Estimating a Dynamic Game of Spatial Competition: The Case of the U.K. Supermarket Industry*

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Abstract

We develop a dynamic model of new store openings and closings with spatial competition and an entry regulator in a continuous-time framework. In the model, opportunities to open a new store or to close an existing store arrive randomly. If a firm receives the opportunity, it decides whether to send an application for opening a store in that location, taking into account both the rivals' future responses as well as the adverse cannibalization effects on own neighboring stores. The regulator either approves or rejects the application, based on the potential effects of the opening on consumer surplus and profits of rival firms. We estimate the model by a two-step method, using data from the U.K. supermarket industry on exact locations and dates of store openings/closings, applications for store opening, and approval decisions by the regulator, together with rich data of consumer choices and consumer locations. In counter-factual analysis, we evaluate the effect of the change in the government planning regulation that took place in 1996 in the U.K.

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1 Introduction

Supermarket stores vary widely in the range of products they offer. The "big-box" format—located in large stores outside of town centres—is attractive for its wide product range. Other formats offer smaller stores and attract consumers in alternative ways: some have strengths in specific product categories such as fresh or frozen produce, others offer low prices on a limited product range, and others offer a convenient geographic location. Many of these alternative formats are located in town centres. The emergence of these distinct formats has led to several public policy questions. One is whether big-box retailing has an adverse impact on small stores and consequently the environmental quality of town centres.[1] Another is whether tight restrictions on development of big box retailers—resulting in a high level of concentration within the big box format—is enough to confer market power in some locations regardless of the presence of outlets from other formats.

In the U.K., environmental and competition policy authorities have taken distinct positions with regard to store format. Environmental planning authorities have a policy of protecting town centres, through restrictions on the development of big box stores. Competition authorities, on the other hand, have claimed that these restrictions can reduce competition within the big-box format, given that shoppers on large trips regard smaller-format stores as poor substitutes for large stores.[2] Until recently these two areas of policy have developed independently and currently there is a debate over how much planning policy should respond to competition as well as environmental policy issues. Competition and environmental policy have – taken together – inhibited the ability of some firms to build more large stores, and these firms have instead developed many small format stores in recent years.

The aim of this paper is to analyze the welfare effects of the current environmental planning policy and evaluate the potential impact on market structure of alternative policies, including a "competition test" proposed by the Competition Commission (CC). Specifically, we will answer the following three questions. The first question is if the current policy ended up suppressing competition between stores by allowing existing stores to exercise higher market power. Second, we will investigate if the current policy induced the preemption incentives to block the opening of big-box stores, resulting in inefficient entry. Finally, we will ask what would happen to the market structure if alternative policies are implemented. In particular, we study the relevance of the competition test based on the idea that the current criterion allows not only preemption but also entry deterrence by incumbents. The competition test suggested by the CC will block proposals that would raise concentration in the neighborhood. In addition, since the local authority myopically decides upon each application under the current policy guideline, we also analyze the implication of introducing a "dynamic regulator" which not only considers the static effects of new store entry but also has dynamic consideration like preemption motives.[3]

To answer these policy questions, we develop a dynamic industry model with multi-stop shopping consumers, multi-store firms, and an entry regulator. Our demand model, based on Schiraldi, Seiler, and Smith (2012), aims to capture the consumers' multi-store shopping behavior which depends on their private shopping costs and the opportunities for shopping benefits in the their local choice set. Specifically, we model

[1] We do not comment on whether protection of town centre stores, and town centre vitality, are worthwhile public policy objectives.

[2] As a consequence, mergers between firms in different formats are treated more leniently than mergers in the largest formats. The most dominant big box retailers Tesco, ASDA and Sainsbury were prohibited from merging with Safeway, which also operated big box stores. Inter-format policy in the U.K.: the merger of ASDA and Netto was permitted, while Tesco and Sainsbury have been allowed to expand market share by acquisition (and new openings) of small-format stores.

[3] The guidelines of the planning policy and the CC Report imply that local authorities do not have dynamic considerations in approval decisions.

consumer demand using a continuos discrete framework where consumers choose the stores to visit and how to allocate expenditure across different categories. The possibility of combining different categories of products across stores gives the extra incentive to consumers to engage in multi-store shopping. Multi-stop shopping consumers can, in principle, substitute demand between the stores they visit depending on price and quality differences. This provides an extra constraint (on top of that which is implied by one stop shopping alone) on the market power of stores.

In the supply side, multi-store firms engage in a dynamic game of new store openings and closings with spatial competition and the regulator in continuous time. In the model, opportunities to open a new store (a large parcel of land suitable for use as a supermarket) or to close an existing store arrive randomly. Once a parcel of land becomes available, the firm which is called upon to move decides whether to file an application for opening a store at that location, taking into account both the rivals' future responses as well as the adverse cannibalization effects on own neighboring stores. The regulator either approves or rejects the application, based on the potential effects of the opening on consumer surplus and profits of rival firms. Similarly, the firm that receives an opportunity to close one of its existing stores decides whether to do so. A Markov Perfect Equilibrium generates sequential openings and closings as well as certain patterns of store dynamics in geographically connected locations.

To estimate the model, we use the exact dates and locations of submission of applications for store openings, data on approve/reject decisions by the regulator, and opening dates in case of approval, together with rich data of consumer choices and consumer locations. Using a "static-dynamic" breakdown, we first estimate the demand model separately from the dynamic game, and recover per-period payoffs from the product market competition. The dynamic game is estimated using a two-step method. In the first stage, exogenous arrival rates, approval probabilities, and conditional choice probabilities are estimated. In the second step, using a forward simulation by Hotz, Miller, Sanders, and Smith (1994), we compute value functions. Finally, we estimate a set of structural parameters, including parameters in the regulator's choice (approve/reject) equation as well as entry costs and application costs, by maximizing the pseudo likelihood function.

In counter-factual analysis, we evaluate the effect of the government regulation policy. Specifically, we separately measure the policy's effects on consumer surplus, profits of superstore rivals, and profits of small-sized stores. The policy is intended to protect profits of stores in town centres, and we can look to see the effect on such stores. We will also simulate the effect of alternative regulatory frameworks on those components and compare its implication on the market structure and consumer surplus.

This paper is the first to recover the objective function of the regulator in the context of competition policy. Timmins (2002) identifies the objective function of water utility regulators in the U.S., and measures efficiency gains obtained from alternative pricing policies. Unlike Timmins (2002), the regulated agents in our application have strategic interactions, and therefore we will explicitly evaluate if (and how) the planning regulation achieved the goal of controlling the market structure in favor of small stores, located in town centres.

Another important contribution is that we incorporate the spatial nature of competition in a dynamic oligopoly, which is an important feature in the U.K. supermarket industry. To deal with the curse of dimensionality problem, as in Aguirregabiria and Ho (2012) and Sweeting (2011), we reduce the state space by aggregating information on the market structure of neighboring locations into a small number of variables (changes in own and rival firms' profits associated with a store opening), which we recover from a rich demand model. This allows us to estimate a dynamic model with spatial competition allowing for firm heterogeneity. In addition, we exploit properties of continuous time, which have been recently studied by Arcidiacono, Bayer, Blevins, and Ellickson (2010) and Doraszelski and Judd (2010) in estimating dynamic games.[4] In their framework, no more than one firm moves simultaneously, which reduces the cost of computing equilibria significantly. Our continuous-time framework, based on Arcidiacono et. al (2010), not only alleviates the curse of dimensionality associated with estimating a dynamic game, but also captures the fact that land availability arrives randomly and continuously. Furthermore, this framework can best exploit the high frequency nature of our data (openings, closings, and application submissions).

This work also relates to several papers that analyze a retail industry. Beresteanu, Ellickson, and Misra (2010) estimate a dynamic game between supermarkets in the U.S. In their model, geographic markets are independent and there is no distinction regarding locations of stores within each market. On the other hand, in our application, locations of stores within the market are important choice and state variables. Aguirregabiria and Vicentini (2012) propose a dynamic model of oligopoly with spatial competition between multi-store retailers. While the structure of spatial competition in our model is similar to theirs, we focus on an application and analyze the role played by the entry regulator. Sadun (2011) analyzes the effect of the planning regulation in the U.K. She uses exogenous variation in local political control across the U.K. to evaluate the causal impact of planning regulation on independent retailers. We can answer related questions by counterfactual analyses.

The rest of the paper is organized as follows. Section 2 briefly summarizes the planning regulation in the U.K. and provides simple statistics on application decisions and store openings/closings. We present the full model of dynamic oligopoly with spatial competition in Section 3 and the estimation procedure in Section 4. A simplified version of the model and estimation results are shown in Section 5.

2 Industry Background

2.1 Planning Regulation

There was a major planning policy change dating from the mid 1990s. From 1996 planning policy was changed resulting in a much more restrictive attitude to the entry of new big box retailing outlets, with the primary aim of protecting town centres, where small (competitor) stores are typically located. To open a store requires planning consent, and the criteria in the new planning guidelines are: (a) a sequential test that asks (for big-box out-of-centre applications) if development at a town centre location was possible instead; (b) a need test that asks if there is a "need" for the extra floorspace conditional on existing floorspace and town population; and (c) a test for whether the development will adversely affect business in the town centre.[5] The planning policy is intended to be much less restrictive for small stores in town centres than for big box outlets out of town. Since the introduction of tighter planning criteria new store formats have changed with a much higher proportion (than previously) of new stores being of small-store format in town centre locations, and a much smaller proportion being big-box format. For the detail of the regulation, see Sadun (2011).

2.2 Data

The data comprises of several datasets. The first dataset is a survey of consumer choices covering the period Oct 2002-Sept 2005. The consumer survey contains information on stores that each consumer visited. For the most important six firms we know the exact store that the consumer visited, and for the other firms, we know

[4] Takahashi (2011) is also one of a few studies that apply a continuous-time framework for estimating a dynamic game.

[5] The criteria for (b) are relatively observable, and (c) can be obtained using our demand model. The criteria for (a) may depend on unobservable factors to do with available sites in the centre.

the firm (but not exact store). Demographic information on the consumers is also recorded, including location, social class, and household size. We aggregate spending into 4 broad product categories detailed in Schiraldi et al. (2012). For each of these categories we compute a price index for each firm. (Firms with heterogeneous store size have more than one price index, depending on store size, to reflect the general pricing practice of setting prices nationally for given size formats). The prices are constructed for each bi-week period using the product-level prices and revenue weights observed in the consumer data. These price indices are computed separately for each of 8 demographic groups (by household size and social class) to reflect their different tastes (see Schiraldi et al. 2012 for a detailed discussion).

The second dataset contains store information in Great Britain, which is provided by Institute for Grocery Distributors (IGD). We have information on all incumbent stores in 2001 as well as new store openings and closings of existing stores during the period 2002-2006. For each observation of stores, we can observe the date of opening/closing, firm identity, store size (floor space), store location (by postcode, which yields an exact grid reference), and the type of location (town centre, out of town centre, etc). For any consumer location in the consumer survey, we can use store locations in the IGD data to compute choice sets of nearby stores.

The third dataset, provided by Glenigan, contains information on planning applications to open a new store. For each application, we have the planning details (firm identity, floor space, and site address) as well as the approval decision (approve/reject) and the exact date of the decision.

Table 1 shows the number of stores in 2005 by firm and store format (size) and market shares for 10 selected firms. As we discuss later, store size is firm's choice variable. There is significant heterogeneity among firms in terms of store sizes and the number of stores they own. For example, the share of ASDA's stores that are large is about 70%. Meanwhile, Co-op has almost 1,600 stores, but in the small format.

There were a large number of openings and closings in the U.K. during the sample period. These figures also differ widely across firms. Table 2 shows the decomposition by firm. Tesco opened new stores most actively. Table 3 shows the regional distributions of openings and closings.

Table 4 summarizes frequencies of approval and rejection during the period 2001-2006. The frequency of rejection is non-negligible. This implies that firms face uncertainty about the regulator's decision when deciding whether to submit an application or not. The number of rejections for the group of firms we call "Discounter" —which are always small—implies that openings of small store are often rejected, too.

For the purposes of defining locations, described in the model in the next section, we use postal sectors, which are small neighborhoods of a few thousand households each. There are around 11,000 of these in Great Britain.

3 Model

This section describes a model of dynamic oligopoly with spatial competition, where firms open and close stores in multiple locations to maximize the discounted expected sum of intertemporal profits. The demand side of the model closely follows that of Schiraldi et al. (2012). We assume that a consumer's problem is static and outcomes in the product market competition do not affect firms' dynamic decisions. Therefore, profits from product market are simply a function of the current state, and can be taken as primitives in estimating the dynamic game of store openings and closures.

3.1 Demand

Consumer i in period t selects shopping choice c comprising either a single store j, or a store pair (j, j') from the nearest J stores to the consumer, which implies a choice set $\mathcal{C}_i$ containing $\binom{J+1}{2}$ distinct choices. The consumer buys q_{jtk} units of category k at individual store $j \in c$ for $k = 1, ..., K$. The price indices paid per unit of these demands depend on the consumer's demographic group $g(i)$: $p_{ijtk} = p_{g(i)jtk}$. The consumer's demands and prices at the store(s) in choice c are:

$$
\begin{aligned}
q_{ctk} &\equiv [q_{jtk}]_{j \in c} \equiv \begin{cases} (q_{jtk}, q_{j'tk}) & \text{if } c = (j, j') \\ (q_{jtk}) & \text{if } c = j \end{cases} \\
p_{ictk} &\equiv [p_{ijtk}]_{j \in c} \equiv \begin{cases} (p_{ijtk}, p_{ij'tk}) & \text{if } c = (j, j') \\ (p_{ijtk}) & \text{if } c = j \end{cases}
\end{aligned}
$$

for $k = 1, .., K$. The consumer's overall bundle of demands and prices at shopping choice c are $q_{ct} \equiv [q_{ctk}]_{k=1}^{K} \equiv (q_{ct1}, ..., q_{ctK})$ and $p_{ict} \equiv [p_{ictk}]_{k=1}^{K} \equiv (p_{ict1}, ..., p_{ictK})$.

The indirect utility function of consumer i with income y_i who chooses c at time t is given by

$$
U_{ict}(p_{ict}, y_i, w_{ict}, x_{ict}, v_{ict}, \varepsilon_{ict}) = \alpha a_{ict}(y_i, p_{ict}, w_{ict}, v_{ict}, \gamma) + \beta' x_{ict} + \varepsilon_{ict}, \tag{1}
$$

where w_{ict} is a set of observable store and consumer variables, $v_{ict} = (v_{ict1}, ..., v_{ictK})$ are unobserved category-specific tastes, ε_{ict} is unobserved taste specific to choice c, and (α, β, γ) are parameters to be estimated. x_{ict} is a list of observed attributes that influence the consumer's utility from shopping choice c, but which do not otherwise affect the quantity chosen (conditional on shopping choice c): it includes the transport and shopping costs of choice c and store attributes that affect the consumer's overall valuation of the shopping experience (e.g., whether the store is in a town centre or not). The portion $a_{ict}()$ is the maximum utility available at choice c given the price list p_{ict}:

$$
a_{ict}(y_i, p_{ict}, w_{ict}, v_{ict}, \gamma) = \max_{q_{ct} \geq 0} \left[\sum_{k=1}^{K} u_{ictk}(q_{ctk}, w_{ict}, v_{ictk}, \gamma) + (y_i - p'_{ict} q_{ct}) \right] \tag{2}
$$

where $u_{ictk}(q_{ctk}, w_{ict}, v_{ictk}, \gamma)$ is the direct utility for category k. The assumption of category-level direct utilities is not essential but simplifies the econometric model and is of limited cost given the broad nature of the category definitions. Conditional on choice c the utility maximizing bundle q_{ctk} that solves the problem in (2) is implied by Kuhn-Tucker complementary slackness conditions:

$$
\left.
\begin{array}{c}
\left(\frac{\partial u_{ictk}}{\partial q_{jtk}} - p_{ijtk} \right) \leq 0 \\
q_{jtk} \geq 0 \\
q_{jtk} \left[\frac{\partial u_{ictk}}{\partial q_{jtk}} - p_{ijtk} \right] = 0
\end{array}
\right\} \Longrightarrow \tilde{q}_{jtk}(p_{ictk}, w_{ict}, v_{ictk}) \text{ for } j \in c \text{ and } k = 1, .., K. \tag{3}
$$

The choice of c satisfies the condition

$$
\begin{aligned}
i \text{ chooses } c &\iff U_{ict}(p_{ict}, y_i, w_{ict,} x_{ict}, v_{ict}, \varepsilon_{ict}) \\
&\geq U_{ic't}(p_{ic't}, y_i, w_{ic't,} x_{ic't}, v_{ic't}, \varepsilon_{ic't}) \text{ for all } c' \in \mathcal{C}_i.
\end{aligned}
$$

Combining discrete and continuous decisions, consumer i's (unconditional) demand in store j for category k at time t is given by:

$$
q_{jtk}(p_{it}, y_i) = \sum_{c \in c_i(j)} \tilde{q}_{jtck}(p_{ictk}, w_{ict}, v_{ictk}) \times 1 \left[\begin{array}{c} U_{ict}(p_{ict}, y_{it}, w_{ict}, x_{ict}, v_{ict}, \varepsilon_{ict}) \\ > U_{ic't}(p_{ic't}, y_{it}, w_{ic't,} x_{ic't}, v_{ic't}, \varepsilon_{ic't}) \end{array} \right]
$$

where 1[] is an indicator function and $c_i(j)$ is the set of choices c in $\mathcal{C}_i$ that contain store j.

The direct utility $u_{ictk}(q_{ctk}, w_{ict}, v_{ictk}, \gamma)$ for category k is assumed to be quadratic in the quantities q_{ctk} bought in the store(s) in choice c:

$$u_{ictk}(q_{ctk}, w_{ict}, v_{ictk}, \gamma) = \frac{1}{\gamma_{2k}}\left[\sum_{j\in c}\gamma_{1ijt}\left(w_{ijt}, v_{ijtk}\right)q_{jtk} - \frac{1}{2}\sum_{j\in c}\sum_{j'\in c}\left[\left(\gamma_{2k}1_{[j'=j]} + \gamma_{3k}1_{[j'\neq j]}\right)q_{jtk}q_{j'tk}\right]\right] \quad (4)$$

where $1_{[.]}$ is an indicator function. The marginal utility of category k at store j depends positively on first order term γ_{1ijt}, which is assumed to be a function of w_{ijt} and v_{ijtk}, and declines with the quantities of k bought at store(s) in c:

$$\frac{\partial u_{ictk}}{\partial q_{jkt}} = \frac{1}{\gamma_{2k}}\left(\gamma_{1ijt}\left(w_{ijt}, v_{ijtk}\right) - \sum_{j'\in c}\left[\gamma_{2k}1_{[j'=j]}q_{j'kt} + \gamma_{3k}1_{[j'\neq j]}q_{j'kt}\right]\right) \quad (5)$$

The parameter γ_{3k} governs the rate at which the marginal utility of k at store j declines in the other store's quantity $q_{j'tk}$ (when there are two stores in c). For interior solutions—where $q_{jtk} > 0$—the Kuhn Tucker conditions (3) imply that marginal utility is equated to the marginal value of expenditure on other goods:

$$\gamma_{1ijt}\left(w_{ijt}, v_{ijtk}\right) - \sum_{j'\in c}(1_{[j'=j]}q_{jtk} + 1_{[j'\neq j]}\gamma_{3k}q_{j'tk}) = \gamma_{2k}p_{ijtk}. \quad (6)$$

When i buys positive k from only one store j then (6) implies

$$q_{jtk} = \gamma_{1ijt}\left(w_{ijt}, v_{ijtk}\right) - \gamma_{2k}p_{ijtk}$$

and when i buys positive k from two stores (j, j') then (6) implies

$$q_{jtk} = \frac{1}{1-\gamma_{3k}\gamma_{3k}}\left[\gamma_{1ijt}\left(w_{ijt}, v_{ijtk}\right) - \gamma_{2k}p_{ijtk} - \gamma_{3k}(\gamma_{1ij't}(w_{ij't}, v_{ij'tk}) - \gamma_{2k}p_{ij'tk})\right] \quad (7)$$

which shows that as $\gamma_3 \in [0,1]$ increases from zero there is an increase in the size of the cross price effects between the two stores in c.

We assume that γ_{1ijt} is a linear function of observable store and consumer variables w_{ijt} and a random taste draw v_{ijtk}

$$\gamma_{1ijt} = \gamma_{10}'w_{ijt} + v_{ijtk} \quad (8)$$

where the term v_{ijtk} is specified as follows:

$$v_{ijtk} = \begin{cases} v_{itk} + \epsilon_{ijctk} & \text{if } c = (j, j') \\ v_{itk} & \text{if } c = (j) \end{cases} \quad (9)$$

where v_{itk} is the consumer's taste for category k and if the consumer visits two stores then ϵ_{ijctk} is an extra disturbance that determines the consumer's split of category demand between the two stores in c; to simplify the estimation of the model this is assumed equal and opposite for j and j': $\epsilon_{ijctk} = -\epsilon_{ij'ctk}$.

The conditions (3) imply for consumer i at time t non-negative conditional demand functions[6] $\tilde{q}_{jctk}(p_{ictk}, w_{ict}, v_{ictk}, \gamma)$ for each $j \in c$ and each $k = 1, ..., K$, which are written as follows: $\tilde{q}_{ctk}(p_{ictk}, w_{ict}, v_{ictk}, \gamma) \equiv [\tilde{q}_{jctk}(p_{ictk}, w_{ict}, v_{ictk}, \gamma)]_{j\in c}$ and $\tilde{q}_{ct}(p_{ict}, w_{ict}, v_{ict}, \gamma) \equiv [\tilde{q}_{ctk}(p_{ictk}, w_{ict}, v_{ictk}, \gamma)]_{k=1}^{K}$.

Substituting these into (2) we have the first portion of (1):

$$a_{ict}(v_{ict}, \gamma) = \left[\sum\nolimits_{k=1}^{K} u_{ick}(\tilde{q}_{ctk}(p_{ickt}, w_{ict}, v_{ickt}, \gamma), v_{ictk}, \gamma) + (y_i - p_{ict}'\tilde{q}_{ct}(p_{ict}, w_{ict}, v_{ict}, \gamma))\right],$$

[6] We assume that standard regularity (coherency) conditions are satisfied by γ_2 and γ_3 (see Amemiya (1974)).

where we make deterministic observable variables (y_i, p_{ict}, w_{ict}) implicit in $a_{ict}()$ because that can be captured by subscripts of a.

Overall, consumer i visiting store combination c at time t obtains utility (again, making observable variables implicit)

$$U_{ict}(v_{ict}, \varepsilon_{ict}) = \alpha a_{ict}(v_{ict}, \gamma) + \beta' x_{ict} + \varepsilon_{ict},$$

The scaling term on ε_{ict} is set to unity and parameter α determines the trade off between a_{ict} and the other influences on store choice given by x_{ict}. ε_{ict} is distributed Type-1 Extreme Value so that conditional on a particular draw v_{ict} the parameters $\theta_D = (\alpha, \beta, \gamma)$ imply the probability of consumer i making the choice c at time t is given by:

$$\begin{aligned} \mathcal{P}_{ict}(\theta_D | v_{it}) &= \Pr(U_{ict}(v_{ict}, \varepsilon_{ict}) > U_{ic't}(v_{ic't}, \varepsilon_{ic't}) \ \forall c' \in \mathcal{C}_i \mid v_{it}) \\ &= \frac{\exp\left(\alpha a_{ict}(v_{ict}, \gamma) - \beta' x_{ct}\right)}{\sum_{c' \in \mathcal{C}_i} \exp\left(\alpha a_{ic't}(v_{ic't}, \gamma) - \beta' x_{c't}\right)} \end{aligned}$$

The unconditional probability $\mathcal{P}_{ict}(\theta_D)$ is given by integrating over the distribution $F(v_{it})$:

$$\mathcal{P}_{ict}(\theta_D) = \Pr\left(U_{ict}(v_{ict}, \varepsilon_{ict}) > U_{ic't}(v_{ict}, \varepsilon_{ic't}) \ \forall c'\right) \tag{10}$$

$$= \int \frac{\exp\left(\alpha a_{ict}(v_{ict}, \gamma) - \beta' x_{ict}\right)}{\sum_{c' \in \mathcal{C}_i} \exp\left(\alpha a_{ic't}(v_{ic't}, \gamma) - \beta' x_{ic't}\right)} dF\left(v_{it}\right)$$

Consumer i's (unconditional) expected demand in j for k is given by summing over shopping choices c in the set $c_i(j) \subset \mathcal{C}_i$ that contain the store j

$$E\left(q_{ijtk} \mid \theta_D\right) = \sum\nolimits_{c \in c_i(j)} \left\{ \int \left[\tilde{q}_{ijctk}(p_{ict}, v_{ictk}) \mathcal{P}_{ict}(\theta_D \mid v_{it})\right] dF\left(v_{it}\right) \right\}.$$

3.2 Dynamic Model of Retail Oligopoly

3.2.1 Environment

N firms and an entry regulator are playing a dynamic game in a spatial environment. There are L locations in the game and consumers are non-uniformly distributed across locations. Firms sequentially open a new store or close an existing store in one of the L locations. The entry regulator can regulate new store openings. There is spatial competition in the sense that firms compete for consumers not only within the boundary of the location, but also across locations. Firms are forward looking and maximize intertemporal profit in an infinite time horizon, accounting for current and future stores configuration. Time is continuous.

For a firm to take any action (both opening and closing), an "opportunity" that is specific to each action must arrive. For example, for a firm to open a new store at a particular location, a parcel of land must be available for the firm in that location, that is of a suitable size and location for a supermarket; such as when a factory or school closes down and becomes available for other uses. Similarly, the opportunity for closing must arrive at a firm in a specific location so that the firm decides if to close a given store or not (this reflects the idea that firms should find a counterpart who buys the site, perhaps the store building too). Following Arcidiacono, Bayer, Blevins, and Ellickson (2010), we assume that these opportunities arrive randomly and follow a Poisson process with exogenous rates.

There are two types of store formats; big and small. We assume that there are two types of firms; B and S, where B can potentially open either a big store or a small store, while S is allowed to open a small store only (This is motivated by the fact that some firms such as ASDA are observed to have the ability to stock a large store, but other firms such as Discounters operate a policy of a narrow product range in all stores). We assume that there are two types of land, small or big. The store size of a firm is automatically determined by the size of the land that the firm acquires. Let N^B and N^S be the numbers of firms of type B and firms of type S in this game, respectively. Note that $N^B + N^S = N$. To open a store, firms are required (by law) to costly file an application to the regulator (social planner). Only if the application is approved, can the applying firm open a store. Thus, when a firm receives an opportunity to open a store, it decides whether to file an application or not. Then, if the application is submitted, the regulator either approves or rejects the application based on the social welfare function that we define later. Once the application is approved, the firm will open a store. On the other hand, stores can be closed without the regulator's approval if a firm has an opportunity to do so.

For expositional simplicity, we introduce the following notation. A triple of variables, denoted by x, fully characterize a parcel of land for store opening and closing:

$$x = (l, f, tc),$$

where l represents location, $f \in \{S, B\}$ denotes the size of the land, and tc is a binary variable that equals one if the land is in town center and zero otherwise. Thus, for example, if the land is for a big store and located in the town center of location l_0, then $x = (l_0, B, 1)$. X denotes the set of x. We sometimes speak of "store x" to denote an opportunity to open (or close) a store at the site characterized by x.

Let ω denote a set of all payoff-relevant and information-relevant state variables in the game. That is, ω completely characterizes the current state of the game.[7] Let Ω be the set of all possible states.

Arrival Process We assume that a land opportunity arrives at rate of $\lambda_{ix}(\omega)$ and an opportunity of closing a store arrives at rate of $\lambda_{ix}^c(\omega)$. These rates potentially differ across firms, types of land, locations, and states. Since arrivals of opportunities are not observed by the researcher, we will state conditions for identification of these rates below. For later analysis, we define $\Lambda_\omega = \sum_i \sum_x (\lambda_{ix}(\omega) + \lambda_{ix}^c(\omega))$.

In our analysis, we use several convenient properties of the Poisson process. First, the waiting time until each event follows the exponential distribution with the arrival rate of the corresponding event. Second, the waiting time until any event follows the exponential distribution with rate Λ_ω. Third, conditional on arrival of any event, the probability that the event is a particular event is given by λ/Λ_ω, where λ is the arrival rate of the particular event.

Firms The instantaneous profit that firm i's stores earn out of sales in location l depends on the payoff relevant variables in both location l and l's neighboring locations. Payoff relevant variables include store configurations and location specific demand conditions. For example, if there are more rival stores in the same location, everything else being equal, a firm's revenue from the location will be smaller (business stealing). If there are more stores owned by the same firm, per-store revenues of the firm will be smaller (cannibalization). On the other hand, if there is more demand in the location, with store configurations being equal, a firm's revenue will be higher. Furthermore, if there are more stores in neighboring locations, everything else being equal, a firm's revenue will be less because some consumers may choose to shop at stores in neighboring locations. Thus, we allow business stealing and cannibalization effects to operate not only within the boundary of the location, but also across locations. This is the source of spatial competition.

[7] At this point, we do not specify if the state space is discrete or continuous. In application, we discretize the state space.

Let $\pi_{il}(\omega)$ denote the instantaneous payoff that firm i earns from location l when the current state is given by ω[8]. We also define $\Pi_i(\omega) = \sum_l \pi_{il}(\omega)$. Assuming that r is a common discount factor, the expected net present value that a firm maximizes is written as

$$\mathbb{E}_{\omega,T}\left[\int_0^\infty \Pi_i(\omega_t) e^{-rt} dt + \sum_{n=1}^\infty e^{-rT_n} \Psi_i(T_n)\right],$$

where T_n is the random time of the n-th event and $\Psi_i(T_n)$ is firm i's one-shot payoff associated with the event that takes place at T_n. We will specify the ingredients in $\Psi_i(T_n)$ later.

There are two types of decisions (actions) for firms. When an opportunity for store opening arrives to a firm, the firm decides whether to send an application or not. We assume that if the application is approved, the firm will open a new store immediately. There is a one-shot payoff associated with each decision. If the firm decides to apply, it receives a one-shot payoff of $\psi_{ix}(\omega) + \epsilon_{i1}$, where $\psi_{ix}(\omega)$ is a commonly observed deterministic payoff and ϵ_{i1} is a random shock. If the firm decides not to apply, it receives a random shock ϵ_{i0}. We assume that ϵ_{i1} and ϵ_{i0} follow the iid type 1 extreme value distribution and are privately observed by firm i upon arrival of the opportunity.

$\psi_{ix}(\omega)$ is given by

$$\psi_{ix}(\omega) = -C - P_{ix}(\omega)\kappa_x,$$

where C is the deterministic application cost, $P_{ix}(\omega)$ is i's belief about the probability that the application is accepted, and κ_x is a deterministic start-up cost for the new store, including building a store, establishing a distribution channel, and so on. Note that κ_x is multiplied by $P_{ix}(\omega)$ because κ_x is incurred only after the application is approved.

Thus, for store x, firm i's decision to send an application is

$$\chi_{ix} : \Omega \times \mathcal{E} \to \mathbb{R}_+,$$

where $\mathcal{E}$ is the support of $(\epsilon_{i1}, \epsilon_{i0})$. We use $\delta_{ix}(\omega)$ to denote the ex-ante probability (i.e., before $(\epsilon_{i1}, \epsilon_{i0})$ realize) that firm i sends an application when an opportunity to do so arrives.

The second type of firms' decision is a closing decision. When an opportunity to close an existing store arrives, the firm who owns the store decides whether to close it or not. If the firm decides to close an existing store, it receives a one-shot additive payoff of $\eta_x + \varepsilon_{i1}$, where η_x is a deterministic scrap value that depends on x and ε_{i1} is a privately observed shock. If the firm decides not to close the store, it receives a private shock ε_{i0}. We assume that ε_{i1} and ε_{i0} follow the iid type 1 extreme value distribution and are observed upon arrival of the opportunity. Thus, firm i's decision on store closing when a store in x is allowed to be closed and when the state is given by ω is written as

$$\chi_{ix}^c : \Omega \times \widetilde{\mathcal{E}} \to \{0, 1\}$$

where $\widetilde{\mathcal{E}}$ is the support of $(\varepsilon_{i1}, \varepsilon_{i0})$. We use $\mu_{ix}(\omega)$ to denote the ex-ante probability (i.e., before $(\varepsilon_{i1}, \varepsilon_{i0})$ realize) that firm i closes the store.

Finally, we summarize firm i's one-shot payoff associated with each event as follows:

[8] This is the total profit earned from all the stores owned by firm i in location l, not a per-store profit in the location.

$$\Psi_i(T_n) = \begin{cases} -C - \kappa_x + \epsilon_i & \text{apply and approved} \\ -C + \epsilon_i & \text{apply but rejected} \\ \epsilon_{i0} & \text{do not apply} \\ \eta_x + \varepsilon_{i1} & \text{close a store} \\ \varepsilon_{i0} & \text{decide not to close a store} \\ 0 & \text{other firm makes a decision} \end{cases}$$

where T_n specifies which of the six events takes place.

Regulator In the current paper, the regulator is one of the players in the game. The regulator approves firms' applications according to its objective function. Let $CS(\omega)$ be the total consumer surplus when the state is ω. $PS_{TC}(\omega)$ and $PS_{OTC}(\omega)$ denote the producer surplus earned in town center and out of town center, respectively. Note that producer surplus include one-shot payoffs Ψ_i associated with store openings and closings. The general form of the regulator's objective function is

$$\mathbb{E}_{\omega,T}\left[\alpha_1 \int_0^\infty CS(\omega_t) e^{-rt} dt + \alpha_2 \int_0^\infty PS_{TC}(\omega_t) e^{-rt} dt + \alpha_3 \int_0^\infty PS_{OTC}(\omega_t) e^{-rt} dt + \sum_{n=1}^{\infty} e^{-rT_n} \Psi_R(T_n)\right] \tag{11}$$

where $\Psi_R(T_n)$ is the regulator's instantaneous payoff of making decisions, which is given by

$$\Psi_R(T_n) = \begin{cases} \epsilon_{R1} & \text{accept} \\ \epsilon_{R0} & \text{reject} \end{cases}.$$

We assume that ϵ_{R1} and ϵ_{R0} follow the iid type 1 extreme value distribution and that they are privately observed by the regulator upon application. The assumption that the regulator privately observes $(\epsilon_{R1}, \epsilon_{R0})$ is justified by the observation in the data that a significant proportion of applications was rejected. With a strictly positive application cost, firms would not apply if there are not stochastic elements in the approval/rejection decision and if they know that their application would be rejected.

It is important to emphasize that the regulator may put different weights on each competitor's profit by the type of the competitor, instead of simply summing up their profits. For example, the regulator puts more emphasis on the change in profits of firms of type S more than that of firms of type B in order to protect small stores in town centres. As we discuss later, this is an empirical question and we estimate $(\alpha_1, \alpha_2, \alpha_3)$. Note that if we set $\alpha_1 = \alpha_2 = \alpha_3 = 1$, the objective function is simply the expected discounted sum of social welfare. We define a mapping φ_{ix} from the state and shocks to a binary variable:

$$\varphi_{ix} : \Omega \times \widetilde{\mathcal{E}} \to \{0,1\}$$

to denote the regulator's decision when firm i applies for opening store x. We use $P_{ix}(\omega)$ to denote the ex-ante probability (i.e., before $(\epsilon_{R1}, \epsilon_{R0})$ realize) that the regulator accepts firm i's application for opening store x.

Expectation Remember that $\delta_{ix}(\omega)$ denotes the ex-ante probability that firm i applies for opening store x, that $\mu_{ix}(\omega)$ is the ex-ante probability that firm i closes store x, and that $P_{ix}(\omega)$ is the ex-ante probability that the regulator approves firm i's application for x. We use σ_i to denote a collection of players' choice probabilities including the regulator:

$$\sigma_i = \begin{cases} \{\delta_{ix}(\omega), \mu_{ix}(\omega)\}_{x \in X, \omega \in \Omega} & \text{if } i = 1, ..., N \\ \{P_{ix}(\omega)\}_{x \in X, \omega \in \Omega} & \text{if } i = R \end{cases},$$

where R stands for the regulator. Let $\boldsymbol{\sigma} = (\sigma_1, ..., \sigma_N, \sigma_R)$.

3.2.2 Value Function and Best Responses

We restrict ourselves to a stationary environment. Thus, we drop the time subscript in what follows.

Firms Payoffs stay constant as long as the state is unchanged. The ex-ante (integrated) value function is then:

$$
\begin{aligned}
V_i(\omega) &= \mathbb{E}_\tau \Bigg[\int_0^\tau e^{-\rho t} \Pi_i(\omega)\, dt \\
&\quad + e^{-\rho\tau} \Bigg\{ \sum_{j=1}^N \sum_x \frac{\lambda_{jx}(\omega)}{\Lambda_\omega} \Big[\delta_{jx}(\omega) \left[P_{jx}(\omega) V_i(\omega + [j,x]) + (1 - P_{jx}(\omega)) V_i(\omega) \right] \\
&\quad + (1 - \delta_{jx}(\omega)) V_i(\omega) + \mathbf{1}_{(j=i)} \left(\delta_{jx}(\omega) \left[-C - P_{jx}(\omega) \kappa_x + \epsilon_{j1}^\delta \right] + (1 - \delta_{jx}(\omega)) \epsilon_{j0}^\delta \right) \Big] \\
&\quad + \sum_{j=1}^N \sum_x \frac{\lambda_{jx}^c(\omega)}{\Lambda_\omega} \Big[\mu_{jx}(\omega) V_i(\omega - [j,x]) + \left(1 - \mu_{jx}(\omega)\right) V_i(\omega) \\
&\quad + \mathbf{1}_{(j=i)} \left(\mu_{jx}(\omega) \left[\eta_x + \varepsilon_{j1}^\mu \right] + \left(1 - \mu_{jx}(\omega)\right) \varepsilon_{j0}^\mu \right) \Big] \Bigg\} \Bigg]
\end{aligned}
$$

where $\omega + [j, x]$ ($\omega - [j, x]$) represents the state which is reached when firm j opens (closes) store x, and ϵ_{i1}^δ (ε_{i1}^μ) is the expected value of the random shock conditional on firm i applying for opening store x (firm i closing the store). ϵ_{i0}^δ and ε_{i0}^μ are defined analogously.

The second and third lines describe the possible change in value due to the arrival of a parcel of land. Specifically, with probability $\frac{\lambda_{jx}(\omega)}{\Lambda_\omega}$ a site becomes available to firm j. With probability $1 - \delta_{jx}(\omega)$, the firm decides not to apply and there is no change in states. With probability $\delta_{jx}(\omega)$, firm j submits an application. Upon submitting an application, the application is approved with probability $P_{jx}(\omega)$ and a new store is opened, and with probability $1 - P_{jx}(\omega)$, the application is rejected. We also add the expected value of the one-shot payoff if firm i is called upon to move. The forth and fifth lines describe the change in value if the possibility of closing a store arises.

By the distributional assumption on ϵ, the probability that firm i applies can be written as

$$
\widetilde{\delta}_{ix}(\omega; \boldsymbol{\sigma}) = \frac{\exp\left(P_{ix}(\omega)\left[V_i(\omega + [i,x]) - V_i(\omega) - \kappa_x\right] - C\right)}{1 + \exp\left(P_{ix}(\omega)\left[V_i(\omega + [i,x]) - V_i(\omega) - \kappa_x\right] - C\right)}. \tag{12}
$$

Similarly if there is a closing, under the assumption that $(\varepsilon_{i1}, \varepsilon_{i0})$ follow the iid type 1 extreme value distribution, the best response probabilities are given by

$$
\widetilde{\mu}_{ix}(\omega; \boldsymbol{\sigma}) = \frac{\exp\left(V_i(\omega - [i,x]) + \eta_x\right)}{\exp\left(V_i(\omega - [i,x]) + \eta_x\right) + \exp\left(V_i(\omega)\right)}. \tag{13}
$$

Regulator Even though we define the regulator's objective function in a general way in equation (11), we assume that the regulator is a "static regulator" in the sense that it does not consider any further state change in the future after the change in state associated with the current application in question. That is, the static regulator does not consider other firms' future responses resulting from his approval decision. In addition, the static regulator does not take any future instantaneous payoff into account. He ignores instantaneous payoffs that he may receive associated with his approval decisions in the future. The static regulator is close to the actual Local Government Authority, as they do not base their decisions on dynamic aspects.[9] In our

[9] As we discussed in Section 2, the criteria in the central planning guidelines almost exclusively talk about direct change in consumer surplus and producer surplus in town center, not about future developments of the market.

counter-factual analysis, however, we explicitly consider a "dynamic regulator" that maximizes the objective function in (11) with $\alpha_1 = \alpha_2 = \alpha_3 = 1$ to discuss an optimal regulation.

Under the assumption of static regulator,

$$
\begin{aligned}
V_R(\omega) &= \mathbb{E}_\omega \left[\alpha_1 \int_0^\infty CS(\omega_t) e^{-rt} dt + \alpha_2 \int_0^\infty PS_{TC}(\omega_t) e^{-rt} dt + \alpha_3 \int_0^\infty PS_{OTC}(\omega_t) e^{-rt} dt \right] \\
&= \alpha_1 \int_0^\infty CS(\omega) e^{-rt} dt + \alpha_2 \int_0^\infty PS_{TC}(\omega) e^{-rt} dt + \alpha_3 \int_0^\infty PS_{OTC}(\omega) e^{-rt} dt \\
&= K \left[\alpha_1 CS(\omega) + \alpha_2 PS_{TC}(\omega) + \alpha_3 PS_{OTC}(\omega) \right],
\end{aligned}
$$

where we normalize $K = 1$. Since it considers the current instantaneous payoff by assumption, the regulator approves an application if and only if

$$\alpha_1 CS(\omega + [i,x]) + \alpha_2 PS_{TC}(\omega + [i,x]) + \alpha_3 PS_{OTC}(\omega + [i,x]) + \epsilon_{R1} > \alpha_1 CS(\omega) + \alpha_2 PS_{TC}(\omega) + \alpha_3 PS_{OTC}(\omega) + \epsilon_{R0}.$$

Thus, the best response approval probability is given by

$$
\begin{aligned}
\widetilde{P}_{ix}(\omega, \boldsymbol{\sigma}) &= \Pr\left(\alpha_1 \Delta CS(\omega) + \alpha_2 \Delta PS_{TC}(\omega) + \alpha_3 \Delta PS_{OTC}(\omega) + \epsilon_{R1} - \epsilon_{R0} > 0 \right) \\
&= \frac{\exp\left(V_R(\omega + [i,x])\right)}{\exp\left(V_R(\omega + [i,x])\right) + \exp\left(V_R(\omega)\right)},
\end{aligned}
\tag{14}
$$

where $\Delta CS(\omega) = CS(\omega + [i,x]) - CS(\omega)$, and $\Delta PS_{TC}(\omega)$ and $\Delta PS_{OTC}(\omega)$ are defined analogously. In the estimation stage, we estimate $(\alpha_1, \alpha_2, \alpha_3)$.[10]

3.2.3 Equilibrium

We consider Markov Perfect Equilibria. Let $V_i^{\sigma_i, \sigma_{-i}}(\omega)$ be the value of firm i in state ω if he follows strategy σ_i and all other players follow strategy σ_{-i}. A Markov Perfect Equilibrium is a Markovian strategy profile $\boldsymbol{\sigma}^*$ such that, for all $i \in \{1, 2, ..., N, R\}$,

$$V_i^{\sigma_i^*, \sigma_{-i}^*}(\omega) \geq V_i^{\sigma_i', \sigma_{-i}^*}(\omega)$$

for all ω and σ_i'. We will focus on symmetric equilibria. The standard argument applies for the existence of equilibria; see Doraszelski and Satterthwaite (2010) and Doraszelski and Judd (2010).

3.2.4 Solving the Model

Since there are many locations in the U.K., the state space (store configurations and demand conditions for all the locations) is extremely large. To solve the model, we impose a series of simplifying assumptions. First, we assume that the firm's decision is decentralized:

Assumption 1 Each firm has a local manager in each location. The manager decides whether to take an action, when an opportunity arrives in his own location.

If we assume that each local manager maximizes its profit earned in that location only, we would not be able to capture cannibalization effects, which are one of the most important elements of the industry. Thus, we assume that business stealing effects and cannibalization effects exist only within a certain distance and that the local manager takes into account the profits earned by the same firm within that distance. To state the next assumption formally, we define $R(l)$ to denote the set of l's neighboring locations, whose payoff relevant variables affect firm's profits in location l. Use m for a typical element of the set; i.e., $m \in R(l)$. Then, we have the following assumption:

[10] This is achieved under an appropriate normalization. Since only a relative size matters, we normalize $\alpha_1 + \alpha_2 + \alpha_3 = 1$.

Assumption 2 The local manager of firm i in location l takes into account the profits of other stores owned by the same firm in locations in $R(l)$, as well as in location l. Let $\Pi_{il}(\omega) = \pi_{il}(\omega) + \sum_{m \in R(l)} \pi_{im}(\omega)$. The local manager of firm i in location l maximizes

$$\mathbb{E}_{\omega,T}\left[\int_0^\infty \Pi_{il}(\omega) e^{-rt} dt + \sum_{n=1}^{\infty} e^{-rT_n} \Psi_{il}(T_n)\right], \tag{15}$$

where the expectation operator is in terms of the evolution of the state and arrivals of opportunities, and Ψ_{il} is defined as before.

Aguirregabiria and Ho (2012) and Sweeting (2011) impose a similar assumption as Assumption 1 to break the dimensionality problem (though in a discrete time setting unlike here). Assumption 2 can be fairly general since we have not yet specified $R(l)$. We believe that how far from location l this set should extend is an empirical question.

Solving the model is still a daunting task, as the set of locations in $R(l)$ could be very large and so is the state space that each local manager faces. Thus, we discuss how to aggregate the information regarding other payoff relevant variables and market structure in the neighboring locations. For any given l, we have many locations $m \in R(l)$ and stores in these locations could all affect the profit of a store in l. Let N_{il}^{own} be the number of stores that firm i has in l. Let $N_{il}^{B,rival}$ ($N_{il}^{S,rival}$) denote the number of big (small) stores in l owned by i's rivals. Let X_l be the K dimensional vector of variables that characterize profitability of location l (e.g., number of households, demographics, etc). For the sake of argument, let

$$\begin{aligned}
N_{iR(l)}^{own} &= \sum\nolimits_{m \in R(l)} N_{im}^{own} \\
N_{iR(l)}^{B,rival} &= \sum\nolimits_{m \in R(l)} N_{im}^{B,rival} \\
N_{iR(l)}^{S,rival} &= \sum\nolimits_{m \in R(l)} N_{im}^{S,rival}
\end{aligned}$$

to denote the total numbers of own stores, big rival stores, and small rival stores, respectively, in the neighboring locations. Also let

$$\underset{(K\times 1)}{X_{R(l)}} = \sum\nolimits_{m \in R(l)} X_m,$$

where we sum up X_l element by element over locations in $R(l)$. If we impose symmetry and anonymity in terms of locations in $R(l)$, then $(N_{iR(l)}^{own}, N_{iR(l)}^{B,rival}, N_{iR(l)}^{S,rival}, X_{R(l)})$ are sufficient statistic for $\{N_{im}^{own}, N_{im}^{B,rival}, N_{im}^{S,rival}, X_m\}_{m \in R(l)}$. However, the state space still consists of

$$\left(N_{il}^{own}, N_{il}^{B,rival}, N_{il}^{S,rival}, X_l, N_{iR(l)}^{own}, N_{iR(l)}^{B,rival}, N_{iR(l)}^{S,rival}, X_{R(l)}\right)$$

which is a $(2K+6)$ dimensional vector. This is an object with a large number of dimensions.

Therefore, to further reduce the size of the state space, we construct the following variables using the demand model:

$$\begin{aligned}
\Delta\Pi_{il}(\omega) &= \Pi_{il}(\omega + [i,x]) - \Pi_{il}(\omega) \\
\overline{\Delta\Pi_{il}^{rival}}(\omega) &= \sum_{j \neq i} \left[\Pi_{jl}(\omega + [i,x]) - \Pi_{jl}(\omega)\right]/(N-1) \\
\overline{\Delta\Pi_{il}^{own}}(\omega) &= \sum_{j \neq i} \left[\Pi_{il}(\omega + [j,x]) - \Pi_{il}(\omega)\right]/(N-1)
\end{aligned}$$

In words, $\Delta\Pi_{il}(\omega)$ measures the change in the total profit earned by all own stores in l and $m \in R(l)$ if a new small store of firm i opens in location l. In a similar way, $\overline{\Delta\Pi_{il}^{rival}}(\omega)$ is defined to measure the change in the total profit earned by rival firms (all $j \neq i$) in l and $m \in R(l)$ if a new small store of firm i opens in location l. Finally, $\overline{\Delta\Pi_{il}^{own}}(\omega)$ measures the average change in profit induced by rival firms in $R(l)$ on the total profit of firm i. Using these variables, derived from the demand model, we impose the following assumption.

Assumption 3 $\left(\Delta\Pi_{il}(\omega), \overline{\Delta\Pi_{il}^{rival}}(\omega), \overline{\Delta\Pi_{il}^{own}}(\omega)\right) \in \mathbb{R}^3$ is the sufficient statistic for X_l and $\{N_{im}^{own}, N_{im}^{B,rival}, N_{im}^{S,rival}, X_m\}_{m\in R(l)}$.

This assumption says that the change in own and rival profits can effectively summarize the market structure in the neighboring locations by aggregating incentives and disincentives of opening a new store in a given location. For example, if the firm has a large number of own stores in $R(l)$, this would be reflected in a large (potentially negative) value of $\Delta\Pi_{il}(\omega)$. Similarly, the importance of the local market structure and local firm network would be reflected in $\overline{\Delta\Pi_{il}^{rival}}(\omega)$ and $\overline{\Delta\Pi_{il}^{own}}(\omega)$.

Thus, the state space we use is

$$\widetilde{\omega} = \left(N_{il}^{own}, N_{il}^{B,rival}, N_{il}^{S,rival}, \Delta\Pi_{il}(\omega), \overline{\Delta\Pi_{il}^{rival}}(\omega), \overline{\Delta\Pi_{il}^{own}}(\omega), l\right)$$

where we keep location index l as the sixth variable in the state to allow λ to be different across locations in a way that cannot be captured by $\left(\Delta\Pi_{il}(\omega), \overline{\Delta\Pi_{il}^{rival}}(\omega), \overline{\Delta\Pi_{il}^{own}}(\omega)\right)$. Remember that factual and counterfactual profits to construct $\left(\Delta\Pi_{il}(\omega), \overline{\Delta\Pi_{il}^{rival}}(\omega), \overline{\Delta\Pi_{il}^{own}}(\omega)\right)$ are calculated from the demand estimates.

4 Estimation

4.1 Demand Side

Parameters $\theta_D = (\alpha, \beta, \gamma)$ are estimated in two steps. In the first stage, γ are obtained using moments based on first order conditions for continuous choice of quantities q_{ijkt}. We use instrumental variables as suggested in Dubin and McFadden (1984) to control for the fact that the regressors include (characteristics of) stores that are chosen by i. In the second step we estimate remaining parameters (α, β) using moments based on the model's predictions for discrete choice $c = (j, j')$, using simulated method of moments (McFadden (1989)). The second step uses estimates of (γ, v_{itk}) that come from the first step.

First step. The first step uses moments based on the error terms in the portion $a()$ of the consumer's utility. We assume at the true parameters γ^0 that the following conditions hold for each k:

$$E(v_{itk}(\gamma^0) \mid z_{1itk}) = 0 \quad (16)$$

$$E(\epsilon_{itk}(\gamma^0) \mid z_{2itk}) = 0 \quad (17)$$

v_{itk} and ϵ_{itk} are defined in (9) and $[z_{1itk}, z_{2itk}]$ are exogenous instruments.

Let $c_k \subseteq c$ denote the set of stores in c at which the consumer has positive demand for category k and let $n(c_k)$ denote the number of such stores. Empirically our time period length and our category definitions are such that $n(c) \geq n(c_k) \geq 1$ for all but a negligible number of observations (i.e. consumers always buy positive quantities of each of the categories in at least one store but do not always buy positive quantities from both stores when c has two stores).

The moment condition (16) uses store averages, for the store(s) in c_k, as follows

$$\bar{q}_{itk} = \frac{1}{n(c_k)} \sum\nolimits_{j\in c_k} q_{ijtk} \qquad \bar{w}_{itk} = \frac{1}{n(c_k)} \sum\nolimits_{j\in c_k} w_{ijtk} \qquad \bar{p}_{itk} = \frac{1}{n(c_k)} \sum\nolimits_{j\in c_k} p_{ijtk}.$$

Substituting these, (8), and (9), into (6) we obtain the expression:

$$v_{itk}(\gamma) = \bar{q}_{itk}\left(1 + \gamma_{3k} 1_{[n(c_k)=2]}\right) - (\gamma_{1k}' \bar{w}_{itk} - \gamma_{2k}\bar{p}_{itk})$$

where ϵ_{itk} disappears (by its symmetry property).

The vector $\bar{w}_{ict}$ contains some exogenous variables that do not need separate instruments—a constant, household characteristics (number of adults and children), and time dummies (year and quarter)—and to identify the parameters on these variables we use the variable as its own instrument. The vector $\bar{w}_{ict}$ also contains some variables that do need instruments—store characteristics (floorspace and firm dummies), price, and the two-store dummy $1_{[n(c_k)=2]}$. These are endogenous in the sense that they are determined by the consumer's shopping choice c so it is possible that v_{itk} is not independent of these; for example a consumer with a large positive draw for v_{itk} may prefer to choose a large store, or a store that has a low price for category k. We follow Dubin and McFadden (1984) and use instruments based on the expected values of the variables $\left(\bar{w}_{itk}, \bar{p}_{itk}, 1_{[n(c_k)=2]}\right)$. To construct these expectations we use probabilities estimated for each c_k using a flexible model of the discrete choice of c_k from the set $\mathcal{C}_i$. We write these probabilities $\hat{\rho}_{ick}(w_i, x_i)$—where w_i and x_i are the observables that enter the utility function (1) through $a(.)$ and directly, respectively—to illustrate that we exploit a kind of "exclusion restriction" implicit in the model: the variables x_i that enter directly determine the shopping choice c_k but have no (direct) effect on the quantities q_{ictk} that are chosen. The variables that enter U_{ict} directly, most notably distance, generate exogenous variation in the characteristics $\left(\bar{w}_{itk}, \bar{p}_{itk}, 1_{[n(c_k)=2]}\right)$ of the shopping choice c. In other words we use the variation between consumers in store choice sets $\mathcal{C}_i$.

The moment condition (17) uses between-store differences for stores in c_k when $n(c_k) = 2$:

$$\Delta q_{itk} = q_{ijtk} - q_{ij'tk}. \qquad \Delta w_{itk} = w_{ijtk} - w_{ij'tk} \qquad \Delta p_{itk} = p_{ijtk} - p_{ij'tk}$$

Substituting (8), and (9), into (6) and taking differences, we obtain:

$$\epsilon_{itk}(\gamma) = \frac{1}{2}\left[\gamma_{1k}' \Delta w_{itk} - \Delta q_{itk}(1 - \gamma_{3k}) - \gamma_{2k}\Delta p_{ijkt}\right].$$

Again the differences depend on the choice c that is chosen by the consumer. We use as instruments z_{2itk} the expected values of the variables $(\Delta w_{ict}, \Delta p_{ijkt})$ using the same probabilities $\hat{\rho}_{ick}(w_i, x_i)$ used in the construction of z_{1itk}. (To implement the differencing, each store is allocated a random number and differencing is such that the larger of these is the first store in the difference). Note that the difference equation is only estimated on the subset of observations for which two stop shopping is observed (i.e. for which $n(c_k) = 2$). To ensure that there is no selection bias that arises from this, the following condition must hold $E(\epsilon_{itk}|z_{2itk}) = E(\epsilon_{itk}|z_{2itk}, 1_{[n(c_k)=2]}) = 0$. This implies that ϵ_{itk} is independent of $1_{[n(c)=2]}$ conditional on z_{2ikt}, which we consider reasonable given that the errors here are in differences rather than levels and the differencing is based on random ordering. The moment conditions in (17) complement those in (16) by adding information on how consumers allocate demand between stores in c_k when $n(c_k) = 2$.

Second step. With $(\hat{\gamma}, \hat{v}_{it}) = (\hat{\gamma}_k, \hat{v}_{itk})_{k=1}^K$ for each k in hand from the first step we now estimate remaining parameters using moments based on the discrete choice c. We use the residuals:

$$d_{ict} - \mathcal{P}_{ict}(\alpha, \beta, \hat{\gamma}|\hat{v}_{it}) \text{ for all } i \text{ and } t, \text{ and all } c \in \mathcal{C}_{it}$$

where

$$d_{ict} = \begin{cases} 1 & \text{if consumer } i \text{ chooses } c \text{ at time } t \\ 0 & \text{otherwise.} \end{cases}$$

and the discrete probability $\mathcal{P}_{ict}()$ is written to make explicit the dependence on the estimates $(\hat{\gamma}, \hat{v}_{it})$ from the first step. Note that while the *category*-specific unobserved taste terms $\hat{v}_{it}$ are estimated in the first step, the observed store-split residuals $\hat{\epsilon}_{it}$ are specific to the chosen shopping choice c and not estimated generally for all stores. We therefore treat the ϵ_{jictk} terms as unknown in the second step). The expression $\mathcal{P}_{ict}(\alpha, \beta, \hat{\gamma}|\hat{v}_{it})$ is as defined in (10), except that we now condition on $\hat{v}_{it}$, so that we have

$$\mathcal{P}_{ict}(\alpha, \beta, \hat{\gamma}|\hat{v}_{it}) = \int \frac{\exp\left(\alpha a_{ict}(\hat{v}_{ict}, \hat{\gamma}, \epsilon_{ict}) - \beta' x_{ict}\right)}{\sum_{c' \in \mathcal{C}_i} \exp\left(\alpha a_{ic't}(\hat{v}_{ict}, \hat{\gamma}, \epsilon_{ict}) - \beta' x_{ic't}\right)} dF\left(\epsilon_{it}\right).$$

We assume that at true parameters $\left(\alpha^0, \beta^0\right)$ the following population moments hold:

$$E\left[d_{ict} - \mathcal{P}_{ict}(\alpha^0, \beta^0, \hat{\gamma}|\hat{v}_{it})\right| z_{3ict}\right] = 0$$

where z_{3ict} is a vector of instruments based on the variables x_{ict}. We simulate the choice probability using R draws to yield $\mathcal{P}^R_{ict}(\theta_D)$ which, for any $R \geq 1$, by construction has the property of being an unbiased estimate. Thus we have sample moment conditions as follows:

$$\sum_i \sum_t \sum_{c \in \mathcal{C}_{it}} \left[d_{ict} - \mathcal{P}^R_{ict}(\theta_D)\right] z_{3ict}. \tag{18}$$

To control for unobserved quality that may be correlated with the price variable, we include firm dummies in both the variables w entering the continuous demand the variables x entering discrete choices; we also include time dummies in w.

4.2 Supply Side

We assume that the length of the time period goes to infinity for the asymptotics of our estimator, while the number of firms N and the number of locations L are fixed. Since all locations are connected indirectly, we can say that one big game was played in the whole U.K. Although there may be multiple equilibria in the model, the Markovian assumption implies that data on a single time series has been generated by one equilibrium (for a discussion, see Pesendorfer and Schmidt-Dengler (2008)). Thus, we can employ a two-step method to estimate parameters of the model.[11]

4.2.1 First Stage

We take estimates of $\pi_{il}(\omega)$ from the demand side in the previous subsection as primitives in the estimation of the dynamic game. Our goal in the first stage is to consistently estimate $(\delta_{ix}(\omega), \mu_{ix}(\omega), P_{ix}(\omega), \lambda_{ix}(\omega), \lambda^c_{ix}(\omega))$ from data on store opening/closing, approval decisions, and time duration between any observed events. Since we do not observe cases in which a firm has an opportunity to take an action but decides not to do so, arrival rates and choice probabilities are not separately identified in the first stage. That is, only the products $\lambda_{ix}(\omega) \times \delta_{ix}(\omega)$ and $\lambda^c_{ix}(\omega) \times \mu_{ix}(\omega)$ are identified for each i, x, and ω. For a discussion, see Arcidiacono et. al (2010). Therefore, in the first stage, we consistently estimate $(\lambda_{ix}(\omega)\delta_{ix}(\omega), \lambda^c_{ix}(\omega)\mu_{ix}(\omega), P_{ix}(\omega))$ by maximum likelihood.

Let $\mathcal{T}$ be the length of the sample period and there are $\mathcal{N}(\mathcal{T})$ *moves* in the data (moves include any observed actions of firms). We use $n \in \{1, ..., \mathcal{N}(\mathcal{T})\}$ to index moves. We consider $\mathcal{N}(\mathcal{T}) \to \infty$ as $\mathcal{T} \to \infty$. Let t_n be the time of the n-th move in the entire game. Let ω_n be the state between t_n and t_{n+1}. Note that the observed applications are also included in the moves. Let $f(t; \lambda)$ denote the PDF of exponential

[11] See also Aguirregabiria and Mira (2007) and Bajari, Benkard, and Levin (2007).

distribution with arrival rate λ evaluated at t. $F(t;\lambda)$ is its CDF. We first consider the contribution of firm i to the likelihood. For any move n, if the move was firm i's applying for opening store x and the application was approved, its contribution to the likelihood is

$$f(t_n - t_{n-1};\lambda_{ix}(\omega)\,\delta_{ix}(\omega))\,P_{ix}(\omega)\prod\nolimits_{x'\in X\backslash x}(1 - F(t_n - t_{n-1};\lambda_{ix'}(\omega)\,\delta_{ix'}(\omega)))$$
$$\times\prod\nolimits_{x\in X}(1 - F(t_n - t_{n-1};\lambda^c_{ix}(\omega)\,\mu_{ix}(\omega))).$$

Next, if the move was a rejection of application that leads to no actual action, then the contribution is

$$f(t_n - t_{n-1};\lambda_{ix}(\omega)\,\delta_{ix}(\omega))\,(1 - P_{ix}(\omega))\prod\nolimits_{x'\in X\backslash x}(1 - F(t_n - t_{n-1};\lambda_{ix'}(\omega)\,\delta_{ix'}(\omega)))$$
$$\times\prod\nolimits_{x\in X}(1 - F(t_n - t_{n-1};\lambda^c_{ix}(\omega)\,\mu_{ix}(\omega))).$$

If the move was firm i's closing store x, the contribution is

$$f(t_n - t_{n-1};\lambda^c_{ix}(\omega)\,\mu_{ix}(\omega))\prod\nolimits_{x'\in X\backslash x}(1 - F(t_n - t_{n-1};\lambda^c_{ix'}(\omega)\,\mu_{ix'}(\omega)))$$
$$\times\prod\nolimits_{x\in X}(1 - F(t_n - t_{n-1};\lambda_{ix}(\omega)\,\delta_{ix}(\omega))).$$

Finally, if the move was by some other firm, then the contribution is

$$\prod\nolimits_{x\in X}(1 - F(t_n - t_{n-1};\lambda_{ix}(\omega)\,\delta_{ix}(\omega)))\prod\nolimits_{x\in X}(1 - F(t_n - t_{n-1};\lambda^c_{ix}(\omega)\,\mu_{ix}(\omega))).$$

Then, $L_n(\boldsymbol{\gamma}|data)$ is the product of all these contributions over N players. The likelihood is the product of these contributions over all n:

$$\mathcal{L}(\boldsymbol{\gamma}|data) = \prod_{n=1}^{\mathcal{N}(\mathcal{T})} L_n(\boldsymbol{\gamma}|data),$$

where $\boldsymbol{\gamma} = \{\lambda_{ix}(\omega)\,\delta_{ix}(\omega),\lambda^c_{ix}(\omega)\,\mu_{ix}(\omega),P_{ix}(\omega)\}_{i,x,\omega}$. Let $\hat{\boldsymbol{\gamma}}$ be the MLE.

4.2.2 Second Stage

The set of parameters estimated in this stage include opening costs κ_x, closing values η_x, application costs C, and parameters in the regulator's choice equation $\boldsymbol{\alpha} = (\alpha_1,\alpha_2,\alpha_3)$. Let $\boldsymbol{\theta} = (\kappa_x,\eta_x,C,\boldsymbol{\alpha})$. In this stage, we calculate the value functions using $\hat{\boldsymbol{\gamma}}$ from the first stage and forward simulation. To simulate value functions, we need to separate arrival rates from choice probabilities, and there are two ways to achieve the goal. First, we use other data source to estimate arrival rates. Second, we arbitrarily pick $\lambda_{ix}(\omega)$ and $\lambda^c_{ix}(\omega)$ in order to perform forward simulation and maximize the objective function in the outer loop over $\lambda_{ix}(\omega)$ and $\lambda^c_{ix}(\omega)$ as well as over $\boldsymbol{\theta}$. That is, the structure of the model and data tell us what combination of $(\lambda_{ix}(\omega),\lambda^c_{ix}(\omega),\boldsymbol{\theta})$ is most likely.[12] Arcidiacono et. al (2010) adopt the second approach. For the sake of argument, assume that we also take the second approach and re-define $\boldsymbol{\theta} = (\kappa_x,\eta_x,C,\boldsymbol{\alpha},\lambda_{ix}(\omega),\lambda^c_{ix}(\omega))$.

The algorithm is as follows:

1. Pick $\boldsymbol{\theta}$.

2. Compute value functions using forward simulation and $\hat{\boldsymbol{\gamma}}$.

3. Calculate the implied probability of application $\delta_{ix}(\omega;\boldsymbol{\theta})$, closing probability $\mu_{ix}(\omega;\boldsymbol{\theta})$ and approval probabilities $P_{ix}(\omega;\boldsymbol{\theta})$.

[12] Even for this approach, we still need to impose at least one normalization on arrival rates; e.g., there exists (x,ω) such that $\lambda_x(\omega) = \bar{\lambda}$ where $\bar{\lambda}$ is a known positive real value.

4. Calculate the pseudo likelihood

$$\mathcal{L}(\boldsymbol{\theta}|data) \equiv \sum_{n=1}^{\mathcal{N}(\mathcal{T})} \ln L_n(\boldsymbol{\theta}, \boldsymbol{\gamma}|data).$$

where $L_n(\boldsymbol{\theta}, \hat{\boldsymbol{\gamma}}|data)$ is calculated using best response probabilities given in (12), (13), and (14). Go back to step 1 until we find the maximizer of $\mathcal{L}(\boldsymbol{\theta}|data)$.

Let $\hat{\boldsymbol{\gamma}}$ be a consistent estimator for $\boldsymbol{\gamma}_0$ and $\sqrt{L}(\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma}_0) \to_d N(0, \Sigma)$. Under the regularity conditions,

$$\sqrt{L}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) \to_d N(0, V) \text{ as } \mathcal{T} \to \infty.$$

where

$$V = \Omega_{\theta\theta}^{-1} + \Omega_{\theta\theta}^{-1}\Omega_{\theta\boldsymbol{\gamma}}\Sigma\Omega_{\theta\boldsymbol{\gamma}}'\Omega_{\theta\theta}^{-1}$$

and $\Omega_{\theta\theta} = E\left(\{\bigtriangledown_\theta s_n\}\{\bigtriangledown_\theta s_n\}'\right)$ and $\Omega_{\theta\boldsymbol{\gamma}} = E(\{\bigtriangledown_\theta s_n\}\{\bigtriangledown_{\boldsymbol{\gamma}} s_n\})'$ with $s_n = \ln L_n(\boldsymbol{\theta}_0, \hat{\boldsymbol{\gamma}}_0|data)$.

5 Results

5.1 Demand Estimation

The data used for stage 1 is summarized in Table 5. In stage 1 we estimate the data on 12,555 consumers and for each of these consumers we draw 12 two-week time periods in Oct 2002 - Oct 2005. The large number of consumers helps in terms of obtaining reasonably precise estimates. The demographic variables in w include the number of adults, the number of children, and an indicator for whether the household is from an "upper" social class. We include quarterly and yearly dummies. The results for stage 1 are summarized in Table 6. The results are done separately for each of the four categories of demand and these are presented by column in the table. The price parameter γ_2 is negative as expected, and the magnitude generates category level price elasticities conditional on choice of store of around -1. As expected the substitution parameter γ_3 falls somewhere in the range between 0 We present a range of other estimates that enter into the deterministic portion of the first order term in quadratic utility (γ_1). The firm dummies, demographic effects, quarterly and year dummies are reported in the lower part of the table.

The parameters from the discrete part of the model are presented in Table 7. The parameters are of the expected sign, *distance* is negative, *distance* interacted with *employment* suggests fully employed people are more sensitive to distance. *Sales area* is positive, suggesting it is worth going further for, and the parameter on *two stores* indicates a shopping cost from visiting more than one store. The *town centre* variable is negative, suggesting perhaps that on average there is some inconvenience associated with a town centre location, such as parking costs. *Same quadrant* which is positive (suggesting that people prefer to combine stores in the same quadrant). The parameter α_0 is large and positive, showing the role of the utility portion $a()$, and implying negative price effects.

The implications for elasticities, and markups, are reported in Table 8. We report elasticities separately for each category of demand. These elasticities allow for two components: an effect on quantity demanded through the continuous demand functions, holding store choice constant, and an effect through store choice. The elasticities are generally in the range -1.5 to -3.5. The high elasticity for M&S and lower elasticity for lower price stores such as ASDA reflects a number of factors. First, a store that is typically part of a two-store shopping pattern, as is true for M&S, tends to have higher elasticities, because people can switch shopping between stores without changing their discrete shopping choice c. This may explain why M&S has a relatively high—and ASDA has a relatively low—own price elasticity. Second, the higher price firms may have higher elasticities because they have higher prices and lower quantities—which in many demand specifications leads to higher elasticities. The markups implied by the demand model—combined with an assumption of multi-product Nash price setting—are also shown in this table. Markups in the range 15%-50% are found.

5.2 Supply Side: An Example

We estimate a simplified version of the model to demonstrate computational feasibility and also to serve as a first step to estimating the full model. First, we drop store sizes and town center status from the analysis. Second, we treat exits as exogenous and exit shocks arrive at a uniform rate λ^c. In addition, assume that the scrap value and application costs are zero. Thus, two endogenous choices are (i) whether a firm sends an application for opening a uniformly-sized store or not, when an opportunity arrives; and (ii) whether the regulator accepts firm's application or not.

The unit of location is postal sector. There are 11,445 postal sectors in the whole U.K. These are small geographic locations of a few thousand households each—a local "village" size of location. We excluded locations where we did not observe any incumbents nor entry during the sample period. As a result, we end up with 5,060 locations. For a given location, we define *center* as the centroid of the location. Any postal sectors within a circle with radius of 10km from the center of l are defined as neighboring locations of l.

While the arrival rate for opportunities of store opening could be different across all locations and across firms, estimating 5,060 arrival rates for each firm is not realistic. Thus, we define 11 regions in the U.K. and assume arrival rates are constant within each region and across firms but different across regions (in principle we can make arrival rates a function of location specific observable covariates). That is, letting λ_l denote the arrival rate of store opening opportunities, we assume $\lambda_l = \lambda_{l'}$ if l and l' belong to the same region.[13]

There are 9 firms in the game. ASDA, Tesco, Sainsbury's, and Morrisons are firms that often open a big size store, so we call them "type B", while Co-op, Discounter, Somerfield, Waitrose, and Others are called "type S". That is, although we do not distinguish store formats for the current specification, we keep firm types so that one type may affect a firm's profit in a different way than another type does.

In this example, we consider nine state variables:

$$\omega = \left(N_{il}^{own}, N_{il}^{B,rival}, N_{il}^{S,rival}, N_{iR(l)}^{own}, N_{iR(l)}^{B,rival}, N_{iR(l)}^{S,rival}, X_l, X_{R(l)}, region\right) \in \Omega$$

where X_l is a scalar variable that captures the potential number of consumers in location l and $X_{R(l)}$ captures the potential number of consumers in $R(l)$. Other variables are defined as in the previous section. To reduce the state space, we discretize the first eight variables as follows:

$$N_{il}^{own} = \begin{cases} 1 & \text{if there is no own store in } l \\ 2 & \text{if there is one or more own store in } l \end{cases}$$

$$N_{il}^{B,rival} = \begin{cases} 1 & \text{if the number of rival stores of type B in } l \text{ is } 0 \\ 2 & \text{if the number of rival stores of type B in } l \text{ is } 1 \\ 3 & \text{if the number of rival stores of type B in } l \text{ is } 2 \\ 4 & \text{if the number of rival stores of type B in } l \text{ is } 3 \text{ or more} \end{cases}$$

[13] 11 regions are North East, North West and Merseyside, Yorkshire and the Humber, East Midlands, West Midlands, Eastern, London, South East, South West, Wales, and Scotland.

$$N_{il}^{S,rival} = \begin{cases} 1 & \text{if the number of rival stores of type S in } l \text{ is } 0 \\ 2 & \text{if the number of rival stores of type S in } l \text{ is } 1 \\ 3 & \text{if the number of rival stores of type S in } l \text{ is } 2 \end{cases}$$

$$N_{iR(l)}^{own} = \begin{cases} 1 & \text{if the number of own stores in } R(l) \text{ is smaller than } 6 \\ 2 & \text{if the number of own stores in } R(l) \text{ is between 6 and 10} \\ 3 & \text{if the number of own stores in } R(l) \text{ is greater than } 10 \end{cases}$$

$$N_{iR(l)}^{B,rival} = \begin{cases} 1 & \text{if the number of rival stores of type B in } R(l) \text{ is smaller than } 11 \\ 2 & \text{if the number of rival stores of type B in } R(l) \text{ is between 11 and 50} \\ 3 & \text{if the number of rival stores of type B in } R(l) \text{ is greater than } 50 \end{cases}$$

$$N_{iR(l)}^{S,rival} = \begin{cases} 1 & \text{if the number of rival stores of type S in } R(l) \text{ is smaller than } 11 \\ 2 & \text{if the number of rival stores of type S in } R(l) \text{ is between 11 and 50} \\ 3 & \text{if the number of rival stores of type S in } R(l) \text{ is greater than } 50 \end{cases}$$

$$X_l = \begin{cases} 1 & \text{if } pop_l \text{ is smaller than 2,001} \\ 2 & \text{if } pop_l \text{ is between 2,001 and 4,000} \\ 3 & \text{if } pop_l \text{ is greater than 4,000} \end{cases}$$

$$X_{R(l)} = \begin{cases} 1 & \text{if } pop_{l,R} \text{ is smaller than 25th percentile} \\ 2 & \text{if } pop_{l,R} \text{ is between 25th and 75th percentiles} \\ 3 & \text{if } pop_{l,R} \text{ is greater than 75th percentile} \end{cases},$$

where pop_l represents the total number of households in l and $pop_{l,R} = \sum_{m \in R(l)} pop_m$. Note that the 25th and 75th percentiles of $pop_{l,R}$ are $26,857$ and $137,497$, respectively. As a result, the size of the state space is $2 \times 4 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 5,832$ for each of 11 regions.

We use a reduced form payoff function. The profit for *each store* is given by

$$\begin{aligned} \pi_{il}(\omega) \quad = \quad & \mathbf{1}\left(N_{il}^{own} = 2\right) \Bigg(\theta^{own} + \sum_{k=2}^{4} \mathbf{1}\left(N_{il}^{B,rival} = k\right) \theta_k^{B,rival} + \sum_{k=2}^{3} \mathbf{1}\left(N_{il}^{S,rival} = k\right) \theta_k^{S,rival} \\ & + \sum_{k=2}^{3} \mathbf{1}\left(N_{iR(l)}^{own} = k\right) \theta_k^{own,R} + \sum_{k=2}^{3} \mathbf{1}\left(N_{iR(l)}^{B,rival} = k\right) \theta_k^{B,rival,R} \\ & + \sum_{k=2}^{3} \mathbf{1}\left(N_{iR(l)}^{S,rival} = k\right) \theta_k^{S,rival,R} + \theta_{pop} \log(pop_l) + \theta_{pop,R} \log(pop_{l,R}) \Bigg). \end{aligned}$$

We assume that the profit function is different between big firms and small firms, so estimate it separately for each type. In addition, we assume that the cost of store opening also differs across different types of firm.

We assume that the regulator's objective function is such that his choice probability is given by

$$P_{ix}(\omega) = \Pr\left(\alpha_0 + \alpha_1 pop_l + \alpha_2 N_l^B + \alpha_3 N_l^S + \epsilon_{R1} - \epsilon_{R0} > 0\right).$$

N_l^B and N_l^S represent the numbers of affected stores owned by big firms and small firms, respectively. Thus, if the existence of more incumbent firms makes an approval less likely, then α_2 and α_3 are expected to be negative. In addition, if the regulator puts different weights on each competitor's profit by the type of the competitor, α_2 and α_3 are expected to be different from each other.

So the set of structural parameters is

$$\boldsymbol{\theta} = \left\{ \left(\theta_i^{own}, \{\theta_{ki}^{B,rival}\}_{k=2}^4, \{\theta_{ki}^{S,rival}, \theta_{ki}^{own,R}, \theta_{ki}^{B,rival,R}, \theta_{ki}^{S,rival,R}\}_{k=2}^3, \theta_{pop,i}, \theta_{pop,R,i}, \kappa_i \right)_{i=B,S}, \alpha_0, \alpha_1, \alpha_2, \alpha_3 \right\}.$$

5.2.1 First Stage

In the first stage, we recover the conditional choice probabilities of firms and the regulator as a flexible function of observable states. Specifically, the conditional choice probability of firm i, $\delta_i(\omega;\boldsymbol{\gamma})$, is modeled as logit:

$$\begin{aligned}\delta_i(\omega;\boldsymbol{\gamma}) \quad = \quad & F\Bigg(\mathbf{1}\left(N_{il}^{own}=2\right)\gamma^{own}+\sum_{k=2}^{4}\mathbf{1}\left(N_{il}^{B,rival}=k\right)\gamma_k^{B,rival}+\sum_{k=2}^{3}\mathbf{1}\left(N_{il}^{S,rival}=k\right)\gamma_k^{S,rival}\\ &+\sum_{k=2}^{3}\mathbf{1}\left(N_{iR(l)}^{own}=k\right)\gamma_k^{own,R}+\sum_{k=2}^{3}\mathbf{1}\left(N_{iR(l)}^{B,rival}=k\right)\gamma_k^{B,rival,R}+\sum_{k=2}^{3}\mathbf{1}\left(N_{iR(l)}^{S,rival}=k\right)\gamma_k^{S,rival,R}\\ &+\sum_{k=2}^{3}\mathbf{1}\left(X_l=k\right)\gamma_k^{pop}+\sum_{k=2}^{3}\mathbf{1}\left(X_{R(l)}=k\right)\gamma_k^{pop,R}+\mathbf{1}\left(i\text{ is big firm}\right)\gamma^{big}\Bigg)\end{aligned}$$

and the approval probability of the regulator when firm i is applying, $P_i(\omega;\boldsymbol{\gamma})$, is also modeled as logit:

$$\begin{aligned}P_i(\omega;\boldsymbol{\gamma}) \quad = \quad & F\Bigg(\widetilde{\gamma}^{const}+\mathbf{1}\left(N_{il}^{own}=2\right)\widetilde{\gamma}^{own}+\sum_{k=2}^{4}\mathbf{1}\left(N_{il}^{B,rival}=k\right)\widetilde{\gamma}_k^{B,rival}+\sum_{k=2}^{3}\mathbf{1}\left(N_{il}^{S,rival}=k\right)\widetilde{\gamma}_k^{S,rival}\\ &+\sum_{k=2}^{3}\mathbf{1}\left(N_{iR(l)}^{own}=k\right)\widetilde{\gamma}_k^{own,R}+\sum_{k=2}^{3}\mathbf{1}\left(N_{iR(l)}^{B,rival}=k\right)\widetilde{\gamma}_k^{B,rival,R}+\sum_{k=2}^{3}\mathbf{1}\left(N_{iR(l)}^{S,rival}=k\right)\widetilde{\gamma}_k^{S,rival,R}\\ &+\sum_{k=2}^{3}\mathbf{1}\left(X_l=k\right)\widetilde{\gamma}_k^{pop}+\sum_{k=2}^{3}\mathbf{1}\left(X_{R(l)}=k\right)\widetilde{\gamma}_k^{pop,R}+\mathbf{1}\left(i\text{ is big firm}\right)\widetilde{\gamma}^{big}\Bigg)\end{aligned}$$

where F is the CDF of the difference between two extreme value shocks and we let $\boldsymbol{\gamma}$ be the set of λ_l, λ^{cl}, and all γ.

The total number of observed moves during the sample period is 1949. Let t_n be the time of the n-th move in the entire game. Let M be the set of locations. For any move n, if the move was firm i's applying for opening a store in l and the application was approved, its contribution to the likelihood is

$$f\left(t_n-t_{n-1};\lambda_l\delta_i(\omega)\right)P_i(\omega)\prod\nolimits_{l'\in M\setminus l}\left(1-F\left(t_n-t_{n-1};\lambda_{l'}\delta_i(\omega)\right)\right)\prod\nolimits_{l\in M}\left(1-F\left(t_n-t_{n-1};\lambda^c\right)\right).$$

Next, if the move was a rejection of application that leads to no actual action, then the contribution is

$$f\left(t_n-t_{n-1};\lambda_l\delta_i(\omega)\right)\left(1-P_i(\omega)\right)\prod\nolimits_{l'\in M\setminus l}\left(1-F\left(t_n-t_{n-1};\lambda_{l'}\delta_i(\omega)\right)\right)\prod\nolimits_{l\in M}\left(1-F\left(t_n-t_{n-1};\lambda^c\right)\right).$$

If the move was firm i's closing store x, the contribution is

$$f\left(t_n-t_{n-1};\lambda^c\right)\prod\nolimits_{l'\in M\setminus l}\left(1-F\left(t_n-t_{n-1};\lambda^c\right)\right)\prod\nolimits_{x\in X}\left(1-F\left(t_n-t_{n-1};\lambda_l\delta_i(\omega)\right)\right).$$

Finally, if the move was by some other firm, then the contribution is

$$\prod\nolimits_{l\in M}\left(1-F\left(t_n-t_{n-1};\lambda_l\delta_i(\omega)\right)\right)\prod\nolimits_{l\in M}\left(1-F\left(t_n-t_{n-1};\lambda^c\right)\right).$$

Then, $L_n(\boldsymbol{\gamma}|data)$ is the product of all these contributions over N players. The likelihood is the product of these contributions over all n:

$$\mathcal{L}(\boldsymbol{\gamma}|data)=\prod_{n=1}^{1949}L_n(\boldsymbol{\gamma}|data),$$

where $\boldsymbol{\gamma}=\{\{\delta_i(\omega),P_i(\omega)\}_{i,\omega},\{\lambda_l\}_l,\lambda^c\}$. Let $\hat{\boldsymbol{\gamma}}$ be the MLE.

5.2.2 Second Stage

We use a forward simulation to approximate the value functions. For any region and initial state ω, we simulate the value functions using $\delta_i(\omega; \hat{\boldsymbol{\gamma}}), P_i(\omega; \hat{\boldsymbol{\gamma}}), \hat{\lambda}_l$, and $\hat{\lambda}^{cl}$. The details of this procedure is given in the Appendix.

Next, we calculate the implied choice probabilities of firms:

$$\widetilde{\delta}_i(\omega; \boldsymbol{\theta}) = \frac{\exp(V_i(\omega + [i,l]) - \kappa_i)}{\exp(V_i(\omega + [i,l]) - \kappa_i) + \exp(V_i(\omega))},$$

where $\omega + [i,l]$ is the state that is reached if firm i opens a new store at l, and the implied acceptance probability is

$$\widetilde{P}_{ix}(\omega; \boldsymbol{\theta}) = \frac{\exp\left(\alpha_0 + \alpha_1 pop_l + \alpha_2 N_l^B + \alpha_3 N_l^S\right)}{1 + \exp\left(\alpha_0 + \alpha_1 pop_l + \alpha_2 N_l^B + \alpha_3 N_l^S\right)}.$$

Following Arcidiacono et al. (2010), we calculate this choice probability only for states that are relevant for estimation. Note that we made the dependence of δ and P on $\boldsymbol{\theta}$ explicit to emphasize that the implied choice probability is a function of structural parameters.

Finally, we form the pseudo likelihood using data and the likelihood function above with $\delta_i(\omega; \boldsymbol{\theta})$ and $P_i(\omega; \boldsymbol{\theta})$ replaced by $\widetilde{\delta}_i(\omega; \boldsymbol{\theta})$ and $\widetilde{P}_i(\omega; \boldsymbol{\theta})$, respectively. Our estimator $\hat{\boldsymbol{\theta}}$ maximizes the pseudo likelihood function.

5.2.3 Results

Table 9 shows the estimate of arrival rates for 11 regions. The arrival rates are around 0.018. This corresponds to an average waiting time of 55 years. This seems long, but remember that we have 5,060 locations and 9 firms. Thus, there are $0.018 \times 5,060 \times 9 = 819.7$ arrivals of an opportunity per year in the whole U.K. The sample period is 5 years, so this means that about 4,100 opportunities arrived during the sample period. We also report the estimate of other parameters in $\boldsymbol{\gamma}$ in Table 10.

Table 11 summarizes the estimate of $\boldsymbol{\theta}$. The first specification is our baseline result, while in the second specification we allow the constant in the regulator's acceptance probability to differ depending on the size of the applying firm. Since all θs in this example are reduced form parameters, it is not straightforward to interpret parameters in the payoff functions. To interpret the magnitude of the entry cost, however, we can think of the following example. Take an average location with 3,156 households in the own location and 108,107 households in the neighboring locations in total (these numbers are sample averages of X_l and $X_{R(l)}$, respectively). Consider one open store owned by a big firm and assume that there are no other stores open (both own and rival stores) in l and $R(l)$. Then, using the first specification, the annual profit earned by the single store in location l is calculated as

$$\int_0^1 \hat{\pi}_{il} e^{-rs} ds = \int_0^1 \left[\hat{\theta}^{own} + \hat{\theta}_{pop} \log(pop_l) + \hat{\theta}_{pop,R} \log(pop_{l,R})\right] e^{-rs} ds = 0.2478.$$

Our estimate for κ for big firms is 1.145, so calculate $1.145/0.248 = 4.617$. Thus, the entry cost is roughly equal to 4.6 years of the total operating profit that the average store earns.

The parameters in the regulator's objective function are also worth mentioning. As we expected, the coefficients on the number of stores owned by big rival firms and small rival firms are both negative. After controlling for the size of population, if more rival firms are affected, the application in question is more likely to be rejected. In addition, the weight placed on the number of big stores is different from that for small stores. The first specification implies that the regulator places more weight on protecting small rivals. This difference in weight, however, disappears once we allow the constant to differ depending on the size of the applying firm in the second specification.

The example has demonstrated the computational feasibility of the approach; we aim to develop the example by incorporating the other features of the model.

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6 Appendix

For simplicity, we assume that big firms have the common value functions and that small firms have the common value functions. Thus, we compute the value of a big firm from the viewpoint of ASDA, and the value of a small firm from the viewpoint of Discounter. Since arrival rates differ across locations, the state space should contain the information for regions; $\bar{\omega} = (\omega, region)$. Note that in the current specification, the state is completely characterized by $\bar{\omega}$. Thus, locations are anonymous once we control for population size and region. The problem of this simplification is that each location has a different number of neighboring locations. For each $\bar{\omega}$, we chose one location whose number of neighboring locations is median in the region. Define a mapping $\Gamma : \bar{\omega} \to \{1, ..., 5060\}$ that reflects our choice of location. We set $r = 0.05$.

1. Fix $\bar{\omega}$. Set the corresponding store configuration. For locations that are not neighbors of $\Gamma(\bar{\omega})$, randomly assign the store configuration.

2. We generate a sequence of events. Let q denote an iteration number.

   (a) Given the store configuration, fill in $\delta_i(\bar{\omega}, \hat{\gamma})$ and $P_i(\bar{\omega}, \hat{\gamma})$ for all i and l.

   (b) For each pair of (i, l), we generate a random draw $u_{i,l}$ from the uniform distribution. Calculate $\tau_{i,l} = -\frac{\log(1-u_{i,l})}{\hat{\lambda}_l \delta_i(\omega, \hat{\gamma})}$, which is the inverse of the exponential distribution. Pick $\tau^{en} = \min\{\tau_{i,l}\}_{\forall i,l}$, $i^{en} = \arg\min_i \{\tau_{i,l}\}_{\forall i,l}$, and $l^{en} = \arg\min_l \{\tau_{i,l}\}_{\forall i,l}$. Also, draw a random number from the uniform distribution $u_{i,l}^{ex}$ for each of all the existing stores in the U.K., calculate $\tau_{i,l}^{ex} = -\frac{\log\left(1-u_{i,l}^{ex}\right)}{\hat{\lambda}^{cl}}$, and let $\tau^{ex} = \min \tau_{i,l}^{ex}$. Let $\widetilde{\tau}_q = \min\{\tau^{en}, \tau^{ex}\}$. That is, $\widetilde{\tau}_q$ is the q-th event time (the time elapsed since the start of the game). $\widetilde{i}_q$ and $\widetilde{l}_q$ are firm and location identities, respectively, that correspond to $\widetilde{\tau}_q$. Let firm $\widetilde{i}_q$ in location $\widetilde{l}_q$ choose his action based on $\delta_i(\bar{\omega}, \hat{\gamma})$. If firm $\widetilde{i}_q$ applies, draw a random number again from the uniform distribution $u_{i,l}^{app}$ and opens a store if $u_{i,l}^{app} \leq P_i(\omega, \hat{\gamma})$. If this event is a closure of a store, then the store is closed with probability one.

   (c) Based on the move made in step 2.b, update the state. Go back to step 2.a. Continue until $\sum_q \widetilde{\tau}_q > T_{\max}$ where $T_{\max}$ is predetermined. Let $\mathcal{N}_q$ be the number of moves observed during $[0, T_{\max}]$.

3. Use $\{\widetilde{\tau}_1, ..., \widetilde{\tau}_{N_\tau}\}$ to calculate

$$V_{il}^{ns}(\bar{\omega}) \approx \sum_{q=1}^{\mathcal{N}_q} \left[\int_{\widetilde{\tau}_{q-1}}^{\widetilde{\tau}_q} e^{-\rho t} \Pi_{il}(\bar{\omega})\, dt + e^{-\rho \widetilde{\tau}_q} \Psi_\omega \right]$$

where

$$\Psi_{\bar{\omega}} = \mathcal{I}\left(\widetilde{i}_q = i, \widetilde{l}_q = l, u < \delta_i(\bar{\omega}, \hat{\gamma})\right)(\epsilon_1 - \kappa_i) + \mathcal{I}\left(\widetilde{i}_q = i, \widetilde{l}_q = l, u \geq \delta_i(\bar{\omega}, \hat{\gamma})\right)\epsilon_0$$

and

$$V_{il}(\bar{\omega}) = \frac{1}{NS} \sum_{ns=1}^{NS} V_{il}^{ns}(\bar{\omega}).$$

We use $T_{\max} = 5$ and $NS = 40$.

Fascia	Store Size Class	Stores #	Store Size Avg.(Sq. Ft)	Market Share Trips	Market Share Expenditure	Spending Per Customer	Range #Lines	price
ASDA		263	45,411	12.49	18.10	25.28		
	L	82	32,020	3.08	4.23	23.96	34,405	1.01
	XL	181	51,477	9.41	13.87	25.71	39,794	1.01
MORRISONS		294	30,661	6.32	8.65	23.93		
	L	261	29,038	5.06	6.81	23.51	36,014	1.04
	XL	33	43,498	1.26	1.84	25.60	28,608	1.03
SAINSBURY'S		502	29,431	11.65	15.44	23.16		
	M	106	6,999	1.00	0.75	13.15	24,405	1.18
	L	145	22,985	3.51	3.78	18.82	36,470	1.19
	XL	251	42,628	7.14	10.91	26.69	42,574	1.20
TESCO		975	23,579	21.28	28.58	23.44		
	M	446	4,391	4.02	3.57	15.52	38,078	1.11
	L	310	26,742	8.64	11.51	23.25	42,759	1.12
	XL	219	58,180	8.62	13.50	27.34	44,956	1.12
DISCOUNTER		484	7,842	6.56	4.52	12.03	18,183	0.82
ICELAND		621	4,863	3.90	2.20	9.83	11,560	1.17
CO-OP		1,599	4,247	7.72	3.49	7.90	24,512	1.26
SOMERFIELD		793	8,608	5.35	3.33	10.88	31,680	1.22
OTHERS		886	9,813	19.26	11.69	10.59	30,453	1.12
M&S		284	8,655	3.35	1.96	10.20	9,749	1.92
WAITROSE		165	19,203	2.14	2.03	16.56	23,493	1.48

Table 1: **Descriptive Statistics: Store & Shopping Characteristics.** For each type of store the number of stores, average store size, market shares and average expenditure is reported. For the four biggest supermarket chains the stores are split into different size bands (see text for more detail). Market shares are calculated based on the total number of trips and alternatively on the consumers' overall expenditure. Market shares and average expenditure are calculated over the sample period 2002-2005.

Firm	Open	Close	Total
ASDA	40	10	50
MORRISONS	94	181	275
SAINSBURY'S	118	20	138
TESCO	581	28	609
DISCOUNTER	24	0	24
WAITROSE	29	5	34
CO-OP	78	1	79
SOMERFIELD	373	64	437
OTHERS	59	150	209
Total	1,396	459	1,855

Table 2: **New Openings and Closings by Firm, 2002-2006.**

Region	Opening	Closing	Sum
North East	47	7	54
North West and Merseyside	165	46	211
Yorkshire and the Humber	99	21	120
East Midlands	37	21	58
West Midlands	159	57	216
Eastern	114	54	168
London	125	33	158
South East	131	56	187
South West	216	66	282
Wales	69	19	88
Scotland	234	79	313
Total	1,396	459	1,855

Table 3: **Frequency of Openings and Closings by Regions, Data and Simulation.**

A: Application Decisions		
Accepted	Super store	269
	Non super store	528
Rejected	Super store	30
	Non super store	128
Total		955

B: Application Decisions by Firm

Firm	Accept	Reject	Total
ASDA	53	19	72
MORRISONS	45	1	46
SAINSBURY'S	40	4	44
TESCO	148	17	165
DISCOUNTER	206	66	272
WAITROSE	12	0	12
CO-OP	15	3	18
SOMERFIELD	12	3	15

Table 4: **Application Decisions, 2002-2006.** Firm identity is sometimes missing, so the total number in this panel does not match the total number in panel A.

Variable	Category	Mean	Std Dev	Variable	Category	Mean	Std Dev
			A: Two Store Dummy				
$1_{[n(c_k)=2]}$	1	0.51	0.50	$1_{[n(c_k)=2]}$	2	0.56	0.50
$1_{[n(c_k)=2]}$	3	0.30	0.46	$1_{[n(c_k)=2]}$	4	0.45	0.49
			B: Quantity Means and Absolute Differences				
$\bar{q}_{itk}$	1	15.08	13.54	$\Delta\bar{q}_{itk}$	1	-0.20	18.57
$\bar{q}_{itk}$	2	14.70	11.14	$\Delta\bar{q}_{itk}$	2	-0.33	16.49
q_{itk}	3	9.81	9.00	Δq_{itk}	3	-0.29	11.21
q_{itk}	4	13.27	11.25	Δq_{itk}	4	-0.27	14.72
			C: Price Means and Absolute Differences				
p_{itk}	1	1.08	0.12	Δp_{itk}	1	0.00	0.17
p_{itk}	2	1.18	0.14	p_{itk}	2	0.00	0.19
p_{itk}	3	1.00	0.11	Δp_{itk}	3	0.00	0.12
p_{itk}	4	1.08	0.15	Δp_{itk}	4	0.00	0.19
			D: Store and Demographic Variables (w_j, h)				
sales area	–	.864	.709	Δsales area	–	1.00	.77
adults	–	2.02	.834	child	–	.66	1.01
upper	–	0.58	0.49	lower	–	0.42	0.49
			#obs: 144,215 ; #hh 12,555; #obs/hh: 12				

Table 5: **Descriptive Statistics for First and Second Order Quadratic Parameters.** The statistics in Panel A are dummies for two stop shopping as defined in the paper. Means in Panels A and B are computed for all consumer-time observations in whcih consumers visit either one or two stores. Differences are computed for consumer-time observations where consumers visit two stores. The statistics in panel D are for all consumer-time-category observations.

Category:	Grocery		Fresh		Household		Meat and Prepared	
	Coef.	Std. Err.	Coef.	Std. Err.	Coef.	Std. Err.	Coef.	Std. Err.
γ_3 (Substitution)	0.454	0.034	0.429	0.025	0.406	0.040	0.499	0.032
γ_1 (Constant)	22.053	2.565	19.270	1.939	14.014	1.605	18.381	1.921
γ_2 (Price)	-12.696	2.059	-9.436	1.404	-7.844	1.386	-11.377	1.526
γ_1 (Floorspace)	0.523	0.181	0.499	0.156	0.569	0.132	0.906	0.159
Firm Effects:								
SAINSBURY	0.590	0.613	3.181	0.438	0.953	0.469	2.282	0.493
MORRISONS	0.064	0.646	1.270	0.585	-0.361	0.465	0.150	0.625
TESCO	1.084	0.679	3.121	0.516	0.992	0.505	1.171	0.589
ASDA	0.402	0.761	1.174	0.620	-0.175	0.514	0.522	0.652
M&S	5.397	1.551	5.563	0.854	2.549	1.240	10.056	1.144
ICELAND	-5.779	0.804	-3.722	0.585	-1.542	1.151	1.270	0.625
WAITROSE	3.773	0.791	5.466	0.600	1.506	0.588	5.418	0.694
DISCOUNTER	-2.829	0.820	-4.974	0.666	-3.064	0.630	-2.957	0.688
SOMERFIELD	-0.780	0.585	-0.622	0.443	-1.445	0.482	-0.004	0.493
CO-OP	-0.186	0.560	-0.175	0.455	0.263	0.508	0.322	0.515
Demographic and Time:								
#Adults	3.612	0.063	3.492	0.051	1.743	0.036	3.648	0.058
#Children	1.986	0.045	2.237	0.037	1.027	0.029	1.573	0.036
Lower Social Class	-0.899	0.073	-1.229	0.062	-0.629	0.055	-0.946	0.081
Quarter 2	-0.282	0.096	-0.835	0.082	-0.148	0.076	0.173	0.085
Quarter 3	1.345	0.099	-1.283	0.086	0.025	0.075	0.072	0.085
Quarter 4	-0.726	0.088	-0.107	0.075	-0.007	0.073	-0.495	0.079
Year 2	0.088	0.080	-0.431	0.078	-0.156	0.077	-0.403	0.072
Year 3	0.353	0.077	0.558	0.067	0.014	0.068	0.058	0.070
#Observations	139,845							
GMM criterion Q(b) =	0.005		0.006		0.003		0.003	

Table 6: **Parameters in Quadratic Utility.** The table reports the parameters estimated using Dubin McFadden instruments for each of four grocery categories. .

Parameter	Variable	Coef	Std. Err
α_0	a(,)	94.84	24.69
β	Distance	1.22	0.04
	Sales area	1.15	0.00
	Two stores	5.39	0.51
	Town centre	-0.57	0.00
	Distance * Employed	-1.60	0.15
	Same Quadrant	0.20	0.00
	TESCO	-0.35	0.02
	SAINSBURY	-0.31	0.01
	ASDA	0.41	0.01
	MORRISONS	.42	0.00
	DISCOUNTER ICELAND	.57	0.00
	M&S	.69	0.00
	CO-OP	-1.32	0.01
	WAITROSE	-0.61	0.01
	#obs	3,200	
	GMM Objective Value:		

Table 7: **Parameters from the Second Step.** The table reports the parameters estimated in the second stage using Simulated GMM.

	Elasticities				Markups			
Category:	1	2	3	4	1	2	3	4
Discounter	-1.75	-1.52	-1.10	-1.64	0.36	0.35	0.51	0.31
ASDA	-1.69	-1.56	-1.30	-1.59	0.31	0.35	0.31	0.28
CO-OP	-1.98	-1.78	-1.65	-1.85	0.33	0.37	0.35	0.31
ICELAND	-1.74	-1.84	-1.50	-1.75	0.39	0.37	0.43	0.40
MORRISONS	-2.01	-1.76	-1.48	-1.72	0.24	0.32	0.29	0.27
M&S	-3.58	-3.08	-2.98	-3.48	0.18	0.22	0.19	0.19
OTHER	-2.36	-2.13	-1.66	-2.01	0.23	0.26	0.31	0.26
SAINSBURY	-2.02	-2.10	-1.43	-1.96	0.24	0.25	0.31	0.24
SOMERFIELD	-2.09	-1.98	-1.63	-1.95	0.29	0.30	0.33	0.29
TESCO	-1.69	-1.55	-1.15	-1.46	0.30	0.35	0.36	0.31
WAITROSE	-2.52	-2.39	-1.66	-2.46	0.20	0.24	0.27	0.19

Table 8: **Own Price Elasticities and Mark Ups.** The table reports the own price elasticites of each of the category firm combinations. The right hand side panel gives Nash Equilibrium markups implied by the system of demands assuming that firms internalise cross elasticites between categories.

Region	Estimate	Std. Err
North East	0.0187	0.0009
North West and Merseyside	0.0151	0.0006
Yorkshire and the Humber	0.0167	0.0007
East Midlands	0.0151	0.0007
West Midlands	0.0187	0.0006
Eastern	0.0185	0.0005
London	0.0196	0.0006
South East	0.0192	0.0005
South West	0.0204	0.0005
Wales	0.0153	0.0009
Scotland	0.0185	0.0005
Closing rate	0.0061	0.0005

Table 9: **Arrival Rate by Regions and Closing Rate**. Standard errors to be added.

Parameter	Estimate	Std. Err	Estimate	Std. Err
constant	–	–	2.6926	0.0810
γ^{own}	-1.3094	0.0479	-1.4405	0.2162
$\gamma_2^{B,rival}$	0.8779	0.0784	0.2165	0.2871
$\gamma_3^{B,rival}$	0.8667	0.0693	-0.3346	0.2645
$\gamma_4^{B,rival}$	-0.9500	0.0685	-0.5901	0.2016
$\gamma_2^{S,rival}$	-0.9094	0.1630	0.0358	0.6878
$\gamma_3^{S,rival}$	0.0632	0.4083	-0.0460	0.9382
$\gamma_2^{own,R}$	-0.8017	0.0761	-0.0491	0.3143
$\gamma_3^{own,R}$	-1.1239	0.1499	0.6876	0.2811
$\gamma_2^{B,rival,R}$	-1.7265	0.0353	-0.1012	0.2924
$\gamma_3^{B,rival,R}$	-0.9334	0.0827	-0.4839	0.2236
$\gamma_2^{S,rival,R}$	0.1090	0.1021	0.2242	0.2348
$\gamma_3^{S,rival,R}$	0.0943	0.1424	-0.3824	0.2595
γ_2^{pop}	-0.0186	0.1579	0.4692	0.2078
γ_3^{pop}	0.3011	0.1058	0.0596	0.3118
$\gamma_2^{pop,R}$	-0.5266	0.0769	-0.5789	0.1891
$\gamma_3^{pop,R}$	-0.2329	0.1080	-0.4845	0.2418
Big firm dummy	0.4455	0.0456	2.1950	0.1434

Table 10: **Parameters in Conditional Choice Probability**.

Parameter	Estimate			
	Specification 1		Specification 2	
	Big firm	Small firm	Big firm	Small firm
κ (opening cost)	1.1450	1.8312	1.1420	1.8308
	Parameters in regulator's choice equation			
α_0 (constant)	2.9817		4.4155	2.3838
α_1 (population)	0.0340		-0.0114	
α_2 (# of big stores)	-0.2830		-0.3463	
α_3 (# of small stores)	-0.5020		-0.3528	
	Parameters in payoff function			
θ^{own}	-0.1573	0.0482	-0.1585	0.0490
$\theta_2^{B,rival}$	0.0961	0.0010	0.0962	0.0011
$\theta_3^{B,rival}$	0.1289	-0.0039	0.1297	-0.0040
$\theta_4^{B,rival}$	-0.0281	0.0007	-0.0278	0.0001
$\theta_2^{S,rival}$	0.0028	0.0143	0.0028	0.0142
$\theta_3^{S,rival}$	-0.0879	-0.0006	-0.0875	-0.0009
$\theta_2^{own,R}$	-0.1285	0.0165	-0.1286	0.0170
$\theta_3^{own,R}$	-0.2172	0.0029	-0.2182	0.0037
$\theta_2^{B,rival,R}$	-0.0523	-0.0145	-0.0523	-0.0147
$\theta_3^{B,rival,R}$	-0.0365	-0.0194	-0.0371	-0.0195
$\theta_2^{S,rival,R}$	0.0592	0.0034	0.0591	0.0034
$\theta_3^{S,rival,R}$	0.0436	0.0228	0.0438	0.0231
θ_{pop}	0.0079	0.0111	0.0080	0.0110
$\theta_{pop,R}$	0.0300	-0.0144	0.0301	-0.0146

Table 11: **Opening Cost and Parameters in Payoff Function.** Standard errors to be added.