Shadow Banking and Bank Capital Regulation

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Abstract

This paper studies the prudential regulation of banks in the presence of a shadow banking system. If banks do not internalize all the costs that risks on deposits create for the economy, imposing capital requirements on them is desirable in principle. We suppose that banks can use the shadow banking system to bypass such capital requirements, albeit at an informational cost. If it is not possible to regulate the shadow banking system at all, then relaxing capital requirements for traditional banks so as to shrink shadow activity may be more desirable than tightening them. Such a tightening creates a surge in shadow banking that may overall increase financial fragility, and reduce welfare. If it is possible to impose a haircut on refinancings in the shadow banking system, then tightening the capital requirements of traditional banks becomes optimal, but makes their shareholders much worse off. They would therefore strongly oppose any shadow-banking regulation.

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1 Introduction

The U.S. banking system now features two components of equal importance, traditional banks and the so-called "shadow banking system." This term refers to the nexus of financial institutions that are not subject to the prudential regulation of banks, and yet are involved in the refinancing of loans. In the shadow banking system, a wide range of institutions, including investment banks, hedge funds, money-market mutual funds, and various sorts of special purpose vehicles match investors and borrowers through long intermediation chains, in which short-term secured debt (repos) plays a crucial role. Pozsar et al. (2010) offer an excellent detailed description of the shadow banking system, and show that its total liabilities have the same order of magnitude as that of the traditional banking sector. Most observers agree that an important force driving the growth of shadow banking is regulatory arbitrage (see, e.g., Gorton and Metrick, 2010, or Pozsar et al., 2010). Many shadow-banking arrangements aim at bypassing bank capital requirements, thereby achieving a higher effective leverage than the one prudential regulation permits in principle. This was particularly evident in the years preceding the 2008 banking crisis (Acharya et al., 2011). It is the collapse of this highly leveraged shadow banking system that caused that of traditional banks in 2007-8 (Gorton and Metrick, 2012).

In the face of the costs that the 2008 banking crisis created for the world economy, a global trend towards imposing heightened capital requirements on traditional banks has emerged. On the other hand, regulatory reforms remain thus far largely silent on shadow banking (Gorton and Metrick, 2010). As argued by Kashyap et al. (2010), this raises the possibility that heightened capital requirements for traditional banks trigger even more regulatory arbitrage than observed in the recent past, thereby inducing a large migration of banking activities towards the unregulated shadow banking system. The increased resilience of the residual traditional banking system may then be more than offset by such a growth of the unregulated sector. This may ultimately lead to an overall more unstable banking industry. The goal of this paper is to develop a framework that is suited to the study of this risk of counterproductive capital requirements.

It has been a long-standing idea that financial innovation is often triggered by new regulatory constraints (see, e.g., Silber, 1983; Miller, 1986). The arbitrage of capital requirements has become in particular an important feature of the banking industry since the implementation of the first Basel accords.¹ Yet, formal models of optimal bank regulation with endogenous financial innovation are lacking. Given the current regulatory agenda, it seems important to develop frameworks for the analysis of prudential regulation in which the possibility of regulatory arbitrage is taken seriously. This paper studies the optimal prudential regulation of banks in the presence of a shadow banking system. Its main goals are to characterize the circumstances under which the endogenous growth of shadow banking activity defeats the purpose of capital requirements, and to devise regulatory responses.

Our framework features two ingredients - the prudential regulation of traditional banks and a shadow banking sytem - that we introduce in two steps. We first write down a simple model of optimal prudential regulation with perfect enforcement. We then study a modification of this model in which banks can bypass prudential regulation at some cost using the shadow banking system.

First, our model of optimal prudential regulation formalizes the stan-

¹Kashyap et al. (2010) survey the evidence.

dard view that capital requirements reduce the negative externalities that bank failures create for the rest of the economy. We posit that banks offer stores of value backed by their assets that facilitate transactions between non-financial agents. Bank shareholders seek to free up capital by issuing as many claims as possible against their risky assets. On one hand, leveraging up this way creates value because shareholders have valuable redeployment options. On the other hand, these claims become riskier stores of value, which creates costs for non-financial agents who transact using them. Bank shareholders do not fully internalize these costs. This implies that banks privately choose excessive leverage, which creates scope for prudential regulation.

Second, we introduce shadow banking as costly regulatory arbitrage in this model. We suppose that banks can secretly re-trade with non-financial agents so as to bypass prudential regulation and increase their leverage, albeit at a cost because they are privately informed about their assets when doing so. We find that there are two locally optimal regulatory responses to such regulatory arbitrage. First, the regulator can tighten capital requirements, which triggers an increase in shadow banking activity but makes banks that are not willing to incur adverse selection costs very safe. Second, the regulator may also prefer to relax regulatory capital requirements so as to bring shadow banking activity back in the spotlight of regulation. While current regulatory reforms seem to trend towards the former solution, the latter one may actually be preferable, particularly clearly so if informational frictions in the shadow banking system are not important.

Third, we add the possibility that the regulator can partially regulate shadow banking by forcing shadow-banking institutions to put their own capital at risk when buying traditional bank's assets. This corresponds for example to imposing lower bounds on haircuts on repos. We show that in this case tightening capital requirements for traditional banks becomes the only socially optimal policy provided this regulatory haircut is sufficiently large. The more illiquid the shadow banking system is, the larger the regulatory haircut needs to be. The increase in social welfare from such a shadow-banking regulation comes however with a sizeable transfer from traditional bank shareholders to non-financial agents. This suggests that shadow-banking regulation, even when socially desirable, would trigger an important lobbying effort by the banking industry.

Related literature

This paper offers a formal model of the "regulatory dialectic" described by Kane (1988), whereby financial regulation spurs financial innovation for regulatory-arbitrage purposes, which in turn affects financial regulation. We characterize the equilibrium resulting from such an interaction between regulation and innovation in the case of bank capital regulation. Martin and Parigi (2011) also develop a model in which regulatory constraints may trigger excessive structured finance. A related earlier literature on regulatory arbitrage also establishes the possibility that capital requirements may actually increase the risk of bank failure: Kim and Santomero (1988) and Rochet (1992) show that this may occur if the capital requirements for various asset classes do not reflect their relative systematic risks. This may lead banks to reshuffle their portfolios away from the efficient frontier, and increase their probability of failure. We complement this literature by deriving the optimal equilibrium regulatory response to regulatory arbitrage.

This paper also relates to the literature on the interaction between banks and markets. Bolton et al. (2010) study the *ex ante* impact on banks of *ex post* adverse selection in the secondary market for their assets. In their setup, the fear of future adverse selection may induce banks to offload their risky assets too early so as to sell them at fair value. This is inefficient because this implies that the suppliers of liquidity to banks hoard less cash to snap up these assets given a lower expected return on them. This in turn reduces the total quantity of valuable risky assets in which banks decide to invest in the first place. By contrast, our setup emphasizes that adverse selection in secondary markets for bank assets may be *ex ante* desirable as it reduces the scope for regulatory arbitrage. Gennaioli, Shleifer, and Vishny (2011) develop a model of shadow banking whereby banks pool their idiosyncratic risks, thereby increasing their systematic exposure, and use the safe part of these recombined portfolios to back the issuance of safe debt. While this is efficient under rational expectations, shadow banking creates large financial instability and systemic risk when agents underestimate the tail of systematic risk. Although their focus is not on regulation and regulatory arbitrage, their theory implies like ours, but for entirely different reasons, that regulating leverage in the shadow banking system is important for financial stability.

The paper is organized as follows. Section 2 develops a simple model of optimal capital requirements for banks. Section 3 studies how the introduction of a shadow banking system affects such capital requirements. Section 4 discusses the regulation of the shadow banking system. Section 5 develops extensions, and Section 6 concludes.

2 A Simple Model of Optimal Capital Requirements

There are 3 dates t = 0, 1, 2. There are three agents: a household, an entrepreneur, and the shareholder of a bank. Agents are risk neutral. They

do not discount future cash flows, and cannot consume negatively. There is a consumption good that is valued by all agents and used as the *numéraire*.

Household. The household receives a date-0 endowment of W units of the numéraire good, where W > 0.

Entrepreneur. The entrepreneur has access to a technology that enables him to produce a second consumption good. Only the household derives utility from consuming his output. Production takes place at date 2. The entrepreneur sets a production scale I_0 - a number of units to be produced at date 2 - at date 0. He can adjust this production scale from I_0 to I_2 at date 2, but this comes at an adjustment cost

$$\frac{k}{2}\left(I_2-I_0\right)^2,$$

where k > 0. Once the scale is set, the production of one unit of output requires an input of c units of the *numéraire* good at date 2, where $c \in (0, 1)$. The household values one unit of output as much as one of the *numéraire* good. The entrepreneur is free to withdraw his human capital: He can always walk away at no cost at date 2, thereby not producing any output. He can use this threat to renegotiate any arrangement, and has all the bargaining power during such renegotiations.

The initial household endowment W must be stored from date 0 to date 2 so that the entrepreneur can produce and trade with the household. The role of the bank is to provide such stores of value.²

Bank. The shareholder of the bank - simply "the bank" henceforth can offer two storage technologies to the household. First, it has access to a risk-free unit-return technology from date t to date t + 1, where $t \in$

 $^{^{2}}$ As in Stein (2012), that claims on bank assets facilitate transactions better than other claims to future consumption is assumed here. In other words, we do not offer a theory of banking, but rather a theory of banking regulation taking for granted this role for bank claims.

 $\{0;1\}$. Second, the bank also owns legacy assets, in the form of a portfolio of outstanding loans that has a promised repayment L > 0 at date 2. The probability that the portfolio performs well is equal to $p \in (0,1)$. A nonperforming portfolio pays off 0. The bank can pledge all or part of the expected repayment pL to the household.

At date 1, the bank may also receive an investment opportunity with probability $q \in (0, 1)$. If it invests x units in this opportunity, it creates a gross return x + f(x) that cannot be pledged to the other agents. The function f satisfies the Inada conditions.³ Following Bolton et al. (2011), non-pledgeability could stem for example from the fact that this is a longterm opportunity that pays off only at some remote date 3 at which only the bank shareholder values consumption. It may also be that this opportunity requires bank monitoring and thus skin in the game. This stark assumption could be relaxed, and we could allow for partial pledgeability adding only some notational complexity.

In sum, this setup simply captures two important functions of banks: issuing liabilities that facilitate transactions, and funding activities that the other agents cannot fund.

The bank makes a take-it-or-leave-it offer to the household for a storage contract.

Information structure. The bank privately learns at date 1 whether its loan portfolio is performing or not. Whether it receives its date-1 investment opportunity f(.) or not is also private information to the bank.

³One could of course add that with a prob. 1 - q the bank receives an opportunity that it can pledge to the household but it would play no role.

We suppose that

$$2W > 3L, \tag{1}$$

$$kL < \min\{c; 1-c\}.$$
 (2)

These conditions are simply meant to simplify the discussion of the entrepreneur's problem. Before solving the model, it is worthwhile discussing its two central features. First, the Modigliani-Miller irrelevance theorem does not hold for the bank because the bank shareholder has access to valuable investment opportunities that he cannot pledge to the other agents. Thus, immobilizing bank capital in outstanding loans comes at an opportunity cost. Leveraging the bank's assets in place by pledging their future cash flows to the household creates value.⁴

Second, the claim that the household holds against the bank plays an important role in this economy because it enables Pareto-improving trades between non-bank agents. The riskiness of this claim creates adjustment costs for the entrepreneur. This is because the entrepreneur cannot commit not to renegotiate prices and quantities, which prevents him from sharing risk with other agents. We now show that these two features create room for a prudential regulation of the bank.

Analysis

The equilibrium is fully characterized by i) the storage contract that the bank offers to the household, ii) the initial capacity choice of the entrepreneur at date 0, iii) the capacity adjustment and pricing decisions of the

 $^{^{4}}$ Consistent with the large empirical evidence documenting that shocks to banks capital affect their lending activities, it seems reasonable to interpret such opportunity costs as one-off costs associated with a transition to a new capital structure rather than permanent flow costs associated with a given level of equity. See Kashyap et al. (2010) for a related discussion.

entrepreneur at date 2.

Consider first the bank decision. As will be established below (Lemma 3), a bank which knows that the loan is performing at date 1 has no way to credibly signal it to the household. Thus, if the bank receives its redeployment opportunity at date 1 and seeks to sell a stake in the loan to the household at this date, this will come at the cost of an adverse selection discount. It is therefore preferable for the bank to pledge the part of the loan it wishes to at date 0, rather than wait until date 1 when the proceeds would be smaller because of adverse selection. A storage contract is thus fully characterized by the fraction $\lambda \in [0, 1]$ of the loan repayment that the bank sells at a fair price to the household at date 0. The residual household savings $W - \lambda pL$ is stored using the risk-free technologies until date 2.

We interpret this fraction λ as the net or risk-adjusted bank leverage. The bank starts out indeed as a 100% equity institution. It then receives W from the household, and has a gross liability W net of "cash on hand" $W - \lambda pL$, the fraction of the endowment that is invested in the safe storage. Thus the bank keeps an equity stake $1 - \lambda$ in its risky assets, refinancing the residual with household net (risky) debt λpL . In other words, the bank shareholder levers up risky assets so as to free up capital for profitable but non-pledgeable alternative projects. Notice that this is frictionless for the bank because the loans are fairly valued by the household and there is no moral hazard. For a given leverage λ , the expected utility of the bank is

$$U_B = pL + qf(\lambda pL). \tag{3}$$

Consider now the entrepreneur's decisions. Given lack of commitment and inalienability of human capital, the entrepreneur behaves as a monopolist that maximizes his *ex post* profits after uncertainty resolves at date 2. Bank leverage λ implies that the date-2 endowment of the household is distributed as

$$\begin{cases} W + (1-p)\lambda L & \text{with prob.} & p \\ W - p\lambda L & \text{with prob.} & 1-p \end{cases}$$

Conditions (1) and (2) imply the following entrepreneur's decisions.

Lemma 1

The entrepreneur chooses an initial capacity $I_0 = W$. If the loan portfolio performs, he then increases his date-2 capacity to $I_2 = W + (1 - p) \lambda L$. He reduces it to $I_2 = W - p\lambda L$ if the portfolio does not perform. He always sets a maximal output price of one per unit. His date-0 expected utility (his expected profit) is therefore

$$U_E = W(1-c) - \frac{k}{2}p(1-p)\lambda^2 L^2.$$
 (4)

Proof. See Appendix.■

Scale adjustment costs imply that the entrepreneur would like to hedge the uncertainty on date-2 demand induced by the riskiness of bank claims. His inability to commit prevents him from doing so, however.⁵ Thus the riskiness of bank claims induces inefficient costs on non-financial agents. If p is large, there are frequent tiny adjustment costs when the loan portfolio performs, and large rare downsizing costs in case of default.

It is transparent from (3) that absent regulatory constraints, the bank would choose maximal leverage $\lambda = 1$, while (4) shows that the entrepreneur's utility decreases w.r.t. λ . This model thus captures parsimoniously the broad terms of the debate on heightened bank capital requirements. On one hand, heightened capital requirements reduce the negative externalities

 $^{{}^{5}}$ Under full commitment, the household could agree to pay for the output above his valuation when the bank performs against the promise that the entrepreneur charges him a price less than 1 for the same quantity if the bank does not perform.

that bank failures impose on the rest of the economy by undermining the role of bank claims as reliable stores of value. On the other hand, allocating more bank equity to the financing of oustanding loans comes at a cost if there is no perfect substitute to this equity capital for the financing of alternative valuable projects.

That banks do not fully internalize the social costs induced by their risk taking creates room for prudential regulation. In general, imposing a capital requirement on the bank - an upper bound on its leverage λ - may increase total surplus. We posit that such a capital requirement is determined by a regulator who observes transactions between the household and the bank at dates 0 and 1, and can therefore enforce such a regulation. The regulator chooses a leverage λ that optimally trades off the adjustment costs of the entrepreneur and the opportunity cost of bank capital so as to maximize:

$$U_E + U_B = pL + qf(\lambda pL) + W(1-c) - \frac{k}{2}p(1-p)\lambda^2 L^2,$$
 (5)

which readily yields

Proposition 2

If

$$qf'(pL) \ge k\left(1-p\right)L,\tag{6}$$

then there is no prudential regulation, and the bank chooses $\lambda = 1$. Otherwise the regulator imposes a capital requirement, or a maximal leverage $\lambda^* < 1$ s.t.

$$qf'(\lambda^* pL) = \lambda^* k \left(1 - p\right) L. \tag{7}$$

The capital requirement is binding.

Proof. First-order condition on (5).

Not surprisingly, the optimal leverage λ^* is larger when the opportunity cost of bank capital is high (q large), and the negative externalities that the bank imposes on the real economy are small (k small). It is also easy to see that an increase in p holding pL fixed leads to a higher optimal leverage. That is, capital requirements should put more weight on riskier assets. We now study how the existence of a shadow banking system modifies these results. Before doing so, it is worthwhile stressing that a number of stark assumptions are only made here for simplicity. The negative bank leverage externalities that we obtain would arise in any environment in which: i) banks do not internalize the entire value generated by the use of their liabilities by non-financial agents, ii) non-financial agents cannot purchase full and fair insurance against the risks associated with bank liabilities. Condition i) is the baseline motivation that is typically brought forward to justify prudential regulation. Regarding condition ii), while deposit insurance goes some way along providing such insurance, the consequences of the current global banking crisis and the costs of its associated bail-outs suggest that this insurance is far from perfect.⁶

3 Shadow Banking and Bank Capital

We introduce shadow banking as follows. We still suppose that the regulator can observe the bank and the household at date 0. We assume, however, that he no longer does so at date 1. Thus, while the regulator can still impose a regulatory capital requirement λ on the bank at date 0, the bank's decision to readjust its leverage at date 1 is beyond the reach of regulation.

⁶Abstracting from any deposit-insurance scheme is for simplicity, and is not crucial to our results. We could introduce a deposit insurance scheme financed by taxation. As is standard, as long as such a scheme comes with deadweight costs or distorts incentives e.g., it leads the bank to increase the riskiness of its loans - capital requirements would still be useful.

On the other hand, such readjustments come at an adverse selection cost to the bank because of its date-1 private information. Date-0 capital regulation is therefore not entirely pointless, but date-1 transactions between the household and the bank undermine it. We deem such date-1 transactions the shadow banking activity. We suppose that the bank cannot commit and behaves in an *ex post* optimal fashion at date 1.

This modelling of shadow banking squares well with the "securitization without risk transfer" phenomenon observed before the subprime crisis, and documented by Acharya et al. (2011). Banks were guaranteeing the vehicles to which they were transferring their loans, so as to free up regulatory capital without offloading their risks. This is exactly the purpose of shadow banking in this section. Banks use the shadow banking system for regulatory arbitrage. They increase their effective leverage beyond the level that prudential regulation allows for.⁷

Notice that shadow banking as modelled here may also describe the "true" sale of loans - with risk transfer - to institutions such as money market mutual funds (MMMFs). As quasi-money instruments, MMMFs are likely to create similar negative externalities when "breaking the buck" to that induced by losses on bank deposits. Under these circumstances, true securitization does also increase the aggregate effective leverage of the issuers of private money in the economy because the institutions involved in the shadow-banking chain (various SPVs and ultimately MMMFs) are not subject to leverage rules. Stein (2012) invokes a similar argument to advocate a symmetric regulatory treatment of banks and shadow banks.

Suppose that the bank has sold a fraction λ of its outstanding loans to the household at date 0. The bank privately observes at date 1 whether

⁷The next section introduces a more socially desirable role for shadow banking beyond pure regulatory arbitrage.

the portfolio is performing or not. For simplicity, we refer to a bank which observes that the portfolio performs at date 1 as a "good" bank, and to a bank which knows that it is worthless as a "bad" bank. The bank has two private motives to secretly trade all or part of its residual stake $1 - \lambda$ at date 1. First, the bank may have received an investment opportunity with probability q. Second, a bad bank may seek to get rid of its now worthless stake. We consider only pooling date-1 equilibria in which the household pays a pooling price at date 1 regardless of the type of the selling bank. The following lemma ensures that this is without loss of generality.

Lemma 3

A good bank with a redeployment opportunity cannot signal its type to the household at date 1.

Proof. See the Appendix.

The pooling price that the household is willing to pay for a unit stake offered by the bank is

$$r = \frac{pq}{1 - p + pq}.$$

It is the probability that the loan performs conditionally on the event that the bank does offer a stake, which occurs if the bank is bad (with prob. 1-p), or if it is good and experiences a preference shock (with prob. pq).

At date 1, a bad bank always mimicks a good bank with a preference shock. Such a good bank trades so as to solve

$$\max_{L' \in [0,(1-\lambda)L]} \lambda pL + rL' + f\left(\lambda pL + rL'\right) + (1-\lambda)L - L'.$$
(8)

In words, the good bank optimally chooses the face value L' of the claim that it sells to the household via the date-1 shadow banking system, while retaining $(1 - \lambda) L - L'$. It is convenient to introduce the function φ defined as

$$\varphi = f'^{-1},$$

a decreasing bijection from $(0, +\infty)$ into $(0, +\infty)$. The first-order condition associated with (8) yields L' as a function of λ .

Lemma 4

If
$$\lambda pL > \varphi\left(\frac{1-p}{pq}\right)$$
, then $L' = 0$.
If $(\lambda p + (1-\lambda)r)L < \varphi\left(\frac{1-p}{pq}\right)$, then $L' = (1-\lambda)L$.
Otherwise,
 $rL' = \varphi\left(\frac{1-p}{pq}\right) - \lambda pL.$
(9)

Proof. See above.

Fom (9), the amount that the bank raises in the shadow banking system, rL', is larger when adverse selection is lower (q large). If q is very small, then the household strongly suspects that the bank is seeking to offload a lemon, and the shadow banking system is not liquid. Conversely, the bank has all the more recourse to shadow banking because λ is small. A more constrained bank is more willing to incur the illiquidity premium caused by adverse selection at date 1. This is the key mechanism through which an increase in capital requirements (a reduction in λ) spurs the development of shadow banking. In sum, (9) shows that heightened regulatory constraints may create liquidity in otherwise information-problematic markets.

Notice that

$$\varphi\left(\frac{1-p}{pq}\right) > pL \to \text{For all } \lambda, \ \left(\lambda p + (1-\lambda)r\right)L < \varphi\left(\frac{1-p}{pq}\right).$$

Thus $L' = (1 - \lambda) L$ for all values of λ from Lemma 4 when $\frac{\varphi\left(\frac{1-p}{pq}\right)}{pL} > 1$. In words, for any capital requirement level, the bank always fully offloads its

entire exposure in the shadow banking market unless it knows that the loan performs and has no redeployment opportunity. This case seems unrealistic and of limited interest, we therefore assume it away from now on by positing:

$$\varphi\left(\frac{1-p}{pq}\right) < pL. \tag{10}$$

We now study how the regulator sets the capital requirement rationally anticipating that this affects shadow banking activity. As a first step, we derive how leverage λ affects the utility of the entrepreneur and that of the bank in the presence of shadow banking. Recall that the utility of the entrepreneur decreases w.r.t. λ absent shadow banking from (4), while that of the bank increases from (3). Fix a given initial bank leverage λ . It is easy to see that if both a bad bank and a good bank with a redeployment opportunity refinance a stake L' in the shadow banking system at date 1, then the date-0 utility of the entrepreneur is

$$U_E = W(1-c) - \frac{k(1-p)}{2} \left[p\lambda^2 L^2 + rL'^2 + 2r\lambda LL' \right], \qquad (11)$$

and that of the bank is

$$U_B = pL + qf\left(p\lambda L + rL'\right). \tag{12}$$

This leads to the following interesting result.

Lemma 5

Whenever the shadow banking system is active - that is, L' > 0, it would be strictly socially preferable that the bank raises instead the entire date-1 cash $p\lambda L + rL'$ at date 0.

Proof. If the bank sells a stake L' in the shadow banking system for rL' at date 1, the variance of the household's demand is from (11):

$$(1-p)\left(p\lambda^2L^2 + rL'^2 + 2r\lambda LL'\right).$$

If the bank were instead to raise the equivalent amount entirely at date 0, it would have to sell a total stake $\lambda L + L''$ such that

$$p\left(\lambda L + L''\right) = \lambda pL + rL'$$

or simply $L'' = \frac{rL'}{p}$. The variance of the household's claim would be in this case

$$p(1-p) \left(\lambda L + L''\right)^2 = (1-p) \left[\left(p\lambda^2 L^2 + rL'^2 + 2r\lambda LL' \right) - r \left(1 - \frac{r}{p} \right) L'^2 \right] < (1-p) \left(p\lambda^2 L^2 + rL'^2 + 2r\lambda LL' \right).$$

The intuition for this result is simple. Since r < p, the bank must transfer more risk to the household when it raises one dollar at date 1 than when it does so at date 0 because it has to pledge a larger fraction of the loan face value. This implies that for a fixed level of bank utility, the entrepreneur's utility decreases with respect to the fraction of the date-1 bank cash that is raised through the shadow banking system.⁸

As a second step, we use Lemma 4 to express the stake L' as a function of λ in (11) and (12). First, if $\lambda \geq \frac{\varphi\left(\frac{1-p}{pq}\right)}{pL}$,

$$U_E = W(1-c) - \frac{k}{2}p(1-p)\lambda^2 L^2,$$

$$U_B = pL + qf(\lambda pL).$$

 $\text{If }\lambda < \frac{\varphi \left(\frac{1-p}{pq}\right) - rL}{(p-r)L},$

$$U_E = W(1-c) - \frac{k(1-p)}{2} \left(r + (p-r)\lambda^2\right) L^2,$$
(13)

$$U_B = pL + qf((\lambda (p-r) + r)L).$$
(14)

⁸Notice that this source of inefficiency would disappear if we assumed q = 1 and therefore r = p. Yet shadow banking would still be inefficient because it would still be the case that the regulator has limited control over effective bank leverage in its presence. There are other interesting features of the model that disappear when q = 1, hence our focus on q < 1.

Otherwise

$$U_E = W(1-c) - \frac{k(1-p)}{2p} \begin{bmatrix} \varphi\left(\frac{1-p}{pq}\right)^2 \\ +\frac{p-r}{r}\left(\varphi\left(\frac{1-p}{pq}\right) - \lambda pL\right)^2 \end{bmatrix}, \quad (15)$$

$$U_B = pL + qf(\varphi\left(\frac{1-p}{pq}\right)).$$
(16)

The case $\lambda \geq \frac{\varphi\left(\frac{1-p}{pq}\right)}{pL}$ corresponds to the situation in which capital requirements are loose and/or adverse selection is important, so that the shadow banking system is inactive (L'=0). The presence of the shadow banking system does not affect utilities U_E and U_B in this case.

The case in which $\lambda < \frac{\varphi\left(\frac{1-p}{pq}\right)-rL}{(p-r)L}$ is the opposite case in which the shadow banking system is as active as it can get $(L' = (1 - \lambda)L)$ because date-0 leverage is low and date-1 trading is liquid. As in the no shadow-banking case, U_E decreases w.r.t. λ while U_B increases in this case. Shadow banking reduces the sensitivity of both utilities to changes in λ , however.⁹ This is because the entire credit risk is ultimately transferred to the house-hold regardless of λ , unless the bank turns out to be good and does not receive a reinvestment opportunity.

The most interesting case is the intermediate one. In this case, from (15), an increase in regulatory leverage λ does actually increase the entrepreneur's utility. Tightening capital requirements increases negative leverage externalities in this case because the reduction in regulatory leverage is more than offset by the induced surge of shadow banking activity. Conversely, from (16), the utility of the bank does not depend on λ in this range of parameters because the bank uses the shadow banking system to undo any regulatory constraint, and invests a fixed amount in its redeployment

⁹Formally, $p\lambda^2$ is replaced with $(p-r)\lambda^2 + r$ in U_E , and $p\lambda$ is replaced with $(p-r)\lambda + r$ in U_B .

opportunity. Figure 1 summarizes the above discussion by depicting how U_E and U_B vary with respect to λ in the presence of the shadow banking system.

Figure 1 Here.

We are now equipped to study optimal capital requirements in the presence of shadow banking.

Proposition 6

i) If

$$k\varphi\left(\frac{1-p}{pq}\right) \le 1,\tag{17}$$

then the shadow banking system is inactive and the optimal capital requirement is λ^* as in Proposition 2. Shadow banking plays no role.

ii) If

$$\frac{pk}{p-r}\left(\varphi\left(\frac{1-p}{pq}\right) - rL\right) \le 1 < k\varphi\left(\frac{1-p}{pq}\right),\tag{18}$$

then the shadow banking system is inactive, the optimal capital requirement is $\overline{\lambda} = \frac{\varphi\left(\frac{1-p}{pq}\right)}{pL} > \lambda^*$. The presence of the (inactive) shadow banking system makes the bank better off but reduces total surplus.

iii) If

$$\frac{pk}{p-r}\left(\varphi\left(\frac{1-p}{pq}\right)-rL\right) > 1,\tag{19}$$

there are two local maxima for total surplus, $\overline{\lambda} = \frac{\varphi\left(\frac{1-p}{pq}\right)}{pL} > \lambda^*$, and $\underline{\lambda} < \lambda^*$. The shadow banking system is inactive at the highest leverage $\overline{\lambda}$, fully active at the lowest $\underline{\lambda}$. In both cases, the bank is better off than absent shadow banking, but total suplus is lower. Either local maximum can be global depending on parameter values.

Proof. See the Appendix.

The various regimes in Proposition 6 are best described by letting the adjustment cost parameter k vary, holding other parameters fixed. In case i), k is sufficiently small *ceteris paribus* that the optimal regulatory leverage is large. The bank is not sufficiently constrained by regulation that it feels the need to incur the trading costs associated with an opaque shadow banking system. Shadow banking is irrelevant in this case.

In case ii) with a larger k, it is no longer so. Shadow banking is a relevant threat. The regulator does not handle this threat with tighter capital requirements. On the contrary, he relaxes the capital requirement $(\overline{\lambda} > \lambda^*)$ up to the point at which the bank will not find any further refinancing in the shadow banking system worthwhile. In other words, the regulator does himself at date 0 what the bank would do anyway at date 1 - increase its effective leverage - in a socially more efficient fashion given the inefficiencies associated with shadow banking highlighted in Lemma 5. As a result, there is no equilibrium shadow-banking activity. Yet the threat of shadow banking is effective as the bank is strictly better off with a higher feasible leverage. Again, total surplus would be higher absent shadow banking under a smaller leverage λ^* .

Finally, in case iii) in which k is the largest, there are two locally optimal capital requirements. First, the regulatory leverage $\overline{\lambda}$ that is optimal in case ii) is still locally optimal, and may or may not be the global optimum. Second, there exists another local optimum $\underline{\lambda} < \lambda^*$ whereby the strategy of the regulator is the polar opposite. Here, shadow banking is as active as it can get. Only an unconstrained bank with good loans stays away from it, while a constrained bank and/or a bank that has bad news about its portfolio fully refinance its assets at date 1 ($L' = (1 - \lambda)L$). The regulator imposes

a very tight regulatory requirement that is effective only if the bank has no shadow banking activity. The effective leverage of a bank that is active in the shadow banking system is ultimately 100%, however, and total welfare strictly lower than absent shadow banking from Lemma 5. Figure 1 depicts the two local maxima in this case iii):

Figure 2 Here.

It is instructive to discuss which of the two locally optimal leverages $\overline{\lambda}$ and $\underline{\lambda}$ is the global one depending on parameter values. Notice first the following:

Corollary 7

If other things equal q is sufficiently large then there is a unique optimal leverage, either λ^* when shadow banking is irrelevant (case i) in Proposition 6) or $\overline{\lambda} > \lambda^*$ when it is an actual threat.

Proof. As $q \to 1$, $r \to p$ and (10) implies that the LHS of (19) is negative. Thus only situations i) and ii) in Proposition 6 apply to this economy.

In words, if the shadow banking system is not too illiquid in the sense that outside finance comes at the same cost for banks at date 1 as at date 0 (r close to p), then it is not even locally optimal to impose tight capital requirements that set off shadow banking activity. Leverage $\overline{\lambda}$ is the unique optimum in this case when the shadow banking threat is relevant. The intuition is simply that the bank is *ex ante* unlikely to not tap the shadow banking system when q is large. We also have

Corollary 8

Suppose $\frac{pk}{p-r}\left(\varphi\left(\frac{1-p}{pq}\right)-rL\right)>1$. The bank is always worse off under leverage $\underline{\lambda}$ than $\overline{\lambda}$.

$$\overline{\lambda} \le \sqrt{\frac{r}{p}},$$
(20)

then U_E , and therefore total surplus are larger at leverage $\overline{\lambda}$ than $\underline{\lambda}$.

If (20) does not hold, total welfare is larger under $\overline{\lambda}$ ($\underline{\lambda}$) if k is sufficiently small (large).

Proof. See the Appendix.

The intuition for these conditions is the following. There are two ways for the regulator to respond to shadow banking. The first strategy consists in letting the bank do at date 0 what it would do anyway at date 1 by setting a loose capital requirement $\overline{\lambda}$. Shadow banking is inactive in this case, which we know is desirable from Lemma 5. This response is optimal provided either $\overline{\lambda}$ or k are sufficiently small. Another response consists in setting a very tight capital requirement. This triggers a large shadow-banking activity $(L' = (1 - \lambda)L)$ that undoes completely such a tight requirement. However, it may be the case that a bank is not capital-constrained and has a good portfolio. In this case, it is unwilling to incur the costs of shadow banking activity, and the entrepreneur benefits from the extremely low bank leverage in these states of the world. Notice, however, that the bank is always worse off under this strategy, so that it cannot be optimal unless: i) k is sufficiently large that bank leverage really hurts the real economy, ii) the leverage $\overline{\lambda} = \frac{\varphi(\frac{1-p}{p_1})}{pL}$ that eliminates shadow banking activity is large.

Comparative statics with respect to portfolio quality

The subprime crisis has arguably shifted views on the riskiness of mortgage portfolios holding their characteristics constant. To study how such a shift affects the equilibria described in Proposition 6, we study the comparative statics of the equilibrium with respect to the riskiness of the portfolio.

If

Suppose an increase in the risk of the portfolio in the sense of second-order stochastic dominance. That is, suppose that, ceteris paribus, p decreases while pL is constant. Condition (9) implies that this reduces rL'. Increased risk raises date-1 adverse selection which leads the bank to reduce its shadow banking activity. This also implies a smaller $\overline{\lambda} = \frac{\varphi\left(\frac{1-p}{pq}\right)}{pL}$. In other words, it is easier to fully eliminate shadow banking when the portfolio is riskier in the sense that it can be done with tighter capital requirements. This of course may symmetrically help explain how optimistic beliefs about housing prices fuelled the growth of shadow banking during the credit boom.

On the other hand, one can see from (26) that the impact of increased risk on the local optimum with shadow banking $\underline{\lambda}$ is ambiguous. In this case, increased risk both reduces the date-1 cash holdings of the bank because of illiquidity, and increases the variance of date-2 demand. Whether $\underline{\lambda}$ should be higher or lower as a result is unclear. The impact of risk increase on condition (20) is also ambiguous because both sides decrease as a result of a risk increase. Thus the impact of portfolio quality on the determination of the globally optimal leverage is indeterminate.

The current debate on bank capital regulation under the lens of this model.

It is interesting to analyze the current evolutions of bank regulation using these results. First, the current size of the U.S. shadow banking system shows that we are clearly not in the situation i) in Proposition 6, but rather in situations ii) or iii). Second, banking regulations worldwide seem to trend towards the strategy of setting significantly tighter capital requirements such as $\underline{\lambda}$, while not very actively regulating shadow banking. As feared by many observers (e.g., Kashyap et al., 2010), the model predicts that this would foster an important shadow banking activity. It also predicts that if it is not possible to implement any shadow banking regulation, then this may be the optimal policy provided in particular that k is sufficiently large.

There is also the more thought-provoking possibility that tightening capital requirements without regulating shadow banking may as in the model be a locally optimal policy, but not the globally optimal one. Conditionally on not being able to regulate shadow banking, it may well be wiser to adopt looser capital requirements ($\overline{\lambda}$ here) until shadow banking activity dries up, and banking comes back entirely under the light of regulation. In short, it is preferable to have a regulated banking system with overly low capital requirements rather than facially large capital requirements on a small traditional banking system, whose stability is threatened anyway by a large unregulated shadow banking sector. It is easy to understand, however, why policymakers who are unable or unwilling to enforce a proper regulation of the shadow banking system would find the adoption of policy $\underline{\lambda}$ preferable to that of policy $\overline{\lambda}$, even if $\overline{\lambda}$ is optimal. The announcement of policy $\overline{\lambda}$ is a clear acknowledgement that shadow banking activity would undermine any traditional banking regulation, while policy λ might be mistaken for a tough regulatory action by observers who underestimate the endogenous growth in shadow banking induced by this policy.

This discussion rests on the premises that any regulation of the shadow banking system is out of reach. We now relax this assumption, and discuss the impact of a partial regulation of the shadow banking system on the above results.

4 Regulating Shadow Banking

The main take-away from Proposition 6 is that, as advocated by many observers, it seems highly desirable that any bank capital reform features also some solvency regulation of the shadow banking system.¹⁰ Several scholars have recently issued proposals for shadow-banking regulation. Kashyap et al. (2010) suggest a security-based regulation, whereby all repo transactions should be subject to a minimal haircut. Such a minimal haircut amounts to imposing a capital requirement in the shadow banking sector given the central role that repos play in its refinancing chains. Gorton and Metrick (2010) recommend to add to such a haircut regulation *institution-based* rules that would introduce some prudential regulation of the institutions that participate in the shadow banking system. The details of these regulations greatly matter in practice, be it only because they determine their enforceability. But assuming that such regulations can be enforced, at a high level they all boil down to imposing a maximal risk-adjusted leverage on the shadow banking sector. Our model of the shadow banking system offers a framework to study the effectiveness of such a regulation. We modify the previous model as follows. We suppose that the trade between the bank and the household at date 1 is intermediated without any friction by a competitive shadow bank who enters the economy at date 1 with the same information as that of the household. We suppose that the regulator, in addition to the maximal leverage λ of the traditional banking sector, also has the ability to impose that the shadow bank finances only a fraction equal to at most $l \in [0, 1)$ of its assets by tapping the household, and purchases the residual 1-l with its own funds. This residual 1 - l corresponds either to a minimal haircut on asset-backed securities or to a capital charge for shadow banking institutions. In other words, we still assume that the regulator cannot observe shadow banking activity after date 0, but that he can at least impose an upper bound l on shadow-banking leverage. This technological constraint

 $^{^{10}{\}rm See,~e.g.,}$ "FSB chief in call to rein in 'shadow banking'," Financial Times, January 15, 2012 9:25 pm.

for the regulator is determined outside the model. For example, going beyond l is not enforceable and would trigger further financial innovation and regulatory arbitrage. The previous section considered the particular case in which l = 1.

We suppose that the Modigliani-Miller theorem holds for the shadow bank. That is, it requires a unit return on its assets regardless of its capital structure. This is an important assumption because it introduces a socially desirable role for shadow banking in addition to plain regulatory arbitrage: Shadow banking is a way to diversify the sources of refinancing for banks. The shadow bank supplies a fairly priced additional capital buffer 1 - l that can absorb losses on loans without upsetting the real economy as losses on the household's claim do. This is in line with the typical arguments that are put forward in favor of securitization. This positive role of shadow banking entails the following.¹¹

Lemma 9

For any regulatory leverage λ ,

- total welfare is decreasing in l, strictly so if L' > 0,

-there exists \underline{l} such that for all $\underline{l} \leq \underline{l}$, total welfare is strictly larger with an active shadow banking system than without.

Proof. The haircut l does not affect the shadow-banking activity of the bank L' nor its utility because it does not affect pricing in the shadow banking system since Modigliani-Miller holds for the shadow bank. The entrepreneur's utility decreases with l other thing being equal because a larger haircut reduces the fraction of the stake L' that is borne by the household, and thus the riskiness of date-2 demand. Thus total welfare

¹¹We still assume that the shadow bank is not indifferent and that it prefers to maximize its leverage to l. It can be due to small gains from leverage that we neglect.

strictly decreases in l when shadow banking is relevant (L' > 0). To see that shadow banking is welfare improving for l sufficiently small, consider the case l = 0. In this case, U_B is the same as in the previous section where l = 1, and thus strictly higher than absent shadow banking provided L' > 0. The entrepreneur has the same utility as absent shadow banking since he does not bear the associated risks. Thus total welfare is strictly larger with shadow banking than without for l = 0, and also for l sufficiently small by continuity.

This version of the model develops a more balanced view of the shadow banking system, whereby regulatory arbitrage motives coexist with genuine optimal financing considerations. The following proposition re-visits Proposition 6 in the case of a sufficiently small l.

Proposition 10

Suppose

$$l < \frac{r}{p}.$$
 (21)

i) If (17) holds, then the shadow banking system is immaterial, and so is its regulation.

ii) Otherwise, the shadow banking system is always fully active $(L' = (1-\lambda)L)$ at the optimal regulatory leverage. The optimal regulatory leverage is at most equal to $\frac{\varphi(\frac{1-p}{pq})-rL}{(p-r)L} < \overline{\lambda}$, and is always strictly smaller than when l = 1.

Total welfare is strictly larger than when l = 1, but the bank is strictly worse off.

Proof. See the Appendix.■

Proposition 10 yields two interesting insights. First, it shows that the regulatory strategy which consists in setting a loose capital requirement $\overline{\lambda}$

so as to shrink the shadow banking sector is no longer relevant. Shadow banking is always fully active at the optimal leverage. The intuition is simple. Recall that the motive for the $\overline{\lambda}$ strategy is that the entrepreneur's utility actually increases w.r.t regulatory leverage λ over some range of λ . This is in turn because r < p implies that the bank transfers more risk to the entrepreneur per dollar raised at date 1 than at date 0. Thus the induced shadow-banking activity more than offsets the direct risk reduction caused by a decrease in λ . If the haircut in the shadow banking system l satisfies (21), then it is no longer the case that the bank transfers more risk to the household per dollar raised in the shadow banking system. As a result, the entrepreneur's utility now always decreases w.r.t. λ . Thus the strategy whereby the regulator tightens capital requirements up to a point where shadow-banking activity is maximal remains the only optimal one. Notice that if the shadow banking system is liquid (r close to p), then the haircut needed to obtain this result is small. The shadow banking system is liquid in particular when the portfolio has low risk (p large other things)equal).

Second, total welfare is strictly higher with such an l than when l = 1from Lemma 9, but the bank is much worse off. Essentially, it is no longer able to use shadow banking as a threat that leads to the adoption of lax capital requirements $\overline{\lambda}$. Welfare is maximized at effective leverage levels that are always lower than when l = 1, possibly much more so when $\overline{\lambda}$ is socially optimal for l = 1. Thus the model predicts that the banking sector should be strongly opposed to shadow-banking regulation. It has a lot to lose from such a regulation even though this would be overall socially desirable.

We do not dwell on the case in which l is larger than $\frac{r}{p}$ because it is less

interesting and less tractable. Essentially, it is possible to see that in this case, there may still exist a local maximum for welfare with a regulatory leverage such that shadow banking is not fully active $(L' < (1 - \lambda)L)$. It is still the case, however, that such a leverage is strictly smaller than $\overline{\lambda}$ for l < 1, and that the shadow banking system is not completely inactive at this leverage.

5 Extensions

5.1 Alternative Sources of Illiquidity in the Shadow Banking System

We introduce adverse selection as the source of date-1 illiquidity because the assumption that lending generates proprietary information for banks is common in the banking literature (Rajan, 1992), and well documented empirically (e.g., Lummer and McConnell, 1989).¹² Some anecdotal evidence (e.g., the Goldman Sachs Abacus transaction) suggests that adverse selection may be a concern in the shadow banking system, but systematic evidence is admittedly not available thus far.

It is important to stress, however, that our results do not live or die on this assumption of date-1 adverse selection. The important feature of the model is that trade at date 1 is more costly than at date 0 (r < p). This could stem from any other sources of illiquidity, such as due diligence or other legal costs, or perhaps model uncertainty. One advantage of adverse selection is that it links simply illiquidity to the quality of the portfolio.

¹²Gorton and Ordonez (2012) also explain liquidity dry-ups in the shadow-banking system with informational arguments.

5.2 Security Design

The support of the loan portfolio payoff $\{0; L\}$ enables us to abstract from security design issues that are not central to our argument. It is still worthwhile discussing the case of a general payoff distribution. With such a distribution, the prudential regulation of banks no longer consists in merely defining the stake sold to the household λ , but more generally in defining an optimal security sold to the household. Standard debt should be the optimal security here, for two reasons. Suppose first that there is no shadow banking activity because the regulator can observe the bank and the household at date 1. In this case, debt minimizes the riskiness of the household's claim for a given expected amount raised at date 0, and is thus socially optimal. Second, consider the situation in Section 3 in which the bank and the household can secretly re-trade at date 1. There is an additional benefit in this case from having issued senior debt at date 0. The bank is left with the most informationally sensitive part of its cash flows at date 1 - the equity tranche - and thus the lemons problem is maximal. This deters shadow banking activity. In sum, in our environment, bank debt is optimal for the standard reason that it is the safest security at date 0, and for the additional reason that the residual claim is the riskiest one, which reduces subsequent regulatory arbitrage at date 1 because of adverse selection.

5.3 Biased Regulator

Suppose that instead of maximizing total surplus, the regulator is biased and chooses λ so as to maximize

$$U_E + \beta U_B, \tag{22}$$

where $\beta > 0$. The regulator makes an inefficient decision when $\beta \neq 1$ because he does not properly compare the costs and benefits of leverage. It is easy to see that maximizing (22) amounts to maximizing total surplus using an adjustment cost $\frac{k}{\beta}$ instead of the true cost parameter k. Thus, if the regulator is captured by the industry ($\beta > 1$), he chooses an excessive leverage, and shadow banking activity would be inefficiently low in equilibrium. Conversely, if the regulator is biased against banks, he may impose tighter capital requirements and set off a larger shadow banking activity than surplus maximization would require.

5.4 Skin in the Game and Lending Standards

One alleged consequence of the growth of the shadow banking system is that it lowered screening standards at origination and therefore the overall quality of loans (see Keys et al., 2010, for some evidence). While this need not be inefficient in principle (Plantin, 2011), many observers argue that lending standards dropped to undesirable levels before the subprime crisis (see, e.g., Geithner and Summers, 2009). Parlour and Plantin (2008) develop such a model of inefficient securitization in which less adverse selection in the secondary market for bank assets may be undesirable *ex ante*. The reason it is so in their paper is that it lowers the bank's incentives to exert screening effort at origination. It is easy to add this ingredient to our setup and see how it amplifies the costs of an unregulated shadow banking system. Suppose that the loan portfolio pays off with probability p if the bank privately exerts effort at date 0 at some utility cost c, and pays off 0 almost surely otherwise. For a given regulatory leverage λ and shadow banking activity L', screening is incentive-compatible iff

$$p(1-\lambda)L - rL' \ge c. \tag{23}$$

The interpretation of (23) is straightforward. Viewed from date 0, the total cash flows that a screening bank expects to receive at dates 1 and 2 are equal to $p(1 - \lambda) L$, while a bank that does not screen receives rL' at date 1 almost surely. Since date-1 cash in case of a shock q is $\lambda pL + rL'$ regardless of the screening effort, incentive-compatibility simply requires that the differential in expected cash flows $p(1 - \lambda) L - rL'$ be greater than c. From Lemma 4, (23) becomes

$$pL \ge \varphi\left(\frac{1-p}{pq}\right) + c \tag{24}$$

when shadow banking activity is not maximal, and

$$(p-r)(1-\lambda)L \ge c$$

when $L' = (1 - \lambda)L$. Notice that (24) does not depend on λ , so that the screening effort is determined only by the primitive parameters of the economy regardless of regulation in the presence of shadow banking. When $L' = (1 - \lambda)L$, incentive-compatibility may or may not require a lower leverage than the regulatory level. Notice that the bank should deliberately adopt such a low leverage if it is socially desirable because it internalizes all the benefits from having proper screening incentives.

6 Concluding Remarks

This paper develops an analysis of the prudential regulation of banks in the presence of a shadow banking system. It builds on a simple model of optimal capital requirements based on the premises that banks do not fully internalize the costs that the riskiness of their liabilities creates for the real economy.

Allowing banks to bypass such capital requirements in an opaque shadow banking system, we find that the optimal regulatory response may be to relax capital requirements so as to bring shadow banking activity back in the spotlight of regulation. It is more effective than a tightening of capital requirements, which spurs destabilizing shadow banking activity.

Adding to this pure regulatory-arbitrage function of shadow banking a genuine optimal financing role, we find that heightened capital requirements for traditional banks become the sole optimal policy provided the shadow sector can also be subject to some form of capital requirements. The associated welfare increase, however, comes with a large transfer from traditional-bank shareholders to non-financial agents. One should therefore expect significant lobbying efforts against the regulation of leverage in the shadow banking system.

Banks in our model offer stores of value that non-financial agents use to transact, but they do not incur any liquidity risk by doing so given the static environment. Liquidity transformation is of course an important function of banks in practice. It implies that relatively small shocks on their net wealth may be amplified by runs and panics, and translate into major insolvency issues. Focussing on pure solvency issues as we do here is a natural first step to gain insights into the interplay of capital requirements and shadow banking. Applying our modelling of costly regulatory arbitrage to environments in which bank illiquidity is also a concern is a natural avenue for future research. The debate on whether traditional capital requirements suffice to address liquidity risk, or whether additional regulations should complement them (and which ones?) is still unsettled. Taking seriously the endogenous growth of shadow banking that such liquidity regulations could potentially induce seems in order when designing them.

7 Appendix

7.1 Proof of Lemma 1

Suppose that the contract storage is characterized by $\lambda \in [0, 1]$. Clearly, it must be that the entrepreneur chooses

$$I_0 \in [W - p\lambda L, W + (1 - p)\lambda L]$$

For such an I_0 , suppose first that the date-2 household demand is $W + (1-p) \lambda L$. Then the entrepreneur chooses $I_2 \in [I_0, W + (1-p) \lambda L]$ so as to maximize

$$I_0(1-c) + (I_2 - I_0)(1-c) - \frac{k(I_2 - I_0)^2}{2}.$$

This is because given a capacity I_2 , it is clearly optimal to set a maximal unit price of 1. Condition (2) implies that the entrepreneur actually chooses $I_2 = W + (1 - p) \lambda L.$

Suppose now that the date-2 household demand is $W - p\lambda L$. Then the entrepreneur chooses $I_2 \in [W - p\lambda L, I_0]$ so as to maximize

$$W - p\lambda L - cI_2 - \frac{k(I_2 - I_0)^2}{2}.$$
 (25)

This is because given a capacity I_2 , it is clearly optimal to set a unit price $\frac{W-p\lambda L}{I_2}$. The entrepreneur may also simply walk away from his assets at no cost if (25) is negative for all $I_2 \in [W - p\lambda L, I_0]$. Condition (2) implies that $I_2 = W - p\lambda L$ maximizes (25) and condition (1) implies that (25) is positive for such an I_2 .

Thus the entrepreneur chooses I_0 so as to maximize

$$(1-c) E\widetilde{D} - \frac{kE\left[\left(\widetilde{D} - I_0\right)^2\right]}{2},$$

where \widetilde{D} is the date-2 household demand. It is therefore optimal to set

$$I_0 = E\tilde{D} = W.$$

7.2 Proof of Lemma 3

Suppose a good bank seeks to signal his type using a sale with recourse. That is, the good bank stores cash K until date 2 and promises to pay it to the household if the loan does not perform. Denoting L' the promised payoff sold to the household at date 1, a mimicking bad bank would then gain a net consumption L' - K at date 1 and give up the stored K at date 2. A necessary condition for no-mimicking is thus L' < K. But this actually reduces the date-1 cash holdings of the good bank, and thus violates its participation constraint.

7.3 **Proof of Proposition 6**

i) Condition (17) means that

$$qf'\left(\frac{\varphi\left(\frac{1-p}{pq}\right)}{pL}pL\right) \ge \frac{\varphi\left(\frac{1-p}{pq}\right)}{pL}k\left(1-p\right)L,$$

or from (7) that $\lambda^* \geq \frac{\varphi\left(\frac{1-p}{pq}\right)}{pL}$. From Lemma 4, this implies that the shadow banking market is therefore inactive at λ^* . Thus the presence of shadow banking is immaterial, and the optimal leverage is still λ^* .

ii) Utilities (13) and (14) imply that total welfare varies as follows when λ describes $\left[0, \frac{\varphi\left(\frac{1-p}{pq}\right) - rL}{(p-r)L}\right]$ (notice that this set may be empty). It admits an interior maximum $\underline{\lambda}$ which solves

$$qf'\left(\left(\underline{\lambda}\left(p-r\right)+r\right)L\right) = \underline{\lambda}k\left(1-p\right)L\tag{26}$$

if $\frac{pk}{p-r}\left(\varphi\left(\frac{1-p}{pq}\right)-rL\right) > 1$, and is maximal at $\lambda = \frac{\varphi\left(\frac{1-p}{pq}\right)-rL}{(p-r)L}$ otherwise. Condition (18) thus implies that social welfare is increasing w.r.t. λ over $\left[0, \frac{\varphi\left(\frac{1-p}{pq}\right)-rL}{(p-r)L}\right]$. It also implies that $\lambda^* < \frac{\varphi\left(\frac{1-p}{pq}\right)}{pL}$, so that social welfare decreases w.r.t. λ over $\left[\frac{\varphi\left(\frac{1-p}{pq}\right)}{pL}, 1\right]$. From (15) and (16) we know that welfare also increases w.r.t. λ over $\left[\frac{\varphi\left(\frac{1-p}{pq}\right)-rL}{(p-r)L}, \frac{\varphi\left(\frac{1-p}{pq}\right)}{pL}\right]$. Thus welfare is maximal for leverage $\overline{\lambda} = \frac{\varphi\left(\frac{1-p}{pq}\right)}{pL}$. Shadow banking is inactive but is a threat that benefits the bank and overall reduces welfare since $\overline{\lambda} > \lambda^*$.

iii) Otherwise, if $\frac{pk}{p-r} \left(\varphi \left(\frac{1-p}{pq} \right) - rL \right) > 1$, then $\underline{\lambda}$ as defined above in (26) also corresponds to a local maximal of total welfare together with $\overline{\lambda}$. To see that $\underline{\lambda} < \lambda^*$, notice that for all $\lambda < 1$,

$$\lambda(p-r) + r > \lambda p.$$

Thus it must be that $\underline{\lambda}$, which solves (26), and λ^* , which solves (7), satisfy:

$$\begin{array}{rcl} \underline{\lambda} & < & \lambda^*, \\ \\ \underline{\lambda}(p-r) + r & > & \lambda^* p. \end{array}$$

7.4 Proof of Corollary 8

First, we have by definition of $\underline{\lambda}$

$$(\underline{\lambda}(p-r)+r)L \le \varphi\left(\frac{1-p}{pq}\right),$$

which establishes that U_B is always larger under $\overline{\lambda}$ than $\underline{\lambda}$.

Second, from (13) and (15), the entrepreneur is better off under λ iff

$$(\underline{\lambda}^2(p-r)+r)pL^2 \ge \varphi \left(\frac{1-p}{pq}\right)^2.$$
(27)

This holds for all $\underline{\lambda}$ if

$$\frac{\varphi\left(\frac{1-p}{pq}\right)}{pL} = \overline{\lambda} \le \sqrt{\frac{r}{p}}.$$
(28)

Whether condition (28) holds or not, it is also easy to see that welfare is larger under $\overline{\lambda}$ than $\underline{\lambda}$ if $\frac{pk}{p-r} \left(\varphi \left(\frac{1-p}{pq} \right) - rL \right)$ is sufficiently close to 1. By continuity welfare at leverage $\underline{\lambda}$ becomes arbitrarily close to welfare at leverage $\frac{\varphi \left(\frac{1-p}{pq} \right) - rL}{(p-r)L}$ in this case, which we know is strictly lower than welfare at $\overline{\lambda}$ since welfare increases w.r.t. λ over $\left[\frac{\varphi \left(\frac{1-p}{pq} \right) - rL}{(p-r)L}, \frac{\varphi \left(\frac{1-p}{pq} \right)}{pL} \right]$.

If (28) does not hold, then if k is sufficiently large, (26) implies in turn that $\underline{\lambda}$ becomes arbitrarily close to 0. In this case only the entrepreneur's utility matters for total welfare. From (27) it is higher under $\underline{\lambda} \approx 0$ than $\overline{\lambda}$ if

$$\left(\frac{\varphi\left(\frac{1-p}{pq}\right)}{pL}\right)^2 > \frac{r}{p}$$

7.5 **Proof of Proposition 10**

i) is obvious.

To establish ii), notice that in the presence of a haircut, the variance of the household's demand for given λ , L' becomes:

$$(1-p)\left(p\lambda^{2}L^{2} + rl^{2}L'^{2} + 2r\lambda lLL'\right).$$
(29)

For $\lambda \in \left[\frac{\varphi\left(\frac{1-p}{pq}\right)-rL}{(p-r)L}, \frac{\varphi\left(\frac{1-p}{pq}\right)}{pL}\right]$, (9) implies that this variance becomes $\frac{(1-p)}{p} \begin{pmatrix} \varphi\left(\frac{1-p}{pq}\right)^2 \\ +\left(\varphi\left(\frac{1-p}{pq}\right)-\lambda pL\right) \begin{pmatrix} \left(\frac{pl^2}{r}-1\right)\varphi\left(\frac{1-p}{pq}\right) \\ -\left(\frac{pl^2}{r}-(2l-1)\right)\lambda pL \end{pmatrix} \end{pmatrix}$ derivating w.r.t. λ yields

$$-2L(1-p)\left[\begin{array}{c} \left(\frac{pl}{r}-1\right)l\varphi\left(\frac{1-p}{pq}\right)\\ -\left(\frac{pl^2}{r}-(2l-1)\right)\lambda pL\end{array}\right],$$

which is positive if $\frac{pl}{r} < 1$. Thus variance increases w.r.t. λ , and U_E therefore decreases over $\left[\frac{\varphi\left(\frac{1-p}{pq}\right)-rL}{(p-r)L}, \frac{\varphi\left(\frac{1-p}{pq}\right)}{pL}\right]$.

This implies that welfare is decreasing w.r.t. λ on the right of $\frac{\varphi\left(\frac{1-p}{pq}\right)-rL}{(p-r)L}$. For $\lambda \leq \frac{\varphi\left(\frac{1-p}{pq}\right)-rL}{(p-r)L}$, we know that $L' = (1-\lambda)L$, and thus variance (29) becomes

$$(1-p)\left[(p-r)\lambda^2 + r(\lambda(1-l)+l)^2\right]L^2.$$

Thus, $\frac{dU_E}{d\lambda}$ is larger in absolute value when l < 1 than when l = 1. This implies that the local maximum of total surplus over $\left(0, \frac{\varphi\left(\frac{1-p}{pq}\right) - rL}{(p-r)L}\right)$, if any, is smaller than $\underline{\lambda}$, and that welfare is increasing over this whole interval for a larger range of values of k than when l = 1.

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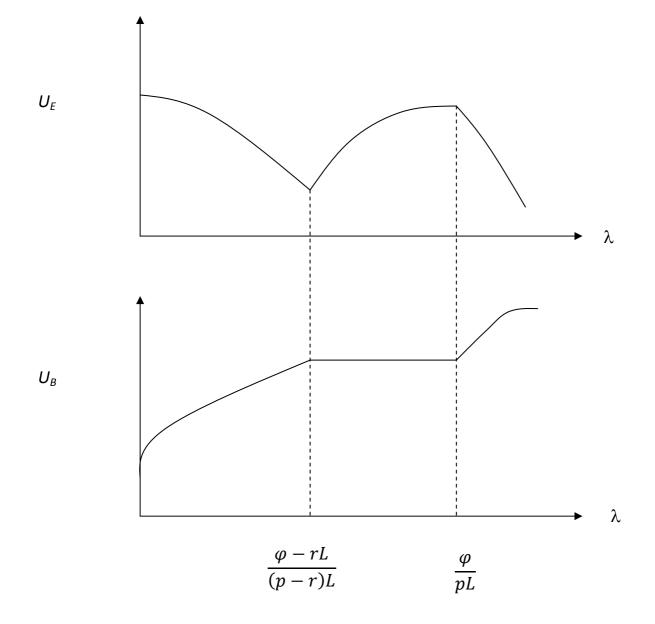


Figure 1: Utilities as a function of leverage in the presence of shadow banking

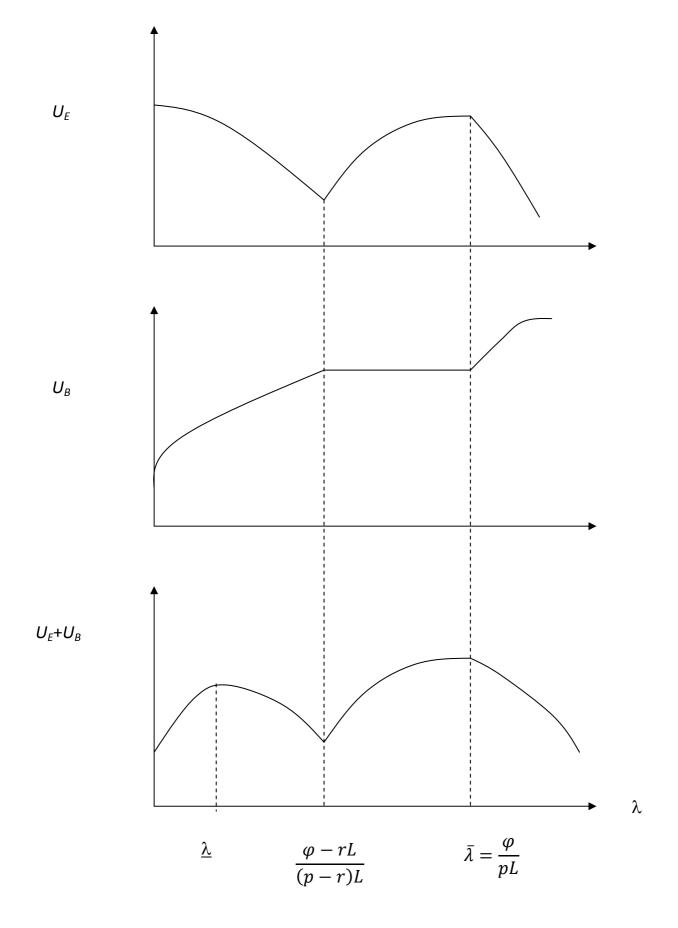


Figure 2: Surplus in Proposition 6 case *iii*)