Do social status seeking behaviors worsen the tragedy of the commons?

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Summary. The present paper considers the exploitation of a common-property, non-renewable resource, by individuals concerned with their social status. Assuming that the social status is reflected by the individuals' relative consumptions, we formalize this motivation by means of a utility function, depending on the individual's actual consumption and on the consumption level he aspires, the latter being related to the consumptions in his reference group. We compare the benchmark cooperative solution with a noncooperative Markov-perfect Nash equilibrium. We confirm, under more general conditions than in the existing literature, that the individuals' concern for social status exacerbates the tragedy of the commons. We finally discuss the policy implications and provide a taxation scheme capable of implementing the cooperative solution as a noncooperative Markov-perfect Nash equilibrium.

JEL Classification. Q30, C61.

1 Introduction

When a nonrenewable resource is owned in common, the economic agents exploiting it may have too little incentives to conserve it, because they fear the units of the resource they leave in situ may actually be extracted by others (for a review, see Dockner et al., 2000, Long, 2010, and Long, 2011). This adverse behavior leads to the so-called tragedy of the commons. In parallel, a recent literature has raised the idea that the concern for social status may amplify the phenomenon, inducing an even faster extraction of the resource (Katayama and Long, 2008; Long and McWhinnie, 2012).

This paper pursues this line of research. With respect to the previous literature, the main contribution is the generality within which the results are established. We use a more general utility function. We measure the individuals' status as a more general function of the others' consumptions. In this setting, focusing on symmetric Markov-perfect Nash equilibria, we confirm that the tragedy of the commons occurs and is exacerbated by status.

Dockner et al. (2000) and Long (2010, 2011) review models of exploitation of a common-property, nonrenewable resource (Bolle, 1986; Long and Shimomura, 1998; Long et al., 1999; McMillan and Sinn, 1984; Sinn, 1984). They compare the benchmark cooperative solution, with a noncooperative open-loop Nash equilibrium or Markov-perfect Nash equilibrium. By definition, the tragedy of the commons occurs when a larger fraction of the resource stock is extracted along the noncooperative equilibrium, compared with the benchmark cooperative solution. In the open-loop version of the game, as shown by Bolle (1986), the nature of the solution depends on how we restrict the set of feasible strategies available to each agent. On the one hand, in the case where only strictly feasible strategies are allowed ([1]), the cooperative extraction path coincides with an open-loop Nash equilibrium of the noncooperative game. On the other hand, in the case where weakly feasible strategies are allowed ([2]), the cooperative extraction path cannot be supported as an open-loop Nash equilibrium of the noncooperative game. Finally, a Markov-perfect Nash-equilibrium implies a faster extraction rate than an open-loop Nash equilibrium, because each player knows that being more conservationist will encourage the others to extract more (Long and Shimomura, 1998; Long et al., 1999). To summarize, in most cases, the literature shows that the exploitation of a common pool fosters an overconsumption of the exhaustible resource, leading to a tragedy of the commons ([3]).

All this literature implicitly assumes that the preferences are independent across individuals. Yet, many social scientists reject this postulate as unrealistic. In particular, they argue that the social status plays a role in the determination of individuals' behaviors (Mac Adams, 1992; Weiss and Fershtman, 1998). In economics, early recognition of this motivation can be found in Smith (1759) and Veblen (1899). The first formal analysis are due to Duesenberry (1949), Liebenstein (1950), Becker (1974) and Pollak (1976).

In a recent survey, Weiss and Fershtman (1998) expound how one can incorporate status considerations in an economic model. Firstly, one needs to measure individuals' social status. Secondly, one needs to specify the benefits associated with a higher social status. This can be done, either directly, by putting

[1] The agents can only choose strategies that allow the other players' plans to be satisfied, given the exhaustion constraint.

[2] The agents are allowed to choose strategies that may frustrate the other players' plans, due to the exhaustion constraint.

[3] Importantly, remark that this literature relies on simple specifications, using isoelactic utility functions.

the associated benefits into the individuals' utility functions, or indirectly, by specifying the channels by which status shifts the individuals' opportunity set.

Following the first approach, Hollander (2001) argues that the behavior of individuals concerned with their social position, can be appropriately described by means of a utility function, the arguments of which are the individual's actual consumption and the consumption level he aspires, the latter being equal to the average consumption in his reference group. This formalization has found applications in economic growth, finance, consumption analysis, among other topics. See, for example, Arrow and Dasgupta (2009), Carroll et al. (1997), Dupor and Liu (2003), Frank (1985), Galí (1994) and Ljungqvist and Uhlig (2000). It has also received supporting experimental and empirical evidence. See, for example, Alpizar et al. (2005), Clark and Oswald (1996), Clark et al. (2008) and Luttmer (2005).

The implications that such motivations have on environmental issues have already been investigated (Brekke and Howarth, 2002; Ng and Wang, 1993; Howarth, 1996). As expected, this literature shows that the concern for social status induces excessive levels of consumption and environmental damages (Ng and Wang, 1993). In response, an optimal policy requires a consumption tax and a pollution tax above the standard pigovian prescription (Howarth, 1996).

Recently, an emerging literature also explored the implications of such motivations on the exploitation of common-property resources, including Katayama and Long (2008) and Long and McWhinnie (2012). Long (2011) points this issue as an interesting future topic for research in the field of dynamic games of natural resources.

Katayama and Long (2008) consider an economy where the individuals manage a capital stock and a nonrenewable resource stock, both under common-property, to produce a final output with a Cobb-Douglas technology. Each individual's preference depends on his consumption and on the average consumption in the community. Preferences are represented by Cobb-Douglas utility functions. For a special parametrization, Katayama and Long (2008) are able to derive a linear Markov-perfect equilibrium. They show that the individuals' concern for relative positions exacerbates overconsumption, with a higher fraction of the capital stock consumed, and lowers the rate of extraction ([4]).

Long and McWhinnie (2012) consider a community of fishermen exploiting a common-property fish stock for a foreign market. They use a standard model of fishery, with the Schaeffer's harvest function and the logistic growth function, except that the fishermen are assumed to care about their relative performance (either profits or harvests). Preferences are represented by isoelastic utility

[4] This last result holds only if extraction is costly. With costless extraction, no impact on the resource exploitation is found.

functions. Long and McWhinnie (2012) show that, in a (open loop) Nash equilibrium steady state, the individuals' concern for relative performance worsens the tragedy of the commons.

This short review outlines the fact that the existing papers give few examples where the individuals' motivation for relative positions exacerbates overconsumption. However, they always rely on simple specifications of both preferences and technologies (i.e., Cobb-Douglas and isoelastic functions).

The present paper extends the literature, by introducing a more general formalization of the individuals' social status and weaker restrictions on the individuals' preference. Precisely, we consider the exploitation of a common-property, non-renewable resource ([5]). Following Katayama and Long (2008) and Long and McWhinnie (2012), we postulate that each individual cares about his consumption and about his social status, the latter depending on his relative position with respect to a consumption standard in his reference group. However, contrary to Katayama and Long (2008) and Long and McWhinnie (2012), we formalize the consumption standard which the individuals aspire as a general function of the others' consumptions and we introduce no restrictions about the shape of the individuals' utility. In this general setting, we are still able to explicitly derive the benchmark cooperative solution and a noncooperative Markov-perfect Nash equilibrium ([6]) ([7]). Our results confirm that a Markov-perfect Nash-equilibrium implies a faster extraction rate than the optimal benchmark solution. This remains true even in the limit case where the individuals do not pay attention to their social status. This means that the tragedy of the commons occurs in a general setting. Taking this case as a benchmark, our results also confirm the general intuition that the individuals' concern for relative positions exacerbates the tragedy of the commons. We finally display a taxation scheme to implement the benchmark cooperative solution as a noncooperative Markov-perfect Nash equilibrium.

The rest of the paper is organized as follows. Section 2 sets out the model. Section 3 analyses the cooperative solution. Section 4 characterizes a Markov-perfect Nash equilibrium. Section 5 discusses the normative implications of our results. Section 6 expounds an optimal taxation scheme. Most proofs are given in the appendix.

[5]In this respect, Katayama and Long (2010) and Long and McWhinnie (2012) use more general models than ours, since we consider neither capital stock nor natural growth of resource. Hence, our generalization is limited to the representation of individuals' social status and preference.

[6]We do not consider open-loop Nash equilibria. The advantage of Markov-perfect Nash equilibria is that they satisfy the condition of subgame perfectness (Dockner et al, 2000).

[7]An anonymous referee remarks that the results presented here can be related with Rincon-Zapatero et al. (1998) and Rincon-Zapatero (2004), dealing with the characterization of Nash equilibria in differential games, by means of a system of partial differential equations. Indeed, they illustrate their results with an application to non-renewable resources games. Although some similarities exist, Rincon-Zapatero et al. (1998) and Rincon-Zapatero (2004) did not obtain the characterization proposed here, due to their focus on finite-horizon problems.

2 The model.

Consider a non renewable resource that can be exploited simultaneously by n consumers. At each instant of time t, let $c_i(t)$ denote consumer i's rate of consumption and $y_i(t)$ the consumption level he aspires. The latter, depending on the rates of consumption in his reference group, reflects consumer i's concern for relative position. More specifically, we let $y_i(t) = \phi_i(c_{-i}(t))$, where the function $\phi_i(\cdot)$ implicitly formalizes how consumer i delineates his reference group and aggregates the rates of consumption within the group ([8]). Let $v(c_i, y_i)$ be individual i's utility function. We postulate that the utility function $v(c_i, y_i)$ is twice continuously differentiable, with $v_1(c_i, y_i) > 0$ and $v_2(c_i, y_i) \leq 0$ ([9]) ([10]).

The following assumptions will be used below.

Assumption 1. (Status) *For all i and $c_{-i}(t)$, we have:*
(a) $\phi_i(c_{-i}(t)) \geq \sum_{j\neq i} c_j(t)/(n-1)$;
(b) if $c_{-i}(t) = (c, ..., c)$, for some c, then $\phi_i(c_{-i}(t)) = c$.

Katayama and Long (2008) and Long and McWhinnie (2012) consider the case where consumer i's aspiration is the average consumption in the rest of the population, i.e. $y_i(t) = \sum_{j\neq i} c_j(t)/(n-1)$. Hence, their specification satisfies Assumption 1. However, Assumption 1 is clearly more general. The first part (a) states that the consumption level $y_i(t)$ that consumer i aspires is at least equal to the average consumption $\sum_{j\neq i} c_j(t)/(n-1)$ among his peer. This condition thus implies that consumer i gives more weight to the upper part of the consumptions' distribution. The second part (b) means that if everyone consumes c in the rest of the population, the consumption level $y_i(t)$ that consumer i aspires is also equal to c. Among others, an appealing specification, satisfying assumption 1, but differing from Katayama and Long (2008) and Long and McWhinnie (2012), is $y_i(t) = \max\{c_j(t); j \neq i\}$.

Assumption 2. (Utility function)
(a) The utility function $v(c_i, y_i)$ is concave.
(b) There exists α, with $0 \leq \alpha < 1$, such that, for all c, we have $\alpha v_1(c, c) + v_2(c, c) = 0$ ([11]).

Assumption 2 is essentially a technical assumption. The first part (a) will be used to prove Lemma 1 below. The second part (b) is implicit in most of the literature. For example, it is in force in Carroll et al. (1997), Dupor and Liu (2003), Galí (1994), Katayama and Long (2008), Ljungqvist and Uhlig (2000) and Long and McWhinnie (2012). This condition will be used to derive

[8] Here, we use the notation: $c_{-i}(t) = (c_1(t), ..., c_{i-1}(t), c_{i+1}(t), ..., c_n(t))$.

[9] Here and below, $v_1(c_i, y_i)$ and $v_2(c_i, y_i)$ denote the first and second derivatives of $v(c_i, y_i)$, respectively.

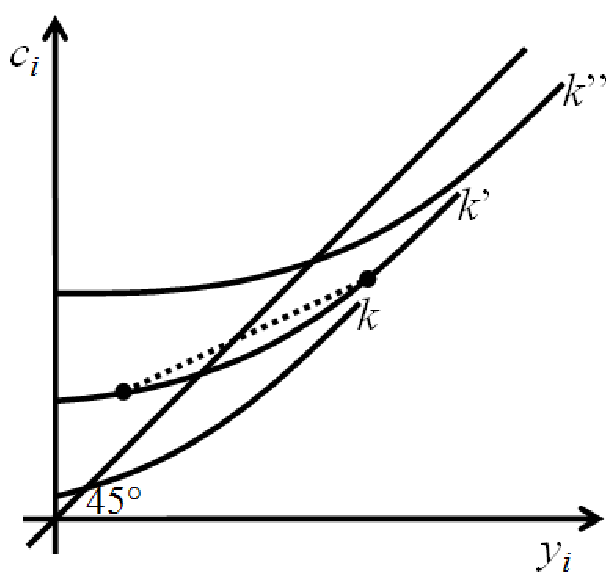
[10] Remark that if $v_2(c_i, y_i) = 0$, for all c_i and y_i, we obtain the standard common-pool model of exploitation of an exhaustible resource, as a special case of our model.

[11] Note that $\alpha = 0$ in the special case where $v_2(c_i, y_i) = 0$, for all c_i and y_i.

Propositions 2 and 3 below. Intuitively, it means that, along the 45-degree line (i.e., the locus where $c_i(t) = y_i(t) = c$, for some c), the consumer i's marginal rate of substitution between consumption $c_i(t)$ and aspiration $y_i(t)$ is constant and lies between 0 and 1. It also implies that the utility is increasing along the 45-degree line, because the negative effect of others' consumption is less than the gains from one's own consumption (Arrow and Dasgupta, 2009) ([12]).

Below, it will be useful to define $u(c) = v(c, c)$, for all c. By construction, the utility function $u(c)$ measures the utility that an individual obtains when consuming c, while the consumption he aspires is also equal to c. Due to Assumption 1.b, this also corresponds to the utility he derives from living in a society where everybody consumes c. Our assumptions above imply that $u'(c) > 0$ and $u''(c) < 0$, for all c ([13]).

The figure below aims at summarizing our assumptions about the individuals' preference. It depicts a familly of indifference curves, i.e., locus where $v(c_i, y_i) = k$, k' and k'', with $k < k' < k''$. The utility is increasing in the N-W direction. At any point, the slope of the indifference curve measures the marginal rate of substitution between consumption $c_i(t)$ and aspiration $y_i(t)$. By assumption 2, the marginal rate of substitution is constant along the 45-degree line. The concavity of the utility function $v(c_i, y_i)$ implies that any segment joining two points of a given indifference curve lies above this indifference curve. The concavity of the utility function $u(c)$ implies that along the 45-degree line, the utility is increasing at a diminishing rate.



[12] In a competitive economy, Arrow and Dasgupta (2009) prove that this condition implies that the socially optimal and market equilibrium paths coincide.

[13] Katayama and Long (2008) and Long and McWhinnie (2012) consider specifications, such that, implicitly, $u(c)$ belongs to the familly of isoelastic utility functions.

Figure 1.

The economy is endowed with a finite stock, x_0, of the resource. The resource state, $x(t)$, evolves according to the ordinary differential equation

$$\dot{x}(t) = -\sum_{i=1}^{n} c_i(t),\ x(0) = x_0. \tag{1}$$

Each consumer i's problem is to choose an individual consumption path $c_i(\cdot)$ to maximize

$$\int_0^\infty v(c_i(t), y_i(t))\, e^{-\delta t} dt, \tag{2}$$

where δ is a common rate of time preference, with $\delta > 0$.

The social objective is to find a vector of individual consumption paths $(c_i(\cdot))_{i=1}^n$, for all i, to maximize

$$\sum_{i=1}^{n} \int_0^\infty v(c_i(t), y_i(t))\, e^{-\delta t} dt. \tag{3}$$

3 Cooperative solution.

We derive here the socially optimal consumption path and discuss its properties. In particular, we show that, from the social viewpoint, no account should be taken of the individuals' concern for relative consumptions.

The following lemma will greatly simplify the determination of the optimal policy, since it implies that we can limit our attention to symmetric consumption paths. To expound this result, for all vector $(c_i(t))_{i=1}^n$ of feasible individual rates of consumption, we define

$$V\left((c_i(t))_{i=1}^n\right) = \sum_{i=1}^{n} v\left(c_i(t), \phi_i(c_{-i}(t))\right),$$

the corresponding sum of the consumers' utilities.

Lemma 1. (Symmetry of the optimal solution.)
For all $(c_i(t))_{i=1}^n$, if there exist $j \neq k$ such that $c_j(t) \neq c_k(t)$, then

$$V\left(\left(\sum_{i=1}^{n} c_i(t)/n\right)_{i=1}^n\right) > V\left((c_i(t))_{i=1}^n\right).$$

The proof is given in the Appendix.

Intuitively, Lemma 1 says that if, along a given consumption path $(c_i(\cdot))_{i=1}^n$, the aggregate consumption $\sum_{i=1}^{n} c_i(t)$ is not distributed equally at some time t, the equal redistribution of the same quantity will increase the consumers' total utility. As a corollary, as this deviation does not challenge the feasibility of the initial consumption path $(c_i(\cdot))_{i=1}^n$, lemma 1 implies, by contradiction, that an optimal consumption path is symmetric.

From Lemma 1, we will derive a socially optimal consumption path, by first substituting $c_i(t) = y_i(t) = c(t)$, for all i and t, into (3), and by then choosing $c(t)$, for all t, to maximize

$$\begin{array}{l} \int_0^\infty nu(c(t))\, e^{-\delta t} dt, \\ \text{where:} \\ \dot{x}(t) = -nc(t)\,,\, x(0) = x_0, \\ c(t) \geq 0,\, x(t) \geq 0. \end{array}$$

To state our result in Proposition 1 below, define $\sigma(c) \equiv -u''(c)\, c/u'(c)$, for all c, the elasticity of the marginal utility $u'(c)$. Assuming integrability of $\sigma(c)$, let $\Theta(c) \equiv \int_0^c \sigma(s)\, ds$, for all c, and assume that $\lim_{c\to\infty} \Theta(c) = \infty$.

Proposition 1. (Optimal policy in Feedback form.)

The optimal policy, given in the feedback form $c_i = f(x)$, for all i, is implicitly defined by

$$\int_0^{f(x)} \sigma(s)\, ds = \delta x/n, \text{ for all } x. \tag{4}$$

The proof is given in the Appendix.

It is clear from Proposition 1 that the optimal policy only depends on the curvature of the marginal utility, $\sigma(\cdot)$, the rate of discount, δ, the stock of the resource, x, and the population size, n.

Importantly, this means as a corollary that the optimal policy is not influenced by the way the individuals measure and value their social status, as formalized here by the functions $\phi_i(\cdot)$, for all i, and $v(\cdot)$. More precisely, proposition 1 shows that any set of functions $\phi_i(\cdot)$, for all i, and $v(\cdot)$, provided that it leaves the utility function $u(\cdot)$ unchanged, will determinate the same optimal policy. In this sense, we can affirm that, from the social viewpoint, no account should be taken of the individuals' concern for relative consumptions.

The figure below illustrates the utilization of Proposition 1. It displays a possible graph of the elasticity of the marginal utility $\sigma(\cdot)$. According to Proposition 1, it is optimal, at any time, that each consumer extracts a quantity c°, such that the surface area below the graph, from the origin to c°, equals $\delta x_0/n$, where δ is the discount rate, x_0 is the current state of the resource and n is the population size.

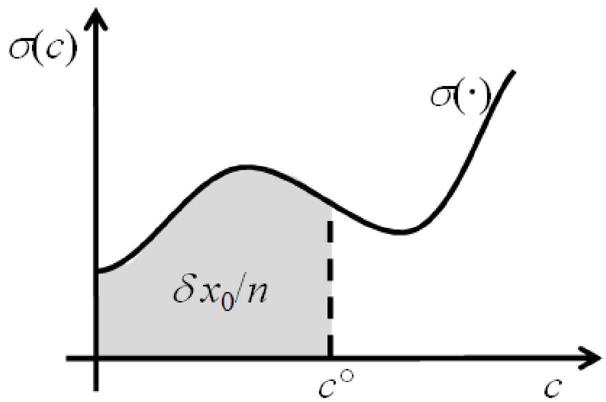


Figure 2.

From Figure 2, we immediatly see that the optimal individual consumption c° does not depend on α, is increasing in δ and x_0, and is decreasing in n and $\sigma(\cdot)$ ([14]). A larger population has an ambiguous effect on the aggregate consumption nc°. However, further investigations make it possible to show the following property.

Property 1. *The optimal aggregate consumption is increasing, constant or decreasing in n, respectively, if the elasticity of marginal utility $\sigma(c)$ is increasing, constant or decreasing, repectively.*

The proof is shown in the appendix.

Figure 3 below helps to understand Property 1. It considers two possible graphs for the elasticity of the marginal utility, one being constant, $\underline{\sigma}(\cdot)$, and the other being increasing, $\sigma_+(\cdot)$. In both cases, the surface area below each graph, from the origin to c°, is assumed equal to $\delta x_0/n$, by construction. Hence, in both cases, the optimal individual consumption is initially equal to c°. Assume now that we double the population size (i.e., $2n$). According to Proposition 1, the new optimal individual consumptions can be found when the surface area below each graph is half what it was initially (i.e., $\delta x_0/2n$). Graphically, this yields the optimal individual consumptions $\underline{c}$ and c_+, respectively corresponding to $\underline{\sigma}(\cdot)$ and $\sigma_+(\cdot)$. Clearly, with the constant elasticity (i.e., $\underline{\sigma}(\cdot)$), the new optimal individual consumption is simply half what it was initially (i.e., $\underline{c} = c^\circ/2$). Therefore, the aggregate optimal consumption is unchanged (i.e., $2n\underline{c} = nc^\circ$). However, with the increasing elasticity, the new optimal individual consumption

[14] Here, in saying that c° is decreasing in $\sigma(\cdot)$, we mean that when comparing two problems only differing with respect to their elasticity of marginal utility, $\sigma_1(\cdot)$ and $\sigma_2(\cdot)$ (say), the optimal consumption c° will be smaller in the second problem when $\sigma_1(c) < \sigma_2(c)$, for all c.

is larger than half what it was initially (i.e., $c_+ > c^\circ/2$). Therefore, the aggregate optimal consumption increases (i.e., $2nc_+ > nc^\circ$).

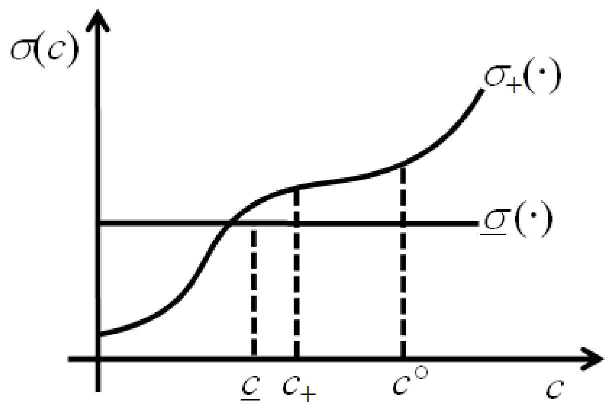


Figure 3.

4 Markov-perfect Nash equilibrium.

In this section, we derive a Markov-perfect Nash equilibrium and discuss its properties. In particular, we prove that it depends on the individuals' preference about their relative consumptions.

A Markovian strategy for individual i is a function s_i, associating resource states x with individual i's consumptions $c_i = s_i(x)$. A vector $S = (s_i)_{i=1}^n$ is called a strategic profile. It is said to be feasible if there exists a unique absolutely continuous state trajectory $x(\cdot)$ satisfying (1), with $c_i(t) = s_i(x(t))$, for all i and t, and if the corresponding individuals' objectives (2), for all i, are well defined (Dockner et al., 2000).

For all feasible strategic profile $S = (s_i)_{i=1}^n$ and initial state x_0, let

$$\begin{array}{l} V^i(S, x_0) = \int_0^\infty v(c_i(t), y_i(t)) e^{-\delta t} dt, \\ \text{where:} \\ \dot{x}(t) = -\sum_{i=1}^n c_i(t), \; x(0) = x_0, \\ y_i(t) = \phi_i(c_{-i}(t)), \\ (c_i(t))_{i=1}^n = (s_i(x(t)))_{i=1}^n . \end{array} \tag{5}$$

A (stationary) Markov-perfect Nash equilibrium is a feasible vector $S^* = (s_i^*)_{i=1}^n$ such that, for all i, s_i and x_0, $V^i(S^*, x_0) \geq V^i((S^*/s_i), x_0)$, with $(S^*/s_i) = \left(s_1^*, ..., s_{i-1}^*, s_i, s_{i+1}^*, ..., s_n^*\right)$ a feasible strategic profile.

We expound our result in Proposition 2 below. It is assumed here that $\sigma(c) > (n-1+\alpha)/n$, for all c ([15]).

Proposition 2. (Markov-perfect Nash equilibrium.)
Let each individual i's strategy $s_i^* = g$, for all i, be implicitly defined by

$$\int_0^{g(x)} (\sigma(s) - (n-1+\alpha)/n)\, ds = \delta x/n, \text{ for all } x. \tag{6}$$

The strategic profile $S^* = (s_i^*)_{i=1}^n$ yields a Markov-perfect Nash equilibrium.

The proof is given in the Appendix.

The figure below illustrates the utilization of Proposition 2. Remember that the elasticity of the marginal utility $\sigma(\cdot)$ is assumed larger than $(n-1+\alpha)/n$. According to Proposition 2, a Markov-perfect Nash equilibrium results if, at any time, each consumer extracts a quantity c^*, such that the surface area between the graphs of the elasticity of marginal utility $\sigma(\cdot)$ and of the horizontal line of ordinate $(n-1+\alpha)/n$, from the origin to c^*, equals $\delta x_0/n$, where δ is the discount rate, x_0 is the current state of the resource and n is the population size.

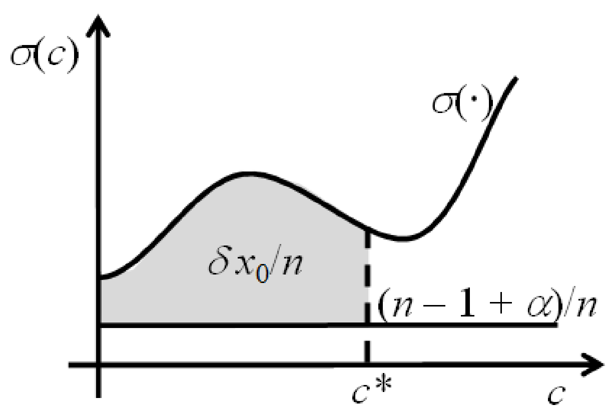


Figure 4.

From Figure 4, we can conclude that the individual Nash equilibrium consumption c^* is increasing in α, δ and x_0, and is decreasing in $\sigma(\cdot)$ ([16]). A larger

[15] As $\alpha < 1$, we have $(n-1+\alpha)/n < 1$. Thus, examples of standard utility functions satisfying this condition are $u(c) = \ln(c)$ and $(c^{1-\mu} - 1)/(1-\mu)$, with $\mu > 1$.

[16] Here, in saying that c^* is decreasing in $\sigma(\cdot)$, we mean that if two problems only differ with respect to their elasticity of marginal utility, $\sigma_1(\cdot)$ and $\sigma_2(\cdot)$ (say), the noncooperative consumption will be smaller in the second problem when $\sigma_1(c) < \sigma_2(c)$, for all c.

population has an ambiguous effect, both at the individual and aggregate levels. Nevertheless, we are still able to derive the following property.

Property 2.

(a) The individual Nash equilibrium consumption is decreasing, constant or increasing in n, respectively, if the elasticity of marginal utility is larger than, equal to, or smaller than one, respectively.

(b) The Nash equilibrium aggregate consumption is increasing in n, if the elasticity of marginal utility is smaller than or equal to one.

(c) The Nash equilibrium aggregate consumption is increasing in n, if the elasticity of marginal utility is non-decreasing.

The proof is shown in the appendix.

The figure below will help to clarify why a larger population has an ambiguous effect. It represents how the individual Nash equilibrium consumption varies, when the resource stock and the population increase *proportionally.* Assume that, in an initial situation, where the resource stock is x_0 and the population size is n, the individual Nash equilibrium consumption is c^*. Hence, by assumption, the surface between $\sigma(\cdot)$ and $(n-1+\alpha)/n$, from the origin to c^*, has measure $\delta x_0/n$, from Proposition 2. Now, consider a final situation, obtained from the initial one, by increasing the resource stock and the population proportionally. Formally, in the final situation, the resource stock x_0' and the population n' are such that $n' > n$ and $x_0'/n' = x_0/n$. From Proposition 2, the resulting individual Nash equilibrium consumption, c^{**}, is found when the surface between $\sigma(\cdot)$ and $(n'-1+\alpha)/n'$, from the origin to c^{**}, has measure $\delta x_0'/n'$. Now, as $x_0'/n' = x_0/n$ by assumption, to obtain c^{**} from c^* in Figure 5, intuitively, one simply has "to slip, remodel and paste the shaded area in the direction of the arrow". It immediatly follows that $c^{**} > c^*$, showing that, when the per capita stock remains constant, a larger population induces a larger individual Nash equilibrium consumption.

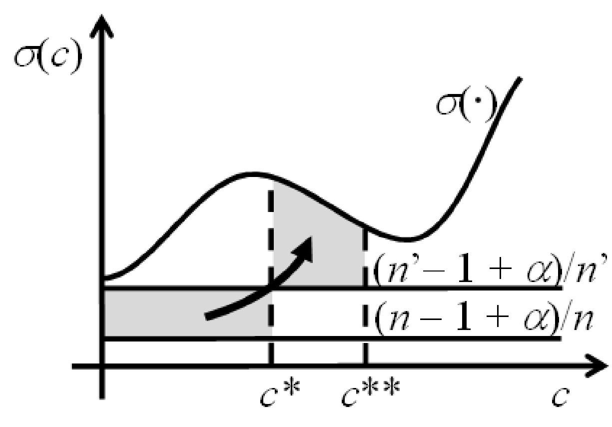


Figure 5.

This suggests a way to decompose the effect of a larger population. On the one hand, there is the strategic effect, dealt with in Figure 5. The per capita stock being constant, a larger population renders its conservation more risky, inducing the individuals to consume the resource more quickly. On the other hand, there is the scarcity effect. Conditional on the population being constant, a smaller stock of the resource induces the individuals to reduce their consumption. Since the two effects go in opposite directions, a larger population has an ambiguous overall effect on the Nash equilibrium consumption. Property 2 shows that the strategic effect dominates when the elasticity of marginal utility is smaller than one, and conversely.

5 Normative implications.

The results above imply that there is overconsumption of the resource at the Markov-perfect Nash equilibrium, compared with the benchmark cooperative solution. Indeed, as the shaded areas in figures 2 and 4 must have same measure, it is not difficult to see that we always have $c^* > c^\circ$, for all possible values of the parameters. We can also remark that the gap between c° and c^* is increasing in $(n-1+\alpha)/n$.

In order to isolate the role played by the individuals' preferences for social status in our results, let us consider the benchmark case where $\alpha = 0$. Then, our framework generalizes the standard model of a common-property, nonrenewable resource exploitation, as reviewed by Dockner et al. (2000) and Long (2010, 2011). It shows the result that the tragedy of the commons is exacerbated by status holds under a general form of the utility function and formalizes that it holds under a more general form of status.

Now, let us compare this benchmark case with a situation where $\alpha > 0$. As $(n-1+\alpha)/n$ is increasing in α, we can conclude that the gap between c° and c^* becomes larger. Hence, our framework generalizes previous results in the literature (Katayama and Long, 2008, and Long and McWhinnie, 2012). It shows the result that the tragedy of the commons is exacerbated by status holds under a general form of the utility function and formalizes that it holds under a more general form of status.

Figure 6 provides a way to decompose graphically the role played by the property regime and the individuals' concern for status, in the overconsumption of the resource. The optimal consumption is c°, obtained when the surface area below $\sigma(\cdot)$ has measure $\delta x_0/n$ (by Proposition 1). The Markov-perfect Nash equilibrium consumption is c^*, obtained when the surface area between $\sigma(\cdot)$ and the line $(n-1+\alpha)/n$ has (the same) measure $\delta x_0/n$ (by Proposition 2).

To isolate the role played by the common-property regime in the transition from c° and c^*, consider anew the special case where $\alpha = 0$. In this situation, the Markov-perfect Nash equilibrium consumption would be $c\prime$, obtained when the surface area between $\sigma(\cdot)$ and the line $(n-1)/n$ has measure (the same) $\delta x_0/n$ (by Proposition 2). To obtain it graphically in Figure 6, intuitively, one simply has "to slip, remodel and paste the shaded area in the direction the arrow". By construction, the transition from c° to c' can be attributed to the tragedy of the commons. The remaining gap between c' to c^{*} is due to the individuals' concern for social status.

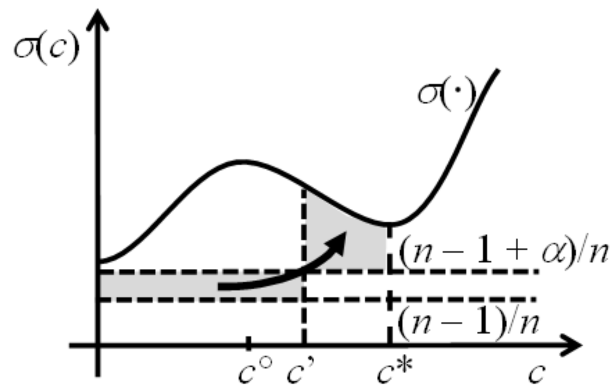


Figure 6.

6 Policy recommandations.

In this section, we design a taxation scheme such that the socially optimal consumption path can be sustained as a Markov-perfect Nash equilibrium. Under this policy, each individual is taxed on the difference between his personal consumption and his socially defined consumption standard, depending on the rates of consumption in the rest of the population. The tax rate is calculated so as to induce the internalization of the social costs of the players' behaviors along the socially optimal consumption path.

At each instant of time t, assume that the social planner assigns to each individual i a consumption standard $z_i(t)$, defined by reference with the rates of consumption among his peers. Formally, we let $z_i(t) = \varphi_i(c_{-i}(t))$ ([17]). Suppose that if $c_{-i}(t) = (c, ..., c)$, for some c, then $\varphi_i(c_{-i}(t)) = c$. Moreover, assume that player i is taxed $\tau(z_i(t))$ on the difference between his consumption $c_i(t)$ and his socially defined consumption standard $z_i(t)$, where $\tau(\cdot)$ is a tax

[17] Of course, the assumption that $\varphi_i(\cdot) = \phi_i(\cdot)$ seems natural. However, by definition, it means that the social planner has perfect information on the individuals' reference groups and on the way they determine their aspiration levels. This seems a very demanding assumption. It is thus worth noticing that this assumption is not necessary to obtain our implementation result.

schedule to be defined below. Under this policy, each consumer i's problem becomes to choose an individual consumption path $c_i(\cdot)$ to maximize

$$\int_0^\infty \begin{bmatrix} v(c_i(t), y_i(t)) \\ -\tau(z_i(t))(c_i(t) - z_i(t)) \end{bmatrix} e^{-\delta t} dt.$$

Passing through the same steps as in section 5, for all feasible strategic profile $S = (s_i)_{i=1}^n$ and initial state x_0, let

$$\begin{array}{l} W^i(S, x_0) = \int_0^\infty \begin{bmatrix} v(c_i(t), y_i(t)) \\ -\tau(z_i(t))(c_i(t) - z_i(t)) \end{bmatrix} e^{-\delta t} dt, \\ \text{where:} \\ \dot{x}(t) = -\sum_{i=1}^n c_i(t), \; x(0) = x_0, \\ y_i(t) = \phi_i(c_{-i}(t)), \\ z_i(t) = \varphi_i(c_{-i}(t)), \\ (c_i(t))_{i=1}^n = (s_i(x(t)))_{i=1}^n . \end{array} \tag{7}$$

A (stationary) Markov-perfect Nash equilibrium is a feasible vector $S^* = (s_i^*)_{i=1}^n$ such that, for all i, s_i and x_0, $W^i(S^*, x_0) \geq W^i((S^*/s_i), x_0)$, with $(S^*/s_i) = \left(s_1^*, ..., s_{i-1}^*, s_i, s_{i+1}^*, ..., s_n^*\right)$ a feasible strategic profile.

Proposition 3 below describes the proposed optimal taxation policy. The result holds under the same conditions as Proposition 1.

Proposition 3. (Optimal taxation scheme)
Assume that each individual i is taxed at the rate

$$\tau(z_i(t)) = (n - 1 + \alpha) u'(z_i(t)) / ((1 - \alpha) n),$$

and plays the strategy $s_i^* = f$, where f is the optimal policy defined in Proposition 1. The strategic profile $S^* = (s_i^*)_{i=1}^n$ yields a Markov-perfect Nash equilibrium.

The proof is given in the Appendix.

7 Conclusion.

The paper considered the exploitation of a common-property, non-renewable resource, by individuals concerned with their social status. We extended results in Katayama and Long (2008), and Long and McWhinnie (2012), assuming a more general formalization of the individuals' social status and preferences. We confirmed that status seeking behaviors exacerbates the tragedy of the commons and we found a taxation scheme to implement the socially optimal consumption path as a Markov-perfect Nash equilibrium.

8 Appendix.

8.1 Proof of lemma 1.

Consider any vector of individual consumptions $C(t) = (c_i(t))_{i=1}^n$. Recall that $V(C(t)) = \sum_{i=1}^n v(c_i(t), \phi_i(c_{-i}(t)))$. Let $\overline{C}(t) = (\overline{c}_i(t))_{i=1}^n = (\sum_{i=1}^n c_i(t)/n)_{i=1}^n$ represent the equal redistribution of $C(t) = (c_i(t))_{i=1}^n$. We need to show that, if $C(t) \neq \overline{C}(t)$, then $V(\overline{C}(t)) > V(C(t))$.

By definition,

$$V\left(\overline{C}(t)\right) = \sum_{i=1}^n v(\overline{c}_i(t), \phi_i(\overline{c}_{-i}(t))). \tag{8}$$

Since the consumption vector $\overline{C}(t) = (\sum_{i=1}^n c_i(t)/n)_{i=1}^n$ is symmetric, we have (by assumption 1.b)

$$\overline{c}_i(t) = \phi_i(\overline{c}_{-i}(t)) = \sum_{i=1}^n c_i(t)/n, \text{ for all } i.$$

Substituting into (8), we get

$$\begin{aligned} V\left(\overline{C}(t)\right) &= \sum_{i=1}^n v\left(\sum_{i=1}^n \frac{c_i(t)}{n}, \sum_{i=1}^n \frac{c_i(t)}{n}\right), \\ &= nv\left(\sum_{i=1}^n \frac{c_i(t)}{n}, \sum_{i=1}^n \frac{c_i(t)}{n}\right). \end{aligned}$$

Now, remark that

$$\left(\sum_{i=1}^n \frac{c_i(t)}{n}, \sum_{i=1}^n \frac{c_i(t)}{n}\right) = \sum_{i=1}^n \frac{1}{n}\left(c_i(t), \sum_{j\neq i} \frac{c_j(t)}{n-1}\right).$$

After substitution, this yields

$$V\left(\overline{C}(t)\right) = nv\left(\sum_{i=1}^n \frac{1}{n}\left(c_i(t), \sum_{j\neq i} \frac{c_j(t)}{n-1}\right)\right).$$

Clearly, if $C(t) \neq \overline{C}(t)$, by concavity of v,

$$v\left(\sum_{i=1}^n \frac{1}{n}\left(c_i(t), \sum_{j\neq i} \frac{c_j(t)}{n-1}\right)\right) > \sum_{i=1}^n \frac{1}{n} v\left(c_i(t), \sum_{j\neq i} \frac{c_j(t)}{n-1}\right),$$

and

$$V\left(\overline{C}(t)\right) > \sum_{i=1}^n v\left(c_i(t), \sum_{j\neq i} \frac{c_j(t)}{n-1}\right).$$

Now, remember that, for all i, v is decreasing in y_i and $\phi_i(c_{-i}(t)) \geq \sum_{j\neq i} c_j(t)/(n-1)$ (by Assumption 1.a). It follows that

$$v\left(c_i(t), \sum_{j\neq i} \frac{c_j(t)}{n-1}\right) \geq v(c_i(t), \phi_i(c_{-i}(t))),$$

for all i, and, summing over i, that

$$\sum_{i=1}^{n} v\left(c_i(t), \sum_{j\neq i} \frac{c_j(t)}{n-1}\right) \geq \sum_{i=1}^{n} v\left(c_i(t), \phi_i(c_{-i}(t))\right).$$

Finally, this allows us to show that

$$V\left(\overline{C}(t)\right) > \sum_{i=1}^{n} v\left(c_i(t), \phi_i(c_{-i}(t))\right) = V(C(t)),$$

which shows lemma 1. ■

8.2 Proof of Proposition 1.

The social problem is to choose $c(\cdot)$ to maximize

$$\begin{array}{l} \int_0^\infty nu(c(t))\, e^{-\delta t} dt, \\ \text{where:} \\ \dot{x}(t) = -nc(t),\ x(0) = x_0, \\ c(t) \geq 0,\ x(t) \geq 0. \end{array}$$

Define the current-value Hamiltonian

$$H(x, c, \lambda) = n(u(c) - \lambda c),$$

where λ is a co-state multiplier associated with the state x.

A feasible control path $c(\cdot)$, with corresponding state trajectory $x(\cdot)$, is optimal if there exists $\lambda(\cdot)$ such that

$$n(u'(c(t)) - \lambda(t)) \leq 0 \text{ and } n(u'(c(t)) - \lambda(t))\, c(t) = 0, \tag{9}$$

$$\dot{\lambda}(t) = \delta\lambda(t), \tag{10}$$

$$\lim_{t\to\infty} e^{-\delta t}\lambda(t)\, x(t) = 0. \tag{11}$$

Now, $f(x)$ being such that

$$\Theta(f(x)) \equiv \int_0^{f(x)} \sigma(s)\, ds = \delta x/n, \text{ for all } x,$$

let $c(t)$, $x(t)$ and $\lambda(t)$, for all t, satisfy

$$\begin{gathered} c(t) = f(x(t)), \\ \dot{x}(t) = -nc(t),\ x(0) = x_0, \\ \dot{\lambda}(t) = \delta\lambda(t),\ \lambda(0) = u'(f(x_0)). \end{gathered}$$

We show below that the proposed control path $c(\cdot)$ is feasible and satisfies conditions (9), (10) and (11). Proposition 1 follows.

1) Feasibility. We first show, in Lemmas 1 and 2, that the proposed control path $c(\cdot)$ is feasible.

Lemma 1. For all $x > 0$, $f(x) > 0$, and $f(0) = 0$.

Proof. If $x = 0$, $f(0) = 0$ follows from $\Theta(0) = 0$. Likewise, $\lim_{x\to\infty} f(x) = \infty$ follows from $\lim_{c\to\infty} \Theta(c) = \infty$. If $0 < x < \infty$, as $\Theta(0) = 0 < \delta x/n < \infty = \lim_{c\to\infty} \Theta(c)$ and $\Theta(c)$ is continuous, there exists $0 < f(x) < \infty$ such that $\Theta(f(x)) = \delta x/n$ (by the intermediate value theorem). As $\Theta(c)$ is increasing (for $\Theta'(c) = \sigma(c) > 0$, for all c), this solution is unique. $\square$

Lemma 2. The individual consumption path $c(\cdot)$ generates a trajectory of the resource stock $x(\cdot)$ such that $x(t) \geq 0$, for all t, and $\lim_{t\to\infty} x(t) = 0$.

Proof. By definition, the individual consumption path $c(\cdot)$ generates a trajectory $x(\cdot)$ such that $\dot{x}(t) = -nf(x(t))$, for all t, with initial condition $x(0) = x_0$. Lemma 2 follows directly from the properties of $f(\cdot)$, which imply that $\dot{x}(t) = -nf(x(t)) < 0$, when $x(t) > 0$, and $\dot{x}(t) = 0$, when $x(t) = 0$. $\square$

2) Necessary conditions. We now check that the necessary conditions (9), (10) and (11) are satisfied.

Let T represent the time of depletion of the resource stock (including the possibility that $T = \infty$).

Lemma 3. Along the path $x(\cdot)$ induced by $f(\cdot)$, the marginal utility $u'(f(x(t)))$ grows at the rate δ, for all $t < T$, and is equal to $u'(0)$, for all $t \geq T$.

Proof. First, remark that $\Theta(f(x)) = \delta x/n$, for all x, implies that $\sigma(f(x)) f'(x) = \delta/n$, for all x (by differentiation).

Define $p(t) = u'(f(x(t)))$, for all t.

For all $t < T$, differentiation yields

$$\dot{p}(t) = u''(f(x(t))) f'(x(t)) \dot{x}(t).$$

Substituting $\dot{x}(t) = -nf(x(t))$ and dividing by $p(t) = u'(f(x(t)))$, we get

$$\frac{\dot{p}(t)}{p(t)} = n\sigma(f(x(t))) f'(x(t)) = \delta.$$

For all $t \geq T$, $x(t) = 0$ (by definition) and $f(0) = 0$ imply that $p(t) = u'(0)$. $\square$

The maximum condition (9) follows directly from lemma 3.

The adjoint equation (10) is satisfied by construction of $\lambda(t)$.

The transversality condition (11) is satisfied since $e^{-\delta t}\lambda(t) = u'(f(x_0))$, by construction, and $\lim_{t\to\infty} x(t) = 0$, by lemma 2. $\blacksquare$

8.3 Proof of property 1.

For all n, let c° satisfy $\Theta(c^\circ) = \delta x_0/n$ and let $C^\circ = nc^\circ$. By differentiation, we obtain

$$\begin{aligned} \frac{dc^\circ}{dn} &= -\frac{\delta x_0}{n^2\sigma(c^\circ)}, \\ \frac{dC^\circ}{dn} &= c^\circ - \frac{\delta x_0}{n\sigma(c^\circ)}. \end{aligned}$$

By the mean value theorem, given that $\Theta(0) = 0$, there exists c, with $0 < c < c^\circ$, such that

$$\Theta(c^\circ) = \sigma(c)\, c^\circ.$$

Hence, we have

$$c^\circ = \frac{\delta x_0}{n\sigma(c)}.$$

By substitution, it follows that

$$\frac{dC^\circ}{dn} = \frac{\delta x_0}{n}\frac{\sigma(c^\circ) - \sigma(c)}{\sigma(c)\,\sigma(c^\circ)},$$

which proves the property. ■

8.4 Proof of Proposition 2.

Consider any player i. Assume that all individuals $j \neq i$ play the Markovian strategy $s_j^* = g(x)$, where $g(x)$ satisfies

$$\Psi(g(x)) \equiv \int_0^{g(x)} (\sigma(s) - (n-1+\alpha)/n)\, ds = \delta x/n, \text{ for all } x.$$

Then, noting that $y_i(t) = \phi_i(g(x(t)), ..., g(x(t))) = g(x(t))$, for all t, by assumption 1.b, the problem of the remaining player i is to find a consumption path, $c_i(\cdot)$, to maximize

$$\begin{aligned} &\textstyle\int_0^\infty v(c_i(t), g(x(t)))\, e^{-\delta t} dt, \\ &\text{where:} \\ &\dot{x}(t) = -(c_i(t) + (n-1)\, g(x(t))),\ x(0) = x_0, \\ &c_i(t) \geq 0,\ x(t) \geq 0. \end{aligned}$$

Define the current-value Hamiltonian

$$H_i(x, c_i, \lambda_i) = v(c_i, g(x)) - \lambda_i(c_i + (n-1)\, g(x)),$$

where λ_i is a co-state multiplier associated with the state x.

A feasible control path $c_i(\cdot)$, with corresponding state trajectory $x(\cdot)$, is optimal if there exists $\lambda_i(\cdot)$ such that

$$v_1(c_i(t), g(x(t))) - \lambda_i(t) \leq 0 \text{ and } (v_1(c_i(t), g(x(t))) - \lambda_i(t))\, c_i(t) = 0, \tag{12}$$

$$\dot{\lambda}_i(t) = (\delta + (n-1)\, g'(x(t)))\, \lambda_i(t) - v_2(c_i(t), g(x(t)))\, g'(x(t)), \tag{13}$$

$$\lim_{t\to\infty} e^{-\delta t} \lambda_i(t)\, x(t) = 0. \tag{14}$$

Let $c_i(t)$ and $x(t)$, for all t, satisfy

$$c_i(t) = g(x(t)),$$
$$\dot{x}(t) = -ng(x(t)),\ x(0) = x_0.$$

We show below that the proposed consumption path $c_i(\cdot) = g(x(\cdot))$ is feasible and we find $\lambda_i(\cdot)$ to satisfy conditions (12), (13) and (14). Proposition 2 follows.

1) Feasibility. It is immediate to adapt lemmas 1 and 2 to show that the proposed control path $c_i(\cdot)$ is feasible.

2) Necessary conditions. Let T represent the time of depletion of the resource stock (including the possibility that $T = \infty$). We first construct $\lambda_i(\cdot)$ to simultaneously satisfy the maximum condition (12) and the adjoint condition (13). We then prove that the transversality condition (14) holds.

For all $t < T$, as $x(t) > 0$, $c_i(t) = g(x(t)) > 0$. Thus, the maximum condition (12) requires that

$$\lambda_i(t) = v_1(c_i(t), g(x(t))) = \frac{1}{1-\alpha} u'(g(x(t))).$$

To show that the adjoint condition (13) holds, differentiate this expression to get

$$\dot{\lambda}_i(t) = \frac{1}{1-\alpha} u''(g(x(t)))\, g'(x(t))\, \dot{x}(t).$$

Substitute $\dot{x}(t) = -ng(x(t))$ and use $\lambda_i(t) = u'(g(x(t))) / (1-\alpha)$ to obtain

$$\dot{\lambda}_i(t) = n\sigma(g(x(t)))\, g'(x(t))\, \lambda_i(t).$$

Now, by differentiation, $\Psi(g(x)) = \delta x/n$, for all x, implies that

$$n\sigma(g(x))\, g'(x) = \delta + (n-1+\alpha)\, g'(x), \text{ for all } x.$$

Thus, after substitution, we can write

$$\dot{\lambda}_i(t) = (\delta + (n-1+\alpha)\, g'(x(t)))\, \lambda_i(t).$$

Finally, using $\lambda_i(t) = v_1(c_i(t), g(x(t)))$ and $v_2(c_i(t), g(x(t))) = -\alpha v_1(c_i(t), g(x(t)))$ (as $c_i(t) = g(x(t))$), we derive

$$\dot{\lambda}_i(t) = (\delta + (n-1) g'(x(t))) \lambda_i(t) - v_2(c_i(t), g(x(t))) g'(x(t)),$$

which is the adjoint equation (13).

For $t \geq T$, as $x(t) = 0$, $c_i(t) = g(0) = 0$. Thus, the maximum condition (12) requires that

$$v_1(0,0) = \frac{1}{1-\alpha} u'(0) \leq \lambda_i(t).$$

Assume that $\lambda_i(t)$ satisfies the adjoint equation (13), with $\lambda_i(T) = v_1(0,0)$. Then, we can show that

$$\lambda_i(t) = \left(\lambda_i(T) - \frac{v_2(0,0) g'(0)}{\delta + (n-1) g'(0)}\right) e^{\left(\delta + (n-1)g'(0)\right)(t-T)} + \frac{v_2(0,0) g'(0)}{\delta + (n-1) g'(0)}.$$

Substituting $v_2(0,0) = -\alpha v_1(0,0) = -\alpha \lambda_i(T)$ and rearranging, we can write

$$\lambda_i(t) = \frac{(\delta + (n-1+\alpha) g'(0)) e^{\left(\delta + (n-1)g'(0)\right)(t-T)} - \alpha g'(0)}{\delta + (n-1) g'(0)} \lambda_i(T).$$

As $e^{\left(\delta + (n-1)g'(0)\right)(t-T)} \geq 1$, this implies that $\lambda_i(t) \geq \lambda_i(T) = v_1(0,0)$, which proves that the maximum condition (12) is true.

To verify the transersality condition (14), we need to separate the cases where T is finite or infinite. If T is finite, the condition is trivially verified, since $x(t) = 0$, for all $t \geq T$. If T is infinite, define $A(t) = e^{-\delta t} \lambda_i(t) x(t)$. By differentiation, we can obtain

$$\frac{\dot{A}(t)}{A(t)} = -\delta + \frac{\dot{\lambda}_i(t)}{\lambda_i(t)} + \frac{\dot{x}(t)}{x(t)}.$$

Using

$$\begin{aligned} \frac{\dot{\lambda}_i(t)}{\lambda_i(t)} &= \delta + (n-1+\alpha) g'(x(t)), \\ \dot{x}(t) &= -n g(x(t)), \end{aligned}$$

we can get

$$\frac{\dot{A}(t)}{A(t)} = (n-1+\alpha) g'(x(t)) - n \frac{g(x(t))}{x(t)}.$$

Now, by definition, for all x, $g(x)$ satisfies $\Psi(g(x)) = \delta x / n$, which is equivalent to

$$\int_0^{g(x)} (n(\sigma(s) - 1) + 1 - \alpha) \, ds = \delta x.$$

A first implication is that (by differentiation with respect to x)

$$\left(n\left(\sigma\left(g\left(x\right)\right)-1\right)+1-\alpha\right)g'\left(x\right)=\delta.$$

A second implication is that there exists c, with $0<c<g\left(x\right)$, such that (by the mean value theorem)

$$\left(n\left(\sigma\left(c\right)-1\right)+1-\alpha\right)g\left(x\right)=\delta x.$$

This piece of information implies that, for all t, there exists c, with $0<c<g\left(x\left(t\right)\right)$, such that

$$\frac{\dot{A}\left(t\right)}{A\left(t\right)}=\left(n-1+\alpha\right)\frac{\delta}{n\left(\sigma\left(g\left(x\left(t\right)\right)\right)-1\right)+1-\alpha}-n\frac{\delta}{n\left(\sigma\left(c\right)-1\right)+1-\alpha}.$$

Now, when t tends to infinity, the resource stock $x\left(t\right)$ converges to 0, implying that both $g\left(x\left(t\right)\right)$ and c (as $0<c<g\left(x\left(t\right)\right)$) converge to 0. From this, we can write

$$\lim_{t\to\infty}\frac{\dot{A}\left(t\right)}{A\left(t\right)}=-\frac{1-\alpha}{n\left(\sigma\left(0\right)-1\right)+1-\alpha}\delta<0,$$

which implies that the transversality condition (14) holds. ■

8.5 Proof of property 2.

For all n, let c^* satisfies $\Theta\left(c^*\right)-\left(n-1+\alpha\right)c^*/n=\delta x_0/n$ and let $C^*=nc^*$. By differentiation, we show

$$\begin{aligned}\frac{dc^*}{dn}&=-\frac{\Theta\left(c^*\right)-c^*}{n\sigma\left(c^*\right)-n+1-\alpha},\\ \frac{dC^*}{dn}&=c^*-\frac{n\left(\Theta\left(c^*\right)-c^*\right)}{n\sigma\left(c^*\right)-n+1-\alpha}.\end{aligned}$$

Let us show the first assertion. By the mean value theorem, given that $\Theta\left(0\right)=0$, there exists c, with $0<c<c^*$, such that

$$\Theta\left(c^*\right)=\sigma\left(c\right)c^*.$$

Assume that $\sigma\left(c\right)>1$, for all c. Then, $\Theta\left(c^*\right)>c^*$, implying that $dc^*/dn<0$. Likewise, one can prove that $dc^*/dn=0$, if $\sigma\left(c\right)=1$, for all c, and $dc^*/dn>0$, if $\sigma\left(c\right)<1$, for all c.

As $dC^*/dn=c^*+dc^*/dn$, the second assertion directly follows from the first one.

Let us now prove the third assertion.

$$n\Theta\left(c^*\right)-nc^*=\delta x_0-\left(1-\alpha\right)c^*,$$

we can write, after substitution

$$\frac{dC^*}{dn} = c^* - \frac{\delta x_0}{n\sigma\left(c^*\right) - n + 1 - \alpha} + \frac{\left(1-\alpha\right)c^*}{n\sigma\left(c^*\right) - n + 1 - \alpha}.$$

Using anew $\Theta\left(c^*\right) = \sigma\left(c\right)c^*$, we can show that

$$c^* = \frac{\delta x_0}{n\sigma\left(c\right) - n + 1 - \alpha},$$

and, after substitution,

$$\frac{dC^*}{dn} = \frac{n\left(\sigma\left(c^*\right) - \sigma\left(c\right)\right)}{\left(n\sigma\left(c\right) - n + 1 - \alpha\right)\left(n\sigma\left(c^*\right) - n + 1 - \alpha\right)}\delta x_0 + \frac{\left(1-\alpha\right)c^*}{n\sigma\left(c^*\right) - n + 1 - \alpha}.$$

It follows that $dC^*/dn > 0$ whenever $\sigma\left(c^*\right) \geq \sigma\left(c\right)$, which proves the property. ■

8.6 Proof of Proposition 3.

Consider any player i. Assume that all individuals $j \neq i$ play the Markovian strategy $s_j^* = f\left(x\right)$, where $f\left(x\right)$ satisfies $\Theta\left(f\left(x\right)\right) = \delta x/n$. Then, noting that $y_i\left(t\right) = \phi_i\left(f\left(x\left(t\right)\right), ..., f\left(x\left(t\right)\right)\right) = f\left(x\left(t\right)\right)$, $z_i\left(t\right) = \varphi_i\left(f\left(x\left(t\right)\right), ..., f\left(x\left(t\right)\right)\right) = f\left(x\left(t\right)\right)$ and $\tau\left(z_i\left(t\right)\right) \equiv \left(n-1+\alpha\right)u'\left(f\left(x\left(t\right)\right)\right)/\left(\left(1-\alpha\right)n\right)$, for all t, the problem of the remaining player i is to find a consumption path, $c_i\left(\cdot\right)$, to maximize

$$\int_0^\infty \left[\begin{array}{c} v\left(c_i\left(t\right), f\left(x\left(t\right)\right)\right) \\ -\tau\left(f\left(x\right)\right)\left(c_i\left(t\right) - f\left(x\left(t\right)\right)\right) \end{array}\right] e^{-\delta t}dt,$$

where:
$\dot{x}\left(t\right) = -\left(c_i\left(t\right) + \left(n-1\right)f\left(x\left(t\right)\right)\right),\ x\left(0\right) = x_0,$
$c_i\left(t\right) \geq 0,\ x\left(t\right) \geq 0.$

Define the current-value Hamiltonian

$$H_i\left(x, c_i, \lambda_i\right) = v\left(c_i, f\left(x\right)\right) - \tau\left(f\left(x\right)\right)\left(c_i - f\left(x\right)\right) - \lambda_i\left(c_i + \left(n-1\right)f\left(x\right)\right)$$

where λ_i is a co-state multiplier associated with the state x.

A feasible control path $c_i\left(\cdot\right)$, with corresponding state trajectory $x\left(\cdot\right)$, is optimal if there exists $\lambda_i\left(\cdot\right)$ such that

$$\begin{aligned} & v_1\left(c_i\left(t\right), f\left(x\left(t\right)\right)\right) - \tau\left(f\left(x\left(t\right)\right)\right) - \lambda_i\left(t\right) \leq 0 \\ & \text{and } \left(v_1\left(c_i\left(t\right), f\left(x\left(t\right)\right)\right) - \tau\left(f\left(x\left(t\right)\right)\right) - \lambda_i\left(t\right)\right)c_i\left(t\right) = 0, \quad (15) \\ & \dot{\lambda}_i\left(t\right) = \left(\delta + \left(n-1\right)f'\left(x\left(t\right)\right)\right)\lambda_i\left(t\right) - v_2\left(c_i\left(t\right), f\left(x\left(t\right)\right)\right)f'\left(x\left(t\right)\right) \\ & \quad + \left(\tau'\left(f\left(x\left(t\right)\right)\right)\left(c_i\left(t\right) - f\left(x\left(t\right)\right)\right) - \tau\left(f\left(x\left(t\right)\right)\right)\right)f'\left(x\left(t\right)\right), \quad (16) \\ & \lim_{t\to\infty} e^{-\delta t}\lambda_i\left(t\right)x\left(t\right) = 0. \quad (17) \end{aligned}$$

Now, $f(x)$ being such that $\Theta(f(x)) = \delta x/n$, for all x, let $c_i(t)$ and $x(t)$, for all t, satisfy

$$c_i(t) = f(x(t)),$$
$$\dot{x}(t) = -nf(x(t)),\ x(0) = x_0.$$

We show below that the proposed consumption path $c_i(\cdot) = f(x(\cdot))$ is feasible and we find $\lambda_i(\cdot)$ to satisfy conditions (15), (16) and (17). Proposition 3 follows.

1) Feasibility. The proposed path, being identical to the socially optimal consumption path, is clearly feasible (see lemmas 1 and 2).

2) Necessary conditions. Let T represent the time of depletion of the resource stock (including the possibility that $T = \infty$). We first construct $\lambda_i(\cdot)$ to simultaneously satisfy the maximum condition (15) and the adjoint condition (16). We then prove that the transversality condition (17) holds.

For all $t < T$, as $x(t) > 0$ (by definition), $c_i(t) = f(x(t)) > 0$. Thus, the maximum condition (12) requires that (using $v_1(c_i(t), f(x(t))) = u'(f(x(t)))/(1-\alpha)$ and $\tau(f(x(t))) = (n-1+\alpha)u'(f(x(t)))/((1-\alpha)n)$)

$$\lambda_i(t) = v_1(c_i(t), f(x(t))) - \tau(f(x(t))) = \frac{1}{n}u'(f(x(t))).$$

To show that the adjoint condition (16) holds, differentiate this expression to get

$$\dot{\lambda}_i(t) = \frac{1}{n}u''(f(x(t)))f'(x(t))\dot{x}(t).$$

Substitute $\dot{x}(t) = -nf(x(t))$ and use $\lambda_i(t) = u'(f(x(t)))/n$ to obtain

$$\dot{\lambda}_i(t) = n\sigma(f(x(t)))f'(x(t))\lambda_i(t).$$

Now, by differentiation, $\Theta(f(x)) = \delta x/n$, for all x, implies that

$$n\sigma(f(x))f'(x) = \delta, \text{ for all } x.$$

Thus, after substitution, we can write

$$\dot{\lambda}_i(t) = \delta\lambda_i(t).$$

Finally, we can verify that this precisely coincides with the adjoint equation (16). Indeed, using $c_i(t) = f(x(t))$, $v_1(c_i(t), f(x(t))) = u'(f(x(t)))/(1-\alpha)$, $v_2(c_i(t), f(x(t))) = -\alpha u'(f(x(t)))/(1-\alpha)$ and $\lambda_i(t) = u'(f(x(t)))/n$, we can show that

$$(n-1)f'(x(t))\lambda_i(t) - v_2(c_i(t), f(x(t)))f'(x(t))$$
$$+ (\tau'(f(x(t)))(c_i(t) - f(x(t))) - \tau(x(t)))f'(x(t)) = 0.$$

For $t \geq T$, as $x(t) = 0$, $c_i(t) = f(0) = 0$. Thus, the maximum condition (15) requires that

$$v_1(0,0) - \tau(0) = \frac{1}{n} u'(0) \leq \lambda_i(t).$$

Assume that $\lambda_i(t)$ satisfies the adjoint equation (16), with $\lambda_i(T) = v_1(0,0) - \tau(0)$. Then, we can show that

$$\lambda_i(t) = \left(\lambda_i(T) - \frac{(v_2(0,0) + \tau(0)) f'(0)}{\delta + (n-1) f'(0)} \right) e^{\left(\delta + (n-1) f'(0)\right)(t-T)} + \frac{(v_2(0,0) + \tau(0)) f'(0)}{\delta + (n-1) f'(0)}.$$

Substituting $v_2(0,0) + \tau(0) = (n-1) \lambda_i(T)$ and rearranging, we can write

$$\lambda_i(t) = \frac{\delta e^{\left(\delta + (n-1) f'(0)\right)(t-T)} + (n-1) f'(0)}{\delta + (n-1) f'(0)} \lambda_i(T).$$

As $e^{\left(\delta + (n-1) f'(0)\right)(t-T)} \geq 1$, this implies that $\lambda_i(t) \geq \lambda_i(T) = v_1(0,0) - \tau(0)$, which proves that the maximum condition (15) is true.

The transversality condition (17) is immediate, since, if T is finite, $x(t) = 0$, for all $t \geq T$, and if T is infinite, then $e^{-\delta t} \lambda_i(t) = \frac{1}{n} u'(f(x_0))$, for all t, and $\lim_{t \to \infty} x(t) = 0$. ■

9 References.

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