

# Strategic Trading and Return Predictability

Bradyn Breon-Drish

Stanford GSB

6 Oct. 2014



## Return momentum and reversals are persistent empirical phenomena

- ▶ It is well-documented that stock return exhibit momentum at short horizons (Jegadeesh and Titman (1993)), which partially reverses at long horizons (Fama and French (1988), Poterba and Summers (1988))
  - ▶ Related: continuation after public news announcements (Bernard and Thomas (1992))
- ▶ Appear in many different asset classes (Moskowitz, Ooi, and Pedersen (2012))
- ▶ Effects persist despite wide knowledge of their existence
- ▶ Evidence that magnitude of predictability is related to belief heterogeneity (Zhang (2006), Verardo (2009))



## Literature review

- ▶ Behavioral theories focus on investor over/under-reaction to news
  - ▶ Daniel, Hirshleifer, and Subrahmanyam (1998)
  - ▶ Hong and Stein (1999)
- ▶ Risk based explanations attribute predictability to changes in risk exposures and/or risk premia
  - ▶ Berk, Green, and Naik (1999)
  - ▶ Sagi and Seasholes (2007)
  - ▶ Vayanos and Wooley (2013)
  - ▶ Albuquerque and Miao (2014)



## Slow aggregation of information?

- ▶ Allen, Morris, and Shin (2006) argue that price drifts can arise due to slow aggregation of info in presence of noise
- ▶ Banerjee, Kaniel, Kremer (2009):
  - ▶ Finite-horizon CARA-Normal model with long-lived rational traders, random walk supply (iid noise trades).
  - ▶ With rational expectations, agents use information in price to correct any predictability that is not due to risk premia

$$\mathbb{E}[p_{t+1} - p_t | p_t - p_{t-1}] \propto \mathbb{E}[Z_{t+1} - Z_t | p_t - p_{t-1}] = 0,$$

if  $Z_{t+1} - Z_t$  iid.

- ▶ More generally, relation depends only on AR coefficient of supply
  - ▶  $0 < \phi_Z < 1$ , change in risk premia partially reverses  $\Rightarrow$  reversals
  - ▶  $\phi_Z > 1$ , change in risk premia continues  $\Rightarrow$  momentum
- ▶ With  $R > 1$ , stationary setting (Wang (1994), Albuquerque and Miao (2014))
  - ▶  $0 < \phi_Z < 1/R \Rightarrow$  reversals
  - ▶  $1/R < \phi_Z < 1 \Rightarrow$  momentum



## Contribution

- ▶ Solve dynamic Kyle (1989) model with asymmetric information
- ▶ Drifts, followed by reversals, occurs naturally when some traders account for price impact
- ▶ Intuition: A strategic, risk averse trader trades slowly towards a 'target inventory' (inventory that would be optimal in a competitive world).
  - ▶ Current inventory enters price as an adjustment to risk premium
  - ▶ Changes in inventory are persistent
    - ▶ If persistence is sufficiently large, leads to positive serial correlation in returns
  - ▶ In the long run, the target inventory reverts to zero (more generally, fixed fraction of supply), eliminating 'mispricing'
    - ▶ Returns reverse in long run



## Setting

Dynamic Kyle (1989)

- ▶ Infinite horizon, discrete time
- ▶ Risky asset dividend stream

$$D_{t+1} = G_t + \sigma_D \varepsilon_{Dt+1},$$

with persistent component

$$G_t = \phi_G G_{t-1} + \sigma_G \varepsilon_{Gt}$$

- ▶ Risky asset in zero net supply
- ▶ Risk-free asset with return  $R = 1 + r$



# Setting

Dynamic Kyle (1989)

- ▶ Informed trader maximizes

$$\mathbb{E}_I \left[ \sum_{t=0}^{\infty} -e^{-\rho t - \alpha_I C_{It}} \right]$$

- ▶ Observes  $G_t$  and accounts for price impact when forming demand schedule
- ▶ Endowed with  $Z_t$  shares of a nontradeable asset with payoffs  $Y_{t+1}$

$$Z_t = \phi_Z Z_{t-1} + \sigma_Z \varepsilon_{Zt}$$

$$Y_{t+1} = \sigma_Y \varphi_{DY} \varepsilon_{Dt+1} + \sigma_Y \sqrt{1 - \varphi_{DY}^2} \varepsilon_{Yt+1},$$

with  $\varphi_{DY} > 0$ .

- ▶ Mass of competitive uninformed traders

$$\mathbb{E}_U \left[ \sum_{t=0}^{\infty} -e^{-\rho t - \alpha_U C_{Ut}} \right]$$

- ▶ Learn about  $G_t$  by observing prices and realized dividends



# Equilibrium

Search for equilibrium in which agents submit linear demand schedules that specify their desired trade

$$x_{It} = \beta_{I1} G_t + \beta_{I2} Z_t + \eta_I D_t + \chi_{I1} \hat{G}_{t-1} + \chi_{I2} \hat{Z}_{t-1} - \gamma_I P_t - \delta_I \theta_{It}$$

$$x_{Ut} = \eta_U D_t + \chi_{U1} \hat{G}_{t-1} + \chi_{U2} \hat{Z}_{t-1} - \gamma_U P_t - \delta_U \theta_{Ut}.$$

where  $\theta_{jt}$  = shares held immediately prior to trade at  $t$ ,

$\hat{G}_{t-1} = \mathbb{E}_{U_{t-1}}[G_{t-1}]$ , and  $\hat{Z}_{t-1} = \mathbb{E}_{U_t}[Z_{t-1}]$ .



## Uninformed investor's problem

Learning from price

- ▶ Under linear conjecture uninformed trader faces flat residual supply schedule

$$P_t = (\gamma_I + \gamma_U)^{-1} \left( \beta_{I1} G_t + \beta_{I2} Z_t + (\eta_I + \eta_U) D_t + (\chi_{I1} + \chi_{U1}) \hat{G}_{t-1} \right. \\ \left. + (\chi_{I2} + \chi_{U2}) \hat{Z}_{t-1} - \delta_I \theta_{It} - \delta_U \theta_{Ut} \right)$$



## Uninformed investor's problem

Learning from price

- ▶ Under linear conjecture uninformed trader faces flat residual supply schedule

$$P_t = \lambda \left( \beta_{I1} G_t + \beta_{I2} Z_t + \eta D_t + \chi_1 \hat{G}_{t-1} + \chi_2 \hat{Z}_{t-1} - \delta_I \theta_{It} - \delta_U \theta_{Ut} \right)$$



## Uninformed investor's problem

Learning from price

- ▶ Under linear conjecture uninformed trader faces flat residual supply schedule

$$P_t = \lambda \left( \beta_{11} G_t + \beta_{12} Z_t + \eta D_t + \chi_1 \hat{G}_{t-1} + \chi_2 \hat{Z}_{t-1} + \delta \theta_{Ut} \right)$$



## Uninformed investor's problem

Learning from price

- ▶ Under linear conjecture uninformed trader faces flat residual supply schedule

$$P_t = \lambda \left( \beta_{11} G_t + \beta_{12} Z_t + \eta D_t + \chi_1 \hat{G}_{t-1} + \chi_2 \hat{Z}_{t-1} + \delta \theta_{Ut} \right)$$

- ▶ Can infer statistic

$$\begin{aligned} S_{Pt} &= \frac{1}{\lambda} P_t - \eta D_t - \chi_1 \hat{G}_{t-1} - \chi_2 \hat{Z}_{t-1} + \delta \theta_{Ut} \\ &= \beta_{11} G_t + \beta_{12} Z_t \end{aligned}$$



# Uninformed investor's problem

## Kalman Filtering

The conditional distribution of  $(G_t, Z_t)$  given  $\{D_t, S_{P,t}\}$  is normal with mean  $(\hat{G}_t, \hat{Z}_t)$  and constant covariance matrix  $\nu = \begin{pmatrix} \nu_G & \nu_{GZ} \\ \nu_{GZ} & \nu_Z \end{pmatrix}$ .

The conditional mean satisfies the following difference equation

$$\begin{pmatrix} \hat{G}_t \\ \hat{Z}_t \end{pmatrix} = \begin{pmatrix} \phi_G & 0 \\ 0 & \phi_Z \end{pmatrix} \begin{pmatrix} \hat{G}_{t-1} \\ \hat{Z}_{t-1} \end{pmatrix} + K \begin{pmatrix} D_t - \hat{G}_{t-1} \\ S_{P,t} - \beta_{I1}\phi_G\hat{G}_{t-1} - \beta_{I2}\phi_Z\hat{Z}_{t-1} \end{pmatrix},$$

where

$$K = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix}.$$



# Uninformed investor's problem

## Dynamic programming

► Let

$$\begin{aligned} \varepsilon_{D,t}^U &= D_t - \hat{G}_{t-1} \\ \varepsilon_{P,t}^U &= S_{P,t} - \beta_{I1}\phi_G\hat{G}_{t-1} - \beta_{I2}\phi_Z\hat{Z}_{t-1}, \end{aligned}$$

which is iid with zero mean and constant covariance matrix.  
Consider state variables

$$\begin{aligned} M_{U,t+1} &= R(M_{U,t} - c_{U,t} - P_t x_{U,t}) + (\theta_{U,t} + x_{U,t})D_{t+1} \\ \theta_{U,t+1} &= \theta_{U,t} + x_{U,t} \\ D_{t+1} &= \hat{G}_t + \varepsilon_{D,t+1}^U \\ \hat{G}_t &= \phi_G\hat{G}_{t-1} + K_{11}(D_t - \hat{G}_{t-1}) + K_{12}(S_{P,t} - \mathbb{E}_{U,t-1}[S_{P,t}]) \\ \hat{Z}_t &= \phi_Z\hat{Z}_{t-1} + K_{21}(D_t - \hat{G}_{t-1}) + K_{22}(S_{P,t} - \mathbb{E}_{U,t-1}[S_{P,t}]) \\ S_{P,t+1} &= \beta_{I1}\phi_G\hat{G}_t + \beta_{I2}\phi_Z\hat{Z}_t + \varepsilon_{P,t+1}^U. \end{aligned}$$



# Uninformed investor's problem

## Dynamic programming

- ▶ Conjecture that the value function takes exponential-quadratic form

$$V_U(M_{Ut}, X_{Ut}) = -\exp\left\{-A_{U0} - A_{UM}M_{Ut} - \frac{1}{2}X'_{Ut}Q_U X_{Ut}\right\},$$

where  $A_{U0}$  and  $A_{UM}$  are constants,  
 $X_{Ut} = (\theta_{Ut}, D_t, \hat{G}_{t-1}, \hat{Z}_{t-1}, S_{Pt})$ , and  $Q_U$  is a symmetric  $5 \times 5$  matrix.



# Uninformed investor's problem

## Dynamic programming

Uninformed investor Bellman equation

$$0 = \max_{x,c} \left\{ -\exp\{-\alpha_U c\} + e^{-\rho} \mathbb{E}_{Ut} [V_U(M_{Ut+1}, X_{Ut+1})] - V_U(\cdot) \right\}.$$

$$\text{s.t. } M_{Ut+1} = R(M_{Ut} - c_{Ut} - P_t x_{Ut}) + (\theta_{Ut} + x_{Ut}) D_{t+1}$$

$$X_{Ut+1} = a_U X_{Ut} + b_U \hat{\varepsilon}_{t+1}^U + a_{Ux} x_{Ut}$$

$$P_t = \lambda \left( \beta_{11} G_t + \beta_{12} Z_t + \eta D_t + \chi_1 \hat{G}_{t-1} + \chi_2 \hat{Z}_{t-1} + \delta \theta_{Ut} \right)$$

Optimal consumption and demand

$$c_{Ut}^* = \text{constant} + \frac{A_{UM} R}{\alpha_U + A_{UM} R} M_{Ut} + \frac{1}{2} X'_{Ut} m_U X_{Ut}$$

$$x_{Ut}^* = \hat{H}'_U X_{Ut},$$

where  $m_U$  is a  $5 \times 5$  symmetric matrix, and  $\hat{H}_U$  is a  $5 \times 1$  vector.





## Uninformed investor's problem

### Dynamic programming

- ▶ Plugging back into the Bellman equation and equating coefficients produces

$$A_{U0} = \frac{1}{r} (\rho - \log(rd_U) + R \log(r/R))$$
$$A_{UM} = \frac{r}{R} \alpha_U,$$

and matrix  $Q_U$  must satisfy

$$Q_U = m_U/R. \quad (1)$$

- ▶ Functional form for price and the statistic  $S_{P_t}$  implies  $x_{U_t}^*$  can be implemented by submitting a demand schedule that infers  $S_{P_t}$  from the equilibrium price

$$x_{U_t}^* = \hat{H}_U X_{U_t} - h_U P_t. \quad (2)$$

This function must match the initial conjecture.

- ▶ Eq. (3) and (4) is system of 20 equations in 20 unknowns



## Informed investor's problem

### Residual supply schedule

- ▶ Under linear conjecture informed trader faces linear residual supply schedule

$$P_t(x_{I_t}) = \gamma_U^{-1} \left( x_{I_t} - \delta_U \theta_{U_t} + \eta_U D_t + \chi_{U1} \hat{G}_{t-1} + \chi_{U2} \hat{Z}_{t-1} \right)$$



## Informed investor's problem

### Residual supply schedule

- ▶ Under linear conjecture informed trader faces linear residual supply schedule

$$P_t(x_{it}) = \underbrace{\gamma_U^{-1}}_{=\lambda_I} \left( x_{it} - \delta_U \theta_{Ut} + \eta_U D_t + \chi_{U1} \hat{G}_{t-1} + \chi_{U2} \hat{Z}_{t-1} \right)$$



## Informed investor's problem

### Residual supply schedule

- ▶ Under linear conjecture informed trader faces linear residual supply schedule

$$P_t(x_{it}) = \lambda_I \left( x_{it} + \delta_U \theta_{it} + \eta_U D_t + \chi_{U1} \hat{G}_{t-1} + \chi_{U2} \hat{Z}_{t-1} \right)$$



## Informed investor's problem

### Residual supply schedule

- ▶ Under linear conjecture informed trader faces linear residual supply schedule

$$P_t(x_{It}) = \lambda_I \left( x_{It} + \delta_U \theta_{It} + \eta_U D_t + \chi_{U1} \hat{G}_{t-1} + \chi_{U2} \hat{Z}_{t-1} \right)$$

- ▶ Consider state variables

$$M_{It+1} = R(M_t - c - P_t(x_{It})x_{It}) + (\theta_{It} + x_{It})D_{t+1} + Z_t Y_{t+1}$$

$$\theta_{It+1} = \theta_{It} + x_{It}$$

$$D_{t+1} = G_t + \sigma_D \varepsilon_{D,t+1}$$

$$\hat{G}_t = \phi_G \hat{G}_{t-1} + K_{11}(D_t - \hat{G}_{t-1}) + K_{12}(S_{P,t} - \mathbb{E}_{U_{t-1}}[S_{P,t}])$$

$$\hat{Z}_t = \phi_Z \hat{Z}_{t-1} + K_{21}(D_t - \hat{G}_{t-1}) + K_{22}(S_{P,t} - \mathbb{E}_{U_{t-1}}[S_{P,t}])$$

$$G_{t+1} = \phi_G G_t + \sigma_G \varepsilon_{G,t+1}$$

$$Z_{t+1} = \phi_Z Z_t + \sigma_Z \varepsilon_{Z,t+1}$$

$$Y_{t+1} = \sigma_Y \left( \varphi_{DY} \varepsilon_{D,t+1} + \sqrt{1 - \varphi_{DY}^2} \varepsilon_{Y,t+1} \right)$$



## Informed investor's problem

### Dynamic programming

- ▶ Conjecture that the value function takes exponential-quadratic form

$$V_I(M_{It}, X_{It}) = -\exp \left\{ -A_{I0} - A_{IM} M_{It} - \frac{1}{2} X'_{It} Q_I X_{It} \right\},$$

where  $A_{I0}$  and  $A_{IM}$  are constants,

$X_{It} = (\theta_{It}, D_t, \hat{G}_{t-1}, \hat{Z}_{t-1}, G_t, Z_t, Y_t)$ , and  $Q_I$  is a symmetric  $8 \times 8$  matrix with zeros in the last row and column.



## Informed investor's problem

### Dynamic programming

Informed investor Bellman equation

$$0 = \max_{x,c} \left\{ -\exp\{-\alpha_I c\} + e^{-\rho} \mathbb{E}_{It} [V_I(M_{It+1}, X_{It+1})] - V_I(\cdot) \right\}.$$

$$\begin{aligned} \text{s.t.} \quad & M_{It+1} = R(M_{It} - c_{It} - P_t(x_{It})x_{It}) + (\theta_{It} + x_{It})D_{t+1} \\ & X_{It+1} = a_I X_{It} + b_I \hat{\varepsilon}_{t+1} + a_{Ix} x_{It} \\ & P_t(x_{It}) = \lambda_I \left( x_{It} + \delta_U \theta_{It} + \eta_U D_t + \chi_{U1} \hat{G}_{t-1} + \chi_{U2} \hat{Z}_{t-1} \right) \\ & S_{Pt} = \frac{1}{\lambda} P_t(x_{It}) - \eta D_t - \chi_1 \hat{G}_{t-1} - \chi_2 \hat{Z}_{t-1} - \delta \theta_{It} \end{aligned}$$

Optimal consumption and demand

$$\begin{aligned} c_{It}^* &= \text{constant} + \frac{A_{IM} R}{\alpha_I + A_{IM} R} M_{It} + \frac{1}{2} X_{It}' m_I X_{It} \\ x_{It}^* &= \hat{H}'_I X_{It}, \end{aligned}$$

where  $m_I$  is a  $8 \times 8$  symmetric matrix, and  $\hat{H}_U$  is a  $8 \times 1$  vector.



## Uninformed investor's problem

### Dynamic programming

- ▶ Plugging back into the Bellman equation and equating coefficients produces

$$\begin{aligned} A_{I0} &= \frac{1}{r} (\rho - \log(rd_I) + R \log(r/R)) \\ A_{IM} &= \frac{r}{R} \alpha_I, \end{aligned}$$

and matrix  $Q_I$  must satisfy

$$Q_I = m_I / R. \quad (3)$$

- ▶ Functional form for price and the statistic  $S_{Pt}$  implies  $x_{Ut}^*$  can be implemented by submitting a demand schedule

$$x_{It}^* = \hat{H}'_I X_{It} - h_I P_t(x_{It}^*). \quad (4)$$

This function must match the initial conjecture.

- ▶ Eq. (3) and (4) is system of 35 equations in 35 unknowns



## Equilibrium existence

- ▶ Equilibrium characterized by solutions to system of equations
  - ▶ In principle, 45 equations
  - ▶ Dimensionality reduced by some analytical manipulations
- ▶ Resort to numerical solutions



## Definition of drifts and reversals

- ▶ Let  $r_t^e = D_t + P_t - RP_{t-1}$ , and  $r_{t,t+k} = \sum_{j=1}^k r_{t+j}^e$
- ▶ Price exhibits momentum (reversal) at horizon  $k$  if

$$\text{Cov}(r_{t+k}^e, r_t^e) > 0 \quad (< 0)$$

- ▶ Alternately, can consider non-cumulative returns

$$\text{Cov}(r_{t,t+k}^e, r_{t-k,t}^e) > 0 \quad (< 0)$$





# Returns

- ▶ Excess returns

$$\begin{aligned}
 r_{t+1}^e &= D_{t+1} + P_{t+1} - RP_t \\
 &= \frac{\sigma_G}{(R - \phi_G)^2} \varepsilon_{Gt+1} + \sigma_D \varepsilon_{Dt+1} \\
 &\quad a_Z (Z_{t+1} - RZ_t) + a_\theta (\theta_{t+1} - R\theta_t),
 \end{aligned}$$

- ▶ Can show that expected excess returns follow 2-factor model

$$\mathbb{E}_t[r_{t+1}^e] = \underbrace{a_Z(\phi_Z - R)Z_t}_{\text{Risk premium due to } Z} + \underbrace{a_\theta(b_Z Z_t + (b_\theta - R)\theta_t)}_{\text{Risk premium due to } \theta_t}$$



# Calibration

$\alpha_I$	4	U = 3/4 of “capital” $\approx$ % held MF & retail
$\alpha_U$	8	
$R$	1.08	CK <sup>1</sup> CK, $\frac{\sigma_D}{\sigma_G} = 1/2$ = Var( $D_t$ )
$\phi_G$	0.5	
$\sigma_G$	2	
$\sigma_D$	1	
$\sigma_Y$	2.33	
$\varphi_{DY}$	0.7	= 1/R
$\phi_Z$	$\frac{1}{1.08}$	
$\sigma_Z$	1	

<sup>1</sup>Campbell and Kyle (1993, CK) estimate a CARA-Normal model using aggregate stock market data.



## Simulated series: $Z_t, \theta_{1t}, \theta_{1t}^{\text{comp}}$

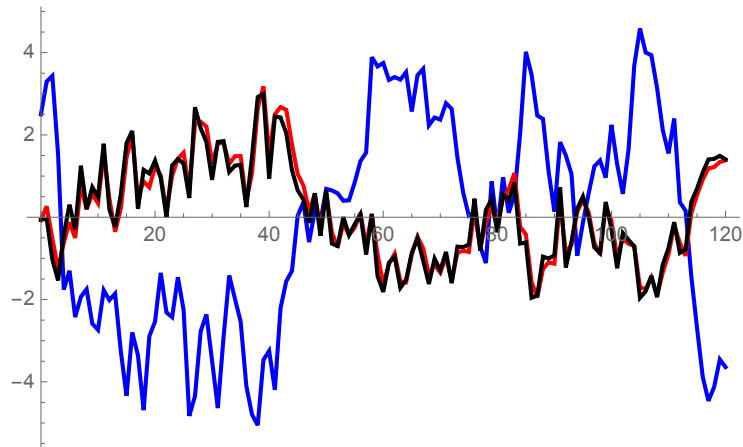


Figure: Endowment of nontradeable asset (blue), informed holdings (black), and informed holdings (red) in competitive setting.

Navigation icons: back, forward, search, etc.

## Momentum/reversals for $k = 1$

At one-period horizon,

$$\begin{aligned} & \text{Cov}(r_{t,t+1}^e, r_{t-1,1}^e) \\ &= a_Z^2 \text{Cov}(Z_{t+1} - RZ_t, Z_t - RZ_{t-1}) \\ & \quad + a_\theta a_Z \text{Cov}(Z_{t+1} - RZ_t, \theta_{1t} - R\theta_{1,t-1}) \\ & \quad + a_\theta a_Z \text{Cov}(\theta_{1,t+1} - R\theta_{1t}, Z_t - RZ_{t-1}) \\ & \quad + a_\theta^2 \text{Cov}(\theta_{1,t+1} - R\theta_{1t}, \theta_{1t} - R\theta_{1,t-1}) \end{aligned}$$

Depends on

- ▶ Autocovariance of ('excess') innovations in  $Z$
- ▶ Lagged cross-covariance of innovations in  $Z_t$  and  $\theta_{1t}$
- ▶ Autocovariance of innovations in  $\theta_{1t}$

Navigation icons: back, forward, search, etc.



## Economic intuition for drifts/reversals at $k = 1$

- ▶ Autocovariance of  $Z_t$ .
  - ▶ Are innovations in quantity of risk  $Z_t$  positively or negatively related to future innovations
  - ▶ In competitive setting: momentum if  $\phi_Z > 1/R$ , reversals if  $\phi_Z < 1/R$ ; sign same at all horizons
  - ▶ In calibrations with  $\phi_Z = 1/R$  this term = 0
- ▶ Autocovariance of  $\theta_{it}$ 
  - ▶ Direct effect of predictability of changes in inventory
  - ▶ In calibrations  $\approx 0$
- ▶ Cross-covariances
  - ▶ Ceteris paribus, nontraded endowment  $\downarrow \Rightarrow$  informed close out short positions  $\Rightarrow r_t^e \uparrow$
  - ▶ As inventories (and hence prices) respond only slowly and  $Z_t$  is highly persistent, this also forecasts a net positive change in risky asset holdings next period and consequently a price increase
  - ▶ However, trading against this 'mispricing' is risky, so uninformed do not completely eliminate, leading to momentum
  - ▶ In the long run,  $Z_t$  reverts to zero, and 'mispricing' due to this shock converges to zero



## Return autocovariances

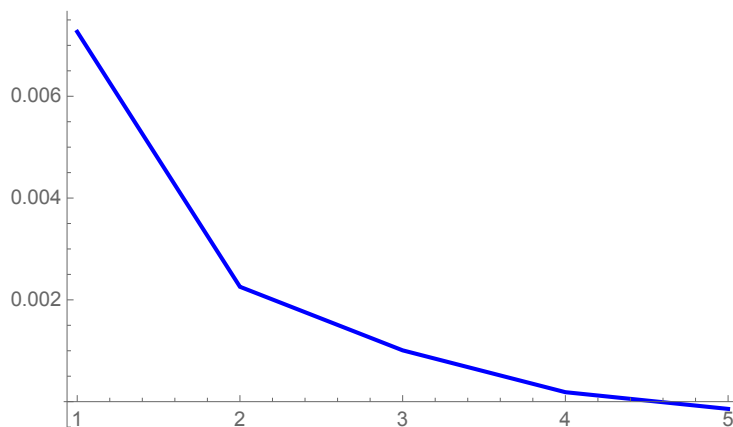


Figure: Autocovariance function of one-period returns



# Conclusion

- ▶ Dynamic, strategic trading model
- ▶ Price drifts/reversals depend on:
  - ▶ Correlation of endowment shocks
  - ▶ Adjustment speed of strategic traders
- ▶ Numerical results suggest return autocorrelations larger when
  - ▶ Information asymmetry is larger
  - ▶ Residual risk is higher