

# Trading and Information Diffusion in Over-the-Counter Markets

(preliminary)

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## Abstract

This paper studies the informational efficiency of over-the-counter markets. We consider an over-the-counter market where dealers trade an asset with a stochastic payoff. Trade is bilateral, and each dealer can simultaneously participate in multiple transactions. The value of the asset is interdependent, and dealers are privately informed about it. Dealers learn additional information from the prices in the transactions they engage in. We show that although dealers trade strategically, the information that is revealed through trading is only distorted by the pattern of bilateral trades. Moreover, information diffuses through the network of trading links, such that the price in any bilateral transaction partially incorporates the private signals of all dealers in the market. In the common-value limit, over-the-counter markets are nearly as informationally efficient as centralized markets. Finally, our comparative static exercises illustrate the implications of these findings for intermediation and price dispersion in over-the-counter markets.

JEL Classifications: G14, D82, D85

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# 1 Introduction

A large proportion of assets is traded in over-the-counter (OTC) markets. The disruption of several of these markets (e.g. credit derivatives, asset backed securities, and repo agreements) during the financial of 2008, has highlighted the role that OTC trades play in the financial system. Typically, OTC markets are strongly concentrated. That is, in nearly every transaction one of the counterparties is one of a handful of large investment banks. Moreover, OTC trades are usually bilateral. This makes it possible for an asset to be traded at different prices at the same time, while market participants observe only a subset of all transaction prices. For this reason, OTC markets are often labeled as opaque. In this context, several important questions arise: How much private information is channeled into prices in this decentralized and non-competitive setting? To what extent participants can learn the private information of others whom they are not trading with? How sensitive the informational properties of OTC markets are to tense market conditions?

To address these questions, in this paper we present a novel way to model trade and information diffusion in OTC markets. In our model, agents trade bilaterally the same risky asset and each agent can participate in multiple transactions with a given subset of other dealers. Trading links connect dealers in a network. Quantities and prices in each bilateral transaction are determined in the equilibrium of a game in which agents take decisions simultaneously and their trading strategies are generalized demand functions. Our approach has several attractive features. First, just as in reality, in our set-up a dealer can trade any quantity of the asset she finds desirable, and understands that her trade may affect transaction prices. Moreover, she can decide to buy a certain quantity at a given price from one counterparty and sell a different quantity at a different price to another. Second, we show that the outcome of the one-shot game corresponds to the steady-state of a dynamic protocol reminiscent of the real world bargaining process in OTC markets.

This structure allows us to derive a number of important analytical results. We show that prices reveal information that is only distorted by the structure of trading links. That is, although dealers understand that their trades affect the equilibrium prices and

allocations, their strategic trading behavior does not affect the informational content of prices. Furthermore, information diffuses through the network, such that the equilibrium price in each transaction partially aggregates the private information of all agents in the economy. When each dealer trades with all the other dealers, over-the-counter markets are as informationally efficient as centralized markets. Finally, we illustrate through various comparative static exercises the implications of these findings for intermediation and price dispersion in over-the-counter markets.

In our main specification, there are  $n$  risk-neutral dealers organized in a dealer network. Intuitively, a link between  $i$  and  $j$  indicates that they are potential counterparties in a trade. There is a single risky asset in zero net supply. The final value of the asset is uncertain and interdependent across dealers<sup>1</sup>. Each dealer observes a private signal about her value, and all dealers have the same quality of information. Since values are interdependent, inferring each others' signals is valuable. Values and signals are drawn from a known multivariate normal distribution and, for simplicity, we assume that the pairwise correlation of any values and signals is the same. In a network, dealers simultaneously choose their trading strategy, taking the other dealers' strategies as given. A dealer's trading strategy is a generalized demand function specifying the quantity of the asset she is willing to trade with each of her potential counterparties depending on the prices that prevail in the transactions she participates in. In each transaction the two dealers trade against an exogenous demand curve. The interpretation is that dealers trade on their own account, as well as on behalf of an uninformed customer base.<sup>2</sup> After all bilateral markets clear, the value of the asset for each dealer is realized. We refer to this structure as the *OTC game*. The OTC game is, essentially, a generalization of the Vives (2011) variant of Kyle (1989) to networks.<sup>3</sup> The main results in the OTC game apply to *any* network.<sup>4</sup>

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<sup>1</sup>In OTC markets, agents may value the same asset differently depending, for instance, on how they use it as collateral, on which technologies to repackage and resell cash-flows they have, or on what risk-management constraints they face. Moreover, differences in asset valuations vary across markets and states.

<sup>2</sup>As we will explain, customers, that is, the exogenous demand curve, plays a technical role in our analysis; it ensures the existence of the equilibrium. This is the same solution as in Vives (2011) and it's alternative is to introduce noise traders as in Kyle (1989).

<sup>3</sup>A useful property of this variant is that were dealers trade on a centralized market, prices would be privately fully revealing. This provides a clear benchmark for our analysis.

<sup>4</sup>We use specific examples only to illustrate how the structure of the dealer network affects the trading outcome.

In real world OTC markets there is no market clearing mechanism which could instantly yield the equilibrium prices and quantities of the OTC game. However, we justify our approach by constructing a quasi-rational, but realistic bargaining dynamic protocol, as follows. Every period each dealer sends a message to her potential counterparties. In each subsequent period, a dealer can update her message given the set of messages she has received in the previous round. Messages can be interpreted, for instance, as quotes that dealers exchange with their counterparties. Upon receiving a set of quotes, a dealer might decide to contact her counterparties with other offers, before trade actually occurs. A rule, that is common knowledge among dealers, maps messages into prices and quantities for each pair of connected dealers. Trade takes place when no dealer wants to significantly revise her message based on the information she receives.

Our main results in the OTC game build on the following key insight. Equilibrium beliefs, prices and quantities in the OTC game are determined in three steps. First, we work-out the equilibrium beliefs in the OTC game. For this, we specify an auxiliary game in which dealers, connected in the same network as in the OTC game, make a best guess of their own value conditional on their signals and the guesses of the other dealers they are connected to. That is, each dealer's strategy specifies her guess as a function of her neighbors' guesses. We label this structure as the *conditional-guessing game* and show that an equilibrium in linear strategies exists for any connected network. Given the equilibrium in the conditional-guessing game, we then provide simple conditions for the existence of the equilibrium in the OTC game. When an equilibrium exists, we establish an equivalence between the beliefs in the OTC game and the beliefs in the conditional-guessing game. Second, we show that the price in a transaction between dealers  $i$  and  $j$  must be a weighted average of the expectations of  $i$  and  $j$ , where the weights depend on the position of these agents in the network. Finally, we derive the equilibrium quantities traded across each link given the distribution of prices and the slope of the exogenous demand.

In addition, we show that the dynamic protocol leads to the same traded prices and quantities as in the one-shot OTC game, when dealers use as an updating rule the equilibrium strategy in the conditional-guessing game, and the rule that maps messages into

prices is the same as the one that maps expectations into prices in the OTC game. Interestingly, even if the updating rule is not necessarily optimal each round it is used in, we show that when trade takes place, dealers could not have done better.

Our model provides testable empirical predictions. For instance, our findings suggest that the information of dealers that are more central in the network of trades affects everyone's belief more strongly, and that correlation across dealers' beliefs decreases with their distance in the network. This has immediate implications for cross-sectional price dispersion. Furthermore, we can use our framework to distinguish between two possible mechanisms of distress in OTC markets. We compare the case when dealers' valuations are more divergent with the case when dealers are more uncertain about their value. Our comparative statics indicate that as dealers are more heterogeneous in their values, prices are less informative, but profits are larger and there is more intermediation and high volume of trading. In contrast, as uncertainty increases and information becomes less precise, prices are less informative, profits decrease and there is less intermediation as volume goes down. Another thought-exercise we can perform to analyze the effect of distressed market conditions on the OTC structure, is to check how the equilibrium is affected by removing a single link from the network. To make this exercise as transparent as possible, we simply compare the circle network with the line network of equal number of dealers. We illustrate that while even those agents learn less who are far from the broken link, the loss in information decreases with the distance from that point. Consistently, with the results before, the broken link has very small effect when the OTC market is close to the common value limit.

### **Related literature**

Most models of OTC markets are based on search (e.g. Duffie, Garleanu and Pedersen (2005); Duffie, Gârleanu and Pedersen (2007), Lagos, Rocheteau and Weill (2008), Vayanos and Weill (2008), Lagos and Rocheteau (2009), Afonso and Lagos (2012), and Atkeson, Eisfeldt and Weill (2012)). The majority of these models do not analyze learning through trade. Important exceptions are Duffie, Malamud and Manso (2009) and Golosov, Lorenzoni and Tsyvinski (2009). Their main focus is the time-dimension of information diffusion either between differentially informed agents, or from homogeneously informed

to uninformed agents. A key assumption in these models is that there exists a continuum of atomistic agents on the market. This assumption implies that as an agent infers her counterparties' information from the sequence of transaction prices, she does not have to consider the possibility that any of her counterparties traded with each other before. Thus, in these models agents can infer an independent piece of information from each bilateral transactions.<sup>5</sup> In contrast, in our model all the meetings take places between a finite set of strategic dealers, but are collapsed in one period. Our results are a direct consequence of the fact that each dealer understands that her counterparties have overlapping information as they themselves have common counterparties, or their counterparties have common counterparties, etc. Our argument is that this insight is potentially crucial for the information diffusion in OTC markets where typically a small number of sophisticated financial institutions are responsible for the bulk of the trading volume. Therefore, we consider that search models and our approach are complementary.

Decentralized trade that takes place in a network has been studied by Gale and Kariv (2007), and Gofman (2011) with complete information and by Condorelli and Galeotti (2012) with incomplete information. These papers are interested in whether the presence of intermediaries affects the efficient allocation of assets, when agents trade sequentially one unit of the asset. Intermediation arises in our model as well. However, we allow a more flexible structure as dealers can trade any quantity of the asset they wish, given the price. Moreover, neither of these papers addresses the issue of information aggregation through trade (Condorelli and Galeotti, 2012, consider a pure private value set-up), which is the focus of our analysis.

Finally, we would like to mention contemporaneous work by Malamud and Rostek (2012) who also use a multi-unit double-auction setup to model a decentralized market. Malamud and Rostek (2012) study allocative efficiency and asset pricing with risk-averse dealers with homogeneous information; their framework allows for trading environments intermediate between centralized and decentralized. In contrast, we study how informa-

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<sup>5</sup>An interesting example of a search model where repeated transactions play a role is Zhu (2012) who analyzes the price formation in a bilateral relationship where a seller can ask quotes from a set of buyers repeatedly. In contrast to our model, Zhu (2012) considers a pure private value set-up. Thus, the issue of information aggregation through trade, which is the focus of our analysis, cannot be addressed in his model.

tion about an asset diffuses through trading with differentially informed, but risk-neutral dealers.

The paper is organized as follows. The following section introduces the model set-up and the equilibrium concept. In Section 3, we describe the conditional-guessing game, and we show the existence of the equilibrium in the OTC game. We characterize the informational content of prices in Section 4. Section 5 provides dynamic foundations for our main specification. In section 6 we illustrate the properties of the OTC game with some simple examples and discusses potential applications.

## 2 A General Model of Trading in OTC Markets

### 2.1 The model set-up

We consider an economy with  $n$  dealers which develops over two periods ( $t = 0, 1$ ). There are two assets: a risky asset and money, and both are redeemable against the only consumption good in the economy. In the first period agents have the opportunity to trade bilaterally, as we describe below. In the second period each dealer consumes the return realized from her portfolio.

Dealers' preferences are represented by the following utility function

$$U_i(q_i) = \theta_i q_i. \tag{1}$$

where  $q_i$  is the the quantity of the risky asset that the dealer holds at the end of the trading period. Each dealer is uncertain about the value of the asset. This uncertainty is captured by  $\theta_i$ , referred to as dealer  $i$ 's value. We assume that  $\theta_i$  is normally distributed with mean 0 and variance  $\sigma_\theta^2$ . Moreover, we consider that values are interdependent across dealers. In particular,  $\mathcal{V}(\theta_i, \theta_j) = \rho \sigma_\theta^2$  for any two agents  $i$  and  $j$ , where  $\mathcal{V}(\cdot, \cdot)$  represents the variance-covariance operator, and  $\rho \in [0, 1]$ . Differences in dealers' values reflect, for instance, differences in usage of the asset as collateral, in technologies to repackage and resell cash-flows, in risk-management constraints.

We assume that each dealer receives a private signal,  $s_i = \theta_i + \varepsilon_i$ , where  $\varepsilon_i \sim$

$IIDN(0, \sigma_\varepsilon^2)$  and  $\mathcal{V}(\theta_j, \varepsilon_i) = 0$ . Note that, for simplicity, we assume that each signal has the same precision.

Dealers seek to maximize their expected utility by trading assets through bilateral transactions. At date  $t = 0$ , each dealer  $i$  can engage in bilateral trades with a subset  $g_i$  of other dealers. Let  $|g_i| = m_i$  be the number of trading counterparties that agent  $i$  has in the network  $g$ . The set of bilateral trades can be represented through a network  $g$  in which a link  $ij$  represents a transaction between dealers  $i$  and  $j$ . Each network is characterized by an adjacency matrix, which is a  $n \times n$  matrix

$$A = (a_{ij})_{ij \in \{1, \dots, n\}}$$

where  $a_{ij} = 1$  if  $i$  and  $j$  have a link and  $a_{ij} = 0$  otherwise. While our main results hold for any network, throughout the paper, we illustrate the results using two types of networks as examples.

**Example 1** *The first type of networks is the family of **circulant networks**. In an  $(n, m)$  circulant network each dealer is connected with  $m/2$  other dealers on her left and  $m/2$  on her right. Note that the  $(n, 2)$  circulant network is the circle and the  $(n, n - 1)$  circulant network is the complete network. (A  $(9, 4)$  circulant network is shown panel (a) of Figure 1.)*

**Example 2** *The second type of networks is the family of **core-periphery networks**. In an  $(n, r)$  core-periphery network there are  $r$  fully connected agents (the core) each of them with links to  $\frac{n-r}{r}$  dealers (the periphery) and no other links exist. Note that the  $(n, 1)$  core-periphery network is an  $n$ -star network where one dealer is connected with  $n - 1$  other dealers. (A  $(9, 3)$  core periphery network is shown in panel (b) of Figure 1.)*

These two types of simple networks allow us to isolate the effect of different features of OTC markets in trade and information diffusion. In a circulant network, we isolate the effects of network density and of distance between dealers in a symmetric setting. In contrast, in a star network, we capture information asymmetries that arise among dealers due to their position in the network.



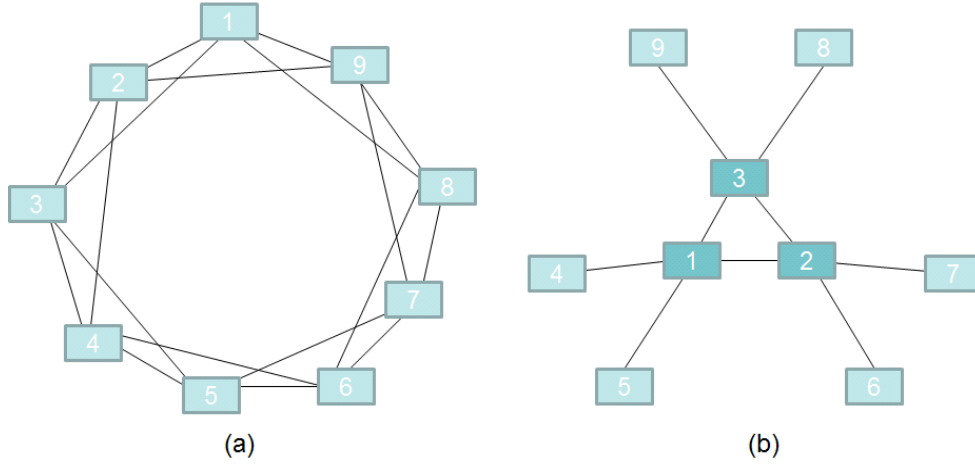


Figure 1: This figures shows two examples of networks. Panel (a) shows a  $(9, 4)$  circulant network. Panel (b) shows a  $(9, 3)$  core-periphery network.

The strategy of a dealer  $i$  is a map from the signal space to the space of generalized demand functions. For each dealer  $i$  with signal  $s_i$ , a generalized demand function is a continuous function  $\mathbf{Q}_i : R^{m_i} \rightarrow R^{m_i}$  which maps the vector of prices,  $\mathbf{p}_{g_i} = (p_{ij})_{j \in g_i}$ , that prevail in the transactions that dealer  $i$  participates in network  $g$  into vector of quantities she wishes to trade with each of her counterparties. The  $j$ -th element of this correspondence,  $Q_i^j(s_i; \mathbf{p}_{g_i})$ , represents her demand when her counterparty is dealer  $j$ , such that

$$\mathbf{Q}_i(s_i; \mathbf{p}_{g_i}) = \left( Q_i^j(s_i; \mathbf{p}_{g_i}) \right)_{j \in g_i}.$$

The final holding of a dealer  $i$  is given by

$$\mathbf{Q}_i^T(s_i; \mathbf{p}_{g_i}) \mathbf{1}. \tag{2}$$

This representation of dealers' demand functions captures an important characteristic of the OTC markets. Namely, the price and the quantity traded in a bilateral transaction are known only by the two counterparties involved in trade and are not revealed to all market participants.

In addition, for each transaction between  $i$  and  $j$  there exists an exogenous downward

sloping demand

$$D(p) = \beta_{ij}p, \quad (3)$$

where  $\beta_{ij} < 0$ . The interpretation is that dealers trade on their own account, as well as on behalf of an uninformed customer base. In our analysis customers play a technical role in our analysis: the exogenous demand (3) ensures the existence of the equilibrium. This is the approach that Vives (2011) takes as well, and it is an alternative to introducing noise dealers as in Kyle (1989). As we show below, while the slope of the exogenous supply affects the equilibrium allocation, it does not have an effect on the equilibrium beliefs, nor the equilibrium prices.

The expected payoff for dealer  $i$  with signal  $s_i$  corresponding to a given strategy profile  $\{\mathbf{Q}_i(s_i; \mathbf{p}_{g_i})\}_{i \in \{1, \dots, n\}}$  is

$$E \left[ \sum_{j \in g_i} Q_i^j(s_i; \mathbf{p}_{g_i}) (\theta_i - p_{ij}) \mid s_i \right]$$

where  $p_{ij}$  are the elements of the bilateral clearing price vector  $\mathbf{p}$  defined by the smallest element of the set

$$\mathbf{P}(\{\mathbf{Q}_i(s_i; \mathbf{p}_{g_i})\}_i, \mathbf{s}) \equiv \left\{ \mathbf{p} \mid Q_i^j(s_i; \mathbf{p}_{g_i}) + Q_i^j(s_j; \mathbf{p}_{g_j}) + \beta_{ij}p_{ij} = 0, \forall ij \in g \right\}$$

by lexicographical ordering<sup>6</sup>, if  $\mathbf{P}$  is non-empty. If  $\mathbf{P}$  is empty, we say the market brakes down and all dealers' payoff is zero. A linear strategy profile insures that the price map  $\mathbf{P}(\cdot, \mathbf{s})$  defines a unique market clearing price vector.

## 2.2 Equilibrium Concept

The environment described above represents a Bayesian game, henceforth the OTC game. The risk-neutral utility and normal information structure allows us to search for a linear equilibrium of this game defined as follows.

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<sup>6</sup>To be more precise, we order the links first by  $i$  and then by  $j$  and order the market clearing price vectors first by their first element, if it happens to be equal by the second element etc. Whether we choose the smallest or any other element by this ordering, or we use an other way to select a unique price vector is immaterial. We just have to ensure that our game is well defined even for strategies which are allowed, but never part of the equilibrium.

**Definition 1** A Linear Bayesian Nash equilibrium of the OTC game is a vector of linear generalized demand functions  $\{\mathbf{Q}_1(s_1; \mathbf{p}_{g_1}), \mathbf{Q}_2(s_2; \mathbf{p}_{g_2}), \dots, \mathbf{Q}_n(s_n; \mathbf{p}_{g_n})\}$  such that  $\mathbf{Q}_i(s_i; \mathbf{p}_{g_i})$  solves the problem

$$\max_{(Q_i^j)_{j \in g_i}} E \left\{ \left[ \sum_{j \in g_i} Q_i^j(s_i; \mathbf{p}_{g_i}) (\theta_i - p_{ij}) \right] | s_i \right\} \quad (4)$$

subject to  $\mathbf{p} = \mathbf{P}(\cdot, \mathbf{s})$  or, such that

$$Q_i^j(s_i; \mathbf{p}_{g_i}) + Q_j^i(s_j; \mathbf{p}_{g_j}) + \beta_{ij} p_{ij} = 0. \quad (5)$$

for any  $i$  and  $j$  that have a link in the network  $g$ .

A dealer  $i$  chooses a demand function for each transaction  $ij$ , in order to maximize her expected profits, given her information,  $s_i$ , and given the demand functions chosen by the other dealers. Moreover, as in Klemperer and Meyer (1989), her demand must be optimal for each realization of the uncertainty in her residual demand in transaction  $ij$ ,  $(-Q_j^i(s_j; \mathbf{p}_{g_j}) - \beta_{ij} p_{ij})$ . From  $i$ 's perspective, the uncertainty in the residual demand arises from the other dealers' signals, as they are reflected in the price vector  $\mathbf{p}_{g_i}$ .

Then, an equilibrium of the OTC game is a fixed point in demand functions.

### 3 The Equilibrium

In this section, we show existence and characterize the equilibrium in the OTC game. The key insight in this section is that equilibrium beliefs, prices and quantities in the OTC game can be determined in three steps. First, we determine the equilibrium beliefs in the OTC game. For this we introduce an auxiliary game, the *conditional-guessing game*, in which dealers simply aim to guess their valuation  $\theta_i$ , without any trade taking place. We then establish an equivalence between the equilibrium beliefs in the conditional-guessing game and the equilibrium beliefs in the OTC game. Second, we derive the equilibrium demand functions of each dealer for each link from the first-order conditions in the OTC game. Finally, we use the bilateral clearing conditions to show that in any linear equilibrium of the OTC game, the price in a transaction between dealers  $i$  and  $j$  must be a weighted

average of dealers'  $i$  and  $j$  expectations of their values  $\theta_i$  and  $\theta_j$ , where the weights depend on the position of these agents in the network.

The conditional guessing game is also a useful benchmark to measure how much information is revealed through trading in the OTC market. In the OTC game, prices play a dual role: on the one hand prices regulate the allocation of the asset among dealers, on the other hand they convey information. Dealers' market power interacts with the allocative function of prices, but not with the information transmission role. Indeed, as the equivalence between beliefs in the OTC game and conditional-guessing game shows, the network structure influences beliefs only to the extent that some agents do not directly learn about the expectation of everyone else.

When describing the conditional-guessing game, we keep the same notation as above. Throughout the paper, we relegate the proofs to the Appendix.

### 3.1 The formation of beliefs: The conditional-guessing game

The conditional guessing game is the non-competitive counterpart of the OTC game. The main difference is that instead of choosing quantities and prices to maximize trading profits, each agent aims to guess her value as precisely as she can. Importantly, agents are not constrained to choose a scalar as their guess. In fact, each dealer is allowed to choose a conditional-guess function which maps the guess of each of her neighbors, into the her guess.

Formally, we define the game as follows. Consider a set of  $n$  agents that are connected in a network  $g$ . The payoff of an agent  $i$  depends on the realization of the asset value  $\theta_i$ . Each agent has private information about her value,  $s_i = \theta_i + \varepsilon_i$ . The joint distribution of signals and values is the same as in the OTC game. Before the uncertainty is resolved, each agent  $i$  makes a guess,  $e_i$ , about the value of the asset,  $\theta_i$ . Her guess is the outcome of a function that has as arguments the guesses of other dealers she is connected to in the network  $g$ . In particular, given her signal, dealer  $i$  chooses a guess function,  $\mathcal{E}_i$ , which maps the vector of guesses of her neighbors,  $\mathbf{e}_{g_i}$ , into a guess  $e_i$ . When the uncertainty is resolved, agent  $i$  receives a payoff

$$-(\theta_i - e_i)^2.$$

**Definition 2** An equilibrium of this game is given by a strategy profile  $(\mathcal{E}_1, \mathcal{E}_2, \dots, \mathcal{E}_n)$  such that each agent  $i$  chooses strategy  $\mathcal{E}_i : R \times R^{m_i} \rightarrow R$  in order to maximize her expected payoff

$$\max_{\mathcal{E}_i} \left\{ -E \left( (\theta_i - e_i)^2 | s_i \right) \right\},$$

where  $e_i$  is the guess that prevails when

$$e_i = \mathcal{E}_i(\mathbf{e}_{g_i}) \tag{6}$$

for all  $i \in \{1, 2, \dots, n\}$ .

An agent  $i$  chooses a guess function that maximize her expected profits, given her information,  $s_i$ , and given the guess functions chosen by the other agents evaluated at the fixed point determined by the set of conditions (6). Moreover, her guess function must be optimal for each realization of the other dealers' signals  $s_j$ , as it is reflected in the vector of guesses  $e_{g_i}$ . Therefore, her optimal guess is then given by

$$e_i = E(\theta_i | s_i, \mathbf{e}_{g_i}). \tag{7}$$

We assume that if a fixed point in (6) does not exist, then dealers don't make any guesses and their profits are zero. Essentially, the set of conditions (6) is the counterpart in the conditional-guessing game of the market clearing condition in the OTC game. Then, an equilibrium of the conditional guessing game is the fixed point in guess functions.

In the next proposition, we state that the guessing game has an equilibrium in any network. We also point out some properties which later will turn out to be useful.

**Proposition 1** In the conditional-guessing game, for any network  $g$ , there exists an equilibrium in linear guess functions, such that

$$\mathcal{E}_i(s_i, \mathbf{e}_{g_i}) = \bar{y}_i s_i + \bar{\mathbf{z}}_{g_i}^T \mathbf{e}_{g_i}$$

for any  $i$ , where  $\bar{y}_i$  is a scalar and  $\bar{\mathbf{z}}_{g_i} = (\bar{z}_{ij})_{j \in g_i}$  is a vector of length  $m_i$ .

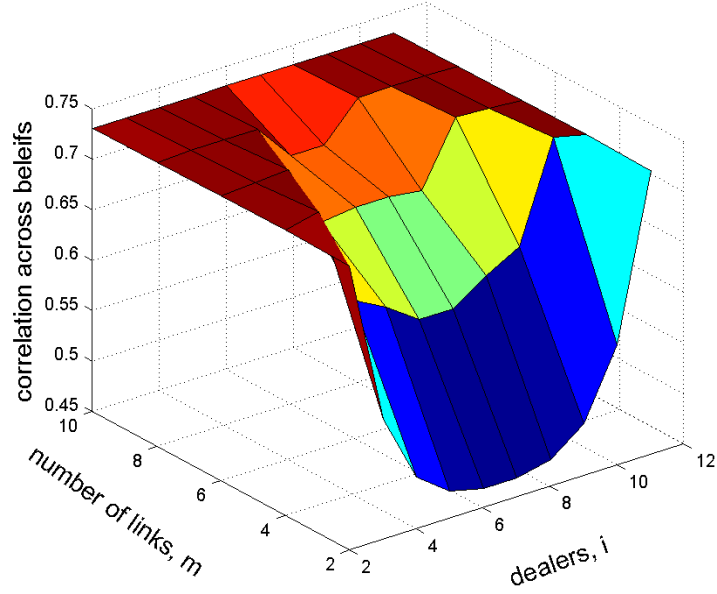


Figure 2: The plot shows the correlation of equilibrium expectations of dealer 1 and dealer  $i$ ,  $\frac{\mathcal{V}(E(\theta_1|s_1, \mathbf{e}_{g_1}), E(\theta_i|s_i, \mathbf{e}_{g_i}))}{\sqrt{\mathcal{V}(E(\theta_1|s_1, \mathbf{e}_{g_1}))\mathcal{V}(E(\theta_i|s_i, \mathbf{e}_{g_i}))}}$ , in each of the  $(11, m)$ -circulant networks.

Furthermore, whenever  $\rho < 1$

1.  $\bar{Y} \in (0, 1]^{n \times n}$ ,
2.  $\lim_{n \rightarrow \infty} \bar{Z}^n = 0$  and  $(I - \bar{Z})$  is invertible, and
3.  $\mathbf{e} = (I - \bar{Z})^{-1} \bar{Y} \mathbf{s}$ ,

where  $\mathbf{e} = (e_i)_{i \in \{1, 2, \dots, n\}}$ ,  $\bar{Y}$  is a matrix with elements  $\bar{y}_i$  on the diagonal and 0 otherwise, and  $\bar{Z}$  is a matrix with elements  $\bar{z}_{ij}$ , when  $i$  and  $j$  ( $\neq i$ ) have a link and 0 otherwise.

As an illustration, Figure 2 depicts the correlation of equilibrium expectations of dealer 1 and dealer  $i$ ,

$$\frac{\mathcal{V}(E(\theta_1|s_1, \mathbf{e}_{g_1}), E(\theta_i|s_i, \mathbf{e}_{g_i}))}{\sqrt{\mathcal{V}(E(\theta_1|s_1, \mathbf{e}_{g_1}))\mathcal{V}(E(\theta_i|s_i, \mathbf{e}_{g_i}))}}$$

in each of the  $(11, m)$ -circulant networks where dealers are ordered by  $i$ . It is apparent that while the correlation is always positive, it decreases with the distance from dealer 1, but increases with the number of links starting from each dealer. This is so, because

dealers learn more when they can condition their guess in more other dealers. In extreme case when the network is complete, the correlation across each dealers' equilibrium beliefs is the same. As we show in the proof of Proposition 1, this indicates that each dealer learns all the useful information that exists in the economy. We return to this issue in the following section.

### 3.2 The OTC game: Equilibrium existence and characterization

In this section we show how we can derive an equilibrium in the OTC game, given the equilibrium of the conditional-guessing game. Our derivation follows Kyle (1989) and Vives (2011) with the necessary adjustments.

We conjecture an equilibrium in demand functions, where the demand function of dealer  $i$  in the transaction with dealer  $j$  is given by

$$Q_i^j(s_i; \mathbf{p}_{g_i}) = b_i^j s_i + (\mathbf{c}_i^j)^T \mathbf{p}_{g_i} \quad (8)$$

for any  $i$  and  $j$ , where  $(\mathbf{c}_i^j) = (c_{ik}^j)_{k \in g_i}$ . We consider that the demand function of an agent  $i$ , when trading with agent  $j$ , depends not only on the price  $p_{ij}$  in the transaction between  $i$  and  $j$ , but also on the prices in the other transactions that  $i$  participates in. This specification allows a dealer to adjust the quantity she wishes to trade in each transaction conditional on all the prices she can trade at. Hence, a dealer is, for instance, able to buy a given quantity at a given price from one counterparty and sell a different quantity at a different price to another.

As it is standard in similar models, we simplify the optimization problem of (4) which is defined over a function space, to finding the functions  $Q_i^j(s_i; \mathbf{p}_{g_i})$  point-by-point. That is, for each realization of the signals,  $\mathbf{s}$ , we solve for the optimal quantity  $q_i^j$  that each dealer  $i$  demands when trading with a counterparty  $j$ . The idea is as follows. Given the conjecture (8) and market clearing (5), the residual inverse demand function of dealer  $i$  in a transaction with dealer  $j$  is

$$p_{ij} = - \frac{b_j^i s_j + \sum_{k \in g_j, k \neq i} c_{jk}^i p_{jk} + q_i^j}{c_{ji}^i + \beta_{ij}}. \quad (9)$$

Denote  $I_i^j \equiv - \left( b_{ij}^i s_j + \sum_{k \in g_j, k \neq i} c_{jk}^i p_{jk} \right) / \left( c_{ji}^i + \beta_{ij} \right)$  and rewrite (9) as

$$p_{ij} = I_i^j - \frac{1}{c_{ji}^i + \beta_{ij}} q_i^j.$$

The uncertainty that dealer  $i$  faces about the signals of others is reflected in the random intercept of the residual inverse demand,  $I_i^j$ , while her capacity to influence the price is reflected in the slope  $-1 / \left( c_{ji}^i + \beta_{ij} \right)$ . Thus, the price  $p_{ij}$  is informationally equivalent to the intercept  $I_i^j$ . This implies that finding the vector of quantities  $\mathbf{q}_i = \mathbf{Q}_i(s_i; \mathbf{p}_{g_i})$  for one particular realization of the signals,  $\mathbf{s}$ , is equivalent to solving

$$\max_{(q_i^j)_{j \in g_i}} E \left[ \sum_{j \in g_i} q_i^j \left( \theta_i + \frac{1}{c_{ji}^i + \beta_{ij}} q_i^j + I_i^j \right) \mid s_i, \mathbf{p}_{g_i} \right]$$

or

$$\max_{(q_i^j)_{j \in g_i}} \sum_{j \in g_i} q_i^j \left( E(\theta_i \mid s_i, \mathbf{p}_{g_i}) + \frac{1}{c_{ji}^i + \beta_{ij}} q_i^j + I_i^j \right).$$

From the first order conditions we derive the quantities  $q_i^j$  for each link of  $i$  and for each realization of  $\mathbf{s}$  as

$$2 \frac{1}{c_{ji}^i + \beta_{ij}} q_i^j = - \frac{1}{c_{ji}^i + \beta_{ij}} I_i^j - E(\theta_i \mid s_i, \mathbf{p}_{g_i}).$$

Then, using the definition of  $I_i^j$  from above, we can find the optimal demand function

$$Q_i^j(s_i; \mathbf{p}_{g_i}) = - (c_{ji}^i + \beta_{ij}) (E(\theta_i \mid s_i, \mathbf{p}_{g_i}) - p_{ij}) \quad (10)$$

for each dealer  $i$  when trading with dealer  $j$ .

For our later results, the chain structure embedded in the definition of  $I_i^j$  is critical. The price  $p_{ij}$  gives information on  $I_i^j$  which embeds information on the prices agent  $j$  trades at in equilibrium. For example, if agent  $j$  trades with agent  $k$  then  $p_{jk}$  affects  $p_{ij}$ . By the same logic,  $p_{jk}$  in turn is affected by the prices agent  $k$  trades at with her counterparties, etc. Therefore,  $p_{ij}$  will aggregate the private information of signals of every agent, dealer  $i$  is indirectly connected to, even if this connection is through several other dealers. At the same time, this information aggregation is not perfect. The exact



nature of this process is in the focus of our analysis.

It is clear that conjecture (8) together with the market clearing condition (4) imply that any market clearing price vector has to be a linear combination of signals, hence, jointly normally distributed. This implies that conditional expectations are linear. In particular, for each agent  $i$  there exists a scalar  $y_i$  and a vector  $\mathbf{z}_{g_i}$  such that

$$E(\theta_i | s_i, \mathbf{p}_{g_i}) = y_i s_i + \mathbf{z}_{g_i}^T \mathbf{p}_{g_i}. \quad (11)$$

where  $\mathbf{z}_{g_i} = (z_{ij})_{j \in g_i}$ . Substituting in (11) into (10) and comparing it to conjecture (8) determines the relationship that exists in equilibrium between the coefficients  $y_i$  and  $z_{ij}$  in the expectation (11) and the coefficients in the demand functions (8). The following Lemma summarizes these relationships.

**Lemma 1** *Suppose that there exists an equilibrium in the OTC game. Then the equilibrium demand functions*

$$Q_i^j(s_i; \mathbf{p}_{g_i}) = b_i^j s_i + (\mathbf{c}_i^j)^T \mathbf{p}_{g_i}$$

must be such that

$$\begin{aligned} b_i^j &= -\beta_{ij} \frac{2-z_{ji}}{z_{ij}+z_{ji}-z_{ij}z_{ji}} y_i \\ c_{ij}^j &= -\beta_{ij} \frac{2-z_{ji}}{z_{ij}+z_{ji}-z_{ij}z_{ji}} (z_{ij} - 1) \\ c_{ik}^j &= -\beta_{ij} \frac{2-z_{ji}}{z_{ij}+z_{ji}-z_{ij}z_{ji}} z_{ik}. \end{aligned}$$

This lemma has interesting implications for the market liquidity of the asset, as it is captured by the slope  $c_{ij}^j$  of dealer's  $i$  demand when trading with dealer  $j$ . A higher value of  $c_{ij}^j$  can be interpreted as the possibility of dealer  $i$  to take a larger position when trading with dealer  $j$ , without having a too large price impact. In our specification, dealer's  $i$  price impact in a transaction depends on whom he is trading with. Even in very symmetric structures, such as the family of circulant networks, the relative position of the dealers in the network affects the sensitivity of their demands to the price, as shown in Figure 3.<sup>7</sup> In particular, the demand of a dealer is less responsive to price changes when she trades with a counterparty with whom she has more information in common. For instance, this

<sup>7</sup>In figure 3, for simplification, we normalize the slope with the size of outside demand and plot  $c_{ij}^j/\beta_{ij}$ , as this ratio is independent of  $\beta_{ij}$ .

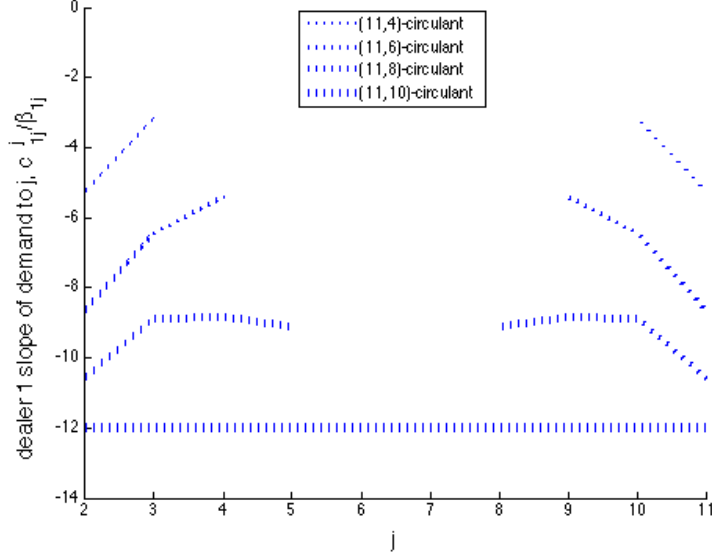


Figure 3: The figure shows the slope,  $\frac{c_{1j}^j}{\beta_{1j}}$  of the demand curve,  $Q_1^j$  submitted by agent 1 for the trade where the counterparty is  $j$  in the  $(11, m)$  circulant networks. Other parameters are  $\sigma_\varepsilon^2 = \sigma_\theta^2 = 1$ ,  $\rho = 0.5$ .

can arise when two dealers have common neighbors. This suggests that the liquidity of asset that is traded over the counter depends on the particular pair of dealers that are transacting.

The following proposition states the main result of this section.

**Proposition 2** *Let  $\bar{Y}$  and  $\bar{Z}$  be the matrices that support an equilibrium in the conditional-guessing game and let  $e_i = E(\theta_i | s_i, \mathbf{e}_{g_i})$  the corresponding equilibrium belief of agent  $i$ . Then, in the OTC game prices and quantities at link  $ij$  are given by*

$$p_{ij} = \frac{(2 - z_{ji}) e_i + (2 - z_{ij}) e_j}{4 - z_{ij} z_{ji}} \quad (12)$$

$$Q_i^j(s_i; \mathbf{p}_{g_i}) = -\frac{2 - z_{ji}}{z_{ij} + z_{ji} - z_{ij} z_{ji}} \beta_{ij} (e_i - p_{ij}) \quad (13)$$

and expectations are given by

$$E(\theta_i | s_i, \mathbf{p}_{g_i}) = e_i = E(\theta_i | s_i, \mathbf{e}_{g_i}) \quad (14)$$

whenever  $\rho < 1$  and the following system

$$\begin{aligned} \frac{y_i}{\left(1 - \sum_{k \in g_i} z_{ik} \frac{2 - z_{ki}}{4 - z_{ik} z_{ki}}\right)} &= \bar{y}_i \\ z_{ij} \frac{\frac{2 - z_{ij}}{4 - z_{ij} z_{ji}}}{\left(1 - \sum_{k \in g_i} z_{ik} \frac{2 - z_{ki}}{4 - z_{ik} z_{ki}}\right)} &= \bar{z}_{ij}, \forall j \in g_i \end{aligned} \quad (15)$$

admits a solution for each  $i \in \{1, \dots, n\}$  such that  $z_{ij} \in (0, 2)$ .

This proposition provides a simple procedure to characterize the equilibrium of the OTC game, given the equilibrium of the conditional guessing game. First, each dealer's expectation about the value of the asset in the OTC game is the same as her expectation in the conditional guessing game. Then, given the coefficients  $\bar{z}_{ij}$  and  $\bar{y}_i$  that define the equilibrium expectation in the guessing game, we can calculate the coefficients  $z_{ij}$  and  $y_i$  that define the equilibrium expectation in the OTC game from (15). Finally, given these coefficients, we can calculate the equilibrium prices and quantities by (12)-(13).

The intuition behind the equivalence of the two games is simple. On the one hand, it is easy to check that given the equivalence of expectations, (14), prices and quantities (12)-(13) are consistent with the first order condition (10) and market clearing conditions (5). On the other hand, each dealer  $i$  understands that the equilibrium price when trading with dealer  $j$  is a weighted average of their beliefs. Therefore, she can infer the belief of her counterparty from the price, given that she knows her own belief. When choosing her generalized demand function, she essentially conditions her expectation about the asset value on the expectations of the other dealers she is trading with. This explains why the equilibrium beliefs in the OTC game are the same as the equilibrium beliefs in the conditional-guessing game.

Proposition 2 proves existence of an equilibrium in the OTC game conditional on the existence of a solution to the system (15). While we do not have reasons to doubt that a solution of this system exists for any connected network, we do not have yet a general proof. The corollary below shows the proof of existence for the special cases of our examples.

**Corollary 1** *There exists an equilibrium in the OTC game for any network in the circulant and the core-periphery family.*

We defer the discussion about the equilibrium features to Section 6, where we illustrate numerically what the model implies about intermediation and profits.

## 4 Price Informational Efficiency

In this section we discuss what the model implies about the informational content of prices. We relate our findings to two benchmarks that we introduce below.

### Benchmark 1: Centralized markets

The centralized market version of our model is the risk-neutral version in Vives (2011). The main difference in his model compared to ours is that in a centralized market agents submit simple demand functions to a market maker and the market clears at a single price. For completeness, we summarize the properties of the equilibrium in centralized markets in the following proposition, but let the reader find the proof in the original version.

**Proposition 3** *Let be  $\rho < 1$ . In a centralized market there is a linear demand function equilibrium if and only if*

$$n - 2 < \frac{n\rho\sigma_\varepsilon^2}{(1 - \rho)(\sigma_\varepsilon^2 + (1 + (n - 1)\rho)\sigma_\theta^2)} \quad (16)$$

where demand functions has the form of

$$Q_i(s_i; p) = bs_i + cp.$$

The price is fully privately revealing in the sense of

$$\mathcal{V}(\theta_i | s_i, p) = \mathcal{V}(\theta_i | \mathbf{s}).$$

**Benchmark 2: Non-strategic information sharing**

In the non-strategic information sharing of our model, any two dealers that have a link, instead of trading, simply exchange their respective signals. This implies that the posterior belief of a dealer  $i$  that has  $m_i$  links in network  $g$  is given by

$$E\left(\theta_i | s_i, (s_j)_{j \in g_i}\right) = \frac{(1 - \rho) \sigma_\theta^2}{(\sigma_\varepsilon^2 + (1 - \rho) \sigma_\theta^2)} \left[ s_i + \frac{\rho \sigma_\varepsilon^2}{(1 - \rho) (\sigma_\theta^2 (1 + m_i \rho) + \sigma_\varepsilon^2)} \sum_{j \in g_i \cup i} s_j \right].$$

This specification is an adaptation of the information percolation models to our setting (see Duffie, Malamud and Manso (2009)). While the assumptions that justify non-strategic information sharing in these models (such as the assumption that the market is populated by a continuum of traders) do not apply in our case, we nevertheless consider that this is a useful benchmark to compare our results against.

Next, we return to the analysis of the OTC game. The first implication of our model concerns the role of prices in aggregating information.

**Proposition 4** *Suppose that there exists an equilibrium in the OTC game, and let  $\rho < 1$ . Then, in any connected network  $g$*

$$(I - \bar{Z})^{-1} \bar{Y} > 0,$$

*and each bilateral price is a linear combination of all signals in the economy, with a positive weight on each signal.*

The intuition for this result is as follows. The price in a bilateral transaction between two dealers is a weighted average of their respective beliefs. Furthermore, the equilibrium belief of each agent depends on her signal and on the prices she can trade at. Thus, any information that is incorporated in the price between dealer  $i$  and dealer  $j$ , is further reflected (with the adequate weights) in the prices in the transactions between dealer  $i$

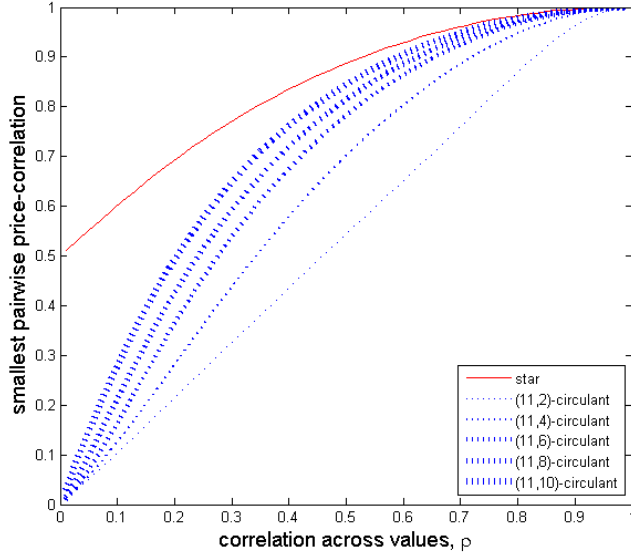


Figure 4: The plot compares the maximal dispersion in prices (as measured by the minimal pairwise correlation across prices) across the star network (solid line) and the circulant network family (dotted lines, thicker lines correspond to larger  $m$  values). Other parameters are  $n = 11$ ,  $\sigma_\varepsilon^2 = \sigma_\theta^2 = 1$ .

and her counterparties and dealer  $j$  and her counterparties, respectively. This process continues, and information diffuses through the trading network.

An equivalent way of stating the result in Proposition 4 is that each dealer's equilibrium belief depends on the private signal of all the other dealers, regardless whether she trades with those dealers or not. In contrast, the belief of a dealer in the non-strategic information sharing benchmark depends only on the private signals of the other dealers with whom she has a link. In this sense, information diffusion is fast in our set up as the information of each dealer reaches all other dealers at least to some extent.

An important observation that follows from this proposition is that different dealers will likely pay different prices for the same asset at the same time. Figure 4 illustrates the maximal price dispersion, measured as the minimal correlation between prices, in each of the  $(11, m)$ -circulant networks.

The second implication of our model that we emphasize is related to the revelation of information into prices. We know from Vives (2011) that in a centralized market the price

is fully privately revealing. We show that the equilibrium in the OTC game has a similar property. This follows from Proposition 2, which shows that the equilibrium beliefs in the OTC game are the same as the equilibrium beliefs in the conditional-guessing game. The equivalence of expectations implies that the information revealed through trading is distorted by the structure of trading links, but not by the dealers' market power.

This property of the equilibrium does not imply that dealers learn all the relevant information in the economy, as it happens in a centralized market. In particular, it follows from Proposition 4 that in a network  $g$ , a dealer  $i$  can use  $m_i$  linear combinations of the vector of signals,  $\mathbf{s}$ , to infer the other  $(n - 1)$  signals apart from her own. Except if she has  $m_i = n - 1$ , this is generally not sufficient for the dealer to learn all the relevant information in the economy. However, when the network is complete, for instance, each dealer learns as much information as if they were trading in a centralized market. In addition, as the following proposition shows, each agent learns (almost) all the valuable information in the market at the common value limit, regardless of the network.

**Proposition 5** *If  $\rho \rightarrow 1$ , then in the OTC game prices are privately fully revealing as*

$$\lim_{\rho \rightarrow 1} (\mathcal{V}(\theta_i | s_i, \mathbf{p}_{g_i}) - \mathcal{V}(\theta_i | \mathbf{s})) = 0.$$

The intuition behind the result is as follows. When values are perfectly correlated, the equilibrium expectation in the conditional-guessing game is the same for each dealer, and is proportional to the average of the  $n$  private signals. This is an equilibrium because when each dealer faces the same uncertainty about the value of the asset, the average of all signals is the best possible guess for everyone. Hence, conditional on her neighbors' guesses, a dealer arrives to the same guess. While the equilibrium in the OTC game collapses when  $\rho = 1$  (a manifestation of the well-known Grossman paradox), we show that the equilibrium beliefs converge to the average of all signals as the correlation across values approaches 1.

We use the result in this proposition to argue in section 6.2 that under normal economic conditions OTC markets are informationally close to their centralized counterpart, while under market stress the information loss due to the decentralized structure is perhaps

much more significant.

## 5 Dynamic Foundations

In real world OTC markets there is no market clearing mechanism which could instantly produce the equilibrium prices and quantities from the complex generalized demand functions of the OTC game. In this section we introduce a quasi-rational, but realistic bargaining dynamic protocol producing the same outcome than our OTC game.

Suppose that time is discrete, and in each period there are two stages: the morning stage and the evening stage. In the evening, each dealer  $i$  sends a message,  $h_{i,t}$ , to all counterparties she has in the network  $g$ . In the morning, each dealer receives these messages. In the following evening, a dealer can update the message she sends, possibly taking into account the information received in the morning. Messages can be interpreted, for instance, as quotes that dealers exchange with their counterparties. Upon receiving a set of quotes, a dealer might decide to contact her counterparties with other offers, before trade actually occurs. The protocol stops if there exists an arbitrarily small scalar  $\delta_i > 0$ , such that  $|h_{i,t} - h_{i,t_\delta}| \leq \delta_i$  for each  $i$ , in any subsequent period  $t \geq t_\delta$ . That, is the protocol stops when no dealer wants to significantly revise her message in the evening after receiving information in the morning.

Importantly, there exists a rule that maps messages into prices and quantities for each pair of dealers that have a link in the network  $g$ , and this rule is common knowledge for all dealers. When the protocol stops, trading takes place at the prices determined by this rules, and quantities are allocated accordingly. No transactions take place before the protocol stops.

Suppose that there exists an equilibrium in the one-shot OTC game. Let dealers use their equilibrium strategy in the conditional guessing game to update each evening the messages they send based on the messages they receive in the morning, such that

$$h_{i,t} = \bar{y}_i s_i + \bar{z}_{g_i}^T \mathbf{h}_{g_i,t-1}, \quad \forall i$$

where  $\mathbf{h}_{g_i,t} = (h_{j,t})_{j \in g_i}$ , and  $\bar{y}_i$  and  $\bar{z}_{ij}$  have been characterized, for any  $i$  and  $j \in g_i$ , in



Proposition 1. Further, consider a rule based on (12) that determines the price between a pair of agents  $ij$  that have a link in the network  $g$  as follows

$$p_{ij,t} = \frac{(2 - z_{ji})}{4 - z_{ij}z_{ji}} h_{i,t} + \frac{(2 - z_{ij})}{4 - z_{ij}z_{ji}} h_{j,t},$$

where the relationship between  $z_{ij}$  and  $\bar{z}_{ij}$  has been characterized in Proposition 2. Given the prices, the quantity that agent  $i$  would receive in the transaction with  $j$  is  $q_{i,t}^j(s_i, \mathbf{p}_{g_i,t})$ , where the the function has been characterized in Corollary 1. As before, agents seek to maximize their expected utility, given their private signal, and the messages they observe.

**Proposition 6** *Let  $\mathbf{h}_t = (h_{i,t})_{i \in \{1,2,\dots,n\}}$  be the vector of messages sent at time  $t$ , and  $\boldsymbol{\mu} = (\mu_i)_{i \in \{1,2,\dots,n\}}$  be a vector of IID  $N(0, \sigma_\mu^2)$  random normal variables. Suppose that  $\rho < 1$ . Then*

1. *If  $\mathbf{h}_t = (I - \bar{Z})^{-1} \bar{Y} \mathbf{s}$ , then  $\mathbf{h}_{t+1} = (I - \bar{Z})^{-1} \bar{Y} \mathbf{s}$ , for any  $t$ .*
2. *If  $\mathbf{h}_{t_0} = (I - \bar{Z})^{-1} \bar{Y} (\mathbf{s} + \boldsymbol{\mu})$ , then there exists a vector of arbitrarily small scalars  $\boldsymbol{\delta} = (\delta_i)_{i \in \{1,2,\dots,n\}}$  such that trading takes place in period  $t_\delta$  and*

$$\left| \mathbf{h}_{t_\delta} - (I - \bar{Z})^{-1} \bar{Y} \mathbf{s} \right| < \frac{1}{2} \boldsymbol{\delta}.$$

3. *If  $\mathbf{h}_{t_0} = (I - \bar{Z})^{-1} \bar{Y} (\mathbf{s} + \boldsymbol{\mu})$ , then there exists a vector of arbitrarily small scalars  $\boldsymbol{\delta} = (\delta_i)_{i \in \{1,2,\dots,n\}}$  such that trading takes place in period  $t_\delta$  and*

$$\left| E(\theta_i | s_i, \mathbf{h}_{g_i,t_0}, \mathbf{h}_{g_i,t_0+1}, \dots, \mathbf{h}_{g_i,t_\delta}) - E(\theta_i | s_i, \mathbf{p}_{g_i}) \right| < \frac{1}{2} \boldsymbol{\delta},$$

where  $\mathbf{p}_{g_i}$  are the equilibrium prices in the one-short OTC game.

Thus, this dynamic protocol leads to the same traded prices and quantities, independently from which vector of messages we start the protocol at. Interestingly, even if this updating rule is not necessarily optimal in every round when it is used, we show that ex-post, when transactions occur, dealers could not do better.

## 6 Discussion

In this section, we illustrate the properties of the OTC game with some simple examples. In the first part, we compare the outcome of the game when it is played in variants of the circulant network or when it is played in a star network.<sup>8</sup> Then, in the second part, we highlight the insights our set-up provides about OTC markets in periods of distress.

### 6.1 Example

In this part, we compare trading and information diffusion in the two simplest possible examples of networks. In particular, we compare the equilibrium when seven dealers with the same realizations of signals trade in a (7,2)-circulant network (i.e. a circle) versus in a 7-star. We use Figure 5 and Table 1 to illustrate the analysis.

In this example, the realization of the value of each of the 7 dealers is  $\theta_i = 0$ , but they receive the signals

$$s_i = i - 4.$$

That is, their signals are ordered as their index is, dealer seven is the most optimistic, dealer 1 is the most pessimistic and dealer 4 is just right. To visualize this, in Figure 5 nodes corresponding to pessimist (optimist) dealers are blue (red). For illustrative purposes, we placed pessimists and optimists on nodes in a way that sometimes dealers with very large informational differences trade. The price,  $p_{ij}$  of each transaction is in the rhombi on the link. The quantities traded by each of the counterparties,  $q_j^j, q_j^i$  and the corresponding profits a given trader makes on the given transaction are in the rectangles near the links. Profits are in brackets. Each number is rounded to the nearest decimal.

For example, when dealers 6 and 3 trade in a circle network, dealer 6 takes the long position of  $q_6^3 = 2.3$  at price  $p_{36} = 0.1$  leading to the loss of

$$q_6^3 (\theta_6 - p_{36}) = (0 - 0.1) (2.3) \approx -0.2$$

---

<sup>8</sup>Note, that Proposition gives a simple numerical procedure finding the equilibrium for any network very fast. (The code is available on request and will be posted on authors' websites soon.) We use this simple examples, not because of computational constraints, but to keep the discussion as intuitive as we can.

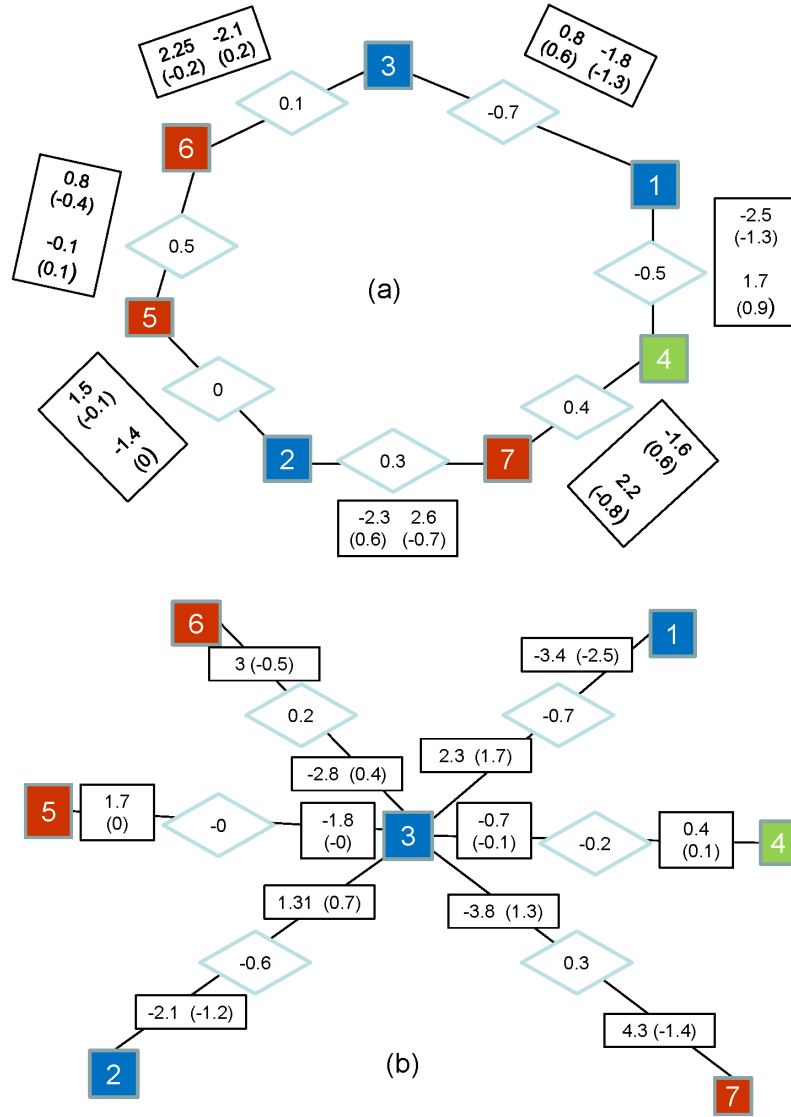


Figure 5: The two graphs illustrate the equilibrium of the OTC game in a  $(7, 2)$  circulant network (panel (a)) and in a 7–star network (panel (b)). In each case,  $\theta_i = 0$  for each dealer and  $s_i = i - 4$ , thus blue (red) nodes denote pessimist (optimist) dealers. Values in the rhombi on each edge are equilibrium prices. The quantity  $q_{ij}$  and the corresponding profit earned (in brackets) by a given dealer in a given trade are in the rectangles. Other parameter values are  $\rho = 0.5$ ,  $\sigma_\theta^2 = \sigma_\varepsilon^2 = 1$ ,  $\beta_{ij} = -10$ .

Dealer	$s_i$	$E(\theta_i   s_i, \mathbf{p}_{g_i})$		$\mathbf{1}^T \mathbf{q}_i$		$\mathbf{1}^T  \mathbf{q}_i $		$\mathbf{1}^T \mathbf{q}_i (\theta_i - \mathbf{p}_{g_i})$	
		circle	star	circle	star	circle	star	circle	star
3	-1	-0.5	-0.3	-1.3	-5.4	2.9	12.7	0.8	4
1	-3	-1.2	-1.3	-4.3	-3.4	4.3	3.4	-2.7	-2.5
4	0	-0.1	-0.1	0.1	0.4	3.3	0.4	1.6	0.1
7	3	1	1.1	4.8	4.3	4.8	4.3	-1.5	-1.4
2	-2	-0.4	-0.9	-3.7	-2.1	3.7	2.1	0.6	-1.2
5	1	0.5	0.3	1.4	1.7	1.6	1.7	0	0
6	2	0.7	0.7	3.1	3	3.1	3	-0.6	-0.5

Table 1: The table summarizes the signal, the expectation, the net position, the gross position and the total profit if each dealer in the two cases when they arranged into a (7,2)-circle network and a 7-star network.

While dealer 3 takes the short position of  $q_3^1 = 0.8$  at the same price leading to a profit of 0.6.

In Table 1, we summarize the signal, the expectation, the net position, the gross position and the total profit if each dealer. Looking at the Figure 5 and Table 1, we can make several intuitive observations.

First, while in most transactions one of the counterparties gain and the other loose, the profits and quantities do not add up to zero. It is because the excess demand or supply is the quantity  $\beta_{ij} p_{ij}$  bought by the consumers.

Second, when a dealer has counterparties both whom are more and less optimistic than her, this dealer will intermediate trades between them. That is, the size of her gross positions will be larger than the size of her net positions. This is the case of dealer 3, 4 and 5 in the circle network. In the star network, dealer 3 is in an ideal position to intermediate trades. This is explains her enormous gross position compared to others in that network.

Third, the profits dealer make, apart from the accuracy of their information, depends heavily on their position in the network. For example, dealer 4 is very lucky in the circle network not only, because her information is accurate, but also because she turns out to be connected with two dealers of extreme opinion. So she can take both a large long and a large short position of almost equal sizes and make a large profit on both. That explains why she makes the largest profit in the circle network. However, in the star network her profit is much smaller than the profit of the central dealer 3, even if the guess of dealer 3

is less accurate.

Forth, the after-trade expectation of each dealer depends both on the shape of the network and her position in it. That is apparent in Table 1 by comparing  $E(\theta_i | s_i, \mathbf{p}_{g_i})$  across networks. In general, the extent of adjustment of their post trade expectations compared to their pre-trade expectations depends on the degree of pessimism and optimism of their counterparties and also on how much their counterparties know. For example, dealer 6 arrives to the same expectation in the two networks because the effect of learning from more agents in the circle network is offset by the effect of learning from the better informed dealer 3 in the star network.

## 6.2 Comparative statics

In this part, we focus on two comparative statics exercises. The scope of the first exercise is to see whether we can use our framework to study different channels that drive changes in volume or price dispersion. For this, as an illustration, we contrast the effect of the information uncertainty,  $\sigma_\varepsilon^2$ , and the effect of the correlation across values,  $\rho$ . With our second exercise we aim to gain insights on how much the performance of OTC markets deteriorates under market stress. For this, we look at the effect of breaking a single link in the network.

### 6.2.1 Value interdependence and information uncertainty

First, we assess in parallel the effect of information uncertainty, as captured by changes in  $\sigma_\varepsilon^2$ , and the effect of heterogeneity in dealers' values, as captured by changes in  $\rho$ . In our framework, the correlation between values,  $\rho$ , plays a dual role. On the one hand, it captures the extent of gains from trade, and, consequently the level of adverse selection in the OTC market. On the other hand, it captures how relevant a dealer finds the information of others for estimating her valuation. In other words, the higher  $\rho$  is the more dealers learn about fundamentals by trading. Thus, dealers have improved information if either the precision of information is high ( $\sigma_\varepsilon^2$  is low), or they are able to infer from trading the signals of other when  $\rho$  is high.

Figure 6 compares the expected dispersion in prices (measured as  $E\left(\frac{\sum_{i,j \in g} (p_{ij} - p_{average})^2}{(mn/2)}\right)$ )

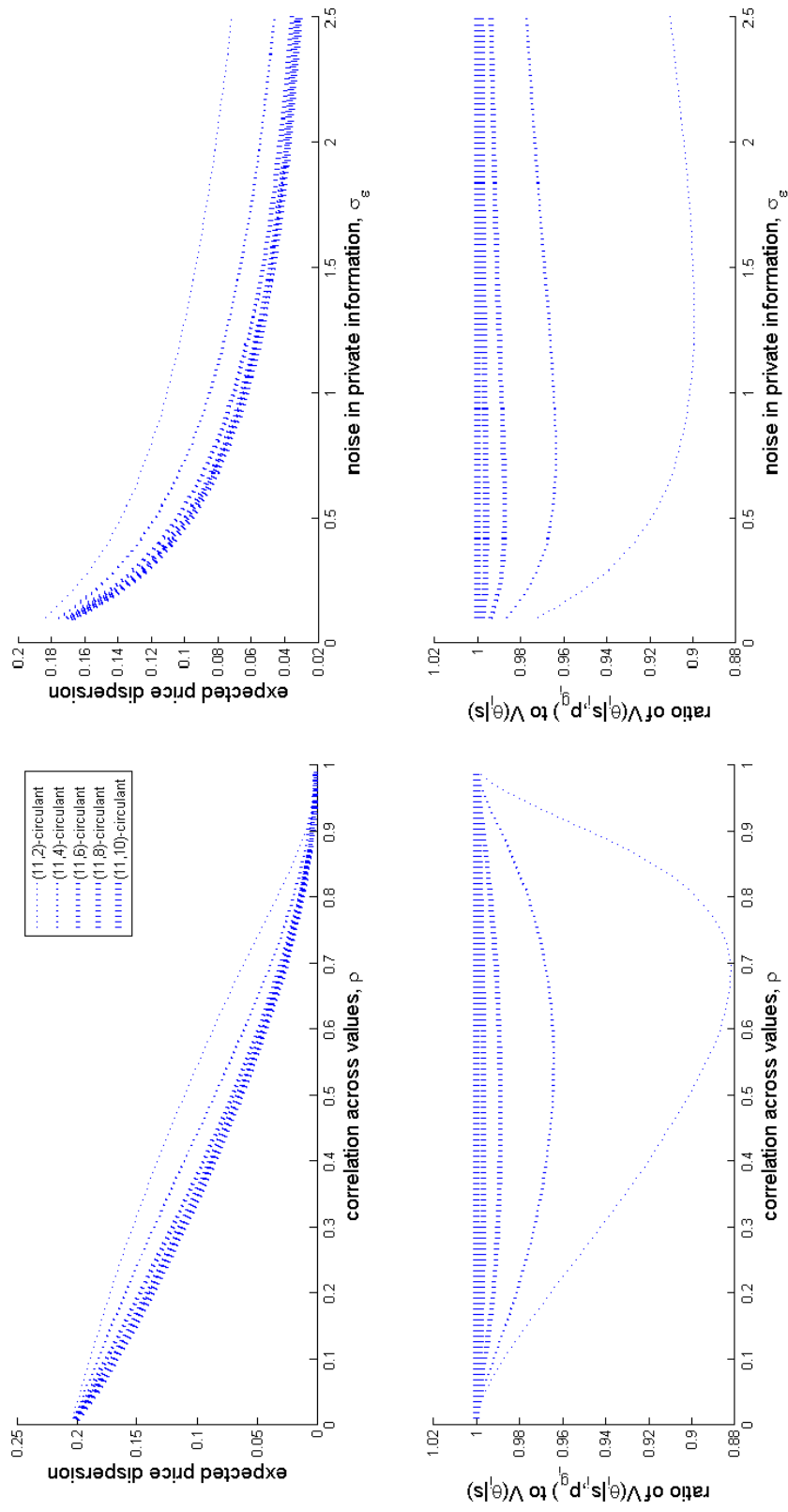


Figure 6: The plots compare the expected dispersion in prices (top panels) and the fraction of information revealed through trading (bottom panels) for the circulant network family (dotted lines, thicker lines correspond to larger  $m$  values). Other parameters are  $n = 11$ ,  $\sigma_\varepsilon^2 = \sigma_\theta^2 = 1$  for the comparative statics on  $\rho$ , and  $n = 11$ ,  $\sigma_\theta^2 = 1$ ,  $\rho = 0.5$  for the comparative statics on  $\sigma_\varepsilon^2$ .

in the upper panels) and the fraction of information revealed through trading (measured as  $\frac{\mathcal{V}(\theta_i|s_i, \mathbf{P}g_i)}{\mathcal{V}(\theta_i|\mathbf{s})}$  in the bottom panels) for the circulant network family (dotted lines, thicker lines correspond to larger  $m$  values) for various levels of  $\rho$  (left two panels) and  $\sigma_\varepsilon^2$  (right two panels). Other parameters are  $n = 11$ ,  $\sigma_\varepsilon^2 = \sigma_\theta^2 = 1$  for the comparative statics on  $\rho$ , and  $n = 11$ ,  $\sigma_\theta^2 = 1$ ,  $\rho = 0.5$  for the comparative statics on  $\sigma_\varepsilon^2$ .

When  $\rho$  is very low, a dealer finds that the signals of other dealers are not very informative for her own value. Therefore, a dealer relies mostly on her signal to estimate her value, which increases price dispersion. As  $\rho$  increases, our measure of the fraction of information revealed through trading declines initially, indicating that the rate at which information becomes relevant for dealers is lower than the rate at which prices reveal information. However, when the correlation across values is close to 1, prices are fully revealing and price dispersion goes down. This is in line with the result from Proposition 5. Both effects are larger for less dense networks as our comparison across circulant networks shows.

As the information uncertainty increases, our measures for price dispersion and for information revelation move in the same direction as when  $\rho$  decreases. This seems counter-intuitive. However, as  $\sigma_\varepsilon^2$  increases, the signals become so noisy that even access to all the information in the economy does not reduce the uncertainty about  $\theta_i$  for any dealer  $i$ . In other words,  $\mathcal{V}(\theta_i|\mathbf{s})$  converges to  $\sigma_\theta^2$ . Prices in this economy are equally uninformative, which explains why fraction of information revealed through trading goes up. At the same time, when  $\sigma_\varepsilon^2$  is small, the information that a dealer has is very precise and will have a higher weight in her belief relative to the other dealers' signals. In this case, price dispersion is high. As,  $\sigma_\varepsilon^2$  increases and agents no longer find the signals useful, price dispersions goes down.

Finally, we check how the total expected profit and expected intermediation changes with the level of correlation across values in networks with different shape and density. The bottom panels in Figure 7 show expected total profit in the  $(11, m)$ -circulant networks as  $\rho$  changes (right panel) and as  $\sigma_\varepsilon^2$  increases (left panel).<sup>9</sup> Expected profits tend to decrease

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<sup>9</sup>We keep the sum  $\sum_i \Sigma_{g_i} \beta_{ij}$  constant. Intuitively, this is equivalent of dividing a constant mass of consumers across the various links in different networks. We do this to make sure that it is not the increasing number of consumers drive the results.

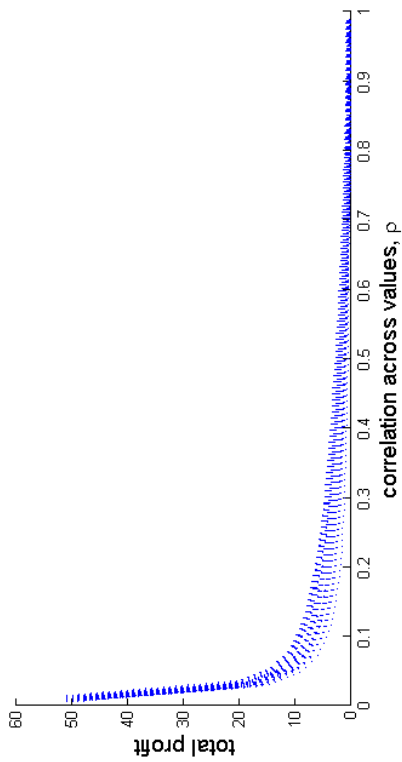
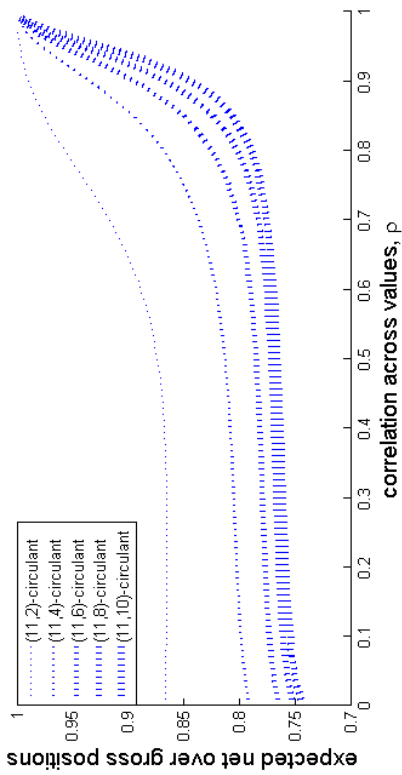
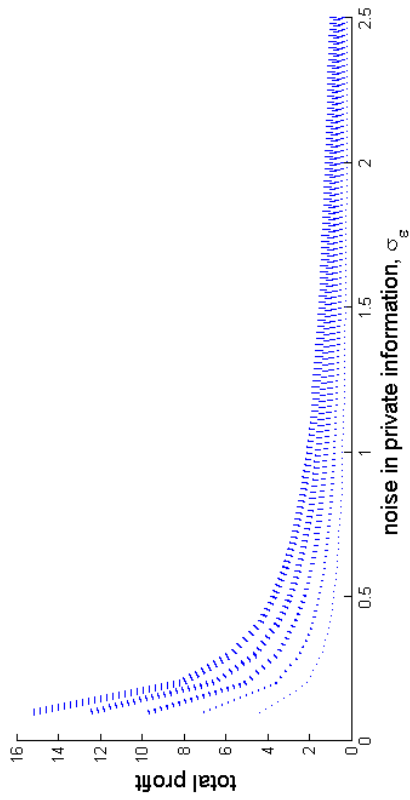
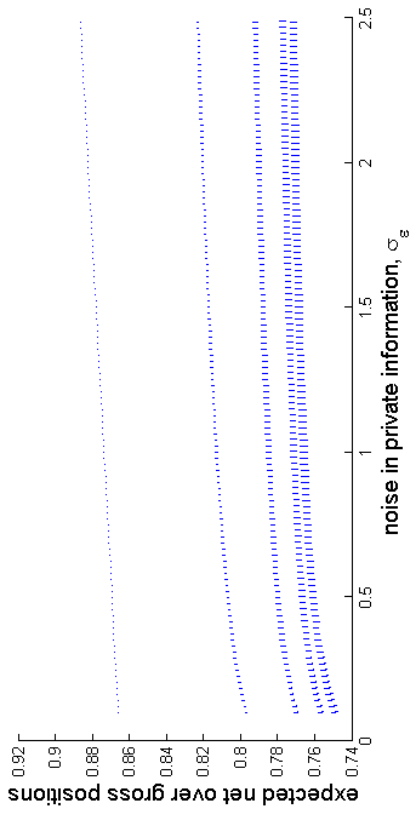


Figure 7: The plots compare the total profits and expected intermediation when dealers trade in  $(11, m)$ -circulant networks as  $\rho$  increases (left panels) and as  $\sigma_\varepsilon^2$  increases (right panels). Other parameters are  $n = 11$ ,  $\sigma_\theta^2 = \sigma_\theta^2 = 1$  for the comparative statics on  $\rho$ , and  $n = 11$ ,  $\sigma_\theta^2 = 1$ ,  $\rho = 0.5$  for the comparative statics on  $\sigma_\varepsilon^2$ . The sum of the coefficients  $\beta_{ij}$  over all links are kept constant across each network at  $-10$ .



with the correlation across values, and when the information uncertainty is higher. The reason is that as correlation decreases (the information uncertainty increases), agents are less (more) worried about adverse selection and trade more (less) aggressively. This enforces the intuition of section 6.1 that intermediating trades is extremely profitable. To see this point better, the top panels show the expected intermediation, that is, the ratio of absolute net positions to total gross positions,

$$E \left( \frac{\left| \sum_{j \in g_i} q_i^j \right|}{\sum_{j \in g_i} |q_i^j|} \right),$$

for each of these networks. The smaller is this measure, the larger is the intermediation level. As we see, intermediation increases in  $m$  in the  $(n, m)$  circulants. The reason is that even in the complete network prices for each transaction are different, and as agents are better informed, they take larger positions. Intermediation levels are relatively high for intermediate level of  $\rho$ . However, intermediation decreases as  $\sigma_\varepsilon^2$  increases, which is consistent with the drop in trading activity during the financial crisis.

### 6.2.2 Breaking a link

In our second exercise, we assess how information revelation changes when a single link is broken. To keep the exercise simple, we take the example of an 11-circle and brake the link between the first and the last agent. Thus, the circle becomes a line.

Figure 8 shows the fraction each agent learns after the brake compared to the amount they learned before the break. That is, our measure is

$$\frac{\mathcal{V}(\theta_i | s_i, \mathbf{p}_{g_i}) |_{line}}{\mathcal{V}(\theta_i | s_i, \mathbf{p}_{g_i}) |_{circle}}.$$

Consistently to Proposition 5 there is almost no effect when  $\rho$  is close to 1. In this case, dealers learn it all independently of the shape of the network. However, when  $\rho$  is at an intermediate level, the effect of the break is large. Furthermore the effect is much smaller for those who are far from the break-point. Under our interpretation of  $\rho$ , this suggests that OTC markets are much more resilient in normal times than in bad times.

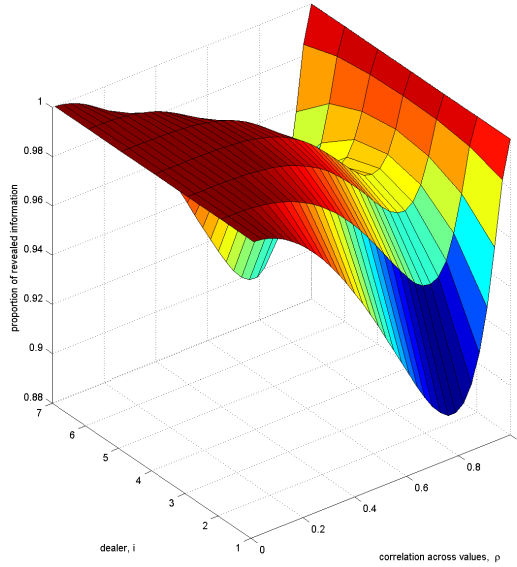


Figure 8: The plot shows the proportion of information revealed in a line-network compared to the corresponding circle network for each agent  $i$ , and various correlations across agent signals,  $\rho$ . Parameter values are  $\sigma_\theta^2 = \sigma_\varepsilon^2 = 1$  and  $n = 7$ .

## 7 Conclusion

In this paper we present a model of strategic information diffusion in over-the-counter markets. In our set-up a dealer can trade any quantity of the asset she finds desirable, and understands that her trade may affect transaction prices. Moreover, she can decide to buy a certain quantity at a given price from one counterparty and sell a different quantity at a different price to another.

We show that the equilibrium price in each transaction partially aggregates the private information of all agents in the economy. In this economy, prices reveal as much information as the structure of trading links allows it. The informational efficiency of prices is the highest in networks where each agent trades with every other agent, or in the common value limit, regardless of the network structure.

## References

- Afonso, Gara, and Ricardo Lagos.** 2012. “Trade Dynamics in the Market for Federal Funds.” Federal Reserve Bank of New York Staff Reports, 549.
- Atkeson, Andrew G., Andrea L. Eisfeldt, and Pierre-Olivier Weill.** 2012. “The Market for OTC Credit Derivatives.” <https://sites.google.com/site/pierreolivierweill/otcderivatives.pdf>.
- Condorelli, Daniele, and Andrea Galeotti.** 2012. “Bilateral Trading in Networks.” <http://privatewww.essex.ac.uk/~dcond/TRADNET.pdf>.
- Duffie, Darrell, Nicolae Garleanu, and Lasse Heje Pedersen.** 2005. “Over-the-Counter Markets.” *Econometrica*, 73(6): 1815–1847.
- Duffie, Darrell, Nicolae Gârleanu, and Lasse Heje Pedersen.** 2007. “Valuation in Over-the-Counter Markets.” *Review of Financial Studies*, 20(6): 1865–1900.
- Duffie, Darrell, Semyon Malamud, and Gustavo Manso.** 2009. “Information Percolation With Equilibrium Search Dynamics.” *Econometrica*, 77(5): 1513–1574.
- Gale, Douglas, and Shachar Kariv.** 2007. “Financial Networks.” *American Economic Review, Papers & Proceedings*, 92: 99–103.
- Gofman, Michael.** 2011. “A Network-Based Analysis of Over-the-Counter Markets.” <https://mywebpace.wisc.edu/gofman/web/JMP/MichaelGofmanJMP.pdf>.
- Golosov, Mikhail, Guido Lorenzoni, and Aleh Tsyvinski.** 2009. “Decentralized Trading with Private Information.” National Bureau of Economic Research, Inc NBER Working Papers 15513.
- Horn, Roger A., and Charles R. Johnson.** 1985. *Matrix analysis*. Cambridge Univ. Press.
- Klemperer, Paul D, and Margaret A Meyer.** 1989. “Supply Function Equilibria in Oligopoly under Uncertainty.” *Econometrica*, 57(6): 1243–77.

- Kyle, Albert S.** 1989. “Informed Speculation with Imperfect Competition.” *Review of Economic Studies*, 56(3): 317–55.
- Lagos, Ricardo, and Guillaume Rocheteau.** 2009. “Liquidity in Asset Markets With Search Frictions.” *Econometrica*, 77(2): 403–426.
- Lagos, Ricardo, Guillaume Rocheteau, and Pierre-Olivier Weill.** 2008. “Crashes and Recoveries in Illiquid Markets.” National Bureau of Economic Research, Inc NBER Working Papers 14119.
- Malamud, Semyon, and Marzena Rostek.** 2012. “Decentralized Exchange.”
- Meyer, Carl D.,** ed. 2000. *Matrix analysis and applied linear algebra*. Philadelphia, PA, USA:Society for Industrial and Applied Mathematics.
- Vayanos, Dimitri, and Pierre-Olivier Weill.** 2008. “A Search-Based Theory of the On-the-Run Phenomenon.” *Journal of Finance*, 63(3): 1361–1398.
- Vives, Xavier.** 2011. “Strategic Supply Function Competition With Private Information.” *Econometrica*, 79(6): 1919–1966.
- Zhu, Haoxiang.** 2012. “Finding a Good Price in Opaque Over-the-Counter Markets.” *Review of Financial Studies*, 25(4): 1255–1285.

## A Appendix

Throughout several of the following proofs we often decompose  $\theta_i$  into a common value component,  $\hat{\theta}$ , and a private value component,  $\eta_i$ , such that

$$\theta_i = \hat{\theta} + \eta_i$$

and

$$s_i = \hat{\theta} + \eta_i + \varepsilon_i$$

with  $\hat{\theta} \sim N(0, \sigma_{\hat{\theta}}^2)$ ,  $\eta_i \sim IID N(0, \sigma_{\eta}^2)$  and  $\mathcal{V}(\eta_i, \eta_j) = 0$ . This implies that

$$(1 - \rho) \sigma_{\hat{\theta}}^2 = \sigma_{\eta}^2$$

Further, we generalize the notation  $\mathcal{V}$  to be the variance-covariance operator applied to vectors of random variables. For instance,  $\mathcal{V}(\mathbf{x})$  represents that variance-covariance matrix of vector  $\mathbf{x}$ , and  $\mathcal{V}(\mathbf{x}, \mathbf{y})$  represents the covariance matrix between vector  $\mathbf{x}$  and  $\mathbf{y}$ .

**Lemma 2** *Consider the jointly normally distributed variables  $(\theta_i, \mathbf{s})$ . Let an arbitrary weighting vector  $\boldsymbol{\omega} > \mathbf{0}$ . Consider the coefficient of  $s_i$  in the projection of  $E(\theta_i | s_i)$ . Adding  $\boldsymbol{\omega}^T \mathbf{s}$  as a conditioning variable, additional to  $s_i$ , decreases the coefficient of  $s_i$ , that is,*

$$\frac{\partial E(\theta_i | s_i, \boldsymbol{\omega}^T \mathbf{s})}{\partial s_i} < \frac{\partial E(\theta_i | s_i)}{\partial s_i}$$

**Proof.** From the projection theorem

$$E(\theta_i | s_i, \boldsymbol{\omega}^T \mathbf{s}) = E(\theta_i | s_i) + \frac{\mathcal{V}(\theta_i, \boldsymbol{\omega}^T \mathbf{s} | s_i)}{\mathcal{V}(\boldsymbol{\omega}^T \mathbf{s} | s_i)} (\boldsymbol{\omega}^T \mathbf{s} - E(\boldsymbol{\omega}^T \mathbf{s} | s_i))$$

consequently

$$\frac{\partial E(\theta_i | s_i, \boldsymbol{\omega}^T \mathbf{s})}{\partial s_i} = \frac{\partial E(\theta_i | s_i)}{\partial s_i} - \frac{\mathcal{V}(\theta_i, \boldsymbol{\omega}^T \mathbf{s} | s_i)}{\mathcal{V}(\boldsymbol{\omega}^T \mathbf{s} | s_i)} \frac{\mathcal{V}(s_i, \boldsymbol{\omega}^T \mathbf{s})}{\mathcal{V}(s_i)}.$$

Thus, it is sufficient to show that  $\mathcal{V}(\boldsymbol{\omega}^T \mathbf{s}, s_i) > 0$  and  $\mathcal{V}(\theta_i, \boldsymbol{\omega}^T \mathbf{s} | s_i) > 0$ . For the former,

we know that

$$\mathcal{V}(\boldsymbol{\omega}^T \mathbf{s}, s_i) = \omega_i (\sigma_\varepsilon^2 + (1 - \rho) \sigma_\theta^2) + \rho \sigma_\theta^2 \boldsymbol{\omega}^T \mathbf{1} > 0$$

Then, we use the projection theorem to show that

$$\begin{aligned} \mathcal{V}(\theta_i, \boldsymbol{\omega}^T \mathbf{s} | s_i) &= \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 \boldsymbol{\omega}^T \mathbf{1} \\ \sigma_\theta^2 \boldsymbol{\omega}^T \mathbf{1} & \mathcal{V}(\boldsymbol{\omega}^T \mathbf{s}) \end{pmatrix} \\ &\quad - \frac{1}{\sigma_\theta^2 + \sigma_\varepsilon^2} \begin{pmatrix} \sigma_\theta^2 \\ (\omega_i (\sigma_\varepsilon^2 + (1 - \rho) \sigma_\theta^2) + \rho \sigma_\theta^2 \boldsymbol{\omega}^T \mathbf{1}) \end{pmatrix} \begin{pmatrix} \sigma_\theta^2 & (\omega_i (\sigma_\varepsilon^2 + (1 - \rho) \sigma_\theta^2) + \rho \sigma_\theta^2 \boldsymbol{\omega}^T \mathbf{1}) \end{pmatrix} \end{aligned}$$

implying that

$$\mathcal{V}(\theta_i, \boldsymbol{\omega}^T \mathbf{s} | s_i) = \sigma_\theta^2 \boldsymbol{\omega}^T \mathbf{1} - \frac{(\omega_i (\sigma_\varepsilon^2 + (1 - \rho) \sigma_\theta^2) + \rho \sigma_\theta^2 \boldsymbol{\omega}^T \mathbf{1}) \sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} = \sigma_\theta^2 \frac{\sigma_\varepsilon^2 + (1 - \rho) \sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} (\boldsymbol{\omega}^T \mathbf{1} - \omega_i) > 0.$$

■

**Lemma 3** Take the jointly normally distributed system  $\begin{pmatrix} \hat{\theta} \\ \mathbf{x} \end{pmatrix}$  where  $\mathbf{x} = \hat{\theta} \mathbf{1} + \boldsymbol{\varepsilon}' + \boldsymbol{\varepsilon}''$ , with the following properties

- $E \begin{pmatrix} \hat{\theta} \\ \mathbf{x} \end{pmatrix} = 0$ ,  $\mathcal{V}(\hat{\theta}, \boldsymbol{\varepsilon}') = 0$  and  $\mathcal{V}(\hat{\theta}, \boldsymbol{\varepsilon}'') = 0$ ;
- $\mathcal{V}(\boldsymbol{\varepsilon}')$  is diagonal, and  $\mathcal{V}(\mathbf{x}) \geq \mathcal{V}(\hat{\theta} \mathbf{1} + \boldsymbol{\varepsilon}')$ .

Then the vector  $\boldsymbol{\omega}$  defined by

$$E(\hat{\theta} | \mathbf{x}) = \boldsymbol{\omega}^T \mathbf{x}$$

has the properties that  $\boldsymbol{\omega}^T \mathbf{1} < 1$  and  $\boldsymbol{\omega} \in (0, 1)^n$ .

**Proof.** By the projection theorem, we have that

$$\boldsymbol{\omega}^T = \mathcal{V}(\hat{\theta}, \mathbf{x}) (\mathcal{V}(\mathbf{x}))^{-1}.$$

Then

$$\mathcal{V}(\hat{\theta}, \mathbf{x}) (\mathcal{V}(\mathbf{x}))^{-1} \leq \mathcal{V}(\hat{\theta}, \hat{\theta}\mathbf{1} + \boldsymbol{\varepsilon}' + \boldsymbol{\varepsilon}'') (\mathcal{V}(\hat{\theta}\mathbf{1} + \boldsymbol{\varepsilon}'))^{-1} = \mathcal{V}(\hat{\theta}, \hat{\theta}\mathbf{1}) (\mathcal{V}(\hat{\theta}\mathbf{1} + \boldsymbol{\varepsilon}'))^{-1}.$$

The inequality comes from the fact that both  $\mathcal{V}(\mathbf{x})$  and  $\mathcal{V}(\hat{\theta}\mathbf{1} + \hat{\boldsymbol{\varepsilon}})$  are positive definite matrixes and that  $\mathcal{V}(\mathbf{x}) \geq \mathcal{V}(\hat{\theta}\mathbf{1} + \hat{\boldsymbol{\varepsilon}})$ . (See Horn and Johnson (1985), Corollary 7.7.4(a)).

Since

$$\mathcal{V}(\hat{\theta}, \hat{\theta}\mathbf{1}) (\mathcal{V}(\hat{\theta}\mathbf{1} + \boldsymbol{\varepsilon}'))^{-1} \mathbf{1} = 1 - \frac{\frac{1}{\sigma_{\hat{\theta}}^2}}{\frac{1}{\mathcal{V}(\boldsymbol{\varepsilon}')}\mathbf{1} + \frac{1}{\sigma_{\hat{\theta}}^2}} < 1,$$

then

$$\boldsymbol{\omega}^T \mathbf{1} < 1$$

which implies that

$$\boldsymbol{\omega} \in (0, 1)^n.$$

■

**Lemma 4** *For any network  $g$ , define a mapping  $F : R^{n \times n} \rightarrow R^{n \times n}$  as follows. Let  $\mathbf{V}$  be an  $n \times n$  matrix with columns  $\mathbf{v}_j$  and*

$$e_j = \mathbf{v}_j^T \mathbf{s}$$

for each  $j = 1, \dots, n$ . The mapping  $F(\mathbf{V})$  is given by

$$\begin{pmatrix} E(\theta_1 | s_1, \mathbf{e}_{g_1}) \\ E(\theta_2 | s_2, \mathbf{e}_{g_2}) \\ \dots \\ E(\theta_n | s_n, \mathbf{e}_{g_n}) \end{pmatrix} = F(\mathbf{V}) \mathbf{s}.$$

Then, the mapping  $F$  is a continuous self-map on the space  $[0, 1]^{n \times n}$ .

**Proof.** Let

$$\mathbf{v}_j^0 = \begin{pmatrix} v_{j1}^0 & v_{j2}^0 & \dots & v_{jn}^0 \end{pmatrix}^T$$

and consider that

$$\begin{aligned} e_j &= (\mathbf{v}_j^0)^T \mathbf{s} \\ &= (\mathbf{v}_j^0)^T (\hat{\theta} \mathbf{1} + \boldsymbol{\varepsilon} + \boldsymbol{\eta}) \end{aligned}$$

where

$$\boldsymbol{\eta} = \begin{pmatrix} \eta_1 & \eta_2 & \dots & \eta_n \end{pmatrix}^T$$

and

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \dots & \varepsilon_n \end{pmatrix}^T$$

Let

$$\hat{e}_j = \frac{e_j}{(\mathbf{v}_j^0)^T \mathbf{1}} = \hat{\theta} + \left( \frac{\mathbf{v}_j^0}{(\mathbf{v}_j^0)^T \mathbf{1}} \right)^T (\boldsymbol{\varepsilon} + \boldsymbol{\eta})$$

and

$$\hat{\mathbf{e}}_{g_i} = (\hat{e}_j)_{j \in g_i}$$

To prove the result, we apply Lemma 3 for each  $E(\theta_i | s_i, \mathbf{e}_{g_i})$ . In particular, for each  $i$ , we construct a vector  $\boldsymbol{\varepsilon}'_{g_i}$  with the first element  $(\varepsilon_i + \eta_i)$  and the  $j$ -th element equal to  $\frac{v_{jj}^0}{(\mathbf{v}_j^0)^T \mathbf{1}} (\varepsilon_j + \eta_j)$  with  $j \in g_i$ , and a vector  $\boldsymbol{\varepsilon}''_{g_i}$  with the first element 0 and the  $j$ -th element equal to  $\left( \frac{\mathbf{v}_j^0}{(\mathbf{v}_j^0)^T \mathbf{1}} \right)^T (\boldsymbol{\varepsilon} + \boldsymbol{\eta} - (\varepsilon_j + \eta_j) \mathbf{1}_j)$  with  $j \in g_i$  ( $\mathbf{1}_j$  is a column vector of 0 and 1 at position  $j$ ). Then, we have that

$$\begin{pmatrix} s_i \\ \hat{\mathbf{e}}_{g_i} \end{pmatrix} = \hat{\theta} \mathbf{1}^T + \boldsymbol{\varepsilon}'_{g_i} + \boldsymbol{\varepsilon}''_{g_i}$$

Below, we show that the conditions in Lemma 3 apply.

First, by construction,  $\boldsymbol{\varepsilon}'_{g_i}$  has a diagonal variance-covariance matrix. Next, we also



show that  $\mathcal{V} \begin{pmatrix} s_i \\ \hat{\boldsymbol{\varepsilon}}_{g_i} \end{pmatrix} \geq \mathcal{V} (\hat{\boldsymbol{\theta}} \mathbf{1}^T + \boldsymbol{\varepsilon}'_{g_i})$  element by element. Indeed

$$\begin{aligned} \mathcal{V}(\hat{\varepsilon}_j) &= \sigma_{\hat{\theta}}^2 + \left( \frac{v_{jj}^0}{(\mathbf{v}_j^0)^T \mathbf{1}} \right)^2 (\sigma_{\varepsilon}^2 + \sigma_{\eta}^2) + \left( \frac{\mathbf{v}_j^0}{(\mathbf{v}_j^0)^T \mathbf{1}} \right)^T V(\boldsymbol{\varepsilon} + \boldsymbol{\eta} - (\varepsilon_j + \eta_j) \mathbf{1}_j) \left( \frac{\mathbf{v}_j^0}{(\mathbf{v}_j^0)^T \mathbf{1}} \right)^T \\ &\quad + \frac{v_{jj}^0}{(\mathbf{v}_j^0)^T \mathbf{1}} \left( \frac{\mathbf{v}_j^0}{(\mathbf{v}_j^0)^T \mathbf{1}} \right)^T \mathcal{V}((\varepsilon_j + \eta_j), (\boldsymbol{\varepsilon} + \boldsymbol{\eta} - (\varepsilon_j + \eta_j) \mathbf{1}_j)) \end{aligned}$$

and

$$\begin{aligned} \mathcal{V}(\hat{\varepsilon}_j, \hat{\varepsilon}_k) &= \sigma_{\hat{\theta}}^2 + \frac{v_{kk}^0}{(\mathbf{v}_k^0)^T \mathbf{1}} \left( \frac{\mathbf{v}_j^0}{(\mathbf{v}_j^0)^T \mathbf{1}} \right)^T \mathcal{V}((\varepsilon_k + \eta_k), (\boldsymbol{\varepsilon} + \boldsymbol{\eta} - (\varepsilon_j + \eta_j) \mathbf{1}_j)) \\ &\quad + \frac{v_{jj}^0}{(\mathbf{v}_j^0)^T \mathbf{1}} \left( \frac{\mathbf{v}_k^0}{(\mathbf{v}_k^0)^T \mathbf{1}} \right)^T \mathcal{V}((\varepsilon_j + \eta_j), (\boldsymbol{\varepsilon} + \boldsymbol{\eta} - (\varepsilon_k + \eta_k) \mathbf{1}_j)) \\ &\quad + \left( \frac{\mathbf{v}_j^0}{(\mathbf{v}_j^0)^T \mathbf{1}} \right)^T \mathcal{V}((\boldsymbol{\varepsilon} + \boldsymbol{\eta} - (\varepsilon_j + \eta_j) \mathbf{1}_j), (\boldsymbol{\varepsilon} + \boldsymbol{\eta} - (\varepsilon_k + \eta_k) \mathbf{1}_j)) \left( \frac{\mathbf{v}_k^0}{(\mathbf{v}_k^0)^T \mathbf{1}} \right), \end{aligned}$$

which implies that

$$\mathcal{V}(\hat{\varepsilon}_j) > \sigma_{\hat{\theta}}^2 + \left( \frac{v_{jj}^0}{(\mathbf{v}_j^0)^T \mathbf{1}} \right)^2 (\sigma_{\varepsilon}^2 + \sigma_{\eta}^2) \quad (17)$$

and

$$\mathcal{V}(\hat{\varepsilon}_j, \hat{\varepsilon}_k) > \sigma_{\hat{\theta}}^2. \quad (18)$$

This is because

$$\mathcal{V}((\varepsilon_j + \eta_j), (\boldsymbol{\varepsilon} + \boldsymbol{\eta} - (\varepsilon_j + \eta_j) \mathbf{1}_j)) = 0$$

and

$$\mathcal{V}(\eta_i, \eta_j) = 0 \text{ and } \mathcal{V}(\varepsilon_i, \varepsilon_j) = 0 \quad \forall i, j.$$

Moreover,

$$\mathcal{V}(s_i, \hat{e}_j) = \sigma_{\hat{\theta}}^2 + \frac{v_{ji}^0}{\left(\mathbf{v}_j^0\right)^T \mathbf{1}} (\sigma_{\varepsilon}^2 + \sigma_{\eta}^2) > \sigma_{\hat{\theta}}^2. \quad (19)$$

From (17), (18), and (19), it follows that  $\mathcal{V}\left(\begin{smallmatrix} s_i \\ \hat{\mathbf{e}}_{g_i} \end{smallmatrix}\right) \geq \mathcal{V}\left(\hat{\theta}\mathbf{1}^T + \varepsilon'_{g_i}\right)$ . Then, for each  $i$  there exists a vector  $\boldsymbol{\omega}_{g_i} = (\omega_{ij})_{j \in \{i \cup g_i\}}^T$  with the properties that  $\boldsymbol{\omega}'_{g_i} \mathbf{1} < \mathbf{1}$  and  $\boldsymbol{\omega}_{g_i} \in (0, 1)^{m_i+1}$ , such that

$$E\left(\hat{\theta} | s_i, \hat{\mathbf{e}}_{g_i}\right) = \boldsymbol{\omega}_{g_i}^T \begin{pmatrix} s_i \\ \hat{\mathbf{e}}_{g_i} \end{pmatrix}.$$

It is immediate that

$$E(\theta_i | s_i, \mathbf{e}_{g_i}) = E(\theta_i | s_i, \hat{\mathbf{e}}_{g_i}) = E\left(\hat{\theta} | s_i, \hat{\mathbf{e}}_{g_i}\right) + E(\eta_i | s_i)$$

where

$$E(\eta_i | s_i) = \frac{\sigma_{\eta}^2}{\sigma_{\hat{\theta}}^2 + \sigma_{\eta}^2 + \sigma_{\varepsilon}^2} s_i.$$

Then, from Lemma 2,

$$v_{ii} = \omega_{ii} + \sum_{k \in g_i} \omega_{ik} \frac{v_{ki}^0}{\left(\mathbf{v}_k^0\right)^T \mathbf{1}} + \frac{\sigma_{\eta}^2}{\sigma_{\hat{\theta}}^2 + \sigma_{\eta}^2 + \sigma_{\varepsilon}^2} = \frac{\partial E(\theta_i | s_i, \mathbf{e}_{g_i})}{\partial s_i} < \frac{\partial E(\theta_i | s_i)}{\partial s_i} < 1 \quad (20)$$

and

$$v_{ij} = \sum_{k \in g_i} \omega_{ik} \frac{v_{kj}^0}{\left(\mathbf{v}_k^0\right)^T \mathbf{1}} < \sum_{k \in g_i} \omega_{ik} < 1, \forall j \in g_i. \quad (21)$$

Thus far, we have used only that  $v_{ij}^0 \geq 0$  (and not that  $v_{ij}^0 \in [0, 1]$ ). This implies that if  $v_{ij}^0 \geq 0$ , then  $v_{ij} \in [0, 1]$ . Then it must be true also that  $\forall v_{ij}^0 \in [0, 1]$ , then  $v_{ij} \in [0, 1]$ . This concludes the proof. ■

**Proof of Proposition 1.**

An equilibrium exists if there exists a matrix  $V$  such that

$$V\mathbf{s} = (\bar{Y} + \bar{Z}V)\mathbf{s},$$

and there exist matrices  $\bar{Y}$  and  $\bar{Z}$  such that

$$F(V)\mathbf{s} = (\bar{Y} + \bar{Z}V)\mathbf{s},$$

where  $F(\cdot)$  is the mapping introduced in Lemma 4. The first condition insures that

$$\mathbf{e} = V\mathbf{s}$$

is a fixed point in (6), and the second condition insures that first order conditions (7) are satisfied.

We construct an equilibrium for  $\rho < 1$  and for  $\rho = 1$  as follows.

**Case 1:**  $\rho < 1$

By Brower's fixed point theorem, the mapping  $F(\cdot)$  admits a fixed point on  $[0, 1]^{n \times n}$ .

Let  $V^* \in [0, 1]^{n \times n}$  be a matrix such that

$$F(V^*) = V^*.$$

Let  $\bar{Y}$  be a diagonal matrix with elements

$$\bar{y}_i = \omega_{ii} + \frac{\sigma_\eta^2}{\sigma_\theta^2 + \sigma_\eta^2 + \sigma_\varepsilon^2}$$

and let  $\bar{Z}$  have elements

$$\bar{z}_{ij} = \begin{cases} \frac{\omega_{ij}}{(\mathbf{v}_j^*)^T \mathbf{1}}, & \text{if } ij \in g \\ 0, & \text{otherwise} \end{cases}$$

where  $\omega_{ii}$  and  $\omega_{ij}$  have been introduced the proof above. Both matrices  $\bar{Y} \geq 0$  and  $\bar{Z} \geq 0$ .

Substituting  $V^*$  in (20) and (21), it follows that

$$V^* = \bar{Y} + \bar{Z}V^*,$$

and since  $F(V^*) = V^*$ , then

$$F(V^*) = \bar{Y} + \bar{Z}V^*.$$

Next we show properties 1-3.

1. From (20) it also follows that

$$\bar{y}_i < \frac{\partial E(\theta_i | s_i, \mathbf{e}_{g_i})}{\partial s_i} < 1.$$

Moreover, as  $\rho < 1$ , then  $\sigma_\eta^2 > 0$ , which implies that  $\bar{y}_i > 0$ . It follows that  $\bar{Y}$  is invertible.

2. We first show that matrix  $V^*$  is nonsingular. For this we construct a matrix  $W^* = \bar{Y}^{-1}(I - \bar{Z})$  and show that  $W^*V^* = I$ . Indeed, the element on the position  $(i, i)$  on the diagonal of  $W^*V^*$  is equal to

$$\begin{aligned} \frac{1}{\bar{y}_i} \left( v_{ii}^* - \sum_{k \in g_i} \bar{z}_{ik} v_{ki}^* \right) &= \frac{1}{\bar{y}_i} \left( \omega_{ii} + \sum_{k \in g_i} \omega_{ik} \frac{v_{ki}^*}{(\mathbf{v}_k^*)^T \mathbf{1}} + \frac{\sigma_\eta^2}{\sigma_\theta^2 + \sigma_\eta^2 + \sigma_\varepsilon^2} - \sum_{k \in g_i} \frac{\omega_{ik}}{(\mathbf{v}_k^*)^T \mathbf{1}} v_{ki}^* \right) \\ &= \frac{1}{\bar{y}_i} \left( \omega_{ii} + \frac{\sigma_\eta^2}{\sigma_\theta^2 + \sigma_\eta^2 + \sigma_\varepsilon^2} \right) \\ &= 1 \end{aligned}$$

while the element on the position  $(i, j)$  off the diagonal of  $W^*V^*$  is equal to

$$\frac{1}{\bar{y}_i} \left( v_{ij}^* - \sum_{k \in g_i} \bar{z}_{ik} v_{kj}^* \right) = \frac{1}{\bar{y}_i} \left( \sum_{k \in g_i} \omega_{ik} \frac{v_{kj}^*}{(\mathbf{v}_k^*)^T \mathbf{1}} - \sum_{k \in g_i} \frac{\omega_{ik}}{(\mathbf{v}_k^*)^T \mathbf{1}} v_{kj}^* \right) = 0$$

where we used again the fact that  $V^*$  is a fixed point in (20) and (21).

Since  $V^*$  is nonsingular, then  $(I - \bar{Z})$  is also nonsingular as

$$(I - \bar{Z}) = \bar{Y} (V^*)^{-1}.$$

Given that  $\bar{Z} \geq 0$  this implies, as shown in Meyer (2000), that the largest eigenvalue of  $\bar{Z}$  is strictly smaller than 1. This is a useful result, as it is sufficient to show that

$$\lim_{n \rightarrow \infty} \bar{Z}^n = 0_{n \times n}$$

and that

$$(I - \bar{Z})^{-1} = \sum_{n=1}^{\infty} \bar{Z}^n.$$

(For both claims see Meyer (2000) pp. 620 & 618.)

3. The equilibrium outcome guess vector is, by construction

$$\mathbf{e} = V^* \mathbf{s}$$

which implies that

$$\mathbf{e} = (I - \bar{Z})^{-1} \bar{Y}.$$

**Case 2:**  $\rho = 1$

Let  $\bar{Y} = 0$  and  $\bar{Z}$  have elements  $\bar{z}_{ij}$  if  $ij \in g$ , and 0 otherwise, with  $\sum_{j \in g_i} \bar{z}_{ij} = 1$ . Let

$V^*$  be a matrix with  $v_{ij}^* = \frac{\sigma_{\theta}^2}{n\sigma_{\theta}^2 + \sigma_{\varepsilon}^2}$  for any  $i$  and  $j$ .

It is straightforward to see that

$$V^* = \bar{Z} V^*.$$

Next we show that

$$F(V^*) = V^*.$$

Next, we show that the matrix  $V^*$  with  $v_{ij}^* = \frac{\sigma_{\hat{\theta}}^2}{n\sigma_{\hat{\theta}}^2 + \sigma_{\varepsilon}^2}$  for any  $i$  and  $j$  satisfies

$$F(V^*) = V^*.$$

By definition, matrix  $V^*$  with  $v_{ij}^* = \frac{\sigma_{\hat{\theta}}^2}{n\sigma_{\hat{\theta}}^2 + \sigma_{\varepsilon}^2}$  is a fixed point of the mapping  $F(\cdot)$ , if

$$e_i = v^* \sum_{j=1}^n s_j, \quad \forall i \in \{1, 2, \dots, n\}$$

then

$$E(\theta_i | s_i, \mathbf{e}_{g_i}) = v^* \sum_{j=1}^n s_j, \quad \forall i \in \{1, 2, \dots, n\}.$$

As  $\rho = 1$ , then  $\theta_i = \hat{\theta}$  for any  $i$  and

$$\begin{aligned} E(\theta_i | s_i, \mathbf{e}_{g_i}) &= E\left(\hat{\theta} \middle| v^* \sum_{i=1}^n s_i\right) \\ &= \frac{1}{v^*} \frac{\sigma_{\hat{\theta}}^2}{n\sigma_{\hat{\theta}}^2 + \sigma_{\varepsilon}^2} v^* \sum_{i=1}^n s_i \\ &= v^* \sum_{i=1}^n s_i. \end{aligned}$$

It follows that

$$F(V^*) = \bar{Z}V^*.$$

This concludes the proof.

■

**Proof of Lemma1.** Taking into account that agents' beliefs have an affine structure as in (11) and identifying coefficients in (10) we obtain that

$$\begin{aligned} b_i^j &= -\left(c_{ji}^i + \beta_{ij}\right) y_i \\ c_{ij}^j &= -\left(c_{ji}^i + \beta_{ij}\right) (z_{ij} - 1) \\ c_{ik}^j &= -\left(c_{ji}^i + \beta_{ij}\right) z_{ik} \end{aligned}$$

for any  $i$  and  $j \in g_i$ . Therefore, for any pair  $ij$  that has a link in the network  $g$ , the following two equations must hold at the same time

$$\begin{aligned} c_{ij}^j &= -(c_{ji}^i + \beta_{ij})(z_{ij} - 1) \\ c_{ji}^i &= -(c_{ij}^j + \beta_{ij})(z_{ji} - 1). \end{aligned}$$

which implies that

$$\begin{aligned} c_{ij}^j &= \frac{(z_{ij} - 1)(z_{ji} - 2)}{z_{ij} + z_{ji} - z_{ij}z_{ji}}\beta_{ij} \\ c_{ji}^i &= \frac{(z_{ji} - 1)(z_{ij} - 2)}{z_{ij} + z_{ji} - z_{ij}z_{ji}}\beta_{ij}. \end{aligned}$$

A simple manipulations shows that

$$c_{ji}^i + \beta_{ij} = \frac{2 - z_{ji}}{z_{ij} + z_{ji} - z_{ij}z_{ji}}\beta_{ij}$$

and

$$\frac{(c_{ji}^i + \beta_{ij})}{c_{ji}^i + c_{ij}^j + 3\beta_{ij}} = \frac{2 - z_{ji}}{4 - z_{ij}z_{ji}}.$$

The latter relationship we have used in the proof of Proposition 2. ■

**Proof of Proposition 2.** We will show that given an equilibrium of the conditional-guessing game and the conditions of the proposition, we can always construct an equilibrium for the OTC game where beliefs are the same, in particular,

$$E(\theta_i | s_i, \mathbf{p}_{g_i}) = E(\theta_i | s_i, \mathbf{e}_{g_i}).$$

To see this, suppose that

$$\mathbf{e}_i = E(\theta_i | s_i, \mathbf{e}_{g_i}) = \mathbf{v}_i \mathbf{s}$$

for each  $i$  is the linear combination of signals which gives the equilibrium guess in the

conditional-guessing game. Then

$$E(\theta_i | s_i, \mathbf{e}_{g_i}) = \bar{y}_i s_i + \sum_{k \in g_i} \bar{z}_{ik} E(\theta_k | s_k, \mathbf{e}_{g_k})$$

for every  $i$ . If the system (15) has a solution, then

$$E(\theta_i | s_i, \mathbf{e}_{g_i}) = \frac{y_i}{\left(1 - \sum_{k \in g_i} z_{ik} \frac{2-z_{ki}}{4-z_{ik}z_{ki}}\right)} s_i + \sum_{k \in g_i} z_{ij} \frac{\frac{2-z_{ij}}{4-z_{ij}z_{ji}}}{\left(1 - \sum_{k \in g_i} z_{ik} \frac{2-z_{ki}}{4-z_{ik}z_{ki}}\right)} E(\theta_k | s_k, \mathbf{e}_{g_k}) \quad (22)$$

holds for every realization of the signals, and for each  $i$ . Using that from Lemma 1

$$\frac{2 - z_{ki}}{4 - z_{ik}z_{ki}} = \frac{(c_{ki}^i + \beta_{ik})}{c_{ki}^i + c_{ik}^k + 3\beta_{ik}},$$

we can rewrite (22) as

$$E(\theta_i | s_i, \mathbf{e}_{g_i}) = y_i s_i + \sum_{k \in g_i} z_{ik} \frac{(c_{ki}^i + \beta_{ik}) E(\theta_i | s_i, \mathbf{e}_{g_i}) + (c_{ik}^k + \beta_{ik}) E(\theta_k | s_k, \mathbf{e}_{g_k})}{c_{ki}^i + c_{ik}^k + 3\beta_{ik}}.$$

Now we show that picking the prices and demand functions

$$p_{ij} = \frac{(c_{ki}^i + \beta_{ik}) E(\theta_i | s_i, \mathbf{e}_{g_i}) + (c_{ik}^k + \beta_{ik}) E(\theta_k | s_k, \mathbf{e}_{g_k})}{c_{ki}^i + c_{ik}^k + 3\beta_{ik}} \quad (23)$$

$$Q_i^j(s_i; \mathbf{p}_{g_i}) = -(c_{ji}^i + \beta_{ij}) (E(\theta_i | s_i, \mathbf{e}_{g_i}) - p_{ij}) \quad (24)$$

is an equilibrium of the OTC game.

First note that this choice implies

$$E(\theta_i | s_i, \mathbf{e}_{g_i}) = y_i s_i + \sum_{k \in g_i} z_{ik} p_{ij} = E(\theta_i | s_i, \mathbf{p}_{g_i}). \quad (25)$$

The second equality comes from the fact that the first equality holds for any realization of signals and the projection theorem determines a unique linear combination with this property for a given set of jointly normally distributed variables. Thus, (24) for each  $ij$  link is equivalent with the corresponding first order condition (10). Finally, (25) also



implies that the bilateral clearing condition between a dealer  $i$  and dealer  $j$  that have a link in network  $g$

$$- (c_{ji}^i + \beta_{ij}) (E(\theta_i | s_i, \mathbf{p}_{g_i}) - p_{ij}) - (c_{ij}^j + \beta_{ij}) (E(\theta_j | s_j, \mathbf{p}_{g_j}) - p_{ij}) + \beta_{ij} p_{ij} = 0$$

is equivalent to (23). That concludes the statement. ■

### Proof of Corollary 1.

#### Case 1: Circulant networks

In circulant networks, we search for equilibria such that beliefs are symmetric, that is

$$z_{ij} = z_{ji}$$

for any pair  $ij$  that has a link in network  $g$ . The system (15) becomes

$$\begin{aligned} \frac{y_i}{\left(1 - \sum_{k \in g_i} z_{ik} \frac{2 - z_{ik}}{4 - z_{ik}^2}\right)} &= \bar{y}_i \\ z_{ij} \frac{\frac{2 - z_{ij}}{4 - z_{ij}^2}}{\left(1 - \sum_{k \in g_i} z_{ik} \frac{2 - z_{ik}}{4 - z_{ik}^2}\right)} &= \bar{z}_{ij} \end{aligned}$$

for any  $i \in \{1, 2, \dots, n\}$ . Working out the equation for  $z_{ij}$ , we obtain

$$\frac{z_{ij}}{2 + z_{ij}} = \bar{z}_{ij} \left(1 - \sum_{k \in g_i} \frac{z_{ik}}{2 + z_{ik}}\right)$$

and summing up for all  $j \in g_i$

$$\sum_{j \in g_i} \frac{z_{ij}}{2 + z_{ij}} = \sum_{j \in g_i} \bar{z}_{ij} \left(1 - \sum_{k \in g_i} \frac{z_{ik}}{2 + z_{ik}}\right).$$

Denote

$$S_i \equiv \sum_{k \in g_i} \frac{z_{ik}}{2 + z_{ik}}.$$

Substituting above and summing again for  $j \in g_i$

$$S_i \left( 1 + \sum_{j \in g_i} \bar{z}_{ij} \right) = \sum_{j \in g_i} \bar{z}_{ij}$$

or

$$S_i = \frac{\sum_{j \in g_i} \bar{z}_{ij}}{\left( 1 + \sum_{j \in g_i} \bar{z}_{ij} \right)}.$$

We can now obtain

$$z_{ij} = \frac{2\bar{z}_{ij}(1 - S_i)}{1 - \bar{z}_{ij}(1 - S_i)}$$

and

$$y_i = \bar{y}_i(1 - S_i).$$

## Case 2: Core-periphery networks

There exist at least one equilibrium of the conditional guessing game such that for all  $i, j$  in the core

$$\bar{z}_{ij} = \bar{z}_{ji} = \bar{z}_c \tag{26}$$

and for all player  $i$  in the periphery

$$\begin{aligned} \bar{z}_{ij} &= \bar{z}_p \\ \bar{z}_{ji} &= \bar{z}_{cp} \end{aligned} \tag{27}$$

Then the system (15) becomes

$$\frac{y_c}{1 - (r-1)z_c \frac{2-z_c}{4-z_c^2} - \frac{n-r}{r} z_{cp} \frac{2-z_p}{4-z_{cp}z_p}} = \bar{y}_c \quad (28)$$

$$z_c \frac{\frac{2-z_c}{4-z_c^2}}{1 - (r-1)z_c \frac{2-z_c}{4-z_c^2} - \frac{n-r}{r} z_{cp} \frac{2-z_p}{4-z_{cp}z_p}} = \bar{z}_c \quad (29)$$

$$z_{cp} \frac{\frac{2-z_{cp}}{4-z_{cp}z_{cp}}}{1 - (r-1)z_c \frac{2-z_c}{4-z_c^2} - \frac{n-r}{r} z_{cp} \frac{2-z_p}{4-z_{cp}z_p}} = \bar{z}_{cp} \quad (30)$$

for agents in the core and

$$\frac{y_p}{1 - z_p \frac{2-z_{cp}}{4-z_pz_{cp}}} = \bar{y}_p \quad (31)$$

$$z_p \frac{\frac{2-z_p}{4-z_pz_{cp}}}{1 - z_p \frac{2-z_{cp}}{4-z_pz_{cp}}} = \bar{z}_p \quad (32)$$

for agents in the periphery.

From equation (32) it is easy to see that

$$z_p = 2\bar{z}_p \quad (33)$$

Substituting this back to the equations (29) and (30) we get

$$z_c \frac{2-z_c}{4-z_c^2} = \bar{z}_c \left( 1 - (r-1)z_c \frac{2-z_c}{4-z_c^2} - \frac{n-r}{r} z_{cp} \frac{2-2\bar{z}_p}{4-2z_{cp}\bar{z}_p} \right) \quad (34)$$

$$z_{cp} \frac{2-z_{cp}}{4-2z_{cp}\bar{z}_p} = \bar{z}_{cp} \left( 1 - (r-1)z_c \frac{2-z_c}{4-z_c^2} - \frac{n-r}{r} z_{cp} \frac{2-2\bar{z}_p}{4-2z_{cp}\bar{z}_p} \right) \quad (35)$$

From (34)

$$z_c \frac{2-z_c}{4-z_c^2} = \frac{\bar{z}_c(4-2z_{cp}\bar{z}_p) - \frac{n-r}{r} z_{cp}\bar{z}_c(2-2\bar{z}_p)}{(1+(r-1)\bar{z}_c)(4-2z_{cp}\bar{z}_p)} \quad (36)$$

Substituting this back to (35) one gets

$$(1+(r-1)\bar{z}_c)z_{cp}^2 + \left( \bar{z}_{cp}(2\bar{z}_p-2)\frac{n-r}{r} - 2-2\bar{z}_p\bar{z}_{cp} - 2(r-1)\bar{z}_c \right) z_{cp} + 4\bar{z}_{cp} = 0 \quad (37)$$

what has got real solution if the discriminant is non-negative for any  $\bar{z}_p, \bar{z}_c, \bar{z}_{cp} \in [0, 1]$ , so

when

$$\left( \bar{z}_{cp}(2\bar{z}_p - 2)\frac{n-r}{r} - 2 - 2\bar{z}_p\bar{z}_{cp} - 2(r-1)\bar{z}_c \right)^2 - 16\bar{z}_{cp} - 16(r-1)\bar{z}_p\bar{z}_{cp} \geq 0 \quad (38)$$

Note that for  $r = 1$  this is the discriminant of the  $n$ -star case. This discriminant is decreasing in  $\bar{z}_p$  if  $\bar{z}_p \in [0, 1]$  and non-negative for  $\bar{z}_p = 1$  therefore the equation has real solution.

Then substituting the solution  $z_{cp}$  back to (36) is linear in  $z_c$  and after the solutions for (28) and (31) follow immediately.

■

**Proof of Proposition 4.** Suppose network  $g$  is connected. This implies that between any two agents  $i$  and  $j$ , there exists a sequence of dealers  $\{i_1, i_2, \dots, i_r\}$  such that  $ii_1 \in g$ ,  $i_k i_{k+1} \in g$ , and  $i_r j \in g$  for any  $k \in \{1, 2, \dots, r\}$ . The sequence  $\{i_1, i_2, \dots, i_r\}$  forms a *path* between  $i$  and  $j$ . The length of this path,  $r$ , represents the distance between  $i$  and  $j$ .

Suppose that an equilibrium exists. From Proposition 1 we know that in equilibrium

$$\mathbf{e} = (I - \bar{Z})^{-1} \bar{Y} \mathbf{s}.$$

and from Proposition 1 we know that

$$(I - \bar{Z})^{-1} \bar{Y} = V^*.$$

Suppose that there exists an equilibrium

$$v_{ij}^* = 0$$

for some  $i$  and  $j$  at distance  $r$  from each other. Then from (21) it follows that

$$\sum_{k \in g_i} \omega_{ik} \frac{v_{kj}^*}{(\mathbf{v}_k^*)^T \mathbf{1}} = 0,$$

and, since  $\omega_{ik} > 0$  for  $\forall i, k \in \{1, 2, \dots, n\}$ , then it must be that

$$v_{kj}^* = 0, \quad \forall k \in g_i.$$

This means that all the neighbors of agent  $i$  place 0 weight on  $j$ 's information. Further, this implies

$$\sum_{l \in g_k} \omega_{il} \frac{v_{lj}^*}{(\mathbf{v}_l^*)^T \mathbf{1}} = 0,$$

and

$$v_{lj}^* = 0, \quad \forall l \in g_k.$$

Hence, all the neighbors and the neighbors of the neighbors of agent  $i$  place 0 weight on  $j$ 's information. We can iterate the argument for  $r$  steps, and show that it must be that any agent at distance at most  $r$  from  $i$  places 0 weight on  $j$ 's information. Since the distance between  $i$  and  $j$  is  $r$ , then

$$v_{jj}^* = 0,$$

which is a contradiction, since (20) must hold and  $\rho < 1$  ( $\sigma_\eta^2 > 0$ ). This concludes the proof. ■

### Proof of Proposition 5.

As  $\rho \rightarrow 1$ , we show that there exists an equilibrium such that

$$\lim_{\rho \rightarrow 1} E(\theta_i | s_i, \mathbf{p}_{g_i}) = v^* \sum_{i=1}^n s_i, \quad \forall i \in \{1, 2, \dots, n\}$$

where  $v^* = \frac{\sigma_\theta^2}{n\sigma_\theta^2 + \sigma_\varepsilon^2}$ .

If there exists an equilibrium in the OTC game, then it follows from the proof of Proposition 1 that

$$E(\theta_i | s_i, \mathbf{p}_{g_i}) = \bar{y}_i s_i + \sum_{k \in g_i} \bar{z}_{ik} E(\theta_k | s_k, \mathbf{p}_{g_k}).$$

or

$$E(\theta_i | s_i, \mathbf{e}_{g_i}) = \bar{y}_i s_i + \sum_{k \in g_i} \bar{z}_{ik} E(\theta_k | s_k, \mathbf{e}_{g_k}).$$

Taking the limit as  $\rho \rightarrow 1$ , and using Lemma 5, we have that

$$\lim_{\rho \rightarrow 1} E(\theta_i | s_i, \mathbf{p}_{g_i}) = \frac{\sigma_\theta^2}{n\sigma_\theta^2 + \sigma_\varepsilon^2} \sum_{i=1}^n s_i.$$

Given that

$$\lim_{\rho \rightarrow 1} E(\theta_i | s_i, \mathbf{p}_{g_i})$$

The conditional variance is

$$\mathcal{V}(\theta_i | s_i, \mathbf{p}_{g_i}) = \sigma_\theta^2 - \mathcal{V}(E(\theta_i | s_i, \mathbf{p}_{g_i}))$$

and taking the limit  $\rho \rightarrow 1$ , we obtain

$$\lim_{\rho \rightarrow 1} \mathcal{V}(\theta_i | s_i, \mathbf{p}_{g_i}) = \sigma_\theta^2 - \left( \frac{\sigma_\theta^2}{n\sigma_\theta^2 + \sigma_\varepsilon^2} \right)^2 n(\sigma_\varepsilon^2 + n\sigma_\theta^2).$$

and

$$\begin{aligned} \lim_{\rho \rightarrow 1} \mathcal{V}(\theta_i | \mathbf{s}) &= \sigma_\theta^2 - \mathcal{V}(E(\hat{\theta} | \mathbf{s})) \\ &= \sigma_\theta^2 - \left( \frac{\sigma_\theta^2}{n\sigma_\theta^2 + \sigma_\varepsilon^2} \right)^2 n(\sigma_\varepsilon^2 + n\sigma_\theta^2) \\ &= \sigma_\theta^2 \frac{\sigma_\varepsilon^2}{n\sigma_\theta^2 + \sigma_\varepsilon^2} \end{aligned}$$

■

### Proof of Proposition 6.

Dealers revise their messages according to the rule that

$$h_{i,t} = \bar{y}_i s_i + \bar{\mathbf{z}}_{g_i}^T \mathbf{h}_{g_i,t-1}, \quad \forall i.$$

or, in matrix form

$$\mathbf{h}_{t+1} = \bar{\mathbf{Y}} \mathbf{s} + \bar{\mathbf{Z}} \mathbf{h}_t.$$

1. Since  $\mathbf{h}_{t_0} = (I - \bar{Z})^{-1} \bar{Y} \mathbf{s}$ , then

$$\begin{aligned}
\mathbf{h}_{t_0+1} &= \bar{Y} \mathbf{s} + \bar{Z} \mathbf{h}_{t_0} \\
&= \bar{Y} \mathbf{s} + \bar{Z} (I - \bar{Z})^{-1} \bar{Y} \mathbf{s} \\
&= \bar{Y} \mathbf{s} + (I - (I - \bar{Z})) (I - \bar{Z})^{-1} \bar{Y} \mathbf{s} \\
&= \bar{Y} \mathbf{s} + (I - \bar{Z})^{-1} \bar{Y} \mathbf{s} - \bar{Y} \mathbf{s} \\
&= (I - \bar{Z})^{-1} \bar{Y} \mathbf{s}
\end{aligned}$$

It follows straightforwardly, from an inductively argument that

$$\mathbf{h}_t = (I - \bar{Z})^{-1} \bar{Y} \mathbf{s}.$$

2. From

$$\mathbf{h}_{t_0+1} = \bar{Y} \mathbf{s} + \bar{Z} \mathbf{h}_{t_0}$$

it follows that

$$\mathbf{h}_{t_0+n} = (I + \bar{Z} + \dots + \bar{Z}^{n-1}) \bar{Y} \mathbf{s} + \bar{Z}^n \mathbf{h}_{t_0}$$

In the limit as  $n \rightarrow \infty$ , from Proposition 1 we know that

$$\lim_{n \rightarrow \infty} \mathbf{h}_{t_0+n} = (I - \bar{Z})^{-1} \bar{Y} \mathbf{s}.$$

This implies that for any vector  $\gamma \in R_+^n$ , there exists an  $n_\gamma$  such that

$$\left| \mathbf{h}_{t_0+n} - (I - \bar{Z})^{-1} \bar{Y} \mathbf{s} \right| < \gamma, \forall n \geq n_\gamma.$$

Fix an arbitrarily small vector  $\gamma$ . Then

$$-\gamma < (I - \bar{Z})^{-1} \bar{Y} \mathbf{s} - \mathbf{h}_{t_0+n_\gamma} < \gamma$$

and

$$-\gamma < \mathbf{h}_{t_0+n} - (I - \bar{Z})^{-1} \bar{Y} \mathbf{s} < \gamma, \forall n \geq n_\gamma.$$

Adding up these two inequalities we have that

$$-2\gamma < \mathbf{h}_{t_0+n} - \mathbf{h}_{t_0+n_\gamma} < 2\gamma, \forall n \geq n_\gamma.$$

This shows that there exists  $\delta = 2\gamma$  and  $t_\delta = t_0 + n_\gamma$  such that

$$|\mathbf{h}_t - \mathbf{h}_{t_\delta}| < \delta, \forall t \geq t_\delta.$$

which implies that the protocol stops at  $t_\delta$ .

3. We start by observing that

$$E(\theta_i | s_i, \mathbf{h}_{g_i, t_0}, \mathbf{h}_{g_i, t_0+1}, \dots, \mathbf{h}_{g_i, t_0+n}) = E(\theta_i | s_i, \mathbf{h}_{g_i, t_0+n}), \forall n \geq 0.$$

Further, in the limit  $n \rightarrow \infty$ , we have that

$$\lim_{n \rightarrow \infty} \mathbf{h}_{t_0+n} = (I - \bar{Z})^{-1} \bar{Y} \mathbf{s} = \mathbf{e},$$

and subsequently

$$\lim_{n \rightarrow \infty} \mathbf{h}_{g_i, t_0+n} = \mathbf{e}_{g_i}, \forall i.$$

Then

$$\lim_{n \rightarrow \infty} E(\theta_i | s_i, \mathbf{h}_{g_i, t_0+n}) = E(\theta_i | s_i, \mathbf{e}_{g_i}) = E(\theta_i | s_i, \mathbf{p}_{g_i}).$$

As above, we can construct  $t_\delta$  such that protocol stops and show that

$$|E(\theta_i | s_i, \mathbf{h}_{g_i, t_0+n}) - E(\theta_i | s_i, \mathbf{p}_{g_i})| < \frac{1}{2}\delta.$$

■