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"Attaining efficiency through Kantian optimization: Utopian economics…or is it?"

by

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Abstract: Although evidence accrues in biology, anthropology and experimental economics that *homo sapiens* is a cooperative species, the reigning assumption in economic theory is that individuals optimize in an autarkic manner (as in Nash and Walrasian equilibrium). I here postulate an interdependent kind of optimizing behavior, called Kantian. It is shown that in simple macro-economic models, when there are negative externalities (such as congestion effects from use of a commonly owned resource) or positive externalities (such as a social ethos reflected in individuals' preferences), Kantian equilibria dominate Nash-Walras equilibria in terms of efficiency. While economists schooled in Nash equilibrium may view the Kantian behavior as utopian, there is some – perhaps much -- evidence that it exists. If cultures evolve through group selection, the hypothesis that Kantian behavior is more prevalent than we may think is supported by the efficiency results here demonstrated.

1. Introduction

Three strands of work in contemporary social science, evolutionary biology, and political philosophy unite in emphasizing this fact: that *homo sapiens* is a cooperative species. In evolutionary biology, this statement is accepted as a premise, and scientists have been interested in explaining how cooperation and 'altruism' may have developed among humans. In economics, there is now a long series of experiments whose results are most easily explained by the hypothesis that individuals are to some degree altruistic. Altruism is to be distinguished from reciprocity: when behaving in a cooperative manner, a reciprocator expects cooperation in return, which will increase his/her net payoff (net, that is, of the original sacrifice entailed in cooperation), while an altruist cooperates without the expectation of a future reciprocating behavior. Many biologists,

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experimental economists, and anthropologists now accept the existence of altruistic as well as reciprocating behavior. A recent summary of the state-of-the-art in experimental economics, anthropology, and evolutionary biology is provided by Bowles and Gintis (2011). See Rabin (2006) for a summary of the evidence from experimental economics. An anthropological view is provided in Henrich and Henrich (2007).

In political philosophy, G.A. Cohen (2010) offers a definition of 'socialism' as a society in which earnings of individuals at first accord with a conception of equality of opportunity that has developed in the last thirty years in political philosophy (see Rawls (1971), Dworkin (1981), Arneson (1989), and Cohen(1989)), but in which inequality in those earnings is then reduced because of the necessity to maintain 'community,' an ethos in which '…people care about, and where necessary, care for one another, and, too, care that they care about one another.' Community, Cohen argues, may induce a society to reduce material inequalities (for example, through taxation) that would otherwise be acceptable according to 'socialist' equality of opportunity. But, Cohen writes:

> …the principal problem that faces the socialist ideal is that we do not know how to design the machinery that would make it run. Our problem is not, primarily, human selfishness, but our lack of a suitable organizational technology: our problem is a problem of design. It may be an insoluble design problem, and it is a design problem that is undoubtedly exacerbated by our selfish propensities, but a design problem, so I think, is what we've got.

An economist reading these words thinks of the first theorem of welfare economics. A Walrasian equilibrium is Pareto efficient in an economy with complete markets, private goods, and the absence of externalities. But under the communitarian ethos, people care about the welfare of others – which induces massive consumption externalities – and so the competitive equilibrium will not, in general, be efficient. What economic mechanism can deliver efficiency under these conditions[2]?

There is an important line of research, conducted by Ostrom (1990) and her collaborators, arguing that, in many small societies, people figure out how to avoid, or

[2] In war-time Britain, many spoke of 'doing their bit' for the war effort – voluntary additional sacrifice for the sake of the common good. (See the wonderful BBC series 'Foyle's War' to understand the pervasiveness of this ethos.) But, if I want to contribute to the common struggle, how *much* extra should I do?

solve, the 'tragedy of the commons.' The 'tragedy' has in common with altruism the existence of an externality which conventional behavior does not properly address[3]. It may be summarized as follows. Imagine a lake which is owned in common by a group of fishers, who each possess preferences over fish and leisure, and perhaps differential skill (or sizes of boats) in (or for) fishing. The lake produces fish with decreasing returns with respect to the fishing labor expended upon it. In the game in which each fisher proposes as her strategy a fishing time, the Nash equilibrium is inefficient: there are congestion externalities, and all would be better off were they able to design a decrease, of a certain kind, in everyone's fishing. Ostrom has studied many such societies, and maintains that many or most of them learn to regulate 'fishing,' without privatizing the 'lake.' Somehow, the inefficient Nash equilibrium is avoided. This example is not one in which fishers care about other fishers (necessarily), but it is one in which cooperation is organized to deal with a negative externality of autarkic behavior.

Ostrom's observations pertain to small societies. In large economies, we observe the evolution of the welfare state, supported by considerable degrees of taxation of market earnings. It is not immediately evident that welfare states are due to a feeling of community (à la Cohen), or simply provide a more conventional public good or a good in which market failures abound (insurance), or reflect reciprocating behavior among citizens (welfare states expand after wars, perhaps as a reward to returning soldiers; see Scheve and Stasavage[in press]). Nevertheless, the large scope of welfare states, especially in Northern Europe, is perhaps most easily explained by a communitarian ethos. Redistributive taxation is, that is to say, to at least some degree a reaction to the material deprivation of a section of society, which others view as undeserved, and desire to redress. Nevertheless, as is well-known, redistributive taxation induces, to some degree at least, allocative inefficiency. The solution is second-best.

Economic theorists are beginning to pay attention to the design problem – that is, how to achieve economic efficiency in a society where people care about other people. Perhaps to say they are 'beginning' to do so is uncharitable: implementation theory,

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[3] In the case of altruism, 'conventional' behavior is market behavior, and in the case of the tragedy of the commons, it is autarkic optimizing behavior in using a resource which is owned in common.

largely initiated with Maskin's (1999) work of thirty years ago, asks whether a social-choice rule can be implemented as the Nash equilibrium of a game. And before Maskin, Leonid Hurwicz pioneered the work on mechanism design, in which he studied the efficiency properties of different economic mechanisms at a highly abstract level. This work, however, did not focus upon the issue of externalities induced by the fact that people care about the welfare of other people.

The most recent contribution which is relevant to this inquiry is that of Dufwenberg, Heidhues, Kirchsteiger, Riedel, and Sobel (2010), whose paper entitled "Other-regarding preferences in general equilibrium," studies, at an abstract level, the veracity of the first and second welfare theorems in the presence of other-regarding preferences. As interesting as the results of this paper are, it is perhaps more interesting, from the viewpoint of the evolution of economic thinking, that their article is the result of combining three independent papers by subsets of the five authors: in other words, the problem of addressing seriously the efficiency consequences of the existence of other-regarding preferences is certainly in the air at present.

In this paper, I wish to offer a partial solution to the problem of economic allocation in the presence of a *social ethos* – I use the term, taken from Bowles and Gintis (2011) -- although 'other-regarding preferences' is a synonym. (Perhaps social ethos includes the kind of second-order preference that G.A. Cohen refers to in defining community, that people care that they care about others, while 'other-regarding preferences' does not.) The 'problem' is that market equilibria are in general Pareto inefficient in the presence of a social ethos, and moreover, redistributive taxation is also inefficient.

In the remainder of this introductory section, I will describe the environment for this inquiry. There will be two important institutions which organize economic activity: the market, in which individuals are paid wages equal to their marginal productivities by a firm which hires labor in order to maximize profits, and a political mechanism, through which a tax rule is chosen, which redistributes income. In general individuals care about the welfare of others as well as their own welfare. There are two aspects to this caring: how individuals choose to *aggregate* individual welfares into social welfare, and the *degree* to which social welfare counts in the individual's preferences. We will assume here that individuals are homogeneous with respect to these two decisions.

An individual of type γ has preferences represented by an *all-encompassing utility function* which might be of the form:

$$U(x(\cdot),E(\cdot),\gamma)=u(x(\gamma),E(\gamma),\gamma)+\theta\exp\int\log[u(x(\tau),E(\tau),\tau)]dF(\tau) \quad (1.1)$$

where $u(\cdot,\cdot,\gamma)$ is the *personal utility function* of type γ, $E(\cdot)$ is a function which describes the effort or labor of individuals of all types, $x(\cdot)$ is a function which defines the amount of output (a single good) allocated to each type, θ is a non-negative number measuring the degree of social ethos, *F* is the distribution of types in the society, and the social-welfare function (in this case) is given by a member of the CES family

$$W^p(u[i])=\left(\int u[i]^p\,dF(i)\right)^{1/p}, \quad (1.2)$$

as $p\to 0$. (It is well-known that the function in (1.2) approaches the exponential of the average of the average of the logarithms as $p\to 0$.) Think of an individual's type as signifying, inter alia, the degree to which effort is costless for him, or his natural talent.

A society in which people do not count the welfare of others is one with *individualistic ethos*: in such a society, $\theta=0$, A society in which they do is one with *social ethos*. Social ethos can be stronger or weaker, as represented by the parameter θ. When $\theta=\infty$, the economy is equivalent to the one in which for everyone, all-encompassing utility is equal to social welfare; this is the purely altruistic economy.

The technology is simple : there is firm which produces a single output from average effort according to a concave , differentiable production function *G.* The value $G(\int E(\gamma)\,dF(\gamma))$ is per capita output of the good when the effort schedule is $E(\cdot)$.

Suppose that production is linear and there are zero profits at competitive equilibrium. A typical allocation rule is the linear-tax rule:

$$x^t(E(\cdot),\gamma)=(1-t)wE(\gamma)+t\int wE(\tau)\,dF(\tau)\ , \quad (1.3)$$

where *w* is the wage paid by the firm and *t* is the tax rate. Under the competitive assumption, the firm pays a wage equal to the marginal product of effort: $w=G'(\bar{E})$, where $\bar{E}\equiv\int E(\gamma)\,dF(\gamma)$.

There are two important kinds of externality here – both positive: the tax system creates positive externalities to individual labor, because in general some of each worker's earnings is redistributed to others, and there are also positive consumption externalities due to social ethos. It is unfortunate that, under classical behavior, individuals ignore the positive externalities induced by their labor. I call this classical behavior *autarkic*, and contrast it with behavior that I call *interdependent.* The equilibrium concept associated with autarkic behavior is *Nash* equilibrium; the concept associated with interdependent behavior is *Kantian* equilibrium. In Nash equilibrium, each person adjusts his action if and only if his situation would approve assuming that others do not adjust theirs. In Kantian equilibrium, a person adjusts his action if and only if his situation would improve if all others adjust their actions *in similar fashion* to the personal adjustment he is contemplating. Definitions will be provided in the next section. There is only one concept of Nash equilibrium, but there are many concepts of Kantian equilibrium, because the phrase 'in similar fashion' can be spelled out in various ways.

Thus, we will have a three-dimensional choice of possible social equilibria. I will fix – for the most part -- two aspects of the society: (a) there is a profit-maximizing firm which hires workers, paying them wages equal to their marginal product; (b) all individuals share the same social welfare function and the same degree (θ) of altruism. There remain three variables of interest: (1) the degree to which individuals count social welfare in their all-encompassing preferences, which will be measured by θ; (2) the linear tax rate t that the polity chooses to redistribute market incomes; and (3) the optimizing behavior of individuals, which can be autarkic (Nash) or interdependent (Kantian)[4]. Thus a *social equilibrium* will be, for the most part, characterized by a triple (θ, t, J) where $\theta \in \mathbb{R}_+$ measures the degree of social ethos, $t \in [0,1]$ is a constant tax rate on earnings, and $J \in \{N, K^+, K^\times\}$ specifies individual optimizing behavior as Nash (N), additive Kantian (K^+), or multiplicative Kantian $(K^\times)$.

[4] A fourth variable of interest might be the social-welfare function that individuals use; but the qualitative nature of the results will not depend on this, so I restrict analysis to the CES family defined in (1.2). We might also wish to generalize beyond linear tax systems – but we will see below this is hardly necessary.

My main focus will be upon behavior: that is, upon how a change in optimizing behavior from autarkic to interdependent (Nash to Kantian) can (or cannot) resolve the inefficiency of competitive markets with taxation. It should be kept in mind that there are two sources of inefficiency: that due to the fact that taxation induces allocative inefficiency, because the marginal rate of transformation between labor and output is unequal to the marginal rate of substitution between labor and output, due to the tax wedge, and to the fact that social ethos engenders externalities not taken into account when individuals optimize autarkically. Just as economists are often asked to accept the idea that the formal concept Nash equilibrium captures a common kind of actual stable point in human economic relations, so I will ask readers to accept, for the sake of argument, that Kantian equilibrium (in its various versions) can capture a kind of social equilibrium. Only at the end of the paper will I contemplate whether Kantian behavior is achievable in human societies, or is simply a utopian idea.

2. Kantian equilibrium

Immanuel Kant proposed the behavioral ethic known as the categorical imperative: take those actions and only those actions which you would have all others emulate[5]. This suggests the following formalization. Let $\{V^{\gamma}(E(\cdot))\}$ be a set of payoff functions for a game played by types γ, where the strategy of each player is a non-negative effort $E(\gamma)$. Thus the payoff of each depends upon the efforts of all. A *multiplicative Kantian equilibrium* is an effort schedule $E^{*}(\cdot)$ such that *nobody would prefer that everybody alter his effort by the same factor.* That is:

$$(\forall\gamma)(\forall r \geq 0)(V^{\gamma}(E^{*}(\cdot)) \geq V^{\gamma}(rE^{*}(\cdot))) . \qquad (2.1)$$

This concept was first introduced in Roemer (1996), and elaborated more fully in Roemer (2010), although in those texts it was simply called 'Kantian equilibrium.'

The remarkable feature of multiplicative Kantian equilibrium is that it resolves the tragedy of the commons. Consider the example given in section 1 of the community

[5] The somewhat more general version of the categorical imperative is that one's behavior should accord with 'universalizable maxims.'

of fishers. At an effort allocation $E(\cdot)$, if each fisher of type γ keeps his catch, then his fish income will be

$$x^f(\gamma, E(\cdot)) = \frac{E(\gamma)}{\int E(\tau)\,dF(\tau)} G(\int E(\tau)\,dF(\tau)).$$

Thus, the fishers' game is defined by the payoff functions:

$$V^\gamma(E(\cdot)) = u(x^f(\gamma, E(\cdot)), E(\gamma), \gamma). \qquad (2.2)$$

It is proved in the two citations given above to Kantian equilibrium (and repeated in Proposition 5 below) that *if a strictly positive effort allocation is a multiplicative Kantian equilibrium, then it is Pareto efficient in the economy* (u, G, F) -- that is, the economy with $\theta = 0$. This is a stronger statement than saying the allocation is efficient in the game $\{V^\gamma\}$: for in the game, only certain types of allocation are permitted – ones in which fish are distributed in proportion to effort expended. But the economy (u, G, F) defines any allocation as feasible, as long as $\int x(\gamma)\,dF(\gamma) \le G(\int E(\gamma)\,dF(\gamma))$. So Kantian behavior, if adopted by individuals, resolves the tragedy of the commons in a strong way. The intuition is that the Kantian counterfactual (that *every* person will expand his labor by a factor *r* if I do so – or so I contemplate) forces each to internalize the externality associated with the congestion effect of his own fishing. It is not obvious that multiplicative Kantian equilibrium will internalize the externality in exactly the right way – to produce efficiency – but it does. In Roemer (2010), it is shown that under rather weak assumptions on the game $\{V^\gamma\}$, non-zero multiplicative Kantian equilibria exist. In particular, this is so for the fisher economy, if *G* and $u(\cdot, \gamma)$ are concave functions[6].

A *proportional solution* in the fisher economy is defined as an allocation $(x(\cdot), E(\cdot))$ with two properties:

(i) $x(\gamma) = x^f(E(\cdot), \gamma)$, and

(ii) $(x(\cdot), E(\cdot))$ is Pareto efficient.

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[6] In Roemer (1996, 2010), the number of players is finite. This case is encompassed by the present notation: the distribution of types *F* can be discrete. The theorems carry over to the continuous case.

The proportional solution was introduced in Roemer and Silvestre (1993), although the concept of multiplicative Kantian equilibrium came later. The proportional solutions of the fisher economy are exactly its positive multiplicative Kantian equilibria (see Roemer (2010)). In the small societies which Ostrom has studied, which are (in the formal sense) usually 'economies of fishers' where each 'keeps his catch,' she argues that internal regulation assigns 'fishing times' that engender a Pareto efficient allocation. If this is so, these allocations are proportional solutions, and therefore (by the theorem just quoted) they are multiplicative Kantian equilibria in the game where participating fishers/hunters/miners propose labor times for accessing a commonly owned resource. This suggests that small societies discover their multiplicative Kantian equilibria. Ostrom (1990), however, does not provide any evidence for the existence of a kind of dynamic process in which fishers propose efforts which converge, via some tâtonnement-like process, to a multiplicative Kantian equilibrium. Knowing the theory of multiplicative Kantian equilibrium, one is tempted to ask whether a 'Kantian ethos' exists in these small societies, which somehow leads to the discovery of the equilibrium.

I now introduce a second version, called *additive Kantian equilibrium.* An effort allocation $E(\cdot)$ is an additive Kantian equilibrium if and only if no individual would have all individuals add (or subtract) the same amount of effort to the present effort. That is:

$$(\forall\gamma)(\forall r \in \mathbb{R})(V^{\gamma}(E(\cdot)) \geq V^{\gamma}(E(\cdot)+r)), \tag{2.3}$$

where $E(\cdot)+r$ is the allocation in which the effort of type γ individuals is $\max(E(\gamma)+r,0)$. (It is assumed that effort is unbounded above but bounded below by zero.) Additive Kantian equilibrium again postulates that each person 'internalizes' the effects of his contemplated change in effort, but now the variation is additive rather than multiplicative.

In the sequel, I will denote the two kinds of Kantian behavior as $K^{\times}$ and K^{+}.

We can easily define a general 'Kantian variation' which includes as special cases additive and multiplicative Kantian equilibrium. We say a function $\varphi : \mathbb{R}^2_+ \rightarrow \mathbb{R}_+$ is a *Kantian variation* if :

$$\forall x \quad \varphi(x,1) = x .$$

Denote by $\varphi[E(\cdot),r]$ the effort schedule $\tilde{E}$ defined by $\tilde{E}(\gamma) = \varphi(E(\gamma),r)$.

Then an effort schedule $E(\cdot)$ is a $\varphi-$Kantian equilibrium if and only if:

$$(\forall\gamma)(V^{\gamma}(\varphi[E(\cdot),r]) \text{ is maximized at } r=1) \quad . \quad (2.4)$$

If we let $\varphi(x,r)=rx$, this reduces to multiplicative Kantian equilibrium; if we let $\varphi(x,r)=x+r-1$, it reduces to additive Kantian equilibrium.

Let $\varphi(x,r)$ be any Kantian variation which is concave in *r*. Then an effort schedule $E(\cdot)$ is a non-zero $\varphi-$Kantian equilibrium if and only if:

$$\forall\gamma \quad \frac{d}{dr}V^{\gamma}(\varphi[E(\cdot),r])\bigg|_{r=1}=0 . \quad (2.5)$$

Eqn. (2.5) follows immediately from definition (2.4), since $V^{\gamma}(\varphi[E(\cdot),r])$ is a concave function of *r,* and hence its maximum, if it is interior, is achieved where its derivative with respect to *r* is zero. Note that both the additive and multiplicative Kantian variations are concave functions of *r.*

3. Pareto efficiency in economies with social ethos

In this section, we characterize interior Pareto efficient allocations in economies where individuals have all-encompassing utility functions given by (1.1), with the more general CES social-welfare function: that is, we assume that:

$$U(x(\cdot),E(\cdot),\gamma)=u(x(\gamma),E(\gamma),\gamma)+\theta\left(\int u(x(\tau),E(\tau),\tau)]^{p}\,dF(\tau)\right)^{1/p} , \quad (3.1)$$

where $u(\cdot,\gamma)$ is concave in $(x(\gamma),E(\gamma))$ and $1\geq p>-\infty$. As usual, the case $p=0$ in generates the formulation in (1.1), and it is assumed that $\theta\geq 0$.

At an allocation $(x^{*}(\cdot),E^{*}(\cdot))$, we write $u(x^{*}(\gamma),E^{*}(\gamma),\gamma)\equiv u[*,\gamma]$, and for the two partial derivatives of *u,* $u_{j}(x^{*}(\gamma),E^{*}(\gamma),\gamma)\equiv u_{j}[*,\gamma]$.

Proposition 1 A strictly positive allocation is Pareto efficient in the economy (U,G,F,θ) if and only if:

$$\text{(a)} \qquad \forall\gamma \quad \frac{u_{2}[*,\gamma]}{u_{1}[*,\gamma]}=-G'(\bar{E}) \text{, and}$$

$$\text{(b)} \qquad \forall\gamma \quad \frac{1}{u_1[*,\gamma]} \geq \frac{\theta(Q^*)^{(1-p)/p} u[*,\gamma]^{p-1} \int u_1[*,\tau]^{-1}\, dF(\tau)}{1+\theta(Q^*)^{(1-p)/p} \int u[*,\tau]^{p-1}\, dF(\tau)},$$

where $Q^* \equiv \int u[*,\gamma]^p\, dF(\gamma)$.

<u>Proof:</u>

Consider the program

$$\max_{K,h(),q()} \int_{\tau\in D} u(x^*(\tau)+h(\tau),E^*(\tau)+q(\tau),\tau)dF(\tau)+\theta F(D)K$$

subject to

$$\forall\gamma \quad u(x^*(\gamma)+h(\gamma),E^*(\gamma)+q(\gamma),\gamma)+\theta K \geq u(x^*(\gamma),E^*(\gamma),\gamma)+\theta K^*$$

$$\forall\gamma \quad x^*(\gamma)+h(\gamma)\geq 0$$

$$\forall\gamma \quad E^*(\gamma)+q(\gamma)\geq 0$$

$$K \leq \left(\int u(x^*(\gamma)+h(\gamma),E^*(\gamma)+q(\gamma),\gamma)^p\, dF(\gamma)\right)^{1/p}$$

$$G\left(\int (E^*(\gamma)+q(\gamma))dF(\gamma)\right) \geq \int (x^*(\gamma)+h(\gamma))dF(\gamma)$$

where D is any set of types of positive measure. Suppose the solution to this program is $h^* \equiv 0, q^* \equiv 0, K = K^*$. ($K$* is the value of the social-welfare function – given in the K constraint in the program -- when $h = q = 0$.) Then $(x^*(),E^*())$ is a Pareto efficient allocation. Since we are studying strictly positive allocations, the second and third sets of constraints at the proposed optimal solution will be slack.

We will show that conditions (a) and (b) of the proposition characterize the * allocations for which this statement is true. Let (h,q,K) be any feasible triple in the above program, for a fixed positive allocation (x^*,E^*). Let $\Delta K = K - K^*$. Then define the Lagrange function:

$$\Delta(\varepsilon)=\int_{\tau\in D} u(x^*(\tau)+\varepsilon h(\tau),E^*(\tau)+\varepsilon q(\tau),\tau)dF(\tau)+\theta F(D)(K^*+\varepsilon\Delta K)+$$
$$\rho\Big(G\big(\int (E^*(\tau)+\varepsilon q(t))\,dF(\tau)-\int (x^*(\tau)+\varepsilon h(\tau))\,dF(\tau)\Big)+\lambda\left(\int u(x^*(\tau)+\varepsilon h(\tau),E^*(\tau)+\varepsilon q(\tau),\tau)^p\, dF(\tau)\right)^{1/p}-$$
$$\lambda\Big(K^*+\varepsilon\Delta K)\Big)+\int B(\gamma)(u(x^*(\tau)+\varepsilon h(\tau),E^*(\tau)+\varepsilon q(\tau),\tau)+\theta\varepsilon\Delta K-u(x^*(\tau),E^*(\tau),\tau))dF(\tau).$$

Suppose there is non-negative function $B(\cdot)$ and non-negative numbers (λ,ρ) for which Δ is maximized at zero. Note $\Delta(0)$ is the value of the objective of the above program, when $h^* \equiv 0 \equiv q^*$ and $K = K^*$, and $\Delta(1)$ equals the value of the objective at (h,q,K) plus some non-negative terms. The claim will then follow. Since Δ is a concave function, it suffices to produce an allocation $(x^*(), E^*())$ for which non-negative (B,λ,ρ) exist such that $\Delta'(0) = 0$.

Compute the derivative of Δ at zero:

$$
\begin{aligned}
&\Delta'(0) = \int_D \left(u_1[*,\gamma]h(\gamma) + u_2[*,\gamma]q(\gamma)dF(\gamma)\right) + \theta F(D)\Delta K + \\
&\rho\left(G'(\int E^*(\tau)\,dF(\tau))\int q(\tau)\,dF(\tau) - \int h(\tau)\,dF(\tau)\right) + \\
&\frac{\lambda}{p}(Q^*)^{(1-p)/p}\, p\int u[*,\gamma]^{p-1}\left(u_1[*,\gamma]h(\gamma) + u_2[*,\gamma]q(\gamma)\right)dF(\gamma) - \\
&\lambda\Delta K + \int B(\gamma)\left(u_1[*,\gamma]h(\gamma) + u_2[*,\gamma]q(\gamma) + \theta\Delta K\right)dF(\gamma).
\end{aligned}
$$

We now gather together the coefficients of $\Delta K, h,$ and q in the above expression and set them equal to zero:

Coefficient of ΔK: $\quad \theta F(D) + \theta\int B(\gamma)\,dF(\gamma) - \lambda = 0, \qquad$ (A1)

Coefficient of $h(\gamma)$: $\quad u_1[*,\gamma]\mathbf{1}_D - \rho + \lambda(Q^*)^{(1-p)/p}u[*,\gamma]^{p-1}u_1[*,\gamma] + B(\gamma)u_1[*,\gamma] = 0, \quad$ (A2)

Coefficient of $q(\gamma)$: $\quad u_2[*,\gamma]\mathbf{1}_D + \rho G'(\bar{E}) + \lambda(Q^*)^{(1-p)/p}u[*,\gamma]^{p-1}u_2[*,\gamma] + B(\gamma)u_2[*,\gamma] = 0, \quad$ (A3)

where $\mathbf{1}_D(\gamma) = \begin{cases} 1, \text{ if } \gamma \in D \\ 0, \text{ if } \gamma \notin D \end{cases}$ and $\bar{E} = \int E^*(\gamma)\,dF(\gamma)$.

By setting all these coefficients equal to zero, and solving for the Lagrange multipliers, we will discover the characterization of the allocation $(x^*(), E^*())$. Note that, at an interior Pareto efficient solution, we must have:

$$\frac{u_2[*,\gamma]}{u_1[*,\gamma]} = -G'(\bar{E}),$$

for this is the statement that the marginal rate of substitution for each type between labor and output is equal to the marginal rate of transformation between labor and output. Therefore write:

$$u_1[*,\gamma] + u_2[*,\gamma] = u_1[*,\gamma]\left(1 + \frac{u_2[*,\gamma]}{u_1[*,\gamma]}\right) = u_1[*,\gamma]\big(1 - G'(\bar{E})\big). \quad \text{(A4)}$$

Now add together the equations for the coefficients of $q(\gamma)$ and $h(\gamma)$, divide this new equation by $1 - G'(\bar{E})$, use equation (A4), and the result is exactly the equation (A2). Therefore, eqn. (A4) has enabled us to eliminate equation (A3): if we can produce non-negative values $(B(\cdot),\lambda,\rho)$ satisfying (A1) and (A2), we are done.

Solve eqn. (A2) for $B(\gamma)$:

$$B(\gamma) = \frac{\rho - u_1[*,\gamma]\mathbf{1}_D - u_1[*,\gamma]\lambda(Q^*)^{(1-p)/p}u[*,\gamma]^{p-1}}{u_1[*,\gamma]} \quad . \quad \text{(A5)}$$

From eqn. (A1), we have $\lambda = \theta F(D) + \theta\int B(\gamma)\,dF(\gamma)$, and substituting the expression for $B(\gamma)$ into this equation, we integrate and solve for λ:

$$\lambda = \frac{\theta\rho\int u_1[*,\gamma]^{-1}\,dF(\gamma)}{1 + \theta(Q^*)^{(1-p)/p}\int u[*,\gamma]^{p-1}\,dF(\gamma)} \quad \text{(A6)}.$$

Eqn. (A5) says that $B(\gamma)$ is non-negative if and only if

$$\rho \geq u_1[*,\gamma](\mathbf{1}_D + \lambda(Q^*)^{(1-p)/p}u[*,\gamma]^{p-1}) \quad \text{(A7);}$$

substituting the expression for λ from (A6) into (A7) yields an inequality in ρ which, by rearranging terms, can be written as:

$$\rho\left(1 - u_1[*,\gamma]\frac{\theta(Q^*)^{(1-p)/p}u[*,\gamma]^{p-1}\int u_1[*,\tau]^{-1}\,dF(\tau)}{1 + \theta(Q^*)^{(1-p)/p}\int u[*,\tau]^{[-1}\,dF(\tau)}\right) \geq u_1[*,\gamma]. \quad \text{(A8)}$$

In sum, we can find non-negative Lagrange multipliers iff we can produce a non-negative number ρ such that (A8) is true for all γ. This can be done iff:

$$\forall\gamma \quad \frac{1}{u_1[*,\gamma]} \geq \frac{\theta(Q^*)^{(1-p)/p} u[*,\gamma]^{p-1} \int u_1[*,\tau]^{-1}\, dF(\tau)}{1+\theta(Q^*)^{(1-p)/p} \int u[*,\tau]^{p-1}\, dF(\tau)},$$

proving the proposition. ■

We deduce some remarks/corollaries from Proposition 1. First, we introduce a *quasi-linear economy* for which the results take a particularly simple and intuitive form. We will use this economy as an example throughout the paper. In the quasi-linear economy, we take

$$u(x,E,\gamma) = x - \frac{E^2}{\gamma}. \qquad (3.2)$$

Corollaries/remarks

1. Let $\theta = 0$. Then condition (b) of Prop. 1 is always true, and so Pareto efficiency reduces to condition (a), which we knew. Indeed, it is obvious that any allocation which is Pareto efficient in the θ-economy (for any θ) must be efficient in the economy with $\theta = 0$. For suppose not. Then the allocation in question is Pareto-dominated by some allocation in the 0-economy. But immediately, that allocation must dominate the original one in the θ-economy, as it causes the social-welfare function to increase (as well as the private part *u* of all-encompassing utility). It is therefore not surprising that the characterization of Prop. 1 says that 'the allocation is efficient in the 0-economy (part (a)) and satisfies a condition which becomes increasingly restrictive as θ becomes larger (part (b)).'

2. Define $PE(\theta)$ as the set of interior Pareto efficient allocations for the $\theta-$ economy. It follows from condition (b) of Prop. 1 that the Pareto sets are nested, that is:

$$\theta > \theta' \Rightarrow PE(\theta) \subset PE(\theta').$$

Hence, denoting the fully altruistic economy by $\theta = \infty$, we have:

$$PE(\infty) = \cap_{\theta \geq 0} PE(\theta).$$

3. Let $\theta \to \infty$; then condition (b) of Prop. 1 reduces to:

$$\forall\gamma \quad \frac{u_1[*,\gamma]^{-1}}{\int u_1[*,\tau]^{-1}\, dF(\tau)} \geq \frac{u[*,\gamma]^{p-1}}{\int u[*,\tau]^{p-1}\, dF(\tau)}. \qquad (3.3)$$

We have:

<u>Corollary 1</u> An interior allocation is efficient in the fully altruistic economy if and only if:

$$\text{(a)}\ \forall\gamma \quad \frac{u_2[*,\gamma]}{u_1[*,\gamma]} = -G'(\bar{E})\ ,$$

and (c) for some $\lambda > 0, \quad \forall\gamma \quad u_1[*,\gamma] = \lambda u[*,\gamma]^{1-p}$.

<u>Proof:</u>

We need only show that (3.3) implies (c). (The converse is obviously true.)

Denote $\lambda = \dfrac{\int u_1[*,\tau]^{-1}\, dF(\tau)}{\int u[*,\tau]^{p-1}\, dF(\tau)}$. Then (3.3) can be written:

$$\forall\gamma \quad u_1[*,\gamma]^{-1} \geq \lambda u[*,\gamma]^{p-1}. \qquad \text{(A9)}$$

Suppose there is a set of types of positive measure for which the inequality in (A9) is slack. Then integrating (A9) gives us:

$$\int u_1[*,\gamma]^{-1}\, dF(\gamma) > \lambda \int u[*,\gamma]^{p-1}\, dF(\gamma)\,,$$

which says $\lambda > \lambda$, a contradiction. Therefore (A9) holds with equality for almost all γ, and the corollary follows. ■

3. Call an allocation *egalitarian* if all utilities are identical. Is there an interior egalitarian efficient allocation as $\theta \to \infty$? From Corollary 1, at such an allocation, we must have:

$$\forall\gamma \quad u_1[*,\gamma]^{-1} = k\,.$$

(By condition (a), $u_2[*,\gamma]$ must be constant as well.)

We show that, in general, such an allocation will not exist. Consider the Cobb-Douglas economy where $u(x,E,\gamma) = x^{\alpha}(\gamma - E)^{1-\alpha}$, and let $G(x) = x$. We first search for an allocation $(x^*(\gamma), E^*(\gamma))$ at which condition (a) of Proposition 1 holds, and utility is constant across types, at level $\bar{u}$. We have:

$$u_1[*,\gamma] = \frac{\alpha\bar{u}}{x(\gamma)}, \quad u_2[*,\gamma] = -\frac{(1-\alpha)\bar{u}}{\gamma - E(\gamma)}, \quad \frac{x(\gamma)}{\gamma - E(\gamma)} = \frac{\alpha}{1-\alpha} \qquad (3.4)$$

where the third equation comes from condition (a) (which says $u_1 + u_2 = 0$).

Consequently, $x(\gamma) = \frac{\alpha}{1-\alpha}(\gamma - E(\gamma))$, and so $\bar{u} = \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}(\gamma - E(\gamma))$. From these two equations, it follows that $x(\gamma)$ is constant, and so $x(\gamma) = \int E(\tau)\,dF(\tau) \equiv \bar{E}$. Integrating the third equation in line (3.4) we have:

$$\bar{E} = \frac{\alpha}{1-\alpha}(\bar{\gamma} - \bar{E}) \text{ and so } \bar{E} = \alpha\bar{\gamma} \text{ implying } x(\gamma) = \alpha\bar{\gamma}.$$

Now the third equation in line (3.3) implies that:

$$E(\gamma) = \gamma - (1-\alpha)\bar{\gamma}. \tag{3.5}$$

But this is only a feasible solution if :

$$\forall\gamma \quad \gamma > (1-\alpha)\bar{\gamma}, \tag{3.6}$$

which is a particular condition on the distribution *F*. In this example, we therefore have:

- if there is an interior, egalitarian allocation which is efficient in the 0-economy, then it is efficient in the $\theta-$ economy, for all θ;
- if *F* fails to satisfy condition (3.6), there is no interior, egalitarian allocation which is efficient in the 0-economy.

The proof of the first bulleted claim is immediate, because we have shown that, at such an allocation, $x(\gamma)$ is constant, and so $u_1[*,\gamma]$ is constant.

This suggests the following conjecture: that if there is an interior, egalitarian allocation that is efficient in the 0-economy, it is efficient in the θ-economy, for all θ.

The conjecture is false, as the following example shows. Let $u(x,E) = 2x^{1/2} - \frac{E}{\gamma}$, let $G(x) = x$. Then efficiency in the 0-economy (condition (a)) is equivalent to $x(\gamma) = \gamma^2$.

An allocation is egalitarian only if $2\gamma - \frac{E(\gamma)}{\gamma} = k$, or $E(\gamma) = 2\gamma^2 - k\gamma$. The production constraint now implies that $2\int\gamma^2\,dF(\gamma) - k\bar{\gamma} = \int\gamma^2\,dF(\gamma)$, or $k = \frac{\int\gamma^2\,dF(\gamma)}{\bar{\gamma}}$. Thus:

$$E(\gamma) = 2\gamma^2 - \gamma\frac{m_2}{\bar{\gamma}},$$

where m_2 is the second moment of F. It remains only to show that $E(\cdot)$, so defined, is positive, which requires that

$$\forall\gamma \quad \gamma > \frac{m_2}{2\bar{\gamma}}.$$

This condition is met, for example, by the uniform distribution on $[1,b]$ where $2(1+b) \geq b^2$. In this case, we have produced a 0-efficient, interior, egalitarian allocation: but $u_1(x(\gamma), E(\gamma), \gamma) = \frac{1}{\gamma}$; so the necessary condition for this allocation to be efficient in the fully altruistic economy fails.

5. We might, then, conjecture that if the egalitarian, 0-efficient, interior allocation is not efficient in the fully altruistic economy, then there are *no* interior allocations that are efficient in the fully altruistic economy. Let us study the existence of such allocations in the economy of remark 4. Let $p = 0$. We continue to have $x(\gamma) = \gamma^2$, from condition (a). By corollary 1, we also have $\lambda u[*,\gamma] = \frac{1}{\gamma}$, or $\lambda(2\gamma - \frac{E(\gamma)}{\gamma}) = \frac{1}{\gamma}$, which implies that $E(\gamma) = 2\gamma^2 - \frac{1}{\lambda}$. Integrating gives $\frac{1}{\lambda} = 2m_2 - \bar{E}$; but using the production constraint, $\bar{E} = m_2$, and so $1/\lambda = m_2$. Hence $E(\gamma) = 2\gamma^2 - m_2$. Thus, we have produced a non-egalitarian efficient allocation in $PE(\infty)$ if F is such that :

$$\forall\gamma \quad \gamma^2 > \frac{m_2}{2}. \quad (3.7)$$

Note the allocation is unique, if it exists (that is, if (3.7) holds). But it is not egalitarian. So there are economies where $PE(\infty) \neq \varnothing$, an efficient, interior allocation in the 0-economy exists, but no egalitarian allocation belongs to $PE(\infty)$. It is perhaps a surprise that efficiency and interiority in the fully altruistic economy do not imply equal utilities, even when the social-welfare function is concave.

6. However, consider the quasi-linear economy. Then $u_1 \equiv 1$. Now corollary 1 implies that *in the quasi-linear economy, the only Pareto efficient interior allocation as* $\theta \to \infty$ *is the equal-utility allocation for which condition* (a) *holds.*

Let us compute this allocation in the quasi – linear economy of (3.2) in which production is linear: $G(x) = x$. Then these conditions reduce to:

(i) $\frac{2E(\gamma)}{\gamma} = 1$, and

(ii) $k = x(\gamma) - \frac{E(\gamma)^2}{\gamma}$, and

(iii) $\int x(\gamma)\,dF(\gamma) = \int E(\gamma)\,dF(\gamma)$.

It is not hard to show that (i), (ii), and (iii) characterize the equal utility allocation:

$$E(\gamma) = \frac{\gamma}{2}, \quad x(\gamma) = \frac{\gamma + \bar{\gamma}}{4}, \text{ where } \bar{\gamma} = \int \gamma\, dF(\gamma).$$

7. Consider the preferences when $p = 0$. In this case, the altruistic part of U is $\exp[\int \log(u[*,\gamma])dF(\gamma)$, and $Q^* = 1$. Therefore condition (b) of Prop. 1 becomes simpler:

$$\forall\gamma \quad \frac{u[*,\gamma]}{u_1[*,\gamma]} \geq \frac{\theta \int u_1[*,\gamma]^{-1}\,dF(\gamma)}{1 + \theta \int u[*,\gamma]^{-1}\,dF(\gamma)}.$$

8. It follows from the example in remark 5 that there are economies for which $PE(\infty) = \emptyset$. Take the example of that remark, and let F be a distribution for which (3.6) fails. For this economic environment, Pareto efficiency can be achieved in the fully altruistic economy only if either some individuals do not consume or some supply zero effort.

4. Kantian equilibrium and Pareto efficiency

This section studies Kantian equilibria in economies with three kinds of property regime, called ‘private ownership,’ ‘the kibbutz’, and ‘socialist.’

A. Private ownership economies

Consider an economy (U,G,F,θ,p), where p specifies the social-welfare function (see (1.2)). Assume, as well, the economy is one of private ownership, and the share of profits going to an individual of type γ is $\sigma(\gamma)$, where $\int \sigma(\gamma)\,dF(\gamma)=1$. Let $t\in[0,1]$ be any affine income tax regime. At an effort schedule $E(\cdot)$, the income of type γ is

$$g(t,\bar{E},\gamma)\equiv(1-t)(G'(\bar{E})E(\gamma)+\sigma(\gamma)\Pi(\bar{E}))+tG(\bar{E}), \quad (4.1)$$

where profits per capita are $\Pi(\bar{E})\equiv G(\bar{E})-G'(\bar{E})\bar{E}$.

An additive Kantian equilibrium under this allocation rule is an effort schedule $E(\cdot)$ which satisfies:

$$\forall\gamma \quad \left.\frac{d}{dr}\right|_{r=0} U(g(t,\bar{E}+r,\gamma),E(\gamma)+r,\gamma)=0. \qquad (4.2)$$

We have:

<u>Proposition</u> <u>2</u>

A. Every interior additive Kantian equilibrium of this economy is independent of θ. In particular, it is characterized by:

$$\forall\gamma \quad \left.\frac{d}{dr}\right|_{r=0} u(g(t,\bar{E}+r,\gamma),E(\gamma)+r,\gamma)=0. \quad (4.3)$$

B. If G is linear, then every interior additive Kantian equilibrium is in $PE(0)$.

C. Let G be strictly concave. Consider the quasi-linear economy of (3.2), and suppose $\sigma(\gamma)=\frac{\gamma}{\bar{\gamma}}$. Then every interior additive Kantian equilibrium is in $PE(0)$. For no other distribution of shares is this true.

<u>Proof:</u>

1. Define $\psi(t,E(\cdot),\gamma)=\left.\frac{d}{dr}\right|_{r=0} u(g(t,\bar{E}+r,\gamma),E(\gamma)+r,\gamma)$. Then condition (4.2) can be written:

$$\psi(t,E(\cdot),\gamma)+\theta\left(\int u(g(t,E(\cdot),\tau)^p\,dF(\tau)\right)^{1/p-1}\left(\int u(g(t,E(\cdot),\tau)^{p-1}\psi(t,E(\cdot),\tau)\,dF(\tau)\right)=0,$$

(B.1)

from which it follows that:

$$\forall\gamma \quad \psi(t, E(\cdot), \gamma) = K,$$

where K is independent of γ. Substituting this back into the (B.1) yields:

$$K + \theta K\left(\int u(g(t, E(\cdot), \tau)^{p}\, dF(\tau)\right)^{1/p-1}\left(\int u(g(t, E(\cdot), \tau)^{p-1}\, dF(\tau)\right) = 0,$$

and so $K = 0$, which proves claim *A*.

2. From part *A,* interior K^{+} equilibrium is characterized by:

$$\forall\gamma \quad u_1[\gamma]\left((1-t)G''(\bar{E})(E(\gamma) - \sigma(\gamma)\bar{E}) + G'(\bar{E})\right) + u_2[\gamma] = 0. \quad \text{(B.2)}$$

If $G'' = 0$, this reduces to :

$$\forall\gamma \quad G'(\bar{E}) = -\frac{u_2[\gamma]}{u_1[\gamma]},$$

the condition for Pareto efficiency in the 0-economy, given that $E(\cdot)$ is interior. This proves part *B*.

3. In the quasi-linear economy of (3.2), the 0-efficient allocations consist in all interior allocations for which

$$E(\gamma) = \frac{\gamma}{2}G'(\bar{E}), \qquad \text{(B.3)}$$

where $\bar{E}$ is defined (by integrating the (B.3)) $\bar{E} = \frac{\bar{\gamma}}{2}G'(\bar{E})$. In the qusi-linear case, (B.2) reduces to:

$$(1-t)G''(\bar{E})(E(\gamma) - \sigma(\gamma)\bar{E}) + G'(\bar{E}) = \frac{2}{\gamma}E(\gamma). \quad \text{(B.4)}$$

If $\sigma(\gamma) = \dfrac{E(\gamma)}{\bar{E}}$, then (B.4) becomes the statement for efficiency in the 0-economy – that is, (B.4) reduces to (B.3). Define $E(\cdot)$ by (B.3): then we have:

$$\sigma(\gamma) = \frac{\gamma}{\bar{\gamma}} = \frac{\gamma G'(\bar{E})}{\bar{\gamma}G'(\bar{E})} = \frac{E(\gamma)}{\bar{E}},$$

as was to be shown.

Finally, note that, from (B.4), if $E(\gamma) \neq \sigma(\gamma)\bar{E}$ for a non-null set of types, the K^{+} allocation is not 0-efficient. This verifies part *C*. ■

Part *A* of the proposition is disappointing, for it says that K^+ behavior can at most repair the allocative inefficiency of taxation – it cannot address the inefficiency due to the altruistic externality. This suggests a division of labor between optimizing economic behavior, on the one hand, and political behavior, on the other. Part *B* tells us that – at least in the case where *G* is linear -- K^+ behavior solves allocative inefficiency in the 0-economy, and if the polity chooses a sufficiently high tax rate, then perhaps the altruistic externality can be addressed. Part *C* addresses a special case; in general (that is, for any quasi-linear personal utility function of the form $u(x,E,\gamma) = x - \varsigma(E,\gamma)$) there will be a singular distribution of ownership shares under which K^+ equilibria with taxation will be 0-efficient.

I wish to address next the claim just made, that by choosing a rate of taxation that is sufficiently high, a society can address the inefficiency due to the altruistic externality. The next proposition studies the special case of the quasi-linear economy of (3.2).

<u>Proposition 3</u>

Consider the economy (U,G,F,θ,p) where $G(x) = x$, $p = 0$, and *u* is given by (3.2).

A. If $t = \frac{1}{2}$, then the K^+ equilibrium is $\theta -$efficient for all $\theta \geq 0$.

B. Let $U^+[t,\theta,\gamma]$ be the all-encompassing utility of an agent of type γ at the K^+ equilibrium for the $\theta -$economy at tax rate *t,* and let $U^N[t,\theta,\gamma]$ be the utility of that agent at the Nash (Walrasian) equilibrium. Suppose that *F* is such that:

$$\log \bar{\gamma} > \int \log(\frac{\gamma}{4} + \frac{\bar{\gamma}}{2}) dF(\gamma) . \quad (4.4)$$

Then, for any $\varepsilon > 0$, there is a number θ^* sufficiently large that:

$$(\forall \theta \geq \theta^*)(\forall \gamma \text{ such that } F(\gamma) < 1-\varepsilon)(U^+(\frac{1}{2},\theta,\gamma) > U^N(\frac{1}{2},\theta,\gamma)) \ . \ (4.5)$$

<u>Proof:</u>

1. By Prop. 2, since G is linear, the K^+ equilibrium is 0-efficient; therefore $E^+(\gamma)=\frac{\gamma}{2}$ is the effort allocation. It follows that $u[E^+,\gamma]=(1-\frac{1}{2})\frac{\gamma}{2}+\frac{1}{2}\frac{\bar{\gamma}}{2}-\frac{\gamma}{4}=\frac{\bar{\gamma}}{4}$: that is, personal utilities are constant over types. Note that part (a) of Corollary 1 holds (as this is simply a statement of 0-efficiency) and part (c) holds since $u_1\equiv 1$ and $u[E^+,\gamma]$ is constant. Hence the allocation defined by $t=\frac{1}{2}$ and $E^+(\cdot)$ is efficient in the fully altruistic economy. Therefore, by Remark 2 after Prop. 1, this allocation is $\theta-$efficient for all θ.

2. Compute that $U^+[\frac{1}{2},\theta,\gamma]=\frac{\bar{\gamma}}{4}+\theta\exp(\int\log[\frac{\bar{\gamma}}{4}]dF(\tau))=(1+\theta)\frac{\bar{\gamma}}{4}$. The Walrasian (Nash) equilibrium at tax rate t is given by $E^N(\gamma)=\frac{(1-t)\gamma}{2}$ and the all-encompassing utility at the Walrasian equilibrium when $t=\frac{1}{2}$ computes to be:

$$U^N[\frac{1}{2},\theta,\gamma]=\frac{\gamma}{16}+\frac{\bar{\gamma}}{8}+\theta\exp\left(\int\log[\frac{\tau}{16}+\frac{\bar{\gamma}}{8}]dF(\tau)\right).$$

Let γ^ε be defined by $F(\gamma^\varepsilon)=1-\varepsilon$. To verify part B, we must show that there exists a number θ^* such that:

$$\gamma<\gamma^\varepsilon\Rightarrow(1+\theta)\frac{\bar{\gamma}}{4}>\frac{\gamma}{16}+\frac{\bar{\gamma}}{8}+\theta\exp\left(\int\log[\frac{\tau}{16}+\frac{\bar{\gamma}}{8}]dF(\tau)\right).$$

This inequality will be true for sufficiently large θ precisely when:

$$\frac{\bar{\gamma}}{4}>\exp\left(\int\log(\frac{\tau}{16}+\frac{\bar{\gamma}}{8})dF(\tau)\right),$$

or $\log\frac{\bar{\gamma}}{4}>\int\log(\frac{\tau}{16}+\frac{\bar{\gamma}}{4})dF(\tau)$, which is equivalent to (4.4), proving part *B*.

■

In words, Proposition 3*B* states that, if condition (4.4) holds for the distribution of types, then for a sufficiently altruistic society, all but a sliver of the most talented agents fare better, in all-encompassing utility, at the K^+ equilibrium, than at the Walrasian equilibrium, when the tax rate is $\frac{1}{2}$. It should be noted that no tax rates greater than $\frac{1}{2}$ are admissible for the economy (3.2), because such tax rates would generate negative utility for some agents at the effort schedule E^+, which is inadmissible because of the definition of the social-welfare function. (Stated more formally, there is no interior K^+ equilibrium in this economy for $t > \frac{1}{2}$ because utility is undefined.)

It is not in general true that for a quasi-linear economy of the form $u(x,E,\gamma) = x - \varsigma(E,\gamma)$, there exists a tax rate such that the K^+ is in $PE(\infty)$: this is an artifact of the quadratic cost function in (3.2). (For instance, there is no such tax rate if $\varsigma(E,\gamma) = \frac{E^3}{\gamma}$.)

To get a better feel for the relationship of agent utilities in the K^+ and Walrasian equilibria, I present some simulations. The economy is the one postulated in Proposition 3; *F* is the lognormal distribution with median 40 and mean 50. This implies that the median type chooses an effort level of 20 in the Walrasian equilibrium when the tax rate is zero. The type at the 91.5th centile of the distribution chooses an effort level of 100; so the pre-tax income ratio of this type to the median is 5:1. Condition (4.4) holds for this distribution, so the conclusion of Prop.3*B* holds.

For given θ, there is a minimal tax rate such that the K^+ equilibrium allocation at that tax rate is in $PE(\theta)$. For the quasi-linear economy, the condition for θ-efficiency at the K^+, given in remark 7, reduces to:

$$\forall\gamma \quad \frac{1}{\theta} + \int\left((1-t)\frac{\tau}{2} + t\frac{\bar{\gamma}}{2} - \frac{\tau}{4}\right)^{-1} dF(\tau) \geq \left((1-t)\frac{\gamma}{2} + t\frac{\bar{\gamma}}{2} - \frac{\gamma}{4}\right)^{-1}.$$

But the right-hand side is maximized at $\gamma = 0$, and so the minimal efficient tax rate is the solution of the following equation:

$$\frac{1}{\theta} + \int\left((1-t)\frac{\tau}{2} + t\frac{\bar{\gamma}}{2} - \frac{\tau}{4}\right)^{-1} dF(\tau) = \frac{2}{t\bar{\gamma}}. \quad (4.6)$$

Table 1 presents the minimal efficient tax rate for values of θ in the interval [1,200], and also social welfare, as measured by the social-welfare function that the citizens use, in the K^+ and Nash-Walras equilibrium at those tax rates.

Theta	min effic tax rate	Soc Wel, Kant	Soc Wel, Nash
1	0.036	10.3744	10.3489
20	0.295294	12.0555	10.7223
30	0.342959	12.2279	10.4841
40	0.373083	12.3167	10.2919
50	0.393735	12.3684	10.142
60	0.408723	12.401	10.0242
70	0.420071	12.4229	9.92993
80	0.428948	12.4383	9.85323
90	0.436074	12.4495	9.78979
100	0.441916	12.4579	9.73653
110	0.44679	12.4644	9.69124
120	0.450917	12.4695	9.6523
130	0.454454	12.4736	9.61848
140	0.45752	12.4769	9.58884
150	0.460201	12.4796	9.56266
160	0.462566	12.4819	9.53938
170	0.464667	12.4838	9.51855
180	0.466546	12.4854	9.4998
190	0.468236	12.4868	9.48284
200	0.469765	12.488	9.46742

Table 1 Minimal tax rates for θ-efficiency, and social welfare (Kant and Nash equilibrium)

We now turn to political economy. Preferences of citizens over tax rates are single-peaked for both additive Kantian and Nash equilibrium allocations, and ideal tax rates are decreasing in γ (as long as $\theta < \infty$). The simplest prediction of the political equilibrium is therefore the ideal tax rate of the voter of median type. Table 2 presents the tax rate at this political equilibrium for both additive Kantian and Nash-Walras economic equilibria, and also presents social welfare in those equilibria.

theta	t-Kant	t-Walras	Soc Wel Kant	Soc Wel Walras
1	0.5	0.1815	12.5	10.9387
10	0.5	0.190097	12.5	10.941
20	0.5	0.190884	12.5	10.941
30	0.5	0.191159	12.5	10.941
40	0.5	0.191299	12.5	10.941
50	0.5	0.191383	12.5	10.941
60	0.5	0.19144	12.5	10.941
70	0.5	0.191481	12.5	10.941
80	0.5	0.191511	12.5	10.941
90	0.5	0.191535	12.5	10.941
100	0.5	0.191554	12.5	10.941
110	0.5	0.19157	12.5	10.941
120	0.5	0.191583	12.5	10.941
130	0.5	0.191594	12.5	10.941
140	0.5	0.191604	12.5	10.941
150	0.5	0.191612	12.5	10.941
160	0.5	0.191619	12.5	10.941
170	0.5	0.191625	12.5	10.941
180	0.5	0.191631	12.5	10.941
190	0.5	0.191636	12.5	10.941
200	0.5	0.191641	12.5	10.941

Table 2 Social welfare at political equilibrium in the K^+ and Nash-Walras regimes

Note that, in table 2, the political equilibrium in the K^+ allocation is always in $PE(\infty)$ (by Prop. 3). It is interesting that the median voter chooses the largest admissible tax rate even when θ is 'small:' indeed, the same voter in Walras equilibrium is discouraged by the allocative inefficiency of taxation, and chooses an ideal tax rate of less than 0.20. However, a value of $\theta = 1$ is not – actually – small: for the citizen is weighting social welfare and her own utility equally. This suggests that we reproduce the exercise reported in table 2, but for values of θ in [0,1]. Table 3 reports the results.

theta	t-Kant	t-Walras	Soc Wel Kant	Soc Wel Walras
0	0.5	0.166667	12.5	10.9273
0.1	0.5	0.169868	12.5	10.9306
0.2	0.5	0.172334	12.5	10.9328
0.3	0.5	0.174294	12.5	10.9344
0.4	0.5	0.175891	12.5	10.9356
0.5	0.5	0.177218	12.5	10.9364
0.6	0.5	0.178338	12.5	10.9371
0.7	0.5	0.179297	12.5	10.9377
0.8	0.5	0.180127	12.5	10.9381
0.9	0.5	0.180853	12.5	10.9384

Table 3 Social welfare at political equilibrium in the K^{+} and Nash-Walras regimes

We note that even if she is completely self-interested ($\theta = 0$), the median type chooses the largest admissible tax rate in the Kantian regime, because the problem of allocative inefficiency, characteristic of Walrasian equilibrium, has been solved.

One might object that the assumption of linear production tilts the comparison in favor of the Kantian regime. Suppose G is strictly concave. Then, at a zero tax rate, the Walrasian equilibrium is 0-efficient, but the additive Kantian equilibrium is not (unless the distribution of profit shares is singular). This suggests that we compare the two regimes for such an example. I choose $G(x,r) = \frac{x^r}{r}$, and retain the quasi-linear utility function of the previous simulations and the lognormal distribution of types. We must choose a distribution of profit shares which generates an additive Kantian equilibrium which is not in $PE(0)$: equal division will do ($\sigma(\gamma) \equiv 1$), by Proposition 2C.

I describe the computational procedure by which the K^{+} equilibrium is computed for various tax rates. The characterization of the effort schedule in K^{+} equilibrium is given in equation (B.4). For the specified production function above, this equation may be solved to yield:

$$E(\gamma,t) = \frac{\bar{E}(t)^{r-1}\gamma(1+(1-r)(1-t))}{2+\gamma(1-r)(1-t)\bar{E}(t)^{r-2}}, \qquad (4.7)$$

where $\bar{E}(t)$ is the integral of $E(\gamma,t)$ dF. Integrating (4.7) and manipulating the result gives an equation in the single unknown $\bar{E}(t)$:

$$1=\int\frac{(1+(1-r)(1-t))\gamma}{2\bar{E}(t)^{2-r}+(1-r)(1-t)\gamma}dF(\gamma)\,. \qquad (4.8)$$

Fixing *r*, we solve (4.8) for $\bar{E}(t)$ numerically, for various values of *t*, and then compute the Kantian equilibrium effort schedule from (4.7). Then we compute social welfare at the various values of *t.*

It is a standard exercise to compute the effort schedule for Walrasian equilibrium. Individual effort is given by $\tilde{E}(\gamma,t)=\frac{(1-t)w\gamma}{2}$, and average effort is given by $\tilde{E}(t)=\frac{(1-t)w\bar{\gamma}}{2}$, where *w* is the Walrasian wage, which solves to be:

$$w=G'(\tilde{E})=\left(\frac{(1-t)\bar{\gamma}}{2}\right)^{(r-1)(2-r)} \quad .$$

Tables 4a and 4b are the analogs of table 3, for the production function stated above, with $r=0.75$ and $r=0.50$. In the first case, the maximum admissible tax rate is about 0.70, because for higher rates, some utilities become negative, and the social-welfare function is undefined. For $r=0.5$, the maximum admissible tax rate is about 0.9. In both cases, it turns out that the ideal tax rate of the median type, in the Kantian regime, is the maximum admissible rate. We see from the tables that the ideal tax rate of the median type, in the Walrasian regime, decreases with *r*. For each value of θ, I compute the ideal tax rate of the median type at the Kantian and Walrasian equilibrium, and report the values of social welfare at those political equilibria.

theta	t-Kant	t-Walras	Soc Wel - Kant	Soc Wel - Walras
0	0.7	0.166667	5.68571	5.42386
0.1	0.7	0.164717	5.68571	5.42415
0.2	0.7	0.16314	5.68571	5.42436
0.3	0.7	0.161839	5.68571	5.42451
0.4	0.7	0.160747	5.68571	5.42463
0.5	0.7	0.159818	5.68571	5.42473
0.6	0.7	0.159018	5.68571	5.4248
0.7	0.7	0.158322	5.68571	5.42486
0.8	0.7	0.157711	5.68571	5.42491
0.9	0.7	0.15717	5.68571	5.42495
1.	0.7	0.156688	5.68571	5.42499

Table 4a Political-equilibrium tax rates and social welfare in Kantian and Walrasian regimes, for the quasi-linear economy with $G(x) = x^{0.75} / 0.75$ and $\sigma(\gamma) \equiv 1$

theta	t-Kant	t-Walras	Soc Wel - Kant	Soc Wel - Walras
0	0.9	0.166667	4.31667	4.28841
0.1	0.9	0.160879	4.31667	4.28933
0.2	0.9	0.156081	4.31667	4.29003
0.3	0.9	0.152041	4.31667	4.29057
0.4	0.9	0.148592	4.31667	4.29099
0.5	0.9	0.145615	4.31667	4.29133
0.6	0.9	0.143019	4.31667	4.2916
0.7	0.9	0.140736	4.31667	4.29183
0.8	0.9	0.138712	4.31667	4.29202
0.9	0.9	0.136906	4.31667	4.29218
1.	0.9	0.135285	4.31667	4.29231

Table 4b Political-equilibrium tax rates and social welfare in Kantian and Walrasian regimes, for the quasi-linear economy with $G(x) = x^{0.5} / 0.5$ and $\sigma(\gamma) \equiv 1$

We see that, even with substantial concavity, the political equilibrium in the Kantian regime dominates that of the Walrasian regime in terms of social welfare, at least for values of θ in $[0,1]$.

I conclude this sub-section with an intuitive remark concerning the efficiency of additive Kantian and Walrasian equilibrium in private ownership economies. In such economies, if the tax rate is *t,* the first-order condition for maximization of utility of the generic citizen in Nash-Walras equilibrium is:

$$u_1(1-t)G'(\bar{E})+u_2=0 \quad \text{or} \quad -\frac{u_2}{u_1}=(1-t)G'(\bar{E})\,, \tag{4.9a}$$

while the condition for the generic citizen in K^+ equilibrium is:

$$-\frac{u_2}{u_1}=G'(\bar{E})+(1-t)G''(\bar{E})(E(\gamma)-\sigma(\gamma)\bar{E})\,. \tag{4.9b}$$

In both economies, the condition for 0-efficiency is $-u_2/u_1=G'(\bar{E})$. Thus, for the Walrasian regime, the wedge between the individual's MRS and the MRT is small when *t* is small, but for the Kantian regime, the wedge is small when *t* is *large.* Hence, the efficiency advantage of the Kantian regime over the Walrasian regime becomes more powerful, the larger the tax rate. If large tax rates are the way that private-ownership economies can attempt to address the inefficiency due to the altruistic externality, the Kantian regime appears to be a better mechanism than the Walrasian one.

B. The kibbutz

The *kibbutz* is an economy in which everyone consumes the same amount of the good. Thus, if $E(\cdot)$ is the effort schedule, then the allocation of output is $x(\gamma)=G(\bar{E})$. Formally, this is also a private-ownership economy with a tax rate of unity, but it is worthwhile to highlight this case in a sub-section of its own. (Some of the Israeli kibbutzim were kibbutz economies, in the early days, although there were no private-ownership property rights.) We have:

<u>Proposition 4</u> For any concave *G:*

A. Every interior additive Kantian equilibrium of the kibbutz economy is independent of θ. In particular, it is characterized by:

$$\forall\gamma \quad \left.\frac{d}{dr}\right|_{r=0} u(G(\bar{E}+r),E(\gamma)+r,\gamma)=0\,. \tag{4.10}$$

B. Every interior additive equilibrium of the kibbutz economy is 0-efficient.

C. For the quasi-linear economy of (3.2), with $G(x) = x^r / r$, the unique interior K^+ kibbutz equilibrium is given by $E(\gamma) = 2^{r-2}\dfrac{\gamma}{\bar{\gamma}^{r-1}}$, and uniform consumption is $G(\bar{E}) = \dfrac{1}{r}\left(\dfrac{\bar{\gamma}}{2}\right)^{r(2-r)}$. Utilities are positive for all types if and only if the upper bound of the support of *F* is less than $\dfrac{2^{r-1}}{r}\bar{\gamma}^{2-r}$.

<u>Proof:</u>

1. The proof of part A mimics the proof of Proposition 2*A*.
2. Expanding (4.9) gives $u_1 G'(\bar{E}) + u_2 = 0$. So if a K^+ equilibrium with constant consumption $G(\bar{E})$ exists, it is 0-efficient.
3. For the quasi-linear economy with the specified family *G*, equation (4.9) becomes

$$\bar{E}^{r-1} = \frac{2E(\gamma)}{\gamma} \text{ or } E(\gamma) = \frac{\gamma}{2}\bar{E}^{r-1}.$$

Integrating this equation, we solve :

$$\bar{E} = \left(\frac{\bar{\gamma}}{2}\right)^{2-r}.$$

Hence, $E(\gamma) = \left(\dfrac{\gamma}{2}\right)\left(\dfrac{\bar{\gamma}}{2}\right)^{(2-r)(r-1)}$. It follows that utilities are positive for all types if and only if $G(\bar{E}) = \dfrac{1}{r}\left(\dfrac{\bar{\gamma}}{2}\right)^{r(2-r)} - \left(\dfrac{\gamma}{2}\right)\left(\dfrac{\bar{\gamma}}{2}\right)^{(2-r)(r-1)} \geq 0$, which reduces to the stated bound. ■

Part *A* of Prop. 4 says that (again), additive K^+ does not address the positive externality due to other-regarding preferences. The kibbutz economy, however, does generate 0-efficient allocations even when *G* is strictly concave. In general, the kibbutz equilibrium will only be $\theta-$ efficient for θ sufficiently close to zero.

C. The socialist economy

Define an economy as *socialist* if the allocation of output is proportional to effort expended[7]: that is, $x(\gamma) = \frac{E(\gamma)}{\bar{E}} G(\bar{E})$. The prototype socialist economy is an economy of fishers where each fisher keeps his catch. For completeness, we prove:

Proposition 5 For any concave G, any interior *multiplicative* Kantian equilibrium is 0-efficient, and is $\theta-$ efficient for θ sufficiently close to zero (if all utilities are positive, so social welfare is defined).

Proof:

1. It remains true that the condition for multiplicative Kantian equilibrium is

$$\forall\gamma \quad \left.\frac{d}{dr}\right|_{r=1} u\left(\frac{rE(\gamma)}{r\bar{E}} G(r\bar{E}), rE(\gamma), \gamma\right) = 0.$$

2. The condition in step 1 expands to:

$$u_1 \cdot \frac{E(\gamma)}{\bar{E}} G'(\bar{E})\bar{E} + u_2 E(\gamma) = 0,$$

which reduces to $u_1 G'(\bar{E}) + u_2 = 0$, the condition for efficiency in the 0-economy.

3. The allocation, if it exists, is $\theta-$ efficient for small enough θ, by Proposition 1*B*, and continuity, if utility is bounded away from zero. ■

As mentioned in the introduction, multiplicative Kantian equilibria exist for a large class of economies.

D. Existence of interior K^+ equilibrium

In this section thus far, I have computed interior additive Kantian equilibria for quasi-linear economies. Here, I present a general existence theorem for additive Kantian equilibria. I prove the theorem for an economy with a discrete distribution of types (i.e., a finite set of types), denoted $\gamma_1, ..., \gamma_n$. Denote an effort distribution by the vector $E = (E_1, ..., E_n)$ where E_i is the effort of type γ_i. Let $g(E_i, \bar{E})$ be the allocation rule,

[7] Classically, under socialism, "From each according to his ability, to each according to his work."

which assigns output to type i as a function of its effort plus average effort. First, define the correspondences:

$$r_i(E) = \{r \mid r \in \arg\max_{\rho} u(g(E_i + \rho, \bar{E} + \rho), E_i + \rho), \gamma_i)\} \quad .$$

Proposition 6 Let $V^i(E_1,...,E_n) = u(g(E_i, \bar{E}), E_i)$, where u_i and g are concave. Suppose there exists numbers $b, B \in \mathbb{R}_{++}$ such that:

$$(b \le E \le B) \Rightarrow (\forall i)(b \le r_i(E)E_i \le B). \qquad (4.11)$$

Then there exists an interior additive Kantian equilibrium for the game $\{V^i\}$.

The proof mimics the proof of Theorem 2 of Roemer (2010), for multiplicative Kantian equilibrium; it is an application of the Kakutani fixed point theorem. The key step is to verify that the sets $r_i(E)$ are convex, for any effort schedule *E*. But this follows directly from the concavity of the functions V^γ, because

$$V^\gamma(E + \lambda\rho_1 + (1-\lambda)\rho_2) = V^\gamma(\lambda(E+\rho_1) + (1-\lambda)(E+\rho_2) \ge \lambda V^\gamma(E+\rho_1) + (1-\lambda)V^\gamma(E+\rho_2). \quad (4.11)$$

5. Generalizations

A. Several kinds of labor

Suppose there are several kinds of effort or labor – let us say, two – and output is given by a function $G(\bar{E}_1, \bar{E}_2)$, where $\bar{E}_j$ is the average amount of labor of kind *j*. Suppose that *G* is homogeneous of degree 1. In the private-ownership economy, profits are zero at Walrasian equilibrium, with any tax rate *t*. Personal utility is defined as a function $u(x(\gamma), E_1(\gamma), E_2(\gamma), \gamma)$, and all-encompassing utility is defined as before. In the private-ownership economy, we have:

$$x(\gamma, t, E_1(\cdot), E_2(\cdot)) = (1-t)\big(w_1 E_1(\gamma) + w_2 E_2(\gamma)\big) + tG(\bar{E}_1, \bar{E}_2),$$

where $w_j = \dfrac{\partial}{\partial E_j} G(\bar{E}_1, \bar{E}_2)$. An additive Kantian equilibrium is now defined as a pair of effort schedules $E_1(\cdot), E_2(\cdot)$ such that:

$$(\forall r, s \in \mathbb{R}_+)(\forall \gamma)(U[E_1(\cdot), E_2(\cdot), \gamma] \ge U[E_1(\cdot) + r, E_2(\cdot) + s, \gamma] \ ; \ (5.1)$$

that is, no individual would like to additively alter the two effort schedules by independent variations. We have:

Proposition 7 Consider the private-ownership economy with several kinds of effort, where production is homogeneous of degree 1 in average efforts.

A. If G is separable[8], then any K^+ additive equilibrium is 0-efficient, for any admissible tax rate t.

B. If G is not separable, then the only K^+ equilibria which are 0-efficient are ones in which :

$$\forall\gamma \quad \frac{E_1(\gamma)}{E_2(\gamma)} = \frac{\bar{E}_1}{\bar{E}_2}. \qquad (5.2)$$

Proof:

1. The analog of Proposition 2A and Proposition 4A remains true: condition (5.1) reduces to the same statement where the personal utility function u is substituted for the all-encompassing utility function U.

2. By concavity of u, the first-order condition for the maximization of u with respect to the variation (r,s) is that the two partial derivatives are zero; that is:

$$\left.\frac{\partial}{\partial r}\right|_{r=0} u(x(\gamma,t,E_1(\cdot)+r,E_2(\cdot)),E_1(\gamma)+r,E_2(\gamma),\gamma)=0 \quad (5.3a)$$

and $$\left.\frac{\partial}{\partial s}\right|_{s=0} u(x(\gamma,t,E_1(\cdot),E_2(\cdot)+s),E_1(\gamma),E_2(\gamma)+s,\gamma)=0 \quad . (5.3b)$$

Expanding these two equations gives:

$$u_1\cdot\big((1-t)(G_{11}E_1(\gamma)+G_1+G_{21}E_2(\gamma))+tG_1\big)+u_2=0 \qquad (5.4a)$$

and $$u_1\cdot\big((1-t)(G_{22}E_2(\gamma)+G_2+G_{12}E_1(\gamma))+tG_2\big)+u_3=0\,. \qquad (5.4b)$$

If G is separable, these equations reduce to:

$$u_1G_1+u_2=0, \quad u_1G_2+u_3=0 \qquad (5.5)$$

which are the conditions for Pareto efficiency at an interior solution.

3. Since G is homogeneous of degree 1, Euler's equation says:

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[8] Define G as separable if $G_{12}=G_{21}\equiv 0$.

$$G(\bar{E}_1, \bar{E}_2) = G_1\bar{E}_1 + G_2\bar{E}_2,$$

and partial differentiation of this equation gives:

$$G_1 = G_{11}\bar{E}_1 + G_1 + G_{21}\bar{E}_2 \Rightarrow G_{11}\bar{E}_1 + G_{21}\bar{E}_2 = 0$$

and $$G_2 = G_{12}\bar{E}_1 + G_{22}\bar{E}_2 + G_2 \Rightarrow G_{12}\bar{E}_1 + G_{22}\bar{E}_2 = 0.$$

Thus, if G is homogeneous of degree 1 and

$$\forall\gamma \quad \frac{E_1(\gamma)}{E_2(\gamma)} = \frac{\bar{E}_1}{\bar{E}_2}, \qquad (5.2)$$

then conditions (5.4a) and (5.4b) reduce to (5.5). However, if (5.2) does not hold for a set of types of positive measure, then the K^+ equilibrium is not 0-efficient. ■

Proposition 7 is a negative result, because the condition (5.2) is false in any interesting economy. Only in the very special case of separable production is the 0-efficiency of K^+ equilibrium preserved. We do have, however:

Proposition 8 Let G be concave. Then any K^+ equilibrium in the kibbutz economy (where each worker receives output $G(\bar{E}_1, \bar{E}_2)$) is 0-efficient.

Proof: Easy, as in Proposition 4.

I next generalize the definition of socialist allocation to the several-labor context. At a pair of effort schedules $(E_1(\cdot), E_2(\cdot))$, aggregate the efforts of individuals by multiplying their efforts by the 'efficiency wages'. Thus, in the socialist economy, the output accruing to an individual of type γ will be:

$$x(\gamma, E_1(\cdot), E_2(\cdot)) = \frac{G_1(\bar{E}_1, \bar{E}_2)E_1(\gamma) + G_2(\bar{E}_1, \bar{E}_2)E_2(\gamma)}{G_1(\bar{E}_1, \bar{E}_2)\bar{E}_1 + G_2(\bar{E}_1, \bar{E}_2)\bar{E}_2} G(\bar{E}_1, \bar{E}_2); \qquad (5.6)$$

if G is homogeneous of degree 1, then by Euler's law, the denominator of the fraction in (5.6) is $G(\bar{E}_1, \bar{E}_2)$ and so:

$$x(\gamma, E_1(\cdot), E_2(\cdot)) = G_1(\bar{E}_1, \bar{E}_2)E_1(\gamma) + G_2(\bar{E}_1, \bar{E}_2)E_2(\gamma). \qquad (5.7)$$

A multiplicative Kantian equilibrium is defined as a pair of effort schedules $(E_1(\cdot), E_2(\cdot))$ such that:

$$(\forall r,s \in \mathbb{R}_+)(\forall \gamma)(U[E_1(\cdot),E_2(\cdot),\gamma] \geq U[rE_1(\cdot),sE_2(\cdot),\gamma]) . \quad (5.8)$$

We have:

Proposition 9 Suppose G is homogenous of degree 1. Let $(E_1(\cdot),E_2(\cdot))$ be a positive multiplicative Kantian equilibrium for the socialist economy such that (5.2) holds. Then the equilibrium is 0-efficient.

Proof:

1. As before, we can replace U with u in the definition (5.8) of multiplicative Kantian equilibrium. The first-order conditions for the maximization required in (5.8) are:

$$\forall \gamma \quad u_1 \cdot \left(G_{11}\bar{E}_1 E_1(\gamma) + G_1 E_1(\gamma) + G_{21}\bar{E}_1 E_2(\gamma)\right) + u_2 E_1(\gamma) = 0 \quad (5.9a)$$

and $\forall \gamma \quad u_1 \cdot \left(G_{12}\bar{E}_2 E_1(\gamma) + G_2 E_2(\gamma) + G_{22}\bar{E}_2 E_2(\gamma)\right) + u_3 E_2(\gamma) = 0 . \quad (5.9b)$

Divide (5.9a) by $E_1(\gamma)$ giving:

$$\forall \gamma \quad u_1 \cdot \left(G_{11}\bar{E}_1 + G_1 + G_{21}\bar{E}_1 \frac{E_2(\gamma)}{E_1(\gamma)} \right) + u_2 = 0 ; \quad (5.10)$$

now replace $E_2(\gamma)/E_1(\gamma)$ by $\bar{E}_2/\bar{E}_1$, invoking (5.2), and note that $G_{11}\bar{E}_1 + G_{21}\bar{E}_2 = 0$ (see step 3 of the proof of Prop. 7). Thus (5.10) reduces to:

$$\forall \gamma \quad u_1 \cdot G_1 + u_2 = 0 . \quad (5.11a)$$

In like manner, (5.9b) reduces to:

$$\forall \gamma \quad u_1 \cdot G_2 + u_3 = 0 . \quad (5.11b)$$

But (5.11ab) are the conditions for 0-efficiency of an interior solution. ■

Again, Proposition 9 is disappointing, because condition (5.2) is not only singular but unnatural. The results of this section must be interpreted as saying that the efficiency of Kantian equilibrium does not extend in a strong way to economies with more than one kind of labor.

B. Other generalizations

I have looked at other possible Kantian variations $\varphi(E,r)$, and do not believe that generalization beyond the additive and multiplicative versions, studied here, is worthwhile. Moreover, I have not found interesting results by using taxation more general than affine. I cannot claim, however, to have exhausted the possibilities for generalization.

6. Is Kantian behavior plausible?

Certainly, parents try to teach Kantian behavior to their children, at least in some contexts. "Don't throw that candy wrapper on the ground: How would you feel if everyone did so?" The golden rule ("Do unto others as you would have them do unto you") is a special case of Kantian ethics. Wishful thinking ("if I do X, then all those who are similarly situated to me will do X"), although a predictive claim, rather than an ethical one, will also induce Kantian equilibrium – if all think that way. This may explain why people vote in large elections, and charitable contributions. So there is some reason to believe that Kantian equilibria are accessible to human societies.

Think about the relationship between the theoretical concept of Nash equilibrium and the empirical evidence (where it exists) that agents play the Nash equilibrium in certain social situations which can be modeled as games. Of course, we do not claim that agents are consciously computing the Nash equilibrium of the game: rather, we believe there is some process by which players *discover* the Nash equilibrium, and once it is discovered, it is stable, given autarkic reasoning. We now know there are many experimental situations in which players in a game do not play the Nash equilibrium. Conventionally, this behavior has been rationalized by proposing that players really have different payoff functions from the ones that the experimenter is trying to induce in them or assumes that they have. Another possibility, however, is that players in these games are playing some kind of Kantian equilibrium. The non-experimental (i.e., real-world) counterpart, as I have said in the introduction, may be the games that the societies which Ostrom has studied are playing. If these games can be modeled as 'fisher' economies, with common ownership of a resource whose use displays negative congestion externalities, and if, as Ostrom contends, these societies figure out how to engender efficient allocations of labor applied to the common resource, then they are discovering the multiplicative Kantian equilibrium of the game. Perhaps Kantian reasoning helps to maintain the equilibrium, if behavior is 'interdependent' and not 'autarkic.' (Ostrom often explains the maintenance of the efficient labor allocation by the use of sanctions and punishments, but that may not be the entire story.)

It should be noted that Kantian ethics, and therefore the behavior they induce, require *less* selflessness than another kind of ethic: putting oneself in the shoes of others. “I should give to the unfortunate, because I could have been unfortunate – indeed, but for the grace of God…” The Kantian ethic says, “I will give to the unfortunate an amount which I would like all others who are similarly situated to me to give.” Assuming that there a social ethos (that is, $\theta > 0$) this kind of reasoning may induce substantial charity – or, in the political case, fiscal redistribution. The Kantian ethic does not require the individual to place herself in the shoes of another. In this sense, it requires a less radical departure from self than ‘grace of God’ reasoning does.

My analysis has studied the consequences of assuming that the optimizing behavior of individuals might not be autarkic, as in Nash equilibrium, but interdependent, as in the various kinds of Kantian equilibrium. To the extent that human societies have prospered by invoking the ability of individuals of our species’ ability to cooperate with others, it is perhaps likely that Kantian reasoning is a cultural adaptation, selected by the evolution of cultures (the classic reference is Boyd and Richerson [1985]). Because we have shown that Kantian behavior can resolve, in many cases, the inefficiency of autarkic behavior, cultures which discover it, and attempt to induce that behavior in their members, will thrive relative to others.

7. Conclusion

I have studied economies with three kinds of property regime: private ownership, the kibbutz, and socialist allocation. In the macro version of the models, where labor is assumed to be only of one kind, Kantian equilibria resolve certain inefficiencies of Nash and Nash-Walras equilibrium. In private-ownership economies, if the production function is linear, then the additive Kantian equilibria with respect to any admissible affine income tax scheme are Pareto efficient when $\theta = 0$. There is a division of labor between Kantian optimizing behavior and political choice: the former guarantees allocative efficiency at any tax rate (for linear production), while political choice can choose a sufficiently redistributive policy to engender efficiency for economies with a considerable degree of social ethos. When production is concave non-linear, then additive Kantian equilibria are not 0-efficient, but combined with median-voter politics, my simulations suggest that they perform better than Nash-Walras equilibria for those societies.

The kibbutz economy involves an extreme kind of solidarity, where each contributes according to his ability (γ), but receives according to his need – here postulated to be uniform, hence uniform consumption. There is some historical evidence for the existence of kibbutz economies – not only in the Israeli communes from which the name is taken, but also in hunter-gatherer societies. In the kibbutz economy, additive Kantian equilibria are 0-efficient for any concave production function, and they will be efficient for some interval of positive θ, which may be quite large, if the distribution of types has a fairly small support. The kibbutz economy is also the 'camping trip' economy discussed by Cohen (2010), although he calls it an instance of 'socialism.'

Socialist allocation, in this paper, is defined to be the distribution of consumption in proportion to effort expended. In the macro version, multiplicative Kantian equilibrium engenders 0-efficiency for any concave *G*. These equilibria will typically be θ – efficient for some interval near zero – and again, the interval can be quite large if the support of the distribution of types is not large.

Finally, I have argued that, in our moral education of children, we often try to invoke Kantian behavior. I have suggested that, rather than explaining deviations from Nash equilibrium in games by positing that individuals have strange preferences, we study whether they are playing some kind of Kantian, rather than Nash, equilibrium. Because Kantian behavior appears to have significant advantages over Nash behavior in terms of efficiency – when there are either positive or negative externalities – we should ask whether cultural evolution has sometimes selected for it.

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