Inattention to Rare Events*

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Abstract

Why were people so unprepared for the global financial crisis, the European debt crisis, and the Fukushima nuclear accident? To address this question, we study a model in which agents make state-contingent plans - think about actions in different contingencies - subject to the constraint that agents can process only a limited amount of information. The model predicts that agents are unprepared in a state when the state has a low probability, the optimal action in that state is uncorrelated with the optimal action in normal times, and actions are strategic complements. We then compare the equilibrium allocation of attention to the efficient allocation of attention. We characterize analytically the conditions under which society would be better off if agents thought more carefully about optimal actions in rare events.

Keywords: rare events, disasters, rational inattention, efficiency. (*JEL*: D83, E58, E60).

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1 Introduction

Recently the world was struck by several events with major consequences: the global financial crisis, the European debt crisis, and the Fukushima nuclear accident. A common feature of these events is that people were unprepared for them. How come virtually no one had thought through what to do if an investment bank like Lehman Brothers collapses? How come virtually no one had thought through what to do if several governments in the euro area find themselves on the brink of default? How come virtually no one had thought through what to do if an earthquake and tsunami disable the cooling system of a nuclear reactor on the Japanese coast? The questions we ask in this paper are: Why were people so unprepared for these events? Under which circumstances will people be unprepared again in the future? Would a social planner want people to be more prepared for these events?

We believe that these are important questions. Had people been prepared to take good action in each of these events, each of these events would have unfolded less dramatically. For example, according to a report by the U.S. Nuclear Regulatory Commission the situation at the Fukushima nuclear power plant would have been substantially less severe if the Tokyo Electric Power Company (Tepco) had taken better actions following the earthquake and tsunami that hit Japan on March 11, 2011.[1] However, being well prepared for each contingency is costly. Therefore, it is unclear whether from an ex-ante perspective a social planner would want people to be more prepared for these events.

To address these questions formally, we study a model in which agents make state-contingent plans (i.e., they think about actions in different contingencies) subject to an information-processing constraint. There are two states. Agents commit today to actions in the different states tomorrow. This assumption captures the idea that decision-making takes time and once the state realizes agents have to act quickly. Therefore, agents need to plan ahead. Agents have a prior over what the optimal action is in each state and they can process additional information. However, agents can process only a *finite amount* of additional information. Subject to this constraint, agents decide how carefully to think about the optimal action in state one and the optimal action in state two.

We embed this decision problem into a setup with a continuum of agents. The payoff of an agent in a state depends on the agent's own action in that state, the mean action in the population

[1] The report was published by the German daily *Tagesspiegel* on its Web site www.tagesspiegel.de.

in that state, and a fundamental in that state. The payoff function is quadratic. The uncertainty about the optimal action in a state is due to uncertainty about the mean action in the population in that state and the fundamental in that state.

We derive the equilibrium allocation of attention and compare it to the efficient allocation of attention. Let us first describe the equilibrium allocation of attention and let us turn to efficiency thereafter.

The model makes the following predictions. If a state is less likely, agents think less carefully about the optimal action in that state, and thus the mean squared difference between the optimal action and the actual action in that state is larger. More precisely, agents equate the probability-weighted expected loss due to suboptimal actions across states. Therefore, the ratio of the expected loss due to suboptimal action in state one to the expected loss due to suboptimal action in state two equals one over the relative probability of state one. For example, if the relative probability of state one is 0.01, then the expected loss due to suboptimal action will be one hundred times larger in state one than in state two. Agents will take on average worse actions in the low probability state.

Furthermore, the correlation of optimal actions across states matters for the quality of actions taken in different states. Suppose one state has a high probability ("normal times") and the other state has a low probability ("unusual times"). Agents will think carefully about the optimal action in normal times and thus will take good actions in normal times. If the optimal action in normal times and the optimal action in unusual times are independent, thinking about the best action in normal times fails to improve actions in unusual times. However, if the optimal action in normal times and the optimal action in unusual times are correlated, thinking about the best action in normal times also improves actions in unusual times. Thus, agents will take on average good actions in the low probability state if the optimal action in the low probability state and the optimal action in the high probability state are highly correlated.

Finally, strategic complementarity in actions makes the allocation of attention more extreme. Suppose again that one state has a high probability ("normal times") and the other state has a low probability ("unusual times") and thus agents think less about the optimal action in unusual times. If actions are strategic complements (i.e., the optimal action in a state is increasing in the mean action in the population in that state), then the fact that other agents are not thinking carefully about the optimal action in unusual times reduces the incentive for an individual agent to think carefully about the optimal action in unusual times. As a result, the larger the degree of strategic complementarity in actions, the less agents think about the optimal action in unusual times. In fact, for a sufficiently high degree of strategic complementarity in actions, agents do not think at all about the optimal action in unusual times. Agents are completely inattentive to the rare event.

Let us look at the recent events from the perspective of the model. Why was Tepco so unprepared for the Fukushima nuclear accident? The model proposes the following answer: Humans have a limited ability to process information and therefore cannot prepare well for every contingency. A level nine earthquake is a low probability event; thinking carefully about how to run a nuclear power plant efficiently in normal times fails to improve actions in times when an earthquake and tsunami disabled the plant's cooling system;[2] and strategic complementarity in actions amplifies the effect of a low probability on the allocation of attention. We think the strategic complementarity in actions in this case arose because companies tend to be punished less if they fail in times when other companies are failing too.

Why were policy-makers, financial institutions, and academics so unprepared for the collapse of Lehman Brothers? The model proposes the following answer: Humans have a limited ability to process information and therefore cannot prepare well for every contingency. Collapse of one of the most important U.S. financial institutions seemed a priori unlikely; and thinking carefully about how to regulate financial institutions in normal times or how to fine-tune open market operations to achieve a desired level of the federal funds rate helps little when confronted with an imminent collapse of Lehman Brothers. Furthermore, we believe there is strategic complementarity in actions: Policy-makers within a government have to push a common agenda to get a bill passed in Congress. The management of a financial institution is punished less if it fails in times when other financial institutions are failing too.[3] Academics like to work on topics that other academics are working

[2] *Financial Times* in its May 7-8, 2011, issue quotes Goshi Hosono, a senior aide to Japan's prime minister, saying "Tepco's job is to deliver a constant supply of electricity – extremely routine work. It is a company for stable times."

[3] In 14 out of 15 leading U.S. and European banks, the chief executive officer in 2010 either was already the CEO before September 2008 (12 out of 15) or was a high ranking insider before September 2008 (2 out of 15). The only financial institution with a CEO in 2010 who was an outsider before September 2008 is Royal Bank of Scotland, effectively nationalized after September 2008. See the June 15, 2011, issue of *Financial Times*. We think this fact supports the idea that the management of a financial institution is punished little when it does poorly at a time when

on, because then those other academics are more likely to be interested in the work. Strategic complementarity in actions makes agents focus even more on one contingency.

Would a planner want people to be more prepared for rare events? To answer this question, we study the following planner problem. The planner can tell agents how to allocate their attention (i.e., the planner can tell agents how carefully to think about the optimal action in state one and the optimal action in state two), but the planner has to respect the agents' information-processing constraint (i.e., the constraint that agents can process only a limited amount of information). The planner maximizes ex-ante utility of the agents. We then ask: Does the equilibrium allocation of attention equal the efficient allocation of attention (i.e., the solution to the planner problem)? In other words, would society be better off if agents allocated their attention differently? Consider the case that the economy is efficient under perfect information, that is, inefficiencies, if any, arise due to agents' limited attention. We characterize analytically the relationship between the equilibrium allocation of attention and the efficient allocation of attention. It turns out that a simple condition on the payoff function of the agents governs the relationship between the equilibrium allocation of attention and the efficient allocation of attention. If the cross derivative of the payoff function with respect to the own action and the average action in the population and the second derivative of the payoff function with respect to the average action in the population sum to zero, the equilibrium allocation of attention equals the efficient allocation of attention. In this case, society cannot do better by creating incentives for agents or forcing agents to allocate their attention differently, for example, by passing a law that requires companies running nuclear power plants to have a precise plan for actions in the case of an earthquake or tsunami. The equilibrium allocation of attention already equals the efficient allocation of attention. Thus, ex-ante utility cannot be increased by changing the allocation of attention. However, if the sum of these second derivatives is negative, the planner would prefer agents to pay *more* attention to the state that they are devoting less attention to. Finally, if the sum of these second derivatives is positive, the planner would prefer agents to pay *even less* attention to the state that they are devoting less attention to.

This paper makes contact with several recent strands of literature. The paper is related to the literature on rational inattention building on Sims (2003).[4] The first main difference to the existing

other financial institutions do poorly too.

[4]For theoretical papers, see Sims (2003, 2006, 2010), Luo (2008), Maćkowiak and Wiederholt (2009, 2010), Van Nieuwerburgh and Veldkamp (2009, 2010), Woodford (2009), Matejka (2010 a,b), Mondria (2010), Paciello (2010),

literature on rational inattention is the application. We study how agents make state-contingent plans. Since agents commit to a contingent plan and have a limited ability to process information, the probability of a state affects the quality of the action taken in that state. The second main difference to the existing literature on rational inattention is that we compare the equilibrium allocation of attention to the efficient allocation of attention. That is, we ask whether society would be better off if agents allocated their attention differently. To the best of our knowledge, no one has done this before.

The paper is also related to Angeletos and Pavan (2007). Angeletos and Pavan (2007) study an economy with a continuum of agents in which each agent observes a noisy private and public signal. The precision of the two signals is exogenous. Actions are a linear function of the two signals and Angeletos and Pavan (2007) refer to the coefficients on the two signals as the "use of information." They then compare the equilibrium use of information to the efficient use of information, where the latter is defined as the one that maximizes ex-ante utility. We find that the condition that governs the relationship between the equilibrium and efficient use of information in Angeletos and Pavan (2007) also governs the relationship between the equilibrium and efficient allocation of attention in our model with an endogenous signal precision.

Hellwig and Veldkamp (2009) study a beauty contest model with information choice. The payoff of an agent depends on his own action, a fundamental, and the mean action in the population. Agents choose the number of signals that they acquire concerning the fundamental. The main differences to our model are that there is only one regime, agents face a fixed cost per signal (instead of a limited amount of attention), and their payoff function is less general. An (unpublished) working paper version of Hellwig and Veldkamp (2009) contains a subsection studying efficiency of information acquisition for a very particular quadratic payoff function.[5] For this payoff function, there exists an equilibrium which is ex ante efficient. This result is consistent with our result concerning efficiency of the equilibrium allocation of attention, because the assumed payoff function satisfies the sufficient condition for ex-ante efficiency described on the previous page.[6]

Paciello and Wiederholt (2011), Tutino (2011), and Yang (2011). For empirical papers, see Maćkowiak, Moench, and Wiederholt (2009), Kacperczyk, Van Nieuwerburgh, and Veldkamp (2011), Melosi (2011), and Coibion and Gorodnichenko (2011).

[5] The payoff of an agent is a linear combination of the squared distance between his own action and the fundamental and the squared distance between his own action and the mean action in the population.

[6] Llosa and Venkateswaran (2011) extend the efficiency result in the working paper version of Hellwig and Veldkamp

This paper also makes contact with the literature on rare large disasters. See for example Barro (2006), Barro, Nakamura, Steinsson, and Ursua (2010), Gabaix (2010), and Gourio (2010). This literature investigates the implications of rare large disasters for asset prices and business cycles. In this literature, agents act perfectly in a rare event. We model agents as acting imperfectly in a rare event. We then investigate how much incentive agents have to prepare for a rare event. If people had been prepared to take good action in historical rare adverse events, these events would have unfolded less dramatically and perhaps would not be called "disasters" today.

The paper is organized as follows. Section 2 presents the model. Section 3 presents the analytical solution of the model when optimal actions are independent across states. Section 4 studies the case of correlated optimal actions. Section 5 compares the equilibrium allocation of attention to the efficient allocation of attention. Section 6 considers an extension: Bayesian learning about the probability of the rare event. Section 7 concludes.

2 Model

We study an economy with a continuum of agents indexed by $i \in [0,1]$. Time is discrete and indexed by $t = 0, 1, 2, \ldots$.

Each period the economy is in one of two regimes. The regime follows a two-state Markov chain. For simplicity, the regime is i.i.d. over time. In the following, we refer to regime one as state one and regime two as state two. Let p_n denote the probability of being in state n. Both states have positive probability, that is, $p_1 > 0$ and $p_2 > 0$.

Every period each agent i commits to a state-contingent plan for the next period. This assumption captures the idea that decision-making takes time and once the state realizes agents have to act quickly. Therefore, agents need to plan ahead. Let $a_{i,t} = (a_{i,t,1}, a_{i,t,2}) \in \mathbb{R}^2$ denote the state-contingent plan that agent i commits to in period $t-1$ for period t; where $a_{i,t,n}$ denotes the action that agent i will take at time t in state n.

Let $\Psi^{n,t}$ denote the cumulative distribution function for action $a_{i,t,n}$ in the cross-section of the population. The payoff of agent i at time t in state n is given by $U^n(a_{i,t,n}, a_{t,n}, z_{t,n})$ where $a_{i,t,n}$ is the action of agent i at time t in state n, $a_{t,n} \equiv \int a_{i,t,n} d\Psi^{n,t}(a_{i,t,n})$ is the mean of individual actions

(2009) to a somewhat more general payoff function and study in detail a price setting application.

in the population, and $z_{t,n}$ is an exogenous payoff-relevant variable. The superscript n indicates that the payoff function may differ across states. For tractability, we assume that U^n is quadratic

$$\begin{aligned} U^n(a_{i,t,n}, a_{t,n}, z_{t,n}) &= U^n(0,0,0) + U^n_{a_i} a_{i,t,n} + U^n_a a_{t,n} + U^n_z z_{t,n} \\ &+ \frac{U^n_{a_i a_i}}{2} a^2_{i,t,n} + \frac{U^n_{aa}}{2} a^2_{t,n} + \frac{U^n_{zz}}{2} z^2_{t,n} \\ &+ U^n_{a_i a} a_{i,t,n} a_{t,n} + U^n_{a_i z} a_{i,t,n} z_{t,n} + U^n_{az} a_{t,n} z_{t,n}. \end{aligned} \tag{1}$$

This assumption can also be viewed as a second-order approximation of any twice differentiable function with these three arguments. We also assume that U^n is concave in its first argument ($U^n_{a_i a_i} < 0$), the exogenous variable $z_{t,n}$ affects the payoff-maximizing action ($U^n_{a_i z} \neq 0$), and the degree of strategic complementarity or substitutability does not exceed one ($-1 < U^n_{a_i a}/U^n_{a_i a_i} < 1$). In the following, we often exploit the fact that the payoff function U^n can be expressed as[7]

$$U^n(a_{i,t,n}, a_{t,n}, z_{t,n}) = U^n\left(a^*_{i,t,n}, a_{t,n}, z_{t,n}\right) + \frac{U^n_{a_i a_i}}{2}\left(a_{i,t,n} - a^*_{i,t,n}\right)^2,$$

where

$$a^*_{i,t,n} \equiv -\frac{U^n_{a_i}}{U^n_{a_i a_i}} - \frac{U^n_{a_i a}}{U^n_{a_i a_i}} a_{t,n} - \frac{U^n_{a_i z}}{U^n_{a_i a_i}} z_{t,n}.$$

Finally, we assume that the coefficients on $a_{t,n}$ and $z_{t,n}$ in the last equation sum to one. This assumption is without loss in generality. If this assumption is not satisfied, one can always redefine the fundamental $z_{t,n}$ by multiplying it with a constant to ensure that this assumption is satisfied. Defining $\delta_n \equiv -U^n_{a_i a_i}/2$, $\varphi_n \equiv -U^n_{a_i}/U^n_{a_i a_i}$ and $\gamma_n \equiv -U^n_{a_i a}/U^n_{a_i a_i}$, the last two equations then become

$$U^n(a_{i,t,n}, a_{t,n}, z_{t,n}) = U^n\left(a^*_{i,t,n}, a_{t,n}, z_{t,n}\right) - \delta_n\left(a_{i,t,n} - a^*_{i,t,n}\right)^2, \tag{2}$$

with

$$a^*_{i,t,n} = \varphi_n + \gamma_n a_{t,n} + (1-\gamma_n) z_{t,n}. \tag{3}$$

For simplicity, the vector of fundamentals $z_t = (z_{t,1}, z_{t,2})$ is i.i.d. over time. Agents have the common prior belief that the vector of fundamentals is i.i.d. over time and that the fundamental in state one and the fundamental in state two are normally distributed with mean zero and covariance

[7] To obtain this result, compute a Taylor expansion of U^n around $a^*_{i,t,n}$ and notice that the first derivative of U^n with respect to $a_{i,t,n}$ evaluated at $a^*_{i,t,n}$ equals zero and the second derivative of U^n with respect to $a_{i,t,n}$ equals $U^n_{a_i a_i}$.

matrix Σ, that is, $z_t = (z_{t,1}, z_{t,2}) \sim i.i.d.N(0, \Sigma)$. There is prior uncertainty about the fundamental in both states and the fundamentals in the two states are not perfectly correlated, that is, Σ is non-singular. One can think of the prior uncertainty about the vector of fundamentals as reflecting uncertainty about how the economy functions at time t in the two states.

Agents can process additional information before committing to a plan. However, agents can process only a *limited amount* of additional information. Processing information about the optimal action in state one and the optimal action in state two in the next period is modeled as receiving a noisy signal concerning the fundamentals in the two states in the next period

$$s_{i,t-1} = \begin{pmatrix} z_{t,1} \\ z_{t,2} \end{pmatrix} + \begin{pmatrix} \varepsilon_{i,t-1,1} \\ \varepsilon_{i,t-1,2} \end{pmatrix},$$

where the noise $(\varepsilon_{i,t-1,1}, \varepsilon_{i,t-1,2})$ is independent of the fundamentals, normally distributed with mean zero and covariance matrix Λ, and independent across individuals and over time. Let $\Omega = \Sigma - \Sigma(\Sigma + \Lambda)^{-1}\Sigma$ denote the posterior covariance matrix of z_t after receiving $s_{i,t-1}$. Following Sims (2003), we model the fact that humans have a limited ability to process information as a constraint on uncertainty reduction, where uncertainty is measured by entropy. That is, each agent faces the following constraint on uncertainty reduction:

$$\frac{1}{2}\log_2\left(\frac{|\Sigma|}{|\Omega|}\right) \leq \kappa,$$

where $|\Sigma|$ denotes the determinant of the prior covariance matrix of z_t and $|\Omega|$ denotes the determinant of the posterior covariance matrix of z_t after receiving $s_{i,t-1}$. The parameter $\kappa > 0$ indexes the ability of an agent to process information. A larger κ means an agent can process more information and can therefore reduce uncertainty by more.

Subject to the information-processing constraint, each agent decides how carefully to think about the optimal action in state one and the optimal action in state two. Agents aim to maximize the expected payoff in the next period. Formally, agent i solves in period $t-1$

$$\max_{\Lambda}\left\{\sum_{n=1}^{2} p_n E\left[U^n\left(a_{i,t,n}, a_{t,n}, z_{t,n}\right)\right]\right\}, \tag{4}$$

subject to

$$a_{i,t,n} = E\left[\varphi_n + \gamma_n a_{t,n} + (1-\gamma_n) z_{t,n} | s_{i,t-1}\right], \tag{5}$$

$$s_{i,t-1} = \begin{pmatrix} z_{t,1} \\ z_{t,2} \end{pmatrix} + \begin{pmatrix} \varepsilon_{i,t-1,1} \\ \varepsilon_{i,t-1,2} \end{pmatrix}, \tag{6}$$

and

$$\frac{1}{2}\log_2\left(\frac{|\Sigma|}{|\Omega|}\right) \leq \kappa, \tag{7}$$

and the restriction that Λ is a positive semidefinite matrix. Objective (4) is the expected payoff in the next period. Equation (5) states that the agent will commit to the best plan given his or her posterior. Equation (6) is the signal and constraint (7) is the information-processing constraint.

Note that the covariance matrix of noise Λ and the posterior covariance matrix of the fundamentals Ω have no subscripts i and t. The reason is that the solution to problem (4)-(7) is the same for each agent i and every period t. This also means that the equilibrium is symmetric and that agents only have to solve this problem once.[8]

3 Analytical solution when optimal actions are independent

When the optimal action in state one and the optimal action in state two are independent, the model can be solved analytically. We use this analytical solution to illustrate how the probability of state one and the degree of strategic complementarity in actions affect the extent to which agents think about the optimal action in state one.

Proposition 1 *Assume that the optimal action in state one and the optimal action in state two are independent (i.e., Σ is diagonal). Consider equilibria of the form $a_{t,n} = \psi_n + \phi_n z_{t,n}$ where ψ_n and ϕ_n are coefficients. Then, each agent decides to receive independent signals about the fundamental in state one and the fundamental in state two (i.e., the covariance matrix of noise Λ solving problem (4)-(7) is diagonal). Furthermore, the information-processing constraint reduces to*

$$\underbrace{\frac{1}{2}\log_2\left(\frac{\Sigma_{11}}{\Omega_{11}}\right)}_{\kappa_1} + \underbrace{\frac{1}{2}\log_2\left(\frac{\Sigma_{22}}{\Omega_{22}}\right)}_{\kappa_2} \leq \kappa,$$

where Σ_{nn} and Ω_{nn} denote the prior and the posterior variance of the fundamental in state n and κ_n denotes the uncertainty reduction about the fundamental in state n. Assume that $\gamma_1 = \gamma_2 \equiv \gamma$.

[8] Note also that we have assumed that signals are normally distributed. One can show that Gaussian signals are optimal given the quadratic objective, the Gaussian prior, and the constraint on entropy reduction. See Sims (2006).

If the parameters κ and γ satisfy $2^{\kappa} > \frac{\gamma}{1-\gamma}$ and $(1-\gamma)\, 2^{\kappa} + \gamma 2^{-\kappa} > 1$, the equilibrium is unique and the attention allocated to thinking about the optimal action in state one equals

$$\kappa_1 = \begin{cases} \kappa & \text{if } \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \geq (1-\gamma)\, 2^{\kappa} + \gamma 2^{-\kappa} \\ \frac{1}{2}\kappa + \frac{1}{2}\log_2(x) & \text{if } \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \in \left[\frac{1}{(1-\gamma)2^{\kappa}+\gamma 2^{-\kappa}}, (1-\gamma)\, 2^{\kappa} + \gamma 2^{-\kappa}\right] \\ 0 & \text{if } \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \leq \frac{1}{(1-\gamma)2^{\kappa}+\gamma 2^{-\kappa}} \end{cases}, \tag{8}$$

where

$$x \equiv \frac{\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} - \frac{\gamma}{1-\gamma}2^{-\kappa}}{1 - \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{\gamma}{1-\gamma}2^{-\kappa}}. \tag{9}$$

Furthermore, for any parameters κ and γ, the set of equilibria is given by the following results: (1) $\kappa_1 = \kappa$ is an equilibrium if $\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \geq (1-\gamma)\, 2^{\kappa} + \gamma 2^{-\kappa}$; (2) $\kappa_1 = 0$ is an equilibrium if $\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \leq \frac{1}{(1-\gamma)2^{\kappa}+\gamma 2^{-\kappa}}$; (3) $\kappa_1 = \frac{1}{2}\kappa + \frac{1}{2}\log_2(x)$ is an equilibrium if $(1-\gamma)\, 2^{\kappa} + \gamma 2^{-\kappa} \geq 1$, $\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \in \left[\frac{1}{(1-\gamma)2^{\kappa}+\gamma 2^{-\kappa}}, (1-\gamma)\, 2^{\kappa} + \gamma 2^{-\kappa}\right]$ and $\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{\gamma}{1-\gamma} < 2^{\kappa}$; or $(1-\gamma)\, 2^{\kappa} + \gamma 2^{-\kappa} \leq 1$, $\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \in \left[(1-\gamma)\, 2^{\kappa} + \gamma 2^{-\kappa}, \frac{1}{(1-\gamma)2^{\kappa}+\gamma 2^{-\kappa}}\right]$ and $\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{\gamma}{1-\gamma} > 2^{\kappa}$; and (4) any $\kappa_1 \in [0, \kappa]$ is an equilibrium if $\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} = \frac{\gamma}{1-\gamma}2^{-\kappa} = 1$.

Proof. See Appendix A. ■

To understand Proposition 1, consider first the simplest case. If $\gamma = 0$, the payoff-maximizing action of an agent depends only on the fundamental not on the average action in the population, that is, actions are neither strategic complements nor strategic substitutes. See equation (3). When Σ and Λ are diagonal, the decision problem (4)-(7) then reduces to

$$\max_{(\kappa_1,\kappa_2)\in\mathbb{R}^2_+} \left[-\sum_{n=1}^{2} p_n\delta_n\Omega_{nn}\right], \tag{10}$$

subject to

$$\Omega_{nn} = \Sigma_{nn}2^{-2\kappa_n}, \tag{11}$$

and

$$\kappa_1 + \kappa_2 = \kappa. \tag{12}$$

The unique solution to this problem is

$$\kappa_1 = \begin{cases} \kappa & \text{if } \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \geq 2^{\kappa} \\ \frac{1}{2}\kappa + \frac{1}{2}\log_2\left(\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\right) & \text{if } \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \in [2^{-\kappa}, 2^{\kappa}] \\ 0 & \text{if } \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \leq 2^{-\kappa} \end{cases}. \tag{13}$$

The attention allocated to thinking about the optimal action in state one is increasing in the agent's information-processing ability, κ, the relative probability of state one, p_1/p_2, the relative cost of a mistake in state one, δ_1/δ_2, and the relative prior variance of the fundamental in state one, Σ_{11}/Σ_{22}. If the probability of state one is sufficiently low, agents do not think at all about the optimal action in state one; if the probability of state one is in an intermediate range, agents think to some extent about the optimal action in both states; and if the probability of state one is sufficiently high, agents think only about the optimal action in state one. Finally, if $\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}} = 1$ agents think to the same extent about the optimal action in state one and the optimal action in state two.

The equilibrium allocation of attention affects the quality of actions taken in the two states. If in equilibrium agents think to some extent about the optimal action in both states (i.e., $0 < \kappa_1 < \kappa$), the mean squared difference between the optimal action and the actual action in state one equals

$$\Omega_{11} = \Sigma_{11}\left(2^{\kappa}\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\right)^{-1}.$$

The mean squared difference between the optimal action and the actual action in state two equals

$$\Omega_{22} = \Sigma_{22}\left(2^{\kappa}\sqrt{\frac{p_2\delta_2\Sigma_{22}}{p_1\delta_1\Sigma_{11}}}\right)^{-1}.$$

This follows from equations (11)-(13). Combining these two equations yields

$$p_1\delta_1\Omega_{11} = p_2\delta_2\Omega_{22}.$$

In words, agents equate the probability-weighted expected loss due to suboptimal action across states. This implies that

$$\frac{\delta_1\Omega_{11}}{\delta_2\Omega_{22}} = \frac{1}{\frac{p_1}{p_2}}. \tag{14}$$

The ratio of the expected loss due to suboptimal action in state one to the expected loss due to suboptimal action in state two equals one over the relative probability of state one. Suppose that state one has a low probability ("unusual times") and state two has a high probability ("normal times"). The model predicts that agents decide to take on average worse actions in state one than in state two. Observing that agents take good actions in normal times does not imply that agents will take good actions in unusual times! Agents may simply be focusing on normal times. Quantitatively, the model predicts that if the probability of state one is 0.01, then the expected loss due to suboptimal action will be ninety nine times larger in state one than in state two.

Next, consider the case of strategic complementarity in actions. Here strategic complementarity in actions means that the payoff-maximizing action of an agent depends positively on the average action in the population, that is, $\gamma > 0$. See equation (3). Strategic complementarity in actions makes the equilibrium allocation of attention more extreme. Whatever agents were paying more attention to in the absence of strategic complementarity, agents are paying even more attention to in the presence of strategic complementarity (if possible, that is, if the allocation of attention in the absence of strategic complementarity was not already a corner solution). This result can be seen from equations (8)-(9). We also illustrate this result with a figure. Figure 1 depicts the equilibrium allocation of attention as a function of $\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}$; for parameters κ and γ satisfying $2^{\kappa} > \gamma/(1-\gamma)$ and $(1-\gamma)\,2^{\kappa}+\gamma 2^{-\kappa} > 1$. These parameter restrictions ensure that the equilibrium allocation of attention is unique (see Proposition 1). In Figure 1 $\gamma = 0$ denotes the case of no strategic complementarity in actions, $\gamma >> 0$ denotes a value of γ close to the value at which $(1-\gamma)\,2^{\kappa} + \gamma 2^{-\kappa} = 1$, and $\gamma > 0$ denotes a value of γ between these two extremes. Pick any value of $\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}$ with the property $\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \neq 1$, for example, a value with $\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} < 1$. In the absence of strategic complementarity in actions ($\gamma = 0$), agents think less about the optimal action in state one than about the optimal action in state two ($\kappa_1 < \frac{1}{2}\kappa$). As the degree of strategic complementarity in actions increases (from $\gamma = 0$ to $\gamma > 0$ or $\gamma >> 0$), agents think even less about the optimal action in state one (κ_1 falls). The reason is the following. When actions are strategic complements, the fact that other agents are not thinking carefully about the optimal action in a state reduces the incentive for an individual agent to think about the optimal action in that state. This effect of strategic complementarity in actions on the equilibrium allocation of attention is well understood in the literature on information choice. See, for example, Maćkowiak and Wiederholt (2009) and Hellwig and Veldkamp (2009). In addition, as the degree of strategic complementarity increases, corner solutions occur more easily. This implies that when the degree of strategic complementarity is high, small changes in the probability of the two states can have large effects on the equilibrium allocation of attention. In fact, as γ approaches the value of γ at which $(1-\gamma)\,2^{\kappa} + \gamma 2^{-\kappa} = 1$, the parameter region in which the equilibrium allocation of attention is an interior solution collapses to a single point. Finally, for a sufficiently high degree of strategic complementarity, there exist multiple equilibria. Namely, whenever $(1-\gamma)\,2^{\kappa} + \gamma 2^{-\kappa} \leq 1$, there exists more than one equilibrium allocation of attention. See Proposition 1.

Strategic substitutability in actions has the opposite effect. Strategic substitutability in actions (i.e., $\gamma < 0$) makes the equilibrium allocation of attention less extreme. This result can again be seen from equations (8)-(9).

In Proposition 1 it is assumed that the degree of strategic complementarity in actions is the same across states, that is, $\gamma_1 = \gamma_2 \equiv \gamma$. In Appendix A we also characterize in closed form the set of equilibria when the degree of strategic complementarity differs across states. Suppose that actions are strategic complements in both states but the degree of strategic complementarity may differ across states. The equations given in Appendix A then imply the following results. There exists a unique equilibrium for all $\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \in \mathbb{R}_{++}$ if and only if

$$\frac{1}{(1-\gamma_1)\left[2^{\kappa}+\frac{\gamma_2}{1-\gamma_2}2^{-\kappa}\right]} < (1-\gamma_2)\left[2^{\kappa}+\frac{\gamma_1}{1-\gamma_1}2^{-\kappa}\right] < \frac{1}{\frac{\gamma_2}{1-\gamma_2}2^{-\kappa}}. \tag{15}$$

The attention allocated to state one then equals

$$\kappa_1 = \begin{cases} \kappa & \text{if } \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \geq (1-\gamma_2)\left[2^{\kappa}+\frac{\gamma_1}{1-\gamma_1}2^{-\kappa}\right] \\ \frac{1}{2}\kappa + \frac{1}{2}\log_2(x) & \text{otherwise} \\ 0 & \text{if } \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \leq \frac{1}{(1-\gamma_1)\left[2^{\kappa}+\frac{\gamma_2}{1-\gamma_2}2^{-\kappa}\right]} \end{cases},$$

where

$$x \equiv \frac{\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} - \frac{\gamma_1}{1-\gamma_1}2^{-\kappa}}{1-\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{\gamma_2}{1-\gamma_2}2^{-\kappa}}.$$

These equations and condition (15) imply that, if the equilibrium allocation of attention is an interior solution, increasing the probability of a state raises the attention allocated to that state. Furthermore, increasing the degree of strategic complementarity in a single state (an experiment we could not do before because we assumed that the degree of strategic complementarity was the same across states) reduces the attention allocated to that state. Finally, when the degree of strategic complementarity differs across states, there are many different ways of increasing the degree of strategic complementarity simultaneously in both states (e.g., one can increase the degree of strategic complementarity by the same absolute amount in both states or by the same percentage amount in both states). Multiplying $\gamma_1/(1-\gamma_1)$ and $\gamma_2/(1-\gamma_2)$ by the same constant $c > 1$, reduces the attention allocated to state one if and only if

$$\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}} < \frac{\frac{\gamma_1}{1-\gamma_1}}{\frac{\gamma_2}{1-\gamma_2}}.$$

When the degree of strategic complementarity is the same across states (i.e., $\gamma_1 = \gamma_2 \equiv \gamma$), this statement reduces to the statement made earlier that if agents allocate less attention to state one, then raising the degree of strategic complementarity in both states reduces the attention allocated to state one.

4 Correlated optimal actions

In this section we relax the assumption that optimal actions are independent across states. We study how the correlation of optimal actions across states affects the quality of actions taken in the different states. We consider the special case of no strategic complementarity in actions, that is, $\gamma_1 = \gamma_2 = 0$. In this case the solution to the decision problem of a single agent is also the solution of the model, because there is no interaction between agents.

The decision problem of a single agent is given by equation (4)-(7). The statement "the optimal action in state one is correlated with the optimal action in state two" means that the matrix Σ appearing in constraint (7) is non-diagonal. We solve the problem (4)-(7) numerically for different values of the covariance between the optimal actions in the two states, Σ_{12}.

It is simplest to understand the solution when one supposes that: (i) state one has a low probability ("unusual times") and state two has a high probability ("normal times"), and (ii) the cost of a mistake in each state is the same, $\delta_1 = \delta_2$. The solution has the following feature: The larger in absolute value the prior correlation of the optimal actions across the two states, the smaller the expected loss in unusual times. To see why this result arises, start in the case when there is independence between the optimal action in normal times and the optimal action in unusual times. Agents then think mostly about the optimal action in normal times, and thinking about the optimal action in normal times gives no information about the optimal action in unusual times. Consequently, the expected loss in unusual times is large. Next, suppose the optimal action in normal times and the optimal action in unusual times are correlated a priori. Now thinking about the best action in normal times gives some information about the best action in unusual times. Consequently, the expected loss in unusual times falls. The stronger the prior correlation, the stronger this effect and thus the smaller the mean squared difference between the actual action and the optimal action in unusual times.

Another feature of the solution is that the posterior correlation of the optimal actions across states is a convex function of their prior correlation. When the prior correlation rises in absolute value, the posterior correlation stays close to zero at first and then swiftly moves to one in absolute value. So long as the prior correlation is not too large in absolute value, it is optimal to behave practically as if the prior correlation were zero.

Consider a numerical example. Suppose that $\Sigma_{11} = \Sigma_{22} = 1$, i.e. the prior variance of the optimal action in each state equals one. Furthermore, let $p_1 = 0.01$ meaning that the probability of unusual times is 0.01. To begin with, suppose that $\Sigma_{12} = 0$, i.e. the optimal actions are independent across the states. In this case, Λ and Ω are diagonal. We choose a value of κ such that the posterior variance of the optimal action in normal times, Ω_{22}, equals 0.01.[9] Then it turns out that the posterior variance of the optimal action in unusual times, Ω_{11}, equals 0.99. Suppose that Σ_{12} rises, i.e. the optimal actions become more and more positively correlated a priori, and all other parameters remain unchanged.[10] Figure 2 shows Ω_{11}, Ω_{22}, and the posterior correlation of the optimal actions, $\Omega_{12}/\sqrt{\Omega_{11}\Omega_{22}}$, as functions of Σ_{12}. Ω_{11} falls and is concave. More prior correlation in the optimal actions implies that agents do better on average in unusual times, but concavity means that this effect sets in slowly.[11] Furthermore, $\Omega_{12}/\sqrt{\Omega_{11}\Omega_{22}}$ rises and is convex. For values of Σ_{12} as large as 0.8, Ω_{12} is as small as 0.1 meaning that it is optimal to behave practically as if Σ_{12} were zero.

5 Efficient allocation of attention

Would society be better off from an ex-ante perspective if agents allocated their attention differently? To answer this question, we study the following planner problem. The planner can tell agents how to allocate their attention (i.e., how carefully to think about the optimal actions in the different states). The planner has to respect the agents' information-processing constraint (i.e., the planner has to respect that agents can process only a limited amount of information). Finally, the planner maximizes ex-ante utility of the agents. The propositions in this section characterize

[9] This value of Ω_{22} means that thinking about the optimal action in normal times reduces the variance of that action by a factor of 100.

[10] Since $\Sigma_{11} = \Sigma_{22} = 1$, Σ_{12} is both the prior covariance of the optimal actions and their prior correlation.

[11] Ω_{22} also falls.

analytically the relationship between the equilibrium allocation of attention and the efficient allocation of attention (i.e., the solution to the planner problem). When the two coincide, ex-ante utility cannot be raised by creating incentives for agents to allocate their attention differently, for example, by passing a law that requires companies running nuclear power plants to have a precise plan for actions in the case of an earthquake or tsunami. The equilibrium allocation of attention already equals the efficient allocation of attention. When the two differ, ex-ante utility can be raised by changing the allocation of attention.

Before stating the planner problem, we derive a simple expression for expected utility in state n, that is, $E\left[U^n\left(a_{i,t,n}, a_{t,n}, z_{t,n}\right)\right]$. The derivation follows closely the derivation of a similar expression in Angeletos and Pavan (2007). Let $\tilde{U}^n\left(a_{t,n}, z_{t,n}\right) \equiv U^n\left(a_{t,n}, a_{t,n}, z_{t,n}\right)$ denote the payoff in state n when all agents take the same action $a_{i,t,n} = a_{t,n}$. It follows from equation (1) that

$$\begin{aligned}\tilde{U}^n\left(a_{t,n}, z_{t,n}\right) = {} & U^n(0,0,0) + \left(U^n_{a_i} + U^n_a\right) a_{t,n} + U^n_z z_{t,n} \\ & + \frac{U^n_{a_i a_i} + 2U^n_{a_i a} + U^n_{aa}}{2} a^2_{t,n} + \frac{U^n_{zz}}{2} z^2_{t,n} + \left(U^n_{a_i z} + U^n_{az}\right) a_{t,n} z_{t,n}. \end{aligned} \tag{16}$$

In the following, we assume that $\tilde{U}^n\left(a_{t,n}, z_{t,n}\right)$ is concave in its first argument, that is,

$$U^n_{a_i a_i} + 2U^n_{a_i a} + U^n_{aa} < 0. \tag{17}$$

Let $a^*_{t,n}$ denote the common action $a_{t,n} \in \mathbb{R}$ that maximizes $\tilde{U}^n\left(a_{t,n}, z_{t,n}\right)$. It follows from equations (16) and (17) that

$$a^*_{t,n} = -\frac{U^n_{a_i} + U^n_a}{U^n_{a_i a_i} + 2U^n_{a_i a} + U^n_{aa}} - \frac{U^n_{a_i z} + U^n_{az}}{U^n_{a_i a_i} + 2U^n_{a_i a} + U^n_{aa}} z_{t,n}. \tag{18}$$

One can show that expected utility in state n equals

$$\begin{aligned} E\left[U^n\left(a_{i,t,n}, a_{t,n}, z_{t,n}\right)\right] = {} & E\left[\tilde{U}^n\left(a^*_{t,n}, z_{t,n}\right)\right] - \frac{\left|U^n_{a_i a_i} + 2U^n_{a_i a} + U^n_{aa}\right|}{2} E\left[\left(a_{t,n} - a^*_{t,n}\right)^2\right] \\ & - \frac{\left|U^n_{a_i a_i}\right|}{2} E\left[\left(a_{i,t,n} - a_{t,n}\right)^2\right]. \end{aligned} \tag{19}$$

The proof is in Appendix B. The last equation implies that expected utility is maximized when all agents take the action $a^*_{t,n}$ for all $z_{t,n}$, that is, $a_{i,t,n} = a^*_{t,n}$ for all $z_{t,n}$. There is a loss in expected utility when the mean action in the population does not move one for one with $a^*_{t,n}$ (the second term on the right-hand side of the last equation) and when there is dispersion in actions (the third term on the right-hand side of the last equation).

When Σ is diagonal and the planner considers equilibria of the form $a_{t,n} = \psi_n + \phi_n z_{t,n}$, the problem of the planner who chooses the allocation of attention of the agents so as to maximize expected utility of the agents reads:

$$\max_{(\kappa_1,\kappa_2)\in\mathbb{R}_+^2} \sum_{n=1}^{2} p_n \left\{ \frac{U^n_{a_i a_i} + 2U^n_{a_i a} + U^n_{aa}}{2} E\left[\left(a_{t,n} - a^*_{t,n}\right)^2\right] + \frac{U^n_{a_i a_i}}{2} E\left[\left(a_{i,t,n} - a_{t,n}\right)^2\right] \right\}, \tag{20}$$

subject to equation (18),

$$a_{t,n} = \psi_n + \phi_n z_{t,n}, \tag{21}$$

$$a_{i,t,n} = (\varphi_n + \gamma_n \psi_n) + (\gamma_n \phi_n + 1 - \gamma_n) \frac{\frac{\Sigma_{nn}}{\Lambda_{nn}}}{\frac{\Sigma_{nn}}{\Lambda_{nn}} + 1} \left(z_{t,n} + \varepsilon_{i,t-1,n}\right), \tag{22}$$

$$\psi_n = \frac{\varphi_n}{1-\gamma_n}, \phi_n = \frac{(1-\gamma_n)\frac{\frac{\Sigma_{nn}}{\Lambda_{nn}}}{\frac{\Sigma_{nn}}{\Lambda_{nn}}+1}}{1-\gamma_n \frac{\frac{\Sigma_{nn}}{\Lambda_{nn}}}{\frac{\Sigma_{nn}}{\Lambda_{nn}}+1}}, \tag{23}$$

$$\frac{\Sigma_{nn}}{\Lambda_{nn}} = 2^{2\kappa_n} - 1, \tag{24}$$

and

$$\kappa_1 + \kappa_2 \leq \kappa. \tag{25}$$

Objective (20) is expected utility of the agents minus $\sum_{n=1}^{2} p_n E\left[\tilde{U}^n\left(a^*_{t,n}, z_{t,n}\right)\right]$, which is a term that the planner cannot affect. Equation (22) follows from equations (5)-(6) and equation (21). Equation (23) follows from equations (21)-(22), the definition of $a_{t,n}$, and the assumption that noise washes out in the aggregate. Equation (24) follows from the definition $\kappa_n \equiv \frac{1}{2}\log_2\left(\frac{\Sigma_{nn}}{\Omega_{nn}}\right)$ and $\Omega_{nn} = \Sigma_{nn} - \Sigma_{nn}\left(\Sigma_{nn} + \Lambda_{nn}\right)^{-1}\Sigma_{nn}$. Finally, constraint (25) is the information-processing constraint of the agents in the case of diagonal Σ and Λ.

In the following, we focus on the case that the economy is efficient under perfect information, that is, the equilibrium actions under perfect information equal the welfare-maximizing actions. It follows from equation (5) and the definition of $a_{t,n}$ that the equilibrium actions under perfect information are given by

$$a_{i,t,n} = \frac{\varphi_n}{1-\gamma_n} + z_{t,n}.$$

The welfare-maximizing actions are given by $a_{i,t,n} = a^*_{t,n}$ where $a^*_{t,n}$ is given by equation (18). The condition that the equilibrium actions under perfect information equal the welfare-maximizing actions thus reads

$$-\frac{U^n_{a_i} + U^n_a}{U^n_{a_i a_i} + 2U^n_{a_i a} + U^n_{aa}} = \frac{\varphi_n}{1-\gamma_n}, \tag{26}$$

and

$$-\frac{U_{a_i z}^n + U_{az}^n}{U_{a_i a_i}^n + 2U_{a_i a}^n + U_{aa}^n} = 1. \tag{27}$$

Substituting equation (18), equations (21)-(24), and equations (26)-(27) into the planner's objective (20) gives

$$\max_{(\kappa_1,\kappa_2)\in\mathbb{R}_+^2} -\sum_{n=1}^{2} p_n \delta_n \Sigma_{nn} \left[\left(1 - 2\gamma_n + \frac{U_{aa}^n}{U_{a_i a_i}^n}\right) \frac{1}{\left(\gamma_n + (1-\gamma_n)\, 2^{2\kappa_n}\right)^2} + \frac{(1-\gamma_n)^2 \left(2^{2\kappa_n} - 1\right)}{\left(\gamma_n + (1-\gamma_n)\, 2^{2\kappa_n}\right)^2} \right], \tag{28}$$

subject to

$$\kappa_1 + \kappa_2 \leq \kappa. \tag{29}$$

Increasing the attention allocated to state n reduces the mean squared difference between the mean action $a_{t,n}$ and the welfare-maximizing action $a_{t,n}^*$ (see the first term in square brackets in the objective), but may increase or decrease the dispersion in actions in state n (see the second term in square brackets in the objective). The reason for the second effect is that at $\kappa_n = 0$ dispersion in actions in state n equals zero and as $\kappa_n \to \infty$ dispersion in actions in state n goes to zero, while for intermediate values of κ_n dispersion in actions is positive.

Finally, in the following, we focus on the case where the degree of strategic complementarity is the same across states and the ratio $U_{aa}^n / U_{a_i a_i}^n$ is the same across states. The planner problem then reduces to:

$$\max_{(\kappa_1,\kappa_2)\in\mathbb{R}_+^2} -\sum_{n=1}^{2} p_n \delta_n \Sigma_{nn} \left[\left(1 - 2\gamma + \frac{U_{aa}}{U_{a_i a_i}}\right) \frac{1}{\left(\gamma + (1-\gamma)\, 2^{2\kappa_n}\right)^2} + \frac{(1-\gamma)^2 \left(2^{2\kappa_n} - 1\right)}{\left(\gamma + (1-\gamma)\, 2^{2\kappa_n}\right)^2} \right], \tag{30}$$

subject to

$$\kappa_1 + \kappa_2 \leq \kappa. \tag{31}$$

The next two propositions state results concerning the relationship between the equilibrium allocation of attention and the efficient allocation of attention.

Proposition 2 *Assume that Σ is diagonal, $\gamma_1 = \gamma_2 \equiv \gamma$, and $2^\kappa > \frac{\gamma}{1-\gamma}$ and $(1-\gamma)\, 2^\kappa + \gamma 2^{-\kappa} > 1$. The equilibrium allocation of attention, denoted κ_1^{equ}, is then given by equation (8). Furthermore, assume that condition (17), conditions (26)-(27) and $\left(U_{aa}^1 / U_{a_i a_i}^1\right) = \left(U_{aa}^2 / U_{a_i a_i}^2\right) \equiv \left(U_{aa} / U_{a_i a_i}\right)$ hold. The efficient allocation of attention, denoted κ_1^{eff}, is then given by the solution to problem (30)-(31). Finally, suppose that the constraint (31) is binding and the problem (30)-(31) is convex.*

Then the following result holds. If $\gamma = (U_{aa}/U_{a_i a_i})$ or $\kappa_1^{equ} = \frac{1}{2}\kappa$, the equilibrium allocation of attention equals the efficient allocation of attention: $\kappa_1^{equ} = \kappa_1^{eff}$.

Proof. See Appendix C. ■

Proposition 2 can be interpreted as a welfare theorem for the allocation of attention. The proposition states conditions under which the equilibrium allocation of attention equals the efficient allocation of attention. The setup is the following: The conditions of Proposition 1 hold; agents take the welfare-maximizing actions under perfect information; there is a certain degree of symmetry across states; and the planner problem is convex. In this case, the equilibrium allocation of attention equals the efficient allocation of attention if either the payoff function has the property that the ratio $-(U_{a_i a}/U_{a_i a_i})$ equals the ratio $(U_{aa}/U_{a_i a_i})$, or in equilibrium agents allocate their attention equally across states (i.e., $\frac{p_1 \delta_1 \Sigma_{11}}{p_2 \delta_2 \Sigma_{22}} = 1$), or both.

A few comments on the setup are in order. The conditions of Proposition 1 imply that there exists a unique equilibrium and a closed form solution for the equilibrium allocation of attention. This simplifies the proof of Proposition 2. The condition that the economy is efficient under perfect information is a natural benchmark. It means that inefficiencies, if any, arise due to limited attention by agents. The requirement that there is a certain degree of symmetry across states will be relaxed later.

The following proposition characterizes the direction of the inefficiency when the payoff function does not have the property $-(U_{a_i a}/U_{a_i a_i}) = (U_{aa}/U_{a_i a_i})$ and in equilibrium agents do no allocate their attention equally across states.

Proposition 3 *Assume that the conditions of Proposition 2 are satisfied. Then the following result holds. If $\gamma \neq (U_{aa}/U_{a_i a_i})$, $\kappa_1^{equ} \neq \frac{1}{2}\kappa$, and $\kappa_1^{equ} \in (0, \kappa)$, the equilibrium allocation of attention differs from the efficient allocation of attention. More precisely, when $\gamma < (U_{aa}/U_{a_i a_i})$ the planner would prefer agents to pay more attention to the state that they are allocating less attention to (i.e., when $\gamma < (U_{aa}/U_{a_i a_i})$ then $0 < \kappa_n^{equ} < \frac{1}{2}\kappa$ implies $\kappa_n^{eff} > \kappa_n^{equ}$). In contrast, when $\gamma > (U_{aa}/U_{a_i a_i})$ the planner would prefer agents to pay even less attention to the state that they are allocating less attention to (i.e., when $\gamma > (U_{aa}/U_{a_i a_i})$ then $0 < \kappa_n^{equ} < \frac{1}{2}\kappa$ implies $\kappa_n^{eff} < \kappa_n^{equ}$).*

Proof. See Appendix D. ■

When agents allocate their attention to some extent to both states, agents do not allocate their attention equally across states, and the payoff function does not have the property $-\left(U_{a_i a}/U_{a_i a_i}\right) = \left(U_{aa}/U_{a_i a_i}\right)$, the equilibrium allocation of attention differs from the efficient allocation of attention. In addition, the direction of the inefficiency can be seen directly from the payoff function. If $-\left(U_{a_i a}/U_{a_i a_i}\right) < \left(U_{aa}/U_{a_i a_i}\right)$ the planner would prefer agents to pay *more* attention to the state that they are devoting less attention to. If $-\left(U_{a_i a}/U_{a_i a_i}\right) > \left(U_{aa}/U_{a_i a_i}\right)$ the planner would prefer agents to pay *even less* attention to the state that they are devoting less attention to.

For example, assume that state one has a low probability ("unusual times") and state two has a high probability ("normal times"). Furthermore, suppose that $\frac{1}{(1-\gamma)2^{\kappa}+\gamma 2^{-\kappa}} < \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} < 1$, implying that in equilibrium agents think to some extent about the optimal action in unusual times, but less than about the optimal action in normal times. Then, if $-\left(U_{a_i a}/U_{a_i a_i}\right) < \left(U_{aa}/U_{a_i a_i}\right)$ the planner would prefer agents to think more carefully about the optimal action in unusual times and focus less on the optimal action in normal times than is the case in equilibrium.

Proposition 2 states two conditions under which the equilibrium allocation of attention equals the efficient allocation of attention. One of the two conditions reads

$$-\frac{U_{a_i a}}{U_{a_i a_i}} = \frac{U_{aa}}{U_{a_i a_i}}. \tag{32}$$

This condition is equivalent to a condition that has already appeared in the literature in a different context. More precisely, this condition is equivalent to the following condition which appears in Angeletos and Pavan (2007):

$$-\frac{U_{a_i a}}{U_{a_i a_i}} = 1 - \left(\frac{U_{a_i a_i}}{U_{a_i a_i}} + 2\frac{U_{a_i a}}{U_{a_i a_i}} + \frac{U_{aa}}{U_{a_i a_i}}\right). \tag{33}$$

Angeletos and Pavan (2007) study an economy with a continuum of agents in which each agent observes a noisy private and public signal. The precision of the two signals is exogenous. Due to the quadratic Gaussian structure of the economy, actions are a linear function of the two signals and Angeletos and Pavan (2007) refer to the coefficients on the two signals as the "use of information." They then compare the equilibrium use of information to the efficient use of information, where the latter is defined as the one that maximizes ex-ante utility. For economies that are efficient under perfect information, it turns out that the equilibrium use of information equals the efficient use of information if and only if condition (33) is satisfied. We thus arrive at the following conclusion. The same condition that governs the relationship between the equilibrium use of information and the efficient use of information in Angeletos and Pavan (2007) also governs the relationship between the equilibrium allocation of attention and the efficient allocation of attention in our model with an endogenous signal precision. Our intuition for this finding is the following. If the use of information is efficient, then the acquisition of information is also efficient, so long as there is no direct externality in the acquisition of information (which is the case here).

Proposition 2 assumes that there is a certain degree of symmetry across states. The degree of strategic complementarity $\gamma_n \equiv -\left(U^n_{a_i a}/U^n_{a_i a_i}\right)$ is assumed to be the same across states and the ratio $\left(U^n_{aa}/U^n_{a_i a_i}\right)$ is assumed to be the same across states. When this symmetry requirement is not satisfied, a sufficient condition for the equilibrium allocation of attention to equal the efficient allocation of attention is that condition (32) holds for each state, that is, $-\left(U^n_{a_i a}/U^n_{a_i a_i}\right) = \left(U^n_{aa}/U^n_{a_i a_i}\right)$ for $n = 1, 2$. The proof is the same as before. The agents' first-order condition then equals the planners' first-order condition, and the conditions for corner solutions are the same for the agents and the planner.

Finally, Proposition 3 which characterizes the direction of the inefficiency when $\gamma \neq (U_{aa}/U_{a_i a_i})$, $\kappa_1^{equ} \neq \frac{1}{2}\kappa$, and $\kappa_1^{equ} \in (0, \kappa)$, does not cover the case of corner solutions. We now cover this case. When $\gamma > (U_{aa}/U_{a_i a_i})$ and $\kappa_1^{equ} = 0$ or $\kappa_1^{equ} = \kappa$, the equilibrium allocation of attention equals the efficient allocation of attention. The reason is simple. The planner would prefer agents to pay even less attention to the state that they are devoting less attention to. However, this is impossible because the equilibrium allocation of attention is already a corner solution. Hence, the equilibrium allocation of attention equals the efficient allocation of attention. They are both corner solutions. When $\gamma < (U_{aa}/U_{a_i a_i})$ and $\kappa_1^{equ} = 0$ or $\kappa_1^{equ} = \kappa$, the equilibrium allocation of attention may equal or differ from the efficient allocation of attention. If the efficient allocation of attention is a corner solution, the two coincide. If the efficient allocation of attention is not a corner solution, the two differ.

6 Extension: Learning the probability of rare events

We have assumed that the probability of the economy being in any given state at any point in time is known. In this section we study a version of the model in which the probability of the economy being in any given state is a random variable.

Consider a random variable X that has a Bernoulli distribution with an unknown parameter p, i.e. X can take only the values 0 and 1, the probabilities are

$$\Pr(X = 1) = p \qquad \text{and} \qquad \Pr(X = 0) = 1 - p,$$

and p itself is a random variable. We think of $X = 1$ as "unusual times" and we think of $X = 0$ as "normal times". Suppose that: (i) agents observe sequentially random variables $X_1, ..., X_s, ...$ that are i.i.d. over time and each has this Bernoulli distribution; (ii) in period 0, the agents' prior distribution of p is a beta distribution with parameters $\alpha > 0$ and $\beta > 0$; and (iii) in every period $t = 1, 2, ...$, agents observe whether $X = 1$ or $X = 0$ and agents update their prior distribution of p. Then the agents' posterior distribution of p given that $X_t = x_t$, $t = 1, ..., s$, is a beta distribution with parameters $\alpha + y$ and $\beta + s - y$, where $y = \sum_{t=1}^{s} x_t$. Furthermore, agents still solve the problem (4)-(7) where the probability of the economy being in any given state has been replaced by the agents' posterior expectation of that probability.[12]

This version of the model matches what we believe are the following features of reality. When a rare event fails to occur for some time, agents tend to underestimate the probability of the rare event. Consequently, agents think even less about the optimal action in the rare event. Furthermore, when the rare event does occur agents tend to increase significantly their estimate of the probability of another rare event. Consequently, an occurrence of the rare event causes a significant reallocation of attention toward thinking about what to do in the rare event.

Consider a numerical example. Suppose that the true value of p is 0.01. In period 0, the agents' prior distribution of p is a beta distribution with parameters $\alpha = 1$ and $\beta = 99$. Note that the agents' prior expectation of p equals the truth, because the prior expectation of p equals $\alpha / (\alpha + \beta) = 0.01$. Let $X_t = 0$ for $t = 1, ..., s-1$, $X_t = 1$ for $t = s$, and $s = 101$. In words, the state turns out to be "normal times" one hundred periods in a row and in period 101 the state turns out to be "unusual times".[13] The agents' posterior expectation of p evolves over time as shown in Figure 3. Note that between period 1 and period 100, the agents' posterior expectation of p falls slowly. Just before the state "unusual times" occurs, the agents' posterior expectation of p equals

[12] This statement is true because the agents' prior distribution of p and the stochastic process $\{X_t\}$ are independent of the stochastic process $\{z_t, \varepsilon_{i,t}\}$.

[13] The probability that "unusual times" fail to occur in one hundred Bernoulli trials with $p = 0.01$ equals about 0.36.

0.005. Agents underestimate the probability of “unusual times” by fifty percent. Furthermore, note that just after “unusual times” the agents’ posterior expectation of p changes by a large amount. The agents’ posterior expectation of p doubles to 0.01. Consequently, the occurrence of the rare event causes a significant reallocation of attention toward thinking about what to do in the rare event.

7 Conclusion

This paper proposes an explanation for why people were so unprepared for the global financial crisis, the European debt crisis, and the Fukushima nuclear accident. The explanation has four features: (1) Humans have a limited ability to process information and therefore cannot prepare well for every contingency. (2) These events seemed a priori unlikely. (3) Thinking carefully about the optimal action in normal times does not improve much actions in those unusual times. (4) Actions are strategic complements. Formally, we study a rational inattention model in which agents decide how carefully to think about optimal actions in different contingencies, subject to an information-processing constraint. We find that agents are unprepared in a state when the state has a low probability, the optimal action in that state is uncorrelated with the optimal action in normal times, and actions are strategic complements. We then use the model to ask the following question: Would society be better off if agents allocated their attention differently? To answer this question, we compare the equilibrium allocation of attention to the efficient allocation of attention. We find that the same condition that governs the relationship between the equilibrium use of information and the efficient use of information in Angeletos and Pavan (2007) governs the relationship between the equilibrium allocation of attention and the efficient allocation of attention in our model with an endogenous information structure.

In the real world, there exists regulation that affects the allocation of attention. For example, Federal Aviation Regulations force passengers on airplanes every time they take a flight to think about the optimal action in the rare event of a water landing. Does this increase ex-ante utility? At the same time, there does not seem to be regulation in Japan that requires companies running nuclear power plants to have a precise plan of what to do when an earthquake and tsunami has disabled a plant’s cooling system. Should this be changed? The efficiency results in this paper help understand when regulation that affects the allocation of attention can improve welfare and when it cannot improve welfare.

The efficiency question asked in this paper - whether the equilibrium allocation of attention equals the efficient allocation of attention - is new to the best of our knowledge; has a clear answer; and could be asked in a wide range of other contexts. For example, one could ask whether the extent to which investors think about payoffs of their assets in different states of the world is efficient.

A Proof of Proposition 1

Step 1: We consider equilibria where the average action in a state is an affine function of the fundamental in that state. Formally, for $n = 1$ and $n = 2$,

$$a_{t,n} = \psi_n + \phi_n z_{t,n}, \tag{34}$$

where ψ_1, ϕ_1, ψ_2, and ϕ_2 are undetermined coefficients that we need to solve for.

Step 2: The information choice problem (4)-(7) can now be stated as follows. Substituting equations (2), (3) and (5) into objective (4), deducting a constant that the agent cannot affect from the objective, and using equation (34) to substitute for $a_{t,n}$ in the objective yields

$$\max_{\Lambda} \left\{ -\sum_{n=1}^{2} p_n \delta_n \left(\gamma_n \phi_n + 1 - \gamma_n\right)^2 \Omega_{nn} \right\}, \tag{35}$$

subject to

$$\Omega = \Sigma - \Sigma \left(\Sigma + \Lambda\right)^{-1} \Sigma, \tag{36}$$

$$\frac{1}{2} \log_2 \left(\frac{|\Sigma|}{|\Omega|} \right) \leq \kappa, \tag{37}$$

and the restriction that Λ is a positive semidefinite matrix. Here Ω_{nn} denotes the posterior variance of the fundamental in state n. Furthermore, using the formula for the determinant of a two-by-two matrix, the information flow constraint (37) can be expressed as

$$\frac{1}{2} \log_2 \left(\frac{\Sigma_{11}\Sigma_{22} - \Sigma_{12}^2}{\Omega_{11}\Omega_{22} - \Omega_{12}^2} \right) \leq \kappa, \tag{38}$$

where Ω_{12} denotes the posterior covariance of the fundamental in the two states.

Step 3: When the optimal action in state one and the optimal action in state two are independent (i.e., $\Sigma_{12} = 0$), it is optimal to receive independent signals concerning the optimal action in state one and the optimal action in state two (i.e., $\Lambda_{12} = 0$). The proof is as follows. First, the information flow constraint (38) is always binding. Second, increasing Ω_{12}^2 for a given Ω_{11} and Ω_{22} raises the information flow on the left-hand side of constraint (38) without improving objective (35). Third, when $\Sigma_{12} = 0$, then $\Omega_{12} = 0$ if and only if $\Lambda_{12} = 0$. Hence, when $\Sigma_{12} = 0$, the solution to the information choice problem (35)-(37) has the property $\Lambda_{12} = 0$. Next, using $\Sigma_{12} = \Lambda_{12} = \Omega_{12} = 0$ the information choice problem (35)-(37) simplifies to

$$\max_{\left(\Lambda_{11}^{-1}, \Lambda_{22}^{-1}\right) \in \mathbb{R}_+^2} \left\{ -\sum_{n=1}^{2} p_n \delta_n \left(\gamma_n \phi_n + 1 - \gamma_n\right)^2 \Omega_{nn} \right\}, \tag{39}$$

subject to

$$\Omega_{nn} = \frac{1}{\frac{\Sigma_{nn}}{\Lambda_{nn}} + 1} \Sigma_{nn}, \tag{40}$$

and

$$\frac{1}{2}\log_2\left(\frac{\Sigma_{11}}{\Omega_{11}}\right) + \frac{1}{2}\log_2\left(\frac{\Sigma_{22}}{\Omega_{22}}\right) \leq \kappa. \tag{41}$$

Let $\kappa_n \equiv \frac{1}{2}\log_2\left(\frac{\Sigma_{nn}}{\Omega_{nn}}\right)$ denote the uncertainty reduction about the fundamental in state n. The information choice problem (39)-(41) can be written as

$$\max_{(\kappa_1,\kappa_2)\in\mathbb{R}_+^2} \left\{ -\sum_{n=1}^{2} p_n \delta_n \left(\gamma_n \phi_n + 1 - \gamma_n\right)^2 \Omega_{nn} \right\}, \tag{42}$$

subject to

$$\Omega_{nn} = 2^{-2\kappa_n} \Sigma_{nn}, \tag{43}$$

and

$$\kappa_1 + \kappa_2 \leq \kappa. \tag{44}$$

The unique solution to this problem is given by

$$\kappa_1 = \begin{cases} \kappa & \text{if } x \geq 2^{\kappa} \\ \frac{1}{2}\kappa + \frac{1}{2}\log_2(x) & \text{if } x \in \left[2^{-\kappa}, 2^{\kappa}\right] \\ 0 & \text{if } x \leq 2^{-\kappa} \end{cases}, \tag{45}$$

where

$$x \equiv \sqrt{\frac{p_1 \delta_1 \left(\gamma_1 \phi_1 + 1 - \gamma_1\right)^2 \Sigma_{11}}{p_2 \delta_2 \left(\gamma_2 \phi_2 + 1 - \gamma_2\right)^2 \Sigma_{22}}}; \tag{46}$$

and

$$\kappa_2 = \kappa - \kappa_1. \tag{47}$$

The optimal uncertainty reduction about the fundamental in state one is an increasing function of κ and x. Finally, it follows from equation (40) and $\kappa_n \equiv \frac{1}{2}\log_2\left(\frac{\Sigma_{nn}}{\Omega_{nn}}\right)$ that the optimal signal precisions are then given by

$$\Lambda_{11}^{-1} = \frac{2^{2\kappa_1} - 1}{\Sigma_{11}}, \tag{48}$$

$$\Lambda_{22}^{-1} = \frac{2^{2\kappa_2} - 1}{\Sigma_{22}}. \tag{49}$$

Step 4: Equations (45)-(47) give the optimal allocation of attention as a function of the parameters of the model and the undetermined coefficients ϕ_1 and ϕ_2. The next step is to solve for the undetermined coefficients ϕ_1 and ϕ_2 as a function of the optimal allocation of attention. Combining results one then obtains the equilibrium of the model. The actions by agent i are given by equation (5). Substituting the guess (34) into equation (5) yields

$$a_{i,t,n} = (\varphi_n + \gamma_n \psi_n) + (\gamma_n \phi_n + 1 - \gamma_n) E[z_{t,n} | s_{i,t-1}].$$

Calculating the conditional expectation in the last equation using equation (6), $\Sigma_{12} = \Lambda_{12} = 0$, and equations (48)-(49) yields

$$a_{i,t,n} = (\varphi_n + \gamma_n \psi_n) + (\gamma_n \phi_n + 1 - \gamma_n) \left(1 - 2^{-2\kappa_n}\right) (z_{t,n} + \varepsilon_{i,t-1,n}).$$

Calculating the mean action in the population gives

$$a_{t,n} = (\varphi_n + \gamma_n \psi_n) + (\gamma_n \phi_n + 1 - \gamma_n) \left(1 - 2^{-2\kappa_n}\right) z_{t,n}.$$

It follows that, for a given allocation of attention (i.e., for a pair κ_1 and κ_2), the guess (34) is correct if and only if

$$\psi_n = \frac{\varphi_n}{1 - \gamma_n}, \tag{50}$$

$$\phi_n = \frac{(1 - \gamma_n)\left(1 - 2^{-2\kappa_n}\right)}{1 - \gamma_n \left(1 - 2^{-2\kappa_n}\right)}. \tag{51}$$

The last two equations give the undetermined coefficients ψ_1, ψ_2, ϕ_1, and ϕ_2 as a function of the allocation of attention κ_1 and κ_2 and the parameters φ_1, φ_2, γ_1, and γ_2.

Step 5: An equilibrium allocation of attention is a pair (κ_1, κ_2) satisfying equations (45)-(47), where ϕ_1 and ϕ_2 are given by equation (51). Using equation (51) to substitute for ϕ_1 and ϕ_2 in equation (46) yields

$$x = \sqrt{\frac{p_1 \delta_1 \Sigma_{11}}{p_2 \delta_2 \Sigma_{22}} \frac{\frac{1-\gamma_1}{1-\gamma_1\left(1-2^{-2\kappa_1}\right)}}{\frac{1-\gamma_2}{1-\gamma_2\left(1-2^{-2\kappa_2}\right)}}}. \tag{52}$$

Thus, an equilibrium allocation of attention is a pair (κ_1, κ_2) satisfying equations (45), (47) and (52). It is useful to distinguish three types of equilibria: (i) the equilibrium allocation of attention has the property $\kappa_1 = 0$, (ii) the equilibrium allocation of attention has the property $\kappa_1 = \kappa$, and (iii) the equilibrium allocation of attention has the property $\kappa_1 = \frac{1}{2}\kappa + \frac{1}{2}\log_2(x)$.

First, turn to an equilibrium with the property $\kappa_1 = 0$. Substituting $\kappa_1 = 0$ and $\kappa_2 = \kappa$ into equation (52) yields

$$x = \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{1-\gamma_1}{1-\gamma_2}\left[1-\gamma_2\left(1-2^{-2\kappa}\right)\right].$$

It follows from the last equation and equation (45) that $\kappa_1 = 0$ is an equilibrium if and only if

$$\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{1-\gamma_1}{1-\gamma_2}\left[1-\gamma_2\left(1-2^{-2\kappa}\right)\right] \leq 2^{-\kappa}.$$

This condition can be stated as

$$\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \leq \frac{1}{(1-\gamma_1)\left[2^{\kappa}+\frac{\gamma_2}{1-\gamma_2}2^{-\kappa}\right]}. \tag{53}$$

Second, consider an equilibrium with the property $\kappa_1 = \kappa$. Substituting $\kappa_1 = \kappa$ and $\kappa_2 = 0$ into equation (52) yields

$$x = \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{1-\gamma_1}{1-\gamma_2}\frac{1}{1-\gamma_1\left(1-2^{-2\kappa}\right)}.$$

It follows from the last equation and equation (45) that $\kappa_1 = \kappa$ is an equilibrium if and only if

$$\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{1-\gamma_1}{1-\gamma_2}\frac{1}{1-\gamma_1\left(1-2^{-2\kappa}\right)} \geq 2^{\kappa}.$$

This condition can be stated as

$$\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \geq (1-\gamma_2)\left[2^{\kappa}+\frac{\gamma_1}{1-\gamma_1}2^{-\kappa}\right]. \tag{54}$$

Third, turn to an equilibrium with the property $\kappa_1 = \frac{1}{2}\kappa + \frac{1}{2}\log_2(x)$. Substituting $\kappa_1 = \frac{1}{2}\kappa + \frac{1}{2}\log_2(x)$ and $\kappa_2 = \kappa - \kappa_1$ into equation (52) yields

$$x = \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{\frac{1-\gamma_1}{1-\gamma_1\left(1-2^{-\kappa}\frac{1}{x}\right)}}{\frac{1-\gamma_2}{1-\gamma_2\left(1-2^{-\kappa}x\right)}}.$$

Rearranging the last equation yields

$$\left[1-\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{\gamma_2}{1-\gamma_2}2^{-\kappa}\right]x = \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}-\frac{\gamma_1}{1-\gamma_1}2^{-\kappa}. \tag{55}$$

If $\left[1-\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{\gamma_2}{1-\gamma_2}2^{-\kappa}\right] \neq 0$, the unique solution to the last equation is

$$x = \frac{\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}-\frac{\gamma_1}{1-\gamma_1}2^{-\kappa}}{1-\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{\gamma_2}{1-\gamma_2}2^{-\kappa}}. \tag{56}$$

Thus, when $\left[1-\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{\gamma_2}{1-\gamma_2}2^{-\kappa}\right]\neq 0$, it follows from the last equation and equation (45) that $\kappa_1=\frac{1}{2}\kappa+\frac{1}{2}\log_2(x)$ is an equilibrium if and only if

$$\frac{\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}-\frac{\gamma_1}{1-\gamma_1}2^{-\kappa}}{1-\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{\gamma_2}{1-\gamma_2}2^{-\kappa}}\in\left[2^{-\kappa},2^{\kappa}\right]. \tag{57}$$

Furthermore, when

$$\left[1-\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{\gamma_2}{1-\gamma_2}2^{-\kappa}\right]>0, \tag{58}$$

condition (57) is equivalent to

$$\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\in\left[\frac{\frac{1}{1-\gamma_1}}{2^{\kappa}+\frac{\gamma_2}{1-\gamma_2}2^{-\kappa}},\frac{2^{\kappa}+\frac{\gamma_1}{1-\gamma_1}2^{-\kappa}}{\frac{1}{1-\gamma_2}}\right]. \tag{59}$$

When

$$\left[1-\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{\gamma_2}{1-\gamma_2}2^{-\kappa}\right]<0, \tag{60}$$

condition (57) is equivalent to

$$\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\in\left[\frac{2^{\kappa}+\frac{\gamma_1}{1-\gamma_1}2^{-\kappa}}{\frac{1}{1-\gamma_2}},\frac{\frac{1}{1-\gamma_1}}{2^{\kappa}+\frac{\gamma_2}{1-\gamma_2}2^{-\kappa}}\right]. \tag{61}$$

Finally, if

$$\left[1-\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{\gamma_2}{1-\gamma_2}2^{-\kappa}\right]=0, \tag{62}$$

equation (55) reduces to

$$\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}=\frac{\gamma_1}{1-\gamma_1}2^{-\kappa}. \tag{63}$$

In summary, if conditions (58)-(59) or conditions (60)-(61) hold, a unique equilibrium with the property $\kappa_1=\frac{1}{2}\kappa+\frac{1}{2}\log_2(x)$ exists and in this equilibrium x is given by equation (56). If conditions (62)-(63) hold, a continuum of equilibria with the property $\kappa_1=\frac{1}{2}\kappa+\frac{1}{2}\log_2(x)$ exist; namely any $\kappa_1\in[0,\kappa]$ is such an equilibrium.

This completes the characterization of equilibria of the form (34). If $\gamma_1=\gamma_2\equiv\gamma$, conditions (53), (54), (58)-(59), (60)-(61) and (62)-(63) and equation (56) reduce to the conditions and equation given in Proposition 1.

B Proof of Equation (19)

Step 1: A Taylor expansion of U^n around $a_{i,t,n} = a_{t,n}$ gives

$$\begin{aligned} U^n\left(a_{i,t,n}, a_{t,n}, z_{t,n}\right) &= U^n\left(a_{t,n}, a_{t,n}, z_{t,n}\right) + \left[U^n_{a_i} + \left(U^n_{a_ia_i} + U^n_{a_ia}\right) a_{t,n} + U^n_{a_iz} z_{t,n}\right]\left(a_{i,t,n} - a_{t,n}\right) \\ &\quad + \frac{U^n_{a_ia_i}}{2}\left(a_{i,t,n} - a_{t,n}\right)^2 . \end{aligned} \tag{64}$$

Let $W^n\left(a_{t,n}, \sigma_{a_{i,t,n}}, z_{t,n}\right)$ denote welfare in state n under a utilitarian aggregator

$$W^n\left(a_{t,n}, \sigma_{a_{i,t,n}}, z_{t,n}\right) \equiv \int U^n\left(a_{i,t,n}, a_{t,n}, z_{t,n}\right) d\Psi^{n,t}\left(a_{i,t,n}\right). \tag{65}$$

Combining the last two equations gives

$$W^n\left(a_{t,n}, \sigma_{a_{i,t,n}}, z_{t,n}\right) = U^n\left(a_{t,n}, a_{t,n}, z_{t,n}\right) + \frac{U^n_{a_ia_i}}{2}\sigma^2_{a_{i,t,n}}, \tag{66}$$

where $\sigma^2_{a_{i,t,n}} \equiv \int \left(a_{i,t,n} - a_{t,n}\right)^2 d\Psi^{n,t}\left(a_{i,t,n}\right)$ denotes the dispersion of individual actions in the population. Next, a Taylor expansion of $W^n\left(a_{t,n}, \sigma_{a_{i,t,n}}, z_{t,n}\right)$ around $a_{t,n} = a^*_{t,n}$ and $\sigma_{a_{i,t,n}} = 0$, where $a^*_{t,n}$ is given by equation (18), yields

$$W^n\left(a_{t,n}, \sigma_{a_{i,t,n}}, z_{t,n}\right) = W^n\left(a^*_{t,n}, 0, z_{t,n}\right) + \frac{U^n_{a_ia_i} + 2U^n_{a_ia} + U^n_{aa}}{2}\left(a_{t,n} - a^*_{t,n}\right)^2 + \frac{U^n_{a_ia_i}}{2}\sigma^2_{a_{i,t,n}}. \tag{67}$$

Here we used equation (66) to compute the first and second derivatives of W^n and exploited the fact that the first derivative of W^n with respect to $a_{t,n}$ evaluated at $a^*_{t,n}$ equals zero.

Step 2: Given any strategy $a_{i,t,n} : \mathbb{R}^2 \to \mathbb{R}$, expected utility in state n is given by

$$E\left[U^n\left(a_{i,t,n}, a_{t,n}, z_{t,n}\right)\right] = \int_{z_t}\int_{s_{i,t-1}} U^n\left(a_{i,t,n}\left(s_{i,t-1}\right), a_{t,n}\left(z_t\right), z_{t,n}\right) dP\left(s_{i,t-1}|z_t\right) dP\left(z_t\right), \tag{68}$$

where $a_{t,n}\left(z_t\right) = \int_{s_{i,t-1}} a_{i,t,n}\left(s_{i,t-1}\right) dP\left(s_{i,t-1}|z_t\right)$. Substituting equation (64) into equation (68) and using equation (66) gives

$$E\left[U^n\left(a_{i,t,n}, a_{t,n}, z_{t,n}\right)\right] = \int_{z_t} W^n\left(a_{t,n}\left(z_t\right), \sigma_{a_{i,t,n}}, z_{t,n}\right) dP\left(z_t\right). \tag{69}$$

Substituting equation (67) into the last equation yields

$$\begin{aligned} E\left[U^n\left(a_{i,t,n}, a_{t,n}, z_{t,n}\right)\right] &= E\left[W^n\left(a^*_{t,n}, 0, z_{t,n}\right)\right] + \frac{U^n_{a_ia_i} + 2U^n_{a_ia} + U^n_{aa}}{2} E\left[\left(a_{t,n} - a^*_{t,n}\right)^2\right] \\ &\quad + \frac{U^n_{a_ia_i}}{2} E\left[\left(a_{i,t,n} - a_{t,n}\right)^2\right]. \end{aligned} \tag{70}$$

Noting that $W^n\left(a^*_{t,n}, 0, z_{t,n}\right) = \tilde{U}^n\left(a^*_{t,n}, z_{t,n}\right)$ gives the desired result.

C Proof of Proposition 2

Step 1: The first two sentences of Proposition 2 follow from Proposition 1. The next two sentences of Proposition 2 follow from the text above Proposition 2.

Step 2: Substituting $\kappa_2 = \kappa - \kappa_1$ into objective (30) and setting the first derivative of the objective with respect to κ_1 equal to zero yields the first-order condition

$$\begin{aligned} & p_1\delta_1\Sigma_{11}\left[\frac{(1-\gamma)^2 2^{2\kappa_1}}{\left[\gamma+(1-\gamma)2^{2\kappa_1}\right]^2} + 2\frac{(1-\gamma)2^{2\kappa_1}}{\left[\gamma+(1-\gamma)2^{2\kappa_1}\right]^3}\left(\frac{U_{aa}}{U_{a_ia_i}}-\gamma\right)\right]2\ln(2) \\ & -p_2\delta_2\Sigma_{22}\left[\frac{(1-\gamma)^2 2^{2(\kappa-\kappa_1)}}{\left[\gamma+(1-\gamma)2^{2(\kappa-\kappa_1)}\right]^2} + 2\frac{(1-\gamma)2^{2(\kappa-\kappa_1)}}{\left[\gamma+(1-\gamma)2^{2(\kappa-\kappa_1)}\right]^3}\left(\frac{U_{aa}}{U_{a_ia_i}}-\gamma\right)\right]2\ln(2) = 0. \end{aligned} \tag{71}$$

Let $F_{\kappa_1=0}$ and $F_{\kappa_1=\kappa}$ denote the value of the left-hand side of equation (71) at $\kappa_1 = 0$ and $\kappa_1 = \kappa$, respectively. When the constraint (31) is binding and the planner problem (30)-(31) is convex, the solution to the planner problem is given by

$$\kappa_1^{eff} = \begin{cases} \kappa & \text{if } F_{\kappa_1=\kappa} \geq 0 \\ \kappa_1^{FOC} & \text{if } F_{\kappa_1=0} > 0 > F_{\kappa_1=\kappa} \\ 0 & \text{if } F_{\kappa_1=0} \leq 0 \end{cases}, \tag{72}$$

where κ_1^{FOC} denotes the unique solution to equation (71) in the case of $F_{\kappa_1=0} > 0 > F_{\kappa_1=\kappa}$.

Step 3: If $\gamma = (U_{aa}/U_{a_ia_i})$, the first-order condition (71) reduces to

$$p_1\delta_1\Sigma_{11}\frac{(1-\gamma)^2\, 2^{2\kappa_1}}{\left[\gamma+(1-\gamma)\,2^{2\kappa_1}\right]^2}2\ln(2) - p_2\delta_2\Sigma_{22}\frac{(1-\gamma)^2\, 2^{2(\kappa-\kappa_1)}}{\left[\gamma+(1-\gamma)\,2^{2(\kappa-\kappa_1)}\right]^2}2\ln(2) = 0. \tag{73}$$

Now the condition $F_{\kappa_1=0} \leq 0$ reads

$$\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \leq \frac{1}{\gamma 2^{-\kappa} + (1-\gamma)\,2^{\kappa}},$$

and the condition $F_{\kappa_1=\kappa} \geq 0$ reads

$$\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \geq \gamma 2^{-\kappa} + (1-\gamma)\,2^{\kappa}.$$

Furthermore, solving equation (73) for κ_1 in the case of $F_{\kappa_1=0} > 0 > F_{\kappa_1=\kappa}$ yields

$$\kappa_1 = \frac{1}{2}\kappa + \frac{1}{2}\log_2\left(\frac{\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} - \frac{\gamma}{1-\gamma}2^{-\kappa}}{1 - \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{\gamma}{1-\gamma}2^{-\kappa}}\right).$$

Hence, if $\gamma = (U_{aa}/U_{a_i a_i})$, the efficient allocation of attention is given by

$$\kappa_1^{eff} = \begin{cases} \kappa & \text{if } \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \geq \gamma 2^{-\kappa} + (1-\gamma)\,2^{\kappa} \\ \frac{1}{2}\kappa + \frac{1}{2}\log_2\left(\frac{\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} - \frac{\gamma}{1-\gamma}2^{-\kappa}}{1-\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{\gamma}{1-\gamma}2^{-\kappa}}\right) & \text{otherwise} \\ 0 & \text{if } \sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} \leq \frac{1}{\gamma 2^{-\kappa}+(1-\gamma)2^{\kappa}} \end{cases}. \tag{74}$$

Comparing equation (74) to equation (8) shows that if $\gamma = (U_{aa}/U_{a_i a_i})$ then $\kappa_1^{equ} = \kappa_1^{eff}$.

Step 4: If $\kappa_1^{equ} = \frac{1}{2}\kappa$, then $\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}} = 1$. See equation (8). Furthermore, when $\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}} = 1$, the first-order condition (71) reduces to

$$\begin{aligned} &\left[\frac{(1-\gamma)^2 2^{2\kappa_1}}{\left[\gamma+(1-\gamma)2^{2\kappa_1}\right]^2} + 2\frac{(1-\gamma)2^{2\kappa_1}}{\left[\gamma+(1-\gamma)2^{2\kappa_1}\right]^3}\left(\frac{U_{aa}}{U_{a_i a_i}} - \gamma\right)\right] \\ -&\left[\frac{(1-\gamma)^2 2^{2(\kappa-\kappa_1)}}{\left[\gamma+(1-\gamma)2^{2(\kappa-\kappa_1)}\right]^2} + 2\frac{(1-\gamma)2^{2(\kappa-\kappa_1)}}{\left[\gamma+(1-\gamma)2^{2(\kappa-\kappa_1)}\right]^3}\left(\frac{U_{aa}}{U_{a_i a_i}} - \gamma\right)\right] = 0. \end{aligned}$$

A solution to the last equation is $\kappa_1^{FOC} = \frac{1}{2}\kappa$. When the planner problem is convex, this implies that $\kappa_1^{eff} = \frac{1}{2}\kappa$. It follows that if $\kappa_1^{equ} = \frac{1}{2}\kappa$ then $\kappa_1^{equ} = \kappa_1^{eff}$.

D Proof of Proposition 3

If $\kappa_1^{equ} \in (0,\kappa)$, then

$$\kappa_1^{equ} = \frac{1}{2}\kappa + \frac{1}{2}\log_2(x), \tag{75}$$

where

$$x \equiv \frac{\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}} - \frac{\gamma}{1-\gamma}2^{-\kappa}}{1-\sqrt{\frac{p_1\delta_1\Sigma_{11}}{p_2\delta_2\Sigma_{22}}}\frac{\gamma}{1-\gamma}2^{-\kappa}}, \tag{76}$$

and

$$x \in \left(2^{-\kappa}, 2^{\kappa}\right). \tag{77}$$

See equations (8)-(9). Let $F_{\kappa_1=\kappa_1^{equ}\in(0,\kappa)}$ denote the value of the left-hand side of the planner's first-order condition (71) at $\kappa_1 = \kappa_1^{equ} \in (0,\kappa)$. Substituting equation (75) into the left-hand side of equation (71) gives

$$\begin{aligned} F_{\kappa_1=\kappa_1^{equ}\in(0,\kappa)} &= p_1\delta_1\Sigma_{11}\left[\frac{(1-\gamma)^2 2^{\kappa}x}{\left[\gamma+(1-\gamma)2^{\kappa}x\right]^2} + 2\frac{(1-\gamma)2^{\kappa}x}{\left[\gamma+(1-\gamma)2^{\kappa}x\right]^3}\left(\frac{U_{aa}}{U_{a_i a_i}} - \gamma\right)\right]2\ln(2) \\ &\quad -p_2\delta_2\Sigma_{22}\left[\frac{(1-\gamma)^2 \frac{2^{\kappa}}{x}}{\left[\gamma+(1-\gamma)\frac{2^{\kappa}}{x}\right]^2} + 2\frac{(1-\gamma)\frac{2^{\kappa}}{x}}{\left[\gamma+(1-\gamma)\frac{2^{\kappa}}{x}\right]^3}\left(\frac{U_{aa}}{U_{a_i a_i}} - \gamma\right)\right]2\ln(2). \end{aligned}$$

Furthermore, equation (76) implies

$$p_1\delta_1\Sigma_{11}\frac{(1-\gamma)^2\,2^\kappa x}{\left[\gamma+(1-\gamma)\,2^\kappa x\right]^2}=p_2\delta_2\Sigma_{22}\frac{(1-\gamma)^2\,\frac{2^\kappa}{x}}{\left[\gamma+(1-\gamma)\,\frac{2^\kappa}{x}\right]^2}.$$

Substituting the last equation into the previous equation gives

$$\begin{aligned} F_{\kappa_1=\kappa_1^{equ}\in(0,\kappa)} &= p_1\delta_1\Sigma_{11}\frac{(1-\gamma)\,2^\kappa x}{\left[\gamma+(1-\gamma)\,2^\kappa x\right]^2} \\ &\quad \left[\frac{2}{\gamma+(1-\gamma)\,2^\kappa x}-\frac{2}{\gamma+(1-\gamma)\,\frac{2^\kappa}{x}}\right]\left(\frac{U_{aa}}{U_{a_ia_i}}-\gamma\right)2\ln(2)\,. \end{aligned} \tag{78}$$

Since $p_1\delta_1\Sigma_{11}>0$, $\gamma\in(-1,1)$, and $x\in(2^{-\kappa},2^\kappa)$, the last expression equals zero if and only if $\frac{U_{aa}}{U_{a_ia_i}}=\gamma$ or $x=1$. Furthermore, when $\frac{U_{aa}}{U_{a_ia_i}}>\gamma$, then $x<1$ implies $F_{\kappa_1=\kappa_1^{equ}\in(0,\kappa)}>0$ while $x>1$ implies $F_{\kappa_1=\kappa_1^{equ}\in(0,\kappa)}<0$. By contrast, when $\frac{U_{aa}}{U_{a_ia_i}}<\gamma$, then $x<1$ implies $F_{\kappa_1=\kappa_1^{equ}\in(0,\kappa)}<0$ while $x>1$ implies $F_{\kappa_1=\kappa_1^{equ}\in(0,\kappa)}>0$. In addition, $x<1$ means $\kappa_1<\frac{1}{2}\kappa$, and $x>1$ means $\kappa_1>\frac{1}{2}\kappa$. See equation (75). Finally, by assumption $\kappa_1^{equ}\in(0,\kappa)$ and the planner problem is convex. Hence, when $\frac{U_{aa}}{U_{a_ia_i}}>\gamma$, then $\kappa_n<\frac{1}{2}\kappa$ implies $\kappa_n^{eff}>\kappa_n^{equ}$. By contrast, when $\frac{U_{aa}}{U_{a_ia_i}}<\gamma$, then $\kappa_n<\frac{1}{2}\kappa$ implies $\kappa_n^{eff}<\kappa_n^{equ}$.

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Figure 1: Attention to state one as function of relative likelihood

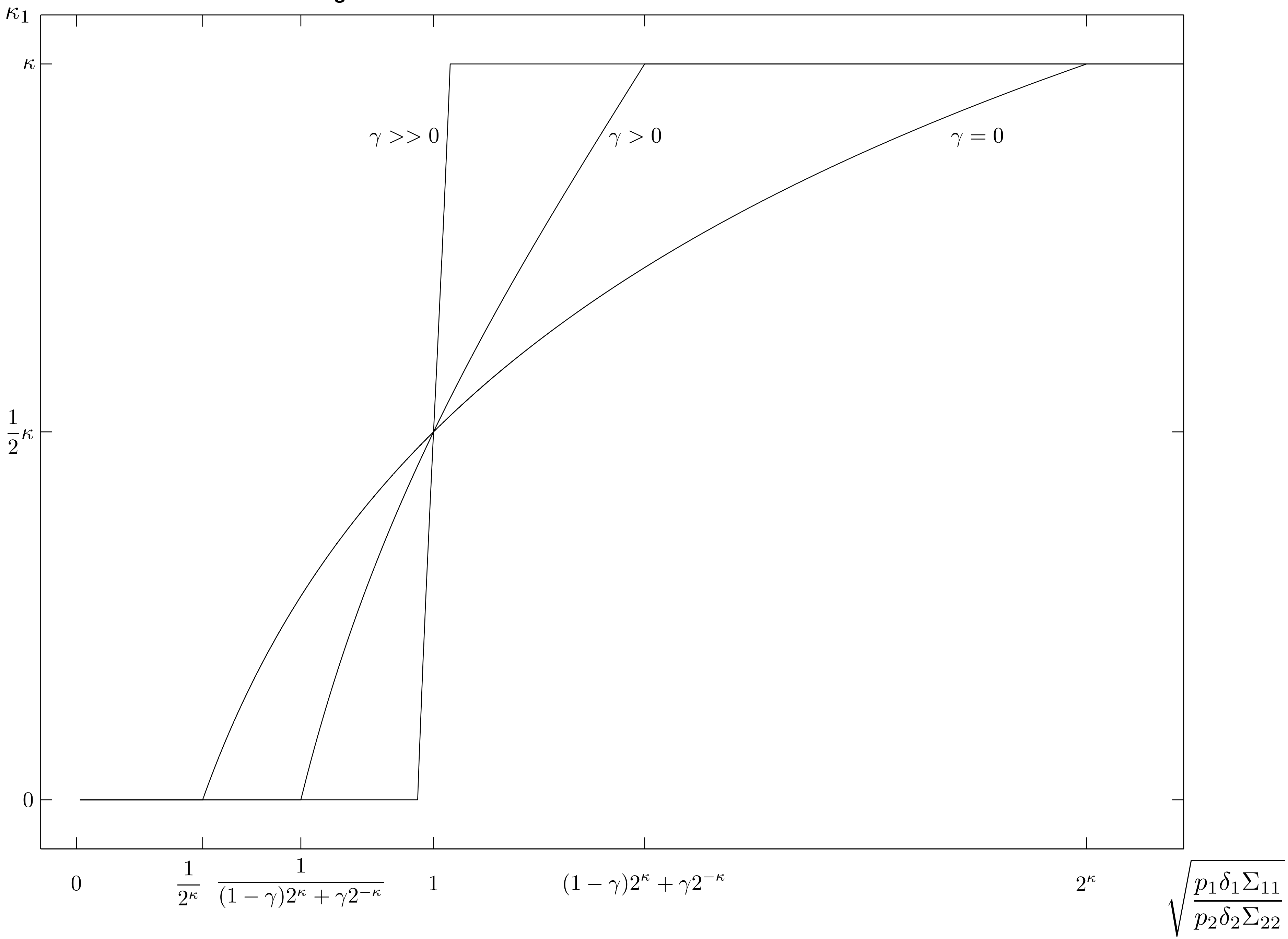
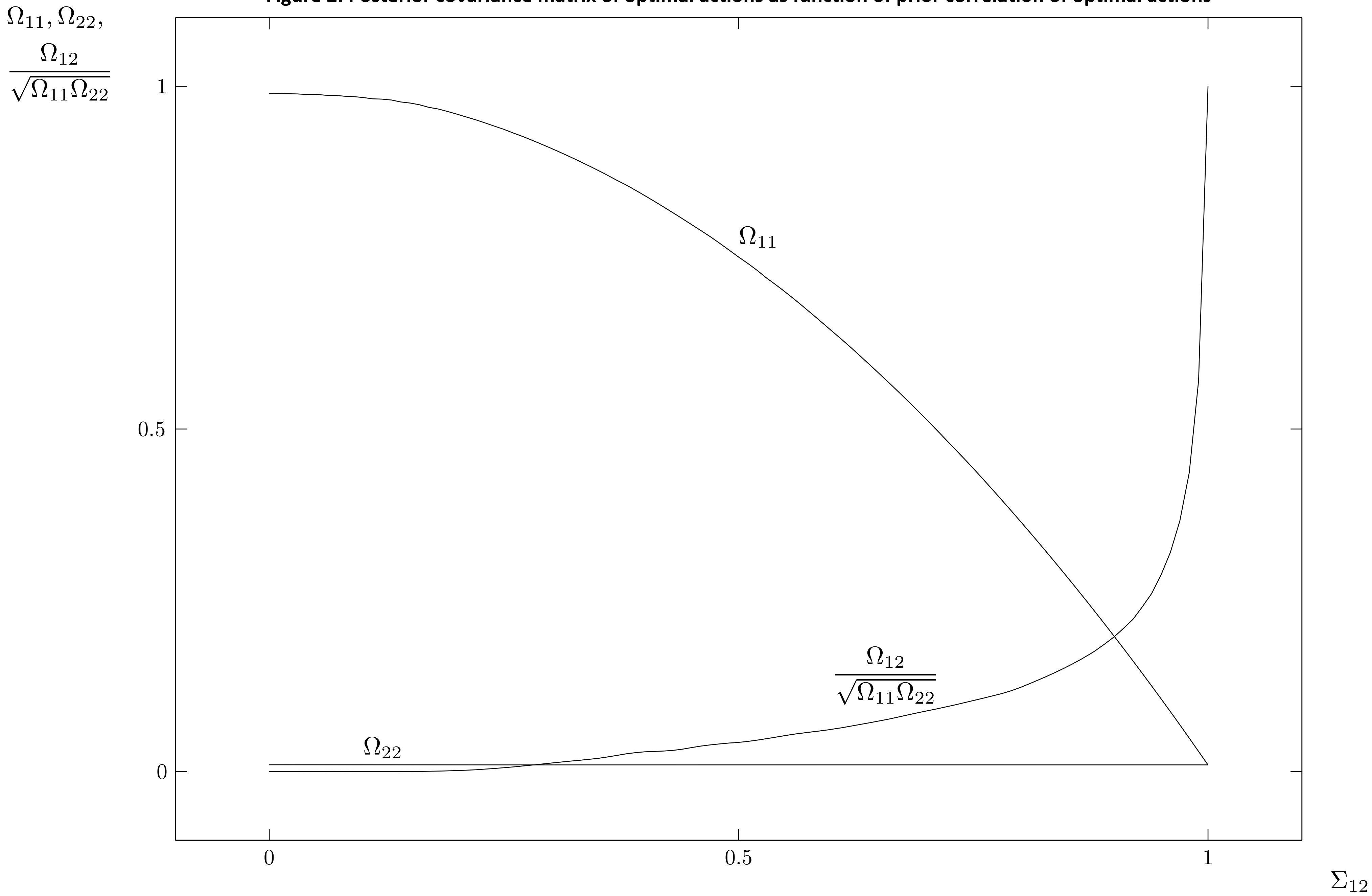


Figure 2: Posterior covariance matrix of optimal actions as function of prior correlation of optimal actions



This figure assumes: $\gamma_1 = \gamma_2 = 0, \delta_1 = \delta_2, \Sigma_{11} = \Sigma_{22} = 1, p_1 = 0.01$

Figure 3: Posterior expectation of the probability of unusual times