

Nonlinear pricing as exclusionary conduct

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Abstract

We offer an exclusionary scenario for quantity rebates and market-share discounts offered by dominant firms, based on the upcoming introduction of a rival good in the market. We explain how the shape of the rebates depends on the incumbents' beliefs about the characteristics of the rival good. When buyers can dispose of unconsumed units at little cost, they might opportunistically purchase unneeded units with the sole purpose of pocketing rebates. We find that such opportunism is never seen in equilibrium and explain how the magnitude of disposal costs affects the shape of optimal price-quantity schedules.

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1 Introduction

In recent years, exclusionary conduct by firms with market power has become a high-priority issue on the agenda of antitrust agencies. For instance, the European Commission has made it clear that the emphasis of its enforcement activities is on “ensuring that undertakings which hold a dominant position do not exclude their competitors by other means than competing on the merits of the products or services they provide.” The U.S. Department of Justice concurs that “whether conduct has the potential to exclude, eliminate, or weaken the competitiveness of equally efficient competitors can be a useful inquiry”, and suggests that this inquiry “may be best suited to particular pricing practices.”¹

It is indeed in the area of pricing behavior that the so-called “equally efficient competitor test” most naturally applies. The test involves the “effective price” offered by the dominant firm, i.e. the price that competitors have to match. A lower effective price thus places more competitive pressure on rivals. The test consists in checking that the effective price covers production costs over the relevant output range. A violation of the test, therefore, is tantamount to a form of below-cost pricing. Such an outcome, however, says nothing about the precise channel by which the competitive process is harmed. The structure of the test—a price-cost comparison—might suggest a predatory scenario, whereby the dominant firm would incur a short-term sacrifice in the hope of later recoupment, but antitrust authorities are reluctant to engage in such a legally difficult route. As a general rule, they avoid being specific about possible “theories of harm”, for fear of weakening their cases in court. On the other hand, jurisprudence, in most countries, imposes a high standard of proof on defendants putting forward efficiency reasons for their conduct.

The purpose of the present article is to offer an exclusionary scenario that accounts for the various, often highly nonlinear, price schedules observed in

¹See, respectively, [European Commission \(2009\)](#) and [U.S. Department of Justice \(2008\)](#). High-profile exclusionary cases involving pricing practices include *Virgin v British Airways* (S.D.N.Y. 1999 and 2nd Cir. 2001), *Concord Boat* (8th Cir. 2000), *Lepage’s v 3M* (3d Cir. 2003) in the United States; *Virgin/British Airways* (2000/74/EC of 14 July 1999), *Michelin* (COMP/E-2/36.041/PO of 20 June 2001), *Tomra* (COMP/E-1/38.113, 2006), and *Intel* (COMP/C-3 /37.990, 2009) in Europe.

practice. The scenario is consistent with the as efficient competitor test and provides a transparent interpretation of the shape and structure of the tariffs in terms of competitive pressure placed on rival firms. The scenario highlights a number of environmental parameters that influence the exclusionary power of price schedules implemented by dominant firms.

Our anticompetitive scenario relies on a simple incumbency model whereby a dominant firm and a buyer agree on a price-quantity schedule knowing that a rival good is about to be introduced on the market. The characteristics of that good, however, are uncertain at the time of the contract, which makes incomplete information a critical ingredient of the exclusionary analysis.²

As standard in the literature, we distinguish two classes of price-quantity schedules, depending on whether the price charged to the buyer can be made contingent upon the quantity she purchases from the rival. When this is the case, the price schedule is said to be “conditional” on the quantity supplied by the rival, which allows in particular for market-share discounts. It turns out that the welfare effects of conditional price schedules can be analyzed under general assumptions on the buyer’s demand. Optimal conditional tariffs derive from a tradeoff between rent extraction and efficiency. Combining insights from [Baron and Myerson \(1982\)](#) and [Aghion and Bolton \(1987\)](#), we find that the rivals sell less than the efficient quantity and that the distortion increases with the rival’s bargaining power vis-à-vis the buyer. For a given level of the rival’s sales, however, the dominant firm does not need to distort its own quantity, because it can directly penalize buyers for supplying from rivals.

The above general analysis does not easily carry over to the case where the dominant firm’s prices cannot be contingent upon the quantity purchased from the rival (“unconditional” price schedules). In this case, the analysis is more involved as the buyer and the incumbent have one instrument to achieve two objectives: (i) solving the rent extraction-efficiency tradeoff, which requires setting marginal price below marginal costs; (ii) preventing the buyer from opportunistically purchasing unneeded units with the sole intention of pocketing the quantity rebates.

²The same timing is studied in [Marx and Shaffer \(1999\)](#), [Marx and Shaffer \(2004\)](#), [Feess and Wohlschlegel \(2010\)](#) under complete information.

To address buyer opportunism, we adopt a demand specification that involves two fundamental parameters: disposal costs and the “contestable share” of the market. This framework is sufficiently rich to reflect the notions that the dominant firm’s product is a “must-have” good and that purchasing unneeded units entails disposal costs for the buyer. The magnitude of disposal costs depends on the characteristics of the traded product, and may vary substantially across industries, as disposing of computer chips, tyres for trucks, or heavy pieces of machineries is likely to entail different costs. As regards the contestable share of the market, it is defined by the European Commission as “how much of a customer’s purchase requirements can realistically be switched to a competitor” within a reasonable time horizon. In practice, the contestable share is a critical ingredient of the as-efficient competitor test as it defines the relevant quantity range over which the price and the cost should be computed. Its determination has proved a highly contentious issue in *Intel*, reinforcing our view that contestable shares are fundamentally uncertain.

Accordingly, we derive optimal price schedules under two-dimensional uncertainty, assuming that both the surplus created by the rival and the share of the buyer’s demand it can address are unknown to the buyer and the dominant firm at the time of the contract. Contrary to most of the literature, we do not put any a priori restriction on the shape of the price-quantity schedule.

To describe the qualitative property of optimal tariffs, we introduce the notion of elasticity of entry, which measures the rival’s sensitivity to competitive pressure. The elasticity is a function of the two-dimensional distribution of uncertainty. It reflects the extent to which more pressure placed on rivals (i.e. lower effective prices) translates into more exclusion. We find that the shape of the optimal tariffs depends on this elasticity. In particular, optimal tariffs are linear only when the elasticity does not vary with the contestable share. Optimal tariffs tend to be concave when the entry elasticity increases with the scale of entry, and hence the dominant firm wants to place less pressure on larger competitors. When disposal costs are sufficiently large and the elasticity is non-monotonic in the contestable share, optimal schedules may exhibit highly nonlinear shapes and admit decreasing parts, as is the case under so-called “retroactive rebates”. Such rebates, also called “all-units discounts”, are granted for all purchased units once

a quantity threshold is reached. They induce downward discontinuities in price-quantity schedules –a pattern that has received much attention from antitrust enforcers.³

Decreasing parts in price-quantity schedules might induce the buyer to purchase inefficiently many units from the dominant company. We find, however, that such buyer opportunism is never seen in equilibrium. The buyer’s temptation to purchase unneeded units depends on the magnitude of disposal costs. As the buyer and the incumbent cannot condition prices on quantities purchased from the competitor, lower disposal costs translate into less competitive pressure on the rival and thus reduce the extent of inefficient foreclosure. In contrast, conditional tariffs provide the buyer and the incumbent with enough flexibility to address separately buyer opportunism and the efficiency-rent tradeoff, regardless of the magnitude of disposal costs. When disposal costs are low, unconditional tariffs, therefore, are potentially less harmful to competition. Both types of non-linear tariffs, however, deserve attention from antitrust enforcers in industries where disposal costs are large.

It is worthwhile connecting the current article with recent works on market-share discounts. In a setting with a dominant firm, a competitive fringe and two retailers, [Inderst and Shaffer \(2010\)](#) show that market-share discounts can be used by the dominant firm to dampen intra- and inter-brand competition. Their anticompetitive scenario, contrary to the one presented here, highlights retail competition and assume complete information. Turning to models with imperfect information, most of the literature has examined how specific forms of pricing perform in discriminating among privately informed buyers. For instance, in a discrete type model, [Kolay, Shaffer, and Ordovery \(2004\)](#) show that all-units discounts are more effective than menus of two-part tariffs in screening out retailers with private information about the state of demand. [Majumdar and Shaffer \(2009\)](#) and [Calzolari and Denicolo \(2013\)](#) introduce market-share discounts. In the former article, a dominant firm resorts to nonlinear pricing to screen a buyer who is informed about the size of demand and who also sells a good provided by a competitive fringe. The latter article addresses the issue in a symmetric duopoly setting, considering both market-share discounts and

³See, among others, [Waelbroeck \(2005\)](#) and [Faella \(2008\)](#).

exclusive contracts. In contrast to these papers, we consider incomplete rather than asymmetric information as the buyer and the incumbent do not know the characteristics of the rival good at the time of the contract.

Overall, the contribution of this article is twofold. First, we explain how the beliefs of incumbent market players about a new, rival good, together with the magnitude of disposal costs, affect the shape of optimal price-quantity schedules. Second, we compare the exclusionary properties of conditional and unconditional properties, relating the welfare effects of market-shared discounts to the issue of ex post buyer opportunism.

The article is organized as follows. Section 2 introduces the model. Section 3 studies conditional price-quantity schedules. Section 4 introduces the issue of buyer opportunism. Section 5 derives optimal unconditional schedules under two-dimensional uncertainty when disposal costs are large. Section 6 explains how the magnitude of disposal costs affects the shape of optimal tariffs and the welfare effects of market-share discounts.

2 The model

A dominant firm, I , competes with a rival, E , to serve a buyer, B . Production costs are assumed to be constant and are denoted by c_E and c_I . If the buyer purchases q_I units of good I and q_E units of good E , she earns a gross profit of

$$V(q_E, q_I; \theta_E) = v_E q_E + v_I q_I - h(q_E, q_I; s_E), \quad (1)$$

where h is a convex function of (q_E, q_I) with first derivatives at $(0, 0)$ equal to zero and with nonnegative cross-derivative to reflect the imperfect substitutability of the two goods. The parameters v_E and v_I reflect the buyer's willingness to pay for the first units of goods E and I . The parameter s_E affects how marginal utilities vary with the quantities purchased. Total surplus is given by

$$W(q_E, q_I; c_E, \theta_E) = V(q_E, q_I; \theta_E) - c_E q_E - c_I q_I = \omega_E q_E + \omega_I q_I - h(q_E, q_I; s_E),$$

where $\omega_E = v_E - c_E \geq 0$ and $\omega_I = v_I - c_I > 0$ are the unit surpluses generated by good E and good I respectively. We denote by $q_E^*(c_E, \theta_E)$ and $q_I^*(c_E, \theta_E)$ the efficient quantities, i.e. the quantities that maximize W .

We consider situations where the characteristics of the new, rival good are not yet known, and hence both the cost c_E and the buyer's taste for the rival good, $\theta_E = (s_E, v_E)$, are uncertain. In contrast, we assume away any informational asymmetry as to the characteristics of the incumbent's good: the parameter v_I is known ex ante.

2.1 Timing of the game

The order of events reflects the incumbency advantage of the dominant firm and the uncertainty as to the characteristics of the rival good:

- First, the buyer and the incumbent design a price-quantity schedule to maximize (and split) their joint expected surplus, denoted by Π_{BI} . At this stage, the buyer and the dominant firm know the production cost and the characteristics of good I , but do not know the production cost c_E and the characteristics θ_E of the new product.
- Next, the buyer and the competitor discover the cost and preference parameters, c_E and θ_E , relative to the rival good.
- Then, the buyer and the competitor, both knowing the terms of the agreement between the buyer and the incumbent, agree on a price and a quantity. This negotiation takes place under complete information and is assumed to be efficient. For example, B and E can use a two-part tariff with slope c_E . We denote by β the competitor's bargaining power, which determines the sharing of the surplus.
- Finally, the buyer purchases from the incumbent.

The buyer and the incumbent may want to let the price of good I depend on the quantity purchased from the rival firm, i.e. to use "conditional price schedules" of the form $T(q_E, q_I)$. However, enforcing a conditional price may be unfeasible (e.g. because the incumbent does not observe q_E) or legally prohibited. Accordingly, we also consider the situation where the firms are restricted to use unconditional price schedules $T(q_I)$. A key issue in the paper is to compare the exclusionary properties and the welfare effects of these two kinds of tariffs, given

the characteristics of the traded goods. Apart from the above distinction, we do not impose any regularity condition on the price schedules.

As regards the timing of negotiation, we assume that the buyer and the dominant firm cannot renegotiate once uncertainty is resolved. (If they could, they would simply agree on the optimal tariff under complete information.) The contribution of the current paper is, on the contrary, to study the shape of the price schedule under incomplete information. We also assume that the buyer and the dominant firm cannot renegotiate after the buyer has purchased from the competitor. In particular, they have a common incentive to renegotiate the quantity of good I whenever q_I does not maximize $W(q_E, q_I; c_E, \theta_E)$, where q_E is the quantity already purchased from the competitor. The ex post efficient, renegotiation-proof quantity of incumbent's good, which maximizes W given q_E , is denoted by $q_I^*(q_E; \theta_E)$. By substitutability, this quantity decreases with q_E .

2.2 Purchase decisions under a given price schedule

The last two stages of the game take place under perfect information, given the price schedule T and the known characteristics of the rival good. The buyer and the rival choose the quantities to maximize their joint surplus

$$S_{BE}(c_E, \theta_E) = \max_{q_E, q_I} V(q_E, q_I; \theta_E) - T(q_E, q_I) - c_E q_E, \quad (2)$$

with no consideration for the incumbent's cost or profit. (If the tariff is not allowed to depend on q_E , $T(q_E, q_I)$ is replaced with $T(q_I)$ in (2) and all subsequent expressions.) Suppose the buyer has purchased q_E units from the competitor. Then she picks $q_I(q_E; \theta_E)$ to maximize

$$\max_{q_I} V(q_E, q_I; \theta_E) - T(q_E, q_I). \quad (3)$$

Whenever the marginal price of an extra unit of good I differs from c_I , the quantity $q_I(q_E; \theta_E)$ chosen by B does not maximize the joint surplus of B and I , i.e. differs from $q_I^*(q_E; \theta_E)$. As explained in greater detail below, this may happen because the schedule T is also designed to extract surplus from the rival, which may involve setting the marginal price below the marginal cost c_I . This would give the buyer an ex post incentive to buy units of good I in excess of $q_I^*(q_E; \theta_E)$.

The quantity purchased from the competitor maximizes

$$S_{BE}(c_E, \theta_E) = \max_{q_E} V(q_E, q_I(q_E; \theta_E); \theta_E) - T(q_E, q_I(q_E; \theta_E)) - c_E q_E, \quad (4)$$

which is equivalent to (2). The buyer and the competitor share the surplus S_{BE} according to their respective bargaining power and outside options. The competitor's outside option is normalized to zero. As to the buyer, she may source exclusively from the incumbent, so her outside option is $V(0, q_I(0; \theta_E); \theta_E) - T(0, q_I(0; \theta_E))$. It follows that the surplus created by the relationship between B and E is given by

$$\Delta S_{BE}(c_E, \theta_E) = S_{BE}(c_E, \theta_E) - [V(0, q_I(0; \theta_E); \theta_E) - T(0, q_I(0; \theta_E))]. \quad (5)$$

Denoting by $\beta \in (0, 1)$ the competitor's bargaining power vis-à-vis the buyer, we derive the competitor's and buyer's profits:

$$\begin{aligned} \Pi_E &= \beta \Delta S_{BE} \\ \Pi_B &= (1 - \beta) \Delta S_{BE} + V(0, q_I(0; \theta_E); \theta_E) - T(0, q_I(0; \theta_E)). \end{aligned} \quad (6)$$

If $\beta = 0$, the competitor has no bargaining power and may be seen as a competitive fringe from which the buyer can purchase any quantity at price c_E . On the contrary, the case $\beta = 1$ happens when the competitor has all the bargaining power vis-à-vis the buyer.

2.3 Second-best equilibrium and inefficiencies

Ex ante, the buyer and the incumbent design the price schedule to maximize their expected joint surplus, equal to the total surplus minus the profit left to the competitor:

$$\mathbb{E}_{c_E, \theta_E} \Pi_{BI} = \mathbb{E}_{c_E, \theta_E} \{W(q_E, q_I; c_E, \theta_E) - \Pi_E\}, \quad (7)$$

where q_E, q_I and Π_E are given by (3), (4), (5) and (6). The sharing of the expected joint surplus between the buyer and the incumbent, and hence the respective bargaining power of each party, play no role in the following analysis.

From the ex ante perspective, the tariff has two purposes: on the one hand, maximizing the expected welfare W ; on the other, extracting rent from the rival,

i.e. making $\Pi_E = \beta \Delta S_{BE}$ as small as possible. Rent extraction is obtained by placing competitive pressure on the rival firm, i.e. leaving it with no other choice than to match low prices or make no sales. In practice, competitive pressure translates into quantity rebates granted to the buyer.

The rent extraction motive is stronger as the competitor’s bargaining power vis-à-vis the buyer rises. For $\beta = 0$, the rival earns no profit and the first-best obtains, which can be checked throughout the paper. For $\beta > 0$, we find two kinds of inefficiencies. First, it may not be in the buyer’s best interest to pick the efficient quantity $q_I^*(q_E; \theta_E)$; she may indeed prefer to pocket the rebates granted by the incumbent and purchase inefficiently many units of good I . Section 4 will consider a particularly severe form of ex post inefficiency, where the buyer might possibly purchase and scrap unneeded units of good I . We call such a behavior opportunistic. Buyer opportunism is anticipated ex ante when designing the price-quantity schedule.

Second, as pointed out by competition authorities, the quantity purchased from the competitor may not be efficient, $q_E < q_E^*$, a phenomenon called “inefficient market foreclosure”. Inefficient foreclosure is complete when $q_E = 0 < q_E^*$, partial when $0 < q_E < q_E^*$. In both cases, the rival is prevented from selling the efficient number of units of good E .

Under complete information, it has been shown by Marx and Shaffer (1999) and Marx and Shaffer (2004) that the second-best allocation is efficient.⁴ In this article, we find partial and/or complete inefficient exclusion when the characteristics of the rival good are uncertain and explain how uncertainty affects the shape of optimal price schedules.

3 Rent-efficiency tradeoff

In this and the next section, the parameter s_E is assumed to be known. The analysis follows standard arguments (e.g. Laffont and Martimort (2002)). The distribution of ω_E given s_E is denoted by $F(.|s_E)$ and is assumed to admit a positive and continuous density function $f(.|s_E)$ on $[\underline{\omega}_E, \bar{\omega}_E]$. To focus on the more interesting cases, we assume $\underline{\omega}_E < \omega_I < \bar{\omega}_E$. The surplus created by the

⁴We recall their results using our notations in Appendix F.

trade between the buyer and the rival, (5), can be rewritten as

$$\Delta S_{BE}(\omega_E) = \max_{q_E \geq 0} \left\{ \omega_E q_E + v_I q_I(q_E) - h(q_E, q_I(q_E)) - T(q_E, q_I(q_E)) - [v_I q_I(0) - h(0, q_I(0)) - T(0, q_I(0))] \right\}, \quad (8)$$

where we have dropped the known value of s_E in the arguments of q_E , q_I and S_{BE} . The derivative of ΔS_{BE} is given by the envelope theorem

$$\frac{\partial \Delta S_{BE}}{\partial \omega_E} = q_E(\omega_E). \quad (9)$$

Using $\Pi_E = \beta \Delta S_{BE}$ and integrating by parts, we get

$$\int_{\underline{\omega}_E}^{\bar{\omega}_E} \Pi_E(\omega_E) f(\omega_E | s_E) d\omega_E = \Pi_E(\underline{\omega}_E) + \beta \int_{\underline{\omega}_E}^{\bar{\omega}_E} q_E(\omega_E) [1 - F(\omega_E | s_E)] d\omega_E.$$

Substituting in (7), we rewrite the buyer-incumbent objective as

$$\mathbb{E}_{\omega_E} \Pi_{BI} = \mathbb{E}_{\omega_E} S^v(q_E, q_I(q_E); \omega_E) - \Pi_E(\underline{\omega}_E), \quad (10)$$

where, following Jullien (2000), we have defined the “virtual surplus” S^v as

$$S^v(q_E, q_I; \omega_E) = W(q_E, q_I; \omega_E) - \beta q_E \frac{1 - F(\omega_E | s_E)}{f(\omega_E | s_E)}. \quad (11)$$

The virtual surplus is the total surplus $W(q_E, q_I; \omega_E)$ adjusted for the informational rents $\beta q_E (1 - F(\omega_E | s_E)) / f(\omega_E | s_E)$ induced by the self-selection constraints. In Appendix A, we show that the second-best quantity maximizes the virtual surplus under the constraint that it is nondecreasing in ω_E . The choice of q_E reflects the tradeoff between efficiency and rent extraction while the choice of q_I relates to ex post efficiency.

We now consider the case where the tariff is allowed to depend on q_E . A two-part tariff with slope c_I , $T(q_E, q_I) = c_I q_I + P(q_E)$, ensures that the buyer picks the ex post efficient quantity, $q_I^*(q_E)$, for any prior choice of q_E . By (8), we see that the rival’s profit, $\Pi_E = \beta \Delta S_{BE}$, depends only on the difference $P(q_E) - P(0)$, which thus governs the efficiency rent tradeoff.

Differentiating the virtual surplus with respect to q_E , we find that the second-best quantity is given by

$$\frac{d}{dq_E} W(q_E(\omega_E), q_I^*(q_E); \omega_E) \leq \beta \frac{1 - F(\omega_E | s_E)}{f(\omega_E | s_E)}, \quad (12)$$

with equality if $q_E > 0$. Given that $W_{q_I}(q_E, q_I^*(q_E); \omega_E) = 0$, the above total derivative in q_E is equal to the partial derivative

$$W_{q_E}(q_E, q_I^*(q_E); \omega_E) = \omega_E - \frac{\partial h}{\partial q_E}(q_E, q_I^*(q_E)) = P'(q_E) \quad (13)$$

where we have used the first-order condition of the buyer's and rival's problem (8). Comparing (12) and (13) yields the optimal quantity $q_E(\omega_E)$:

$$P'(q_E(\omega_E)) = \beta \frac{1 - F(\omega_E | s_E)}{f(\omega_E | s_E)}. \quad (14)$$

To interpret the optimality conditions, it is convenient to introduce the notion of elasticity of entry, which reflects the rival's sensitivity to competitive pressure for a given level of unit surplus ω_E :

$$\varepsilon(\omega_E | s_E) = \frac{\omega_E f(\omega_E | s_E)}{1 - F(\omega_E | s_E)}. \quad (15)$$

To illustrate, consider the linear tariff $T(q_E, q_I) = c_I q_I + \left[\omega_E - \frac{\partial h}{\partial q_E}(0, q_I^*(0)) \right] q_E$, under which a rival makes positive sales if and only if it brings unit surplus above ω_E .⁵ Increasing ω_E by 1% decreases the number of active rivals by $\varepsilon(\omega_E | s_E)\%$. In the remainder of the paper, we maintain the following assumption regarding the distribution of ω_E given s_E .

Assumption 1. *For any given s_E , the elasticity of entry, $\varepsilon(\omega_E | s_E)$, is nondecreasing in ω_E . Moreover, if $\bar{\omega}_E = \infty$, the upper bound of $\varepsilon(\omega_E | s_E)$ as ω_E rises is greater than one, for all s_E .*

The monotonicity of the elasticity of entry holds in particular when the hazard rate $f/(1-F)$ is nondecreasing in ω_E , a usual assumption in the nonlinear pricing literature. The above analysis yields the following proposition, formally proved in Appendix A.

Proposition 1. *When s_E is known ex ante, the conditional tariff $T(q_E, q_I)$ that maximizes the buyer and incumbent's joint profit is given by $c_I q_I + P(q_E)$, with*

$$\frac{\omega_E - \frac{\partial h}{\partial q_E}(q_E, q_I^*(q_E))}{\omega_E} = \frac{\beta}{\varepsilon(\omega_E | s_E)} \quad (16)$$

⁵This follows from the first-order condition of the buyer-entrant problem (8) at $q_E = 0$.

for positive values of q_E . The quantity purchased from the dominant firm is *ex post* efficient while that purchased from the rival is distorted downwards for $\omega_E < \bar{\omega}_E$, undistorted at $\bar{\omega}_E$ when $\bar{\omega}_E < \infty$, and increasing in ω_E . If the hazard rate is nondecreasing, the price schedule is increasing and concave in q_E .

The efficiency-rent tradeoff leads to more inefficient exclusion as the rival's bargaining power, β , rises and the elasticity of entry, ε , falls.

Proposition 1 builds a bridge between the literatures on market foreclosure and nonlinear pricing. Equation (16) shows an analogy with the textbook monopoly pricing formula. The buyer and the incumbent jointly act like a monopoly towards the rival, setting $P(q_E)$ to extract rent at the cost of reducing the extent of entry: $q_E < q_E^*$. When the elasticity ε is high, the buyer and the incumbent cannot easily extract rents and the rival sells more units. When β is high, the rival has a strong bargaining power vis-à-vis the buyer, which makes rent extraction a more serious issue and pushes towards reducing q_E .

Aghion and Bolton (1987) interpreted the difference $P(q_E) - P(0)$ as a penalty for breach of contract. They assumed that the buyer's demand was supplied entirely by a single supplier, so the purchase decision was "extensive". In contrast, we allow the buyer to split her purchase requirements between the two suppliers and find that inefficient foreclosure may be complete or partial: $0 \leq q_E < q_E^*$. We interpret the difference $P(q_E) - P(0)$ as rebates lost when supplying from the competitor. The presence of these rebates implies a form of below-cost pricing. Specifically, the average incremental price of the "last" units of good I (units between $q_I^*(q_E)$ and $q_I^*(0)$) is lower than the production cost:

$$\frac{T(0, q_I^*(0)) - T(q_E, q_I^*(q_E))}{q_I^*(0) - q_I^*(q_E)} = c_I - \frac{P(q_E) - P(0)}{q_I^*(0) - q_I^*(q_E)} < c_I.$$

The above price-cost comparison is reminiscent of the "as-efficient competitor test" mentioned in the introduction. In the next section, we present the precise form of the test advocated by the European Commission, which involves a particular value for q_E , called "size of the contestable demand".

Quadratic example With $h(q_E, q_I; s_E) = q_E^2/2 + q_I^2/2 + \sigma(s_E)q_E q_I$, $0 \leq \sigma(s_E) < 1$, the second-best quantities purchased from both suppliers are

$$q_E = q_E^* - \frac{\beta\omega_E}{(1 - \sigma^2)\varepsilon} \quad \text{and} \quad q_I = q_I^* + \frac{\beta\sigma\omega_E}{(1 - \sigma^2)\varepsilon},$$

with $q_E^* = (\omega_E - \sigma\omega_I)/(1 - \sigma^2)$ and $q_I^* = (\omega_I - \sigma\omega_E)/(1 - \sigma^2)$. The penalty scheme is given by $P(q_E) - P(0) = (1 - \sigma^2)(q_E^*q_E - q_E^2/2)$ for $q_E \leq q_E^*$.

4 “Must-have” good and disposal costs

When the price schedule is *not* allowed to depend on q_E , the tariff $T(q_I)$ governs the choice of the quantities purchased from both suppliers. Ex ante, the buyer and the dominant firm have only one instrument to manage buyer opportunism and solve the rent-efficiency tradeoff. The analysis, therefore, is more complex for unconditional tariffs $T(q_I)$ than for conditional tariffs $T(q_E, q_I)$, and cannot be carried out in the above general model. Hereafter, we specialize to a framework with inelastic buyer demand, where buyer opportunism translates into a simple constraint on the marginal price. Then the rent-efficiency tradeoff can be solved within the limits allowed by this constraint.

The restriction that total demand is inelastic entails no limitation given the purpose of our analysis because, as already mentioned, we are not interested in quantity distortions caused by inefficient bilateral bargaining,⁶ but in how nonlinear pricing by the dominant firm alters the split of the buyer’s purchase requirements between the two suppliers. Hereafter, total demand is normalized to one.

In Section 4.1, we specify a particular form for the buyer preferences, introducing the size of the contestable demand and the level of disposal costs. We apply the analysis of Section 3 to solve the rent-efficient tradeoff and derive the optimal conditional tariff $T(q_E, q_I)$. In Section 4.2, we address the buyer opportunism problem and derive the optimal unconditional tariff $T(q_I)$.

⁶Recall that we assume away any bilateral inefficiency (e.g. asymmetric information) between the buyer and each of the two suppliers. In particular, the buyer and the incumbent would, in the absence of a rival, have no reason to distort the traded quantity. Similarly, we assume throughout the article that the negotiation between the buyer and the rival takes place under perfect information and is efficient (see Section 2.1).

4.1 Contestable market share and disposal costs

In the remainder of the paper, we assume a particular form for the buyer's utility $V(q_E, q_I; \theta_E)$, where $\theta_E = (s_E, v_E)$ represents the characteristics of the rival good. Under this specification, the parameter s_E is interpreted as the size of the constable market, s_E . The utility function also depends on an industry-specific parameter, called the magnitude of disposal costs and denoted by γ . In Appendix B, we present a formal expression for $V(q_E, q_I; \theta_E)$ and check that the assumptions of Section 2 are satisfied, i.e. the utility function is concave and the two goods are substitutes. We now describe the qualitative properties of the utility function that will be used in the following analysis.

The size of the contestable market s_E is the fraction of the buyer's demand that the rival can address within the relevant time period. If the buyer purchases less units of good E than s_E and less units of both goods together than her total requirements, she enjoys utility $V(q_E, q_I; \theta_E) = v_E q_E + v_I q_I$. In other words, the function h introduced in (1) is assumed to be identically zero on the set $q_E \leq s_E$ and $q_I \leq 1 - q_E$.

We allow the buyer to purchase more units than she needs, but assume that she incurs a cost γ per unconsumed units. In some industries, the buyer is able to resell unused items on a secondary market, see the discussion in Section 6. To account for that possibility, we allow γ to be negative, but assume that reselling entails a productive inefficiency, i.e. the total costs $\gamma + c_I$ and $\gamma + c_E$ are always nonnegative. In particular, units of good E beyond s_E cannot be utilized by the buyer and must be disposed of at the per-unit cost γ , hence $\partial V / \partial q_E = -\gamma$ for $q_E > s_E$. As a result, to save on disposal and production costs, the buyer and the rival, who negotiate under perfect information, never trade more than s_E units. It follows that the inequality $q_E \leq s_E$ holds for all values of (c_E, θ_E) .

Let us now consider the possibility that the buyer purchases more units than her total requirements, i.e. buys units of good I in excess of $1 - q_E$, for given $q_E \leq s_E$. If $v_E > v_I$, the buyer would dispose of these extra units of good I at per-unit cost γ , hence $\partial V / \partial q_I = -\gamma$ in this region. If $v_I > v_E$ and $q_E > 0$, the buyer would consume some of the extra units of good I and dispose of units of good E instead, hence $\partial V / \partial q_I = v_I - v_E - \gamma$.

Efficiency Given $q_E \leq s_E$, a unit of good I below $1 - q_E$ generates positive surplus ω_I . From the above analysis, we see that units above this threshold generate a surplus of $-c_I - \gamma$ if $v_E > v_I$ and $\omega_I - v_E - \gamma$ if $v_I > v_E$, hence a negative surplus when $\omega_I < \omega_E$. It follows that the ex post efficient quantity is given by

$$q_I^*(q_E; \theta_E) = 1 - q_E(c_E, \theta_E)$$

for all (c_E, θ_E) with $\omega_E > \omega_I$. Regarding ex ante efficiency, we observe that a unit of good E below s_E generates positive surplus ω_E , while a unit above s_E generates only disposal and production costs. It follows that the efficient quantities are given by

$$(q_E^*(c_E, \theta_E), q_I^*(c_E, \theta_E)) = \begin{cases} (s_E, 1 - s_E) & \text{if } \omega_E > \omega_I \\ (0, 1) & \text{if } \omega_E < \omega_I. \end{cases} \quad (17)$$

Inefficient foreclosure occurs when $0 \leq q_E < s_E$ while $\omega_E > \omega_I$. Inefficient foreclosure is complete or partial according to whether the above inequality $0 \leq q_E$ holds as an equality or is strict.

Rent-efficiency tradeoff under one-dimensional uncertainty We maintain the assumption that the size of the contestable demand is known ex ante and allow the price schedule to depend on q_E , so buyer opportunism is not an issue: Ex post efficiency, $q_I = q_I^*$, is guaranteed by $T(q_E, q_I) = c_I q_I + P(q_E)$. The virtual surplus is thus given as in Section 3 by (11). When $\omega_E < \omega_I$, efficiency and rent extraction both push towards $q_E = 0$ and $q_I = 1$. When $\omega_E > \omega_I$, we evaluate the virtual surplus at the ex post efficient quantity $q_I^* = 1 - q_E$:

$$S^v(q_E, q_I^*; c_E, \theta_E) = \omega_I + q_E \left(\omega_E - \omega_I - \beta \frac{1 - F(\omega_E | s_E)}{f(\omega_E | s_E)} \right). \quad (18)$$

Lemma 1. *When the price schedule can depend on q_E , the second-best quantity is given by*

$$q_E(s_E, \omega_E) = \begin{cases} 0 & \text{if } \omega_E \leq \hat{\omega}_E(s_E) \\ s_E & \text{otherwise,} \end{cases}$$

where $\hat{\omega}_E(s_E) \in (\omega_I, \bar{\omega}_E)$ is the unique solution to

$$\frac{\hat{\omega}_E(s_E) - \omega_I}{\hat{\omega}_E(s_E)} = \frac{\beta}{\varepsilon(\hat{\omega}_E(s_E) | s_E)}. \quad (19)$$

The fraction of efficient types that are inactive increases with the rival's bargaining power vis-à-vis the buyer and decreases with the elasticity of entry.

Proof. When demand is inelastic, the maximization of the virtual surplus yields a corner solution: $q_E = s_E$ if $s^v > 0$ and at $q_E = 0$ if $s^v < 0$, with $s^v(s_E, \omega_E) = \omega_E[1 - \beta/\varepsilon(\omega_E|s_E)] - \omega_I$. This quantity is positive if and only if

$$\frac{\omega_E - \omega_I}{\omega_E} > \frac{\beta}{\varepsilon(\omega_E|s_E)}.$$

The left-hand side increases in ω_E , and the right-hand side is non-increasing in ω_E by the first part of Assumption 1, which yields the uniqueness of a solution (19). Moreover, the virtual surplus per unit is negative for $\omega_E = \omega_I$ and positive for $\omega_E = \bar{\omega}_E$ when $\bar{\omega}_E < \infty$. If $\bar{\omega}_E = \infty$, the second-part of Assumption 1 guarantees that s^v is positive for high values of ω_E . Hence the existence of a solution to equation (19) lying between ω_I and $\bar{\omega}_E$. Straightforward comparative statics shows that $\hat{\omega}_E$ increases with β and decreases with ε . \square

Equation (16) is the analog of (19) when demand is inelastic. As above, we find that the rent-efficiency tradeoff leads to below-cost pricing. Indeed, if the rival generates a unit surplus below $\hat{\omega}_E(s_E)$, it does not make any sales and thus gets no rent. The penalty scheme $P(q_E)$ is adjusted to leave no rent to the marginal competitor: $\Pi_E = \beta\Delta S_{BE}(s_E, \hat{\omega}_E) = 0$. The surplus created by the buyer and the rival trading s_E units is

$$\Delta S_{BE}(c_E, \theta_E) = V(s_E, 1 - s_E; \theta_E) - c_E s_E - T(s_E, 1 - s_E) - [V(0, 1; \theta_E) - T(0, 1)],$$

where the difference in buyer gross utilities is given by

$$V(s_E, 1 - s_E; \theta_E) - V(0, 1; \theta_E) = (v_E - v_I)s_E.$$

Considering the marginal competitor, $\omega_E = \hat{\omega}_E$, we find $(\hat{\omega}_E - v_I)s_E + T(0, 1) - T(s_E, 1 - s_E) = 0$. In line with the general results of Section 3, the average price for the contestable units, called “effective price” by the European Commission, is lower than the production cost of those units:

$$\frac{T(0, 1) - T(s_E, 1 - s_E)}{s_E} = v_I - \hat{\omega}_E < c_I. \quad (20)$$

Below-cost pricing is due to the rebates lost by the buyer if she supplies the contestable part of her demand from the rival firm.

Although the buyer can split her purchase between between the two suppliers, inefficient foreclosure only consists of full foreclosure as in [Aghion and Bolton \(1987\)](#). This, however, is no longer true in the more realistic case where s_E is ex ante uncertain, as we will see [Section 5](#). Moreover, the rent-efficiency tradeoff described in [Proposition 1](#) and [Lemma 1](#) is not affected by the buyer opportunism problem or by the magnitude of disposal costs because we have allowed the price schedule used by the incumbent to depend on the quantity purchased from the rival.

4.2 Buyer opportunism

We now turn to unconditional price schedules. In particular, we explain how the possibility of buyer ex post opportunism affects the design of a schedule $T(q_I)$. Assume that the incumbent subsidizes the purchase of good I to the point that the marginal price $T'(q_I)$ is below $-\gamma$ in some interval. The buyer would purchase the corresponding units from the dominant firm even if she does not need them. She would indeed find it optimal to dispose of the units at cost γ and to pocket the subsidy. Over-purchasing would be ex post profitable because the negative price would outweigh the disposal cost.

Yet the buyer and the dominant firm would soon realize that this outcome is suboptimal from an ex ante point of view. Anticipating the opportunistic behavior of the buyer, they would modify the above schedule, offering the buyer, together with the quantity \hat{q}_I at price $T(\hat{q}_I)$, the possibility to buy less units than \hat{q}_I , say $q_I \leq \hat{q}_I$, in return for a payment slightly below $T(\hat{q}_I) + \gamma(\hat{q}_I - q_I)$. This change would avoid useless production and disposal costs, without affecting the profit left to the competitor.

A symmetric reasoning shows that it is never optimal ex ante to sell units above the buyer's reservation price, v_I . The buyer and the dominant firm should always grant the buyer the opportunity to purchase as many units as she wants at a price slightly below v_I . The next proposition, proved in [Appendix C](#), shows that the buyer and the dominant firm are better off committing to a price sched-

ule with marginal price between $-\gamma$ and v_I . The main point to be checked is that this requirement does not raise the rent left to the rival.

Proposition 2. *The buyer and the dominant firm are better off using a tariff with marginal price between $-\gamma$ and v_I . The quantity purchased from the dominant firm is ex post efficient: $q_I = q_I^*(q_E; \theta_E) = 1 - q_E$, for any (c_E, θ_E) .*

Proposition 2 guarantees that the buyer purchases the number of units corresponding to her total requirements.⁷ We are thus able to focus attention on the split of the buyer's requirements between the two suppliers.

We now introduce the notion of super-efficiency. We say that the rival firm is *super-efficient* if and only if $\omega_E > v_I + \gamma$. When γ tends to $-c_I$, super-efficiency becomes equivalent to standard efficiency. When disposal costs are infinite, there are no super-efficient rivals.

Corollary 1. *When the tariff is not allowed to depend on q_E , super-efficient rivals serve all of the contestable demand.*

Proof. Since the quantity purchased from the incumbent is ex post efficient and the quantity purchased from the rival is lower than s_E (see the beginning of Section 4.1), we can write the surplus in the buyer-rival relationship can be written as

$$\begin{aligned} S_{BE}(s_E, \omega_E) &= \max_{q_E \leq s_E} V(q_E, q_I^*(q_E; \theta_E); \theta_E) - T(q_I^*(q_E; \theta_E)) - c_E q_E \\ &= \max_{q_E \leq s_E} \omega_E q_E + v_I(1 - q_E) - T(1 - q_E). \end{aligned} \quad (21)$$

The maximand in (21) increases in q_E on the interval $[0, s_E]$. Indeed its derivative is given by

$$\omega_E - v_I + T'(1 - q_E) \geq \omega_E - v_I - \gamma,$$

which is positive if the rival firm is super-efficient. \square

As γ tends to $-c_I$, the condition that all units of good I are sold at a price above $-\gamma$ represents a stronger constraint. At the limit $\gamma = -c_I$, the condition

⁷As mentioned at the end of Section 2.1, it also implies that the buyer and the incumbent have no joint incentive to renegotiate the quantity q_I once the buyer has purchased q_E from the rival.

$T'(q_I) \geq -\gamma$ leaves no scope for anticompetitive exclusion. The desire to avoid buyer opportunism dominates the rent extraction motive. On the other hand, as γ tends to $+\infty$, buyer opportunism becomes a less severe problem. The buyer and the dominant firm can more easily exploit their incumbency advantage to extract rents from the rival, even when it is very efficient.

Proposition 3. *If the price schedule cannot be made contingent upon q_E , the second-best quantity purchased from the competitor is s_E when $\omega_E \geq \min(\hat{\omega}_E, v_I + \gamma)$ and zero otherwise, where $\hat{\omega}_E$ is given by (19). The buyer and the incumbent may use a two-tariff with slope $\max(v_I - \hat{\omega}_E, -\gamma)$.*

Proof. Considering an unconditional tariff $T(q_I)$ and ignoring first the issue of buyer opportunism, we maximize the virtual surplus as explained in Lemma 1. Rivals with $\omega_E < \hat{\omega}_E$ are not active, and hence earn zero profit.

When $\hat{\omega}_E < v_I + \gamma$, the buyer opportunism issue does not affect the rent-efficient tradeoff. The buyer and the incumbent may set the price of contestable units at $v_I - \hat{\omega}_E$, as in (20), without generating buyer opportunism because this price is above $-\gamma$.

When $v_I + \gamma < \hat{\omega}_E$, the above marginal price would induce the buyer to purchase too many units from the incumbent. We know from Corollary 1 that it is optimal for the buyer and the incumbent to let super-efficient rivals serve all of the contestable demand. This is done by setting the marginal price at $-\gamma$. Only super-efficient rivals are active, earning $\beta(\omega_E - v_I - \gamma)s_E$. \square

When the price schedule cannot depend on q_E , we again find below-cost pricing at the margin. Indeed, denoting by $p^e(s_E)$ the average price of the contestable units (“effective price”)

$$p^e(s_E) = \frac{T(1) - T(q_I^*(s_E))}{1 - q_I^*(s_E)} = \frac{T(1) - T(1 - s_E)}{s_E}, \quad (22)$$

we find

$$p^e(s_E) = \max(v_I - \hat{\omega}_E, -\gamma) < c_I. \quad (23)$$

The effective price of the contestable units, $p^e(s_E)$, can be negative but cannot be lower than $-\gamma$ because a price below $-\gamma$ would trigger buyer opportunism, which is suboptimal. Compared to the situation where the incumbent can use

a conditional price schedule, the extent of inefficient exclusion is reduced if and only if $v_I + \gamma < \hat{\omega}_E$. This inequality holds when disposal costs are low, the rival has strong bargaining power vis-à-vis the buyer, and the elasticity of entry is low. Under this circumstance, the possibility of buyer opportunism limits the exclusionary power of unconditional tariffs. On the other hand, when $v_I + \gamma \geq \hat{\omega}_E$, the ability to condition the price on q_E does not change the second-best equilibrium.

At the second-best, the rival either serves all of the contestable demand or is inactive. It follows that the optimal allocation can be implemented with a two-part tariff (whose slope is the effective price given (23)). This is no longer true when the size of contestable demand is uncertain, the situation to which we now turn.

5 The shape of optimal price schedules

Building on the one-dimensional analysis of Section 4, we now introduce uncertainty about the size of the contestable demand, s_E . This parameter, as the utility v_E , depends on the characteristics of the rival good, which are not yet known when the buyer and the incumbent agree on the price schedule. The cumulative distribution function of s_E , denoted by G , is assumed to admit a positive and continuous density function g on $[\underline{s}_E, \bar{s}_E]$. Under uncertainty about both s_E and v_E , the expected virtual surplus is given by

$$\iint s^v(s_E, \omega_E) q_E(s_E, \omega_E) dF(\omega_E | s_E) dG(s_E),$$

where $s^v(s_E, \omega_E) = \omega_E [1 - \beta / \varepsilon(\omega_E | s_E)] - \omega_I$ is the unit virtual surplus introduced in Section 4.1. Recall that the virtual surplus is positive for $\omega_E > \hat{\omega}_E(s_E)$ and negative for $\omega_E < \hat{\omega}_E(s_E)$, where $\hat{\omega}_E(s_E)$ is given by (19).

As s_E is unknown ex ante, incentive compatibility must be checked along this second dimension. According to (21), the quantity purchased from the rival must be nondecreasing in s_E . Solving the problem separately for each s_E yields an allocation that respects or violates this monotonicity constraint, depending on how the elasticity of entry $\varepsilon(\omega_E | s_E)$ varies with s_E . The next lemma, proved

in Appendix D, relates the variations of $\varepsilon(\omega_E|s_E)$ in s_E to the primitives of the model.

Lemma 2. *The random variables s_E and ω_E are independent if and only if the elasticity of entry, $\varepsilon(\omega_E|s_E)$, does not depend on s_E . When the elasticity of entry increases (decreases) with s_E , ω_E first-order stochastically decreases (increases) with s_E .*

When the elasticity of entry is nondecreasing in the size of the contestable demand (Section 5.1), the buyer and the incumbent want to induce more entry for larger rivals, which essentially results in concave tariffs. Larger rivals sell more units and the monotonicity constraint regarding s_E is satisfied. The other cases require a more careful analysis because solving the problem for each s_E separately does not yield an incentive compatible allocation. In Appendix E, we provide a constructive method to build the second-best allocation and the corresponding price schedule. In Sections 5.2 and 5.3, we explain qualitatively how the shape of the optimal unconditional price schedule depends on the variations of the elasticity of entry, considering successively the cases of a decreasing and of a non-monotonic elasticity of entry.

To concentrate on the role of the heterogeneity about contestable demand, we assume in this section that disposal costs are very large, thus ruling out the issue of buyer opportunism. In Section 6, we will explain how optimal price schedules are modified when disposal costs are low.

5.1 Concave price schedules

When s_E and ω_E are independent, the threshold $\hat{\omega}_E(s_E)$ is flat, as represented on Figure 1a. For each size of the contestable market, the problem is the same form as in Section 4. It follows that the optimal price schedule is a two-part tariff with slope $v_I - \hat{\omega}_E$, as shown on Figure 1b.

From now on, we consider cases where the elasticity of entry varies with s_E and show that two-part tariffs are no longer optimal: the optimal tariff must exhibit some curvature. We start with the case where the elasticity increases with s_E : larger competitors, i.e. competitors with a larger contestable demand, are more sensitive to competitive pressure. Under this circumstance, the efficiency-

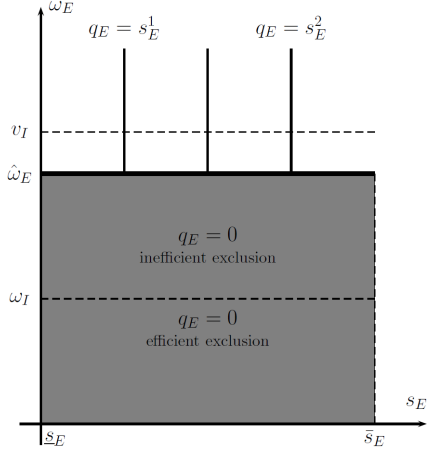


Figure 1a: Quantity purchased from the rival in the (s_E, ω_E) plan

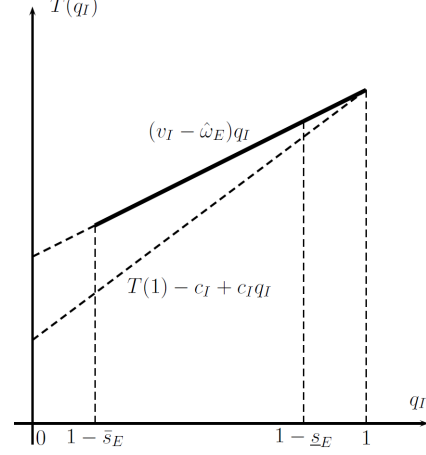


Figure 1b: Optimal price schedule (case $v_I > \hat{\omega}_E$)

rent tradeoff leads the buyer and the incumbent to place *less* competitive pressure on larger competitors.

Proposition 4. *When the elasticity of entry $\varepsilon(\omega_E|s_E)$ increases with s_E , the effective price, $p^e(q_E) = [T(1) - T(1 - q_E)]/q_E$, increases with q_E . The price schedule is concave in a neighborhood of $q_I = 1$. It is globally concave if $\hat{\omega}_E$ is concave or moderately convex in s_E . The equilibrium features inefficient exclusion. Partial foreclosure is not present.*

Proof. When $\varepsilon(\omega_E|s_E)$ increases with s_E , the threshold $\hat{\omega}_E$ given by (19) decreases with s_E , see Figure 2a. Solving the problem separately for each s_E , the buyer and the incumbent set the effective price $p^e(s_E)$ at $v_I - \hat{\omega}_E(s_E)$, which increases in s_E . The rival makes no sales if $\omega_E < \hat{\omega}_E(s_E)$ and serves all the contestable demand if $\omega_E > \hat{\omega}_E(s_E)$. The quantity q_E increases with s_E . Using (23), we recover the price schedule as

$$T(q_I) = T(1) - c_I + c_I q_I + (\hat{\omega}_E - \omega_I)(1 - q_I),$$

where $\hat{\omega}_E$ is evaluated at $q_E = 1 - q_I$. To prove concavity in a neighborhood of $q_I = 1$, we differentiate the above expression twice with respect to q_I , which yields $T''(q_I) = 2\hat{\omega}'_E + (1 - q_I)\hat{\omega}''_E$. The term $\hat{\omega}'_E$, which is negative for any q_I , tends to make the tariff concave. Assuming that $\hat{\omega}''_E(0)$ is not infinite, we get $T''(1) = 2\hat{\omega}'_E(0) < 0$, hence the concavity at the top. \square

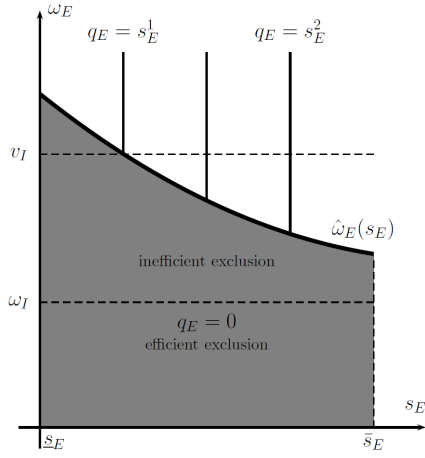


Figure 2a: Second best with $\varepsilon(\omega_E|s_E)$ increasing in s_E

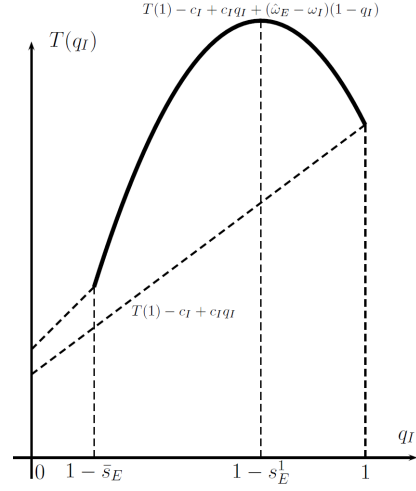


Figure 2b: Optimal price schedule with $s_E = 0$ and $v_I < \hat{\omega}_E(0)$

Proposition 4 assumes that the elasticity of entry is nondecreasing in the size of the contestable demand. According to Lemma 2, this assumption implies that rivals with larger s_E tend to generate a lower surplus ω_E and hence are more sensitive to competitive pressure. The buyer and the dominant firm therefore exert less pressure on larger rivals, and the optimal effective price $p^e(q_E) = [T(1) - T(1 - q_E)]/q_E$ increases with q_E . In the case shown on Figure 2b, $\hat{\omega}_E(0) > v_I$, the effective price $p^e(q_E)$ is negative for small values of q_E , which gives the buyer strong incentives to supply exclusively from the dominant firm.

Competition agencies tend to take a more negative stance on highly nonlinear price schedules than on simple quantity rebates, which are often presumed to be economically justified. The analysis of this section, however, shows that concavity does not necessarily reflect economies of scale. Apparently innocuous, nondecreasing and concave, price schedules may yield anticompetitive market foreclosure.

5.2 Convex price schedules

We now turn to the case where the elasticity of entry $\varepsilon(\omega_E|s_E)$ decreases with s_E . Under this circumstance, the efficiency-rent tradeoff leads the buyer and the incumbent to place *more* competitive pressure on larger competitors: the threshold

$\hat{\omega}_E(s_E)$ is monotonically increasing in s_E .

If q_E were equal to s_E above this threshold and zero below, the quantity purchased from the rival would locally decrease with s_E , which is impossible. Hence the presence of bunching along the s_E -dimension. Following the constructive method presented in Appendix E, we find bunching intervals such as the horizontal interval $[A, C]$ represented on Figure 3a. A rival whose type belongs to $[A, C]$ sells s_E^1 units, where s_E^1 denotes the left extremity of the bunching interval. The unit virtual surplus $s^v(s_E, \omega_E)$ is positive (resp. negative) on $[A, B]$ (resp. $[B, C]$), as $\omega_E > \hat{\omega}_E(s_E)$ (resp. $\omega_E < \hat{\omega}_E(s_E)$) in this region. The buyer and the incumbent must leave a positive rent to rivals in $[B, C]$ because they cannot prevent them from selling less than their contestable demand. The virtual surplus must be zero in expectation on bunching segments

$$\int_{s_E^1}^{\bar{s}_E} s^v(s, \omega_E) f(\omega_E|s) g(s) ds = 0.$$

Proposition E.1 provides three different conditions under which the above equation defines an increasing relationship between ω_E and s_E^1 , denoted by $\omega_E = \Psi(s_E^1)$ on Figure 3a. These sufficient conditions imply mild restrictions on the distribution of ω_E , the range of the elasticity of entry and the bargaining power β (see Appendix E).⁸ The light-shaded area on the figure represents the set of types for which the competitor is partially foreclosed: $0 < q_E(s_E, \omega_E) = s_E^1 < s_E$.

Proposition 5. *Assume that $\varepsilon(\omega_E|s_E)$ decreases with s_E . Then the optimal tariff is convex. The equilibrium outcome exhibits inefficient exclusion, in the form of both full and partial foreclosure.*

Proof. For all $s_E \in [\underline{s}_E, \bar{s}_E]$, $\omega_E = \Psi(s_E)$ and $s_E' > s_E$, the solution of the buyer-competitor problem (21) is interior for (s_E', ω_E) . Absent two-dimensional bunching, and the solution, $q_E = s_E$, is given by the first-order condition $T'(1 - s_E) = v_I - \Psi(s_E)$ or $T'(q_I) = v_I - \Psi(1 - q_I)$, which increases with q_I because Ψ is an increasing function. We conclude that the price-quantity schedule T is convex. The analysis holds under two-dimensional bunching, with the minor

⁸When none of these conditions holds, the quantity purchased from the rival may be constant on two-dimensional regions in the (s_E, ω_E) -space. Two-dimensional bunching is studied in supplementary section I.5.

difference the price schedule is locally non-differentiable (it admits a convex kink). \square

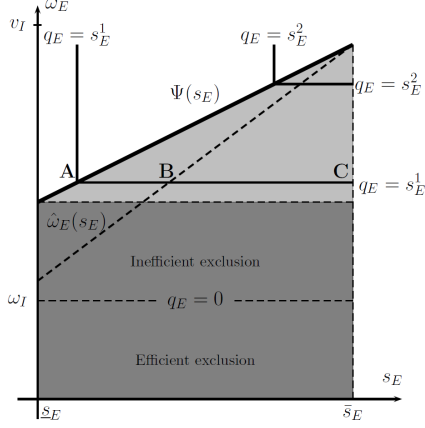


Figure 3a: Rent-efficiency threshold $\hat{\omega}_E$ (dashed), second-best threshold $\Psi(s_E)$ (bold) with decreasing elasticity

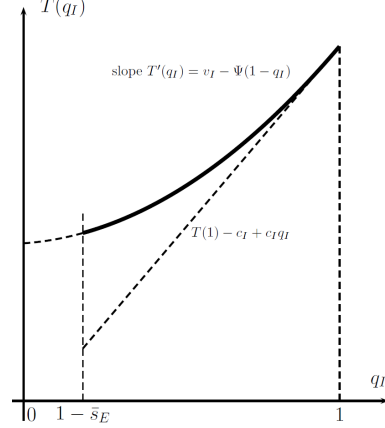


Figure 3b: Optimal price schedule with $\underline{s}_E = 0$ and $v_I > \hat{\omega}_E(1)$.

The price schedule plays the role of a barrier to expansion. When the price schedule $T(q_I)$ is convex in q_I , the objective of the buyer-rival pair, $(\omega_E - v_I)q_E - T(1 - q_E)$, is concave in q_E . The buyer and the rival compare the surplus created by an extra unit of good E , ω_E , with the surplus foregone by consuming one unit less of good I , $v_I - T'(1 - q_E)$. Rivals whose types lie in the light-shaded triangle represented on Figure 3a are induced to sell q_E units with $q_E < s_E$ and $\omega_E = v_I - T'(1 - q_E)$. This quantity q_E depends on ω_E but not on s_E . Such rivals are partially foreclosed from the market.

Defendants in antitrust litigation commonly put forward that the alleged abuse did not prevent competitors from achieving a sizeable share of the market. Our analysis points out that antitrust enforcers are right to discard this line of defense as a positive market share is not incompatible with (partial) anticompetitive foreclosure.

5.3 “Retroactive” price schedules

We now show that the buyer and the incumbent use “retroactive” or “all-units” rebates if a simple condition on the elasticity of entry is satisfied. Such rebates

are granted if the buyer reaches a quantity threshold and apply to *all* units purchased, not only to units above the threshold. They induce downwards discontinuities in price-quantity schedules. Figure 5a shows the most simple retroactive price schedule. The slopes of the two segments correspond to the unit prices that are applied to all units depending on whether or not the quantity threshold \bar{q}_I is attained.

We assume that the elasticity of entry is first decreasing then increasing as the size of the contestable demand rises: competitors with intermediate size are less sensitive to competitive pressure than competitors with small or large size. Under this circumstance, the efficiency-rent tradeoff leads the buyer and the incumbent to place strong competitive pressure on competitors with intermediate size and less pressure on small or large competitors: $\hat{\omega}_E(s_E)$ is inverted U-shaped.

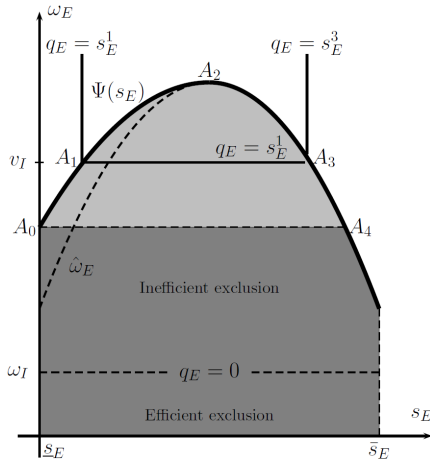


Figure 4a: Rent-efficiency tradeoff $\hat{\omega}_E(s_E)$ (dashed), second-best threshold (solid) with U-shaped elasticity

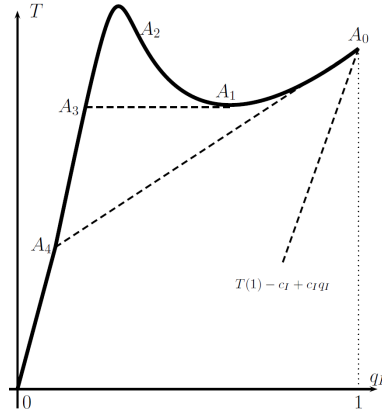


Figure 4b: Optimal price schedule with $s_E = 0$ and $\bar{s}_E = 1$

We rely on Figures 4a and 4b to explain the shape of the optimal price schedule in this instance. Above the solid curve on Figure 4a, the competitor serves all of the contestable demand. In the light-shaded area below the solid curve, the quantity purchased from the competitor does not depend on the size of the contestable market. For instance, a rival whose type lies on the horizontal segment $[A_1, A_3]$ sells s_E^1 units. On such an interval, the unit virtual surplus is

negative at the right of the dashed line $\omega_E = \hat{\omega}_E(s_E)$ and positive at its left.

Between A_0 and A_2 , the equation of the light-shaded area's upper boundary, $\omega_E = \Psi(s_E)$, follows from the condition that the expected virtual surplus is zero on the bunching intervals such as the interval $[A_1, A_3]$. As seen in Section 5.2, the quantity chosen by the buyer and the competitor is an interior solution of their surplus maximization and is therefore given by the first-order condition: $T'(1 - s_E) = v_I - \Psi(s_E)$; the price-quantity schedule T is convex in this region, see Figure 4b.

Between A_2 and A_4 , we recover the tariff by expressing that the quantity purchased from the rival is constant on the bunching segments. For example, if the rival is at A_3 , the buyer-rival pair is indifferent between buying s_E^1 or s_E^3 : $(\omega_E - v_I)s_E^1 - T(1 - s_E^1) = (\omega_E - v_I)s_E^3 - T(1 - s_E^3)$. As $T(1 - s_E^1)$ is known, one can infer $T(1 - s_E^3)$. At points A_1 and A_3 , we have $\omega_E = v_I$, and hence $T(1 - s_E^1) = T(1 - s_E^3)$. It is readily confirmed that $T'' = 0$ at A_2 , i.e. T has an inflexion point.

Thus, an U-shaped elasticity of entry leads to a price-quantity schedule that is neither globally concave nor globally convex. The decreasing part of the price schedule gives the buyer a strong incentive to supply from the incumbent beyond the point A_1 .

When the distribution of types is continuous, the optimal price schedule is continuous. If instead the size of the contestable demand takes a finite number of values, a price schedule with a retroactive rebate, such as the one superimposed on Figure 5a, is optimal. Specifically, suppose that the support of s_E consists of three points $s_E^1 < s_E^2 < s_E^3$ and that the distribution of ω_E given s_E is such that $\hat{\omega}_E(s_E^2) > \max(\hat{\omega}_E(s_E^1), \hat{\omega}_E(s_E^3))$. Then the rent-efficiency tradeoff leads the buyer and the incumbent to place more (less) competitive pressure on the rivals with contestable demand s_E^2 (s_E^1 and s_E^3). This can be done with the schedule shown on Figure 5b. Critical on this figure are the slopes of the three dashed lines, which reflect the pressure put at each level.⁹ Rivals with contestable market share s_E^2 or s_E^3 serve all of the contestable demand when they generate a high surplus ω_E , sell s_E^1 units when they generate a moderate surplus (partial

⁹The quantity threshold (the discontinuity point in the schedule) can take any value between $1 - s_E^2$ and $1 - s_E^1$.

foreclosure) or are inactive when they generate a low surplus.

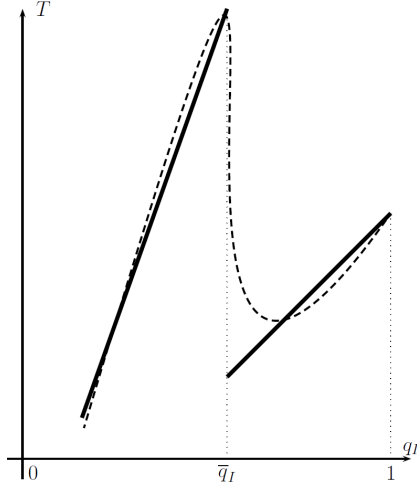


Figure 5a: Retroactive rebate approximating an optimal price schedule

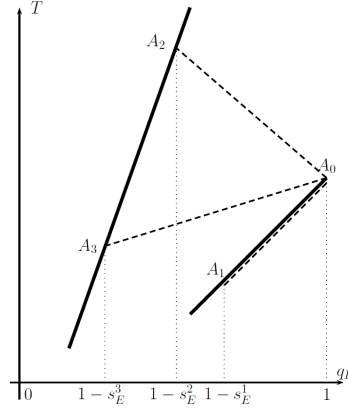


Figure 5b: Retroactive rebate when the contestable demand takes three values

The analysis of this section has assumed that disposal costs are very large. Indeed, suppose that the magnitude of disposal costs, γ , is smaller than the absolute value of the slope between A_1 and A_2 on Figure 5b. This is the case, for instance, under free disposal ($\gamma = 0$). Then the buyer who has purchased s_E^2 units from the rival finds it ex post optimal to purchase more units of good I than the efficient quantity, namely $1 - s_E^1 > q_I^* = 1 - s_E^2$. As formally stated in Proposition 2, the possibility of buyer opportunism makes it suboptimal for the buyer and the incumbent to offer marginal prices below $-\gamma$. The minimum level of the disposal costs that sustains the price schedule represented on Figure 4b is the opposite of the slope at the inflexion point A_2 .

6 The role of disposal costs

We now investigate how the magnitude of disposal costs affects the equilibrium outcome. Purchased units for which the buyer has no use might entail zero or even negative cost if they can be freely stored, disposed of, or resold on secondary markets. In some industries, however, unused items are heavy, voluminous, or dangerous, and thus are difficult to store or dispose of. Moreover, they may have

buyer-specific characteristics that make them difficult to resell.¹⁰

As explained above, the possibility of buyer opportunism under an unconditional price schedule constrains the marginal price to be above $-\gamma$, where γ denotes the magnitude of disposal costs (Proposition 2). As a result, super-efficient rivals, i.e. rivals with $\omega_E \geq v_I + \gamma$, serve all of the contestable demand at the second-best optimum (Corollary 1).

Consider for instance the case of free disposal, $\gamma = 0$. On Figure 4a, the portion of the light-shaded area above the segment $[A_1, A_3]$ is irrelevant: the curve $\omega_E = \Psi(s_E)$ must be truncated and replaced with $\min(\Psi(s_E), v_I + \gamma)$. On Figure 4b, the portion of the price schedule between A_1 and A_3 is irrelevant as well, and can be replaced with a flat part between these two points. More generally, for an arbitrary level of γ , the truncation procedure and its consequences on the optimal unconditional tariff are represented on Figures 6a and 6b. Between B_1 and B_2 , the marginal price is $-\gamma$.

Similarly if $\hat{\omega}_E(0) > v_I + \gamma$, the threshold $\hat{\omega}_E(s_E)$ on Figure 2a must be replaced with $v_I + \gamma$ for low values of s_E . The effective price $p^e(s_E)$ is replaced with $-\gamma$ in this region. It follows that the price schedule $T(q_I)$ is linear for high values of q_I . When $\hat{\omega}_E(\bar{s}_E) > v_I + \gamma$ on Figure 3a, the threshold $\hat{\omega}_E$ and the marginal price $T'(q_I)$ must be replaced with respectively $v_I + \gamma$ and $-\gamma$ for high s_E . In this case, the price schedule $T(q_I)$ is linear for low values of q_I .

Proposition 6. *When the buyer and incumbent are not allowed to condition the price schedule on the quantity purchased from the competitor, their expected profit, $\mathbb{E}\Pi_{BI}$, is nondecreasing and total welfare is non-increasing in the magnitude of disposal costs.*

For very large disposal costs, the second-best allocation is the same whether or not the price schedule can be made contingent upon quantities purchased from the rival.

Proof. The monotonicity of $\mathbb{E}\Pi_{BI}$ in γ follows from Proposition 2. Indeed, the constraint that the marginal price should not be below $-\gamma$ is milder as γ rises.

As seen in Section 5, the optimal allocation when disposal costs are very large is characterized by the boundary line $w_E = \Psi(s_E)$ above which the competitor

¹⁰Disposal costs also depend on the seller's ability to impose or to prevent particular uses of the purchased units and on the buyer's ability to avoid monitoring by the dominant firm.

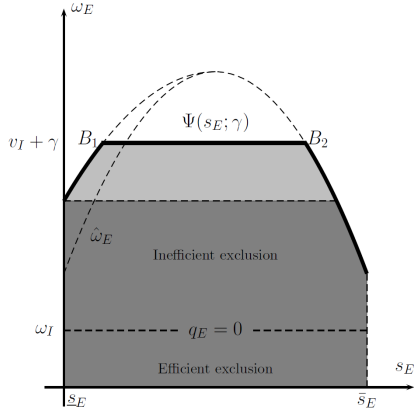


Figure 6a: Truncated second-best threshold (solid) with U-shaped elasticity

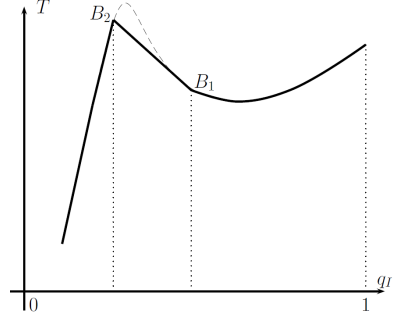


Figure 6b: Optimal price schedule

serves all of the contestable demand, see the solid curves on Figures 2a, 3a and 4a. Assuming away the complications of two-dimensional bunching,¹¹ the function Ψ derives from the condition that the expected virtual surplus is zero on horizontal bunching intervals.

For an arbitrary value of the disposal costs, we know from Corollary 1 that super-efficient competitors $\omega_E > v_I + \gamma$ serve all of the contestable demand at the second-best optimum. In other words, the boundary line is necessarily located below the horizontal line $\omega_E = v_I + \gamma$. Accordingly we replace $\Psi(s_E)$ with $\min(\Psi(s_E), v_I + \gamma)$. This truncation respects the bunching conditions on horizontal intervals and hence maximizes the expected virtual surplus on the set $\omega_E \leq v_I + \gamma$. Moreover, along the truncated boundary line $\omega_E = \min(\Psi(s_E), v_I + \gamma)$, we have, using (21)

$$T(1) - T(1 - s_E) = (v_I - \omega_E)s_E + \Delta S_{BE}(s_E, \omega_E).$$

Differentiating with respect to s_E and observing the terms coming from ω_E cancel out by the envelope theorem, we get

$$T'(1 - s_E) = v_I - \omega_E + \frac{\partial \Delta S_{BE}}{\partial s_E} \geq v_I - \omega_E \geq -\gamma,$$

where we have used the monotonicity of ΔS_{BE} in s_E , which follows from (21). The truncation procedure therefore yields an allocation that maximizes the expected virtual surplus and a price schedule that respects the constraint $T' \geq -\gamma$.

¹¹Proposition E.1 provides mild conditions that rule out two-dimensional bunching.

The above truncation does not change the boundary line when there are no super-efficient competitors, $\bar{\omega}_E \leq v_I + \gamma$, or when the efficiency-rent tradeoff requires that super-efficient competitors serve all of the contestable demand, i.e. $\hat{\omega}_E(s_E) \leq v_I + \gamma$ for all s_E . On the other hand, suppose that $\hat{\omega}_E(s_E) > v_I + \gamma$ for some value of s_E . Then the construction of Ψ under $\gamma = +\infty$ shows that the maximum of Ψ is larger than $v_I + \gamma$: the constraint $T' \geq -\gamma$ is binding and the possibility of buyer opportunism under finite disposal costs lowers the buyer-incumbent pair's profit. \square

When the buyer and the incumbent cannot condition prices on quantities purchased from the competitor, the possibility of ex post buyer opportunism under finite disposal costs limits the competitive pressure placed on the rival, thus protecting super-efficient competitors from exclusion. Lower disposal costs reduce the extent of inefficient foreclosure. In the polar case where $\gamma = -c_I$, the constraint $T'(q_I) \geq -\gamma$ leaves no scope for anticompetitive exclusion.

In contrast, as explained in Section 4.1, conditional tariffs $T(q_E, q_I)$ provide enough flexibility to address separately buyer opportunism and the efficiency-rent tradeoff, irrespective of the magnitude of disposal costs. In particular, such tariffs make it possible and profitable for the buyer and the incumbent to exclude super-efficient competitors. When disposal costs are very large, however, a ban on conditional tariffs would make no difference as buyer opportunism is absent by assumption.

Overall, the exclusionary effects of nonlinear pricing by dominant firms depend on the shape of the tariffs and on the magnitude of disposal costs. When disposal costs are low, unconditional tariffs are potentially less harmful to competition. Both types of nonlinear tariffs, however, deserve attention from antitrust enforcers in industries where disposal costs are large.

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Appendix

A Proof of Proposition 1

The surplus gain ΔS_{BE} , given by (8), is convex in ω_E because it is the upper bound of a family of functions that depend linearly on ω_E . It follows that ΔS_{BE} is almost everywhere differentiable and that its derivative, βq_E , is nondecreasing in ω_E . Conversely, Fenchel duality theory shows that any convex function ΔS_{BE} can be written as (8) for a certain tariff T . The problem is thus equivalent to maximizing (10) subject to the monotonicity constraint on q_E .

By concavity of the expected virtual surplus S^v , the optimal quantity is given by the first-order condition (12), which, combined with (14), yields the characterization (16). We must also find the profit $\Pi_E(\underline{\omega}_E)$ left to the least efficient rival. If the optimal quantity is zero for that type, $q_E(\underline{\omega}_E) = 0$, then $\Pi_E(\underline{\omega}_E) = \Delta S_{BE}(\underline{\omega}_E) = 0$ by (8). Otherwise, if $q_E(\underline{\omega}_E) > 0$, the less efficient rival's profit, $\Pi_E(\underline{\omega}_E)$, is set to zero by an appropriate choice of the difference $P(q_E(\underline{\omega}_E)) - P(0)$.

We now check that $\omega_E - \partial h / \partial q_E(q_E, q_I^*(q_E))$ decreases in q_E . Differentiating $W_{q_I}(q_E, q_I^*(q_E); \omega_E) = 0$ with respect to q_E yields

$$W_{q_E q_I} + (q_I^*)'(q_E) W_{q_I q_I} = 0.$$

Differentiating the left-hand side of (13) with respect to q_E , we get

$$\frac{d}{dq_E} W_{q_E}(q_E, q_I^*(q_E); \omega_E) = W_{q_E q_E} + W_{q_E q_I} (q_I^*)' = W_{q_E q_E} - \frac{(W_{q_E q_I})^2}{W_{q_I q_I}},$$

which is negative by concavity of W . The left-hand side of (16) is therefore decreasing in q_E , which yields uniqueness of a solution to Equation (16). The solution exists and is positive if and only if $\omega_E - \partial h / \partial q_E(0, q_I^*(0)) > \beta \omega_E / \varepsilon(\omega_E | S_E)$, which is true for ω_E close enough to $\bar{\omega}_E$ by Assumption 1. Otherwise, the optimal quantity is at the corner $q_E = 0$.

As the left-hand (right-hand) side of (16) increases with (is non-increasing in) ω_E , we find that q_E increases with ω_E , hence the monotonicity constraint

on q_E is respected. Under the slightly stronger assumption that $f/(1 - F)$ is nondecreasing, equation (14) then yields the concavity of the price schedule in q_E .

The quantity purchased from the rival is distorted downwards because W is concave in q_E and W_{q_E} is positive at the second-best optimum. The higher the rival's bargaining power vis-à-vis the buyer β , and the smaller the elasticity ε , the more severe the downward distortion of q_E .

B Buyer's utility under disposal costs

We first present a formal expression for the utility function $V(q_E, q_I; \theta_E)$ used in Sections 4, 5 and 6. Recall that $\theta_E = (s_E, v_E)$ represents the characteristics of good E , namely the size of the contestable demand s_E and the buyer's valuation v_E . The magnitude of disposal costs in the concerned industry is given by γ .

Having purchased quantities q_E and q_I from the rival and the dominant firm, the buyer chooses consumption levels x_E and x_I so as to maximize

$$V(q_E, q_I; \theta_E) = \max_{(x_E, x_I) \in X(q_E, q_I)} v_E x_E + v_I x_I - \gamma(q_E - x_E) - \gamma(q_I - x_I)$$

where the set $X(q_E, q_I)$ is defined by the constraints $x_E \leq q_E$, $x_E \leq s_E$, $x_I \leq q_I$, and $x_E + x_I \leq 1$: the buyer cannot consume more of each good than the purchased quantity, more of both goods together than her total requirement, and more of good E than the contestable demand.

Next, we prove the concavity of V in the vector (q_E, q_I) . Consider two vectors $q^0 = (q_E^0, q_I^0)$, $q^1 = (q_E^1, q_I^1)$ and a weight α with $0 \leq \alpha \leq 1$. Let $x^0 = (x_E^0, x_I^0)$ and $x^1 = (x_E^1, x_I^1)$ denote the consumption levels chosen by the buyer after having purchased q_0 and q_1 . By linearity, $\alpha x^0 + (1 - \alpha)x^1$ belongs to the set $X(\alpha q^0 + (1 - \alpha)q^1)$. The result follows immediately.

Finally, we prove that the two goods are substitutable, formally V is supermodular. Consider again $q^0 = (q_E^0, q_I^0)$, $q^1 = (q_E^1, q_I^1)$. Let q^\wedge and q^\vee be the componentwise minimum and maximum of q^0 and q^1 . We want to prove that

$$V(q^\wedge) + V(q^\vee) \leq V(q_0) + V(q_1). \quad (\text{B.1})$$

Let x^\wedge and x^\vee denote optimal consumption levels associated with q^\wedge and q^\vee . We have $x^\wedge \leq x^\vee$ component by component. Suppose, with no loss of generality, that $q_E^\vee = q_E^0$, and hence $x_E^\vee \leq q_E^0$.

- If $q_I^\vee = q_I^0$, we have $x_I^\vee \leq q_I^0$. We set $x_0 = (x_E^\vee, x_I^\vee)$ and $x_1 = (x_E^\wedge, x_I^\wedge)$ and observe that $x_i \in X(q^i)$, which yields (B.1).
- If $q_I^\vee = q_I^1$, we set $x_0 = (x_E^\vee, x_I^\wedge)$ and $x_1 = (x_E^\wedge, x_I^\vee)$, check again that $x_i \in X(q^i)$, which yields (B.1).

C Proof of Proposition 2

The proof follows from two lemmas.

Lemma C.1. *Starting from any tariff T , we can find a tariff \hat{T} such that the marginal price \hat{T}' is greater than or equal to $-\gamma$ and the surplus of the buyer-incumbent pair is not lower under \hat{T} than under T . The buyer never purchases more than her total requirements: $q_I(q_E; \theta_E) \leq 1 - q_E$ for any q_E .*

Proof. Starting from any tariff T , we define \hat{T} as

$$\hat{T}(q_I) = \inf_{q \geq q_I} T(q) + \gamma(q - q_I). \quad (\text{C.1})$$

The tariff \hat{T} is affine with slope $-\gamma$ in regions where the lower bound in (C.1) is reached at $q > q_I$. Formally, we have: $\hat{T}(q_I) = T(q_I) + (\gamma - \lambda)(q - q_I)$, where q is a solution to the above problem and λ is the Lagrange multiplier associated to the constraint $q \geq q_I$. The envelope theorem yields $\hat{T}'(q_I) = \lambda - \gamma \geq -\gamma$.

First we check that the buyer and the rival choose the same quantity q_E under the tariffs T and \hat{T} . Let $U(q_E)$ and $\hat{U}(q_E)$ denote the buyer's net utility if she has purchased units q_E units from the competitor under T and \hat{T} :

$$U(q_E) = \max_{q_I} V(q_E, q_I) - T(q_I) \quad \text{and} \quad \hat{U}(q_E) = \max_{q_I} V(q_E, q_I) - \hat{T}(q_I).$$

As $\hat{T} \leq T$, we have: $\hat{U} \geq U$. Suppose that, under \hat{T} , it is optimal for the buyer to purchase \hat{q}_I from the incumbent if she has purchased q_E from the competitor. By construction of \hat{T} , there exists $q_I \geq \hat{q}_I$ such that $\hat{T}(q_I)$ equals or is arbitrarily

close to $T(q_I) + \gamma(q_I - \hat{q}_I)$. Observing that buying an extra unit of good I cannot deteriorate the buyer's utility by more than $-\gamma$, i.e. $\partial V/\partial q_I \geq -\gamma$, we get:

$$\hat{U}(q_E) = V(\hat{q}_I, q_E) - \hat{T}(\hat{q}_I) = V(\hat{q}_I, q_E) - \gamma(q_I - \hat{q}_I) - T(q_I) \leq V(q_I, q_E) - T(q_I).$$

It follows that $\hat{U}(q_E) \leq U(q_E)$, and hence $\hat{U}(q_E) = U(q_E)$ for any q_E . To decide on the quantity q_E , the buyer and the rival maximize $U(q_E) - c_E q_E$ under tariff T and $\hat{U}(q_E) - c_E q_E$ under tariff \hat{T} . As the two objectives coincide, they agree on the same quantity under the two tariffs: $q_E(c_E, \theta_E) = \hat{q}_E(c_E, \theta_E)$ for any c_E, θ_E . For the same reason, the rival's profit, $\beta \Delta S_{BE} = \beta[U(q_E) - U(0) - c_E q_E]$, is the same under T and \hat{T} .

Second, we check that under tariff \hat{T} the buyer may purchase less than $1 - q_E$ from the incumbent and that the total welfare is not lower under \hat{T} than under T . Let q_E and q_I denote the purchased quantities under tariff T . As $\hat{T}(q_I) \leq T(q_I)$, the buyer may always choose to purchase the same quantity from the incumbent ($\hat{q}_I = q_I$) under the tariffs T and \hat{T} :

$$U(q_E) = \tilde{U}(q_E) = V(q_E, q_I) - T(q_I) \leq V(q_E, q_I) - \hat{T}(q_I).$$

Now consider the special case where $q_I > 1 - q_E$. As explained at the end of Section 4.1, purchasing one extra unit of good I in the region where $q_I > 1 - q_E$ decreases the buyer utility by γ if $v_E > v_I$ or if $q_E = 0$ and by $v_I - v_E - \gamma$ if $v_I > v_E$ and $q_E > 0$. In the latter case, the buyer would indeed consume the extra unit of good I and dispose of a unit of the rival good instead. Yet this latter case is impossible here because the buyer and the rival would reduce q_E in the first place, thus improving their joint surplus $V(q_E, q_I) - T(q_I) - c_E q_E$. We conclude that $\partial V/\partial q_I = -\gamma$ in this region. Under tariff \hat{T} , the buyer is better off purchasing $\hat{q}_I = 1 - q_E$ rather than $q_I > 1 - q_E$ from the incumbent. This is because she saves $\gamma(q_E + q_I - 1)$ in terms of disposal costs and loses no more than the same amount in terms of price subsidy.¹² The change from q_I to \hat{q}_I does not decrease the total surplus. On the contrary, it avoids inefficient production and

¹²If the tariff is affine with slope $-\gamma$ in the corresponding region, the buyer is actually indifferent between purchasing q_I and $1 - q_E$ from the incumbent. To break the indifference, we use $\hat{T}(q_I) = \inf_{q \geq q_I} T(q) + \gamma'(q - q_I)$, for γ' slightly lower than γ . The buyer then strictly prefers $1 - q_E$ to $q_I > 1 - q_E$, for any (c_E, θ_E) .

disposal costs:

$$W(q_E, \hat{q}_I) = V(q_E, \hat{q}_I) - c_E q_E - c_I \hat{q}_I \geq V(q_E, q_I) - c_E q_E - c_I q_I = W(q_E, q_I).$$

In sum, the change from T to \hat{T} does not alter the competitor's profit and does not decrease the total surplus. We conclude from (7) that the change does not decrease the expected payoff of the buyer-incumbent coalition, and that $q_I \leq 1 - q_E$ for any (c_E, θ_E) . \square

Lemma C.2. *Starting from any tariff T , we can find a tariff \tilde{T} such that the marginal price \tilde{T}' is lower than or equal to v_I and the surplus of the buyer-incumbent pair is not lower under \tilde{T} than under T . We conclude that the buyer never purchases less than her total requirements: $q_I(q_E; \theta_E) \geq 1 - q_E$ for any q_E .*

Proof. The proof is very similar to that of Lemma C.1. See Appendix G. \square

Taken together, Lemmas C.1 and C.2 yield ex post efficiency: $q_I = q_I^*(q_E; \theta_E) = 1 - q_E$ for all c_E, θ_E .

D Elasticity of entry and distribution of uncertainty

In this section, we prove Lemma 2. The elasticity of entry varies with s_E in the same way as the hazard rate h given by

$$h(\omega_E | s_E) = \frac{f(\omega_E | s_E)}{1 - F(\omega_E | s_E)}.$$

We have

$$\int_{\underline{\omega}_E}^{\omega_E} h(x | s_E) dx = -\ln[1 - F(\omega_E | s_E)].$$

If the elasticity of entry does not depend on (increases with, decreases with) s_E , the same is true for the hazard rate, and hence also for the cdf $F(\omega_E | s_E)$, which yields the results.¹³

¹³The variable ω_E first-order stochastically decreases (increases) with s_E if and only if $F(\omega_E | s_E)$ increases (decreases) with s_E .

E Derivation of the optimal quantity function

In the above section, the optimal price schedule has been obtained by solving the rent-efficiency tradeoff separately for each size of the contestable market. This method, however, does not in general yield an incentive compatible allocation. To illustrate, suppose that the ERT line is as shown on Figure 7. The solution to the relaxed problem, which is zero below this line and s_E above, is not incentive compatible. Indeed, the rival of type $B = (\omega_E, s_E)$ is inactive and earns zero profit, while the rival $A = (\omega_E, s'_E)$, $s'_E < s_E$, serves all of the contestable demand. It follows that rival B has an incentive to sell s'_E and mimic rival A . Hence, in this example, solving the relaxed problem does not yield the second-best allocation.

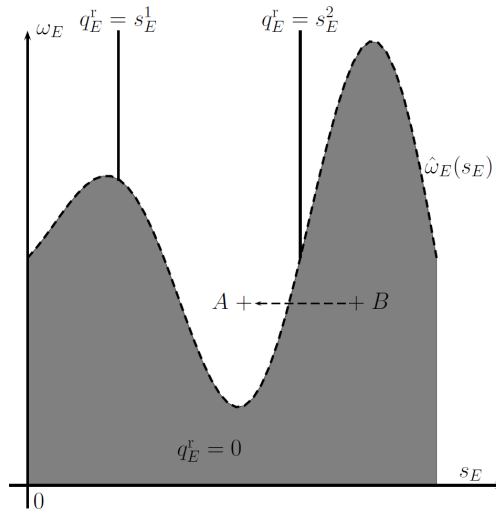


Figure 7: ERT line (dashed). Here, the relaxed solution is not implementable.

We now characterize implementable quantity functions and offer a heuristic derivation of second-best allocations. The main idea is that configurations like that of Figure 7 give rise to partial foreclosure, for which an appropriate first-order condition must be derived. We do not insist on the mathematical resolution of the problem, which is relegated in a technical appendix available from the authors.¹⁴ The readers interested only by the qualitative results regarding the

¹⁴Deneckere and Severinov (2009) propose a method for solving a more general class of problems, which relies on a characterization of “isoquants”. We exploit here the particular shape of these curves, see in particular Figure 8 below.

shape of optimal price schedules should proceed directly to Section 5.2.

Implementable quantity functions The buyer and the competitor maximize their joint surplus, knowing the unconditional price schedule $T(q_I)$ agreed upon by the buyer and the incumbent. They choose a quantity q_E that depends on the competitor's characteristics, (s_E, ω_E) , which gives rise to a "quantity function" $q_E(s_E, \omega_E)$. Assume infinite disposal costs ($\gamma = +\infty$) and relying on Proposition 2, we can restrict attention to price schedules whose marginal does not exceed v_I . A quantity function $q_E(s_E, \omega_E)$ is implementable with an unconditional price schedule if and only if there exists a function $T(q_I)$ satisfying $T'(q_I) \leq v_I$ such that $q_E(s_E, \omega_E)$ is solution to (??) for all (s_E, ω_E) .

As q_E is nondecreasing in ω_E , there exists, for any $s_E > 0$, a threshold $\Psi(s_E)$ such that the buyer supplies all contestable units from the competitor, $q_E(s_E, \omega_E) = s_E$, if and only if $\omega_E > \Psi(s_E)$. We define the *boundary line* $\omega_E = \Psi(s_E)$ associated to the quantity function $q_E(s_E, \omega_E)$ by

$$\Psi(s_E) = \inf\{x \in [\underline{\omega}_E, \bar{\omega}_E] \mid q_E(x, s_E) = s_E\},$$

with the convention $\Psi(s_E) = \bar{\omega}_E$ when the above set is empty. Above the boundary line, $q_E(s_E, \omega_E)$ equals s_E ; below that line, it is independent on s_E .

Boundary line and quantity function As shown on Figure 8, an implementable quantity function $q_E(., .)$ is entirely described by the associated boundary line. The bunching sets, i.e. the sets on which the quantity $q_E(s_E, \omega_E)$ is constant, are determined by the boundary line. They can be of three types: (i) vertical lines above points on the boundary line where that line decreases (e.g. $q_E = s_E^3$ and $q_E = s_E^4$ on the Figure); (ii) "L"-shaped unions of vertical lines above and horizontal lines above and at the right of points where the boundary line increases (e.g. $q_E = s_E^1$, $q_E = s_E^2$ and $q_E = s_E^5$); (iii) two-dimensional areas whose left border is vertical, being included either in the ω_E -axis (then $q_E = 0$, see the shaded area on Figure 8) or in a vertical part of the boundary line (see the light shaded area on Figure 10b).

Partial foreclosure Increasing parts of the boundary function thus translate into horizontal bunching segments or into two-dimensional bunching areas, and

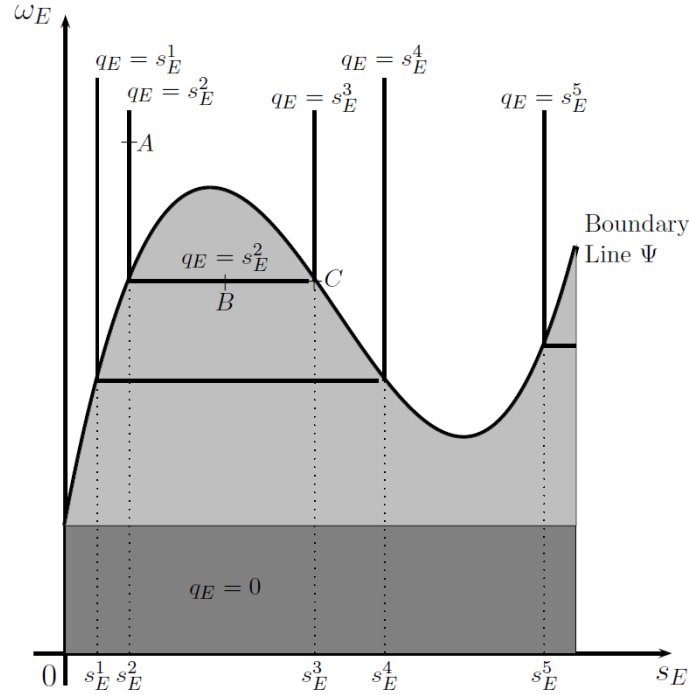


Figure 8: Implementable quantity function (isolines)

hence into partial foreclosure: $0 < q_E(s_E, \omega_E) < s_E$ for some types located below the boundary. (For instance, type B on Figure 8 sells $q_E = s_E^2$, which is lower than the size of its contestable market.) In such regions, the constraint $q_E \leq s_E$ is slack: increasing s_E does not allow the competitor to enter at a larger scale and q_E does not depend on s_E .

Shape of the boundary line and curvature of the tariff In Appendix H, we explain how to recover the price schedule T from the boundary function Ψ and we link the shape of the price schedule to that of the boundary line. Flat parts of the boundary line correspond to linear parts of the tariff (see Figure 1a and 1b) and increasing parts of the boundary line correspond to convex parts of the tariff (see Figures 3a and 3b, or the interval A_1A_3 on Figures 4a and 4b). In both cases, the constraint $q_E \leq s_E$ in the buyer-competitor pair's problem (??) is not binding.

In contrast, the curvature of the tariff may change along decreasing parts of the boundary: the tariff is concave near local maxima of the boundary line and convex near local minima. Local maxima of the boundary line thus correspond

to inflection points of the tariff. An example is the point $A3$ on Figures 4a and 4b.

Construction of the optimal allocation We now explain intuitively how to derive the optimal boundary line $\omega_E = \Psi(s_E)$ from the ERT line $\omega_E = \hat{\omega}_E(s_E)$.

Consider a point (s_E, ω_E) above the ERT line. If the virtual surplus is always positive at the right of this point, there is no objection to setting $q_E = s_E$. In contrast, if the virtual surplus is negative at the right of this point, setting $q_E = s_E$ implies that q_E will have to be positive in an area where the virtual surplus is negative. By a standard ironing procedure, we show that the expected virtual surplus on horizontal bunching segments is zero. Denoting by $[AB]$ such a segment (see Figure 9b), we get

$$\mathbb{E}(s^v \mid [AB]) = 0, \quad (\text{E.1})$$

with the boundary conditions that the virtual surplus is positive at A and zero at B . This leads to the following construction of the optimal boundary line $\omega_E = \Psi(s_E)$. We first draw the ERT line $\omega_E = \hat{\omega}_E(s_E)$. For $s_E = \bar{s}_E$, we set $\Psi(\bar{s}_E) = \hat{\omega}_E(\bar{s}_E)$. Then we consider lower values of s_E . If the ERT line decreases at \bar{s}_E , the boundary coincides with the ERT line, as long as it remains decreasing. When the ERT line starts increasing (possibly at \bar{s}_E), we know that there is horizontal bunching. Equation (E.1) provides a unique value for $\Psi(s_E)$. If the candidate boundary hits the ERT line at some value of s_E , it must be on a decreasing part of that line and, from that value on, the optimal boundary coincides with the ERT line (as long as $\hat{\omega}_E$ remains decreasing). Proposition E.1 in Appendix E presents three different sets of sufficient conditions under which the above construction indeed yields the optimal allocation.¹⁵

Proposition E.1. *Assume that one of the following conditions holds:*

1. *The conditional density $f(\omega_E | s_E)$ is nondecreasing in ω_E ;*
2. *The hazard rate, $f/(1 - F)$, is nondecreasing in ω_E and β , $\underline{\varepsilon}$ and $\bar{\varepsilon}$ satisfy*

$$\beta \leq 4\underline{\varepsilon}\bar{\varepsilon}/(\Delta\varepsilon)^2; \quad (\text{E.2})$$

¹⁵When none of the three sufficient conditions holds, the increasing parts of the optimal boundary line may have vertical portions, generating two-dimensional bunching areas. A vertical ironing procedure is thus needed (see Appendix I.5).

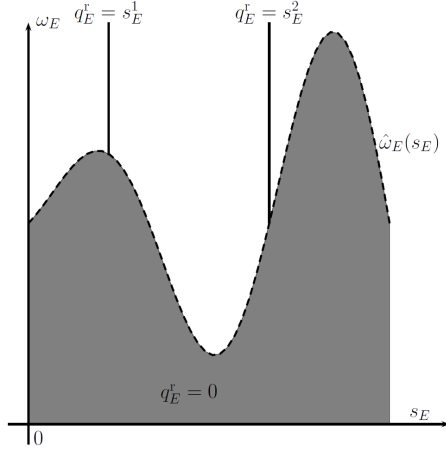


Figure 9a: The relaxed solution locally decreases with s_E .

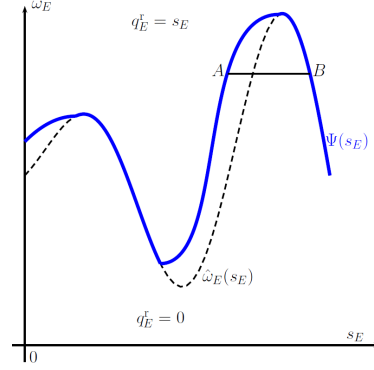


Figure 9b: ERT line $\hat{\omega}_E$ (dashed). Optimal boundary Ψ (solid)

3. The elasticity of entry is nondecreasing in ω_E (Assumption 1) and β , $\underline{\varepsilon}$ and $\bar{\varepsilon}$ satisfy

$$\beta \leq \frac{\bar{\varepsilon}}{1 + (1 + \Delta\varepsilon)^2 / 4\underline{\varepsilon}}. \quad (\text{E.3})$$

Then the complete problem can be solved separately for each ω_E . The optimal boundary line Ψ lies above the ERT line, $\Psi \geq \hat{\omega}_E$, and can be constructed from the following properties:

1. $\Psi(1) = \hat{\omega}_E(1)$;
2. Its non-increasing parts coincide with the ERT line;
3. Its increasing parts are defined by equation (E.1).

Proof. See Appendix I. □

The sufficient conditions of Proposition E.1 are fairly mild. A first sufficient condition is f being nondecreasing in ω_E . A second set of sufficient conditions is the hazard rate $f/(1 - F)$ being nondecreasing in ω_E and the range of the entry elasticity being not too wide (condition (E.2)). A third set of sufficient conditions consists of the elasticity of entry being nondecreasing in ω_E , as stated in Assumption 1,¹⁶ and of another condition on the range of ε , (E.3), more

¹⁶Assumption 1 is weaker than f or $f/(1 - F)$ being nondecreasing in ω_E .

restrictive than (E.2). Technically, the conditions (E.2) and (E.3) involve the rival's bargaining power, β , and the minimum and maximum values of ε in the whole distribution of types, $\underline{\varepsilon}$ and $\bar{\varepsilon}$. Even the stronger condition (E.3) is not very restrictive, in the sense that it allows for a wide range $[\underline{\varepsilon}, \bar{\varepsilon}]$.¹⁷

¹⁷For instance, if the rival's bargaining power, β , equals one, the elasticity of entry may vary freely between $\underline{\varepsilon} = 1.2$ and $\bar{\varepsilon} = 3.98$, or between $\underline{\varepsilon} = 5$ and $\bar{\varepsilon} = 26.64$. If β equals .75, then the elasticity of entry may vary freely between $\underline{\varepsilon} = 1.2$ and $\bar{\varepsilon} = 5.99$, or between $\underline{\varepsilon} = 5$ and $\bar{\varepsilon} = 33.59$.

Supplementary material, for online publication

F Complete information

When the price schedule is allowed to depend on both quantities, B and I commit on a two-part tariff in q_I , whose constant part depends on the quantity purchased from the rival: $T(q_E, q_I) = c_I q_I + P(q_E)$. The linear part induces the efficient choice $q_I^*(q_E, \theta_E)$ ex post, neutralizing buyer opportunism. The constant part, $P(q_E)$, is used to extract all the surplus created by the rival. Specifically, the incumbent imposes a “penalty” $P(q_E^*) - P(0)$ for supplying from the competitor. Setting this penalty at $W(q_E^*, q_I^*) - W(0, q_I^*(0))$ guarantees that $\Pi_E = \Delta S_{BE} = 0$.

When the price schedule depends on q_I only, there is a tension between buyer opportunism and rent extraction. Placing too much competitive pressure on the rival, in practice granting generous quantity rebates, may indeed attract the buyer, who is tempted to actually purchase the corresponding units from the incumbent. It may therefore be optimal to let the rival earn a positive profit at the second-best equilibrium under complete information (see Appendix F).

Lemma F.1 (Marx and Shaffer, 2004). *Under complete information, the buyer purchases the efficient quantities q_E^* and q_I^* from both suppliers. If the tariff is allowed to depend on both quantities, then all the surplus is extracted from the rival: $\Pi_E = 0$. If the tariff is function of q_I only, then the rival earns*

$$\Pi_E = \beta \lim_{\hat{q}_I \rightarrow +\infty} \max_{q_E} [V(q_E, \hat{q}_I) - c_E q_E - V(0, \hat{q}_I)] \geq 0. \quad (\text{F.1})$$

As a preliminary observation, notice that, for any \hat{q}_I , the value of the maximum term in (F.1) is nonnegative by construction. Moreover, this maximum value is non-increasing in \hat{q}_I . Indeed its derivative, given by the envelope theorem, satisfies:

$$\frac{d}{d\hat{q}_I} \max_{q_E} [V(q_E, \hat{q}_I) - c_E q_E - V(0, \hat{q}_I)] = V_{q_I}(q_E, \hat{q}_I) - V_{q_I}(0, \hat{q}_I) \leq 0. \quad (\text{F.2})$$

The limit in (F.1) is therefore the lower bound of the maximum term as \hat{q}_I varies. The rival earns a positive profit under complete information if and only if this lower bound is positive. This is the case with the utility function introduced in

Section 4.1 and a super-efficient rival. Indeed, in this circumstance, the maximum term in (F.1) is constant and equal to $(\omega_E - v_I - \gamma)s_E$ for any $\hat{q}_I \leq 1$. It follows that super-efficient rivals earn a positive profit, $\Pi_E = \beta(\omega_E - v_I - \gamma)s_E > 0$, at the second-best optimum under complete information.

The proof of Lemma F.1 proceeds in two steps. First, we derive a lower bound for the rival's profit. Second, we find a tariff such that the chosen quantities are efficient and the lower bound for the rival's profit is attained.

Step 1.- Let T be any price schedule. Let (q_E, q_I) be the chosen quantities under tariff T , as defined in (2). We also set $\hat{q}_I = q_I(0)$, where the function $q_I(\cdot)$ is defined by (3), and $\hat{T} = T(0)$. For any \hat{q}_E , we have

$$S_{BE}(\hat{q}_E) = V(\hat{q}_E, q_I(\hat{q}_E)) - c_E \hat{q}_E - T(q_I(\hat{q}_E)) \geq V(\hat{q}_E, \hat{q}_I) - c_E \hat{q}_E - \hat{T},$$

hence, using the definition of $S_{BE}(q_E)$:

$$S_{BE}(q_E) \geq \max_{\hat{q}_E} V(\hat{q}_E, \hat{q}_I) - c_E \hat{q}_E - \hat{T}.$$

and

$$\Pi_E = \beta(S_{BE}(q_E) - S_{BE}(0)) \geq \beta \max_{\hat{q}_E} V(\hat{q}_E, \hat{q}_I) - c_E \hat{q}_E - V(0, \hat{q}_I).$$

We have seen above that the value of the maximum term is non-increasing in \hat{q}_I . We conclude that: $\Pi_E \geq \beta L$. We have thus found an upper bound for the buyer-incumbent pair's profit:

$$\Pi_{BI} \leq W(q_E, q_I) - \Pi_E \leq W(q_E^*, q_I^*) - \beta L.$$

Step 2.- We now show that we can find a tariff such that the chosen quantities are q_E^* and q_I^* and the rival's profit equals or is arbitrarily close to βL .

Let \hat{q}_I be such that the maximum term in (F.1) equals or is arbitrarily close to L . Let \hat{q}_E be such that $V(\hat{q}_E, \hat{q}_I) - c_E \hat{q}_E$ is maximal and $V(\hat{q}_E, \hat{q}_I) - c_E \hat{q}_E - V(0, \hat{q}_I)$ equals or is arbitrarily close to L . We have: $V_{q_E}(\hat{q}_E, \hat{q}_I) - c_E \leq 0$, with equality when $\hat{q}_E > 0$.

First we observe that $\hat{q}_I > q_I^*$ and $\hat{q}_E \leq q_E^*$. Indeed, the derivative in (F.2) evaluated at q_I^* is given by $V_{q_I}(q_E^*, q_I^*) - V_{q_I}(0, q_I^*)$ which is negative because $q_E^* > 0$ by assumption. This shows that $\hat{q}_I > q_I^*$. It follows that

$$0 \leq V_{q_E}(\hat{q}_E, \hat{q}_I) - c_E \leq V_{q_E}(\hat{q}_E, q_I^*) - c_E,$$

which yields $\hat{q}_E \leq q_E^*$.

We now define a tariff T up to an additive constant by the following properties: T is linear on the interval $[0, \hat{q}_I)$ with slope c_I and the difference $T(\hat{q}_I) - T(q_I^*)$ is given by

$$V(q_E^*, q_I^*) - T(q_I^*) - c_E q_E^* = V(\hat{q}_E, \hat{q}_I) - T(\hat{q}_I) - c_E \hat{q}_E. \quad (\text{F.3})$$

Using the definition of (q_E^*, q_I^*) and $\hat{q}_I > q_I^*$, we get

$$\begin{aligned} T(\hat{q}_I) - T(q_I^*) &= c_I(\hat{q}_I - q_I^*) \\ &\quad + \{[V(\hat{q}_E, \hat{q}_I) - c_I \hat{q}_I - c_E \hat{q}_E] - [V(q_E^*, q_I^*) - c_E q_E^* - c_I q_I^*]\} \\ &< c_I(\hat{q}_I - q_I^*). \end{aligned}$$

We conclude that the above-defined tariff T jumps downwards at \hat{q}_I .

Now we check that the buyer, having purchased q_E^* from the rival, strictly prefers purchasing q_I^* than \hat{q}_I from the incumbent:

$$V(q_E^*, q_I^*) - T(q_I^*) > V(q_E^*, \hat{q}_I) - T(\hat{q}_I). \quad (\text{F.4})$$

Indeed, the inequality (F.4) is equivalent, after replacing $T(\hat{q}_I) - T(q_I^*)$ with its value from (F.3), to

$$V(q_E^*, \hat{q}_I) - V(\hat{q}_E, \hat{q}_I) < c_E(q_E^* - \hat{q}_E),$$

which follows from the concavity of V in q_E and $V_{q_E}(\hat{q}_E, \hat{q}_I) \leq c_E$. Next, we check that the buyer, having purchased \hat{q}_E from the rival, strictly prefers purchasing \hat{q}_I than q_I^* from the incumbent:

$$V(\hat{q}_E, \hat{q}_I) - T(\hat{q}_I) > V(\hat{q}_E, q_I^*) - T(q_I^*). \quad (\text{F.5})$$

Indeed the inequality (F.5) is equivalent to

$$V(q_E^*, q_I^*) - V(\hat{q}_E, q_I^*) > c_E(q_E^* - \hat{q}_E),$$

which follows from the concavity of V in q_E and $V_{q_E}(q_E^*, q_I^*) = c_E$. It follows that there exists $\bar{q}_E \in (\hat{q}_E, q_E^*)$ such that the buyer, having purchased q_E from the rival, purchases q_I from the incumbent with

$$q_I = q_I(q_E) = \begin{cases} \hat{q}_I & \text{if } q_E \leq \bar{q}_E \\ q_I^*(q_E) & \text{if } q_E \geq \bar{q}_E, \end{cases}$$

where $q_I^*(q_E)$ is given by

$$V_{q_I}(q_E, q_I^*(q_E)) = c_I. \quad (\text{F.6})$$

The surplus function $S_{BE}(q_E) = V(q_E, \hat{q}_I) - c_E q_E - T(\hat{q}_I)$ is concave on $[0, \bar{q}_E]$ with a local maximum at \hat{q}_E . It has a local minimum and a convex kink at \bar{q}_E , because

$$S'_{BE}((\bar{q}_E)^-) = V_{q_E}(\bar{q}_E, \hat{q}_I) - c_E < V_{q_E}(\bar{q}_E, q_I^*) - c_E = S'_{BE}((\bar{q}_E)^+).$$

For $q_E > \bar{q}_E$, the surplus function is given by $S_{BE}(q_E) = V(q_E, q_I^*(q_E)) - c_E q_E - T(q_I^*(q_E))$. Its the first derivative is given by the envelope theorem:

$$S'_{BE}(q_E) = V_{q_E}(q_E, q_I^*(q_E)) - c_E.$$

Differentiating (F.6) yields the first derivative of $q_I^*(q_E)$. We then derive the second derivative of S_{BE} for $q_E > \hat{q}_E$

$$S''_{BE}(q_E) = V_{q_E, q_E} - (V_{q_E, q_I})^2 / V_{q_I, q_I},$$

which is negative by concavity of V . It follows that S_{BE} is concave for $q_E > \bar{q}_E$. The function has another local maximum at q_E^* . Thanks to (F.3), the values of the two local maxima of the function $S_{BE}(\cdot)$ are equal. The difference between this common maximal value and $S_{BE}(0)$ is equal to L by construction, which achieves the proof of the lemma.

G Proof of Lemma C.2

We start from any price schedule T . Let \tilde{T} be defined by

$$\tilde{T}(q_I) = \inf_{q \leq q_I} T(q) + v_I(q_I - q). \quad (\text{G.1})$$

The tariff \tilde{T} is derived from the tariff T as follows. When the incumbent offer q units at price $T(q)$, he also offers to sell more units than q , say $q_I > q$, at price $T(q) + v_I(q_I - q)$. The additional units are offered at the monopoly price v_I . By construction, the slope of \tilde{T} is lower than or equal to v_I .

Let $\tilde{U}_B(q_E)$ denote the buyer's net utility after she has purchased q_E units from the competitor under the price schedule \tilde{T}

$$\tilde{U}_B(q_E) = \max_{q_I} V(q_E, q_I) - \tilde{T}(q_I). \quad (\text{G.2})$$

As $\tilde{T} \leq T$, we have: $\tilde{U}_B \geq U_B$. Suppose that, under \tilde{T} , it is optimal for the buyer to purchase \tilde{q}_I from the incumbent if she has purchased q_E from the competitor. By construction of \tilde{T} , there exists $q_I \leq \tilde{q}_I$ such that $\tilde{T}(\tilde{q}_I)$ equals or is arbitrarily close to $T(q_I) + v_I(\tilde{q}_I - q_I)$. We have:

$$\begin{aligned}\tilde{U}_B(q_E) &= V(q_E, \tilde{q}_I) - \tilde{T}(\tilde{q}_I) &= V(q_E, \tilde{q}_I) - T(q_I) - v_I(\tilde{q}_I - q_I) \\ &= V(q_E, q_I) - T(q_I),\end{aligned}\tag{G.3}$$

which implies $\tilde{U}_B(q_E) \leq U_B(q_E)$, and hence $\tilde{U}_B(q_E) = U_B(q_E)$ for all q_E . As the problem of the buyer-competitor pair depends only on the functions $U_B(\cdot)$ and $\tilde{U}_B(\cdot)$, they agree on the same quantity q_E and the competitor earns the same profit, $\beta\Delta S_{BE}$, under T and \tilde{T} for all (c_E, s_E, v_E) .

We now examine the quantity purchased from the incumbent. Suppose that the buyer, having purchased q_E from the competitor, chooses to purchase q_I from the incumbent under the original price schedule T . As $\tilde{T}(q_I) \leq T(q_I)$, the buyer may choose to purchase the same quantity from the incumbent under the new tariff \tilde{T} :

$$U_B(q_E) = \tilde{U}_B(q_E) = V(q_E, q_I) - T(q_I) \leq V(q_E, q_I) - \tilde{T}(q_I).$$

Yet, under the tariff \tilde{T} , if $q_I < 1 - q_E$, the buyer may as well choose to purchase $1 - q_E$ from the incumbent. Indeed, by definition of \tilde{T} , we have $\tilde{T}(1 - q_E) \leq T(q_I) + v_I(1 - q_E - q_I)$ and hence

$$\begin{aligned}U_B(q_E) = \tilde{U}_B(q_E) &= V(q_E, q_I) - T(q_I) \\ &\leq V(q_E, q_I) + v_I(1 - q_E - q_I) - \tilde{T}(1 - q_E) \\ &= V(q_E, 1 - q_E) - \tilde{T}(1 - q_E).\end{aligned}\tag{G.4}$$

As $v_I > c_I$, the change from q_I to $1 - q_E > q_I$ increases the total surplus:

$$\begin{aligned}W(q_E, 1 - q_E) &= V(q_E, 1 - q_E) - c_E q_E - c_I(1 - q_E) \\ &= V(q_E, q_I) - c_E q_E - c_I q_I + (v_I - c_I)(1 - q_E - q_I) \\ &\geq W(q_E, q_I).\end{aligned}\tag{G.5}$$

In sum, the change from T to \tilde{T} does not alter the rival's profit and does not decrease the total surplus. We conclude from (7) that the change does not decrease the expected payoff of the buyer-incumbent coalition.

H Implementable quantity functions

H.1 From the boundary line to the quantity function

Because the quantity function $q_E(s_E, \omega_E)$ is nondecreasing in s_E and constant below the boundary, we have:

$$q_E(s_E, \omega_E) = \begin{cases} \min\{x \leq s_E \mid \Psi(y) \geq \omega_E \text{ for all } y \in [x, s_E]\} & \text{if } \Psi(s_E) > \omega_E, \\ s_E & \text{if } \Psi(s_E) \leq \omega_E. \end{cases} \quad (\text{H.1})$$

For type *A* (resp. *B*) on Figure 8, we have $\Psi(s_E) < \omega_E$ (resp. $\Psi(s_E) > \omega_E$) and the solution of the problem (??) is unique and equal to s_E^2 . In contrast, type *C* is indifferent between s_E^2 and s_E^3 and, by convention, is assumed to choose s_E^3 . In other words, when (??) has multiple solutions, equation (H.1) selects the highest.

Lemma H.1. *A quantity function $q_E(., .)$ is implementable if and only if there exists a boundary function $\Psi(.)$ defined on $[0, 1]$ such that (H.1) holds.*

We prove here the sufficient part of Lemma H.1. Starting from any boundary function Ψ defined on $[0, 1]$, we define the quantity function $q_E(s_E, \omega_E)$ by equation (H.1), and the surplus gain $\Delta S_{BE}(s_E, \omega_E)$ by

$$\Delta S_{BE}(s_E, \omega_E) = \int_{\underline{\omega}_E}^{\omega_E} q_E(s_E, x) dx.$$

We observe that the functions thus defined $q_E(s_E, \omega_E)$ and $\Delta S_{BE}(s_E, \omega_E)$, are nondecreasing in both arguments, and the latter function is convex in ω_E . Next, we notice that the expression $(\omega_E - v_I)q_E(s_E, \omega_E) - \Delta S_{BE}(s_E, \omega_E)$ is constant on q_E -isolines. Indeed, both $q_E(., \omega_E)$ and $\Delta S_{BE}(., \omega_E)$ are constant on horizontal isolines (located below the boundary Ψ). On vertical isolines (above the boundary), $\Delta S_{BE}(s_E, .)$ is linear with slope s_E , guaranteeing, again, that the above expression is constant. We may therefore define $T(q)$, up to an additive constant, by

$$T(1) - T(1 - q) = (v_I - \omega_E)q + \Delta S_{BE}(s_E, \omega_E), \quad (\text{H.2})$$

for any (s_E, ω_E) such that $q = q_E(s_E, \omega_E)$. Equation (H.2) unambiguously defines $T(1) - T(1 - q)$ on the range of the quantity function $q_E(., .)$. This range

contains zero, but may have holes when $\bar{\omega}_E$ is finite and Ψ is above $\bar{\omega}_E$ on some intervals. Specifically, if Ψ is above $\bar{\omega}_E$ on the interval $I = [s_E^1, s_E^2]$, then q_E does not take any value between s_E^1 and s_E^2 . In this case, we define T by imposing that it is linear with slope $v_I - \bar{\omega}_E$ on the corresponding interval: $T(1 - s_E^1) - T(1 - q) = (v_I - \bar{\omega}_E)(q - s_E^1)$ for $q \in I$.

We now prove that the buyer and the competitor, facing the above defined tariff T , agree on the quantity $q_E(s_E, \omega_E)$. We thus have to check that

$$\Delta S_{BE}(s_E, \omega_E) \geq (\omega_E - v_I)q' + T(1) - T(1 - q') \quad (\text{H.3})$$

for any $q' \leq s_E$. When q' is the range of the quantity function, we can write $q' = q_E(s'_E, \omega'_E)$ for some (s'_E, ω'_E) , with $q' \leq s'_E$. Observing that $q' = q_E(q', \omega'_E)$ and using successively the monotonicity of ΔS_{BE} in s_E and its convexity in ω_E , we get:

$$\begin{aligned} \Delta S_{BE}(s_E, \omega_E) &\geq \Delta S_{BE}(q', \omega_E) \\ &\geq \Delta S_{BE}(q', \omega'_E) + (\omega_E - \omega'_E)q', \end{aligned}$$

which, after replacing $T(1) - T(1 - q')$ with its value from (H.2), yields (H.3). To check (H.3) when q' is not in the range of the quantity function (q' belongs to a hole $[s_E^1, s_E^2]$ as explained above), use (H.3) at s_E^1 and the linearity of the tariff between s_E^1 and q' .

H.2 From the boundary function to the price schedule

Lemma H.2. *The shape of the boundary function Ψ and the curvature of the price schedule T are linked in the following way:*

1. *If Ψ is increasing (resp. constant) around s_E , then the tariff is strictly convex (resp. linear) around $1 - s_E$.*
2. *If Ψ decreases and is concave around s_E , then the tariff is concave around $1 - s_E$.*
3. *If Ψ decreases and is convex around s_E and s_E is close to a local minimum of Ψ , then the tariff is convex around $1 - s_E$.*

4. If Ψ has a local maximum at s_E , then the tariff has an inflection point at $1 - s_E$.

Proof. First, suppose that Ψ is nondecreasing on a neighborhood of s_E . Let s'_E slightly above s_E . Then $q_E = s_E$ is an interior solution of the buyer-rival pair's problem (??) for s'_E and $\omega_E = \Psi(s_E)$. It follows that the first order condition $\Psi(s_E) - v_I + T'(1 - s_E) = 0$ holds, implying property 1 of the lemma. The property holds when Ψ has an upward discontinuity at s_E , in which case the tariff has a convex kink at $1 - s_E$. To illustrate, Figures 10a and 10b consider the case where the boundary line is a nondecreasing step function with two pieces.

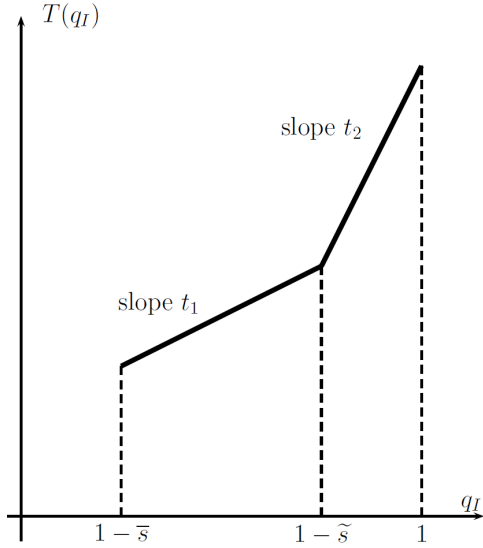


Figure 10a: Convex kink in the price schedule

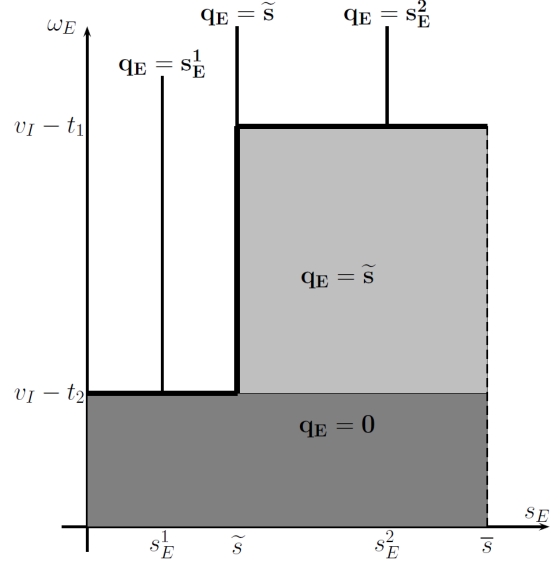


Figure 10b: Two-step increasing boundary line

Next, suppose that the boundary line decreases around s_E . Here we assume that Ψ is twice differentiable. We denote by $[\sigma(s_E), s_E]$ the set of value s'_E such that $q_E(s'_E, \omega_E) = \sigma(s_E)$, where $\omega_E = \Psi(s_E)$. The buyer-rival surplus $\Delta S_{BE}(s_E, \omega_E)$ is convex and hence continuous in ω_E . It can be computed slightly below or above $\Psi(s_E)$. At $(s_E, \Psi(s_E))$, the buyer and the rival are indifferent between quantities s_E and $\sigma(s_E)$:

$$\Delta S_{BE}(s_E, \Psi(s_E)) = [\Psi(s_E) - v_I] \sigma(s_E) - T(1 - \sigma(s_E)) = [\Psi(s_E) - v_I] s_E - T(1 - s_E).$$

Differentiating and using the first-order condition at $\sigma(s_E)$ yields

$$T'(1 - s_E) = -\Psi'(s_E)[s_E - \sigma(s_E)] - \Psi(s_E) + v_I.$$

Differentiating again yields

$$T''(1 - s_E) = \Psi''(s_E)[s_E - \sigma(s_E)] + \Psi'(s_E)[2 - \sigma'(s_E)]. \quad (\text{H.4})$$

In the above equation, the two bracketed terms are nonnegative (use $\sigma' \leq 0$), and the slope Ψ' is negative by assumption, which yields item 2 of the lemma. Around a local minimum of Ψ , Ψ' is small, and the first term is positive, hence property 3. Property 4 follows from items 1 and 2. \square

I Proof of Proposition E.1

In Section I.1, we offer a convenient parametrization of horizontal bunching intervals. In Section I.2, we state and prove a one-dimensional optimization result, which serves to maximize the expected virtual surplus for a given level of ω_E . In Section I.3, we rewrite the complete problem as the maximization of the expected virtual surplus under monotonicity constraints. In Section I.4, we show that these constraints are not binding under fairly mild conditions. In Section I.5, we address the case where the monotonicity constraint are binding and two-dimensional bunching occurs.

I.1 Parameterizing horizontal bunching intervals

Consider an implementable quantity function q_E . For any ω_E , the function of one variable $q_E(\cdot, \omega_E)$ is nondecreasing on $[0, 1]$, being either constant or equal to the identity map: $q_E = s_E$. By convention, we call regions where it is constant “odd intervals”, and regions where $q_E = s_E$ “even intervals”.

We are thus led to consider any partition of the interval $[0, 1]$ into “even intervals” $[s_{2i}, s_{2i+1})$ and “odd intervals” $[s_{2i+1}, s_{2i+2})$, where (s_i) is a finite, increasing sequence with first term zero and last term one.¹⁸ We associate to any such

¹⁸ For notational consistency, we denote the first term of the sequence by $s_0 = 0$ if the first interval is even and by $s_1 = 0$ if the first interval is odd. Similarly, we denote the last term by $s_{2n} = 1$ if the last interval is odd and by $s_{2n+1} = 1$ if the last interval is even.

partition the function of one variable that coincides with the identity map on even intervals, is constant on odd intervals, and is continuous at odd extremities. We denote by K the set of the functions thus obtained.

For any implementable quantity function q_E , the functions of one variable, $q_E(\cdot, \omega_E)$, belong to K for all ω_E . Conversely, any quantity function such that $q_E(\cdot, \omega_E)$ belong to K for all ω_E is implementable if and only if even (odd) extremities do not increase (decrease) as ω_E rises. Hereafter, we call the conditions on the extremities the “monotonicity constraints”.

Even (odd) extremities constitute decreasing (increasing) parts of the boundary line. Odd intervals, $[s_{2i+1}, s_{2i+2})$, constitute horizontal bunching segments, or, more precisely, the horizontal portions of the L-shaped bunching regions.

I.2 A one-dimensional optimization result

In this section, we maximize a linear integral functional on the above-defined set K .

Lemma I.1. *Let $a(\cdot)$ be a continuous function on $[0, 1]$. Then the problem*

$$\max_{r \in K} \int_0^1 a(s)r(s) ds$$

admits a unique solution r^ characterized as follows. For any interior even extremity s_E^{2i} , the function a equals zero at s_E^{2i} and is negative (positive) at the left (right) of s_E^{2i} . For any interior odd extremity s_E^{2i+1} , the function a is positive at s_E^{2i+1} and satisfies*

$$\int_{s_E^{2i+1}}^{s_E^{2i+2}} a(s) ds = 0. \quad (\text{I.1})$$

If $a(1) > 0$, then $r^(s) = s$ at the top of the interval $[0, 1]$. If $a(1) < 0$, then r^* is constant at the top of the interval.*

Proof. Letting $I(r) = \int_0^1 a(x)r(x) dx$, we have

$$I(r) = \sum_i \int_{x_{2i}}^{x_{2i+1}} xa(x) dx + \sum_i x_{2i+1} \int_{x_{2i+1}}^{x_{2i+2}} a(x) dx,$$

where the index i in the two sums goes from either $i = 0$ or $i = 1$ to either $i = n - 1$ or $i = n$, in accordance with the conventions exposed in Footnote 18.

Differentiating with respect to an interior even extremity yields

$$\frac{\partial I}{\partial x_{2i}} = a(x_{2i}) \cdot [x_{2i-1} - x_{2i}].$$

The first-order condition therefore imposes $a(x_{2i}^*) = 0$. The second-order condition for a maximum shows that a must be negative (positive) at the left (right) of x_{2i}^* .

Differentiating with respect to an interior odd extremity yields

$$\frac{\partial I}{\partial x_{2i+1}} = \int_{x_{2i+1}}^{x_{2i+2}} a(x) dx.$$

The first-order condition therefore imposes $\int_{x_{2i+1}^*}^{x_{2i+2}^*} a(x) dx = 0$. The second-order condition for a maximum imposes that a is nonnegative at x_{2i+1}^* .

If $a(1) > 0$, then it is easy to check that $r^*(x) = x$ at the top, namely on the interval $[x_{2n}^*, x_{2n+1}^*]$ with x_{2n}^* being the highest zero of the function a and $x_{2n+1}^* = 1$. If the function a admits no zero, it is everywhere positive and hence $r^*(x) = x$ on the whole interval $[0, 1]$.

If $a(1) < 0$, then r^* is constant at the top, namely on the interval $[x_{2n-1}^*, x_{2n}^*]$, with $x_{2n}^* = 1$ and $\int_{x_{2n-1}^*}^1 a(x) dx = 0$. If the integral $\int_y^1 a(x) dx$ remains negative for all y , then r^* is constant and equal to zero on the whole interval $[0, 1]$. \square

I.3 Solving the complete problem

The complete problem consists in maximizing the expected virtual surplus subject to the even (odd) extremities being nonincreasing (nondecreasing). The latter conditions are called hereafter the ‘‘monotonicity constraints’’.

Applying Lemma I.1 with $a(s_E) = s^v(s_E, \omega_E)$ for any given ω_E , we find that the virtual surplus is zero at candidate even extremities: $s^v(x_{2i}(\omega_E), \omega_E) = 0$ and is negative (positive) at the left (right) of these extremities. In other words, candidate even extremities belong to decreasing parts of the ERT line. Thus, as regards even extremities, the monotonicity constraints are never binding.

Lemma I.1 also implies that the virtual surplus is positive at odd extremities. These extremities therefore lie above the ERT line. By the first-order

condition (I.1), the expected virtual surplus is zero on horizontal bunching intervals:

$$\mathbb{E}(s^v|H) = 0, \quad (\text{I.2})$$

where H is a horizontal bunching interval with extremities s_E^{2i+1} and s_E^{2i+2} . The virtual surplus on a bunching interval is first positive, then negative as s_E rises, and its mean on the interval is zero. The segment $[AB]$ on Figure 9b is an example of horizontal bunching interval (in fact the horizontal part of an “L”-shaped bunching set). Unfortunately, the first-order condition (I.2) does not imply that candidate odd extremities $s_{2i+1}(\omega_E)$ are nondecreasing in ω_E : odd extremities might decrease with ω_E in some regions, generating two-dimensional bunching.

I.4 Sufficient conditions

We now check that each of the three conditions mentioned in Proposition E.1 is sufficient for the odd extremities $s_E^{2i+1}(\omega_E)$ to be nondecreasing in ω_E .

We can restrict attention to efficient rivals, $\omega_E \geq \omega_I$.¹⁹ We rewrite equation (I.2) as $A(s_E^{2i+1}, \omega_E) = 0$ with

$$\begin{aligned} A(s_E^{2i+1}, \omega_E) &= \int_{s_E^{2i+1}}^{s_E^{2i+2}} s^v(s, \omega_E) f(\omega_E|s) g(s) \, ds \\ &= \int_{s_E^{2i+1}}^{s_E^{2i+2}} [(\omega_E - \omega_I) f(\omega_E|s) - \beta(1 - F(\omega_E|s))] g(s) \, ds. \end{aligned}$$

The function A is nonincreasing in s_E^{2i+1} , as the virtual surplus is nonnegative at this point:

$$\frac{\partial A}{\partial s_E^{2i+1}}(s_E^{2i+1}, \omega_E) = -s^v(s_E^{2i+1}, \omega_E) f(\omega_E|s_E^{2i+1}) g(s_E^{2i+1}) \leq 0.$$

Differentiating with respect to ω_E , we get

$$\frac{\partial A}{\partial \omega_E}(s_E^{2i+1}, \omega_E) = \int_{s_E^{2i+1}}^{s_E^{2i+2}} [(\omega_E - \omega_I) f'(\omega_E|s) + f(\omega_E|s) + \beta f(\omega_E|s)] g(s) \, ds,$$

¹⁹For $\omega_E < \omega_I$, the virtual surplus is negative for all s_E and the solution is $q_E = 0$ for all s_E .

where we denote by f' the derivative of f in ω_E .

When f is nondecreasing in ω_E , or $f' \geq 0$, we have $\partial A / \partial \omega_E \geq 0$, and hence the odd extremities are nondecreasing in ω_E . We now examine successively the cases where the hazard rate is nondecreasing in ω_E (a weaker condition than $f' \geq 0$) and the elasticity of entry is nondecreasing in ω_E (an even weaker condition).

I.4.1 Assuming that the hazard rate does not decrease in ω_E

We now assume that the hazard rate, $f/(1-F)$, is nondecreasing in ω_E , which can be expressed as $f' \geq -\varepsilon f / \omega_E$. Using $\omega_E \geq \omega_I$, we find that

$$\begin{aligned} \frac{\partial A}{\partial \omega_E} &\geq \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left[-(\omega_E - \omega_I) \frac{\varepsilon}{\omega_E} + 1 + \beta \right] f(\omega_E|s) g(s) ds \\ &= \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left\{ \varepsilon \left[\frac{\omega_I}{\omega_E} - 1 + \frac{\beta}{\varepsilon} \right] + 1 \right\} f(\omega_E|s) g(s) ds. \end{aligned}$$

On a horizontal interval H , the variable ω_E is constant, and only the elasticity ε may vary. Hence, the first order condition (I.2) yields: $\mathbb{E}(1 - \beta/\varepsilon | H) = \omega_I/\omega_E$. The right-hand side of the above inequality is equal, up to a positive multiplicative constant, to

$$1 - \text{cov} \left(\varepsilon, 1 - \frac{\beta}{\varepsilon} \middle| H \right).$$

We now look for a sufficient condition for this expression to be nonnegative for any distribution of ε . Noting $m = \mathbb{E}(\varepsilon|H)$ the expectation of ε on H , the condition can be rewritten as

$$\mathbb{E} \left[(\varepsilon - m) \left(1 - \frac{\beta}{\varepsilon} \right) \middle| H \right] \leq 1.$$

The function $(\varepsilon - m)(1 - \beta/\varepsilon)$ is convex in ε . We denote by $[\underline{\varepsilon}, \bar{\varepsilon}]$ the support of the distribution of ε . For given values of $\underline{\varepsilon}$, $\bar{\varepsilon}$ and $m = \mathbb{E}(\varepsilon|H)$, the expectation of this convex function is maximal when the distribution of ε has two mass points at $\underline{\varepsilon}$ and $\bar{\varepsilon}$, associated with the respective weights $\frac{\bar{\varepsilon}-m}{\bar{\varepsilon}-\underline{\varepsilon}}$ and $\frac{m-\underline{\varepsilon}}{\bar{\varepsilon}-\underline{\varepsilon}}$. We thus need to make sure that

$$(\bar{\varepsilon} - m)(\underline{\varepsilon} - m) \left(1 - \frac{\beta}{\underline{\varepsilon}} \right) + (m - \underline{\varepsilon})(\bar{\varepsilon} - m) \left(1 - \frac{\beta}{\bar{\varepsilon}} \right) \leq \bar{\varepsilon} - \underline{\varepsilon},$$

for any $m \in [\underline{\varepsilon}, \bar{\varepsilon}]$. The left-hand side of the above inequality is maximal for $m = (\underline{\varepsilon} + \bar{\varepsilon})/2$. It follows that the inequality holds for all $m \in [\underline{\varepsilon}, \bar{\varepsilon}]$ if and only if the condition (E.2) is satisfied.

I.4.2 Assuming that the elasticity of entry does not decrease in ω_E

We now assume that the $\varepsilon(\omega_E|s_E)$ is nondecreasing in ω_E , as stated in Assumption 1. We have:

$$\frac{\partial \varepsilon(\omega_E|s_E)}{\partial \omega_E}(s_E^{2i+1}, \omega_E) = \frac{\partial}{\partial \omega_E} \left[\frac{\omega_E f(\omega_E|s_E)}{1 - F(\omega_E|s_E)} \right] \geq 0$$

which can be rewritten as $f' \geq -(1 + \varepsilon)f/\omega_E$. Using $\omega_E \geq \omega_I$, we find that

$$\frac{\partial A}{\partial \omega_E} \geq \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left[\frac{\omega_I}{\omega_E} - \varepsilon \left(1 - \frac{\beta}{\varepsilon} - \frac{\omega_I}{\omega_E} \right) \right] f(\omega_E|s)g(s) ds.$$

On a horizontal interval H , the variable ω_E is constant, and only the elasticity ε may vary. Hence, the first order condition (I.2) yields: $\mathbb{E}(1 - \beta/\varepsilon | H) = \omega_I/\omega_E$. The right-hand side of the above inequality is equal, up to a positive multiplicative constant, to

$$\mathbb{E} \left(1 - \frac{\beta}{\varepsilon} \middle| H \right) - \text{cov} \left(\varepsilon, 1 - \frac{\beta}{\varepsilon} \middle| H \right).$$

We now look for a sufficient condition for this expression to be nonnegative for any distribution of ε . Noting $m = \mathbb{E}(\varepsilon|H)$ the expectation of ε on H , the condition can be rewritten as

$$\mathbb{E} \left[(\varepsilon - m - 1) \left(1 - \frac{\beta}{\varepsilon} \right) \middle| H \right] \leq 0.$$

The function $(\varepsilon - m - 1)(1 - \beta/\varepsilon)$ is convex in ε . We denote by $[\underline{\varepsilon}, \bar{\varepsilon}]$ the support of the distribution of ε . For given values of $\underline{\varepsilon}$, $\bar{\varepsilon}$ and $m = \mathbb{E}(\varepsilon|H)$, the expectation of this convex function is maximal when the distribution of ε has two mass points at $\underline{\varepsilon}$ and $\bar{\varepsilon}$, associated with the respective weights $\frac{\bar{\varepsilon} - m}{\bar{\varepsilon} - \underline{\varepsilon}}$ and $\frac{m - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}$. We thus need to make sure that

$$(\bar{\varepsilon} - m)(\underline{\varepsilon} - m - 1) \left(1 - \frac{\beta}{\underline{\varepsilon}} \right) + (m - \underline{\varepsilon})(\bar{\varepsilon} - m - 1) \left(1 - \frac{\beta}{\bar{\varepsilon}} \right) \leq 0, \quad (\text{I.3})$$

for any $m \in [\underline{\varepsilon}, \bar{\varepsilon}]$. The above function is the sum of two quadratic functions of m . The first is convex with roots $\underline{\varepsilon} - 1$ and $\bar{\varepsilon}$; the second is concave with roots

$\underline{\varepsilon}$ and $\bar{\varepsilon} - 1$. Both quadratic functions have zero derivative at $m = (\underline{\varepsilon} + \bar{\varepsilon} - 1)/2$. The sum of the two functions is concave as $\underline{\varepsilon} < \bar{\varepsilon}$.

When $\bar{\varepsilon} \leq \underline{\varepsilon} + 1$, the concave quadratic function is negative on the interval $[\underline{\varepsilon}, \bar{\varepsilon}]$, and hence the inequality (I.3) holds on that interval. When $\bar{\varepsilon} > \underline{\varepsilon} + 1$, we need to make sure that the maximum value of the concave quadratic function is lower than the minimum value of the convex quadratic function. This is the case if and only if

$$\left(1 - \frac{\beta}{\bar{\varepsilon}}\right) (\Delta\varepsilon - 1)^2 \leq \left(1 - \frac{\beta}{\underline{\varepsilon}}\right) (\Delta\varepsilon + 1)^2.$$

which is equivalent to (E.3).

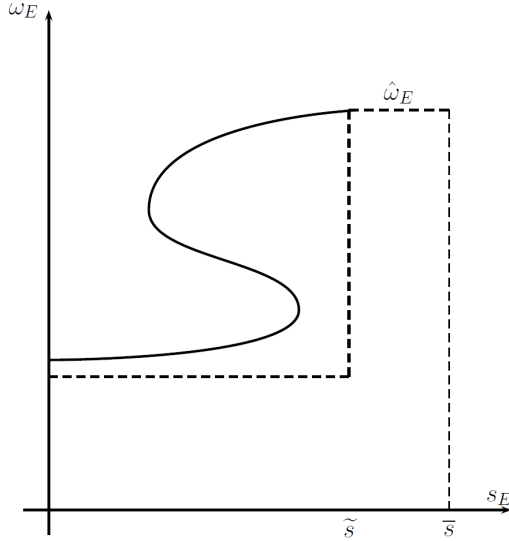


Figure 11a: ERT line (dashed). Non-monotonic odd extremities (solid line)

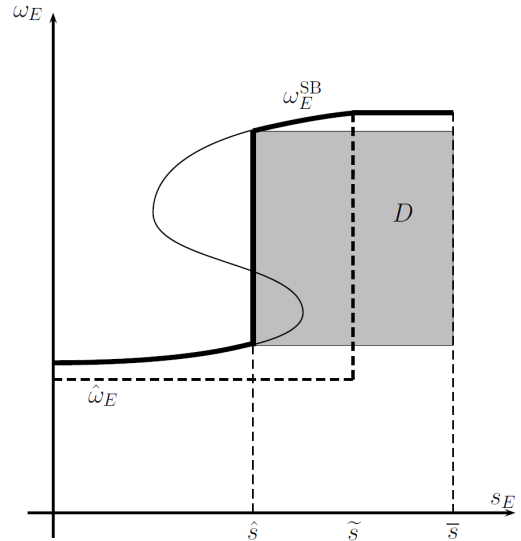


Figure 11b: Two-dimensional bunching area: $q_E = \hat{s}$ on D .

I.5 Two-dimensional bunching

When none of the above sufficient conditions holds, it may happen that solving the problem separately for each ω_E yields odd extremities (left extremities of horizontal bunching segments) that are non-monotonic with ω_E , as represented on Figure 11a. Such a line does not define a boundary function $\Psi(s_E)$. This means that the monotonicity constraints are binding and that the optimal boundary line has an increasing vertical portion, generating a two-dimensional

pooling area. An example of such an area is the shaded region D pictured on Figure 11b, on which the quantity is constant. The value of the constant (\hat{s} on the picture) is determined by the first-order condition

$$\mathbb{E}(s^v|D) = 0.$$

This example has been constructed by assuming that (i) ω_E follows a Pareto distribution conditionally on s_E , for all s_E ; (ii) the elasticity of entry takes two values, $\underline{\varepsilon}$ and $\bar{\varepsilon}$, with a large difference $\bar{\varepsilon} - \underline{\varepsilon}$; (iii) small rivals are very sensitive to the competitive pressure placed by the incumbent (their elasticity is $\bar{\varepsilon}$) and large rivals are much less sensitive (their elasticity is $\underline{\varepsilon}$). Hence the increasing ERT line with two pieces.