Optimal energy transition and taxation of non-renewable resources

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Abstract

This paper investigates the optimal taxation path of a non-renewable resource in the presence of an imperfect substitute renewable resource. We present an optimal growth model and characterize the social optimum and the decentralized equilibrium. We show that the economy gradually reduces the share of non-renewable resource and converges to a steady state in which it uses only the renewable resource. The decentralized economy converges to the same steady state as the social optimum in terms of capital stock and consumption whether there is a regulator intervention or not. What matters for welfare, however, is the speed at which the economy approaches the clean state that determines the level of damages to the environment. We obtain the optimal taxation rule and show that its time profile can be either always increasing, decreasing or U-shaped depending on the initial state of the economy as well as the properties of the clean state that it converges. Finally we provide some simulation results to illustrate these theoretical findings.

Keywords: energy transition, optimal taxation, non-renewable resource, renewable resource, imperfect substitution, simultaneous resource use.

JEL-Classification: Q43, Q38, Q30, Q20

1 Introduction

Energy transition refers to the process in which the economy substitutes renewable resources for non-renewables over time and eventually approaches a green, zero-carbon state. What matters for energy transition is not only the current level of policy measures but also their planned time paths. When the policies are suboptimal, in terms of their levels and paths, the speed of transition can be too slow or too rapid. The consequence of a too slow transition is that the environment will be damaged more than the socially optimal level. Similarly, when the transition is too rapid, the substitution costs which depend on the degree of substitution between resources will outweigh the environmental benefits. These facts raise the importance of studying the optimal policy measures for a decentralized economy that goes through the course of energy transition.

What is the optimal time profile of taxes on non-renewable resources to induce the economy to use the optimal transition path to clean technologies? Should we tax low at the beginning and tighten it over time so that we let the economy develop while giving the firms increasingly higher incentive to substitute the renewables? Or should we tax very high and loosen it over time so that we initially stimulate the use of renewables? In this paper, we approach these classical questions from two different points: First, we look for the main channels that determine the optimal tax rate and its path. Second, we take into account the existence of an imperfect substitute renewable resource and we analyze the role of the degree of substitution between non-renewable and renewable resources.

We show that, consistent with the recent studies such as van der Ploeg and Withagen (2014), Golosov et al. (2014), the current level and the time path of optimal tax rate depends on the initial state of the economy which is defined by the level of capital stock, the level of cumulative non-renewable resource extraction and the state of the environment. Moreover, we emphasize another point that is rather neglected in the literature: the taxation path also depends on the properties of the final (clean) state that the economy converges which is determined by the cost of renewables and the degree of substitution between non-renewable and renewable resources.

The final state of the economy will use only the renewable resources thus suffer from the technological characteristics that make non-renewable and renewable resources imperfect substitutes. These technological characteristics are the technical constraints, the geographical constraints and the differences in the opportunity costs of using renewable resources in production. Here a question arises: are the renewable resources capable of sustaining today's level of economic activities by themselves? On the one hand, if the renewable resources are a good substitute for the non-renewables, the growth can keep on, the capital stock and consumption can increase over time as it is considered in many studies. In this case the optimal tax rate has an increasing time profile. On the other hand, if the renewable resources are not a good substitute for the non-renewables, we may well be over-producing and over-consuming today and hence need to reduce the level of capital stock, production and consumption over time to sustain a clean economy in the long run. In this case, the optimal tax is initially set to a very high level and decreases over time. A similar comparison can be made for a given degree of substitution between the resources and two economies with different capital stocks. For the economy with a capital stock that is lower than its long run - clean state value, the optimal tax is initially low and increasing, hence not creating a burden for growth while stimulating energy transition over time. However, for the economy which accumulated capital higher than its long term value, hence over-producing and over-consuming, the optimal tax is initially set to a high value and decreases over time. These results highlight several different dimensions in the policy making.

The debate on the optimal taxation path of non-renewable resources was pioneered by Sinclair (1994) and Ulph and Ulph (1994). Both studies agree on the time profile of taxation is all that matters on the extraction path of non-renewable resources. On the one hand, Sinclair claims that in order to postpone the current extractions and smooth the consumption of non-renewables, the ad-valorem tax rate has to be initially set to a high value and should fall over time. On the other hand, Ulph and Ulph show cases where the tax rate should first rise and then fall. Recently, van der Ploeg and Withagen (2014) identify the conditions under which the optimal tax rate rises or decreases by establishing four different regimes of energy use depending on the initial stocks of oil and capital. They consider also the role of a renewable resource which is a perfect substitute of oil. Golosov et al. (2014) also contribute the debate by establishing that the optimal tax rate should be proportional to output and they show that whether the optimal tax rises or falls depends on the output growth rate and the increase rate of non-renewable resource price.[1] But these studies neglect the course of gradual and smooth transition to a clean economy as well as the presence of the renewable resources that the economy uses simultaneously with non-renewables.[2]

The framework in this paper is an optimal growth model close to van der Ploeg and Withagen (2014).[3] We consider an economy that consists of households, final good producing firms and resource extraction firms. The firms use capital and energy to produce the final good. Energy can be obtained by using non-renewable and renewable resources which are imperfect substitutes. The renewable resource price is exogenously given and fixed while the non-renewable resource price is endogenously determined by the extraction cost and the rent of resource. The extraction cost of non-renewable resource increases as the economy extracts more of it (à la Heal (1976)). Moreover, the extraction of non-renewable resource damages the environment in an irreversible way.[4] In this framework, we characterize the optimal transition path and then the decentralized equilibrium path. Studying the two, we obtain the optimal taxation rule that induces the decentralized equilibrium to

[1]Under some specific assumptions on the utility, production and damage functions and on the accumulation dynamics of capital and pollution.

[2]For example, van der Ploeg and Withagen (2014), by considering non-renewable and renewable resources as perfect substitutes, show transition towards a cleaner economy but with instantaneous switches between the different energy use regimes. Golosov et al. (2014) consider many non-renewable and renewable resources being used simultaneously and in their numerical example they use a CES-type of energy production, but they do not closely investigate the optimal transition path and the role of renewable resources.

[3]Our framework is also close to Vardar (2013), but in this paper we consider a decentralized economy. In addition, we simplify it by neglecting the existence of a perfect substitute backstop and considering that only the two imperfect substitute resources are available.

[4]See Tahvonen and Withagen (1996) and Toman and Withagen (2000) for a comparison of policy outcomes with reversible and irreversible pollution accumulation dynamics.

the social optimum.

The results show that it is always optimal to use both non-renewable and renewable resources simultaneously. Since these resources are imperfect substitutes, there exist situations in which a resource is rational to use even though it is the expensive one.[5,6] Furthermore, as the extraction cost increases it is optimal to reduce the share of non-renewables and substitute renewables in production over time. As the non-renewable resource use approaches zero, the economy converges to a steady state in which only the renewable resource is used. Remarkably, the decentralized economy converges to the same steady state in terms of capital and consumption whether there is a regulator intervention or not. What matters for welfare, however, is the speed at which the economy approaches the clean state - the energy transition. The optimal energy transition depends on the level of environmental damages that the society is ready to accept in the long run.

In the laissez-faire economy the profit-motivated firms do not internalize the environmental damages of non-renewable resource extraction. Although the laissez-faire equilibrium path also converges to the clean state in the long run, the households consume more and the firms extract the non-renewable resource rapidly, thus damage the environment more and faster compared to the optimal transition path. The energy transition is slower than the optimum in the absence of regulation. The regulator can correct this market failure by taxing non-renewable resource extraction.

The optimal tax rate is equal to the present value of all future marginal damages caused by the current non-renewable resource extraction. Its formula includes the endogenous net rental rate of capital, the marginal damages to the environment and the marginal utility of consumption. Accordingly, the optimal tax rate depends on the endogenous variables such as the capital stock, consumption, cumulative extraction and the non-renewable resource price, as well as the exogenous factors such as the renewable resource price and the degree of substitution between non-renewable and renewable resources. We investigate how these different factors affect the optimal tax rate. For example, a larger capital stock leads to a lower interest rate -hence a higher value of the future- which in turn makes the environmental quality more valuable and thus raises the optimal tax rate. Greater capital stock also gives the firms more incentive for extraction to fuel a larger economy that has to be corrected by a higher tax rate. Similarly, a higher level of consumption will lead to a lower marginal utility of consumption, more satisfied and fulfilled households will care more about the environmental damages and thus the optimal tax rate will rise. As a general rule, we show that the wedge on non-renewable resource extraction should be tightened when the firms have more

[5]Imperfect substitution captures the technical or geographical constraints in substitution possibilities. For instance, some resources require specific geographical properties and the firms cannot use them even though they are the cheapest. Similarly, in some industries it may take time for new technologies to be adopted, hence the cheap resource cannot be used immediately.

[6]This result is consistent with the historical data on resource use. See Mattusch (2008) who shows that the non-renewable resource use for energy dates back to 371 and 287 BC and until industrial revolution nearly all energy used was renewable.

incentive to use it.

The time profile of optimal taxation depends on the initial state, especially the initial capital stock, and it is either always increasing, decreasing or U-shaped.[7,8] For instance, if the initial capital stock is lower than its long term - steady state value, the tax rate always increases over time. In contrast, if it is greater than its steady state value, the optimal tax is initially set to a very high rate and it falls over time as the economy consumes the over-accumulated capital stock. This result differs from a large number of studies in the literature.

The renewable resource plays a crucial role on both the level and the time profile of optimal taxation. Expensive renewable resources increase the incentive of the firms to extract more non-renewables. Therefore, the optimal tax rate rises and its time profile shifts up when the price of renewable gets higher. The role of the degree of substitution between non-renewable and renewable resources depends on the time period. A strong degree of substitution leads the economy to benefit from the cheaper resource by allocating it in high proportions, and when the price of non-renewable resource exceeds the price of renewable the economy rapidly substitutes the renewable.[9] Consequently, when the degree of substitution is stronger, the optimal tax rate is initially set to a greater value and its time profile is higher in the short and medium run but lower in the long run.

When these results taken as a whole, this paper also relates to the wide literature on the optimal taxation of non-renewable resources with the emphasis on its time profile. Early studies such as Withagen (1994) showed that the socially optimal extraction path consumes less than the laissez-faire path. Hoel and Kverndokk (1996) considered increasing extraction costs and pollution with natural absorption and showed that the tax rate should first rise then fall. Farzin (1996) and Farzin and Tahvonen (1996) showed the taxation path may either be arbitrary, rising or first falling then rising over time. In the last decade, studies such as Goulder and Mathai (2000), Schou (2002), van der Zwaan et al. (2002), Grimaud and Rougé (2005), Groth and Schou (2007), Grimaud and Rougé (2008) and Grimaud et al. (2011) investigated the roles of technical progress, directed technical change, innovation, learning-by-doing and endogenous growth. Belgodere (2009) emphasized that the time path of optimal tax may differ and the replacement of renewables may change the outcome dramatically. There were also other approaches such as Groom et al. (2005) on the role of discount rate and Daubanes and Grimaud (2010) on the role of international heterogenities. More recently, Aghion et al. (2012) stated that increasing taxes are needed to allow the clean tech-

[7] The time profile of optimal taxation's dependence on the initial state is also emphasized in **?**, van der Ploeg and Withagen (2012b, 2014).

[8] Despite that this result is similar to Farzin and Tahvonen (1996), the mechanism leading to it is different. Farzin and Tahvonen consider a depreciating carbon stock in the atmosphere together with irreversible carbon accumulation which leads to different taxation profiles. In the present paper, however, we only consider that non-renewable resource extraction damages the environment in an irreversible way but we may have different taxation profiles according to different initial capital stock and initial cumulative extraction.

[9] As in the Herfindahl (1967) principle.

nologies to overtake the dirty ones. van der Ploeg and Withagen (2012a,b, 2013) and van der Ploeg (2014) studied the relationship between taxes, backstop technology and the Green Paradox. Rezai et al. (2012) presented a comparison of the results on taxation with additive and multiplicative damages and Gaudet and Lasserre (2013) provided an analytical overview of the different types of taxes on non-renewable resources.

The remainder of the paper is structured as follows. Section 2 introduces the model framework, preferences and technology. Section 3 characterizes the social optimum. Section 4 presents the decentralized framework, characterizes the equilibrium path and establishes the optimal taxation rule. Section 5 presents the results of the simulations and Section 6 concludes.

2 The model

Time is continuous and infinite. We consider an infinitely-lived representative household that gains utility by consuming the final good. The economy has two sectors: final good production and resource extraction. In the final good production sector, capital is used with non-renewable and renewable resources which are imperfect substitutes. The extraction of the non-renewable resource damages the environment in an irreversible way which in turn reduces the total welfare.

The instantaneous total welfare ($V(.)$) consists of the household's utility from consumption ($U(.)$) and non-renewable resource extraction's damage to the environment ($D(.)$). We consider the following separable form:

$$V(C, Z) = U(C) - D(Z)$$

where C denotes consumption of the final good and Z denotes cumulative extraction of the non-renewable resource ($Z_t = Z_0 + \int_{s=0}^{t} E_{ds} ds$ with E_{dt} is the instantaneous quantity of extraction). Following the standard practice, the utility of consumption is increasing and strictly concave in C ($U_C(C) > 0$ and $U_{CC}(C) < 0$) and the damage of cumulative extraction is increasing and strictly convex in Z ($D_Z(Z) > 0$ and $D_{ZZ}(Z) > 0$)[10]. These properties are satisfied in the following form for $V(.)$:

$$V(C, Z) = \frac{C^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} - \frac{\phi_d}{2} Z^2 \text{ , } \sigma > 0 \text{ and } \phi_d > 0$$

Production of the final good requires capital (K) and energy (E). Energy is obtained from non-renewable (dirty) (E_d) and renewable (clean) (E_c) resources which are imperfect substitutes. The function $H(.)$ captures the imperfect substitution and it is in CES form, $H(E_d, E_c) = (\gamma_d E_d^{1-\frac{1}{\epsilon}} + \gamma_c E_c^{1-\frac{1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}}$ where $\gamma_d > 0$ and $\gamma_c > 0$ represent the weights of non-renewable and renewable re-

[10]In the rest of the text the subscript for a function denotes its derivative respect to a variable or argument of the function. For example, $f_1(.)$ denotes the derivative of function f respect to its first argument and $f_x(.)$ denotes the derivative of function f respect to the variable x.

sources in production respectively with $\gamma_d + \gamma_c = 1$.[11] The parameter ϵ denotes the degree of substitution; since we consider imperfect substitution it requires to assume $\epsilon > 1$. The production function $F(.)$ is Cobb-Douglas, $F(K, E) = K^\alpha E^\beta$ where $\alpha > 0$ and $\beta > 0$ are the output elasticities of capital and energy in production respectively, and $\alpha + \beta \leq 1$. Embedding the different types of resources for energy in the production function leads to the following form:

$$F(K, H(E_d, E_c)) = K^\alpha((\gamma_d E_d^{1-\frac{1}{\epsilon}} + \gamma_c E_c^{1-\frac{1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}})^\beta$$

The extraction cost depends on the level of cumulative extraction (Z) and it is increasing and strictly convex in Z ($G_Z(Z) > 0$ and $G_{ZZ}(Z) > 0$). We consider the following specification:

$$G(Z) = \frac{\phi_g}{2} Z^2 \ , \phi_g > 0$$

3 Social optimum

The social planner solves the following problem:

$$\max_{\{C_t, E_{dt}, E_{ct}\}} \int_{t=0}^{\infty} e^{-\rho t}(U(C_t) - D(Z_t))dt$$

$$\dot{K}_t = F(K_t, H(E_{dt}, E_{ct})) - G(Z_t)E_{dt} - \pi_c E_{ct} - C_t \tag{1}$$

$$\dot{Z}_t = E_{dt} \tag{2}$$

$$C_t, E_{dt}, E_{ct} \geq 0 \ \forall t$$

$$\text{with } K_0 > 0 \text{ and } Z_0 > 0 \text{ are given.}$$

where the price of renewable resource is exogenously given and denoted as π_c. The current-value Hamiltonian function associated to this problem is:

$$\mathcal{H}_t^{SO} = U(C_t) - D(Z_t) + \lambda_t \left(F(K_t, H(E_{dt}, E_{ct})) - G(Z_t)E_{dt} - \pi_c E_{ct} - C_t\right) - \mu_t E_{dt}$$

where λ_t denotes the co-state variable associated to capital and is interpreted as the shadow value of capital. Similarly, μ_t denotes the co-state variable associated to cumulative extraction and is interpreted as the shadow value of non-renewable resource.

[11]The use of CES functional form is a proper way to capture the imperfect substitution between resources. It leads to different marginal productivities for resources which is a key point for optimal allocation. For instance, making a unit investment in renewables by installing a new solar panel in a region where there is not enough sunshine and making a unit investment in non-renewables by building a new well in a resource-rich region will not have the same effect on aggregate production. The function $H(.)$ allows us to capture these realities. These technological characteristics are embedded in the degree of substitution parameter (ϵ).

The necessary conditions for an optimum are:

$$U_C(C_t) = \lambda_t \tag{3}$$

$$E_{dt} \geq 0, \qquad \lambda_t \left(F_2(K_t, H(E_{dt}, E_{ct}))H_{E_d}(E_{dt}, E_{ct}) - G(Z_t)\right) - \mu_t \leq 0 \tag{4}$$

$$E_{ct} \geq 0, \qquad F_2(K_t, H(E_{dt}, E_{ct}))H_{E_c}(E_{dt}, E_{ct}) - \pi_c \leq 0 \tag{5}$$

$$\dot{\lambda}_t = (\rho - F_1(K_t, H(E_{dt}, E_{ct})))\,\lambda_t \tag{6}$$

$$\dot{\mu}_t = \rho\mu_t - \lambda_t G_Z(Z_t)E_{dt} - D_Z(Z_t) \tag{7}$$

$$\lim_{t\to+\infty} e^{-\rho t}\lambda_t = 0 \tag{8}$$

$$\lim_{t\to+\infty} e^{-\rho t}\mu_t = 0 \tag{9}$$

together with the equations (1) and (2). In order to proceed on resolution, we define the price of non-renewable resource as follows:

$$\pi_{dt} = G(Z_t) + \mu_t/\lambda_t \tag{10}$$

This definition states that the price of non-renewable resource is endogenously determined by the extraction cost and the shadow value of non-renewable resource in capital units, in other words, the rent of resource. By taking the time derivative of (10) and using (2), (3), (6) and (7), we obtain the law of motion of π_d given by:

$$\dot{\pi}_{dt} = F_1(K_t, H(E_{dt}, E_{ct}))\,(\pi_{dt} - G(Z_t)) - D_Z(Z_t)/U_C(C_t) \tag{11}$$

Equation (11) is the modified Hotelling rule. Using (10) we can rewrite condition (4) as:

$$E_{dt} \geq 0, \qquad F_2(K_t, H(E_{dt}, E_{ct}))H_{E_d}(E_{dt}, E_{ct}) - \pi_{dt} \leq 0 \tag{4'}$$

Conditions (4′) and (5) are complementary slackness (c.s.) conditions and they show that a type of resource will be used if its marginal productivity is equal to its price. Since non-renewable and renewable resources are imperfect substitutes the economy always uses both simultaneously, that is, the equalities in conditions (4′) and (5) always hold. This property allows us to solve $F_2(.)H_{E_d}(.) = \pi_d$ and $F_2(.)H_{E_c}(.) = \pi_c$ and obtain the optimal amounts of non-renewable and renewable resources ($E^*_{dt}(K_t, \pi_{dt}, \pi_c)$ and $E^*_{ct}(K_t, \pi_{dt}, \pi_c)$). Using these results together with conditions $(1 - 11)$ we find the differential equation system in $(K_t, Z_t, C_t, \pi_{dt})$. Optimal trajectories should satisfy the following:

$$\dot{K}_t = F(K_t, H(E^*_{dt}(.), E^*_{ct}(.))) - G(Z_t)E^*_{dt}(.) - \pi_c E^*_{ct}(.) - C_t \tag{12}$$

$$\dot{Z}_t = E^*_{dt}(K_t, \pi_{dt}, \pi_c) \tag{13}$$

$$\dot{C}_t / C_t = \sigma(F_1(K_t, H(E^*_{dt}(.), E^*_{ct}(.))) - \rho) \tag{14}$$

$$\dot{\pi}_{dt} = F_1(K_t, H(E^*_{dt}(.), E^*_{ct}(.)))(\pi_{dt} - G(Z_t)) - D_Z(Z_t)/U_C(C_t) \tag{15}$$

As the price of non-renewable resource increases the economy reduces its share in production, and eventually converges to a regime in which it uses only the renewable resource. To find the state that the economy converges, we need to compute the marginal productivity of energy since the optimal resource allocation and its path depends on it. In the optimum, it reduces to $F_2(K_t, H(E^*_{dt}(.), E^*_{ct}(.))) = (\gamma_d^\epsilon \pi_{dt}^{1-\epsilon} + \gamma_c^\epsilon \pi_c^{1-\epsilon})^{\frac{1}{1-\epsilon}}$ which leads us to define the *energy price index* as follows:

Definition 1. *Let π_H be the energy price index given by:*

$$\pi_{Ht}(\pi_{dt}, \pi_c) = (\gamma_d^\epsilon \pi_{dt}^{1-\epsilon} + \gamma_c^\epsilon \pi_c^{1-\epsilon})^{\frac{1}{1-\epsilon}}$$

The energy price index has a limit for a given renewable price. As the non-renewable price increases it tends to a constant, $\lim_{\pi_{dt} \to +\infty} \pi_{Ht}(\pi_{dt}, \pi_c) = \pi_c \gamma_c^{\frac{\epsilon}{1-\epsilon}}$. The economy that simultaneously uses the non-renewable and renewable resources will asymptotically converge to the regime in which it uses only the renewable resource with the following conditions:

$$\lim_{t \to +\infty} \pi_{dt} - G(Z_t) = 0 \tag{16}$$

$$\lim_{t \to +\infty} \pi_{Ht}(\pi_{dt}, \pi_c) = \pi_c \gamma_c^{\frac{\epsilon}{1-\epsilon}} \tag{17}$$

$$\lim_{t \to +\infty} C^S(K_t, Z_t) - C^R(K_t) = 0 \tag{18}$$

Condition (16) is derived from the definition of non-renewable resource price (10). It means that the shadow value of non-renewable resource must approach zero, thus the resource rent must vanish over time. Condition (17) ensures that the energy price index approaches its limit, thus the non-renewable use approaches zero. In condition (18), $C^S(.)$ and $C^R(.)$ denote the optimal consumption as a function of the state variables in simultaneous use regime and only renewable use regime respectively. This condition is to ensure that the state (K, Z) and co-state (λ, μ) variables of the optimal control problem cannot jump, thus the economy will have a continuous path of consumption, capital and resource use over time.

In the regime that the economy converges, the production function reduces to $F(K, H(E_d =$

$0, E_c)) = \hat{F}(K, E_c) = K^{\alpha} E_c^{\beta} \gamma_c^{\frac{\beta\epsilon}{\epsilon-1}}$. Using condition (5), we obtain the optimal amount of renewable resource use, $E_c^{\star}(K)$. Finally, conditions (3, 5 and 6) allow us to obtain the differential equation system in (K, C). Optimal trajectories of the asymptotic clean regime should satisfy the following:

$$\dot{K}_t = F(K_t, H(0, E_{ct}^{\star}(K_t))) - \pi_c E_{ct}^{\star}(K_t) - C_t \tag{19}$$

$$\dot{C}_t / C_t = \sigma(F_1(K_t, H(0, E_{ct}^{\star}(K_t))) - \rho) \tag{20}$$

$$\dot{Z}_t = 0 \tag{21}$$

$$\dot{\pi}_{dt} = 0 \tag{22}$$

As the extraction of non-renewable resource approaches zero, the dynamics of endogenous variables in equations (12 – 15) will approach the above differential equation system (19 – 22). This system has a stationary point (K^{ss}, C^{ss}) which can be obtained by solving the following equations:

$$F_1(K^{ss}, H(0, E_c^{\star}(K^{ss}))) = \rho \tag{23}$$

$$C^{ss} = F(K^{ss}, H(0, E_c^{\star}(K^{ss}))) - \pi_c E_c^{\star}(K^{ss}) \tag{24}$$

The system given in equations (19 – 20) has a unique trajectory that leads to the steady state (K^{ss}, C^{ss}). This unique trajectory allows us to find the optimal consumption rule in the asymptotic regime, $C^R(K)$, which we referred in condition (18).

Proposition 1. *The economy gradually reduces the share of the non-renewable resource and eventually converges to a regime in which it uses only the renewable. There exists a unique optimal path* $\{K_t, Z_t, C_t, \pi_{dt}\}_{t=0}^{\infty}$ *starting from any initial state* $K_0 > 0, Z_0 > 0$ *that follows the dynamics in equations (12-15) and satisfies conditions (16-18). This path converges to the steady state* (K^{ss}, C^{ss}) *given in (23,24).*

Proof. See Appendix A1. □

According to Proposition 1, the optimal path of non-renewable and renewable resource use is $\left\{E_{dt}^{*}(K_t, \pi_{dt}, \pi_c), E_{ct}^{*}(K_t, \pi_{dt}, \pi_c)\right\}_{t=0}^{\infty}$. Consequently, the optimal path of renewable use converges to $\{E_c^{\star}(K_t)\}_{t=0}^{\infty}$.

4 Equilibrium analysis

Let us turn to the equilibrium analysis. We assume that there are large number of final good producing firms and resource extraction firms which produce with the same technology and there is perfect competition in all markets. We first investigate the optimal behavior of agents and then characterize the equilibrium. This will allow us to study the effects of taxes on the decentralized economy as well as to obtain the optimal taxation rule that leads to the socially optimal transition path.

4.1 Household

The representative household solves:

$$\begin{aligned} \max_{\{C_t\}} \int_{t=0}^{\infty} & e^{-\rho t}(U(C_t))dt \\ s.t.\ \dot{K}_t &= r_t K_t + \Pi_t + T_t - C_t \\ C_t &\geq 0\ \forall t \\ \text{with } K_0 &> 0 \text{ is given.} \end{aligned} \tag{25}$$

where r_t is the net rental rate of capital, Π_t profits from the ownership of final good producing firms and resource extraction firms. T_t denotes the government transfers and it is equal to the total tax revenues.

The current-value Hamiltonian function associated to this problem is given by:

$$\mathcal{H}_t^H = U(C_t) + \lambda_t \left(r_t K_t + \Pi_t + T_t - C_t\right)$$

First order conditions for optimality are:

$$U_C(C_t) = \lambda_t \tag{26}$$

$$\dot{\lambda}_t/\lambda_t = \rho - r_t \tag{27}$$

$$\lim_{t\to+\infty} e^{-\rho t}\lambda_t = 0 \tag{28}$$

Solving (26) and (27) gives the following well-known Ramsey rule for consumption:

$$\dot{C}_t/C_t = \sigma(r_t - \rho) \tag{29}$$

4.2 Final good producing firm

The representative final good producing firm aims to maximize its profits for given input prices. The programme of the firm is:

$$\max_{\{K_t, E_{dt}, E_{ct}\}} F(K_t, H(E_{dt}, E_{ct})) - r_t K_t - \pi_{dt} E_{dt} - \pi_c E_{ct}$$

In the case where the resource prices are positive and finite ($\pi_{dt} \in (0, +\infty)$ and $\pi_c \in (0, +\infty)$) the firm uses both type of resources simultaneously. The first order conditions are:

$$F_1(K_t, H(E_{dt}, E_{ct})) = r_t \tag{30}$$

$$F_2(K_t, H(E_{dt}, E_{ct}))H_{E_d}(E_{dt}, E_{ct}) = \pi_{dt} \tag{31}$$

$$F_2(K_t, H(E_{dt}, E_{ct}))H_{E_c}(E_{dt}, E_{ct}) = \pi_c \tag{32}$$

By solving (31) and (32) we obtain the optimal amount of non-renewable and renewable resources ($E^\star_{dt}(K_t, \pi_{dt}, \pi_c)$ and $E^\star_{ct}(K_t, \pi_{dt}, \pi_c)$). As a result, the condition for firm profit maximization reduces to:

$$F_1(K_t, H(E^\star_{dt}(.), E^\star_{ct}(.))) = r_t \tag{33}$$

4.3 Non-renewable resource extracting firm

The representative non-renewable resource extracting firm maximizes the discounted value of its intertemporal profits by taking into account the tax rate and the extraction cost which increases by cumulative extraction. We introduce the taxation of non-renewable resource as the amount paid per unit of extraction. This application can be considered as a wedge on non-renewable resource extraction in this decentralized economy. The extraction firm solves the following problem:

$$\begin{aligned} \max_{\{E_{dt}\}} \int_{t=0}^{\infty} e^{-R_t}((\pi_{dt} - \tau_t)E_{dt} - G(Z_t)E_{dt})dt \\ s.t.\ \dot{Z}_t = E_{dt} \\ E_{dt} \geq 0\ \forall t \\ \text{with } Z_0 > 0 \text{ is given.} \end{aligned} \tag{34}$$

where τ_t denotes the per unit tax rate of the resource at time t and R_t denotes cumulative interest rate as $R_t = \int_0^t r_s ds$. The current-value Hamiltonian function associated to this problem is:

$$\mathcal{H}^{EX}_t = (\pi_{dt} - \tau_t)E_{dt} - G(Z_t)E_{dt} - \mu_t E_{dt}$$

The first order conditions for optimality will be as follows:

$$\pi_{dt} = G(Z_t) + \tau_t + \mu_t \tag{35}$$

$$\dot{\mu}_t = r_t\mu_t - G_Z(Z_t)E_{dt} \tag{36}$$

$$\lim_{t\to+\infty} e^{-R_t}\mu_t = 0 \tag{37}$$

In equation (35), the taxation appears as a driver of the non-renewable resource price together with the extraction cost and resource rent. We solve (35) and (36) to obtain the optimal law of motion of π_d:

$$\dot{\pi}_{dt} = \dot{\tau}_t + r_t(\pi_{dt} - G(Z_t) - \tau_t) \tag{38}$$

The relationship in equation (38) is the Hotelling rule for the market economy. It depends on the net rental rate of capital, extraction cost and taxation. It also shows that the regulator has to determine both the level and the time profile of taxation in order to control the resource extraction. Note that it reduces to the standard Hotelling rule in the absence of taxation and extraction cost.

4.4 Equilibrium

The competitive equilibrium definition we consider is as follows:

Definition 2. *Given the time profile of taxation $\{\tau_t\}_{t=0}^{\infty}$, initial capital stock (K_0) and cumulative extraction (Z_0), the intertemporal competitive equilibrium is such that*

i the time profiles of consumption $\{C_t\}_{t=0}^{\infty}$, capital stock $\{K_t\}_{t=0}^{\infty}$ and net rental rate of capital $\{r_t\}_{t=0}^{\infty}$ maximize the discounted value of household's intertemporal utility, thus (25) and (29) hold for each t,

ii the time profiles of capital stock $\{K_t\}_{t=0}^{\infty}$, non-renewable resource price $\{\pi_{dt}\}_{t=0}^{\infty}$, net rental rate of capital $\{r_t\}_{t=0}^{\infty}$, non-renewable resource use $\{E_{dt}\}_{t=0}^{\infty}$ and renewable resource use $\{E_{ct}\}_{t=0}^{\infty}$ maximize the final good producing firm's profit at each instant, thus (33) holds for each t,

iii the time profiles of net rental rate of capital $\{r_t\}_{t=0}^{\infty}$, non-renewable resource price $\{\pi_{dt}\}_{t=0}^{\infty}$, non-renewable resource use $\{E_{dt}\}_{t=0}^{\infty}$ and cumulative extraction $\{Z_t\}_{t=0}^{\infty}$ maximize the discounted value of extraction firm's intertemporal profits, thus (34) and (38) hold for each t.

Using equations (25, 29, 33, 34, 38) we obtain the differential equation system in (K, Z, C, π_d) that denotes the law of motion for endogenous variables in the intertemporal equilibrium. This system is given by:

$$\dot{K}_t = F(K_t, H(E^{\star}_{dt}(.), E^{\star}_{ct}(.))) - G(Z_t)E^{\star}_{dt}(.) - \pi_c E^{\star}_{ct}(.) - C_t \tag{39}$$

$$\dot{Z}_t = E^{\star}_{dt}(K_t, \pi_{dt}, \pi_c) \tag{40}$$

$$\dot{C}_t / C_t = \sigma(F_1(K_t, H(E^{\star}_{dt}(.), E^{\star}_{ct}(.))) - \rho) \tag{41}$$

$$\dot{\pi}_{dt} = \dot{\tau}_t + F_1(K_t, H(E^{\star}_{dt}(.), E^{\star}_{ct}(.)))(\pi_{dt} - G(Z_t) - \tau_t) \tag{42}$$

Let us first consider the decentralized equilibrium in the absence of taxation. We define the laissez-faire economy as the tax rate on non-renewable resource being zero for all t ($\tau_t = 0\ \forall t$).

Proposition 2. *In the laissez-faire economy, there exists a unique equilibrium path that is given by $\{K_t, Z_t, C_t, \pi_{dt}\}_{t=0}^{\infty}$ starting from any initial state $K_0 > 0, Z_0 > 0$ that follows the dynamics in equations (39-42) and satisfies conditions (16-18). This path converges to the steady state (K^{ss}, C^{ss}) given in (23,24).*

Proof. See Appendix A2. □

As the price of non-renewable resource increases, the firms gradually reduce the share of non-renewable resource and the dynamics of endogenous variables in equations (39-42) approaches (19-22). Proposition 2 shows that the final state is not affected by the absence of regulation. Sooner or later the market economy converges to the clean production state as well. However, the speed of transition, which also determines the level of environmental damages, is driven by the regulator intervention.

This is an interesting result concerning the welfare implications of public policy on the energy transition. The market economy rationally responds to the increasing extraction cost of non-renewable resource, therefore the firms reduce non-renewable resource extraction and allocate more renewables in production over time also on the equilibrium path. On this equilibrium path, since the negative externalities of extraction are not internalized, the households consume more and the firms extract the non-renewable resource faster and thus damage the environment more and faster in the absence of regulation. The regulator can correct this market failure by introducing taxation on non-renewable resource extraction. Even though the final state that the economy converges is identical, public policy induces the decentralized economy to the optimal transition path, hence leads to a higher welfare level.

4.5 Optimal taxation of non-renewable resource

We characterized the equilibrium path of the decentralized economy in the absence of taxation in Proposition 2. The optimal path of taxation is the one that induces this equilibrium path to be equivalent to the social optimum which we characterized in Proposition 1. In order to obtain the optimal taxation rule, we consider the social optimum given in equations (12-15) and compare it with the decentralized equilibrium given in equations (39-42). The first three equations are equivalent in both system of differential equations. The taxation scheme, therefore, is optimal if (42) is equivalent to (15), that is:

$$\dot{\tau}_t^{\star} + F_1(.)(\pi_{dt} - G(Z_t) - \tau_t^{\star}) = F_1(.)(\pi_{dt} - G(Z_t)) - D_Z(Z_t)/U_C(C_t) \tag{43}$$

Rearranging (43) gives the following law of motion for the optimal tax rate:

$$\dot{\tau}_t^\star = F_1(K_t, H(E_{dt}^*(.), E_{ct}^*(.)))\tau_t^\star - D_Z(Z_t)/U_C(C_t) \tag{44}$$

Now we can write the optimal tax rate which is given in Proposition 3 as follows:

Proposition 3. *The optimal tax rate of non-renewable resource at a given time t is:*

$$\tau_t^\star = \int_t^\infty e^{-\int_t^s F_1(K_u, H(E_{du}^*(K_u,\pi_{du},\pi_c), E_{cu}^*(K_u,\pi_{du},\pi_c)))du} D_Z(Z_s)/U_C(C_s)ds \tag{45}$$

When the tax profile is $\{\tau_t^\star\}_{t=0}^\infty$, there exists a unique equilibrium path which is identical to the optimal path that converges to the steady state (K^{ss}, C^{ss}) given in (23,24).

Proof. Equation (45) is a direct conclusion of solving equation (44). The fact that the equilibrium path is identical to the optimal path is guaranteed by the comparison of (12-15) and (39-42) and taking into account (43). The rest of the proof follows the same procedure of Proposition 1 in Appendix A1. □

Proposition 3 shows that the optimal tax rate is a forward-looking variable. One unit of non-renewable resource should be taxed at a rate which is equivalent to the present value of all future marginal damages occurred by itself. The determinants of the optimal tax rate are the net rental rate of capital (marginal productivity of capital), the marginal damage to the environment and the marginal utility of consumption. The formula for the optimal taxation on the non-renewable resource (45) has several similarities to the well-known literature. It, however, has new ingredients (such as the degree of substitution between resources and the resource prices) that provide useful and new insights about the different channels that affect the optimal taxation.

How does the optimal tax rate change with respect to the endogenous variables? For this, we investigate the signs of partial derivatives which are given in Proposition 4:[12]

Proposition 4. *The optimal tax rate is increasing in* (*i*) *the level of consumption* ($\partial\tau_t^\star/\partial C_t > 0$), (*ii*) *the stock of capital* ($\partial\tau_t^\star/\partial K_t > 0$), (*iii*) *cumulative extraction* ($\partial\tau_t^\star/\partial Z_t > 0$) *and* (*iv*) *the price of non-renewable resource* ($\partial\tau_t^\star/\partial\pi_{dt} > 0$).

[12]Even though we have a differential equation system in which the dynamics of variables evolve endogenously, this application provides useful information to identify how each individual variable drives the optimal taxation rule.

Proof. For notational ease, let $r(K_t, \pi_{dt}) = F_1(K_t, H(E^*_{dt}(K_t, \pi_{dt}, \pi_c), E^*_{ct}(K_t, \pi_{dt}, \pi_c)))$. Using (45):

(i) $\dfrac{\partial \tau^\star_t}{\partial C_t} = -e^{-r(K_t,\pi_{dt})} D_Z(Z_t) \dfrac{U_{CC}(C_t)}{(U_C(C_t))^2} > 0$ since $D_Z(Z_t) > 0$ and $U_{CC}(C_t) < 0$.

(ii) $\dfrac{\partial \tau^\star_t}{\partial K_t} = -e^{-r(K_t,\pi_{dt})} r_K(K_t, \pi_{dt}) \dfrac{D_Z(Z_t)}{U_C(C_t)} > 0$ since $r_K(K_t, \pi_{dt}) < 0$, $D_Z(Z_t) > 0$ and $U_C(C_t) > 0$.

(iii) $\dfrac{\partial \tau^\star_t}{\partial Z_t} = e^{-r(K_t,\pi_{dt})} \dfrac{D_{ZZ}(Z_t)}{U_C(C_t)} > 0$ since $D_{ZZ}(Z_t) > 0$ and $U_C(C_t) > 0$.

(iv) $\dfrac{\partial \tau^\star_t}{\partial \pi_{dt}} = -e^{-r(K_t,\pi_{dt})} r_{\pi_d}(K_t, \pi_{dt}) \dfrac{D_Z(Z_t)}{U_C(C_t)} > 0$ since $r_{\pi_d}(K_t, \pi_{dt}) < 0$, $D_Z(Z_t) > 0$ and $U_C(C_t) > 0$

□

Proposition 4 asserts that there are four endogenous channels that affect the optimal tax rate of the non-renewable resource. We call the first channel as the *fulfillment effect*: as the household gets more satisfaction in consumption, she will care more about the environment. The household's marginal utility of consumption falls as the level of consumption rises which in turn increases the household's care for the environment for a given level of cumulative extraction. As a result, the optimal tax rate of non-renewable resource increases with the level of consumption.

The second channel is the *discounting effect*: changes in the net rental rate of capital alters the interest rate. The net rental rate of capital falls as the stock of capital increases which in turn leads to a lower interest rate. The value of the future becomes higher, thus the care about the environment rises as well. Therefore the optimal tax rate increases with the stock of capital.

One remark about the role of the capital stock on the optimal tax rate is worth to be mentioned. An economy with a larger stock of capital indeed requires a higher amount of energy to fuel the production, thus there will be stronger incentive for the firms to extract more non-renewable resource. To correct this incentive, the regulator should tighten the wedge on extraction and thus increase the tax rate on the non-renewable resource.

The third channel is the *direct environment effect*: more cumulative extraction makes the marginal damage to the environment to be higher. The value of one unit of extraction's marginal damage rises which in turn increases the optimal tax rate. The more the cumulative extraction is, the worse the environmental status is, and therefore the higher the optimal tax rate is.

The fourth channel is the *non-renewable price effect*: an increase in the non-renewable price decreases the net rental rate of capital thus the interest rate falls. The value of future rises due to the lower discounting, which is similar to the discounting effect. Therefore the optimal tax rate increases with the price of non-renewable resource.

These results show that the time profile of optimal tax rate is either always increasing, decreasing or U-shaped depending on the initial state of the economy. We know that the capital stock and consumption both rise over time if the initial capital stock is less than its steady state value, or vice versa. Besides, the non-renewable resource price and cumulative extraction are always increasing by definition. Therefore, the optimal tax rate is always increasing over time if the initial capital stock is less than its steady state value. If the initial capital stock is too large (greater than its steady state value), however, the tax rate will have a decreasing and U-shaped time profile. We shall investigate the different time profiles of optimal taxation in the numerical analysis section.

Let us now turn to the effect of the exogenous factors. What are the roles of the renewable resource and the degree of substitution on the optimal taxation of non-renewable resource? Proposition 5 addresses this question as follows:

Proposition 5. *For a given level of consumption, capital stock, cumulative extraction and non-renewable resource price,* (*i*) *the optimal tax rate is decreasing in the degree of substitution between non-renewable and renewable resources* ($\partial\tau_t^\star/\partial\epsilon < 0$) *and* (*ii*) *it is increasing in the price of renewable resource* ($\partial\tau_t^\star/\partial\pi_c > 0$)*.*

Proof. For notational ease, let $\hat{r}(\epsilon,\pi_c) = F_1(K_t, H(E^*_{dt}(.), E^*_{ct}(.)))$. Again using (45):

(*i*) $\dfrac{\partial\tau_t^\star}{\partial\epsilon} = -e^{-\hat{r}(\epsilon,\pi_c)}\hat{r}_\epsilon(\epsilon,\pi_c)\dfrac{D_Z(Z_t)}{U_C(C_t)} < 0$ since $\hat{r}_\epsilon(\epsilon,\pi_c) > 0$ (for $\epsilon > 0$), $D_Z(Z_t) > 0, U_C(C_t) > 0$.

(*ii*) $\dfrac{\partial\tau_t^\star}{\partial\pi_c} = -e^{-\hat{r}(\epsilon,\pi_c)}\hat{r}_{\pi_c}(\epsilon,\pi_c)\dfrac{D_Z(Z_t)}{U_C(C_t)} > 0$ since $\hat{r}_{\pi_c}(\epsilon,\pi_c) < 0$, $D_Z(Z_t) > 0, U_C(C_t) > 0$.

□

A strong degree of substitution between non-renewables and renewables allows the economy to benefit from price differences and allocate the cheap resource in high amounts in production. In the optimum, for a given stock of capital and menu of resource prices, the total energy use rises if the degree of substitution is stronger. This leads the marginal productivity of capital to rise and so does the interest rate. As a consequence of the discounting effect, the optimal tax rate falls if the degree of substitution is stronger.

Higher renewable price, in contrast, reduces the net rental rate of capital hence the interest rate falls. It also gives incentive to firms to extract a larger amount of the non-renewable resource, hence the wedge on extraction should be tightened. Therefore, again due to the discounting effect, the optimal tax rate increases with the price of renewable resource.

One should note that the analysis in Proposition 5 are for the optimal tax rate at a given time taking into account other variables are constant. They reveal how each individual factor affects the current tax rate. Considering the time path of the optimal taxation, all the endogenous variables' paths are modified according to a change in one of these structural factors. For example, a stronger degree of substitution may lead to a higher initial non-renewable price determination as well as to a higher initial level of consumption on the socially optimal path. In this case the impact on the optimal tax rate will depend on whether the negative channel (higher degree of substitution) or the positive channels (non-renewable price effect and fulfillment effect) dominate. The effect of these changes are not straightforward and we shall investigate these in detail by using numerical analysis in the following section.

5 Numerical analysis

This section illustrates the theoretical results we obtained in the previous sections. The aim is to investigate the differences between the social optimum and the laissez-faire equilibrium paths, the different time profiles of optimal taxation according to the initial state of the economy, the role of the renewable resource and the role of the degree of substitution between non-renewable and renewable resources.

5.1 Calibration

The elasticity of capital in production is set to $\alpha = 0.2$ and the elasticity of energy in production to $\beta = 0.1$. We set equal weights to the non-renewable and renewable resources in production, that is $\gamma_d = \gamma_c = 0.5$. The discount rate is set to $\rho = 0.02$ and the elasticity of intertemporal substitution to $\sigma = 0.5$, hence the relative risk aversion coefficient to be $1/\sigma = 2$. The renewable resource price is $\pi_c = 4$, the parameter of marginal cost of extraction is $\phi_g = 0.1$ and the parameter of marginal damages to the environment is $\phi_d = 0.0003$. Finally, the degree of substitution is $\epsilon = 3$ when we consider it low, and $\epsilon = 10$ when we consider it high.[13]

5.2 Social optimum vs. Laissez-faire

We investigate the differences between the social optimum and laissez-faire equilibrium path which are illustrated in fig. 1.[14] In the laissez-faire, the damages of extraction are not internalized by the firms and there is no regulator intervention. The firms thus extract a larger amount of the non-renewable resource compared to the social optimum which results in less renewable use over time (fig. 1(a,d)). Indeed, the reason of higher amount of extraction is that the price of non-renewable resource is lower and rises slower due to the absence of taxation (fig. 1(b,e)). In addition, lower

[13]This calibration setting is similar to the one in van der Ploeg and Withagen (2014) and Vardar (2013).

[14]In particular, the solid lines represent the case which is characterized in Proposition 1, and the dashed lines represent a specific case of the equilibrium paths without taxation on the non-renewable resource, which is characterized in Proposition 2.

price and higher extraction cost leads to a lower rent of non-renewable resource over time in the laissez-faire case (fig. 1(e)).

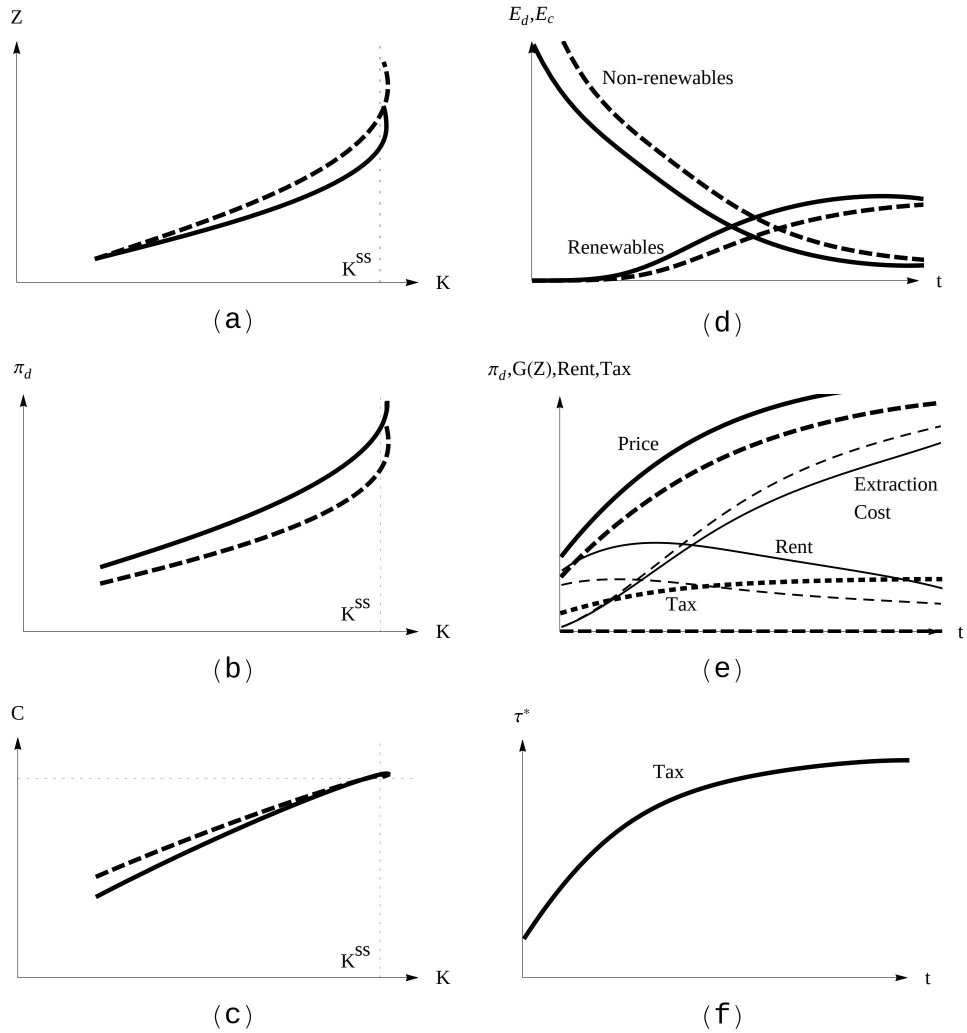


Figure 1: Social optimum (solid lines) vs. Laissez-faire (dashed lines)

Despite the fact that there is no regulation, the market economy converges to the same steady state as the socially optimal path. The consumption is higher in the short and medium run, but converges to the socially optimal level in the long run (fig. 1(c)). Notwithstanding larger amount of extraction in the laissez-faire case, the firms reduce the share of non-renewable resource and the market economy also converges to a steady state in which it uses only the renewable resource. The speed of this transition, however, is slower compared to the socially optimal one (fig. 1(a,c,d)). Accordingly, the damages to the environment are higher which leads to a lower level of welfare. This is an illustration of the fact that taxation of non-renewable resources does not affect the final state of the economy. However, it drives the speed at which the decentralized economy approaches the clean state - the energy transition.

5.3 Time profile of optimal taxation

The speed of transition to the clean state, which depends on the taxation profile, determines the level of environmental damages hence the welfare outcome. For the specific example of the initial state in fig. 1(f), the optimal tax rate monotonically increases and approaches a constant value. But is this the case for all possible initial states? The answer is no. The optimal tax rate can be either increasing, decreasing or U-shaped depending on the initial capital stock and cumulative extraction.

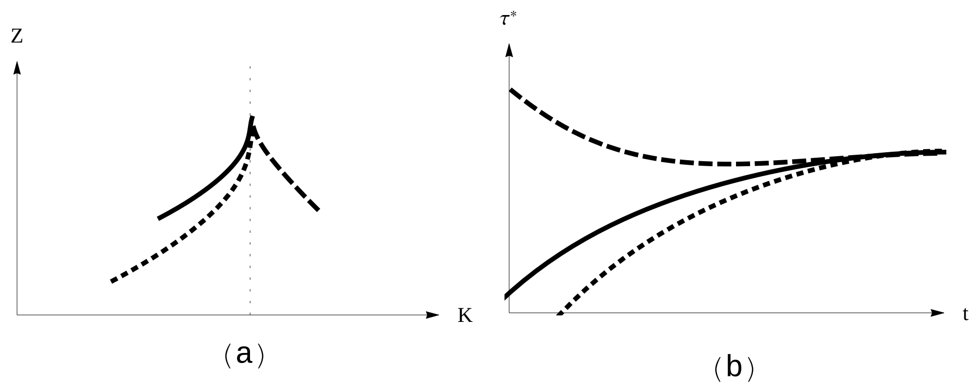


Figure 2: Different time profiles of the optimal tax rate

Fig. 2 illustrates some examples of different taxation profiles. The initial state of the economy plays a crucial role on the time profile of the optimal tax rate. In the case where the initial capital stock is lower than its steady state level, the economy is always accumulating capital and increasing consumption. The four (positive) channels, which were introduced in Proposition 4, affect the optimal tax rate and thus it always increases over time. The initial optimal tax rate and its time profile rises if the initial capital stock as well as the initial cumulative extraction are larger, and vice versa. In contrast, if the initial capital stock is greater than its steady state value, the households consume the over-accumulated capital which leads to a decreasing capital and consumption over time. The initial consumption level is too high and the initial net rental rate of capital is too low. Thus the optimal tax rate is initially high due to the fulfillment and discounting effects. As the capital stock melts away and consumption decreases, the optimal tax rate also decreases over time. This path may have a U-shaped profile if the direct environment effect and non-renewable price effect dominate the other two at a future date since the non-renewable price and cumulative extraction are always increasing. Another interesting result is that the optimal tax rate approaches a unique constant value for any given initial state (as the economy converges to the unique steady state in the long run).

5.4 The role of the degree of substitution between resources

We investigate the role of the degree of substitution by using fig. 3. We depict two cases: strong degree of substitution (the solid lines) and weak degree of substitution (the dashed lines).

When the degree of substitution is strong, the firms extract a greater amount of non-renewable resource until its price reaches the price of renewable, then they use more renewable resource compared to the weak degree of substitution case. Eventually the market economy converges to a steady state with a larger capital stock and higher consumption (fig. 3(a,c,d)). The effect on the price of non-renewable resource depends on the time period. In the case of strong degree of substitution, the price of non-renewable is initially set to a higher value and rises more rapidly in the short and medium run. But it rises slower in the long run as the extraction reduces swiftly after the renewable becomes the cheaper resource. Consequently, the non-renewable resource is cheaper in the long run when the degree of substitution is strong (fig. 3(b,e)).

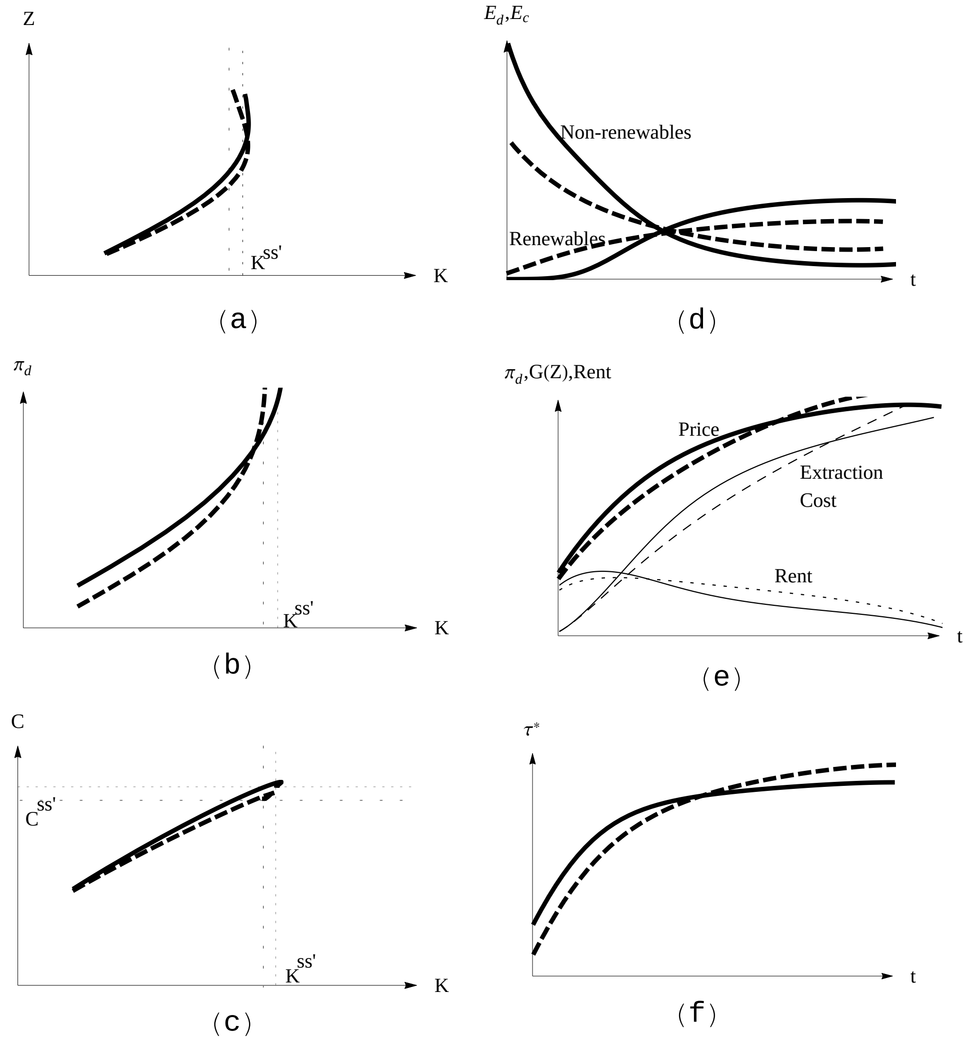
Due to these dynamics, the effect of the degree of substitution on the time profile of optimal taxation also depends on the time period. With strong degree of substitution, greater initial consumption and higher non-renewable price increases the initial optimal tax rate due to the aforementioned channels. These channels dominate the negative effect of the degree of substitution which was introduced in Proposition 5. Accordingly, when the degree of substitution is strong, the optimal tax rate is higher in the short and medium run but lower in the long run (fig. 3(f)). This also demonstrates the fact that the wedge on non-renewable resource use must be tightened when the firms have more incentive to extract it.

6 Conclusion

It is well known that the time profile of taxation on non-renewable resources is as important as its current level. The regulator has to decide on both to control the resource extraction. But there is still an ongoing debate on the shape of the time profile of these policy measures.

In this paper we developed an optimal growth model to investigate the optimal taxation of a non-renewable resource which is an imperfect substitute to a renewable resource. This framework allowed us to investigate the determinants of the optimal tax rate and its time profile in a decentralized economy that goes through a gradual transition to a clean, zero-carbon state.

We showed that the decentralized economy converges to the clean state in terms of capital and consumption (the same state as the socially optimal one) in the long run whether there is a regulator intervention or not. What matters for welfare, however, is the speed at which the economy approaches there - the energy transition, which determines the level of environmental damages. The policy problem is to induce the economy to follow the paths leading to the level



Note: The solid lines represent the strong degree of substitution case ($\epsilon = 10$) and the dashed lines represent the weak degree of substitution case ($\epsilon = 3$).

Figure 3: The role of the degree of substitution on optimal taxation

of environmental damages that the society is ready to accept in the long run. In the laissez-faire economy, the profit motivated firms do not internalize the environmental damages that they are causing, thus they extract a greater amount of the non-renewable resource and the households consume more compared to the optimal transition path. Consequently, the speed of transition to clean economy is slower in the absence of regulation. The regulator can correct this market failure by introducing taxation on non-renewable resource extraction.

We identified the factors that affect the optimal tax rate such as the capital stock, consumption, cumulative extraction, the resource prices and the degree of substitution between non-renewable and renewable resources. The endogenous net rental rate of capital, the marginal utility of consumption and the marginal damages to the environment are the key components of the optimal tax rate. On the time profile of optimal taxation, the initial state of the economy is all that matters. If the initial capital stock is lower than its long term value then the optimal tax is always increasing over time. However, if the initial capital stock is too large -greater than its steady state value- then the optimal tax will have a decreasing and U-shaped time profile as the over-accumulated capital is consumed and the capital stock and consumption diminish. Accordingly, the social cost of suboptimal policies can be in many forms. For instance, let's take the example of a constant tax rate. It can initially be too high that it becomes an obstacle for short term development. In the long term, however, it can remain insufficient for leading the renewable resources to overtake the non-renewables.

The results also showed that the renewable resources play a crucial role on the taxation of non-renewables. More expensive renewables will require the regulator to increase the tax rate on non-renewables. In contrast, the technological improvements in the renewable technologies that reduce the costs will lead the regulator to loosen the policy measures on non-renewable resource use. Furthermore, if the degree of substitution between non-renewable and renewable resources becomes stronger, the optimal tax rate rises in the short and medium run, but it will be lower in the long run. Therefore, the regulator has to tighten the policy measures on non-renewable resource use in the short run as a response to the technological innovations that improve the substitutability of resources.

The framework in the present paper can be extended towards several directions. Further research includes taking into account technological progress that improves the efficiencies of both non-renewable and renewable resources as well as changes the degree of substitution between resources. In addition, considering reversible pollution together with irreversible pollution, a more realistic global carbon cycle, will improve the results. Finally, incorporating the present modeling into empirical applications can lead to more realistic policy suggestions.

Appendix

A1: Proof of Proposition 1

The Jacobian of the system in (12 − 15) can be written as:

$$\mathbf{J} = \begin{bmatrix} F_K - G(Z)E^*_{dK} - \pi_c E^*_{cK} & -G_Z E^*_d & -1 & F_{\pi_d} - G(Z)E^*_{d\pi_d} - \pi_c E^*_{c\pi_d} \\ E^*_{dK} & 0 & 0 & E^*_{d\pi_d} \\ \sigma C F_{1K} & 0 & \sigma(F_1 - \rho) & \sigma C F_{1\pi_d} \\ F_{1K}(\pi_d - G(Z)) & -(F_1 G_Z + D_{ZZ}/U_C) & D_Z U_{CC}/(U_C)^2 & F_{1\pi_d}(\pi_d - G(Z)) + F_1 \end{bmatrix} \tag{A.1}$$

We want to prove that the steady state given in (23, 24) has the saddle point properties for the system in (12 − 15). For that, first we evaluate the Jacobian at the steady state. Note that at the steady state $E_d^* = 0$, $\pi_d = G(Z)$ and $F_1 = \rho$. Except the trivial implications of these values on the Jacobian, it is worth showing that:

$$\mathbf{J}_{11}^{ss} = F_1 + (F_2 H_{E_d} - G(Z))E_{dK}^* + (F_2 H_{E_c} - \pi_c)E_{cK}^* = \rho$$
$$\text{and } \mathbf{J}_{14}^{ss} = (F_2 H_{E_d} - G(Z))E_{d\pi_d}^* + (F_2 H_{E_c} - \pi_c)E_{c\pi_d}^* = 0$$
$$\text{since } F_1 = \rho\ ,\ F_2 H_{E_d} = \pi_d = G(Z) \text{ and } F_2 H_{E_c} = \pi_c$$

Accordingly, the Jacobian evaluated at the steady state is as follows:

$$\mathbf{J}^{ss} = \begin{bmatrix} \rho & 0 & -1 & 0 \\ E_{dK}^* & 0 & 0 & E_{d\pi_d}^* \\ \sigma C^{ss} F_{1K} & 0 & 0 & \sigma C^{ss} F_{1\pi_d} \\ 0 & -(F_1 G_Z + D_{ZZ}/U_C) & D_Z U_{CC}/(U_C)^2 & \rho \end{bmatrix} \tag{A.2}$$

The characteristic equation associated with the Jacobian $\mathbf{J}^{ss}$ is given by:

$$\xi^4 - (\mathrm{Tr}\mathbf{J}^{ss})\xi^3 + M_2\xi^2 - M_3\xi + \det \mathbf{J}^{ss} = 0 \tag{A.3}$$

where M_2 and M_3 are the sum of all diagonal second and third order minors of $\mathbf{J}^{ss}$, respectively. One can show that:

$$\mathrm{Tr}\mathbf{J}^{ss} = 2\rho \text{ and } - M_3 + \rho M_2 - \rho^3 = 0 \tag{A.4}$$

Theorem 1 in Dockner (1985) shows that if the equations in (A.4) are satisfied then one can write the four roots of the characteristic equation in (A.3) as follows:[15]

$$\xi_{1,2,3,4} = \frac{\rho}{2} \pm \sqrt{\left(\frac{\rho}{2}\right)^2 - \frac{\Omega}{2} \pm \frac{1}{2}\sqrt{\Omega^2 - 4\det \mathbf{J}^{ss}}} \tag{A.5}$$

where $\Omega = M_2 - \rho^2$ and for $\mathbf{J}^{ss}$ it can be written as:[16]

$$\Omega = M_2 - \rho^2 = \begin{vmatrix} \partial\dot{K}/K & \partial\dot{K}/C \\ \partial\dot{C}/K & \partial\dot{C}/C \end{vmatrix} + \begin{vmatrix} \partial\dot{Z}/Z & \partial\dot{Z}/\pi_d \\ \partial\dot{\pi}_d/Z & \partial\dot{\pi}_d/\pi_d \end{vmatrix} + \begin{vmatrix} \partial\dot{C}/C & \partial\dot{C}/\pi_d \\ \partial\dot{\pi}_d/C & \partial\dot{\pi}_d/\pi_d \end{vmatrix} \tag{A.6}$$

[15] See Dockner (1985) and Dockner and Feichtinger (1991) Appendix 1 for derivation.

[16] Note that the explicit expression for Ω given in (A.6) is slightly different from Dockner (1985), Dockner and Feichtinger (1991) and Tahvonen (1991) due to the differences in the Jacobian. However, the relevant point to apply the theorem relies on satisfying (A.4) so that we can write the roots as in (A.5). Using the definition of Ω, which is the sum of all diagonal second order minors of $\mathbf{J}^{ss}$ minus ρ^2, we obtain the explicit expression of Ω in (A.6) for our problem.

In Theorem 3 of Dockner (1985) and Theorem of Tahvonen (1991) it is stated that when the determinant of the Jacobian is positive and Ω is negative the stationary point has the saddle point properties. Therefore, to prove that the steady state is a (local) saddle point, it is now sufficient to show that $\det \mathbf{J}^{ss} > 0$ and $\Omega < 0$. First, let us compute the determinant:

$$\det \mathbf{J}^{ss} = \underbrace{(\overbrace{F_1 G_Z}^{>0} + \overbrace{D_{ZZ}/U_C}^{>0})}_{>0}\Big(\underbrace{\overbrace{E^*_{d\pi_d}}^{<0}\ \overbrace{\sigma C^{ss}}^{>0}\ \overbrace{F_{1K}}^{<0}}_{>0} - \underbrace{\overbrace{E^*_{dK}}^{>0}\ \overbrace{\sigma C^{ss}}^{>0}\ \overbrace{F_{1\pi_d}}^{<0}}_{<0}\Big) > 0 \tag{A.7}$$

Equation (A.7) shows that the determinant of $\mathbf{J}^{ss}$ is always positive due to the assumptions on $U(.)$, $F(.)$, $D(.)$, $G(.)$, as well as the optimal non-renewable use $E^*_d(.)$ which is obtained by (4′) and (5). To complete the proof, we compute the value of Ω given in (A.6) as follows:

$$\Omega = \underbrace{\sigma C F_{1K}}_{<0} + \underbrace{\overbrace{E^*_{d\pi_d}}^{<0}\ \overbrace{(F_1 G_Z + D_{ZZ}/U_C)}^{>0}}_{<0} - \underbrace{\overbrace{\sigma C^{ss} F_{1\pi_d}}^{<0}\ \overbrace{D_Z U_{CC}/(U_C)^2}^{<0}}_{>0} < 0 \tag{A.8}$$

The result in (A.8) which shows that Ω is always negative, together with the result in (A.7), ensures that the characteristic equation in (A.3) consists of two roots with positive real parts and two roots with negative real parts. There are two two-dimensional manifolds which contain the steady state with one of them being stable. If the solution starts on this manifold then the path will asymptotically approach the steady state. For a given initial state $K_0 > 0$ and $Z_0 > 0$, it is possible to choose initial values C_0 and π_{d0} such that the corresponding paths approach the steady state as $t \to \infty$. Moreover, when the conditions in Mangasarian sufficiency theorem (Mangasarian (1966)) are satisfied, the saddle point path is the optimal infinite time solution. The concavity of Hamiltonian is clearly satisfied due to the assumptions on $U(.)$, $F(.)$, $D(.)$ and $G(.)$, therefore, the path leading to the saddle point is the optimal infinite time solution.

A2: Proof of Proposition 2

In the absence of taxation, the Jacobian of the equilibrium system in (39 – 42) evaluated at the steady state can be written as:

$$\hat{\mathbf{J}}^{ss} = \begin{bmatrix} \rho & 0 & -1 & 0 \\ E^*_{dK} & 0 & 0 & E^*_{d\pi_d} \\ \sigma C^{ss} F_{1K} & 0 & 0 & \sigma C^{ss} F_{1\pi_d} \\ 0 & -F_1 G_Z & 0 & \rho \end{bmatrix} \tag{A.9}$$

One can show that $\hat{\mathbf{J}}^{ss}$ in (A.9) also satisfies the equations in (A.4). By following the same procedure in the proof of Proposition 1, we can compute det $\hat{\mathbf{J}}^{ss} > 0$ and $\hat{\Omega} < 0$, hence show that the steady state is a (local) saddle point for the equilibrium system as well as the path leading to the saddle point is the optimal infinite time solution.

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