# Fiscal Policy in an Expectations Driven Liquidity Trap*

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This version: April 2012, First version: April 2010

**Abstract**

In the basic New Keynesian model in which the monetary authority operates a Taylor rule, multiple rational expectations equilibria arise, some of which display all the features of a liquidity trap. We show that a loss in confidence can set the economy on a deflationary path that eventually prevents the monetary authority from adjusting the interest rate and can lead to potentially very large output drops. Contrary to a line of recent papers, we describe equilibria in which demand stimulating policies become less effective in a liquidity trap than in normal circumstances. In contrast, supply side policies, such as cuts in labor income taxes, become more powerful. We show that these results also hold for local deviations from rational expectations.

**Keywords**: Liquidity trap, fiscal policy, sunspots, confidence shocks
**JEL Classification**: E3, E6, E62

*We are grateful to comments received from Florin Bilbiie, Gauti Eggertson, Espen Moen, Assaf Razin, Karl Shell, Pedro Teles and from participants at various seminars and conferences. The responsibility for any errors and omissions is entirely ours.

## 1 Introduction

By what magnitude does output increase in response to temporary changes in government purchases of goods and services or to changes in various taxes? This central question in macroeconomics has received renewed attention following the Great Recession. Many central banks responded to the crisis by aggressively cutting short term interest rates. This policy led to unprecedented low levels of nominal short term interest rates and forced policy makers to reach for alternative stabilization instruments, including fiscal policy interventions. Unfortunately, there exists little empirical evidence on whether fiscal policies implemented under such conditions are especially effective or not.[1] There is even less evidence on the relative merits of demand and supply oriented policies. For this reason, it is pertinent to use economic theory to shed light on the issue and this is the goal of this paper.

We study the dynamics of an economy in a liquidity trap, i.e. a situation of zero nominal interest rates and depressed output levels, caused by a loss of confidence. The analysis is cast in a standard New Keynesian model with Calvo price setting frictions. Monetary policy follows an interest rate rule responsive to inflation consistent with local equilibrium determinacy when inflation is near the target. However, because of the zero lower bound, globally there exist multiple rational expectations (RE) equilibria, see Benhabib, Schmitt-Grohé and Uribe (2001a,b, 2002). The lower bound on short term nominal interest rates implies a non-monotonic relationship between consumption growth and expected inflation: consumption growth depends negatively on inflation for sufficiently high levels of inflation, but the relationship becomes positive when monetary policy hits the lower bound. Because of this kink, there exist RE equilibria in which pessimistic expectations bring the economy into a temporary liquidity trap. Such a loss in confidence is deflationary, sends real interest rates soaring and causes drops in output and welfare.

The first contribution of this paper is to describe a class of RE equilibria in which a transitory confidence loss can cause reductions in economic activity that are much larger than in the equilibria

[1] Almunia, Bénétrix, Eichengreen, O'Rourke and Rua (2010) find large multipliers associated with defense spending in the 1930s. Ramey (2011) instead finds no evidence that the multiplier was larger during 1939-1949.

with more permanent deflation discussed in Benhabib, Schmitt-Grohé and Uribe (2001a,b, 2002). The reason is that the expectation of a future recovery fuels intertemporal substitution and makes firms reluctant to cut prices which implies that output must drop in equilibrium. This finding is consistent with earlier studies, such as Eggertson and Woodford (2003), Christiano, Eichenbaum and Rebelo (2011) and Eggertson (2011), that show that a binding zero lower bound can imply large welfare reducing output losses. One result in this literature is that the output declines are larger when prices are more flexible. We show that large reductions in economic activity can occur not only after fundamental shocks to aggregate demand, but also as a consequence of nonfundamental shifts in expectations. In our liquidity trap equilibria however, more price flexibility leads to smaller output losses.

The second contribution of this paper is to show that the effects of fiscal policy interventions in a liquidity trap are generally ambiguous. We construct equilibria in which supply side oriented fiscal policy interventions, such as cuts in labor income taxes, provide more output stimulus in liquidity traps than during normal times. On the other hand, fiscal policies that stimulate demand, such as increased government spending or transitory cuts in consumption taxes, become less successful in raising output at the zero lower bound. This may seem counterintuitive as the main problem in a liquidity trap is the paradox of thrift and weak demand. However, in our setting a policy of demand (supply) stimulus requires a larger (smaller) drop in income to eliminate excess savings in equilibrium. Our examples contradict several recent studies arguing that, when interest rates are kept constant, higher government spending *must* raise expected future inflation, lower real interest rates and therefore boost private spending. Eggertson (2011), Christiano et al. (2011) and Woodford (2011) argue that the marginal spending multiplier has to be larger and can be well above one in a liquidity trap.[2] Similarly, Eggertson (2011) shows that transitory cuts in consumption taxes become more effective, whereas labor tax cuts become contractionary. We show that these previous results do not follow automatically from the fact that monetary policy is not responsive, but are sensitive

[2] Cogan, Cwik, Taylor and Wieland (2010) find only a modest increase in the spending multiplier. Woodford (2011) argues that this result derives from their assumptions about the duration of the government spending stimulus.

to assumptions about the nature of the shock that drives the economy into a liquidity trap and about how changes in fiscal policy affect expectations.

The arguments in Christiano et al. (2011) and Eggertson (2011) and others favor spending policies over income tax policies for stabilization in liquidity traps and lend support to governments that have engaged in spending increases in the recent recession. Our analysis shows that the issue of the (relative) merits of different fiscal policy interventions cannot be settled without a deeper understanding of the causes of liquidity traps. Within the context of the New Keynesian model analyzed by Christiano et al. (2011), Eggertson (2011), and Woodford (2011), a key determinant of whether a liquidity trap is a fundamental or belief-driven equilibrium phenomenon is the expected duration of the shock inducing zero nominal interest rates. If agents expect interest rates to remain at zero for a sufficiently extended period, the model dynamics are characterized by indeterminacy. If marginal changes in fiscal policy have no direct effect on the expected duration of the liquidity trap, then existing results on the zero lower bound fiscal multipliers are reversed. The historical experiences with prolonged periods of near zero interest rates support the relevance of studying the dynamics in the indeterminacy region of the model.

It is well known at least since Sargent and Wallace (1975) that under an interest rate rules RE monetary models can display equilibrium indeterminacy. Some researchers dismiss the possibility of indeterminate dynamics as mere theoretical curiosities, see for instance McCallum (2003). One potential justification is that when the RE assumption is replaced with simple recursive learning schemes, the dynamics of the resulting model often do not converge to the 'undesirable' RE equilibria. We show that this is indeed the case for the belief-driven liquidity trap equilibria we describe in this paper. However, Evans, Guse and Honkapohja (2008) show that even under learning, the existence of the deflationary RE equilibria discussed by Benhabib, Schmitt-Grohé and Uribe (2001a,b, 2002) can have strong effects on the dynamics of output and inflation in response to shocks to expectations. We show that the same is true for our class of RE equilibria with transitory liquidity traps. Moreover, in numerical simulations the transitional dynamics due to learning turn out to have little influence on our results at least for local deviations from rational expectations.

Several recent studies have looked at optimal monetary policy in the face of fundamental shocks and binding constraints on nominal interest rates, e.g. Eggertson and Woodford (2003), Adam and Billi (2006) and Werning (2011). An important question in the design of optimal policies is how to rule out undesirable equilibria and avoid expectations driven liquidity traps.[3] Benhabib, Schmitt-Grohé and Uribe (2002) propose switches to non Ricardian policy regimes should agents coordinate on pessimistic expectations. Appendix A of this paper extends this proposal to the class of equilibria with temporary liquidity traps described in this paper. Atkeson, Chari and Kehoe (2010) propose regime switching monetary rules that implement the intended equilibrium by ensuring that if the average choice of private agents deviates from that in the intended equilibrium, the reversion policies imply a best response of each individual agent that is different from the average choice. In this paper we abstract from such sophisticated monetary policies. As Christiano et al. (2011), Eggertson (2011) and many others, we study an environment where monetary policy adheres to a Taylor rule.

## 2 Expectations Driven Liquidity Traps in the New Keynesian framework

### 2.1 The Environment

We consider a New Keynesian model with four types of agents: A large number of identical infinitely lived households; competitive final goods producers; monopolistically competitive intermediate goods firms that set prices subject to nominal rigidities; and a government that is in charge of fiscal and monetary policies.

[3]Atkeson, Chari and Kehoe (2010) and Cochrane (2011) reiterate that the Taylor principle is neither necessary nor sufficient for uniqueness. Cochrane (2011) reviews and criticizes various proposals to eliminate indeterminacy in the New Keynesian model.

**Households** Households maximize utility subject to a sequence of budget constraints and a no Ponzi game restriction:

$$\begin{aligned}
\mathcal{U} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \omega_t \left( U(c_t, l_t) + V(m_t) \right) && (1) \\
(1+\tau_{c,t}) P_t c_t + M_t + \frac{B_t}{1+i_t} &\leq (1-\tau_{n,t}) W_t (1-l_t) + B_{t-1} + M_{t-1} + \Upsilon_t + T_t && (2) \\
\lim_{s\to\infty} \mathbb{E}_t \frac{B_{t+s}}{(1+i_t)\cdots(1+i_{t+s})} &\geq 0 && (3)
\end{aligned}$$

$\mathbb{E}_t x_s$ denotes $E(x_s|I_t)$ where $I_t$ is the date $t$ information set, $\beta \in (0,1)$ is the subjective discount factor, $c_t$ is final goods consumption, $l_t \in [0,1]$ is leisure, $m_t = M_t/P_t$ are real money balances and $\omega_t > 0$ is a taste shock. $P_t$ is the final goods price level, $\tau_{c,t}$ is a sales tax, $B_t$ are purchases of one period nominal discount bonds and $i_t$ is the nominal interest rate. $\tau_{n,t}$ is a proportional labor income tax, and $W_t$ is the nominal wage. $\Upsilon_t$ denotes dividends and $T_t$ are government transfers. We assume that $U$ and $V$ are increasing and strictly concave and that:

$$\lim_{m\to\infty} \frac{V_m(m)}{U_c(c,l)} < 0 \ , \ \forall c, l \geq 0$$

where $V_m = \partial V(m)/\partial m$. This ensures finite real money demand at zero short term nominal interest rates. In equilibrium, the short-term nominal interest rate needs to be nonnegative, $i_t \geq 0 \, \forall t$, to guarantee budget sets are bounded, i.e. to avoid that agents can make arbitrarily large profits by choosing arbitrarily large money holdings financed by issuing bonds.

The optimality conditions are

$$\begin{aligned}
\frac{U_l(c_t, l_t)}{U_c(c_t, l_t)} &= \frac{(1-\tau_{n,t}) W_t}{(1+\tau_{c,t}) P_t} && (4) \\
U_c(c_t, l_t) &= \beta(1+i_t)\mathbb{E}_t \left[ \frac{\omega_{t+1}}{\omega_t} \frac{(1+\tau_{c,t}) P_t}{(1+\tau_{c,t+1}) P_{t+1}} U_c(c_{t+1}, l_{t+1}) \right] && (5) \\
\frac{V_m(m_t)}{U_c(c_t, l_t)} &= \frac{i_t}{1+i_t} \frac{1}{(1+\tau_{c,t})} && (6)
\end{aligned}$$

where $U_x := \partial U(c,l)/\partial x$ for $x = c,l$. Finally, optimal decisions must obey the transversality condition

$$\lim_{s\to\infty} \mathbb{E}_t \left[ \frac{B_{t+s} + M_{t+s}}{(1+i_t)\cdots(1+i_{t+s})} \right] = 0 \tag{7}$$

**Final Goods Sector** Final goods firms are competitive and produce an identical good by aggregating a continuum of intermediate goods purchased at prices $P_{it}$. The technology is

$$y_t = \left( \int_0^1 y_{it}^{1-1/\eta} di \right)^{1/(1-1/\eta)}, \ \eta > 1 \tag{8}$$

where $y_{it}$ is the input of intermediate good of variety $i$. Cost minimization implies:

$$y_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\eta} y_t \tag{9}$$

$$P_t = \left( \int_0^1 P_{it}^{1-\eta} di \right)^{1/(1-\eta)} \tag{10}$$

The final goods are used either for private or government consumption. The economy wide resource constraint is

$$y_t \geq c_t + g_t \tag{11}$$

where $g_t$ denotes government purchases of the final good.

**Intermediate Goods Sector** There is a continuum of price setting monopolistically competitive intermediate goods firms. Their technology is

$$y_{it} = n_{it} \tag{12}$$

where $n_{it}$ denotes producer $i$'s use of labor services. Each period, whether a firm can reset the price of its product is determined by a Poisson process with arrival rate $(1-\xi) \in (0,1]$. A firm $i$ that receives the opportunity to reset the price in period $t$ chooses $P_{it}^*$ to maximize

$$\mathbb{E}_t \sum_{s=t}^{\infty} \xi^{s-t} Q_{t,s} \left(P_{it}^* - (1-\tau_r) W_s\right) \left( \frac{P_{it}^*}{P_s} \right)^{-\eta} y_s \tag{13}$$

$Q_{t,s} = \beta^{s-t}\left(U_c(c_s,l_s)/U_c(c_t,l_t)\right)\left(P_t/P_s\right)$ is the stochastic discount factor and $\tau_r$ is a proportional employment cost subsidy. We assume $\tau_r = 1/\eta$ to eliminate the monopoly pricing distortion. The first order condition for $P^*_{it}$ is:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \xi^{s-t} Q_{t,s}\left[\left(P^*_{it} - W_s\right) y_{is}\right] = 0 \tag{14}$$

Since all firms that are given the chance to reset the price face identical optimization problems, they all set the same price, $P^*_t$. Consequently, using the law of large numbers, we can express the aggregate price index as

$$P_t^{1-\eta} = \xi P_{t-1}^{1-\eta} + (1-\xi)\, P_t^{*1-\eta} \tag{15}$$

Letting $n_t = \int_0^1 n_{it} di$ and equalizing supply and demand for intermediate good $i$ and aggregating across firms, aggregate output can be expressed as:

$$y_t = v_t^{-1} n_t, \tag{16}$$

where $v_t = \int_0^1 \left(P_{it}/P_t\right)^{-\eta} di \geq 1$ is a price dispersion term determined recursively as

$$v_t = \xi \pi_t^{\eta} v_{t-1} + (1-\xi)\, p_t^{*-\eta} \tag{17}$$

where $\pi_t = P_t/P_{t-1}$ is inflation and $p^*_t = P^*_t/P_t$. Price dispersion is an inefficiency wedge that arises due to the price setting friction. Its minimum value, $v_t = 1$, is reached when either prices are fully flexible or in equilibria in which the price level is constant.

**Government** We specify monetary policy by an interest rate rule

$$1 + i_t = \phi\left(\frac{\pi_t}{\tilde{\pi}}\right) \tag{18}$$

where $\tilde{\pi} \geq 1$ is an inflation target. We assume that $\phi(1) = \beta^{-1}\tilde{\pi}$ and that $\phi(\cdot) \geq 1$ for all $\pi_t$ such that the nominal interest rate satisfies the zero bound on interest rates, and that $\phi'(\cdot)$ is sufficiently large when $i_t > 0$ to ensure local determinacy in the neighborhood of $\tilde{\pi}$.[4] Below a critical value of $\pi_t$, the monetary authority implements a zero nominal interest rate.[5]

Fiscal policy involves a choice of taxes, government spending, and debt. The government's budget constraint is given as

$$\frac{B_t}{1+i_t} = B_{t-1} - M_t + M_{t-1} + D_t \tag{19}$$

where $D_t$ is the deficit in period $t$

$$D_t = P_t g_t + T_t + \frac{1}{\eta} W_t n_t - \left(\tau_{c,t} P_t c_t + \tau_{n,t} W_t \left(1 - l_t\right)\right) \tag{20}$$

Unless mentioned otherwise, we assume that fiscal policies are Ricardian, in the sense that they always satisfy equation (7).

### 2.2 Equilibrium Analysis

Let $w_t = W_t/P_t$, $b_t = B_t/P_t$, $t_t = T_t/P_t$, $d_t = D_t/P_t$.

**Equilibrium Definition** A competitive rational expectations equilibrium is a stochastic sequence of allocations $(c_t,\, n_t,\, l_t,\, y_t)_{t=0}^{\infty}$, prices $(\pi_t,\, w_t,\, p_t^*,\, v_t)_{t=0}^{\infty}$, monetary policies $(i_t,\, m_t)_{t=0}^{\infty}$, and fiscal policies $(b_t,\, d_t,\, g_t,\, \tau_{c,t},\, \tau_{n,t},\, t_t)_{t=0}^{\infty}$ such that (i) households maximize utility subject to all constraints, (ii) final goods producers maximize profits, (iii) intermediate goods producers maximize profits, (iv) monetary policy is guided by the interest rate rule, (v) fiscal policies are consistent with the government budget constraint, and (vi) goods, asset and labor markets clear, for given initial conditions $b_{-1}$, $m_{-1} \geq 0$ and $v_{-1} \geq 1$, a law of motion for $\omega_t$ and specifications of fiscal policies.

[4] Coibion and Gordonichenko (2011) show that, at positive trend inflation levels $\tilde{\pi} > 1$ local determinacy may require the central bank to raise interest rates more than one for one with inflation.

[5] In practice central banks may stop responding to inflation at strictly positive interest rates, but this is not important for our analysis. Neither is it important that (18) does not include the output gap, expected inflation or other observable macroeconomic variables. What is important is that we ignore possible unconventional monetary policy measures at the lower bound.

Market clearing requires

$$n_t = 1 - l_t \tag{21}$$

$$y_t = c_t + g_t \tag{22}$$

Given fiscal policies and a law of motion of $\omega_t$, the equilibrium sequences for output, inflation and price dispersion $(y_t, \pi_t, v_t)_{t=0}^{\infty}$ are solutions to the following system of nonlinear stochastic difference equations:

$$1 = \beta\phi\left(\frac{\pi_t}{\bar{\pi}}\right)\mathbb{E}_t\left[\frac{\omega_{t+1}}{\omega_t}\frac{(1+\tau_{c,t})}{(1+\tau_{c,t+1})\pi_{t+1}}\frac{U_c(y_{t+1}-g_{t+1}, 1-v_{t+1}y_{t+1})}{U_c(y_t-g_t, 1-v_t y_t)}\right] \tag{23}$$

$$p_t^*\pi_t = \frac{\mathbb{E}_t\sum_{s=t}^{\infty}(\beta\xi)^{s-t}\omega_s\frac{U_l(y_s-g_s,1-v_s y_s)}{1-\tau_{n,s}}\left(\prod_{j=0}^{s-t}\pi_{t+j}\right)^{\eta}y_s}{\mathbb{E}_t\sum_{s=t}^{\infty}(\beta\xi)^{s-t}\omega_s\frac{U_c(y_s-g_s,1-v_s y_s)}{1+\tau_{c,s}}\left(\prod_{j=0}^{s-t}\pi_{t+j}\right)^{\eta-1}y_s} \tag{24}$$

$$v_t = \xi\pi_t^{\eta}v_{t-1} + (1-\xi)\,p_t^{*-\eta} \tag{25}$$

for an initial condition $v_{-1}$, where $p_t^*$ is implicitly determined by

$$1 = \xi\pi_t^{\eta-1} + (1-\xi)\,p_t^{*1-\eta} \tag{26}$$

Equation (23) is the equilibrium version of the intertemporal Euler equation combined with the interest rate rule and with equation (16). Equation (24) is the equilibrium version of the condition for the optimal reset price. Equation (25) is the law of motion for price dispersion.

We deviate from the common practice of loglinearizing the equilibrium conditions in instead focus on solutions to the system of nonlinear equations in (23)-(25).[6] We do this for two reasons: First, our arguments are based on the global indeterminacy that arises because of the zero lower bound and it is therefore natural to study the global dynamics. Second, expectations driven liquidity

[6]See Wolman (2005), and Evans, Guse and Honkapohja (2008) for exceptions. In subsequent work, Christiano and Eichenbaum (2012) and Braun, Körber and Waki (2012) confirm the key results of this paper in the nonlinear New Keynesian model with Rotemberg adjustment costs.

traps can only exist when the economy is sufficiently far away from the (usual) steady state such that local linear approximation can become numerically inaccurate. In addition, deflationary equilibria may generate significant price dispersion, which is a source of persistence and inefficiency that is assumed away in local approximations around a zero inflation steady state. In Section 4 , we show that all our results carry over qualitatively to loglinearized versions of the model.

To facilitate the global analysis we focus exclusively on Markovian equilibria generated from recursion of a state space system of the form

$$u_t = f(s_t) \tag{27}$$

$$s_t = h(s_{t-1}) + \mu\varepsilon_t, \; s_0 \text{ given} \tag{28}$$

where $s_t$ denotes the vector of state variables, $u_t$ is the inflation/output vector, $\varepsilon_t$ contains random innovations to the exogenous stochastic processes in the state vector and $\mu$ is a selection vector. The state variables include price dispersion and the exogenous forcing process for $\omega_t$ when it is active.

**Steady States** We begin by studying the steady state properties of a deterministic version of the model. We set $\omega_t = 1$ for all $t$ and, for simplicity, ignore fiscal policy for now and set $g_t = \tau_{c,t} = \tau_{n,t} = 0$ for all $t \geq 0$. A steady state $(s,u)$ is a fixed point of (28) such that $s = h(s)$ and $u = f(s)$. As discussed by Benhabib, Schmitt-Grohé and Uribe (2001a,b, 2002), when fiscal policy is Ricardian and monetary policy follows an interest rate rule subject to a lower bound, there generally exist two different steady states.[7] Note from the Euler equation in (23) that the steady state real interest rate has to equal $1/\beta$ in order for consumption to be constant. The lower bound on the interest rate implies that this condition can hold for two different combinations of nominal interest rates and inflation. Intuitively, the zero lower bound prevents the central bank from eliminating market clearing at deflationary levels.

[7]With endogenous labor supply, more than two steady states can exist depending on the properties of labor supply. Because this is not the mechanism that generates multiple equilibria in this paper, we ignore this possibility.

The first steady state, which we refer to as the intended steady state $(\pi^I, y^I, v^I)$, has inflation at the target level $\pi^I = \tilde{\pi}$ and a positive nominal interest rate. Output and price dispersion are implicitly determined by

$$\frac{U_l(y^I, 1-v^I y^I)}{U_c(y^I, 1-v^I y^I)} = \frac{1-\xi\beta\tilde{\pi}^{\eta}}{1-\xi\beta\tilde{\pi}^{\eta-1}}\left(\frac{1-\xi}{1-\xi\tilde{\pi}^{\eta-1}}\right)^{\frac{1}{\eta-1}} \quad , \quad v^I = \frac{1-\xi}{1-\xi\tilde{\pi}^{\eta}}\left(\frac{1-\xi\tilde{\pi}^{\eta-1}}{1-\xi}\right)^{\frac{\eta}{\eta-1}} \tag{29}$$

For most of the analysis, we set $\tilde{\pi} = 1$ so that the government pursues price stability. In that case, there is no price dispersion in the intended steady state, $v^I = 1$. This implies that $y^I = y^E$ where $y^E$ equals the efficient (flexible price) level of output determined implicitly by the condition $U_l(y^E, 1-y^E)/U_c(y^E, 1-y^E) = 1$. The intended steady state with a zero inflation target therefore acts as a useful welfare benchmark (ignoring real money holdings).

There exists a second, unintended, steady state $(\pi^U, y^U, v^U)$, in which the nominal interest rate is at the lower bound, i.e. $\phi(\pi/\tilde{\pi}) = 1$, and there is deflation $\pi^U = \beta$. As in the intended steady state, the real interest rate equals $1/\beta$. With declining price levels, firms that can reset the price of their good set a relative price below unity, $p^{*U} < 1$. The unintended steady state levels of output and price dispersion are determined by

$$\frac{U_l(y^U, 1-v^U y^U)}{U_c(y^U, 1-v^U y^U)} = \frac{1-\xi\beta^{1+\eta}}{1-\xi\beta^{\eta}}\left(\frac{1-\xi}{1-\xi\beta^{\eta-1}}\right)^{\frac{1}{\eta-1}} , \quad v^U = \frac{1-\xi}{1-\xi\beta^{\eta}}\left(\frac{1-\xi\beta^{\eta-1}}{1-\xi}\right)^{\frac{\eta}{\eta-1}} \tag{30}$$

Output in the unintended steady state differs from the efficient output level, $y^U \neq y^E$. Price dispersion drives a wedge between the marginal utility of consumption and leisure and $U_l(y^U, 1-v^U y^U) < U_c(y^U, 1-v^U y^U)$. If the marginal rate of substitution between leisure and consumption is increasing in output, this translates to consumption, labor supply and output levels below the efficient level. However, as we will show below, the discrepancy $y^U - y^E$ tends to be small quantitatively relative to the output loss that can occur in a temporary liquidity trap. With flexible prices, or indexation to lagged inflation, the unintended steady state still exists but output and consumption are at efficient levels. Abstracting from utility derived from real money balances, the welfare loss from being in a perpetual liquidity trap comes exclusively from price dispersion.

**Sunspot Equilibria** The multiplicity of steady states is a strong indicator for the existence of sunspot equilibria.[8] In a sunspot equilibrium, agents condition their expectations on an information set that contains an extrinsic random variable, or sunspot (see Shell (1977) and Cass and Shell (1983)), that otherwise has no impact on fundamentals. We interpret the sunspot as indicating exogenous variations in confidence or sentiment.[9] We denote confidence by $\psi_t$ and assume it evolves according to a $n$-state discrete Markov chain, $\psi_t \in [\psi_1, ..., \psi_n]$ with an associated probability transition matrix $R$. Equilibrium dynamics are still described by the system of the form (27)-(28), but the vector of state variables contains the confidence variable $\psi_t$, i.e. $s_t = [v_{t-1}, \omega_t, \psi_t]$.

Formally, a Markov sunspot equilibrium is a Markov competitive equilibrium defined by a pair of functions $f(s_t)$ and $h(s_t)$ for which $f([v_{t-1}, \omega_t, \psi_t = \psi_i]) \neq f([v_{t-1}, \omega_t, \psi_t = \psi_j])$ and $h([v_{t-1}, \omega_t, \psi_t = \psi_i]) \neq h([v_{t-1}, \omega_t, \psi_t = \psi_j])$ for $i \neq j$, where $i, j = 1, ..., n$. Therefore, output and inflation are stochastic processes whose values depend on the realization of the random variable $\psi_t$. Sunspot solutions to (23)-(25) contrast with minimal state variable solutions that are only a function of $v_{t-1}$ and $\omega_t$.

Fluctuations in confidence allow for temporary liquidity traps during which output may drop far below the unintended steady state level of output. The main reason derives from the real interest rate adjustment needed to ensure market clearing. Suppose agents grow pessimistic and expect a temporary but persistent drop in income leading to lower desired consumption. Nominal rigidities and market clearing require output and prices to fall. Since pessimism is temporary the price setters are reluctant to cut prices but, if the zero lower bound binds, any fall in prices produces a temporary increase in the real rate that triggers intertemporal substitution effects. Because saving must

[8] This is because sunspot equilibria usually exist near distinct steady states. Eliminating the second steady state, for instance by targeting deflation $\tilde{\pi} = \beta$, however is not sufficient to rule out sunspot fluctuations. We analyze sunspot equilibria that are generally far away from the steady states.

[9] See Benhabib and Farmer (2000) for an excellent survey of macroeconomic models with indeterminacy and sunspot equilibria.

be zero in equilibrium, the higher real rates require a further drop in output and stronger price declines, which again increase real interest rates and lower consumption, etc. This downward spiral ends when output and wealth have fallen sufficiently to discourage saving and the real interest rate equates consumption to output. Because of lower output, the initial loss of confidence can become a self fulfilling prophecy consistent with rational expectations. Locally, the monetary authority can prevent this downwards spiralling savings glut by lowering nominal interest rates sufficiently to offset the real rate increases. Globally, however, it is unable to do so because of the zero lower bound.

**A Two State Example** Suppose the sunspot variable $\psi_t$ follows a two state Markov chain with transition matrix $R$,

$$\psi_t \in [\psi_O, \psi_P] \ , \ R = \begin{bmatrix} 1 & 0 \\ 1-q & q \end{bmatrix} \ , \ 0 < q < 1 \tag{31}$$

The first state, $\psi_O$, is the intended state where (relative) optimism prevails. The second state, $\psi_P$, is characterized by pessimism. Once sentiments are pessimistic, the probability of continued pessimism in the next period is given by $q$. The optimistic state is assumed to be absorbing. We focus on this two state case with one absorbing state because it simplifies the intuition and because it facilitates comparison with Christiano et al. (2011), Eggertson (2011) and Woodford (2011), whose liquidity trap inducing shock has the exact same stochastic properties. It is straightforward to allow for transitions from the optimistic state to the pessimistic state, in which case output and inflation in the optimistic state no longer converge to the intended steady state.

The simple stochastic structure permits a graphical representation of the equilibrium dynamics. Assume that $\omega_t = 1$ for all $t$ and that the only non-zero fiscal policy instrument is the constant employment subsidy, i.e. $g_t = \tau_{n,t} = \tau_{c,t} = 0$ for all $t$. Let $\pi_P$, $y_P$ and $v_P$ be the values of inflation, output and price dispersion that the economy converges to conditional upon $\psi_t = \psi_P$ (the fixed points of $f([v_{t-1}, \psi_t = \psi_P])$ and $h([v_{t-1}, \psi_t = \psi_P])$). Furthermore, let $\pi'_O$, $y'_O$ and $v'_O$ denote the values obtained from evaluating $f([v_P, \psi_t = \psi_O])$ and $h([v_P, \psi_t = \psi_O])$. These are the values of inflation, output and price dispersion immediately after returning to optimism from the pessimistic state $[v_P, \psi_P]$. Evaluating equations (23) and (24) at the point $\pi_P$, $y_P$ and $v_P$ yields:

$$U_c(y_P, 1 - v_P y_P) = \beta\phi\left(\frac{\pi_P}{\tilde{\pi}}\right)\left[\frac{q}{\pi_P}U_c(y_P, 1 - v_P y_P) + \frac{1-q}{\pi'_O}U_c(y'_O, 1 - v'_O y'_O)\right] \quad (32)$$

$$p^*_P = \frac{(1 - \beta\xi q \pi_P^{\eta-1})}{(1 - \beta\xi q \pi_P^{\eta})}\left(\Lambda_P \frac{U_l(y_P, 1 - v_P y_P)}{U_c(y_P, 1 - v_P y_P)} + (1 - \Lambda_P) p^{*\prime}_O \pi'_O\right) \quad (33)$$

where $0 < \Lambda_P < 1$ is a complicated function of expectations of future inflation and output levels and $p^*_P, p^{*\prime}_O$ are linked to $\pi_P$, $\pi'_O$ through equation (26). Since $p^{*\prime}_O$, $\pi'_O$, $y'_O$, $v'_O$ as well as $\Lambda_P$ are functions of $v_P$ and $v_P$ is related to $\pi_P$ through $(1 - \xi\pi_P^{\eta})v_P = (1 - \xi)\, {p_P^*}^{-\eta}$, the above equations describe two relationships between inflation and output that can be graphed in the two dimensional plane. We will refer to these relationships as $(\pi, y)^{EE}$ and $(\pi, y)^{AS}$, respectively, where $(\pi, y)^{EE}$ are the combinations of $\pi$ and $y$ that are consistent with condition (32) and $(\pi, y)^{AS}$ are those consistent with (33). Intersections describe possible limit points to which the economy may converge while $\psi_t = \psi_P$.

Figure 1 depicts two possible cases that can arise for identical preference and policy parameters, but different values of the parameter $q$. There is always an intersection that corresponds to $y^I$ and $\pi^I$. This cannot be a limit point of a sunspot equilibrium, since the equilibrium outcomes in that case are identical across realizations of $\psi_t$. For a range of values of the parameter $q$, there exists a second intersection at $\pi_P$ and $y_P$ that is characterized by deflation and zero nominal interest rates. The reason is that the $(\pi, y)^{EE}$ schedule implies a downward sloping relationship between output and inflation for sufficiently high levels of inflation, but becomes upward sloping for levels of inflation for which the zero lower bound is binding.

The left panel of Figure 1 depicts a situation for a value of $q$ for which the intended steady state is the unique steady state. The right panel shows a case with a different value of $q$ for which a second intersection point does exist. As $q$ tends to zero, the upward sloping part of the $(\pi, y)^{EE}$ becomes steeper, while the $(\pi, y)^{AS}$ curve becomes flatter. For low enough values of $q$, a second intersection does not exist. Intuitively, if the expected duration of the pessimistic state is short, agents cannot rationally expect a sufficient amount of deflation that would lead to a binding lower bound on the short term nominal interest rate. For $q$ larger than a certain critical value, the equilibrium conditions support a sunspot limit point characterized by depressed output levels, deflation and zero nominal interest rates. For $q \to 1$, the second intersection converges to the unintended steady state $\pi^U$ and $y^U$.

At any second intersection, the $(\pi, y)^{AS}$ curve is steeper than the $(\pi, y)^{EE}$ curve, which is a necessary condition for the existence of the sunspot equilibria we study. This determinacy condition on the relative slopes of $(\pi, y)^{AS}$ and $(\pi, y)^{EE}$ happens to be the exact opposite of the parameter restrictions of Christiano et al. (2011) and Eggertson (2011) required to generate a liquidity trap outcome after a discount factor or interest rate spread shock. Suppose that there is a shock to preferences $\omega_t$ that evolves according to a Markov process with transition matrix $R$ in (31). An increase in $\mathbb{E}_t \omega_{t+1}/\omega_t$ shifts the $(\pi, y)^{EE}$ schedule to the left. If $\mathbb{E}_t \omega_{t+1}/\omega_t$ increases enough and $q$ is sufficiently low, it can generate a liquidity trap in equilibrium, as illustrated in the left panel of Figure 2. In such a liquidity trap, the $(\pi, y)^{EE}$ curve must be steeper then the $(\pi, y)^{AS}$ curve. However, if the expected duration of the regime is too long, as in the right panel of Figure 2, a liquidity trap cannot arise.[10] The difference in slopes of the two schedules between the right panel of Figure 1 and the left panel of Figure 2 is the reason why, as we discuss below, policy interventions lead to different outcomes depending on the type of shock, fundamental or nonfundamental, generating the liquidity trap.

**Sunspots in a Calibrated Model** The discussion above ignores transitional dynamics due to price dispersion. If the Taylor principle holds, the dynamics in the neighborhood of the intended steady state are locally uniquely determined. On the other hand, the limit point $(\pi_P, y_P, v_P)$ is a sink such that generally the transitional dynamics in the pessimistic state are not uniquely determined. We now compute the dynamics restricting attention to equilibria in which the only state variables are $v_t$ and $\psi_t$. Given the requirement that the dynamics are recursive in this state space, we always found a unique nonlinear equilibrium path to the point $(\pi_P, y_P, v_P)$.

[10]Of course the $\omega_t$-shock can be of the opposite sign, shifting the $(\pi, y)^{EE}$ curve to the right such that two intersections exist. This is the extension considered recently by Braun, Körber and Waki (2012). If a sunspot selects the lower intersection, this case is qualitatively identical to the pure sunspot driven liquidity trap without discount factor shock.

We impose the following functional forms:

$$U(c_t, l_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \theta \frac{l_t^{1-\kappa} - 1}{1-\kappa}, \ \sigma, \theta, \kappa > 0 \tag{34}$$

$$\phi\left(\frac{\pi_t}{\tilde{\pi}}\right) = \max\left(\frac{\pi_t^{\phi_\pi}}{\beta}, 1\right), \ \phi_\pi > 1 \tag{35}$$

The policy rule in (35) assumes a price stability target and the restriction $\phi_\pi > 1$ guarantees local determinacy in the neighborhood of the intended steady state. The max operator ensures that the policy rule satisfies the zero lower bound. We assume that there are no fundamental sources of uncertainty, i.e. $\omega_t = 1$ for all $t$. We set the parameter values to $\beta = 0.99$, $\kappa = 2.65$, $\sigma = 1$, $\eta = 10$, $\phi_\pi = 1.5$ and $\xi = 0.65$. The value for $\beta$ implies an annual real interest rate in the intended steady state of 4 percent. The calibrated value of $\xi$ means that firms can adjust prices approximately once every three quarters, see for instance Nakamura and Steinsson (2008). The value of $\eta = 10$ implies a markup of 11 percent in the intended steady state. $\theta$ is chosen so that households spend 30 percent of their time endowment working in the intended steady state . The value of $\kappa$ implies a Frisch elasticity of around 0.75, which is in the range of values deemed realistic by labor economists.

Figure 3 depicts the equilibrium path assuming the transition matrix for $\psi_t$ given in (31) with $q = 0.80$.[11] At date 0, the economy is in the pessimistic state displaying a liquidity trap. Confidence is regained and interest rates become positive at (the stochastic) date T. The initial distribution of prices is characterized by $v_{-1} = 1$. The black solid (broken) horizontal lines denote the equilibrium values at the intended (unintended) steady state. Output in the upper left panel of Figure 3 is plotted in percentage deviation of the intended steady state level. The other panels display the annual inflation rate, the level of price dispersion, and the annual level of short term nominal interest rate. Starting from the initial state $(v_{-1}, \psi_P)$, output and inflation converge to the sunspot limit point. As long as deflation persists, the price dispersion increases but the transitional dynamics turn out to

[11] The functions $h(\cdot)$ and $f(\cdot)$ are approximated numerically by piecewise linear functions obtained from time iteration of a recursive version of the system in (23)-(25). Different equilibria (for a fixed value of $q$) can be found by varying the starting point in the iterations. Matlab programs are available on the authors' website.

be relatively unimportant. Until pessimism turns to optimism at date $T$, output and consumption are about 0.9% below the efficient level, whereas the annual rate of inflation is 6% below the (zero) target. In the unintended steady state, output is only about 0.2% below the efficient level.

Figure 4 displays equilibrium paths for a different calibration where $\sigma = 0.7$ and $\xi = 0.82$ (the other parameters remain the same). Hence, savings are more interest rate elastic, which flattens the $(\pi, y)^{EE}$ schedule. The lower value of $\sigma$ also makes marginal cost less elastic with respect to output and the higher value of $\xi$ implies more rigid prices, both of which flatten the $(\pi, y)^{AS}$ schedule. In the unintended steady state, output is about 1% below the efficient level because of price dispersion. In the temporary liquidity trap, there is a 6% gap with the efficient level of output in the short run, whereas the annual rate of inflation is 8.5% below the target. For other parameter values the output loss in a liquidity trap can be even larger. The largest deviations from the intended steady state are obtained when the $(\pi, y)^{AS}$ and $(\pi, y)^{EE}$ curves are similar in slope.[12]

**Sensitivity Analysis** Figure 5 plots the levels of output and inflation in the liquidity trap limit for alternative parameter values of $\sigma$, $\xi$, $q$ and $\kappa$. We center the sensitivity analysis around the parameter values of the benchmark calibration, i.e. $\sigma = 1$, and $\xi = 0.65$. The other parameters are the same as before. In the figure, the squares denote the benchmark parameter values. The graphs display output and inflation levels in the sunspot limit points obtained from the system in (32)-(33).

The first row of Figure 5 shows that the output drop is decreasing in $\sigma$. A lower value of $\sigma$ flattens the $(\pi, y)^{EE}$ curve and strengthens intertemporal substitution in response to price declines. It also makes marginal cost less elastic with respect to output and hence flattens the $(\pi, y)^{AS}$ curve. Therefore, a lower value of $\sigma$ implies a larger fall in income and a more modest increase in the real interest rate in a liquidity trap. The second row of Figure 5 illustrates that output losses are increasing in the degree of price stickiness. This finding contrasts sharply with the case of a funda-

[12]This also true when the liquidity trap is generated by a fundamental shock, as in Christiano et al. (2011) and Eggertson (2011). The key difference is that here the slope of the AS curve needs to approach the slope of the EE curve from below, and not vice versa.

mentals driven liquidity trap, see for instance Christiano et al. (2011) and Werning (2011). When firms expect to be able to reset prices in the future with lower probability, they are less willing to accommodate demand through reductions in their current price and instead reduce production. But this requires stronger deflation and real interest increases to equate consumption with output. As the value of $\xi$ increases, the slope of the $(\pi, y)^{AS}$ curve approaches the slope of the $(\pi, y)^{EE}$ curve and output drops grow very large. Above a critical value of $\xi$ (in this case approximately 0.82), the sunspot limit point ceases to exist. As $\xi$ approaches zero, output converges to the flexible price efficient output level, $y_P \to y^E$ , while inflation approaches $\pi_P \to \beta q/(1-\beta(1-q))$.

The third row of Figure 5 shows the range of the persistence parameter $q$ for which a sunspot equilibrium exists. In this case the critical value of $q$ is approximately 0.56. The longer pessimism is expected to prevail, the smaller are the output losses and levels of deflation during the liquidity trap. The temporary nature of the confidence crisis creates intertemporal substitution motives and makes firms reluctant to cut prices. Higher values of $q$ flatten the $(\pi, y)^{EE}$ curve and steepen the $(\pi, y)^{AS}$ curve. For $q \to 1$, inflation and output levels in a liquidity trap approach the levels of the unintended steady state, $y^U$ and $\pi^U$. Finally, the last row in Figure 5 shows the effect the elasticity of labor supply. A higher Frisch elasticity (lower $\kappa$) flattens the $(\pi, y)^{AS}$ curve as marginal cost becomes less elastic with respect to output and therefore leads to stronger declines in output and inflation.

We conclude that expectations driven liquidity traps can arise under relatively weak conditions. Roughly speaking, the requirement is that the $(\pi, y)^{EE}$ curve in (32) is flatter than the $(\pi, y)^{AS}$ curve in (33), which is consistent with wide range of plausible parameter values. Temporary expectations driven liquidity traps are also very likely to exist in more complicated monetary models as long as the monetary authority operates an interest rate target subject to a lower bound. If there exists an inflation-output trade off, a liquidity trap induced by a loss in confidence will automatically be associated with potentially very large drops in output and welfare.[13]

[13]Braun, Körber and Waki (2012) and Christiano and Eichenbaum (2012) replicate our results for a model with Rotemberg adjustment costs. In Mertens and Ravn (2011a,b) , we show that when the New Keynesian model is augmented with financial frictions, the output losses in an expectations driven liquidity trap become significantly larger.

Blanchard, Dell'Ariccia and Mauro (2010) and others have suggested raising inflation targets to alleviate constraints on monetary policy. Figure 6 depicts inflation and output in the intended steady state as well as in a liquidity trap for different values of the inflation target $\tilde{\pi}$. Larger deviations from a nonzero target generate more price dispersion, which lowers output relative to the efficient level. In a liquidity trap, a higher inflation target makes price setters more reluctant to cut prices and therefore flattens the $(\pi, y)^{AS}$ schedule. As long as the $(\pi, y)^{AS}$ remains steeper then the $(\pi, y)^{EE}$ curve, for a sunspot of given persistence $q$, this results in larger output drops and more deflation in a liquidity trap. Another important effect, however, is that the value of the inflation target impacts on the range of values of $q$ for which expectations driven liquidity traps can exist. Figure 7 shows the combinations of the probability $q$ and the inflation target $\tilde{\pi}$ for which there exist two intersections as in panel $b$ of Figure 1. Higher inflation targets raise the critical value of $q$ for which two equilibria exists. In general, however, higher inflation targets alone will not succeed in ruling out the possibility of expectations driven liquidity traps, while output losses and deflation become more pronounced when they occur. In our calibrated example, an inflation target as high as 7 percent still permits expectations driven liquidity traps for values of $q$ at least 0.70.

## 3 Fiscal Policy in a Liquidity Trap

Given the existence of equilibria in which pessimism brings the economy into a recession and forces the monetary authority to lower interest rates to their lowest possible levels, it is interesting to examine how changes in fiscal policy affect equilibrium outcomes. Since the key problem in a deflationary liquidity trap is weak demand, either because of a fundamental demand shock or a loss of confidence, a natural policy response is to increase public sector demand. Christiano et al. (2011), Eggertson (2011) and Woodford (2011) examine fiscal stimulus in a liquidity trap generated by a fundamental demand shock such as $\omega_t$ and argue that demand stimulating policies must have stronger effects on output when nominal rates are constant. Eggertson (2011), for example, finds a multiplier after a marginal increase in government spending of 2.3 in a liquidity trap compared to 0.3 when the short term interest rate is positive. The intuition is that, when interest rates are at the lower bound, temporary but persistent expansionary fiscal policy lowers the real interest rates (due to the impact on inflation expectations) and crowds in consumption. On the other hand, interventions intended to stimulate the supply side, such as cuts in labor taxes, are necessarily counterproductive in a liquidity trap. In Eggertson (2011), the output multiplier of a labor income tax rate cut is mildly positive when the interest rate is positive, but negative in a liquidity trap. This is because in a liquidity trap supply stimuli lower inflation expectations, increase real rates and crowd out private consumption.

The same fiscal policy interventions can also be analyzed in expectations driven liquidity trap equilibria. One complication in the presence of equilibrium multiplicity is that without further assumptions the outcome of comparative statics exercises are also not uniquely determined. In our model, equilibria are selected by the Markov process for $\psi_t$ and can be indexed by the parameter $q$. In this section we derive results under the assumption that the value of $q$ is invariant to policy changes. This assumption is the most natural for a comparison with Christiano et al. (2011) and Eggertson (2011) and seems a reasonable benchmark given that we will consider only marginal policy interventions. More generally however, nothing precludes agents' sentiments from being directly changed by fiscal policy interventions, for instance through changes in the value of $q$ or by causing a jump to the intended equilibrium. For the purpose of demonstrating how the existing results in the literature can be overturned, the assumption of constant $q$ is sufficient.

When a confidence shock brings about zero nominal interest rates and agents' sentiments, i.e. the process for $\psi_t$ does not change, fiscal stimulus in the form of government spending may not be very successful in raising demand. In particular, if the economy remains in the liquidity trap, output may respond adversely to an increase in government spending depending upon how changes in fiscal policies impact on real interest rates. Ultimately, whether fiscal stimulus increases or lowers real interest rates depends on the relative slope of the $(\pi, y)^{AS}$ and $(\pi, y)^{EE}$ schedules. Because the relative slopes must be different depending on whether a liquidity trap arises from a fundamental demand shock or from a loss of confidence, the effects of marginal demand and supply side policies have essentially the opposite sign.

To see this graphically, consider Figure 8. The first row depicts the effects of policy induced shifts in aggregate demand or supply in a liquidity trap that is generated by an initial large leftward shift of the $(\pi, y)^{EE}$ schedule (from 1 to 2). In this case expansionary demand policies (from 2 to 3, left panel) are inflationary and the output effect is positive and can be very large. At the same time, an increase in $(\pi, y)^{AS}$ (from 2 to 3, right panel) only leads to more deflation and lower output. The second row in Figure 8 depicts the case where the economy is in an expectations driven liquidity trap (point 2). In this case, conditional upon no change in sentiments, expansionary demand policies (from 2 to 3, left panel) further depress output and lead to more deflation. In contrast, expansionary supply policies increase output and lead to more moderate deflation (from 2 to 3, right panel).

The graphical representation of the effects of policy changes is incomplete because it ignores the fact that fiscal interventions such as increased government spending have simultaneous demand and supply effects. We therefore quantify the multipliers associated with the various fiscal policy instruments in numerical solutions of the model. We do this by looking at small perturbations of the equilibrium paths. Our approach to computing the fiscal multipliers is conceptually identical to Eggertson (2011) and Christiano et al. (2011). Multipliers at positive interest rates are found by analyzing the effect of small changes in fiscal policy in the neighborhood of the intended steady state. The multipliers in a liquidity trap are derived by considering small changes in fiscal policy in the liquidity trap state of the sunspot equilibrium, keeping constant the parameter $q$. One difference with most of the literature is that we compute the multipliers based on the nonlinear solution of the functions $f(\cdot)$ and $h(\cdot)$, whereas previous studies rely on linear approximations. In the neighborhood of the intended steady state, both approaches yield the same numbers for the fiscal multipliers. For larger deviations from the intended steady state, such as required to generate zero interest rates, the multipliers may be quantitatively different as a result of nonlinearities. We stress however that it is not the nonlinear approximation, but the nature of the shock that triggers a liquidity trap that is the reason why our results are qualitatively different from Eggertson (2011) and Christiano et al. (2011), see also Section 4 .

**Spending multipliers** Let $(y_t)_{t=0}^{\infty}$ be an equilibrium path for output in the model where government spending is constant, i.e. $g_t = g$. Next, let $(y_t(\delta))_{t=0}^{\infty}$ be an equilibrium path where government spending starts at $g+\delta$ where $\delta > 0$ and in subsequent periods, spending remains at $g+\delta$ with probability $p_g$ and returns to $g$ with probability $1-p_g$. Once spending has returned to $g$, it remains at that level forever. The marginal spending multiplier in period $t$ is computed as

$$m_t^g = \lim_{\delta \to 0} \frac{y_t(\delta) - y_t}{\delta} \tag{36}$$

In the case of the sunspot equilibrium, we impose that spending is $g+\delta$ in the liquidity trap state and $g$ when interest rates are positive. This means that the spending process is perfectly correlated with the sunspot, or alternatively that spending is automatically adjusted in response to the liquidity trap state. Again, this approach corresponds exactly to the setup of Eggertson (2011) and Christiano et al. (2011). The level of government spending $g$ is set to be 20% of output in the intended steady state. Figure 9 depicts the spending multipliers for different values of $\xi$ , $p_g = q$, $\sigma$ and $\kappa$. The parametrization is centered around the same values as before, including $\xi = 0.65$ and $\sigma = 1$. The multipliers are very similar for different levels of price dispersion, so the figures only plot their values at the intended steady state and the sunspot limit point.

The multiplier in the neighborhood of the intended steady state is about 0.55 for the benchmark parameter values. It is always positive and, for the range of parameters we consider, smaller than one because of crowding out. The multiplier falls as consumption becomes more interest rate sensitive (lower $\sigma$), as the spending increase more persistent (higher $q$), as prices less sticky (lower $\xi$) or labor supply less elastic (higher $\kappa$). For all parameter values, the spending multiplier at the zero bound is smaller than the multiplier at the intended steady state with a positive nominal interest rate. For the benchmark liquidity trap it is roughly 0.35. The multiplier usually remains positive in a liquidity trap despite a shift in $(\pi, y)^{EE}$ (as in the left panel of the second row in Figure 8) because there is also an outward shift of the aggregate supply schedule due to a wealth effect on labor supply. The liquidity trap multiplier is always smaller than under positive short term nominal interest rates, because the increase in spending leads to higher real interest rates and crowding out. For parameter values where the output drop in a liquidity trap is the largest, the spending multiplier declines the most. This happens when the slopes of the $(\pi, y)^{AS}$ and $(\pi, y)^{EE}$ curves become similar, which occurs for low values of the persistence $q$ or high degrees of price stickiness. For the lowest values of $q$, the liquidity trap multiplier becomes mildly negative.

**Tax multipliers** We compute the multipliers associated with temporary changes in sales and labor income taxes as

$$m_t^{\tau} = -\lim_{\delta \to 0} \frac{y_t(\delta) - y_t}{y_t \delta} \tag{37}$$

where $\{y_t\}_{t=0}^{\infty}$ is the equilibrium path with a constant tax rate $\tau$ and $\{y_t(\delta)\}_{t=0}^{\infty}$ is an equilibrium path where the tax rate starts at $\tau + \delta$ where $\delta > 0$ and in subsequent periods, the tax rate remains at $\tau + \delta$ with probability $p_\tau$ and returns to $\tau$ with probability $1 - p_\tau$. To be precise, $m_t^{\tau}$ is the tax semi-elasticity of output, i.e. it is the percent change in output associated with a marginal decrease in the tax rate. We use the same parameters as before but set $\tau_c = 0.10$ and $\tau_n = 0$ when computing the sales tax multipliers, and $\tau_c = 0$, $\tau_n = 0.25$ when computing the labor income tax multipliers.

Figure 10 depicts the sales tax multipliers, whereas Figure 11 plots the labor tax multipliers for different values of $\xi$ , $p_\tau = q$ , $\sigma$ and $\kappa$. The effects of a temporary decrease in sales taxes are qualitatively very similar to an increase in government spending. There is a rightward shift in the $(\pi, y)^{EE}$ schedule because current consumption becomes cheaper relative to future consumption, while the $(\pi, y)^{AS}$ curve shifts to the right because of intertemporal substitution of labor supply. For the benchmark parameters, the sales tax multiplier is 0.4. The effect is larger for higher elasticities of intertemporal substitution of consumption and labor supply, more temporary tax cuts and a larger degree of price stickiness. For all parameter values, the effects of sales tax cuts are reduced in a liquidity trap driven by confidence loss. As was the case for spending increases, the rightward shift in $(\pi, y)^{EE}$ leads to more deflation crowding out consumption. In the benchmark case, the sales tax multiplier drops to 0.27. The impact on the multiplier can be much larger when the $(\pi, y)^{AS}$ and $(\pi, y)^{EE}$ slopes are close, which is the case when $q$ declines and $\xi$ grows large.

A labor tax cut instead becomes more powerful in the confidence driven liquidity trap. A labor tax cut raises aggregate supply through labor supply, but leaves the $(\pi, y)^{EE}$ relationship unchanged. At positive nominal interest rates, this leads to lower prices, which the monetary authority accommodates by a nominal rate cut. For the benchmark case, the labor tax multiplier is approximately 0.45. The output effect becomes larger when intertemporal substitution effects are stronger, the tax cut is more temporary and prices are more flexible. In a liquidity trap, the tax multiplier is always larger than at positive interest rates. For the benchmark case, the tax multiplier is 0.65. Again, the difference in multipliers grows larger for parameter values that lead to the largest drops in output, which occurs when $(\pi, y)^{AS}$ and $(\pi, y)^{EE}$ have similar slopes.

These results confirm a general consensus among economists that the effects of fiscal policy depend on the monetary policy reaction. However, many economists have argued that increased spending must generate larger effects in a liquidity trap. The analysis above provides a counterexample to this assertion. It highlights the importance of knowing the cause of the downturn for determining the relative merits of different fiscal policy interventions. Two important caveats are in order: First, our results are based on the assumption that the expected duration of the liquidity trap is not changed by the policy intervention. With alternative assumptions on how policy changes affect expectations, multipliers can be both smaller or larger than those we report. Second, the fiscal multipliers in this section are for marginal policy changes only. The average effect of any non infinitesimal policy intervention may be much different from its marginal effect. For instance, a fiscal measure may succeed in avoiding or shortening the liquidity trap in which case it becomes all the more desirable.[14]

## 4 Learning, Liquidity Traps and Fiscal Multipliers

Any policy analysis conducted in the New Keynesian framework relies importantly on the assumption of rational expectations (RE). Many have advocated for verifying the robustness to alternative assumptions regarding the formation of expectations, and in particular to small deviations from

[14] This point was stressed by Erceg and Lindé (2010)

strict rationality. A common criterion in this regard is expectational (E-) stability of RE equilibria, which is closely related to the local stability properties under simple recursive learning schemes, such a least squares or (small) constant gain learning.[15] E-stability has been used to evaluate policy rules, e.g. Bullard and Mitra (2002), or for selecting among multiple RE equilibria, e.g. McCallum (2003). In this section we investigate the E-stability properties of the nonfundamental RE equilibria discussed above and investigate the dynamics when learning agents make expectational errors.

**A Linearized Model** In order to facilitate the analysis, we examine a version of the model where the equilibrium conditions are first linearized and only the zero lower bound nonlinearity is preserved. We assume the functional forms (34)-(35) with a price stability target $\tilde{\pi} = 1$ and $\sigma = 1$. For simplicity we abstract from sales taxes, i.e. $\tau_{c,t} = 0$ for all $t$, as well as from preference shocks $\omega_t = 1$ for all $t$. Linearizing the equilibrium conditions in (23)-(25) around the intended steady state yields

$$\hat{y}_t - s_g \hat{g}_t = \hat{y}^e_{t+1} - s_g \hat{g}^e_{t+1} - (1 - s_g)\left(\beta(i_t - r) - \hat{\pi}^e_{t+1}\right) \quad (38)$$

$$\hat{\pi}_t = \frac{(1-\beta\xi)(1-\xi)}{\xi}\left(\left(\frac{\kappa y^I}{1-y^I} + \frac{1}{1-s_g}\right)\hat{y}_t - \frac{s_g}{1-s_g}\hat{g}_t + \frac{\tau_{n,t}-\tau_n}{1-\tau_n}\right) + \beta\hat{\pi}^e_{t+1} \quad (39)$$

$$i_t = \max\left(\frac{1-\beta}{\beta} + \frac{\phi_\pi \hat{\pi}_t}{\beta}, 0\right) \quad (40)$$

where $\hat{y}_t$ ($\hat{g}_t$) denotes the percentage deviation of output (government spending) from the intended steady state value, $\hat{\pi}_t$ is the inflation rate, $s_g$ and $\tau_n$ are the government spending-output ratio and labor tax rate in the intended steady state, and $r = 1/\beta - 1$ is the natural interest rate. The notation $x^e_{t+1}$ denotes the period $t$ expectation of $x_{t+1}$.

Consider first the case of rational expectations without government spending ($s_g = 0$). As before, we consider equilibria in which the economy starts in a state $\psi_P$ and transitions to an absorbing state $\psi_O$ with a constant probability $1 - q$. The linearization eliminates transitional dynamics induced by price dispersion. Therefore RE solutions for output and inflation are now described by scalars

[15] See Marcet and Sargent (1989) and Evans and Honkapohja (2001).

$(\hat{\pi}_O, \hat{y}_O, \hat{\pi}_P, \hat{y}_P)$. Just as in the nonlinear model there may exist, apart from the intended equilibrium $(\hat{\pi}_O, \hat{y}_O, \hat{\pi}_P, \hat{y}_P) = 0$, also a second equilibrium in which $(\hat{\pi}_O, \hat{y}_O) = 0$ whereas the pair $(\hat{\pi}_P, \hat{y}_P)$ is a nonzero solution to the following linearized versions of equations (32) and (33):

$$\hat{y}_P = q(\hat{y}_P + \hat{\pi}_P) + \beta r - \beta \max\left(\frac{1-\beta}{\beta} + \frac{\phi_\pi \hat{\pi}_P}{\beta}, 0\right) \tag{41}$$

$$\hat{\pi}_P = \rho \hat{y}_P + \beta q \hat{\pi}_P \tag{42}$$

where $\rho = \frac{(1-\beta\xi)(1-\xi)}{\xi}\left(\frac{\kappa y^I}{1-y^I} + 1\right) > 0$. This system has a solution for all $q \in (q^*, 1]$ that is given by

$$\hat{\pi}_P = -\frac{1-\beta}{\Delta} < 0, \tag{43}$$

$$\hat{y}_P = -\frac{(1-\beta q)(1-\beta)}{\rho\Delta} < 0 \tag{44}$$

where $\Delta = q - (1-q)\frac{1-\beta q}{\rho} < 1$. For the zero constraint on $i_t$ to bind, it is required that $0 < \Delta < \phi_\pi$. Since $\phi_\pi > 1$ and $\Delta < 1$ the second inequality is redundant and the critical value $q^*$ is the smallest root of $q^* - (1-q^*)\frac{1-\beta q^*}{\rho} = 0$. As in the nonlinear model, a nonzero solution to this system corresponds to a second intersection of the, now linearized, $(\pi, y)^{EE}$ and $(\pi, y)^{AS}$ schedules in (41)-(42). The requirement that $\Delta > 0$ corresponds to the condition on the relative slopes of these schedules discussed before. Thus, the expectation driven liquidity traps present in the nonlinear model have a natural counterpart in the linearized systems typically used in the literature.

**E-stability** We now consider the E-stability properties of the model. We assume that agents perfectly know the $\psi_O$ state but may make small expectational errors in the $\psi_P$ state. This means we can focus on the following system

$$\hat{y}_{P,t} = q(\hat{y}^e_{P,t+1} + \hat{\pi}^e_{P,t+1}) + \beta r - \beta \max\left(\frac{1-\beta}{\beta} + \frac{\phi_\pi \hat{\pi}_{P,t}}{\beta}, 0\right) \tag{45}$$

$$\hat{\pi}_{P,t} = \rho \hat{y}_{P,t} + \beta q \hat{\pi}^e_{P,t+1} \tag{46}$$

In a neighborhood of the RE solution (43)-(44), the E-stability condition is that the eigenvalues of

$$q \begin{bmatrix} 1 & 1 \\ \rho & \rho+\beta \end{bmatrix} \tag{47}$$

are less then one in absolute value. It is straightforward to verify that the largest eigenvalue of (47) is given by $q/q^*$. Therefore the indeterminacy condition $q \in (q^*, 1]$, or equivalently $\Delta > 0$, implies automatically that the sunspot equilibria are not E-stable. In contrast, the condition for E-stability in a small neighborhood of the intended steady state solution is simply the Taylor principle: $\phi_\pi > 1$.[16] The fact that the RE solution is not E-stable means that when agents make the slightest expectational error, output and inflation dynamics diverge away from the RE liquidity trap equilibrium under certain learning schemes. Depending on the errors made, output and inflation either converge asymptotically towards the intended steady state or do not converge at all, that is until there is an exogenous switch to the $\psi_O$ state at which point inflation and output jump directly to the intended steady state.

**Dynamics with Adaptive Expectations** On the grounds of E-stability, Christiano and Eichenbaum (2012) argue that nonfundamental liquidity traps can perhaps be dismissed as curiosities. However, as shown by Evans, Guse and Honkapohja (2008), the presence of the permanent liquidity trap RE equilibria of Benhabib et al. (2001a,b) can have profound implications for output and inflation dynamics even with recursive learning. We now show that the same is true for the case of temporary liquidity traps. Consider the following constant gains learning rules:

$$\hat{\pi}^e_{P,t+1} = \hat{\pi}^e_{P,t} + \gamma\left(\hat{\pi}_{P,t-1} - \hat{\pi}^e_{P,t}\right) \tag{48}$$

$$\widehat{y}^e_{P,t+1} = \widehat{y}^e_{P,t} + \gamma\left(\widehat{y}_{P,t-1} - \widehat{y}^e_{P,t}\right) \tag{49}$$

[16]These findings are closely related to Bullard and Mitra (2002). Evans and Honkapohja (2005) and Evans, Guse and Honkapohja (2008) show that the permanent liquidity traps of Benhabib et al. (2001a,b) are not E-stable, which can be seen as a special case of our model corresponding to $q = 1$.

where $0 < \gamma < 1$ is a gain parameter and $\hat{\pi}_{P,-1}, \hat{y}_{P,-1}, \hat{\pi}^e_{P,0}$ and $\hat{y}^e_{P,0}$ are given. In our setting, constant gains learning is identical to classical adaptive expectations. We assume that the economy starts in the $\psi_P$ state and that $\hat{\pi}_{P,-1} = \hat{y}_{P,-1} = 0$. The dynamics under learning are given by a sequence of temporary equilibria determined by (45)-(46), the laws of motion for expectations in (48)-(49), and the following initializations of expectations

$$\hat{\pi}^e_{P,0} = -\frac{1-\beta}{\Delta}(1+\varepsilon_\pi) \ , \ \ \varepsilon_\pi \sim N(0,\sigma_\varepsilon) \tag{50}$$

$$\hat{y}^e_{P,0} = -\frac{(1-\beta q)(1-\beta)}{\rho\Delta}(1+\varepsilon_y) \ , \ \ \varepsilon_y \sim N(0,\sigma_\varepsilon) \tag{51}$$

The random variables $\varepsilon_\pi$ and $\varepsilon_y$ determine period 0 expectational errors that are proportional to the 'correct' values under rational expectations. We use the same parameter values as in the baseline calibration above, i.e. $\beta = 0.99$, $\rho = 0.45$, $\phi_\pi = 1.5$ and $q = 0.80$. The gain parameter is set to $\gamma = 0.10$ and the standard deviation of the expectational errors is $\sigma_\varepsilon = 0.10$.

Figure 12 shows velocity plots illustrating the expectational dynamics under learning conditional on the $P$-state. The figures also depicts trajectories for three different initializations of expectations in the neighborhood of the liquidity trap RE outcome (blue circle). All three trajectories converge asymptotically to the intended steady state RE outcome (red circle). The figure on the left depicts the case with a zero bound on interest rates (blue when it is binding, red when not). For comparison, the right figure shows the dynamics for the same initial conditions but without the zero bound constraint, i.e. permitting negative nominal rates. The dynamics around the RE liquidity trap are locally saddle-path stable, whereas the dynamics around the intended RE is a locally stable spiral. The presence of the temporary RE liquidity trap has important effects on the dynamics with adaptive expectations. Because the agents make relatively small errors, expectations converge only very slowly towards the intended steady state and may even be attracted to the RE liquidity trap in the short run. In the numerical example, it takes approximately 50 quarters before learning causes an endogenous exit from the zero lower bound, at which point the probability of an exogenous exit because of a switch to the $\psi_O$ state is virtually one. Without the zero bound, agents make large expectational errors and spiral towards the intended RE much faster. For initial expectations in a neighborhood of those in a temporary RE liquidity trap, the learning dynamics are not very important in the short run.

Figure 13 shows the objectively expected output paths under rational and adaptive expectations, which can be interpreted as impulse responses to a confidence shock in period 0. For the learning model, we simulated trajectories by randomly sampling the initial expectational errors $\varepsilon_\pi$ and $\varepsilon_y$. The figure shows the median and 0.95 percentiles of the resulting distribution of impulse responses. The two panels compare the output responses under learning with and without the zero bound constraint for the same distribution of initial expectations. First, it is evident that the median response under learning coincides almost exactly with the rational expectations outcome. Moreover, the distribution of output paths is centered around the RE path, and the output responses under learning simply scale with the initial shock to expectations. In other words, the unstable transitional dynamics due to learning have little influence over the expected duration of the liquidity trap. This is despite the fact that we chose a value for the gain parameter that is much higher than typical in the learning literature.[17] Second, when monetary policy is not constrained by the zero bound, the output paths under adaptive expectations remain fairly close to the intended state despite the large shocks to expectations. This shows how even a simple Taylor rule is very successful in insulating the economy from adverse expectational shocks when negative nominal interest rates are allowed. In contrast, with the zero lower bound, monetary policy cannot prevent significant output drops caused by sufficiently adverse shocks to confidence. For the calibrated example, the output deviation is about -0.7%, which differs from the -0.9% output deviation in the nonlinear model because of the linearization. As in the nonlinear analysis, output drops can be far larger for slight changes in parameters that bring the value of $\Delta$ closer to zero. As Evans, Guse and Honkapohja (2008), we conclude that the possibility of destabilizing expectational shocks due to the zero lower bound remains highly relevant with learning dynamics.

[17] For instance, Evans, Guse and Honkapohja (2008) assume a value of $\gamma = 1/30$. Eusepi and Preston (2011) calibrate $\gamma = 0.002$ based on data from the Survey of Professional Forecasters. Results are very similar for least squares learning, i.e. with a decreasing gain.

**Fiscal Multipliers with Adaptive Expectations** As a final exercise, we investigate whether our results on fiscal policy interventions in an expectations driven liquidity trap change under adaptive expectations. For brevity, we focus on spending and labor tax changes only and compute multipliers as in the nonlinear model. We also assume that expectations about future spending and taxes are always correct. For the spending multiplier, we first fix initial expectation errors $(\varepsilon_y, \varepsilon_\pi)$, and compute the output path in state $\psi_P$ under learning for a constant level of government spending with $s_g = 0.20$. For the same initial expectation errors we compute the output path in which spending is marginally higher and is perfectly correlated with the Markov process for $\psi_t$. Tax multipliers are computed in a similar fashion by also setting $\tau_n = 0.25$.

Figure 14 shows the multipliers over time in the RE equilibria of the linearized model as well as for adaptive expectations using $\varepsilon_y = \varepsilon_\pi = -0.05$. By construction, the multipliers around the intended steady state in the linearized model are constant and identical to those of the nonlinear model. The multipliers around the RE liquidity trap are also constant and quantitatively slightly different from the nonlinear model because of the linearization. The qualitative difference between the standard and liquidity trap multipliers in the nonlinear model extends naturally to the linearized model: the liquidity trap spending (tax) multiplier is smaller (larger) than the standard spending (tax) multiplier. The multipliers under adaptive expectations change over time because of transitional learning dynamics. Initially the multipliers are in a neighborhood of those in the RE liquidity trap. As agents update expectations, the multipliers diverge slowly and jump discretely to a neighborhood of the standard RE multiplier when the nominal interest rate becomes positive (around the 50th quarter). In the long run the multipliers converge to the standard RE values. We conclude that the unstable transitional learning dynamics following small errors in expectations do not overturn the qualitative predictions derived from the RE model in the short run, which is what is relevant given the transitory nature of the liquidity traps.

## 5 Conclusion and Directions for Further Research

In monetary models where the central bank operates an interest rate rule, there are equilibria in which the zero bound on short term nominal interest rates is occasionally binding. If there exists an inflation-output trade off, as in the New Keynesian model, temporary liquidity traps may occur in equilibrium during which economic activity is severely depressed. Losses in confidence lead to a downward spiral of increased savings, deflation and output drops that is aggravated by intertemporal substitution effects and forward looking price setting behavior when the crisis is expected to be temporary. We have shown that attempts to raise demand through fiscal policy may well become less effective in an expectations driven liquidity trap, whereas supply side stimulus can become more potent. These findings provide a counterexample to existing results on the effects of fiscal policy in a zero interest rate environment recently derived in the context of the New Keynesian model.

In this paper, we do not take a stance on whether current and past experiences of (near) zero interest rates are best described by fundamental shocks or self fulfilling changes in confidence. We simply point out that while both scenarios can lead to large recessions and deflation, they have different implications for ex post policy responses. We have also shown that the possibility of belief-driven liquidity traps increases with their expected duration. Historically, episodes of near zero nominal interest rates have been very prolonged, and therefore their potentially self fulfilling nature deserves to be taken seriously.

There are several interesting avenues for future research. To the extent that empirical research can uncover differences in the effects of fiscal policy in and outside of a liquidity trap, it is possible to discriminate between the two liquidity trap scenarios empirically. The possibility of expectations driven liquidity traps also introduces new considerations relevant for the choice of a numerical inflation target. Another important policy question is how to eliminate expectations driven liquidity traps through policy ex ante. Benhabib, Schmitt-Grohé and Uribe (2002) propose monetary and fiscal policies that violate the households' transversality conditions along candidate equilibrium paths with strong deflation. Under their strategy, the government manages to prevent the unintended deflationary steady state equilibrium by threatening to implement a fiscal stimulus package consisting of a severe increase in the deficit should the inflation rate become sufficiently low. In appendix A, we show how their proposed rule can be extended to the case of temporary liquidity traps as long as the government threatens to increase the deficit at a sufficiently high rate. In addition to possible practical objections to a commitment to unsustainable deficits, a potential complication arises when liquidity traps may be triggered not only by a loss in confidence, but also by a fundamental shock. Unless the deficit rule can be made contingent on the type of shock, this fiscal strategy becomes inconsistent with the existence of an equilibrium. Correia, Fahri, Nicolini and Teles (2011) instead show how locally the appropriate choice of consumption and labor income taxes can implement the same allocation that would be achieved if nominal interest rates could be reduced following a negative fundamental shock. A similar systematic tax policy may well prove to be successful in ruling out expectations driven liquidity traps as well. Alternatively, Atkeson, Chari and Kehoe (2010) describe sophisticated monetary policies that implement the intended competitive equilibrium uniquely in a linear version of our model (in which there are no endogenous state variables) by switching to an appropriate monetary growth rule. We leave it for future research to construct such policies in nonlinear settings with endogenous state variables.

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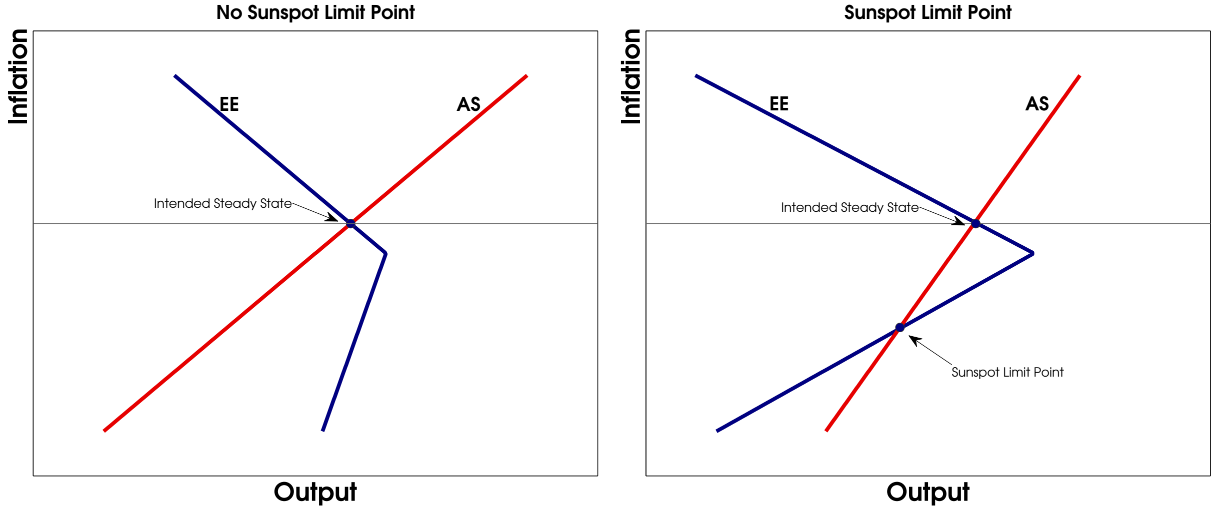


**Figure 1** An Expectations Driven Liquidity Trap

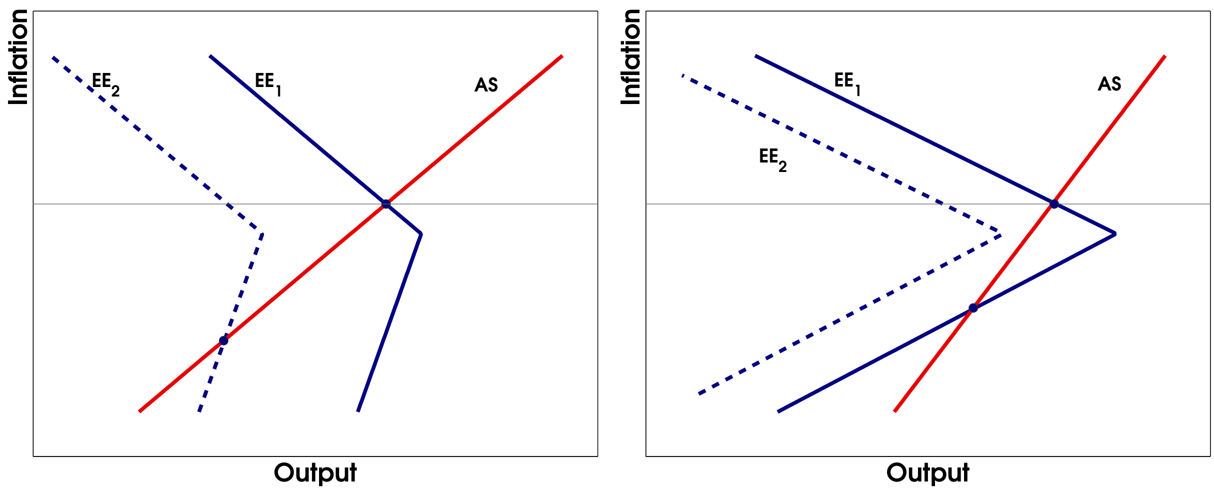


**Figure 2** Liquidity Trap After a Fundamental EE Shock

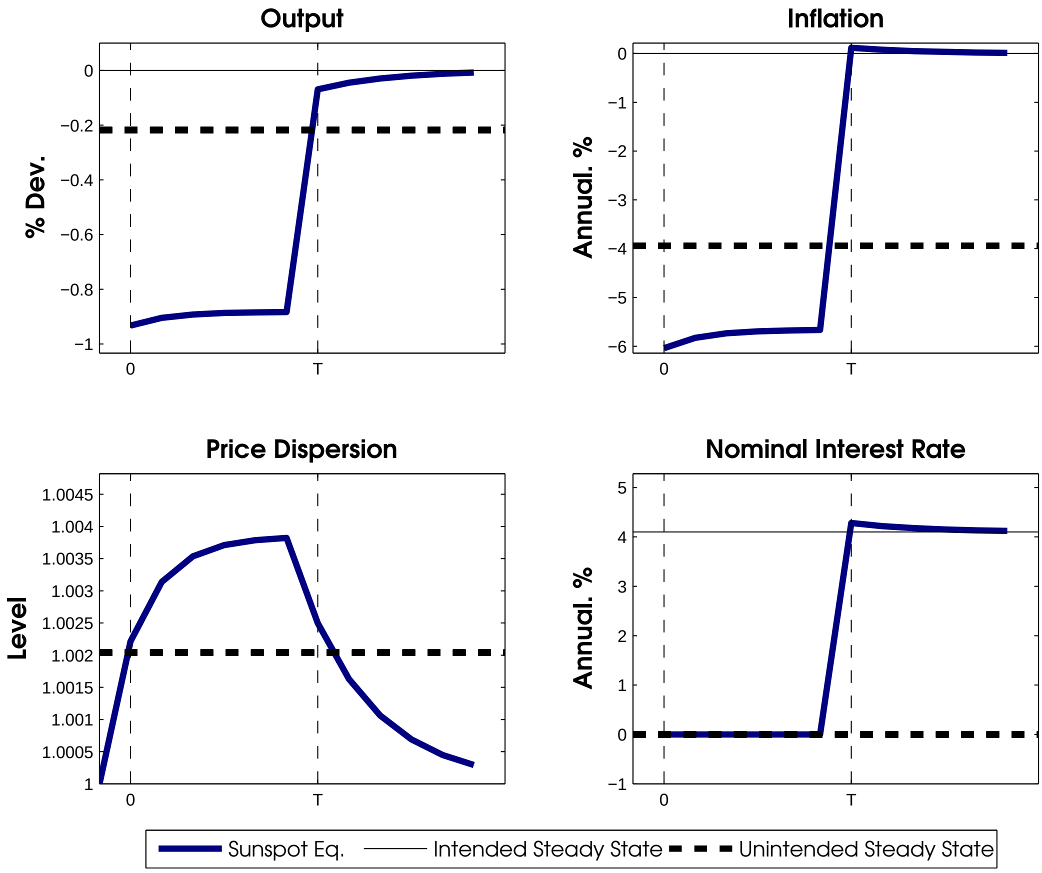


**Figure 3** Dynamics in a Expectations Driven Liquidity Trap ($\xi = 0.65$, $\sigma = 1$)

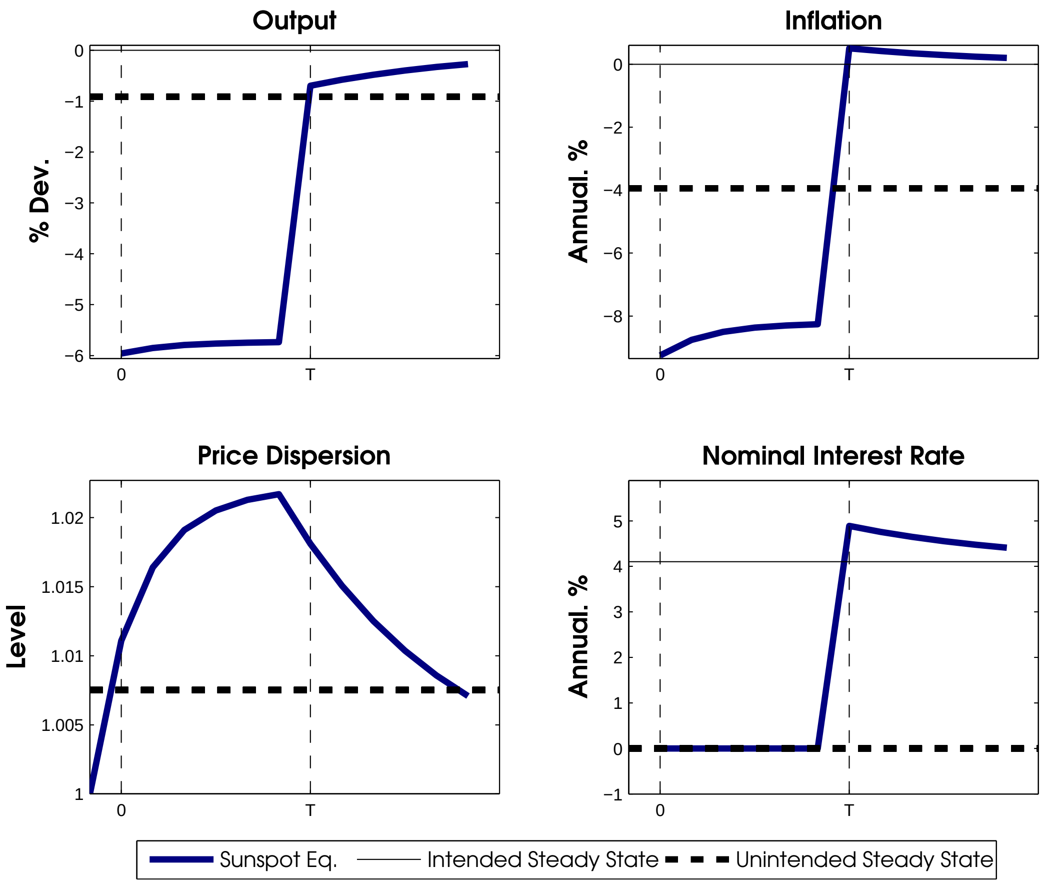


**Figure 4** Dynamics in a Expectations Driven Liquidity Trap ($\xi = 0.82$, $\sigma = 0.7$)

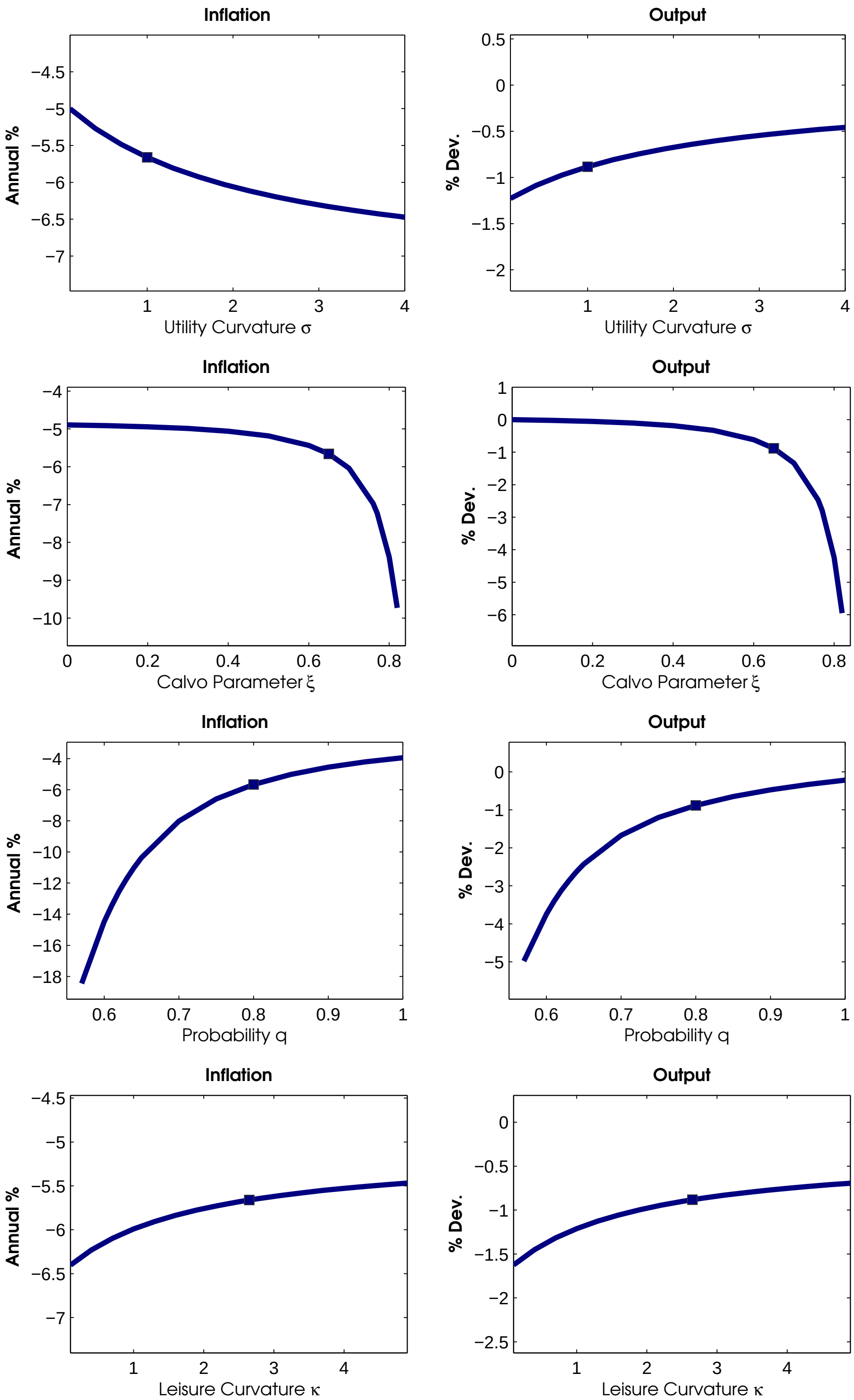


**Figure 5** Deflation and Output Loss in a Expectations Driven Liquidity Trap: Sensitivity Analysis

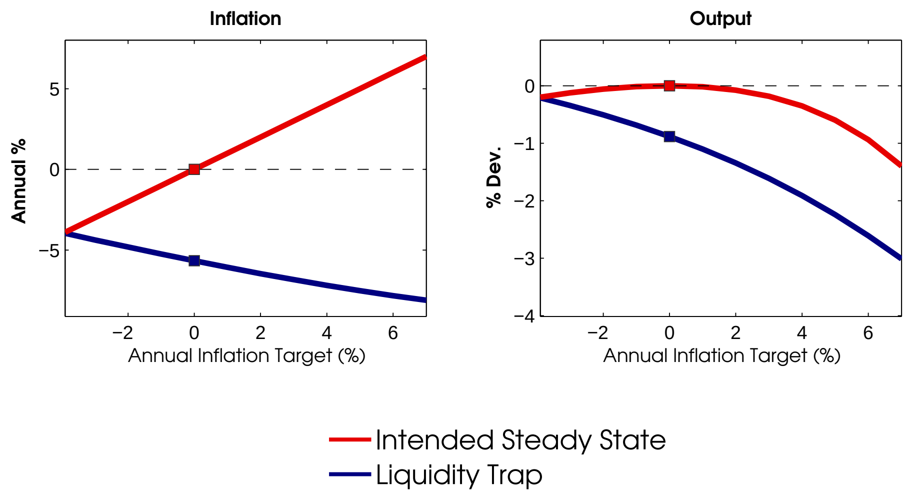


**Figure 6** Inflation and Output in the Intended Steady State and in an Expectations Driven Liquidity Trap: Different Inflation Targets

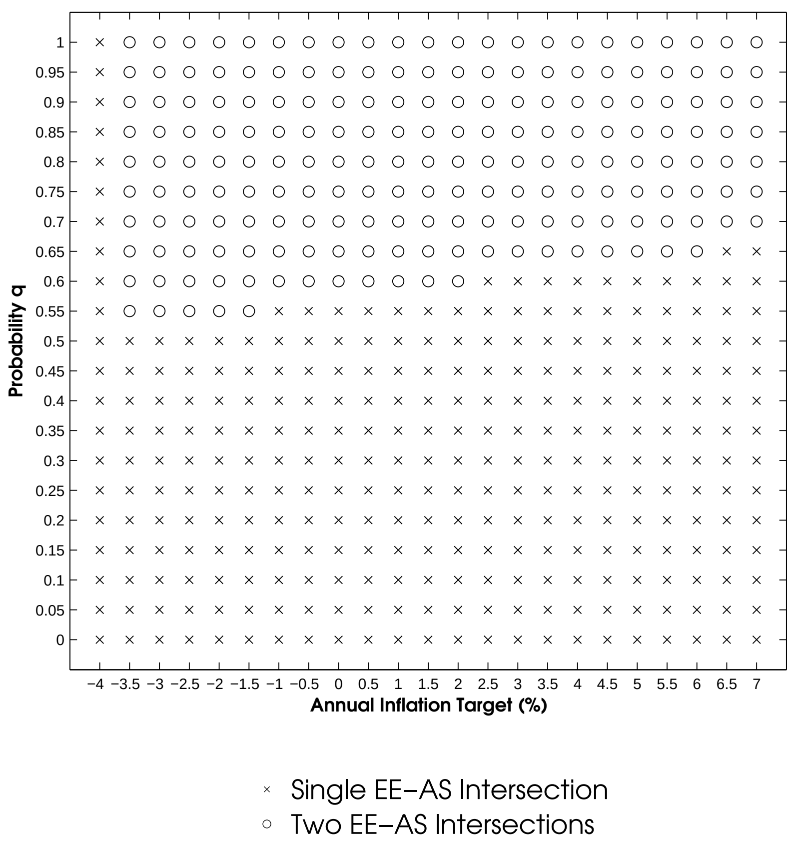


**Figure 7** Expectations Driven Liquidity Traps and the Inflation Target

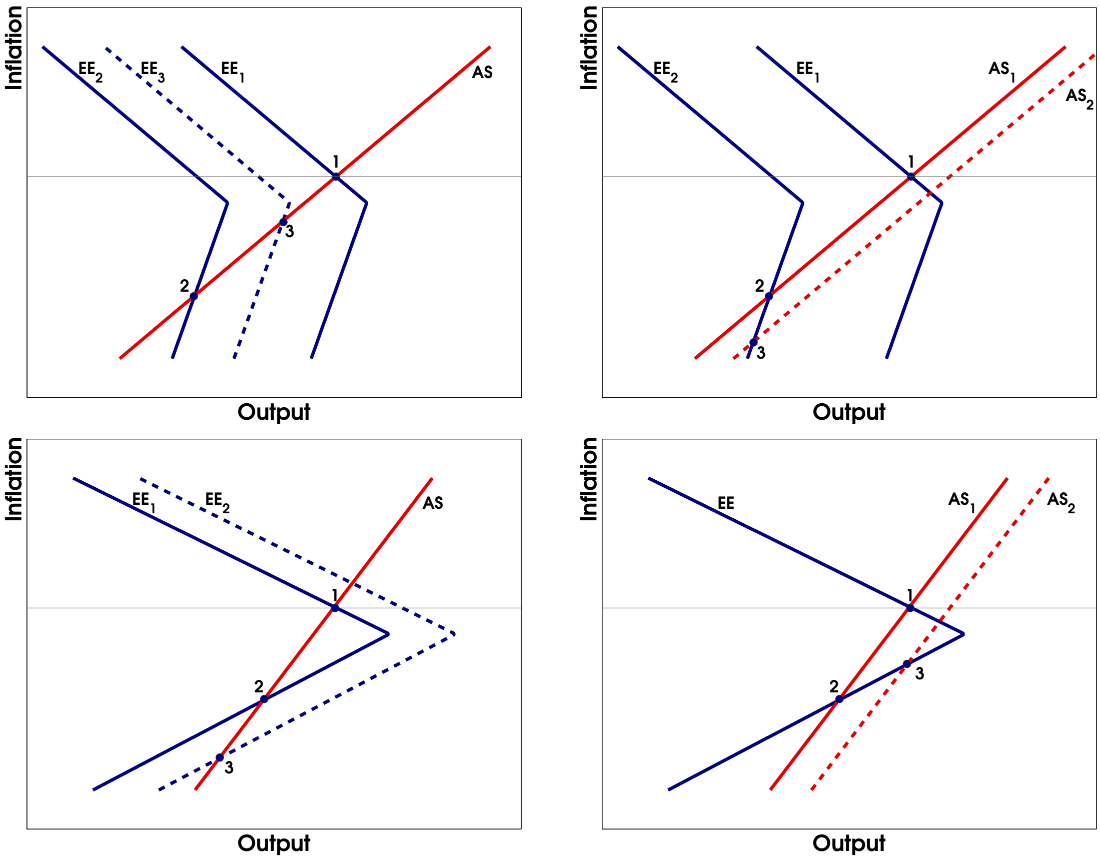


**Figure 8** The Effect of Demand and Supply Policies in Liquidity Traps

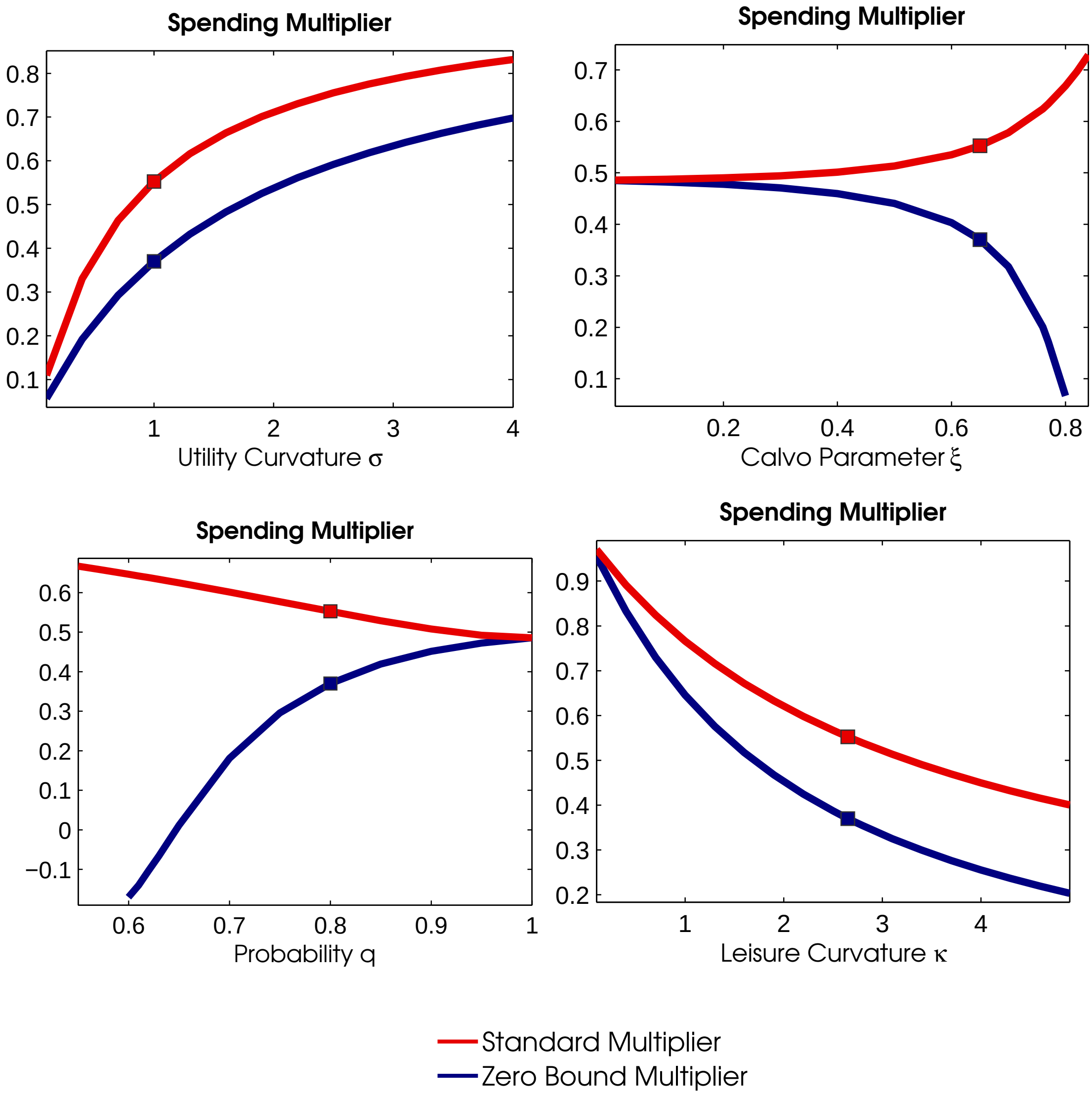


**Figure 9** Marginal Spending Multipliers

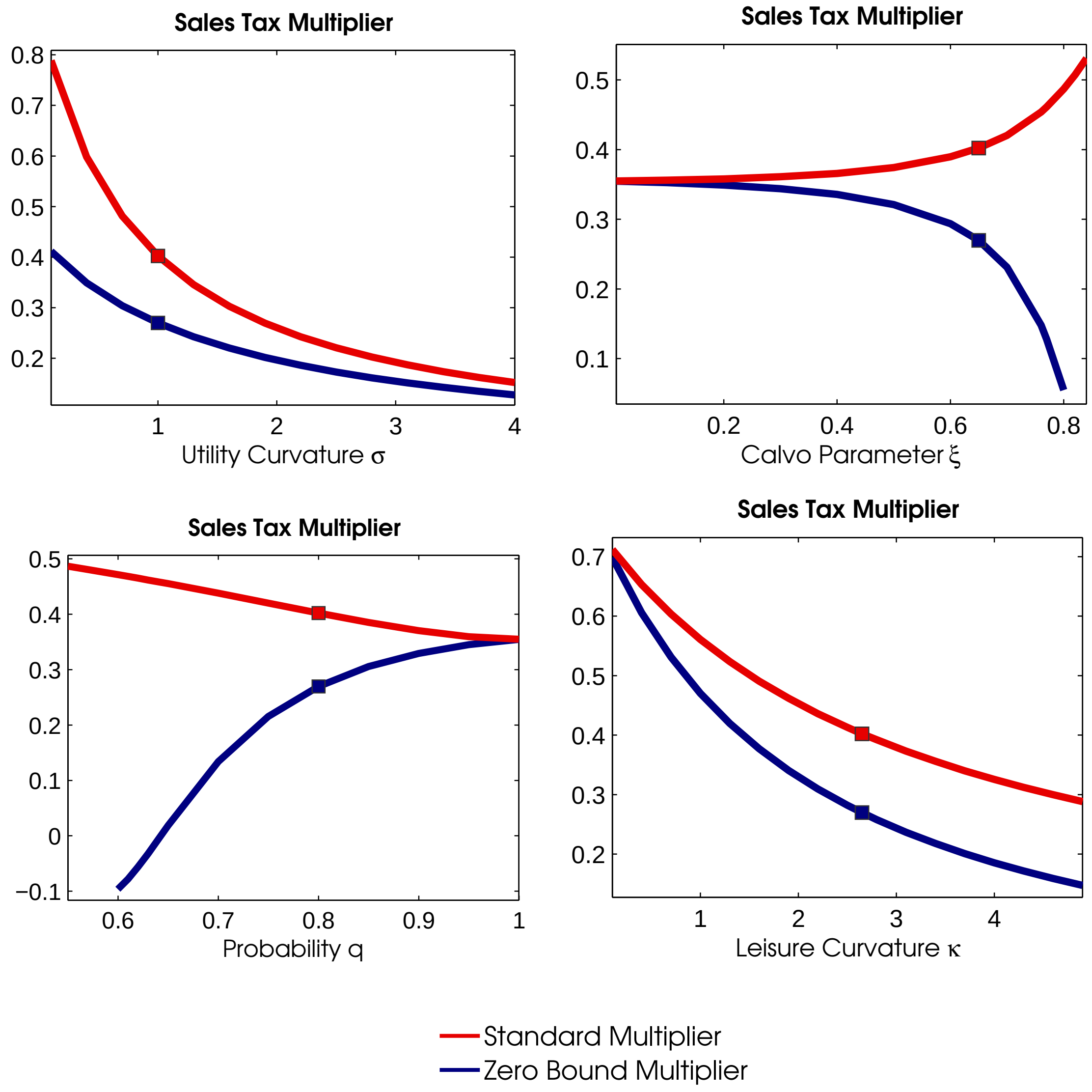


**Figure 10** Marginal Sales Tax Multipliers

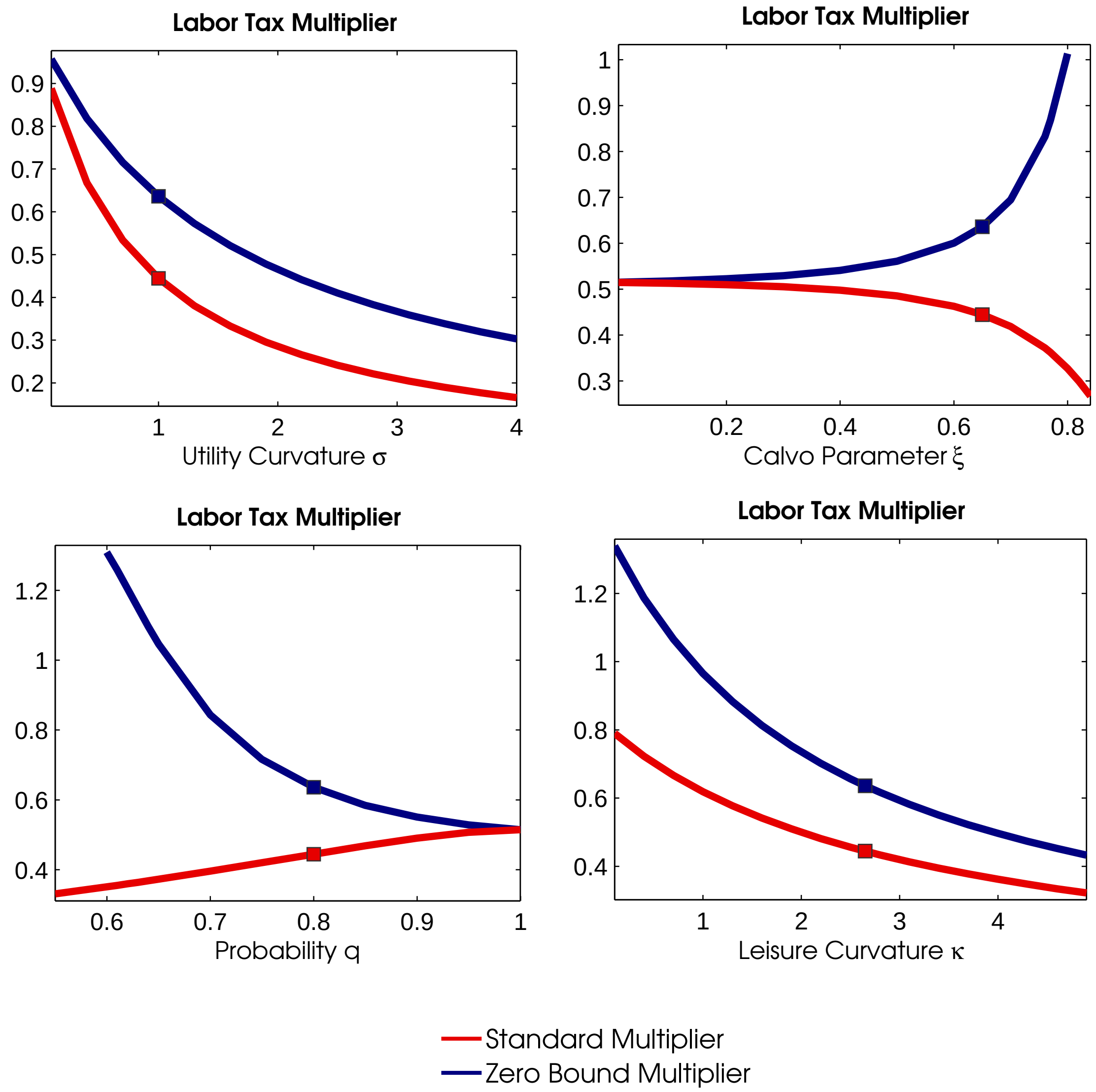


**Figure 11** Marginal Labor Tax Multipliers

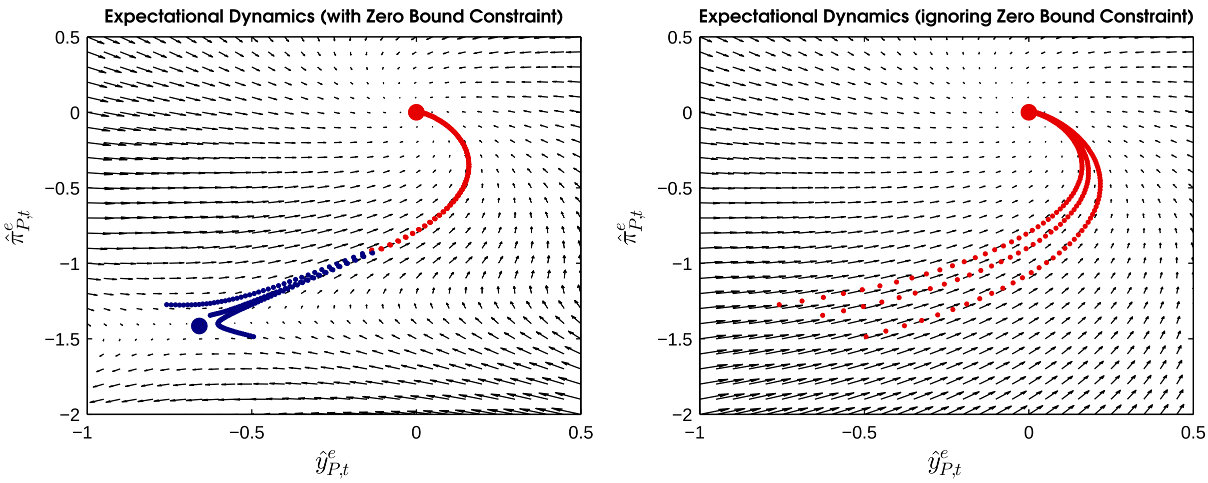


**Figure 12** Expectational Dynamics with Adaptive Expectations
Circles denote RE equilibria. Blue (red) indicates zero (strictly positive) nominal interest rate.

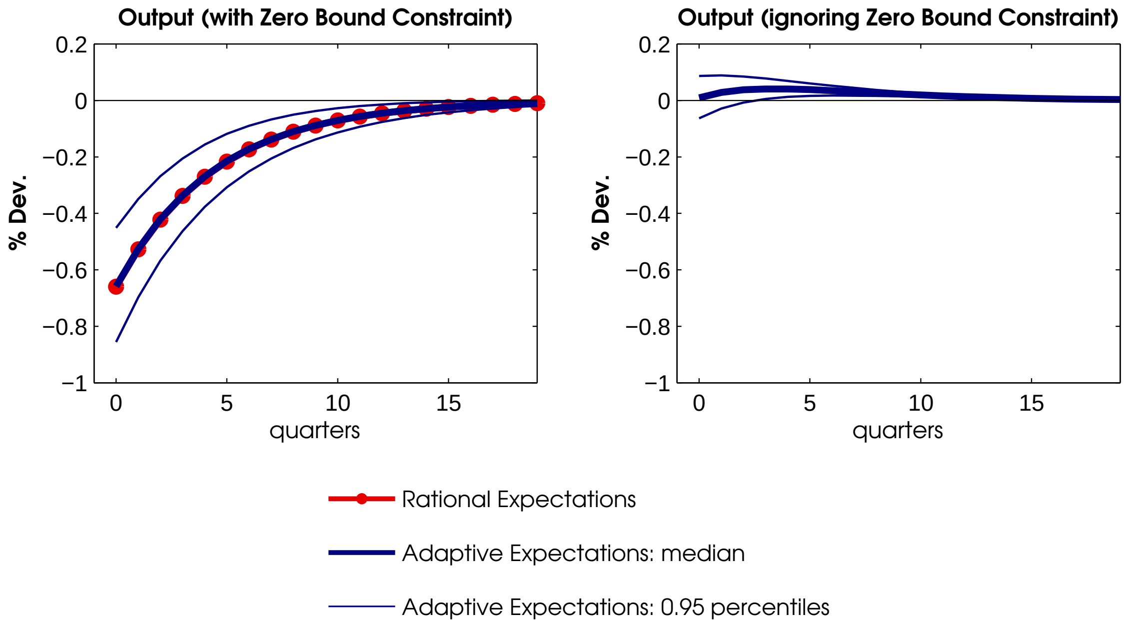


**Figure 13** Impulse Responses to a Confidence Shock

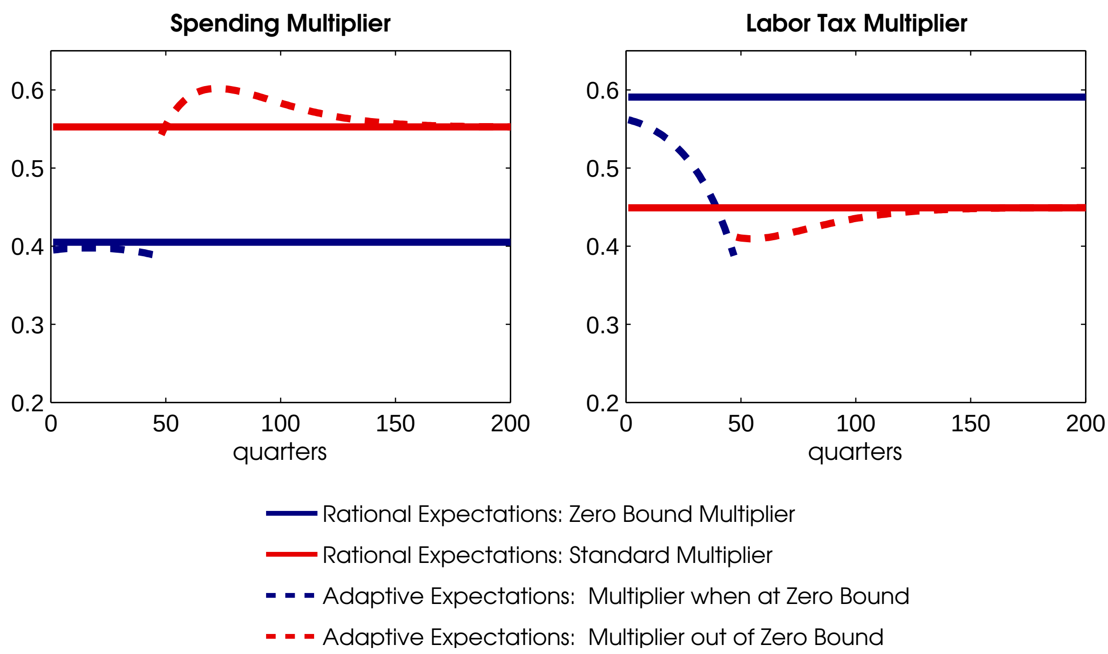


**Figure 14** Fiscal Multipliers under Rational and Adaptive Expectations

## A Using Fiscal Policy To Avoid Liquidity Traps

This appendix examines an extension of the proposal in Benhabib, Schmitt-Grohé and Uribe (2002) to avoid expectations driven liquidity traps which relies on violating the transversality condition on the end of time stock of household wealth whenever deflationary expectations arise for nonfundamental reasons.

Define $a_t = (B_t + M_t)/P_t$ as total real government liabilities, and express the government budget constraint in (19)- (20) as

$$a_t = \frac{1+i_t}{\pi_t} a_{t-1} + \tilde{d}_t \tag{52}$$

where $\tilde{d}_t = (1+i_t)d_t - i_t m_t$ is the real primary deficit including seigniorage. The household optimality condition in (7) requires intertemporal fiscal solvency, and government policies in equilibrium must be such that

$$\lim_{t\to\infty} \mathbb{E}_t \left[ a_{t+s} \frac{\pi_t}{1+i_t} \cdots \frac{\pi_{t+s}}{1+i_{t+s}} \right] = 0 \tag{53}$$

When this transversality condition holds, the net present value of current and all future tax and seigniorage revenues equals current outstanding debt and the net present value of all current and future expenditures. In order to rule out the unintended steady state, Benhabib, Schmitt-Grohé and Uribe (2002) propose fiscal rules of the type:

$$\tilde{d}_t = \varkappa(\pi_t)\, a_{t-1} \tag{54}$$

Analogous to Benhabib, Schmitt-Grohé and Uribe (2002), consider the following policy:

$$\varkappa(\tilde{\pi}) < 1/\beta \ , \ \ \varkappa\left(\pi^U\right) > 1/\beta \tag{55}$$

where $\pi^U$ is the inflation rate in the unintended steady state. It follows from the government budget constraint in (52) that:

$$a_{t+s} = \Pi_{j=0}^{s} \left( \frac{1+i_{t+j}}{\pi_{t+j}} + \varkappa(\pi_{t+j}) \right) a_{t-1} \tag{56}$$

such that the transversality condition can be expressed as

$$\lim_{s\to\infty} \mathbb{E}_t \left[ \Pi_{j=0}^{s} \left( \varkappa(\pi_{t+j}) \frac{\pi_{t+j}}{1+i_{t+j}} \right) \right] a_{t-1} = 0 \tag{57}$$

In candidate equilibrium paths that converge to the intended steady state, $\pi_t/(1+i_t) \to \beta$ and $\pi_t \to \tilde{\pi}$ and since $\varkappa(\tilde{\pi}) < 1/\beta$, the transversality condition is satisfied. For candidate equilibrium paths that converge to the unintended steady state, $\pi_t/(1+i_t) \to \beta$ and $\pi_t \to \pi^U = \beta$ and since $\varkappa\left(\pi^U\right) > 1/\beta$, the transversality condition does not hold, unless $a_{t-1} = 0$. Therefore, the fiscal policy in (55) can rule out equilibria that converge to the unintended steady state. Under this fiscal strategy, the government prevents the unintended deflationary equilibrium by threatening to drastically increase the deficit should the inflation rate become sufficiently low.

A similar fiscal strategy can be devised in order to rule out temporary liquidity traps driven by a sunspot with stochastic properties given in (31). In this case, the threat to the deficit must be such that

$$\lim_{s\to\infty} \left[ \Pi_{j=0}^{s} \left( q\varkappa(\pi_{t+j}) \frac{\pi_{t+j}}{1+i_{t+j}} \right) \right] a_{t-1} \neq 0 \tag{58}$$

If $\pi_P$ is the inflation rate in the sunspot limit point for a given persistence $0 < q < 1$, the requirement on fiscal policy to rule out the sunspot equilibrium is modified:

$$\varkappa(\pi_P) > \frac{1}{q\pi_L} > 1/\beta \tag{59}$$

Recall that sunspot equilibria exists for all $q$ greater than a certain critical value. To rule out all these sunspot equilibria, the condition in (59) must hold for $q$ approaching this critical value. Therefore, the government could also avoid temporary liquidity traps as long as it commits to sufficiently large increases in the deficit in response to deflationary pressures.