The possibility of ideological bias in structural macroeconomic models

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A richer model

Introducing the people

- Add people's expectations in addition to gvt policy:

$$
\begin{aligned}
y &= -\beta i + \alpha g + u_0 + \theta v \\
i &= p + y \\
y &= \delta p - \mu p^e + v
\end{aligned}
$$

- $i =$ interest rate, $p=$ price level, and $p^e =$expected price level
- AD shock u_0, AS shock v.
- $0 \leq \mu \leq \delta$.

A richer model

Rewriting it

$$y = -bp^e + ag + u + \rho v; \quad (1)$$

$$p = \frac{\mu}{\delta}p^e - \frac{v}{\delta} + \frac{y}{\delta}. \quad (2)$$

- a, b, and ρ are composite parameters given by

$$a = \frac{\alpha\delta}{\delta + \beta(1+\delta)};$$

$$b = \frac{\beta\mu}{\delta + \beta(1+\delta)} \leq \mu;$$

$$\rho = \frac{\beta + \theta\delta}{\delta + \beta(1+\delta)} \geq \frac{b}{\mu}.$$

- $u = \frac{\delta}{\delta+\beta(1+\delta)} u_0$.

A richer model (3)

- People and government observed a signal

$$z = \omega u + \varepsilon.$$

- Again, $\omega^2 \sigma_u^2 + \sigma_\varepsilon^2 = 1$.
- Ex-ante information set (for forming expectations) $= \{z\}$,
- Ex-post information set (for validating the model) $= \{y, p, z\}$.

The perceived model

- Plausibility conditions: all parameters nonnegative and $0 \leq \hat{\mu} \leq \hat{\delta}$.
- Any $(\hat{a}, \hat{b}, \hat{\rho})$ that satisfies $\hat{\rho} \geq \frac{\hat{b}}{\hat{\mu}}$ and $\hat{b} \leq \hat{\mu}$ can be matched by some $(\hat{\alpha}, \hat{\beta}, \hat{\theta})$
- I assume that the theorist can directly set $(\hat{a}, \hat{b}, \hat{\rho})$, and add $\hat{\rho} \geq \frac{\hat{b}}{\hat{\mu}}$ and $\hat{b} \leq \hat{\mu}$ to the plausibility conditions.

I will proceed as follows. First, I solve for the equilibrium, given the model used by the people and the level of government spending. Second, I derive the optimal government policy. Third, I spell out the autocoherence conditions that the model must satisfy.

Solving for p and y

- Reduced form for p is

$$p = \frac{\mu}{\delta} p^e - \frac{v}{\delta} - \frac{b}{\delta} p^e + \frac{a}{\delta} g + \frac{u}{\delta} + \frac{\rho v}{\delta}.$$

- But people believe that:

$$p = \frac{\hat{\mu}}{\hat{\delta}} p^e - \frac{\hat{v}}{\hat{\delta}} - \frac{\hat{b}}{\hat{\delta}} p^e + \frac{\hat{a}}{\hat{\delta}} g + \frac{\hat{u}}{\hat{\delta}} + \frac{\hat{\rho}\hat{v}}{\hat{\delta}}.$$

- Allows to compute p^e

$$p^e = \frac{1}{\hat{\delta} + \hat{b} - \hat{\mu}} \hat{E}(u \mid z) + \frac{\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}} g. \tag{3}$$

- Then solve for p and y

$$y = -\frac{b}{\hat{\delta} + \hat{b} - \hat{\mu}} \hat{E}(u \mid z) + \left(a - \frac{b\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}} \right) g + u + \rho v. \tag{4}$$

$$p = \frac{\mu - b}{\delta(\hat{\delta} + \hat{b} - \hat{\mu})} \hat{E}(u \mid z) + \left(\frac{a}{\delta} + \frac{\hat{a}(\mu - b)}{\delta(\hat{\delta} + \hat{b} - \hat{\mu})} \right) g + \frac{u}{\delta} + \frac{\rho - 1}{\delta} v. \tag{5}$$

Optimal government policy

- Upon realization of z, the government sets g so as to minimize

$$\hat{E}(y^2 + \varphi g^2 \mid z) = \hat{E}(y^2 \mid z) + \varphi g^2.$$

- No credibility problem; the FOC

$$\frac{\hat{d}y}{\hat{d}g}\hat{E}(y \mid z) + \varphi g = 0. \tag{6}$$

- $\frac{\hat{d}y}{\hat{d}g}$ = perceived reduced form Keynesian multiplier (PRFKM)

The correct RFKM

$$\frac{dy}{dg} = a - \frac{b\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}}. \tag{7}$$

- CRFKM depends on both CM and PM
- CRFKM not identified because g proportional to z

Why does PM matter?

This is because part of the expansionary effect of government spending is dissipated by greater inflationary expectations, which in turn generate greater inflation and a contractionary response of the interest rate. For example, the more people believe that government policy is effective (the greater $\hat{a}$), the more they think it will be inflationary, and the smaller the Keynesian multiplier given a. For the same reason, the more people believe the output/inflation trade-off is unfavorable (the smaller $\hat{\delta}$), the smaller $\frac{dy}{dg}$.

- The government uses the perceived model to compute the (RF) Keynesian multiplier.
- Just replace a and b with $\hat{a}$ and $\hat{b}$, in (7)

$$\frac{\hat{d}y}{\hat{d}g} = \frac{\hat{a}(\hat{\delta} - \hat{\mu})}{\hat{\delta} + \hat{b} - \hat{\mu}}. \tag{8}$$

- This then alloes to compute g,

$$g = \gamma z,$$

$$\gamma = -\hat{a}\frac{(\hat{\delta} - \hat{\mu})^2}{\varphi\left(\hat{\delta} + \hat{b} - \hat{\mu}\right)^2 + \hat{a}^2\left(\hat{\delta} - \hat{\mu}\right)^2}\hat{\omega}\hat{\sigma}_u^2 < 0. \tag{9}$$

Properties of government policy

- $|\gamma|$ is larger,
  - The more people believe in a favorable "long-term" phillips curve, i.e. the greater $\hat{\delta} - \hat{\mu}$
  - The more they believe the interest response of aggregate demand is low, i.e. the smaller $\hat{b}$
- Effect of $\hat{a}$ ambiguous:"income effect" and "substitution" effect
  - for small $\hat{a}$, substitution effect dominates

What is next?

- Compute the correct and perceived reduced form models
- Use these to compute the moments of the observables
- Match these moments $==>$ Autocoherence conditions
- Compute the economist's welfare as function of perceived parameters
- Find the perceived parameters that maximize it subject ton AC conditions

The correct reduced form model

Observable	Expression
Output	$y = a_{yu}u + a_{y\varepsilon}\varepsilon + \rho v$
Price	$p = a_{pu}u + a_{p\varepsilon}\varepsilon + \frac{\rho-1}{\delta}v$
Coefficients	**Expression**
a_{yu}	$-\frac{b}{\hat{\delta}+\hat{b}-\hat{\mu}}\omega\hat{\omega}\hat{\sigma}_u^2 + \omega\gamma(a - \frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}}) + 1$
$a_{y\varepsilon}$	$-\frac{b}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}\hat{\sigma}_u^2 + \gamma(a - \frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}})$
a_{pu}	$\frac{\mu-b}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\omega\hat{\omega}\hat{\sigma}_u^2 + \omega\gamma\left(\frac{a}{\delta} + \frac{\hat{a}(\mu-b)}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\right) + \frac{1}{\delta}$
$a_{p\varepsilon}$	$\frac{\mu-b}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\hat{\omega}\hat{\sigma}_u^2 + \left(\frac{a}{\delta} + \frac{\hat{a}(\mu-b)}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\right)\gamma$

Table 1 – The correct reduced form model

The perceived reduced form model

Observable	Expression
Output	$y = \hat{a}_{yu}\hat{u} + \hat{a}_{y\varepsilon}\hat{\varepsilon} + \hat{\rho}\hat{v}$
Price	$p = \hat{a}_{pu}\hat{u} + \hat{a}_{p\varepsilon}\hat{\varepsilon} + \frac{\hat{\rho}-1}{\hat{\delta}}v$

Coefficients	Expression
$\hat{a}_{yu}$	$-\frac{\hat{b}}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}^2\hat{\sigma}_u^2 + \gamma\hat{\omega}\frac{\hat{a}(\hat{\delta}-\hat{\mu})}{\hat{\delta}+\hat{b}-\hat{\mu}} + 1$
$\hat{a}_{y\varepsilon}$	$-\frac{\hat{b}}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}\hat{\sigma}_u^2 + \gamma\frac{\hat{a}(\hat{\delta}-\hat{\mu})}{\hat{\delta}+\hat{b}-\hat{\mu}}$
$\hat{a}_{pu}$	$\frac{\hat{\mu}-\hat{b}}{\hat{\delta}(\hat{\delta}+\hat{b}-\hat{\mu})}\hat{\omega}^2\hat{\sigma}_u^2 + \frac{\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}\gamma + \frac{1}{\hat{\delta}}$
$\hat{a}_{p\varepsilon}$	$\frac{\hat{\mu}-\hat{b}}{\hat{\delta}(\hat{\delta}+\hat{b}-\hat{\mu})}\hat{\omega}\hat{\sigma}_u^2 + \frac{\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}}\gamma$

Table 2 – The perceived reduced form model

A flavor of what is going on

$$a_{yu} = -\frac{b}{\hat{\delta}+\hat{b}-\hat{\mu}}\omega\hat{\omega}\hat{\sigma}_u^2 + \omega\gamma(a - \frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}}) + 1, \tag{10}$$

has three components.

- The constant 1 captures the direct effect of the aggregate demand shock on output.
- $\omega\gamma(a - \frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}})$ is typically negative and captures the stabilizing effect of fiscal policy.
  - $a =$ direct effect of fiscal policy,
  - $-\frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}} =$ dissipation through price expectations
    - This reaction is stronger, the greater the perceived effect of fiscal policy on output ($\hat{a}$), the greater the actual effect of interest rates on output (b), and the more "unfavorable" the perceived Phillips curve (the greater $\hat{\mu}$ and the smaller $\hat{\delta}$).
- $-\frac{b}{\hat{\delta}+\hat{b}-\hat{\mu}}\omega\hat{\omega}\hat{\sigma}_u^2 =$ effect of the direct reaction of price expectations to the signal about the demand shock. ("monetary component")

Price block irrelevance

- Joint distribution of p and z is observed
- $==>$ AC imply that $\hat{E}(p \mid z) = E(p \mid z)$.
- Unique REE conditional on gov. policy
- One important consequence is

Proposition 2 — The autocoherence conditions imply

$$\hat{\delta} - \hat{\mu} = \delta - \mu.$$

Corollary – Given $\hat{a}$, and $\hat{b}$, γ is independent of the choice of $\hat{\delta}$ and $\hat{\mu}$, and so is the equilibrium.

Why?

- Output effect of government spending only depends on $\delta - \mu$
- To match AC conditionsm the economist is forced to reveal it.
- The demand signal z is not polluted by the supply shock
- It acts as an *instrumental variable* allowing agents to infer $\delta - \mu$ from $cov(y, z)$ and $cov(p, z)$

The revealed price block case

- Assume δ, μ, ω, σ_u are known.
- One can show that AC implies matching RF responses:

$$\begin{aligned} a_{y\varepsilon} &= \hat{a}_{y\varepsilon}, a_{yu} = \hat{a}_{yu}, a_{p\varepsilon} = \hat{a}_{p\varepsilon}, \\ a_{pu} &= \hat{a}_{pu}, \rho = \hat{\rho}. \end{aligned}$$

- They boil down to a trade-off between $\hat{a}$ and $\hat{b}$:

$$(\hat{b} - b)\omega\sigma_u^2 = \gamma\left[(\hat{a} - a)(\delta - \mu) + \hat{a}b - a\hat{b}\right]; \tag{11}$$

$$\gamma = -\hat{a}\frac{(\delta - \mu)^2}{\varphi\left(\delta + \hat{b} - \mu\right)^2 + \hat{a}^2\left(\delta - \mu\right)^2}\omega\sigma_u^2. \tag{12}$$

The quasi-Lucas approximmation

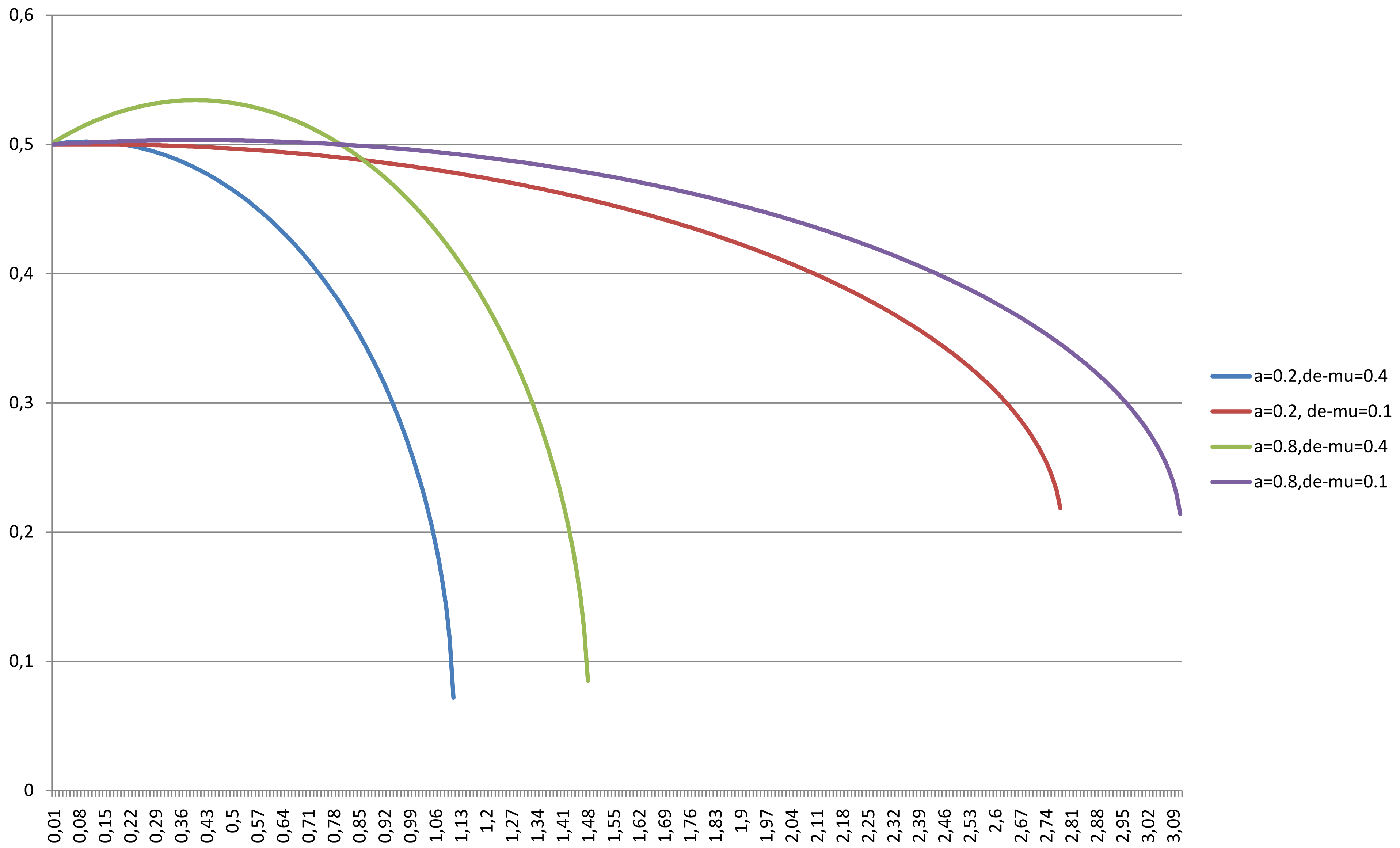
- Assume $\delta - \mu << 1$. Then trade-off is equivalent to

$$\hat{b} \approx b - \frac{\hat{a}(\hat{a} - a)}{\varphi b}(\delta - \mu)^2. \tag{13}$$

This trade-off has the following properties

- $(\hat{b} - b)(\hat{a} - a) < 0$
  - The more g has a large perceived impact on y, the lower the perceived impact of i.
- The trade-off is flatter, the smaller $\delta - \mu$, the greater φ and the greater b.
  - $==>$The more the theoretical effect of interest rates must be close to the actual one, and the more arbitrary the theoretical impact of government spending.
- See figure 1 for the more general case

Figure 1 -- The a/b trade-off



Why?

$$\hat{a}_{yu} = -\frac{\hat{b}}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}^2\hat{\sigma}_u^2 + \gamma\hat{\omega}\frac{\hat{a}(\hat{\delta}-\hat{\mu})}{\hat{\delta}+\hat{b}-\hat{\mu}} + 1.$$

Consider an increase in $\hat{a}$ and hold γ constant. Then the output response a_{yu} is unchanged (conditional REE uniqueness). On the other hand, people will believe that it has fallen, since they think that the direct expansionary effect of fiscal policy (which outweighs its indirect contractionary effect through inflation expectations) is now stronger. This is captured by the fiscal component in $\hat{a}_{yu}$, $\gamma\hat{\omega}\frac{\hat{a}(\hat{\delta}-\hat{\mu})}{\hat{\delta}+\hat{b}-\hat{\mu}}$, which, since $\hat{\mu} < \hat{\delta}$, clearly falls in algebraic value as $\hat{a}$ goes up. This discrepancy would invalidate the model empirically unless $\hat{b}$ is changed so as to restore the equality between the actual and perceived elasticity of output to demand shocks. The dominant effect of a reduction in $\hat{b}$ (in a quasi-Lucas economy) is to increase the algebraic value of the perceived monetary component of $\hat{a}_{yu}$, given by $-\frac{\hat{b}}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}^2\hat{\sigma}_u^2$;the lower $\hat{b}$, the lower the perceived output response to interest rates, and the lower the perceived stabilizing effect of monetary reactions to demand shocks. This effect raises the perceived response of output to demand shocks, thus restoring the model's autocoherence. This explains why there is a negative trade-off between $\hat{a}$ and $\hat{b}$. Since $\hat{b}$ is the interest elasticity of output, this means that experts face a trade-off between believing in fiscal policy effectiveness versus believing in monetary policy effectiveness. An economist who would underpredict both elasticities would also underpredict output volatility and could not empirically validate his model.

The expert's optimal model

- Again, quasi-dictatorship holds

$$\gamma = \bar{\gamma} = -a \frac{(\delta - \mu)^2}{\bar{\varphi}(\delta + b - \mu)^2 + a^2 (\delta - \mu)^2} \omega \sigma_u^2. \tag{14}$$

- Typically,

$$\frac{d\hat{a}}{d\bar{\varphi}} < 0.$$

- More conservative economists will understate the impact of public interest rates and accordingly, to remain autocoherent, overstate that of interest rates.
- Furthermore, again $\varphi = \bar{\varphi} \Longrightarrow \gamma = \bar{\gamma}$.
  - Since autocoherence imposes rational inflation expectations, there is no scope for manipulating the public and an economist aligned with the government cannot do better than reveal the truth.

The expert's optimal model (2)

$\bar{\varphi}$	$a = 0.2, \delta - \mu = 0.4$		$a = 0.2, \delta - \mu = 0.1$	
	$\hat{a}$	$\hat{b}$	$\hat{a}$	$\hat{b}$
0.08	1.1^*	0.117	1.78	0.43
0.4	0.39	0.48	0.34	0.498
$0.8 = \varphi$	0.2	0.5	0.2	0.5
1.2	0.13	0.502	0.13	0.5
1.6	0.1	0.502	0.1	0.5

Variant 2: price block not revealed

- Assume now that

$$z = \omega u - \lambda v.$$

- Solve again the model as before
- z is no longer a valid instrument $==> \delta - \mu$ not revealed
- For simplicity assume a and b are known

Variant 2: the true RF model

Observable	Expression
Output	$y = a_{yu} u + a_{yv} v$
Price	$p = a_{pu} u + a_{pv} v$
Coefficients	**Expression**
a_{yu}	$1 - b\omega\hat{c} + \omega\gamma(a - \frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}})$
a_{yv}	$\rho + b\lambda\hat{c} - \gamma\lambda(a - \frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}})$
a_{pu}	$\omega\gamma\left(\frac{a}{\delta} + \frac{\hat{a}(\mu-b)}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\right) + (\mu - b)\frac{\omega\hat{c}}{\delta} + \frac{1}{\delta}$
a_{pv}	$\frac{\rho-1}{\delta} - (\mu - b)\frac{\lambda\hat{c}}{\delta} - \lambda\gamma\left(\frac{a}{\delta} + \frac{\hat{a}(\mu-b)}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\right).$

Table 4 – The correct reduced form model, Variant B

Variant 2: the perceived RF model

Observable	Expression
Output	$y = \hat{a}_{yu}\hat{u} + \hat{a}_{yv}\hat{v}$
Price	$p = \hat{a}_{pu}\hat{u} + \hat{a}_{pv}\hat{v}$

Coefficients	Expression
$\hat{a}_{yu}$	$1 - \hat{b}\hat{\omega}\hat{c} + \hat{\omega}\gamma\frac{\hat{a}\left(\hat{\delta}-\hat{\mu}\right)}{\hat{\delta}+\hat{b}-\hat{\mu}}$
a_{yv}	$\hat{\rho} + \hat{b}\hat{\lambda}\hat{c} - \gamma\hat{\lambda}\frac{\hat{a}\left(\hat{\delta}-\hat{\mu}\right)}{\hat{\delta}+\hat{b}-\hat{\mu}}$
a_{pu}	$\hat{\omega}\gamma\frac{\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}} + (\hat{\mu} - \hat{b})\frac{\hat{\omega}\hat{c}}{\hat{\delta}} + \frac{1}{\hat{\delta}}$
a_{pv}	$\frac{\hat{\rho}-1}{\hat{\delta}} - (\hat{\mu} - \hat{b})\frac{\hat{\lambda}\hat{c}}{\hat{\delta}} - \hat{\lambda}\gamma\frac{\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}}.$

Table 5 – The perceived reduced form model, Variant B

The quasi-correct approximation

- Assume parameters are close to truth
- Then AC conditions can be linearized as

$$v = \Delta\hat{\delta}.q,$$

where $v = (\Delta(\hat{\delta} - \hat{\mu}), \Delta\hat{\lambda}, \Delta\hat{\sigma}_v, \Delta\hat{\omega}, \Delta\hat{\sigma}_u, \Delta\hat{\rho})'$

- Quasi-dictator then selects parameters such that

$$\Delta\hat{\delta} = m\Delta\varphi,$$

where

$$m = \frac{\frac{\partial\gamma}{\partial\varphi}}{(\nabla_v\gamma).q}. \tag{15}$$

Some characteristics of a theory

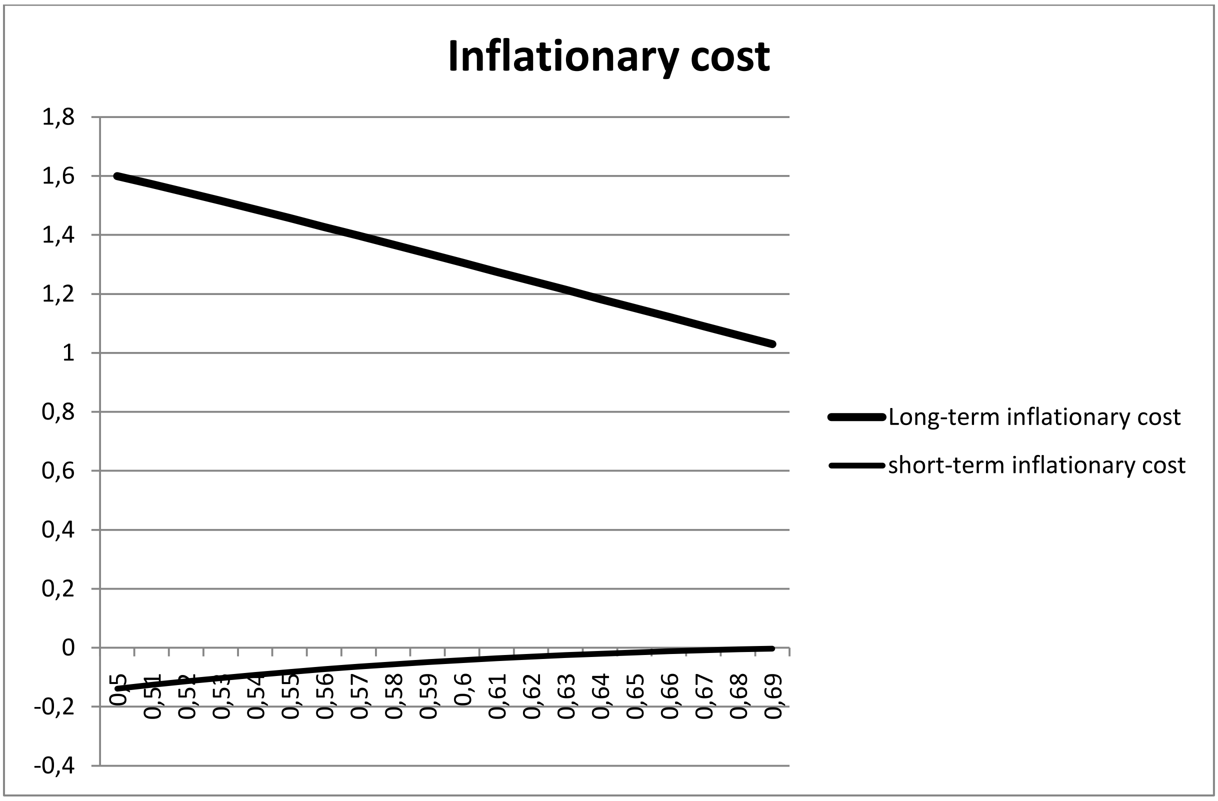
- The short-term inflationary cost of output (STC). This is equal to $1/\hat{\delta}$.
- The long-term inflationary cost of output (LTC), equal to $1/(\hat{\delta}-\hat{\mu})$.
- The relative importance of supply shocks (RIS), equal to $\frac{\hat{\sigma}_v^2}{\hat{\sigma}_u^2}$.
- The supply-intensity of the price indicator (SIP), equal to $\hat{\lambda}^2\hat{\sigma}_v^2$.
- The share of output fluctuations explained by supply shocks (SSO); given by $\frac{\hat{a}_{yv}^2\hat{\sigma}_v^2}{\hat{a}_{yv}^2\hat{\sigma}_v^2+\hat{a}_{yu}^2\hat{\sigma}_u^2}$.
- For each of these parameters, its *ideological sensitivity* is defined as its derivative with respect to φ.

Ideological sensitivities

Parameter	Ideological sensitivity
STC	$-m/\delta^2$
LTC	$-mq_1/(\delta-\mu)^2$
RIS	$\frac{2\sigma_v}{\sigma_u^2}m(q_3-\frac{\sigma_v}{\sigma_u}q_5)$
SIP	$2m(\lambda\sigma_v^2 q_2+\lambda^2\sigma_v q_3)$

Table 5 – Ideological sensitivities of key perceived parameters (Figures 4 to 9)

Figure 4 – Ideological sensitivities, a = 0.7 ; b = 0.5 ; ω = 1; λ=1; σ_u^2 = 0.5; δ = 0.7 ; ρ = 1.



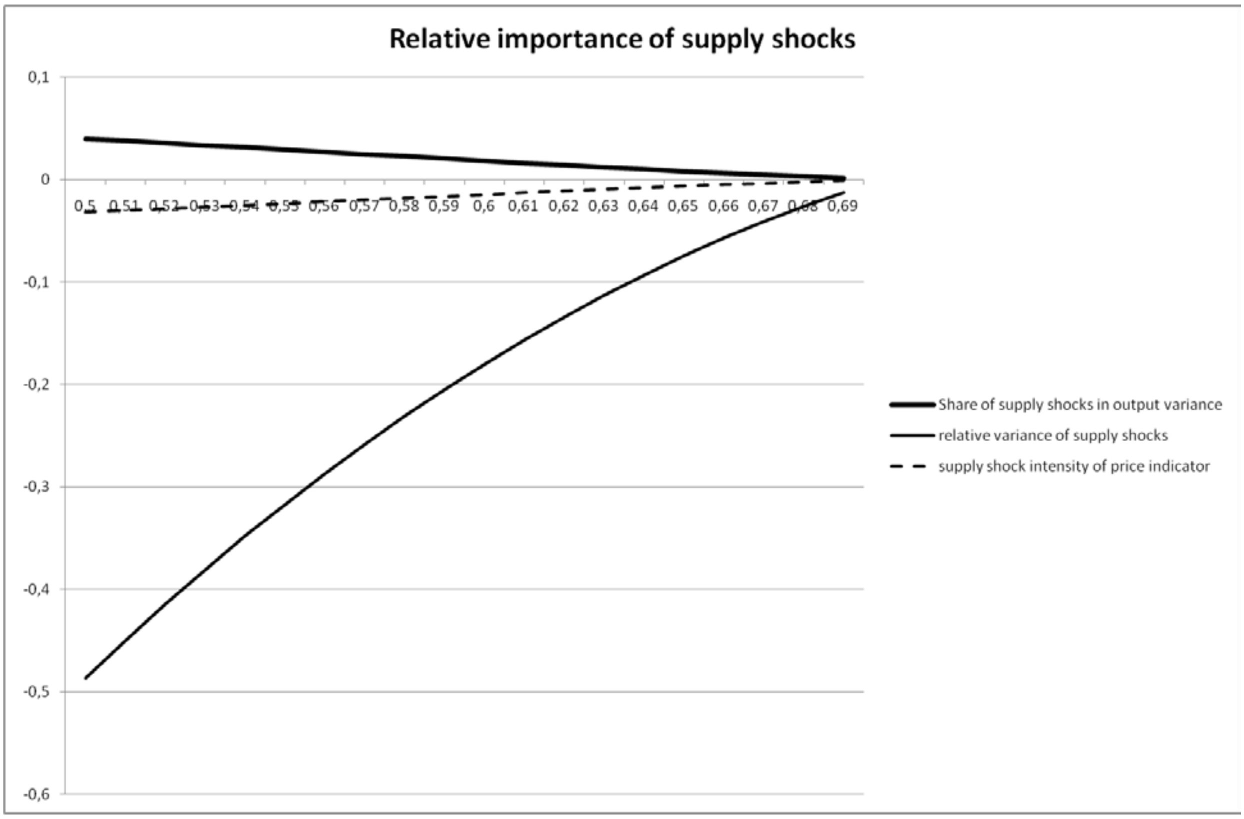
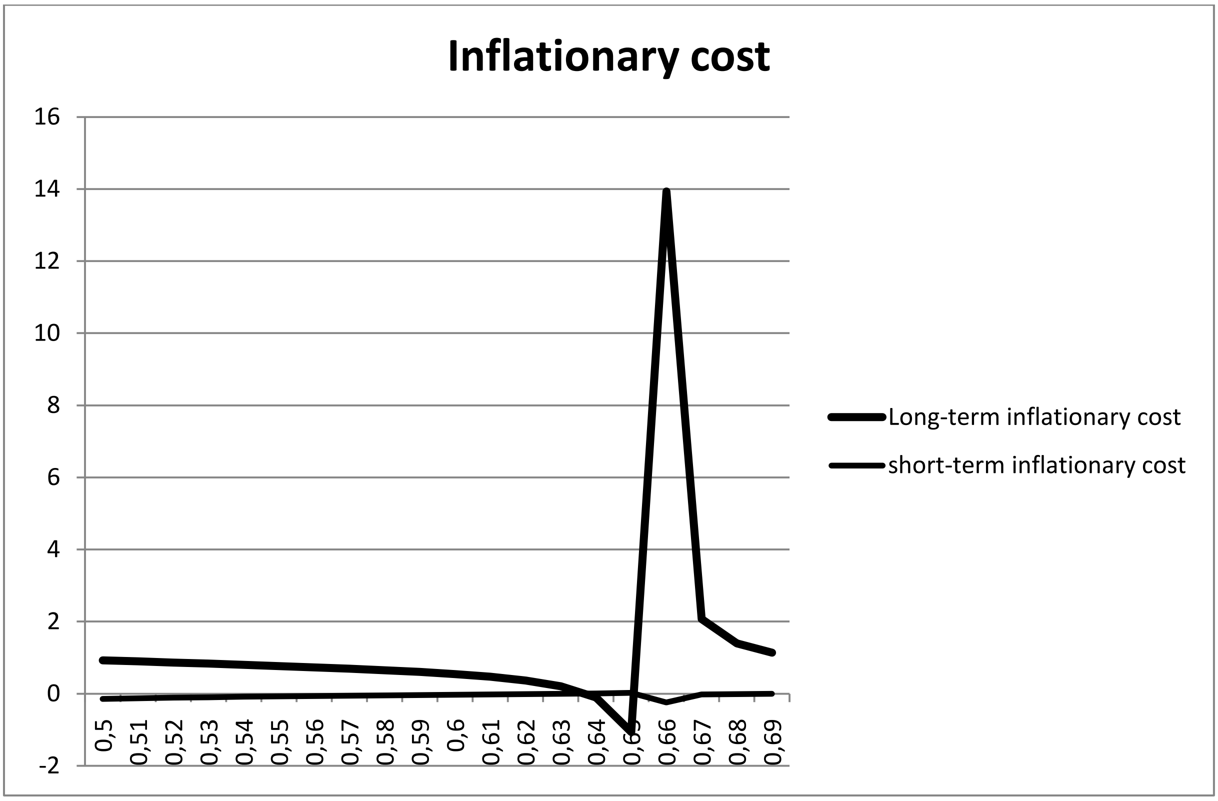


Figure 5 – Ideological sensitivities, a = 0.7 ; b = 0.5 ; ω = 1; λ=1; σ_u^2 = 0.9; δ = 0.7 ; ρ = 1.



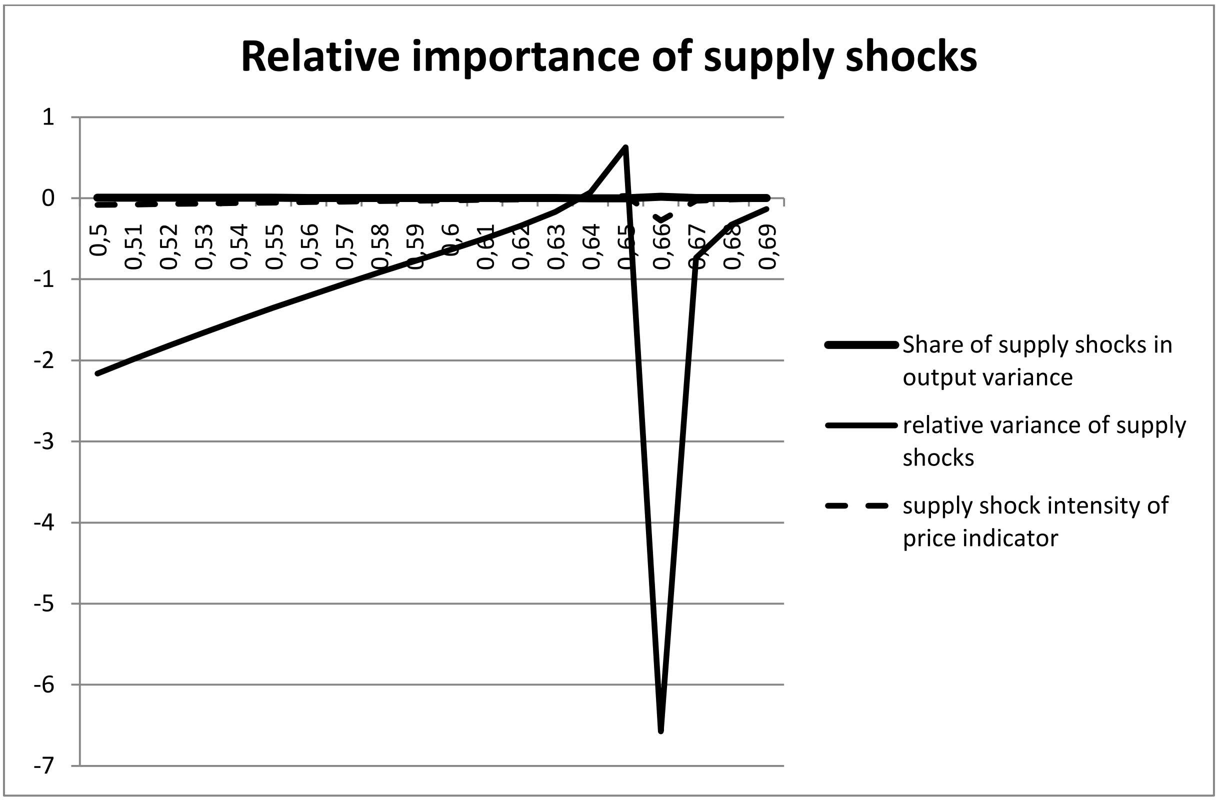
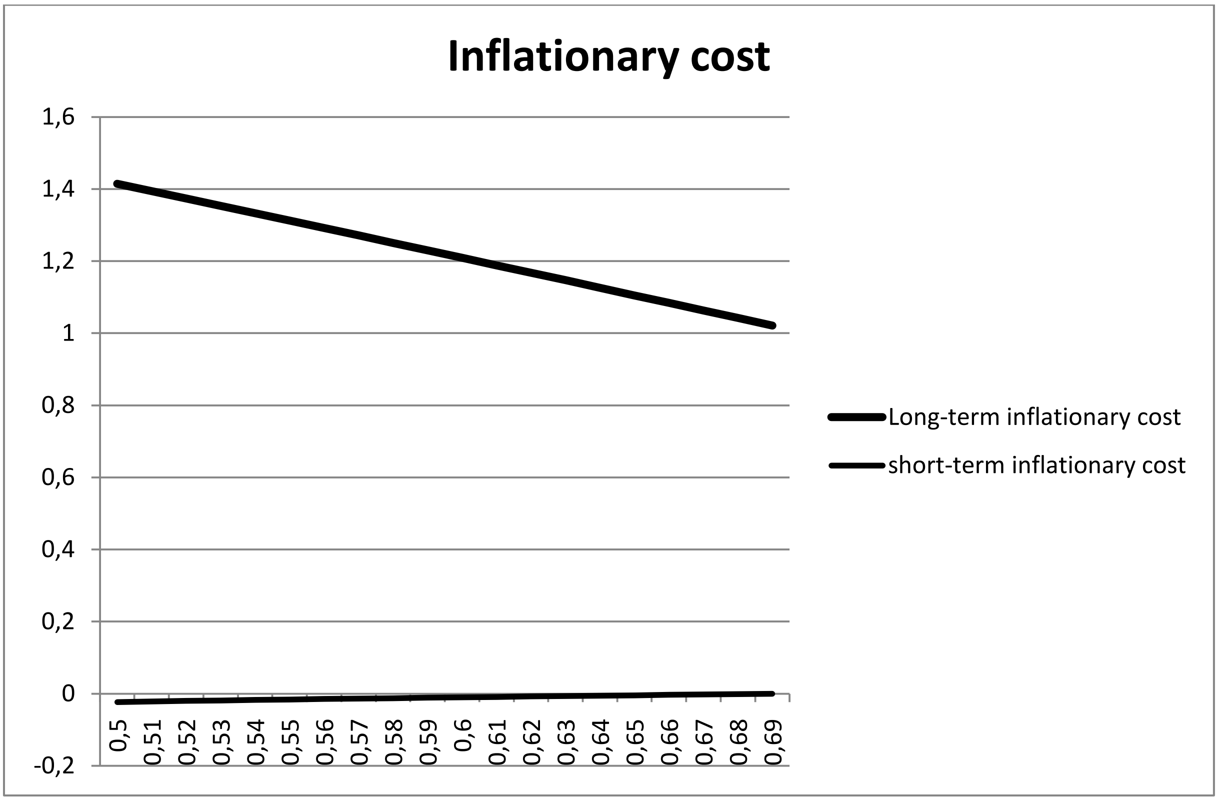


Figure 6 – Ideological sensitivities, a = 0.7 ; b = 0.5 ; ω = 1; λ=1; σ_u^2 = 0.1; δ = 0.7 ; ρ = 1.



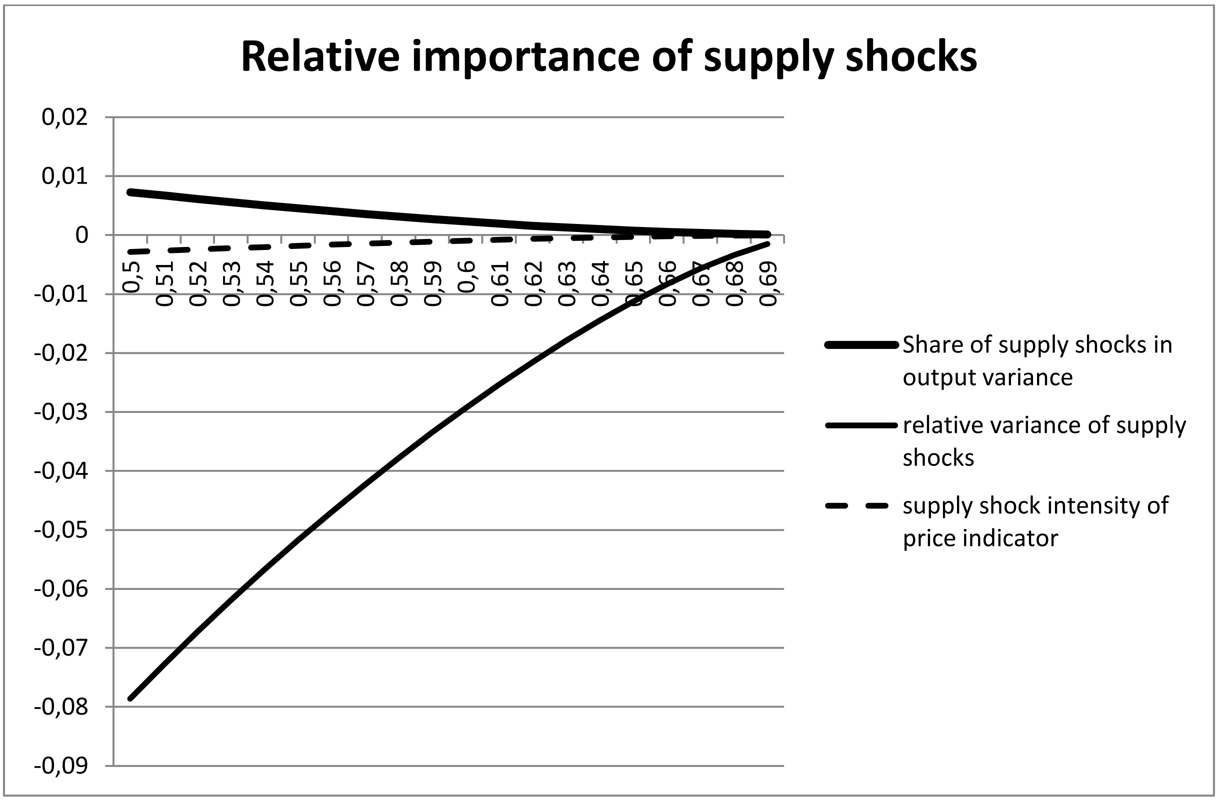
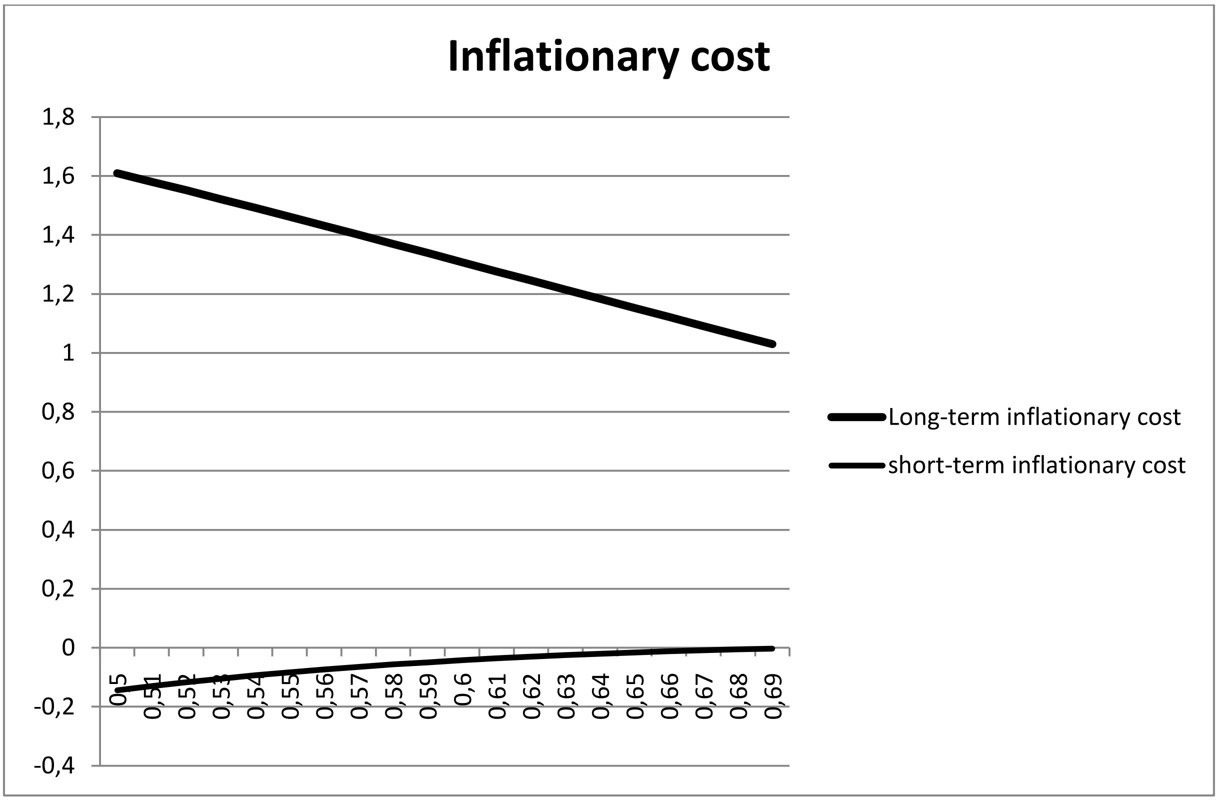


Figure 7 – Ideological sensitivities, a = 1 ; b = 0.5 ; ω = 1; λ=1; σ_u^2 = 0.5; δ = 0.7 ; ρ = 1.



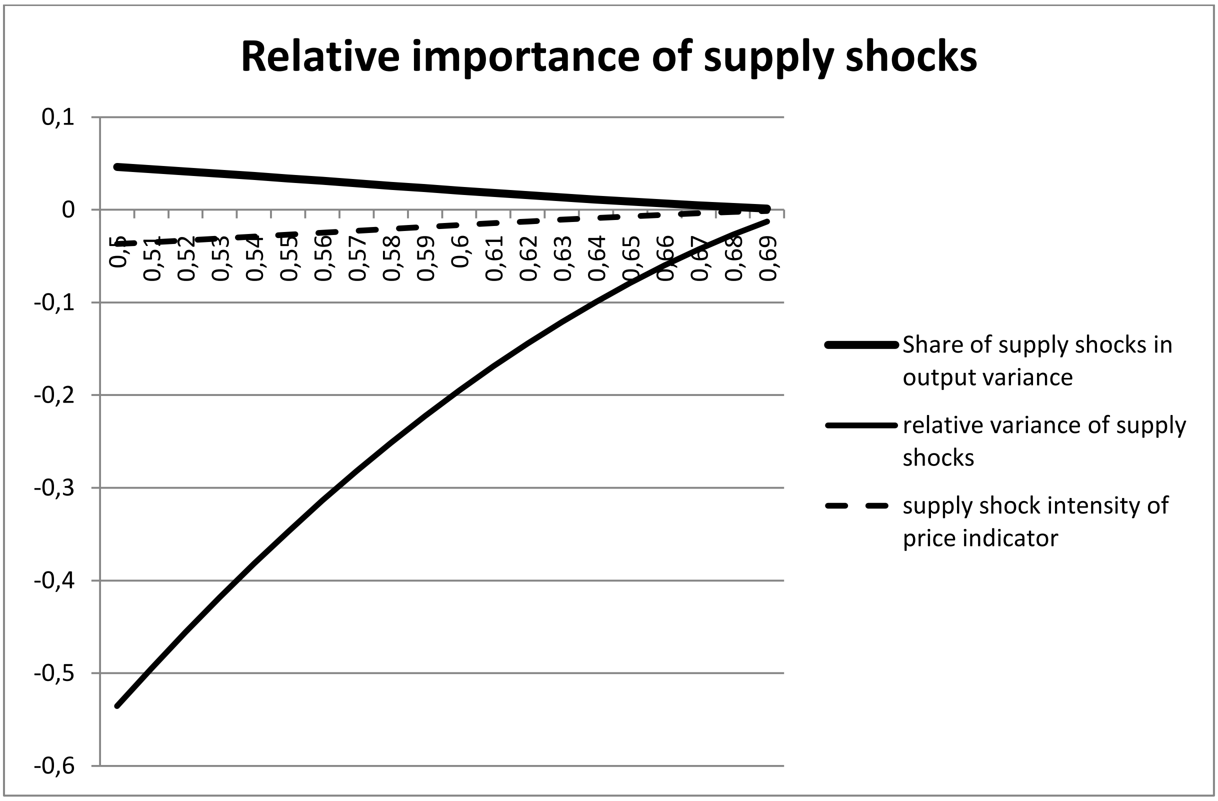
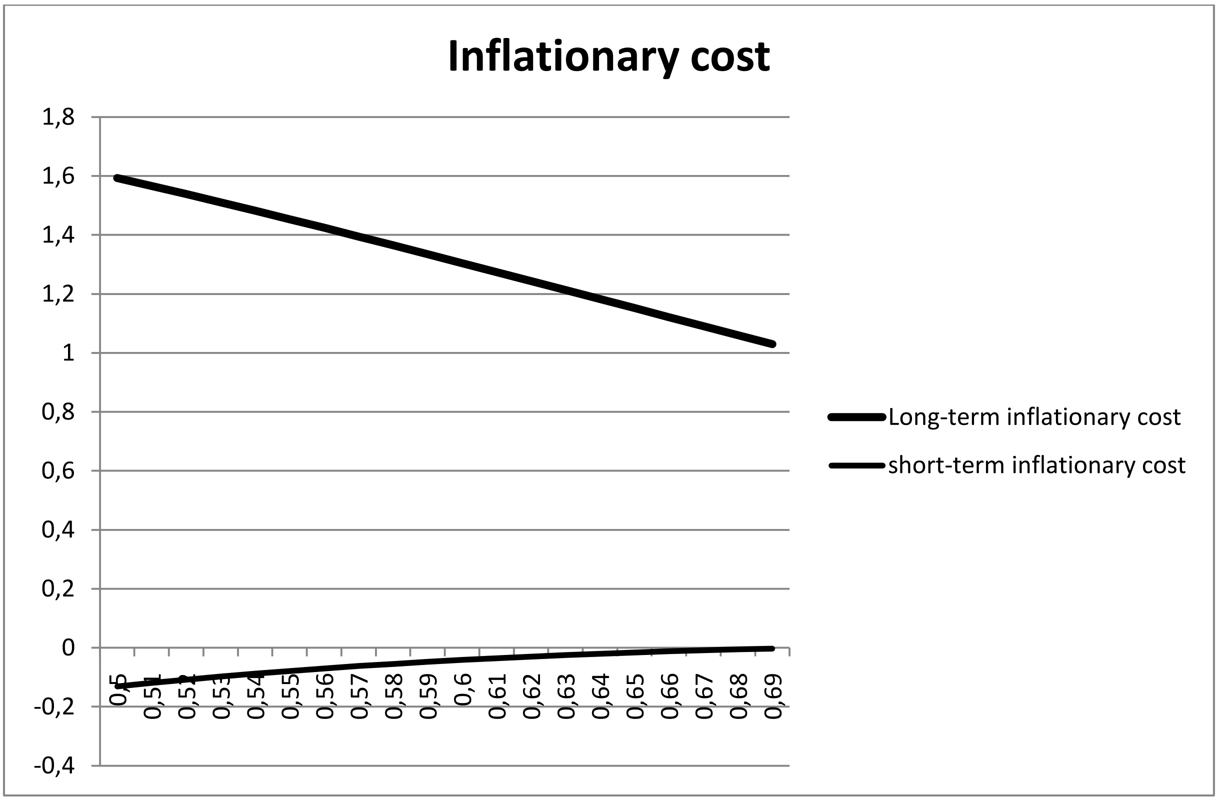


Figure 8 – Ideological sensitivities, a = 0.3 ; b = 0.5 ; ω = 1; λ=1; σ_u^2 = 0.5; δ = 0.7 ; ρ = 1.



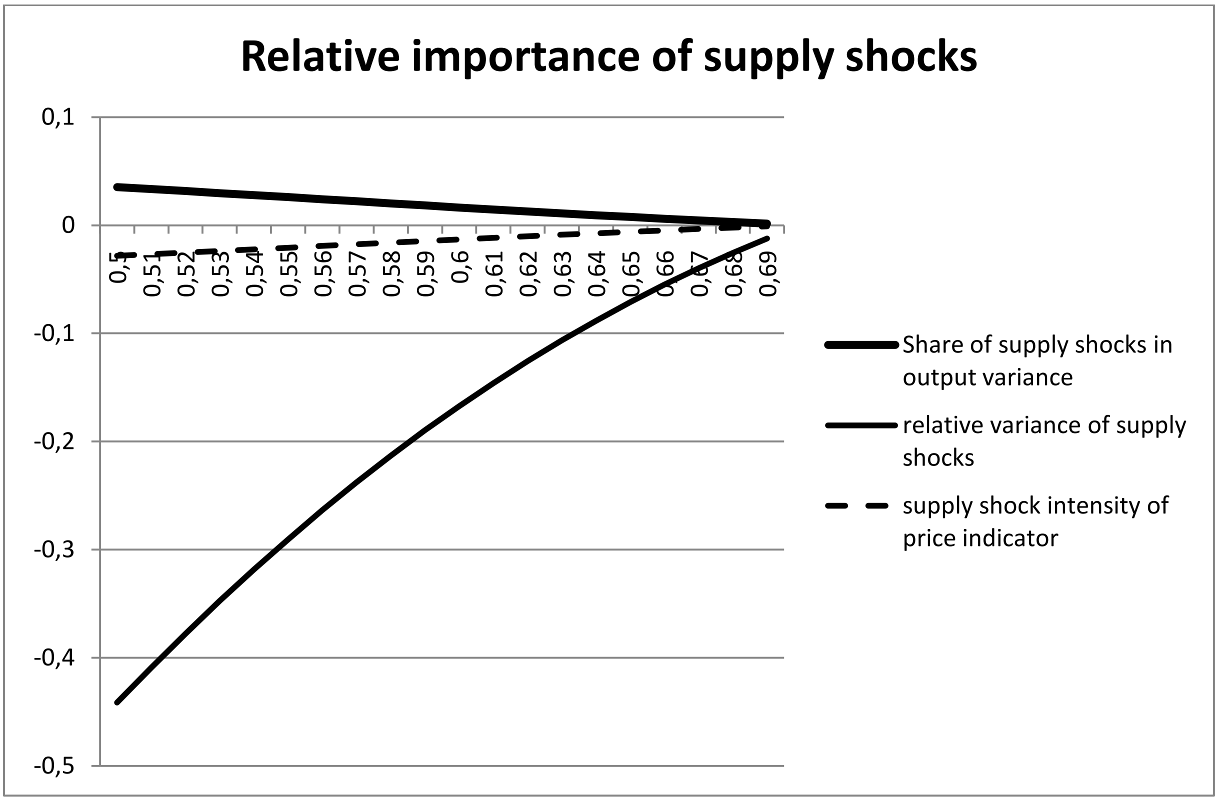
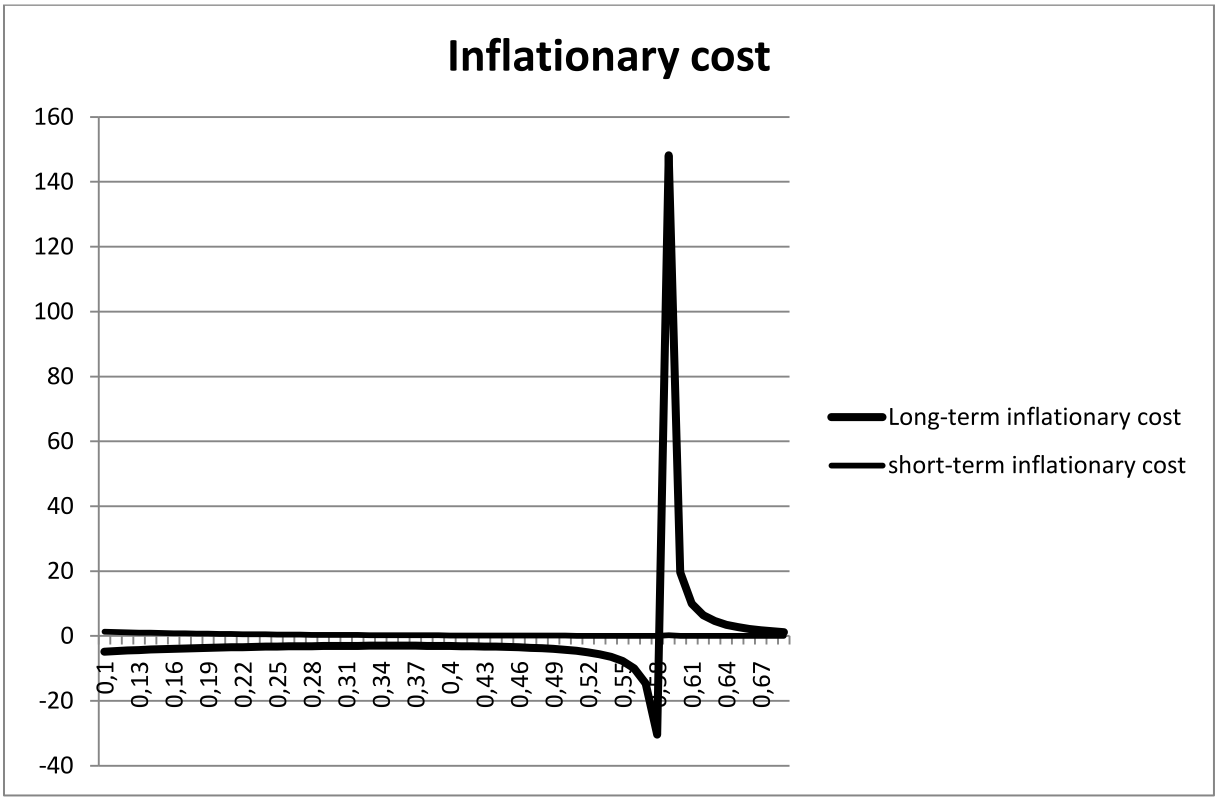
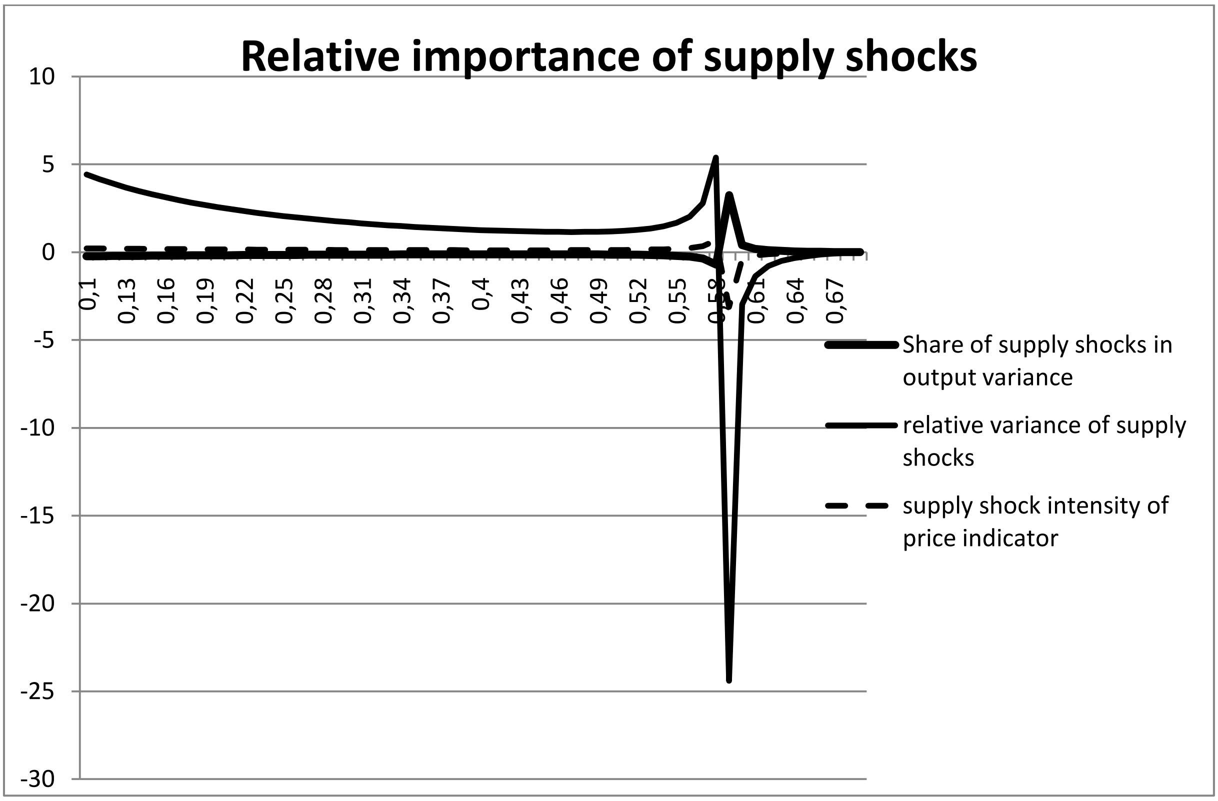


Figure 9 – Ideological sensitivities, a = 0.7 ; b = 0.1 ; ω = 1; λ=1; σ_u^2 = 0.5; δ = 0.7 ; ρ = 1.





Which pattern?

- Typically, the ideological sensitivity of LTC is positive
- For example, the STC is (paradoxically) always negative except on Figure 9.
- But the ideological sensitivity of STC is always small (ie we are close to the truth)
- SSO typically has a positive ideological sensitivity
- Not true for RIS
- The economy can be "critical" (no effect of ideology, ideological sensitivities become infinite)