

# **Bias Transmission and Variance Reduction in Two-Stage Quantile Regressions**

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## **Abstract:**

In this paper, we integrate the structural approach and the fitted-value approach of the literature on quantile regression with endogeneity problems, including for the non-constant effect case. First, we derive the asymptotic distribution and the variance-covariance matrix of two-stage quantile estimators under very general conditions on both error terms and exogenous variables. Second, we exhibit a 'bias transmission property' from the asymptotic representation of our estimator. Third, we show how structural and fitted-values approaches can be integrated, providing some natural independence hypotheses for instruments. Fourth, we elicit the possibility of non-constant effects models with the fitted-value approach, a situation sometimes believed to be ruled out with this approach. Fifth, when the above independence hypotheses are not met, the fitted-value approach allows for the estimation of models that cannot be identified with the structural approach. Sixth, we introduce a new technique to improve the efficiency of two-stage quantile regressions. Using a reformulation of the dependent variable, we built a trade-off between an asymptotic bias confined to the intercept (or to coefficients of secondary interest) and a reduction of the variance of the other coefficients estimator. Monte Carlo simulation results show the excellent performance of our variance reduction approach.

**JEL Codes:** C13, C30.

**Key Words:** Two-Stage Estimation, Variance Reduction, Quantile Regression, Asymptotic Bias.

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