Vertical Integration and Innovation*

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Abstract

We studied the impact of vertical integration on investment incentives and social welfare when both upstream and downstream firms make innovative investments. First, we show that vertical integration has larger impact on upstream investments than on downstream investments. When upstream inputs are perfect substitutes, vertical integration leads to upstream crowd-out effect and has no effect on downstream investments. Second, it is only beneficial for firms to integrate when both upstream and downstream innovations are important. And firms may engage in too much integration when only upstream innovation is needed. Third, vertical integration increases social welfare when both upstream and downstream innovation matter; when only upstream innovation is important, vertical integration increases social welfare only if there is strong product differentiation.

Key Words: Vertical Integration, Innovation, Complementarity

JEL Classification: D4, L1, L4

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1 Introduction

In a number of vertically related industries, innovative investments take place at both upstream and downstream levels. For instance, in the value chain of pharma industry, upstream firms, which include biotech companies, research institutions and etc, devote their investments to the discovery and development of potential drugs; and downstream firms (Vertically integrated Big Pharmas consist a large part of the downstream market, however generic manufactures have become an more and more important part.) focus their investment on manufacturing and marketing of approved drugs.

The global pharma industry is going through a consolidation process in recent years. Although horizontal mergers between Big Pharmas have attracted most of attention, (For instance, Pfizer has acquired WarnerCLambert, Pharmacia, Wyeth and King Pharmaceuticals since 2000. Other mergers include Merck/Schering-Plough, Teva/Barr, and etc.) vertical integration has become an important part of the consolidation process.[1] The most notable acquisitions include Merck's 5.4 billion euros bid for Millipore and Astellas Pharma's 3.5 billion dollars bid for OSI Pharmaceuticals. More recently, the world third largest pharmaceutical company Roche has attempted to acquire the two leading gene-testing companies Illumina and Life Technologies. Not only does vertical integration happen in the developed markets such as North America, Europe and Japan, it is also becoming more and more important in emerging markets like China. In 2012, China Pharmaceutical Group acquired the research and production capacity of Robust Sun Holdings for 1.2 billion dollars.[2] At the same time of the surge of vertical integration, we also witness an increase of outsourcing R&D activities in the pharmaceutical industry. Outsourcing of preclinical development is projected to increase by up to 50% in the next few years.[3]

A closer look at this two seemingly controversial phenomena suggest a deeper reason for the two trends in the global pharmaceutical industry. On one hand, vertical integration happens mostly for biotech firms; on the other hand, outsourcing is observed mostly in the more traditional ways of drug discovery, i.e. chemistry and molecule based drug discovery. In the traditional drug discovery process, most innovation happens in the discovery stage, while downstream development and manufacturing technologies are rather mature. However, for the newer biotech discovery process, innovations in the whole value chain are required.

We build a simple model to justify this hypothesis. We present a model with duopoly in both upstream and downstream markets. Upstream firms make investments first; after observing the outcomes of upstream investments, downstream firms make their investments. Then successful upstream and downstream firms bargain over the price of the input.

Our first result shows that vertical integration has asymmetric impact on upstream in-

[1]According to the Global Pharma and Biotech M&A Report 2012 of IMAP, 6 out of the 15 largest transactions in 2011 are R&D driven.

[2]The current wave of backward integration is partly driven by the "patent cliff"faced by Big Pharmas, who are in deep needs for new drugs to fill their pipelines. "Patent Cliff"refers to the situation that Big Pharmas are losing the patent protection for their drugs, after which they will face the competition from generic manufacturers.

[3]Asia Pharma R&D Outsourcing Congress.

vestment and downstream investment. In the situation when upstream inputs are perfect substitutes for downstream firms, when only upstream innovation is needed for the industry, vertical integration leads to upstream crowd-out effect: integration increases the investment incentive of the integrated upstream firm and decreases that of the independent one. The crowd-out effect results from better coordination inside the integrated entity. This is because in case when both upstream firms make successful investments, they only compete for the independent downstream firm but not for the integrated downstream firm. This leaves the integrated upstream firm positive profit when upstream firms compete a-la-Bertrand to supply downstream firms, but the independent upstream firm only gets zero profit. However, when only downstream innovation is needed for the final product, vertical integration has no effect on downstream investment incentives. This is simply because upstream firms are always present and price competition drives the input price down to zero. The general idea behind the different impacts of vertical integration on upstream investment and downstream investment is that upstream competition is stronger than downstream competition. Therefore, upstream firms benefit more from vertical integration; and thus vertical integration has larger impact on upstream investment than on downstream investment.

When both upstream and downstream innovations are needed for the final product, the scenario changes in two ways. First, downstream competition is not always present, and thus the upstream firm cannot catch the whole profit of downstream market even when he is the only upstream innovator. Second, when upstream firms and downstream firms bargain after the realization of the outcomes of investments, we encounter the classical hold-up problem.

The interaction between upstream and downstream innovations lead to some new effects of vertical integration. Firstly, vertical integration also affects downstream investment incentives. This is the situation when the integrated upstream firm is the only upstream innovator. In this case, the integrated downstream firm can get the whole benefit of his innovation whenever he is successful, as a result of the resolution of the hold-up problem inside the integrated entity. But the independent downstream firm is still subject to such hold-up problem. And thus the integrated downstream firm has larger investment incentive, which also crowds out the investment of the independent downstream firm.

Secondly, the upstream crowd-out effect still exists and is strengthened by the necessity of downstream investment. The same effect as when only upstream innovation is needed is still present. The integrated upstream firm gets positive profit even when both upstream firms make successful investments. There are two more effects induced by the presence of downstream investments. First, when there is only one successful upstream firm, the profit for an integrated upstream monopolist is higher than that of a separated one. This results from the benefit of hold-up resolution inside the integrated entity, and it further increases the investment incentive of the integrated upstream firm. Second, the integrated upstream firm still gets positive profit even if he failed in investment, which tends to reduce his investment incentive. We show that the net effect of these two is to further improve the investment incentive of the integrated upstream firm. Hence, downstream investment amplifies the upstream crowd-out effect of vertical integration.

We also consider the effect of vertical integration on social welfare. When only downstream innovation is needed, vertical integration has no effect on social welfare. When only upstream innovation is needed, improved investment incentive of the integrated upstream firm may be socially excessive. The intuition is simple: given that one upstream firm has already made a successful investment, the social value from the investment of a second upstream firm may be less than the private gain. And therefore, when only upstream innovation matters, vertical integration may lead to overall over-investment and hurts social welfare. This is the case when downstream product differentiation is limited, or when upstream investment cost is not too large. However, when both upstream and downstream innovations are needed, we show that vertical integration actually increases social welfare, since investment incentives under vertical separation is insufficient due to hold-up problems.

Lastly, we consider firms' incentive to integrate vertically. When only downstream innovation is needed, firms are indifferent between integration and separation. As integration has no effect on downstream investment incentives, there is no gain from integration. When only upstream innovation is needed, there is gain from integration due to the crowd-out effect. Thus, firms have unilateral incentive to integrate vertically. However, as independent firms are hurt by the integration, they also have incentives to counter-integrate. The result that all firms integrate, but all firms are hurt by the integration. In this case, there is over-integration in the market. If we consider a repeated interaction between firms, then the only outcome may be that all firms remain separated. When both upstream and downstream innovations are needed, investments are insufficient under separation as a result of hold-up problems. Hence, integration is beneficial to firms, and this benefit still dominates even when all firms are integrated.

The analysis in this paper sheds light on the recent development of the global pharmaceutical industry. In the newly biotech drug development process, since innovations are important throughout the value chain, firms choose to vertically integrate. In the traditional molecule drug development process, as investment concentrates more and more on the upstream discovery, firms choose vertical separation. As we show in the extension, allowing firms to contract for upstream innovation, they can do even better than vertical separation, which may explain the trend of outsourcing in the traditional drug discovery.

Our results can also be applied to other industries and cases where innovation plays an important role. For instance, in the satellite navigation industry, Tele Atlas and Navteq are the two main firms in the upstream market for navigable digital map databases. In 2008, Tele Altas and Navteq were subsequently acquired by two main downstream firms TomTom and Nokia. Several works have been done concerning this case, for instance, Hombert et al (2012) considers the effect of such vertical integration on the pricing of upstream firms to independent downstream navigation service companies; Allain et al (2011) considers the potential information leakage and imitation problem following the integration. However, both papers consider the downstream foreclosure effect; our analysis shows that the integration may also affect upstream investment incentives, and such effects should not be neglected. Also in the smartphone and tablet industry, industry giants Google and Microsoft both integrate to the design stage of the industry, rather than stay only as operating system provider.

Our paper contributes to the literature on the foreclosure effect of vertical integration, which dates back to the seminal paper of Ordover, Saloner and Salop (1990). Subsequent contributions which resolve the commitment issue in this paper include Choi and Yi (2000), Chen and Riordan (2007), and etc. Most of the literatures along this line focus on the downstream foreclosure effect, and our result can be interpreted as the upstream foreclosure effect of vertical integration. In addition, such downstream foreclosure effect may benefit the independent upstream firm. Since vertical integrated upstream firm may restrict his supply to the independent downstream firm, the independent upstream firm may then enjoy some market power vis-a-vis the independent downstream firm. However, taken into consideration of upstream investment, vertical integration may instead hurt the independent upstream firm.

With a focus on the role of investments, most of previous works have either only upstream investments or only downstream investments. Bolton and Whinston (1993) studied the effect of vertical integration on downstream investments. In their model, whether there is downstream competition ex post exogenously depends on whether there are one or two units of input available. On the contrary, competition is endogenously determined by downstream investment in our model, and vertical integration softens downstream competition. Buehler and Schmutzler (2008) also studies the effect of vertical integration on downstream investments in a successive oligopoly model. Chen and Sappington (2010) studied upstream investment incentive with a monopolistic upstream firm, they show that vertical integration generally enhances upstream innovation under downstream Cournot competition but may diminish upstream innovation with downstream Bertrand competition. Since there is no upstream competition, upstream foreclosure does not exist there. Brocas (2003) also studies the effect of vertical integration on upstream investment, however the focus in her paper is mainly downstream switching cost. Our paper is a first attempt to the problem with both upstream and downstream investments.

Our paper is also related to some literatures discussing the effect of integration on innovation of complementary products. Farrell and Katz (2000) shows that integration into a complementary product market allows a monopolist to extract more rent from the market where he dominates. Schmidt (2009) studies how vertical integration of an patent holder affect the contractual terms between upstream patent holders and downstream producers. In these papers, the complementarity is between horizontally related products. In our paper, such complementarity is vertical.

The paper proceeds as follows: We present the basic framework in Section 2; Section 3 studies two benchmarks, when only upstream innovation or only downstream innovation is needed for the final product; Section 4 presents the results with both upstream and downstream innovation matter for the final product; We discuss the welfare implications in Section 5; Section 6 provides some extensions and discussion; Section 7 concludes. All proofs are shown in the Appendix.

2 The Framework

Players and Market Interaction The industry consists of an upstream market and a downstream market. There are two upstream firms U_A and U_B, and they supply inputs for two downstream firms D_1 and D_2. Each $D_j, j = 1, 2$ demands only one unit of the input. We assume that once D_j fails to trade with both $U_i, i = A, B$, he has no alternative source for the input. Similarly, if U_i fails to trade with both downstream firms, she has no other ways to access the final consumer market.

Product Market The payoff to each downstream firm depends on whether there is competition in the final good market. For each downstream firm D_j, if he is the only active one in the downstream market, he gets profit Δ; if both downstream firms are active, each D_j can only get a profit of δ. We assume that $0 < 2\delta < \Delta$, so that competition dissipates part of the profit but not all of it. Thus the payoff for downstream firms is described in the following Table 1, where "A" and "NA" indicate whether D_j is active or not active in the downstream market. Therefore, if U_i is the only upstream innovator, the industry profit is maximized when she only sells to one of the two downstream firms.

Table 1: Product Market Payoffs

D_1 \ D_2	A	NA
A	δ,δ	Δ,0
NA	0,Δ	0,0

Technology A successful final product may need innovative investment from both upstream and downstream firms. Each upstream firm U_i can make investment in innovation, which in case of success enables her to supply the input; otherwise, U_i has no successful innovation and has to stay out of the market. We model the upstream innovation in the following way: for a given level of investment e_i, U_i succeeds in innovation with probability e_i and fails with the complementary probability $1 - e_i$. The cost of investment is the same for both upstream firms, which is given by $C_U(e)$. We assume that there is no marginal cost of production. Thus, the total cost for an upstream firm is the fixed cost of investment. Furthermore, there is no capacity constraint or any other shocks that may constrain the production of U_i, and each U_i can supply both downstream firms if she is willing to.

Each downstream firm D_j also makes investment in innovation, which in case of success allows him to transform the input to the final product on a one-to-one basis at zero cost. As the upstream market, we model downstream investment as follows: each downstream firm D_j makes an investment d_j which succeeds with probability d_j and fails with probability $1 - d_j$. The cost of investment is $C_D(d)$. We consider non-channel specific investment, and as long as one firm is successful, he is able to trade with any other successful firms.

For simplicity, we assume that the cost function of upstream and downstream investments are quadratic, and satisfy the additional condition which guarantees that the profit functions are well-defined and the solutions are interior.

Assumption 1. *$C_U(e) = \frac{1}{2}c_u e^2$ and $C_D(d) = \frac{1}{2}c_d d^2$; moreover, $c_u \geq \Delta$ and $c_d \geq \Delta$.*

Timing of the Game Throughout the paper, we study the investment game with time-line as follows:

- *Stage 1: Upstream Investment* Upstream firms choose their investments $e_i, i = A, B$, and the outcomes of upstream investments realize;

- *Stage 2: Downstream Investment* Observing the outcomes of upstream investments, downstream firms choose their investments $d_j, i = 1, 2$, and the outcomes of downstream investments realize;

- *Stage 3: Bargaining* Successful upstream firms and downstream firms bargain over the terms of inputs supply; and inputs are delivered according to all agreements between firms;

- *Stage 4: Final Product Market*: Final product market and payoff to downstream firms realize.

We make two comments about the time-line. First, we assume that the downstream firms invest after observing the outcomes of upstream investments. This fits a number of situations. For instance, downstream distributors put effort in marketing after upstream firms have successfully introduced some new products; downstream developers start to develop final products after upstream innovators have made successful discoveries. Moreover, with such sequential investment, we are able to eliminate some socially wasteful downstream investment. Since if both upstream firms failed in investment, there would be no value of downstream investment. Second, we assume that bargaining happens after the realization of outcomes of both upstream and downstream investments. This allows us to study the interaction between upstream and downstream investments. If bargaining happens in an interim stage, i.e. the successful upstream innovators bargain with downstream firms after the realization of outcomes of upstream investments but before downstream firms making any investments, then downstream investment incentives won't be affected by the bargaining process or whether there is vertical integration.

Bargaining We propose a simple bargaining procedure between (successful) upstream firms and (successful) downstream firms. There can be four scenarios: both upstream and downstream market can be either monopoly or duopoly. We assume that when there is bilateral monopoly or bilateral duopoly, with equal probability, either upstream firms or downstream firms are chosen to make offers. For instance, if upstream firms were chosen, they make offers to downstream firms, downstream firms make acceptance or rejection decisions; inputs are delivered according to accepted offers, and game ends. It is the same if downstream firms are chosen to make offers.[4] However, when there is monopoly on one market and duopoly on the other market, we assume that it is the monopolist who make offers to the other two firms.

We assume that an offer is simply a price for one unit of input,[5] and whenever a firm is indifferent between accepting and rejecting an offer, he chooses to accept. All offers and acceptance decisions are publicly observable.

[4]In the case of bilateral duopoly, the probability of whether it is the upstream firms or downstream firms that make offers is channel independent. In other words, the bargaining power is at market level instead of at firm level.

[5]We discuss the situation when exclusive offers are allowed in Section 6.

3 Two Benchmarks: One Side Innovation

To distinguish the main forces at work, we start with two benchmarks where only one-side innovation, either upstream or downstream, is needed for the final product. We study the interaction between upstream and downstream innovation in the next section. In these two benchmarks, the timing of investments does not matter, since investments only happen in one market. When only upstream innovation is needed, vertical integration improves the coordination between the integrated firms, which increases the investment incentive of the integrated upstream firm and decreases that of the independent upstream firm. And firms may engage in over-integration. On the other hand, when only downstream innovation is needed, vertical integration has no effect on downstream investment incentives.

3.1 Only Upstream Innovation is Needed

We start with the first benchmark where only upstream innovation is needed for the final product. In this situation, as long as the upstream investment is successful, the downstream firms can transform the inputs into final products. That is to say downstream firms do not directly contribute to the value of the final product, they specialize in developing channels to final consumers. We begin with the case when all firms remain separated.

3.1.1 Vertical Separation

Since downstream investment is not needed, both downstream firms are viable for the successful upstream innovator. We need to specify the bargaining outcome depending on whether only one upstream firm or both upstream firms have made successful investments. The result of the bargaining is summarized as Lemma 1.

Lemma 1. *The payoff matrix for upstream firms when only upstream innovation is need is given by where "S"and "F"indicate whether U_i succeeds or fails in investment.*

Table 2: Upstream Payoffs under Separation

U_A \ U_B	S	F
S	0,0	Δ,0
F	0,Δ	0,0

Proof. Clearly, when both upstream firms fail in investment, they both get zero profit. Two cases remain: when only one upstream firm succeeds who then acts as an upstream monopolist; and when both upstream firms succeed and upstream competition emerges.

Case 1: Only one upstream firm succeeds, suppose it is U_A.

In this case, it is U_A who is to make offers, then U_A can always guarantee a payoff of Δ by making offers (Δ, p) and $p > \Delta$ to the two downstream firms respectively, which will be accepted by the first one and rejected by the second one. And thus U_A gets profit Δ, which is the maximum profit he can extract from downstream market.

Case 2: Both upstream firms succeed in investment.

If the downstream firms make offers, they will ask for zero price to both upstream firms. And no downstream firm can be better off by making other offers. The reason is simple: the upstream firm is willing to accept any non-negative offers. If it is the upstream firms that make offers, the best they can do is to offer zero price, since otherwise the other upstream firm will always undercut. Therefore, when both upstream firms are successful, they each get zero profit under vertical separation. □

When only one upstream firm has successful innovation, the upstream monopolist gets the whole downstream market profit Δ.[6] When both upstream firms obtain successful innovation, competition between them drives down the input price to zero.

Then for each upstream firm U_i, the problem is to choose an investment level to maximize her expected profit

$$e_i^* = argmax_{e_i}\{e_i(1-e_{i'})\Delta - C_U(e_i)\}, i' \neq i$$

which gives us the best response of U_i, given the investment level of $U_{i'}$

$$e_i^* = {C_U'}^{-1}((1-e_{i'})\Delta), i' \neq i$$

It is easy to see that $\frac{\partial e_i^*}{\partial e_{i'}} = -\frac{\Delta}{C_U''(e_i^*)} < 0$, and thus the investments of the two upstream firms are strategic substitutes. As $U_{i'}$ increases investment, it is more likely that U_i cannot recoup the benefit of his innovation and then U_i has less incentive to invest. Moreover, from Assumption 1, it is clear that $-1 < \frac{\partial e_i^*}{\partial e_{i'}} < 0$ and then there exists a unique equilibrium in the investment game e_U, which is given by the solution of the equation

$$e_U \text{ solves } \Delta(1-e) = C_U'(e)$$

3.1.2 Partial Vertical Integration

Suppose now there is partial vertical integration, i.e. only one upstream firm and one downstream firm integrate, without loss of generality, we assume that U_A and D_1 now integrate, while U_B and D_2 remain separated.

Compared to the case of vertical separation, there are a few differences. First, if the integrated entity $U_A - D_1$ was the only successful firm in the upstream market, U_A is an upstream monopolist and can extract all downstream market profit Δ. We assume that in this case U_A would only supply his own downstream affiliate D_1 and D_2 is excluded from the market.

Second, when the independent upstream firm U_B is the only one who has successful upstream innovation, we do not model any information leakage risk, which would give the independent downstream firm D_2 certain advantage vis-a-vis the integrated downstream firm D_1. For example, U_B may face the risk that if she supplies inputs to D_1, D_1 may leak some key information to U_A, and then U_A would also have successful innovation and compete with U_B. This would make D_1 as an inferior downstream firm in the perspective of U_B, which grants

[6]This is also the case even when it is the downstream firms that make offers, if exclusive offers are allowed.

D_2 a stronger bargaining position. This type of information leakage problem has been studied in a few other papers such as Allain et al(2011) and Chen(2011). Our focus here is not on how vertical integration affects the information flows in the industry, hence we assume away any information problem. Moreover, in our setting of sequential moves, upstream innovation happens before the actual production of final product, and thus such backward information flow won't be a big concern. Therefore, when U_B is the sole upstream innovator, she is still able to extract the whole downstream market profit Δ due to downstream competition.

Third, when both upstream firms succeed in investment, the integrated downstream firm D_1 can guarantee a zero input price from U_A, which means that U_B cannot make positive profit from supplying D_1. And then upstream competition drives the input price for D_2 down to zero. Thus the profit for U_B is zero while U_A can still catch part of downstream market profit, which is δ as both D_1 and D_2 are active in the downstream market.

In sum, the payoff matrix for $U_A - D_1$ and U_B is given by Table 3.

Table 3: Upstream Payoffs under Integration

$U_A - D_1$ \ U_B	S	F
S	δ,0	Δ,0
F	0,Δ	0,0

Then the upstream firms choose e^1_{VI} and e^2_{VI} such that

$$e^1_U = argmax_{e_1}\{e_1(1-e_2)\Delta + e_1 e_2 \delta - C_U(e_1)\}$$

and

$$e^2_U = argmax_{e_2}\{e_2(1-e_1)\Delta - C_U(e_2)\}$$

Under Assumption 1, it is easy to see that $\frac{\partial e^1_U}{\partial e_2} = -\frac{\Delta-\delta}{C''_U(e^1_U)} \in (-1,0)$, and $\frac{\partial e^2_U}{\partial e_1} = -\frac{\Delta}{C''_U(e^2_U)} \in (-1,0)$. Hence the upstream investment game under vertical integration has a unique solution which solves the two best response functions,

$$\begin{cases} C'_U(e^1_U) & = \Delta(1-e^2_U) + \delta e^2_U \\ C'_U(e^2_U) & = \Delta(1-e^1_U) \end{cases} \tag{1}$$

Compared to the situation under vertical separation, we have the following proposition,

Proposition 1. *The integrated upstream firm invests more than the independent upstream firm. Indeed, we have* $e^1_U > e_U > e^2_U$.

Proof. See Appendix. □

The proposition is shown in Figure 1. The equilibrium investment is determined by the intersection of the two best response curves $BR^1(e_2)$ and $BR^2(e_1)$. The best response function for the independent upstream firm U_B is not affected by integration. However, integration of U_A and D_1 leads to a clockwise rotation of the best response function of U_A, which clearly

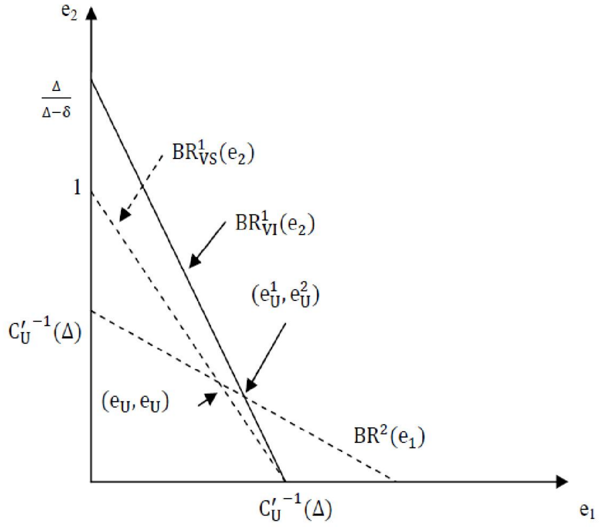

Figure 1: Equilibrium Upstream Investment under Vertical Separation and Vertical Integration

shows that the integrated upstream firm U_A invests more than under vertical separation while the independent upstream firm U_B invests less.

The stronger investment incentive of the integrated upstream firm originates from the better coordination inside the integrated entity $U_A - D_1$. Under vertical integration, when both upstream firms make successful investment, they only compete for the independent downstream firm; and thus the integrated upstream firm is still able to earn positive profit rather than zero under vertical separation, which increases her incentive to invest. Since upstream investments are strategic substitutes, the independent upstream firm invests less because there is a larger chance that the integrated firm would have successful innovation.

3.2 Only Downstream Innovation is Needed

Now we turn to the case when only downstream innovation is needed for the final product, i.e. the input is sort of generic product which cannot be used directly by the final consumers. Downstream firms need to make investments which enable them to transform the inputs to final products in case of success.

Clearly, in this case, the presence of upstream competition drives the input price down to zero. And therefore, each downstream firm can keep the whole benefit of his innovation Δ in case when he is the sole innovator in downstream market. When both downstream firms are successful, competition dissipates part of the profit and each D_j gets δ. Thus the payoff to downstream firms is described as Table 1.

When U_A and D_1 integrates (without loss of generality), no firm can do better than when they are separated. When only D_1 succeeds, he gets Δ; when only D_2 succeeds, upstream competition leads to zero price for the input. When both downstream firms succeed, the integrated entity $U_A - D_1$ retrieves δ from its downstream affiliates; while competition leads to zero input price for D_2. Therefore, the payoff to $U_A - D_1$ and D_2 is again given by Table 1. Thus, we have the following result.

Proposition 2. *When only downstream innovation is needed, downstream investment incentives are not affected by vertical integration.*

We denote d as the downstream investment in this case, and we have $d = \frac{\Delta}{c_d+\Delta-\delta}$. The difference between the situation when only upstream innovation is needed and when only downstream innovation is needed originates from the fact that: the inputs produced by two upstream firms are homogenous and thus perfect substitutes for downstream firms; while downstream firms' innovations are differentiated. From the perspective of the final product market, product differentiation is due to the innovation of downstream firms rather than that of the upstream firms. In other words, the upstream competition is stronger than the competition in the downstream market.

3.3 The General Case

The extreme divergence in the two benchmarks results from the perfect substitution of upstream inputs. However, the general logic is that upstream competition is stronger than downstream competition, and vertical integration has a larger impact on upstream innovation than on downstream innovation.

Consider the general case where upstream inputs are also differentiated, but not channel specific. Specifically, if one upstream firm sells to both downstream firms, then each downstream firm gets profit δ_D; if each upstream firm sells to one downstream firm, then each downstream firm gets profit $\delta_D + \delta_U$. Thus, δ_D measures downstream differentiation, and δ_U measures upstream differentiation. We prove the following result in the Appendix.

Proposition 3. *If downstream is more differentiated than upstream, then vertical integration has a larger impact on upstream innovation than downstream innovation; if upstream is more differentiated, then vertical integration has the same impact on both upstream and downstream innovations.*

Proof. See Appendix. □

In the special case where there is no upstream differentiation, we have the result as in our benchmarks: vertical integration only affects upstream investments but not downstream investments. The above proposition highlights two channels that vertical integration can affect investment incentives. The first channel is through better coordination inside the integrated entity. This is better understood when upstream differentiation is relatively weak. In this case, each firm can only get the benefit of differentiation which is contributed by his own product. Vertical integration allows each firm to also benefit from the differentiation that is created by his affiliate. Therefore, upstream firms benefit more since downstream firms are more differentiated.

The second channel is through bargaining. This is clearly shown in the situation when upstream is more differentiated. In this case, the firm that makes offer can obtain the value of differentiation from both upstream and downstream. However, the probability that a firm is going to make an offer is only 0.5. And thus vertical integration eliminates such bargaining friction, which improves investment incentives. Notice that even if upstream firms contribute to all the differentiation ($\delta_D = 0$), they cannot get the whole benefit of their innovation. This is due to the fact that downstream firms are sort of bottleneck, as they have the access to final consumers. Thus downstream firms are granted a stronger bargaining position. Moreover, since each downstream firm only needs one unit of input, they won't compete for the same supplier; while upstream firms may want to compete against each other for selling to the same downstream firm. Hence, upstream competition is stronger than downstream competition, which makes upstream firms benefit more from vertical integration. This result is similar to de Fontenay and Gans (2005), where they also showed that firms facing stronger competition have stronger incentives to integrate. Their results originates from the fact that industry profit is different under different market structure. However, in our paper, the maximum industry profit is the same under both vertical separation and vertical integration.

3.4 Incentives to Integrate

In this section, we study how the analysis above shapes firms' incentives to integrate vertically. When only downstream innovation is needed, it is obvious that firms are indifferent between integrate or not. When only upstream innovation is needed, it is natural to ask how U_B and D_2 would respond if U_A and D_1 integrate.

It is clear from the above analysis that when only upstream innovation is needed, the resulting crowd-out effect of vertical integration hurts the independent upstream firm, which implies that the independent upstream firm also has incentive to integrate with the independent downstream firm. This is indeed the case. The joint profit of U_B and D_2 is

$$\pi_{U_B - D_2} = e_2(1 - e_1)\Delta + e_2 e_1 \delta - \frac{1}{2} c_u e_2^2$$

which is decreasing in e_1. Under full integration, the upstream investment is $e_U^{FI} = \frac{\Delta}{c_u + \Delta - \delta}$. Given $e_1 = e_U^{FI}$, the joint profit of U_B and D_2 is maximized at $e_2 = e_U^{FI}$ also. Moreover, $\pi_{U_B - D_2}$ is decreasing in e_1. Hence, given e_2, the joint profit for U_B and D_2 is higher when $U_A - D_1$ invests e_U^{FI} rather than e_U^1, since $e_U^1 > e_U^{FI}$. Therefore, following the integration of U_A and D_1, U_B and D_2 also have incentives to integrate.

Such counter-strategy lowers the profitability of the first integration $U_A - D_1$, which may lead to the situation that even the first integration is not profitable. Under vertical separation, the upstream investment is given by $e_U = \frac{\Delta}{c_u + \Delta}$; The joint profit of $U_A - D_1$ when both upstream firms invest e is given by

$$\pi_{U_A - D_1} = e(1 - e)\Delta + e^2 \delta - \frac{1}{2} c_u e^2$$

which is decreasing if $e > \tilde{e} = \frac{\Delta}{c_u + 2(\Delta - \delta)}$.

Since $e_U^{FI} > e_U > \tilde{e}$, $U_A - D_1$ is worse-off if their integration is followed by the integration of $U_B - D_2$. Therefore, when only upstream innovation is needed, firms have excessive incentives to integrate. And they may fall into a prisoner's dilemma: both firms want to integrate, however, they are hurt when both are actually integrated. The reason behind this result is that upstream firms are over-investing at the presence of upstream competition, and vertical integration increases the investment incentives of both firms, which makes the over-investment problem even worse.

Alternatively, if we consider a market where firms interact repeatedly: at each period, the firms choose whether to integrated or not. Then for patient enough firms, we may have a collusive-style market outcome such that all firms remain separated. As we show in the extension, if we allow firms to contract on upstream innovation, they can do even better than just remain separated.

4 Interaction Between Upstream and Downstream Innovations

Based on the analysis of the previous section, we now study the situation when both upstream and downstream innovations are needed for the final product. This situation fits a number of settings: upstream firms may be manufacturers who need to make investment in order to develop high quality product, while downstream retailers need to make investment in services in order to improve relationship with consumers; upstream firms may consist of patent holders who make investment to develop new ideas, while downstream developers invest to develop the ideas into final products. There is a strong complementarity between upstream and downstream innovations, in the sense that there is no value for the final product without either upstream innovation or downstream innovation.

Different from the section above, when there is only one upstream firm who succeeds in investment, upstream competition is not present and downstream investment incentives are different from what has been analyzed above. We start with the analysis in this situation.

4.1 Downstream Investment with Upstream Monopolist

Given that only one upstream firm is successful, without loss of generality, suppose it is U_A. We first study the downstream investment incentives when all firms are separated.

4.1.1 Vertical Separation

When all firms are separated, in the continuation game each downstream firm chooses his investment independently from the other. We make two remarks here. First, we maintain the time-line as before, i.e. the payoff to each firm is realized through ex post bargaining, which we follow the classic property right literatures. The downstream investment is not contractible, and the upstream monopolist cannot write a contract with downstream firms before any downstream investment happens. The bargaining takes place after the outcomes of downstream investments are realized and observed by all firms.

Second, in the previous section, when only upstream innovation is needed for the final product, downstream competition is always present. Therefore, as long as the upstream monopolist makes successful innovation, she is able to extract the whole downstream market profit. When downstream innovation is needed for the final product, downstream competition is not guaranteed. When only one downstream firm obtains successful innovation, there is in fact an upstream monopolist and a downstream monopolist. Our simple bargaining procedure implies that in this case the upstream firm and the downstream firm each gets $\frac{\Delta}{2}$ (Each firm gets Δ when he is chosen to make offers, which happens with probability $\frac{1}{2}$). However, when both downstream firms succeed, downstream competition is again present, and the upstream monopolist is able to catch the whole downstream market profit Δ and each downstream firm ends up with zero profit. There is a major difference between our model and the model of Bolton and Whinston(1993): in their model, downstream competition is exogenously determined by whether there are one or two units of input available; however, in the present model, ex ante downstream competition always exists and ex post downstream competition is endogenously determined by the investment of each downstream firm.

The payoff matrix for the two downstream firms can be summarized as

Table 4: Downstream Payoffs under Separation

D_1 \ D_2	S	F
S	0,0	$\frac{\Delta}{2}$,0
F	0,$\frac{\Delta}{2}$	0,0

Then for each downstream firm, they choose d_j^* such that

$$d_j^* = argmax_{d_j}\{d_j(1-d_{j'})\frac{\Delta}{2} - C_D(d_j)\}, j' \neq j$$

The same reasoning as the previous section shows that under Assumption 1, there exists a unique equilibrium in the investment stage ($d_1 = d_2 = d_{VS}$) which is given by the solution to the following equation

$$C'_D(d_{VS}) = (1 - d_{VS})\frac{\Delta}{2}$$

4.1.2 Vertical Integration

Now suppose that the upstream monopolist U_A integrates with one of the two downstream firms D_1. The game is the same as vertical separation except in the bargaining stage. First, when the independent downstream firm D_2 is the only one who succeeds in downstream innovation, the bargaining outcome is such that $U_A - D_1$ and D_2 each gets $\frac{\Delta}{2}$; Second, when D_1 is the sole innovator in downstream market, the integrated entity $U_A - D_1$ retains the whole downstream market profit Δ. And thus D_1 is able to get the whole benefit of his investment. Third, when both downstream firms succeed, there is no actual downstream competition. The upstream monopolist only supplies her downstream affiliate. The payoff matrix is given by

Then under vertical integration, the problem for $U_A - D_1$ and D_2 is to choose d_1^* and d_2^* such that

$$d_1^* = argmax_{d_1}\{d_1\Delta + (1-d_1)d_2\frac{\Delta}{2} - C_D(d_1)\}$$

Table 5: Downstream Payoffs under Integration

$U_A - D_1$ \ D_2	S	F
S	Δ,0	Δ,0
F	$\frac{\Delta}{2}, \frac{\Delta}{2}$	0,0

and

$$d_2^* = argmax_{d_2}\{d_2(1-d_1)\frac{\Delta}{2}) - C_D(d_2)\}$$

The unique equilibrium (d_{VI}^1, d_{VI}^2) in the investment stage under Assumption 1 is given by the solution to the following equations

$$\begin{aligned} C_D'(d_{VI}^1) &= \Delta - \tfrac{1}{2}\Delta d_{VI}^2 \\ C_D'(d_{VI}^2) &= \tfrac{\Delta}{2} - \tfrac{1}{2}\Delta d_{VI}^1 \end{aligned} \tag{2}$$

Proposition 4. *The integrated downstream firm invests more than the independent downstream firm, indeed we have $d_{VI}^1 > d_{VS} > d_{VI}^2$.*

Proof. The proof is similar to the proof of Proposition 1, we thus omit the detailed proof. □

Lemma 2 can be shown in the Figure 2. The best response function for the independent downstream firm D_2 is not affected by vertical integration. However, the best response function of the integrated downstream firm is pushed outward by vertical integration. There are two effect of vertical integration on downstream investments, first, the integrated downstream firm is able to catch all the benefit of his innovation whenever he is successful, no matter whether the independent downstream firm succeeds or not. This boosts the investment incentive of the integrated firm. Second, even if the integrated downstream firm fails in innovation, the integrated entity $U_A - D_1$ still gets $\frac{\Delta}{2}$ if the independent downstream firm succeeds, which tends to decrease the investment incentive of the integrated downstream firm. Starting from the symmetric equilibrium as in the vertical separation case, the net effect of vertical integration is to increase the investment incentive of the integrated downstream while leave that of the independent downstream firm unchanged. Since downstream investments are strategic substitutes, higher investment incentive from the integrated downstream firm crowds out the incentive of the independent firm.

Lemma 2. *If there is only one successful upstream firm, in the continuation game, the profit for the integrated upstream monopolist is higher than the separated one.*

Proof. See Appendix. □

Under vertical separation, there is serious under-investment problem: each downstream firm can only get half of the benefit $(\Delta/2)$ from his innovation even when he is the sole innovator in the downstream market, while the social benefit in this case is at least Δ. Under vertical integration, the insufficient incentive problem persists for the independent downstream firm; however, the hold-up problem is solved for the integrated entity. This increases the profit of the

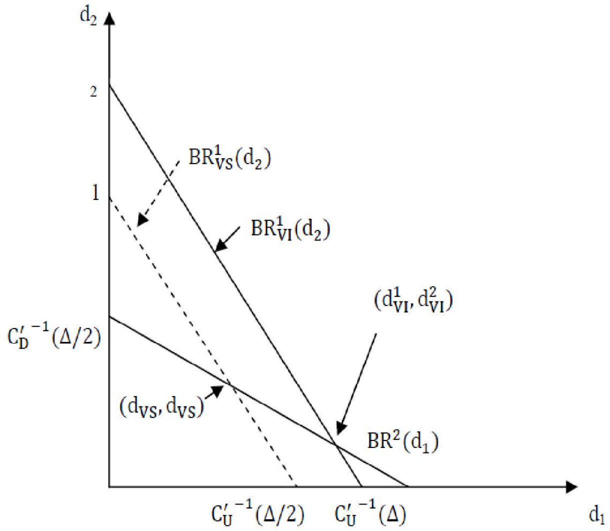

Figure 2: Equilibrium Downstream Investment under Vertical Separation and Vertical Integration

integrated upstream monopolist. Moreover, the integrated entity gets additional benefit from the crowd-out of the investment from the independent downstream firm. Denote the profit for a vertically separated upstream monopolist as π_{VS}, and the profit for an integrated upstream monopolist as π_{VI}, the lemma shows that $\pi_{VI} > \pi_{VS}$.

4.2 Upstream Investment–Vertical Separation

Now we turn to the investment incentives of upstream firms when all firms remain separated. When only one upstream firm succeeds, the subgame goes as in Section 4.1, the upstream monopolist gets continuation payoff π_{VS}. When both upstream firms obtain successful innovation, the subgame goes as Section 3.2, where each upstream firm gets zero continuation payoff. Therefore, the payoff matrix for the upstream firms at the investment stage is given by

Table 6: Upstream Payoffs under Separation

U_A \ U_B	S	F
S	0,0	π_{VS},0
F	0,π_{VS}	0,0

Under Assumption 1, there is a unique equilibrium (e_{VS}, e_{VS}) in the investment game which is given by the solution to the following equation,

$$C_U'(e_{VS}) = (1 - e_{VS})\pi_{VS}$$

4.3 Upstream Investment–Vertical Integration

Suppose now U_A and D_1 integrate. When the independent upstream firm U_B is the only one who succeeds in upstream innovation, the subgame goes as Section 4.1.1, where the profit for

U_B is π_{VS}. However, in this circumstance, even though U_A does not have a successful upstream innovation, she can still get positive profit from her downstream affiliate when D_1 is the sole downstream innovator. We denote this profit as π_{VI}^F, which is

$$\pi_{VI}^F = d_{VI}^2(1-d_{VI}^1)\frac{\Delta}{2} - \frac{1}{2}c_d(d_{VI}^2)^2$$

When the integrated upstream firm U_A is the sole upstream innovator, the subgame goes as Section 4.1.2. The profit for the upstream monopolist U_A is given by π_{VI}. As the previous lemma shows, we have $\pi_{VI} > \pi_{VS}$.

When both upstream firms make successful investment, the subgame goes as Section 3.2. However, the profit for the two upstream firms are different: the independent upstream firm U_B gets zero profit due to competition from U_A; while the integrated firm can still get positive profit by supplying her downstream affiliates. We denote this profit as π_{VI}^D, which is given by

$$\pi_{VI}^D = d^2\delta + d(1-d)\Delta - \frac{1}{2}c_d d^2$$

where d is the downstream investment when only downstream innovation is needed.

Then the payoff matrix for upstream firms in the investment stage is given by

Table 7: Upstream Payoffs under Integration

$U_A - D_1$ \ U_B	S	F
S	π_{VI}^D,0	π_{VI},0
F	π_{VI}^F,π_{VS}	0,0

Then the unique equilibrium in the upstream investment stage is given by

$$\begin{cases} C_U'(e_{VI}^1) & = \pi_{VI} - (\pi_{VI} + \pi_{VI}^F - \pi_{VI}^D)e_{VI}^2 \\ C_U'(e_{VI}^2) & = \pi_{VS} - \pi_{VS}e_{VI}^1 \end{cases} \tag{3}$$

then the following proposition can be easily shown

Proposition 5. *The integrated upstream firm invests more than the independent upstream firm, indeed we have $e_{VI}^1 > e_{VS} > e_{VI}^2$.*

Proof. See Appendix. □

The effect of vertical integration is three-fold: first, there is better coordination inside the integrated entity $U_A - D_1$. This is the case of our benchmark in Section 3.1. When both upstream firms have successful innovation, they only compete for the independent downstream firm. This increases the investment incentive for the integrated upstream firm and decreases the incentive of the independent upstream firm.

Second, the elimination of hold-up problem between U_A and D_1 further increases the investment incentive of the integrated upstream firm, which results from the fact that $\pi_{VI} > \pi_{VS}$. This effect is only present when downstream innovation is also needed for the product: an integrated upstream monopolist has larger continuation payoff than an independent upstream monopolist. In this sense, downstream innovation amplifies the crowd-out effect of vertical integration on the independent upstream firm. In other words, at the presence of downstream innovation, the upstream firm benefits even more from vertical integration.

Third, the combination of upstream innovation and downstream innovation gives rise to an additional effect. This originates from the fact that the integrated upstream firm obtains positive profit even when he fails in upstream investment, since the downstream affiliate D_1 still gets a profit of $\Delta/2$ when he is the only successful downstream innovator. This tends to reduce the investment incentive of the integrated upstream firm. The above proposition shows that this effect is dominated, and the overall effect of vertical integration is a result of crowd-out of the investment the independent upstream firm.

4.4 Incentives to Integrate

It is clear from the above analysis that, the integration of U_A and D_1 benefits $U_A - D_1$ while hurts both the independent upstream firm U_B and the independent downstream firm D_2. Hence U_B may also have incentive to integrate with the independent downstream firm D_2. There are indeed joint gains from such integration: first, when U_B is the only upstream innovator, they jointly get higher profit since the investment of D_1 will be crowded out by D_2 now. Second, when both upstream firms succeed or only U_A succeeds, the joint profit of U_B and D_2 does not change. Therefore, U_B and D_2 indeed have incentives to integrate. Specifically, the joint profit of U_B and D_2 under separation is

$$\begin{aligned}\pi^{VS}_{U_B-D_2}(e_A,e_B) = \;& e_A e_B[d^2\delta + d(1-d)\Delta - \tfrac{1}{2}c_d d^2] \\ &+e_B(1-e_A)[d_{VS}\Delta + d_{VS}(1-d_{VS})\tfrac{\Delta}{2} - \tfrac{1}{2}c_d d_{VS}^2] \\ &+e_A(1-e_B)[d^2_{VI}(1-d^1_{VI})\tfrac{\Delta}{2} - \tfrac{1}{2}c_d(d^2_{VI})^2] - \tfrac{1}{2}c_u e_B^2\end{aligned}$$

and the joint profit under full integration is

$$\begin{aligned}\pi^{FI}_{U_B-D_2}(e_A,e_B) = \;& e_A e_B[d^2\delta + d(1-d)\Delta - \tfrac{1}{2}c_d d^2] \\ &+e_B(1-e_A)[d^1_{VI}\Delta + d^2_{VI}(1-d^1_{VI})\tfrac{\Delta}{2} - \tfrac{1}{2}c_d (d^1_{VI})^2] \\ &+e_A(1-e_B)[d^2_{VI}(1-d^1_{VI})\tfrac{\Delta}{2} - \tfrac{1}{2}c_d(d^2_{VI})^2] - \tfrac{1}{2}c_u e_B^2\end{aligned}$$

It is easy to check that $\pi^{VS}_{U_B-D_2}(e_A,e_B) < \pi^{FI}_{U_B-D_2}(e_A,e_B)$ for any given e_A and e_B. Furthermore, we have $\pi^{VS}_{U_B-D_2}(e^1_{VI},e^2_{VI}) < \pi^{FI}_{U_B-D_2}(e^1_{VI},e^2_{VI}) < \pi^{FI}_{U_B-D_2}(e_{FI},e_{FI})$. The first inequality reflects the standalone gain from the resolution of hold-up problem; and the second inequality shows the gain from counter crowd-out.

As a result, the counter-merger of $U_B - D_2$ reduces the joint profit of $U_A - D_1$. However, the joint profit of U_A and D_1 is still higher than that under vertical separation. To see this, the joint profit under vertical separation is

$$\begin{aligned}\pi^{VS}_{U-D} = \;& e^2_{VS}[d^2\delta + d(1-d)\Delta - \tfrac{1}{2}c_d d^2] \\ &+e_{VS}(1-e_{VS})\underbrace{[d_{VS}\Delta + d_{VS}(1-d_{VS})\Delta - \frac{1}{2}c_d d^2_{VS} - \frac{1}{2}c_d d^2_{VS}]}_{\pi^{ind}_{VS}} \\ &-\tfrac{1}{2}c_u e^2_{VS}\end{aligned}$$

and the joint profit under full integration is

$$
\begin{aligned}
\pi_{U-D}^{FI} = \;& e_{FI}^2[d^2\delta + d(1-d)\Delta - \tfrac{1}{2}c_d d^2] \\
& + e_{FI}(1-e_{FI})\underbrace{[d_{VI}^1\Delta + d_{VI}^2(1-d_{VI}^1)\Delta - \frac{1}{2}c_d(d_{VI}^1)^2 - \frac{1}{2}c_d(d_{VI}^2)^2]}_{\pi_{FI}^{ind}} \\
& -\tfrac{1}{2}c_u e_{FI}^2
\end{aligned}
$$

It is easy to check that $\pi_{VS}^{ind} < \pi_{FI}^{ind}$. When upstream investment is relatively small, the joint profit is mainly determined by the first order benefit $e_{VS}\pi_{VS}^{ind}$ and $e_{FI}\pi_{FI}^{ind}$. Hence, the joint profit is higher under full integration than vertical separation. We provide a proof and an example in Appendix B.

Therefore, when both upstream and downstream innovations are needed for the final product, we are likely to observe merger waves. This is in contrast to the situations when only one side innovation is needed, where full integration either has no effect (only downstream innovation) or hurts both firms (only upstream innovation). In other words, with only upstream innovation, the benefit from vertical integration totally comes from the crowd-out effect, which is detrimental if there is connter-integration; with both upstream and downstream innovations, the benefit of vertical integration comes from both improved efficiency and crowd-out effect, which is still beneficial even with counter-integration. Full integration may still lead to over-investment with respect to industry profit maximization, however such cost is smaller than that resulting from under-investment due to hold-up problem.

4.5 Industry Overview

The above analysis indicates that in an innovation-driven industry, we are likely to see more integration when both upstream and downstream innovations matter for the final product. In other words, in an industry where investment cost is more skewed to one side, either upstream or downstream, vertical integration may not be a common practice.

We show this in an example. We assume that upstream and downstream investment cost functions are quadratic, with $c_u + c_d = 2$. Downstream product market is characterized by the Hotelling line as before, with $v = 1$ and $t = 0.1$. The following figure shows the relative benefit of full integration with respect to separation, as the cost structure varies.

As we can see, as $c_d \to 0$, we are in the situation where only upstream innovation is needed, where full integration is strictly dominated. As $c_u \to 0$ ($c_d \to 2$), we approach the situation with only downstream innovation, and firms are indifferent between integration and separation.[7] Integration is beneficial and the equilibrium market structure features full integration only when both innovations are important, especially when upstream innovation is important.

[7] As can be seen, when c_u approaches zero but is still positive, full integration is strictly dominated by separation. This is because when upstream investment cost is small, there is significant upstream over-investment, and integration exacerbates such problem.

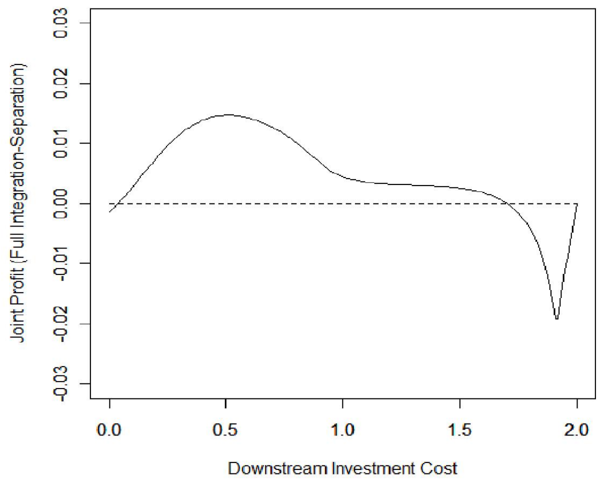

Figure 3: Relative Benefit of Full Integration

5 Welfare Implications

Now we turn to the welfare effect of vertical integration. To do this, we need to specify the downstream final product market. We proceed with the simple Hotelling-line model. The two downstream firms are located at the end point of a Hotelling line with length 1. A representative consumer is randomly located on the line according to the uniform distribution, and the consumer has valuation v for the product. In addition, the consumer incurs a transportation cost which is t per unit distance. We assume that v is large enough compared to t, i.e. $v > 2t$, hence the market is fully covered no matter there is one or two downstream firms. Basic result of the simple Hotelling model shows that, when there is only one downstream firm, he charges price $v - t$ and the profit $\Delta = v - t$; when there are two downstream firms, they charge the same price $p = t$ and get profit $\delta = \frac{t}{2}$.

5.1 One Side Investment

When only downstream innovation is needed, vertical integration has no effect on social welfare. This is simply because upstream competition leads to zero input price, which does not alter the nature of downstream competition.

When only upstream innovation is needed, the welfare effect of vertical integration comes from two aspects. On the cost side, the total cost of investment increases; On the benefit side, first, the probability that there will be at least one upstream innovation is higher under vertical integration; second, there is social gain from product differentiation.

The welfare function is actually given by

$$W = (e_1 + e_2 - e_1 e_2)(v - \frac{t}{2}) + e_1 e_2 \frac{t}{4} - \frac{1}{2} c_u e_1^2 - \frac{1}{2} c_u e_2^2$$

Thus, if investment cost is high, the welfare is largely determined by the first order gain $(e_1 + e_2)(v - \frac{t}{2})$, which is higher under vertical integration. This is due to the fact that the

Best response curve has slope less than 1, and thus total investment is higher under vertical integration. However, if investment cost is low, whether vertical integration increases social welfare critically depends on the extent of product differentiation.

Proposition 6. *Consider the situation where only upstream innovation is needed, for large investment cost, social welfare is always higher under vertical integration; for small investment cost, if product differentiation is weak (t is small), social welfare is lower under (partial) vertical integration than under vertical separation; if product differentiation is strong (t is big), (partial) vertical integration improves social welfare.*

Proof. See Appendix. □

Under vertical separation, the upstream firm is not able to capture the benefit of product differentiation; under vertical integration, the integrated firm is able to catch some of this benefit, which boosts his investment incentive. However, the private gain may exceed the social gain and that may lead to over-investment and lower social welfare. From the social planner's perspective, when there is only one downstream firm, the social surplus from the final product market is $v - \frac{t}{2}$. When there are two downstream firms, this social surplus becomes $v - \frac{t}{4}$. And thus, given that one firm has already made successful investment, the social gain of a second successful firm is $\frac{t}{4}$; while the private gain of the second firm is $\frac{t}{2}$, which is larger than the social gain. Therefore, when the product differentiation in the downstream market is weak, the welfare gain from more upstream investment is weak; and the negative effect of over-investment dominates. When there is stronger product differentiation in the downstream market, the positive effect of higher upstream investment dominates and vertical integration increases social welfare. A numerical example of the welfare effect is shown below in Figure 4(where we set $v = 1$).

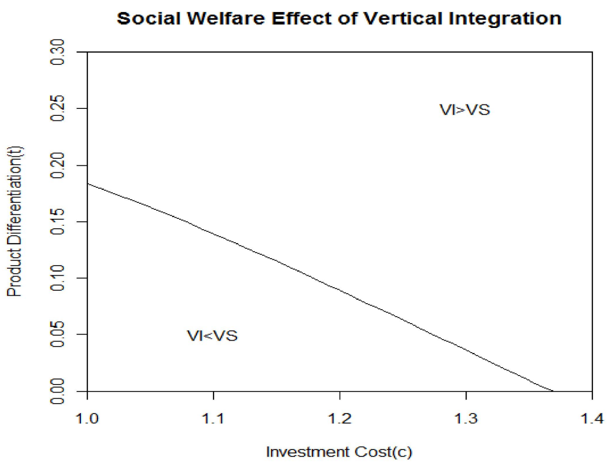

Figure 4: Welfare Effect of Vertical Integration

Remark In a slightly different model, we have an even stronger result. Suppose the locations of downstream firms are not fixed at the end points of the Hotelling line before innovative investment. Instead, the active downstream firms can choose their location after the investment and bargaining stage. This fits the situation where downstream innovations themselves are not differentiated, it is downstream marketing strategies that differentiate the downstream firms. In this modified model, if only one downstream firm is active, he would choose to locate at the mid-point of the Hotelling line, and get profit $\Delta = v - \frac{t}{2}$; if both downstream firms are active in the market, they would differentiate themselves by locating at the end points of the Hotelling line and each gets profit $\delta = \frac{t}{2}$. However, in this modified Hotelling model, the social surplus in the downstream market when there is one downstream firm active is $v - \frac{t}{4}$; and it remains to be the same $v - \frac{t}{4}$ when there are two downstream firms active.

Therefore, in this case, if the social planner can only control the upstream investment but not downstream firms' marketing strategies, there is no social gain from a second upstream innovation if one upstream firm has already made successful investment. However, the private gain for a second upstream innovation is $\frac{t}{2}$, which generates excessive upstream investment incentives. Notice that in this different model of downstream market, there is still gain from vertical integration: as an upstream monopolist is not able to catch all consumer surplus from the downstream market, the upstream investment incentive is insufficient under vertical separation. Vertical integration can improve the investment incentives. However, the range of parameters that vertical integration increases social welfare shrinks in this modified model, as the excessive investment incentive is stronger.

5.2 Both Upstream and Downstream Investments

Now consider the situation when both upstream and downstream innovations are needed. Denote W_D as the social welfare in the continuation game when both upstream firms succeed in investment; similarly, denote W_{VS}^M and W_{VI}^M as the social welfare in the continuation game when there is only one successful upstream firm, who is vertically separated or vertically integrated respectively. And thus the total social welfare under vertical separation is given by

$$W_{VS} = e_{VS}^2 W_D + 2e_{VS}(1 - e_{VS})W_{VS}^M - 2C_U(e_{VS})$$

and the social welfare under vertical integration is given by

$$W_{VI} = e_{VI}^1 e_{VI}^2 W_D + e_{VI}^1(1 - e_{VI}^2)W_{VI}^M + e_{VI}^2(1 - e_{VI}^1)W_{VS}^M - C_U(e_{VI}^1) - C_U(e_{VI}^2)$$

As before, when the investment levels are relatively low, the welfare is approximately

$$W_{VS} \approx 2e_{VS}W_{VS}^M \text{ and } W_{VI} \approx e_{VI}^1 W_{VI}^M + e_{VI}^2 W_{VS}^M$$

It is clear that we have $e_{VI}^1 + e_{VI}^2 > 2e_{VS}$. Moreover, we have $W_{VS}^M < W_{VI}^M$[8], and thus the social welfare under vertical integration is greater than that under separation. The main source of the positive welfare effect of vertical integration is the resolution of the hold-up problem between the integrated upstream and downstream firms. This increases social welfare in two ways: Firstly, the social welfare is higher when the upstream monopolist is vertically integrated than when it is vertically separated, which is $W_{VS}^M < W_{VI}^M$. Thus, the upstream crowd-out effect means that it is more likely to be the integrated upstream firm that is going to be the sole innovator in

[8] A proof of this point is in Appendix B.

the upstream market. Secondly, the total probability that there will be at least one successful product in the market is larger, which is $e_{VI}^1 + e_{VI}^2 > 2e_{VS}$. This is because the total amount of investment in both upstream and downstream level increases.[9] In a word, when both upstream and downstream innovations are important for the final product, the investment incentives are generally insufficient due to the hold-up problem under vertical separation. Vertical integration partially overcomes this problem and pushes the investment levels towards social optimum.

Moreover, when both upstream and downstream innovations are needed, full integration further increases social welfare. The social welfare under full integration is

$$W_{FI} = e_{FI}^2 W_D + 2e_{FI}(1 - e_{FI})W_{VI}^M - 2C_U(e_{FI})$$

Again, this welfare can be approximated by $W_{FI} \approx 2e_{FI}W_{VI}^M$, which is higher than W_{VI} as a result of $2e_{FI} > e_{VI}^1 + e_{VI}^2$. The further welfare improvement from full integration comes from the fact that hold-up problem is now totally resolved inside each integrated entity. A numerical example of the welfare effect is shown below in Figure 5, where we set $v = c_u = c_d = 1$.

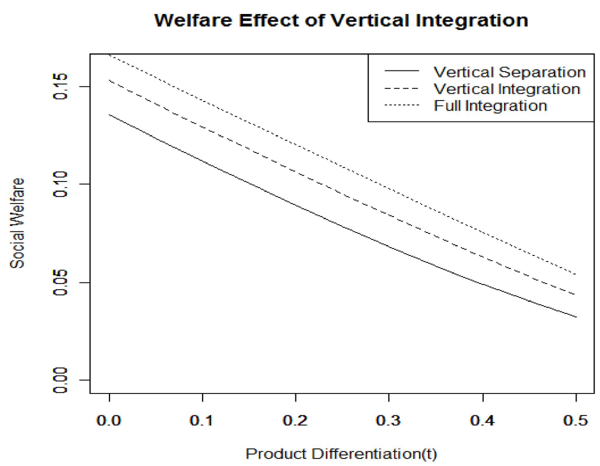

Figure 5: Welfare Effect of Vertical Integration With Both Upstream and Downstream Innovation

6 Discussion and Extension

6.1 Robustness

For the robustness of our results, we discuss two main issues. First, concerning the timing and observability of investment outcomes, we assume that upstream investments occur before downstream investments and bargaining happens after all investment outcomes realize. The key assumption is that bargaining happens ex post, while the observability and the timing of investments do not affect our main results. This is clearly the case when only one-side innovation is needed for the final product. When both upstream and downstream innovations

[9]This is because the best response curves have slope less than 1, and thus the reductions in investments from independent firms do not fully offset the increases from the integrated firms.

are needed, the driving force for our results is the resolution of hold-up problem inside the integrated entity. This benefit from vertical integration remains no matter whether investment outcomes are observable when the firms make their investment decisions. However, by assuming that downstream firms invest after observing the outcomes of upstream investments, we remove some potential social welfare loss from wasteful downstream investments when no upstream firm succeeds. If downstream firms have to make investments before observing upstream investment outcomes, vertical integration has the further advantage from better coordinating investment decisions inside the integrated entity.

The second simplification we made in our model is that downstream firms have unit demand, and all offers in the bargaining stage are publicly observable. Secret offers may reduce the profit of an upstream monopolist, as we encounter the classic commitment problem. And thus there is additional incentive to integrate vertically if offers are not observable. When downstream firms have elastic demand (as the case where downstream market is characterized by Cournot competition or differentiated Bertrand competition), vertical integration has additional foreclosure effect on independent firms which has been extensively analyzed in the literatures. Our results are robust to such more general settings, since resolution of coordination problems inside the integrated entity remains true in these settings. However, relying on unit demand and observable offers, we can focus on the effect of vertical integration through the channel of investment.

6.2 Exclusive Dealing

In the above analysis, we assume the offers that a firm can make are simply price quotes. Now we briefly discuss the situation when exclusive dealing offers are allowed. Firms can make offers with exclusive dealing clause, i.e. an offer can be either a simple price quote p, or (p, E) which requires exclusive relationship.[10] Exclusive dealing has no effect on the payoffs when there is bilateral monopoly or when there is downstream monopoly. When there is upstream monopoly, exclusive dealing has the further advantage that the upstream monopolist is able to catch the whole industry profit even if it is the downstream firms that make offers. In this case, the profit for the upstream monopolist is 2δ instead of Δ without exclusive dealing. Competition with exclusive dealing drives up the input price to Δ and allows the upstream monopolist to extract the whole industry profit.[11]

The main difference is the situation where there is bilateral duopoly. When all firms are separated, exclusive dealing does not change the payoffs if upstream firms make offers. In this case, upstream competition leads to zero input price. If downstream firms make offers, input price will again be zero without exclusive dealing. Since any non-negative offers will be accepted, there is no reason for downstream firms to make positive offers. However, both downstream firms asking for zero price is no longer an equilibrium if exclusive dealing offers are allowed. Given that D_2 asks for zero price to both upstream firms, D_1 can profitably deviate by

[10] The exclusive requirement is not mutual exclusive, i.e. the firm that makes offer can make two exclusive dealing offers, however the firm that makes acceptance decisions can only accept one exclusive dealing offer.

[11] In this case, each downstream firm offers (Δ, E) is the unique equilibrium.

making an exclusive dealing offer ($\frac{\epsilon}{2} > 0, E$) to each upstream firm. Then both upstream firms are willing to take the exclusive dealing offer, and D_2 is excluded from the market. D_1 gets profit $\Delta - \epsilon$, which is higher than δ if he also made zero price offers. We show in Appendix B that in this case, there is no symmetric equilibrium as defined in the proof of Proposition 4, and downstream firms end up with zero profit in the asymmetric equilibrium. Hence, the possibility of exclusive dealing offers intensifies downstream competition and benefits the upstream firms.

When U_A and D_1 integrate, the integrated entity $U_A - D_1$ can use exclusive dealing offers to exclude the independent upstream firm or the independent downstream firm. As emphasized by Chen and Riordan (2007), the combination of vertical integration and exclusive dealing can lead to ex post cartelization. For instance, when it is downstream firms that make offers to upstream firms, the highest price that the independent downstream firm D_2 can offer U_B is δ. And the integrated entity $U_A - D_1$ can make an exclusive offer $(\delta + \epsilon, E)$ to U_B, which will be accepted by U_B and then D_2 is excluded from the market. $U_A - D_1$ gets profit $\Delta - \delta - \epsilon$, which is higher than when exclusive dealing is not allowed (δ). Therefore, when exclusive dealing is allowed, upstream crowd-out effect still exists, and vertical integration also affects downstream investments.

6.3 Information Disclosure by Upstream Firms

In our model, we assume that a final product necessitates both upstream and downstream innovation; but upstream innovation and downstream innovation are independent in the sense that the downstream innovation does not require any information or actual delivery of upstream innovation. All that downstream firms need to know is whether there is successful upstream innovation or not. In this subsection, we relax this assumption and assume that downstream firms need information about the upstream innovation in order to make successful investment. We focus on quadratic cost function.

We modify the game as follows,

- *Upstream Investment Stage*: Each U_i makes investment decision e_i; if both firms fail, the game ends;
- *Information Disclosure Stage*: The successful upstream firm decides whether to disclose the information about the innovation to both downstream firms or only one of them;
- *Downstream Investment Stage*: Each D_j makes investment decision d_j if he receives information from the upstream firm; if both firms fail, the game ends;
- *Bargaining Stage*: The successful upstream firm(s) and successful downstream firm(s) bargain over the price of the input;
- *Payoff Stage*: Payments are made and inputs are delivered if any agreement is reached; downstream market realizes and game ends.

In the game above, downstream investment needs information about the upstream innovation but not the actual delivery of the input. In other words, if both upstream firms have successful innovation, the downstream firm is free to choose any upstream supplier no matter from where he gets the necessary information for investment. We assume that when the upstream firm is indifferent between disclose information and not disclose any information, he chooses to disclose. Then the subgame is the same as in Section 3 if both upstream firms make successful investment under either vertical separation or vertical integration. Because even though the integrated upstream firm may refrain from disclosing information to the independent downstream firm, the upstream competitor would disclose such information.

When it turns out to be the case that there is a monopolistic upstream innovator, the incentive to disclose may differ depending on whether the upstream monopolist is vertically integrated or not. Under vertical separation, the profit for the upstream monopolist is π_{VS} if he discloses the information to both downstream firms; when he only discloses to one downstream firm, it is easy to know that downstream investment is $d = \frac{\Delta}{2c_d}$, and the profit for the upstream monopolist is $\pi^1_{VS} = \frac{\Delta^2}{4c_d}$.

When the upstream monopolist is vertically integrated with one of the two downstream firms, the profit is π_{VI} if he also discloses the information to the independent downstream firm. When he refrains from disclosing the information, downstream investment is given by $d = \frac{\Delta}{c_d}$ and the profit for the integrated upstream monopolist is $\pi^1_{VI} = \frac{\Delta}{2c_d}$. Suppose there is a cost K related to disclosing information to a second downstream firm. Such cost may be related to the risk of information leakage, where it is only a private cost but not social cost; or the cost may be about how to convey the information correctly to the downstream firms, then it is also a social cost. To show our main insight, we assume such cost is only a private cost. The next proposition shows that an integrated upstream firm has less incentive to disclose information to both downstream firms.

Proposition 7. *There exists a range of value $K \in (\underline{K}, \bar{K})$ such that the separated upstream monopolist discloses information to both downstream firms, while the integrated upstream monopolist does not disclose to the independent downstream firm.*

Proof. See Appendix. □

Under vertical separation, the upstream monopolist has more incentive to disclose information to a second downstream firm. First, due to our assumptions on the cost function, downstream competition does not lower the total level of downstream investment; second, when both downstream firms obtain successful innovation, the payoff for the upstream monopolist is now Δ rather than $\frac{\Delta}{2}$. However, under vertical integration, by disclosing information to the independent downstream firm, the integrated upstream firm has to balance the benefit of lowering investment cost (since now she makes less investment) and the loss of profit when the independent downstream firm is the only downstream innovator (since now the integrated firm can only get $\frac{\Delta}{2}$ rather than Δ).

With respect to welfare, if $K \in (\underline{K}, \bar{K})$, the social welfare is the same as in the previous section under vertical separation. However, the result is different under vertical integration when the integrated upstream firm is the only upstream innovator. Since in this case, the integrated upstream firm would choose to only disclose information to her downstream affiliate.

It can be easily shown that if downstream investment cost is relatively high, the welfare under vertical integration is lower than that under vertical separation if there is only one upstream innovator.

Proposition 8. *When there is only one upstream innovator, if* $K \in (\underline{K}, \bar{K})$*, there exists a* $\bar{c_d} > \Delta$ *such that social welfare is higher under vertical separation than vertical integration if downstream investment cost is large enough, i.e. if* $c_d > \bar{c_d}$*.*

Proof. See Appendix. □

Therefore, when downstream investment cost is low, the benefit from the elimination of hold-up problem inside the integrated entity outweighs the cost from excluding the independent downstream firm; however, when downstream cost is high, vertical separation delivers higher social welfare as now promoting investment incentive becomes the main issue. Taken into consideration upstream innovation as well, the result still holds. Social welfare is higher under vertical integration if downstream investment cost is low, and higher under vertical separation otherwise.

6.4 Interim Bargaining

In the discussion above, we assume that bargaining between upstream firms and downstream firms happens after all outcomes of investments have already realized and been observed by all firms. The main insights in our paper still hold if bargain happens in an interim stage, i.e. upstream firms bargain with downstream firms after the outcomes of upstream investments realized but before downstream firms make any investment. This would be the case if downstream innovation needs the actual delivery of the input. In this situation, hold-up problem does not exist for downstream firms; however vertical integration still affects the investment incentives of upstream firms, i.e. the upstream crowd-out effect still exists.

When both upstream firms obtain successful innovation, the integrated upstream firm gets positive profit while the independent upstream firm gets zero profit. When only one of the two upstream firms succeeds, the integrated upstream monopolist is able to catch a larger part of downstream market profit even though now the bargaining does not affect downstream investment incentives. This is because the integrated upstream monopolist holds a stronger bargaining position vis-a-vis the downstream firms, since the outside option for the integrated upstream firm is higher. Therefore, an integrated upstream monopolist can recoup a larger profit from his innovation.

6.5 Contracting for Innovation–Outsourcing

We have assumed through the analysis that the investments are not contractible, and bargaining takes place after the outcomes of all investments are observed. The question we addressed in this section is whether upstream and downstream firms can do better by signing a contract before investment, rather than integrate vertically. We consider a simple set of contract, where the downstream firm pays the upstream firm in exchange for exclusivity of the innovation once the upstream firm succeeds in investment, and the payment is conditional on whether the upstream firm is the only one that succeeds. When only downstream innovation is needed, such contracts are redundant and vertical integration has no effect on downstream investment.

When both upstream and downstream innovations are needed, integration is a dominant strategy for each pair of upstream and downstream firms, and integration benefits all firms. Then the question is whether such contracts can do better than integration. When both upstream firms are successful, the maximum that a downstream firm is willing to pay is π_{VI}^D, which is the same that an integrated upstream firm would get. When only one upstream firm is successful, the profit for the integrated downstream firm is

$$\pi_{VI} = (\frac{\Delta}{4c_d^2 - \Delta^2})^2(12c_d^3 + \Delta^3 - 8c_d^2\Delta - \frac{1}{2}c_d\Delta^2)$$

if the downstream firm has an exclusive contract with the successful upstream firm, it is easy to check that the profit for the downstream firm is

$$\pi_E = \frac{\Delta^2}{2c_d}$$

simple calculation shows that $\pi_E < \pi_{VI}$. Therefore, the profit for the downstream firm is higher under integration than under exclusive contract, which means that the exclusive contract cannot provide stronger upstream investment incentive than integration. And hence firms cannot benefit from such contracts.

When only upstream innovation is needed, integration is a dominant strategy for each pair of firms; however, as shown before, integration in this case hurts firms. Consider a repeated game, where in each period firms can choose whether to integrate or not; then if firms are patient enough, we can have a collusive-like equilibrium where no firm integrates, and all firms integrate once a pair of upstream and downstream firms integrate. The question then is whether firms can do better than vertical separation by signing exclusive contracts in this repeated game.

Consider D_1 makes such an offer to U_A: he pays U_A an amount of p_1 if only U_A succeeds; and he pays p_2 if both upstream firms succeed; in return, U_A only sells to D_1 whenever he succeeds. We consider a symmetric equilibrium in the sense that D_2 makes similar offer to U_B. Then the optimal offer that downstream firms make is the solution to

$$\begin{aligned} max_{p_1,p_2} \quad & e_1(1-e_2)(\Delta - p_1) + e_1e_2(\delta - p_2) \\ s.t. \quad & e_1 = max_{e_1} e_1(1-e_2)p_1 + e_1e_2p_2 - \tfrac{1}{2}c_ue_1^2 \\ & e_2 = max_{e_2} e_2(1-e_1)p_1 + e_1e_2p_2 - \tfrac{1}{2}c_ue_2^2 \\ & e_1(1-e_2)p_1 + e_1e_2p_2 - \tfrac{1}{2}c_ue_1^2 \geq \tfrac{1}{2}c_u(\tfrac{\Delta}{c_u+\Delta})^2 \\ & e_1(1-e_2)(\Delta - p_1) + e_1e_2(\delta - p_2) \geq \delta(\tfrac{\Delta}{c_u+\Delta})^2 \end{aligned}$$

The first two constraints are the Individual Rationality constraints which say that upstream firms choose their investment optimally; the last two constraints are the Incentive Compatibility constraints which imply that the firms should be better than when they are separated. For simplicity, consider the case where we have $c_u = \Delta = 1$. Then the two IR constraints mean that

$$e_1 = e_2 = \frac{p_1}{1 + p_1 - p_2}$$

the upstream IC constrain means that

$$p_1 + p_2 \geq 1$$

thus the best offer that downstream firms can offer is $p_1 + p_2 = 1$; however, substitute this into the downstream IC constraint, it shows that the IC constraint holds with equality. Therefore, with this optimal contract, downstream firms are indifferent between making such offer and remaining separated. Nonetheless, firms can do better if we allow the offers to include a fixed transfer f that is independent of the upstream investment outcomes. In this case, the previous problem becomes

$$\begin{aligned} max_{p_1,p_2} \quad & e_1(1-e_2)(\Delta - p_1) + e_1 e_2(\delta - p_2) - f \\ s.t. \quad & e_1 = e_2 = \tfrac{p_1}{1+p_1-p_2} \\ & e_1(1-e_2)p_1 + e_1 e_2 p_2 - \tfrac{1}{2} c_u e_1^2 + f \geq \tfrac{1}{8} \\ & e_1(1-e_2)(\Delta - p_1) + e_1 e_2(\delta - p_2) - f \geq \tfrac{\delta}{4} \end{aligned}$$

It is easy to check that the profit maximizing investment $e_1 = e_2 = \frac{1}{3-2\delta}$ can be achieved by setting $f = \frac{1}{8} - \frac{1}{2(3-2\delta)^2}$ and $p_2 + 2(1-\delta)p_1 = 1$.

7 Conclusion

In this paper, we studied the effect of vertical integration on the investment incentives of upstream and downstream firms. When only upstream innovation is needed for the industry, vertical integration may lead to overall over-investment and decreases social welfare. The crowd-out effect of vertical integration is strengthened when downstream innovation is also needed. However, vertical integration also promotes both upstream and downstream investment. The overall impact of vertical integration on social welfare turns out to be positive. Our results suggest that when evaluating the impact of vertical integration, especially in industries with intensive innovation, the exact nature between upstream and downstream investment may be a key point in the decision. Studying the impact of vertical integration in a more general bargaining environment, or in the presence of other forms of complementarity between upstream and downstream innovation might be interesting avenues for future research. In addition, there has been a number of empirical papers studying the effect of horizontal mergers on innovation, it would also be valuable to explore empirically the theory we developed in this paper.

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8 Appendix

8.1 Appendix A

8.1.1 Proof of Proposition 1

Rewrite Equation (1) as

$$\begin{cases} C_U'(e_U^1) & = \Delta(1-e_U^2) + \alpha e_U^2 \\ C_U'(e_U^2) & = \Delta(1-e_U^1) \end{cases}$$

When $\alpha = 0$, the solution corresponds to the investment level under vertical separation; when $\alpha = \delta$, it is the solution under vertical integration. It is clear to see that

$$\frac{\partial e_U^1}{\partial \alpha} = \frac{e_U^2}{C_U''(e_U^1)}$$

which is always positive under Assumption 1. Therefore, we must have $e_U^1 > e_U$. Furthermore, we have $C_U'(e_U^2) = \Delta(1-e_U^1) < \Delta(1-e_U) = C_U'(e_U)$, and thus $e_U^2 < e_U$.

8.1.2 Proof of Proposition 3

To see the effect of vertical integration, we need to determine the payoff-matrix for the firms that make investments.

In both cases (when only upstream innovation is needed or only downstream innovation is needed), if both firms fail in investment, the payoffs for all firms are zero. Similarly, if only one firm succeeds in investment, the firm is able to extract the industry profit Δ.

The situation is different if both firms are successful, where we have bilateral duopoly. For simplicity, we assume that when $U_A(D_1)$ is indifferent between the offers of $D_1(U_A)$ and $D_2(U_B)$, $U_A(D_1)$ chooses the offer of $D_1(U_A)$. Similar assumption applies for U_B and D_2. Moreover, we assume that when multiple equilibria exist, the two firms that make offers coordinate on the most profitable one.

The case when it is the downstream firms who make offers is simple. Each downstream firm asks $p = 0$ to both upstream firms is an equilibrium, and each downstream firm gets profit $\delta_D + \delta_U$. There is no profitable deviation for either downstream firm.

Now we turn to the situation when it is the upstream firms who make offers. Denote O_{ij} as the offer made by upstream firm $U_i, i = A, B$ to downstream firm $D_j, j = 1, 2$. We focus on a particular type of equilibrium, which we call symmetric equilibrium. A symmetric equilibrium is such that $O_{A1} = O_{B2}$, and $O_{A2} = O_{B1}$.[12]

Case 1: $\delta_U \leq \delta_D$

This is the case when downstream is more differentiated than upstream. Suppose (p_1, p_2) is offered by U_A in equilibrium, then by symmetry U_B offers (p_2, p_1).

Claim 1: In equilibrium, the two downstream firms choose different suppliers, and $p_1 = p_2 + \delta_U$.

To see the first part, w.l.o.g, suppose both downstream firms choose U_A. Then we must have $\delta_D - p_1 \geq \delta_D + \delta_U - p_2$, and $\delta_D - p_2 > \delta_D + \delta_U - p_1$, which are contradictory. And thus the two downstream firms choose different suppliers. Assuming that in equilibrium U_A supplies D_1, and U_B supplies D_2.[13] Then we must have $\delta_D + \delta_U - p_1 \geq \delta_D - p_2$, which gives us $p_1 - p_2 \leq \delta_U$. Profit maximization for U_A means that we must have $p_1 = p_2 + \delta_U$.

According to Claim 1, the offer of U_A can be simplified to $(p+\delta_U, p)$, similarly for U_B. Then each upstream firm gets a profit of $p + \delta_U$, and each downstream firm gets a profit of $\delta_D - p$. We have the following result.

Claim 2: In equilibrium we must have $p \leq \delta_U$.

First, any $p > \delta_D$ cannot be an equilibrium, as the profit of the downstream firms will be negative. Suppose $\delta_D \geq p > \delta_U$, given the offer of U_B as $(p, p + \delta_U)$, by offering $(p - \frac{\epsilon}{2}, p - \frac{\epsilon}{2})$ to both firms, U_A can get a profit of $2p - \epsilon > p + \delta_U$. To see this, accepting this offer from U_A, each downstream firm gets a profit of $\delta_D - (p - \frac{\epsilon}{2}) > \delta_D - p$.

Then we can show that any $0 \leq p \leq \delta_U$ can be an equilibrium. Given that U_B offers $(p, p+\delta_U)$, the above argument implies that U_A cannot be better off by attracting both downstream

[12]As what makes clear later, firms would maximally differentiate themselves in equilibrium. And thus the two downstream firms won't choose the same supplier in equilibrium. Our notion of symmetric equilibrium means that the two downstream firms receive the same offer from their suppliers.

[13]This simplification does not matter up to a relabeling of firms.

firms if $p \leq \delta_U$. Moreover, U_A cannot extract more than $p + \delta_U$ from D_1, since otherwise D_1 would accept the offer of U_B. If U_A tries to attract D_2, he has to offer a price lower than $p+\delta_U$. Hence, there is no profitable deviation for U_A.

By our assumption that the two upstream firms can coordinate on the most profitable equilibrium, there is an equilibrium in which U_A offers $(p_{A1} = 2\delta_U, p_{A2} = \delta_U)$ and U_B offers $(p_{B1} = \delta_U, p_{B2} = 2\delta_U)$. Each upstream firm gets profit $2\delta_U$, and each downstream firm gets profit $\delta_D - \delta_U$.

Case 2: $\delta_U > \delta_D$

As the first case, it cannot be the case that both downstream firms choose the same supplier. In an equilibrium with simple price offers, we can focus on such situation that U_A offers $(p + \delta_U, p)$, where $p \leq \delta_D$. We show that $(\delta_D + \delta_U, \delta_D)$ is an equilibrium and it is the profit maximizing equilibrium for the upstream firms. First, no upstream firm can do better since there is no way to exclude one downstream firm from the market. Moreover, no upstream firm has profitable deviation. Given that the offer of U_B is $(\delta_D, \delta_D + \delta_U)$, decreasing either price cannot make U_A better off. Increasing the offer to D_1 will lose this downstream firm. Increasing the offer to D_2 has no effect.

Now we turn to the effect of vertical integration on investment incentives. When downstream firms are more differentiated than upstream firms (Case 1), if only upstream innovation is needed, the payoff matrix is shown Table 8. It is clear that the effect of vertical integration on upstream investment incentives can be measured by δ_D.

Table 8: Only Upstream Innovation is Needed

(a) Vertical Separation

U_A \ U_B	S	F
S	δ_U,δ_U	Δ,0
F	0,Δ	0,0

$\rightarrow$

(b) Vertical Integration

$U_A - D_1$ \ U_B	S	F
S	$\delta_U + \delta_D$,δ_U	Δ,0
F	0,Δ	0,0

If only downstream innovation is needed, the payoff matrix is given by Table 9. And the effect of vertical integration on downstream investment can be measured by δ_U. Since in Case 1, $\delta_U \leq \delta_D$, the effect of vertical integration has a larger effect on upstream innovation than on downstream innovation. In the special case where $\delta_U = 0$ as in our benchmarks, vertical integration only affects upstream investments but not downstream investments.

Table 9: Only Downstream Innovation is Needed

(a) Vertical Separation

D_1 \ D_2	S	F
S	δ_D,δ_D	Δ,0
F	0,Δ	0,0

$\rightarrow$

(b) Vertical Integration

$D_1 - U_A$ \ D_2	S	F
S	$\delta_U + \delta_D$,δ_D	Δ,0
F	0,Δ	0,0

When upstream firms are more differentiated than downstream firms (Case 2), the payoff matrix for the innovating firms is given by the same Table 10, no matter when only upstream innovation is needed or only downstream innovation is needed. And vertical integration has the same effect on upstream investments and downstream investments.

Table 10: When Upstream is More Differentiated

(a) Vertical Separation

$D_1(U_A)$ \ $D_2(U_B)$	S	F
S	$\frac{\delta_D+\delta_U}{2}, \frac{\delta_D+\delta_U}{2}$	Δ,0
F	0,Δ	0,0

$\rightarrow$

(b) Vertical Integration

$U_A - D_1$ \ $D_2(U_B)$	S	F
S	$\delta_U + \delta_D, \frac{\delta_D+\delta_U}{2}$	Δ,0
F	0,Δ	0,0

8.1.3 Proof of Lemma 3

With quadratic cost function, we have

$$d_{VS} = \frac{\Delta}{2c_d + \Delta}$$

under vertical separation; and under vertical integration we have

$$d_{VI}^1 = \frac{\Delta(4c_d - \Delta)}{4c_d^2 - \Delta^2} \text{ and } d_{VI}^2 = \frac{2\Delta(c_d - \Delta)}{4c_d^2 - \Delta^2}$$

Thus,

$$\pi_{VS} = \frac{\Delta^2}{2c_d + \Delta}$$

and

$$\pi_{VI} = \frac{\Delta^2}{(4c_d^2 - \Delta^2)^2}(12c_d^3 - 8c_d^2\Delta - \frac{1}{2}c_d\Delta^2 + \Delta^3)$$

Therefore, after simplification, we have

$$\pi_{VI} - \pi_{VS} = \frac{\Delta^2}{(4c_d^2 - \Delta^2)^2}(4c_d^3 + \frac{3}{2}c_d\Delta^2 - 4c_d^2\Delta)$$

which is always positive since $c_d \geq \Delta$.

8.1.4 Proof of Proposition 5

The best response function for the independent upstream firm is unchanged after integration. The best response function for the integrated upstream firm is given by

$$C_U'(e_{VI}^1) = \pi_{VI} - (\pi_{VI} + \pi_{VI}^F - \pi_{VI}^D)e_{VI}^2$$

which can be obtained from an outward shift and rotation of the best response function under vertical separation. Then to prove the result, the only thing we need to show is that $\pi_{VI}^F < \pi_{VI}^D$, which guarantees that the rotation does not offset the effect of ourward shift. With quadratic cost function, we have

$$\pi_{VI}^F = \frac{1}{2}c_d(\frac{\Delta}{2c_d + \Delta})^2$$

and

$$\pi_{VI}^D = \frac{1}{2}c_d(\frac{\Delta}{c_d + \Delta - \delta})^2$$

Clearly we have $\pi_{VI}^F < \pi_{VI}^D$, since $2c_d + \Delta > c_d + \Delta - \delta$.

8.1.5 Proof of Proposition 6

As $v = 1$, we focus on the case where $t \in [0, 0.5]$. Under vertical separation, upstream investment is given by

$$e_U = \frac{\Delta}{c+\Delta} = \frac{1-t}{2-t}$$

the corresponding social welfare is given by

$$w_{VS} = e_U^2(1-\frac{t}{4}) + 2e_U(1-e_U)(1-\frac{t}{2}) - e_U^2 = \frac{1}{(4(t-2)^2}(-t^3+6t^2-13t+8)$$

Under vertical integration, upstream investments are

$$e_U^1 = \frac{\Delta(c_u-(\Delta-\delta))}{c_u^2-\Delta(\Delta-\delta)} = \frac{(1-t)\frac{3t}{2}}{1-(1-t)(1-\frac{3t}{2})}$$

and

$$e_U^2 == \frac{\Delta(c_u-\Delta)}{c_u^2-\Delta(\Delta-\delta)} = \frac{(1-t)t}{1-(1-t)(1-\frac{3t}{2})}$$

the corresponding social welfare is then

$$\begin{aligned} w_{VI} &= e_U^1 e_U^2(1-\tfrac{t}{4}) + (e_U^1(1-e_U^2)+e_U^2(1-e_U^1))(1-\tfrac{t}{2}) - \tfrac{1}{2}(e_U^1)^2 - \tfrac{1}{2}(e_U^2)^2 \\ &= \tfrac{1}{(2(3t-5)^2}(-6t^3+27t^2-46t+25) \end{aligned}$$

We have

$$w_{VS} - w_{VI} = \frac{t}{4(3t^2-11t+10)^2}(3t^4-18t^3+34t^2-22t+3)$$

The solution of the above equation tells us that

$$w_{VS} - w_{VI} > 0 \text{ if } 0 < t < \hat{t}$$

and

$$w_{VS} - w_{VI} < 0 \text{ if } \hat{t} < t < 0.5$$

where $\hat{t} = 1 - \frac{\sqrt{6}}{3} \approx 0.18$.

8.1.6 Proof of Proposition 7

It suffices to show that $\pi_{VS} - \pi_{VS}^1 > \pi_{VI} - \pi_{VI}^1$, which is equivalent to show $\pi_{VI}^1 - \pi_{VS}^1 > \pi_{VI} - \pi_{VS}$. With quadratic cost function, we have

$$\pi_{VI}^1 - \pi_{VS}^1 = \frac{\Delta^2}{4c_d}$$

and

$$\pi_{VI} - \pi_{VS} = \frac{\Delta^2}{(4c_d^2-\Delta^2)^2}(4c_d^3 + \frac{3}{2}c_d\Delta^2 - 4c_d^2\Delta)$$

After simplification, we have

$$(\pi_{VI}^1 - \pi_{VS}^1) - (\pi_{VI} - \pi_{VS}) = \frac{\Delta^2}{4c_d(4c_d^2-\Delta^2)^2}(16c_d^3\Delta + \Delta^4 - 14c_d^2\Delta^2)$$

which is always positive since $c_d > \Delta$.

Let $\underline{K} = \pi_{VI} - \pi_{VI}^1$ and $\bar{K} = \pi_{VS} - \pi_{VS}^1$, the for $K \in (\underline{K}, \bar{K})$, we have $\pi_{VS} - \pi_{VS}^1 - K > 0$, while $\pi_{VI} - \pi_{VI}^1 - K < 0$.

8.1.7 Proof of Proposition 8

With quadratic cost function, when only one upstream makes successful innovation, if she is vertically separated, the welfare is

$$W_{VS}^{M} = (\frac{\Delta}{2c_d + \Delta})^2(\Delta + 3c_d)$$

If she is vertically integrated, then only the integrated downstream firm makes investment, which is given by $d_1 = \frac{\Delta}{c_d}$, and thus the social welfare is

$$W_{VI}^{1} = \frac{\Delta^2}{2c_d}$$

Then we have

$$W_{VS}^{M} - W_{VI}^{1} = \frac{\Delta^2}{4c_d(2c_d + \Delta)^2}(2c_d^2 - 2c_d\Delta - \Delta^2)$$

It is clear that $2c_d^2 - 2c_d\Delta - \Delta^2$ is increasing in c_d, and it is negative when $c_d = \Delta$ and positive for c_d big enough. Therefore, there exists a $\bar{c}_d$ such that when $c_d < \bar{c}_d$, $W_{VS}^{M} < W_{VI}^{1}$; and when $c_d > \bar{c}_d$, $W_{VS}^{M} > W_{VI}^{1}$.

8.2 Appendix B

8.2.1 Full Integration When Both Innovations are Needed

We show in this proof that the joint profit of $U_A - D_1$ is higher under full integration than under vertical separation. Denote e_{VS} and e_{FI} as the upstream investments under vertical separation and full integration respectively, and denote d as the downstream investment when both upstream firms are successful. The joint profit of $U_A - D_1$ under vertical separation is

$$\begin{aligned}
\pi_{U_A-D_1}^{VS} = \;& e_{VS}(1-e_{VS})[d_{VS}\Delta + d_{VS}(1-d_{VS})\tfrac{\Delta}{2} - \tfrac{1}{2}c_d d_{VS}^2] \\
& + e_{VS}^2[d^2\delta + d(1-d)\Delta - \tfrac{1}{2}c_d d^2] \\
& + (1-e_{VS})e_{VS}[d_{VS}(1-d_{VS})\tfrac{\Delta}{2} - \tfrac{1}{2}c_d d_{VS}^2] \\
& - \tfrac{1}{2}c_u e_{VS}^2
\end{aligned}$$

Similarly, the joint profit of $U_A - D_1$ under full integration is

$$\begin{aligned}
\pi_{U_A-D_1}^{FI} = \;& e_{FI}(1-e_{FI})[d_{VI}^1\Delta + d_{VI}^2(1-d_{VI}^1)\tfrac{\Delta}{2} - \tfrac{1}{2}c_d(d_{VI}^1)^2] \\
& + e_{FI}^2[d^2\delta + d(1-d)\Delta - \tfrac{1}{2}c_d d^2] \\
& + (1-e_{FI})e_{FI}[d_{VI}^2(1-d_{VI}^1)\tfrac{\Delta}{2} - \tfrac{1}{2}c_d(d_{VI}^2)^2] \\
& - \tfrac{1}{2}c_u e_{FI}^2
\end{aligned}$$

We prove for the case where $c_u = c_d = \Delta$, then we show in a numerical example that the joint profit is higher under full integration also for $c_u > \Delta$ and $c_d > \Delta$. When $c_u = c_d = \Delta$, $d_{VS} = \frac{1}{3}$, $d_{VI}^1 = 1$, $d_{VI}^2 = 0$, $d = \frac{\Delta}{2\Delta - \delta}$, $e_{VS} = \frac{1}{4}$, and $e_{FI} = \frac{1}{3-(\frac{\Delta}{2\Delta-\delta})^2}$. As $\delta \to 0$, we have $e_{FI} \to \frac{4}{11}$. And then $\pi_{U_A-D_1}^{VS} = \frac{23}{384}\Delta$, and $\pi_{U_A-D_1}^{FI} = \frac{8}{121}\Delta$. Then it is easy to see that $\pi_{U_A-D_1}^{FI} > \pi_{U_A-D_1}^{VS}$. Therefore, for δ relatively small, the joint profit for $U_A - D_1$ is higher under full integration than under vertical separation.

As in the analysis of welfare effect, we consider downstream market as a Hotelling line, and assume that $v = 1$. Then we have $\Delta = 1 - t$, and $\delta = \frac{t}{2}$. As we can see from Figure 6, the joint profit of $U_A - D_1$ is higher under full integration (for cost higher than Δ, and for any $t < 0.5$). Thus, anticipating that $U_B - D_2$ would counter-integrate, U_A and D_1 still have incentives to integrate, since the benefit from better coordination dominates.

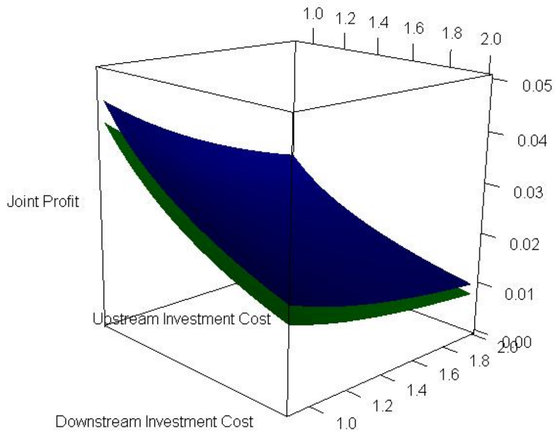

Figure 6: The Joint Profit of $U_A - D_1$ ($t = 0.2$), Blue is full integration, green is vertical separation

8.2.2 Welfare Effect When Both Innovations are Needed

In this subsection, we prove $W_{VS}^M < W_{VI}^M$.

$$W_{VS}^M = (2d_{VS} - d_{VS}^2)(v - \frac{t}{2}) - cd_{VS}^2$$

and

$$W_{VI}^M = (d_{VI}^1 + d_{VI}^2 - d_{VI}^1 d_{VI}^2)(v - \frac{t}{2}) - \frac{1}{2}c_d(d_{VI}^1)^2 - \frac{1}{2}c_d(d_{VI}^2)^2$$

Using the fact that $v - \frac{t}{2} = \Delta + \delta$, we have

$$W_{VI}^M - W_{VS}^M = \frac{1}{2}c_d(\frac{\Delta}{(4c_d^2 - \Delta^2)^2}(\Delta(4c_d^2 - 8c_d\Delta + 5\Delta^2) + 8\delta(2c_d^2 - 2c_d\Delta + \Delta^2))$$

which is always positive.

8.2.3 The Case of Exclusive Dealing

We study the pure strategy equilibrium in the bilateral duopoly case when it is the downstream firms that make offers. Firstly, we show that simple price offers cannot be an equilibrium, and the equilibrium offers must have exclusive dealing clause. Secondly, we show that there is no symmetric equilibrium, where $O_{A1} = O_{B2}$ and $O_{A2} = O_{B1}$. As before, we make the tie-breaking assumption that when U_A is indifferent between the offers of D_1 and D_2, he chooses the offer of D_1; similarly, U_B prefers the offer of D_2 when he is indifferent.

Claim 1: Equilibrium offers much contain exclusive dealing clauses.

We need to show that both downstream firms making simple price offers cannot be an equilibrium. Since any non-negative offers will be accepted by upstream firms, both downstream firms get a profit of δ. Hence, we must have $p_{Aj} + p_{Bj} \leq \delta$, where p_{Aj} and p_{Bj} are the offers made by D_j to U_A and U_B respectively. Then $D_{j'}$ can profitably deviate by make two exclusive dealing offers $(p_{Aj} + \frac{\epsilon}{2}, E)$ and $(p_{Bj} + \frac{\epsilon}{2}, E)$ to the two upstream firms respectively. Then both U_A and U_B will take the exclusive dealing offer, and the profit for $D_{j'}$ is $\Delta - p_{Aj} - p_{Bj} - \epsilon$, which is higher than δ.(Since $\Delta > 2\delta$ and $p_{Aj} + p_{Bj} \leq \delta$)

Claim 2: There exists no symmetric equilibrium.

Consider a symmetric equilibrium with both exclusive dealing offers, i.e. D_1 offers $\{(p_1, E), (p_2, E)\}$ to U_A and U_B respectively. Then we must have $max\{p_1, p_2\} \leq \delta$. To see this: it cannot be the case that both upstream firms accept the offers from the same downstream firm due to our tie-breaking assumption. And thus each downstream firm must get profit δ, which means that the offered price cannot be higher than δ. Hence, given that D_1 makes such offers, D_2 can deviate by offering $\{(p_1 + \frac{\epsilon}{2}, E), (p_2 + \frac{\epsilon}{2}, E)\}$, which will be accepted by both upstream firms and give D_2 a profit of $\Delta - p_1 - p_2 - \epsilon > \delta - p_1$. Therefore, there is no symmetric equilibrium with both exclusive dealing offers. Similar argument also indicates that there is no symmetric equilibrium where each downstream firm makes an exclusive dealing offer to one upstream firm and a simple price offer to the other upstream firm.

Thus there are only asymmetric equilibria, and any such equilibrium must have the following property.

Claim 3: In any equilibrium, one downstream firm is excluded from the market.

If both downstream firms are active in the market, each of them is supplied by one upstream firm and gets profit δ. And thus, it must be the case that all four prices offered by downstream firms are no larger than δ, which in turn means that each downstream firm has incentive to overbid and exclude the other downstream firm as the argument in Claim 2.

Claim 4: In any equilibrium, we must have $p_{Aj} + p_{Bj} = \Delta$ for the active downstream firm D_j; and $min\{p_{Aj}, p_{Bj}\} \geq \delta$.

Suppose D_1 is active and D_2 is excluded from the market. For the first part, if not, D_2 can profitably deviate by offering slightly higher prices to both upstream firms and thus exclude D_1. For the second part, if not, suppose $p_{B1} < \delta$, then D_2 can profitably deviate by offering $p_{B1} + \epsilon$ to U_B.

Therefore, in any pure strategy equilibrium, both downstream firms earn zero profit. And the industry profit is shared between the two upstream firms. However, pure strategy equilibrium may fail to exist. For instance, suppose D_1 is active in the equilibrium, and D_1 offers $\{(p, E), (\Delta - p, E)\}$ to U_A and U_B, where $\delta \leq p \leq \Delta - \delta$. Then the best response of D_2 is to offer $\{(p_1 \leq p, \cdot), (p_2 < \Delta - p, \cdot)\}$. However, given that D_2 offers $p_2 < \Delta - p$ to U_B, the best response for D_1 is not to offer $\Delta - p$ to U_B but rather to offer $p_2 + \epsilon$ to U_B. To circumvent such circumstances, we can restrict the price quote to be discrete numbers with equal distance ϵ, i.e. prices can only be $\Delta, \Delta - \epsilon, \Delta - 2\epsilon, ...$ Then D_1 offers $\{(p, E), (\Delta - p, E)\}$ to U_A and

U_B, and D_2 offers $\{(p, \cdot), (\Delta - p - \epsilon, \cdot)\}$ is a pure strategy equilibrium.