Natural Resources, R&D and Economic Growth*

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Abstract

This paper studies the long-run economic impact of natural resources by constructing a Schumpeterian endogenous growth model that incorporates an upstream resource intensive sector. Natural resources are extracted, processed and utilized to produce intermediate capital goods which are essential inputs for producing a final consumption good. R&D activities are targeted at improving the quality of existing intermediate products. In this context, we characterize balanced growth paths and examine the issues of sustainability and long-run growth associated with these equilibrium solution trajectories. The analysis is conducted through the comparison of the two natural resource types: renewable versus non-renewable. It is shown that negative growth is possible, however, only applied to an economy that is endowed with non-renewable resources. To escape from stagnant growth, it is essential to have a strong innovative sector. This paper also identifies conditions under which growth is larger with renewable resources than with their non-renewable counterparts and vice versa.
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1 Introduction

Prior to the twentieth century, natural resources, usually comprising primary commodities, played a pivotal role in world trade. Many countries, such as Aus-

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tralia, the United States, and Canada, benefited greatly from significant primary commodity exports in the early stages of their economic development (North and Thomas, 1973; Auty and Mikesell, 1998). However, since the turn to the twentieth century, natural resources have often been treated as less important than labour and capital in generating economic growth and development. In fact, natural resource abundance may be harmful to the economic development of low and middle income countries due to the so-called 'resource curse' puzzle (Nankani, 1979; Sachs and Warner, 1997, 2001). One of the reasons is that activities in natural resource sector may crowd out physical capital, human capital, and other more technologically advanced activities (e.g. investment in high-tech manufacturing sectors) which reduce the rate of technological progress, the main driver of output growth in the last century.

Despite the potential harmful impact of natural resources on growth, Jones (2002) indicates that having very few natural resources does not alleviate the negative impact of the resource depletion rate on economic performance. Instead, the issue is how to find an effective use of resources given their role in the production function. In that respect, technological progress could make natural resources 'virtually unlimited' (Tisdell, 1990). Higher productivity and greater rates of innovation could reduce the rate of resource exploitation that may be harmful for future growth (Robson, 1980; Perkins et al., 2006). In other words, technological progress may strengthen the sustainability of a country by enhancing its ability to overcome resource scarcity over time.[1]

In this paper, we attempt to answer the questions on the role of technological progress and natural resources in affecting output growth through the lens of modern Schumpeterian growth theory. To that end, we construct a model of endogenous growth with creative destruction and natural resources. Upon attaining balanced growth paths, we analyze key properties of these equilibrium paths and derive conditions under which the economy obtains permanent positive growth. We also compare the rates of growth across different types of resources.

In greater details, the model has two factors of production, labour and natural resources, and four sectors, primary (or resource production), research, intermediate good production, and final consumption good production. The primary sector uses labour to process raw natural resources into materials. Here, both types of resources are considered. Unlike non-renewable resources, renewable resources have the capacity to grow in size over time to provide productive input to the intermediate good sector. However, the size of the resource stock cannot be enlarged without bound. Specifically, it is endogenously determined by the rate of extraction and the intrinsic growth of the resources themselves. The R&D sector hires labour to improve the efficiency of production inputs. The intermediate good sector purchases designs created in the R&D sector and

[1] However, there are also some skeptical views on the role of technological progress in abating the impact of natural resources on economic performance. For instance, Tisdell (1990) points out that the speed of technological improvement may not be enough to offset the decreasing availability of natural resources. Technological change may also raise consumption of natural resources due to the so-called 'Jervons Paradox' (Alcott, 2005).

employs labour together with processed materials obtained from the primary sector to produce intermediate products which are essential for the production of a final consumption good in the final good sector. The analysis is conducted for both cases of renewable and non-renewable resources.

Our main results obtained from the model are as follows. For each type of resources, there exists an optimal balanced growth path. Along these balanced growth paths, while the dynamics of renewable resources do not affect output growth, those of non-renewable resources decelerate it. This is because renewable resources will be optimally extracted to their maximum yield at which the extraction rate is equal to the natural growth of resources resulting in a zero growth of resources at optimal. By contrast, as the extraction of non-renewable resources reduces the economy's resource wealth, this negative effect needs to be offset by the final output produced. However, output growth under renewable resources does not always dominate that under non-renewable resources. Rather, the ordering of growth rates depends on factors such as the rate of time preference, the instrinsic growth of renewable resources, and the productivity of research activity. The intuition is that an expanding resource sector attracts more labour to its production activity leaving less labour to research. Likewise, a more impatient society will extract non-renewable resources faster and, thus, dampen the resource stock more quickly. In order to escape from possible negative growth triggered by non-renewable resources, the research sector must be sufficiently productive to make up for the fall in resource wealth.

Linking to the relevant literature, previous studies often consider natural resources, innovation, and growth separately; either between resource abundance and economic growth (e.g. Sachs and Warner, 1995; Lederman and Maloney, 2007) or between innovation and economic performance (e.g. Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992). Hence, it remains an open question as whether natural resources actually play a significant role in enhancing or inhibiting standards of living over time and to what extent and what direction, technological improvement could affect this process. To the best of our knowledge, Grimaud and Rouge (2003), Lafforgue (2008), Peretto (2008, 2012), Peretto and Valente (2011) are among the few recent attempts to fill this gap. However, these models only focus on non-renewable resources while this paper extends its investigation to renewable resources as well. In particular, we consider whether the economy behaves differently under different resource types.

The rest of the paper is organized as follows. Section 2 provides basic setting of the model. Section 3 is devoted to characterizing growth equilibrium paths of the economy. In particular, we consider the long-run implications of natural resources on output growth and the issue of sustainability. Section 4 ends the paper with some concluding remarks.

2 The model

2.1 *The final goods sector*

Final consumption good Y is homogeneously produced and sold on a competitive market. There are a large number of identical firms whose production technology is the following:

$$Y_t = Q^{1-\alpha} \int_0^1 A_{it} x_{it}^{\alpha} di, \ \alpha \in (0,1) \tag{1}$$

where Q is a fixed production factor like land or water surface (which is normalized to 1 for simplicity), x_{it} is intermediate good of vintage i that is indexed on a unit interval, and A_{it} is a productivity parameter attached to the latest version of the intermediate good i.

The final good is taken as a numeraire so that $P_Y = 1$. The final good producers are price takers in the input and output markets and their profit maximization problem is:

$$\max \ \pi_{Yt} = Y_t - \int_0^1 p_{xit} x_{it} di$$

where p_{xit} denotes the price of intermediate good i at time t. This gives the (inverse) demand function for each intermediate good as follows:

$$p_{xit} = \alpha A_{it} x_{it}^{\alpha-1}, \ \forall i \in [0,1] \tag{2}$$

This equation says that in a competitive market, each intermediate good receives its marginal product in terms of the final consumption good.

2.2 *The intermediate goods sector*

This sector is monopolistically competitive. Goods are available at time t in a continuum of different varieties indexed on a unit interval. Each intermediate producer faces the following production technology:

$$x_{it} = \frac{M_{it}^{\beta} L_{it}^{1-\beta}}{A_{it}}, \ \beta \in [0,1], \ \forall i \in [0,1] \tag{3}$$

Here, L_{it} is labour employment in industry i at time t and M_{it} is the use of processed natural resource materials. The Cobb - Douglas function of M_{it} and L_{it} is deflated by A_{it} to reflect the fact that successive vintages of the intermediate product, which embody increasingly complex technology, require increasing resources to produce. Following Romer (1990) and Grossman and Helpman (1991), assume that each intermediate good embodies a design created in the research sector and is protected by a patent law. Because no firm can produce an intermediate product without the consent of the patent holder of the design, each intermediate firm is a monopolist of the product it produces.

Profit maximization problem for the representative monopolist i is:

$$\max \ \pi_{xit} = p_{xit} x_{it} - p_{mt} M_{it} - w_t L_{it}$$

subject to the demand equation (2) and production technology equation (3). In this formulation, p_{mt} is the price of one unit of processed material and w_t is the wage rate paid to one unit of labour. The first order conditions with respect to M_{it} and L_{it} deliver:

$$p_{mt} = \frac{\alpha^2 \beta A_{it} x_{it}^{\alpha}}{M_{it}}$$

$$w_t = \frac{\alpha^2 (1-\beta) A_{it} x_{it}^{\alpha}}{L_{it}}$$

Rearranging and summing over i gives:

$$M_t = \frac{\alpha^2 \beta Y_t}{p_{mt}} \tag{4}$$

$$L_{xt} = \frac{\alpha^2 (1-\beta) Y_t}{w_t} \tag{5}$$

where $M_t = \int_0^1 M_{it} di$ is the aggregate stock of materials and $L_{xt} = \int_0^1 L_{it} di$ is the total labour employment used for producing intermediate goods. Plugging these results into equation (3) yields:

$$x_{it} = \left(\frac{M_t^{\beta} L_{xt}^{1-\beta}}{Y_t}\right)^{\frac{1}{1-\alpha}}$$

This implies $x_{it} = x_t, \forall i$. This means that when intermediate firms are identical and face the same input costs, they produce the same amount of output. Using this result, the production function in (1) can now be rewritten as:

$$Y_t = A_t^{1-\alpha} (M_t^{\beta} L_{xt}^{1-\beta})^{\alpha} \tag{6}$$

where $A_t = \int_0^1 A_{it} di$ is the economy's aggregate knowledge level.[2]

The representative intermediate firm's flow of profit will be:

$$\pi_{xit} = \alpha A_{it} x_{it}^{\alpha} - \alpha^2 \beta A_{it} x_{it}^{\alpha} - \alpha^2 (1-\beta) A_{it} x_{it}^{\alpha} = \alpha(1-\alpha) A_{it} x_{it}^{\alpha} \tag{7}$$

This means that operating profit to each monopolist is proportional to his technology level A_{it}. Substituting the value of Y_t given in (6) into the equation for x_t gives:

$$x_t = \frac{M_t^{\beta} L_{xt}^{1-\beta}}{A_t}$$

Plugging this value into the profit function for each intermediate firm in (7) yields:

$$\pi_{xit} = \alpha(1-\alpha) A_{it} \frac{M_t^{\alpha\beta} L_{xt}^{\alpha(1-\beta)}}{A_t^{\alpha}} \tag{8}$$

[2] A_t is also equal to the economy's average technology level as the number of intermediate industries is indexed on a unit interval.

2.3 *The research sector*

As mentioned above, the production of each intermediate good requires the purchase of specific design made from the research sector. This sector is assumed competitive so that any firm or individual can conduct R&D activities provided that benefits exceed the costs. A successful vertical innovation creates a better version of an existing intermediate product and replaces it in the final good production. Because the design is protected by the patent law, the successful innovator can reap the monopoly profits until the next successful innovator occurs in that industry.

With access to the stock of knowledge, research firms use labour to develop new blueprints. Assume that at any point in time, an R&D firm that hires one unit of labour is successful in discovering the next higher quality product with a Poisson arrival rate $\lambda > 0$. Innovations happen such that product of vintage τ makes the product of previous vintage $\tau - 1$ obsolete and then replaces it in production. Each time, when an innovation is successful, the aggregate knowledge level is improved as the following:

$$A_\tau = \mu A_{\tau-1}, \ \mu > 1, \ \forall\tau \tag{9}$$

A same amount is spent on vertical R&D in each industry because the prospective payoff is the same in each industry. If L_{rt} is the total amount of labour devoted to doing research then the expected value of A at time $t + \triangle t$ is:

$$E(A_{t+\triangle t}) = \lambda L_{rt}\triangle t\mu A_t + (1 - \lambda L_{rt}\triangle t)A_t = A_t + \lambda(\mu - 1)L_{rt}A_t\triangle t$$

Rearranging and taking the time limit gives:

$$\dot{A}_t = \lim_{\triangle t \to 0} \frac{E(A_{t+\triangle t}) - A_t}{\triangle t} = \lambda(\mu - 1)L_{rt}A_t \tag{10}$$

Under the assumption of free entry, new firms will enter until all profit opportunities are exhausted. Hence, the level of labour employed in research is determined by the arbitrage condition which equates the marginal cost of an extra unit of labour, w_t, to its expected marginal benefit λV_t where V_t is the value of a vertical innovation:

$$\lambda V_t = w_t \tag{11}$$

As the market for design is competitive, the value of vertical innovation at date t will be bid up to the expected present value of future operating profits to be earned by the incumbent intermediate monopolist before being replaced by the next innovator in the industry. The time until replacement is distributed exponentially with parameter $I_t = \lambda L_{rt}$ which is the rate of successful innovation arrival. As a result, the value of vertical innovation is:

$$V_t = \int_t^\infty \pi_{xt\tau} e^{-\int_t^\tau (r_s + I_s)ds} d\tau \tag{12}$$

where r_s is the instantaneous interest rate at date s, and $\pi_{xt\tau}$ is the flow of operating profit at date τ to any firm in the sector whose technology is of vintage t. The instantaneous discount rate contains the interest rate and the rate of creative destruction I_s which captures the probability of being displaced by a new innovator.

Following Caballero and Jaffe (1993) and Howitt and Aghion (1998), assume that the leading-edge technology parameter $A_t^{\max} \equiv \max\{A_{it}, \forall i \in [0,1]\}$ is available to any successful innovator. Growth of this leading-edge technology parameter is due to knowledge spillovers produced by innovations. Similar to Howitt and Aghion (1998), it can be shown that the ratio of the leading-edge technology $A_t^{\max}$ to the average technology A_t will be constant. Indeed, each innovation replaces a randomly chosen A_{it} with the leading edge $A_t^{\max}$. As innovation occurs at rate $I_t = \lambda L_{rt}$ per product and the average change across innovating sectors is $A_t^{\max} - A_t$ so:

$$\dot{A}_t = \lambda L_{rt}(A_t^{\max} - A_t)$$

Dividing both sides by A_t gives:

$$\frac{\dot{A}_t}{A_t} = \lambda L_{rt}\left(\frac{A_t^{\max}}{A_t} - 1\right)$$

This together with (10) implies that $\frac{A_t^{\max}}{A_t} = \mu$. Therefore, using (8), the flow of profit at date τ to the innovator who performed a vertical innovation at date t is:

$$\pi_{xt\tau} = \alpha(1-\alpha)\frac{A_{t\tau}^{\max}}{A_{t\tau}} M_{t\tau}^{\alpha\beta} L_{xt\tau}^{\alpha(1-\beta)} A_{t\tau}^{1-\alpha} = \alpha(1-\alpha)\mu M_{t\tau}^{\alpha\beta} L_{xt\tau}^{\alpha(1-\beta)} A_{t\tau}^{1-\alpha} \quad (13)$$

2.4 *The primary or resource sector*

Assume that the resources are owned by households in the economy. This assumption is important to guarantee that households care about dynamics of resources when maing their resource harvest decision. Following Schaefer (1957), assume at each point in time, the amount of materials extracted is:

$$M_t = BL_{mt}R_t \quad (14)$$

where L_{mt} represents labour input in the resource sector, R_t is the stock of resources, and B is the productivity of resource production. This equation indicates that harvest production exhibits increasing returns to scale to all production factors. Harvest output not only depends on labour employment but also on the existing stock of resources.

The dynamics of the stock of resources are as follows:

$$\dot{R}_t = f(R_t) - M_t \quad (15)$$

Here, $f(R_t)$ is the natural growth of the resources that takes the following logistic growth form:

$$f(R_t) = \eta R_t\left(1 - \frac{R_t}{\overline{R}}\right),\ \eta \geq 0 \quad (16)$$

In this formulations, $\overline{R}$ is the carrying capacity of the environment and η represents the intrinsic growth rate of resources. When $\eta > 0$, the natural resources are renewable and when $\eta = 0$, they are non-renewable. It can be seen from (15) that when $f(R_t) > M_t$, the natural growth of resources is greater than the amount of resources extracted so the resource stock will rise. On the contrary, when $M_t > f(R_t)$, the resource stock will fall. The stock of resources will stay constant when $M_t = f(R_t)$. This implies a possible sustainable yield for the economy when the same level of resource stock is unchanged. If we denote R_0 as the initial stock of resources then the stock at time t is given by:

$$R_t = R_0 + \int_0^t \dot{R}_\nu d\nu = R_0 + \int_0^t \left[f(R_\nu) - M_\nu\right] d\nu$$

The resource constraint requires that $\int_0^\infty \left[f(R_t) - M_t\right] dt \leq R_0$.

The resource processing firms maximize their lifetime profit $\int_t^\infty \pi_{m\tau} e^{-\int_t^\tau r_s ds} d\tau$, $\forall t$ subject to the dynamics of resource stock given in (15) where π_{mt} is the flow of instantaneous profit:

$$\pi_{mt} = p_{mt} M_t - w_t L_{mt} \tag{17}$$

2.5 *Consumers' behaviour*

Assume constant population and normalize the size of population to 1 for simplicity ($L = 1$). There are a large number of households each of which contains one infinitely lived agent. Each agent supplies one unit of labour to the market and earns the wage rate w_t. The representative household's lifetime utility takes the following form:

$$U = \int_0^\infty \log(C_t).e^{-\rho t} dt \tag{18}$$

where $\rho > 0$ is the rate of time preference and C_t is the aggregate consumption. Households derive utility from consumption only and there is no preference for leisure.

As households earn income from assets (e.g. a financial wealth including claims on firm ownership that yields a rate of return of r_t), labour, and dividends distributed from firms, their budget constraint is:

$$\dot{a_t} = r_t a_t + w_t + \pi_{Yt} + \hat{\pi}_{xt} + \pi_{mt} - C_t \tag{19}$$

In this equation, $r_t a_t$ is the interest income from renting out the asset a_t at interest rate r_t, w_t is the labour income, and π_{Yt}, $\hat{\pi}_{xt}$, π_{mt} are profits distributed to households from firms producing final goods, intermediate goods, and resource materials respectively. The representative household will maximize utility given in (18) subject to the constraint given in (19).

3 Equilibrium of the market economy

Assume full employment for simplicity. Hence, the labour market equilibrium requires that:

$$L_{xt} + L_{rt} + L_{mt} = 1 \tag{20}$$

3.1 Existence and characterization of the steady state equilibrium

At each point in time, intermediate good producers borrow an amount of $w_t L_{rt}$ from households in the financial market to finance research activities. When an innovation is successful, the intermediate good producers use their profit π_{xt} to make interest payment so $\pi_{xt} = r_t a_t + \hat{\pi}_{xt}$. Observe that on aggregate $\pi_{Yt} = (1-\alpha)Y_t$; $\hat{\pi}_{xt} = \pi_{xt} - r_t a_t = \alpha(1-\alpha)Y_t - r_t a_t$; $\pi_{mt} = \alpha^2 \beta Y_t - w_t L_{mt}$; and $C_t = Y_t$. Plugging these into (19) and using (5) then (20) we obtain $\dot{a}_t = w_t - w_t L_{mt} - w_t L_{xt} = w_t L_{rt}$ which is the intermediate good producers' borrowing to finance the R&D activities. The households' utility maximization exercise described in (18) and (19) provides the usual condition on consumption growth:

$$g_C = \frac{\dot{C}_t}{C_t} = r_t - \rho \tag{21}$$

Definition 1. *An equilibrium of this economy is an infinite sequence of quantity allocations,* $\{C_t, Y_t, A_t, R_t, a_t, M_t, x_t, L_{xt}, L_{mt}, L_{rt}, \pi_{Yt}, \pi_{xt}, \pi_{mt}\}_{t=0}^{\infty}$, *and prices,* $\{p_{xt}, p_{mt}, w_t, r_t\}_{t=0}^{\infty}$, *such that consumers, final goods producers, intermediate firms, and research firms maximize their objective functions taking prices as given and all markets clear.*

In this section, we focus on equilibrium paths where all variables grow at constant rates or balanced growth paths (BGPs). In particular, we analyze BGPs which are defined as follows:

Definition 2. *A BGP is an equilibrium path where all variables grow at a constant rate and the allocations of labour across the intermediate goods, resource, and the R&D sectors are also constant.*

Specifically, along these BGPs, L_{xt}, L_{mt}, L_{rt} are all constant; R_t, A_t, C_t, and Y_t grow at constant rates g_R, g_A, g_C, and g_Y respectively; and interest rate $r_t = r$, $\forall t$. As a matter of convenience, the time index will now be dropped for those variables that do not vary over time.

From (12) and (13), as soon as $[r + \lambda L_r - \alpha\beta g_R - (1-\alpha)g_A] > 0$, the value of a vertical innovation is:

$$V_t = \alpha(1-\alpha)\mu A_t^{1-\alpha}(M_t^{\beta} L_x^{1-\beta})^{\alpha} \int_0^{\infty} e^{-[r+\lambda L_r - \alpha\beta g_R - (1-\alpha)g_A]\tau} d\tau = \frac{\alpha(1-\alpha)\mu Y_t}{[r+\lambda L_r - \alpha\beta g_R - (1-\alpha)g_A]}$$

This together with the arbitrage condition in (11) yield the following:

$$\frac{\lambda\alpha(1-\alpha)\mu Y_t}{[r+\lambda L_r-\alpha\beta g_R-(1-\alpha)g_A]}=w_t \tag{22}$$

Proposition 1 *Assume $\mu\lambda \geq \frac{\alpha\rho}{1-\alpha}$, then for each type of resources, there exists a unique equilibrium BGP in which the growth rates of output, technology, consumption, and resources are constant and the allocations of labour across different sectors take constant values.*

Proof. See Appendix.
As shown in the proof, the condition $\mu\lambda \geq \frac{\alpha\rho}{1-\alpha}$ is required to guarantee the existence of equilibrium BGPs. For renewable resources, the long-run values of our interested variables are:

$$L_x = \tfrac{\eta(1-\beta)}{4B\beta}.\tfrac{(1-\tilde{w}^2)}{\tilde{w}}$$

$$L_m = \tfrac{\eta(1-\tilde{w})}{2B}$$

$$L_r = \tfrac{(1-\alpha)\mu\eta}{4B\alpha\beta}.\tfrac{(1-\tilde{w}^2)}{\tilde{w}} - \tfrac{\rho}{\lambda}$$

$$g_A = \lambda(\mu-1)L_r(\tilde{w})$$

$$g_Y = g_C = (1-\alpha)\lambda(\mu-1)L_r(\tilde{w})$$

$$g_R = 0$$

where $\tilde{w}$ is the unique solution to the equation below:

$$\tfrac{\eta(1-\tilde{w})}{2B} + \tfrac{\eta(1-\beta)}{4B\beta}.\tfrac{(1-\tilde{w}^2)}{\tilde{w}} + \tfrac{(1-\alpha)\mu\eta}{4B\alpha\beta}.\tfrac{(1-\tilde{w}^2)}{\tilde{w}} - \tfrac{\rho}{\lambda} = 1$$

Observe that since $\tilde{w} \in (0,1]$ and it solves the above equation so the allocation of labour satisfies that condition that $0 < L_m(\tilde{w})$, $L_x(\tilde{w})$, $L_r(\tilde{w}) < 1$. Output and consumption growth is driven by growth of technological knowledge. Because $g_Y = g_C = (1-\alpha)g_A$, the rate of growth of technological knowledge is greater than that of output (as well as consumption).

Similarly, the BGP for non-renewable resources is characterized by the following:

$$L_x = \tfrac{(1-\beta)\rho}{B\beta}.\tfrac{(1-\hat{w})}{\hat{w}}$$

$$L_m = \tfrac{(1-\hat{w})\rho}{B}$$

$$L_r = \tfrac{(1-\alpha)\mu\rho}{B\alpha\beta}.\tfrac{(1-\hat{w})}{\hat{w}} - \tfrac{\rho}{\lambda}$$

$$g_A = \lambda(\mu - 1)L_r(\hat{w})$$

$$g_Y = g_C = (1-\alpha)\lambda(\mu - 1)L_r(\hat{w}) - \alpha\beta BL_m(\hat{w})$$

$$g_R = -BL_m(\hat{w})$$

in which $\hat{w}$ uniquely solves the following equation:

$$\tfrac{(1-\hat{w})\rho}{B} + \tfrac{(1-\beta)\rho}{B\beta}.\tfrac{(1-\hat{w})}{\hat{w}} + \tfrac{(1-\alpha)\mu\rho}{B\alpha\beta}.\tfrac{(1-\hat{w})}{\hat{w}} - \tfrac{\rho}{\lambda} = 1$$

An obvious remark is that given $\hat{w} \in [0, 1]$ and it is a solution to the equation above then $0 < L_x,\ L_m,\ L_r < 1$. Output and consumption growth is driven by the dynamics of technology and natural resources. Because, natural resources affect output and consumption growth negatively, technology grows at a faster rate than output and consumption.

3.2 Properties of the steady state equilibrium path

In this part, the impact of variations of different parameters of the model on L_x, L_m, L_r and g_Y, g_C, g_A (especially on g_Y will be analyzed) for each type of resources.

Proposition 2 *Key properties of the equilibrium BGPs: other things equal, output growth is increasing in the productivity of the R&D sector (λ and μ) but decreasing in the rate of time preference (ρ). While an increase in the productivity of the resource sector (B) is always growth enhancing under renewable resources, it is growth enhancing under non-renewable resources if:*

$$\frac{(1-\alpha)^2(\mu-1)\mu(\lambda+\rho)}{[\alpha+(1-\alpha)\mu]^2} - \rho > 0$$

In particular, when ρ is small, or either λ or μ is large then this sufficient condition is satisfied.

Proof. See Appendix.

The impact of λ, μ, ρ, and B on the rate of growth of the economy can be explained through market mechanisms. An increase in either λ or μ implies a more productive R&D sector which induce R&D firms to employ more workers. According to (10), the rate of growth of technology, g_A, increases. Technological progress will result in higher output growth as $g_Y = (1-\alpha)g_A$. From the demand side, more research in the R&D sector means more borrowing from R&D firms to finance their research activities which, in turn, boosts up the interest rate. An increase in interest rate entails a higher rate of consumption growth and, hence, a higher rate of output growth.

An increase in ρ means consumers relatively prefer present consumption to future consumption so they will lend less money and thus r will rise. Because R&D firms need to borrow money to finance their research in the first place,

L_r falls and so do g_A and g_Y. From the demand side, as ρ increases more than r, consumers prefer present consumption to future consumption so they are not interested in increasing future consumption. Hence, $g_C = g_Y$ decreases.

A change in B affects output growth for both types of resources. An increase in B makes the harvest production of natural resources more productive. Because resource firms optimize their extraction over time, they will reduce the labour used. As a result, there will be more labour for the production of intermediate products and R&D activities. Technological progress will induce higher output growth for the case of renewable resources as resource extraction is fully offset by their natural growth. However, for the case of non-renewable resources, whether output growth is higher or not depends on whether technological change is able to generate enough growth to make up for the amount of natural resources that has been depleted (μ and λ must be sufficiently large or ρ must be sufficiently small).

All these results highlight the role of R&D activities in driving economic growth. If technological innovation is strong, natural resources will be turned into good use and an improvement in the productivity of the resource sector enhances growth. If technological progress is weak (μ is close to 1), such a change will only result in a harmful growth impact of non-renewable resources.

Proposition 3 *Along the equilibrium BGPs, under renewable resources, output growth is always non-negative. Under non-renewable resources, if further assume* $\lambda \geq \frac{\alpha\rho}{1-\alpha}$, *then output growth may be negative if* $\mu \to 1$.

Proof. See Appendix.

This proposition highlights the connection between output growth, technological change, and natural resource dynamics. Because long-run growth of renewable resources is zero, output growth is solely and positively determined by technological progress which is non-negative by construction. However, it is negatively affected by the speed of non-renewable resource extraction. This is because the extraction of non-renewable resources reduces the economy's wealth which, in turn, reduces growth. More importantly, growth will be negative if the speed of resource depletion is more than the rate of change of technology. This scenario of stagnant growth happens when the magnitude of technological improvement due to innovation is small (μ is close to 1). Note that, here, we still need the condition $\lambda \geq \frac{\alpha\rho}{1-\alpha}$ to make sure that the BGP for non-renewable resources exists.

3.3 Comparison of output growth rates

In this sub-section, we compare the rates of growth of output for different types of natural resources. This is done through some propositions below:

Proposition 4 *Along the equilibrium BGPs, if additionally, the intrinsic growth of renewable resources is smaller than the rate of time preference* ($\eta < \rho$), *output growth is higher under renewable resources than under non-renewable resources* ($g_Y(\tilde{w}) > g_Y(\hat{w})$).

Proof. See Appendix.

This proposition says that other things equal, renewable resources only result in higher output growth if the intrinsic growth of natural resources is smaller than the rate of time preference. The intuition is as follows. When the intrinsic growth of renewable resources, η, is high, more labour is attracted to the (renewable) resource sector leading to a contraction of the research sector so output growth is reduced. To some extent, this effect is similar to that of an increase in B as previously analyzed under Proposition 2. When the rate of time preference, ρ, is high, as consumers are more impatient, more (non-renewable) resources will be extracted and output growth will be reduced. Which factor reduces growth more depends on the comparison between them.

Proposition 5 *There exists $\bar{\lambda} > 0$ such that, for any $\lambda > \bar{\lambda}$, one can find $\eta^*(\lambda)$ which has the following property:*

$$\eta < \eta^*(\lambda) \Rightarrow g_Y(\tilde{w}) > g_Y(\hat{w})$$

$$\eta > \eta^*(\lambda) \Rightarrow g_Y(\tilde{w}) < g_Y(\hat{w})$$

$$\eta = \eta^*(\lambda) \Rightarrow g_Y(\tilde{w}) = g_Y(\hat{w})$$

Proof. See Appendix.

Let us make some comments on this proposition which is concerned with the parameter capturing the natural growth of renewable resources. As output growth is always non-negative under renewable resources but maybe negative under non-renewable resources, it is only sensible to compare the rate of growth of output under these two types of resources when growth is in its non-negative range. To guarantee this, the productivity of knowledge production (characterized by λ) must be sufficiently high ($\lambda > \bar{\lambda}$). Within this range, while growth of output stays unchanged under non-renewable resources, it varies with the value of η under renewable resources. Along the BGP of renewable resources, the economy will optimally extract renewable resources up to its maximum yield level. An increase in η implies that relatively more investment (in terms of labour allocation) will be undertaken in the primary sector as this sector becomes relatively more productive. As a result, there will be less labour allocated to knowledge production so output growth decreases. At first, when η is small (for a given λ above its threshold value $\bar{\lambda}$), output growth is higher under renewable resources than their non-renewable counterparts. When it reaches some value η^* the two growth rates are equal. When $\eta > \eta^*$, the two growth rates reverse their initial orderings.

Proposition 6 *There exists $\bar{\rho}$ such that for any $\rho < \bar{\rho}$ then $g_Y(\tilde{w}) < g_Y(\hat{w})$.*

Proof. See Appendix.

Similar to what is shown in Proposition 5, renewable resources do not necessarily dominate their non-renewable counterparts in terms of output growth rate generated. However, what matters now is the rate of time of preference, ρ. Because households always follow their optimal rule of extracting renewable resources at the maximum yield, ρ (the degree of impatience) seems to matter more for an economy facing non-renewable resources than renewable resources. Other things equal, when ρ is small enough (below some threshold, in this case, $\bar{\rho}$), households are very patient because they value future consumption relatively more than current consumption so they extract relatively little non-renewable resources. The relative reduction in resource production activity leads to a relative increase in knowledge production for the corresponding economy. As a result, output growth will enjoy a higher rate.

4 Conclusions

In this paper, we have considered a simple endogenous growth with creative destruction. We considered balanced growth paths for different types of natural resources: renewable and non-renewable. We showed that under optimal conditions, at the steady state, the dynamics of non-renewable resources hamper output growth while those of renewable resources are not a real concern. We then indicated that equilibrium growth will be positive if parameters capturing the efficiency of the R&D sector are sufficiently high.

We also found that an increase in the productivity of the research sector has an additional positive effect on the long-run rate of output growth. A more patient society chooses a low harvesting rate and reaches a higher long-run output growth. An increase in the productivity of resource extraction is always growth enhancing with renewable resources. It is growth enhancing with non-renewable resources only if the economy is patient enough or the research sector is productive enough.

In comparing long-run rates of output growth, renewable resources always result in a positive rate of growth. Despite a possible triggering of a negative growth rate, non-renewable resources are able to induce a higher growth rate under some conditions involving the intrinsic growth of renewable resources, the productivity of research activities, and/or the rate of time preference.

According to Gylfason et al. (1999), natural resource endowment is a mixed blessing. Whether growth will be negative, positive, lower, or higher is an endogenously determined outcome reflecting people's choice. What matters most is how to use these resources in the most effective way. In that respect, the model presented in this paper, although simple, provides a good starting point for considering the long-run economic implications of natural resource dynamics and the efficiency of their management. A possible extension could be the consideration of horizontal innovation in parallel with vertical innovation activities. It would also be interesting to investigate the transitional dynamics of the model.

Appendix

Proof of Proposition 1

(i) When natural resources are renewable ($\eta > 0$)

From (14)-(16), we have $g_R = \eta(1 - \frac{R_t}{\bar{R}}) - BL_m$. Given that L_m is constant in steady state, g_R will be constant if and only if R_t is constant or $\dot{R}_t = 0$. This implies $R = \frac{\bar{R}}{\eta}(\eta - BL_m)$ and $g_R = 0$. Because the stock of resources is non-negative, $L_m \leq \frac{\eta}{B}$. Given that $L_m \leq 1$ (total labour devoted to the resource sector cannot exceed the total labour force), the constraint on L_m will be $L_m \leq \min(1, \frac{\eta}{B})$.

From resource processing firms' profit in (17), after substituting $R = \frac{\bar{R}}{\eta}(\eta - BL_m)$ and maximizing with respect to the choice variable L_m we get:

$$L_m = \frac{\eta(1-\tilde{w})}{2B}$$

where $\tilde{w} = \frac{w_t}{\bar{R}Bp_{mt}} \in (0,\, 1]$.[3] This yields $R = \frac{\bar{R}}{2}(1 + \tilde{w})$ and $M = BL_mR = \frac{\eta\bar{R}}{4}(1 - \tilde{w}^2)$. For R being constant, $\tilde{w}$ must be constant. In other words, p_{mt} and w_t must both grow at a same rate or $g_w = g_{p_m}$. As a result, M is also constant.

From (4) and (5) we have:

$$Y_t = \frac{Mp_{mt}}{\alpha^2\beta} = \frac{\eta\bar{R}}{4\alpha^2\beta}p_{mt}(1 - \tilde{w}^2)$$

$$L_x = \frac{\eta(1-\beta)}{4B\beta}.\frac{(1-\tilde{w}^2)}{\tilde{w}}$$

These imply $g_Y = g_w = g_{p_m}$. Because M and L_x are constant along the BGP, according to (6), $g_Y = (1-\alpha)g_A$. Also along the BGP, $g_Y = g_C = r - \rho$ from (21) so $r = \rho + (1-\alpha)g_A$. Therefore, (22) becomes:

$$L_r = \frac{(1-\alpha)\mu\eta}{4B\alpha\beta}.\frac{(1-\tilde{w}^2)}{\tilde{w}} - \frac{\rho}{\lambda}$$

Imposing labour market clearing condition given in (20) with a note that L_x, L_m, L_r are all decreasing in $\tilde{w}$, we have:

$$\frac{\eta(1-\tilde{w})}{2B} + \frac{\eta(1-\beta)}{4B\beta}.\frac{(1-\tilde{w}^2)}{\tilde{w}} + \frac{(1-\alpha)\mu\eta}{4B\alpha\beta}.\frac{(1-\tilde{w}^2)}{\tilde{w}} - \frac{\rho}{\lambda} = 1 \qquad (23)$$

We now investigate conditions to be imposed so that $0 \leq L_m,\, L_x,\, L_r \leq 1$. Let $\bar{a} \in (0,\, 1]$ satisfy the following:

$$\frac{(1-\alpha)\mu\eta}{4B\alpha\beta}.\frac{(1-\bar{a}^2)}{\bar{a}} = \frac{\rho}{\lambda}$$

[3] It should be noted that since R is constant, there is no intertemporal issue. Hence, maximizing the lifetime profit is equivalent to maximizing instantaneous profit at each point in time.

The LHS of this equation is a decreasing function with respect to $\bar{a}$. It approaches $+\infty$ when $\bar{a}$ tends to 0 and equals to 0 when $\bar{a} = 1$. Thus, there exists a unique solution $\bar{a}_0 \in (0, 1]$ to this equation. For $\tilde{w} < \bar{a}_0$ we have $\frac{(1-\alpha)\mu\eta}{4B\alpha\beta}.\frac{(1-\tilde{w}^2)}{\tilde{w}} > \frac{\rho}{\lambda}$. Since the function $\frac{\eta(1-\tilde{w})}{2B} + \frac{\eta(1-\beta)}{4B\beta}.\frac{(1-\tilde{w}^2)}{\tilde{w}}$ is also decreasing in $\tilde{w}$, approaching $+\infty$ when $\tilde{w}$ tends to 0, if the LHS of (23) is smaller than or equal to 1 when $\tilde{w} = \bar{a}_0$ we can conclude that this equation has a unique solution $\tilde{w} < \bar{a}_0$. In this case, $0 \leq L_m(\tilde{w}),\ L_x(\tilde{w}),\ L_r(\tilde{w}) \leq 1$.

We have:

$$\frac{(1-\bar{a}_0^2)}{\bar{a}_0} = \frac{\rho}{\lambda}.\frac{4B\alpha\beta}{(1-\alpha)\mu\eta}$$

Therefore:

$$\frac{\eta(1-\bar{a}_0)}{2B} + \frac{\eta(1-\beta)}{4B\beta}.\frac{(1-\bar{a}_0^2)}{\bar{a}_0} + \frac{(1-\alpha)\mu\eta}{4B\alpha\beta}.\frac{(1-\bar{a}_0^2)}{\bar{a}_0} - \frac{\rho}{\lambda} = \frac{\eta(1-\bar{a}_0)}{2B} + \frac{\rho(1-\beta)\alpha}{(1-\alpha)\mu\lambda}$$

Since $\bar{a}_0$ satisfies the equation:

$$\bar{a}_0^2 + \frac{4B\alpha\beta\rho}{(1-\alpha)\mu\eta\lambda}\bar{a}_0 - 1 = 0$$

then $\bar{a}$ satisfy $1 - \bar{a}_0 = \frac{4B\alpha\beta\rho}{(1-\alpha)\mu\eta\lambda}.\frac{\bar{a}_0}{1+\bar{a}_0} \leq \frac{2B\alpha\beta\rho}{(1-\alpha)\mu\eta\lambda}$ as $\frac{\bar{a}_0}{1+\bar{a}_0} \leq \frac{1}{2}$ (with equality when $\bar{a}_0 = 1$). Therefore:

$$\frac{\eta(1-\bar{a}_0)}{2B} + \frac{\eta(1-\beta)}{4B\beta}.\frac{(1-\bar{a}_0^2)}{\bar{a}_0} + \frac{(1-\alpha)\mu\eta}{4B\alpha\beta}.\frac{(1-\bar{a}_0^2)}{\bar{a}_0} - \frac{\rho}{\lambda} \leq \frac{\rho\alpha}{(1-\alpha)\lambda\mu}$$

One can conclude that if $\frac{\rho\alpha}{(1-\alpha)\lambda\mu} \leq 1$ then $0 \leq L_m(\tilde{w}),\ L_x(\tilde{w}),\ L_r(\tilde{w}) \leq 1$. Using these results, growth rates of interested parameters can be calculated, for example, $g_A = \lambda(\mu-1)L_r(\tilde{w})$, $g_Y = g_C = (1-\alpha)\lambda(\mu-1)L_r(\tilde{w})$.

(ii) When natural resources are non-renewable ($\eta = 0$)

From (14)-(16), we have $g_R = -BL_m$. From (4)-(6) and (14), we have $w_t = \alpha^2(1-\beta)A_t^{1-\alpha}L_m^{\alpha\beta}R_t^{\alpha\beta}L_x^{\alpha(1-\beta)-1}$ and $p_{mt} = \alpha^2\beta A_t^{1-\alpha}L_m^{\alpha\beta-1}R_t^{\alpha\beta-1}L_x^{\alpha(1-\beta)}$. Define $\hat{w} = \frac{w_t}{Bp_{mt}R_t}$. It can be seen that $\hat{w} \geq 0$ and is constant along the BGP.

Now we turn to resource processing firms' profit maximization problem. The current value Hamiltonian function can be established as follows:

$$H = p_{mt}BL_mR_t - w_tL_m - \lambda_tBL_mR_t$$

where λ_t is the co-state variable. The optimality conditions are:

$$p_{mt}BR_t(1-\hat{w}) - \lambda_tBR_t = 0$$

$$\dot{\lambda}_t = r\lambda_t - p_{mt}BL_m + \lambda_tBL_m$$

$$\lim_{t\to\infty} e^{-rt}\lambda_t R_t = 0$$

together with the dynamic equation $\dot{R}_t = -BL_m R_t$. These lead to the following result:

$$g_{p_m} = r + (1 - \tfrac{1}{1-\hat{w}})BL_m$$

From (21), along the BGP we have $g_Y = g_C = r - \rho$. In addition, from (4) we have $g_Y = g_{p_m} + g_R$. Substituting these into the above equation gives:

$$L_m = \tfrac{(1-\hat{w})\rho}{B}$$

Clearly, as $L_m \geq 0$ we need $\hat{w} \leq 1$. Also from (4) and (5), after some simple calculations, we have:

$$L_x = \tfrac{1-\beta}{\beta}.\tfrac{L_m}{\hat{w}} = \tfrac{(1-\beta)\rho}{B\beta}.\tfrac{(1-\hat{w})}{\hat{w}}$$

From (6) and (14) we get $g_Y = (1-\alpha)g_A + \alpha\beta g_R$. Using this result and also noting $g_Y = r - \rho$, from (22) together with (4) we have:

$$L_r = \tfrac{(1-\alpha)\mu L_m}{\alpha\beta\hat{w}} - \tfrac{\rho}{\lambda} = \tfrac{(1-\alpha)\mu\rho}{B\alpha\beta}.\tfrac{(1-\hat{w})}{\hat{w}} - \tfrac{\rho}{\lambda}$$

Now using the equilibrium condition for the labour market in (20) we have:

$$\frac{(1-\hat{w})\rho}{B} + \frac{(1-\beta)\rho}{B\beta}.\frac{(1-\hat{w})}{\hat{w}} + \frac{(1-\alpha)\mu\rho}{B\alpha\beta}.\frac{(1-\hat{w})}{\hat{w}} - \frac{\rho}{\lambda} = 1 \qquad (24)$$

Obviously, when $\hat{w} \in [0,\ 1]$ then $L_m \geq 0$ and $L_x \geq 0$. In order to have $L_r \geq 0$, we need $\hat{w} \leq \frac{(1-\alpha)\mu\lambda}{B\alpha\beta+(1-\alpha)\mu\lambda}$. Clearly, $\frac{(1-\alpha)\mu\lambda}{B\alpha\beta+(1-\alpha)\mu\lambda} \leq 1$ so the condition $\hat{w} \leq 1$ is automatically satisfied. Therefore, as soon as (24) has a solution $\hat{w} \in [0,\ \frac{(1-\alpha)\mu\lambda}{B\alpha\beta+(1-\alpha)\mu\lambda}]$ then it is sufficient to have $0 \leq L_x,\ L_m,\ L_r \leq 1$.

The LHS of (24) is decreasing in $\hat{w}$, approaching $+\infty$ when $\hat{w} \to 0$ and equal to $\frac{B\alpha^2\beta(1-\beta)\rho+\alpha(1-\alpha)\lambda\mu\rho}{(1-\alpha)\lambda\mu[B\alpha\beta+(1-\alpha)\lambda\mu]}$ when $\hat{w} = \frac{(1-\alpha)\mu\lambda}{B\alpha\beta+(1-\alpha)\mu\lambda}$. We have $\frac{B\alpha^2\beta(1-\beta)\rho+\alpha(1-\alpha)\lambda\mu\rho}{(1-\alpha)\lambda\mu[B\alpha\beta+(1-\alpha)\lambda\mu]} \leq \frac{\alpha\rho}{(1-\alpha)\mu\lambda}$ (equality happens when $\beta = 0$). Hence, if $\frac{\alpha\rho}{(1-\alpha)\mu\lambda} \leq 1$, the equation renders a unique positive solution $\hat{w} \in [0,\ \frac{(1-\alpha)\mu\lambda}{B\alpha\beta+(1-\alpha)\mu\lambda}]$ which allows us to compute equilibrium values characterizing the BGP such as $L_x(\hat{w}), L_m(\hat{w}), L_r(\hat{w}), g_A = \lambda(\mu-1)L_r(\hat{w})$, and $g_Y = g_C = (1-\alpha)\lambda(\mu-1)L_r(\hat{w}) - \alpha\beta BL_m(\hat{w})$.

Proof of Proposition 2

(i) For renewable resources ($\eta > 0$)

An increase in either μ or λ will shift the graph of the LHS of (23) upward while the RHS remains the same resulting in a higher equilibrium value of $\tilde{w}$. Because L_x and L_m are apparently decreasing in $\tilde{w}$ and $L_r = 1 - L_x - L_m$ so L_r increases which induces higher technological change and higher output growth because

$g_Y = (1-\alpha)\lambda(\mu-1)L_r$. However, an increase in ρ will shift the graph of the LHS downward resulting in a lower equilibrium value of $\tilde{w}$ and meaning higher L_x and L_m. As a result, L_r is lower which implies a lower output growth.

The impact of an increase in B is not immediately clear. However, we claim that L_r will increase and hence g_Y. First, when B increases, the graph of the LHS of (23) shifts downward. This implies that $\tilde{w}$ decreases. Now assume that $\frac{1}{B}\frac{1-\tilde{w}^2}{\tilde{w}}$ decreases. Since $\frac{1+\tilde{w}}{\tilde{w}}$ increases (because $\tilde{w}$ decreases, it would imply that $\frac{1}{B}(1-\tilde{w})$ decreases as well since $\frac{1}{B}\frac{1-\tilde{w}^2}{\tilde{w}} = \frac{1}{B}(1-\tilde{w}).\frac{1+\tilde{w}}{\tilde{w}}$. As a result, the LHS of (23) will decrease while its RHS remains unchanged which entails a contradiction. Hence, $\frac{1}{B}\frac{1-\tilde{w}^2}{\tilde{w}}$ increases and L_r increases. Therefore, g_Y rises.

(ii) For non-renewable resources ($\eta = 0$)

Output growth is given by $g_Y = (1-\alpha)\lambda(\mu-1)L_r(\hat{w}) - \alpha\beta B L_m(\hat{w})$. An increase in either μ or λ will shift the graph of the LHS of (24) upward while the RHS remains unchanged resulting in a higher equilibrium value of $\hat{w}$. Because L_x and L_m are decreasing in $\hat{w}$ and $L_r = 1 - L_x - L_m$ so L_r increases while both L_x and L_m decrease. This guarantees that g_Y increases.

The impact of an increase in ρ is not that straightforward. To better examine the impact we rewrite (24) as follows:

$$\frac{(1-\hat{w})}{B} + \frac{(1-\beta)}{B\beta}.\frac{(1-\hat{w})}{\hat{w}} + \frac{(1-\alpha)\mu}{B\alpha\beta}.\frac{(1-\hat{w})}{\hat{w}} = \frac{1}{\lambda} + \frac{1}{\rho} \tag{25}$$

The LHS of (25) is decreasing in $\hat{w}$. When ρ increases, its RHS moves downward. Hence $\hat{w}$ increases. We claim that L_m increases or, equivalently, $(1-\hat{w})\rho$ increases. Suppose that is not true, then from (24), $\frac{\rho(1-\hat{w})}{\hat{w}}$ must increase. Since $(1-\hat{w})\rho$ decreases, it would imply that $\hat{w}$ decreases, which is a contradiction. Hence, L_m increases. Now either L_x increases or decreases. In the first case, L_r decreases (because both L_m and L_x have increased). In the second case, it must be that $\frac{\rho(1-\hat{w})}{\hat{w}}$ decreases. Since $L_r = \frac{(1-\alpha)\mu\rho}{B\alpha\beta}.\frac{(1-\hat{w})}{\hat{w}} - \frac{\rho}{\lambda}$, one can see that L_r decreases. Therefore, g_Y decreases.

Let us consider the impact of an increase of B. For that, again consider equation (24). First, it is obvious that $\hat{w}$ will decrease. We define the following:

$$B = \frac{\lambda\rho(\Phi + \hat{w})(1-\hat{w})}{(\lambda+\rho)\hat{w}} \tag{26}$$

where

$$\Phi = \frac{1-\beta}{\beta} + \frac{(1-\alpha)\mu}{\alpha\beta} \tag{27}$$

Then output growth is:

$$\begin{aligned} g_Y &= (1-\alpha)\lambda(\mu-1)\left[\frac{(1-\alpha)\mu\rho}{B\alpha\beta}.\frac{1-\hat{w}}{\hat{w}} - \frac{\rho}{\lambda}\right] - \alpha\beta\rho(1-\hat{w}) \\ &= (1-\alpha)\lambda(\mu-1)\left[\frac{(1-\alpha)\mu(\lambda+\rho)}{\lambda\alpha\beta(\Phi+\hat{w})} - \frac{\rho}{\lambda}\right] - \alpha\beta\rho(1-\hat{w}) \end{aligned}$$

This result is obtained by using (26). Now differentiating this relation with respect to B we get:

$$\begin{aligned}\frac{\partial g_Y}{\partial B} &= -\frac{\partial \hat{w}}{\partial B}\left[\frac{(1-\alpha)^2(\mu-1)\mu(\lambda+\rho)}{\alpha\beta(\Phi+\hat{w})^2}-\alpha\beta\rho\right] \\ &\geq -\frac{\partial \hat{w}}{\partial B}\left[\frac{(1-\alpha)^2(\mu-1)\mu(\lambda+\rho)}{\alpha\beta(\Phi+1)^2}-\alpha\beta\rho\right]\end{aligned}$$

The inequality result in the second line is deduced given that $w \leq 1$ and $\frac{\partial \hat{w}}{\partial B} < 0$ (note that an increase in B shifts the graph of the LHS of (24) downward while its RHS stays unchanged resulting in a smaller value of $\hat{w}$). Using (27) we have:

$$1+\Phi = \frac{\alpha+(1-\alpha)\mu}{\alpha\beta}$$

Substituting this result into the above inequality delivers:

$$\frac{\partial g_Y}{\partial B} \geq -\alpha\beta\frac{\partial \hat{w}}{\partial B}\left[\frac{(1-\alpha)^2(\mu-1)\mu(\lambda+\rho)}{[\alpha+(1-\alpha)\mu]^2}-\rho\right]$$

Hence, if

$$\left[\frac{(1-\alpha)^2(\mu-1)\mu(\lambda+\rho)}{(\alpha+(1-\alpha)\mu)^2}-\rho\right] > 0 \Rightarrow \frac{\partial g_Y}{\partial B} > 0 \tag{28}$$

Obviously, when ρ is small or λ is large then (28) is satisfied. When μ converges to infinity, we get:

$$\frac{(1-\alpha)^2(\mu-1)\mu(\lambda+\rho)}{[\alpha+(1-\alpha)\mu]^2} \to \lambda+\rho$$

The conclusion follows. When μ is close to 1, the expression inside the brackets of the RHS of the equation for $\frac{\partial g_Y}{\partial B}$ becomes negative implying $\frac{\partial g_Y}{\partial B} \leq 0$.

Proof of Proposition 3

(i) For renewable resources ($\eta > 0$)

Because $g_Y = (1-\alpha)\lambda(\mu-1)L_r$ and $L_r \geq 0$, it is obvious that $g_Y \geq 0$.

(ii) For non-renewable resources ($\eta = 0$)

We have:

$$g_Y = (1-\alpha)\lambda(\mu-1)\left[\tfrac{(1-\alpha)\mu\rho}{B\alpha\beta}.\tfrac{1-\hat{w}}{\hat{w}}-\tfrac{\rho}{\lambda}\right]-\alpha\beta\rho(1-\hat{w})$$

We claim that when μ is close to 1 then g_Y will be negative. Indeed, one can see that when μ tends to 1, $\hat{w}$ converges to $0 < \hat{w}^* \leq \frac{(1-\alpha)\lambda}{B\alpha+(1-\alpha)\lambda} < 1$ and $L_r(\hat{w}) \to L_r(\hat{w}^*) \leq 1$ under the assumption $\lambda \geq \frac{\alpha\rho}{1-\alpha}$. Thus, $g_Y \to -\alpha\beta\rho(1-\hat{w}^*) < 0$.

Proof of Proposition 4

From (23) and (24), we get:

$$\eta\left[\frac{1-\tilde{w}}{2}+\Phi.\frac{1-\tilde{w}^2}{4\tilde{w}}\right]=\rho\left[(1-\hat{w})+\Phi.\frac{(1-\hat{w})}{\hat{w}}\right] \tag{29}$$

Now assume $g_Y(\tilde{w}) \leq g_Y(\hat{w})$ which means $L_r(\tilde{w}) < L_r(\hat{w})$ or:

$$\frac{\eta(1-\tilde{w}^2)}{4\tilde{w}} < \rho\frac{1-\hat{w}}{\hat{w}} \tag{30}$$

This together with (29) implies the following simultaneous condition:

$$\frac{\eta}{2}(1-\tilde{w}) > \rho(1-\hat{w}) \tag{31}$$

$$\Leftrightarrow \tilde{w} < \left(1-\frac{2\rho}{\eta}\right)+\frac{2\rho}{\eta}\hat{w} \tag{32}$$

These two simultaneous conditions (30) and (31) give:

$$\rho\frac{1-\hat{w}}{\hat{w}} > \frac{\eta(1-\tilde{w}^2)}{4\tilde{w}} = \frac{\eta(1-\tilde{w})(1+\tilde{w})}{4\tilde{w}} > \frac{\eta}{4}.\frac{2\rho}{\eta}(1-\hat{w})\frac{1+\tilde{w}}{\tilde{w}}$$

or

$$\hat{w} < \frac{2\tilde{w}}{1+\tilde{w}} \tag{33}$$

From (32) and (33) we have:

$$\tilde{w} < (1-\tfrac{2\rho}{\eta})+\tfrac{4\rho}{\eta}.\tfrac{\tilde{w}}{1+\tilde{w}}$$

which is equivalent to:

$$\tilde{w}^2-\frac{2\rho}{\eta}\tilde{w}-\left(1-\frac{2\rho}{\eta}\right) \quad < \quad 0 \tag{34}$$

Let $H(\tilde{w})=\tilde{w}^2-\frac{2\rho}{\eta}\tilde{w}-(1-\frac{2\rho}{\eta})$. We have $H'(\tilde{w})=2\tilde{w}-\frac{2\rho}{\eta}$ and $H''(\tilde{w})=2>0$. Hence, $H(\tilde{w})$ has a global minimum at $\frac{\rho}{\eta}$ because $H'(\frac{\rho}{\eta})=0$. If $\rho>\eta$ then $\frac{\rho}{\eta}>1$. In addition, $H(0)>0$ and $H(1)=0$ meaning that $H(z)>0, \forall z\in(0,1)$. This contradicts with (34). We conclude that $g_Y(\tilde{w}) > g_y(\hat{w})$.

Proof of Proposition 5

Let $\Omega=\frac{(1-\alpha)^2\lambda\mu(\mu-1)}{B\alpha\beta}, \Phi=\frac{1-\beta}{\beta}+\frac{(1-\alpha)\mu}{\alpha\beta}$ and $\triangle = g_Y(\tilde{w})-g_Y(\hat{w})$. From (24), $\hat{w}$ depends on ρ but is independent of η. Using (29), we obtain:

$$\triangle \quad = \quad \frac{\Omega}{\Phi}\left[\rho(1-\hat{w})-\frac{\eta}{2}(1-\tilde{w})\right]+\alpha\beta\rho(1-\hat{w})$$

or

$$\triangle = \rho(1-\hat{w})\left[\frac{\Omega}{\Phi}+\alpha\beta\right] - \frac{\Omega\eta}{2\Phi}(1-\tilde{w})$$

From (23), observe that $\tilde{w}$ increases when η increases. We claim that $\eta(1-\tilde{w})$ increases too. Indeed, if not, when η increases, together both $\eta(1-\tilde{w})$ and $\frac{\eta(1-\tilde{w}^2)}{\tilde{w}}$ decrease meaning that the LHS of (23) decreases while its RHS remains unchanged and a contradiction occurs. Another observation is that $\triangle$ decreases when η increases.

Let $l = \lim_{\eta\to+\infty}\eta(1-\tilde{w})$. From (23), we get:

$$l = \frac{2B(\lambda+\rho)}{\lambda(\Phi+1)}$$

Hence, when η goes to infinity,

$$\triangle \to (1-\hat{w})\rho\left[\frac{\Omega}{\Phi}+\alpha\beta\right] - \frac{\Omega B(\lambda+\rho)}{\Phi\lambda(1+\Phi)} \tag{35}$$

Note that (24) is equivalent to

$$(1-\hat{w}) = \frac{B(\lambda+\rho)}{\lambda\rho(1+\frac{\Phi}{\hat{w}})}$$

Hence, (35) becomes:

$$\begin{aligned}
\triangle \to z &= \frac{B(\lambda+\rho)}{\lambda}\left[\frac{\frac{\Omega}{\Phi}+\alpha\beta}{(1+\frac{\Phi}{\hat{w}})} - \frac{\Omega}{\Phi(1+\Phi)}\right] \\
&= \frac{\beta(\lambda+\rho)}{\lambda}\left[\frac{\Omega+\alpha\beta(1+\Phi)-\frac{\Omega}{\hat{w}}}{(1+\frac{\Phi}{\hat{w}})(1+\Phi)}\right] \\
&= \frac{\beta(\lambda+\rho)}{\lambda}\left[\frac{\Omega+\alpha+(1-\alpha)\mu-\frac{\Omega}{\hat{w}}}{(1+\frac{\Phi}{\hat{w}})(1+\Phi)}\right]
\end{aligned}$$

Let $\bar{w} = \frac{\Omega}{\alpha+(1-\alpha)\mu+\Omega}$. It is obvious that $z < 0 \Leftrightarrow \hat{w} < \bar{w}$.

Equation (24) can be rewritten as: $G(\hat{w}) = 1 + \frac{\rho}{\lambda}$, where

$$G(x) = \frac{(1-x)\rho}{B} + \frac{(1-\beta)\rho}{B\beta}.\frac{(1-x)}{x} + \frac{(1-\alpha)\mu\rho}{B\alpha\beta}.\frac{(1-x)}{x}$$

In computing $G(\bar{w})$, tedious calculations give:

$$G(\bar{w}) = \frac{[\alpha+(1-\alpha)\mu]\rho}{B}\left[\frac{1}{[\Omega+\alpha+(1-\alpha)\mu]} + \frac{\Phi}{\Omega}\right]$$

When $\lambda \to +\infty$, we have $G(\bar{w}) \to 0 < 1$. That means for any λ large enough then $G(\bar{w}) < G(\hat{w})$. This is equivalent to $\hat{w} < \bar{w}$ since $G(.)$ is decreasing. Choose $\bar{\lambda}$ such that $G(\bar{w}) < G(\hat{w})$ for any $\lambda > \bar{\lambda}$. Therefore, for any $\lambda > \bar{\lambda}$, we have:
(i) From (23), when $\eta \to 0$, then $\tilde{w} \to 0$. This implies $\triangle \to \rho(1-\hat{w})\left[\frac{\Omega}{\Phi} + \alpha\beta\right] > 0$.
(ii) $\lim_{\eta\to+\infty} \triangle = z < 0$.
Since $\triangle$ is decreasing in η, the conclusion follows.

Proof of Proposition 6

Again, let $\Omega = \frac{(1-\alpha)^2\lambda\mu(\mu-1)}{B\alpha\beta}, \Phi = \frac{1-\beta}{\beta} + \frac{(1-\alpha)\mu}{\alpha\beta}$ and $\triangle = g_Y(\tilde{w}) - g_Y(\hat{w})$. As in the proof of Proposition 5:

$$\triangle = \rho(1-\hat{w})\left[\frac{\Omega}{\Phi} + \alpha\beta\right] - \frac{\Omega\eta}{2\Phi}(1-\tilde{w})$$

From (23), we obtain that $\hat{w} \to 0$ when $\rho \to 0$ and $\lim_{\rho\to 0}\frac{\rho}{\hat{w}} = \frac{B}{\Phi}$. Define $\tilde{w}(0) = \lim_{\rho\to 0}\tilde{w}$. Equation (29) now becomes:

$$\eta\left[\frac{1-\tilde{w}(0)}{2} + \Phi.\frac{1-\tilde{w}(0)^2}{4\tilde{w}(0)}\right] = \frac{1}{B}$$

We have $\tilde{w}(0) \in (0,1)$ and

$$\lim_{\rho\to 0}\triangle = -\frac{\Omega\eta}{2\Phi}(1-\tilde{w}(0)) < 0$$

The proof is complete.

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