The possibility of ideological bias in structural macroeconomic models*

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ABSTRACT: This paper studies the trade-offs that an expert with ideological biases faces in designing his model. I assume the perceived model must be autocoherent, in that its use by all agents delivers a self-confirming equilibrium. The exercise is carried in the context of a simplified AS-AD model, where in principle the expert can influence policy by manipulation six key parameters: the Keynesian multiplier, the interest elasticity of aggregate demand, the response of output to actual and expected inflation in the Phillips curve, and the variances of supply and demand shocks. Typically, a larger reported Keynesian multiplier is favored by more left-wing economists, as is a flatter inflation output trade-off.

But an important aspect of the analysis is that autocoherence conditions imply constraints and trade-offs between parameters. For example a larger reported Keynesian multiplier must be associated with a lower interest elasticity of aggregate demand for the economists's model to match the data. Also, some parameters or some combinations of parameters must be truthfully revealed for the expert to remain autocoherent. These are the parameters that are "identified" from the empirical moments of the distribution of observables. This illustrates the tight link between parameter identification and the scope for bias that is generated by the autocoherence conditions.

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Keywords – Political Economy, Autocoherence, Experts, Macroeconomic modelling, Ideology, Bias

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1 Introduction

Can ideological bias pervade economic modelling, and yet act in such a way that prevailing models remain consistent with the data? Such biases may explain ongoing controversies among macroeconomists about key structural parameters, in particular (i) the size of the Keynesian multiplier, (ii) the slope of the aggregate supply curve, and (iii) the nature of the shocks that drive business cycles.

These controversies are well known to our profession. Here is an excerpt from a web reading list (http://homepage3.nifty.com/ronten/crisis-readings.htm) about the financial crisis:

> The Spending Multiplier Debate
>
> Positive: Romer and Bernstein: The Job Impact of the American Recovery and Reinvestment Plan; Krugman: The Conscience of a Liberal; Getting fiscal; War and Non-Remembrance; Don't know much about history; Paul Krugman recommends us to learn more from Japan's experiences; Adam P. Posen (1998) Fiscal

Policy Works When It Is Tried in Restoring Japan's Economic Growth.

Skeptical: Mankiw: Fiscal Policy Puzzles; *Spending and Tax Multipliers, How Not to Stimulate the Economy; John Taylor: Why Permanent Tax Cuts Are the Best Stimulus; Cogan, Cwik, Taylor and Wieland (2009): New Keynesian versus Old Keynesian Government Spending Multipliers; Smets-Wouters (2003) Model; Hall and Woodford: Measuring the Effect of Infrastructure Spending on GDP with a comment by Robert Gordon.

It is clear that these author's beliefs are natural matches for their political preferences[1]. This suggests that people seem to adopt views about underlying parameters that are conducive to the policies they would otherwise favor

[1] Here is an excerpt from wikipedia:

> Romer: "Christina Romer (née Duckworth; born December 25, 1958) is the Class of 1957 Garff B. Wilson Professor of Economics at the University of California, Berkeley and the out-going Chair of the Council of Economic Advisers in the Obama administration. She resigned from her role on the Council of Economic Advisers on September 3, 2010. After her nomination and before the Obama administration took office, Romer worked with economist Jared Bernstein to co-author the administration's plan recovery from the 2008 recession. In a January 2009 video presentation, she discussed details of the job creation package that the Obama administration submitted to Congress."
>
> Krugman: "In a review for The New York Times, Pulitzer prize-winning historian David M. Kennedy stated, "Like the rants of Rush Limbaugh or the films of Michael Moore, Krugman's shrill polemic may hearten the faithful, but it will do little to persuade the unconvinced"
>
> Mankiw: "He returned to politics when he was appointed by President George W. Bush as Chairman of the Council of Economic Advisors in May 2003."
>
> Taylor: "He has been active in public policy, serving as the Under Secretary of the Treasury for International Affairs during the first term of the George W. Bush Administration"

for ideological reasons.[2] Similarly, it is also true, historically, that left-wing economists have leaned toward a "flat" Phillips curve, while conservatives prefer it steep or even vertical. In the eighties, when the RBC literature was being developed, "fresh water" conservatives emphasized the quantitative role of supply shocks, while "salt water" social democrats thought that demand shocks were the driving force of economic fluctuations.

Since macroeconomic models play such an important role in the formulation of policies and in the formation of expectations, it is important to understand the production process for these models. In this paper, I assume that the intellectuals in charge of developing the theory are self interested, and I studies the trade-offs that they face in designing their model. As in Saint-Paul (2011), I assume the perceived model must be *autocoherent*, in that its use by all agents delivers a self-confirming equilibrium (as in Fudenberg and Levine (2003,2007) and Sargent (2008)). This means that the economist must choose perceived parameter values such that the moments of the observables, predicted using the perceived model as a data generating process, must match the actual moments in an equilibrium where policies and expectations are determined using the perceived model but the economy actually evolves according to the correct one[3]. The exercise is carried in the context of a simplified AS-AD model, where in principle the expert can influence policy by manipulating six key parameters:

-The response of aggregate demand to government expenditure

-The response of aggregate demand to interest rates

-The response of output to actual inflation in the Phillips curve

[2] To some extent, Fuchs et al. (1999) report similar findings.

[3] Therefore there is no way to prove that the model is actually incorrect. If the public knew the economists's preferences and if the true model were drawn from some known meta-probability distribution, the public could reverse engineer the true model from the perceived one, as in the cheap talk literature (Crawford and Sobel, 1982). However this is ruled out here and I assume throughout the paper that the expert is trusted.

-The response of output to expected inflation

-The variance of supply shocks

-The variance of demand shocks

Do we expect the economists's political preferences to influence those parameters in the directions predicted by the above discussion? The answer is a rough yes. For example, a larger reported Keynesian multiplier is favored by more left-wing economists, because it induces the government to pursue more activist policies, which they like better than conservatives. Similarly, a flatter inflation output trade-off will increase the perceived efficiency of activist policies, and left-wing economists will also want to favor those configurations.

But an important aspect of the analysis is that autocoherence conditions imply constraints and trade-offs between parameters. For example a larger reported Keynesian multiplier must be associated with a lower interest elasticity of aggregate demand for the economists's model to match the data, otherwise the covariance between output and an observed leading indicator of activity will be missed. Consequently, being over-optimistic about fiscal policy implies being over-pessimistic about monetary policy. Similarly, the economist must often run against his preferences for the short-term Phillips curve or the relative variance of supply shocks for the autocoherence constraints to be met.

Furthermore, some parameters or some combinations of parameters must be truthfully revealed for the expert to remain autocoherent. These are the parameters that are "identified" from the empirical moments of the distribution of observables that the economist must match. In the simple example below, agents base their expectations on a signal of the underlying demand shock, and that signal is orthogonal to the supply disturbance in the Phillips curve. This allows private agents to implicitly estimate the correct long-run slope of the Phillips curve by using that signal as an instrument; in other words, the long-run slope of the Phillips curve is implicitly revealed by the equilibrium moments of the observables. As it turns out, only that long-run slope matters for policy, and it is then impossible for economists to influence policy through the perceived Phillips curve parameters. I then extend the model to allow for this possibility, by assuming that the agents' signal is polluted by the supply shock. This illustrates the tight link between parameter identification and the scope for bias that is generated by the autocoherence conditions.

Another insight delivered by the analysis below is the possibility of what I label as criticality. For some parameter values of the true model, the autocoherence conditions imposed on the perceived model may make it locally impossible for the expert to influence policy. In such a case even small deviations between the expert's ideological preferences and the government will lead to large differences in the reported parameter values.

The present paper is related to several strands of literature. The idea of self-confirming equilibrium has been extensively analyzed in a series of papers by Fudenberg and Levine (2003,2009). It has also been applied to the political economy literature by Piketty (1995), on which Bénabou and Ok (2001), and Alesina and Angeletos (2005) build, in the context of a simple problem of redistribution where incorrect beliefs on the output elasticity of effort may be sustained. On the other hand, Sargent (2008) has been most prominent in analyzing the possibility that authorities may use an incorrect model[4]. This paper's main contribution is two-fold. First, it provides a positive theory of macroeconomic modelling in a political economy context. Second, it proposes a systematic approach to the characterization of self-confirming equilibria in a world where agents make use of a macroeconomic model to set their actions,

[4]See also Hansen et al. (2006), Sargent et al. (2006), Buera et al. (2011). Related work also includes Farmer (1991) and Evans and Honkapohja (2003) in the learning context. King and Watson (1994) illustrate how the same data can be subjected to a "Keynesian" vs. a "Monetarist" reading.

which is based on the moment-matching autocoherence conditions[5].

2 An AS-AD model

The economy is driven by a standard AS-AD structure. Two kinds of agents make decisions: the government and the people. The government sets government expenditures, while the people's decisions depend on their inflationary expectations. Both use a perceived model which will be determined by a single self-serving economist. Therefore, we will distinguish between the correct model (CM), and the perceived one (PM), whose parameters are denoted with a hat[6].

The model consists of three equations:

$$\begin{aligned} y &= -\beta i + \alpha g + u_0 + \theta v \\ i &= p + y \\ y &= \delta p - \mu p^e + v \end{aligned}$$

The endogenous variables are y, output, g, public expenditure, i, the interest rate, p, the price level, and p^e, the expected price level (to make the discussion more realistic I will interchangeably refer to p as the inflation rate). Therefore the model is closed if a rule for forming expectations and a policy rule are added to these three structural equations.

The economy is subjected to an aggregate demand shock u_0 and an aggregate supply shock v. The first equation is an "IS" curve, the second one

[5] The present paper is part of a larger research project by the author. Saint-Paul (2011) is a simplified exposition of the main issues along with some empirical evidence. In Saint-Paul (2012a), the formal properties of autocoherent models in a general linear context are analyzed. Saint-Paul (2012b) studies a game between competing schools of though each with its own set of followers.

[6] Note that I assume that the perceived model has the same specification as the correct one, only that the underlying parameters differ. This is mostly for convenience, and it turns out that in the model discussed below the economist cannot do better by drawing from a wider set of models. However this need not be the case in a more general context.

can be interpreted as either an LM curve or a Taylor rule[7], and the third equation is an aggregate supply (or Phillips) curve. Note that the supply shock also affects aggregate demand. This makes it harder to identify the true model's parameters and raises the expert's degrees of freedom in designing his model. There is no dearth of theoretical mechanisms for supply shocks to affect aggregate demand as well; in most models greater productivity will change investment and consumption plans through its relative price and wealth effects. The coefficients of the interest rate equation are assumed to be common knowledge and normalized to one for simplicity.

I assume $0 \leq \mu \leq \delta$. Roughly, δ can be interpreted as the slope of the short-run Phillips curve and $\delta - \mu$ as the slope of the long-run Phillips curve. If $\delta = \mu$, we have a Lucas supply curve, and there is no long-run trade-off between output and inflation. If $\mu = 0$, we have an old fashioned Phillips curve which ignores expectations. The output-inflation trade-off is more "favorable", the greater δ and the smaller μ. Intuitively, we might expect more "progressive" experts to favor models with large values of δ and small values of μ. The other parameters of interest are α, referred to as the "demand Keynesian multiplier" (DKM), and β, the interest elasticity of aggregate demand. These two parameters are nonnegative.

Eliminating interest rates, the model can be re-expressed as the following recursive form:

$$y = -bp^e + ag + u + \rho v; \tag{1}$$

$$p = \frac{\mu}{\delta}p^e - \frac{v}{\delta} + \frac{y}{\delta}. \tag{2}$$

[7] In this static model, we do not have to worry about the actual values of the coefficients of the interest rate rule. Here they are normalized to one to save on notation.

Here, a, b, and ρ are composite parameters given by

$$\begin{aligned} a &= \frac{\alpha\delta}{\delta+\beta(1+\delta)}; \\ b &= \frac{\beta\mu}{\delta+\beta(1+\delta)} \leq \mu; \\ \rho &= \frac{\beta+\theta\delta}{\delta+\beta(1+\delta)} \geq \frac{b}{\mu}. \end{aligned}$$

To save on notation, the aggregate demand shock is redefined as $u = \frac{\delta}{\delta+\beta(1+\delta)}u_0$.

Both expectations and government policy are formed upon observing a signal of the demand shock,

$$z = \omega u + \varepsilon,$$

where ε is noise. I assume

$$(u, v, \varepsilon) \sim N(0, \begin{pmatrix} \sigma_u^2 & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & \sigma_\varepsilon^2 \end{pmatrix}).$$

Furthermore, to simplify on notation I will impose the normalization

$$\omega^2\sigma_u^2 + \sigma_\varepsilon^2 = 1.$$

After the equilibrium is realized, people observe the output level y and the price level p. Given that the monetary policy rule is known and the interest rate only depends on p and y, there is no additional information in observing the interest rate.

Thus, we distinguish between two information sets: The information set prevailing when expectations and government policy are formed, which is given by $\{z\}$, and the information set which determines the data against which any credible model must be validated. That information set is given by $\{y, p, z\}$.[8] Government spending is also observed but since it will be proportional to z, with a slope parameter which is common knowledge, that knowledge is redundant.

The perceived model must satisfy the plausibility conditions that all its parameters are nonnegative and that $0 \leq \hat{\mu} \leq \hat{\delta}$. Since, given the other parameters, any plausible target value for $(\hat{a}, \hat{b}, \hat{\rho})$ that satisfies $\hat{\rho} \geq \frac{\hat{b}}{\hat{\mu}}$ and $\hat{b} \leq \hat{\mu}$ can be matched by an appropriate choice of $(\hat{\alpha}, \hat{\beta}, \hat{\theta})$, I will consider that the theorist can directly set the three composite parameters $(\hat{a}, \hat{b}, \hat{\rho})$, and accordingly add the inequalities $\hat{\rho} \geq \frac{\hat{b}}{\hat{\mu}}$ and $\hat{b} \leq \hat{\mu}$ to the set of plausibility conditions.

I will proceed as follows. First, I solve for the equilibrium, given the model used by the people and the level of government spending. Second, I derive the optimal government policy. Third, I spell out the autocoherence conditions that the perceived model must satisfy. Finally, I derive the optimal autocoherent model from the point of view of a self-interested economist.

3 Solution

3.1 Solving for p and y.

The first step in solving for the equilibrium consists in computing p^e. Substituting (1) into (2) we get that

$$p = \left(\frac{\mu - b}{\delta}\right) p^e + \left(\frac{\rho - 1}{\delta}\right) v + \frac{a}{\delta} g.$$

People believe that the following relationship holds instead:

[8] Note that I require the model to match those data despite that it will be used prior to their realization. While the model is one-shot, I want it to take into account the fact that the people's forecasting model will be used repeatedly and therefore must match the data.

$$p = \left(\frac{\hat{\mu} - \hat{b}}{\hat{\delta}}\right) p^e + \left(\frac{\hat{\rho} - 1}{\hat{\delta}}\right) \hat{v} + \frac{\hat{a}}{\hat{\delta}} g.$$

Note the hats on u and v : the realization of the shocks that would be inferred from the people's model differ from the actual ones, unless the model is correct.

To obtain p^e we take expectations on both sides, *using the conditional distributions generated by the* ***perceived*** *model.* I denote again by a hat this expectation. We get that

$$p^e = \frac{1}{\hat{\delta} + \hat{b} - \hat{\mu}} \hat{E}(u \mid z) + \frac{\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}} g. \tag{3}$$

Substituting into (1), we get a reduced form equation for output

$$y = -\frac{b}{\hat{\delta} + \hat{b} - \hat{\mu}} \hat{E}(u \mid z) + \left(a - \frac{b\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}}\right) g + u + \rho v. \tag{4}$$

Plugging (3) and (4) into (2), we then get

$$p = \frac{\mu - b}{\delta(\hat{\delta} + \hat{b} - \hat{\mu})} \hat{E}(u \mid z) + \left(\frac{a}{\delta} + \frac{\hat{a}(\mu - b)}{\delta(\hat{\delta} + \hat{b} - \hat{\mu})}\right) g + \frac{u}{\delta} + \frac{\rho - 1}{\delta} v. \tag{5}$$

3.2 Optimal government policy

As in Saint-Paul (2011), the government wants to stabilize output and government spending. Its objective function is

$$\min \hat{E}(y^2 + \varphi g^2),$$

where φ is a parameter which captures how conservative the government is. I could also allow for the government to stabilize prices, but since the government can only react to demand shocks – there is no supply signal at the time of setting policy – that additional objective is similar to stabilizing output, and I ignore it for simplicity.[9]

Upon realization of the signal z, the government sets g so as to minimize

$$\hat{E}(y^2 + \varphi g^2 \mid z) = \hat{E}(y^2 \mid z) + \varphi g^2.$$

I assume g is observed at the time of setting inflationary expectations. Therefore, there is no credibility problem and the government will internalize the entire feedback effect of fiscal stimulus on output through inflationary expectations and its monetary policy response when setting its policy[10]. Therefore, the first-order condition is

$$\frac{\hat{d}y}{\hat{d}g}\hat{E}(y \mid z) + \varphi g = 0. \tag{6}$$

The derivative $\frac{\hat{d}y}{\hat{d}g}$ is the perceived reduced form Keynesian multiplier (RFKM), which is different from the impact multiplier (IKM), itself equal to a and perceived as $\hat{a}$. Its correct value can be obtained from (4):

$$\frac{dy}{dg} = a - \frac{b\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}}. \tag{7}$$

The true RFKM not only depends on the true model but also on the perceived one. This is because part of the expansionary effect of government spending is dissipated by greater inflationary expectations, which in turn generate greater inflation and a contractionary response of the interest rate. For example, the more people believe that government policy is effective (the

[9]One could extend the model by assuming that a signal of the supply shock is also observed. Responding to that signal would involve a trade-off between price stability and output stability. In this paper the focus is instead on price/output stability vs. government expenditure stability.

[10]In Saint-Paul (2011), I discuss how parameter manipulation can be a way for a "benevolent" economist to provide the government with a commitment device.

greater $\hat{a}$), the more they think fiscal stimulus will be inflationary, and the smaller the RFKM for any given value of a. For the same reason, the more people believe the output/inflation trade-off is unfavorable (the smaller $\hat{\delta}$), the smaller $\frac{dy}{dg}$.

The government uses the perceived model to compute the Keynesian multiplier. To get the perceived multiplier, one just has to replace a and b with $\hat{a}$ and $\hat{b}$, respectively, in (7), getting

$$\frac{\hat{d}y}{\hat{d}g} = \frac{\hat{a}(\hat{\delta} - \hat{\mu})}{\hat{\delta} + \hat{b} - \hat{\mu}}. \tag{8}$$

To compute g, we can compute $\hat{E}(y \mid z)$ by applying hatted expectations to (4), yielding

$$\hat{E}(y \mid z) = \frac{\hat{a}(\hat{\delta} - \hat{\mu})}{\hat{\delta} + \hat{b} - \hat{\mu}} g + \frac{\hat{\delta} - \hat{\mu}}{\hat{\delta} + \hat{b} - \hat{\mu}} \hat{E}(u \mid z). \tag{9}$$

By Bayes' law, we have

$$\hat{E}(u \mid z) = \frac{\hat{\omega}\hat{\sigma}_u^2}{\hat{\omega}^2\hat{\sigma}_u^2 + \hat{\sigma}_\varepsilon^2} z.$$

As will be shown below, autocoherence implies that $Ez^2 = \omega^2\sigma_u^2 + \sigma_\varepsilon^2 = \hat{E}z^2 = \hat{\omega}^2\hat{\sigma}_u^2 + \hat{\sigma}_\varepsilon^2 = 1$. To simplify notations I will make use of this right away. Then

$$\hat{E}(u \mid z) = \hat{\omega}\hat{\sigma}_u^2 z. \tag{10}$$

Substituting (10),(9), and (8) into (6), we eventually get

$$g = \gamma z,$$

where

$$\gamma = -\hat{a} \frac{(\hat{\delta} - \hat{\mu})^2}{\varphi\left(\hat{\delta} + \hat{b} - \hat{\mu}\right)^2 + \hat{a}^2\left(\hat{\delta} - \hat{\mu}\right)^2} \hat{\omega}\hat{\sigma}_u^2 < 0. \tag{11}$$

Inspection of this formula reveals that government activism is larger, i.e. $|\gamma|$ is larger,

- The more people believe in a favorable "long-term" Phillips curve, i.e. the greater $\hat{\delta} - \hat{\mu}$
- The more they believe the interest response of aggregate demand is low, i.e. the smaller $\hat{b}$

As for the effect of the perceived IKM $\hat{a}$, there is an "income effect" and a"substitution" effect, implying that γ is not monotonic in $\hat{a}$. For small values of $\hat{a}$, the substitution effect dominates; a more efficient fiscal policy generates greater activism. For large values of $\hat{a}$, though, the income effect dominates: the government takes advantage of an increase in $\hat{a}$ to reduce its activism, since that increase has a direct favorable impact on the degree of stabilization which is being achieved.

Note that the Keynesian multipliers are not identified, because g is endogenous and always proportional to z. If there was a random, exogenous component to g, and if that component were observable, it would make it possible to identify the Keynesian multipliers. That is, the vector space spanned by g and z would be of dimension 2 instead of 1. Here, though, people cannot disentangle the sensitivity of output to government spending from the direct effect of demand shocks. Similar considerations arise in Sargent (2008) and Fudenberg and Levine (2003, 2007). This underidentification would still hold in richer models provided that the number of parameters is large enough relative to the dimension of the observables space.

3.3 The reduced form model

The preceding subsection allows to compute the variables of interest p and y as a function of the realization of the shocks u, v and ε. This solution determines the reduced form model, which is summarized in Table 1. Then, by replacing non hatted parameters (other than γ) by their hatted counterparts, one can compute the reduced form perceived model, which is reported in Table 2. These expressions introduce composite coefficients that capture the response of output and prices to the demand shock u and the error ε.

For example, the coefficient a_{yu} captures the response of output to the demand shock:

$$a_{yu} = -\frac{b}{\hat{\delta} + \hat{b} - \hat{\mu}} \omega \left(\hat{\omega} \hat{\sigma}_u^2 + \hat{a} \gamma \right) + \omega \gamma a + 1, \tag{12}$$

has three components. The constant 1 captures the direct effect of the aggregate demand shock on output. The term $\omega\gamma a$ is the direct contribution of offsetting stabilization policy, which is the product of the effect of the demand shock on the signal (ω), the policy response to the signal (γ), and the impact Keynesian multiplier (a). The first term $-\frac{b}{\hat{\delta}+\hat{b}-\hat{\mu}} \omega \left(\hat{\omega} \hat{\sigma}_u^2 + \hat{a} \gamma \right)$,captures the effect of the signal on inflationary expectations, which in turn affect inflation and output through the reaction of monetary policy. This response of inflationary expectations has two components. First, a direct contribution of the public's perceived demand shock, captured by the first term $\hat{\omega} \hat{\sigma}_u^2$ in the parenthesis. Second, a contribution of the government's stabilizing reaction to the signal, captured by the second term $\hat{a}\gamma$ which is negative (this is the cross effect of monetary and fiscal policy). The greater the perceived IKM ($\hat{a}$), the larger the perceived inflationary consequences of an expansion, and the lower the actual RFKM[11]. This offsetting effect is larger, the greater the actual effect of interest rates on output (b), and the more "unfavorable" the perceived Phillips curve (the greater $\hat{\mu}$ and the smaller $\hat{\delta}$).

[11] The RFKM can become negative if $\hat{a}$ is large enough. Then fiscal expansions will be contractionary because people overestimate their expansionary effects. This cannot happen if $\hat{a} = a$.

Observable	Expression
Output	$y = a_{yu}u + a_{y\varepsilon}\varepsilon + \rho v$
Price	$p = a_{pu}u + a_{p\varepsilon}\varepsilon + \frac{\rho-1}{\delta}v$
Coefficients	**Expression**
a_{yu}	$-\frac{b}{\hat{\delta}+\hat{b}-\hat{\mu}}\omega\hat{\omega}\hat{\sigma}_u^2 + \omega\gamma(a - \frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}}) + 1$
$a_{y\varepsilon}$	$-\frac{b}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}\hat{\sigma}_u^2 + \gamma(a - \frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}})$
a_{pu}	$\frac{\mu-b}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\omega\hat{\omega}\hat{\sigma}_u^2 + \omega\gamma\left(\frac{a}{\delta} + \frac{\hat{a}(\mu-b)}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\right) + \frac{1}{\delta}$
$a_{p\varepsilon}$	$\frac{\mu-b}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\hat{\omega}\hat{\sigma}_u^2 + \left(\frac{a}{\delta} + \frac{\hat{a}(\mu-b)}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\right)\gamma$

Table 1 – The correct reduced form model

Observable	Expression
Output	$y = \hat{a}_{yu}\hat{u} + \hat{a}_{y\varepsilon}\hat{\varepsilon} + \hat{\rho}\hat{v}$
Price	$p = \hat{a}_{pu}\hat{u} + \hat{a}_{p\varepsilon}\hat{\varepsilon} + \frac{\hat{\rho}-1}{\hat{\delta}}v$
Coefficients	**Expression**
$\hat{a}_{yu}$	$-\frac{\hat{b}}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}^2\hat{\sigma}_u^2 + \gamma\hat{\omega}\frac{\hat{a}(\hat{\delta}-\hat{\mu})}{\hat{\delta}+\hat{b}-\hat{\mu}} + 1$
$\hat{a}_{y\varepsilon}$	$-\frac{\hat{b}}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}\hat{\sigma}_u^2 + \gamma\frac{\hat{a}(\hat{\delta}-\hat{\mu})}{\hat{\delta}+\hat{b}-\hat{\mu}}$
$\hat{a}_{pu}$	$\frac{\hat{\mu}-\hat{b}}{\hat{\delta}(\hat{\delta}+\hat{b}-\hat{\mu})}\hat{\omega}^2\hat{\sigma}_u^2 + \frac{\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}\gamma + \frac{1}{\hat{\delta}}$
$\hat{a}_{p\varepsilon}$	$\frac{\hat{\mu}-\hat{b}}{\hat{\delta}(\hat{\delta}+\hat{b}-\hat{\mu})}\hat{\omega}\hat{\sigma}_u^2 + \frac{\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}}\gamma$

Table 2 – The perceived reduced form model

4 Autocoherence conditions

The reduced form models can then be used to derive the autocoherence conditions. The perceived model must correctly predict the joint distribution of the observables. As all variables are Gaussian and it is common knowledge that their mean is zero, the autocoherence property requires that the variance-covariance matrix of $(y, p, z)'$ computed using that perceived model matches the actual one. This determines six independent autocoherence conditions that are derived in the Appendix (equations (22)-(27)). There are nine parameters: $(\hat{a}, \hat{b}, \hat{\rho}, \hat{\sigma}_u^2, \hat{\delta}, \hat{\mu}, \hat{\sigma}_v^2, \hat{\omega}, \hat{\sigma}_\varepsilon^2)$ and therefore three degrees of freedom.

Since the joint distribution of p and z is observed, the autocoherence conditions always imply that $\hat{E}(p \mid z) = E(p \mid z)$. In other words, in equilibrium expectations about the observables are rational in the usual sense[12]. If government policy were fixed, we could then solve for a unique rational expectations equilibrium (REE) for model (1)-(2) in the usual way. All autocoherent models would then be equivalent in that they deliver the same REE equilibrium[13], leaving no room for the economists to manipulate outcomes. However, government policy does depend on the perceived model, because to set its optimal policy the government must know structural parameters (in particular the multiplier a) that are not identified from the joint distribution of (p, y, z). This opens the possibility for the expert to manipulate government policy. Contrary to the people who only need to form expectations about the (ex-post) observable p, the government needs to make an inference about the unobservable demand shock u.

However, not all parameters can be used to manipulate policy. The autocoherence conditions imply that the parameters of the Phillips curve are useless for pursuing an agenda.

Proposition 1 — The autocoherence conditions imply

$$\hat{\delta} - \hat{\mu} = \delta - \mu.$$

Proof – See Appendix.

Corollary – Given $\hat{a}$, and $\hat{b}$, γ is independent of the choice of $\hat{\delta}$ and $\hat{\mu}$, and so is the equilibrium.

Proof – Immediate from (11).

[12] Algebraically, we have that $E(p \mid z) = a_{pu}E(u \mid z) + a_{p\varepsilon}E(u \mid z)$ $= (a_{pu}\omega\sigma_u^2 + a_{p\varepsilon}\sigma_\varepsilon^2)z$. Similarly, $\hat{E}(p \mid z) = (\hat{a}_{pu}\hat{\omega}\hat{\sigma}_u^2 + \hat{a}_{p\varepsilon}\hat{\sigma}_\varepsilon^2)z$. Therefore, the condition $E(p \mid z) = \hat{E}(p \mid z)$ is equivalent to $a_{pu}\omega\sigma_u^2 + a_{p\varepsilon}\sigma_\varepsilon^2 = \hat{a}_{pu}\hat{\omega}\hat{\sigma}_u^2 + \hat{a}_{p\varepsilon}\hat{\sigma}_\varepsilon^2$, i.e. autocoherence condition (24) in the appendix.

[13] Again, this can be checked algebraically. See Appendix.

The policy maker cares about the ultimate effect of output of government spending, which only depends on price formation through the difference between the output response to prices δ and its (adverse) response to price expectations μ. But to match the covariances between output and the demand signal and prices and the demand signal, the economist is forced to reveal this difference. Thus given $\hat{a}$ and $\hat{b}$, he cannot influence policy through the design of the price block of his model.[14] Intuitively, this is because the demand signal z, which is not polluted by the supply shock, acts as an instrumental variable allowing agents to infer $\delta - \mu$ from $cov(y, z)$ and $cov(p, z)$, two empirical moments that must be correctly predicted by the perceived model.

Since there is little room for the perceived Phillips curve to be used by the expert to influence outcomes, in what follows I will assume that δ and therefore μ are known. In the subsequent section I will discuss a case where Proposition 1 does not hold and policy can be influenced through the perceived Phillips curve parameters.

5 The price block is revealed

In this section I assume δ and μ are known. Furthermore, to simplify the analysis, I will also assume that ω and σ_u are known. Note that the only way for the expert to affect the perceived values of a and b is then through the perceived underlying demand Keynesian multiplier $\hat{\alpha}$ and interest elasticity of aggregate demand $\hat{\beta}$. For the sake of simplicity, in the following discussion I will assimilate a change in $\hat{a}$ with a change in $\hat{\alpha}$ in the same direction, and similarly for $\hat{b}$ and $\hat{\beta}$.

[14] Remember, though, that $\hat{a}$ and $\hat{b}$ are themselves composite parameters and their expression depends on $\hat{\delta}$ and $\hat{\mu}$. While given $\hat{\delta}$ and $\hat{\mu}$, any target for those parameters can be reached by picking the appropriate $\hat{\alpha}$ and $\hat{\beta}$, if for example α is known it may be necessary to choose a particular value of $\hat{\delta}$ to get the desired value of $\hat{a}$.

5.1 Simplifying the autocoherence conditions

Under our assumptions, it must be that $\hat{\omega} = \omega, \hat{\sigma}_u = \sigma_u$, $\hat{\delta} = \delta$, and $\mu = \hat{\mu}$. It is then shown in the Appendix that in such a case, autocoherence implies that the perceived reduced form model must match the correct reduced form model, that is:

$$a_{y\varepsilon} = \hat{a}_{y\varepsilon},\ a_{yu} = \hat{a}_{yu},\ a_{p\varepsilon} = \hat{a}_{p\varepsilon},\ a_{pu} = \hat{a}_{pu},\ \rho = \hat{\rho}.$$

Nevertheless, because the correct reduced form coefficients themselves depend on beliefs, through the government policy parameter γ, it does not follow that the perceived structural model should be the same as the correct one. And which perceived model is picked matters, because different perceived models will lead to different stabilization policies and thus different outcomes.

5.2 The trade-off between the fiscal and monetary output responses

Experts are left with only one degree of freedom in designing their model, which is captured by a trade-off between $\hat{a}$ and $\hat{b}$, the perceived effects on output of government spending and price expectations[15]. This trade-off is defined by the following formulae:

[15] This degree of freedom comes from the fact that in this special case, one autocoherence condition becomes redundant. Thus one degree of freedom is left despite that the number of free parameters has been reduced to the number of autocoherence conditions.

Why is one autocoherence condition redundant here? Basically, if one only imposes that $\hat{\omega} = \omega, \hat{\sigma}_u = \sigma_u$, one can derive a condition involving $\hat{\delta}$ of which δ is a solution, although other values may also be solution in principle. Thus the condition $\hat{\delta} = \delta$ is almost endogenously derived from $\hat{\omega} = \omega, \hat{\sigma}_u = \sigma_u$. Imposing it rules out some other values of $\hat{\delta}$ but is redundant as long as δ is selected as the solution to the nonlinear equation which determines $\hat{\delta}$.

$$(\hat{b}-b)\omega\sigma_u^2 = \gamma\left[(\hat{a}-a)(\delta-\mu)+\hat{a}b-a\hat{b}\right]; \tag{13}$$

$$\gamma = -\hat{a}\frac{(\delta-\mu)^2}{\varphi\left(\delta+\hat{b}-\mu\right)^2+\hat{a}^2(\delta-\mu)^2}\omega\sigma_u^2. \tag{14}$$

Eliminating γ between these two yields a cubic equation for $\hat{b}$, as a function of $\hat{a}$, which can be solved analytically, although numerical analysis is necessary to find out how $\hat{b}$ varies with $\hat{a}$ and the other parameters. Whenever there are three values of $\hat{b}$ that solve this equation, the largest root was selected. Given the requirement that $\hat{b} > 0$, if that largest root is negative, then there is no plausible autocoherent model for this value of $\hat{a}$.

But much can be learned by considering the following approximation. Assume this is a "quasi-Lucas" economy, that is, $\delta-\mu << 1$. Then (14) is equivalent to

$$\gamma \approx -\hat{a}\frac{(\delta-\mu)^2}{\varphi\hat{b}^2}\omega\sigma_u^2 \tag{15}$$

and substituting it into (13) we get

$$\hat{b} \approx b-\frac{\hat{a}(\hat{a}-a)}{\varphi b}(\delta-\mu)^2. \tag{16}$$

This trade-off has the following properties

- $(\hat{b}-b)(\hat{a}-a) < 0$ and for $\hat{a} > a/2$, $d\hat{b}/d\hat{a} < 0$. Thus, the more the economist claims that government spending has a large impact on output, the lower the theoretical impact of interest rates. The only exception is if $\hat{a}$ is very low compared to a.

- The trade-off is flatter, the smaller $\delta-\mu$, the greater φ and the greater b. That is, the more the government is averse to stabilization, the less favorable the Phillips curve, and the greater the true impact of interest rates, the more the theoretical effect of interest rates must be close to the actual one, and the more arbitrary the theoretical impact of government spending.

5.3 Interpreting the autocoherence trade-off

How can we make sense of these effects? In order to understand them we can focus on how $\hat{a}$ and $\hat{b}$ affect output's reaction to demand shocks, as captured by the value of a_{yu} and its perceived counterpart[16]

$$\hat{a}_{yu} = -\frac{\hat{b}}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}^2\hat{\sigma}_u^2 + \gamma\hat{\omega}\frac{\hat{a}(\hat{\delta}-\hat{\mu})}{\hat{\delta}+\hat{b}-\hat{\mu}} + 1.$$

As stated above, autocoherence implies that the perceived model must correctly predict this elasticity. Furthermore, we also know that conditional on γ the economy must be at its REE. Therefore the true value of a_{yu} only depends on the perceived model through the policy parameter γ. Consider an increase in $\hat{a}$ and hold γ constant (the effect of the change in γ is more complex and discussed below). Then the equilibrium is unchanged and so is the output response a_{yu}. On the other hand, people will believe that a_{yu} has fallen, since they think that the direct expansionary effect of fiscal policy (which outweighs its indirect contractionary effect through inflation expectations) is now stronger. This is captured by the fiscal component in $\hat{a}_{yu}$, $\gamma\hat{\omega}\frac{\hat{a}(\hat{\delta}-\hat{\mu})}{\hat{\delta}+\hat{b}-\hat{\mu}}$, which, since $\hat{\mu} < \hat{\delta}$ and $\gamma < 0$, clearly falls in algebraic value as $\hat{a}$ goes up. This discrepancy would invalidate the model empirically unless $\hat{b}$ is changed so as to restore the equality between the actual and perceived elasticity of output to demand shocks. The dominant effect of a reduction in $\hat{b}$ (in a quasi-Lucas economy) is to increase the algebraic value of the perceived monetary component of $\hat{a}_{yu}$, given by $-\frac{\hat{b}}{\hat{\delta}+\hat{b}-\hat{\mu}}\hat{\omega}^2\hat{\sigma}_u^2$.[17] The lower $\hat{b}$, the

[16] In this equation, the cross-effect of monetary and fiscal policy has been aggregated into the "fiscal" component rather than the "monetary" one, unlike in equation (12).

[17] $\hat{b}$, also appears in the fiscal component but in a quasi-lucas economy this contribution

lower the perceived output response to interest rates, and the lower the perceived stabilizing effect of monetary reactions to demand shocks. This effect raises the perceived response of output to demand shocks, thus restoring the model's autocoherence. Since $\hat{b}$ is the interest elasticity of output, this means that experts face a trade-off between believing in fiscal policy effectiveness versus believing in monetary policy effectiveness. An economist who would underpredict both elasticities would also underpredict output volatility and could not empirically validate his model.

This is the key mechanism accounting for the negative trade-off between $\hat{a}$ and $\hat{b}$ over most of the relevant range. To understand why we get a positive trade-off for low values of $\hat{a}$, we now need to reintroduce the contribution of the change in γ into the discussion. An increase in $\hat{a}$ also increases $|\gamma|$, the degree of fiscal activism. This magnifies the discrepancy between the perceived and actual fiscal components of a_{yu}–because government expenditures are now more reactive to the demand shock signal.[18] This discrepancy is negative if $\hat{a} > a$, i.e. people expect more fiscal stabilization than actually happens. In this case, the increase in γ further widens the gap between actual and perceived fiscal components, thus reinforcing the negative required response of $\hat{b}$ to the increase in $\hat{a}$. On the other hand, if $\hat{a} < a$, the discrepancy is positive: people expect greater volatility of output coming from the fiscal component than in reality. While the direct effect of a greater $\hat{a}$ tends to make this discrepancy less positive, the indirect effect on γ which magnifies the difference tends to make it larger. For $\hat{a} < a/2$ this effect dominates, which explains why $d\hat{b}/d\hat{a} > 0$ in this zone.

Note also that the lower $|\gamma|$, the less reactive the discrepancy between

is very small since that component is proportional to $(\hat{\delta} - \hat{\mu})^3$.

[18]The perceived fiscal component is $\gamma\hat{\omega}\frac{\hat{a}(\hat{\delta}-\hat{\mu})}{\hat{\delta}+\hat{b}-\hat{\mu}}$. The actual one is obtained by adding the cross effect of fiscal and monetary policy to the direct effect of fiscal policy in (12). Hence it is equal to $-\frac{b}{\hat{\delta}+\hat{b}-\hat{\mu}}\omega\hat{a}\gamma + \omega\gamma a$. Both components are multiplicative in γ, so the difference between the two is larger, the greater the absolute value of γ.

the actual and perceived fiscal components is to an increase in $\hat{a}$. Thus, the lower the required adjustment in $\hat{b}$ and the flatter the trade-off[19]. In turn,$|\gamma|$ is greater, the more favorable the output-inflation trade-off – the larger $\delta - \mu$ – and the smaller the welfare cost φ of fiscal volatility. This explains why the trade-off is flatter, the smaller $\delta - \mu$ and the greater φ.[20]

Figure 1 depicts numerical simulations of the actual trade-off for four different sets of the parameters a and $\delta - \mu$ (Note that the trade-off only depends on δ and μ through the difference $\delta - \mu$).[21] The results are very similar to what the above discussion based on the quasi-Lucas economy suggests. For $\hat{a} > a/2$ the trade-off is decreasing and concave. It stops at a maximum value of $\hat{a}$ beyond which the plausibility condition $\hat{b} > 0$ is violated. In most cases this corresponds to a catastrophe, mathematically speaking, in that the number of roots of the cubic equation defining $\hat{b}$ falls from 3 to 1 in such a way that the two largest roots disappear. Because of this discontinuity, the curves on Figure 1 stop before hitting the horizontal axis. As in the quasi-Lucas case, the trade-off is flatter, the less favorable the Phillips curve, i.e. the smaller $\delta - \mu$. Furthermore, it shifts up and its slope becomes larger algebraically when a, the actual Keynesian multiplier, goes up, which is also implied by (16).

[19] In the limit case where $\gamma = 0$, there is no variation is fiscal policy that would allow to identify a, and the only unidentified parameter that affects the output elasticity to demand shock is b, through the monetary component. Thus, in that limit case, $\hat{b} = b$ and $\hat{a}$ is arbitrary.

[20]

One can also discuss the effect of b, the true interest elasticity of output, on the auto-coherence trade-off. The equilibrium output response a_{yu} falls more with $\hat{a}$, the greater b. This is because the greater b, the greater the stabilizing effects of the monetary response to inflation. This reduces the reduction in $\hat{b}$ that is needed to offset an increase in $\hat{a}$, since the correct output response to demand shock that one has to match is now lower. Consequently, a greater value of b makes the trade-off between $\hat{a}$ and $\hat{b}$ flatter.

[21] The other parameters in Figure 1 are $b = 0.5$, $\varphi = 0.8$, $\omega = 1, \sigma_u^2 = 0.1, \sigma_v^2 = 0.5$.

5.4 The optimal model

We now turn to the choice of the model by the expert and discuss how this choice is affected by his political preferences. As in Saint-Paul (2011), I assume his objective is $\bar{W} = \min \hat{E}(y^2 + \bar{\varphi} g^2)$. In equilibrium, this is equal to (ignoring constants that are independent of the perceived model)

$$\bar{W} = a_{yu}^2 \sigma_u^2 + a_{y\varepsilon}^2 \sigma_\varepsilon^2 + \bar{\varphi}\gamma^2. \tag{17}$$

Since the reduced form elasticities a_{yu} and $a_{y\varepsilon}$ only depend on the perceived model through γ, as long as the point chosen on the $(\hat{a}, \hat{b})$ trade-off is interior, the corresponding value of γ is the one that would be obtained by directly maximizing $\bar{W}$ with respect to γ. In other words, unless plausibility constraints force him into an corner solution, the intellectual is a *quasi-dictator*, meaning that his preferred value of γ is the one that would prevail if the intellectual were setting policy using the right model:[22]

$$\gamma = \bar{\gamma} = -a \frac{(\delta - \mu)^2}{\bar{\varphi}\left(\delta + b - \mu\right)^2 + a^2\left(\delta - \mu\right)^2} \omega \sigma_u^2. \tag{18}$$

This equality allows us to find out how the perceived model depends on the economist's preferences. From this equality we have

$$\frac{d\hat{a}}{d\bar{\varphi}} = \frac{\partial\bar{\gamma}/\partial\bar{\varphi}}{\partial\gamma/\partial\hat{a} + \partial\gamma/\partial\hat{b}.d\hat{b}/d\hat{a}},$$

where the derivative $d\hat{b}/d\hat{a}$ is taken along the autocoherence trade-off between $\hat{b}$ and $\hat{a}$. We know that $\partial\gamma/\partial\hat{b} > 0, \partial\bar{\gamma}/\partial\bar{\varphi} > 0$, and $\partial\gamma/\partial\hat{a} < 0$ if the substitution effect dominates. Then, in the 'normal' part of the trade-off

[22] This can again be checked algebraically. The crucial autocoherence condition $\frac{\hat{\omega}\hat{\sigma}_u^2 + \gamma\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}} = \frac{\omega\sigma_u^2 + \gamma a}{\delta + b - \mu}$ implies that $a_{yu} = \frac{\omega\gamma a(\delta - \mu)}{\delta + b - \mu} + 1 - \frac{b\omega^2\sigma_u^2}{\delta + b - \mu}$ and that $a_{y\varepsilon} = \frac{\gamma a(\delta - \mu)}{\delta + b - \mu} - \frac{b\omega\sigma_u^2}{\delta + b - \mu}$. Substituting these expressions into (17) and deriving the first-order conditions with respect to γ delivers (18).

where $d\hat{b}/d\hat{a} < 0$, we have that $\frac{d\hat{a}}{d\bar{\varphi}} < 0$. More conservative economists will understate the impact of public expenditures and accordingly, to remain autocoherent, overstate that of interest rates. Furthermore, if the economist's preferences are aligned with that of the government, then the correct model is revealed, since by using it the government will then select $\gamma = \bar{\gamma}$. Since autocoherence imposes rational inflation expectations, there is no scope for manipulating the public and an economist aligned with the government cannot do better than reveal the truth.

Table 3 presents numerical simulations for various values of $\bar{\varphi}$, the degree of conservatism of the economist (the parameter values are the same as in Figure 1 and in particular $b = 0.5$). It confirms that the more conservative the economist, the lower his theoretical IKM $\hat{a}$, and the larger the interest elasticity of output $\hat{b}$. Note also that a corner solution prevails for very progressive economists: the largest plausible value of a is selected.

$\bar{\varphi}$	$a = 0.2, \delta - \mu = 0.4$		$a = 0.2, \delta - \mu = 0.1$	
	$\hat{a}$	$\hat{b}$	$\hat{a}$	$\hat{b}$
0.08	1.1*	0.117	1.78	0.43
0.4	0.39	0.48	0.34	0.498
$0.8 = \varphi$	0.2	0.5	0.2	0.5
1.2	0.13	0.502	0.13	0.5
1.6	0.1	0.502	0.1	0.5

$a = 0.8, \delta - \mu = 0.4$		$a = 0.8, \delta - \mu = 0.1$	
$\hat{a}$	$\hat{b}$	$\hat{a}$	$\hat{b}$
1.48*	0.08	3.11*	0.21
1.29	0.33	1.53	0.48
0.8	0.5	0.8	0.5
0.55	0.53	0.54	0.502
0.42	0.534	0.4	0.503

Table 3 –Ideological preferences and the expert's preferred perceived model.

$*$ = maximum possible value.

6 Manipulation of the Phillips curve parameters

While economists' opinions about the Keynesian multiplier differ, so is the case with the parameters of the Phillips curve. But in the preceding model, these parameters cannot be manipulated in a way that matters for policy.[23]

I now study an example where the signal z upon which forecasts are based does not allow to identify the slope of the Phillips curve $\delta - \mu$. That is, I assume that z is now an aggregate of the demand and supply shock:

$$z = \omega u - \lambda v.$$

Again I assume $\lambda, \omega > 0$. The signal z is interpreted as a signal about the aggregate price level. Thus this signal goes up with demand shocks but down with supply shocks. Furthermore, as the signal is polluted by the supply shock, it is no longer a valid instrument for estimating $\delta - \mu$. This quantity can no longer be inferred from the observed moments, and therefore autocoherence no longer compels the expert to reveal it.

I impose the following normalization:

$$E(z^2) = \omega^2 \sigma_u^2 + \lambda^2 \sigma_v^2 = 1.$$

To solve the model we now note that[24] $\hat{E}(u \mid z) = \hat{\omega}\hat{\sigma}_u^2 z$ and $\hat{E}(v \mid z) = -\hat{\lambda}\hat{\sigma}_v^2 z$. Performing the same steps as in section 3.1 and using those expressions, we get that

$$p^e = \frac{\hat{a}}{\hat{\delta} + \hat{b} - \hat{\mu}} g + \hat{c} z,$$

[23] Again that is not exactly true because a and b depend on δ, so that the economist could distort δ in addition to α and β to target $\hat{a}$ and $\hat{b}$, despite that this extra degree of freedom is not needed.

[24] This again anticipates on the autocoherence condition $E(z^2) = \hat{E}(z^2) = 1$.

with

$$\hat{c} = \frac{\hat{\omega}\hat{\sigma}_u^2 - \hat{\lambda}(\hat{\rho}-1)\hat{\sigma}_v^2}{\hat{\delta}+\hat{b}-\hat{\mu}}.$$

Therefore the solution is

$$y = u + \rho v - b\hat{c}z + \left(a - \frac{b\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}}\right) g \tag{19}$$

$$p = \frac{\mu - b}{\delta}\hat{c}z + \left(\frac{a}{\delta} + \frac{\hat{a}(\mu - b)}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\right) g + \frac{u}{\delta} + \frac{\rho - 1}{\delta} v.$$

How is government policy determined in this variant of the model? Conditions (7) and (8) as well as the FOC (6) still hold. But applying hatted expectations to both sides of (19) we now get

$$\hat{E}(y \mid z) = \frac{\hat{a}(\hat{\delta}-\hat{\mu})}{\hat{\delta}+\hat{b}-\hat{\mu}} g + \left[\hat{\omega}\hat{\sigma}_u^2 \frac{\hat{\delta}-\hat{\mu}}{\hat{\delta}+\hat{b}-\hat{\mu}} - \hat{\lambda}\hat{\sigma}_v^2 \left(\hat{\rho} - \frac{(\hat{\rho}-1)\hat{b}}{\hat{\delta}+\hat{b}-\hat{\mu}}\right)\right] z.$$

Consequently optimal fiscal policy is now given by $g = \gamma z$, with

$$\gamma = \hat{a}\frac{\hat{\lambda}\hat{\sigma}_v^2 \left[\hat{\rho}\left(\hat{\delta}-\hat{\mu}\right)^2 + \hat{b}\left(\hat{\delta}-\hat{\mu}\right)\right] - (\hat{\delta}-\hat{\mu})^2\hat{\omega}\hat{\sigma}_u^2}{\varphi\left(\hat{\delta}+\hat{b}-\hat{\mu}\right)^2 + \hat{a}^2\left(\hat{\delta}-\hat{\mu}\right)^2} \lesseqgtr 0. \tag{20}$$

Note that the sign of γ depends on the relative importance of supply and demand shocks. If supply shocks are perceived to be more important ($\hat{\sigma}_v^2$ large enough relative to $\hat{\sigma}_u^2$), an indication of price pressure ($z > 0$) signals a contraction and will be met with expansionary policies ($\gamma > 0$).[25]

The model's new solution is now given in Tables 4 and 5.

[25] Clearly, this could change if a price stability objective were added to the government's utility function.

Observable	Expression
Output	$y = a_{yu}u + a_{yv}v$
Price	$p = a_{pu}u + a_{pv}v$
Coefficients	**Expression**
a_{yu}	$1 - b\omega\hat{c} + \omega\gamma(a - \frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}})$
a_{yv}	$\rho + b\lambda\hat{c} - \gamma\lambda(a - \frac{\hat{a}b}{\hat{\delta}+\hat{b}-\hat{\mu}})$
a_{pu}	$\omega\gamma\left(\frac{a}{\delta} + \frac{\hat{a}(\mu-b)}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\right) + (\mu - b)\frac{\omega\hat{c}}{\delta} + \frac{1}{\delta}$
a_{pv}	$\frac{\rho-1}{\delta} - (\mu - b)\frac{\lambda\hat{c}}{\delta} - \lambda\gamma\left(\frac{a}{\delta} + \frac{\hat{a}(\mu-b)}{\delta(\hat{\delta}+\hat{b}-\hat{\mu})}\right).$

Table 4 – The correct reduced form model, Variant B

Observable	Expression
Output	$y = \hat{a}_{yu}\hat{u} + \hat{a}_{yv}\hat{v}$
Price	$p = \hat{a}_{pu}\hat{u} + \hat{a}_{pv}\hat{v}$
Coefficients	**Expression**
$\hat{a}_{yu}$	$1 - \hat{b}\hat{\omega}\hat{c} + \hat{\omega}\gamma\frac{\hat{a}\left(\hat{\delta}-\hat{\mu}\right)}{\hat{\delta}+\hat{b}-\hat{\mu}}$
a_{yv}	$\hat{\rho} + \hat{b}\hat{\lambda}\hat{c} - \gamma\hat{\lambda}\frac{\hat{a}\left(\hat{\delta}-\hat{\mu}\right)}{\hat{\delta}+\hat{b}-\hat{\mu}}$
a_{pu}	$\hat{\omega}\gamma\frac{\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}} + (\hat{\mu} - \hat{b})\frac{\hat{\omega}\hat{c}}{\hat{\delta}} + \frac{1}{\hat{\delta}}$
a_{pv}	$\frac{\hat{\rho}-1}{\hat{\delta}} - (\hat{\mu} - \hat{b})\frac{\hat{\lambda}\hat{c}}{\hat{\delta}} - \hat{\lambda}\gamma\frac{\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}}.$

Table 5 – The perceived reduced form model, Variant B

There are again six autocoherence solutions and nine parameters. In contrast to the previous section, I will now assume for simplicity that the key parameters of the output block are common knowledge: $\hat{a} = a$ and $\hat{b} = b$. The autocoherence conditions now leave us with one degree of freedom: they define a 1-dimensional manifold in a 7-dimensional space. Rather than solving those highly nonlinear equations, I linearize the system of autocoherence conditions locally around the correct model. For such "quasi-correct" models, the autocoherence conditions are thus a straight line in that space. Define $\Delta\hat{\delta} = \hat{\delta} - \delta << 1$ and similarly for other parameters. Then we can reexpress the AC conditions in the following fashion

$$v = \Delta\hat{\delta}.q,$$

where $v = (\Delta(\hat{\delta} - \hat{\mu}), \Delta\hat{\lambda}, \Delta\hat{\sigma}_v, \Delta\hat{\omega}, \Delta\hat{\sigma}_u, \Delta\hat{\rho})'$ and q is a 6-dimensional vector whose ith coefficient gives us the slope of the trade-off between $\hat{\delta}$ and the ith parameter in v.[26] Of special interest is the first coefficient of q since it defines the set of slope parameters of the long-run Phillips curve that the economist may promote to influence policy while remaining autocoherent.

The algebraic steps to derive the q vector are described in the Appendix, and these formulas can be used to numerically compute q in a given economy.

Which point is going to be selected by the economist along this autocoherence locus? Again, he will be a quasi-dictator and it is natural, given our approximation, to assume that his preferences differ only marginally from those of the government: $\bar{\varphi} = \varphi + \Delta\varphi$, $\Delta\varphi << 1$. Let γ_0 be the value of γ prevailing if the perceived model is correct, then the target value of γ for the economist is given by $\tilde{\gamma} \approx \gamma_0 + \frac{\partial\gamma}{\partial\varphi}\Delta\varphi = \gamma_0 + \Delta\tilde{\gamma}$. On the other hand, the value of γ pursued by the government given the perceived model can be expressed as $\gamma \approx \gamma_0 + (\nabla_v\gamma).v = \gamma_0 + \Delta\gamma$, where $\nabla_v\gamma$ is the appropriate vectors of derivatives[27]. They and $\frac{\partial\gamma}{\partial\varphi}$ are computed in the Appendix. The economist will pick the model that satisfies $\Delta\gamma = \Delta\tilde{\gamma}$, implying that the perceived model can be summarized by a relationship between $\Delta\hat{\delta}$ and $\Delta\varphi$:

$$\Delta\hat{\delta} = m\Delta\varphi,$$

where

$$m = \frac{\frac{\partial\gamma}{\partial\varphi}}{(\nabla_v\gamma).q}. \tag{21}$$

I will now use those results to analyze the structure of the perceived model and how it depends on the underlying parameters of the economy. In order

[26] Given the particular importance of the parameter $\delta - \mu$, I prefer to use $\hat{\delta} - \hat{\mu}$ rather than $\hat{\mu}$.

[27] There is no contribution of $\Delta\hat{\delta}$ in the differentiation of γ with respect to the perceived parameters once one also differentiates with respect to the parameters in v, since $\hat{\delta}$ only appears through $\hat{\delta} - \hat{\mu}$.

to organize the discussion, I will focus on five intuitive characteristics of a theory:

1. The short-term inflationary cost of output (STC). This is equal to $1/\hat{\delta}$.
2. The long-term inflationary cost of output (LTC), equal to $1/(\hat{\delta} - \hat{\mu})$.
3. The relative importance of supply shocks (RIS), equal to $\frac{\hat{\sigma}_v^2}{\hat{\sigma}_u^2}$.
4. The supply-intensity of the price indicator (SIP), equal to $\hat{\lambda}^2 \hat{\sigma}_v^2$.
5. The share of output fluctuations explained by supply shocks (SSO); given by $\frac{\hat{a}_{yv}^2 \hat{\sigma}_v^2}{\hat{a}_{yv}^2 \hat{\sigma}_v^2 + \hat{a}_{yu}^2 \hat{\sigma}_u^2}$.

For each of these parameters, its *ideological sensitivity* is defined as its derivative with respect to φ. A positive ideological sensitivity means that the parameter goes up, the more conservative the economist. The greater the absolute value of ideological sensitivity, the more the parameter will deviate from its true value as a result of the economist's own agenda (and, intuitively, we expect economists with different ideological positions to disagree more about that parameter). The expressions for the ideological sensitivities are given by the following Table.

Parameter	Ideological sensitivity
STC	$-m/\delta^2$
LTC	$-mq_1/(\delta - \mu)^2$
RIS	$\frac{2\sigma_v}{\sigma_u^2} m(q_3 - \frac{\sigma_v}{\sigma_u} q_5)$
SIP	$2m(\lambda \sigma_v^2 q_2 + \lambda^2 \sigma_v q_3)$
SSO	See Appendix

Table 6 – Ideological sensitivities of key perceived parameters

Figures 2 to 7 report ideological sensitivities, as μ varies, for 5 sets of values for the other parameters. We observe the following:

- Typically, the ideological sensitivity of LTC is positive: more conservative economists will report a higher inflationary cost of output in the long run. This makes sense as it will deter activist stabilization policies. However, there are exceptions: on Figure 7 where b is quite low (b=0.1), LTC has a positive sensitivity only if μ is large enough, i.e. on the right of the asymptote.

- However, for other parameters, things are less clear-cut. For example, the STC's ideological sensitivity is always negative except on Figure 7. A conservative economist wants to downplay the efficiency of stabilization through public expenditures, but cannot act on all margins simultaneously because he is bound by the autocoherence conditions. This sometimes forces him to appear progressive on some fronts, as is the case for the short-term inflationary effects of inflation.

- Nevertheless, a pattern emerges: the ideological sensitivity of STC is always small, implying that the truth is almost revealed about δ regardless of the economist's ideological position, while there is much more ideological polarization with respect to the value of μ. A conservative economist will overemphasize the negative impact of inflationary expectations on output, in a way reminiscent of Friedman (1968) and Lucas (1972, 1973), while the left-wing economist will produce models that understate μ, in a fashion not unlike that of Akerlof, Dickens and Perry (2000).

- We also note that in many simulations the share of output fluctuations explained by supply shocks has a positive ideological sensitivity – conservative economists will predict that supply shocks play a bigger role in output fluctuations; however this does not happen because of the RIS, which tends to have a negative sensitivity, but through the perceived responses of output to these shocks $\hat{a}_{yu}$ and $\hat{a}_{yv}$. An exception arises when σ_u is very large (Figure 3), or b very low (Figure 7). In

Figure 7, the conservative economist believes in a mildly more favorable Phillips curve for μ low, but also promotes the view that supply shocks are relatively important. If μ is high, the pattern is similar to the other figures.

- An economy can be "critical", meaning that the denominator of (21) is close to zero. This happens on Figure 7 around $\mu \approx 0.59$, and on Figure 3 around $\mu = 0.66$. In a critical economy, parameters happen to be such that ideology is uninfluential. To compensate for that and act as quasi-dictators, economists will tend to pick very large deviations between the perceived and actual parameters: ideological sensitivities become very large, as captured by the asymptotes in our figures. This in fact means that our approximation is no longer valid; still small ideological deviations have large effects on the prevailing view of the world. Intuitively, an economy is critical if parameter values are such that, locally, the autocoherence locus (a manifold in the perceived parameters space) is included in an iso-policy locus (i.e., a set of parameters that all deliver the same value of γ). Here, this is true locally: the vector q, which tells us the direction where the perceived model must move to remain autocoherent, is orthogonal to the policy gradient $\nabla_v \gamma$, and therefore autocoherence implies local policy invariance. Note, however, that the result that ideological deviations become large around a critical economy would be overturned if there was some convex cost to the expert of deviating from the truth; economists would then no longer be quasi-dictators and in a critical economy, the benefits of manipulation would be negligible relative to the costs of deviating from the truth. Instead of becoming infinite, ideological sensitivities would then fall to zero in a critical economy[28].

[28] This can be seen by looking at the following reduced form optimization problem:

7 Conclusion

Using a simple, plausible, example, we have explored the ideological biases that may arise in a macroeconomic model formulated by a self-interested expert when models are required to match the data, as captured by the autocoherence conditions. The predictions of this paper's "meta-model" match casual observations about the real world positions of economists regarding key parameters. In particular, more conservative economists will tend to produce lower values for the Keynesian multipliers and less favorable output-inflation trade-offs, as well as, possibly, put more emphasis on supply shocks. Nevertheless, another lesson of the analysis is that some concessions generally have to be made in order to preserve autocoherence.

Admittedly the present paper is only a first step in analyzing those issues. In particular, real world economists probably do not know the correct model and are not cynical in reporting information which they know is false. What is needed is a theory of how ideological biases pervade the scientific hypotheses that are formulated in the gradual process of building a theory. Another important direction for further research is to allow for different policy regimes to alternate, for example due to changes in the government. In principle, then, to remain plausible a model would have to be autocoherent in each of those regimes, which will increase the number of autocoherence conditions and reduce the set of autocoherent models, possibly down to a singleton containing the correct model only. In other words, regime change plays the role of a natural experiment which increases a model's falsifiability. On the other hand, it would also be natural, then, to, assume that each regime only delivers a finite number of observations, which in itself increases the

---

$\min_{\hat{\theta}} c(\hat{\theta}-\theta)^2 + (s\hat{\theta}-\varphi)^2$, where θ is the true parameter value, $\hat{\theta}$ the perceived one, $s\hat{\theta}$ the outcome (up to a constant), φ the target outcome, and c the cost of deviating from the truth. The optimal value of $\hat{\theta}$ is $\frac{c\theta+s\varphi}{c+s^2}$, with a radically different behavior around $s=0$ (criticality) depending on whether c is positive vs. zero.

dimension of the set of autocoherent models, since the predicted moments would now have to lie within some confidence interval of the observed ones, rather than match them exactly as assumed in the present paper.

REFERENCES

Akerlof George A., William T. Dickens, and George L. Perry, "Near-Rational Wage and Price Setting and the Long-Run Phillips Curve", *Brookings Papers on Economic Activity*, Vol. 2000, No. 1 (2000), pp. 1-60

Alesina, Alberto and George-Marios Angeletos (2005), "Fairness and Redistribution", *American Economic Review*, 95(4), 960-980

Bénabou, Roland and Efe Ok (2001), "Social Mobility and the Demand for Redistribution: The Poum Hypothesis", *Quarterly Journal of Economics,* 116, No. 2, 447-487

Buera, Francisco, Alexander Monge-Naranjo, and Giorgio Primiceri (2011), "Learning the wealth of nations", *Econometrica*, Vol. 79, No. 1, 1–45

Crawford, Vincent and Joel Sobel (1982), "Strategic information transmission", *Econometrica*

Evans, George and Seppo Honkapohja (2003), "Expectational Stability of Stationary Sunspot Equilibria in a Forward-looking Linear Model", *Journal of Economic Dynamics and Control,* 28(1), 171-81

Farmer, Roger (1991), "The Lucas critique, policy invariance, and multiple equilibria", *Review of Economic Studies,* 58(2), 321-332

Friedman, Milton (1968), "he Role of Monetary Policy", *The American Economic Review*, 58, 1, 1-17

Fuchs, Victor, Alan Krueger and James M. Porterba (1998), "Economists' views about parameters, values, and policies: Survey results in labor and public economics", *Journal of Economic Literature,* 36, 3, 1387-1425

Fudenberg. D. and D.K. Levine (1993) "Self-Confirming Equilibrium" *Econometrica*, 61 , 523-546.

———————————— (2009), "Self-confirming equilibrium and the Lucas critique", *Journal of Economic Theory*

Hansen Lars Peter, Thomas J. Sargent, Gauhar Turmuhambetova, Noah Williams (2006), "Robust control and model misspecification" *Journal of*

Economic Theory 128, 45 – 90

King, Robert G. and Mark Watson (1994), "The post-war U.S. Phillips curve: a revisionist econometric history", *Carnegie-Rochester Conference Series on Public Policy,* 41, 157-219

Marcet, Albert and Juan-Pablo Nicolini (2003) "Recurrent hyperinflation and learning", *American Economic Review*

Lucas, Robert (1972), "Expectations and the neutrality of money", *Journal of Economic Theory*

——————- (1973), "International evidence on the output-inflation trade-off", *American Economic Review*

Piketty, Thomas (1995) : "Social mobility and redistributive politics", *Quarterly journal of economics*, 110, 3, 551-584.

Saint-Paul, Gilles (2011), "Toward a Political Economy of Macroeconomic Thinking", Working paper, NBER, CEPR and TSE. Presented at the International Seminar on Macroeconomics, Malta, june 2011.

——————— (2012a): "Exploring the Properties of Autocoherent models", working paper, forthcoming

———————- (2012b): "Intellectual influence and macroecomic outcomes", working paper, forthcoming

Sargent, Thomas (2008), "Evolution and Intelligent Design", *American Economic Review,* 98(1), 5-37

Sargent, Thomas, Noah Williams, and Tao Zha (2006): "Shocks and Government Beliefs: The Rise and Fall of American Inflation", *American Economic Review*, 96(4), 1193-1224

8 APPENDIX

8.1 Derivation of the autocoherence conditions

1. Variance of z

$$\begin{aligned} Ez^2 &= 1 \\ &= \omega^2\sigma_u^2 + \sigma_\varepsilon^2 \\ &= \hat{E}z^2 \\ &= \hat{\omega}^2\hat{\sigma}_u^2 + \hat{\sigma}_\varepsilon^2. \end{aligned} \tag{22}$$

2. Covariance between z and y

$$\begin{aligned} Eyz &= a_{yu}\omega\sigma_u^2 + a_{y\varepsilon}\sigma_\varepsilon^2 \\ &= \hat{E}yz \\ &= \hat{a}_{yu}\hat{\omega}\hat{\sigma}_u^2 + \hat{a}_{y\varepsilon}\hat{\sigma}_\varepsilon^2 \end{aligned}$$

Using (22), (23) can be rewritten

$$\begin{aligned} (a_{y\varepsilon}\omega + 1)\omega\sigma_u^2 + a_{y\varepsilon}(1-\omega^2\sigma_u^2) &= (\hat{a}_{y\varepsilon}\hat{\omega} + 1)\hat{\omega}\hat{\sigma}_u^2 + \hat{a}_{y\varepsilon}(1-\hat{\omega}^2\hat{\sigma}_u^2), \\ &\Longleftrightarrow \\ \omega\sigma_u^2 + a_{y\varepsilon} &= \hat{\omega}\hat{\sigma}_u^2 + \hat{a}_{y\varepsilon}. \end{aligned} \tag{23}$$

3. Covariance between z and p

$$\begin{aligned} Epz &= a_{pu}\omega\sigma_u^2 + a_{p\varepsilon}\sigma_\varepsilon^2 \\ &= \hat{E}pz \\ &= \hat{a}_{pu}\hat{\omega}\hat{\sigma}_u^2 + \hat{a}_{p\varepsilon}\hat{\sigma}_\varepsilon^2. \end{aligned}$$

Using similar steps as above, we can see that this is equivalent to

$$\frac{\omega\sigma_u^2}{\delta} + a_{p\varepsilon} = \frac{\hat{\omega}\hat{\sigma}_u^2}{\hat{\delta}} + \hat{a}_{p\varepsilon}. \tag{24}$$

4. Covariance between y and p

$$\begin{aligned} Epy &= a_{yu}a_{pu}\sigma_u^2 + a_{y\varepsilon}a_{p\varepsilon}\sigma_\varepsilon^2 + \frac{\rho(\rho-1)}{\delta}\sigma_v^2 \\ &= (a_{y\varepsilon}\omega + 1)(\frac{1}{\delta} + \omega a_{p\varepsilon})\sigma_u^2 + a_{y\varepsilon}a_{p\varepsilon}\sigma_\varepsilon^2 + \frac{\rho(\rho-1)}{\delta}\sigma_v^2 \\ &= \left(\frac{1}{\delta} + \omega a_{p\varepsilon} + a_{y\varepsilon}\omega\right)\sigma_u^2 + a_{y\varepsilon}a_{p\varepsilon} + \frac{\rho(\rho-1)}{\delta}\sigma_v^2 \\ &= \hat{E}py \\ &= \left(\frac{1}{\hat{\delta}} + \hat{\omega}\hat{a}_{p\varepsilon} + \hat{a}_{y\varepsilon}\hat{\omega}\right)\hat{\sigma}_u^2 + \hat{a}_{y\varepsilon}\hat{a}_{p\varepsilon} + \frac{\hat{\rho}(\hat{\rho}-1)}{\hat{\delta}}\hat{\sigma}_v^2. \end{aligned} \tag{25}$$

5. Variance of y

$$\begin{aligned} Ey^2 &= a_{yu}^2\sigma_u^2 + a_{y\varepsilon}^2\sigma_\varepsilon^2 + \rho^2\sigma_v^2 \\ &= (a_{y\varepsilon}\omega + 1)^2\sigma_u^2 + a_{y\varepsilon}^2(1 - \omega^2\sigma_\varepsilon^2) + \rho^2\sigma_v^2 \\ &= (1 + 2a_{y\varepsilon}\omega)\sigma_u^2 + a_{y\varepsilon}^2 + \rho^2\sigma_v^2 \\ &= \hat{E}y^2 \\ &= (1 + 2\hat{a}_{y\varepsilon}\hat{\omega})\hat{\sigma}_u^2 + \hat{a}_{y\varepsilon}^2 + \hat{\rho}^2\hat{\sigma}_v^2. \end{aligned} \tag{26}$$

6. Variance of p.

Note that this autocoherence condition can always be matched by picking the right value of $\hat{\sigma}_\eta^2$, regardless of the other parameters of the perceived model. I write it for the record.

$$
\begin{aligned}
Ep^2 &= a_{pu}^2\sigma_u^2 + a_{p\varepsilon}^2\sigma_\varepsilon^2 + \frac{(\rho-1)^2}{\delta^2}\sigma_v^2 + \sigma_\eta^2 \\
&= (\frac{1}{\delta^2} + \frac{2a_{p\varepsilon}\omega}{\delta})\sigma_u^2 + a_{p\varepsilon}^2 + \frac{(\rho-1)^2}{\delta^2}\sigma_v^2 \\
&= \hat{E}p^2 \\
&= (\frac{1}{\hat{\delta}^2} + \frac{2\hat{a}_{p\varepsilon}\hat{\omega}}{\hat{\delta}})\hat{\sigma}_u^2 + \hat{a}_{p\varepsilon}^2 + \frac{(\hat{\rho}-1)^2}{\hat{\delta}^2}\hat{\sigma}_v^2. \qquad (27)
\end{aligned}
$$

8.2 Proof of that equilibrium coincides with rational expectations for a given value of γ

Note that $\frac{\omega\sigma_u^2}{\delta} + a_{p\varepsilon} = \frac{\omega\sigma_u^2}{\delta} + \frac{\gamma a}{\delta} + \frac{\mu-b}{\delta}\left[\frac{\hat{\omega}\hat{\sigma}_u^2+\gamma\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}}\right]$ and that $\frac{\hat{\omega}\hat{\sigma}_u^2}{\hat{\delta}} + \hat{a}_{p\varepsilon} = \frac{\hat{\omega}\hat{\sigma}_u^2+\gamma\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}}$. Therefore, condition (24) is equivalent to $\frac{\hat{\omega}\hat{\sigma}_u^2+\gamma\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}} = \frac{\omega\sigma_u^2+\gamma a}{\delta+b-\mu}$. Next, note that all the hatted terms in $a_{p\varepsilon}$, a_{pu}, $a_{y\varepsilon}$ and a_{yu} can be grouped in the ratio $\frac{\hat{\omega}\hat{\sigma}_u^2+\gamma\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}}$. Since that ratio must be equal to $\frac{\omega\sigma_u^2+\gamma a}{\delta+b-\mu}$, if γ is exogenous, none of those coefficients depend on the perceived model. Consequently, the equilibrium is unique and must be identical to the REE equilibrium.

8.3 Proof of Proposition 1

As proved in footnote 13, we know that condition (24) is equivalent to

$$
\frac{\hat{\omega}\hat{\sigma}_u^2+\gamma\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}} = \frac{\omega\sigma_u^2+\gamma a}{\delta+b-\mu}. \qquad (28)
$$

Using the definition of $a_{y\varepsilon}$ and $\hat{a}_{y\varepsilon}$, we can rewrite (23) as

$$
\omega\sigma_u^2 + \gamma a - b\left(\frac{\hat{\omega}\hat{\sigma}_u^2+\gamma\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}}\right) = \hat{\omega}\hat{\sigma}_u^2 + \gamma\hat{a} - \hat{b}\left(\frac{\hat{\omega}\hat{\sigma}_u^2+\gamma\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}}\right)
$$

Replacing $\hat{\omega}\hat{\sigma}_u^2 + \gamma\hat{a}$ with $\frac{\omega\sigma_u^2+\gamma a}{\delta+b-\mu}(\hat{\delta}+\hat{b}-\hat{\mu})$ and rearranging, we indeed get $\hat{\delta}-\hat{\mu} = \delta - \mu$.

QED

8.4 The price block revealed case

Assume $\hat{\omega} = \omega, \hat{\sigma}_u = \sigma_u$, and $\hat{\delta} = \delta$. We know from Proposition 2 that $\hat{\mu} = \mu$. From (22) we get

$$\hat{\sigma}_\varepsilon^2 = \sigma_\varepsilon^2.$$

From (23) we get

$$a_{y\varepsilon} = \hat{a}_{y\varepsilon}, \tag{29}$$

Similarly, for (24) to hold we need

$$a_{p\varepsilon} = \hat{a}_{p\varepsilon}. \tag{30}$$

This in turn implies $a_{yu} = \hat{a}_{yu}$ and $a_{pu} = \hat{a}_{pu}$.

Finally, (25) and (26) yield

$$\begin{aligned} \frac{\rho(\rho - 1)}{\delta}\sigma_v^2 &= \frac{\hat{\rho}(\hat{\rho} - 1)}{\hat{\delta}}\hat{\sigma}_v^2; \\ \hat{\rho}^2\hat{\sigma}_v^2 &= \rho^2\sigma_v^2 \end{aligned}$$

The solution to this system is

$$\begin{aligned} \rho &= \hat{\rho}; \\ \hat{\sigma}_v^2 &= \sigma_v^2. \end{aligned}$$

From (28) we get

$$\omega\sigma_u^2(\hat{b} - b) = \gamma\left[\hat{a}(\delta + b - \mu) - a(\hat{\delta} + \hat{b} - \hat{\mu})\right]. \tag{31}$$

Recall, from (11), that

$$\gamma = -\hat{a}\frac{(\delta-\mu)^2}{\varphi\left(\delta+\hat{b}-\mu\right)^2+\hat{a}^2\left(\delta-\mu\right)^2}\omega\sigma_u^2. \tag{32}$$

Substituting, we get the cubic equation that has been solved numerically:

$$(\hat{b}-b)\left(\varphi\left(\delta+\hat{b}-\mu\right)^2+\hat{a}^2\left(\delta-\mu\right)^2\right)+\hat{a}(\delta-\mu)^2\left[(\hat{a}-a)(\delta-\mu)+\hat{a}b-a\hat{b}\right]=0.$$

Finally, the above conditions trivially imply that the remaining condition (27) holds.

8.5 Linearization of the AC conditions in variant B

The six AC conditions are

$$\begin{aligned}
1 &= \hat{\omega}^2\hat{\sigma}_u^2+\hat{\lambda}^2\hat{\sigma}_v^2; && (33)\\
a_{yu}\omega\sigma_u^2-a_{yv}\lambda\sigma_v^2 &= \hat{a}_{yu}\hat{\omega}\hat{\sigma}_u^2-\hat{a}_{yv}\hat{\lambda}\hat{\sigma}_v^2; && (34)\\
a_{pu}\omega\sigma_u^2-a_{pv}\lambda\sigma_v^2 &= \hat{a}_{pu}\hat{\omega}\hat{\sigma}_u^2-\hat{a}_{pv}\hat{\lambda}\hat{\sigma}_v^2; && (35)\\
a_{yu}a_{pu}\sigma_u^2+a_{yv}a_{pv}\sigma_v^2 &= \hat{a}_{yu}\hat{a}_{pu}\hat{\sigma}_u^2+\hat{a}_{yv}\hat{a}_{pv}\hat{\sigma}_v^2; && (36)\\
a_{yu}^2\sigma_u^2+a_{yv}^2\sigma_v^2 &= \hat{a}_{yu}^2\hat{\sigma}_u^2+\hat{a}_{yv}^2\hat{\sigma}_v^2; && (37)\\
a_{pu}^2\sigma_u^2+a_{pv}^2\sigma_v^2 &= \hat{a}_{pu}^2\hat{\sigma}_u^2+\hat{a}_{pv}^2\hat{\sigma}_v^2. && (38)
\end{aligned}$$

Using the definitions in Table 4 and 5 to rearrange (35), and defining $c=\frac{\omega\sigma_u^2-\lambda(\rho-1)\sigma_v^2}{\delta+b-\mu}$, we see that (35) is equivalent to

$$\hat{c}+\frac{\gamma\hat{a}}{\hat{\delta}+\hat{b}-\hat{\mu}}=c+\frac{\gamma a}{\delta+b-\mu}. \tag{39}$$

This expression can be conveniently substituted into the expressions in Tables 4 and 5 to reduce the number of hatted parameters that appear. We get the following:

Coefficients	Expression
a_{yu}	$1+\omega\gamma a - b\omega(c+\frac{\gamma a}{\delta+b-\mu})$
a_{yv}	$\rho-\gamma\lambda a + b\lambda\left(c+\frac{\gamma a}{\delta+b-\mu}\right)$
a_{pu}	$\frac{1}{\delta}+(\mu-b)\frac{\omega}{\delta}\left(c+\frac{\gamma a}{\delta+b-\mu}\right)+\frac{\gamma a\omega}{\delta}$
a_{pv}	$\frac{\rho-1}{\delta}-(\mu-b)\frac{\lambda}{\delta}\left(c+\frac{\gamma a}{\delta+b-\mu}\right)-\frac{\gamma a\lambda}{\delta}$
$\hat{a}_{yu}$	$1+\hat{\omega}\gamma\hat{a} - \hat{b}\hat{\omega}(c+\frac{\gamma a}{\delta+b-\mu})$
$\hat{a}_{yv}$	$\hat{\rho}-\gamma\hat{\lambda}\hat{a} + \hat{b}\hat{\lambda}\left(c+\frac{\gamma a}{\delta+b-\mu}\right)$
$\hat{a}_{pu}$	$\frac{1}{\hat{\delta}}+(\hat{\mu}-\hat{b})\frac{\hat{\omega}}{\hat{\delta}}\left(c+\frac{\gamma a}{\delta+b-\mu}\right)+\frac{\gamma\hat{a}\hat{\omega}}{\hat{\delta}}$
$\hat{a}_{pv}$	$\frac{\hat{\rho}-1}{\hat{\delta}}-(\hat{\mu}-\hat{b})\frac{\hat{\lambda}}{\hat{\delta}}\left(c+\frac{\gamma a}{\delta+b-\mu}\right)-\frac{\gamma\hat{a}\hat{\lambda}}{\hat{\delta}}$

From now on we will take into account that $\hat{a}=a$ and $\hat{b}=b$. Using this Table we can then compute $\Delta\hat{a}_{yu}=\hat{a}_{yu}-a_{yu}$, etc.[29] We get

$$\begin{aligned}
\Delta\hat{a}_{yu} &\approx \left(\frac{\gamma a(\delta-\mu)}{\delta+b-\mu}-bc\right)\Delta\hat{\omega}; \\
\Delta\hat{a}_{pu} &\approx -\frac{\Delta\hat{\delta}}{\delta^2}\left(1+(\mu-b)\omega c+\frac{\gamma a\omega\delta}{\delta+b-\mu}\right)+\frac{\Delta\hat{\mu}}{\delta}\omega(c+\frac{\gamma a}{\delta+b-\mu}) \\
&\quad +\frac{\Delta\hat{\omega}}{\delta}\left((\mu-b)c+\frac{\gamma a\delta}{\delta+b-\mu}\right); \\
\Delta\hat{a}_{yv} &\approx \Delta\hat{\rho}+\Delta\hat{\lambda}\left(bc-\frac{\gamma a(\delta-\mu)}{\delta+b-\mu}\right); \\
\Delta\hat{a}_{pv} &\approx \frac{\Delta\hat{\rho}}{\delta}-\frac{\Delta\hat{\delta}}{\delta^2}\left(\rho-1-(\mu-b)\lambda c-\frac{\gamma a\lambda\delta}{\delta+b-\mu}\right) \\
&\quad -\frac{\Delta\hat{\mu}}{\delta}\lambda(c+\frac{\gamma a}{\delta+b-\mu})-\frac{\Delta\hat{\lambda}}{\delta}\left((\mu-b)c+\frac{\gamma a\delta}{\delta+b-\mu}\right).
\end{aligned}$$

[29] Note that a small deviation between the perceived and correct model changes γ marginally, hence a_{yu} is different from its value under the correct model, and thus $\Delta\hat{a}_{yu}$ is not equal to the difference between $\hat{a}_{yu}$ and the value of a_{yu} under the correct model.

We can also compute

$$\begin{aligned}\Delta\hat{c} \;\approx\; & -\frac{\omega\sigma_u^2-\lambda(\rho-1)\sigma_v^2}{(\delta+b-\mu)^2}\left(\Delta\hat{\delta}-\Delta\hat{\mu}\right)\\ & +\frac{1}{\delta+b-\mu}\left[\sigma_u^2\Delta\hat{\omega}+2\omega\sigma_u\Delta\hat{\sigma}_u-\lambda\sigma_v^2\Delta\hat{\rho}-(\rho-1)\sigma_v^2\Delta\hat{\lambda}-2\lambda(\rho-1)\sigma_v\Delta\hat{\sigma}_v\right].\end{aligned}$$

Finally, substituting (39) into (34) and rearranging using the definitions in Tables 4 and 5 we get the following:

$$\left(c+\frac{\gamma a}{\delta+b-\mu}\right)\left[\hat{\delta}-\hat{\mu}-\delta+\mu\right]=\hat{\lambda}\hat{\sigma}_v^2-\lambda\sigma_v^2. \tag{40}$$

Next, we differentiate (33)-(38), substituting (39) and (40) for (35) and (34) respectively, and replacing $\Delta\hat{a}_{yu}$,etc.,as well as $\Delta\hat{c}$ by their expressions above. We get six linear equations that are expressed as

$$A.(\Delta(\hat{\delta}-\hat{\mu}),\Delta\hat{\lambda},\Delta\hat{\sigma}_v,\Delta\hat{\omega},\Delta\hat{\sigma}_u,\Delta\hat{\rho})'=\Delta\hat{\delta}.w,$$

where the nonzero coefficients of $A: 6\times 6$, and $w: 6\times 1$ are the following:

$A_{12}=\lambda\sigma_v^2$; $A_{13}=\lambda^2\sigma_v$; $A_{14}=\omega\sigma_u^2$; $A_{15}=\omega^2\sigma_u$.

$A_{21}=c+\frac{\gamma a}{\delta+b-\mu}$; $A_{22}=-\sigma_v^2$; $A_{23}=-2\lambda\sigma_v$.

$A_{31}=-\frac{\omega\sigma_u^2-\lambda(\rho-1)\sigma_v^2}{(\delta+b-\mu)^2}-\frac{\gamma a}{(\delta+b-\mu)^2}$; $A_{32}=-\frac{(\rho-1)\sigma_v^2}{\delta+b-\mu}$; $A_{33}=-\frac{2\lambda(\rho-1)\sigma_v}{\delta+b-\mu}$; $A_{34}=\frac{\sigma_u^2}{\delta+b-\mu}$; $A_{35}=\frac{2\omega\sigma_u}{\delta+b-\mu}$; $A_{36}=-\frac{\lambda\sigma_v^2}{\delta+b-\mu}$.

$A_{41}=\left(c+\frac{\gamma a}{\delta+b-\mu}\right)\left(\frac{\lambda a_{yv}\sigma_v^2-\omega a_{yu}\sigma_u^2}{\delta}\right)$; $A_{42}=a_{pv}\sigma_v^2(bc-\frac{a\gamma(\delta-\mu)}{\delta+b-\mu})-\frac{a_{yv}\sigma_v^2}{\delta}\left((\mu-b)c+\frac{a\gamma\delta}{\delta+b-\mu}\right)$; $A_{43}=2a_{yv}a_{pv}\sigma_v$; $A_{44}=a_{pu}\sigma_u^2(\frac{a\gamma(\delta-\mu)}{\delta+b-\mu}-bc)+\frac{a_{yu}\sigma_u^2}{\delta}\left((\mu-b)c+\frac{a\gamma\delta}{\delta+b-\mu}\right)$; $A_{45}=2a_{yu}a_{pu}\sigma_u$; $A_{46}=a_{pv}\sigma_v^2+\frac{a_{yv}}{\delta}\sigma_v^2$.

$A_{52}=a_{yv}\sigma_v^2(bc-\frac{a\gamma(\delta-\mu)}{\delta+b-\mu})$; $A_{53}=a_{yv}^2\sigma_v$; $A_{54}=a_{yu}\sigma_u^2(\frac{a\gamma(\delta-\mu)}{\delta+b-\mu}-bc)$; $A_{55}=a_{yu}^2\sigma_u$; $A_{56}=a_{yv}\sigma_v^2$.

$A_{61}=\left(c+\frac{\gamma a}{\delta+b-\mu}\right)\left(\frac{\lambda a_{pv}\sigma_v^2-\omega a_{pu}\sigma_u^2}{\delta}\right)$; $A_{62}=-\frac{a_{pv}\sigma_v^2}{\delta}\left((\mu-b)c+\frac{a\gamma\delta}{\delta+b-\mu}\right)$; $A_{63}=a_{pv}^2\sigma_v$; $A_{64}=\frac{a_{pu}\sigma_u^2}{\delta}\left((\mu-b)c+\frac{a\gamma\delta}{\delta+b-\mu}\right)$; $A_{65}=a_{pu}^2\sigma_u$; $A_{66}=\frac{a_{pv}\sigma_v^2}{\delta}$.

$$w_4 = \frac{\sigma_u^2 a_{yu}}{\delta^2}\left(1+(\mu-b)\omega c+\frac{\gamma a\omega\delta}{\delta+b-\mu}\right)+\left(c+\frac{\gamma a}{\delta+b-\mu}\right)\left(\frac{\lambda\sigma_v^2 a_{yv}-\omega\sigma_u^2 a_{yu}}{\delta}\right)$$
$$+\frac{\sigma_v^2 a_{yv}}{\delta^2}\left(\rho-1-(\mu-b)\lambda c-\frac{\gamma a\lambda\delta}{\delta+b-\mu}\right);$$
$$w_6 = \frac{\sigma_u^2 a_{pu}}{\delta^2}\left(1+(\mu-b)\omega c+\frac{\gamma a\omega\delta}{\delta+b-\mu}\right)+\left(c+\frac{\gamma a}{\delta+b-\mu}\right)\left(\frac{\lambda\sigma_v^2 a_{pv}-\omega\sigma_u^2 a_{pu}}{\delta}\right)$$
$$+\frac{\sigma_v^2 a_{pv}}{\delta^2}\left(\rho-1-(\mu-b)\lambda c-\frac{\gamma a\lambda\delta}{\delta+b-\mu}\right).$$

In the above, γ is computed at the correct model: $\gamma = \gamma_0$. From there we can compute $q = A^{-1}w$.

To compute the coefficient m in (21) we use (20) and note that

$$\frac{\partial\gamma}{\partial\varphi} = -\gamma\frac{(\delta+b-\mu)^2}{\varphi(\delta+b-\mu)^2+a^2(\delta-\mu)^2}$$

and that

$$\begin{aligned}
\nabla_v\gamma &= \left(\frac{\partial\gamma}{\partial(\hat\delta-\hat\mu)},\frac{\partial\gamma}{\partial\hat\lambda},\frac{\partial\gamma}{\partial\hat\sigma_v},\frac{\partial\gamma}{\partial\hat\omega},\frac{\partial\gamma}{\partial\hat\sigma_u},\frac{\partial\gamma}{\partial\hat\rho}\right)' \\
&= \frac{a}{\varphi(\delta+b-\mu)^2+a^2(\delta-\mu)^2}\begin{pmatrix} 2\lambda\sigma_v^2\rho(\delta-\mu)+b\lambda\sigma_v^2-2(\delta-\mu)\omega\sigma_u^2 \\ \sigma_v^2\left(\rho(\delta-\mu)^2+b(\delta-\mu)\right) \\ 2\sigma_v\lambda\left(\rho(\delta-\mu)^2+b(\delta-\mu)\right) \\ -(\delta-\mu)^2\sigma_u^2 \\ -2(\delta-\mu)^2\omega\sigma_u \\ (\delta-\mu)^2\lambda\sigma_v^2 \end{pmatrix} \\
&\quad -\frac{\gamma}{\varphi(\delta+b-\mu)^2+a^2(\delta-\mu)^2}\begin{pmatrix} (2\varphi(\delta+b-\mu)+2a^2(\delta-\mu)) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.
\end{aligned}$$

In particular, the above derivations imply the following expression for SSO:

$$SSO = \frac{\begin{array}{c}2a_{yu}^2\sigma_u^2\sigma_v^2 a_{yv}(mq_6+mq_2(bc-a\gamma\frac{\delta-\mu}{\delta+b-\mu}))+2a_{yu}^2\sigma_u^2\sigma_v a_{yv}^2 mq_3 \\ +2a_{yu}\sigma_u^2\sigma_v^2 a_{yv}^2 mq_4(bc-a\gamma\frac{\delta-\mu}{\delta+b-\mu}))-2a_{yu}^2\sigma_u\sigma_v^2 a_{yv}^2 mq_5\end{array}}{(a_{yv}^2\sigma_v^2+a_{yu}^2\sigma_u^2)^2}.$$

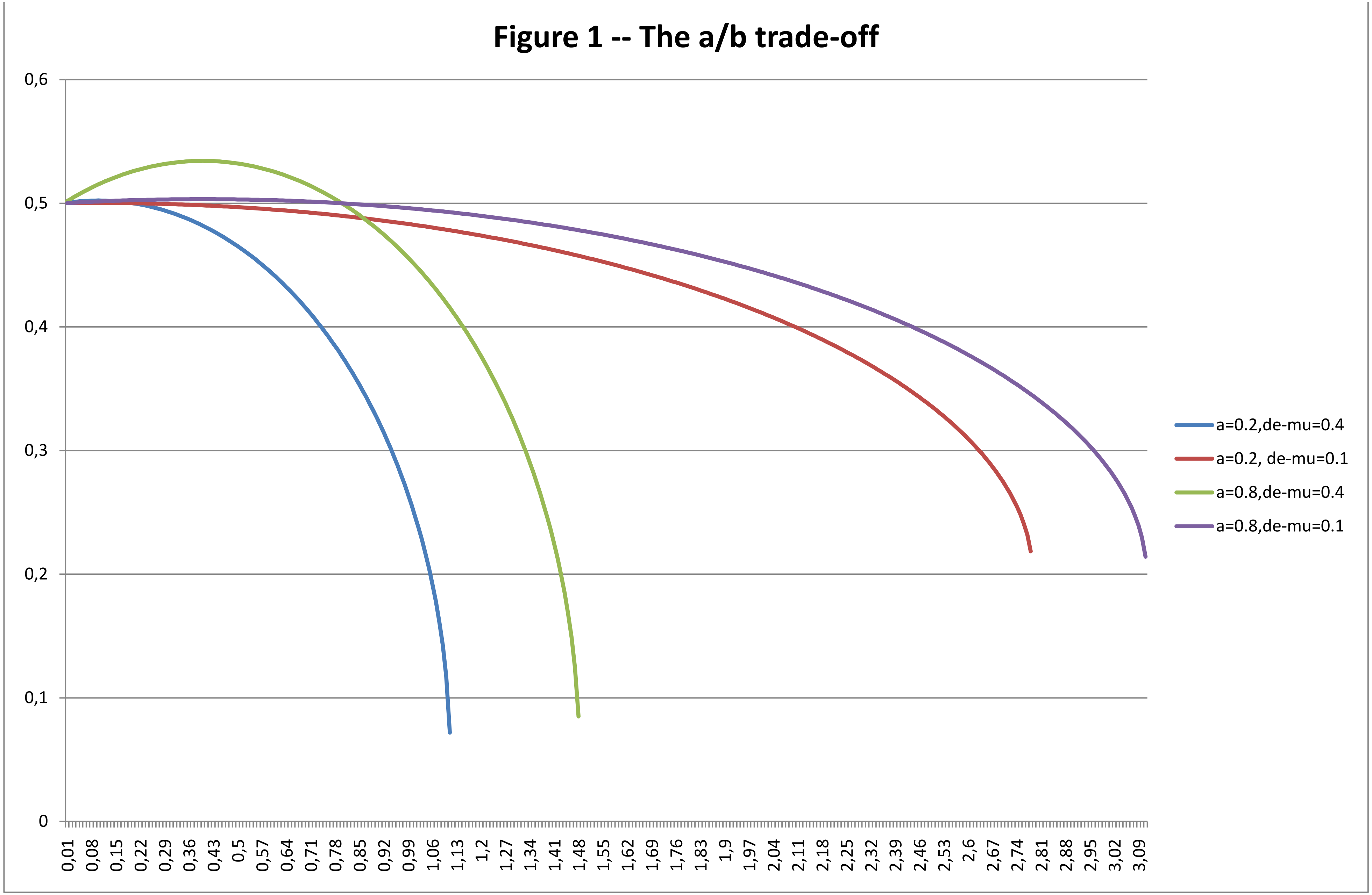
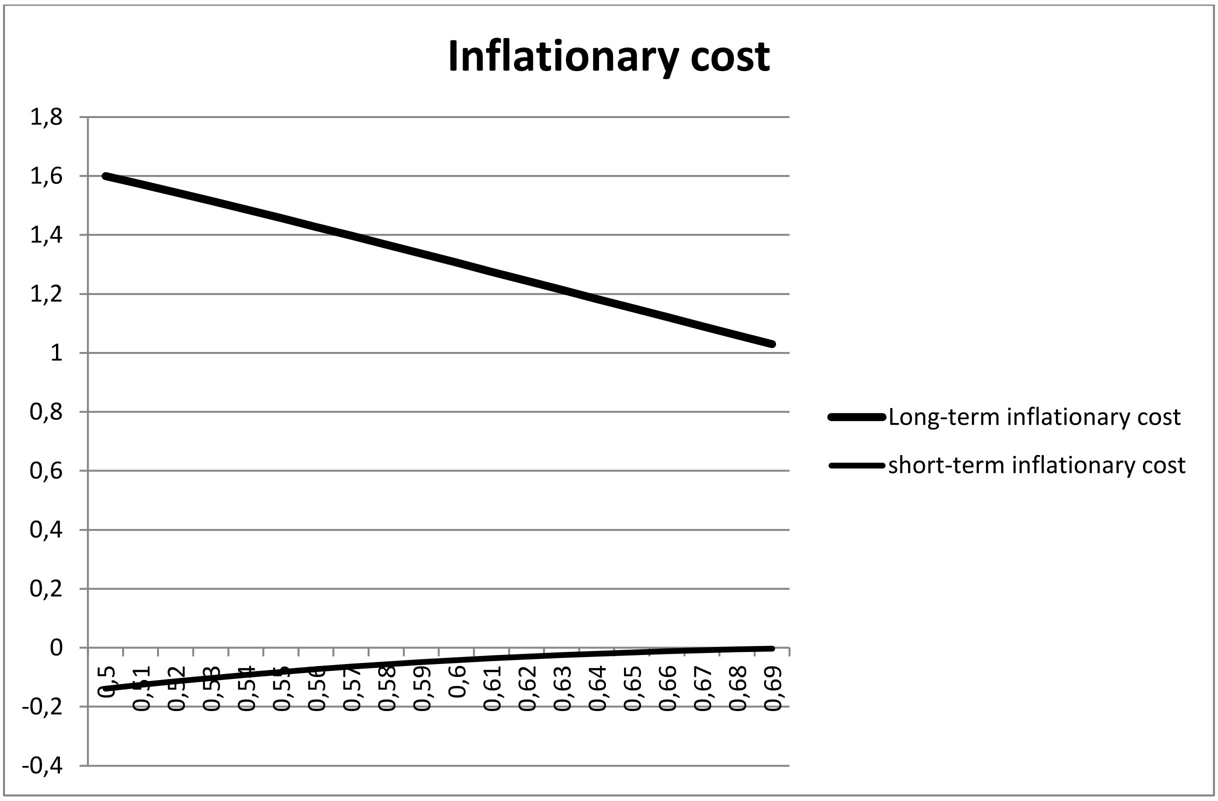


Figure 1 -- The a/b trade-off
0,6
0,5
0,4
0,3
0,2
0,1
0
a=0.2,de-mu=0.4
a=0.2, de-mu=0.1
a=0.8,de-mu=0.4
a=0.8,de-mu=0.1
0,01 0,08 0,15 0,22 0,29 0,36 0,43 0,5 0,57 0,64 0,71 0,78 0,85 0,92 0,99 1,06 1,13 1,2 1,27 1,34 1,41 1,48 1,55 1,62 1,69 1,76 1,83 1,9 1,97 2,04 2,11 2,18 2,25 2,32 2,39 2,46 2,53 2,6 2,67 2,74 2,81 2,88 2,95 3,02 3,09

Figure 2 – Ideological sensitivities, a = 0.7 ; b = 0.5 ; ω = 1; λ=1; σ_u^2 = 0.5; δ = 0.7 ; ρ = 1.



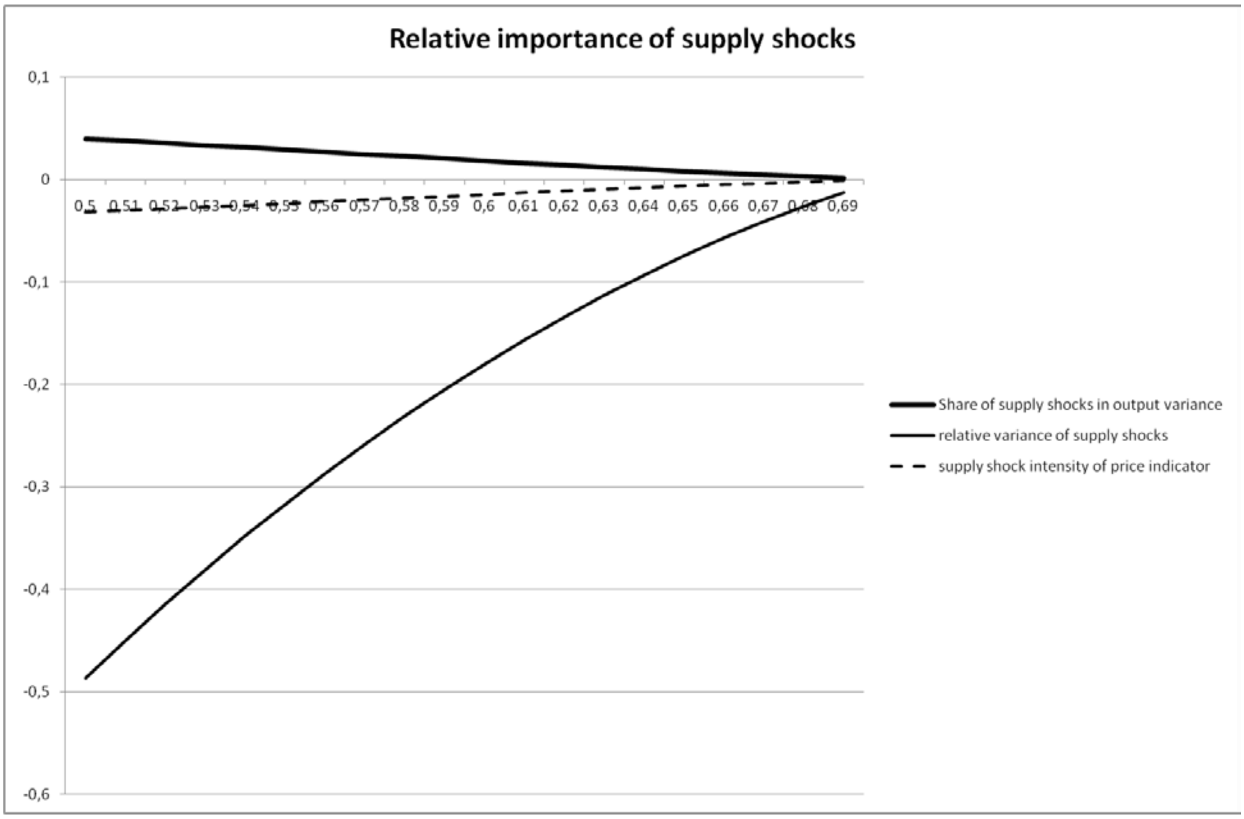
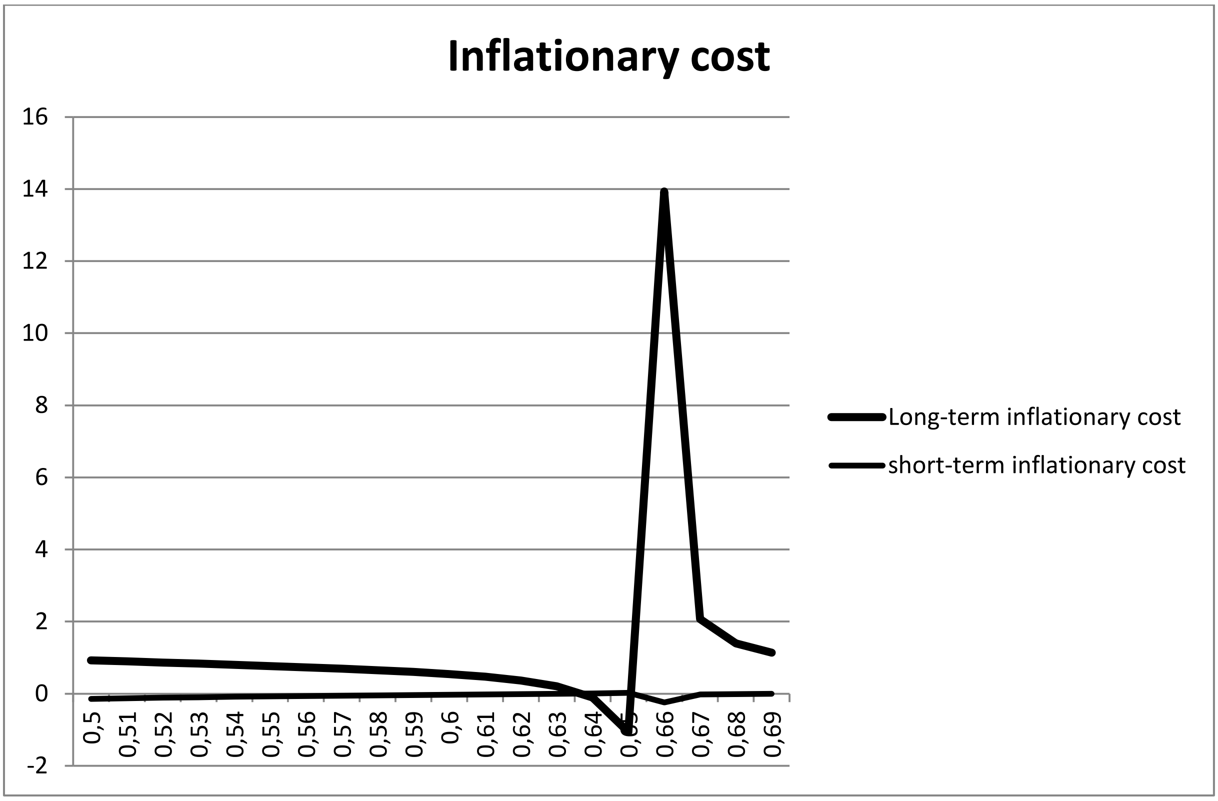


Figure 3 – Ideological sensitivities, a = 0.7 ; b = 0.5 ; ω = 1; λ=1; σ_u^2 = 0.9; δ = 0.7 ; ρ = 1.



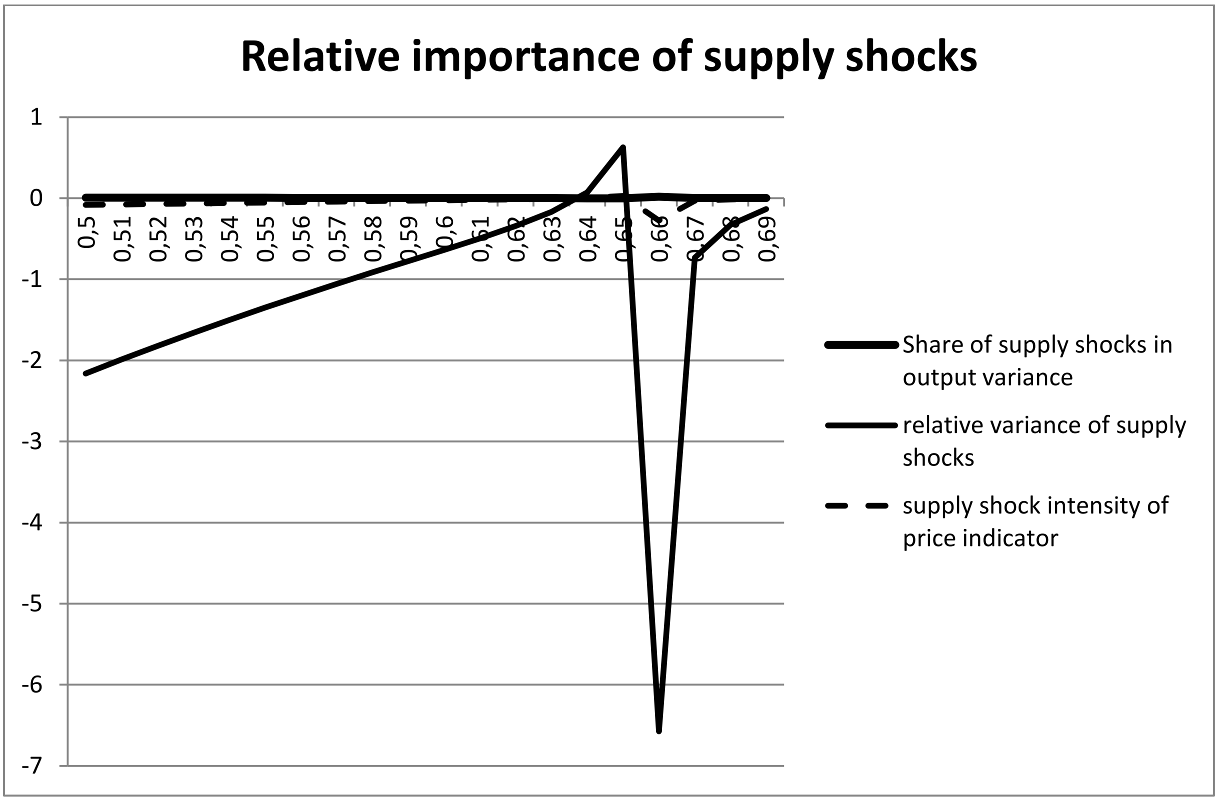
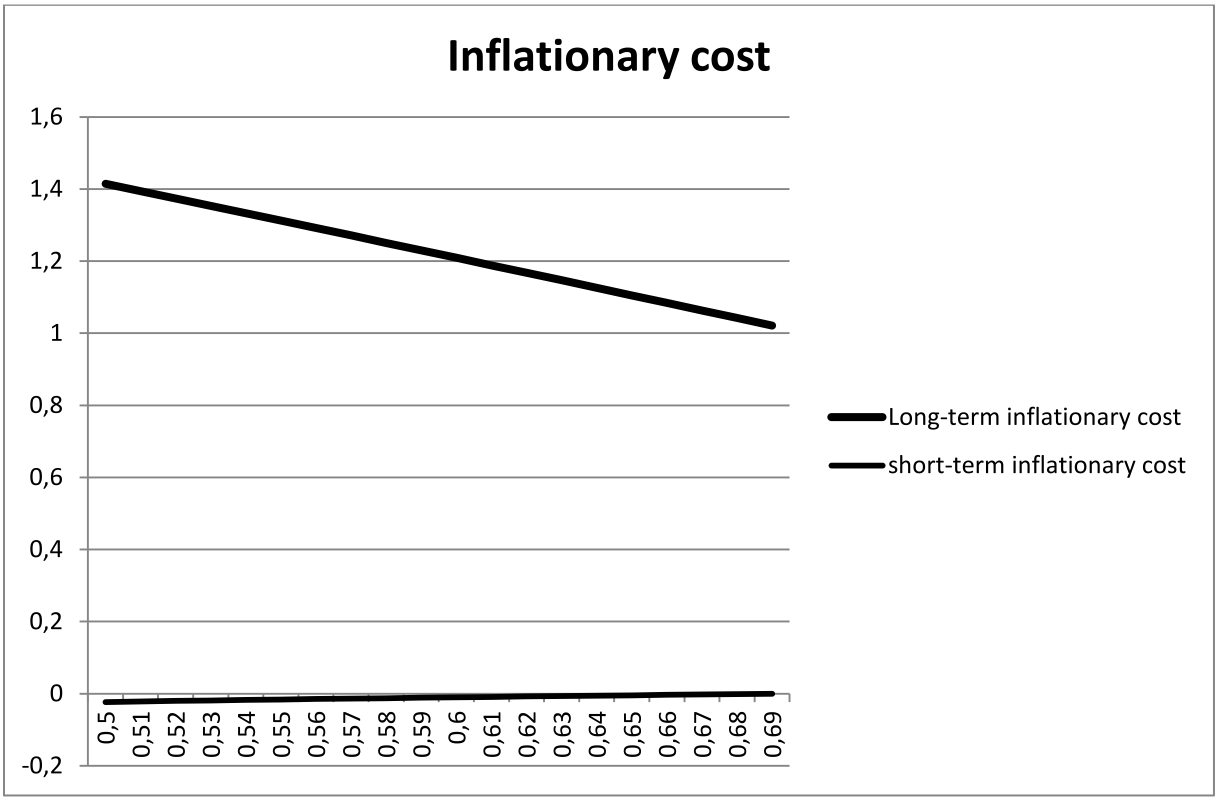


Figure 4 – Ideological sensitivities, a = 0.7 ; b = 0.5 ; ω = 1; λ=1; σ_u^2 = 0.1; δ = 0.7 ; ρ = 1.



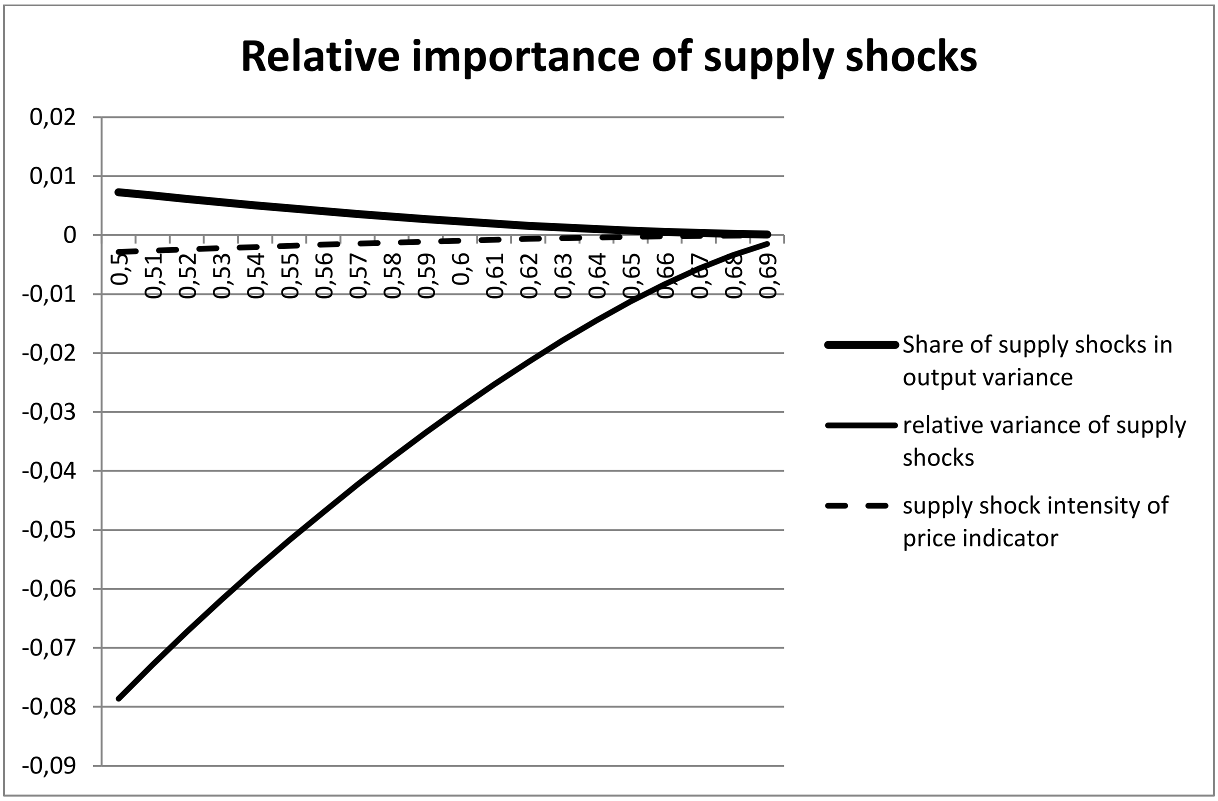
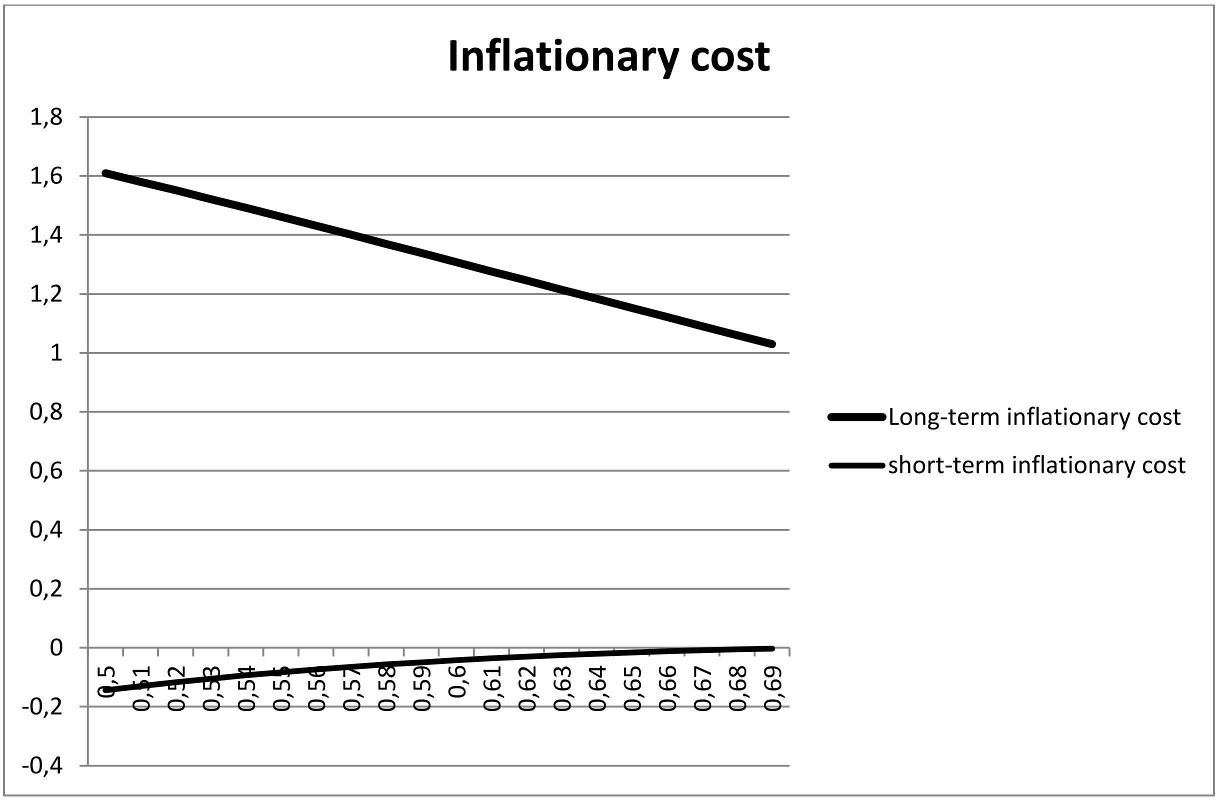


Figure 5 – Ideological sensitivities, a = 1 ; b = 0.5 ; ω = 1; λ=1; σ_u^2 = 0.5; δ = 0.7 ; ρ = 1.



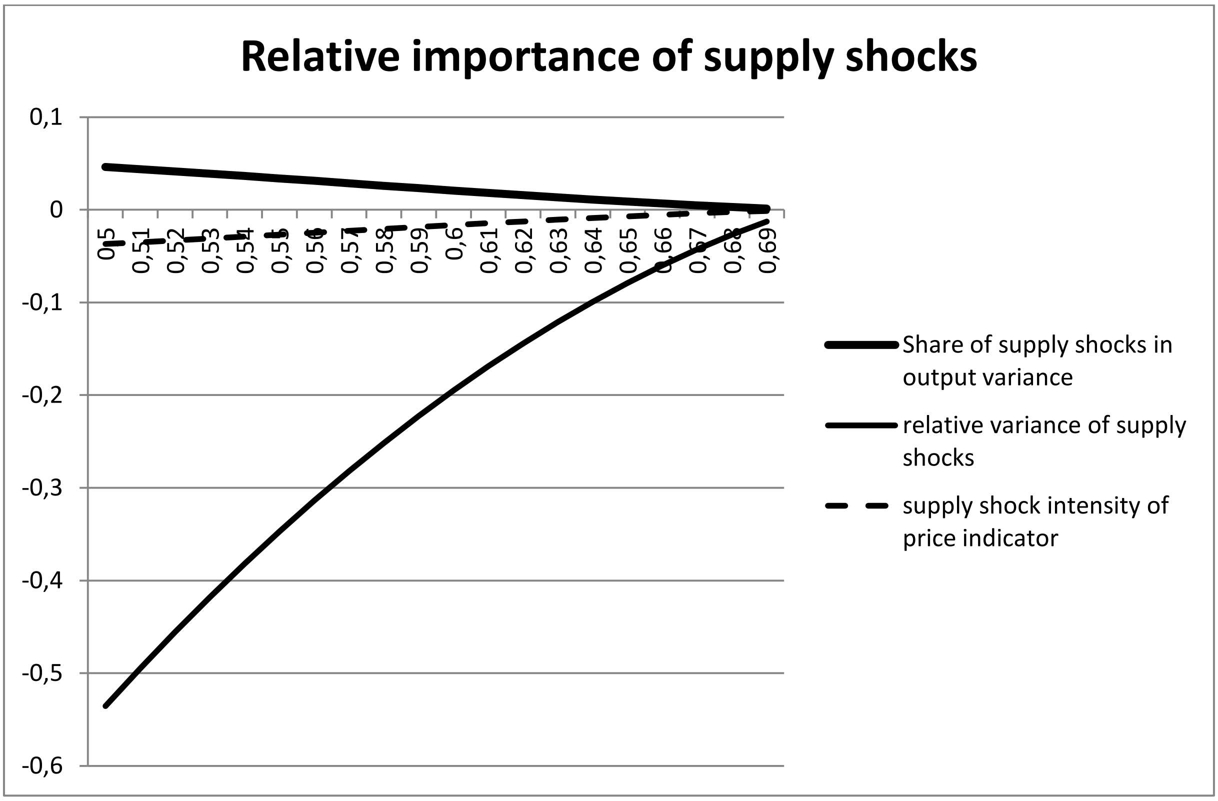
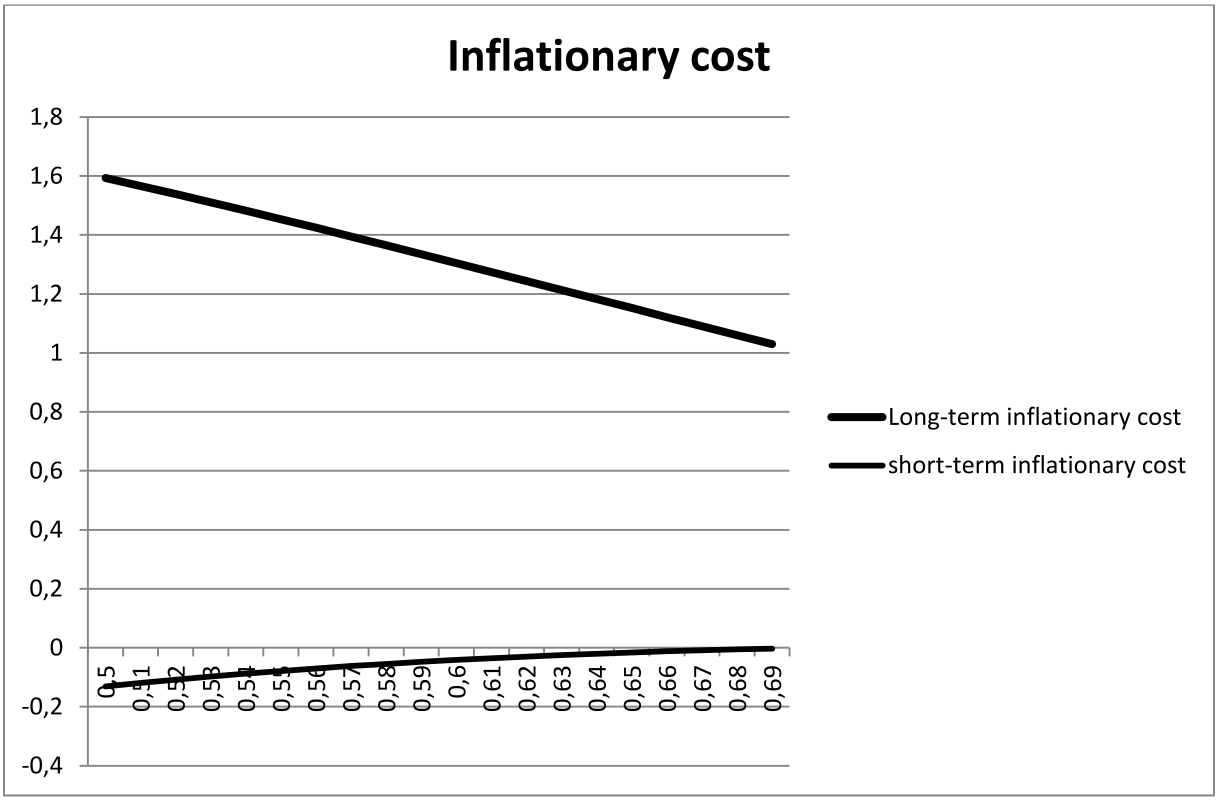


Figure 6 – Ideological sensitivities, a = 0.3 ; b = 0.5 ; ω = 1; λ=1; σ_u^2 = 0.5; δ = 0.7 ; ρ = 1.



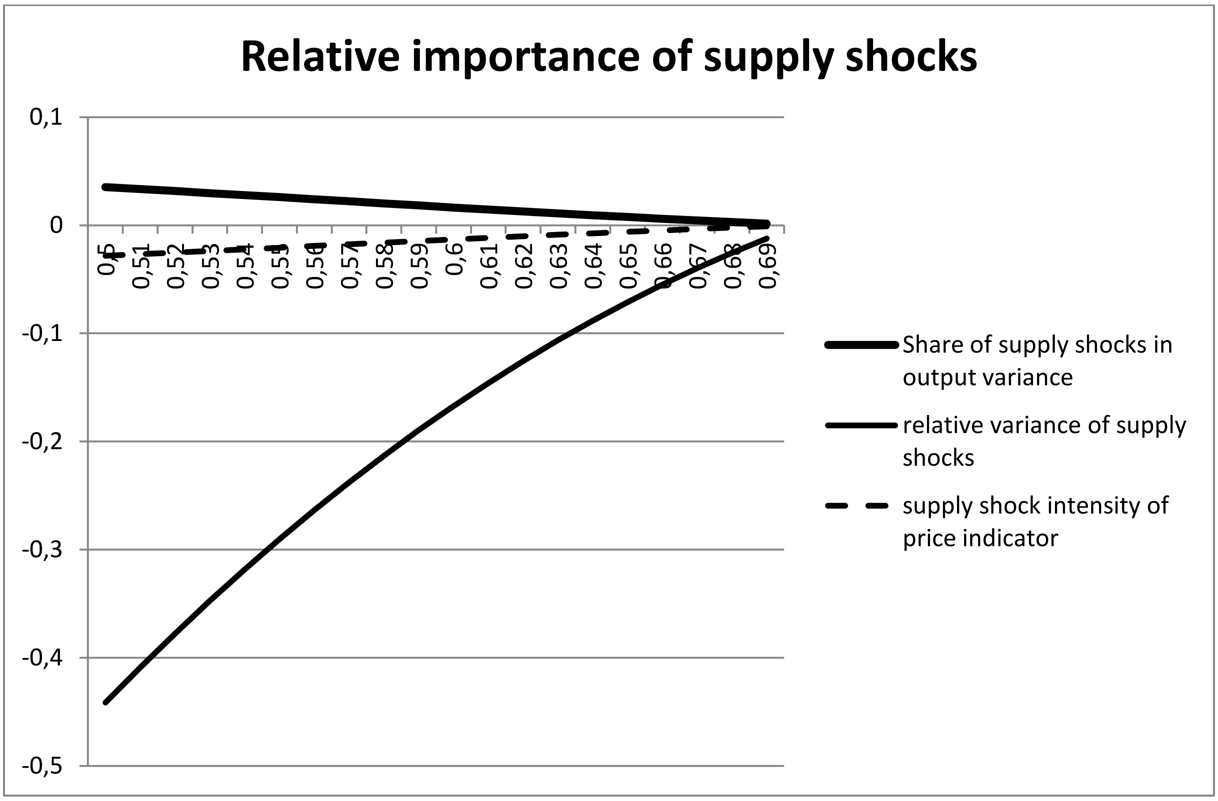


Figure 7 – Ideological sensitivities, a = 0.7 ; b = 0.1 ; ω = 1; λ=1; σ_u^2 = 0.5; δ = 0.7 ; ρ = 1.