Bilateral Trading in Networks*

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Abstract

In many markets, ranging from markets for agricultural goods in developing countries to financial markets, goods flow via a sequence of intermediated trade before reaching final customers, information is asymmetric, and trading opportunities are incomplete. We study a dynamic model of bargaining in networks with asymmetric information that captures these three features. We show that the equilibrium price dynamic is non-monotonic and that traders who intermediate the object arise endogenously and attain a profit. This profit depends on their network location. Inefficiencies may arise in equilibrium, but as the time horizon goes to infinity and traders become perfectly patients, equilibrium becomes ex-post efficient.

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1 Introduction

In many cases sellers can only reach final customers with the help of intermediaries. Consider, for example, markets for agricultural goods in developing countries. In these markets local producers access only a limited number of intermediaries, products are exchanged for cash between intermediaries en route from local producers, prices are negotiated bilaterally and market information is poor and dispersed across market participants (see Fafchamps and Minten (1999),(2001)). Goods flow from local farmers to final consumers via a sequence of intermediated trade. Intermediation, bilateral negotiations and asymmetric information are also prominent features of financial markets, especially over-the-counter markets.[1] Markets for artworks and collectibles, and markets for event tickets are other examples where intermediated resale plays a crucial role.

How does the underlying network structure affect the way prices are set by market participants? What are the network locations that provide larger payoffs to traders? In what circumstances will trade be efficient, despite asymmetric information and incompleteness of trading relationships? We develop a dynamic model of trade with resale under asymmetric information that provide answers to these questions.

Understanding how networks affect the terms of trade and market efficiency is a central and open question in economics. Most of the existing work has focused on the exchange of goods in buyer-seller networks with random matching, complete information and absence of resale (e.g., Calvo-Armengol (2003), Coromina-Bosch (2004), Polanski (2007), Manea (2011),). A handful of papers departs from the random-matching assumption. Kranton and Minheart (2001) consider a static model with asymmetric information. Blume et al. (2009), Nava (2009), Gale and Kariv (2007), (2009) study complete information models where resale is possible. We share with this literature the general approach of using networks to model connections between markets.[2] Our major contribution to this literature is to examine, for

[1]Financial products such as foreign currencies, swaps, forward rate agreements, and exotic options are almost always traded in over-the-counter markets. These securities are subject to counter-party risk and therefore bonds of trusts between firms are particularly important and naturally give raise to trading networks. See Allen and Babus (2009) for a survey on networks in financial markets.

[2]This is in contrast to the random-matching approach pioneered by Rubinstein and Wolinsky (1985), and new models of trading based on random matching, such as Duffie, Garleanu and Pedersen (2005),

the first time, a dynamic model of trade in the presence of asymmetric information.[3]

In our model there are a number of local markets. Each local market is controlled by a risk-neutral trader who has exclusive access to the market. Traders are located in a network (geographical or relational) and can only negotiate with traders to whom they are connected. One arbitrary trader owns a single valuable object. The demand for the object in each local market can be either low or high, is independent from the demand in other markets, and can only be observed by the local trader. Bargaining is modeled as follows. In each round the current owner makes a take-it-or-leave-it offer to a connected trader that she chooses. The trader who receives the offer either accepts or rejects it. When a trader acquires ownership, she can either sell it to her local market and extracts the full surplus in that market, in which case the game ends, or start a new round of trade. The network is commonly known and the time horizon can be either finite or infinite.

This model attempts to capture in the simplest way the interaction of the following three features: (i) incompleteness of trading opportunities, which we model by assuming that exchange is bilateral and takes place on a network; (ii) dispersed market information, which we model by introducing asymmetric of information about local demand and (iii) resale, which we model using a dynamic game.

The set of weak-Markov perfect Bayesian equilibria (henceforth “equilibria”) that we characterize has a simple structure (Propositions 1-3). A trader who acquires the object, sells it to her local market whenever the demand in the market (henceforth also “her value”) is high. In contrast, when her value is low, she engages in a sequence of offers to other connected traders, until the object is sold. All her offers, but the last, come at prices that only traders with high value are willing to accept. We refer to these offers as *consumption offers* because, once accepted, the object is sold to the local market (henceforth also “consumed”). Unless there is no time left, consumption offers are followed by an offer that is accepted by all types of traders (low and high value). We refer to these offers as *resale offers*, because they come at a price equal to the expected revenue that the trader obtains from reselling to other

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Satterthwaite and Shneyerov (2007) and Golosov, Lorenzoni and Tsyvinski (2009).

[3]Gofman (2011) also consider trade with asymmetric information but does not explicitly model the underlying bargaining protocol.

traders.[4] The equilibrium is arbitrage-free because traders always consume when their value is high and never pay less than their resale value when their value is low.

This characterization clarifies how the underlying network structure determines equilibrium prices. We show that the equilibrium sequence of demanded prices is non-monotonic in time (Proposition 4). While resale offers follow a decreasing trend, consumption offers spike above subsequent and earlier resale offers. Resale offers are decreasing in time because, as time passes, all traders become more pessimistic about the total expected demand in the network. Refused consumption offers represent bad news about the value of the object. As a consequence the value of the object (conditional on still being on the market) must decrease in time. Prices in consumption offers spike, as sellers are attempting to exploit their positional power in the network to appropriate the surplus of connected traders with high value.[5]

We then investigate how the network location of a trader affects her payoffs. We call *dealers* those traders who get at least one resale offer with positive probability. We call *clients* those traders who obtain only consumption offers. Traders with low value are never able to obtain a profit from trading. Traders with high value make a positive profit if, and only if, they are dealers. In that case they are able to acquire the object at a price lower than their value and consume it. Dealers arise endogenously, depending on their network architecture and on their expected demand.[6] Roughly speaking, traders on the periphery of the network, and those with high expected values, become clients. Traders who provide access to valuable areas of the network become dealers and earn a rent on their location.

In addition, we show that traders who become dealers earlier obtain a payoff advantage over later dealers (Proposition 5). Despite earlier dealers purchase the object at a higher price than later dealers, they acquire the good with an higher probability. The second effect dominates the first. The decline in price only incorporates the reduction in the expected demand of traders who have rejected consumption offers. The decline in the probability of

[4]This is different from the private value of a trader, which is the value of reselling in the local market.

[5]Our characterization implies that the equilibrium sequence of transaction prices is declining in time, with the exception of the last transaction price that may spike.

[6]This is in contrast to other models of trading where there is an exogenous distinction between buyers, sellers and intermediaries (see for example Rubinstein and Wolinsky (1987) and Blume et al. (2009)).

receiving a resale offer also incorporates the probability that earlier dealers may have high-value and consume the object. An immediate consequence of this result is that the payoff of different dealers is affected by their location in the network: if a trader is *essential* in connecting another trader to the initial owner, than the former obtains a higher expected payoff (provided they have the same type).[7] We view this result as providing a micro-foundation for the prominent sociological theory of structural holes (see Burt (1992)). The theory postulates that individuals or firms who have a bridging position in a social/economic network (i.e., fill structural holes) tend to obtain a payoff advantage over others.[8]

We then address our last question about efficiency. When valuations are public information, efficient outcomes are attained regardless of the network architecture (Proposition 6). Under incomplete information, inefficiencies are unavoidable when traders discount the future. More interestingly, even when traders become perfectly patient, ex-post inefficient outcomes arise generically in finite-horizon games as a result of the combination of asymmetric information and incompleteness of trading opportunities (Proposition 7). The presence of a deadline (e.g., because the object is perishable or has an expiration date) provides commitment power to sellers and plays a key role in generating inefficiencies. Finally, in spirit of the Coase conjecture, we show that as the horizon of the game becomes infinite (i.e., the object is durable) and discounting vanishes, any sequence of finite-horizon weak-Markov PBE converges to an ex-post efficient equilibrium (Proposition 8).

In addition to the literature on networks and markets our paper relates to the literature on sequential bargaining with asymmetric information, e.g., Fudenberg and Tirole (1983), Fudenberg, Levine and Tirole (1985), Sobel and Takahshi (1983), Cramton (1984), Gul, Sonnenchein and Wilson (1986), Chatterjee and Samuelson (1987). In all these models once a buyer acquires the object consumption takes place and the game ends. Asked prices naturally decline. In our environment a seller may face multiple buyers and each buyer may

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[7] A trader i is essential to connect j to the initial owner if trader i lies in every path connecting j to the initial owner.

[8] This theory has been applied to different environments such as organizations and R&D collaboration between firms. In applying his theory to markets and networks of firms, Burt states that "producers with networks rich in structural holes can negotiate favourable terms in their transactions with suppliers and customers, and so should enjoy higher rates of return in their investments".

acquire the object to resell it.[9] As a consequence, bilateral trading strategies become more complicate and our price dynamic more articulate.

We use a simpler, two-value, informational structure than in most of the papers mentioned above. This choice is forced upon us by the intricacies that resale introduces. Zheng (2002) studies a dynamic mechanism design game with resale. The initial seller and all the buyers are part of a complete network and valuations are drawn from continuous distributions. His main results concern the attainability of Myerson's second-best revenue. To achieve tractability he must restrict attention to a special class of distributions.[10] Our two-value assumption makes the problem tractable as it guarantees that when a buyer and seller negotiate only the valuation of the buyer is uncertain. Despite this restriction, we achieve great generality in modeling the resale market.[11]

Section 2 introduces the model. Section 3 develops a simple example that illustrates the main economic insights of this paper. Section 4 presents our main results. Section 5 discusses efficiency and section 6 concludes the main text. Appendix A contains all the proofs. Appendix B discusses multiplicity of equilibria and Appendix C develops an example with multiple types and illustrates the complications that arise.

2 Model

A trading network is an undirected and connected graph $G = (N, E)$, where $N = \{1, \ldots, n\}$ is the set of traders and $E \subseteq 2^{N \times N}$ represents potential trading relationships.[12] The existence

[9] In Fudenberg, Levine and Tirole (1987) a seller with known valuation faces multiple buyers but once the object is sold there is no resale.

[10] Commenting on the difficulties that he encountered, Zheng notes that "extending the positive result beyond the special case is difficult if not impossible".

[11] There are other papers that study the effect of resale opportunities on market outcomes, e.g. Calzolari and Pavan (2006), Haile (2003), Garratt and Troger (2006), Krishna and Hifalir (2008) and Jehiel and Moldovanu (1999). All these papers also focus on a restricted value space, on a special resale protocol, or on complete information.

[12] A network is *undirected* if $ij \in E$ implies $ji \in E$. A *path* between i to j is a non-empty graph where the set of vertices is $\{i, b_1, \ldots, b_m, j\} \subseteq N$ and the set of edges is $\{ib_1, b_1b_2, \ldots, b_mj\} \subseteq E$. A network is *connected* if there is a path between every pair of traders.

of an edge ij in E indicates that traders i and j can trade. There are two goods, money, that everyone owns in large quantity, and an indivisible object, initially owned by trader 1. Each trader i is risk-neutral and has a binary private monetary evaluation for the object, $v_i \in \{v_L, 1\}$, where $0 \leq v_L < 1$.[13] Traders are ex-ante heterogeneous and values are independently distributed. The common prior probability that $v_i = 1$ is $\pi_i \in (0, 1)$.

Trading takes place in time, starting from period $t = 0$. The game consists of T rounds of trade; the horizon is infinite if $T = \infty$. An arbitrary round t develops in three stages:[14]

1st. The current owner of the object makes a *take-it-or-leave-it* offer at price p to one of her neighbors i.[15] The game proceeds to the second stage.

2nd. Trader i decides whether to *accept* or to *reject* the offer. In case of rejection the game proceeds to the third stage. In case of acceptance treader i becomes the new owner and she transfers an amount p of money to the seller. The game proceeds to the third stage.

3rd. The current owner of the object decides whether to *consume* the object or *not consume.* The game ends if the object is consumed. Otherwise, unless $t = T$, the game proceeds to the first stage of round $t + 1$.

Traders discount the future at a common rate $\delta \in (0, 1)$; when we consider games with finite horizon we allow for $\delta = 1$. If agent i consumes the object in period t and pays p_i his utility is $\delta^t(v_i - p_i)$, while if she does not consume the object but pays p_i his utility is $-\delta^t p_i$.

We assume that all actions are observed by all traders, and that everything, but the private values, is common knowledge.[16] The quadruple $\langle G, \boldsymbol{\pi}, T, \delta \rangle$ represents a *network trading*

[13]We work with a two-type model to avoid the complications that arise in bargaining with two-sided asymmetric information. For the same reason, we also need to assume that the high-value v_H is the same for all traders. We note however that our results are robust to the extension where v_L is heterogenous across players. We discuss the implications of relaxing the two-value restriction in Appendix C.

[14]The timing has been selected with the aim of minimizing notation. Nothing would change if a seller were able to consume before making an offer.

[15]To keep the strategy space compact we maintain that prices must be selected from $[0, M]$, where M is large enough. A seller can always decide to let one round pass by making an implausibly high offer.

[16]The assumption that offers are observed by everyone allows us to work with a common posterior. However

game, which is a multi-stage extensive form game with observed actions and independent types. The strategy of each trader specifies an action for both the high and low-value at each non terminal public history h where she is playing. A system of beliefs specifies, for all h, a profile of common posterior probabilities $\boldsymbol{\mu}(h) = (\mu_1(h), \ldots, \mu_n(h))$, where $\mu_i(h)$ indicates the probability that player i has value one. We will often write $\boldsymbol{\mu}^t = \langle \boldsymbol{\mu}^t_{-i}, \mu^t_i \rangle$ for the profile of beliefs at the beginning of a round t, omitting reference to the particular history.

The adopted solution concept is *weak-Markov perfect Bayesian equilibrium.* A perfect Bayesian equilibrium (henceforth PBE) is a strategy profile and a belief system such that the strategies are sequentially rational given the belief system, and the belief system is consistent with Bayesian updating on path and off-path whenever possible (see Fudenberg and Tirole (1991)). In a weak-Markov PBE the strategy of the seller can depend on the previous history, whereas the acceptance and consumption strategy of a trader depends only on her private information, the price demanded, the identity of the seller, the state of beliefs and the number of rounds left (when the horizon is finite).[17]

3 Illustrative Example

We develop an example with $n = 5$ traders connected in the network depicted in figure 1. We set $T = 4$, $v_L = 0$ and $\delta = 1$. Trader 1 is the initial owner and the profile of initial beliefs is $\boldsymbol{\pi} = (0, 1/3, 1/2, 1/3, 2/3)$. Despite this is a specific network trading game, the equilibrium properties we shall emphasize are general properties, as shown in Section 4.

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this assumption is not needed to sustain equilibria that are in pure strategy along the equilibrium path. This is the case for our illustrative example in section 3.

[17] In a strong-Markov PBE the seller's strategy only depends on the state of beliefs and the number of rounds left. It is well know that strong-Markov PBE do not always exist in bargaining games with incomplete information, see Fudenberg, Levine and Tirole (1983). In fact, it may be necessary that the probability of acceptance of some trader is constant over some interval of prices. In this case the seller posterior will be the same after each of the offers in the interval is refused. But for the probability of acceptance to be constant, the seller's next offer will have to depend on the current one.

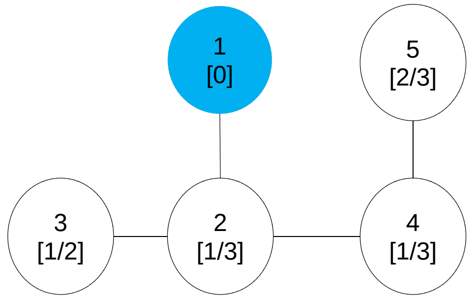


Figure 1: A trading network: trader 1 is the owner and $\boldsymbol{\pi} = (0, 1/3, 1/2, 1/3, 2/3)$.

3.1 Equilibrium with complete information.

As a benchmark, we briefly discuss the case of complete information. Assume that the profile of values is given and is common knowledge. In this case, all subgame-perfect equilibrium outcomes are Pareto efficient. Furthermore, if there is at least a trader with value one, the object is traded at price 1 and the initial seller extracts all the surplus. For instance, assume that trader 5 has high value and all other traders have low value. In equilibrium the good flows from trader 1 to trader 5, via trader 2 and trader 4, each transaction occurs at price 1, and trader 5 consumes the object.

3.2 Equilibrium under incomplete information.

Consider now the incomplete information case. We first present the equilibrium path and then provide a brief description of the main steps involved in its construction. We conclude with the description of the main equilibrium properties.

Summary of the Equilibrium Path. In the first period trader 1 offers the object to trader 2 at her resale value 5/6. Trader 2 purchases the object and consumes it if she has high-value. Otherwise, in the second period of trade, trader 2 asks price 1 to trader 3. Trader 3 accepts the offer and consumes the object if, and only if, she has high-value. Otherwise, trader 2 remains the owner and, in the third period of trade, she offers the object to 4 at trader 4's resale value, which is 2/3. Trader 4 buys the object regardless of her value. Finally, if trader 4 has a high-value she consumes the object, otherwise, in the last round of trade, she offers the good to trader 5 at a price 1. Trader 5 accepts the offer and consumes the good if she has high-value, otherwise the game ends.

Equilibrium Construction. Trader 1 has low-value and therefore she sells the object to trader 2. To determine the price that trader 1 asks we need to determine trader 2's strategy when she receives an offer from 1 in the first round.

First, suppose that agent 2 is a low-value trader. In this case the only reason for purchasing the object is to resale it to other traders. Hence, agent 2's willingness to pay is the expected payoff that she obtains in the continuation game in which she owns the object, there are three rounds of trade left, and the beliefs are $\boldsymbol{\mu}^2 = (0, 0, 1/2, 1/3, 2/3)$.[18] We term this payoff trader 2's *resale value*, we denote it by r_2, and we derive it next.

In the continuation game, illustrated in figure 2(a), there is only one equilibrium. Trader 2 asks price $p_3^2 = 1$ to trader 3, who accepts the offer if she has high-value and rejects it otherwise.[19] In case of rejection, trader 2 makes an offer to agent 4 at a price which is equal to trader 4's resale value, that is $r_4 = 2/3$. Trader 4 accepts this offer regardless of her value (the reason is explained in the next paragraph). Once trader 4 becomes owner she consumes the object if she has high-value. Otherwise, she asks a price of $p_5^4 = 1$ to trader 5 in the last round of trade. Trader 5 accepts the offer only if she has high-value. We obtain:

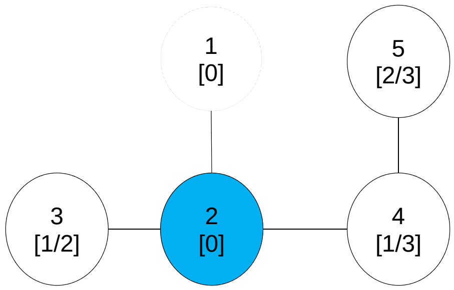
$$r_2 = \Pr[3 \text{ accepts } p_3^2] \times p_3^2 + \Pr[3 \text{ rejects } p_3^2] \times \Pr[4 \text{ accepts } r_2] \times r_4 = \frac{1}{2} + \frac{1}{2} \times \frac{2}{3} = \frac{5}{6}.$$

Second, suppose that trader 2 has high-value, and take as given the strategy of trader 2 when she has low-value. By rejecting a price below her resale value, trader 2 signals that she has high-value. In this case her expected payoff is zero because, whenever trader 1 is certain that trader 2 has value one, trader 1's optimal strategy is to ask a price above one until the last round and then ask a price of one to trader 2. Next, consider an offer that is above trader 2's resale value r_2. Given low-value trader 2's strategy, if trader 2 rejects an offer above r_2 all traders will believe that the probability that trader 2 has value one lies somewhere in the interval $[0, 1/3]$. Hence, the expected payoff that trader 2 obtains by rejecting that offer is

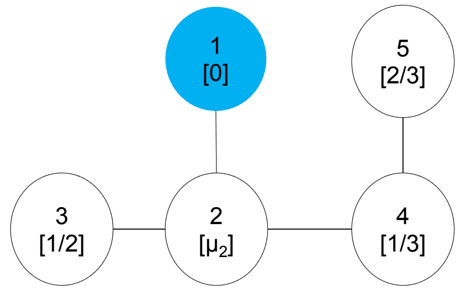
[18] Since in equilibrium high-value traders always consume the object, once trader 2 makes an offer it becomes common knowledge that she has a low value, i.e., $\mu_2^2 = 0$

[19] If trader 3 has high value she accepts any offer up to one (and then consumes the object) because by rejecting she will never receive another offer in the remaining rounds of trade.

the expected payoff that high-value trader 2 obtains in the continuation game where trader 1 is the owner, there are three rounds of trade left, and beliefs are $\boldsymbol{\mu}^2 = (0, \mu_2^2, 1/2, 1/3, 2/3)$, for some $\mu_2^2 \in [0, 1/3]$. This payoff, which we denote $V_2(\mu_2^2)$, is derived as follows.



(a) Three rounds left, trader 2 is the owner



(b) Three rounds left, trader 1 is the owner.

Figure 2: Two different continuation games.

In the continuation game, illustrated in figure 2(b), there is only one equilibrium payoff for trader 2. Trader 1 asks for trader 2's resale value, which is 2/3. Trader 2 accepts the offer regardless of her type and consumes the object if she has high value. Otherwise she asks a price of 1 to trader 3 and then to trader 4, and each of these offers is accepted only by high-value traders. Concluding, if high-value trader 2 rejects a price above her resale value r_2, she will buy the object in the subsequent round at a price of 2/3 and will then consume, obtaining a net payoff of $V_2(\mu_2^2) = 1/3$, for all $\mu_2^2 \in [0, 1/3]$.

It is now possible to verify that a high-value trader 2 strictly prefers to reject every price above $r_2 = 5/6$, because rejection would allow her to buy at a lower price in the subsequent round. Because trader 2 will reject any offer above 5/6 regardless of her value, and because waiting one round provides player 1 with a payoff of 2/3, we can conclude that in equilibrium trader 1 will make an offer to trader 2 at price 5/6.

Main Equilibrium Properties. There are four important properties that emerge from the description of the equilibrium path, which we now discuss in turn. As we shall show in section 4, even if we have derived these properties in an example where the horizon is finite and agents are perfectly patient, these are robust equilibrium properties of network trading games with finite and infinite horizon, and with impatient agents.

(1) Types of offers. There are two types of offers along the equilibrium path: *resale offers* and *consumption offers*. A resale offer is at a price that is accepted both by a high-value and low-value trader. Trader 2 receives a resale offer in period 1 and trader 4 receives a resale offer in period 3. A consumption offer is at a price that makes a high-value trader indifferent between accepting and rejecting the offer, whereas a low-value trader strictly prefers to reject the offer. Trader 3 receives a consumption offer in period 2 and trader 5 receives a consumption offer in period 4.

(2) Types of traders. There are two types of traders in equilibrium: *clients* and *dealers*. A client receives only consumption offers. Trader 3 and trader 5 are clients. A dealer receives a resale offer with positive probability. Trader 2 and trader 4 are dealers.

(3) Price Dynamics. Consider the sequence of equilibrium offers $(r_2, p_3^2, r_4, p_5^4) = (5/6, 1, 2/3, 1)$. Figure 3 illustrates the pattern of prices over time; the bold points in the figure are the prices of resale offers, the other points are the prices of consumption offers (the point at time zero is trader 1's resale value). The prices associated to resale offers are declining along the sequence of equilibrium offers. This reflects the fact that later dealers are more pessimistic about the profitability of selling the object. However, the price sequence is in general non-monotonic, because each consumption offer is at a price which is higher than every price asked in future resale offers. This reflects the idea that dealers are able to exploit their local market power against some of their directly connected traders.[20]

(4) Payoff ranking among traders. Clients and low-value dealers earn zero profits. Dealers with high-value obtain a positive expected profit from buying at a price lower than their valuation and consuming. Furthermore, earlier dealers obtain a higher expected payoff than later dealers. Despite earlier dealers acquire the object at a higher price than later dealers, the former have a higher probability of acquiring the good than the latter. The

[20]In this example, all consumption offers come at a price of 1. Observation 3 of Appendix B provides an example of an equilibrium where some consumption offers are below 1.

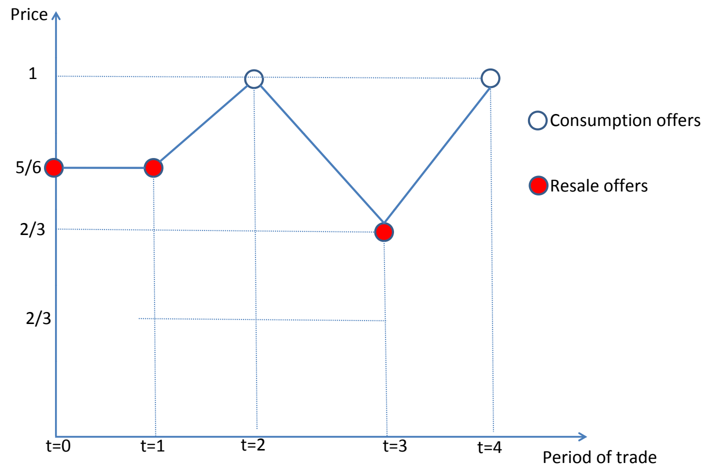


Figure 3: Price Dynamics.

second effect dominates the former. The decline of the price associated to resale offers incorporates only the decline in the expected demand due to the rejection of consumption offers, whereas the difference in the probability of acquiring a good also accounts for the possibility that dealers consume the good themselves. In our example, high-value trader 2 would acquire the object for sure at price 5/6, obtaining a payoff of 1/6. High-value trader 4 would acquire the object with probability 1/6 and, in that event, she would pay a price of 2/3, obtaining an expected payoff of $1/6(1-2/3) = 1/9$.

We conclude with two remarks. First, the equilibrium play with incomplete information generates a number of features that are in stark contrast with the equilibrium properties under complete information. Notably, with asymmetric information the sequence of demanded prices are not constant and dealers obtain a positive profit. Second, even though in this example the equilibrium outcome is ex-post efficient (i.e., the object is consumed by a high-value trader, if any exists), this does not hold in general. Section 5 provides general insights on the efficiency of equilibrium outcomes.

4 Characterization of Equilibria

Our characterization is recursive. We characterize equilibrium strategies at an arbitrary period t by taking as given continuation payoffs accruing from period $t+1$ onward. When the horizon is finite, we assume that all continuation payoffs at $T+1$ are equal to zero.

In a weak Markov PBE the acceptance and consumption strategy of a trader depends only on her private information, the price asked, the identity of the seller and on the number of rounds left. This implies that the equilibrium continuation payoff of a seller, at the beginning of each round, depends only on her private information, the state of beliefs, and the number of rounds left. We call *resale value* the continuation payoff of a low-value seller $i \in N$ at the beginning of round t, when $\mu_i^t = 0$ and the other beliefs are determined by previous history. We denote this quantity with R_i^t, omitting reference to beliefs.

We write the continuation payoff of a trader who rejects an offer as a function of the state of beliefs that can be induced by her action. In particular, omitting reference to the current history, $V_i^{t+1}(\mu_i)$ indicates the continuation payoff of a high-value trader if she refuses the current offer at t, given that the belief about her value in $t+1$ will be μ_i, while all other posterior beliefs, $\boldsymbol{\mu}_{-i}^t$, remain constant given the past history. Finally, omitting explicit reference to the seller, beliefs and the number of rounds left, let $a_j(p, v_j)$ indicate the acceptance strategy of trader j with value v_j if she gets an offer at price p.

Proposition 1. *All weak-Markov PBE of any network trading game with finite horizon satisfy the following conditions.*

LOW-VALUE TRADER. *In every round t (whenever applicable) a low-value trader i:*

1-L. Makes an offer (j, p) that maximizes:

$$E_{v_j}[a_j(p, v_j)]p + (1 - E_{v_j}[a_j(p, v_j)]) \max\{v_L, \delta R_i^{t+1}\}.$$

2-L. Accepts an offer at price p if $p \leq \max\{v_L, \delta R_i^{t+1}\}$ and rejects otherwise.

3-L. Consumes if $v_L > \delta R_i^{t+1}$, does not consume if $v_L < \delta R_i^{t+1}$, and otherwise plays either of the two.

HIGH-VALUE TRADER. *In every round t (whenever applicable) a high-value trader i:*

1-H. Makes an offer to one of his neighbors at a price equal or higher than 1.

2-H. Accepts an offer at price p if $p \leq \max\{v_L, \delta R_i^{t+1}\}$; otherwise plays one of the following:

- *accepts the offer if $p \leq 1 - \delta V_i^{t+1}(0)$;*
- *rejects the offer if $p \geq 1 - \delta V_i^{t+1}(\mu_i^t)$;*

– *accepts with probability* λ *if there exists a* $\lambda \in (0,1)$ *such that*

$$p = 1 - \delta V_i^{t+1}(\mu_i^{t+1}), \text{ where } \mu_i^{t+1} = \frac{(1-\lambda)\mu_i^t}{1-\lambda\mu_i^t}.$$

3-H. Consumes the object.

The first part of proposition 1 characterizes the equilibrium behavior of low-value traders. The willingness to pay of a low-value trader is the maximum between her consumption value and her discounted resale value (2-L).[21] A low value trader always acquires the object to resell it and will make an offer that maximizes her expected revenues, taking into account that once she manages to sell the object her continuation payoff would be zero (1-L).[22] A low-value trader will consume only if the discounted resale, after having attempted an offer, falls below her consumption value v_L (3-L).

The second part of the proposition characterizes equilibrium strategies of high-value traders. Points 1-H and 3-H jointly state that high-value traders always consume, because they cannot profit from re-selling the good, even though it may be possible for them to reacquire it later at a lower price. The price differential that a high-value trader may accrue by selling today and buying in the future will not compensate her for the expected loss suffered in case of consumption by some other trader. In equilibrium there are no arbitrage opportunities.

2-H describes the equilibrium acceptance strategy of high-value trader i. Rejecting an offer at a price lower than or equal the willingness to pay of a low value trader, $\max\{v_L, \delta R_i^{t+1}\}$, cannot be a best reply for trader i, as that would signal that she has high-value for the object. Therefore, consider an offer at price $p > \max\{v_L, \delta R_i^{t+1}\}$. Recall that if trader i has low-value, she rejects such offer. In an equilibrium where the high-value trader accepts, a rejection signal that trader i has low-value and trader i obtains a continuation payoff of $V_i^{t+1}(0)$. Hence, accepting the offer is compatible with equilibrium play if, and only if, the

[21] Generically (i.e., if there is no other offer that gives the same payoff to the seller), it is not possible that a low value trader refuses an offer at $\max\{v_L, \delta R_i^{t+1}\}$; otherwise the seller would offer a slightly lower price.

[22] In the equilibrium path, when $v_L > 0$, a low-value trader who acquires the object will always try to resell it. Suppose, in fact, that a low-value trader i acquires the object in t and consumes it. Then, she must have received an offer at price v_L in t. However, this is not possible since, following 3-L, the trader selling to i would have consumed in $t-1$.

payoff from acquiring the object and consuming it, that is $1 - p$, is above $\delta V_i^{t+1}(0)$. In contrast, consider an equilibrium where the high value trader i rejects the offer. In this case, upon rejection, the beliefs about trader i remain unchanged and his expected payoff is $\delta V_i^{t+1}(\mu_i^t)$. Hence, we must have $1 - p \leq \delta V_i^{t+1}(\mu_i^t)$. Finally, trader i may randomize between acceptance and rejection in such a way that, given the belief that she induces (and the ensuing continuation), she becomes indifferent between accepting and rejecting. Note that when $V_i^{t+1}(\mu_i^t) < V_i^{t+1}(0)$, and trader i receives an offer at a price in $(1 - \delta V_i^{t+1}(0), 1 - \delta V_i^{t+1}(\mu_i^t)]$, then there is no equilibrium where the high value trader i plays a pure strategy.

Proposition 1 characterizes a set of properties of all weak-Markov PBE of network trading games with finite horizon. This characterization is also informative of equilibrium behavior in network trading games with infinite horizon. Indeed, since the game we are analyzing has a compact strategy space and is continuous at infinity, any sequence of weak-Markov PBE equilibria generated as $T \to \infty$ will have a limit equilibrium, which is a weak-Markov PBE equilibrium of the infinite horizon game.[23] Observe that the strategies outlined in proposition 1 depend on the remaining time only through continuation payoffs.

Remark 1. *Fix G,$\boldsymbol{\pi}$ and δ, and let e_T be a sequence of weak-Markov PBE for $T \to \infty$. Each sequence e_T has a limit point, which is a weak-Markov PBE of the infinite horizon game and satisfies the properties of proposition 1.*

Remark 2. *All our forthcoming results also apply to these limit equilibria. Hence, in order to avoid lengthy qualifications, whenever we say "every equilibrium" we mean, unless otherwise specified, (i) every equilibrium of the finite horizon game and (ii) every equilibrium of the infinite horizon game that is the limit of a sequence of finite horizon equilibria as $T \to \infty$.*

Dynamic bargaining games with more than two players have typically multiple weak-Markov PBE (e.g., Fudenberg, Levine and Tirole (1987)). While proposition 1 illustrates a number of properties that hold for all weak-Markov PBE, we need to impose a further restriction if we wish to characterize in greater details the price-dynamic and the effect of a trader's network location on her equilibrium payoff. To understand why we cannot

[23]See Fudenberg and Levine (1991), Fudenberg, Levine and Tirole (1985), Sobel and Takahshi (1983), Cramton (1984). As it is common in these games there may be weak-Markov PBE of the infinite horizon game that do not satisfy proposition 1.

provide sharp predictions on prices and utilities without further restrictions consider, for instance, the case where $V_i^{t+1}(\mu_i^t) > V_i^{t+1}(0) > R_i^{t+1}$. Whenever a price in the range $(1 - \delta V_i^{t+1}(\mu_i^t), 1 - \delta V_i^{t+1}(0))$ is asked to i, both acceptance and rejection are best replies. Hence, trader i's equilibrium behavior at any price in that range is essentially unrestricted. For example, for any three prices in that interval, which may be arbitrarily close, we can construct an equilibrium where trader i rejects the lowest and highest, but accepts the middle one. Appendix B discusses extensively the issue of equilibrium multiplicity in the context of an example. In the remaining of this section, in order to impose some discipline on prices, we focus an a class of weak-Markov PBE that satisfies the following regularity property.

Definition 1. *A weak-Markov PBE is* regular *if every high-value trader i accepts, in any round t, all offers that come at prices less than or equal to $1 - \delta V_i^{t+1}(0)$.*

We remark that in a regular weak-Markov PBE, hereinafter *regular equilibrium*, we do not require that all traders adopt the same acceptance strategy. Moreover, for regularity to potentially restrict the set of weak-Markov PBE outcomes it must be the case that, for prices below $1 - \delta V_i^{t+1}(0)$, there exist best-replies of high value trader i that are different from pure acceptance.[24]

Proposition 2. *A regular equilibrium exists in every network trading game. Furthermore, for any given N, π, δ and T, there are networks (i.e., sets E) for which every weak-Markov PBE is a regular equilibrium.*

In the proof of Proposition 2 we show that when the network is a star, regardless of whether the owner is the central player or a peripheral player, every equilibrium is regular.[25] The arguments developed in the proof can be extended to other class of networks, such as

[24]This occurs only when the continuation payoff of a high value player after rejecting an offer that does not change the beliefs that other traders have about her value, is higher than the continuation payoff of a high value player after rejecting an offer that makes other traders believe that she has a low value, $V_i^{t+1}(\mu_i^t) > V_i^{t+1}(0)$. While this is counterintuitive, we cannot rule it out, as shown in Appendix B. We conjecture however that this phenomenon can be generically eliminated when the horizon is large enough – hence the regularity condition eliminated – by introducing small transaction costs that prevent sellers from being indifferent among different options. See Appendix B for more details.

[25]A star is a network where one player (the center) has a link with each other node (peripheral nodes) and there are no other links.

the line where the initial owner is one of the end agents. The next proposition establishes a property of prices demanded in regular equilibria that is key for our forthcoming results.

Proposition 3. *In every regular equilibrium each offer made to trader i in round t, which is accepted with strictly positive probability, is either:*

- *A* resale offer*: an offer at a price which equals the resale value of i;*
- *A* consumption offer*: an offer at a price higher than trader i's resale value and that makes high value trader i indifferent between accepting and rejecting the offer, i.e., $1 - p = \delta V_i(\mu_i^{t+1})$, where μ_i^{t+1} are the equilibrium updated beliefs in case of refusal.*

We note that in every weak-Markov PBE an offer is either a resale offer or an offer that a low-value trader always rejects. The additional, key property, that holds in regular equilibria is that every offer to trader i above her resale value makes the high-value i indifferent between accepting the offer and rejecting it.

4.1 Price dynamics

In a regular equilibrium the sequence of offers and exchanges develops as follows. Whenever a trader acquires the object she consumes it if she has high-value. If she has low-value she makes a sequence of offers until the object is sold (unless at some point the resale value of the object is lower than her consumption value). In particular, the owner starts by making a sequence of consumption offers to some of her neighbors. If these offers are rejected she makes a resale offer which is accepted for sure. Hence, all possible public histories of the game can be summarized by a list of consumption offers and resale offers.

We call *trading chain* a generic public history of offers that may arise in the event in which all traders have low-value. Formally, the trading chain is a list $(p_1^1, p_2^1 \dots, r_2^1, p_1^2, p_2^2 \dots, r_3^1, \dots)$, where p_i^s indicates the ith consumption offer made by the sth seller, and r_s^{s-1} indicates the resale offer that the $(s-1)$th seller makes to the sth seller. If along the equilibrium path the s-th seller has always a *unique* optimal offer, the resale offer that seller $s-1$ asks to agent s is:

$$r_s^{s-1} = \delta\alpha_1 p_1^s + \delta^2(1-\alpha_1)\alpha_2 p_2^s + \dots + \prod_{i=1}^{k}(1-\alpha_k)\delta^k r_{s+1}^s, \tag{1}$$

where α_k indicates the probability that the kth consumption offer made by the sth seller is accepted. In words, the resale value of a trader who acquires the object is equal to the expected discounted sale price.

In general, a seller could be indifferent among different optimal offers, in which case equilibria exist where the seller randomizes. In these equilibria the trading chain is not unique, and all offers in the chain follow a stochastic process determined by the sellers' equilibrium strategies. Equation 1 must hold for every realization of the random variables. Otherwise, seller s would be making a suboptimal offer at some point.

The following result provides a sharp description of the dynamics of discounted prices asked in equilibrium. Whenever a comparison among two offers in arbitrary rounds t and $t' > t$ is made, let $k = t - t'$.

Proposition 4. *In every regular equilibrium:*

1. *The discounted price asked in resale offers is decreasing along the trading chain:* $r^s_{s+1} \geq \delta^k r^{s'}_{s'+1}$ *for all* $s \leq s'$.

2. *The discounted price asked in every consumption offer is greater than the discounted price asked in every subsequent resale offer:* $p^s_i > \delta^k r^{s'}_{s'+1}$ *for all* $s \leq s'$*;*

3. *For every pair of resale offers that come at different discounted prices, there exists a consumption offer in between the two that comes at a discounted price which is strictly higher than the discounted prices asked in both resale offers.*

To understand the first part of proposition 4 consider two consecutive resale offers, the first at period t and the second at period $t + x$. The trader who receives the resale offer at $t + x$ knows that all consumption offers from period $t + 1$ to period $t + x - 1$ have been rejected. Since when a trader rejects a consumption offer all other traders update downward the beliefs that she has a high consumption value, the trader who receives the resale offer at period $t + x$ is more pessimistic about the profitability of reselling the good as compared to the trader who receives the resale offer at period t. Hence, the discounted price asked in resale offers declines overtime.[26]

[26] If the game has finite horizon a shorter deadline tends, ceteris paribus, to depress resale prices as the opportunities for selling the object necessarily decrease.

The result that discounted prices in consumption offers are above discounted prices in subsequent resale offers reflects the ability of sellers to use their local bargaining power to demand a high price to some of their neighboring traders, before passing the object to another dealer. To gain an intuition for this phenomenon let's consider a special case in which a seller s makes a consumption offer to trader i at price p_i and, if rejected, a resale offer at price r_j to trader $j \neq i$ who, in turn, makes a sequence of consumption offers and then a resale offer to i at price r_i.

We know that the consumption offer to trader i leaves her indifferent between accepting the offer and rejecting it. Abstracting from discounting:

$$1 - p_i = \Pr(\xi)[1 - r_i] \qquad \Longleftrightarrow \qquad p_i = 1 - \Pr(\xi) + \Pr(\xi) r_i$$

where $\Pr(\xi)$ is the probability that trader i receives her resale offer after refusing the consumption offer. This is the probability that trader j does not consume the object, times the probability that all those traders who receive a consumption offer from j do not accept such offer. We can then conclude that the resale value of j, r_j, is bounded from above by $1 - \Pr(\xi) + \Pr(\xi) r_i = p_i$, as every consumption offer is no higher than 1.[27]

The last part of the proposition remarks the non-monotonicity in the price demanded over time. It follows by combining two observations. First, equilibrium discounted resale values decline over time (part 1 of proposition 4). Second, in equilibrium, the discounted resale value of an agent equals the expected discounted price at which she will sell the object (equation 1).

We finally note that, while demanded prices are non-monotonic in time, actual transaction prices decline as time passes, possibly with the exception of the last accepted offer. In fact, after an offer is accepted, trading will only continue if a low-value trader acquires the object. Hence, all accepted offers, except the last one, must be resale offers.

[27]More formally, let $1 - \mu_j$ be the probability that trader j does not consume the object, and let α_j the probability that all those who obtain consumption offers from j do not accept the offer. Hence: $p_i = 1 - \Pr(\xi) + \Pr(\xi) r_i = 1 - \alpha_j(1 - \mu_j) + \alpha_j(1 - \mu_j) r_i^{s'} \geq 1 - \alpha_j + \alpha_j r_i$. Next, note that $1 - \Pr(E) + \Pr(E) r_i^{s'}$ is an upper bound for r_j, because $r_j \leq 1 - \alpha_j + \alpha_j r_i$.

4.2 Payoff ranking

In order to discuss equilibrium payoffs we classify all traders (except the initial owner) into three categories.

Definition 2. *In a given equilibrium we say that a trader is A)* inactive *if, with probability one, she receives no offers; B) a* client *if she is active and the probability of getting a resale offer is zero; C) a* dealer *if there is a positive probability of obtaining at least one resale offer.*

With this taxonomy in mind, the following corollary is a direct consequence of proposition 1 and proposition 3.

Corollary 1. *In every regular equilibrium*[28]

1. *Every trader (except for the initial owner) is either inactive, client or dealer.*
2. *A trader obtains a strictly positive expected payoff if, and only if, she is the initial owner, or she has high-value and she is a dealer.*

Part one of the corollary follows trivially from proposition 1. The fact that the initial seller makes a strictly positive payoff is evident. All low-value traders make zero expected payoff because in every weak-Markov PBE no offer is ever made at a price below the willingness to pay of a low-value trader. To see that clients obtain zero profits consider the last offer that a client obtains in equilibrium. By proposition 3 this offer must leave the high-value trader indifferent between accepting and rejecting it, and since it is the last offer she receives, it must come at a price of one. This implies that all previous offers must be at a price of one as well. Finally, high-value dealers obtain a positive expected payoff because, with some probability, they obtain an offer at their resale value. Since every earlier consumption offer must keep them indifferent between accepting and rejecting, such consumption offers must be at a price strictly below one. Hence, at the beginning of the game, a dealer with high-value expects to make a positive payoff.

[28]Part 1 holds for every weak-Markov PBE (not necessarily regular) part 1 of the corollary holds. Furthermore, in every weak-Markov PBE, low-value dealers obtain zero expected payoffs, whereas the initial owner and high-value dealers obtain a strictly positive expected payoff. It is possible to construct examples of non-regular equilibria where clients obtain strictly positive expected payoff.

Because dealers are the only traders, other than the initial seller, who make a positive profit, we now examine how the position of a dealer in the equilibrium trading chain affects her payoff.

Proposition 5. *In every regular equilibrium:*

1. *If all offers to trader j are preceded with certainty by a resale-offer to trader i, then the discounted expected utility of a high-value trader i is strictly greater than that of a high-value trader j.*

2. *If all resale-offers to trader j are preceded with certainty by a resale-offer to trader i and $\pi_i \geq \pi_j$, then the discounted expected utility of a high-value trader i is strictly greater than that of trader j.*

The main intuition behind the result is that despite later traders pay lower prices than earlier traders, this price differential does not compensate them for the decrease in the probability of obtaining the offer. The decline in price only offsets the expected demand of clients, but does not incorporate the possibility that dealers may consume the object.[29]

Proposition 5 and corollary 1 have sharp implications on the relation between the location of traders in the trading network and their payoffs, as shown in the next corollary. We say that trader j is *essential* for trader i if j belongs to every path connecting i to the initial owner. A trading network G is a *tree* if there is only one path between every pair of traders. A trader is an *end-trader* if she is connected only to another trader and she is not the initial owner.

Corollary 2. *In every regular equilibrium*

1. *Every end-trader obtains zero profit.*

2. *If trader j is essential for trader i, then high-value trader j obtains a higher expected discounted profit than high-value trader i.*

[29] In Appendix B we develop an example of a non-regular equilibrium where proposition 5 does not hold. This equilibrium is eliminated by the introduction of small transaction costs.

3. *If the trading network is a tree, then in every path starting from the initial owner the expected discounted payoff of high-value traders in the path declines with their distance from the initial owner.*

The corollary points out the importance of the location of a trader in a trading network. It emphasizes that traders who are essential in connecting other traders to the initial owner, obtain a payoff advantage. As we discussed in the introduction, this result provides a micro foundation for the prominent sociological theory of structural holes (see Burt (1992)), which postulates that individuals who have a bridging position in a social/economic network tend to obtain a payoff advantage.

5 Efficiency

This section discusses the efficiency properties of weak-Markov perfect equilibria – not necessarily regular. We first show that equilibrium outcomes are Pareto efficient under complete information, regardless of the architecture of the network. We then note that under incomplete information ex-post inefficiencies arise generically whenever there is a deadline and/or discounting. We conclude by showing that as the horizon of the game becomes infinite and discounting vanishes equilibrium outcomes are ex-post efficient.

We start with some definitions. A *feasible outcome* of the game is an allocation of the goods (object and money) to the traders that is achievable within T periods of trade. An outcome is *Pareto efficient* if it is feasible and there is no alternative feasible outcome that would make all traders weakly better off and one trader strictly better off. Absent discounting, an outcome is Pareto efficient if, and only if, the object is consumed by a high-value trader, whenever at least one such trader exists and is reachable within T rounds of trade. When $0 < \delta < 1$ efficiency requires that the object is either consumed by the initial seller or consumed by the high-value trader that is closest to the initial seller. Under incomplete information, an outcome is a mapping from the set of all possible profiles of values into the set of possible feasible outcomes. Following Holmstrom and Myerson (1983), we say that an outcome is *ex-post efficient* if it is Pareto efficient in the classic sense for every profile of values.

Our first result illustrates that, in network trading games, network incompleteness does not give rise to inefficiencies as long as there is complete information.

Proposition 6. *Consider a network trading game where traders' valuations are common knowledge. Every subgame-perfect equilibrium outcome is Pareto efficient.*[30]

In every subgame perfect equilibrium, high-value traders accept every price up to one. Low-value traders accept every price up to the maximum between their consumption value and their resale value. The resale value of a low-value trader directly connected to a high-value trader is equal to one. Traders who are separated from high-value traders by one other agent have a resale value of δ, and so on. Hence, the seller's neighbor with the highest resale value corresponds to the trader who has the shortest path to a high-value trader. We can then establish that a seller consumes if her consumption value is greater than the highest resale value in her neighbor. Otherwise, the seller approaches the neighbor with the highest resale value and offer the object at her resale value. These considerations imply that every equilibrium outcome is Pareto efficient.[31]

We now consider the case of incomplete information. We first note that under discounting ex-post inefficiencies are unavoidable. An initial seller with a low-value must consume the object immediately whenever high-value traders are sufficiently faraway in the network, whereas, if such traders are sufficiently close, the object should flow to one of such high-value agents following the shortest path. It is clear that, regardless of the network architecture, under asymmetric information there is no equilibrium that achieves an ex-post efficient outcome.[32] Hence, we focus on the limit case where agents become perfectly patient, i.e., $\delta \to 1$. In this case, an equilibrium outcome converges to an ex-post efficient outcome if, and only if, along the equilibrium path the object is consumed by a high value trader, if any exists.

Even when discounting vanishes, ex-post inefficient equilibrium outcomes may be present in finite horizon games as a result of the combination of asymmetric information and in-

[30]We remind the reader that when we say "every subgame-perfect equilibrium" we mean every subgame-perfect equilibrium of the finite horizon game and every subgame-perfect equilibrium of the infinite horizon game that is the limit of a sequence of finite horizon subgame-perfect equilibria as $T \to \infty$ (see Remark 1).

[31]This result would hold even if valuations were drawn from some arbitrary set.

[32]Discounting may generate other types of inefficiencies, for example due to delays generated by the seller in the attempt to price-discriminate.

completeness of trading opportunities. In particular, inefficiencies may take place when a trader i, with high *expected* value, provides monopolistic access to another trader j, with low *expected* value. In that case, the object may never reach agent j, even when she is the only trader with a high-value for the object. This happens because when trader i has high enough expected value (relative to the expected value of j) the seller prefers to ask her a price of one rather than her resale value. The ability of the seller to convince trader i to accept a price of one rests on the possibility of making that offer close enough to the deadline. In fact, when the deadline approaches, the seller can credibly threaten trader i that if she rejects the offer she will not receive any offer at a lower price in the future. The example below illustrates this point.

Example 1. *There are three traders, trader* 1 *is the initial owner, who has a link with trader* 2*, who has a link with trader* 3 *(i.e.* $N = \{1, 2, 3\}$ *and* $E = \{12, 23\}$*). The initial profile of prior beliefs is* $\boldsymbol{\pi} = (0, \pi, 1/2)$*. We set* $\pi > 1/2$*,* v_L *sufficiently low,* $T = 2$ *(nothing changes if* $T > 2$*) and* $\delta = 1$*.*

In this example, every equilibrium is payoff equivalent and has the following structure: with probability λ *trader* 1 *ask a price above one in the first round and in the second round asks a price of one to trader* 2*, and with the remaining probability trader* 1 *asks a price of one to trader* 2 *in the first period, and if the offer is rejected, trader* 1 *consumes the object. In both cases, trader* 2 *accepts the offer of* 1 *if and only if she has high value. Hence, the equilibrium outcome is ex-post inefficient: trader* 3 *does not consume the object in the event in which she is the only trader with high-value.*

The inefficiency emphasized by example 1 is typical in network trading games with incomplete information and a deadline. In fact, the next proposition shows that ex-post inefficient equilibrium outcomes occur for an open set of priors in every situation in which the initial owner is not directly connected to all other traders.

Proposition 7. *Let* $\delta = 1$ *and* $\infty > T \geq n - 1$*. There exists (at least) one ex-post efficient equilibrium outcome for every* $\boldsymbol{\pi} \in (0, 1)^{n \times n}$ *if, and only if, the initial seller is connected to all other traders.*

If the initial owner is linked to all other agents, the seller asks a price of one to each trader, sequentially, and, if they all reject the respective offer, the initial owner consumes the object.

The equilibrium outcome is ex-post efficient. However, whenever there is a trader who is essential to connect the initial owner to another set of traders, one can find profiles of initial beliefs which are sufficiently optimistic about the value of the essential trader so that the good will never reach the traders in the latter set even if agents are perfectly patient, leading to ex-post inefficient equilibrium outcomes.

The inefficiency highlighted in proposition 7 depends on the assumption that the game has finite horizon. Let $\Gamma^{\infty}(\delta, G, \boldsymbol{\pi})$ indicate the set of all weak-Markov PBE of the *infinite-horizon* game that are limit points of a sequence of weak-Markov PBE with finite T, for given $(\delta, G, \boldsymbol{\pi})$.

Proposition 8. *There exists a $\delta^* < 1$ such that for all $\delta > \delta^*$ every equilibrium in $\Gamma^{\infty}(\delta, G, \boldsymbol{\pi})$ has the property that the object is consumed by a high value trader, if any. Therefore, as $\delta \to 1$ the equilibrium outcome of any sequence of equilibrium outcomes in $\Gamma^{\infty}(\delta, G, \boldsymbol{\pi})$ converges to an ex-post efficient outcome.*

Despite private information and incompleteness of trading opportunities, as long as traders are sufficiently patient, resale ensures that the object will end up in the hand of the "right" consumer. This result has the flavor of the Coase conjecture, but the analogy is only partial. A seller loses bargaining power against a dealer because her outside option of consuming provides strictly lower payoff than selling to the low-value dealer (gap case). However, a seller do not lose bargaining power against a client because a seller has always the option of consuming, and this provides a payoff no lower than selling to a low-value client (no-gap case).[33]

6 Conclusions

We study a dynamic model of bargaining with asymmetric information, where traders are located in a network. We provide three main results. First, we show that the price dynamic is non-monotonic. Second, we show that dealers, who intermediate the object, arise endogenously and earn a profit. The rents that dealers earn are determined by their position

[33]For an analysis of outside options in bargaining with incomplete information refer to Fudenberg, Levine and Tirole (1987), De Fraja and Muthoo (2000) and Board and Pycia (2011).

in the trading chain, which in turn depends on their position in the network. Finally, we provide insights on the extent to which the combination of asymmetric information and the incompleteness of trading relationships determine inefficient equilibrium outcomes.

In our model, sellers can only bargain with a single trader at a time. Further research may focus on multilateral trading mechanisms (e.g., an auction restricted to buyers in the seller's neighborhood). Taking as given bilateral negotiations, we assume that sellers and buyers have no formal commitment power and cannot contract on future actions. A natural extension would be to allow the seller to propose a contract where the price depends on the consumption choice of the buyer. Finally, the trading network is exogenous in our analysis. A preliminary investigation of models where trading networks are endogenous can be found in Condorelli and Galeotti (2012).

Appendix A: Proofs

Proof of Proposition 1. We prove the proposition by induction. We first show that all equilibria in round T have the desired properties. We then show that these properties hold in every weak-Markov PBE of every game starting in round t, given that they hold in every weak-Markov PBE of every game starting from $t+1$ onward.

First consider round T. Recall that all continuation payoffs are zero and so consuming in the last stage of round T is the only best reply (3-L and 3-H); also 1-L and 1-H follow directly. Consider now 2-L. Since $R_i^{T+1} = 0$, if a low value player rejects an offer she gets a payoff of zero, whereas if she accepts an offer at a price of p she gets $v_L - p$. So, a low value player rejects any offer strictly above v_L, accepts any offer strictly below v_L and she is indifferent between accepting and rejecting an offer that equals v_L. Finally, consider 2-H. If a high value trader rejects an offer she gets a payoff of zero, whereas by accepting an offer at p she gets $1 - p$. Hence, a high value trader accepts every price strictly below one, reject every price strictly above one, and he is indifferent between accepting and rejecting a price of 1. We now that there is no equilibrium where a high value player rejects an offer at a price of one with strictly positive probability, because the seller would have an incentive to slightly undercut that price.

Second, assume that properties 1-L, 2-L, 3-L and 1-H, 2-H, 3-H hold for every weak-

Markov PBE of every game starting in round $t+1$. The remaining of the proof shows that these properties hold for every game starting at t. All payoffs are time-t payoffs.

Consider 3-H. Consuming at the end of round t provides payoff of 1 and is always strictly optimal for a high-value trader. Otherwise, according to our induction hypothesis, she will consume in $t+1$, obtaining δ. Next, consider 3-L. Consuming provides v_L to a low-value trader, while δR_i^{t+1} is the continuation value from not consuming. Hence, consuming is the only best reply if $v_L > \delta R_i^{t+1}$, not consuming is the only best reply if $v_L < \delta R_i^{t+1}$, and the trader is indifferent in case of equality.

Consider now 2-L. Suppose low value trader i accepts an offer in round t at a price p. If she consumes in the last stage of round t she gets v_L. If she does not consume, by the induction hypothesis she makes an offer at the beginning of round $t+1$. Bayesian updating imply that $\mu_i^{t+1} = 0$, and therefore her continuation payoff is δR_i^{t+1}. Hence, accepting the offer in round t at price p gives trader i a payoff of $\max\{v_L, \delta R_i^{t+1}\} - p$. Rejecting the offer provides a continuation payoff which is weakly greater than zero. Hence, if $p > \max\{v_L, \delta R_i^{t+1}\}$ the only best reply of i is to reject such offer.

We now show that if $p < \max\{v_L, \delta R_i^{t+1}\}$ the only best reply of i is to accept such offer. For a contradiction, suppose that trader i refuses with positive probability some offer at price p, with $p < \max\{v_L, \delta R_i^{t+1}\}$. Since accepting p gives a strictly positive payoff to i, to reject such offer is a best reply only if the continuation payoff of i from refusing is strictly greater than zero. We now show that this is impossible. To see this, we write the continuation payoff of trader i, if she refuses, as:

$$\sum_{e \in E} \Pr[e] \delta^{t(e)} \left(\max\{v_L, \delta R_i(e)\} - p(e) \right),$$

where E is the set of events under which i will receive an offer at price $p(e)$ that she will accept, $R_i(e)$ is the resale value of trader i when she becomes owner after accepting offer $p(e)$ and $t(e)$ is the number of periods passed between the period in which event e occurs and t. Since the above continuation must be strictly greater than zero there must exist some $e \in E$ such that

$$\max\{v_L, \delta R_i(e)\} - p(e) > 0.$$

In other words, trader i must receive, with some probability, an offer at a price strictly below $\max\{v_L, \delta R_i(e)\}$. However, note that when e is realized, by our induction hypothesis, trader

i accepts all prices lower or equal to $\max\{v_L, \delta R_i(e)\}$, regardless of her value. Hence, because by the induction hypothesis the continuation payoff of the seller will be zero if the offer is accepted, the seller offering $p(e)$ could strictly improve her payoff by slightly increasing the price, which contradiction the hypothesis that the seller is making an optimal offer (i.e., 1-L). To conclude the argument for case 2-L, observe that trader i is indifferent between accepting and rejecting an offer at a price $p = \max\{v_L, \delta R_i^{t+1}\}$. However, refusing the offer cannot be part of an equilibrium if the seller would obtain a strictly lower payoff by demanding a price higher than $\max\{v_L, \delta R_i^{t+1}\}$ or by making an offer to another agent. In fact, in these cases the seller would want to slightly undercut the price $p = \max\{v_L, \delta R_i^{t+1}\}$.

Consider next 1-L. To see that in equilibrium every low-value seller must maximize the stated objective, it is sufficient to note that the continuation payoff of every low-value trader who has sold an object is zero. This follows because, by our induction hypothesis, only low value traders do not consume, and therefore, in equilibrium, once an agent does not consume everyone believes that she has value zero. Furthermore, by our induction hypothesis, a low value trader gets a payoff of zero, i.e., a trader i will never receive an offer at a price below $\max\{v_L, \delta R_i^{t+1}\}$.

Consider now 2-H. Note first that since the low type i always accepts an offer below or equal to $\max\{v_L, \delta R_i^{t+1}\}$, there is no equilibrium where the high type refuses such an offer with positive probability. Otherwise she would signal that she has high value and get zero payoff. Next, suppose that i receives an offer at a price p such that $p > \max\{v_L, \delta R_i^{t+1}\}$. Recall that a low value trader always rejects such an offer. Suppose that $1 - p \geq \delta V_i^{t+1}(0)$, i.e. the continuation value of i given that everyone beliefs she is a low type. Then accepting is a best reply because by rejecting she induces $\mu_i^{t+1} = 0$ and therefore she obtains $\delta V_i^{t+1}(0)$, while if she accepts and consumes she obtains $1-p$. Suppose that $1-p \leq \delta V_i^{t+1}(\mu_i^t)$. In this case rejecting is a best reply because acceptance provides payoff $1 - p$, which is below the continuation value from rejection given belief updating. Finally accepting with probability λ is a best reply only if the trader is indifferent between acceptance and rejection, given that upon rejection beliefs are updated according to Bayes rule.

We finally consider 1-H. If a high value trader i offers a price $p \geq 1$ to one of his neighbors she obtains a payoff of 1. In fact, an offer at $p > 1$ is always rejected and, by 3-H, trader i consumes at the end of the round. An offer at $p = 1$ is either accepted, in which case the trader who buys the object consume at the end of the round and so trader i gets 1, or it is rejected, in which case trader i consumes at the end of the round at gets 1. We now note that the continuation payoff of trader i at the beginning of round t is lower than or equal to 1. This follows because, for any given history, the sum of the expected (over the two types) continuation payoffs of all traders cannot exceed $(1 - \prod_i (1 - \mu_j^t)) \leq 1$ (i.e., the overall surplus). Hence, if the high-value of i is making an expected payoff strictly greater than one, there must be some type of some player making a strictly negative payoff. This is impossible, as every trader can guarantee herself zero payoff by refusing all offers.

■

Proof of Proposition 2. We first show that a regular equilibrium always exist. We do this by induction. We show that a regular equilibrium exists in the last round and then we show that we can construct a regular equilibrium for a game starting in round t taking as given the existence of at least one regular equilibrium for any game starting in round $t+1$. As standard, we use the one-shot deviation principle for dynamic incomplete information games (see Hendon et al. (1996)).

The equilibria we construct is as follows. The strategy profile of every trader satisfies 1-L, 2-L, 1-H and 3-H defined in Proposition 1 at every information set. Furthermore, in case 3-L we assume that the low-value trader consumes, and in case 2-H we assume that the probability that a high value trader accepts an offer in round t at a price p is the following:

$$\begin{cases} 1 & \textit{if } p \leq L = \max\{\delta R_i^{t+1}, 1 - \delta V_i^{t+1}(0)\} \\ \lambda_i(p) & \textit{if } L < p^t \leq U = \max\{L, 1 - \delta V_i^{t+1}(\mu_i^t)\} \\ 0 & \textit{if } p > U \end{cases} \tag{2}$$

Note that the strategy profile specified in 2 satisfies the regularity condition in definition 1. To see that the strategy profile outlined above constitute an equilibrium when the game starts in round T is straightforward. For a game starting in round T and for any potential seller, we have $V_i^{T+1}(\mu_i) = R_i^{T+1} = 0$ for all μ_i and for all i. Hence, it is a best reply for a high-value trader to accept every offer up to a price of one.

It remains to show that the specified strategy is a best reply in round t, taking as given the continuation equilibria in round $t+1$. What we need to show is that, for every price in

$(1-\delta V_i^{t+1}(0), 1-\delta V_i^{t+1}(\mu_i^t)]$ (whenever this interval is not empty or contains a single point), there must exist a $\mu_i^* < \mu_i^t$ such that $p = 1 - \delta V_i^{t+1}(\mu_i^*)$. In fact, when i receives an offer in the range $(1-\delta V_i^{t+1}(0), 1-\delta V_i^{t+1}(\mu_i^t))$ the only best reply is to randomize and for this to be possible there must exists a $\mu_i^* < \mu_i^t$ such that $p = 1 - \delta V_i^{t+1}(\mu_i^*)$. Existence of μ_i^* is guaranteed because the equilibrium payoff correspondence of i, as a function of the prior beliefs, has a closed graph (see Fudenberg and Tirole (1991)).

We now show that for every N, δ and T, when E is a star, then every weak-Markov perfect equilibrium is regular. First, assume that the center is the owner of the object. In equilibrium she will make a take it or leave it offer to all the connected trader in a given sequence. This is the only equilibrium, which is regular. Next, consider the case of a periphery trader being the initial owner. He will then approach the center in some round t. For the statement to hold is enough that we show that in any weak-Markov perfect equilibrium if the center, i, receives an offer at t, then $V_i^{t+1}(0) \geq V_i^{t+1}(\mu_i^t)$. In fact, we know that if the condition above applies and i acquires the object, then the continuation is regular. Observe that if at beginning of round t trader j is the owner it must be the case that $\mu_j^t = 0$. When j is the owner at t and $\mu_i^t = 0$, then trader j offers the object to i at a price $p = \max\{v_L, \delta R_i^{t+1}\}$, trader i accepts and consumes and so $V_i^{t+1}(0) = \delta(1 - \max\{v_L, \delta R_i^{t+2}\})$. If $\mu_i^{t+1} = \mu_i^t > 0$ then in period $t+1$ agent $i-1$ makes an offer $p \geq \max\{v_L, \delta R_i^{t+2}\}$ which is accepted with positive probability. So, $V_i^{t+1} = \delta(1-p) \leq V_i^{t+1}(0)$. ■

Proof of Proposition 3. The first part of proposition 3 follows directly from proposition 1. We now prove the second part of proposition 3. Consider an offer to i in period t at a price $p > \delta R_i^{t+1}$ and suppose that this offer is accepted with strictly positive probability. If trader i is randomizing among acceptance and rejection, then he must be indifferent between the two options and the proof follows. Suppose then that trader i accepts the offer with probability one. Since acceptance with probability one is a best reply only if $1 - p \geq \delta V_i^{t+1}(0)$, the price asked must be below or equal to $1-\delta V_i^{t+1}(0)$. If it is equal to $1-\delta V_i^{t+1}(0)$ then the proposition follows. So, consider the case in which the price asked is strictly below $1 - \delta V_i^{t+1}(0)$. We now show that this contradicts that the seller is optimizing, i.e., contradiction with $1 - L$ of proposition 1. In fact, since trader i accepts every price below or equal $1 - \delta V_i^{t+1}(0)$ (by the regularity condition) it follows that, conditional on acceptance, the continuation of the seller is independent of the price asked $p \leq 1 - \delta V_i^{t+1}(0)$; but therefore the seller would be better off raising the price to $1 - \delta V_i^{t+1}(0)$. ■

Proof of Proposition 4. Part 1 and 3 of proposition 4 follow immediately from part 2 and equation 1. We prove part 2 of proposition 4 by induction. The Proposition holds at $T-1$, because every resale offer at T will be at zero. Suppose that the conjecture holds at every $t' > t$ and consider a consumption offer at price p_i at time t.

First we introduce some notation. Following the consumption offer (i, p_i) from seller s in round t, many different trading chains can ensue in equilibrium. Among all those trading chains, we consider the following set of trading chains: each of the trading chain in the set differs from the other in at least one offer, and each of the trading chain leads to a seller different from s becoming the next owner. Note that if s remains the owner in all cases, then the statement is trivially true. Suppose there are k of these trading chains. For trading chain $x \in \{1, \ldots, k\}$, denote with $o^x = \{i_1^x, \ldots, i_m^x\}$ the set of traders receiving a consumption offer from s in the trading chain before the first resale offer. Let d^x denote the first trader in the chain x that receives a resale offer (the first future seller after s). For each $y = 1, \ldots, m$, let $r(i_y^x)$ indicate the equilibrium probability that i_y^x refuses the consumption offer. Let $r(o^x)$ indicate the equilibrium probability that all consumption offers to $\{i_1^x, \ldots, i_m^x\}$ are rejected. Finally, let $p(i_y^x)$ denote the *discounted* (relative to time t) price of the consumption offer to i_y^x, $R(d^x)$ the *discounted* (relative to time t) resale value of d_x when she receives the resale offer, and $\Pr(o^x)$ indicate the probability that trading chain x ensues in equilibrium.

The proof is organized in four observations.

First observation. Seller s, starting from $t+1$, expects to obtain the same continuation payoff along each of the trading chain $x = 1, \ldots, k$. That is, for each $x, y \in \{1, \ldots, k\}$:

$$[1 - r(i_1^x)]p(i_1^x) + \cdots + r(o^x)R(d^x) = [1 - r(i_1^y)]p(i_1^y) + \cdots + r(o^y)R(d^y). \tag{3}$$

Condition 3 is a standard indifference condition. If that condition did not hold, it would not be possible for both trading chains to occur with strictly positive probability in equilibrium, because seller s would find profitable to deviate at some point where the two trading chains differ.

Second observation. For every $x = 1, \ldots, k$, since $r(o^x) \leq 1$, it is immediate that

$$R(d^x) \leq (1 - r(o^x)) + r(o^x)R(d^x). \tag{4}$$

We then obtain that for every $x, y \in \{1, \ldots, k\}$:

$$R(d^x) \leq (1 - r(o^y)) + r(o^y)R(d^y), \tag{5}$$

where the inequality follows by using the indifferent condition in 3 and, for any path y, majorizing every discounted price asked before $R(d^y)$ with 1.

Third observation is the following. A trading chain $x \in \{1, .., k\}$ is such that either: (i) after d^x has received a resale offer there is a positive probability that in the future trader i receives a resale offer, or (ii) this probability is zero. Assign to the trading chains which satisfy (i) index $\{1, ..., z\}$, whereas assign indexes $\{z+1, ..., k\}$ to those that satisfy (ii). If no path satisfying (i) exists then the offer in period t must be at a price $p_i = 1$ and the result would be trivially true. So, $z \geq 1$.

Next, for each $x = 1, \ldots, z$, let $o_1^x, \ldots, o_h^x$ be the set of paths starting from d^x and leading to a resale offer to i (recall that there maybe multiple path following a given history x). As before, $r(o_j^x)$ represents the probability that all elements of the path o_j^x, excluding future sellers (i.e. dealers) but possibly including i, refuse their offers. Then, denote the discounted resale price to i following path o_j^x by $R(i_j^x)$ and the probability that the path o_j^x ensues as $\Pr(o_j^x)$. We know that, because we can majorize every price with 1, for every $x = 1, \ldots, z$ and $j = 1, \ldots, m$, the following holds:

$$R(d^x) \leq 1 - r(o_j^x) + r(o_j^x)R(i_j^x)$$

Therefore we conclude, using the previous observations, that for every $x = 1, \ldots, k$ and every $y = 1, \ldots, z$ and every $j = 1, ..., m$ we have:

$$R(d^x) \leq 1 - r(o^y)r(o_j^y) + r(o^y)r(o_j^y)R(i_j^y). \tag{6}$$

Fourth observation. We show that the discounted consumption offer that i obtains at t is $p_i \geq (1 - r(o_j^y)r(o^y)) + r(o^y)r(o_j^y)R(i_j^y)$ for at least one path $y = 1, \ldots, z$ (who recall passes via d^y and then reaches i, where y may also be i).

We know that $1 - p_i$ must equal the continuation payoff of i given that he rejects the offer. The latter is equal to the probability that i obtains a resale offer in the future, times the net profit in that case, which is equal to 1 minus the price she pays (her resale value).[34]

[34] Note that if trader i receives another consumption offer before receiving the resale, then at that con-

Formally, let k be the number of rounds separating the resale offer to i from the consumption offer, we have:

$$\begin{aligned} 1-p_i &= \sum_{j=1}^{z} \Pr(o^j) r(o^j) \Pr(v_{d^y}=0) \left[\sum_{w=1}^{h(j)} \Pr(o^j_w) r(o^j_w)(\delta^k - R(i^j_w)) \right] \\ &\leq \sum_{j=1}^{z} \Pr(o^j) r(o^j) \left[\sum_{w=1}^{h(j)} \Pr(o^j_w) r(o^j_w)(1 - R(i^j_w)) \right] \\ &\leq \max_{j=1,\ldots,z} r(o^j) \left[\max_{w=1,\ldots,h} r(o^j_w)(1-R(i^j_w)) \right] \end{aligned}$$

where the first inequality follows because we have removed the probability that dealers consume which is below one and we have substituted δ^k with 1, and the second inequality follows because we have selected the maximum value among all considered trading chains.

Rewriting the above equation we obtain that there exists a $y = 1...k$ and $j = 1...m$ so that $p_i \geq (1 - r(o^y)r(o^y_j)) + r(o^y)r(o^y_j)R(i^y_j)$. This, together with inequality 6 completes the proof of the proposition. ■

Proof of Proposition 5. Consider all different histories of offers starting from the initial seller and in which trader i gets a resale offer for the first time and name these different *discounted* resale values $R_i(1), \ldots, R_i(k)$ in rounds $t_1, \ldots, t_k$. If this set is empty, than by assumption both i and j makes a zero payoff and the result holds. So suppose it is not empty. Let the path x be the path that leads to resale offer $R_i(x)$, where $x = 1, \ldots, k$. We denote by o^x the traders in path x who receives an offer, excluding both trader i and trader j. $r(o^x)$ indicates the probability that all traders in o^x refuse the respective consumption offer or, they do not consume if they have received a resale offer; $r(i^x)$ and $r(j^x)$ denote the probability that i and j refuse an offer along the path x, whenever they are included in the path (otherwise set these numbers equal to one). Finally, $\Pr(o^x)$ is the probability that the path occurs in equilibrium.

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sumption offer she must be indifferent between accepting and rejecting, where rejecting equals the probability that she receives an offer at the resale value in the future. In the next formula, for sake of exposition, we consider the case where if i rejects the consumption offer, in every path in which she receives a resale offer, she does not receive other consumption offers. The other case can be proved in the same way by taking into account the aforementioned argument, and so details are not provided.

Let t_x be the round in which the resale offer $R_i(x)$ occurs. We can write the expected utility of trader 1 at the beginning of the game, condition on having value 1, as follows:

$$U_i(1) = \sum_x \Pr(o^x) r(o^x) r(j^x) \left(\delta^{t_x} - R_i(x)\right).$$

Next, for each path x, let's consider the different paths that start when i receives the resale offer at the price $R_i(x)$ and that lead to a resale offer to j. If this set is empty then the result follows. Suppose it is not empty. Call the prices of these resale offers to j as $R_j^x(1), \ldots, R_j^x(l)$, occurring in rounds $t_1^x, \ldots, t_l^x$. For each $R_j^x(y)$ let o_y^x indicate that set of traders in the path that leads to the resale offer to j, excluding j himself. Let $\Pr(o_y^x)$ the probability that the path occurs in equilibrium, conditional on path x occurring. Let $r(o_y^x)$ be the probability that all elements in the path refuse the offer (or in case of sellers that they have value zero conditional on the path occurring).

Let $U_j(1)$ indicate the interim utility of a trader j with value 1 at the start of the game and let q indicate the number of rounds elapsing between the resale offer to i and the offer to j. We know that:

$$U_j(1) = \sum_{x=1}^{k} \Pr(o^x) r(o^x)(1-\pi_i) \left[\sum_{y=1}^{l} \Pr(o_y^x) r(o_y^x)(\delta^{t_y^x} - R_j^x(y))\right].$$

We now note that for every $x = 1, \ldots, k$, with associated $y = 1, ..., l$ we have

$$R_i(x) \leq (1 - r(o_y^x))\delta^{t_x} + r(o_y^x) R_j^x(y),$$

given the indifference condition and the fact that we can majorize every price with δ^{t_x}. Hence:

$$\delta^{t_x} - R_i(x) \geq r(o_y^x)(\delta^{t_x} - R_j^x(y)).$$

Observe that the fact that j might receive a consumption offer before his resale along the path o_y^x may only relax the bound. So,

$$U_i(1) \geq \sum_x \Pr(o^x) r(o^x) r(j^x) \left[\sum_y \Pr(o_y^x) r(o_y^x)(\delta^{t_x} - R_j^x(y))\right].$$

Now note that in the first part of the statement of the proposition, by assumption trader j never receives an offer before trader i, which implies that $r(j^x) = 1$; this fact and the fact that $\delta^{t_x} > \delta^{t^x_y}$ allow us to conclude that

$$U_i(1) \geq \sum_x \Pr(o^x) r(o^x) \left[\sum_y \Pr(o^x_y) r(o^x_y)(\delta^{t_x} - R^x_j(y)) \right] > U_j(1).$$

To prove the second part of the statement, note that $r(j^x) \geq 1 - \pi_j$, because if j receives an offer before agent i such offer is, by assumption, a consumption offer, and a low value trader rejects a consumption offer with probability 1. This fact and the assumption that $\pi_i \geq \pi_j$ are sufficient to conclude that $U_i(1) > U_j(1)$. ■

Proof of Proposition 6. Let $d(i, j|G)$ be the geodesic distance between i and j and let $d^*(i) = \arg\min_{j:v_j=1} d(i, j|G)$. Consider the following strategy profile. In period $t = 1, .., T$, if trader i receives an offer (i, p) then she accepts the offer if she has high value and $p \leq 1$. If she has low value she accepts if and only if $p \leq v_L$ or $p \in (v_L, \delta^{d^*(i)}]$ and $d^*(i) \leq T - t - 1$; otherwise she rejects the offer. If trader i is the owner in period t then she consumes whenever $v_i = 1$ or $v_i = v_L$ and $v_L \geq \delta^{d^*(j)}$ for all j connected to i and such that $d^*(j) \leq T - t - 1$. Otherwise, trader i asks a price of $\delta^{d^*(j)}$ to trader $j = \arg\min_{j' \in N_i(G)} d^*(j')$.

To verify that this strategy profile constitutes a subgame perfect equilibrium, one can proceed by induction with respect to t, starting from $t = T$. It is also immediate to verify that the equilibrium outcome is Pareto Efficient. Furthermore, every other subgame perfect equilibrium has a strategy profile that is different from the one above only when there are indifference. A simple argument shows that by breaking such indifference in an alternative way does not change the properties that the equilibrium outcome is Pareto efficient. Finally, since our game is continuous at infinity, any sequence of subgame-perfect equilibria generated as $T \to \infty$ has a limit point, which is a subgame-perfect equilibrium of the infinite horizon game, see Fudenberg and Levine (1991). ■

Proof of Proposition 7. Suppose G is such that the initial owner is linked to all other traders, and let $\delta = 1$ and $T \geq n - 1$. The following is an ex-post efficient equilibrium: trader 1 asks a price of 1 to each of his neighbors sequentially, each of the trader accepts the offer if and only if they have high value, and, if they all reject the offer, trader 1 consumes the good.

Next, let $i = 1$ be the initial seller and let $N_1(G)$ indicate the set of traders in G connected to trader 1. Suppose that G is such that $N_1(G) \neq N \setminus \{1\}$. We show that for every T, there exists some profile of $\boldsymbol{\pi}$ such that under T and $\boldsymbol{\pi}$, every equilibrium outcome is ex-post inefficient. If $T < |N_1(G)|$ (where $|\cdot|$ indicates the cardinality of the set), then every equilibrium is ex-post inefficient because some traders cannot be reached. So, let $T \geq |N_1(G)$, and assume that it is sufficiently large. Let $\pi_j = \pi_H$ for all $j \in N_1(G)$ and let $\pi_j = \pi_L$ for all $j \in G/N_1(G)$. The initial seller can always play the following strategy: wait till $|N_1(G)|$ periods are left and then she asks a price of 1 to each of her neighbors in sequence. This strategy will provide an ex-ante payoff of $1 - (1 - \pi_H)^{|N_1(G)|}$.

Suppose now that there exists an equilibrium outcome that is ex-post efficient. Then, there must exist at least one trader j linked to the initial seller who, at some time t, receives an offer at a price $\max\{v_L, R_j^{t+1}\}$ with probability one. Since j is getting at t an offer at a price of $\max\{v_L, R_j^{t+1}\}$ with probability 1, the highest offer that she can accept in every period $t' < t$ is at a price:

$$p^* = 1 - (1 - \pi_L)^{n-|N_1(G)|-2}(1 - \pi_H)^{|N_1(G)|-1}(1 - \pi_L) < 1$$

Hence, an upper bound to the expected payoff that the initial seller can make in every ex-post efficient equilibrium is:

$$1 - (1 - \pi_H)^{|N_1(G)|-1} + (1 - \pi_H)^{|N_1(G)|-1}\pi_H p^* + (1 - \pi_H)^{|N_1(G)|}[1 - (1 - \pi_L)^{n-|N_1(G)|-1}].$$

Substituting the upper bound for p^* in the expression above, after some elaboration we get that the upper bound in revenue from the strategy above is lower than $1 - (1 - \pi_H)^{|N_1(G)|}$ whenever

$$1 - (1 - \pi_L)^{n-|N_1(G)|-1}\left[1 + \pi_H(1 - \pi_H)^{|N_1(G)|-2}\right] < 0,$$

which is always satisfies for sufficiently small π_L. When the condition above holds the initial seller is better off by asking a price of one to all her neighbors and an inefficiency arises. ■

Proof of Proposition 8. Consider a weak-Markov PBE equilibrium of an infinite horizon trading game belonging to $\Gamma^\infty(\delta, G, \boldsymbol{\pi})$. Suppose, for a contradiction, that for all $\delta < 1$ the equilibrium outcome is such that, with positive probability, the object is never consumed or it is consumed by a low-value trader when an high-value trader is still present with positive probability (i.e., when $\mu_i > 0$ for some i at the end of the game).

This implies that there exists some time t' and a non-empty set of traders $N' \subseteq N$ such that with positive probability: (i) $\mu_i^t > 0$ for all $i \in N'$ and $t \geq t'$; (ii) $\mu_i^t = 0$ for all $i \notin N'$

and $t \geq t'$.[35] Call this statement (a).

Our **first observation** is that, if (a) holds for all $\delta < 1$, then there must exists at least one agent $i \in N'$ who, with positive probability along the equilibrium path, receives an infinite number of offers.

Suppose, for a contradiction that (a) holds for all $\delta < 1$, but with probability one, all agents in N' receive a finite number of offers. This implies that there must exist a round after which no agent in $i \in N'$ receives any further offer and, since (a) holds, $\mu_i^t > 0$ for all subsequent rounds. In turn, this implies that, in case no trader outside N' has value one, the game must end with consumption by a low-value trader. In fact, if no trader who has positive probability of being value one ever obtains any further offer, we must have that $R_i^t = v_L$ for every i and every subsequent t. Hence $v_L > \delta R_i^t$ and consumption must take place. We now show that for δ large enough we must have $\delta R_i^t > v_L$, a contradiction.

First, note that, given that the horizon is infinite, there is always a chain of resale offers that can bring the object next to any trader in the network. Second, if $\delta R_i^{t+1} \leq v_L$ for all δ, it must be the case that all traders in N' would reject any offer strictly above v_L whenever such offer were made to them. In fact, if some trader i in N' were to accept an offer at price $v_L + \epsilon$ with positive probability λ whenever made, then a seller connected to i would, for sufficiently large δ, have a strict incentive to make that offer. This is because her discounted resale including the offer would be $(v_L + \epsilon)\lambda + \delta(1 - \lambda)v_L$, which would be strictly above v_L for high enough δ. Given this, and the fact that the object has time to reach any trader, we can always select δ high enough in a way that the resale of some player would be above the value from consuming, v_L.

Our first observation implies that if (a) holds then there is a non-empty subsets $N^* \subset N'$ such that all agents in N^* receive an infinite number of offers with positive probability. Our **second observation** is that, with some positive probability, an equilibrium path must possibly ensue where each agent in N^* never receive an offer at price below or equal δR_i^{t+1}. Otherwise, she would always either acquire the object and consume (which is incompatible with the hypothesis that she receives an infinite number of offers with positive probability) or she would resell, which is incompatible with his posterior being strictly positive in all

[35]Note that this formulation allows for the possibility that the game ends at some point because of consumption, in which case beliefs are assumed to remain constant.

subsequent rounds (i.e., with statement (a)). Hence, with positive probability, every agent $i \in N^*$ receive an infinite sequence of offers, all at prices strictly higher than δR_i^{t+1}.

Our **third observation** is that when a high-value agent $i \in N^*$ is faced with an infinite sequence of offers at price above δR_i^{t+1}, there is a positive probability that she will accept each of these offers with some probability in $(0, 1)$ (that may be different across offers at different time). Assume that this was not the case. Then, along the equilibrium path it would happen with probability one that, at some point, she accepts or rejects an offer for sure. However, in this case we contradicts either that $i \in N^*$ or that her posterior is $\mu_i^t > 0$ for every period t (contradicting (a)).

Our three observations together imply that if (a) holds then after some period t', along the equilibrium path, with positive probability, some agent $i \in N^*$ finds herself at a point after which she receives an infinite number of offers that she rejects with probability 1 if she is a low-value, and she accepts with some probability $\lambda^t \in (0, 1)$ if she has a high-value. We now show that, for δ sufficiently close to 1, this contradicts sequential rationality, and therefore (a) cannot hold.

For simplicity we consider the case in which there is only one player i which belongs in N^*, and there is a single seller making offers to her, but the proof extends to the more general case. Hence, let $\lambda^t \in (0, 1)$ be the probability with which the offer in t is accepted by i, let s be the seller making offers to i, and assume that $t \geq t'$. $R_s^t(\mu_i^t)$ indicate the continuation value of the seller when she has posterior μ_i^t about i. At the beginning of period t, the continuation payoff of s is at most (we are assuming that he makes the offer to i at price 1):

$$R_s^t(\mu_i^t) \leq \mu_i^t \lambda^t + (1 - \mu_i^t \lambda^t)\delta R_s^{t+1}(\mu_i^{t+1}),$$

Sequential rationality on the side of the seller requires that for all t:

$$R_s^t(\mu_i^t) \geq v_L > 0. \tag{7}$$

Next, note that in equilibrium, μ_i^{t+1} is derived via Bayes rule using λ^t. This implies:

$$\mu_i^{t+1} = \frac{(1 - \lambda^t)\mu_i^t}{1 - \lambda^t \mu_i^t} < \mu_i^t,$$

where the inequality follows because $\lambda^t \in (0, 1)$ and because if i has a low value she rejects the offer with probability 1. Since i receives an infinite number of such offers, there are two possibilities. First, $\lim_{t\to\infty} \mu_i^t \to 0$. If this is the case then

$$\lim_{t\to\infty} R_s^t(\mu_i^t) = R_s^t(0) \leq \delta R_s^t(0),$$

and so $R_s^t(0) = 0$, which contradicts optimality condition 7. Two, $\lim_{t\to\infty} \mu_i^t = \epsilon$, with $\epsilon > 0$; this is possible only if $\lim_{t\to\infty} \lambda^t = 0$, which again implies $\lim_{t\to\infty} R_s^t(\mu_i^t) = R_s^t(0) \leq \delta R_s^t(0)$, and so $R_s^t(0) = 0$, a contradiction with optimality condition 7. ■

Appendix B: Equilibrium multiplicity

In this section we illustrate with an example the consequences of having multiple equilibria. The example clarifies the restriction imposed by the regularity condition introduced in definition 1 (observation 1). It shows that non-regular equilibria, when they exist, may not satisfy some of the equilibrium properties we have discussed in section 4.1 and section 4.2 (observation 2). Finally, it shows that the introduction of small transaction costs rules out, generically, the existence of non-regular equilibria (observation 3).

The example considers the trading network G depicted in figure 4. There are $n = 5$ traders, $T = 6$ rounds of trade, $\delta = 1$, $v_L = 0$ and the initial profile of beliefs is $\boldsymbol{\pi} = (0, 1/3, 1/3, 2/3, 2/3)$.

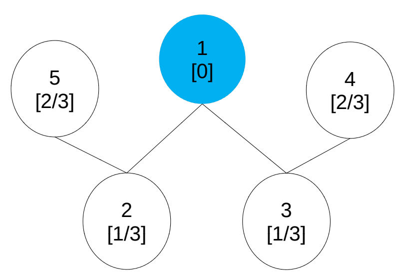
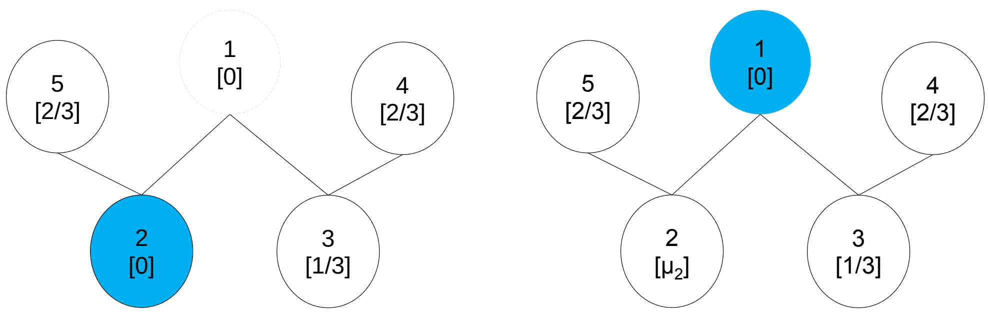


Figure 4: Trading network (Appendix B): trader 1 is the initial owner.

Trader 1 has a low-value and therefore she will make an offer to one of her neighbor. The optimal offer will depend on the acceptance strategy of 2 and 3. We restrict attention, without loss of generality, to the case in which in the first period trader 1 makes an offer to trader 2.

Following proposition 1, to determine the equilibrium strategy of low value trader 2 we need to derive the resale value of trader 2. In this case, the resale value of trader 2 is unique and equal to $R_2^2 = 8/9$. In fact, in the continuation game in figure 5(a) trader 2 will make a consumption offer at a price of 1 to trader 5, and in case of rejection she will sell the object to trader 1 at trader 1's resale value, which, at that point, is equal to 2/3. Hence, the strategy of low-value 2 when she obtains an offer in the first period from trader 1 is to accept every offer at a price below R_2^2 and to reject offers at higher prices.



(a) Five rounds of trade left and the owner is trader 2.

(b) Five rounds of trade left and the owner is trader 1.

Figure 5: Two different continuation games.

To determine the equilibrium behavior of trader 2 when she has high-value we need to determine $V_2^2(\mu_2)$ for all $\mu_2 \in [0, 1/3]$. That is, we need to characterize equilibria in the continuation game in figure 5(b). Note that, regardless of the belief about 2, i.e. $\mu_2 \in [0, 1/3]$, trader 1 is indifferent between the following two equilibrium paths:

path 1. Trader 1 makes a resale offer to trader 2 at 8/9, who accepts the offer. In this case high-value trader 2 gets a payoff of 1/9.[36]

path 2. Trader 1 makes a resale offer to trader 3, which is also equal to 8/9. In the ensuing continuation equilibrium, trader 2 acquires the good when both trader 3 and trader 5

[36]If trader 2 has a low-value she will ask a price of 1 to trader 5 and, upon rejection, will sell the object to trader 1 at her resale value which is, at that point, 2/3. So, trader 2's expected profit (or trader 2's resale value) is $2/3 + 1/3 \times 2/3 = 8/9$.

have low-value, which happens with probability 2/9 and in that case she pays a price of 2/3; her expected profit is therefore 2/27.[37]

Since trader 1 is indifferent between the two paths, the following strategy is optimal: in the continuation game in figure 5(b) trader 1 "plays" path 1 with probability $\sigma(\mu_2)$, and path 2 with the remaining probability. Hence, for $\mu_2 \in [0, 1/3]$, we have that:

$$V_2^2(\mu_2) = \frac{1}{9}\sigma(\mu_2) + \frac{2}{27}[1 - \sigma(\mu_2)] \in \left[\frac{1}{9}, \frac{2}{27}\right].$$

Note that $R_2^2 \leq 1 - V_2^2(\mu_2)$ and that $V_2^2(\mu_2)$ is increasing in $\sigma(\mu_2)$. We now make two observations.

Observation 1. If the strategy of trader 1 is such that $\sigma(0) \geq \sigma(\mu_2)$ for all $\mu_2 \in [0, 1/2]$, then $V_2^2(0) \geq V_2^2(\mu_2)$ for all $\mu_2 \in [0, 1/2]$ and every weak-Markov PBE supported under the postulated trader 1's strategy is regular.

Observation 2. If the strategy of trader 1 is so that $\sigma(0) < \sigma(\mu_2)$, for some $\mu_2 \in (0, 1/3]$, then there may be weak-Markov PBE which are non-regular. We now construct one of them. Suppose $\sigma(0) = 0$ and $\sigma(\mu_2) = 1$ for all $\mu_2 \in (0, 1/3]$. Then $V_2^2(0) = \frac{2}{27} < V_2^2(\mu_2) = \frac{1}{9}$ for all $\mu_2 \in (0, 1/3]$. The following non-regular strategy of high value trader 2 is a best reply: accept every price $p \leq 8/9 + \epsilon$, where $\epsilon > 0$ and small, and reject every other price. Symmetrically, we can construct a best reply of trader 3 when facing an offer from trader 1 in the first period so that high value trader 3 accepts an offer from trader 1 if, and only if, comes at a price no higher than 8/9.

Given the specified strategy of trader 2 and trader 3, trader 1 strictly prefers to ask a price of $8/9+\epsilon$ to trader 2 in the first period. Trader 2 accepts if she has high-value and then consumes the object. If trader 2 has a low-value, she rejects the offer; in this case trader 1 offers the good to trader 3 at her resale value 8/9 and the continuation equilibrium described in path 2 above ensues. In this equilibrium, trader 2 receives an offer above his resale and such that if she has high-value she strictly prefers to accept; this cannot happen in a regular

[37] Indeed, once accepted the offer from trader 1, trader 3 consumes if she has high value, otherwise she asks a price of 1 to trader 4, who accepts the offer if she has high-value. Upon rejection, trader 3 resales the object to trader 1 who then asks a price of 2/3 to trader 2.

equilibrium. Furthermore, note that trader 3 and trader 2 are both dealers, and trader 3 becomes a dealer before trader 2. However, it is easy to compute that the expected utility of high-value trader 2 is higher than the expected utility of high-value trader 3. Again, this is not possible in a regular equilibrium.

Observation 3. Non-regular equilibria are induced by the indifference that trader 1 has between offers. A small introduction of trader specific transaction costs will break this indifference leading to a unique equilibrium outcome. Suppose that if trader 1 sells to trader 2 then trader 1 has to pay a transaction cost of τ_{12}, while if trader 1 sells to trader 3, then she pays a transaction cost of τ_{13}, where $\tau_{12} > \tau_{13}$ and they are both very small. There are no other transaction costs in the economy. Note that now, in the continuation game depicted in figure 5(b), trader 1 strictly prefers path 2 to path 1. Hence, if in the first period trader 1 makes an offer to trader 2 and the offer is rejected, in the second period trader 1 will make a resale offer to trader 3, i.e., $\sigma(\mu_2) = 0$ for all $\mu_2 \in [0, 1/3]$. So, in this context, the introduction of small transaction costs leads to a unique equilibrium, which is a regular equilibrium. The equilibrium path is the following: trader 1 makes a consumption offer to trader 2 at $p_2^1 = 1 - V_2^2(0) = 25/27$, and, upon rejection, trader 1 asks the resale value (which is approximately) $r_3^1 = R_3^3 = 8/9$ to trader 3, who accepts the offer. If trader 3 has low value then she makes a consumption offer to trader 4 at a price of $p_4^3 = 1 - V_4^4 = 1$, and, if rejected, she resells back the good to trader 1 at $r_1^3 = R_1^5 = 2/3$, who, in turn, sells it to trader 2 at $r_2^1 = R_2^6 = 2/3$. If trader 2 has a high value she consumes; otherwise she asks a price of $p_5^2 = 1$ to trader 5.

Appendix C: Multiple values

We consider an example where there are three traders located in a line, i.e., $N = \{1, 2, 3\}$ and $E = \{12, 23\}$. We let $T = 3$, $\delta = 1$ and assume that there are three possible types, $(1, v, 0)$, with corresponding priors $(\pi_i^h, \pi_i^m, \pi_i^l)$. For simplicity we further assume that the initial owner has value zero (i.e., $\pi_1^l = 1$) and that $\pi_i^h = \pi_i^m = \pi_i^l = 1/3$ for $i = 2, 3$. Finally we assume that $v = 1/2 + \epsilon$ for some small but positive ϵ. We now describe the equilibrium path using five observations.

First observation. Suppose trader 2 becomes the owner at the end of the first round (or the second). If she has high-value, trader 2 consumes and obtains a payoff of 1. If she is medium-value, then she waits one round, asks a price of one to trader 3 in the last round, and she consumes if the offer is rejected. Because in the last round a trader accepts every price up to her valuation, the continuation payoff of the medium-value trader 2 is $1/3 + 2v/3$. If trader 2 has low-value, she makes immediately an offer at price v to trader 3, and trader 3 accepts if she has high or medium-value (note that $2v/3 > 1/3$ because $v > 1/2$). Hence the continuation payoff of low-value trader 2 is $2v/3$.

Next, observe that in the construction above both low-value 2 and medium-value 2 try to resell the good to trader 3 at price 1. However, in equilibrium, low-type 2 makes an immediate offer at price v, whereas medium value trader 2 waits that the deadline approaches before making an offer to trader 3 at price 1. In fact, there is no equilibrium in which the medium-value 2 asks a price of one immediately and such an offer is accepted by the high-value 3. If that were the case then the low-value 2 would mimic the high-value and ask a price of one as well; subsequently lowering the price to v in case of rejection. The fact that signalling occurs also in the selling stage represents an additional complexity which is absent in the two-type model.

Second observation. The strategy of trader 2 when she receives an offer from trader 1 in the second round is summarized in Figure 6. This strategy represents a best reply for trader 2 for the following reasons. First, since $2v/3$ is the resale value of 2's low-type, we can specify that everyone accepts prices below $2v/3$ by assuming that an out-of-equilibrium rejection signals high-value. Consider now an offer at a price $p > 1/3 + 2v/3$. If trader 2 follows the strategy and rejects such offer, then beliefs do not change and trader 1 (who is for sure low-value) will ask in the last round a price $v < p$. Hence, by refusing, all types of trader 2 are better off. Lastly, for prices in the range $(2v/3, 1/3 + 2v/3)$ we know that the refusal would signal that 2 has low-value. Hence, following a rejection from 2, trader 1 will not make any other offer to 2. Therefore accepting is a best-reply for 2.

Given the strategy outlined above, and assuming that the posterior beliefs that 2 is high-value and medium-value are respectively μ^h and μ^m, at the beginning of the second round trader 1 will behave as follows. She will ask either price $2v/3$ (i.e., the resale value of the low-value), or $1/3 + 2v/3$ (i.e. the resale value of the medium-value), or will wait one round

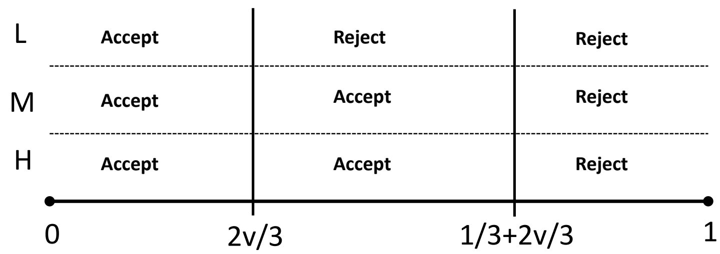


Figure 6: Strategy of 2 with offer at t=2.

and then ask a price of 1 (in the final round player 2 will accept all offers up to 1 if she has value one). Which offer is optimal for trader 1 depends on the beliefs μ^h and μ^m. Observe that waiting is optimal for trader 1 whenever $\mu^h \geq 2/3$; when $\mu^h + \mu^m = \frac{2v}{2v+1}$ trader 1 is indifferent between the resale value of the medium-value trader and the resale value of the low value-trader. If posteriors were equal to prior beliefs, the initial owner would find it optimal to ask price $1/3 + 2v/3$.

Third observation. Let's now consider the strategy of player 2 when she receives an offer from trader 1 in the first round. This is summarized in Figure 7. First observe that, for the same reasons outlined in our second observation, accepting every offer below $2v/3$ and rejecting every offer above $1/3+2v/3$, regardless of her type, is a best reply. However, there is no pure strategy which is compatible with equilibrium play, when the offer is at a price in the range $(2v/3, 1/3+2v/3)$.[38]

We now construct a mixed strategy equilibrium where: (a) the low-value trader always rejects the offer with probability one, (b) the medium-value trader and the high-value trader accept the offer with the same probability λ, and, (c) upon rejection, trader 1 is indifferent

[38] It cannot be the case that the low type refuses the offer and the high and medium type accepts the offer; for otherwise, both the medium value and high value trader strictly prefers to refuse the offer, signal low valuation and get an offer at a price equal to the resale of the low-type in the next round. The other cases can be ruled out with similar arguments.

between asking the resale value of trader 2's low-value, $2v/3$, and the resale value of trader 2's medium-value, $1/3+2v/3$. This construction mimics the construction of a mixed strategy in the model with two-types.

Following our second observation above, for trader 1 to be indifferent between asking a price of $2v/3$ and asking a price of $1/3+2v/3$ upon rejection of his first offer, we must have that (i) $\mu^h \leq 2/3$ and (ii) $\mu^h + \mu^m = \frac{2v}{2v+1}$. Because the medium-value and the high-value accept with the same probability λ, Bayes rule imply that

$$\mu^h = \mu^m = \frac{1-\lambda}{2(1-\lambda)+1}.$$

Therefore $\mu^h + \mu^m = \frac{2(1-\lambda)}{2(1-\lambda)+1}$, and by setting $1-\lambda = v$ we guarantee that both conditions (i) and (ii) above are satisfied.

Finally, in equilibrium, the seller must randomize in the subsequent round between asking price $2v/3$ and price $1/3+2v/3$, in such a way that both the high-value trader 2 and the medium-value trader 2 are indifferent between accepting and rejecting the offer in the first period. Let $\gamma(p)$ be the probability with which trader 1 plays price $2v/3$ given the rejection of a price $p \in (2v/3, 1/3+2v/3)$. To make the medium-value of trader 2 indifferent we must have

$$\frac{1}{3} + \frac{2}{3}v - p = (\frac{1}{3} + \frac{2}{3}v - \frac{2}{3}v)\gamma(p),$$

or equivalently, $\gamma(p) = 1+2v-3p$. And it is easy to check that $\gamma(p)$ also makes the high-value of trader 2 indifferent between accepting and rejecting.

Fourth observation. Given the strategy of trader 2, the initial owner in the first round will either ask the resale value of the low-value trader 2, $2v/3$, or the resale value of the medium-value trader 2, $1/3+2/3v$, or she will wait one round. Given the specified priors, the optimal strategy for trader 1 is to demand a price $1/3+2/3v$ in the first round.

Summarizing. In the first round trader 1 asks the continuation payoff of the medium-value of trader 2. Trader 2 rejects if she has low-value, while she accepts with probability $1-v$ if she has either a high or medium-value. In case of rejection, in the second round trader 1 asks again $1/3+2/3v$ with probability one. Trader 2 rejects if she has low-value, but she accepts if she has either high or medium-value. In the former case, in the last round

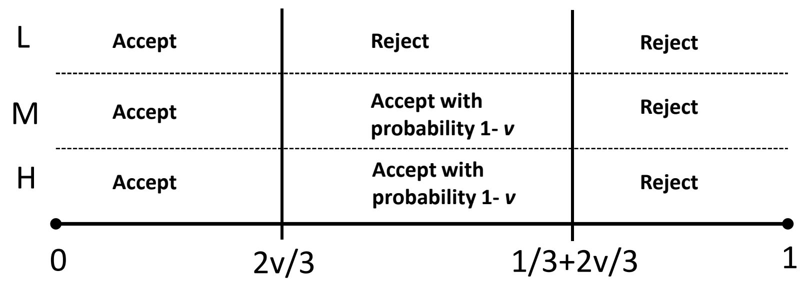


Figure 7: Strategy of 2 with offer at t=1.

trader 1 consumes the object. In the latter case there are two possibilities. If trader 2 has high-value, she consumes the object. If trader 2 has medium-value, then she demands a price of 1 to trader 3, who accepts the offer if she has high-value. Otherwise, trader 2 consumes.

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