

Workforce or Workfare? The Optimal Use of Work Requirements when Labor is Supplied along the Extensive Margin

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Abstract:

“Workforce or Workfare? The Optimal Use of Work Requirements when Labor is Supplied along the Extensive Margin”

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This paper explores the use of workfare as part of a tax mix when labor supply responses are along the extensive margin. In an economy where the government has a priori chosen any tax-and-benefit schedule, we show that, despite their common goal of providing additional incentives for individuals to enter the labor force, workfare and an earned income tax credit are at odds with each other. In the presence of an optimal nonlinear income tax, we also show that introducing unproductive workfare is always suboptimal when individuals face the same disutility of being on workfare. When this disutility is heterogeneous, unproductive workfare may be a useful policy tool. We also provide a sufficient condition for productive workfare to be optimal.

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1 Introduction

Economic downturns, like the recent so-called Great Recession, can place strain on public programs that provide transfers to individuals. On the one hand, increased levels of unemployment add to the need for welfare and related benefits. On the other, governments tend to face considerable budget deficits, making funds increasingly scarce. It is little wonder, therefore, that the proper design of benefit programs recently resurfaced on the policy agenda (OECD, 2009). In Britain, for example, legislation was tabled in February, 2011 with the intent of completely overhauling that country's benefits programs (Government of Great Britain, 2011). Against this backdrop, it seems appropriate to revisit the role of workfare — the practice of requiring those who receive public benefits to spend time on some mandated activity — in the policy mix. Indeed, Quaid (2002) shows that policy makers often turn their attention to workfare whenever substantial welfare reform is contemplated.

In this article, we offer some results on when and how workfare can fit into the policy mix and argue that the case for workfare is weakened if net wage subsidies, such as earned income tax credits (EITC), are part of the policy mix. We do so in a model of labor supply choice along the extensive margin in the tradition of Diamond (1980) and more recent elaborations by Saez (2002) and Choné and Laroque (2005).

Private individuals in our basic model differ along two dimensions, labor market skill and preference for leisure. The labor market is perfectly competitive so that market wages are the respective skill levels. Given these wages and the tax-transfer program adopted by the government, individuals choose whether to work or not work. All workers are assumed to provide a fixed quantity of labor supply. The tax-transfer program might include the requirement that anyone who does not work must spend time in workfare activities. Given heterogeneity in preferences for market work, some portion of workers of each skill type (those with the least taste for leisure) choose to work. The others remain out of the labor force and receive public benefits. When taxes are optimally set, the government chooses an income tax schedule for those in work, the level of the public

benefit and the intensity of workfare to maximize a utilitarian social welfare function subject to a government budget constraint. Most of our results generalize to any Paretian social welfare function.

We derive conditions under which the addition of a completely unproductive workfare requirement to an arbitrary tax-transfer scheme is welfare-improving. Of course, these conditions are expressed in terms of the costs and benefits of workfare. One cost of workfare is the direct utility losses of those who must do the required work. But workfare has behavioral effects, too. By increasing the *ceteris paribus* cost of remaining out of the labor force, workfare induces workforce participation. To a first order, this is a matter of indifference to marginal participants. However, increasing the size of the workforce affects the public budget. If marginal participants pay a net tax, workfare eases the public budget constraint and generates gains that can be weighed against the direct utility loss. However, if marginal participants receive a net wage subsidy, as would be the case if the tax-transfer system features an earned income tax credit, then the behavioral effect of workfare actually worsens the public budget. Thus, the presence of an earned income tax credit weakens the case for workfare when labor supply choices are along the extensive margin.

We also consider the question of when workfare can be a useful addition to an optimal tax-transfer scheme. Most of the existing literature on workfare as part of an optimal tax-transfer mix follows Besley and Coate (1992, 1995) by modeling labor supply with variable hours of work; that is, it is developed in the so-called intensive margin model. In this framework, workfare helps to dissuade highly productive workers from claiming benefits because it is the highly productive that have the highest opportunity cost of time. As Brett (1998) notes, this channel is not available when claimants are out-of-work because the opportunity cost of voluntarily unemployed labor is the marginal rate of substitution between labor and consumption, which does not depend on labor market productivity.

We show that work requirements are never optimal in our basic model. This result

obtains for essentially the same reason as in the utility maintenance version of the Besley and Coate (1995) analysis. If there is a positive amount of required work, the social planner can reduce workfare requirements and the benefit level simultaneously in such a way as to leave the out-of-work equally well off. Moreover, the reduction in benefit level saves resources. It is also possible to give a screening interpretation. In the extensive margin model, the opportunity cost of leisure does not depend on productivity, just as is the case for those out-of-work in Brett (1998). Moreover, the welfare benefit already provides an instrument to screen over preferences for leisure. There remains no screening role for workfare.

We then allow for productive workfare, as in Brett (1998), and show that the condition he derived for the optimal introduction of workfare for the out-of-work in an otherwise intensive labor supply model extends to the pure extensive margin case.

Cuff (2000) introduces preference heterogeneity in an optimal tax framework with intensive labor supply responses. In some variants of her model, it is desirable to design workfare schemes for individuals with a low preference for leisure. For the obvious reason, workfare helps to screen out would-be claimants with a higher preference for leisure. Cuff also notes that the same desire to screen out the “lazy” provides the government with a reason to offer marginal wage subsidies.¹ Thus, along the intensive margin, workfare and wage subsidies push in the same direction and it is reasonable to expect the two policies to coexist. Our tax reform results show that this affinity between wage subsidies and workfare disappears when labor is supplied along the extensive margin.

In order to determine whether Cuff’s results on optimal taxation remain valid when labor is supplied along the extensive margin, we develop another variant of our basic model in which we allow for heterogeneity in distaste for required work. We note that it is not possible to directly disentangle preference heterogeneity over market work from preference heterogeneity over required work in the standard intensive margin models

¹Due to the countervailing force of incentive effects operating along the skill dimension, the optimal tax system might feature either marginal wage taxes or marginal wage subsidies.

because the distaste for required work typically varies with the amount of market work undertaken. In our extensive labor supply model, we can differentiate between these two sources of heterogeneity by introducing a third parameter, which directly measures the distaste for workfare, to the Diamond (1980) model. For most of our analysis, we make no assumptions concerning the correlations among the various dimensions of individual heterogeneity.

We show that, unlike the situation with diversity of tastes for market work alone, there are circumstances under which workfare is a useful addition to an optimal tax-transfer scheme when people differ in tastes for required work. The Besley-Coate argument for reducing any positive level of required work breaks down in the presence of preference heterogeneity. It is possible to construct a simultaneous reduction in required work and the social benefit that leaves social welfare unchanged while initially saving resources. However, because all persons out of work receive the same benefit regardless of their respective distastes for workfare, the preceding policy change will make some beneficiaries — those with the highest distaste for workfare — better off. Consequently, some of these individuals will leave the labor force. These workers then stop paying taxes (when they do not face an EITC), thereby hurting the government budget balance. Depending on the distribution of tastes and on the form of the social welfare function, this loss in revenue may be sufficient to undo the direct savings from the reduced benefit.² We provide examples where workfare is optimal, and not optimal, in presence of preference heterogeneity.

The remainder of the article is organized as follows. In the next section, we introduce and analyze our basic model, which is a straightforward extension of the Diamond (1980) model of optimal taxation. We carry out tax reform analysis in Section 3 and characterize optimal policies in Section 4. In Section 5, we discuss briefly the case of pro-

²Depending on the social welfare function, there may also be beneficiaries who are made worse off by the changes described in this paragraph. These people enter the labor force and pay more taxes. For workfare to be optimal, the revenue gains must outweigh both the revenue loss from this channel and the direct budgetary savings due to the decrease in benefits.

ductive workfare. Section 6 contains our analysis for the case of heterogeneous distaste for required work. Concluding remarks are found in Section 7. The proofs of our results are contained in Appendix A. Appendix B provides a detailed example of a planner's problem with a maxi-min utility social objective function.

2 The Basic Model

All individuals in the economy are capable of working in the market. They can choose to work, but not the number of hours they work. Those who enter the market produce n units of a composite consumption good c . Individuals differ in their market productivity along some interval $[\underline{n}, \bar{n}]$ with $0 < \underline{n} < \bar{n} \leq \infty$. The labor market is perfectly competitive, so that n also measures the before-tax income of the individual.

The government can observe a worker's before-tax income and implement a tax schedule $t(n)$. A worker's consumption is equal to after-tax income and is given by $c(n) = n - t(n)$.³ Negative taxes are interpreted as subsidies. In addition to the tax schedule, the government provides a welfare benefit of b units of the consumption good.

Receipt of the welfare benefit is conditional on the individual engaging in some required activity, denoted by a variable $r \in [0, \infty)$ that reflects some combination of the duration, intensity, and unpleasantness of the required activity. Unlike market work, required work is measured on a continuous scale. This captures the ability of the government to design the workfare task(s). Because the government has considerable latitude to vary the types and duration of workfare activities, continuity of r seems reasonable.

Individuals' utilities are increasing in consumption and decreasing in an index of the onerousness of labor. For market work, this onerousness is measured by a variable $m \in [\underline{m}(n), \bar{m}(n)]$, where this interval may vary by skill type. Taking the tax-transfer into account, a worker experiences utility level $u(c(n), m)$, where the function u is increasing

³This set of informational assumptions is consistent with Diamond (1980). Allowing an agent to work in any occupation that requires a skill below her type opens the possibility of monotonicity constraints and pooling. Choné and Laroque (2011) present sufficient conditions for the absence of pooling at the optimum, without workfare.

in its first argument, decreasing in its second argument, continuously differentiable and strictly concave. Someone outside the labor market experiences utility $u(b, r)$. In this basic version of the model, we assume that all individuals find workfare equally onerous; in other words, r is the same for everyone. We assume that r is measured in such a way that $r = 0$ when there are no work requirements. Under these conditions, individuals differ along two dimensions and can be characterized by an ordered pair (m, n) . The population is described by a continuous distribution function $F(m, n)$ with a density $f(m, n)$.

Given the policies adopted by the government, individuals decide whether to work or not. An individual works if

$$u(c(n), m) \geq u(b, r). \quad (1)$$

For each n , those individuals with the lowest values of m choose to work, while those with a relatively higher preference for leisure remain outside the labor force. Indeed, for each n , there exists a critical value $m^*(c(n), b, r)$ such that the workers with skill type n are exactly those that have $m < m^*(c(n), b, r)$. This critical value is determined by the equation

$$u(c(n), m^*(c(n), b, r)) = u(b, r). \quad (2)$$

It follows immediately from the properties of the utility function that $m^*(c(n), b, r)$ — and, consequently the size of the workforce — increases when $c(n)$ or r increases or when b decreases. The greater is after-tax income from employment, the more desirable is work. The more unpleasant the workfare activity, the less desirable is being out-of-work. In either case, the relative return to working increases, thereby encouraging labor market participation. The greater is the public welfare benefit, the more desirable is remaining out of the labor market, so participation falls. We assume that $m^*(c(n), b, r) \in (\underline{m}, \overline{m})$ for all skill types n , so that small changes in program parameters always have some effect on participation.⁴ Alternatively, our analysis carries through with minor modifications under the proviso that it pertains to those skill types whose participation is affected by

⁴One way to guarantee this property is to assume that $m \in (-\infty, \infty)$.

the policies modeled in this article.

Following Diamond (1980), the government maximizes a utilitarian social welfare function.⁵ Given the participation decisions, social welfare can be written

$$\mathcal{W}(c, b, r) = \int_{\underline{n}}^{\bar{n}} \left[\int_{\underline{m}(n)}^{m^*(c(n), b, r)} u(c(n), m) f(m, n) dm + \int_{m^*(c(n), b, r)}^{\bar{m}(n)} u(b, r) f(m, n) dm \right] dn, \quad (3)$$

where c denotes the entire consumption schedule. To focus on the issue of optimal redistribution, we assume that the sole motives for taxation are to finance the public welfare benefit and to cover an exogenous revenue requirement R . Additionally, so as to not conflate other effects of workfare with its role in influencing labor market participation, we initially assume that workfare is completely unproductive. Under these conditions, total revenue raised by the government can be written

$$\mathcal{R}(c, b, r) = \int_{\underline{n}}^{\bar{n}} \left[\int_{\underline{m}(n)}^{m^*(c(n), b, r)} [n - c(n)] f(m, n) dm - \int_{m^*(c(n), b, r)}^{\bar{m}(n)} b f(m, n) dm \right] dn; \quad (4)$$

the budget constraint of the government is

$$\mathcal{R}(c, b, r) = R. \quad (5)$$

3 Tax Reform Analysis

The government has three policy tools at its disposal: the tax schedule $t(n)$, or equivalently the consumption schedule $c(n)$; the benefit level b ; and the work requirement r . Beginning at an arbitrary, possibly sub-optimal tax-and-benefit schedule without workfare, we ask when it might be possible for the introduction of workfare to improve social welfare. As in Ahmad and Stern (1991, p. 61), we define the marginal cost, in terms of social welfare loss, per unit of revenue raised by changing a policy instrument. For required work, this marginal cost is given by the formula

$$\lambda_r = - \frac{\frac{\partial \mathcal{W}}{\partial r}}{\frac{\partial \mathcal{R}}{\partial r}}. \quad (6)$$

⁵Our results extend to more general Bergson-Samuelson social welfare functions.

In order to simplify the statement of resulting formulas, we define the participation tax for individuals of type n , denoted $\tau(n)$, to be the sum of the tax paid when employed and the public welfare benefit, which is withdrawn upon taking up employment. Formally,

$$\tau(n) = t(n) + b = n - c(n) + b. \quad (7)$$

When $\tau(n)$ is negative, employment of individuals of type n is subsidized by the tax system and we say that there is a participation subsidy. Such a subsidy is commonly referred to as an earned income tax credit (EITC). When the net tax on employment is positive, it is common to refer to the tax system as a negative income tax (NIT) scheme.

Using (3), (4), and (7), the marginal cost of increasing tax revenue via increasing r is given by⁶

$$\lambda_r = - \frac{u_l(b, 0) \int_{\underline{n}}^{\bar{n}} \int_{m^*(n)}^{\bar{m}(n)} f(m, n) dm dn}{\int_{\underline{n}}^{\bar{n}} \tau(n) \frac{\partial m^*(n)}{\partial r} f(m^*(n), n) dn}. \quad (8)$$

The numerator in (8) is negative. It captures the welfare loss due to the disutility of work felt by those engaged in the required work. The denominator can be of any sign. An increase in the intensity of required work increases the size of the workforce, which is captured by an increase in m^* .⁷ The total increase in the workforce of skill type n is given by $(\frac{\partial m^*}{\partial r})f(m^*, n)$. Multiplying this change in the workforce by the net tax on working for that type, $\tau(n)$, gives the change in tax revenue received from workers of skill type n . Integrating over all skill types provides the total effect on the public budget. If the skill type n pays a participation tax, this type contributes a positive amount to the denominator in (8). If skill type n receives a participation subsidy then this revenue effect is negative. In this event, workfare induces more people to enter the labor force, which increases the amount of public funds needed to finance the EITC scheme. If marginal workfare participants receive, on average, a participation subsidy the denominator in

⁶The derivatives of $u(\cdot, \cdot)$ with respect to its first and second arguments are denoted by u_c and u_l , respectively.

⁷To a first order, these marginal participants experience no change in utility when entering the workforce because they were previously indifferent between working and remaining out of the labor force.

(8) is negative, thereby yielding $\lambda_r < 0$. In this circumstance, an increase in workfare is completely counter-productive. It both makes people worse off, all else equal, and reduces government revenue. This discussion is summarized in Proposition 1.

Proposition 1. *The introduction of an unproductive workfare program at low intensity reduces social welfare if marginal participants receive, on average, a participation subsidy.*

Proposition 1 is robust to some of the types of modifications to the government objective functions often found in the literature. For example, the income maintenance approach to workfare in Besley and Coate (1992, 1995) and the reduction of poverty in after-tax income approach to nonlinear income taxes found in Kanbur et al. (1994) both put zero social weight on the disutility of labor. Nevertheless, when workfare actually creates added pressure on public budgets, it cannot be optimal in these models, and we could replace the “if” in Proposition 1 with the phrase “if and only if.” The sign of the numerator in (8), and with it the simple statement of Proposition 1, may change if work itself is seen as a social good, as in Moffit (2006). We consider the case of productive workfare, as in Brett (1998), in Section 5 below.

4 Optimal Policies

The government’s decision problem is to choose $c(n)$, b and r to maximize the social welfare criterion (3), subject to the resource constraint (5).

The goal of our analysis is to examine whether, or under what conditions, workfare is a desirable addition to the policy mix. Thus, we proceed in two steps. First, we consider the optimal tax-transfer mix (the choice of $c(n)$ and b) for an arbitrary intensity of workfare. Second, we ask if, starting from no workfare — that is, $r = 0$ — a small increase in the intensity of workfare enhances social welfare. The first step is formally identical to the existing literature on optimal taxation with work choice along the extensive margin.

The results from this literature that we need below are summarized in Lemma 1 below. Additionally, we denote by $g(n)$ the (average and endogenous) marginal social

welfare weight given to workers of skill n , expressed in terms of public funds. Formally,

$$g(n) \equiv \frac{1}{\lambda F(m^*, n)} \int_{\underline{m}(n)}^{m^*(c(n), b, r)} \frac{\partial u(c(n), m)}{\partial c(n)} f(m, n) dm, \quad (9)$$

where λ is the Lagrange multiplier of the budget constraint (5).

Lemma 1 (Saez, 2002). *The optimal participation tax for individuals of skill type n is negative (equivalently, there is a participation subsidy) exactly when $g(n) > 1$.*

The proof of the lemma is in Appendix A. The condition governing optimality of an EITC scheme (i.e., a negative participation tax) relates the average marginal social welfare weight of workers of skill n (expressed in public funds) to one. This welfare weight represents the dollar equivalent value for the government of distributing an extra dollar uniformly to workers of type n . When this value is larger (lower) than one, then these workers should receive a subsidy (should pay taxes) $\tau(n) < 0 (> 0)$. The intensity of required work might influence this condition through the cross-derivatives of the function u (see (9)). If the marginal utility of consumption increases as workfare becomes more intense, there is a tendency for g to increase, thereby moving the optimal tax-transfer scheme in the direction of an EITC. However, given the focus of the second step of our analysis, which is to examine the advisability of adding a small workfare requirement, we will carry out our subsequent analysis under the assumption that $r = 0$. By so doing, we can treat the issue of whether there is an EITC or an NIT as fixed by the optimum without workfare.

The tax reform analysis of the previous section implies that workfare can never be optimal in combination with an EITC. We now show in Proposition 2 that completely unproductive workfare is never optimal in the extensive margin model, even with a participation tax.

Proposition 2. *Introducing unproductive workfare is always welfare-decreasing in the presence of an optimal tax-transfer scheme.*

The intuition behind Proposition 2 is straightforward. Assume, contrary to the proposition, that there is some positive intensity of workfare at the optimum. We can keep the utility of people on workfare, $u(b, r)$, unaffected by a small decrease in workfare and a reduction in welfare benefit chosen so as to keep these people on the same indifference curve. Because this change does not affect the opportunities in the labor market, there is no effect on the cut-off equation (2). Hence, the policy adjustment has no effect on participation decisions, nor any effect on the utility levels of workers and non-workers. It does, however, save resources by decreasing the aggregate amount of welfare payments.

5 Productive Workfare

The model of the previous section is based on simplifying assumptions about the nature of workfare. In this section, as in Brett (1998), we allow for workfare to have some direct productivity. We now imagine the workfare variable r as some measure of the time spent on workfare. Suppose that each unit of time spent in required activities produces γ units of output.⁸ This modification has no direct impact on utility functions or the social welfare function. It does, however, affect the government budget. Instead of being described by (4), net tax revenue is now given by

$$\mathcal{R}^\gamma(c, b, r) = \int_{\underline{n}}^{\bar{n}} \left[\int_{\underline{m}(n)}^{m^*(c(n), b, r)} [n - c(n)] f(m, n) dm + \int_{m^*(c(n), b, r)}^{\bar{m}(n)} [\gamma r - b] f(m, n) dm \right] dn. \quad (10)$$

From (10), it is clear how to modify Proposition 1 to account for productive workfare. Along the extensive margin and in the presence of an EITC, workfare is useful from a tax reform perspective if and only if its output is enough to compensate for the disutility of work of program participants and to cover the increases in the EITC.

We now turn to the desirability of workfare when an optimal income tax is available. In a model with intensive labor supply choice, Brett (1998) shows that workfare for the out-of-work is optimal if and only if the marginal product of the required activity is sufficient to compensate for the marginal disutility of work for program participants.

⁸We imagine that γ is measured net of any administrative costs of workfare.

The analog to this result in the pure extensive margin model is given by Proposition 3.

Proposition 3. *Introducing productive workfare is welfare increasing in the presence of an optimal tax-transfer scheme if and only if*

$$\gamma > -\frac{u_l(b, 0)}{u_c(b, 0)}. \quad (11)$$

Thus, exactly as in Brett (1998, p.615, Proposition 4), workfare is used at an optimum if and only if it produces enough output to compensate program participants for their disutility of required work. The intuition for this result is straightforward. In the first instance, a one-unit increase in required work delivers an additional γ in revenue. Program participants can be compensated for this work by a $-u_l(b, 0)/u_c(b, 0)$ increase in the benefit level b . If this compensation takes place, nobody experiences a change in utility, so social welfare is unchanged and participation decisions are unaffected. However, the compensation is a drain on the public budget. If (11) is satisfied, this drain on the public budget is more than offset by the direct increase in revenue.

6 Three-Dimensional Heterogeneity Among Individuals

In this section, we relax the assumption that the intensity of workfare, in terms of utility, is identical for everyone. Specifically, we posit a third dimension of heterogeneity among individuals, namely in how they experience workfare. We show that this alternative is of no consequence to our qualitative results on tax reform, but that completely unproductive workfare may be part of an optimal tax-transfer system in some circumstances.

Just as individuals may differ in their costs of market work, they might also differ in their distaste for publicly-required work. To account for this possibility, we again interpret the workfare variable r as an objective measure of required work (for example, its duration). We then posit a preference parameter k that measures the intensity of distaste for this activity and k takes values along some interval $[\underline{k}, \bar{k}]$ with $\underline{k} > 0$. The utility level for a person that is unemployed — and, therefore, participating in workfare

— is given by $u(b, kr)$.⁹ Individuals are endowed with an ordered triple of characteristics (m, n, k) . The joint distribution of these three characteristics is denoted $F(m, n, k)$ and the associated density, $f(m, n, k)$. Apart from continuity, we make no assumptions on the joint distribution function. Thus, the model admits an arbitrary correlation structure among individual characteristics. For ease of notation only, we assume that the support of the distribution is a cube $[\underline{m}, \bar{m}] \times [\underline{n}, \bar{n}] \times [\underline{k}, \bar{k}]$. Individuals that are indifferent between market work and public welfare benefits are described by the equation

$$u(c(n), m) = u(b, kr). \quad (12)$$

The indifference condition (12) implicitly defines an $m^* = \varphi(k, r, c(n), b)$ above which individuals leave the labor force. For a fixed tax-transfer scheme, we suppress the dependence of this locus on the parameters of the tax-transfer system $c(n)$ and b and write $m = \psi(k, r)$, which is an upward-sloping locus in (k, m) space.¹⁰ Individuals above and to the left of this locus remain out of the labor market, because these people either find market work relatively more costly in terms of utility or find workfare relatively less onerous. The mass of workers of skill type n that choose market work is given by¹¹

$$\int_{\underline{k}}^{\bar{k}} \int_{\underline{m}}^{\psi(k, r)} f(m, n, k) dm dk. \quad (13)$$

An increase in r shifts the locus $m = \psi(k, r)$ upward, leading to increased participation in market work. To see this, applying the Implicit Function Theorem to (12) yields

$$\frac{\partial \psi}{\partial r} = \frac{ku_l(b, kr)}{u_l(c(n), m)} > 0. \quad (14)$$

From this point, the tax-reform analysis of Section 3 carries forward, with slightly

⁹The model of Section 2 corresponds to the special case of $k = 1$ for all individuals.

¹⁰By the Implicit Function Theorem, $\frac{\partial \psi}{\partial k} = \frac{ru_l(b, rk)}{u_l(c(n), m)} > 0$.

¹¹We are implicitly assuming that the locus $\psi(k, r)$ intersects the line $k = \underline{k}$ above $m = \underline{m}$, so that for each skill type there is some sufficiently low value of m that induces market work. If this is not the case, we can reformulate our analysis by parameterizing the locus as $k = \eta(m, r)$ and reversing the order of integration in (13) and all subsequent integrals. Our results do not change.

more cumbersome notation. The utilitarian social welfare function can be written

$$\mathcal{W} = \int_{\underline{n}}^{\bar{n}} \int_{\underline{k}}^{\bar{k}} \left[\int_{\underline{m}}^{\psi(k,r)} u(c(n), m) f(m, n, k) dm + \int_{\psi(k,r)}^{\bar{m}} u(b, kr) f(m, n, k) dm \right] dk dn. \quad (15)$$

The budget constraint is

$$\int_{\underline{n}}^{\bar{n}} \int_{\underline{k}}^{\bar{k}} \left[\int_{\underline{m}}^{\psi(k,r)} [n - c(n)] f(m, n, k) dm - \int_{\psi(k,r)}^{\bar{m}} b f(m, n, k) dm \right] dk dn = R. \quad (16)$$

Apart from the appearance of ψ in place of m^* and integration over the variable k , equations (15) and (16) are identical to the corresponding equations (3) and (5) in Section 2. Starting from an arbitrary tax-transfer scheme with no workfare, the revenue effect of a marginal increase in r is

$$\frac{d\mathcal{R}}{dr} = \int_{\underline{n}}^{\bar{n}} \int_{\underline{k}}^{\bar{k}} \tau(n) \frac{\partial \psi(k, 0)}{\partial r} f(\psi(k, 0), n) dm dk dn, \quad (17)$$

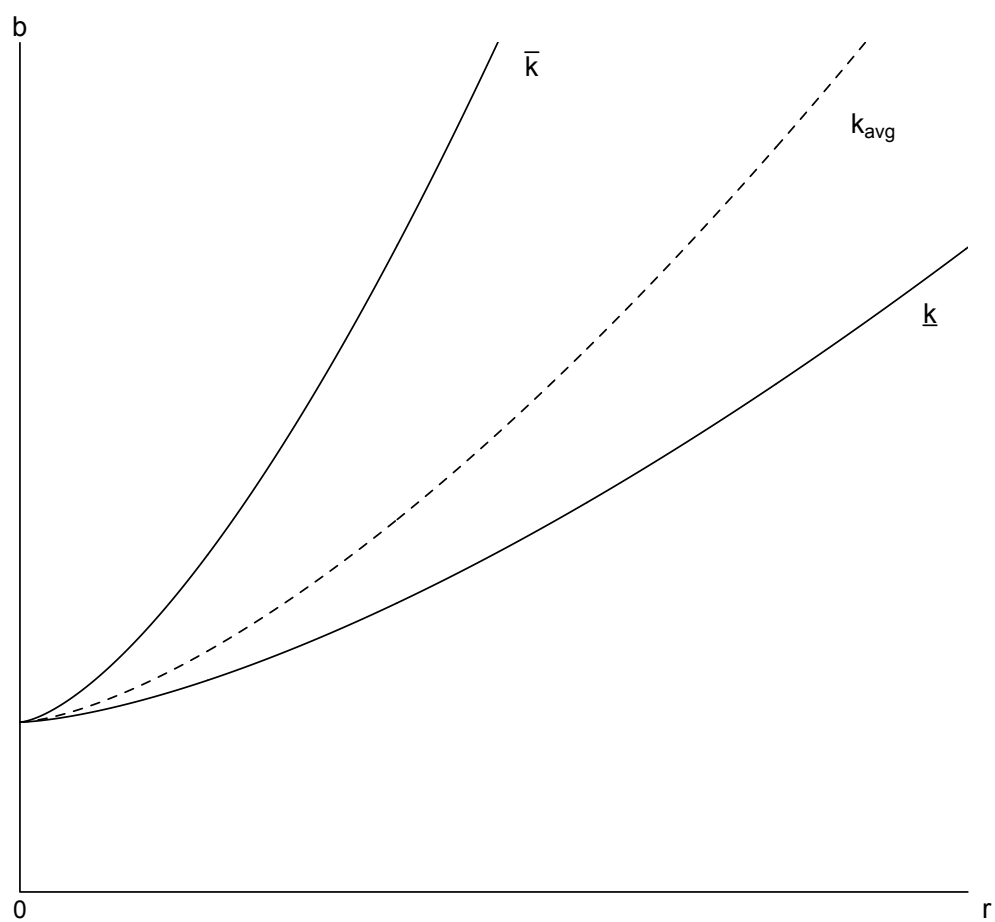
which is positive (negative) if marginal participants face, on average, a participation tax (subsidy). The direct welfare effect of introducing workfare is negative, and is given by

$$\frac{d\mathcal{W}}{dr} = \int_{\underline{n}}^{\bar{n}} \int_{\underline{k}}^{\bar{k}} \int_{\psi(k,0)}^{\bar{m}} k u_l(b, 0) f(m, n, k) dm dk dn \quad (18)$$

Thus, Proposition 1 carries over in this extended version of the model.

Unlike the situation described in Section 4, completely unproductive workfare may play a role in an optimal tax-transfer scheme when there is heterogeneity in the distaste for workfare. The forces at play are illustrated in Figure 1, where indifference curves are drawn in (r, b) -space. A larger disutility of workfare implies a steeper indifference curve. Assume that, starting from an economy with no workfare, $r = 0$, the government slightly increases r . Because people have heterogeneous (non-observable) distastes for workfare, k , there is no rise in the welfare benefit b that would allow the government to keep the utility of all individuals on workfare, $u(b, rk)$, unaffected. Rather, the government can compensate for the aggregate welfare loss by increasing b . This corresponds to a move along the indifference curve for some interior type, labeled k_{avg} in Figure 1. This

Figure 1: Effects of $r \uparrow$ with Heterogeneous Disutility of Workfare



compensating increase in b unambiguously deteriorates tax revenue but does not modify the participation decisions of people with distaste for workfare k_{avg} . However, because everyone gets the same welfare benefit, b , this average compensation is too large for people whose k is below k_{avg} and this average compensation is too small for people whose k is above k_{avg} . As a result, some people whose k is below k_{avg} stop working on the labor market and enter workfare, further reducing tax revenue (assuming that $\tau(n) > 0$). On the other hand, some people whose k is above k_{avg} move into the labor market and start paying taxes. When this positive impact on tax revenue offsets the previous two negative effects, workfare is a useful complement to the optimal tax mix. The formal trade-off among these forces is described in Proposition 4.

Proposition 4. *Introducing unproductive workfare may be welfare increasing in the presence of an optimal tax-transfer scheme when the disutility of workfare is heterogeneous provided that*

$$\begin{aligned}
- \int_{\underline{k}}^{\bar{k}} k \frac{u_l(b, 0)}{u_c(b, 0)} \Gamma(k) dk &< \int_{\underline{k}}^{\bar{k}} \int_{\underline{n}}^{\bar{n}} \frac{\partial \varphi(k, 0, c(n), b)}{\partial r} \tau(n) f(m^*, n, k) dn dk \\
&- \frac{u_l(b, 0)}{u_c(b, 0)} \frac{\int_{\underline{k}}^{\bar{k}} k \Gamma(k) dk}{\int_{\underline{k}}^{\bar{k}} \Gamma(k) dk} \int_{\underline{k}}^{\bar{k}} \int_{\underline{n}}^{\bar{n}} \frac{\partial \varphi(k, 0, c(n), b)}{\partial b} \tau(n) f(m^*, n, k) dn dk \quad (19)
\end{aligned}$$

where

$$\Gamma(k) = \int_{\underline{n}}^{\bar{n}} \int_{\varphi(k, 0, c(n), b)}^{\bar{m}(n)} f(m, n, k) dm dn \quad (20)$$

is the mass of workers of taste parameter k who choose not to work in the no-workfare optimum.

The left-hand side of (19) is the (absolute value of the) revenue loss owing to the compensation term described above. This term is the integral over all out-of-work k -types of their respective marginal rates of substitution between workfare and the benefit level. The right-hand side is the aggregate change in tax revenue, which, in accordance with the preceding discussion, may be of either sign. The first term on the right-hand side is the change in tax revenue due to an incremental increase in required work. Naturally, such

an increase induces greater labor force participation in market work. The second term on the right-hand side captures the countervailing effect of the aggregate compensation via an increase in the welfare benefit. The amount of this compensation per unit of increase in workfare for each benefit recipient is given by the expression outside of the integral in this term. Each unit of compensation induces a change in tax revenue given by the final integral in (19).

While easy to interpret, it would take quite a bit of information to operationalize the condition (19). It contains several endogenous variables, notably the mass of workers of each preference type and the participation taxes at each skill level in the no-workfare optimum. The integrals on the right-hand side of (19) are the aggregate responses of tax revenue to increases in required work and the benefit level, respectively. Estimates of the latter are available. (See, for example, Chetty et al. (2011).) The former is less well-studied.

When the random variables m , n , and k are statistically independent, the right-hand side of (19) is zero, and we have the following Corollary.

Corollary 1. *Introducing unproductive workfare is welfare-decreasing in the presence of an optimal tax-transfer scheme when the disutility of workfare is heterogeneous and the random variables m , n and k are statistically independent.*

The no-workfare optimum essentially screens individuals of each skill type n into market workers versus benefit receivers according to their disutility of market work, m . When k is statistically independent from these variables, the proportion of each k -type that works in the market does not depend on m and n , nor does the proportion of each k -type among marginal labor market participants; that is, both $\Gamma(k)$ and $f(m^*, n, k)$ are proportional to $p(k)$, the density of type k . The former implies,

$$\frac{\int_{\underline{k}}^{\bar{k}} k\Gamma(k)dk}{\int_{\underline{k}}^{\bar{k}} \Gamma(k)dk} = \mu_k, \quad (21)$$

the mean of k . Consequently, the amount of benefit compensation is proportional to the marginal rate of substitution for the mean k -type. The latter implies that the change

in tax revenue owing to behavioral responses of k -type individuals to a one-unit benefit increase is proportional to $p(k)$. Thus, the overall response of tax revenue from k -type individuals is proportional to $\mu_k p(k)$. Specifically, this response is given by

$$- \left[\frac{u_l(b, 0)}{u_c(b, 0)} \frac{\partial \varphi}{\partial b} \tau(n) z(m^*) q(n) \right] \mu_k p(k), \quad (22)$$

where $z(m)$ and $q(n)$ are the densities of m and n , respectively. The revenue effect of the initial increase in r can be expressed in terms similar to those in (22). It follows from (12) and the Implicit Function Theorem that

$$\frac{\partial \varphi(k, 0, c(n), b)}{\partial r} = k \frac{u_l(b, 0)}{u_c(b, 0)} \frac{\partial \varphi(k, 0, c(n), b)}{\partial b}. \quad (23)$$

Hence, the change in tax revenue collected from k individuals is given by

$$\left[\frac{u_l(b, 0)}{u_c(b, 0)} \frac{\partial \varphi}{\partial b} \tau(n) z(m^*) q(n) \right] k p(k). \quad (24)$$

Integrating over all k , the total revenue effect is proportional to $\int_{\underline{k}}^{\bar{k}} (k - \mu_k) p(k) dk = 0$. Thus, with statistical independence, the revenue effects due to changes in participation of different k -types cancel each other out. All that remains is the direct cost to the public budget of the compensatory increase in the welfare benefit. For this reason, workfare is unambiguously welfare-decreasing.

6.1 The Case of a Maxi-Min Objective

The case for workfare is somewhat strengthened under maxi-min where individuals who are considered as the least-well off by the government are those who are the most keen to work on workfare, i.e. those whose $k = \underline{k}$. This social judgment is reminiscent of the “laziness” criterion used in Cuff (2000) in the sense that the planner wishes to redistribute towards individuals with a low distaste for required work. In this case, the social welfare function can then be written as

$$u(b, \underline{k}r). \quad (25)$$

Under the latter criterion, the utility of people characterized by $k = \underline{k}$ is not affected by an increase in r and a compensating rise in b along the \underline{k} -indifference curve in Figure

1. As in the utilitarian case, the rise in b creates a budget loss for the government. However, people with $k > \underline{k}$ would see their well-being reduced if they chose workfare. This implies an increase in their participation in the labor market. Their reduction of well-being is not taken into account in the maxi-min criterion, while their participation increase unambiguously improves tax revenue. It is then possible that this increase in tax revenue outweighs the budget loss implied by the rise in b . A formal statement of this trade-off is given in the following Proposition.

Proposition 5. *Introducing unproductive workfare may be welfare increasing in the presence of an optimal tax-transfer scheme when the disutility of workfare is heterogeneous and social welfare is given by (25) provided that*

$$-\underline{k} \frac{u_l(b, 0)}{u_c(b, 0)} \int_{\underline{k}}^{\bar{k}} \Gamma(k) dk < \int_{\underline{k}}^{\bar{k}} (k - \underline{k}) \int_{\underline{n}}^{\bar{n}} \frac{u_l(b, 0)}{u_l(c(n), m^*)} \tau(n) f(m^*, n, k) dn dk. \quad (26)$$

The left-hand side of (26) is exactly the cost of giving enough compensation, which must be given to all k -types, to make individuals characterized by \underline{k} indifferent between workfare and the previous no-workfare situation. It might happen that the (positive) right-hand side offsets the compensation costs so that workfare is a useful complement to the optimal income tax schedule. We also note that, consistent with the discussion preceding Proposition 5, the right-hand side of (26) is always positive, regardless of the correlation (or lack thereof) among the dimensions of heterogeneity. This is because the increase in the welfare benefit needed to neutralize the effects of workfare on social welfare is smaller for this objective function than it is under utilitarianism. Consequently, the induced reductions in participation, and the associated losses in tax revenue, are smaller. Appendix B provides a specification of utility functions and distribution functions for which introducing workfare can be welfare-improving. In our example, the dimensions of heterogeneity are statistically independent.

For alternative objective functions that put no weight on the disutility of required work, introducing workfare creates no direct social losses. Consequently, unproductive workfare is welfare-improving exactly when the additional market participation it induces

leads to increased government revenue. This will be the case when the no-workfare optimum does not include an EITC for any skill type. As shown by Choné and Laroque (2005), there is never an EITC under maxi-min. Thus, unproductive workfare would be useful if the government's objective is simply to maximize the level of welfare benefit. On the other hand, because EITCs can optimally arise under objectives that also take into account the consumption of labor market participants, workfare may or may not be welfare-improving in all situations in which the disutility of required work is ignored. The case for workfare is strengthened when the utility of at least some people is enhanced by workfare, as suggested by Solow (1998).

It is also possible to extend Cuff's argument that rules out optimal workfare when a distaste for work is viewed as arising from disability. In that case, the government might wish to maximize the utility of a benefit recipient of type \bar{k} . Introducing a work requirement and providing enough compensation to type \bar{k} individuals would make being out of work more attractive for all other k -types, leading to participation effects that exacerbate the revenue losses due to providing compensation.

In contrast to our model, there is no way to distinguish between distaste for required work and distaste for market work in the Cuff (2000) intensive margin model because the utility function is defined over total — both required and market — hours worked. Thus, her work is silent on the issue of which type of preference heterogeneity is required to generate a role for unproductive workfare. When viewed together, the results of this section and Proposition 2 indicate that heterogeneity of the disutility of required work is a pre-condition for the optimality of workfare in a model with labor supply along the extensive margin. Heterogeneous tastes for market work alone is not enough.

7 Conclusion

The informal arguments for workfare and earned income tax credits are quite similar. Both policy instruments are designed to provide additional incentives for individuals to enter the labor force. As such, one might expect these two policies to be complementary.

Indeed, in the work of Cuff (2000), there is some support for this idea in the existing literature on workfare when labor supply decisions are made along the intensive margin. In this article, we have shown, contrary to these notions, that there is a natural antipathy between workfare and earned income tax credits when labor supply decisions are along the extensive margin. In so doing, we have highlighted the need to carefully examine the margin along which policy operates when making policy recommendations. Additionally, when deciding whether workfare ought to be part of the policy mix one needs to take due account of what other policies are part of the mix.

Our tax reform results are robust to the specification of the costs of workfare. What is crucially important in our analysis is that workfare raises the net benefits of working and discourages enrollment in public welfare programs. Solow (1998) points out three forces that might counterbalance the effects highlighted in this article: workfare might be productive in itself; utility might be increasing in workfare rather than decreasing due to an increased sense of self-reliance among the recipients of public welfare benefits; and workfare might increase the number of welfare claimants as any stigma attached to welfare receipt might be reduced. In Section 5, we have already suggested how our results can be adjusted to account for productive workfare. The other two of Solow's forces have the potential to change the magnitude or sign of the welfare and revenue effects highlighted in this article, but we conjecture that the basic logic of the policy reform problem is unaltered.

We find that a case for workfare as part of an optimal tax-transfer scheme cannot emerge when everyone has the same distaste for required work. When individuals differ in their taste for required work, the case we build for workfare is far from universal. The maxi-min objective function we consider represents the best case for maximizing the revenue gains from implementing a (small) workfare program augmented by a benefit increase sufficient to neutralize the direct welfare losses due to workfare. As our example in Appendix B shows, workfare may or may not be welfare-improving, depending on model parameters.

Models of labor supply along the extensive margin may yield further results into workfare. For example, adapting to the extensive margin the model of Blumkin et al. (2013) in which workfare serves a deterrent to income misreporting and welfare fraud may provide extra insights. In addition, Brett (2005) showed that a role for unproductive workfare can emerge in an intensive margin model when second-order incentive compatibility conditions of the optimal income tax problem bind so that workers of different productivities are bunched. Under some circumstance, workfare can help to separate types within the bunch. It is possible that adding workfare, and perhaps heterogeneity of the disutility of required work, to the extensive margin analysis of Choné and Laroque (2011) that allows workers to earn less than their full potential might uncover interactions between workfare and the monotonicity constraints that arise in their model. It might also prove interesting to consider workfare in a model with labor supply responses along both the intensive and extensive margins, like the one formulated by Jacquet et al. (2013). As in any short article, we leave open many interesting questions.

Appendix A Proofs

Proof of Lemma 1. As in Diamond (1980), the Lagrangian associated with the optimal tax problem is

$$\begin{aligned} \mathcal{L}(c(n), b, \lambda; r) = & \int_{\underline{n}}^{\bar{n}} \left[\int_{\underline{m}(n)}^{m^*(c(n), b, r)} u(c(n), m) f(m, n) dm + \int_{m^*(c(n), b, r)}^{\bar{m}(n)} u(b, r) f(m, n) dm \right] dn \\ & + \lambda \left\{ \int_{\underline{n}}^{\bar{n}} \left[\int_{\underline{m}(n)}^{m^*(c(n), b, r)} [n - c(n)] f(m, n) dm - \int_{m^*(c(n), b, r)}^{\bar{m}(n)} b f(m, n) dm \right] dn - R \right\} \end{aligned} \quad (\text{A.1})$$

The first-order condition with respect to $c(n)$ yields:

$$\begin{aligned} \int_{\underline{m}(n)}^{m^*(c(n), b, r)} \left[1 - \frac{1}{\lambda} \frac{\partial u(c(n), m)}{\partial c(n)} \right] f(m, n) dm \\ = [n - c(n) + b] \frac{\partial m^*(c(n), b, r)}{\partial c(n)} f(m^*, n). \end{aligned} \quad (\text{A.2})$$

Using (7) and (9), and dividing both sides by $F(m^*, n)$, (A.2) can be rewritten as

$$1 - g(n) = \tau(n) \frac{\partial m^*(c(n), b, r)}{\partial c(n)} \frac{f(m^*, n)}{F(m^*, n)}. \quad (\text{A.3})$$

From (A.3) we see that $\tau(n)$ takes the sign of $1 - g(n)$, as stated in Lemma 1. \square

Proof of Proposition 2. Consider the optimal tax-transfer scheme with $r = 0$. From (A.1), the optimal choice of the benefit level b satisfies the first order condition

$$\begin{aligned} \int_{\underline{n}}^{\bar{n}} \int_{m^*(c(n), b, r)}^{\bar{m}(n)} u_c(b, 0) f(m, n) dm dn - \lambda \int_{\underline{n}}^{\bar{n}} \int_{m^*(c(n), b, r)}^{\bar{m}(n)} f(m, n) dm dn \\ + \lambda \int_{\underline{n}}^{\bar{n}} [n - c(n) + b] \frac{\partial m^*(c(n), b, 0)}{\partial b} f(m^*, n) dn = 0. \end{aligned} \quad (\text{A.4})$$

Applying the Implicit Function Theorem to (2) yields

$$\frac{\partial m^*(c(n), b, 0)}{\partial b} = \frac{u_c(b, 0)}{u_l(c(n), m^*)} \quad \text{and} \quad \frac{\partial m^*(c(n), b, 0)}{\partial r} = \frac{u_l(b, 0)}{u_l(c(n), m^*)}. \quad (\text{A.5})$$

Multiplying both sides of (A.4) by $u_l(b, 0)/u_c(b, 0)$ and using (A.5) yields

$$\begin{aligned} u_l(b, 0) \int_{\underline{n}}^{\bar{n}} \int_{m^*(c(n), b, r)}^{\bar{m}(n)} f(m, n) dm dn - \lambda \frac{u_l(b, 0)}{u_c(b, 0)} \int_{\underline{n}}^{\bar{n}} \int_{m^*(c(n), b, r)}^{\bar{m}(n)} f(m, n) dm dn \\ + \lambda \int_{\underline{n}}^{\bar{n}} [n - c(n) + b] \frac{\partial m^*(c(n), b, 0)}{\partial r} f(m^*, n) dm dn = 0. \end{aligned} \quad (\text{A.6})$$

We now show that a small increase in r from zero reduces the value of the Lagrangian when (A.4) is satisfied, so that introducing workfare is welfare-decreasing. To that end, differentiating (A.1) with respect to r yields:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial r} = u_l(b, r) \int_{\underline{n}}^{\bar{n}} \int_{m^*(c(n), b, r)}^{\bar{m}(n)} f(m, n) dm dn \\ + \lambda \int_{\underline{n}}^{\bar{n}} [n - c(n) + b] \frac{\partial m^*(c(n), b, r)}{\partial r} f(m^*, n) dn. \end{aligned} \quad (\text{A.7})$$

Subtracting (A.6) from (A.7), evaluating at $r = 0$, and simplifying yields

$$\frac{\partial \mathcal{L}}{\partial r} = \lambda \frac{u_l(b, 0)}{u_c(b, 0)} \int_{\underline{n}}^{\bar{n}} \int_{m^*(c(n), b, r)}^{\bar{m}(n)} f(m, n) dm dn. \quad (\text{A.8})$$

The right hand side of (A.8) is negative because $\lambda > 0$, $u_l(b, 0) < 0$, and $u_c(b, 0) > 0$. \square

Proof of Proposition 3. The proof proceeds as for Proposition 2, with small modifications to account for the output produced by required work. With productive workfare, the Lagrangian is

$$\begin{aligned} \mathcal{L}(c(n), b, \lambda; r) = & \int_{\underline{n}}^{\bar{n}} \left[\int_{\underline{m}(n)}^{m^*(c(n), b, r)} u(c(n), m) f(m, n) dm + \int_{m^*(c(n), b, r)}^{\bar{m}(n)} u(b, r) f(m, n) dm \right] dn \\ & + \lambda \left\{ \int_{\underline{n}}^{\bar{n}} \left[\int_{\underline{m}(n)}^{m^*(c(n), b, r)} [n - c(n)] f(m, n) dm + \int_{m^*(c(n), b, r)}^{\bar{m}(n)} [\gamma r - b] f(m, n) dm \right] dn - R \right\} \end{aligned} \quad (\text{A.9})$$

The first order condition for the optimal choice of b becomes

$$\begin{aligned} \int_{\underline{n}}^{\bar{n}} \int_{m^*(c(n), b, r)}^{\bar{m}(n)} u_c(b, r) f(m, n) dm dn - \lambda \int_{\underline{n}}^{\bar{n}} \int_{m^*(c(n), b, r)}^{\bar{m}(n)} f(m, n) dm dn \\ + \lambda \int_{\underline{n}}^{\bar{n}} [n - c(n) + b - \gamma r] \frac{\partial m^*(c(n), b, r)}{\partial b} f(m^*, n) dn = 0. \end{aligned} \quad (\text{A.10})$$

No modifications are required to (A.5), and the analog to (A.6) is

$$\begin{aligned} u_l(b, r) \int_{\underline{n}}^{\bar{n}} \int_{m^*(c(n), b, r)}^{\bar{m}(n)} f(m, n) dm dn - \lambda \frac{u_l(b, r)}{u_c(b, r)} \int_{\underline{n}}^{\bar{n}} \int_{m^*(c(n), b, r)}^{\bar{m}(n)} f(m, n) dm dn \\ + \lambda \int_{\underline{n}}^{\bar{n}} [n - c(n) + b - \gamma r] \frac{\partial m^*(c(n), b, r)}{\partial r} f(m^*, n) dn = 0. \end{aligned} \quad (\text{A.11})$$

The derivative of the Lagrangian with respect to r is now

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial r} = [u_l(b, r) + \lambda \gamma] \int_{\underline{n}}^{\bar{n}} \int_{m^*(c(n), b, r)}^{\bar{m}(n)} f(m, n) dm dn \\ + \lambda \int_{\underline{n}}^{\bar{n}} [n - c(n) + b - \gamma r] \frac{\partial m^*(c(n), b, r)}{\partial r} f(m^*, n) dn. \end{aligned} \quad (\text{A.12})$$

Adding (A.11) to (A.12), evaluating at $r = 0$, and simplifying yields

$$\frac{\partial \mathcal{L}}{\partial r} = \left[\frac{u_l(b, 0)}{u_c(b, 0)} + \gamma \right] \lambda \int_{\underline{n}}^{\bar{n}} \int_{m^*(c(n), b, r)}^{\bar{m}(n)} f(m, n) dm dn. \quad (\text{A.13})$$

The right hand side of (A.13) is positive exactly when (11) is satisfied. \square

Proof of Proposition 4. Consider the optimal tax-transfer scheme with $r = 0$. Maximizing of (15) subject to (16), the optimal choice of the benefit level b satisfies the first order condition

$$\begin{aligned} \int_{\underline{n}}^{\bar{n}} \int_{\underline{k}}^{\bar{k}} \int_{\varphi(k,r,c(n),b)}^{\bar{m}(n)} u_c(b,0) f(m,n,k) dm dk dn - \lambda \int_{\underline{n}}^{\bar{n}} \int_{\underline{k}}^{\bar{k}} \int_{\varphi(k,r,c(n),b)}^{\bar{m}(n)} f(m,n,k) dm dk dn \\ + \lambda \int_{\underline{n}}^{\bar{n}} \int_{\underline{k}}^{\bar{k}} [n - c(n) + b] \frac{\partial \varphi(k,r,c(n),b)}{\partial b} f(m^*,n,k) dk dn = 0. \end{aligned} \quad (\text{A.14})$$

For any k , we can define the mass of non-employed people as

$$\Gamma(k) \equiv \int_{\underline{n}}^{\bar{n}} \int_{\varphi(k,r,c(n),b)}^{\bar{m}(n)} f(m,n,k) dm dn. \quad (\text{A.15})$$

Applying the implicit function theorem to (12) yields (14) and

$$\frac{\partial \varphi}{\partial b} = \frac{u_c(b,r)}{u_l(c(n),m^*)}. \quad (\text{A.16})$$

Multiplying both sides of (A.14) by $\left[u_l(b,0) \int_{\underline{k}}^{\bar{k}} k \Gamma(k) dk \right] / \left[u_c(b,0) \int_{\underline{k}}^{\bar{k}} \Gamma(k) dk \right]$ and using (14), (A.15), and (A.16) yields

$$\begin{aligned} u_l(b,0) \int_{\underline{k}}^{\bar{k}} k \Gamma(k) dk - \lambda \frac{u_l(b,0)}{u_c(b,0)} \int_{\underline{k}}^{\bar{k}} k \Gamma(k) dk \\ + \lambda \frac{\int_{\underline{k}}^{\bar{k}} k \Gamma(k) dk}{\int_{\underline{k}}^{\bar{k}} \Gamma(k) dk} \int_{\underline{n}}^{\bar{n}} \int_{\underline{k}}^{\bar{k}} [n - c(n) + b] \frac{u_l(b,0)}{u_l(c(n),m^*)} f(m^*,n,k) dk dn = 0. \end{aligned} \quad (\text{A.17})$$

We now show that a small increase in r from zero does not unambiguously reduce the value of the Lagrangian when (A.14) is satisfied. To that end, differentiating the Lagrangian with respect to r yields:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial r} = \int_{\underline{n}}^{\bar{n}} \int_{\underline{k}}^{\bar{k}} \int_{\varphi(k,r,c(n),b)}^{\bar{m}(n)} k u_l(b,kr) f(m,n,k) dm dk dn \\ + \lambda \int_{\underline{n}}^{\bar{n}} \int_{\underline{k}}^{\bar{k}} \frac{\partial \varphi(k,r,c(n),b)}{\partial r} [n - c(n) + b] f(m^*,n,k) dk dn. \end{aligned} \quad (\text{A.18})$$

When evaluated at $r = 0$, this expression can be rewritten as

$$u_l(b,0) \int_{\underline{k}}^{\bar{k}} k \Gamma(k) dk + \lambda \int_{\underline{k}}^{\bar{k}} k \int_{\underline{n}}^{\bar{n}} \frac{u_l(b,0)}{u_l(c(n),m^*)} [n - c(n) + b] f(m^*,n,k) dn dk. \quad (\text{A.19})$$

where (14) and (A.15) have been used. Subtracting (A.17) from (A.19) and simplifying yields

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial r} \Big|_{r=0} &= \lambda \int_{\underline{k}}^{\bar{k}} k \frac{u_l(b, 0)}{u_c(b, 0)} \Gamma(k) dk \\ &+ \lambda \int_{\underline{k}}^{\bar{k}} \left[k - \frac{\int_{\underline{k}}^{\bar{k}} k \Gamma(k) dk}{\int_{\underline{k}}^{\bar{k}} \Gamma(k) dk} \right] \int_{\underline{n}}^{\bar{n}} \frac{u_l(b, 0)}{u_l(c(n), m^*)} [n - c(n) + b] f(m^*, n, k) dn dk. \end{aligned} \quad (\text{A.20})$$

The first term in the right hand side of (A.20) is negative; when increasing r , the increase in welfare benefit b costs money. The second term in the right hand side is proportional to the average marginal rate of substitution between workfare and benefit over all those out of work. If r increases by one unit, this is the amount by which b has to be increased so as to on average compensate for the welfare loss. It is multiplied by λ to express things in terms of public funds. This term might be positive, depending on how the behavioral responses vary with k .

Equation (19) follows from substituting (A.16) and (14) into (A.20) and distributing across the large square bracket. \square

Proof of Corollary 1. When $r = 0$, the cut-off m^* is determined by the equation $u(c(n), m) = u(b, 0)$, so that m^* does not depend on k . This observation allows us to factor terms in m^* outside of integrals over k in the calculations that follow. Now, statistical independence allows us to write

$$f(m, n, k) = z(m)q(n)p(k). \quad (\text{A.21})$$

Substituting (A.21) into the right-hand side of (19) yields

$$\left\{ \int_{\underline{n}}^{\bar{n}} \frac{u_l(b, 0)}{u_l(c(n), m^*)} [n - c(n) + b] z(m^*)q(n) dn \right\} \left\{ \int_{\underline{k}}^{\bar{k}} \left[k - \frac{\int_{\underline{k}}^{\bar{k}} k \Gamma(k) dk}{\int_{\underline{k}}^{\bar{k}} \Gamma(k) dk} \right] p(k) dk \right\}. \quad (\text{A.22})$$

We now show that the term in the second set of braces in (A.22) is zero, so that the entire right-hand side of (A.20), and therefore the right-hand side of (19), vanishes. To

that end, substituting (A.21) into (20) yields

$$\Gamma(k) = \left[\int_{\underline{n}}^{\bar{n}} [1 - Z(m^*)] q(n) dn \right] p(k), \quad (\text{A.23})$$

where Z is the cumulative distribution function associated with the density z . Using (A.23) reduces the term in the second set of braces in (A.22) to

$$\begin{aligned} & \int_{\underline{k}}^{\bar{k}} k p(k) dk - \frac{\left[\int_{\underline{n}}^{\bar{n}} [1 - Z(m^*)] q(n) dn \right] \int_{\underline{k}}^{\bar{k}} k p(k) dk}{\left[\int_{\underline{n}}^{\bar{n}} [1 - Z(m^*)] q(n) dn \right] \int_{\underline{k}}^{\bar{k}} p(k) dk} \\ &= \int_{\underline{k}}^{\bar{k}} k p(k) dk - \int_{\underline{k}}^{\bar{k}} k p(k) dk \quad \left(\text{because } \int_{\underline{k}}^{\bar{k}} p(k) dk = 1 \right), \end{aligned} \quad (\text{A.24})$$

which clearly vanishes. \square

Proof of Proposition 5. We consider the problem in which the government maximizes (25), i.e., the utility of benefit recipients whose $k = \underline{k}$, under the government budget constraint (16).

The first-order condition with respect to b can be written as:

$$\begin{aligned} u_c(b, \underline{k}r) - \lambda \int_{\underline{n}}^{\bar{n}} \int_{\underline{k}}^{\bar{k}} \int_{\varphi(k, r, c(n), b)}^{\bar{m}(n)} f(m, n, k) dm dk dn \\ + \lambda \int_{\underline{n}}^{\bar{n}} \int_{\underline{k}}^{\bar{k}} [n - c(n) + b] \frac{\partial \varphi(k, r, c(n), b)}{\partial b} f(m^*, n, k) dk dn = 0, \end{aligned} \quad (\text{A.25})$$

where λ is the Lagrange multiplier on the budget constraint. Differentiating the Lagrangian with respect to r yields:

$$\underline{k}u_l(b, \underline{k}r) + \lambda \int_{\underline{n}}^{\bar{n}} \int_{\underline{k}}^{\bar{k}} [n - c(n) + b] \frac{\partial \varphi(k, r, c(n), b)}{\partial r} f(m^*, n, k) dk dn. \quad (\text{A.26})$$

When evaluated at $r = 0$, this expression can be rewritten as

$$\underline{k}u_l(b, 0) + \lambda \int_{\underline{k}}^{\bar{k}} k \int_{\underline{n}}^{\bar{n}} \frac{u_l(b, 0)}{u_l(c(n), m^*)} [n - c(n) + b] f(m^*, n, k) dn dk, \quad (\text{A.27})$$

where (14) has been used. Multiplying (A.25) by $\underline{k}u_l(b, 0)/u_c(b, 0)$ and, after using (A.16), subtracting the result from (A.27) yields

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial r} \Big|_{r=0} &= \lambda \underline{k} \frac{u_l(b, 0)}{u_c(b, 0)} \int_{\underline{k}}^{\bar{k}} \Gamma(k) dk \\ &\quad + \lambda \int_{\underline{k}}^{\bar{k}} (k - \underline{k}) \int_{\underline{n}}^{\bar{n}} \frac{u_l(b, 0)}{u_l(c(n), m^*)} [n - c(n) + b] f(m^*, n, k) dn dk. \end{aligned} \quad (\text{A.28})$$

Although not necessary for the proof, we note that when $r = 0$, $n - c(n) + b > 0$. The first-order condition of the Lagrangian with respect to $c(n)$ allows us to write, after simplification:

$$\int_{\underline{m}(n)}^{\varphi(k, r, c(n), b)} f(m, n, k) dm = [n - c(n) + b] \frac{\partial \varphi(k, r, c(n), b)}{\partial c(n)} f(m^*, n, k). \quad (\text{A.29})$$

Since the left hand side of (A.29) and $\partial \varphi(k, r, c(n), b) / \partial c(n)$ are both positive, $n - c(n) + b > 0$. The latter result is standard in the extensive margin model (Choné and Laroque, 2005). Therefore, the second term in the right hand side of (A.28) is positive because $k > \underline{k}$ for all k except $k = \underline{k}$. \square

Appendix B Maxi-min with Three-Dimensional Heterogeneity

We now provide an example to show that workfare may be welfare-improving in the presence of an optimal tax-transfer scheme for the model considered in Section 6.1.

By specifying functional forms for utility and distribution functions, we show that (A.28) can take on any sign. Specifically, we set $f(m, n, k) = z(m)q(n)p(k)$, so that all the three components of an individual's type are statistically independent. Moreover, we posit

$$u(c, m) = \ln(c) - m \quad \text{and} \quad u(b, kr) = \ln(b) - h(kr), \quad \text{with} \quad h(l) = 0.5(l + 1)^2 - 0.5. \quad (\text{A.30})$$

The form of the disutility function h is chosen so that $h(0) = 0$ and $h'(0) = 1$, normalizations that simplify several calculations below. These functional form assumptions on utility are sufficient to imply that

$$\varphi(k, r, c(n), b) = \ln(c(n)) - \ln(b) + h(kr) \quad \text{and} \quad \varphi(k, 0, c(n), b) = \ln(c(n)) - \ln(b), \quad \forall k. \quad (\text{A.31})$$

It now follows from (A.15) that

$$\Gamma(k) = p(k) \int_{\underline{n}}^{\bar{n}} [1 - Z(\ln(c(n)) - \ln(b))] q(n) dn \quad (\text{A.32})$$

whenever $r = 0$.

We also assume that the support of $p(k)$ is an interval $[1, \bar{k}]$. Now substituting (A.32) into (A.28) and using (A.30) yields

$$\begin{aligned} \left. \frac{1}{\lambda} \frac{\partial \mathcal{L}}{\partial r} \right|_{r=0} &= -b \int_{\underline{n}}^{\bar{n}} [1 - Z(\ln(c(n)) - \ln(b))] q(n) dn \\ &\quad + \int_{\underline{n}}^{\bar{n}} [n - c(n) + b] z(\ln(c(n)) - \ln(b)) q(n) dn \int_{\underline{k}}^{\bar{k}} (k - 1) p(k) dk. \end{aligned} \quad (\text{A.33})$$

In order to compute a value for the right-hand side of (A.33), one must first compute the optimal $c(n)$ and b for the case of $r = 0$; that is, one must solve the optimal tax-transfer problem without workfare. As a first step in this process, we note that (A.29) reduces to

$$q(n)p(k)Z(\ln(c(n)) - \ln(b)) = [n - c(n) + b]u_c(c(n), m)z(\ln(c(n)) - \ln(b))q(n)p(k), \quad (\text{A.34})$$

which, using (A.30) is equivalent to

$$\frac{Z(\ln(c(n)) - \ln(b))}{z(\ln(c(n)) - \ln(b))} = \frac{n - c(n) + b}{c(n)}. \quad (\text{A.35})$$

This equation proves to be especially tractable when the disutility of market work follows a logistic distribution on $(-\infty, \infty)$, so that

$$z(m) = \frac{e^m}{(1 + e^m)^2} \quad \text{and} \quad Z(m) = \frac{e^m}{1 + e^m}. \quad (\text{A.36})$$

Under assumption (A.36), (A.35) becomes

$$1 + e^{(\ln(c(n)) - \ln(b))} = \frac{n - c(n) + b}{c(n)}, \quad (\text{A.37})$$

which reduces to

$$\frac{b + c(n)}{b} = \frac{n - c(n) + b}{c(n)}. \quad (\text{A.38})$$

(A.38) defines the optimal $c(n)$ in terms of b . The functional form assumptions we have made render $c(n)$ independent of $q(n)$, conditional on b . Rearranging (A.38) gives a quadratic equation in $c(n)$ that can be solved to yield¹²

$$c(n) = \sqrt{2b^2 + nb} - b. \quad (\text{A.39})$$

The optimal benefit level b can be found by substituting (A.39) into the budget constraint, which then becomes one equation in the single unknown b . Given the independence assumption and a further assumption that $R = 0$, the budget constraint (16) reduces to

$$\int_{\underline{n}}^{\bar{n}} [n - c(n) + b] Z(m^*) q(n) dn - b = 0; \quad (\text{A.40})$$

that is, the participation taxes on the working are sufficient to finance the benefit. (We use the property that m^* is independent of k to factor the integral on left-hand side of (A.40) outside of the integral over k in (16). The remaining factor in k becomes $\int_{\underline{k}}^{\bar{k}} p(k) dk = 1$.) Because substituting (A.39) into (A.40) produces an equation that is difficult to solve analytically, we use computer techniques to complete our calculations. The steps we follow are:

Step 1: Specify $q(n)$. In our calculations, we use a $\Gamma(2, 2)$ distribution, shifted from its typical support of $[0, \infty)$ to support $[5, \infty)$.

Step 2: Generate (A.40) and solve for the optimal b , say b^* .

Step 3: Substitute b^* into (A.39) to find the optimal consumption levels, say $c^*(n)$.

Step 4: Specify $p(k)$. In our calculations, we allow $p(k, a)$, where a is a parameter that changes the shape of $p(k)$. We choose a $\beta(0.5, a)$ distribution, transformed from the standard support of $[0, 1]$ onto the interval $[1, 10]$. For this specification, the mass of k shifts more toward $\underline{k} = 1$ as the parameter a increases.

Step 5: For each value of a , substitute everything into (A.33).

¹²The other root provided by the quadratic formula can be shown to be negative, so we ignore it.

Following this procedure gives Figure 2, which displays that there are, indeed, values of the parameter a for which introducing workfare increases social welfare. We note also that as the mass of $p(k)$ moves closer to \underline{k} (as a increases), the case for workfare weakens because the compensation afforded to \underline{k} -type individuals is almost sufficient for a larger fraction of the population, thereby dampening the positive revenue effect.

We also include in Figure 3 the R computer code used to produce our results.

Figure 2: Effects of $r \uparrow$ on Maxi-min Social Welfare

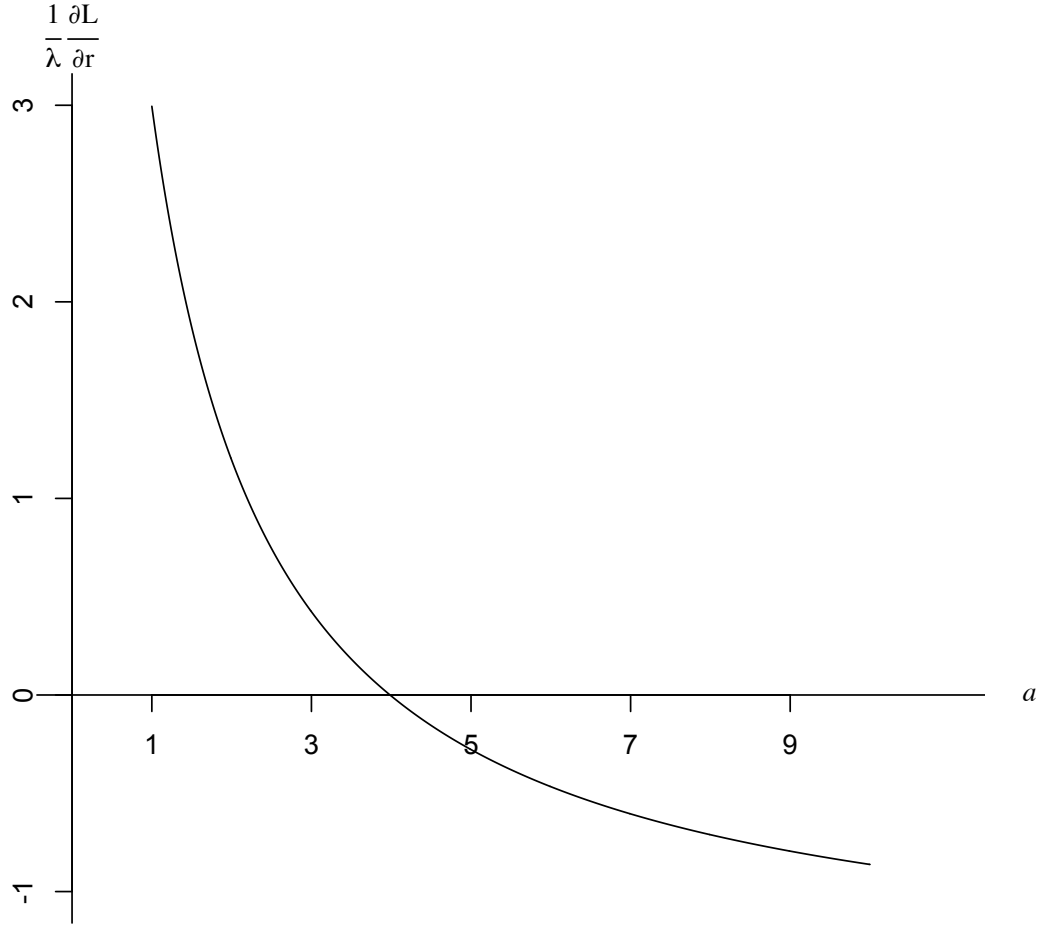


Figure 3: R Code Used for the Computations Graphed in Figure 2

```
# Step 1: Setting up utility and distributions of n and m
C <- function(n,b) {sqrt(2*b^2 + n*b)-b}
v <- function (C) {log(C)}
u <- function (b) {log(b)}
Z <- function (m) {exp(m) / (1+exp(m))}
z <- function (m) {exp(m) / ((1+exp(m))^2)}
q1 <- function (n) {dgamma(n,2,2)}
q <- function (n) {q1(n-nlow)}
nlow <- 5

# Step 2: Coding the budget constraint and finding the optimal b
j <- function (n,b) {(n-C(n,b) + b)*Z( v(C(n,b)) - u (b) )*q(n)}
s <- function(b) {integrate(j,nlow,Inf,b)$value-b}
bstar <- uniroot(s,lower=0.1, upper=10)$root

# Step 3: Back Substitution
cstar <- function (n) {C(n, bstar)}

# Step 4: Specify the distribution of k
p <- function(k,a){(gamma(a+0.5)*(k-1)^(-0.5)*(10-k)^(a-1))/
(gamma(0.5)*gamma(a)*9^(a-0.5))}

# Step 5: Computing 1/lambda times dLdr
wneg <- function (n) {-bstar*(1-G(v(cstar(n))-u(bstar)))*q(n)}
wpos <- function(n) {(n-cstar(n)+bstar)*g(v(cstar(n))-u(bstar))*q(n)}
Lneg <- integrate(wneg, nlow, Inf)$value
Lpos <- integrate(wpos, nlow, Inf)$value
kweight <- function(k,a){p(k,a)*(k-1)}
Lk <- function(a){integrate(kweight,1,10,a)$value}
lambdaInvdLdr <- function (a) {Lneg + Lpos*Lk(a)}
```

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