] \tag{37}$$

In words, if the consumer chooses to pay a price exceeding the producer price p, he will get a subsidy from the government. Intuitively, the difference $T\left(x\left(\theta\right)\right) - px\left(\theta\right)$ can be thought of as sin taxes collected by the government. The equation (37) then tells that in expectation (or in the long run) the government returns all the taxes to the consumer (however, notice that the subsidy does not depend on realized consumption $x\left(\theta\right)$). Then in expectation the consumer will pay the (net) price

$$E_\theta\left[T\left(x\left(\theta\right)\right)\right] - S\left(T\right) = E_\theta\left[T\left(x\left(\theta\right)\right)\right] - \left(E_\theta\left[T\left(x\left(\theta\right)\right)\right] - pE_\theta\left[x\left(\theta\right)\right]\right) = pE_\theta\left[x\left(\theta\right)\right]$$

or an expected net price p per unit of consumption. Hence, this subsidy scheme cancels out all (re)distributional effects of the payment scheme $T\left(x\left(\theta\right)\right)$. Then only the allocative - or corrective - effects of the scheme $T\left(x\left(\theta\right)\right)$ remain in this benchmark case.

Now, plugging (9) and (37) into (8) shows that, ex ante, the consumer maximizes

$$E_\theta\left[V\left(\theta\right)\right] = E_\theta\left[\theta v\left(x\left(\theta\right)\right) - \delta h\left(x\left(\theta\right)\right) - px\left(\theta\right)\right] \tag{38}$$

The consumer's objective function is therefore exactly the same as the planner's objective function (for a particular consumer i); see expression (36).

Solving benchmark cases 1 and 2 We solve the optimization problem(s) in two steps: In the first we just treat (38), or alternatively and equivalently (36), as an unconstrained maximization problem, with $x(\theta)$ as a (sequence of) choice variable(s). The first-order conditions are

$$\theta v'(x^o(\theta)) - \delta h'(x^o(\theta)) - p = 0 \text{ for all } \theta. \tag{39}$$

In words, with each preference shock realization θ, one would like to choose, and implement, the first-best (ex ante) optimal consumption level $x^o(\theta)$.

In the second step, we check, whether, and to what extent, the first-best optimal solution can be implemented, given the constraints (26), (29), (11) and (12).

First, it is easy to see that the solution $x^o(\theta)$ has the property

$$\frac{dx^o(\theta)}{d\theta} = -\frac{v'(x^o(\theta))}{\theta v''(x^o(\theta)) - \delta h''(x(\theta))} \geq 0$$

Hence, the second-order incentive constraint (29) is satisfied.

Second, combining the first-order optimality constraint (39) and the first-order incentive constraint, we get

$$T^{o\prime}(x^o(\theta)) = \theta v'(x^o(\theta)) - \beta\delta h'(x^o(\theta)) = (1-\beta)\,\delta h'(x^o(\theta)) + p \tag{40}$$

(where T^o is the non-linear pricing scheme, that implements the first-best). It is easy to see that

$$T^{o\prime}(x^o(\theta)) = (1-\beta)\,\delta h'(x^o(\theta)) + p \geq p.$$

Hence, the pricing constraint (11) is satisfied.

Finally, the second pricing constraint (12) is not binding, for preference realization θ, if

$$T'(x^o(\theta)) = (1-\beta)\,\delta h'(x^o(\theta)) + p \leq q$$

while the constraint is binding, if

$$T'(x^o(\theta)) = (1-\beta)\,\delta h'(x^o(\theta)) + p > q.$$

If the constraint (12) is binding, the best one can do is to set (this could be proved more formally by setting a Kuhn-Tucker optimization problem with inequality constraints)

$$T'(x(\theta)) = q \tag{41}$$

so that the realized consumption level is $x(\theta) = x^q(\theta)$, and $x^q(\theta) (= x^*(\theta, \beta; q))$ is implicitly given by

$$\theta v'(x^q(\theta)) - \beta\delta h'(x^q(\theta)) - q = 0. \tag{42}$$

To sum up, realized consumption is given by

$$x(\theta) = \max\{x^o(\theta), x^q(\theta)\}$$

(where $x^o(\theta)$ is implicitly given by (39) and $x^q(\theta)$ is implicitly given by (42) and the pricing scheme is given by

$$T'(x(\theta)) = \min\{(1-\beta)\delta h'(x^o(\theta)) + p, q\}.$$

Furthermore, since, i) $x^o(\theta)$ is increasing in θ, ii) and $h'(x)$ is increasing in x, it is easy to see that the marginal price $T'(x^o(\theta))$ is increasing in θ. Hence the constraint (12) is not binding for low shock realizations of $\theta < \theta^q$, and it is binding for high shock realizations $\theta > \theta^q$, where the critical value θ^q is implicitly defined by

$$T^{o\prime}(x^o(\theta^q)) = q \Leftrightarrow (1-\beta)\delta h'(x^o(\theta)) = q - p. \tag{43}$$

Then the results so far can be re-expressed as follows: Realized consumption is

$$x(\theta) = \begin{cases} x^o(\theta) & \text{for } \theta < \theta^q \\ x^q(\theta) & \text{for } \theta \geq \theta^q \end{cases} \tag{44}$$

and the pricing scheme is

$$T'(x(\theta)) = \begin{cases} (1-\beta)\delta h'(x^o(\theta)) & \text{for } \theta < \theta^q \\ q & \text{for } \theta \geq \theta^q \end{cases}.$$

Finally, it is worth noting that the pricing scheme can be presented with no explicit reference to shock realizations θ

$$T'(x) = \begin{cases} (1-\beta)\delta h'(x) & \text{for } x < x^q \\ q & \text{for } x > x^q \end{cases} \tag{45}$$

where x^q is implicitly defined by

$$(1-\beta)\,\delta h'(x^q) = q.$$

Hence, if the social planner wants to implement the allocation (44) for a particular consumer i, it needs information on i) the consumer's self-control problems (the consumer's β), and the consumer's harm function ($h'(x)$). The planner does *not* have know what kind of preference uncertainty the consumer faces.

A final key finding is that if secondary market trade is prohibitively expensive (q is very high, and the pricing constraint (12) is never binding), the non-linear personalized pricing scheme implements the first-best outcome in the benchmark case.

Main case: both allocative and redistributive effects (Proof of Proposition 1) Let us proceed to analyzing a more realistic case, where the redistributive effects of non-linear pricing are not cancelled out by personalized subsidies. Here we simply assume that all consumers get the same lump-sum subsidy

$$S = E_i\left[E_\theta\left[x_i(\theta)\right]\right]$$

where expectations are taken over consumers i, as well as over preference shock realizations θ.

As a first step, we eliminate $T(x(\theta))$ from the ex ante self's objective: plugging (32) into (9) yields

$$V(\theta) = \int_{\underline{\theta}}^{\theta} v\left(x\left(\widetilde{\theta}\right)\right) d\widetilde{\theta} - (1-\beta)\,\delta h(x(\theta)) + \widehat{w}(\underline{\theta})$$

and

$$\begin{aligned} E_\theta\left[V(\theta)\right] &= \int_{\underline{\theta}}^{\overline{\theta}} \left[\int_{\underline{\theta}}^{\theta} v\left(x\left(\widetilde{\theta}\right)\right) d\widetilde{\theta} - (1-\beta)\,\delta h(x(\theta)) + \widehat{w}(\underline{\theta})\right] f(\theta)\,d\theta \\ &= \int_{\underline{\theta}}^{\overline{\theta}} \left[v(x(\theta))\frac{1}{\lambda(\theta)} - (1-\beta)\,\delta h(x(\theta))\right] f(\theta)\,d\theta + \widehat{w}(\underline{\theta}) \end{aligned} \tag{46}$$

where

$$\lambda(\theta) = \frac{f(\theta)}{1-F(\theta)}$$

is the hazard rate. The second form of (46) follows since (integration by parts)

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} v\left(x\left(\widetilde{\theta}\right)\right) d\widetilde{\theta} f(\theta)\, d\theta = \int_{\underline{\theta}}^{\overline{\theta}} v(x(\theta))(1 - F(\theta))\, d\theta = \int_{\underline{\theta}}^{\overline{\theta}} v(x(\theta)) \frac{1}{\lambda(\theta)} f(\theta)\, d\theta$$

Now, the objective function (46) consists of two parts: the integral

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[v(x(\theta)) \frac{1}{\lambda(\theta)} - (1-\beta)\,\delta h(x(\theta)) \right] f(\theta)\, d\theta$$

over all types, and the term

$$\widehat{w}(\underline{\theta}) = \underline{\theta} v(x(\underline{\theta})) - \beta\delta h(x(\underline{\theta})) - T(x(\underline{\theta})) \tag{47}$$

that involves the utility of the *ex post* self with the lowest possible preference shock realization $\underline{\theta}$. Tackling the second part is easy: $\widehat{w}(\underline{\theta})$ is maximized if minimize the payments $T(x(\underline{\theta}))$. Given the constraint (11) this then implies $T'(x) = p$ for all $x \leq x(\underline{\theta})$. (Notice: Here we assume that there is no mass point at $\theta = \underline{\theta}$. If there is a mass point, also the first term in (46), the integral, affects the optimal pricing schedule at the lower lower boundary $\theta = \underline{\theta}$.)

We still want to eliminate (the derivatives of) the price schedule from the constraints (11) and (12). We first conjecture that when the constraint (11) or the constraint (12) holds, we have $\frac{dx}{d\theta} > 0$ (below we will verify the conjecture). Now the incentive constraint (26) implies

$$T'(x(\theta)) = \theta v'(x(\theta)) - \beta\delta h'(x(\theta))$$

Then plugging this equation into (11) yields

$$\theta v'(x(\theta)) - \beta\delta h'(x(\theta)) - p \geq 0 \tag{48}$$

and into (12) yields

$$\theta v'(x(\theta)) - \beta\delta h'(x(\theta)) - q \leq 0 \tag{49}$$

Totally differentiating (48) or (49) reveals that (when either of the constraints binds)

$$\frac{dx}{d\theta} = \frac{v'(x(\theta))}{\beta\delta h''(x(\theta)) - \theta v''(x(\theta))} > 0$$

and our conjecture was indeed correct. Hence if either (48) or (49) binds, the second-order incentive constraint (29) does not bind: we can use the equations (48) and (49) to characterize the ranges in the type (θ) space where the constraints on the curvature of the price schedule curvature (or constraints on unit price) apply.

Finally notice that if (48) binds, (49) does not bind: plugging $\theta v'(x(\theta)) - \beta\delta h'(x(\theta)) = p$ into (49) yields $p - q < 0$. Likewise, if (49) binds, (48) does not bind: plugging $\theta v'(x(\theta)) - \beta\delta h'(x(\theta)) = q$ into (48) yields $q - p > 0$.

Maximization problem. Now we want to maximize

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[v(x(\theta)) \frac{1}{\lambda(\theta)} - (1-\beta)\delta h(x(\theta)) \right] f(\theta)\, d\theta \tag{50}$$

subject to the second-order incentive constraint (29), and the (modified) pricing constraints (48) and (49). We also know that at each point θ in the type space, at most one of these constraints binds.

We conduct our analysis in three steps. In the first step, we solve the problem (50) disregarding the constraints. In the second step, we check weather the candidate solution satisfies the second-order incentive condition ($\frac{dx}{d\theta} \geq 0$). If this is the case, the consumer's choice is characterized by a combination of the candidate solution, and the pricing constraints (48) and (49). If candidate solution does not satisfy the second-order incentive constraint (29), we move to step three, and solve problem, taking on board the constraint $\frac{dx}{d\theta} \geq 0$.

Step 1

We want to maximize (50), taking $x(\theta)$ as the (sequence of) control variable(s).The first-order conditions are of the form

$$v'(x^u(\theta)) \frac{1}{\lambda(\theta)} - (1-\beta)\delta h'(x^u(\theta)) = 0 \tag{51}$$

The expression (51) gives our unconstrained candidate solution $x^u(\theta)$. (Since (50) is concave in $x(\theta)$, the second-order condition is satisfied.)

Step 2

Next we check whether or not the candidate solution (51) satisfies the constraints (29), (48) and (49). We begin with second-order incentive constraint $\frac{dx}{d\theta} \geq 0$. From (51) we get

$$\frac{v'(x(\theta))}{(1-\beta)\delta h'(x(\theta))} = \lambda(\theta) \tag{52}$$

and differentiating (52) gives

$$\left(\frac{(1-\beta)\,\delta h'\left(x\left(\theta\right)\right) v''\left(x\left(\theta\right)\right)-v'\left(x\left(\theta\right)\right)(1-\beta)\,\delta h''\left(x\left(\theta\right)\right)}{\left[(1-\beta)\,\delta h'\left(x\left(\theta\right)\right)\right]^{2}}\right)\frac{dx\left(\theta\right)}{d\theta}=\lambda'\left(\theta\right) \tag{53}$$

From (53) one can easily see that

$$\frac{dx\left(\theta\right)}{d\theta}\geq 0 \text{ iff } \lambda'\left(\theta\right)\leq 0 \tag{54}$$

In words, for (51) to be a valid solution, the hazard rate $\lambda\left(\theta\right)$ has to be non-increasing. Also notice that for there to be a separating equilibrium, with each type $\theta\in[\theta_1,\theta_2]$ having a different consumption level, the hazard rate has to be decreasing $\frac{d\lambda(\theta)}{d\theta}<0$. If the hazard rate is constant $\lambda'\left(\theta\right)=0$, there is bunching in equilibrium, with all types $\theta\in[\theta_1,\theta_2]$ getting the same level of sin good consumption.

Solution, when the second-order incentive constraint (29) is satisfied

When $\lambda'\left(\theta\right)\leq 0$, the second-order incentive constraint (*29)* is satisfied. However, we still have to check the pricing constraints (48) and (49). Using the candidate solution (51) and the first-order incentive constraint (26) we can see that

$$T^{u\prime}\left(x^{u}\left(\theta\right)\right)=\theta v'\left(x^{u}\left(\theta\right)\right)-\beta\delta h'\left(x^{u}\left(\theta\right)\right)=\left[(1-\beta)\,\theta\lambda\left(\theta\right)-\beta\right]\delta h'\left(x^{u}\left(\theta\right)\right) \tag{55}$$

is the slope of the pricing scheme that implements the candidate solution. Now if $p\leq T'\left(x^{u}\left(\theta\right)\right)\leq q$ the candidate solution is indeed valid; otherwise one of the constraints (48) and (49) binds.

To sum up, the solution to the consumer's (ex ante) maximization problem is given by the consumption scheme

$$x\left(\theta\right)=\begin{cases} x^{*}\left(\theta,\beta\right) & \text{if } T^{u\prime}\left(x^{u}\left(\theta\right)\right)<p \\ x^{u}\left(\theta\right) & \text{if } p\leq T^{u\prime}\left(x^{u}\left(\theta\right)\right)\leq q \\ x^{**}\left(\theta,\beta\right) & \text{if } T^{u\prime}\left(x^{u}\left(\theta\right)\right)>q \end{cases} \tag{56}$$

and the pricing scheme

$$T'\left(x\left(\theta\right)\right)=\begin{cases} p & \text{if } T^{u\prime}\left(x^{u}\left(\theta\right)\right)<p \\ T^{u\prime}\left(x^{u}\left(\theta\right)\right) & \text{if } p\leq T^{u\prime}\left(x^{u}\left(\theta\right)\right)\leq q \\ q & \text{if } T^{u\prime}\left(x^{u}\left(\theta\right)\right)>q \end{cases} \tag{57}$$

Since the marginal price $T^{u\prime}\left(x^{u}\left(\theta\right)\right)$, given by (55), varies with consumption, the solution (56), (57) *cannot* be typically implemented with a system of sin licenses, but a more general non-linear personalized pricing scheme is needed.

Hence, if $\lambda'(\theta) < 0$, the consumer would not typically choose the system of sin licenses among all non-linear personalized pricing schemes. Exceptions to this rule arise in two special cases: i) $T^{u\prime}(x^u(\theta)) \leq p$ for all θ, or ii) $T^{u\prime}(x^u(\theta)) \geq q$ for all θ. In these special cases, the solution (56), (57) can be implemented with sin licenses: i) If $T^{u\prime}(x^u(\theta)) \leq p$ for all θ, the consumer chooses a sin license quota $y \geq x^*\left(\overline{\theta}, \beta\right)$, which always allows him to buy his entire consumption at the minimum price, p. In particular, one can clearly see that for a perfectly rational consumer, $(\beta = 1)$, $T^{u\prime}(x^u(\theta)) = -\delta h'(x^u(\theta)) < 0 < p$ for all θ. Hence, sin licenses with a quota $y \geq x^*\left(\overline{\theta}, \beta = 1\right) = x^o\left(\overline{\theta}, \beta = 1\right)$ implement the best possible allocation and pricing scheme for a perfectly rational consumer (who does not value commitment and wants to minimize monetary costs). ii) If $T^{u\prime}(x^u(\theta)) \geq q$ for all θ, the consumer can implement the solution (56), (57) with sin licenses, by choosing a (small) quota $y = x^{**}(\underline{\theta}, \beta)$, so that the consumer's ex post choice will be always based on the maximum (unit) price q, making realized consumption as small as possible. For example (sophisticated) consumers with severe self-control problems (β close to 0) may want to implement this maximum feasible commitment, minimum feasible consumption, mechanism. Clearly, the special case $T^{u\prime}(x^u(\theta)) \geq q$ for all θ can only arise, if $q < \infty$.

Step 3

If the hazard rate is increasing

$$\lambda'(\theta) > 0$$

the condition (54) is not satisfied. Then (51) is not a valid solution, and we have to take explicitly into account the second-order incentive constraint $\frac{dx(\theta)}{d\theta} \geq 0$. Next, we show that under these circumstances the consumer's preferred allocation and pricing scheme can be implemented with a system of sin licenses.

Now, the ex ante consumer maximizes (50), subject to (29), (48) and (49). We also know that at each point θ in the type space, exactly one of these constraints binds. Then the ex ante consumer's optimal solution must take one of the following forms:

i) The ex ante consumer's optimal solution is such that the constraint (48) binds for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. The consumer can implement this solution with sin licenses, by choosing a large quota $y \geq x^*\left(\overline{\theta}, \beta; p\right)$ which always allows him to buy his entire consumption at the minimum price, p.

ii) The constraint (49) binds for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. The consumer can implement this maximum-commitment solution with sin licenses, by choosing a small quota $y =$

$x^{**}(\underline{\theta}, \beta; q)$, so that the consumer's ex post choice will be always based on the maximum (unit) price q, making realized consumption as small as possible.

iii) a)The constraint (48) binds for small preference shock realizations $\theta < \theta_1$,b) (29) binds for intermediate realizations $\theta \in [\theta_1, \theta_2]$ and c) (49) binds for large realizations $\theta > \theta_2$, where θ_1 and θ_2 are (endogenous) threshold values and $\theta_2 > \theta_1$. The above-stated order of the three regimes a), b) and c) can be easily inferred by noting that consumption must be non-decreasing in the preference shock $\frac{dx(\theta)}{d\theta} \geq 0$, and by inspecting the constraints (48) and (49): The constraint (48) is binding for low levels of the preference shock and low levels of consumption; the constraint (49) is binding for high levels of the preference shock and high levels of consumption; the regimes where the constraints (48) and (49) bind do not overlap (that is, $\theta_1 < \theta_2$), and between these two regimes (a and c), the third constraint (29) binds. From these same properties, one can also conclude that each regime a), b) and c) can occur (at most) once, on a single subinterval of the type (θ) space. (Notice, however, that it is possible that $\theta_1 < \underline{\theta}$ or/and $\theta_2 > \overline{\theta}$. Then (48) or/and (49) is/are not effectively binding.) The consumer can implement this solution with sin licenses, by choosing an intermediate-sized quota $x^{**}(\underline{\theta}, \beta; q) < y < x^{*}\left(\overline{\theta}, \beta; p\right)$.

Alternatives i) and ii) are corner solutions: they involve either maximum flexibility, minimum monetary costs and no commitment (i), or maximum feasible commitment (ii). These corner solutions are implemented by choosing either a (very) large sin license quota (i) or a (very) small quota (ii).

Alternative iii) is an interior solution. It involves maximum flexibility and minimum monetary costs at low values of the preference shock and low levels of consumption (a), but maximum feasible commitment at high values of the preference shock and high consumption levels (c). In addition, it involves a constant level of consumption (y) for intermediate preference shock realizations (b). Next we shall study more carefully, how the sin license quota y is determined in a putative interior solution. In particular, we are going to show that the first-order conditions characterizing the choice y are the same as those we already encountered above, when we studied the system of sin licenses.

We proceed as follows. We adopt

$$g(\theta) = \frac{dx(\theta)}{d\theta}$$

as the control variable, while $x(\theta)$ is the state variable. We then use standard methods of optimal control. The *Lagrangian* associated with the problem takes the form

$$\mathcal{L} = H + \eta_1(\theta)\left[\theta v'(x) - \beta\delta h'(x(\theta)) - p\right] - \eta_2(\theta)\left[\theta v'(x) - \beta\delta h'(x(\theta)) - q\right]$$

where H is the *Hamiltonian*

$$H = \left[v(x(\theta))\frac{1}{\lambda(\theta)} - (1-\beta)\delta h(x(\theta))\right] f(\theta) + \mu(\theta) g(\theta)$$

and $\mu(\theta)$, $\eta_1(\theta)$ and $\eta_2(\theta)$ are Lagrange multipliers associated with the constraints (29), (48), and (49), respectively.

Optimality conditions from this exercise are the following:

i) The first-order condition with respect to the control variable g takes the form

$$\begin{array}{c} \mu(\theta) = 0 \text{ if } g(\theta) > 0 \\ \mu(\theta) < 0 \text{ if } g(\theta) = 0 \end{array} \tag{58}$$

ii) The law of motion of the Lagrangian multiplier $\mu(\theta)$ is

$$\frac{d\mu(\theta)}{d\theta} = -H_x = \left[-v'(x(\theta))\frac{1}{\lambda(\theta)} + (1-\beta)\delta h'(x(\theta))\right] f(\theta), \text{ if } g(\theta) = 0 \tag{59}$$

iii) The Kuhn-Tucker condition associated with the constraint (48)

$$\begin{aligned} \eta_1(\theta) &= 0 \text{ if } \theta v'(x) - \beta\delta h'(x(\theta)) - p > 0 \\ \eta_1(\theta) &> 0 \text{ if } \theta v'(x(\theta)) - \beta\delta h'(x(\theta)) - p = 0 \end{aligned} \tag{60}$$

iv) The Kuhn-Tucker condition associated with the constraint (49)

$$\begin{aligned} \eta_2(\theta) &= 0 \text{ if } \theta v'(x) - \beta\delta h'(x(\theta)) - q < 0 \\ \eta_2(\theta) &> 0 \text{ if } \theta v'(x) - \beta\delta h'(x(\theta)) - q = 0 \end{aligned} \tag{61}$$

The putative solution boils down to

$$x(\theta) = \begin{cases} x^*(\theta, \beta; p) & \text{for } \theta < \theta_1 \\ y & \text{for } \theta_1 \leq \theta \leq \theta_2 \\ x^{**}(\theta, \beta; q) & \text{for } \theta > \theta_2 \end{cases}$$

where y, θ_1 and θ_2 are determined in the following way: First, since the first-order conditions (58) imply that

$$\mu(\theta_1) = \mu(\theta_2) = 0$$

we must have

$$\int_{\theta_1}^{\theta_2} \frac{d\mu(\theta)}{d\theta} d\theta = 0$$

Then using (59), we get the condition

$$\int_{\theta_1(y;\beta)}^{\theta_2(y;\beta)} \left[(1-\beta)\,\delta h'(y) - v'(y)\,\frac{1}{\lambda(\theta)}\right] f(\theta)\, d\theta = 0 \tag{62}$$

Finally, at the lower boundary θ_1 the pricing constraint (60) holds as an equality, while at the upper boundary θ_2 the pricing constraint (61) holds as an equality. Hence we have

$$\begin{aligned} \theta_1(y;\beta)\, v'(y) - \beta\delta h'(y) - p &= 0 && (63) \\ \theta_2(y;\beta)\, v'(y) - \beta\delta h'(y) - q &= 0 && (64) \end{aligned}$$

Notice that since $q > p$, the equations (63) and (64) imply that $\theta_2 > \theta_1$, as claimed.

To sum up, y, θ_1 and θ_2 are determined by (62), (63) and (64). Two comments are in order. First, it is possible that $\theta_1 < \underline{\theta}$ or/and $\theta_2 > \overline{\theta}$. Then the constraint (48) or/and the constraint (49) is/are not effectively binding for the consumer.

Second, the equations (62), (63) and (64) are *necessary* conditions that characterize a *putative* interior solution. In particular, below we shall show that (62) boils down to the first-order condition (x), characterizing an interior solution with sin licenses (while (63) and (64) are identical to (y) and (z)). As discussed above in Appendix x, the second-order condition associated with a putative interior solution does not necessarily hold, when secondary market trade is possible and q is not very large (see expression w). To put it in another way, one should remember that the consumer's optimal choice does not necessarily involve an interior solution (alternative iii) in the list presented above), but it can be a corner solution (alternative i) or ii) in the list).

Interpreting the condition (62)

Next we shall show how to interpret the condition (62). This will also allow us to better link the analysis of the non-linear mechanism to our earlier analysis of sin licenses.

First we analyze a special case with $\theta_2 = \overline{\theta}$. Essentially, there are no secondary markets, or the consumer is never tempted by secondary markets. Remember that

$\lambda(\theta) = \frac{f(\theta)}{1-F(\theta)}$. The condition (62) can be re-expressed as

$$\begin{aligned} v'(y)\int_{\theta_1}^{\overline{\theta}}(1-F(\theta))\,d\theta-(1-\beta)\,\delta h'(y)\,(1-F(\theta_1)) &= 0 \Leftrightarrow \\ v'(y)\left[\overline{\theta}-\theta_1-\int_{\theta_1}^{\overline{\theta}}F(\theta)\,d\theta\right]-(1-\beta)\,\delta h'(y)\,(1-F(\theta_1)) &= 0 \qquad (65) \end{aligned}$$

Next notice that since $\int_{\theta_1}^{\overline{\theta}}\theta f(\theta)\,d\theta = \overline{\theta}-\theta_1 F(\theta_1)-\int_{\theta_1}^{\overline{\theta}}F(\theta)\,d\theta$, we have $-\int_{\theta_1}^{\overline{\theta}}F(\theta)\,d\theta = \int_{\theta_1}^{\overline{\theta}}\theta f(\theta)\,d\theta-\overline{\theta}+\theta_1 F(\theta_1)$. Plugging this into (65) yields

$$\begin{aligned} v'(y)\left[\int_{\theta_1}^{\overline{\theta}}\theta f(\theta)\,d\theta-\theta_1(1-F(\theta_1))\right]-(1-\beta)\,\delta h'(y)\,(1-F(\theta_1)) &= 0 \Leftrightarrow \\ v'(y)\,[E[\theta \mid \theta \geq \theta_1]-\theta_1]-(1-\beta)\,\delta h'(y) &= 0 \qquad (66) \end{aligned}$$

(notice that $E[\theta \mid \theta \geq \theta_1] = \frac{\int_{\theta_1(y;\beta)}^{\overline{\theta}}\theta f(\theta)d\theta}{1-F(\theta_1)}$). Next, from (63) we get

$$-\theta_1 v'(y)+\beta\delta h'(y) = -p$$

and plugging this into (66) yields

$$E[\theta \mid \theta \geq \theta_1(y;\beta)]\,v'(y)-\delta h'(y)-p = 0$$

But this is just the condition (14) characterizing the optimal choice of sin licenses, that we derived above in Appendix A1.

Interpreting the result (62) in the more general case with $\theta_2 < \overline{\theta}$.

The condition (62) can be re-expressed as

$$\begin{aligned} v'(y)\int_{\theta_1}^{\theta_2}(1-F(\theta))\,d\theta-(1-\beta)\,\delta h'(y)\,(F(\theta_2)-F(\theta_1)) &= 0 \Leftrightarrow \\ v'(y)\left[\theta_2-\theta_1-\int_{\theta_1}^{\theta_2}F(\theta)\,d\theta\right]-(1-\beta)\,\delta h'(y)\,(F(\theta_2)-F(\theta_1)) &= 0 \qquad (67) \end{aligned}$$

Next notice that since $\int_{\theta_1}^{\theta_2}\theta f(\theta)\,d\theta = \theta_2 F(\theta_2)-\theta_1 F(\theta_1)-\int_{\theta_1}^{\theta_2}F(\theta)\,d\theta$, we have $-\int_{\theta_1}^{\theta_2}F(\theta)\,d\theta =$

$\int_{\theta_1}^{\theta_2} \theta f(\theta)\, d\theta - \theta_2 F(\theta_2) + \theta_1 F(\theta_1)$ Plugging this into (67) yields

$$v'(y)\left[\int_{\theta_1}^{\theta_2} \theta f(\theta)\, d\theta + \theta_2 (1 - F(\theta_2)) - \theta_1 (1 - F(\theta_1))\right] - (1-\beta)\,\delta h'(y)\,(F(\theta_2) - F(\theta_1)) = 0 \Leftrightarrow$$
$$(F(\theta_2) - F(\theta_1))\{\{E[\theta \mid \theta_1 \le \theta \le \theta_2] - \theta_1\}\, v'(y) - (1-\beta)\,\delta h'(y)\}$$
$$+ (1 - F(\theta_2))(\theta_2 - \theta_1)\, v'(y) = 0 \tag{68}$$

Next, from (63) we get

$$-\theta_1 v'(y) + \beta\delta h'(y) = -p$$

while (63) and (64) together yield

$$(\theta_2 - \theta_1)\, v'(y) = q - p$$

Plugging these results into (68) gives

$$[F(\theta_2(y;\beta)) - F(\theta_1(y;\beta))]\,[E[\theta \mid \theta_2(y;\beta) \ge \theta \ge \theta_1(y;\beta)]\, v'(y) - \delta h'(y) - p]$$
$$+ [1 - F(\theta_2(y;\beta))]\,(q - p) = 0$$

But this is just the first-order condition (20) characterizing the choice of sin licenses, when there is potential secondary market trade. Finally, remember that (20) is a necessary condition for an interior optimum, but not a sufficient condition; in particular the second-order condition may not hold.

A4. Proof of Proposition 2

(i) Marginal sin tax on top of sin licenses. Remember that, from the social point of view, consumer i's welfare is given by

$$\begin{aligned} W_i(p,q) \; = \; & \int_{\underline{\theta}}^{\theta_1(y;\beta,p,q)} [\theta v(x^*(\theta,\beta;p)) - \delta h(x^*(\theta,\beta;p)) - x^*(\theta,\beta;p)]\, dF(\theta) \\ & + \int_{\theta_1(y;\beta,p,q)}^{\theta_2(y;\beta,p,q)} [\theta v(y) - \delta h(y) - x(y)]\, dF(\theta) \\ & + \int_{\theta_2(y;\beta,p,q)}^{\overline{\theta}} [\theta v(x^{**}(\theta,\beta;q)) - \delta h(x^{**}(\theta,\beta;q)) - x^{**}(\theta,\beta;q)]\, dF(\theta). \end{aligned}$$

Assume that initially $\tau_1 = 0$, so that $p = 1$ (with sin licenses the consumer can buy the sin good at the producer price), and that $0 < \tau_2 \le k$ (and $q = 1 + \tau_2$). Let us now analyze what happens to consumer welfare, when p is raised by a small amount $d\tau_1$ (but q remains constant).

$$\begin{aligned}
\frac{\partial W_i(p,q)}{\partial \tau} &= \frac{\partial W_i(p,q)}{\partial p} = \int_{\underline{\theta}}^{\theta_1(y;\beta,p,q)} \left[\theta v'\left(x^*(\theta,\beta;p)\right) - \delta h'\left(x^*(\theta,\beta;p)\right) - 1\right] \frac{\partial x^*}{\partial p} dF(\theta) \\
&\quad + \int_{\theta_1(y;\beta,p,q)}^{\theta_2(y;\beta,p,q)} \left[\theta v'(y) - \delta h'(y) - 1\right] \frac{\partial y}{\partial p} dF(\theta) \\
&= -\int_{\underline{\theta}}^{\theta_1(y;\beta,p,q)} \left[(1-\beta)\,\delta h'\left(x^*(\theta,\beta;p)\right)\right] \frac{dx^*}{\partial p} dF(\theta) \\
&\quad + \left[F(\theta_2) - F(\theta_1)\right]\left\{E\left[\theta \mid \theta_1 \le \theta \le \theta_2\right] v'(y) - \delta h'(y) - 1\right\} \frac{dy}{dp}
\end{aligned}$$

Next, using the consumer's first-order condition, related to the choice of the sin license quota y,

$$\begin{gathered}
\left[F(\theta_2(y;\beta)) - F(\theta_1(y;\beta))\right]\left\{E\left[\theta \mid \theta_2(y;\beta) \ge \theta \ge \theta_1(y;\beta)\right] v'(y) - \delta h'(y) - 1\right\} \\
+ \left[1 - F(\theta_2(y;\beta))\right](q-1) = 0
\end{gathered}$$

we get

$$\begin{aligned}
& \left[F(\theta_2) - F(\theta_1)\right]\left\{E\left[\theta \mid \theta_1 \le \theta \le \theta_2\right] v'(y) - \delta h'(y) - 1\right\} \\
= & -\left[1 - F(\theta_2(y;\beta))\right]\tau_2
\end{aligned}$$

Hence,

$$\frac{\partial W_i(p,q)}{\partial p} = -\int_{\underline{\theta}}^{\theta_1(y;\beta,p,q)} \left[(1-\beta)\,\delta h'\left(x^*(\theta,\beta;p)\right)\right] \frac{\partial x^*}{\partial p} dF(\theta) - \left[1 - F(\theta_2(y;\beta))\right] \frac{dy}{dp}\tau_2 \tag{69}$$

Next, we need to analyze $\frac{dy}{dp}$. Totally differentiating equations (5), (7) and (20) yields

$$A \begin{bmatrix} d\theta_1 \\ d\theta_2 \\ dy \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ (1 - F(\theta_1)) \end{bmatrix} dp$$

where

$$A = \begin{bmatrix} v'(y) & 0 & \theta_1 v''(y) - \beta\delta h''(y) \\ 0 & v'(y) & \theta_2 v''(y) - \beta\delta h''(y) \\ (1-\beta)\,\delta h'(y)\, f(\theta_1) & \left[\tau_2 - (1-\beta)\,\delta h'(y)\right] f(\theta_2) & \left[F(\theta_2) - F(\theta_1)\right]\left[E\left[\theta \mid \theta_2 \ge \theta \ge \theta_1\right] v''(y) - \delta\right. \end{bmatrix}$$

For (θ_1, θ_2, y) to be a maximum, we must have $|A| < 0$.

Next, use Cramer's rule $\frac{dy}{dp} = \frac{|A_p|}{|A|}$ where

$$A_p = \begin{bmatrix} v'(y) & 0 & 1 \\ 0 & v'(y) & 0 \\ (1-\beta)\,\delta h'(y)\, f(\theta_1) & [\tau_2 - (1-\beta)\,\delta h'(y)]\, f(\theta_2) & (1 - F(\theta_1)) \end{bmatrix}$$

so that

$$\begin{aligned} |A_p| &= [v'(y)]^2 (1 - F(\theta_1)) - v'(y)(1-\beta)\,\delta h'(y)\, f(\theta_1) \\ &= v'(y)\, f(\theta_1) \left[\frac{1}{\lambda(\theta_1)} v'(y) - (1-\beta)\,\delta h'(y) \right] \end{aligned}$$

where $\frac{1}{\lambda(\theta_1)} v'(y) - (1-\beta)\,\delta h'(y) > 0$, by the first-order optimality conditions from the consumer's problem. Hence,

$$\frac{dy}{dp} = \frac{|A_p|}{|A|} < 0$$

and from (69) we can conclude that $\frac{\partial W_i(p,q)}{\partial p} > 0$. Hence, introducing a small sin tax on top of sin licenses improves welfare.

(ii) *Marginal sin licenses on top of a sin tax.* Assume that a sin tax τ_1 is in place, and consumer i's consumption is given by $x^*(p; \theta_i, \beta_i)$, where $p = 1 + \tau_1$. From the social point of view (i.e. disregarding any distributional effects), the consumer's welfare is given by

$$\widehat{W}_i = \int_{\underline{\theta}}^{\overline{\theta}} [\theta v(x^*(\theta;\beta)) - \delta h(x^*(\theta;\beta)) - x^*(\theta;\beta)]\, dF(\theta)$$

Next assume that a system of sin licenses is introduced on top of the sin tax: with a license the consumer can buy sin goods at per unit price p, while without the license, the per unit price is $q = 1 + \tau_2 > p$. Then, from the social point of view, consumer i's welfare is given by

$$\begin{aligned} W_i &= \int_{\underline{\theta}}^{\theta_1(y;\beta,p,q)} [\theta v(x^*(\theta;\beta)) - \delta h(x^*(\theta;\beta)) - x^*(\theta;\beta)]\, dF(\theta) \\ &+ \int_{\theta_1(y;\beta,p,q)}^{\theta_2(y;\beta,p,q)} [\theta v(y) - \delta h(y) - x(y)]\, dF(\theta) \\ &+ \int_{\theta_2(y;\beta,p,q)}^{\overline{\theta}} [\theta v(x^{**}(\theta;\beta)) - \delta h(x^{**}(\theta;\beta)) - x^{**}(\theta;\beta)]\, dF(\theta) \end{aligned}$$

Then from the social point of view, the introduction of sin licenses has the welfare effect equal to

$$\begin{aligned}\Delta W_i &= \int_{\theta_1(y;\beta,p,q)}^{\theta_2(y;\beta,p,q)} \left[\theta v(y) - \delta h(y) - y - \left[\theta v\left(x^*(\theta,\beta;p)\right) - \delta h\left(x^*(\theta,\beta;p)\right) - x^*(\theta,\beta;p)\right]\right] dF(\theta) \\ &\quad + \int_{\theta_2(y;\beta,p,q)}^{\overline{\theta}} \left\{\theta v\left(x^{**}(\theta,\beta;q)\right) - \delta h\left(x^{**}(\theta,\beta;q)\right) - x^{**}(\theta,\beta;q)\right. \\ &\quad \left. - \left[\theta v\left(x^*(\theta,\beta;p)\right) - \delta h\left(x^*(\theta,\beta;p)\right) - x^*(\theta,\beta;p)\right]\right\} dF(\theta)\end{aligned}$$

Next, let us study a system of *marginal* sin licenses, such that $\tau_2 = \tau_1 + d\tau$ and $q = p + d\tau$, where $d\tau$ is (very) small. Then $\theta_1,\ \theta_2$ and y are given by

$$d\theta_{21} \equiv \theta_2 - \theta_1 = \frac{q-p}{v'(y_i)} = \frac{d\tau}{v'(y_i)} \tag{70}$$

$$\left[\theta_{1,i} + \frac{1}{\lambda(\theta_{1,i})}\right] v'(y_i) - \delta h'(y_i) - p = 0 \tag{71}$$

$$\theta_{1,i} v'(y_i) - \beta_i \delta h'(y_i) - p = 0 \tag{72}$$

Combining (71) and (72) we get.

$$\frac{1}{\lambda_i(\theta_{1,i})} v(y_i) - (1-\beta_i)\,\delta h'(y_i) = 0 \tag{73}$$

$$\widehat{A}\begin{bmatrix} d\theta_1 \\ dy \end{bmatrix} = \begin{bmatrix} \delta h'(y_i) \\ -\delta h'(y_i) \end{bmatrix} d\beta$$

where

$$\widehat{A} = \begin{bmatrix} v'(y_i) & \theta_{1,i} v''(y_i) - \beta_i \delta h''(y_i) \\ -\frac{\lambda_i'(\theta_{1,i})}{[\lambda_i(\theta_{1,i})]^2} & \frac{1}{\lambda_i(\theta_{1,i})} v''(y_i) - (1-\beta_i)\,\delta h''(y_i) \end{bmatrix}$$

and

$$\left|\widehat{A}\right| = v'(y_i)\left[\frac{1}{\lambda_i(\theta_{1,i})} v''(y_i) - (1-\beta_i)\,\delta h''(y_i)\right] + \frac{\lambda_i'(\theta_{1,i})}{[\lambda_i(\theta_{1,i})]^2}\left(\theta_{1,i} v''(y_i) - \beta_i \delta h''(y_i)\right) < 0$$

On the other hand

$$\widehat{A}_{\theta_1} = \begin{bmatrix} \delta h'(y_i) & \theta_{1,i} v''(y_i) - \beta_i \delta h''(y_i) \\ -\delta h'(y_i) & \frac{1}{\lambda_i(\theta_{1,i})} v''(y_i) - (1-\beta_i)\,\delta h''(y_i) \end{bmatrix}$$

$$\widehat{A}_y = \begin{bmatrix} v'(y_i) & \delta h'(y_i) \\ -\frac{\lambda_i'(\theta_{1,i})}{[\lambda_i(\theta_{1,i})]^2} & -\delta h'(y_i) \end{bmatrix}$$

so that

$$\begin{aligned} \left|\widehat{A}_{\theta_1}\right| &= \delta h'(y_i)\left[\left(\frac{1}{\lambda_i(\theta_{1,i})} + \theta_1\right) v''(y_i) - \delta h''(y_i)\right] < 0 \\ \left|\widehat{A}_y\right| &= \delta h'(y_i)\left[\frac{\lambda_i'(\theta_{1,i})}{[\lambda_i(\theta_{1,i})]^2} - v'(y_i)\right] \end{aligned}$$

Hence, we know that

$$\frac{d\theta_{1,i}}{d\beta_i} = \frac{\left|\widehat{A}_{\theta_1}\right|}{\left|\widehat{A}\right|} > 0$$

Intuitively, consumers with severe self-control problems (small β_i) value commitment (in the form of the higher price q) more than relatively rational consumers (with high β_i).

Next, the welfare effect of sin licenses can be re-expressed as

$$\begin{aligned} \Delta W_i &= -\left[\theta v'\left(x^*\left(\theta_1; p, \beta\right)\right) - \delta h'\left(x^*\left(\theta_1; p, \beta\right)\right) - 1\right] \frac{dx^*\left(\theta_1; p, \beta\right)}{d\theta} dF\left(\theta_1\right) d\theta_{21} \\ &\quad \int_{\theta_2(y;\beta,p)}^{\overline{\theta}} \left[\theta v'\left(x^*\left(\theta; p, \beta\right)\right) - \delta h'\left(x^*\left(\theta; q, \beta\right)\right) - 1\right] \frac{dx^*\left(\theta; p, \beta\right)}{dp} dF\left(\theta\right) d\tau \\ &= \left[\left(1-\beta\right)\delta h'\left(x^*\left(\theta; q, \beta\right)\right) - \tau_1\right] \frac{dx^*\left(\theta_1; p, \beta\right)}{d\theta} dF\left(\theta_1\right) \frac{d\tau}{v'(y)} \\ &\quad - \int_{\theta_1(y;\beta,p)}^{\overline{\theta}} \left[\left(1-\beta\right)\delta h'\left(x^*\left(\theta; q, \beta\right)\right) - \tau_1\right] \frac{dx^*\left(\theta; p, \beta\right)}{dp} dF\left(\theta\right) d\tau \end{aligned}$$

However, the first effect is of an order of magnitude smaller than the second effect, and we can ignore it. Hence we can write

$$\Delta W_i = -\int_{\theta_1(y;\beta,p)}^{\overline{\theta}} \left[\left(1-\beta\right)\delta h'\left(x^*\left(\theta, \beta; p, \right)\right) - \tau_1\right] \frac{dx^*\left(\theta, \beta; p\right)}{dp} dF\left(\theta\right) d\tau \tag{74}$$

Finally, integrating over all consumers ($i \in I$) gives the aggregate welfare effect of marginal sin licenses

$$\Delta W = -\int_{i\in I}\int_{\theta_1(y;\beta,p)}^{\overline{\theta}} \left[\left(1-\beta_i\right)\delta h'\left(x^*\left(\theta_i, \beta_i; p, \right)\right) - \tau_1\right] \frac{dx^*\left(\theta_i, \beta_i; p\right)}{dp} dF\left(\theta\right) dG\left(i\right) d\tau \tag{75}$$

Next, let us analyze the welfare effects of increasing the sin tax τ_1 by a small amount $d\tau$. (Here we assume that no sin licenses are in place.)

$$\begin{aligned}\Delta\widehat{W}_i &= \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta' v\left(x^*\left(\theta_i, \beta_i; p\right)\right) - \delta h'\left(x^*\left(\theta, \beta_i; p\right)\right) - 1\right] \frac{dx^*\left(\theta_i, \beta_i; p\right)}{dp} dF_i\left(\theta_i\right) d\tau \\ &= -\int_{\underline{\theta}}^{\overline{\theta}} \left[\left(1-\beta\right)\delta h'\left(x^*\left(\theta_i, \beta_i; p\right)\right) - \tau_1\right] \frac{dx^*\left(\theta_i, \beta_i; p\right)}{dp} dF_i\left(\theta_i\right) d\tau\end{aligned}$$

Integrating over all consumers yields

$$\Delta\widehat{W} = -\int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}} \left[\left(1-\beta_i\right)\delta h'\left(x^*\left(\theta_i, \beta_i; p\right)\right) - \tau_1\right] \frac{dx^*\left(\theta_i, \beta_i; p\right)}{dp} dF_i\left(\theta_i\right) dG\left(i\right) d\tau \quad (76)$$

The final step in the proof is to compare the expressions (75) and (76), and to show that $\Delta\widehat{W} > 0$ implies $\Delta W > 0$. To do so, we rewrite (75) and (76) as follows:

$$\begin{aligned}\Delta W &= \left[\tau_1 - \int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}} \left(1-\beta_i\right)\delta h'\left(x^*\left(\theta_i, \beta_i; p\right)\right) d\omega\left(\theta, i\right)\right] \times \\ &\quad \int_{i\in I}\int_{\theta_{1,i}\left(y_i;\beta_i,p\right)}^{\overline{\theta}} \frac{dx^*\left(\theta_i, \beta_i; p\right)}{dp} dF_i\left(\theta_i\right) dG\left(i\right) d\tau \qquad (77)\end{aligned}$$

and

$$\begin{aligned}\Delta\widehat{W} &= \left[\tau_1 - \int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}} \left(1-\beta_i\right)\delta h'\left(x^*\left(\theta_i, \beta_i; p\right)\right) d\widehat{\omega}\left(\theta, i\right)\right] \times \\ &\quad \int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}} \frac{dx^*\left(\theta_i, \beta_i; p\right)}{dp} dF\left(\theta\right) dG\left(i\right) d\tau \qquad (78)\end{aligned}$$

The weighting / density functions $d\omega\left(\theta, i\right)$ and $d\widehat{\omega}\left(\theta, i\right)$ are defined as follows

$$d\omega\left(\theta, i\right) = \begin{cases} \dfrac{\frac{dx^*\left(\theta_i,\beta_i;p\right)}{dp} dF_i(\theta) dG(i)}{\int_{i\in I}\int_{\theta_{1,i}\left(y_i;\beta_i,p\right)}^{\overline{\theta}} \frac{dx^*\left(\theta_i,\beta_i;p\right)}{dp} dF(\theta) dG(i) d\tau} & \text{if } \theta_i \geq \theta_{1,i}\left(y_i; \beta_i, p\right) \\ 0 & \text{if } \theta_i < \theta_{1,i}\left(y_i; \beta_i, p\right) \end{cases},$$

and

$$d\widehat{\omega}\left(\theta, i\right) = \frac{\frac{dx^*\left(\theta_i,\beta_i;p\right)}{dp} dF_i\left(\theta\right) dG\left(i\right)}{\int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}} \frac{dx^*\left(\theta_i,\beta_i;p\right)}{dp} dF_i\left(\theta\right) dG\left(i\right) d\tau}$$

Clearly, $\int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}} d\omega(\theta,i) = \int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}} d\widehat{\omega}(\theta,i) = 1$.

Now, we can prove that $\Delta\widehat{W} > 0$ implies $\Delta W > 0$. First,it is easy to see that the distribution $\omega(\theta,i)$ stochastically dominates the distribution $\widehat{\omega}(\theta,i)$: basically $\omega(\theta,i)$ is obtained from $\widehat{\omega}(\theta,i)$ by left-truncation, i.e. small values of θ (that is values $\theta < \theta_{1,i}(y_i;\beta_i,p)$) have been left out of the distribution $\omega(\theta,i)$; moreover the truncation point is $\theta_{1,i}(y_i;\beta_i,p)$ is higher, ceteris paribus, for relatively rational consumers (high β_i) than for consumers with severe self-control problems (low β_i). Second, the self-control wedge $(1-\beta_i)\delta h'(x^*(\theta_i,\beta_i;p))$ is clearly increasing in the preference shock θ_i, and in the severity of self-control problems $\rho_i \equiv (1-\beta_i)$. Third, combining steps one and two, one can conclude that

$$\left[\tau_1 - \int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}} (1-\beta_i)\delta h'(x^*(\theta_i,\beta_i;p))\,d\omega(\theta,i)\right] < \left[\tau_1 - \int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}} (1-\beta_i)\delta h'(x^*(\theta_i,\beta_i;p))\,d\widehat{\omega}(\theta,i)\right] \tag{79}$$

Fourth, since $\int_{i\in I}\int_{\theta_{1,i}(y_i;\beta_i,p)}^{\overline{\theta}} \frac{dx^*(\theta_i,\beta_i;p)}{dp} dF_i(\theta_i)\,dG(i)\,d\tau < 0$ and $\int_{i\in I}\int_{\underline{\theta}}^{\overline{\theta}} \frac{dx^*(\theta_i,\beta_i;p)}{dp} dF(\theta)\,dG(i)\,d\tau < 0$, we can conclude that $\Delta\widehat{W} > 0$ implies $\Delta W > 0$. This follows from (79).

A5. Proof of Proposition 3

According to Proposition 1, a consumer would adopt the hybrid system of sin taxes and sin licenses, even when he could choose any non-linear pricing scheme, with a minimum unit price $p = 1+\tau_1$, and a maximum unit price $q = 1+\tau_2$. In particular, the original linear sin tax $\tau \in [\tau_1,\tau_2]$ belongs to this menu of available pricing schemes. Since the consumers are sophisticated, we then know that for each consumer i

$$\Delta V_i \equiv V_i(\tau_1,\tau_2) - V_i(\tau) \geq 0 \tag{80}$$

where

$$\begin{aligned} V_i(\tau_1,\tau_2) &= E_{\theta_i}\left[\theta_i v(x(\theta_i,\beta_i;\tau_1,\tau_2)) - \delta h(x(\theta_i,\beta_i;\tau_1,\tau_2))\right. \\ &\quad \left. - (1+\tau_1)x(\theta_i,\beta_i;\tau_1,\tau_2) - (\tau_2-\tau_1)x^s(\theta_i,\beta_i;\tau_1,\tau_2)\right] + S_i \end{aligned} \tag{81}$$

is (expected) indirect utility under the hybrid scheme, and

$$V_i(\tau) = E_{\theta_i}\left[\theta_i v(x(\theta_i,\beta_i;\tau)) - \delta h(x(\theta_i,\beta_i;\tau)) - (1+\tau)x(\theta_i,\beta_i;\tau)\right] + S_i \tag{82}$$

is (expected) indirect utility under the linear sin tax. (Notice that transfers S_i are the same under both schemes; the consumer is choosing the pricing scheme for himself, not for the whole economy.)

Unlike the consumer, the social planner does not care about the fiscal costs of the different pricing schemes, and the social welfare criterion only involves consumption allocations. In particular, the consumer's choice of the hybrid system, over the system of linear sin taxes, improves social welfare, if and only if $\Delta W_i > 0$, where

$$\Delta W_i \equiv W_i(\tau_1, \tau_2) - W_i(\tau) \tag{83}$$

and

$$W_i(\tau_1, \tau_2) = E_{\theta_i}\left[\theta_i v\left(x\left(\theta_i, \beta_i; \tau_1, \tau_2\right)\right) - \delta h\left(x\left(\theta_i, \beta_i; \tau_1, \tau_2\right)\right) - x\left(\theta_i, \beta_i; \tau_1, \tau_2\right)\right] \tag{84}$$

is the socially relevant welfare measure under the hybrid system, while

$$W_i(\tau) = E_{\theta_i}\left[\theta_i v\left(x\left(\theta_i, \beta_i; \tau\right)\right) - \delta h\left(x\left(\theta_i, \beta_i; \tau\right)\right) - x\left(\theta_i, \beta_i; \tau\right)\right] \tag{85}$$

is the welfare measure under sin taxes.

All the above considerations concern the welfare implications of the choices made by an individual consumer i. However, to evaluate the policy reform, we need an economywide welfare metric. As an initial step, we take the expression (80), and sum over all consumers i. We get

$$\Delta V = E_i\left[\Delta V_i\right] \geq 0 \tag{86}$$

However, this is not the final metric, since the consumers' choices include fiscal considerations (and one consumer's fiscal gain is another consumer's loss). To obtain the socially relevant welfare measure, we have to consider the expression (83). Summing over all consumers i yields

$$\Delta W = E_i\left[\Delta W_i\right] \tag{87}$$

If $\Delta W > 0$, the reform is welfare-improving.

Now the question is: Under what conditions does $E_i\left[\Delta V_i\right] \geq 0$ (expression (86)) imply $\Delta W > 0$? To answer this question, we use (80) - (87), and derive the formula

$$\Delta W - \Delta V = \tau_1 \overline{x}^h(\tau_1, \tau_2) + (\tau_2 - \tau_1)\,\overline{x}^s(\tau_1, \tau_2) - \tau \overline{x}(\tau) \tag{88}$$

where $\overline{x}^h(\tau_1, \tau_2) = E_i[E_{\theta_i}[x(\theta_i, \beta_i; \tau_1, \tau_2)]]$ is aggregate consumption under the hybrid scheme $\overline{x}^s(\tau_1, \tau_2) = E_i[E_{\theta_i}[x^s(\theta_i, \beta_i; \tau_1, \tau_2)]]$ is aggregate consumption, purchased at price $q = 1 + \tau_2$ under the hybrid scheme, and $\overline{x}(\tau) = E_i[E_{\theta_i}[x(\theta_i, \beta_i; \tau)]]$ is aggregate consumption under the system of sin taxes. A notable and useful feature of the formula (88) is that it does not depend on utility functions, harm functions, measures of self-control problems etc. It only involves observable measures of aggregate consumption and tax rates.

Now, if

$$\Delta W - \Delta V \geq 0 \tag{89}$$

we know that $\Delta V \geq 0$ implies $\Delta W \geq 0$. To further interpret the condition (89), notice that (88) can be re-expressed as

$$\Delta W - \Delta V = TR(\tau_1, \tau_2) - TR(\tau)$$

where

$$TR(\tau_1, \tau_2) = \tau_1 \overline{x}^h(\tau_1, \tau_2) + (\tau_2 - \tau_1)\overline{x}^s(\tau_1, \tau_2)$$

is aggregate tax revenue under the hybrid system, and

$$TR(\tau) = \tau \overline{x}(\tau)$$

is aggregate tax revenue under the system of linear sin taxes. Hence, the reform is guaranteed to improve social welfare, if aggregate tax revenue is not reduced.

Acknowledgements

We would like to thank participants at the 5th Nordic Conference on Behavioral and Experimental Economics, 11th Journées Louis-André Gérard-Varet, as well as seminar participants at HECER for comments on an earlier version of the paper. We also thank Sören Blomquist, Botond Köszegi, Jukka Pirttilä and Matti Tuomala for helpful discussions and comments. Kotakorpi would like to thank the Academy of Finland for financial support.

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