ADAPTIVE ESTIMATION IN FUNCTIONAL LINEAR MODELS

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Abstract

We consider the nonparametric estimation of the slope function in functional linear regression, where scalar responses are modeled in dependence of random functions. The theory in this presentation covers both the estimation of the slope function or its derivatives (global case) as well as the estimation of a linear functional of the slope function (local case). We propose an estimator of the slope function which is based on a dimension reduction and additional thresholding. Moreover, in the local case replacing the unknown slope function by this estimator we obtain a plug-in estimator of the value of a linear functional evaluated at the slope. It is shown that in both the global and the local case these estimators can attain minimax optimal rates of convergence. The estimator of the slope function, however, requires an optimal choice of a tuning parameter with regard to certain characteristics of the slope function and the covariance operator associated with the functional regressor. As these are unknown in practice, we investigate a fully data-driven choice of the tuning parameter which combines model selection and Lepski's method and is inspired by the recent work of Goldenshluger and Lepski [2011]. It is shown that the adaptive estimator with data-driven choice of the dimension parameter can attain the lower minimax risk bound in the global case up to a constant and in the local case up to a logarithmic factor, and this over a variety of classes of slope functions and covariance operators.

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