Bank Capital Requirements for Tail Risk: The Internal Agency Problem Matters!

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Abstract

The top executives of banks made an important "contribution" to the 2007-09 financial crisis. Given short-term performance incentives, they were engaged in excessive risk-taking in order to receive immediate gains, without regard for the long-term consequences of management practices adopted. In order to curb the risk-taking propensity of bank executives, many scholars and policymakers now call for mandatory regulation of executive pay. However, it is still unclear how the optimal incentive compensation structure should look. In this paper we show that there is no need for the direct regulation of executive pay. In fact, the excessive risk-taking can be prevented implicitly, through the *incentive mandatory recapitalization rule*, which takes care of the internal agency problem between bank shareholders and bank managers.

Keywords: capital requirements, tail risk, recapitalization, incentive compensation, moral hazard

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"If banks realized that they would be forced to replace lost capital in a timely fashion, then they would have greater incentive to manage risk properly..."
Calomiris and Herring (2011)

1 Introduction

The excessive risk-taking of top bank executives has been pointed out as one of the key ingredients of the recent financial crisis (see, for instance, *Stiglitz (2010)*, *Achariya et al.(2009))*. There is now a convincing empirical support for the fact that equity-based compensations have made bank executives interested in pushing up bank equity value at any price in order to reap gains *(Chen et al. (2006), Williams et al. (2008), Vallascas and Hagendorff (2010))*.[1] At the same time, generous severance pay (so-called "golden handshakes") were working like insurance against a reverse fortune. Taken together, an opportunity to extract gains from the financial market, supported by a safe "landing" in the case of bad luck, have made excessive risk-taking very appealing. Nevertheless, while the risk profile of a bank depends on the executives' strategy, shareholders are supposed to be able to shape executive incentives through incentive compensation arrangements. Thus, observing existing compensation practices alongside a high tolerance of bank shareholders towards excessive risk-taking behaviors of their executives, one may conclude that the existing regulatory framework has failed to provide bank shareholders with sustainable incentives for prudent risk management.

The recognition of this fact brought to life two distinct initiatives: (i) to impose higher capital requirements on banks *(Bhagat and Bolton (2011), Fama (2010), Admati at al. (2011)*[2]) and (ii) to introduce a mandatory regulation of executive pay *(Bebchuk and Spamann (2010), Bolton et al. (2010))*. While the debates over the raise of capital requirements are mostly fed by the fear to harm private interests of bankers, a mandatory regulation of executive pay invokes a wider range of concerns. The first problem is that regulators do not dispose all necessary information and thus has a limited ability for the efficient design and enforcement of executive compensation. Second, it is still unclear how the optimal incentive compensation structure

[1] For instance, *Bebchuk at al. (2010)* evaluate that the top executives of Bear Stearns and Lehman Brothers realized about $2 bln, by unloading shares and options during 2000-2008.

[2] *Admati et al. (2011)* argue that, actually, equity capital is not socially expensive. They provide a detailed discussion of related issues, showing that most objections against stronger capital standards are based on the confusion between capital and liquidity requirements and other erroneous views.

should look. Finally, practical evidence suggests that economic agents always find a way to get around regulation if their incentives diverge from regulatory purposes. Thus, it seems that the explicit regulation of executive pay without the regulation of shareholder incentives would be a waste of regulatory resources.

In this study we propose an *implicit* mechanism to regulate risk-taking incentives of bank executives, by designing capital requirements which will induce shareholders to shape executive compensation in a way to promote prudent risk management at their banks. Allowing for the internal agency problem between bank shareholders and managers, our design of capital requirements contrasts with a commonly used approach which treats a bank as a black box, without taking into consideration its governance issues.

Another crucial feature of capital requirements designed in this study is that they are intended to deal with *tail risk* characterized by infrequent but devastating losses. Tail risk may result from imprudent lending and investment strategies, the abuse of securitization, the excessive reliance on short-term debt funding, poor trading discipline, fraudulent accounting practices and other internal misbehaviors in banking. While the existing theoretical literature on capital regulation is mostly focused on portfolio risk related with random fluctuations of asset return (see, for instance, *Brattachariya (2002)*, *Décamps et al. (2004)*, *Koziol and Lowrenz (2012)*), it seems that tail risk should deserve more regulatory attention since it represents a far greater danger.[3] Indeed, the 2007-2009 financial crisis as a whole was a fruit of materialized tail risk accumulated within the financial system.

In line with the incentive approach to capital regulation, the purpose of capital requirements in our study is to prevent the bank from taking tail risk and thus to avoid large losses. The existing capital regulation literature (see, for instance, *Brattachariya (2002), Décamps et al. (2004), Rochet (2004))* has shown that *portfolio risk* can be prevented through the mandatory *liquidation* rule, which can be easily interpreted in terms of the minimum capital ratio.[4] However, the *liquidation* rule appears to be ill-suited for dealing with *tail risk*: even in the absence of internal agency problems, it fails to prevent the bank from taking tail risk when bank asset value approaches the liquidation point. In fact, since in the neighborhood of the liquidation threshold the bank default is more likely to be triggered by random fluctuations of asset return rather than by infrequent large losses, the bank will engage in tail risk in order to increase the asset growth rate and to move away from the

[3]A collapse of Barings Bank in 1995 and the failure of Royal Bank of Scotland (RBS) in 2008 illustrate the extreme harm that can be caused by materialized tail risk.

[4]Bank liquidation should be viewed as the expropriation of current shareholders, i.e., there is no "physical" liquidation of the bank.

liquidation point.

Taking into account a specific nature of tail risk, we propose to design capital requirements in the form of the incentive *recapitalization* rule. A recapitalization rule will exclude the possibility of bank failure because of portfolio risk. At the same time, a tail risk realization will increase the likelihood of further mandatory recapitalizations and thus will raise the expected recapitalization costs. Thus, it is possible to set a recapitalization rule in a way to induce shareholders to refrain from taking tail risk. It is worth noting that a costly recapitalization is well studied within the context of the liquidity management literature, where attention is paid to the impact of capital issuance costs on the time and the scale of recapitalizations *(Décamps et al. (2011), Rochet and Villeneuve (2011))*. The capital regulation literature considers the option to recapitalize while analysing the bank decision to maintain capital buffer *(Peura and Keppo (2003), Milne and Walley (2002))*. However, to the best of our knowledge, a potential incentive effect of mandatory recapitalizations has not been explored before.

To illustrate our proposals, we build a simple continuous-time model in the principal-agent framework, where a senior manager has a reversible choice between prudent and imprudent risk management strategies. An imprudent risk management strategy exposes the bank to tail risk, while a prudent risk management strategy implies no tail risk but generates lower expected return. In an attempt to reproduce executive behaviors preceding the financial crisis, we assume that the manager has a possibility to extract private benefits while engaging in manufacturing of tail risk. In such a context, in order to prevent the bank manager from adopting imprudent risk management strategy, the regulator should anticipate a design of the optimal incentive contract with the manager and to incorporate it into the capital requirements' design.

In line with a recent literature on dynamic moral hazard, we use a recursive technique to define the optimal contract design. Thus, the cheapest incentive compensation scheme can be inferred from the minimum incentive-compatible contract continuation value. The optimal incentive contract is linear on bank asset value and expires as soon as the bank asset value reaches a recapitalization point. A surprising result we obtain is that the manager cannot be left with empty hands: at the dismissal, he should receive a positive terminal pay-off which arises ex-post as a necessary element of the optimal incentive contract.

We exploit the model framework in order to examine the impact of bonus taxes on risk-taking incentives of banks. In the aftermath of the 2007-09 financial crisis, several European countries introduced a tax on performance bonuses of bank top management in order "to encourage banks to consider their capital position and to make appropriate risk-adjustments when settling the level of bonus payments."[5] We show that bonus taxes appear to be inappropriate for dealing with excessive risk-taking at banks, since they increase agency costs and thus reduce shareholder incentives for prudent risk management. However, choosing the lesser of two evils, it would be better to impose bonus taxes on bank shareholders (as was done in UK, for example), rather than on bank managers (as was done in France).

It is worth noting that we are not the first to point out the need to consider the internal agency problem in capital regulation design. The study of *Bris and Cantale (2004)* addresses this issue in the context of portfolio risk management, examining the impact of capital requirements on the effort choice of a self-interested bank manager in a discrete time framework. They come to the conclusion that optimal capital regulation, which does not take into account the internal agency problem, leads to a socially-unoptimal *lower* level of risk at the bank. Built in the context of tail risk, our model provides a contrary evidence. Moreover, we explicitly show how to adapt the optimal design of capital regulation for the presence of internal agency problems.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3 we design the optimal mandatory recapitalization policy when there is no internal agency problem at the bank. Section 4 presents the optimal mandatory recapitalization policy, allowing for the internal agency problem and discusses the effect of bonus taxes. Section 5 concludes. All proofs are provided in Appendix B.

2 The model

2.1 Tail risk and the double agency problem

We consider a risk-neutral bank protected by limited liability. The bank is financed by a constant volume of insured deposits, D, and incurs a continuous payment rD to depositors, where r denotes a riskless interest rate. Bank assets continuously generate a cash-flow δx_t, where x_t denotes a publicly observable asset value.

The bank faces two kinds of risks: portfolio risk, which reflects minor fluctuations of bank asset value, and tail risk, which implies the infrequent but large losses of bank assets. Large losses caused by a tail risk realization follow a Poisson process $\{N_t\}_{t\geq 0}$ with intensity λ. A large loss destroys a fraction $(1-\alpha)x_t$ of bank assets, where $\alpha \in (0,1)$ is a constant coefficient reflecting a proportion of remaining assets after the large loss realization. However, large losses can be avoided due to internal preventive efforts which

[5] The UK Pre-Budget Report on 9 December 2009.

require instantaneous costs $gx_t dt$, where $g < \lambda(1-\alpha)$. In a broad sense, preventive efforts against tail risk can be viewed as prudent risk management which might imply a promotion of trading discipline, information security, prudent investment and credit strategies.

Let $e_t \in \{1, 0\}$ be an indicator of preventive efforts against tail risk. Then, bank asset's value follows the process:[6]

$$dx_t = (\mu - e_t g)x_t dt + \sigma x_t dW_t - (1-e_t)(1-\alpha)x_t dN_t, \tag{1}$$

where μ denotes the expected return on assets, σ is the asset return volatility and $\{W_t\}_{t\geq 0}$ is a standard Brownian process.

The bank is run by a manager who is responsible for preventive efforts but has discretion over his actions. The contract with the manager should specify: (i) a contract termination rule x_T;[7] (ii) an asset-based remuneration $R(x) \geq 0$ for $x \geq x_T$.[8] The manager is protected by limited liability and has no initial wealth which could be pledged. Following *Holmström and Tirole (1997)* and *Biais et al. (2010)*, we assume that shirking allows the manager to collect private benefits $B(x) = b_0 + b_1 x$.[9] The constant part of private benefits can be interpreted as the opportunity cost of leisure, whereas the variable part might reflect the proceeds from exercising stock options and selling shares, given that adopted tail-risk management strategy can boost equity growth in a short term. Moreover, larger banks are likely to be more generous in offering stock-based compensations, so that their managers are able to extract more gains through private tradings on the financial market. Tail risk materialization, however, makes shirking verifiable *ex-post*, so that shareholders can commit to fire the manager without any terminal pay if a large loss occurs. Conforming to the Maximum Punishment Principle (see *Laffont and Martimort (2002)*), such a commitment will allow shareholders to discourage the manager from shirking at a minimum cost.

[6] Expression (1) captures a trade-off between a faster asset growth and asset safety, formalizing a so-called "search for yield" strategy of banks in the period prior to the crisis. Thus, an aggressive mortgage lending strategy may increase the asset growth rate in a short term but will inevitably lead to large losses in the long term.

[7] Since we are dealing with a stationary problem, a termination rule x_T will determine the expected contract duration.

[8] We could consider a non-zero reservation wage for the manager in order to care about the existing competition for top managers in banking sector. However, the focus of this paper is made on prudent risk management and not on the value creation, which strongly depends on the specific managerial talent.

[9] The manager's moral hazard problem may have an alternative interpretation in terms of personal costs of efforts without changing our results. Specifically, we could assume that the manager incurs private costs $b_0 + b_1 x$ when exerting preventive efforts against tail risk and bears no private costs when shirking.

Since large losses at the bank may inflict negative externalities on the rest of banking sector (especially, in the case when they lead to bank failures),[10] the objective of banking regulator is to induce the bank to exert permanent preventive efforts against tail risk, given that there are two layers of agency problem: one between the regulator and bank shareholders, and other between bank shareholders and the bank manager.

In order to create appropriate incentives for preventive efforts against tail risk, we propose to use a mandatory recapitalization policy which relies on two regulatory tools: mandatory recapitalization rule x_R and recapitalization multiplier $s > 1$, such that sx_R is a target bank asset value after the mandatory recapitalization. Thus, bank shareholders will be obliged to inject new equity capital $(s-1)x_R$ each time the bank asset value hits the mandatory recapitalization trigger x_R from above. However, if the bank asset value suddenly falls below the recapitalization threshold, the bank should be liquidated by the regulator, since a sudden violation of capital requirements would bear evidence of shirking.[11] This rule is used *ex-ante* in order to create pressure on bank shareholders and to induce them to refrain from taking tail risk. However, it will never be used ex-post, since capital requirements will eliminate tail risk.

As in *Décamps et al. (2011)*, we allow for two types of recapitalization costs: proportional costs ξ which are imposed on each unit of capital raised and lump-sum costs f. Recapitalization costs may reflect taxes, expert and registration costs of the new equity issue, as well as the asset restructuring costs.

2.2 Why do we need mandatory recapitalizations?

To justify the need for capital regulation in the described set-up and to show why it should be implemented under the form of mandatory recapitalizations, consider first the optimal strategy of bank shareholders when there is neither internal agency problem nor capital regulation.

Maximizing equity value, bank shareholders optimally decide on whether to undertake preventive efforts against tail risk. This decision is driven by the

[10] For example, the failure and the subsequent nationalization of RBS in 2008 was a direct consequence of the chain of large losses. These losses have origins in the bank's management strategy focused on the aggressive asset growth (via the expansion of subprime loans and investment in asset-backed securities) regardless of inherent tail risk.

[11] Equity expropriation is a maximum feasible penalty in the context of limited liability of bank shareholders. The incentive effect can be produced with a lower penalty but this would result in a higher incentive recapitalization rule and thus would reduce bank equity value.

instantaneous costs of preventive efforts on the one hand and the expected loss of equity value on the other. We introduce a second order differential operator $A_e f(x)$ such that:

$$A_e f(x) = 1/2\sigma^2 x^2 f''(x) + (\mu - eg)xf'(x) - rf(x), \tag{2}$$

where $e \in \{0,1\}$ and $f(x)$ is any contingent claim.

Then, the maximization problem of bank shareholders can be stated as follows:

$$\max_{e_t \in \{0,1\}} \{A_{e_t} E_{NA}(x_t) - (1-e_t)\lambda(E_{NA}(x_t) - E_{NA}(\alpha x_t)) + \delta x_t - rD\} = 0, \tag{3}$$

where x_t is given by (1) and $E_{NA}(x)$ denotes bank equity value in the absence of the internal agency problem.

Consequently, bank shareholders are interested in promoting preventive efforts against tail risk, while the expected negative jump of equity value caused by shirking will exceed instantaneous costs of preventive efforts:

$$\lambda(E_{NA}(x) - E_{NA}(\alpha x)) \geq gxE'_{NA}(x) \tag{4}$$

Let x_e^* be a critical threshold which makes the incentive constraint (4) binding and let $x_L^S < x_e^*$ denote a threshold:

$$x_L^S = \frac{\gamma_2}{\gamma_2 - 1}\frac{r}{r+\lambda}D, \tag{5}$$

where $\gamma_2 < 0$ is a root of $1/2\sigma^2\gamma(\gamma-1) + \mu\gamma = r + \lambda$.

Result 1 *In the absence of regulatory control and internal agency problems, the optimal strategy of bank shareholders is characterized as follows:*

- *(i) to exert preventive efforts against tail risk for* $x_t \geq x_e^*$*;*
- *(ii) to liquidate the bank as soon as* $x_t = x_L^S$*.*

To see the intuition of the above result, consider first the optimal choice between a costly recapitalization and bank liquidation. Recall that equity value can be viewed as a current discounted value of shareholders' profits plus a liquidation/recapitalization option. In the context of limited liability and no regulation, shareholders have a strong interest to liquidate the bank when equity capital is negative, so that the liquidation option has a positive value. The option associated with recapitalizations is negatively valued, since each recapitalization implies an outflow of shareholder wealth. Then, whatever is a chosen effort strategy, in the absence of regulation bank shareholders will never recapitalize the bank on their own and will strategically default at x_L^S. However, in the neighborhood of the liquidation point a moral hazard problem emerges as a consequence of the conflict between portfolio risk and tail risk. The point is that near the liquidation point the bank failure is more likely to be caused by asset return volatility rather than by a tail risk realization, i.e., the liquidation threshold will be reached following a continuous decline rather than following a sudden negative jump of bank asset value. As a result, the bank will stop exerting preventive efforts against tail risk in order to raise the asset growth rate and to move away from the liquidation point. In such a context, capital regulation is essential to prevent moral hazard.

Result 2 *Any mandatory* ***liquidation*** *rule* $x_L > D$ *associated with a positive capital ratio is unable to prevent tail risk at the bank.*

Faced with any mandatory *liquidation* rule $x_L > D$, the bank will engage in tail risk in the neighborhood of x_L. This can be explained by the "liquidation-escape" effect described above: in order to avoid the loss of equity because of portfolio risk, in the neighborhood of the liquidation point shareholders will focus on faster asset growth to the detriment of asset safety. Moreover, under the *liquidation* rule $x_L > D$, a liquidation option has a negative value for shareholders, which makes them even less sensitive to tail risk. *This means that a simple increase of capital requirements would be insufficient to discourage banks from "manufacturing tail risk".*

As it is pointed out by several recent studies (see, for example, *Perotti et al. (2011)*), new regulatory tools are required in order to keep control over tail risk. A mandatory recapitalization could be seen as one of them. Indeed, the effect of the mandatory *recapitalization* rule will differ from the effect of *liquidation* rule. A crucial point is that, *under the mandatory recapitalization rule, portfolio risk cannot provoke the bank default.* At the same time, a tail risk realization in the absence of preventive efforts will raise the likelihood of mandatory recapitalizations, increasing the expected value of further recapitalization costs. This will make shareholders more sensitive to tail risk, improving their incentives for preventive efforts.

Thus, in order to ensure permanent preventive efforts at the bank, the mandatory recapitalization rule should be designed in such a way that: (i) in the absence of preventive efforts against tail risk, the expected losses of equity value would exceed instantaneous gains from shirking; (ii) equity value at the mandatory recapitalization point must be sufficiently high, so that shareholders will optimally prefer to recapitalize the bank rather than to be punished by equity expropriation.

3 Capital regulation when there is no internal agency problem: a benchmark

In order to track the impact of the internal agency problem on capital regulation, we first design the incentive mandatory recapitalization policy in a setting where the interests of the bank manager are perfectly aligned with the interests of bank shareholders.

Let us formally define the regulatory problem. The regulator is looking for the optimal combination of recapitalization rule x_R and recapitalization multiplier $s > 1$ which will ensure preventive efforts at the bank for $\forall x \geq x_R$, maximizing bank value. Note that, in contrast to bank liquidation or a public bail-out, mandatory recapitalizations will be taken in charge by bank shareholders and, therefore, will not generate any social costs. Then, a maximization of bank value will be equivalent to the maximization of bank equity value, so that for any current bank asset value $x_0 > x_R$ the regulatory problem can be stated as follows:

$$\underset{x_R,s}{\text{Max}} \quad E_{NA}(x_0, x_R, s) \geq 0 \text{ s.t.}$$

$$\lambda(E_{NA}(x, x_R, s) - I_{x \geq x_R/\alpha} E_{NA}(\alpha x, x_R, s)) \geq g x E'_{NA}(x, x_R, s) \text{ for } \forall x \geq x_R$$

where $E_{NA}(x, x_R, s)$ is given in Appendix A.1 and $I_{x \geq x_R/\alpha}$ is an indicator function.[12]

Proposition 1 *In the absence of internal agency problems, the regulator can prevent the bank from taking tail risk, by using the optimal mandatory recapitalization policy* $\{s^*, x_R^B(s^*)\}$ *which implies:*[13]

- *(i) a recapitalization threshold* $x_R^B(s)$*:*

$$x_R^B(s) = \frac{(1 - s^{\beta_2})\lambda D + f(\lambda - \beta_2 g)}{(1 - s^{\beta_2})(\lambda - g) - \xi(s-1)(\lambda - \beta_2 g)} > D, \tag{6}$$

where $\beta_2 < 0$ *is a root of* $1/2\sigma^2\beta(\beta - 1) + (\mu - g)\beta = r$.

- *(ii) a recapitalization multiplier* $s^* = \arg\min x_R^B(s)$.

[12]Indeed, if the large loss occurs when $x \in [x_R, x_R/\alpha)$, bank asset value will suddenly fall below x_R and the bank will be liquidated by the regulator.

[13]The regulator can enforce mandatory recapitalizations by the threat of shareholders' expropriation. Since the mandatory recapitalization rule is designed to ensure a strictly positive value of bank equity, shareholders will prefer to recapitalize the bank rather than to be deprived of equity.

Since bank equity value is decreasing on x_R, for any given recapitalization multiplier $s > 1$ its maximum can be attained under the minimum feasible recapitalization rule $x_R^B(s)$, such that $\lambda E_{NA}(x, x_R, s) = gxE'_{NA}(x, x_R, s)$ at $x = x_R$.[14] It can be easily shown that $E(x, x_R^B(s), s) > 0$ at $x = x_R^B(s)$, which means that $x_R^B(s)$ corresponds to a positive capital ratio. Moreover, faced with mandatory recapitalization rule $x_R^B(s)$, the bank will not undertake a voluntary recapitalization at any $x > x_R^B(s)$, since recapitalization costs would always exceed the expected growth of equity value resulted from capital injections.[15] Given that bank equity value is decreasing on recapitalization rule $x_R^B(s)$ and the latter is a convex function of s, the solution of the above regulatory problem will be uniquely defined by $s^* = \arg\min x_R^B(s)$.

In order to illustrate the optimal mandatory recapitalization policy, we resort to numerical simulations. For the parameter set $D = 1$, $r = 0.04$, $\mu = 0.035$, $g = 0.005$, $\sigma = 0.2$, $f = [0.1 \times 10^{-4}, 0.1 \times 10^{-3}]$, $\xi = [0.01, 0.1]$,[16] the optimal recapitalization rule varies in the range of $x_R^B(s^*) \in (1.12, 1.21)$ and the optimal recapitalization multiplier is of the order of $s^* \in (1.01, 1.08)$. For example, for $f = 0.1 \times 10^{-3}$ and $\xi = 0.1$ we obtain $x_R^B(s^*) = 1.21$ and $s^* = 1.02$. This corresponds to the minimum capital ratio of 17.4% and the post-recapitalization capital ratio of 18,9% respectively.

The optimal recapitalization multiplier results from the trade-off between two opposite effects generated by fixed and proportional recapitalization costs: s^* is increasing on f and decreasing on ξ. Fixed recapitalization costs make shareholders willing to raise as much funds as possible in order to postpone further mandatory recapitalizations, whereas proportional costs reduce recapitalization capacity.[17] The optimal recapitalization rule $x_R^B(s^*)$ is increasing on both fixed and proportional recapitalization costs. Indeed, given significant recapitalization costs, the bank equity must be strong enough in order to make shareholders willing to prefer costly recapitalizations to bank liquidation. Moreover, $x_R^B(s^*)$ is increasing on asset volatility, since higher σ exacerbates a trade-off between tail risk and portfolio risk, aggravating a moral hazard problem.

[14] We show in Appendix that, if recapitalization rule ensures the incentive constraint for $x \in [x_R, x_R/\alpha)$, it automatically ensures the incentive constraint for any $x > x_R/\alpha$.

[15] In practice, shareholders are unwilling to undertake voluntary recapitalizations not only because of the private costs they incur, but also because of the fear that it might be perceived as a negative signal about the bank financial health. Making a recapitalization mandatory, however, might partially mitigate this signaling effect.

[16] Empirical estimations realized for the set of U.S. firms provide the following values of average marginal issuance costs: 2.8% in Gomes (2001), 5.1% in Altinkiliç and Hansen (2000), 10.7% for small firms and 5% for large firms in Hennessy and Whited (2007).

[17] *Décamps et al. (2011)* point out similar effects in the liquidity management framework, where the firm's cash flow evolves as the arithmetic Brownian motion.

4 Capital regulation under the internal agency problem

Now we turn to the set-up which allows for internal agency problems at the bank. The manager has a different perception of tail risk than shareholders, since his objective deviates from the maximization of equity value. Thus, besides the instantaneous cost $gx_t dt$ taken out from bank asset value, the real cost of tail risk prevention for shareholders will include an incentive compensation of the manager. In such a context, shareholders are faced with two strategic decisions: (i) whether to induce the manager to exert preventive efforts against tail risk? (ii) if so, what should be the optimal contract which will ensure continuous preventive efforts in the least costly way? We start by answering the second question and define the optimal incentive contract with the manager. Then, allowing for the optimal incentive contract, we consider the initial problem of bank shareholders and build the optimal mandatory recapitalization policy, which will induce shareholders to compensate the manager for preventive efforts.

4.1 The optimal incentive contract

Assume that bank shareholders want to prevent large losses for $\forall x \geq x_R$ and let examine the manager's incentives for preventive efforts. The manager maximizes contract continuation value, $K(x_t)$, which is contingent on the current bank asset value x_t and represents the expected value of total future gains from the managerial position, including eventual private benefits:

$$K(x_t) = E_{x_t}\left[\int_t^{\tau_T} e^{-r(\tau-t)}(R(x_\tau) + (1-u_\tau)B(x_\tau))d\tau\right], \tag{7}$$

where $u_\tau \in \{0,1\}$ is an indicator of the manager's preventive efforts, $x \geq x_T$ follows (1) and τ_T is the first time when the bank's asset value reaches a contract termination rule x_T.

Shirking will have an ambiguous effect on the manager's wealth. On the one hand, it increases contract continuation value due to the saved cost $gx_t dt$ of preventive efforts and brings private benefits $B(x_t)$. On the other hand, the manager risks loosing his position (and, consequently, the expected value of further payoffs) with probability λdt in a small period of time dt. Then, the manager's maximization problem can be stated as follows:

$$\max_{u_t \in \{0,1\}} \{A_{u_t} K(x_t) + R(x_t) - (1-u_t)(\lambda K(x_t) - B(x_t))\} = 0 \tag{8}$$

The manager will exert preventive efforts, while the expected loss of contract continuation value will exceed the instantaneous gain from shirking:

$$\lambda K(x) \geq gxK'(x) + B(x) \tag{9}$$

Thus, for the current bank asset value x_0, the optimization problem of bank shareholders can be stated as follows:

$$\min_{R(x)\geq 0, x_T \geq x_R} K(x_0) \ s.t. \ (9), \tag{10}$$

where $K(x)$ is given by (7) with $u_\tau = 1$.

The optimal incentive contract can be inferred from the contract continuation value which makes the incentive constraint (9) binding:

$$K(x) = \frac{b_0}{\lambda} + \frac{b_1}{\lambda - g}x + C_0 x^{\frac{\lambda}{g}}, \tag{11}$$

where C_0 is a constant depending on the contract termination rule x_T.

Lemma 1 *If x_T is not specified, the incentive contract does not exist.*

When the termination rule is not specified in the contract, i.e., the manager is allowed to keep his position forever, the minimum feasible incentive contract continuation value (11) would be negative for relatively high asset values. It means that an incentive compensation scheme should contain punishments, whereas the limited liability of the manager precludes the implementation of such a contract.

Assuming that x_T exists and replacing the minimum incentive $K(x)$ from (11) into (8) with $u_t = 1$, we immediately get the minimum incentive remuneration. Thus, the overall contract design depends on the boundary condition at x_T.

Proposition 2 *The optimal incentive contract which will induce the manager to refrain from manufacturing tail risk for $\forall x \geq x_R$ implies:*

- *(i) a contract termination rule $x_T^* = x_R$;*
- *(ii) an asset-based remuneration $R^*(x) = r\frac{b_0}{\lambda} + \delta\frac{b_1}{\lambda-g}x$, when $x > x_R$;*
- *(iii) a terminal payoff $R_T^*(x_R) = \frac{b_0}{\lambda} + \frac{b_1}{\lambda-g}x_R$, when $x = x_R$.*

The corresponding contract continuation value will be linear on asset value:

$$K^*(x) = \frac{b_0}{\lambda} + \frac{b_1}{\lambda - g}x \tag{12}$$

Thus, the bank manager should be replaced each time the bank faces mandatory recapitalization, since any $x_T^* > x_R$ would impose larger incentive compensation spendings on bank shareholders. Given $x_T^* = x_R$, the only feasible way to ensure continuous preventive efforts of the manager at minimum cost is to repay him the remaining contract continuation value $R_T^*(x_R)$ at the contract termination date. Surprisingly, this result is consistent with a practice of "golden handshakes". However, without this terminal payoff the manager would not have sufficient incentives to refrain from manufacturing tail risk. Indeed, for any $R_T < R_T^*(x_R)$, we would have $K(x) < 0$ for relatively high asset values, i.e., the incentive contract would not exist. The intuition is simple: in the current set-up the contract is never worthless for the manager, so that he has no interest to resign voluntarily. Then, forced to leave at the recapitalization point, he should be compensated by the amount equal to the expected value of further contract payoffs he could get in the continuation case. However, it is unoptimal to provide any $R_T > R_T^*(x_R)$: in this case it would be too costly for bank shareholders to compensate the manager, when x_t becomes relatively high.

The optimal incentive compensation $R^*(x)$ is linear on bank asset value.[18] It can be interpreted as the sum of a fixed salary, rb_0/λ, and a current bonus, $\delta b_1 x/(\lambda - g)$, which can be paid in the form of cash and/or non-cash compensation. A fixed salary, conditioned on the manager's opportunity costs b_0, can be viewed as an exogenous compensation component determined by the labor market forces. On the contrary, the magnitude of current bonuses depends on the manager's ability to generate private gains on the financial market, b_1, which can be (at least, to some extent) controlled by shareholders. In practice, top executives are partially remunerated by equity-based compensation. In the context of our model, this would create a self-amplifying mechanism: rewarding the manager with stock options and stocks, shareholders would raise his sensitivity to the side gains which can be extracted from the financial market. In order to induce prudent risk management, shareholders will have to increase the variable part of the incentive compensation, which will again raise the manager's incentives for risk-taking and so forth. Consequently, it seems to be reasonable to impose restrictions on

[18] The advantage of linear incentive contracts, which are quite frequent in the principal-agency literature, is their robustness. In our model the linearity of compensation scheme is driven by the linearity of private benefits.

equity-based compensation policies at banks in order to make managers less sensitive to the short-term reactions of the financial market. This will help to simultaneously reduce the magnitude of current bonuses, as well as the scale of terminal payoffs.

4.2 The optimal mandatory recapitalization policy

Now, given the optimal incentive contract with the manager, we turn back to the initial problem of bank shareholders faced with a decision on whether to compensate the manager for preventive efforts or to assume tail risk exposure. Given the internal agency problem, bank shareholders are interested in inducing the manager to exert preventive efforts against tail risk while

$$\lambda(E(x, x_R, s) - I_{x \geq x_R/\alpha} E(\alpha x, x_R, s)) \geq g x E'(x, x_R, s) + R^*(x), \tag{13}$$

where $R^*(x) = r\frac{b_0}{\lambda} + \delta\frac{b_1}{\lambda - g}x$ and $E(x, x_R, s)$ is a bank equity value constructed under the optimal incentive contract with the manager (see Appendix A.2).

Then, for any current bank asset value $x_0 > x_R$, the regulatory problem can be stated as follows:

$$\underset{x_R, s}{\text{Max}} \quad E(x_0, x_R, s) \geq 0 \quad \text{s.t. (13) for } x \geq x_R \tag{14}$$

For any given recapitalization multiplier s, let $x_R^A(s)$ denote a minimum feasible recapitalization rule, which ensures the incentive condition (13) for $x \geq x_R^A(s)$:

$$x_R^A(s) = \frac{(1 - s^{\beta_2})(\lambda D + b_0 + r b_0/\lambda) + (\lambda - \beta_2 g)(f + b_0/\lambda)}{(1 - s^{\beta_2})\left((\lambda - g)\left(1 - \frac{b_1}{\lambda - g}\right) - \frac{\delta b_1}{\lambda - g}\right) - \left(\xi(s-1) + \frac{s b_1}{\lambda - g}\right)(\lambda - \beta_2 g)} \tag{15}$$

and let $s^{**} = \arg\min x_R^A(s)$.

Proposition 3 *Given the internal agency problem at the bank, the regulator can prevent tail risk, by using mandatory recapitalization policy* $\{s^{**}, x_R^A(s^{**})\}$.

In order to evaluate the impact of internal agency problems on capital regulation, we resort to numerical simulations, using parameter $b_0 = 0$, $r = 0.04$, $\mu = 0.035$, $\sigma = 0.2$, $\alpha = 0.8$, $\lambda = 0.05$, $g = 0.005$, $f = 0.1 \times 10^{-3}$, $\xi = 0.1$ and considering different levels of $b_1 \in [0.1 \times 10^{-5}, 0.1 \times 10^{-3}]$. The obtained minimum recapitalization and post-recapitalization ratios are of the order of $17.4\% - 20\%$ and $19.7\% - 29.2\%$ respectively. A comparison of these outcomes with estimations obtained in the benchmark case makes it clear that *the incentive recapitalization policy which does not allow for the internal agency problem would be unable to prevent the bank from taking tail risk*. Thus, the internal agency problem matters and should be taken into account when designing new capital regulation framework.

It is worth noting that agency costs can be reduced due to internal random audits, which consist in a spot check on the manager's preventive efforts. If an audit uncovers no preventive efforts, the manager has to be fired without getting any terminal pay. Since internal random audits will increase the probability of loosing contract continuation value in the case of shirking, the manager's efforts can be ensured by lower incentive compensation. If instantaneous random audit costs are not too high, random audits may help to reduce the total costs of preventive efforts for shareholders and thus would improve their incentives for prudent risk management, allowing the regulator to reduce the mandatory recapitalization rule. Moreover, since a true bank asset value might be opaque for shareholders (especially when the manager has a possibility to manipulate financial statements in order to enjoy higher rewards), random audits may also imply a verifications of bank asset value. This would discourage the manager from misreporting, allowing shareholders to implement the optimal incentive contract.

4.3 The impact of bonus taxes

Using previous results, we are going to examine the effect of bonus taxes on risk-taking incentives of the bank.

Let a tax rate τ be applied to the variable part of managerial compensation. Assume first that bonus taxes should be paid by bank shareholders. In this case, bonus taxes have no impact on the manager's incentives, so that bank shareholders can induce the manager to exert preventive efforts by using the optimal incentive contract defined in Proposition 2. However, the total cost of incentive contract for bank shareholders in the presence of taxes will be given by

$$K^*(x) + \tau \frac{b_1}{\lambda - g} x, \tag{16}$$

where $K^*(x)$ is a minimum incentive contract continuation value in a tax-free world given in (12).

Consider now an alternative setting, where bonus taxes should be paid by the bank manager.[19] In order to have incentives for tail risk prevention, after the tax levy the manager should be left at least with the same wealth

[19] We assume that bonus taxes are paid immediately after receiving a compensation.

than in a tax-free world. Then, the minimum incentive contract continuation value that should be offered to the manager will be given by

$$K^*(x) + \frac{\tau}{(1-\tau)} \frac{b_1}{\lambda - g} x \tag{17}$$

The first evidence that can be drown from this analysis is that bonus taxes appear to be inappropriate for dealing with excessive risk-taking at banks, which contradicts to some official proposals (see, for instance, the UK Pre-Budget Report on 9 December 2009). Increasing a real cost of preventive efforts and, thereby, reducing shareholders' incentives for prudent risk management, the implementation of bonus taxes in the context of our model will require to set a higher mandatory recapitalization rule.[20]

It is also easy to see that a real cost of preventive efforts for bank shareholders in the case, when bonus taxes are imposed on the manager, would be higher as compared to the case, when bonus taxes are imposed on bank shareholders. Thus, choosing the lesser of two evils, it would be better to levy bonus taxes from bank shareholders, rather than from bank managers. This corroborates with a practice adopted by UK government in 2009 when bonus tax was imposed on the owners of financial institutions. On the contrast, the 2009 bonus tax proposals in France were targeting directly bank management, which appears to be counterproductive.

5 Conclusion

This study is an attempt to rethink the approach to bank capital regulation in response to huge incentive distortions revealed by the 2007-09 financial crisis. An important point to be addressed withing a context of forthcoming regulatory reforms is the internal agency problem between bank shareholders and bank managers, which largely contributed to excessive risk-taking in banking sector. Moreover, banking regulators should pay more attention to the tail-risk exposure of banks, since it can cause far greater damages than asset volatility.

In this study we provide a unified theoretical framework to deal with both issues and propose a design of the mandatory recapitalization policy, which struggles with the tail risk origination. We show how, through the appropriate choice of mandatory recapitalization parameters, the regulator can provide bank shareholders with incentives to put in place the incentive compensation scheme which will discourage the bank manager to engage

[20] In fact, the effect of bonus tax is equivalent to the effect produced by the raise of b_1.

in the manufacturing of tail risk. Since internal agency problems make it costly for bank shareholders to promote prudent risk-taking behaviors of bank managers, shareholders must have a larger stake in the game. This might be used as a justification for more stringent capital requirements for systemically important banks, which are characterized by both severe agency problems and a predisposition to large losses. However, as we have shown, a simple increase of capital requirements would not prevent banks from manufacting tail risk. For this reason, we suggest implementing capital regulation in the form of incentive-based recapitalization policy.

Mandatory recapitalizations can possibly be viewed as a solution for TBTF banks. Inducing these banks to replenish their capital in a timely fashion before they get into troubles, would help to avoid costly public bailouts. In is worth noting that mandatory recapitalizations represent a superior alternative to deleveraging, which is often used by banks to restore their capital ratios. While deleveraging is realized through a partial liquidation of assets and thus reduces a credit capacity of banks, mandatory recapitalization implies an expansion of bank balance sheet and theoretically cannot lead to a credit crunch. Moreover, the experience of the recent financial crisis has shown that a massive deleveraging of banks can lead to huge systemic consequences through the mechanism of fire sales (see, for instance, *Acharya and Viswanathan (2011)*, *Adrian and Shin (2008)*). Thus, from the standpoint of a social planner, it would be better to induce banks to inject fresh capital, instead of allowing them to cut leverage in order to comply with capital requirements.

We conclude with a discussion on whether capital requirements should be reduced if the bank acquires insurance policy against tail risk? Actually, only the advanced approach of the Basel II capital requirements considers insurance policy as a risk mitigation tool, allowing for the reduction of mandatory capital. However, it seems that a greater reliance on the insurance protection may aggravate the problem of moral hazard. The point is that insurance policy allows banks to transfer the risk without struggling with risk origination, i.e., it helps to reallocate risks but cannot prevent their accumulation within the financial system. Moreover, in the context of a systemic crisis, insurance companies themselves may experience serious financial problems,[21] being unable to provide loss coverage. Thus, even though an access to insurance may be beneficial for bank shareholders (i.e., it might be cheaper to buy insurance policy rather than to compensate the manager for preventive efforts), prudent risk management would be *the only durable solution* from the per-

[21] This happened to AIG, one of the biggest players of the world insurance market, bailed out by Federal Reserve Bank and U.S. Treasury in 2008.

spective of social welfare. Banks can be allowed to buy insurance protection against some external risks (like external frauds, hacking attacks, natural disasters), since insurance policy will not promote moral hazard and risk accumulation in this case. At the same time, regulators should induce banks to struggle with the internal risk origination. As we have shown in this paper, this can be realized by means of the incentive mandatory recapitalization policy which allows for internal agency problems at banks.

Appendix A. Evaluation of contingent claims

A.1. Equity value in the benchmark case

Let x_R denote any arbitrary recapitalization rule. At the recapitalization point bank equity value satisfies a boundary condition

$$E_{NA}(x_R) = E_{NA}(sx_R) - (1+\xi)(s-1)x_R - f \geq 0$$

Thus, for any given recapitalization rule x_R and preventive efforts against tail risk, equity value follows:

$$E_{NA}(x, x_R, s) = -\left[\frac{f + \xi(s-1)x_R}{1 - s^{\beta_2}}\right]\left(\frac{x}{x_R}\right)^{\beta_2} + x - D, \tag{A1}$$

where $\beta_2 < 0$ is a root of $1/2\sigma^2\beta(\beta-1) + (\mu - g)\beta = r$.

A.2. Equity value under the optimal incentive contract

Given the optimal incentive contract with the manager, a boundary condition at x_R can be written as follows:

$$E(x_R) = E(sx_R) - (1+\xi)(s-1)x_R - f - R_T^*(x_R),$$

where $R_T^*(x_R) = \frac{b_0}{\lambda} + \frac{b_1}{\lambda - g}x_R$ is the optimal terminal payoff offered to the dismissed manager. Thus, the equity value allowing for the optimal incentive compensation scheme, follows:

$$E(x, x_R, s) = -\left[\frac{f + \frac{b_0}{\lambda} + \left(\xi(s-1) + \frac{sb_1}{\lambda - g}\right)x_R}{1 - s^{\beta_2}}\right]\left(\frac{x}{x_R}\right)^{\beta_2} + \left(1 - \frac{b_1}{\lambda - g}\right)x - D - \frac{b_0}{\lambda} \tag{A2}$$

A.3. Equity value under the jump process

Let x_L be any arbitrary liquidation rule. Assume there is no agency problem and the bank never exerts preventive efforts against tail risk. Let denote $x_i = x_L/\alpha^i, i = 1.. + \infty$. We denote $E_J(x)$ bank equity value under the jump-diffusion process. On each interval $[x_i, x_{i+1})$ it satisfies the equation:

$$\frac{1}{2}\sigma^2 x^2 E''_{Ji}(x) + \mu x E'_{Ji}(x) - (r+\lambda)E_{Ji}(x) + \delta x - rD = -\lambda E_{J(i-1)}(\alpha x) \tag{A3}$$

A general solution of the above equation is given by:

$$E_{Ji}(x) = C_{i,1}x^{\gamma_1} + C_{i,2}x^{\gamma_2} + A_{i,1}x + A_{i,0} \tag{A4}$$

where coefficients $C_{i,1}$, $C_{i,2}$, $A_{i,1}$, $A_{i,0}$ will be defined iteratively and $\gamma_1 > 0$, $\gamma_2 < 0$ are the roots of

$$1/2\sigma^2\gamma(\gamma-1) + \mu\gamma = r + \lambda \tag{A5}$$

Replacing (A4) into (A3) and allowing for $\delta = r + \lambda - \mu$[22] we get:

$$-\delta A_{i,1}x-(r+\lambda)A_{i,0}+\delta x-rD = -\lambda(\alpha^{\gamma_1}C_{i-1,1}x^{\gamma_1}+\alpha^{\gamma_2}C_{i-1,2}x^{\gamma_2}+A_{i-1,1}\alpha x+A_{i-1,0}) \tag{A6}$$

Consequently, we have:

$$C_{i-1,1} = -C_{i-1,2}\alpha^{\gamma_2-\gamma_1}x^{\gamma_2-\gamma_1} \tag{A7}$$

$$A_{i,1} = \frac{\delta + \alpha\lambda A_{i-1,1}}{\delta} = \frac{\delta}{\delta - \alpha\lambda}\left[1 - \left(\frac{\alpha\lambda}{\delta}\right)^{i+1}\right] \tag{A8}$$

$$A_{i,0} = \frac{-rD + \lambda A_{i-1,0}}{r+\lambda} = -D\left[1 - \left(\frac{\lambda}{r+\lambda}\right)^{i+1}\right] \tag{A9}$$

Then, the equity value on each interval $[x_i, x_{i+1})$ can be rewritten as follows:

$$E_{Ji}(x) = C_{i,2}x^{\gamma_2}(1 - \alpha^{\gamma_2-\gamma_1}) + A_{i,1}x + A_{i,0} \tag{A10}$$

Since a large loss on $[x_L, x_1)$ will lead to the bank default, equity value on $[x_L, x_1)$ satisfies the equation:

$$\frac{1}{2}\sigma^2x^2E''_{J0}(x) + \mu xE'_{J0}(x) - (r+\lambda)E_{J0}(x) + \delta x - rD = 0 \tag{A11}$$

Using expression (A10) and boundary condition $E_J(x_L) = 0$, we get:

$$E_{J0}(x) = \left(\frac{r}{r+\lambda}D - x_L\right)\left(\frac{x}{x_L}\right)^{\gamma_2} + x - \frac{r}{r+\lambda}D \tag{A12}$$

Let denote:

$$C_{0,2} = \left(\frac{r}{r+\lambda}D - x_L\right){x_L}^{-\gamma_2} \ , \ A_{0,1} = 1 \ , \ A_{0,0} = -\frac{r}{r+\lambda}D \tag{A13}$$

[22] By the absence of arbitrage.

Given matching conditions $E_{Ji}(x_i) = E_{J(i-1)}(x_i)$, coefficients $C_{i,2}, i = 2.. + \infty$ can be uniquely defined as follows:

$$C_{i,2} = C_{i-1,2} + ((A_{i-1,1} - A_{i,1})x_i + A_{i-1,0} - A_{i,0})x_i^{-\gamma_2}(1 - \alpha^{\gamma_2 - \gamma_1})^{-1} \quad \text{(A14)}$$

and coefficient $C_{1,2}$ is given by:

$$C_{1,2} = \left(C_{0,2} + ((A_{0,1} - A_{1,1})x_1 + A_{0,0} - A_{1,0})x_1^{-\gamma_2}\right)(1 - \alpha^{\gamma_2 - \gamma_1})^{-1} \quad \text{(A15)}$$

Appendix B. Proofs

Lemma 2 *Consider* $f_1(x) = a_0 + a_1 x + a_2 x^{a_3}$ *such that* $f_1'(x) > 0$, $a_3 < 0$ *and* $f_2(x) = \theta_1 x + \theta_2$ *such that* $\theta_1 > 0$. *For* $\forall x_0 > 0$ *and* $\phi > 1$ *, if* $f_1(\phi x_0) - f_1(x_0) \leq f_2(x_0)$*, it follows that* $f_1(\phi x) - f_1(x) < f_2(x)$ *for* $\forall x > x_0$.

□ Consider two alternative cases:

- If $a_2 > 0$, then $f_1''(x) > 0$ and $f_1'''(x) < 0$. We have $f_1'(\phi x) - f_1'(x) > 0$ and $f_2'(x) > 0$. However, since $f'''(x) < 0$, we have $f_1''(\phi x) - f_1''(x) < 0$, whereas $f_2''(x) = 0$. This means that $f_2(x)$ is growing at a higher pace than $f_1(\phi x) - f_1(x)$.
- If $a_2 < 0$, then $f_1''(x) < 0$. We have $f_1'(\phi x) - f_1'(x) < 0$, while $f_2'(x) > 0$.

Then, given $f_1(\phi x_0) - f_1(x_0) \leq f_2(x_0)$, in both cases we have $f_1(\phi x) - f_1(x) < f_2(x)$ for $\forall x > x_0$.■

Lemma 3 *For any* $\theta < 0$ *and* $g < \lambda(1-\alpha)$*, it follows that* $\lambda(1-\alpha^{\theta}) - \theta g < 0$.

□ Since $g < \lambda(1-\alpha)$ and $\lambda(1-\alpha^{\theta}) - \theta g$ is increasing on g, we get:

$$\lambda(1-\alpha^{\theta}) - \theta g < \lambda(1-\alpha^{\theta}) - \theta\lambda(1-\alpha) = f(\alpha) \tag{A16}$$

Since $f(1) = 0$ and $f'(\alpha) = \theta\lambda(1-\alpha^{\theta-1}) > 0$ for $\alpha \in (0,1)$, we have $f(\alpha) < 0$ for $\alpha \in (0,1)$. Hence, $\lambda(1-\alpha^{\theta}) - \theta g < f(\alpha) < 0$. ■

Proof of Result 1

Consider the optimal strategy of bank shareholders when there is nether internal agency problem nor capital regulation. Bank shareholders optimally choose (i) a liquidation/recapitalization rule; (ii) the level of preventive efforts, $e_t \in \{0,1\}$, against tail risk.

Let $E_J(x)$ denote bank asset value constructed in the absence of preventive efforts (see Appendix A.3). Let x_e^* denote a critical threshold such that:

$$\lambda(E_J(x) - E_J(\alpha x)) = gxE_J'(x) \tag{A17}$$

Then the optimal shareholders' strategy in the absence of any regulation can be defined as follows:

$$E(x) = \begin{cases} E_J(x) & x \in [x_L^S, x_e^*] \\ (E_J(x_e^*) - x_e^* + D)\left(\frac{x}{x_e^*}\right)^{\beta_2} + x - D & x > x_e^* \end{cases}$$

where $\beta_2 < 0$ is a root of $1/2\sigma^2\beta(\beta-1) + (\mu - g)\beta = r$.

The optimal liquidation rule x_L^S results from $\frac{\partial E_{J0}(x)}{\partial x_L} = 0$ and is given by:

$$x_L^S = \frac{\gamma_2}{\gamma_2 - 1}\frac{r}{r+\lambda}D \tag{A18}$$

1) Let check that the bank will never recapitalize at any $x > x_L^S$. For any $x > 0$ a recapitalization is unoptimal if:

$$E(sx) - E(x) < (1+\xi)(s-1)x + f \tag{A19}$$

By construction, this condition holds for $x = x_L^S$. Then, by Lemma 2, it will hold for any $x > x_L^S$.

2) It remains to verify that the optimal effort strategy is:

$$e_t = \begin{cases} 0 & x \in [x_L^S, x_e^*] \\ 1 & x > x_e^* \end{cases} \tag{A20}$$

First we show that $e_t = 1$ for $x > x_e^*$ and thus x_e^* is unique. The corresponding necessary condition implies:

$$\lambda(E(x) - E(\alpha x)) > gxE'(x) \text{ for } x > x_e^* \tag{A21}$$

Replacing equity value into (A21), we get:

$$\left(E_J(x_e^*) - x_e^* + D\right)\left(\lambda(1-\alpha^{\beta_2}) - \beta_2 g\right)\left(\frac{x}{x_e}\right)^{\beta_2} + (\lambda(1-\alpha) - g)x > 0 \tag{A22}$$

By Lemma 3, we have $\lambda(1-\alpha^{\beta_2}) - \beta_2 g < 0$. Consider two possible cases:

- $E_J(x_e^*) < x_e^* - D$: in this case the left-hand side of (A22) is always positive.
- $E_J(x_e^*) > x_e^* - D$: taking the first derivative of the left part of (A22), it is easy to see that it is increasing on x. Since the condition (A22) is binding for $x = x_e^*$, it will hold for $\forall x > x_e^*$.

We now check that it is unoptimal to exert preventive efforts in the neighborhood of x_L^S. On $[x_L, x_L/\alpha)$ the necessary condition for no efforts take the form $\lambda E_J(x) < gxE_J'(x)$. Replacing equity value, we get:

$$(\lambda - \gamma_2 g)\left(\frac{r}{r+\lambda}D - x_L^S\right)\left(\frac{x}{x_L^S}\right)^{\gamma_2} + (\lambda - g)x - \lambda\frac{r}{r+\lambda}D < 0 \tag{A23}$$

For $x = x_L^S$ the above condition is binding and, since the first term in the left part of (A23) is a decreasing and convex function of x, condition (A23) will hold in the neighborhood of x_L^S. Consequently, in the absence of any regulation, the bank will optimally stop exerting preventive efforts for $x \to x_L$. But since x_e^* is unique, $e_t = 0$ for $[x_L^S, x_e^*]$.

Proof of Result 2

Assume that there is no internal agency problem at the bank. Consider any arbitrary liquidation rule $x_L > D$ and let show that it cannot ensure continuous preventive efforts.

Assume that the bank will continuously exert efforts for $x \geq x_L$. Then, bank equity value would follow:

$$E_{NA}(x, x_L) = (D - x_L)\left(\frac{x}{x_L}\right)^{\beta_2} + x - D, \tag{A24}$$

where β_2 is a negative root of $1/2\sigma^2\beta(\beta - 1) + (\mu - g)\beta = r$.

Under continuous preventive efforts, bank equity value should satisfy the incentive condition of bank shareholders. Since for $x \in [x_L, x_L/\alpha)$ a materialized tail risk will trigger bank liquidation, the incentive condition of bank shareholders can be rewritten as follows:

$$\lambda E_{NA}(x, x_L) \geq gxE'_{NA}(x, x_L) \tag{A25}$$

Allowing for equity value (A24), the above condition transforms to:

$$(\lambda - \beta_2 g)(D - x_L)\left(\frac{x}{x_L}\right)^{\beta_2} + (\lambda - g)x - \lambda D \geq 0 \tag{A26}$$

For $x = x_L$ we get:

$$g\beta_2(x_L - D) - gx_L < 0 \tag{A27}$$

Therefore, any liquidation rule $x_L > D$ is unable to ensure preventive efforts in the neighborhood of the liquidation point.

Proof of Proposition 1

Consider the regulatory problem in the set-up, free of internal agency problems:

$$\underset{x_R, s}{\text{Max}} \;\; E_{NA}(x_0, x_R, s) \geq 0 \text{ s.t.}$$

$$\lambda(E_{NA}(x,x_R,s) - I_{x\geq x_R/\alpha}E_{NA}(\alpha x,x_R,s)) \geq gxE'_{NA}(x,x_R,s) \text{ for } \forall x \geq x_R$$

where $x_0 > x_R$ is a current asset value, $E_{NA}(x_0, x_R, s)$ is given by (A1) and $I_{x\geq x_R/\alpha}$ is an indicator function.

For any recapitalization multiplier $s > 1$, consider the minimum incentive recapitalization trigger $x_R^B(s)$ such that $\lambda E_{NA}(x, x_R, s) = gxE'_{NA}(x, x_R, s)$ at $x = x_R^B(s)$:

$$x_R^B(s) = \frac{(1-s^{\beta_2})\lambda D + f(\lambda - \beta_2 g)}{(1-s^{\beta_2})(\lambda - g) - \xi(s-1)(\lambda - \beta_2 g)} \tag{A28}$$

Let show that the pair $s^* = \arg\min x_R^B(s)$ and $x_R^B(s^*)$ is a solution of the above maximization problem.

P1.1. Incentive-compatibility

For any arbitrary recapitalization multiplier $s > 1$, let check that $x_R^B(s)$ ensures the incentive constraint of bank shareholders for $\forall x \geq x_R^B(s)$. Replacing equity value (A1) into the incentive condition, for $x \in [x_R^B(s), x_R^B(s)/\alpha)$ we must have:

$$-(\lambda - \beta_2 g)\left(\frac{f + \xi(s-1)x_R^B(s)}{1 - s^{\beta_2}}\right)\left(\frac{x}{x_R^B(s)}\right)^{\beta_2} + (\lambda - g)x - \lambda D \geq 0 \tag{A29}$$

Since the above condition is binding for $x = x_R^B(s)$ and its left-hand side is increasing on x, it holds for $\forall x \in [x_R^B(s), x_R^B(s)/\alpha)$. For $x \geq x_R^B(s)/\alpha$, we must have:

$$-(\lambda(1-\alpha^{\beta_2}) - \beta_2 g)\left(\frac{f + \xi(s-1)x_R^B(s)}{1 - s^{\beta_2}}\right)\left(\frac{x}{x_R^B(s)}\right)^{\beta_2} + (\lambda(1-\alpha) - g)x \geq 0 \tag{A30}$$

By Lemma 3, $\lambda(1-\alpha^{\beta_2}) - \beta_2 g < 0$ and thus (A30) always holds.

P1.2. Feasibility

Now, we verify that (i) it is optimal to recapitalize the bank at $x_R^B(s)$, rather than to lose equity; (ii) given $x_R^B(s)$, a voluntary recapitalization at any $x > x_R^B(s)$ is unoptimal.

First, we check that $E_{NA}(x, x_R^B(s), s) > 0$ at $x = x_R^B(s)$. Let $x_R^O(s)$ denote a critical recapitalization threshold such that $E_{NA}(x, x_R^O(s), s) = 0$ at $x = x_R^O(s)$:

$$x_R^O(s) = \frac{(1-s^{\beta_2})D + f}{1 - s^{\beta_2} - \xi(s-1)} > D \tag{A31}$$

Multiplying and dividing $x_R^O(s)$ by $(\lambda - g)$ we can show that:

$$x_R^O(s) = \frac{(1 - s^{\beta_2})D(\lambda - g) + f(\lambda - g)}{(1 - s^{\beta_2})(\lambda - g) - \xi(s-1)(\lambda - g)} < \frac{(1 - s^{\beta_2})\lambda D + f(\lambda - \beta_2 g)}{(1 - s^{\beta_2})(\lambda - g) - \xi(s-1)(\lambda - \beta_2 g)} = x_R^B(s) \tag{A32}$$

Since $x_R^B(s) > x_R^O(s)$, it follows that equity value is strictly positive at $x_R^B(s)$. Moreover, given $x_R^O(s) > D$, we have $(x_R^B(s) - D)/x_R^B(s) > 0$, i.e., a corresponding capital ratio is always positive.

Second, we check that there is no other recapitalization rule that could strictly increase equity value. For any $x > x_R^B(s)$ the following condition must hold:

$$E_{NA}(sx, x_R^B(s), s) - E_{NA}(x, x_R^B(s), s) < (\xi + 1)(s - 1)x + f \tag{A33}$$

By the construction of equity value (A1), condition (A33) is binding at $x = x_R^B(s)$. Then, by Lemma 2, it holds for any $x > x_R^B(s)$.

P1.3. Optimality

Finally, let show that $\{s^*, x_R^B(s^*)\}$ is the optimal recapitalization policy. Note that equity value $E_{NA}(x, x_R, s)$ is decreasing on x_R. Then, by construction, $x_R^B(s)$ solves the above maximization problem for any $s > 1$. Numerical simulations realized show that $x_R^B(s)$ is a convex function of s. Hence, there exists a unique $s^* = \arg\min x_R^B(s)$, so that $\{s^*, x_R^B(s^*)\}$ is the solution of the regulatory maximization problem.

Proof of Lemma 1

Consider contract continuation value provided by the binding incentive constraint (9):

$$K(x) = \frac{b_0}{\lambda} + \frac{b_1}{\lambda - g}x + C_0 x^{\frac{\lambda}{g}} \tag{A34}$$

where C_0 is an arbitrary constant.

Assume that the contract termination rule x_T does not exist, so that the manager is never dismissed. Since at the recapitalization point the manager does not incur any private costs, a boundary condition for contract continuation value at $x = x_R$ is given by $K(x_R) = K(sx_R)$. Then,

$$K(x) = \frac{b_0}{\lambda} + \frac{b_1}{\lambda - g}x - \frac{(s-1)}{(s^{\frac{\lambda}{g}} - 1)}\frac{b_1}{\lambda - g}x_R\left(\frac{x}{x_R}\right)^{\frac{\lambda}{g}} \tag{A35}$$

Since $\lambda/g > 1$, we have $\lim_{x \to +\infty} K(x) \to -\infty$. Hence, if x_T is not specified, the incentive contract does not exist.

Proof of Proposition 2

Assume that shareholders have incentives to compensate the manager for preventive efforts. Thus, the maximization problem of bank shareholders can be stated as follows:

$$\min_{R(x)\geq 0, x_T \geq x_R} K(x)$$

$$\lambda K(x) \geq gxK'(x) + B(x)$$

where $K(x)$ is defined in (7) and I is an indicator function.

The minimum incentive contract $\{R(x), x_T\}$ can be inferred from the minimum incentive contract continuation value, which makes the incentive constraint binding:

$$K^*(x) = \frac{b_0}{\lambda} + \frac{b_1}{\lambda - g}x + C_0 x^{\frac{\lambda}{g}} \tag{A36}$$

where C_0 is a constant which depends on x_T.

By Lemma 1, a contract termination rule must be finite. Assume that $x_T > x_R$. In this case, the manager's preventive efforts can be ensured by 2 separate incentive contracts, designed for $x \geq x_T$ and for $x \in [x_R, x_T]$ respectively.

For $x \geq x_T$, the minimum incentive contract continuation value is:

$$K_1(x) = \frac{b_0}{\lambda} + \frac{b_1 x}{\lambda - g} + \left(R_T(x_T) - \frac{b_0}{\lambda} - \frac{b_1}{\lambda - g}x_T\right)\left(\frac{x}{x_T}\right)^{\frac{\lambda}{g}}, \tag{A37}$$

where $R_T(x_T)$ is any arbitrary terminal payoff.

Note that the only way to ensure the manager's preventive efforts for $\forall x \geq x_T$ at a minimum cost is to set:

$$R_T^*(x_T) = \frac{b_0}{\lambda} + \frac{b_1}{\lambda - g}x_T \tag{A38}$$

Indeed, for any $R_T < R_T^*(x_T)$, we would have $\lim_{x\to+\infty} K_1(x) \to -\infty$. At the same time, for any $R_T > R_T^*(x_T)$, $K_1(x)$ will grow at a higher pace than the expected bank asset cash-flow, so bank shareholder will not compensate the manager for preventive efforts when $x \to \infty$.

Under $R_T^*(x_T)$ given by (A38), $K_1(x)$ becomes:

$$K_1(x) = \frac{b_0}{\lambda} + \frac{b_1}{\lambda - g}x, \tag{A39}$$

and the minimum incentive remuneration for $x > x_T$ is given by:

$$R^*(x) = r\frac{b_0}{\lambda} + \delta\frac{b_1}{\lambda - g}x \tag{A40}$$

For $x \in [x_R, x_T]$, the minimum incentive contract continuation value is:

$$K_2(x) = K_0 - \frac{(s-1)}{(s^{\frac{\lambda}{g}}-1)} \frac{b_1}{\lambda - g} x_R \left(\frac{x}{x_R}\right)^{\frac{\lambda}{g}} + \frac{b_0}{\lambda} + \frac{b_1}{\lambda - g} x, \tag{A41}$$

where the constant K_0 results from $K_1(x_T) = K_2(x_T)$:

$$K_0 = \frac{(s-1)}{(s^{\frac{\lambda}{g}}-1)} \frac{b_1}{\lambda - g} x_R \left(\frac{x_T}{x_R}\right)^{\frac{\lambda}{g}}, \tag{A42}$$

Since for $x \in [x_R, x_T)$ we have $K_2(x) > K_1(x)$, it would be optimal to set $x_T = x_R$, so that the optimal contract continuation value will coincide with $K_1(x)$. Consequently, the optimal incentive compensation scheme will imply the incentive compensation $R^*(x)$ and the terminal payoff $R_T^*(x_T)$ given by (A37) and (A38) respectively.

Proof of Proposition 3

Allowing for internal agency problems between bank shareholders and the bank manager, the regulator faces the problem:

$$\underset{x_R, s}{\text{Max}} \quad E(x_0, x_R, s) \geq 0 \text{ s.t.}$$

$$\lambda(E(x, x_R, s) - I_{x \geq x_R/\alpha} E(\alpha x, x_R, s)) \geq gxE'(x, x_R, s) \text{ for } \forall x \geq x_R$$

where $x_0 > x_R$ is a current bank asset value, $E(x_0, x_R, s)$ is given by (A2) and $I_{x \geq x_R/\alpha}$ is an indicator function.

For any recapitalization multiplier $s > 1$, consider the minimum incentive-compatible recapitalization rule $x_R^A(s)$ such that $\lambda E(x, x_R, s) = gxE'(x, x_R, s)$ at $x = x_R^A(s)$:

$$x_R^A(s) = \frac{(1-s^{\beta_2})(\lambda D + b_0 + rb_0/\lambda) + (f + b_0/\lambda)(\lambda - \beta_2 g)}{(1-s^{\beta_2})\left((\lambda - g)\left(1 - \frac{b_1}{\lambda - g}\right) - \frac{\delta b_1}{\lambda - g}\right) - \left(\xi(s-1) + \frac{s b_1}{\lambda - g}\right)(\lambda - \beta_2 g)} \tag{A43}$$

Let show that the pair $s^{**} = \arg\min x_R^A(s)$ and $x_R^A(s^{**})$ is a solution of the above maximization problem.

P3.1. Incentive-compatibility

For any arbitrary recapitalization multiplier $s > 1$, let check that $x_R^A(s)$ ensures the incentive constraint of bank shareholders for $\forall x \geq x_R^A(s)$. Replacing equity value (A2) into the incentive condition, for $x \in [x_R^A(s), x_R^A(s)/\alpha)$ we must have:

$$-(\lambda-\beta_2 g)H(s)\left(\frac{x}{x_R^A(s)}\right)^{\beta_2}+\left((\lambda-g)\left(1-\frac{b_1}{\lambda-g}\right)-\frac{\delta b_1}{\lambda-g}\right)x \geq \lambda D+b_0+\frac{rb_0}{\lambda} \tag{A44}$$

where $H(s)$ denote:

$$H(s)=\frac{f+\frac{b_0}{\lambda}+\left(\xi(s-1)+\frac{sb_1}{\lambda-g}\right)x_R^A(s)}{1-s^{\beta_2}}$$

Since condition (A44) is binding for $x = x_R^A(s)$ and its left-hand side is increasing on x, it holds for $\forall x \in [x_R^A(s), x_R^A(s)/\alpha)$. For $x \geq x_R^A(s)/\alpha$, we must have:

$$-(\lambda(1-\alpha^{\beta_2})-\beta_2 g)H(s)\left(\frac{x}{x_R^A(s)}\right)^{\beta_2}+\left((\lambda(1-\alpha)-g)\left(1-\frac{b_1}{\lambda-g}\right)-\frac{\delta b_1}{\lambda-g}\right)x \geq \frac{rb_0}{\lambda} \tag{A45}$$

By Lemma 3, $\lambda(1-\alpha^{\beta_2})-g\beta_2<0$ and thus (A45) holds when b_0 and b_1 are not too large.

P3.2. Feasibility

For any given $s > 1$, it is easy to show that $x_R^A(s) > x_R^B(s)$, so that $E(x, x_R^A(s), s) > 0$ at $x = x_R^A(s)$. Therefore, given $x_R^A(s)$, shareholders will optimally prefer to recapitalize the bank rather than to be deprived of equity. At the same time, in order to ensure that faced with $x_R^A(s)$ shareholders will not recapitalize the bank at any $x > x_R^A(s)$, we must have:

$$E(sx, x_R^A(s), s) - E(x, x_R^A(s), s) < (\xi+1)(s-1)x + f + R_T(x), \tag{A46}$$

where $R_T(x) = b_0/\lambda + b_1 x/(\lambda-g)$ is a terminal payoff to the manager.

By the construction of equity value (A2), (A46) is binding at $x = x_R^A(s)$. Then, by Lemma 2, it holds for any $x > x_R^A(s)$.

P3.3. Optimality

Finally, let show that $\{s^{**}, x_R^A(s^{**})\}$ is the optimal recapitalization policy. Since $E(x, x_R, s)$ is decreasing on x_R, $x_R^A(s)$ solves the regulatory maximization problem for any $s > 1$. According numerical simulations realized, $x_R^A(s)$ is a convex function of s. Hence, there exists a unique $s^{**} = \arg\min x_R^A(s)$ such that $\{s^{**}, x_R^A(s^{**})\}$ is the solution of the regulatory maximization problem.

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