Urban Quantities and Amenities

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Abstract

We use a frictionless neoclassical general-equilibrium model to explain cross-metro variation in population density and other urban quantities based on 3 broad amenity types: quality of life, productivity in tradeables, and productivity in non-tradeables. Analytically, we demonstrate the dependence of quantities on amenities through substitution possibilities in consumption and production. Our calibrated model predicts large elasticities, consistent with variation in U.S. data, and some empirical estimates of local labor supply. From only differences in wages and housing costs, we explain half of the variation in density, especially through quality-of-life amenities. We also show density density information can provide or refine measures of land value and local productivity.

Keywords: Population density, productivity, quality of life.

JEL Numbers: H2, H4, J3, Q5, R1

1 Introduction

Why people live where they do is a central issue of urban economics. Researchers and policymakers are deeply interested in whether people follow jobs or jobs follow people, or more technically, whether populations locate more in areas with amenities benefiting firms, who provide employment opportunities, or in areas with amenities directly benefiting households [Blanco (1963); Borts and Stein (1964)]. There is also growing appreciation that people may follow housing, as areas amenable to housing construction more easily accommodate growing populations [Glaeser, Gyourko and Saks (2006), Saks (2008)].

Researchers have employed a wide variety of approaches to study location decisions. As in Carlino and Mills (1987), many authors have used dynamic simultaneous-equation regressions.[1] In general, this approach has yielded inconclusive results. Authors have used the Roback (1982) theoretical framework to estimate the relationship between population and housing [Glaeser, Gyourko and Saks (2006)] and to estimate how cities' traded-good sector affects the non-traded-good sector [Moretti (2010)]. Other authors have used dynamic discrete-choice models to study household location decision [e.g. Kennan and Walker (2011)]. While undoubtedly valuable, the empirical estimates discussed above often require ad-hoc assumptions or ignore potentially interesting general equilibrium effects.

Given the empirical challenges, some authors have used simulated models to explain location decisions. Desmet and Rossi-Hansberg (2012) simulate a monocentric city model and explain the city size distribution using efficiency, amenities, and frictions. Rappaport (2008a, 2008b) calibrates a two-city Roback model to demonstrate the importance of quality of life and productivity for explaining population density. His approach relies more upon computational methods than our own, which we describe in detail below, and is removed from the observed distribution of population and prices across cities. These simulations have improved our understanding of population flows, but they often lack clear analytic results and so can lack transparency at times.

We employ a different strategy to explain location decisions. We interpret purely cross-sectional

[1] See Hoogstra, Florax, and Dijk 2005 for an interesting meta-analysis of this literature.

data on quantities and prices through a frictionless neoclassical model of production and consumption. This framework characterizes metropolitan areas in terms of their local amenities to households, i.e. quality of life, and firms, i.e. productivity, as in Roback and Albouy (2009). While this framework is frequently used to explain variation in local wages and housing costs, very few have used it to examine variation in local quantities, even though it implicitly predicts this variation. Throughout, we highlight the importance of general equilibrium feedback effects. We also use the analytic model to consider the impact of agglomeration economies and congestion costs on household location decisions.

We make a number of contributions which further our understanding of household location decisions. We demonstrate that, under reasonable parameterizations, the neoclassical Roback model predicts very large population responses to amenities. In general, quantities respond to amenities by almost an order of magnitude more than prices. As can be seen from our analytic expressions, variation in urban quantities has a first-order dependence on elasticities of substitution in consumption and production, unlike urban prices. We also show that quality of life explains significantly more population density than does productivity in traded-goods, such as automobiles or software.

Empirically, large elasticities agree with United States population density cross-sectional variation, which is much larger than the variation in prices and wages, as seen in Figure 1. Through our calibrated model, we find that housing-cost and wage levels – which yield measures of quality of life and trade-productivity, as in Albouy (2009) – predict half of the population density variation seen across 276 metro areas. The observed variation suggests that some elasticities of substitution are less than one, implying quantities slightly less responsive than in a Cobb-Douglas economy. Furthermore, we use the density measures to infer productivity in non-tradeables, as well as refine measures of trade-productivity and land values. Our calibrated model also sheds light on commonly estimated elasticities of labor supply and demand; our estimated elasticity of labor supply closely matches estimates from Bartik (1991) or Notowidigo (2012), while our elasticity of labor demand is much larger.

In general, our estimates suggest denser cities have greater amenity levels, especially in tradeable and non-tradeable production. This is less the case in the Pacific and Mountain divisions, where density is low relative to quality of life and trade-productivity levels, implying low levels of home-productivity. New York City has the highest overall level of amenities, followed by San Francisco, Chicago, Los Angeles, and Philadelphia. Cities in Texas, including El Paso and Houston, have the highest levels of home productivity, while non-metro Colorado and Cape Cod have the lowest levels.

The rest of the paper is organized as follows: Section 2 introduces the model. Section 3 calibrates the model, provides numerical results, and discusses identification. Section 4 provides new estimates of trade and home-productivity. Section 5 estimates long-run elasticities of labor demand and supply and housing supply. Section 6 concludes.

2 Prices, Quantities, and Amenities

2.1 Model Set-up

To explain how prices and quantities vary with amenity levels across cities, we use the three-equation general-equilibrium model of Albouy (2009), which adds federal taxes to the three-equation Roback (1982) model. The national economy contains many cities, indexed by j, which trade with each other and share a homogenous population of mobile households. Households consume a numeraire traded good, x, and a non-traded "home" good, y, with local price, p^j.[2] Households live and work in the same city. All input and output markets are perfectly competitive. In addition, all prices and quantities are uniform within a given city, though they vary across cities.

Cities differ exogenously in three general attributes, each of which is an index meant to summarize the effect of amenities on households and firms: (i) quality of life, Q^j, raises household utility; (ii) trade-productivity, A_X^j, lowers costs in the traded-good sector, and (iii) home-productivity, A_Y^j, lowers costs in the home-good sector.[3]

[2] In application, the price of the home good is equated with the cost of housing services. Non-housing goods are considered to be a composite commodity of traded goods and non-housing home goods.

[3] All of these attributes depend on a vector of natural and artificial city amenities, $\mathbf{Z}^j = (Z_1^j, ..., Z_K^j)$, $Q^j = \widetilde{Q}\left(\mathbf{Z}^j\right)$,

Firms produce traded and home goods out of land, capital, and labor. Within a city, factors receive the same payment in either sector. Land, L, is heterogenous across cities, immobile, and receives a city-specific price, r^j. Each city's land supply, $L^j(r)$, may depend positively on r^j, with a finite positive elasticity $\varepsilon^j_{L,r}$. In principle, land supply can vary on two different margins. At the extensive margin, an increase in land supply corresponds to a growing city boundary. At the intensive margin, an increase in land supply takes the form of employing previously unused land within a city's border. Capital, K, is fully mobile across cities and receives the price $\bar{\imath}$ everywhere. The supply of capital in each city, K^j, is perfectly elastic at this price. The national level of capital may be fixed or depend on $\bar{\imath}$. Households, N, are fully mobile, have identical tastes and endowments, and each supplies a single unit of labor. Household size is fixed. Wages, w^j, may vary across cities because households care about local prices and quality of life. The total number of households is $N^{TOT} = \sum_j N^j$, which may be fixed or determined by international migration.

Households own identical diversified portfolios of land and capital, which pay an income $R = \frac{1}{N_{TOT}} \sum_j r^j L^j$ from land and $I = \frac{1}{N_{TOT}} \sum_j \bar{\imath} K^j$ from capital. Total income, $m^j = R + I + w^j$, varies across cities only as wages vary. Out of this income households pay a federal income tax, τm^j, which is redistributed in uniform lump-sum payments, T.[4] Household preferences are modeled by a utility function, $U(x, y; Q^j)$, which is quasi-concave over x, y, and Q^j. The expenditure function for a household in city j is $e(p^j, u; Q^j) \equiv \min_{x,y}\{x + p^j y : U(x, y; Q^j) \geq u\}$. Assume Q enters neutrally into the utility function and is normalized so that $e(p^j, u; Q^j) = e(p^j, u)/Q^j$, where $e(p^j, u) \equiv e(p^j, u; 1)$.[5]

Operating under perfect competition, firms produce traded and home goods according to the functions $X^j = A^j_X F^j_X(L^j_X, N^j_X, K^j_X)$ and $Y^j = A^j_Y F^j_Y(L^j_Y, N^j_Y, K^j_Y)$, where F_X and F_Y are con-

$A^j_X = \widetilde{A_X}\left(\mathbf{Z}^j\right)$, and $A^j_Y = \widetilde{A_Y}\left(\mathbf{Z}^j\right)$. For a consumption amenity, e.g. safety or clement weather, $\partial\widetilde{Q}/\partial Z_k > 0$; for a trade-production amenity, e.g. navigable water or agglomeration economies, $\partial\widetilde{A_X}/\partial Z_k > 0$; for a home-production amenity, e.g. flat geography or the absence of land-use restrictions, $\partial\widetilde{A_Y}/\partial Z_k > 0$. It is possible that a single amenity affects more than one attribute or affects an attribute negatively.

[4]In general, results are robust to elastic labor and land supply so long as the new units supplied are equivalent to the old units (Roback 1980). Furthermore, results do not change significantly with international capital flows or if federal tax revenues are used to purchase tradable goods.

[5]The assumption on the form of the expenditure function is equivalent to assuming that the utility function is homothetic over x, y. Both ensure that quality of life only affects the optimal consumption ratio y/x through prices.

cave and exhibit constant returns to scale, and A_X^j and A_Y^j are assumed to be Hicks-Neutral. Unit cost in the traded-good sector is $c_X(r^j, w^j, \bar{\imath}; A_X^j) \equiv \min_{L,N,K}\{r^j L + w^j N + \bar{\imath} K : A_X^j F(L, N, K) = 1\}$. Similar to the relationship between quality of life and the expenditure function, let $c_X(r^j, w^j, \bar{\imath}; A_X^j) = c_X(r^j, w^j, \bar{\imath})/A_X^j$, where $c_X(r^j, w^j, \bar{\imath}) \equiv c_X(r^j, w^j, \bar{\imath}; 1)$. A symmetric definition holds for the unit cost, c_Y, in the home-good sector.

2.2 Equilibrium Conditions

For any city, the entire system consists of sixteen equations in sixteen unknown variables, with three exogenous parameters, Q^j, A_X^j, A_Y^j, three endogenous prices, p^j, w^j, r^j, and thirteen endogenous quantities, x^j, y^j, X^j, Y^j, N^j, N_X^j, N_Y^j, L^j, L_X^j, L_Y^j, K^j, K_X^j, K_Y^j. Below we characterize the equilibrium, first for prices, then for per-capita consumption quantities, and lastly for production quantities. Throughout, we adopt the "open city" approach and take nationally determined variables, $\bar{u}, \bar{\imath}, I, R, T$, as given for any individual city.[6]

2.2.1 Price Conditions

Since households are fully mobile, they must receive the same utility across all inhabited cities. Higher prices or lower quality of life are compensated with greater after-tax income,

$$[e(p^j, \bar{u}) + \tau m^j - T]/Q^j = m^j, \tag{1}$$

[6]Because of this approach, we ignore the constraint which balances the federal government's budget, $\tau \sum_j N^j m^j + T \sum_j N^j = 0$.

where $\bar{u}$ is the level of utility attained nationally by all households.[7] Firms earn zero profits in equilibrium. For given output prices, more productive cities must pay higher rents and wages,

$$c_X(r^j, w^j, \bar{\imath})/A_X^j = 1 \tag{2}$$

$$c_Y(r^j, w^j, \bar{\imath})/A_Y^j = p^j. \tag{3}$$

Equations (1), (2), and (3) simultaneously determine the city-level prices p^j, r^j, and w^j for each city as implicit functions of three attributes Q^j, A_X^j, and A_Y^j. In equilibrium, this provides a one-to-one mapping between unobserved city attributes and potentially observable prices, although in practice, land prices are not observed reliably.

2.2.2 Consumption Conditions

Given prices w^j and p^j, the budget constraint and the optimal consumption condition,

$$x^j + p^j y^j = (1-\tau)m^j + T \tag{4}$$

$$(\partial U/\partial y) \,/\, (\partial U/\partial x) = p^j, \tag{5}$$

implicitly determine the consumption quantities x^j and y^j. Because utility is constant, consumption quantities are determined through Hicksian demands with the relative price p^j and utility $\bar{u}$. Amenities affect consumption quantities through prices. For a given p^j, higher levels of Q^j lower the amount of x^j and y^j consumed, although the ratio y^j/x^j is unchanged since Q^j enters the expenditure function neutrally. Given Q^j, a household changes its consumption ratio, y^j/x^j, in response to a change in the price ratio, $p^j/1$, according the the elasticity of demand, $\sigma_D \equiv -\frac{d\ln(y/x)}{d\ln(p)} \geq 0$.

[7]The model generalizes easily to a case with heterogenous workers that supply different fixed amounts of labor if these workers are perfect substitutes in production, have identical homothetic preferences, and earn equal shares of income from labor. Additionally, the mobility condition need not apply to all households, but only a sufficiently large subset of mobile marginal households (Gyourko and Tracy 1989). See Albouy (2008) for discussion on how the model's predictions change with multiple household types with different preferences and labor skills.

2.2.3 Production Conditions

Levels of output X^j, Y^j, employment N^j, N_X^j, N_Y^j, capital K^j, K_X^j, K_Y^j, and land L^j, L_X^j, L_Y^j are determined by nine equations in the production sector. The first six express factor demands as a function of output, factor prices, and productivity using Shepard's Lemma.

$$\partial c_X / \partial w = A_X^j N_X^j / X^j \tag{6}$$

$$\partial c_X / \partial r = A_X^j L_X^j / X^j \tag{7}$$

$$\partial c_X / \partial i = A_X^j K_X^j / X^j \tag{8}$$

$$\partial c_Y / \partial w = A_Y^j N_Y^j / Y^j \tag{9}$$

$$\partial c_Y / \partial r = A_Y^j L_Y^j / Y^j \tag{10}$$

$$\partial c_Y / \partial i = A_Y^j K_Y^j / Y^j \tag{11}$$

The next three equations express the local resource constraints for labor, land, and capital. The equations impose that all factors are fully employed.

$$N^j = N_X^j + N_Y^j \tag{12}$$

$$L^j = L_X^j + L_Y^j \tag{13}$$

$$K^j = K_X^j + K_Y^j \tag{14}$$

In addition, we have an equation determining the supply of land as a function of the rental price,

$$L^j = L(r^j), \tag{15}$$

with elasticity $\varepsilon_{L,r}$. The open city and constant-returns assumptions imply that all of the model's quantity predictions increase one-for-one with the quantity of land. If the available land in city j doubled, then population and capital would migrate inwards such that, in the new equilibrium, all of the prices and per-capita quantities would return to the initial equilibrium while the aggregate quantities would increase by the same amount as the increase in land supply. Our assumption that prices and quantities are uniform within a city is also key to this result. We can simplify our analysis by focusing on population density, N^j/L^j. We do this by assuming that land supply is fixed, $\varepsilon_{L,r} = 0$.[8]

The last constraint requires all home-goods to be consumed locally.

$$Y^j = N^j y^j \tag{16}$$

This expression requires information about consumption from the previous subsection and allows quality of life to affect quantities independently of the relevant prices p^j, w^j and r^j. This expression also captures the fundamental difference between the traded- and non-traded good. The market clearing equation for tradable output is eliminated by Walras' Law.

2.3 Log-Linearization

The above section described a system of sixteen nonlinear equations. In order to solve the model, we log-linearize the system to express a particular city's price and quantity differentials in terms of its amenity differentials, each relative to the national average.[9] These differentials are expressed in logarithms so that, for any variable z, $\hat{z}^j \equiv \ln z^j - \ln \bar{z} \cong (z^j - \bar{z})/\bar{z}$ approximates the percent difference in city j of z relative to the average $\bar{z}$.[10] We assume that Q^j, A_X^j, and A_Y^j are continuous variables.

To complete the log-linearization, we need several economic parameters, all defined for the national average. For households, denote the share of gross expenditures spent on the traded and home good as $s_x \equiv x/m$ and $s_y \equiv py/m$; denote the share of income received from land, labor, and capital income as $s_R \equiv R/m$, $s_w \equiv w/m$, and $s_I \equiv I/m$. For firms, denote the cost share of land,

[8]This discussion also highlights the important assumption of land being fully employed in equilibrium. If cities varied in the amount of land utilization, then N^j/L^j would not be a consistent measure of population density across cities. This rules out changes in land supply at both the the extensive and intensive margins.

[9]In Appendix A, we provide results from a nonlinear simulation of the model.

[10]Letting $\mathbb{E}$ be the expectations operator over cities, then $\mathbb{E}[\hat{z}^j] = 0$.

labor, and capital in the traded-good sector as $\theta_L \equiv rL_X/X$, $\theta_N \equiv wN_X/X$, and $\theta_K \equiv \bar{\imath}K_X/X$; denote equivalent cost shares in the home-good sector as ϕ_L, ϕ_N, and ϕ_K. Finally, denote the share of land, labor, and capital used to produce traded goods as $\lambda_L \equiv L_X/L$, $\lambda_N \equiv N_X/N$, and $\lambda_K \equiv K_X/K$. Assume the home-good is more cost-intensive in land relative to labor than the traded-good, both absolutely, $\phi_L \geq \theta_L$, and relatively, $\phi_L/\phi_N \geq \theta_L/\theta_N$, implying $\lambda_L \leq \lambda_N$.

After log-linearizing the system, we obtain the following system of sixteen equations in sixteen unknowns. We can then solve for prices and quantities as linear functions of the three city attributes. The first three equations describe prices.

$$-s_w(1-\tau')\hat{w}^j + s_y\hat{p}^j = \hat{Q}^j \tag{1*}$$

$$\theta_L\hat{r}^j + \theta_N\hat{w}^j = \hat{A}_X^j \tag{2*}$$

$$\phi_L\hat{r}^j + \phi_N\hat{w}^j - \hat{p}^j = \hat{A}_Y^j \tag{3*}$$

Prices depend only upon amenities and cost and expenditure shares. Importantly, prices do not display a first-order dependence on elasticities of substitution.

Two equations describe consumption quantities.

$$s_x\hat{x}^j + s_y\left(\hat{p}^j + \hat{y}^j\right) = (1-\tau)s_w\hat{w}^j \tag{4*}$$

$$\hat{x}^j - \hat{y}^j = \sigma_D\hat{p}^j \tag{5*}$$

Equation (5*) shows that per-capita consumption quantities depend directly on the elasticity of demand. This elasticity determines the household's flexibility in consumption. A higher value of σ_D corresponds to more flexible consumption, so that a household responds more strongly to differences in the price p^j of the home-good. Even though our model contains homogenous households, one can think of higher values of σ_D as approximating preference heterogeneity with sorting across cities in which households with stronger tastes for y choose to live in areas with low prices p.[11] In short, preference heterogeneity aggregates into a more flexible representative

[11] Roback (1980) provides discussion along these lines.

household.

The next six equations describe production quantities. These expressions make use of partial (Allen-Uzawa) elasticities of substitution. Each sector has three partial elasticities of substitution in production for each combination of two factors, where $\sigma_X^{LN} \equiv (\partial^2 c/\partial w \partial r) \,/\, (\partial c/\partial w \cdot \partial c/\partial r)$ is the partial elasticity of substitution between labor and land in the production of X. Symmetric definitions hold for other partial elasticities. Because productivity differences are Hicks-neutral, they do not affect these elasticities of substitution. These elasticities depend on local prices, and so are defined for each city. In practice, we assume that the elasticities are constant across cities.[12] We also assume that $\sigma_X^{NK} = \sigma_X^{KL} = \sigma_X^{LN} \equiv \sigma_X$, and similarly for the home-good sector, as with a constant elasticity of substitution production function.

A higher value of σ_X corresponds to more flexible production of the traded-good. With a single traded good, firms can vary their production only by changing inputs. In the case of multiple traded goods, firms could still make changes on this intensive margin, but they could also change the product, which is akin to an extensive margin of variation. If cities specialize in production, then land-intensive goods will be produced in areas with low quality of life while labor-intensive goods will be produced in areas with high quality of life, as Roback (1980) discusses. In the context of our model, one can interpret such specialization via a larger σ_X. A higher value of σ_Y means that the home-good, which we equate with housing, can produced more densely as firms are able to

[12] We discuss potential complications arising from this below.

shift their inputs away from land and towards labor and capital.

$$\hat{N}_X^j = \hat{X}^j - \hat{A}_X^j + \theta_L \sigma_X^{LN} \left(\hat{r}^j - \hat{w}^j\right) - \theta_K \sigma_X^{NK} \hat{w}^j \tag{6*}$$

$$\hat{L}_X^j = \hat{X}^j - \hat{A}_X^j + \theta_N \sigma_X^{LN} (\hat{w}^j - \hat{r}^j) - \theta_K \sigma_X^{KL} \hat{r}^j \tag{7*}$$

$$\hat{K}_X^j = \hat{X}^j - \hat{A}_X^j + \theta_L \sigma_X^{KL} \hat{r}^j + \theta_N \sigma_X^{NK} \hat{w}^j \tag{8*}$$

$$\hat{N}_Y^j = \hat{Y}^j - \hat{A}_Y^j + \phi_L \sigma_Y^{LN} (\hat{r}^j - \hat{w}^j) - \phi_K \sigma_Y^{NK} \hat{w}^j \tag{9*}$$

$$\hat{L}_Y^j = \hat{Y}^j - \hat{A}_Y^j + \phi_N \sigma_Y^{LN} (\hat{w}^j - \hat{r}^j) - \phi_K \sigma_Y^{KL} \hat{r}^j \tag{10*}$$

$$\hat{K}_Y^j = \hat{Y}^j - \hat{A}_Y^j + \phi_L \sigma_Y^{KL} \hat{r}^j + \phi_N \sigma_Y^{NK} \hat{w}^j \tag{11*}$$

We also have the resource constraints for labor, land, and capital. As can be seen from equations (13*) and (15*), assuming $\varepsilon_{L,r} = 0$ implies a one percent increase in land devoted to traded-good production requires a $(1 - \lambda_L)/\lambda_L$ percent decrease in land devoted to home-good production. On the other hand, an increase in labor devoted to traded-good production does not require a corresponding adjustment to labor devoted to home-good production.

$$\hat{N}^j = \lambda_N \hat{N}_X^j + (1 - \lambda_N) \hat{N}_Y^j \tag{12*}$$

$$\hat{L}^j = \lambda_L \hat{L}_X^j + (1 - \lambda_L) \hat{L}_Y^j \tag{13*}$$

$$\hat{K}^j = \lambda_K \hat{K}_X^j + (1 - \lambda_K) \hat{K}_Y^j \tag{14*}$$

$$\hat{L}^j = \varepsilon_{L,r} \hat{r}^j \tag{15*}$$

We finally have the home-good consumption constraint, which relates population density and housing density, the two main urban quantities of interest.

$$\hat{N}^j + \hat{y}^j = \hat{Y}^j \tag{16*}$$

2.4 Solving the Model

We use (1*), (2*), and (3*) to solve for price differentials in terms of amenity differentials.

$$s_R\hat{r}^j = \frac{1}{1-\frac{\lambda_L}{\lambda_N}\tau'}\left[\hat{Q}^j + \left(1-\frac{1}{\lambda_N}\tau'\right)s_x\hat{A}_X^j + s_y\hat{A}_Y^j\right] \tag{17}$$

$$s_w\hat{w}^j = \frac{1}{1-\frac{\lambda_L}{\lambda_N}\tau'}\left(-\frac{\lambda_L}{\lambda_N}\hat{Q}^j + \frac{1-\lambda_L}{\lambda_N}s_x\hat{A}_X^j - \frac{\lambda_L}{\lambda_N}s_y\hat{A}_Y^j\right) \tag{18}$$

$$s_y\hat{p}^j = \frac{1}{1-\frac{\lambda_L}{\lambda_N}\tau'}\left[\frac{\lambda_N-\lambda_L}{\lambda_N}\hat{Q}^j + (1-\tau')\frac{1-\lambda_L}{\lambda_N}s_x\hat{A}_X^j - (1-\tau')\frac{\lambda_L}{\lambda_N}s_y\hat{A}_Y^j\right] \tag{19}$$

Higher quality of life leads to higher land and home-good prices but lower wages. Higher trade-productivity increases all three prices, while higher home-productivity increases land prices but decreases wages and the home-good price.

With our assumptions on land supply and usage, we interpret $\hat{N}^j$ as the population density differential. We can solve for $\hat{N}^j$ in terms of exogenous amenities and parameters by using the log-linearized system described above. However, the closed form solution is complex and offers little intuition that cannot be gained from our discussion above. Equations (5*)-(12*) demonstrate that $\hat{N}^j$ will depend on three elasticities of substitution. This dependence is a key difference between prices and quantities; as a result, estimating the relationship between quantities and amenities is more difficult than estimating the relationship for prices. We can define reduced-form elasticities and express $\hat{N}^j$ as

$$\hat{N}^j = \varepsilon_{N,Q}\hat{Q}^j + \varepsilon_{N,A_X}\hat{A}_X^j + \varepsilon_{N,A_Y}\hat{A}_Y^j, \tag{20}$$

where the elasticity of population density with respect to quality of life is given by $\varepsilon_{N,Q}$; ε_{N,A_X} and ε_{N,A_Y} are defined similarly. We interpret $\varepsilon_{N,Q}$ as the causal effect of an exogenous difference in consumption amenities, $\hat{Q}$, on population density, $\hat{N}$.[13] In (20), we capture the general-equilibrium effect of amenities on population density.

To clarify the purpose of equation (20), we provide the closed-form expression of the reduced-

[13] We consider feedback between quantities and amenities below.

form elasticity for quality of life,

$$\begin{aligned}\varepsilon_{N,Q} = \Bigg[& \sigma_D \left(\frac{s_x[\lambda_N - \lambda_L]^2}{s_y \lambda_L [\lambda_N - \lambda_L \tau]} \right) + \sigma_X \left(\frac{s_w \lambda_N \lambda_L + s_R \lambda_L^2}{s_w s_R [\lambda_N - \lambda_L \tau]} \right) \\ & + \sigma_Y \left(\frac{s_w s_R \lambda_L [\lambda_N - \lambda_L] - s_y s_w \lambda_N^2 \lambda_L - s_y s_R \lambda_N \lambda_L^2}{s_y s_w s_R \lambda_N [\lambda_N - \lambda_L \tau]} \right) \\ & + \frac{\lambda_N - \lambda_L}{\lambda_N} + \frac{-s_w s_R \lambda_N + s_y s_w \lambda_N^2 [1 + \varepsilon_{L,r}] + s_y s_R \lambda_L^2}{s_y s_w s_R \lambda_N [\lambda_N - \lambda_L \tau]} \Bigg].\end{aligned}$$

Above-average quality of life affects a city's population density through four key channels. First, there is substitution in per-capita consumption away from y^j, as governed by σ_D. Second, traded-good firms produce more labor-intensive goods, as governed by σ_X. Third, more capital and labor is used per unit of land to produce denser housing, as governed by σ_Y. Finally, households consume less of both goods to compensate for higher quality of life. Note that we have described all of these channels in our discussion of equations (1*)-(17*). Equation (20) is an attractive reduced-form representation of the general-equilibrium elasticities.

We can also consider the reduced-form relationship between amenities and other quantities. For example, we can characterize the relationship between amenities and the housing stock as

$$\hat{Y}^j = \varepsilon_{Y,Q} \hat{Q}^j + \varepsilon_{Y,A_X} \hat{A}_X^j + \varepsilon_{Y,A_Y} \hat{A}_Y^j.$$

2.5 The Effect of Quantity Feedback on Amenities

So far, we have considered amenities to be exogenous. We now present simple extensions to allow quantities impact amenities.

We first consider trade-productivity A_X^j which increases in the level of output, X^j. Define A_{X0}^j as city j's "natural advantage" in producing traded-output, possibly reflecting location near established transit lanes or moderate climate. We can decompose trade-productivity as $A_X^j = A_{X0}^j (X^j)^\alpha$, where $\alpha \geq 0$ is the reduced-form agglomeration elasticity. Note that the relationship between traded-output and amenities in the absence of feedback is $\hat{X}^j = \varepsilon_{X,Q} \hat{Q}^j + \varepsilon_{X,A_X} \hat{A}_X^j +$

$\varepsilon_{X,A_Y}\hat{A}_Y^j$. Quantity feedback reinforces a city's natural advantage,

$$\hat{X}^j = \frac{1}{1-\alpha\varepsilon_{X,A_X}}[\varepsilon_{X,Q}\hat{Q}^j + \varepsilon_{X,A_X}\hat{A}_{X0}^j + \varepsilon_{X,A_Y}\hat{A}_Y^j]$$

The feedback of X^j on A_X^j affects all other quantities in the model. For example, the new relationship between population density and amenities is simply

$$\hat{N}^j = \varepsilon_{N,Q}\hat{Q}^j + \varepsilon_{N,A_X}\hat{A}_{X0}^j + \varepsilon_{N,A_Y}\hat{A}_Y^j + \varepsilon_{X,A_X}\hat{X}^j,$$

which again shows that agglomeration reinforces the impact of amenities.

We can also extend the model to consider congestion costs. Specifically, define Q_0^j as city j's natural quality of life, which corresponds largely to geography and climate. We decompose quality of life as $Q^j = Q_0^j(N^j)^{-\gamma}$, where $\gamma \geq 0$ represents a congestion cost. Using the same process as above, we obtain

$$\hat{N}^j = \frac{1}{1+\gamma\varepsilon_{N,Q}}\left[\varepsilon_{N,Q}\hat{Q}_0^j + \varepsilon_{N,A_X}\hat{A}_X^j + \varepsilon_{N,A_Y}\hat{A}_Y^j\right],$$

which shows that congestion costs limit the effect of attractive amenities on population density growth. The new relationship between the housing stock and amenities is

$$\hat{Y}^j = \varepsilon_{Y,Q}\hat{Q}_0^j + \varepsilon_{Y,A_X}\hat{A}_X^j + \varepsilon_{Y,A_Y}\hat{A}_Y^j - \varepsilon_{Y,Q}\gamma\hat{N}^j$$

These two examples demonstrate that feedback effects of quantities on amenities can increase or decrease the response of population density to amenities.

3 Calibration, Data, and Identification

3.1 Parameter Choices

We calibrate the model using the data described below and national-level parameters. Starting with income shares, Krueger (1999) argues that s_w is close to 75 percent. Poterba (1998) estimates that the share of income from corporate capital is 12 percent, so s_I should be higher and is taken as 15 percent. This leaves 10 percent for s_R, which is roughly consistent with estimates in Keiper et al. (1961) and Case (2007).[14]

Turning to expenditure shares, Albouy (2008), Moretti (2008), and Shapiro (2006) find that housing costs approximate non-housing cost differences across cities. The cost-of-living differential is $s_y \hat{p}^j$, where $\hat{p}^j$ equals the housing-cost differential and s_y equals the expenditure share on housing plus an additional term which captures how a one percent increase in housing costs predicts a $b = 0.26$ percent increase in non-housing costs.[15] In the Consumer Expenditure Survey (CEX), the share of income spent on shelter and utilities, s_{hous}, is 0.22, while the share of income spent on other goods, s_{oth}, is 0.56, leaving 0.22 spent on taxes or saved (Bureau of Labor Statistics 2002).[16] Thus, our coefficient on the housing cost differential is $s_y = s_{hous} + s_{oth} b = 0.22 + 0.56 \times 0.26 = 36$ percent. This leaves s_x at 64 percent.

We choose the cost shares to be consistent with the expenditure and income shares above. θ_L appears small: Beeson and Eberts (1986) use a value of 0.027, while Rappaport (2008a, 2008b) uses a value of 0.016. Valentinyi and Herrendorff (2008) estimate the land share of tradables at 4 percent, although their definition of tradables differs from the one here. We use a value of 2.5 percent for θ_L here. Following Carliner (2003) and Case (2007), the cost-share of land in home-goods, ϕ_L, is taken at 23.3 percent; this is slightly above values from McDonald (1981), Roback (1982), and Thorsnes (1997) to account for the increase in land cost shares over time described

[14]The values Keiper reports were at a historical low. Keiper et al. (1961) find that total land value was found to be about 1.1 times GDP. A rate of return of 9 percent would justify using $s_R = 0.10$. Case (2007), ignoring agriculture, estimates the value of land to be \$5.6 trillion in 2000 when personal income was \$8.35 trillion.

[15]See Albouy (2008) for details.

[16]Utility costs account for one fifth of s_{hous}, which means that without them this parameter would be roughly 0.18.

by Davis and Palumbo (2007). Together the cost and expenditure shares imply λ_L is 17 percent, which appears reasonable since the remaining 83 percent of land for home goods includes all residential land and much commercial land; the cost and expenditure shares also agree with s_R at 10 percent.[17] Finally, we choose the cost shares of labor and capital in both production sectors. As separate information on ϕ_K and θ_K does not exist, we set both cost shares of capital at 15 percent to be consistent with s_I. Accounting identities then determine that θ_N is 82.5 percent, ϕ_N is 62 percent, and λ_N is 70.4 percent.

The federal tax rate, when combined with relevant variation in wages with state tax rates, produces an approximate marginal tax rate, τ, of 36.1 percent. Details on this tax rate, as well as housing deductions, are discussed in Appendix E.3.

We also must determine three elasticities of substitution. Following Albouy (2009b), we initially set $\sigma_D = \sigma_X = \sigma_Y = 0.667$. We provide sensitivity analysis surrounding our elasticities of substitution below.

A few potential complications deserve special attention. First, incorrect parameter values might bias our estimates presented below. As mentioned above, the parameters come from a variety of sources and are generally estimated across different years, geographies, and industries. Second, the log-linearized model is most accurate for small deviations from the national average. Population density and amenities

Furthermore, the elasticity of traded-good production, σ_X, might vary at different levels of aggregation. Specifically, the national elasticity might be larger than the city-level elasticity because of greater flexibility at the national level of production. We estimate the impact of this in Table 2. Our estimates also might contain error due to certain modeling assumptions, e.g. frictionless household relocation. We do not adjust for misspecification error. Our model most appropriately describes a long-run equilibrium, where moving costs or other frictions likely have little impact. Fi-

[17]These proportions are roughly consistent with other studies. In the base calibration of the model, 51 percent of land is devoted to actual housing, 32 percent is for non-housing home goods, and 17 percent is for traded goods, including those purchased by the federal government. Keiper et al. (1961) find that about 52.5 of land value is in residential uses, a 22.9 percent in industry, 20.9 percent in agriculture. Case (2007), ignoring agriculture, finds that in 2000 residential real estate accounted for 76.6 percent of land value, while commercial real estate accounted for the remaining 23.4 percent.

nally, the elasticity of home-good production may vary across cities. For example, home-producers in coastal cities might find it more difficult to substitute away from capital or labor towards land. We do not incorporate city-specific production elasticities into our model. If σ_Y varies among cities, the misspecification error will appear in our productivity estimates.

3.2 Reduced-Form Elasticities

When $\sigma_D = \sigma_X = \sigma_Y = 0.667$, the calibrated model yields

$$\hat{N}^j = 8.96\hat{Q}^j + 2.16\hat{A}_X^j + 2.63\hat{A}_Y^j. \tag{21}$$

The model predicts that a city with quality of life which is one percent higher than the national average will have 8.96 percent higher population density. A city with one percent higher trade-productivity will have 2.16 percent higher density; a similar difference in home-productivity leads to a 2.63 percent difference in density.

Normalizing the amenities by the relevant household expenditure shares clarifies the relative importance of amenities.[18]

$$\begin{aligned}\hat{N}^j =& \varepsilon_{N,Q}\hat{Q}^j + \frac{\varepsilon_{N,A_X}}{s_x} s_x \hat{A}_X^j + \frac{\varepsilon_{N,A_Y}}{s_y} s_y \hat{A}_Y^j \\ =& \, 8.96\hat{Q}^j + 3.37 s_x \hat{A}_X^j + 7.31 s_y \hat{A}_Y^j \end{aligned} \tag{22}$$

Now a \$1 difference in $\hat{Q}^j$ is comparable to a \$1 difference in $\hat{A}_X^j$ or $\hat{A}_Y^j$. This formulation highlights the importance of home-productivity. The results that we present below will follow the form of equation (22).

The model easily allows us to study how federal taxes distort the impact of amenities on house-

[18] Recall that we assume that quality of life enters the utility and expenditure functions neutrally.

hold location decisions. After setting $\tau = 0$, we estimate

$$\hat{N}^j = 7.09\hat{Q}^j + 5.81 s_x \hat{A}_X^j + 7.55 s_y \hat{A}_Y^j.$$

Trade-productivity becomes much more important in the absence of federal taxes. To understand this result, note that households benefit from trade-productivty through higher wage income, which is taxed. Households in high wage cities pay more in federal taxes, τm^j, than they receive in lump-sum rebates, T. The federal income tax thus places a wedge between the value of local amenities and a household's location decision.

If we fix the size of each city's housing stock by setting $\sigma_Y = 0$, then we obtain

$$\hat{N}^j = 3.61\hat{Q}^j + 1.23 s_x \hat{A}_X^j + 3.18 s_y \hat{A}_Y^j.$$

Households respond much less to all amenities when the housing stock cannot adjust freely to match demand. These estimates might be more accurate in predicting population flows to negative shocks in the spirit of Glaeser and Gyourko (2005), who highlight the importance of durable housing stocks for population flows.

Setting $\sigma_D = 0$ fixes household housing consumption across cities. In this case, we estimate

$$\hat{N}^j = 8.19\hat{Q}^j + 2.62 s_x \hat{A}_X^j + 8.16 s_y \hat{A}_Y^j.$$

Households respond less to quality of life and trade-productivity, but more to home-productivity. A city's productivity in building housing is almost as important as the consumption amenities it offers to households.

With a Cobb-Douglas economy, $\sigma_D = \sigma_X = \sigma_Y = 1$, we obtain

$$\hat{N}^j = 12.66\hat{Q}^j + 5.06 s_x \hat{A}_X^j + 10.94 s_y \hat{A}_Y^j$$

The elasticities here are significantly larger than what we obtain in our baseline specification but follow the same qualitative pattern.

In Table 2, we explore the sensitivity of our reduced-form estimates to the elasticities of substitution. In general, we find that moderate changes in the elasticities of substitution have a relatively large impact on the reduced-form elasticities. In particular, σ_Y has a very large impact on reduced-form elasticities. So far we have assumed a fixed land supply, $\varepsilon_{L,r} = 0$. Table 2 shows that city expansion on the extensive margin leads to even larger reduced-form elasticities.[19]In a frictionless model, housing adjustment is a critical component of how households respond to amenities.

We have presented results for only one urban quantity, population density. In Table 3 we list the reduced-form elasticities for all endogenous prices and quantities.

3.3 Feedback of Quantities on Amenities

We can easily extend the model to allow for agglomeration economies. We use $\alpha = 0.02$ as the agglomeration elasticity. Table 4A presents the reduced-form elasticities under the standard tax treatment, while Table 4B presents the elasticities under a geographically neutral income tax. In particular, we obtain the following result for population density in the case with taxes,

$$\hat{N}^j = 9.38\hat{Q}^j + 3.61 s_x \hat{A}^j_{X0} + 8.39 s_y \hat{A}^j_Y. \tag{23}$$

As expected, agglomeration strengthens the response of population density to amenities. Tables 7A and 7B allow for easy calculation of the response of other endogenous variables to amenities under agglomeration.

We use $\gamma = 0.05$ as the conglomeration cost elasticity. Table 5A presents the reduced-form elasticities under the standard tax treatment, while Table 5B presents elasticities under a neutral

[19]Note that when we allow $\varepsilon_{L,r} > 0$, then we can no longer interpret $\hat{N}^j$ as the population density differential, but instead only as the population differential. The results in Table 2 do not suggest that population density rises or falls in response to elastic land supply. In general, the source of the land supply (i.e. on the extensive or intensive margin) matters.

income tax. We obtain the following estimate for population density with taxes,

$$\hat{N}^j = 6.19\hat{Q}_0^j + 2.33 s_x \hat{A}_{X0}^j + 5.53 s_y \hat{A}_Y^j. \tag{24}$$

As expected, congestion costs reduce the population response to amenities.

3.4 Data

We define cities at the Metropolitan Statistical Area (MSA) level using 1999 OMB definitions. We treat consolidated MSAs as a single city (e.g. San Francisco is combined with Oakland and San Jose) and also create one non-MSA area for each state. There are 325 geographic areas, of which 276 are MSAs and 49 are non-MSA areas.[20] We use the 5-percent sample of 2000 United States Census from Ruggles et. al (2004) to calculate wage and housing price differentials.[21] Population density data comes from the 2000 Census. We calculate density at the census tract level and then create a MSA level density value using population weights. All of our empirical results below use MSA population weights.

3.5 Identification of Amenity and Land Values

With accurate data on all price differentials, $\hat{r}^j, \hat{w}^j, \hat{p}^j$, and knowledge of national economic parameters, we can estimate amenity differentials, $\hat{Q}^j, \hat{A}_X^j, \hat{A}_Y^j$, with equations (1*)-(3*). As can be seen in equation (1*), we can identify quality of life using only data on wages and housing prices. Equations (2*) and (3*) demonstrate the importance of reliable land rent data for identifying both trade- and home-productivity. Unfortunately, reliable land rent data is not readily available. Three possible solutions to this challenge emerge. First, as in Albouy and Ehrlich (2012), one could attempt to infer $\hat{r}^j$ by using recent transaction purchase data. Several conceptual and empirical challenges arise from this approach. Second, one could assume constant home-productivity across cities, $\hat{A}_Y^j = 0$, and estimate trade-productivity as in Albouy (2009b). The resulting estimates of

[20] New Jersey has no non-MSA area.

[21] See Appendix D for more details on the calculation of wage and price differentials.

trade-productivity are not severely biased.[22] The third approach, which we adopt in this paper, is to use population density data to jointly identify trade- and home-productivity.

By combining equations (2*) and (3*), we can infer the costs faced by tradable firms,

$$\frac{\theta_L}{\phi_L}\hat{p}^j + \left(\theta_N - \phi_N \frac{\theta_L}{\phi_L}\right)\hat{w}^j = \hat{A}_X^j - \frac{\theta_L}{\phi_L}\hat{A}_Y^j. \tag{25}$$

We define the left-hand side of equation (25) as relative cost. Assuming constant home-productivity $\hat{A}_Y^j = 0$, we can construct an initial estimate of trade-productivity as in equation (25) using parameters and data on wages and housing prices. Below, we will use wage and housing price data to assess the performance of the model.

Combining equations (1*) and (20) yields

$$\hat{N}^j - \varepsilon_{N,Q}[\underbrace{s_y\hat{p}^j - s_w(1-\tau)\hat{w}^j}_{\hat{Q}^j}] = \varepsilon_{N,A_X}\hat{A}_X^j + \varepsilon_{N,A_Y}\hat{A}_Y^j. \tag{26}$$

We define the left-hand side of equation (26) as excess density, i.e. density not explained by quality of life. Without population density data, equation (26) is under-identified. But by bringing in population density data, we have two equations in two unknown variables, $\hat{A}_X^j$ and $\hat{A}_Y^j$. Because we are exactly identified, our amenity estimates will *perfectly predict* population densities given our parameter choices.

Lastly, we can use the above system out how to infer differences in land values across cities. Our inferred measure is increasing in excess density and home-good prices, and falling in wages.

$$\hat{r}^j = \frac{1}{\theta_L\varepsilon_{N,A_X} + \phi_L\varepsilon_{N,AY}}\left\{\left[\hat{N}^j - \varepsilon_{N,Q}\left(s_y\hat{p}^j - s_w(1-\tau)\hat{w}^j\right)\right] + \varepsilon_{N,A_Y}\hat{p}^j - \left[\theta_N\varepsilon_{N,A_X} + \phi_N\varepsilon_{N,AY}\right]\hat{w}^j\right\}$$

It is easy to see that inferred land values are unambiguously increasing in raw density, without factoring in quality of life. However, it then depends on the parameter values whether or not

[22] This point is seen directly in equation (25) below after noting that $\theta_L << \phi_L$.

inferred land values are increasing or decreasing in home-good prices or wages.

3.6 Calibration Analysis

As discussed above, we can assume constant home-productivity and then estimate $\hat{Q}^j, \hat{A}_X^j$ using data on $\hat{w}^j, \hat{p}^j$. The model then predicts $\hat{N}^j$ for each city in the absence of home-productivity differences. We use this procedure to assess our initially chosen elasticities of substitution. Specifically, we choose a combination of $\sigma_D, \sigma_X, \sigma_Y$. Then for each city, we calculate the difference between the observed and predicted population density differential $\hat{N}^j$. We define the prediction error as the (MSA population-weighted) variance of these differences. We graph the results of this exercise in Figure 2.[23] Each of the curves represents the prediction error as a function of the elasticities of substitution. If, for simplicity's sake, we restrict $\sigma_D = \sigma_X = \sigma_Y$, then we minimize the prediction error when the elasticities equal roughly 0.667, which is our initial specification. The figure shows that particularly low and high values of the elasticities of substitution increase prediction error. In particular, a Cobb-Douglas specification $\sigma_D = \sigma_X = \sigma_Y = 1$ is too elastic. The takeaway from this exercise is that our initial specification corresponds almost exactly to the "optimal" choice of elasticities of substitution for the constant home-productivity model. We assume $\sigma_D = \sigma_X = \sigma_Y = 0.667$ for all results below.

What drives location decisions in the constant home-productivity model? We can answer this question using a straightforward variance decomposition, which we present in Table 6. Quality of life explains more than half of the total variance in predicted population density, even though the variance of trade-productivity is an order of magnitude larger than the variance of quality of life. With constant home-productivity, relatively small differences in quality of life explain a large amount of the population distribution. In other words, the frictionless neoclassical model predicts that "jobs follow people" much more than "people follow jobs." The other key takeaway from Table 6 is that wage and housing prices explain nearly half of the observed variance in population

[23] Obviously, there are several more possible specifications for our elasticities of substitution. We have considered many different specifications and have not found any dramatic improvements over the results presented here.

density. Specifically, the variance of the predicted population density divided by the variance of observed population density equals approximately 0.49.

4 Trade and Home-Productivity Estimates

We now present estimates from equations (25) and (26). Figure 3 displays relative cost and excess density along the axes.[24] The figure includes iso-productivity curves for trade, home, and total-productivity through the origin, where total productivity is defined as $s_x\hat{A}_X + s_y\hat{A}_Y = 0$. In order to understand how we will estimate home-productivity, consider the linear fit line in Figure 3. Given the trade-productivity (approximated by relative cost) in a city, we infer positive home-productivity if there is above-average excess density. For example, New York lies above the linear fit line. Roughly speaking, the solutions from equations (25) and (26) take above-average excess density and report above-average home-productivity. San Francisco, which has below average excess density given its trade-productivity, will receive a low value of home-productivity. Some Texas cities, like McAllen and El Paso, have high excess density even though their unadjusted density is relatively low. The key for home-productivity inference is whether observed population density exceeds what the model predicts given quality of life and relative cost.

This discussion clarifies two important points about our estimate of home-productivity. First, the measure reflects the existing stock of housing that a city had in 2000. Older cities, like New York, Chicago, and Philadelphia, all have high home-productivity. We can explain part of this by noting that these cities have been built up over the past century, when building and land use regulations were less restrictive. Second, home-productivity partly represents a residual measure of population density.[25] The results should be interpreted accordingly.

The estimation procedure outlined above also refines estimates of trade-productivity over those provided in Albouy (2009). Cities with high relative costs and high levels of excess density are inferred to have high levels of trade-productivity. However, equation (25) shows that home-

[24]Note that we can estimate both of these items using data on population density, wages, and housing prices.

[25]Recall that we can estimate $\hat{Q}^j$ perfectly and $\hat{A}_X^j$ quite well with only wage and housing price data.

productivity reduces trade-productivity estimates via cost reductions. Table 8 compares the trade-productivity estimates between the constant home-productivity case and the procedure used here. In addition, Table 8 lists the population density, quality of life, and home-productivity for all metropolitan and non-metropolitan areas.

Figure 4 uses the same data as Figure 3, but now solves the system (25)-(26) for trade- and home-productivity. New York and San Francisco clearly have the highest trade-productivity. Philadelphia, Chicago, and Los Angeles have similar levels of relatively high trade-productivity. New York also has the highest level of home-productivity. Cities in Texas, including El Paso and San Antonio, have high levels of home-productivity as well. Overall, New York is the most productive city. San Francisco, which is the second most valuable city, is not a leader in productivity due to its relatively low home-productivity.[26] Figure 4 also includes indifference curves for quality of life adjusted density and wages plus housing costs. Holding quality of life constant, trade-productivity and home-productivity must move in opposite directions to keep population density constant. Holding quality of life constant, home-productivity must rise faster than trade-productivity to keep wages and housing costs constant.

With our estimates of trade- and home-productivity in hand, we can now explore household location determinants in the fully specified model. In Table 7A, we decompose the variance of observed (which now equals predicted) population density. In comparing quality of life and trade productivity, we note a similar outcome as in Table 6. In fact, the ratio of variance explaned by quality of life to variance explained by trade-productivity is larger in Table 7A than in Table 6. The relatively large fraction of variance explained by home-productivity suggests that there remains some portion of household location decisions which our simple model does not explain.

[26]Some of these findings appear to conflict with recent work by Albouy and Ehrlich (2012), who use data on land values to infer productivity in the housing sector, which comprises most of the non-tradable sector. While the two approaches largely agree on which large areas have high housing productivity, the land values approach suggests that larger, denser cities generally have lower, rather than higher housing productivity. This apparent contradiction actually highlights what the two methodologies infer differently. Productivity measures based on land values provide a better insight into the marginal cost of increasing the housing supply, by essentially inferring the replacement cost. Productivity measures based on density are more strongly related to the average cost of the housing supply, thereby reflecting the whole history of building in a city. The distinction matters particularly for cities with older housing built on the easiest terrain in the decades prior to the diffusion of residential land-use regulations when factor prices were relatively low.

Nevertheless, quality of life and trade productivity explain nearly half of the total variation in population density.

What would happen to population density if federal taxes were made geographically neutral? We can use our estimates of quality of life, trade- and home-productivity, along with the calibrated model, to predict prices and quantities (including population density) for each city in the absence of distortionary federal income taxes.[27] Table 7B presents the variance decomposition of the geographically netural tax counterfactual. Trade-productivity now explains a larger fraction of population density than does quality of life. As described above, federal taxes introduce a wedge between trade-productivity and the benefits that households receive by locating in productive cities. Eliminating the geographic distortion in the tax code would allow households to benefit more from highly productive cities.

5 General Equilibrium Elasticities

Our model also allows us to shed light on commonly estimated elasticities of local labor demand or housing supply. The thought experiment underlying empirical estimates is almost universally a partial equilibrium one. Our model produces fully general equilibrium results. The adjustments underlying these elasticities might take place over the course of decades and generations. For example, the elasticities account for sectoral labor shifting caused by an productivity changes. A major insight from the following exercise is that the source of the shock underlying a partial equilibrium price change matters a great deal.

5.1 Elasticity of Local Labor Demand

We can change the equilibrium population density of a city through a "supply" shock via $\hat{Q}$, a "demand" shock via $\hat{A}_X$, or a "housing" shock via $\hat{A}_Y$. Each shock leads to higher population density: higher quality of life makes a city more attractive to households; higher trade-productivity

[27] Because we estimate amenities using observed density, wage, and housing price data, we cannot estimate amenities in the absence of distortionary federal taxes.

increases the marginal product of labor; higher home-productivity makes a city more affordable for households. Using a demand shock, we estimate the elasticity of local labor supply as

$$\frac{\partial \hat{N}}{\partial \hat{w}} = \frac{\partial \hat{N}/\partial \hat{A}_X}{\partial \hat{w}/\partial \hat{A}_X} \approx \frac{2.165}{1.090} \approx 1.99$$

A one percent increase in the price of labor leads to a 1.99 percent increase in the amount of labor supplied. This estimate is quite close to empirical estimates from Bartik (1991) and Notowidigo (2012). Under a supply shock, we estimate the elasticity of local labor demand as

$$\frac{\partial \hat{N}}{\partial \hat{w}} = \frac{\partial \hat{N}/\partial \hat{Q}}{\partial \hat{w}/\partial \hat{Q}} \approx \frac{8.953}{-0.359} \approx -24.94,$$

while under a housing shock, we estimate an elasticity as

$$\frac{\partial \hat{N}}{\partial \hat{w}} = \frac{\partial \hat{N}/\partial \hat{A}_Y}{\partial \hat{w}/\partial \hat{A}_Y} \approx \frac{2.885}{-0.117} \approx -24.66.$$

These estimates depend only on the parameters listed in Table 1. The calibrated model suggests that labor demand is much more elastic than labor supply. Wages are much less responsive to changes in quality of life or home-productivity than to changes in trade-productivity.[28]

5.2 Elasticity of Local Housing Supply

We estimate the relationship between amenities and housing density as[29]

$$\hat{Y} = 5.97\hat{Q} + 2.41 s_x \hat{A}_X + 8.19 s_y \hat{A}_Y. \tag{27}$$

Comparing (21) and (27), we see that amenities exert an influence on housing density which is qualitatively similar to their influence on population density.

As with population density, we can change the equilibrium housing density through three dif-

[28] This basic idea comes directly from equation (18).

[29] When land supply is fixed, the total home-good differential represents a housing density differential.

ferent amenities. Higher quality of life and trade-productivity both lead to higher demand for aggregate housing. Higher home-productivity corresponds to a shift in aggregate housing supply. Under a demand shock from $\hat{Q}$, we estimate the elasticity of local housing supply as

$$\frac{\partial\hat{Y}}{\partial\hat{p}} = \frac{\partial\hat{Y}/\partial\hat{Q}}{\partial\hat{p}/\partial\hat{Q}} \approx \frac{5.966}{2.542} \approx 2.35.$$

Under a demand shock from $\hat{A}_X$, we estimate the elasticity of local housing supply as

$$\frac{\partial\hat{Y}}{\partial\hat{p}} = \frac{\partial\hat{Y}/\partial\hat{A}_X}{\partial\hat{p}/\partial\hat{A}_X} \approx \frac{1.539}{1.607} \approx 0.96$$

Housing supply is considerably more elastic when the demand shock comes via quality of life as opposed to trade-productivity. We also can estimate an elasticity of housing demand by considering a supply shock via $\hat{A}_Y$,

$$\frac{\partial\hat{Y}}{\partial\hat{p}} = \frac{\partial\hat{Y}/\partial\hat{A}_Y}{\partial\hat{p}/\partial\hat{A}_Y} \approx \frac{2.951}{-0.172} \approx -17.16.$$

When we fix land supply, housing demand is much more elastic than housing supply. As with the labor elasticities, we find that the source of the price shift matters significantly.

The calculations above show that the frictionless neoclassical model generates own-price demand elasticities which are roughly an order of magnitude larger than supply elasticities.

6 Conclusion

Under plausible specifications of substitution elasticities, matching a neoclassical general equilibrium model with reasonable parameter estimates generates exceptionally large elasticities of population density with respect to amenities. The model also generates extremely large elasticities of local labor demand, whie the elasticity of labor supply closely matches existing empirical estimates. Our model reflects the interrelationship between urban quantities and prices and connects both to amenities in consumption and production. Urban quantities depend particularly on substitution elasticities and the complementarity of amenities.

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Table 1: Calibrated Parameters

Parameter Name	Notation	Calibrated Value
Cost and Expenditure Shares		
Home-good expenditure share	s_y	0.36
Income share to land	s_R	0.10
Income share to labor	s_w	0.75
Traded-good cost share of land	θ_L	0.025
Traded-good cost share of labor	θ_N	0.825
Home-good cost share of land	ϕ_L	0.233
Home-good cost share of labor	ϕ_N	0.617
Share of land used in traded good	λ_L	0.17
Share of labor used in traded good	λ_N	0.70
Tax Parameters		
Average marginal tax rate	τ	0.361
Average deduction level	δ	0.291
Structural Elasticities		
Elasticity of substitution in consumption	σ_D	0.667
Elasticity of traded-good production	σ_X	0.667
Elasticity of home-good production	σ_Y	0.667
Elasticity of land supply	$\varepsilon_{L,r}$	0.0

Table 2: Sensitivity Analysis

	$\varepsilon_{N,Q}$	ε_{N,A_X}	ε_{N,A_Y}
σ_D	1.14	0.72	-0.08
σ_X	1.95	0.47	0.64
σ_Y	8.01	2.05	3.38
$\varepsilon_{L,r}$	11.85	4.01	3.86

Table 2 describes the effect on reduced-form elasticities of increasing each structural elasticity by one. For example, increasing σ_D by 1 increases $\varepsilon_{N,Q}$ by 1.14.

TABLE 3: Base Elasticities

A: With Taxes			
	$\hat{Q}$	$\hat{A}_X$	$\hat{A}_Y$
$\hat{r}$	11.85	4.01	3.86
$\hat{w}$	-0.36	1.09	-0.12
$\hat{p}$	2.54	1.61	-0.17
$\hat{x}$	-0.44	0.35	-0.04
$\hat{y}$	-1.99	-0.62	0.07
$\hat{N}$	8.95	2.16	2.88
$\hat{L}$	0.00	0.00	0.00
$\hat{K}$	8.62	2.86	2.78
$\hat{X}$	9.16	3.34	2.93
$\hat{Y}$	5.97	1.54	2.95
$\hat{N}_X$	9.40	2.27	3.01
$\hat{N}_Y$	7.90	1.88	2.58
$\hat{L}_X$	1.25	0.33	0.36
$\hat{L}_Y$	-0.24	-0.06	-0.07
$\hat{K}_X$	9.16	3.00	2.93
$\hat{K}_Y$	7.66	2.61	2.50
B: Neutral Taxes			
	$\hat{Q}$	$\hat{A}_X$	$\hat{A}_Y$
$\hat{r}$	10.00	6.40	3.60
$\hat{w}$	-0.30	1.02	-0.11
$\hat{p}$	2.15	2.12	-0.23
$\hat{x}$	-0.55	0.79	-0.08
$\hat{y}$	-0.92	-0.90	0.10
$\hat{N}$	7.09	3.72	2.72
$\hat{L}$	0.00	0.00	0.00
$\hat{K}$	6.86	4.38	2.61
$\hat{X}$	7.01	4.80	2.78
$\hat{Y}$	5.17	2.81	2.81
$\hat{N}_X$	7.21	3.79	2.85
$\hat{N}_Y$	6.81	3.55	2.40
$\hat{L}_X$	0.34	0.20	0.37
$\hat{L}_Y$	-0.06	-0.04	-0.07
$\hat{K}_X$	7.01	4.47	2.78
$\hat{K}_Y$	6.61	4.23	2.33

Each value in Table 3 represents the partial effect that a one-percent increase in each amenity has on each price or quantity, i.e. $\partial\hat{r}/\partial\hat{Q} = 11.845$. The values in panel A are derived using the parameters in Table 1. The values in panel B are derived using $\tau = 0$.

Table 4: Agglomeration Elasticities

	A: With Taxes		
	$\hat{Q}$	$\hat{A}_{X0}$	$\hat{A}_Y$
$\hat{r}$	12.64	4.29	4.11
$\hat{w}$	-0.14	1.17	-0.05
$\hat{p}$	2.86	1.72	-0.07
$\hat{x}$	-0.38	0.37	-0.01
$\hat{y}$	-2.11	-0.66	0.03
$\hat{N}$	9.38	2.31	3.02
$\hat{L}$	0.00	0.00	0.00
$\hat{K}$	9.18	3.07	2.95
$\hat{X}$	9.81	3.58	3.14
$\hat{Y}$	6.27	1.65	3.05
$\hat{N}_X$	9.84	2.44	3.15
$\hat{N}_Y$	8.27	2.02	2.70
$\hat{K}_X$	9.75	3.22	3.12
$\hat{K}_Y$	8.17	2.80	2.67
$\hat{L}_X$	1.32	0.35	0.38
$\hat{L}_Y$	-0.25	-0.07	-0.07
	B: Neutral Taxes		
	$\hat{Q}$	$\hat{A}_{X0}$	$\hat{A}_Y$
$\hat{r}$	10.99	7.08	3.99
$\hat{w}$	-0.14	1.13	-0.05
$\hat{p}$	2.47	2.35	-0.10
$\hat{x}$	-0.43	0.88	-0.04
$\hat{y}$	-1.05	-1.00	0.04
$\hat{N}$	7.66	4.12	2.95
$\hat{L}$	0.00	0.00	0.00
$\hat{K}$	7.54	4.85	2.88
$\hat{X}$	7.75	5.31	3.07
$\hat{Y}$	5.61	3.11	2.98
$\hat{N}_X$	7.80	4.19	3.08
$\hat{N}_Y$	7.36	3.93	2.62
$\hat{K}_X$	7.70	4.95	3.05
$\hat{K}_Y$	7.26	4.68	2.59
$\hat{L}_X$	0.37	0.22	0.39
$\hat{L}_Y$	-0.07	-0.04	-0.07

Endogenous productivity: $A_X^j = A_{X0}^j (X^j)^\alpha$, $\alpha = 0.02$

Table 5: Congestion Cost Elasticities

A: With Taxes			
	$\hat{Q}_0$	$\hat{A}_X$	$\hat{A}_Y$
$\hat{r}$	8.19	3.13	2.67
$\hat{w}$	-0.25	1.11	-0.08
$\hat{p}$	1.76	1.42	-0.42
$\hat{x}$	-0.31	0.38	0.01
$\hat{y}$	-1.37	-0.47	0.26
$\hat{N}$	6.19	1.49	1.99
$\hat{L}$	0.00	0.00	0.00
$\hat{K}$	5.96	2.22	1.92
$\hat{X}$	6.33	2.66	2.02
$\hat{Y}$	4.12	1.09	2.36
$\hat{N}_X$	6.49	1.57	2.08
$\hat{N}_Y$	5.46	1.29	1.79
$\hat{K}_X$	6.33	2.32	2.02
$\hat{K}_Y$	5.29	2.04	1.74
$\hat{L}_X$	0.87	0.23	0.24
$\hat{L}_Y$	-0.16	-0.04	-0.04
B: Neutral Taxes			
	$\hat{Q}_0$	$\hat{A}_X$	$\hat{A}_Y$
$\hat{r}$	7.38	5.03	2.60
$\hat{w}$	-0.22	1.06	-0.08
$\hat{p}$	1.58	1.83	-0.44
$\hat{x}$	-0.41	0.87	-0.03
$\hat{y}$	-0.68	-0.78	0.19
$\hat{N}$	5.23	2.74	2.00
$\hat{L}$	0.00	0.00	0.00
$\hat{K}$	5.07	3.44	1.93
$\hat{X}$	5.17	3.84	2.07
$\hat{Y}$	3.82	2.10	2.29
$\hat{N}_X$	5.32	2.80	2.13
$\hat{N}_Y$	5.03	2.62	1.72
$\hat{K}_X$	5.17	3.51	2.07
$\hat{K}_Y$	4.88	3.32	1.67
$\hat{L}_X$	0.25	0.16	0.34
$\hat{L}_Y$	-0.05	-0.03	-0.06

Congestion costs: $Q^j = Q_0^j (N^j)^{-\gamma}$, $\gamma = 0.05$

Table 6: Variance Decomposition, Two Amenity

		Fraction of variance explained by		
	Variance	Quality of Life	Trade Productivity	Covariance
	(1)	(2)	(3)	(4)
Predicted	0.394	0.533	0.306	0.159

Table 4 presents the variance decomposition of predicted population density using data on wages and house prices only.

Table 7: Variance Decomposition, Three Amenity

A: Observed Population Density and Prices. Var = 0.770
Fraction of variance explained by

	$Cov(\hat{Q},\cdot)$	$Cov(\hat{A}_X,\cdot)$	$Cov(\hat{A}_Y,\cdot)$
$Cov(\cdot,\hat{Q})$	0.272	·	·
$Cov(\cdot,\hat{A}_X)$	0.141	0.097	·
$Cov(\cdot,\hat{A}_Y)$	-0.193	0.225	0.457

B: Counterfactual Density and Prices. Var=1.357
Fraction of variance explained by

	$Cov(\hat{Q},\cdot)$	$Cov(\hat{A}_X,\cdot)$	$Cov(\hat{A}_Y,\cdot)$
$Cov(\cdot,\hat{Q})$	0.077	·	·
$Cov(\cdot,\hat{A}_X)$	0.139	0.331	·
$Cov(\cdot,\hat{A}_Y)$	-0.072	0.295	0.230

Panel A presents the variance decomposition using data on population density, wages, and house prices. Panel B presents the variance decomposition under geographically neutral income taxes. Both panels use the same amenity estimates.

Table 8: List of Metropolitan and Non-Metropolitan Areas Ranked by Home-Productivity

Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}_X^j$	Restricted $\hat{A}_X^j$	$\hat{A}_Y^j$
New York, Northern New Jersey, Long Island, NY-NJ-CT-PA	2.28523	0.0286	0.2093	.2634856	.5060188
Philadelphia-Wilmington-Atlantic City, PA-NJ-DE-MD	.9575418	-0.0402	0.0958	.1340165	.3564562
El Paso, TX	.3861507	-0.0412	-0.1641	-.1259174	.3558702
Brownsville-Harlingen-San Benito, TX	.0481333	-0.0572	-0.2207	-.1849878	.3328457
Buffalo-Niagara Falls, NY	.4483389	-0.0545	-0.0417	-.0064377	.329387
Laredo, TX	.5243353	-0.0081	-0.1938	-.1589002	.3257235
Houston-Galveston-Brazoria, TX	.3230614	-0.0720	0.0455	.0753663	.2789998
Chicago-Gary-Kenosha, IL-IN-WI	1.190532	0.0050	0.1312	.1608603	.2767427
Reading, PA	.4030689	-0.0463	-0.0174	.0120069	.2745963
Las Vegas, NV-AZ	.6837181	-0.0252	0.0572	.0841677	.2521167
New Orleans, LA	.6861223	0.0048	-0.0654	-.0384133	.2515957
San Antonio, TX	.2224163	-0.0386	-0.0965	-.0698271	.2492898
Erie, PA	.1525482	-0.0351	-0.1141	-.0895668	.2287987
Odessa-Midland, TX	-.1541067	-0.0634	-0.1363	-.1119673	.2271902
Bakersfield, CA	.1888946	-0.0628	0.0205	.044793	.2268038
Champaign-Urbana, IL	.4351563	-0.0093	-0.0798	-.0560049	.2217358
Bloomington-Normal, IL	.1262726	-0.0614	0.0027	.0257217	.2152528
Altoona, PA	-.0815354	-0.0451	-0.1576	-.1347595	.212741
Syracuse, NY	-.0805975	-0.0693	-0.0562	-.033413	.2123943
Pittsburgh, PA	.118914	-0.0474	-0.0544	-.0316637	.2119814
Sioux City, IA-NE	-.2202788	-0.0595	-0.1608	-.1380867	.2118777
McAllen-Edinburg-Mission, TX	-.5405861	-0.0789	-0.2283	-.2056133	.2114266
Dallas-Fort Worth, TX	.3183636	-0.0444	0.0469	.0680772	.1974107
Fargo-Moorhead, ND-MN	-.1145111	-0.0395	-0.1739	-.1527975	.1972049
Corpus Christi, TX	.0737842	-0.0335	-0.1055	-.0848093	.1931017
Beaumont-Port Arthur, TX	-.5255133	-0.1081	-0.0696	-.049239	.1903867
Decatur, IL	-.3853498	-0.0889	-0.0796	-.0595734	.1869849
Milwaukee-Racine, WI	.5731577	-0.0088	0.0374	.0570333	.1833934
Miami-Fort Lauderdale, FL	.9640421	0.0409	0.0146	.0341226	.1817606
Toledo, OH	.1126589	-0.0410	-0.0373	-.0180386	.1798594
Washington-Baltimore, DC-MD-VA-WV	.683143	-0.0127	0.1163	.1350248	.1750921
Detroit-Ann Arbor-Flint, MI	.3461803	-0.0472	0.1080	.1264656	.1719332
Allentown-Bethlehem-Easton, PA	.3084377	-0.0223	-0.0051	.0127458	.1665831
Memphis, TN-AR-MS	-.0442558	-0.0595	-0.0127	.0050173	.1656674
Lubbock, TX	.0748619	-0.0090	-0.1614	-.1440711	.1616055
Omaha, NE-IA	.1359619	-0.0186	-0.0841	-.06741	.1554167

South Bend, IN	-.0942627	-0.0469	-0.0724	-.0558536	.1546162
Utica-Rome, NY	-.3753005	-0.0639	-0.1254	-.1093547	.1501521
Cleveland-Akron, OH	.3290514	-0.0159	0.0063	.0220215	.1469824
Rochester, NY	.0190762	-0.0410	-0.0287	-.0133072	.1438314
Binghamton, NY	-.3016821	-0.0537	-0.1232	-.1078462	.1429658
Lafayette, IN	.2360758	-0.0060	-0.0687	-.0536155	.1405323
Rockford, IL	-.2372352	-0.0689	-0.0243	-.0094077	.1386504
Scranton–Wilkes-Barre–Hazleton, PA	-.0484406	-0.0271	-0.1064	-.0918143	.1361892
Providence-Fall River-Warwick, RI-MA	.5912651	0.0140	0.0219	.0362946	.1342562
Jamestown, NY	-.6325198	-0.0794	-0.1566	-.1422868	.1338638
Elmira, NY	-.414919	-0.0610	-0.1322	-.1178383	.1336799
Wichita, KS	-.1867949	-0.0476	-0.0786	-.0645386	.1313574
Albany-Schenectady-Troy, NY	-.0258933	-0.0414	-0.0256	-.011832	.1283934
Lincoln, NE	.329759	0.0216	-0.1220	-.1082566	.1283073
Pueblo, CO	.0060262	-0.0029	-0.1616	-.1485231	.1223479
Columbus, OH	.1558117	-0.0282	0.0125	.0256099	.1222642
Muncie, IN	-.2723809	-0.0431	-0.1224	-.1093768	.1213457
Provo-Orem, UT	.4471087	0.0193	-0.0478	-.0348536	.1210581
Stockton-Lodi, CA	.528828	-0.0021	0.0830	.0956708	.1181374
Amarillo, TX	-.0278507	-0.0099	-0.1424	-.1297768	.1181062
St. Louis, MO-IL	.0518211	-0.0335	-0.0073	.0053165	.1180516
Modesto, CA	.3889059	-0.0084	0.0502	.0624303	.1140307
Waterloo-Cedar Falls, IA	-.132249	-0.0229	-0.1290	-.1169513	.1128643
Lexington, KY	-.1503869	-0.0330	-0.0950	-.0829472	.1123206
Davenport-Moline-Rock Island, IA-IL	-.2150651	-0.0414	-0.0877	-.0757843	.1108069
Minneapolis-St. Paul, MN-WI	.2029213	-0.0316	0.0671	.0788453	.1095167
Phoenix-Mesa, AZ	.5067466	0.0118	0.0300	.0415929	.1077507
Fresno, CA	.2324709	-0.0082	-0.0135	-.0019895	.1075737
Columbus, GA-AL	-.4880663	-0.0553	-0.1516	-.1400692	.1072824
Oklahoma City, OK	-.1461316	-0.0204	-0.1347	-.1233957	.1050488
Kokomo, IN	-.6175839	-0.1096	0.0293	.0396908	.0965739
Louisville, KY-IN	-.0163084	-0.0232	-0.0467	-.0366148	.0937779
Cincinnati-Hamilton, OH-KY-IN	-.0068022	-0.0381	0.0199	.0299539	.0935781
Victoria, TX	-.6230851	-0.0738	-0.1039	-.0948376	.0841669
Saginaw-Bay City-Midland, MI	-.4886423	-0.0739	-0.0349	-.0263711	.079897
St. Joseph, MO	-.3472844	-0.0256	-0.1681	-.1596266	.0786807
Waco, TX	-.4387012	-0.0473	-0.1184	-.1101471	.0771257
Peoria-Pekin, IL	-.3992892	-0.0614	-0.0409	-.0327383	.076618
Kansas City, MO-KS	-.1252519	-0.0366	-0.0150	-.0068974	.075246

Rochester, MN	-.3253532	-0.0613	-0.0032	.0046953	.0740497
Tulsa, OK	-.2796107	-0.0316	-0.1036	-.0957309	.0729766
State College, PA	.2922625	0.0364	-0.1195	-.1117945	.0720046
Lansing-East Lansing, MI	-.2182456	-0.0464	-0.0084	-.0009783	.0692506
Boston-Worcester-Lawrence, MA-NH-ME-CT	.7968019	0.0342	0.1284	.1357242	.0683087
Tampa-St. Petersburg-Clearwater, FL	.1087955	0.0026	-0.0544	-.0474067	.0651656
Los Angeles-Riverside-Orange County, CA	1.249761	0.0812	0.1502	.1569763	.0634813
Duluth-Superior, MN-WI	-.6763655	-0.0694	-0.1157	-.1090326	.062516
Abilene, TX	-.2557217	0.0036	-0.2234	-.2167182	.0622989
Salt Lake City-Ogden, UT	.3935055	0.0261	-0.0147	-.0081528	.061412
Des Moines, IA	-.0847455	-0.0216	-0.0366	-.0301734	.0603558
Dayton-Springfield, OH	-.1455098	-0.0301	-0.0296	-.0231789	.0602668
Evansville-Henderson, IN-KY	-.4577099	-0.0468	-0.1045	-.0980137	.0601523
Grand Rapids-Muskegon-Holland, MI	-.2361218	-0.0439	-0.0100	-.0038981	.0573304
Fort Wayne, IN	-.5326287	-0.0629	-0.0670	-.0609858	.0561799
Norfolk-Virginia Beach-Newport News, VA-	.2098869	0.0268	-0.0950	-.0889341	.056178
Grand Forks, ND-MN	-.7002264	-0.0461	-0.2098	-.2041357	.0532274
Topeka, KS	-.3493396	-0.0243	-0.1372	-.1315619	.0526932
Indianapolis, IN	-.1833521	-0.0391	0.0031	.0085998	.0514207
Terre Haute, IN	-.6763202	-0.0597	-0.1387	-.1332446	.0507915
Youngstown-Warren, OH	-.5119416	-0.0524	-0.0904	-.0851075	.0490883
Sioux Falls, SD	-.229807	-0.0059	-0.1456	-.1408578	.0439603
Visalia-Tulare-Porterville, CA	-.086887	-0.0164	-0.0356	-.0308907	.0439264
Spokane, WA	.0105509	0.0079	-0.0904	-.0857898	.0433484
Lancaster, PA	-.0021449	-0.0108	-0.0174	-.0128362	.0423493
Springfield, MA	.1429743	0.0024	-0.0033	.001093	.0414337
San Angelo, TX	-.4866839	-0.0249	-0.1774	-.1732655	.0382254
Appleton-Oshkosh-Neenah, WI	-.1850983	-0.0214	-0.0516	-.0475299	.0377581
Janesville-Beloit, WI	-.3952251	-0.0504	-0.0189	-.0155602	.0312168
La Crosse, WI-MN	-.3554412	-0.0200	-0.1260	-.1227562	.0307129
York, PA	-.2773376	-0.0323	-0.0356	-.0325027	.028489
Springfield, IL	-.4350782	-0.0386	-0.0821	-.0790846	.0281281
Bryan-College Station, TX	.0633749	0.0267	-0.1222	-.1191633	.0281198
Williamsport, PA	-.4712907	-0.0308	-0.1296	-.1266871	.0269975
Richmond-Petersburg, VA	-.2281304	-0.0333	-0.0061	-.0032359	.0268742
Johnstown, PA	-.9111058	-0.0625	-0.2010	-.1981275	.0264227
Yuma, AZ	-.1213466	0.0015	-0.1005	-.0976616	.0263575
Harrisburg-Lebanon-Carlisle, PA	-.2222067	-0.0292	-0.0197	-.016924	.0262063
Canton-Massillon, OH	-.3222547	-0.0243	-0.0828	-.0802185	.023838

Hartford, CT	.0865778	-0.0263	0.1197	.1218906	.0203582
Shreveport-Bossier City, LA	-.5831715	-0.0418	-0.1237	-.1216929	.0186002
Bismarck, ND	-.9184533	-0.0483	-0.2495	-.2477077	.0169085
Green Bay, WI	-.0928975	-0.0107	-0.0220	-.0202243	.0161871
Lake Charles, LA	-.7208816	-0.0639	-0.0845	-.0833693	.0109671
Sharon, PA	-.6100441	-0.0325	-0.1507	-.1504862	.0021221
Honolulu, HI	0	0.2036	0.0571	0	0
Non-metro, HI	0	0.1265	0.0126	0	0
Non-metro, AK	0	0.0116	0.0372	0	0
Anchorage, AK	0	0.0232	0.0772	0	0
Sheboygan, WI	-.3024393	-0.0186	-0.0616	-.0617215	-.001035
Lima, OH	-.7869979	-0.0625	-0.1032	-.1033558	-.0014477
Richland-Kennewick-Pasco, WA	-.4395986	-0.0510	0.0106	.010442	-.0019367
Kalamazoo-Battle Creek, MI	-.6018697	-0.0563	-0.0368	-.037393	-.0057681
West Palm Beach-Boca Raton, FL	.2352861	0.0175	0.0456	.0449492	-.0062766
Sacramento-Yolo, CA	.433597	0.0328	0.0750	.0742749	-.0072132
Owensboro, KY	-.7016693	-0.0410	-0.1439	-.1447629	-.0077083
Huntington-Ashland, WV-KY-OH	-1.071797	-0.0742	-0.1774	-.178206	-.0078358
Orlando, FL	-.0567962	0.0063	-0.0366	-.0378199	-.0109518
Savannah, GA	-.3067237	-0.0106	-0.0801	-.0814664	-.0124286
Wichita Falls, TX	-.6010185	-0.0077	-0.2262	-.2277354	-.0140978
Iowa City, IA	.1030325	0.0345	-0.0724	-.0741353	-.015861
Mansfield, OH	-.7223283	-0.0480	-0.1096	-.1115244	-.0177241
Macon, GA	-.8435723	-0.0684	-0.0789	-.0809414	-.0193693
Bloomington, IN	-.0152942	0.0321	-0.1102	-.1124025	-.0208564
Birmingham, AL	-.5607096	-0.0468	-0.0340	-.0362991	-.0218629
Jacksonville, FL	-.2650929	-0.0091	-0.0514	-.0539105	-.0233245
Jackson, MI	-.7221109	-0.0643	-0.0342	-.0366876	-.0233369
Baton Rouge, LA	-.4652739	-0.0306	-0.0534	-.0559597	-.0242581
Merced, CA	-.2152248	-0.0122	-0.0128	-.0154728	-.0250396
Cedar Rapids, IA	-.2662044	-0.0021	-0.0784	-.0811376	-.0251342
Albany, GA	-.860005	-0.0632	-0.0990	-.1017335	-.0257786
Billings, MT	-.3294488	0.0135	-0.1693	-.172252	-.0271357
Lafayette, LA	-.8738462	-0.0568	-0.1298	-.1327303	-.0272423
Roanoke, VA	-.469801	-0.0170	-0.1066	-.1096053	-.0279536
Steubenville-Weirton, OH-WV	-1.02337	-0.0580	-0.1890	-.1922471	-.0307371
Lewiston-Auburn, ME	-.4354692	-0.0076	-0.1234	-.1268854	-.0322411
Wheeling, WV-OH	-1.026247	-0.0576	-0.1887	-.1922342	-.0329024
Madison, WI	.3332752	0.0531	-0.0179	-.0214981	-.0334092

Austin-San Marcos, TX	.0687546	0.0164	0.0143	.010575	-.0349459
Boise City, ID	-.1853286	0.0102	-0.0770	-.0808233	-.0355713
Eau Claire, WI	-.6133419	-0.0261	-0.1203	-.1243938	-.0385608
Dubuque, IA	-.6671287	-0.0242	-0.1500	-.1543345	-.0405638
Jackson, MS	-.6270529	-0.0314	-0.0990	-.1035512	-.0424646
Melbourne-Titusville-Palm Bay, FL	-.3595625	-0.0000	-0.1036	-.108304	-.0435372
Atlanta, GA	-.2910109	-0.0320	0.0631	.0582452	-.0450328
Yakima, WA	-.2873999	-0.0089	-0.0291	-.0340531	-.0466091
Tucson, AZ	.1217321	0.0522	-0.0909	-.0960166	-.0480639
Denver-Boulder-Greeley, CO	.4667839	0.0545	0.0657	.0600663	-.0523667
Non-metro, ND	-1.113298	-0.0405	-0.2622	-.2685271	-.0590416
Pocatello, ID	-1.041509	-0.0615	-0.1410	-.1474399	-.0598014
Augusta-Aiken, GA-SC	-.8954359	-0.0567	-0.0928	-.0992447	-.0600048
St. Cloud, MN	-.8594629	-0.0482	-0.1102	-.1166887	-.0608784
Albuquerque, NM	.1132034	0.0491	-0.0635	-.070067	-.0609031
Fort Smith, AR-OK	-1.010234	-0.0447	-0.1938	-.2003328	-.0613459
Elkhart-Goshen, IN	-.707327	-0.0428	-0.0594	-.0660949	-.0626701
Charleston, WV	-.9167963	-0.0521	-0.1169	-.12372	-.0633341
Springfield, MO	-.5594393	0.0032	-0.1754	-.1825934	-.0670718
Danville, VA	-1.087645	-0.0574	-0.1627	-.1703033	-.0711861
Gainesville, FL	-.3012467	0.0244	-0.1337	-.1416619	-.074201
Columbia, MO	-.4007436	0.0227	-0.1638	-.1723978	-.080263
Lawton, OK	-.9423505	-0.0161	-0.2530	-.2616441	-.0806974
Mobile, AL	-.6757585	-0.0160	-0.1282	-.1369891	-.0821807
Monroe, LA	-.8673664	-0.0357	-0.1328	-.1417352	-.0837585
Portland-Salem, OR-WA	.240806	0.0472	0.0374	.0283968	-.0844767
Hattiesburg, MS	-.9100014	-0.0287	-0.1796	-.1887074	-.0849453
Non-metro, PA	-1.0573	-0.0529	-0.1455	-.1547492	-.0864575
Montgomery, AL	-.5784808	-0.0034	-0.1241	-.1337116	-.0898736
Huntsville, AL	-.9080492	-0.0548	-0.0617	-.0714873	-.091073
Enid, OK	-1.041137	-0.0317	-0.2187	-.2285127	-.0915362
Pine Bluff, AR	-1.127605	-0.0534	-0.1677	-.1776054	-.0921278
Biloxi-Gulfport-Pascagoula, MS	-.8175875	-0.0264	-0.1352	-.1451404	-.0929305
Colorado Springs, CO	.0604443	0.0553	-0.0661	-.0761663	-.0936749
Lakeland-Winter Haven, FL	-.7590152	-0.0227	-0.1190	-.1293014	-.09579
Jackson, TN	-1.073083	-0.0628	-0.0979	-.1081956	-.0960981
Lawrence, KS	-.2396966	0.0381	-0.1291	-.1394593	-.0969793
Reno, NV	.2627501	0.0530	0.0435	.032989	-.0980274
Yuba City, CA	-.394301	0.0087	-0.0660	-.0773233	-.1057248

Columbia, SC	-.5657874	-0.0073	-0.0759	-.0875116	-.1080138
Casper, WY	-.8326231	-0.0019	-0.2191	-.2308507	-.1098936
Tuscaloosa, AL	-.7135378	-0.0135	-0.0986	-.1116781	-.1219
Parkersburg-Marietta, WV-OH	-1.393989	-0.0720	-0.1696	-.1827344	-.1229434
Pensacola, FL	-.6762272	0.0027	-0.1462	-.1594492	-.1235687
Little Rock-North Little Rock, AR	-.698691	-0.0108	-0.0996	-.1129256	-.1240764
Greenville, NC	-.7659174	-0.0216	-0.0846	-.0980478	-.1251284
Las Cruces, NM	-.6380898	0.0192	-0.1900	-.2037726	-.1281331
Seattle-Tacoma-Bremerton, WA	.3242277	0.0608	0.0949	.0802736	-.1363539
Daytona Beach, FL	-.5620614	0.0195	-0.1439	-.1585508	-.1366871
Tyler, TX	-.8843547	-0.0253	-0.1056	-.1203205	-.1378048
Chattanooga, TN-GA	-.9697257	-0.0347	-0.1051	-.119941	-.1385471
Fayetteville, NC	-.5661589	0.0281	-0.1784	-.1933158	-.1388553
Tallahassee, FL	-.4506846	0.0218	-0.0981	-.1130513	-.139211
Wausau, WI	-1.066127	-0.0494	-0.0865	-.1015052	-.1402135
San Diego, CA	.8719581	0.1226	0.0976	.0825433	-.1402226
Houma, LA	-1.193882	-0.0539	-0.1229	-.1382529	-.1428943
Cheyenne, WY	-.414065	0.0563	-0.2170	-.2325102	-.1444526
Alexandria, LA	-1.127252	-0.0308	-0.1734	-.1898125	-.1531511
Nashville, TN	-.5389	-0.0010	-0.0160	-.0330452	-.1589256
Benton Harbor, MI	-.9375473	-0.0295	-0.0808	-.0979629	-.1600664
Gadsden, AL	-1.445749	-0.0691	-0.1505	-.1677425	-.1609934
Texarkana, TX-Texarkana, AR	-1.556447	-0.0679	-0.1997	-.2175172	-.1660015
Longview-Marshall, TX	-1.360086	-0.0571	-0.1491	-.1672254	-.1690904
Decatur, AL	-1.355786	-0.0719	-0.0853	-.1034276	-.169273
Auburn-Opelika, AL	-.9503742	-0.0153	-0.1316	-.1498269	-.169637
Non-metro, NY	-1.246195	-0.0501	-0.1234	-.1416185	-.1702956
Killeen-Temple, TX	-.6452676	0.0402	-0.2200	-.2382578	-.170305
Fort Pierce-Port St. Lucie, FL	-.6197869	0.0111	-0.0779	-.0968482	-.1767803
Charlotte-Gastonia-Rock Hill, NC-SC	-.6600319	-0.0127	0.0067	-.0125962	-.1798679
Fort Walton Beach, FL	-.3827377	0.0620	-0.1744	-.1937211	-.1800475
Portland, ME	-.2392584	0.0511	-0.0598	-.0793457	-.1822975
Knoxville, TN	-.9367148	-0.0110	-0.1248	-.1444028	-.1824984
Charleston-North Charleston, SC	-.5344716	0.0249	-0.0821	-.1020343	-.1863371
Joplin, MO	-1.215934	-0.0108	-0.2456	-.2658848	-.1889587
Athens, GA	-.7290603	0.0158	-0.1253	-.1459244	-.1926658
Greensboro–Winston Salem–High Point, NC	-.8483993	-0.0156	-0.0494	-.0701273	-.1930203
Non-metro, RI	-.0940434	0.0401	0.0705	.049717	-.1941734
Charlottesville, VA	-.3291245	0.0538	-0.0897	-.1109236	-.1982224

Sarasota-Bradenton, FL	-.1384917	0.0661	-0.0457	-.0674536	-.2028493
Raleigh-Durham-Chapel Hill, NC	-.5031091	0.0105	0.0177	-.0041405	-.2039352
San Francisco-Oakland-San Jose, CA	1.209271	0.1378	0.2889	.2666445	-.2081352
Panama City, FL	-.7231189	0.0257	-0.1378	-.1603233	-.2105179
Non-metro, KS	-1.487791	-0.0346	-0.2397	-.2623977	-.211829
Sumter, SC	-1.390925	-0.0372	-0.1825	-.2053206	-.2130241
Great Falls, MT	-.9568921	0.0364	-0.2828	-.3059563	-.2156483
Salinas (Monterey-Carmel), CA	.8634857	0.1370	0.1445	.1212501	-.216916
Non-metro, OH	-1.386654	-0.0520	-0.1109	-.1343023	-.2186089
Fort Collins-Loveland, CO	-.0445725	0.0794	-0.0320	-.0556689	-.2204746
Non-metro, WV	-1.522943	-0.0421	-0.2097	-.2335687	-.2224839
Florence, AL	-1.397713	-0.0417	-0.1491	-.1732441	-.2254196
Pittsfield, MA	-.6885415	0.0135	-0.0496	-.0737628	-.2254271
Cumberland, MD-WV	-1.433931	-0.0400	-0.1712	-.1955082	-.2264021
Greenville-Spartanburg-Anderson, SC	-1.149242	-0.0306	-0.0784	-.1027147	-.2266005
Fort Myers-Cape Coral, FL	-.4543439	0.0486	-0.0844	-.1087108	-.2268935
Non-metro, IL	-1.518542	-0.0517	-0.1543	-.1791688	-.2317956
Lynchburg, VA	-1.331154	-0.0311	-0.1401	-.1658348	-.2405762
Anniston, AL	-1.578977	-0.0461	-0.1897	-.2157075	-.2426608
Dothan, AL	-1.532595	-0.0404	-0.1865	-.2128823	-.2465344
Eugene-Springfield, OR	-.1585051	0.0883	-0.0837	-.1101517	-.2465874
Chico-Paradise, CA	-.4443775	0.0531	-0.0668	-.0934505	-.2489652
Glens Falls, NY	-1.200676	-0.0204	-0.1085	-.1354954	-.2514766
Rapid City, SD	-.9482528	0.0329	-0.2115	-.2385321	-.2522185
Clarksville-Hopkinsville, TN-KY	-1.275032	-0.0043	-0.2061	-.2332569	-.2538936
Non-metro, IN	-1.500182	-0.0499	-0.1129	-.1407004	-.2598897
Fayetteville-Springdale-Rogers, AR	-1.065683	0.0046	-0.1316	-.1599007	-.2640834
Johnson City-Kingsport-Bristol, TN-VA	-1.484617	-0.0284	-0.1800	-.2089377	-.2700084
Bangor, ME	-1.364958	-0.0177	-0.1690	-.1979196	-.2701102
Non-metro, NE	-1.5918	-0.0207	-0.2557	-.2850641	-.2742227
Burlington, VT	-.4526969	0.0654	-0.0822	-.1117866	-.2763209
Florence, SC	-1.606173	-0.0494	-0.1309	-.1612378	-.2826878
Non-metro, IA	-1.553909	-0.0270	-0.1921	-.2229731	-.2878987
Sherman-Denison, TX	-1.44886	-0.0280	-0.1366	-.1676825	-.2898143
Non-metro, MS	-1.956306	-0.0657	-0.2149	-.2459241	-.2899799
Jonesboro, AR	-1.650891	-0.0256	-0.2380	-.2691773	-.2911434
New London-Norwich, CT-RI	-.7648541	0.0059	0.0514	.0194305	-.2981185
Non-metro, LA	-1.845686	-0.0578	-0.1784	-.2107666	-.3024214
Non-metro, MN	-1.735236	-0.0471	-0.1630	-.196024	-.3084637

Non-metro, OK	-1.830395	-0.0338	-0.2548	-.2883861	-.3136311
Punta Gorda, FL	-.8586937	0.0489	-0.1432	-.1771205	-.3168141
Non-metro, ID	-1.256371	0.0121	-0.1735	-.207579	-.3176243
Jacksonville, NC	-1.094279	0.0513	-0.2544	-.2889087	-.322375
Non-metro, TX	-1.848386	-0.0433	-0.2057	-.2406386	-.3262365
Corvalis, OR	-.4744354	0.0806	-0.0808	-.1159397	-.3278399
Dover, DE	-1.326541	-0.0093	-0.0861	-.1224371	-.3394162
Medford-Ashland, OR	-.4250332	0.0946	-0.0987	-.1351417	-.3399621
Goldsboro, NC	-1.508752	-0.0074	-0.1762	-.2126989	-.3407777
Non-metro, NM	-1.481898	0.0022	-0.2020	-.2386284	-.3419448
Flagstaff, AZ-UT	-1.088802	0.0305	-0.1294	-.166575	-.3473693
Non-metro, WY	-1.402036	0.0067	-0.1650	-.2030406	-.3548632
Santa Barbara-Santa Maria-Lompoc, CA	.7126831	0.1758	0.1246	.0856784	-.3628299
Ocala, FL	-1.581646	-0.0103	-0.1661	-.2049529	-.3629081
Non-metro, UT	-1.315183	0.0098	-0.1239	-.1629005	-.3643525
Non-metro, KY	-2.085953	-0.0570	-0.1926	-.2324691	-.3719968
Non-metro, MD	-1.441416	-0.0225	-0.0373	-.077201	-.372028
Rocky Mount, NC	-1.639932	-0.0241	-0.1143	-.1547982	-.3776701
Redding, CA	-1.011372	0.0415	-0.0737	-.1158261	-.392666
Non-metro, CT	-1.122088	-0.0067	0.0780	.0356649	-.3949834
Non-metro, WI	-1.761495	-0.0280	-0.1199	-.1628875	-.4016096
Grand Junction, CO	-.5175143	0.1143	-0.1336	-.1767228	-.401961
Bellingham, WA	-.7008425	0.0738	-0.0379	-.0819699	-.4107988
Non-metro, VA	-1.908293	-0.0309	-0.1625	-.206538	-.4109614
Hickory-Morganton-Lenoir, NC	-1.623926	-0.0083	-0.1236	-.1677264	-.4116969
Non-metro, MI	-1.864408	-0.0379	-0.1078	-.1522571	-.4146196
Non-metro, MO	-2.048186	-0.0225	-0.2513	-.2961268	-.4184402
Non-metro, NV	-1.409143	-0.0107	0.0048	-.0407018	-.4246427
Wilmington, NC	-.9144512	0.0715	-0.1036	-.1494037	-.4270377
Naples, FL	-.4242264	0.0952	0.0273	-.0186618	-.4287027
Missoula, MT	-.9054939	0.1006	-0.2081	-.2547995	-.4355366
Non-metro, WA	-1.185817	0.0375	-0.0671	-.114417	-.4417068
Myrtle Beach, SC	-1.401878	0.0381	-0.1478	-.1967352	-.4569009
Non-metro, SD	-2.035704	0.0008	-0.2788	-.3283534	-.4624381
Non-metro, CA	-.9819576	0.0590	-0.0173	-.0679527	-.4727517
Non-metro, MT	-1.461077	0.0594	-0.2356	-.2866258	-.4763296
Non-metro, AR	-2.267291	-0.0282	-0.2372	-.288823	-.4821928
Non-metro, GA	-2.219047	-0.0403	-0.1463	-.1993227	-.4947404
Non-metro, FL	-1.823466	0.0100	-0.1669	-.220296	-.4983841

Asheville, NC	-1.34261	0.0583	-0.1319	-.1862243	-.5070552
Non-metro, SC	-2.203423	-0.0329	-0.1397	-.1950061	-.5157019
Non-metro, OR	-1.376679	0.0617	-0.1132	-.1710958	-.5405855
Non-metro, TN	-2.469647	-0.0376	-0.1891	-.2483896	-.553502
Non-metro, NC	-2.163671	-0.0131	-0.1476	-.2069737	-.5544938
San Luis Obispo-Atascadero-Paso Robles, CA	-.4577061	0.1245	0.0766	.016841	-.557736
Santa Fe, NM	-.6409403	0.1266	-0.0167	-.0764559	-.5580029
Non-metro, AL	-2.760724	-0.0675	-0.1885	-.248667	-.5613214
Non-metro, AZ	-1.788697	0.0371	-0.1628	-.2235987	-.5677376
Non-metro, ME	-2.003817	0.0274	-0.1836	-.24734	-.594465
Non-metro, MA	-1.376062	0.0634	-0.0422	-.1059272	-.5946526
Non-metro, VT	-1.775137	0.0733	-0.1648	-.2361799	-.6660584
Non-metro, DE	-2.322081	0.0102	-0.0731	-.1506421	-.723881
Non-metro, NH	-2.059247	0.0422	-0.0820	-.1597158	-.7253312
Barnstable-Yarmouth (Cape Cod), MA	-1.111379	0.1215	0.0464	-.0326311	-.7379338
Non-metro, CO	-2.333411	0.1121	-0.0936	-.2013844	-1.006168

Table 1: See text for estimation procedure. $\hat{A}_X^j$ corresponds to the trade-productivity estimates obtained using wage, housing price, and density data, while Restricted $\hat{A}_X^j$ corresponds to trade-productivity estimates obtained using wage and housing price data plus the constant home-productivity assumption.

Figure 1: Distribution, 2000

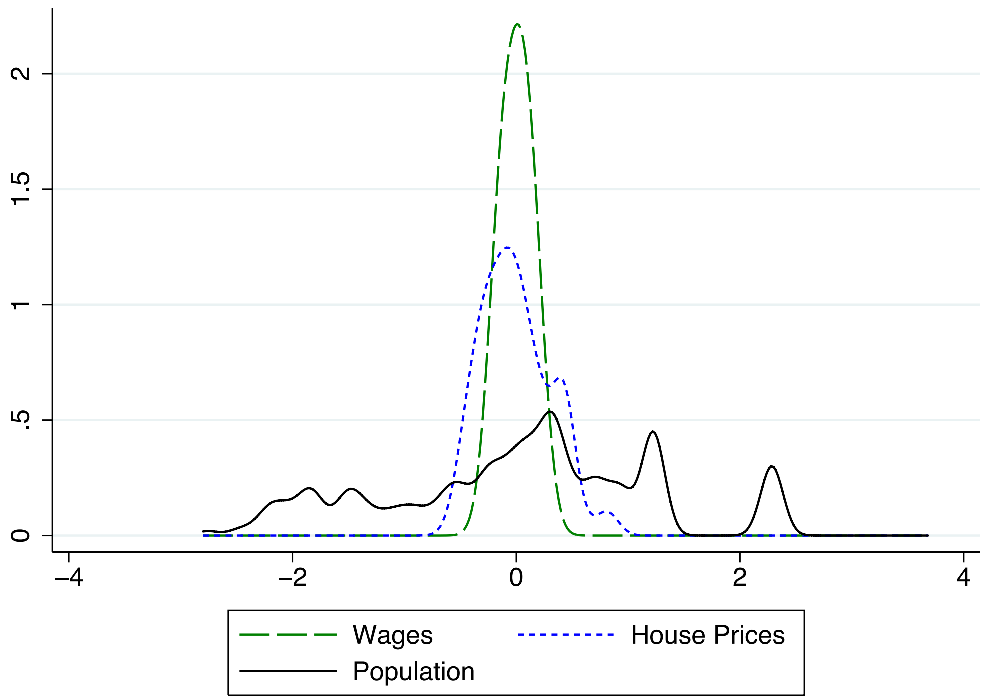


Figure 2: Error in Fitting Pop. Density using $\hat{Q}$ and $\hat{A}_X$ Only

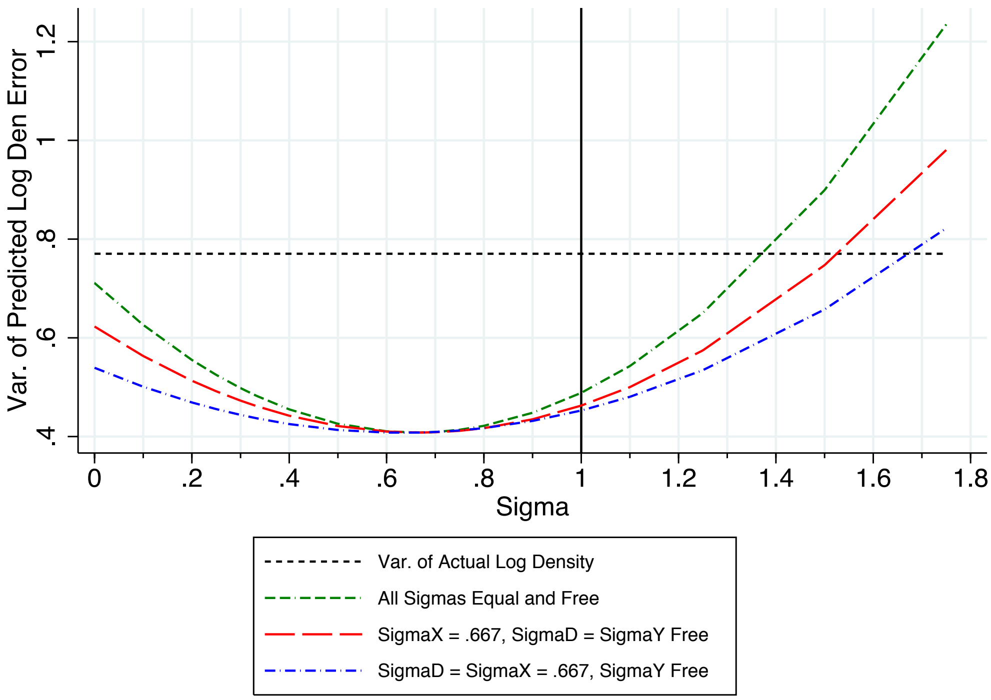


Figure 3: Excess Density and Relative Cost Estimates, 2000

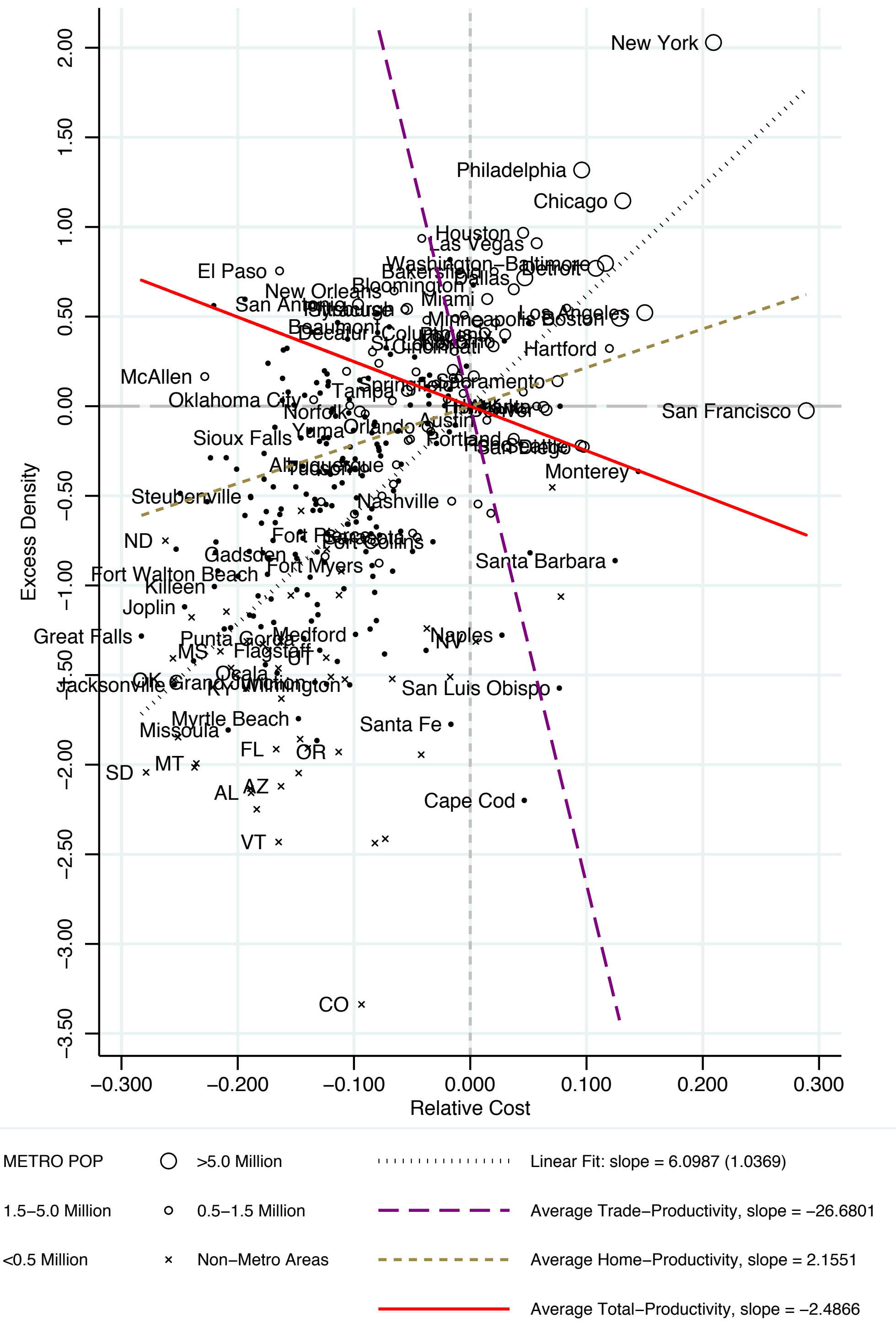


Figure 4: Trade- and Home-Productivity Estimates, 2000

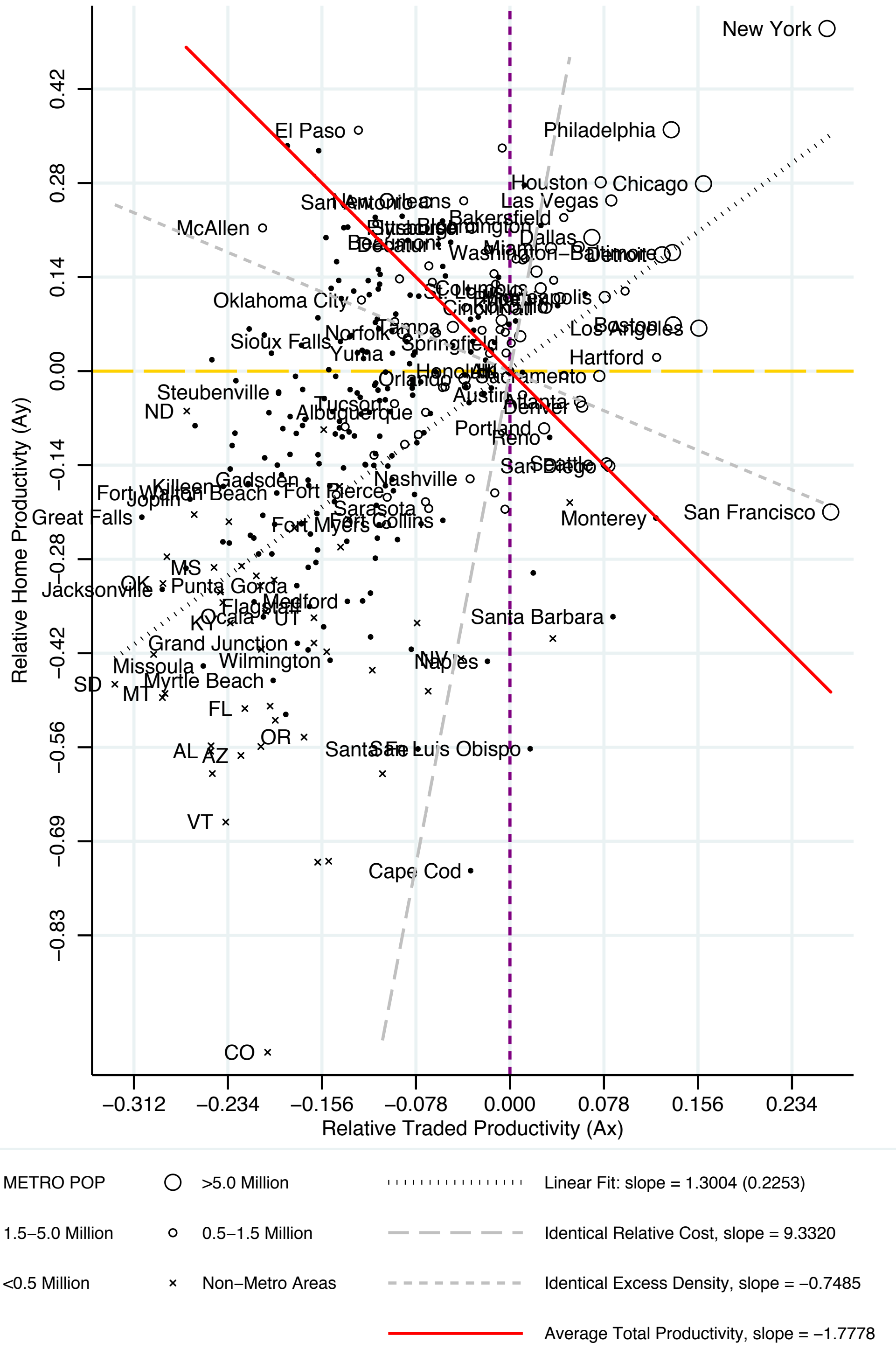


Figure 5: Estimated Amenity Distribution, 2000

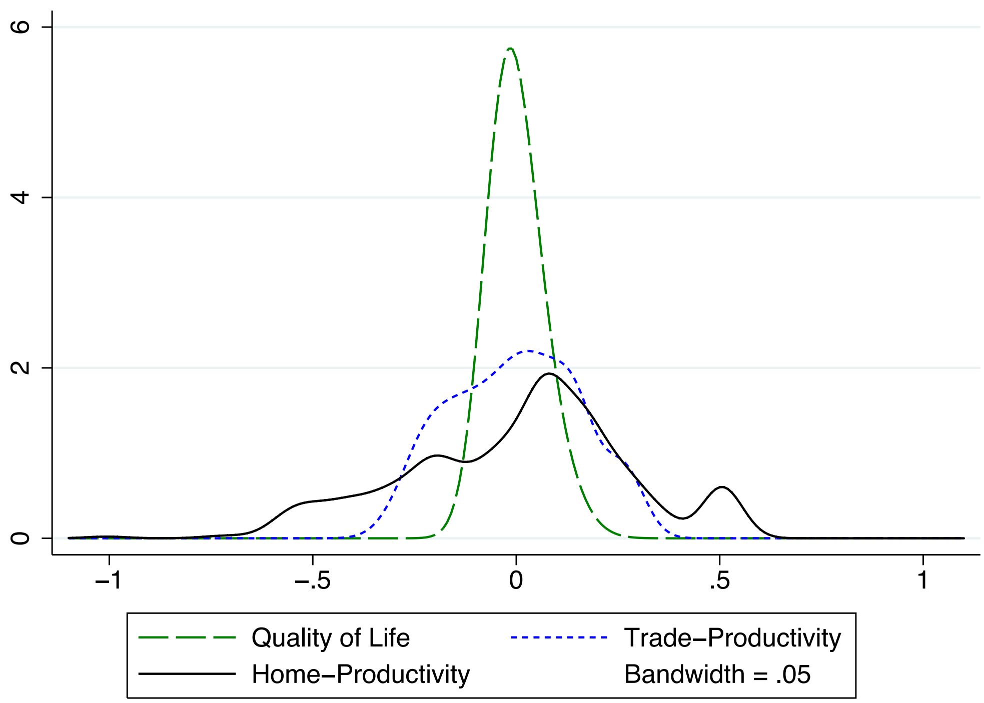


Figure 6: Nonlinear Reduced-Form Elasticities, Single Amenity

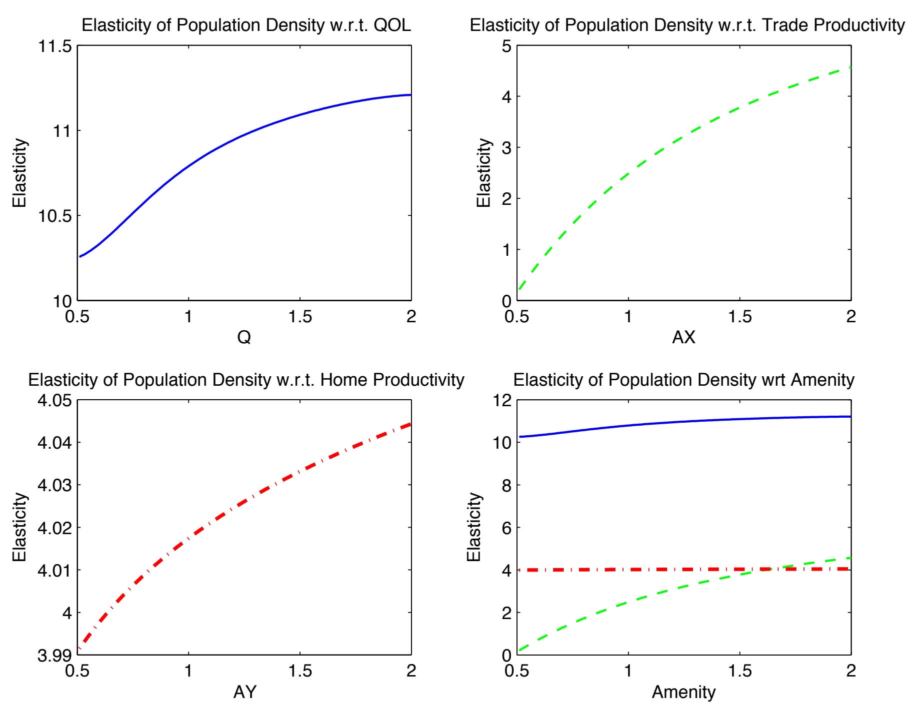
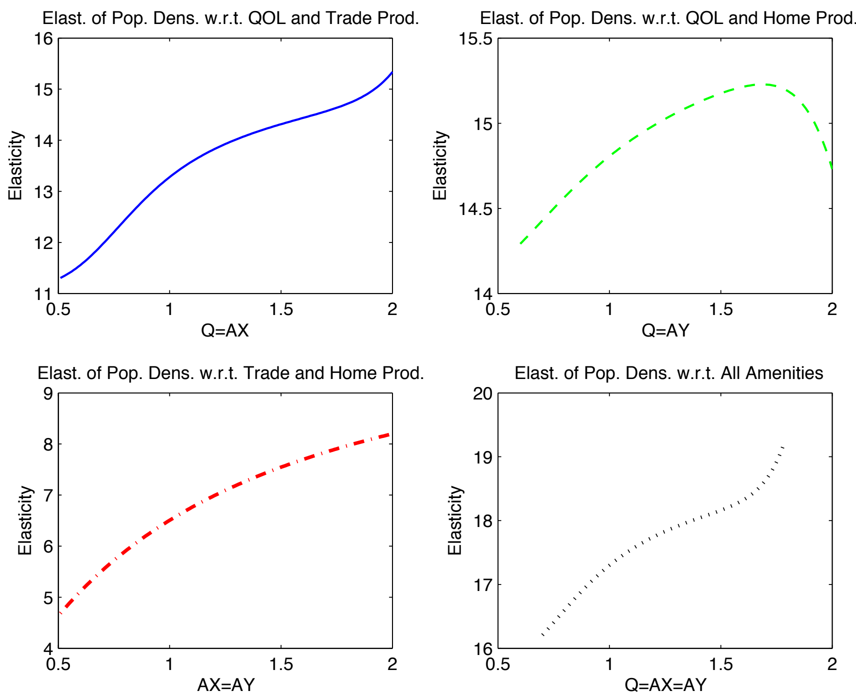


Figure 7: Nonlinear Reduced-Form Elasticities, Multiple Amenities



Appendix - Not for Publication

A Nonlinear Simulation

We employ a two-step simulation method to solve the model without log-linearization. Let the utility function be $U(x, y; Q) = Qx^{1-s_y}y^{s_y}$, which implies that $\sigma_D = 1$. Also define the production function in the traded-good sector to be $X = A_X L_X^{\theta_L} N_X^{\theta_N} K_X^{1-\theta_L-\theta_N}$, which implies that $\sigma_X = 1$. The production function for the home-good sector is defined similarly. This is a Cobb-Douglas economy.

We first consider a "large" city with amenity values normalized so that $Q = A_X = A_Y = 1$. We fix the amount of land and population. We additionally normalize the value of $\bar{\iota}$ and solve (1)-(16) plus a capital constraint, $K_X+K_Y = K$, for fifteen unknown variables, $\bar{u}, w, r, p, x, y, X, Y, N_X, N_Y, L_X, L_Y, K_X, K_Y, K$. After solving for these variables, we verify that the values of s_R, s_w, λ_L, and λ_N match those in Table 1 given our values for $s_y, \theta_L, \theta_N, \phi_L$, and ϕ_K. We also obtain values for $\bar{u}$, R, and I.

We then consider a "small" city, which we endow with land equal to $\frac{1}{1{,}000{,}000}$ of the large city's land.[30] Unlike the large city, the population for the small city is endogenous. We solve the same system as for the large city, but we now solve for $w, r, p, x, y, X, Y, N_X, N_Y, L_X, L_Y, K_X, K_Y, N, K$. By default, $Q = A_X = A_Y = 1$. Varying the values of Q, A_X, and A_Y for the small city and then solving the system yields elasticities of quantities and prices with respect to amenities, e.g. $\frac{\Delta N}{N}\frac{Q}{\Delta Q}$.

For the log-linearized model, we estimate $\hat{N} = \varepsilon_{NQ}\hat{Q} + \varepsilon_{NA_X}\hat{A}_X + \varepsilon_{NA_Y}\hat{A}_Y$ using OLS. Importantly, the estimated coefficients are average elasticities, taken across all cities using population weights. In general, each elasticity is itself a nonlinear function. From the nonlinear model, we can calculate the function. From the linear model, we only see the average elasticity evaluated at observed density levels. We use the nonlinear simulation to ensure that our log-linearized results are reasonable.

When $\sigma_D = \sigma_X = \sigma_Y = 1$ in the log-linearized model, we obtain $\hat{N} = 12.66\hat{Q} + 3.24\hat{A}_X + 3.94\hat{A}_Y$. For the nonlinear simulation, we obtain $\hat{N} = 10.82\hat{Q} + 2.68\hat{A}_X + 4.02\hat{A}_Y$. We cannot determine whether the (relatively small) disparity comes from differences in the responsiveness of quantities to amenities, e.g. $\Delta N/\Delta Q$, or the levels of quantities and amenities, e.g. Q/N, but the different methods yield consistent results. Figure 5 graphs the elasticity of population density with respect to amenities, where the bottom-right panel collects the other three graphs.

Figure 6 graphs the elasticity with respect to amenities, where we now change multiple amenities, in equal amounts, at the same time. The top right panel is particularly interesting. Increasing quality of life and home-productivity both decrease wages; higher quality of life increases the cost of housing, while home-productivity reduces the cost. One explanation for the graph is that, for $Q = A_Y > 1.7$, quality of life dominates home-productivity, driving home prices up and reducing the increase in population density. The bottom-right panel shows that population density responds very strongly, and on a similar order of magnitude as our log-linearized model predicts, to a change in all three amenities.

[30] We do this following Rappaport (2008a) to avoid any feedback effect from the small city to the large one.

B System of Equations

After log-linearizing, we can represent our system of equations in matrix form as:

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ \lambda_N & 0 & 0 & 0 & 1-\lambda_N & 0 & 0 & 0 & -1 \\ 0 & \lambda_L & 0 & 0 & 0 & 1-\lambda_L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \hat{N}_X \\ \hat{L}_X \\ \hat{K}_X \\ \hat{X} \\ \hat{N}_Y \\ \hat{L}_Y \\ \hat{K}_Y \\ \hat{Y} \\ \hat{N} \end{bmatrix} = \begin{bmatrix} (\sigma_X - 1)\hat{A}_X - \sigma_X \hat{w} \\ (\sigma_X - 1)\hat{A}_X - \sigma_X \hat{r} \\ (\sigma_X - 1)\hat{A}_X \\ (\sigma_Y - 1)\hat{A}_Y + \sigma_Y(\hat{p} - \hat{w}) \\ (\sigma_Y - 1)\hat{A}_Y + \sigma_Y(\hat{p} - \hat{r}) \\ (\sigma_Y - 1)\hat{A}_Y + \sigma_Y \hat{p} \\ 0 \\ \varepsilon_{L,r} \\ -s_x \sigma_D \hat{p} - \hat{Q} \end{bmatrix}$$

The quantities on the right-hand side of the system are observed from the data and model parameters.

C Estimating Trade- and Home-Productivity Differentials

Rearranging (**??**) and (25), we have a system of two equations in two unknowns, $\hat{A}_X$ and $\hat{A}_Y$. This system can be represented in matrix form as:

$$\begin{bmatrix} 1 & \frac{-\theta_L}{\phi_L} \\ \varepsilon_{N,A_X} & \varepsilon_{N,A_Y} \end{bmatrix} \begin{bmatrix} \hat{A}_X \\ \hat{A}_Y \end{bmatrix} = \begin{bmatrix} \frac{\theta_L}{\phi_L}\hat{p} + (\theta_N - \phi_N \frac{\theta_L}{\phi_L})\hat{w} \\ \hat{N} - \varepsilon_{N,Q}\hat{Q} \end{bmatrix}$$

Again, the quantities on the right-hand side of the system are observed from the data and model parameters.

D Data and Estimation

United States Census data from the 2000 Integrated Public-Use Microdata Series (IPUMS), from Ruggles et al. (2004), are used to calculate wage and housing price differentials. The wage differentials are calculated for workers ages 25 to 55, who report working at least 30 hours a week, 26 weeks a year. The MSA assigned to a worker is determined by their place of residence, rather than their place of work. The wage differential of an MSA is found by regressing log hourly wages on individual covariates and indicators for which MSA a worker lives in, using the coefficients on these MSA indicators. The covariates consist of

- 12 indicators of educational attainment;
- a quartic in potential experience, and potential experience interacted with years of education;
- 9 indicators of industry at the one-digit level (1950 classification);

- 9 indicators of employment at the one-digit level (1950 classification);
- 4 indicators of marital status (married, divorced, widowed, separated);
- an indicator for veteran status, and veteran status interacted with age;
- 5 indicators of minority status (Black, Hispanic, Asian, Native American, and other);
- an indicator of immigrant status, years since immigration, and immigrant status interacted with black, Hispanic, Asian, and other;
- 2 indicators for English proficiency (none or poor).

All covariates are interacted with gender.

This regression is first run using census-person weights. From the regressions a predicted wage is calculated using individual characteristics alone, controlling for MSA, to form a new weight equal to the predicted wage times the census-person weight. These new income-adjusted weights are needed since workers need to be weighted by their income share. The new weights are then used in a second regression, which is used to calculate the city-wage differentials from the MSA indicator variables. In practice, this weighting procedure has only a small effect on the estimated wage differentials.

Housing price differentials are calculated using the logarithm reported gross rents and housing values. Only housing units moved into within the last 10 years are included in the sample to ensure that the price data are fairly accurate. The differential housing price of an MSA is calculated in a manner similar to wages, except using a regression of the actual or imputed rent on a set of covariates at the unit level. The covariates for the adjusted differential are

- 9 indicators of building size;
- 9 indicators for the number of rooms, 5 indicators for the number of bedrooms, number of rooms interacted with number of bedrooms, and the number of household members per room;
- 2 indicators for lot size;
- 7 indicators for when the building was built;
- 2 indicators for complete plumbing and kitchen facilities;
- an indicator for commercial use;
- an indicator for condominium status (owned units only).

A regression of housing values on housing characteristics and MSA indicator variables is first run using only owner-occupied units, weighting by census-housing weights. A new value-adjusted weight is calculated by multiplying the census-housing weights by the predicted value from this first regression using housing characteristics alone, controlling for MSA. A second regression is run using these new weights for all units, rented and owner-occupied, on the housing characteristics fully interacted with tenure, along with the MSA indicators, which are not interacted. The house-price differentials are taken from the MSA indicator variables in this second regression. As with the wage differentials, this adjusted weighting method has only a small impact on the measured price differentials.

E Additional Tax Issues

E.1 Deduction

Tax deductions are applied to the consumption of home goods at the rate $\delta \in [0, 1]$, so that the tax payment is given by $\tau(m - \delta py)$. With the deduction, the mobility condition becomes

$$\begin{aligned} \hat{Q}^j &= (1 - \delta\tau')s_y\hat{p}^j - (1 - \tau')s_w\hat{w}^j \\ &= s_y\hat{p}^j - s_w\hat{w}^j + \frac{d\tau^j}{m} \end{aligned}$$

where the tax differential is given by $d\tau^j/m = \tau'(s_w\hat{w}^j - \delta s_y p^j)$. This differential can be solved by noting

$$\begin{aligned} s_w\hat{w}^j &= s_w\hat{w}_0^j + \frac{\lambda_L}{\lambda_N}\frac{d\tau^j}{m} \\ s_y\hat{p}^j &= s_y\hat{p}_0^j - \left(1 - \frac{\lambda_L}{\lambda_N}\right)\frac{d\tau^j}{m} \end{aligned}$$

and substituting them into the tax differential formula, and solving recursively,

$$\begin{aligned} \frac{d\tau^j}{m} &= \tau' s_w\hat{w}_0^j - \delta\tau' s_y\hat{p}_0^j + \tau'\left[\delta + (1-\delta)\frac{\lambda_L}{\lambda_N}\right] \\ &= \tau'\frac{s_w\hat{w}_0^j - \delta s_y\hat{p}_0^j}{1 - \tau'\left[\delta + (1-\delta)\lambda_L/\lambda_N\right]} \end{aligned}$$

We can then solve for the tax differential in terms of amenities:

$$\frac{d\tau^j}{m} = \tau'\frac{1}{1 - \tau'\left[\delta + (1-\delta)\lambda_L/\lambda_N\right]}\left[(1-\delta)\left(\frac{1-\lambda_L}{\lambda_N}s_x\hat{A}_X^j - \frac{\lambda_L}{\lambda_N}s_y A_Y^j\right) - \frac{(1-\delta)\lambda_L + \delta\lambda_N}{\lambda_N}\hat{Q}^j\right]$$

This equation demonstrates that the deduction reduces the dependence of taxes on productivity and increases the implicit subsidy for quality-of-life.

E.2 State Taxes

The tax differential with state taxes is computed by including an additional component based on wages and prices relative to the state average, as if state tax revenues are redistributed lump-sum to households within the state. This produces the augmented formula

$$\frac{d\tau^j}{m} = \tau' \left(s_w \hat{w}^j - \delta\tau' s_y \hat{p}^j \right) + \tau'_S [s_w(\hat{w}^j - \hat{w}^S) - \delta_S s_y(\hat{p}^j - \hat{p}^S)] \tag{A.1}$$

where τ'_S and δ_S are are marginal tax and deduction rates at the state-level, net of federal deductions, and $\hat{w}^S$ and $\hat{p}^S$ are the differentials for state S as a whole relative to the entire country.

E.3 Calibration of Tax Parameters

The federal marginal tax rate on wage income is determined by adding together federal marginal income tax rate and the effective marginal payroll tax rate. TAXSIM gives an average marginal federal income tax rate of 25.1 percent in 2000. In 2000, Social Security (OASDI) and Medicare (HI) tax rates were 12.4 and 2.9 percent on employer and employee combined. Estimates from Boskin et al. (1987, Table 4) show that the marginal benefit from future returns from OASDI taxes is fairly low, generally no more than 50 percent, although only 85 percent of wage earnings are subject to the OASDI cap. HI taxes emulate a pure tax (Congressional Budget Office 2005). These facts suggest adding 37.5 percent of the Social Security tax and all of the Medicare tax to the federal income tax rate, adding 8.2 percent. The employer half of the payroll tax (4.1 percent) has to be added to observed wage levels to produce gross wage levels. Overall, this puts an overall federal tax rate, τ' , of 33.3 percent tax rate on gross wages, although only a 29.2 percent rate on observed wages.

Determining the federal deduction level requires taking into account the fact that many households do not itemize deductions. According to the Statistics on Income, although only 33 percent of tax returns itemize, they account for 67 percent of reported Adjusted Gross Income (AGI). Since the income-weighted share is what matters, 67 percent is multiplied by the effective tax reduction given in TAXSIM, in 2000 of 21.6 percent. Thus, on average these deductions reduce the effective price of eligible goods by 14.5 percent. Since eligible goods only include housing, this deduction applies to only 59 percent of home goods. Multiplying 14.5 percent times 59 percent gives an effective price reduction of 8.6 percent for home goods. Divided by a federal tax rate of 33.3 percent, this produces a federal deduction level of 25.7 percent.

State income tax rates from 2000 are taken from TAXSIM, which, per dollar, fall at an average marginal rate of 4.5 percent. State sales tax data in 2000 are taken from the Tax Policy Center, originally supplied by the Federation of Tax Administrators. The average state sales tax rate is 5.2 percent. Sales tax rates are reduced by 10 percent to accommodate untaxed goods and services other than food or housing (Feenberg et al. 1997), and by another 8 percent in states that exempt food. Overall state taxes raise the marginal tax rate on wage differences within state by an average of 5.9 percentage points, from zero points in Alaska to 8.8 points in Minnesota.

State-level deductions for housing expenditures, explicit in income taxes, and implicit in sales taxes, should also be included. At the state level, deductions for income taxes are calculated in an equivalent way using TAXSIM data. Furthermore, all housing expenditures are deducted from the sales tax. Overall this produces an average effective deduction level of $\delta = 0.291$.