Macro-prudential Policy in a Fisherian Model of Financial Innovation*

Javier Bianchi
University of Wisconsin and NYU

Emine Boz
IMF

Enrique G. Mendoza
University of Maryland and NBER

April 2012

Abstract

The interaction between credit frictions, financial innovation, and a switch from optimistic to pessimistic beliefs played a central role in the 2008 financial crisis. This paper develops a quantitative general equilibrium framework in which this interaction drives the financial amplification mechanism to study the effects of macro-prudential policy. Financial innovation enhances the ability of agents to collateralize assets into debt, but the riskiness of this new regime can only be learned over time. Beliefs about transition probabilities across states with high and low ability to borrow change as agents learn from observed realizations of financial conditions. At the same time, the collateral constraint introduces a pecuniary externality, because agents fail to internalize the effect of their borrowing decisions on asset prices. Quantitative analysis shows that the effectiveness of macro-prudential policy in this environment depends on the government's information set, the tightness of credit constraints and the pace at which optimism surges in the early stages of financial innovation. The policy is least effective when the government is as uninformed as private agents, credit constraints are tight, and optimism builds quickly.

Keywords: Financial crises, financial innovation, macro-prudential regulation, Bayesian learning
JEL Codes: D62, D82, E32, E44, F32, F41

*This paper was prepared for the Twelfth IMF Annual Research Conference. We are grateful for comments by Dan Cao, Stijn Claessens, Pierre-Olivier Gourinchas, Ayhan Köse, Paolo Pesenti, and participants at the 12th IMF Annual Research Conference, the 2011 Research Conference of the Reserve Bank of New Zealand, the 2011 Quantitative Macro Workshop of the Reserve Bank of Australia, and the 7th ECB-Federal Reserve Board International Research Forum on Monetary Policy. Correspondence: javier.bianchi@nyu.edu, EBoz@imf.org and mendozae@econ.umd.edu.

"*I think we will have continuing danger from these markets and that we will have repeats of the financial crisis. It may differ in details, but there will be significant financial downturns and disasters attributed to this regulatory gap over and over* until we learn from experience." (Brooksley Born, Aug. 28, 2009 interview for *FRONTLINE: The Warning*)

1 Introduction

Policymakers have responded to the lapses in financial regulation in the years before the 2008 global financial crisis and the unprecedented systemic nature of the crisis itself with a strong push to revamp financial regulation following a "macro-prudential" approach. This approach aims to focus on the macro (i.e. systemic) implications that follow from the actions of credit market participants, and to implement policies that influence behavior in "good times" in order to make financial crises less severe and less frequent. The design of macro-prudential policy is hampered, however, by the need to develop models that are reasonably good at explaining the macro dynamics of financial crises and at capturing the complex dynamic interconnections between potential macro-prudential policy instruments and the actions of agents in credit markets.

The task of developing these models is particularly challenging because of the fast pace of financial development. Indeed, the decade before the 2008 crash was a period of significant financial innovation, which included both the introduction of a large set of complex financial instruments, such as collateralized debt obligations, mortgage backed securities and credit default swaps, and the enactment of major financial reforms of a magnitude and scope unseen since the end of the Great Depression. Thus, models of macro-prudential regulation have to take into account the changing nature of the financial environment, and hence deal with the fact that credit market participants, as well as policymakers, may be making decisions lacking perfect information about the true riskiness of a changing financial regime.

This paper proposes a dynamic stochastic general equilibrium model in which the interaction between financial innovation, credit frictions and imperfect information is at the core of the financial transmission mechanism, and uses it to study its quantitative implications for the design and effectiveness of macro-prudential policy. In the model, a collateral constraint limits the agents ability to borrow to a fraction of the market value of the assets they can offer as collateral. Financial innovation enhances the ability of agents to "collateralize," but also introduces risk because of the possibility of fluctuations in collateral requirements or loan-to-value ratios.

We take literally the definition of financial innovation as the introduction of a truly new financial regime. This forces us to deviate from the standard assumption that agents formulate rational expectations with full information about the stochastic process driving fluctuations in credit conditions. In particular, we assume that agents learn (in Bayesian fashion) about the transition probabilities of financial regimes only as they observe regimes with high and low ability to borrow over time. In the long run, and in the absence of new waves of financial innovation, they learn the true transition probabilities and form standard rational expectations, but in the short run agents' beliefs display waves of optimism and pessimism depending on their initial priors and on the market conditions they observe. These changing beliefs influence agents' borrowing decisions and equilibrium asset prices, and together with the collateral constraint they form a financial amplification feedback mechanism: optimistic (pessimistic) expectations lead to over-borrowing (under-borrowing) and increased (reduced) asset prices, and as asset prices change the ability to borrow changes as well.

Our analysis focuses in particular on a learning scenario in which the arrival of financial innovation starts an "optimistic phase," in which a few observations of enhanced borrowing ability lead agents to believe that the financial environment is stable and risky assets are not "very risky." Hence, they borrow more and bid up the price of risky assets more than in a full-information rational expectations equilibrium. The higher value of assets in turn relaxes the credit constraint. Thus, the initial increase in debt due to optimism is amplified by the interaction with the collateral constraint via optimistic asset prices. Conversely, when the first realization of the low-borrowing-ability regime is observed, a "pessimistic phase" starts in which agents overstate the probability of continuing in poor financial regimes and overstate the riskiness of assets. This results in lower debt levels and lower asset prices, and the collateral constraint amplifies this downturn.

Macro-prudential policy action is desirable in this environment because the collateral constraint introduces a pecuniary externality in credit markets that leads to more debt and financial crises that are more severe and frequent than in the absence of this externality. The externality exists because individual agents fail to internalize the effect of their borrowing decisions on asset prices, particularly future asset prices in states of financial distress (in which the feedback loop via the collateral constraint triggers a financial crash).

There are several studies in the growing literature on macro-prudential regulation that have examined the implications of this externality, but typically under the assumption that agents form rational expectations with full information (e.g. Lorenzoni (2008), Stein (2011), Bianchi (2011), Bianchi and Mendoza (2010), Korinek (2010), Jeanne and Korinek (2010), Benigno, Chen, Otrok, Rebucci, and Young (2010)). In contrast, the novel contribution of this paper is in that we study the effects of macro-prudential policy in an environment in which the pecuniary externality is influenced by the interaction of the credit constraint with learning about the riskiness of a new financial regime. The analysis of Boz and Mendoza (2010) suggest that taking this interaction into account can be important, because they found that the credit constraint in a learning setup produces significantly larger effects on debt and asset prices than in a full-information environment with the same credit constraint. Their study, however, focused only on quantifying the properties of the decentralized competitive equilibrium and abstracted from normative issues and policy analysis.

The policy analysis of this paper considers a social planner under two different informational assumptions. First, an *uninformed planner* who has to learn about the true riskiness of the new financial environment, and faces the set of feasible credit positions supported by the collateral values of the competitive equilibrium with learning. We start with a baseline scenario in which private agents and the planner have the same initial priors and thus form the same sequence of beliefs, and study later on scenarios in which private agents and the uninformed planner form different beliefs. Second, an *informed planner* with full information, who therefore knows the true transition probabilities across financial regimes, and faces a set of feasible credit positions consistent with the collateral values of the full-information, rational expectations competitive equilibrium.[1]

We compute the decentralized competitive equilibrium of the model with learning (DEL) and contrast this case with the above social planner equilibria. We then compare the main features of these equilibria, in terms of the behavior of macroeconomic aggregates and asset pricing indicators, and examine the characteristics of macro-prudential policies that support the allocations of the planning problems as competitive equilibria. This analysis emphasizes the potential limitations of macro-prudential policy in the presence of significant financial innovation, and highlights the relevance of taking into account informational frictions in evaluating the effectiveness of macro-prudential policy.

The quantitative analysis indicates that the interaction of the collateral constraint with optimistic beliefs in the DEL equilibrium can strengthen the case for introducing macro-prudential regulation compared with the decentralized equilibrium under full information (DEF). This is because, as Boz and Mendoza (2010) showed, the interaction of these elements produces larger

[1]The assumption that the planners face a pricing function for collateral that corresponds to a competitive equilibrium is in line with the concept of conditional or financial efficiency defined by Kehoe and Levine (1993) and applied by Lustig (2000) to the setting of a credit market with collateral.

amplification both of the credit boom in the optimistic phase and of the financial crash when the economy switches to the bad financial regime. The results also show, however, that the effectiveness of macro-prudential policy varies sharply with the assumptions about the information set and collateral pricing function used by the social planner. Moreover, for the uninformed planner, the effectiveness of macro-prudential policy also depends on the tightness of the borrowing constraint and the pace at which optimism builds in the early stages of financial innovation.

Consider first the uninformed planner. For this planner, the undervaluation of risk weakens the incentives to build precautionary savings against states of nature with low-borrowing-ability regimes over the long run, because this planner underestimates the probability of landing on and remaining in those states. In contrast, the informed planner assesses the correct probabilities of landing and remaining in states with good and bad credit regimes, so its incentives to build precautionary savings are stronger. In fact, the informed planner's optimal macro-prudential policy features a precautionary component that lowers borrowing levels at given asset prices, and a component that influences portfolio choice of debt v. assets to address the effect of the agents' mispricing of risk on collateral prices.

It is important to note that even the uninformed planner has the incentive to use macro-prudential policy to tackle the pecuniary externality and alter debt and asset pricing dynamics. In our baseline calibration, however, the borrowing constraint becomes tightly binding in the early stages of financial innovation as optimism builds quickly, and as a result macro-prudential policy is not very effective (i.e. debt positions and asset prices differ little between the DEL and the uninformed planner). Intuitively, since a binding credit constraint implies that debt equals the high-credit-regime fraction of the value of collateral, debt levels for the uninformed social planner and the decentralized equilibrium are similar once the constraint becomes binding for the planner. But this is *not* a general result.[2] Variations in the information structure in which optimism builds more gradually produce outcomes in which macro-prudential policy is effective even when the planner has access to the same information set. On the other hand, it is generally true that the uninformed planner allows larger debt positions than the informed planner because of the lower precautionary savings incentives.

[2]It is also important to note that this result is not due to the fact that the uninformed planner faces the same collateral pricing function as DEL. Working under the same pricing assumption in a model with full information, but using a different calibration of collateral coefficients, Bianchi and Mendoza (2010) found that the planner supports very different debt allocations and asset prices than the decentralized equilibrium.

We also analyze the welfare losses that arise from the pecuniary externality and the optimism embedded in agents' subjective beliefs. The losses arising due to their combined effect are large, reaching up to 7 percent in terms of a compensating variation in permanent consumption that equalizes the welfare of the informed planner with that of the DEL economy. The welfare losses attributable to the pecuniary externality alone are relatively small, in line with the findings reported by Bianchi (2011) and Bianchi and Mendoza (2010), and they fall significantly at the peak of optimism.

Our model follows a long and old tradition of models of financial crises in which credit frictions and imperfect information interact. This notion dates back to the classic work of Fisher (1933), in which he described his debt-deflation financial amplification mechanism as the result of a feedback loop between agents' beliefs and credit frictions (particularly those that force fires sales of assets and goods by distressed borrowers). Minsky (1992) is along a similar vein. More recently, macroeconomic models of financial accelerators (e.g. Bernanke, Gertler, and Gilchrist (1999), Kiyotaki and Moore (1997), Aiyagari and Gertler (1999)) have focused on modeling financial amplification but typically under rational expectations with full information about the stochastic processes of exogenous shocks.

The particular specification of imperfect information and learning that we use follows closely that of Boz and Mendoza (2010) and Cogley and Sargent (2008a), in which agents observe regime realizations of a Markov-switching process without noise but need to learn its transition probability matrix. The imperfect information assumption is based on the premise that the U.S. financial system went through significant changes beginning in the mid-90s as a result of financial innovation and deregulation that took place at a rapid pace. As in Boz and Mendoza (2010), agents go through a learning process in order to "discover" the true riskiness of the new financial environment as they observe realizations of regimes with high or low borrowing ability.

Our quantitative analysis is related to Bianchi and Mendoza (2010)'s quantitative study of macro-prudential policy. They examined an asset pricing model with a similar collateral constraint and used comparisons of the competitive equilibria vis-a-vis a social planner to show that optimal macro-prudential policy curbs credit growth in good times and reduces the frequency and severity of financial crises. The government can accomplish this by using Pigouvian taxes on debt and dividends to induce agents to internalize the model's pecuniary externality. Bianchi and Mendoza's framework does not capture, however, the role of informational frictions interacting with frictions in financial markets, and thus is silent about the implications of differences in the information sets of policy-makers and private agents.

Our paper is also related to Gennaioli, Shleifer, and Vishny (2010), who study financial innovation in an environment in which "local thinking" leads agents to neglect low probability adverse events (see also Gennaioli and Shleifer (2010)). As in our model, the informational friction distorts decision rules and asset prices, but the informational frictions in the two setups differ.[3] Moreover, the welfare analysis of Gennaioli, Shleifer, and Vishny (2010) focuses on the effect of financial innovation under local thinking, while we emphasize the interaction between a fire-sale externality and informational frictions.

Finally, our work is also related to the argument developed by Stein (2011) to favor a cap and trade system to address a pecuniary externality that leads banks to issue excessive short-term debt in the presence of private information. Our analysis differs in that we study the implications of a form of model uncertainty (i.e. uncertainty about the transition probabilities across financial regimes) for macro-prudential regulation, instead of private information, and we focus on Pigouvian taxes as a policy instrument to address the pecuniary externality.

The rest of the paper is organized as follows: Section 2 describes the model. Section 3 conducts the quantitative analysis comparing the decentralized competitive equilibrium with the various planning problems. Section 4 provides the main conclusions.

2 A Fisherian Model of Financial Innovation

The setup of the model's competitive equilibrium and learning environment is similar to Boz and Mendoza (2010). The main difference is that we extend the analysis to characterize social planning problems under alternative information sets and collateral pricing functions.

2.1 Decentralized Competitive Equilibrium

The economy is inhabited by a continuum of identical agents who maximize a standard constant-relative-risk-aversion utility function. Agents choose consumption, c_t, holdings of a risky asset

[3] In the model of Gennaioli et al. agents ignore part of the state space relevant for pricing risk by assumption, assigning zero probability to rare negative events, while in our setup agents always assign non-zero probability to all the regimes that are part of the realization vector of the Markov switching process of financial regimes. However, agents do assign lower (higher) probability to tight credit regimes than they would under full information rational expectations when they are optimistic (pessimistic), and this lower probability is an outcome of a Bayesian learning process. Moreover, learning yields equilibrium asset pricing functions in future dates, after learning progresses, that agents did not consider possible with the beliefs of previous dates.

k_{t+1} (i.e. land), and holdings of a one-period discount bond, b_{t+1}, denominated in units of the consumption good. Land is a risky asset traded in a competitive market, where its price q_t is determined, and is in fixed unit supply. Individually, agents see themselves as able to buy or sell land at the market price, but since all agents are identical, at equilibrium the price clears the land market with all agents choosing the same land holdings.

Bonds carry an exogenous price equal to $1/R$, where R is an exogenous gross real interest rate. Thus, the model can be interpreted as a model of a small open economy, in which case b represents the economy's net foreign asset position and R is the world's interest rate, or as a partial equilibrium model of households or a subset of borrowers in a closed economy, in which case b represents these borrowers' net credit market assets and R is the economy's risk free real interest rate. Under either interpretation, the behavior of creditors is not modeled from first principles. They are simply assumed to supply of funds at the real interest rate R subject to the collateral constraint described below.

The bond market is imperfect because creditors require borrowers to post collateral that is "marked to market" (i.e. valued at market prices). In particular, the collateral constraint limits the agents'debt (a negative position in b) to a fraction κ of the market value of their individual land holdings.[4] The collateral coefficient κ is stochastic and follows a Markov regime-switching process. Information is imperfect with respect to the true transition probability matrix governing the evolution of κ, and the agents learn about it by observing realizations of κ over time. We will model learning so that in the long-run the agents' beliefs converge to the true transition probability matrix, at which point the model yields the same competitive equilibrium as a standard rational-expectations asset pricing model with a credit constraint.

Agents operate a production technology $\varepsilon_t Y(k_t)$ that uses land as the only input, and facing a productivity shock ε_t. This shock has compact support and follows a finite-state, stationary Markov process about which agents are perfectly informed.

The agents' preferences are given by:

$$E_0^s \left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right]. \tag{1}$$

[4]This constraint could follow, for example, from limited enforcement of credit contracts, by which creditors can only confiscate a fraction κ of the value of a borrower's land holdings. In actual credit contracts, this constraint resembles loans subject to margin calls or loan-to-value limits, value-at-risk collateralization and mark-to-market capital requirements.

E^s is the subjective conditional-expectations operator that is elaborated on further below, β is the subjective discount factor, and σ is the coefficient of relative risk aversion.

The budget constraint faced by the agents is:

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R_t} = q_t k_t + b_t + \varepsilon_t Y(k_t) \tag{2}$$

The agents' collateral constraint is:

$$-\frac{b_{t+1}}{R_t} \leq \kappa_t q_t k_{t+1} \tag{3}$$

Using μ_t for the Lagrange multiplier of (3), the first-order conditions of the agents optimization problem are given by:

$$u'(t) = \beta R E_t^s \left[u'(t+1)\right] + \mu_t \tag{4}$$

$$q_t(u'(t) - \mu_t \kappa_t) = \beta E_t^s \left[u'(t+1)\left(\varepsilon_{t+1} Y_k(k_{t+1}) + q_{t+1}\right)\right] \tag{5}$$

A *decentralized competitive equilibrium with learning* (DEL) is a sequence of allocations $[c_t, k_{t+1}, b_{t+1}]_{t=0}^{\infty}$ and prices $[q_t]_{t=0}^{\infty}$ that satisfy the above conditions, using the agents' beliefs about the evolution of κ to formulate expectations, together with the collateral constraint (3) and the market-clearing conditions for the markets of goods and assets:

$$\begin{aligned} c_t + \frac{b_{t+1}}{R_t} &= b_t + \varepsilon_t Y(k_t) \\ k_t &= 1 \end{aligned}$$

The *decentralized competitive equilibrium with full information* (DEF) is defined in the same way, except that expectations are formulated using the true transition distribution of κ.

2.2 Learning Environment

Expectations in the payoff function (1) are based on Bayesian beliefs agents form based on initial priors and information they observe over time. We model learning following closely Boz and Mendoza (2010) and Cogley and Sargent (2008a). Hence, we provide here only a short description and refer the interested reader to those other articles for further details.

The stochastic process of κ follows a classic two-point regime-switching Markov process. There are two realizations of κ, a regime with high ability to borrow κ^h and a regime with low ability to borrow κ^l. The "true" regime-switching Markov process has continuation transition probabilities defined by F^a_{hh} and F^a_{ll}, with switching probabilities given by $F^a_{hl} = 1 - F^a_{hh}$ and $F^a_{lh} = 1 - F^a_{ll}$. Hence, learning in this setup is about forming beliefs regarding the distributions of the transition probabilities F^s_{hh} and F^s_{ll} by combining initial priors with the observations of κ that arrive each period. After observing a sufficiently long and varied set of realizations of κ^h and κ^l, agents learn the true regime-switching probabilities of κ. Modeling of learning in this fashion is particularly useful for representing financial innovation as the introduction of a brand-new financial regime for which there is no data history agents could use to infer the true transition distribution of κ, while maintaining a long-run equilibrium that converges to a conventional rational expectations equilibrium.

Agents learn using a beta-binomial probability model starting with exogenous initial priors. Take as given a history of realizations of κ that agents observe over T periods, $\kappa^T \equiv \{\kappa_0, \kappa_1, ..., \kappa_{T-1}, \kappa_T\}$, and initial priors, F^s, of the distributions of F^s_{hh} and F^s_{ll} for date $t = 0$, $p(F^s)$. Bayesian learning with beta-binomial distributions yields a sequence of posteriors $\{f(F^s \mid \kappa^t)\}_{t=1}^T$.

To understand how the sequence of posteriors is formed, consider first that at every date t, from 0 to T, the information set of the agent includes κ^t as well as the possible values that κ can take (κ^h and κ^l). This means that agents also know the number of times a particular regime has persisted or switched to the other regime (i.e. agents know the set of counters $\left[n_t^{hh}, n_t^{hl}, n_t^{ll}, n_t^{lh}\right]_{t=0}^T$ where each n_t^{ij} denotes the number of transitions from state κ^i to κ^j that have been observed prior to date t).[5] These counters, together with the priors, form the arguments of the Beta-binomial distributions that characterize the learning process. For instance, the initial priors are given by $p(F^s_{ii}) \propto (F^s_{ii})^{n_0^{ii}-1}(1 - F^s_{ii})^{n_0^{ij}-1}$. As in Cogley and Sargent (2008a), we assume that the initial priors are independent and determined by n_0^{ij} (i.e. the number of transitions assumed to have been observed prior to date $t = 1$).

The agents' posteriors about F^s_{hh} and F^s_{ll} have Beta distributions as well. The details of how they follow from the priors and the counters are provided in Cogley and Sargent (2008a) and Boz and Mendoza (2010). The posteriors are of the form $F^s_{hh} \propto Beta(n_t^{hh}, n_t^{hl})$ and $F^s_{ll} \propto Beta(n_t^{lh}, n_t^{ll})$,

[5] The number of transitions across regimes is updated as follows: $n_{t+1}^{ij} = n_t^{ij} + 1$ if both $\kappa_{t+1} = \kappa^j$ and $\kappa_t = \kappa^i$, and $n_{t+1}^{ij} = n_t^{ij}$ otherwise.

and the posterior means satisfy:

$$E_t[F^s_{hh}] = n^{hh}_t/(n^{hh}_t + n^{hl}_t), \quad E_t[F^s_{ll}] = n^{ll}_t/(n^{ll}_t + n^{lh}_t) \tag{6}$$

This is a key result for the solution method we follow, because, as will be explained later in this Section, the method relies on knowing the evolution of the posterior means as learning progresses.

An important implication of (6) is that the posterior means change only when that same regime is observed at date t. Since in a two-point, regime-switching setup continuation probabilities also determine mean durations, it follows that the beliefs about both the persistence and the mean durations of the two financial regimes can be updated only when agents actually observe κ^l or κ^h.

2.3 Learning, Debt and Price Dynamics after Financial Innovation

The potential for financial innovation to lead to significant underestimation of risk can be inferred from the evolution of the posterior means. Consider in particular an experiment in which financial innovation is defined as the arrival of a brand new environment in which credit conditions can shift between κ^h and κ^l. By construction, this implies starting the learning process from values of n^{ij}_0 that are close to zero.[6] Given this assumption and the conditions mapping counters of regime realizations into posterior means (eq. (6)), it follows that the first sequence of realizations of κ^h generates substantial optimism (i.e. a sharp increase in $E_t[F^s_{hh}]$ relative to F^a_{hh}).[7] Moreover, it also follows that the magnitude of the optimism that any subsequent sequence of realizations of κ^h generates will be smaller than in the initial optimistic phase. Intuitively, this is because it is only after observing the first switch to κ^l that agents rule out the possibility of κ^h being an absorbent state. Similarly, the first realizations of κ^l generate a pessimistic phase, in which $E_t[F^s_{ll}]$ is significantly higher than F^a_{ll}, so the period of optimistic expectations is followed by a period of pessimistic expectations.

Following Boz and Mendoza (2010), the effects of the above optimistic beliefs on debt and land prices that result from the interaction between the collateral constraint and learning can be explained intuitively by combining the Euler equations on land and bonds (equations (4) and (5))

[6] Recall that n^{ij}_0 are counters of the number of times a regime has been observed before learning starts. A truly new environment would have $n^{ij}_0 = 0$, but since the binomial distribution is not defined for $n^{ij}_0 = 0$, n^{ij}_0 close to zero provides the best approximation to a truly new regime.

[7] From (6), if $n^{ij}_0 = 0.1$ for $i, j = h, l$ and we observe five quarters of κ^h, $E_t[F^s_{hh}]$ rises from 0.5 at $t = 0$ to 0.98 at $t = 5$, while $E_t[F^s_{ll}]$ remains unchanged at 0.5.

to obtain an expression for the model's land premium, $E_t^s[R^q_{t+1+i}]$, and then solving forward for the price of land in Equation (5).

Defining $R^q_{t+1} \equiv (\varepsilon_{t+1} Y_k(t+1) + q(t+1)/q(t)$, the expected land premium one-period ahead is given by:

$$E_t^s \left[R^q_{t+1} - R \right] = \frac{(1-\kappa_t)\mu_t - cov_t^s(\beta u'(c_{t+1}), R^q_{t+1})}{E_t^s \left[\beta u'(c_{t+1}) \right]} \tag{7}$$

This land premium rises in every state in which the collateral constraint binds because of a combination of three effects: the increased excess return on land due to the shadow value of the collateral constraint (which is limited to the fraction $(1-\kappa_t)$ of μ_t because the fraction κ_t of land can be collateralized into debt), the lower covariance between marginal utility and land returns, and the increased expected marginal utility of future consumption. The latter two effects occur because the binding credit constraint hampers the agents' ability to smooth consumption and tilts consumption towards the future.

Consider now a state at date t in the initial optimistic phase of financial innovation in which the collateral constraint binds even at κ^h. Compare first what the land premium would look like in the DEL of the learning economy ($E_t^s[R^q_{t+1} | \kappa_t^h = \kappa^h, \mu_t > 0]$) v. the DEF of the perfect information economy ($E_t^a[R^q_{t+1} | \kappa_t^h = \kappa^h, \mu_t > 0]$). If beliefs are optimistic (i.e. $E_t[F^s_{hh}] > F^a_{hh}$), agents assign lower probability to the risk of switching to κ^l at $t+1$ (which has higher land returns because the constraint is more binding for κ^l than for κ^h) than they would under perfect information. This lowers the expected land premium in the learning model because agents' beliefs put more weight on states with lower land returns.

To see how this affects asset prices, consider the forward solution of q_t:

$$q_t = E_t^s \left[\sum_{j=0}^{\infty} \left(\prod_{i=0}^{j} \left(\frac{1}{E_t^s[R^q_{t+1+i}]} \right) \right) \varepsilon_{t+1+j} Y_k(k_{t+1+j}) \right]. \tag{8}$$

This expression shows that the lower land returns that follow financial innovation when learning leads to optimistic beliefs, either at date t or expected along the equilibrium path for any future date, translate into higher land prices at t (and higher than under full information). But if the constraint was already binding at t with κ^h, and κ^h is the current state, the value of collateral rises and agents borrow more. In addition, as collateral values rise the constraint becomes relatively less binding (i.e. μ_t falls), but this puts further downward pressure on land premia (see eq. (8), which in turn puts further upward pressure on land prices. Hence, optimistic beliefs and the credit constraint interact to amplify the total upward effects on credit and prices. Notice, however, this feedback process is nonlinear, because it depends on the equilibrium dynamics of beliefs, land prices and μ. For example, if the constraint becomes nonbinding as prices rise, at that point the amplification mechanism would stop.

When the first observation of κ^l arrives after the initial spell of $\kappa^{h\prime}s$ that followed financial innovation, the opposite process is set in motion, and this process is characterized by the classic Fisherian deflation mechanism. Observing the first realization of κ^l leads agents to update their regime counters, and hence the posterior mean for the low-credit regime in (6) turns pessimistic (i.e. $E_t[F^s_{ll}] > F^a_{ll}$), so they assign excessive probability to staying in κ^l.[8] This increases the expected land premium because now agents' beliefs put more weight on states with higher land returns, and the higher expected premia lower asset prices relative to full information. As asset prices fall, and if κ^l is the current state, the collateral constraint becomes even more binding, which triggers a Fisherian deflation and fire sales of assets, which in turn put further upward pressure on land premia and downward on land prices as μ rises, and agents continue to put higher probability in these states with even higher land returns and lower land prices.

2.4 Recursive Anticipated Utility Competitive Equilibrium

The fact that this learning setup involves learning from and about an exogenous variable (κ) allows us to solve for the equilibrium dynamics following a two-stage solution method. In the first stage, we use the Bayesian learning framework to generate the agents' sequence of posterior means determined by (6). In the second stage, we characterize the agents' optimal plans as a recursive equilibrium by adopting Kreps's Anticipated Utility (AU) approach to approximate dynamic optimization with Bayesian learning. The AU approach focuses on combining the sequences of posterior means obtained in the first stage with chained solutions from a set of "conditional" AU optimization problems (AUOP).[9] Each of these problems solves what looks like a standard optimization

[8] Again starting from $n^{ij}_0 = 0.1$ for $i, j = h, l$ and observing κ^h the first five quarters and κ^l the sixth quarter, $E_t[F^s_{ll}] = 0.5$ for $t = 0$ *to* 5 and then rises to 0.917 at $t = 6$.

[9] Cogley and Sargent (2008b) show that the AU approach is significantly more tractable than full Bayesian dynamic optimization and yet produces very similar quantitative results, unless risk aversion coefficients are large. The full Bayesian optimization problem uses not just the posterior means but the entire likely evolution of posterior density functions to project the effects of future κ realizations on beliefs. This problem runs quickly into the curse of dimensionality because it requires carrying the counters $\left[n^{hh}_t, n^{hl}_t, n^{ll}_t, n^{lh}_t\right]^T_{t=0}$ as additional state variables. It follows from this argument that one can also interpret AU optimization as a form of bounded rationality.

problem with full information and rational expectations, but using the posterior means of each date t instead of the true transition probabilities (see Boz and Mendoza (2010) for further details).

The AU competitive equilibrium in recursive form is constructed as follows. Consider the date-t AUOP. At this point agents have observed κ_t, and use it to update their beliefs so that (6) yields $E_t[F^s_{hh}]$ and $E_t[F^s_{ll}]$. Using this posterior means, they construct the date-t beliefs about the transition probability matrix across financial regimes $E^s_t[\kappa'|\kappa] \equiv \begin{bmatrix} E_t[F^s_{hh}] & 1-E_t[F^s_{hh}] \\ 1-E_t[F^s_{ll}] & E_t[F^s_{ll}] \end{bmatrix}$. The solution to the date-t AUOP is then given by policy functions $(b'_t(b,\varepsilon,\kappa), c_t(b,\varepsilon,\kappa), \mu_t(b,\varepsilon,\kappa))$ and a pricing function $q_t(b,\varepsilon,\kappa)$ that satisfy the following recursive equilibrium conditions:

$$u'\left(c_t(b,\varepsilon,\kappa)\right) = \beta R\left[\sum_{\varepsilon'\in E}\sum_{\kappa'\in\{\kappa^h,\kappa^l\}} E^s_t[\kappa'|\kappa]\pi(\varepsilon'|\varepsilon)u'(c_t(b',\varepsilon',\kappa'))\right] + \mu_t(b,\varepsilon,\kappa) \tag{9}$$

$$q_t(b,\varepsilon,\kappa)\left[u'(c_t(b,\varepsilon,\kappa)) - \mu_t(b,\varepsilon,\kappa)\kappa\right] = \beta\left[\sum_{z'\in Z}\sum_{\kappa'\in\{\kappa^h,\kappa^l\}} E^s_t[\kappa'|\kappa]\pi(\varepsilon'|\varepsilon)u'(c_t(b',\varepsilon',\kappa'))\left[\varepsilon'Y(1) + q_t(b',\varepsilon',\kappa')\right]\right] \tag{10}$$

$$c_t(b,\varepsilon,\kappa) + \frac{b'_t(b,\varepsilon,\kappa)}{R} = \varepsilon Y(1) + b \tag{11}$$

$$\frac{b'_t(b,\varepsilon,\kappa)}{R} \geq -\kappa q_t(b,\varepsilon,\kappa)1 \tag{12}$$

The time subscripts that index the policy and pricing functions indicate the date of the beliefs used to form the expectations, which is also the date of the most recent observation of κ (date t). Notice that these equilibrium conditions already incorporate the market clearing condition of the land market.

It is critical to note that solving for date-t policy and pricing functions means solving for a full set of optimal plans over the entire (b,ε,κ) domain of the state space and conditional on date-t beliefs. Thus, we are solving for the optimal plans agents "conjecture" they would make over the infinite future acting under those beliefs. For characterizing the "actual" equilibrium dynamics to match against the data, however, the solution of the date-t AUOP determines optimal plans for date t only. This is crucial because beliefs change as time passes, and each subsequent κ_t is observed, which implies that the policy and pricing functions that solve each AUOP also change.

The model's recursive AU equilibrium is defined as follows:

Definition Given a T-period history of realizations $\kappa^T = (\kappa_T, \kappa_{T-1}, ..., \kappa_1)$, a recursive AU competitive equilibrium for the economy is given by a sequence of decision rules $[b'_t(b, \varepsilon, \kappa), c_t(b, \varepsilon, \kappa), \mu_t(b, \varepsilon, \kappa)]_{t=1}^T$ and pricing functions $[q_t(b, \varepsilon, \kappa)]_{t=1}^T$ such that: (a) the decision rules and pricing function for date t solve the date-t AUOP conditional on $E_t^s[\kappa'|\kappa]$; (b) $E_t^s[\kappa'|\kappa]$ is the conjectured transition probability matrix of κ produced by the date-t posterior density of F^s determined by the Bayesian passive learning as defined in (6).

Intuitively, the complete solution of the recursive equilibrium is formed by chaining together the solutions for each date-t AUOP. For instance, the sequence of equilibrium bond holdings that the model predicts for dates $t = 1, ..., T$ is obtained by chaining the relevant decision rules as follows: $b_2 = b'_1(b, \varepsilon, \kappa),\ b_3 = b'_2(b, \varepsilon, \kappa), ...,\ b_{T+1} = b'_T(b, \varepsilon, \kappa)$.

2.5 Conditionally Efficient Planners' Problems

We examine macro-prudential policy by studying two versions of an optimal policy problem faced by a benevolent social planner who maximizes the agents' utility subject to the resource constraint and the collateral constraint. The key difference between these planners' problems and the DEL is that the former internalize the effects of borrowing decisions on the market price of assets that serve as collateral.

We follow Bianchi and Mendoza (2010) in considering that the planners face the same borrowing ability at every given state as agents in a competitive equilibrium. This implies that the planner is required to implement the same pricing function for the valuation of collateral as in a decentralized equilibrium (i.e. we do not allow the planner to manipulate the current price of land at a particular state of nature). The planners, however, can alter future values of land by choosing the amount of debt in the economy. In particular, the planners internalize that when the economy has a larger amount of debt, a negative shock triggering the collateral constraint leads to a lower asset price and a further tightening of collateral constraints via the Fisherian deflation.[10]

The assumption that the collateral pricing function faced by the planners corresponds to the pricing function of a competitive equilibrium is in line with the concept of conditional or financial efficiency defined by Kehoe and Levine (1993) in their analysis of endogenous debt limits, and studied by Lustig (2000) in the context of a credit market with collateral. As Bianchi and Mendoza

[10] In contrast, if a planner can manipulate the collateral pricing function, the planner would internalize not only how the choice of debt at t affects the land price at $t + 1$, but also how it affects land prices and the tightness of the collateral constraint in previous periods.

(2010) argued, there are several advantages of this formulation for the analysis of macro-prudential policy in models with collateral constraints. First, this formulation makes the planners' optimization problem time-consistent, which guarantees that macro-prudential policy, if effective, improves welfare across all states and dates in a time-consistent fashion. Second, it allows for a simpler characterization and decentralization based on the use of Pigouvian taxes on debt and dividends, as we explain below. Third, even with this constrained notion of efficiency, correcting the fire-sale externality can lead to a sharp reduction in the probability and the severity of financial crises (see again Bianchi and Mendoza).

The two planner problems we construct are based on the information set assumed for the government. First, we define an uninformed planner (SP1) as one who is subject to a similar learning problem as private agents. This planner observes the same history κ^T and starts learning off date-0 priors that may or may not be the same as those of the private sector. Because of the conditional efficiency assumption, SP1 prices collateral using the DEL's collateral pricing functions ($q_t^{DEL}(b,\varepsilon,\kappa)$), which ensures that SP1 faces the same set of feasible credit positions as private agents in the DEL. Second, we construct a fully informed planner (SP2) as a planner who knows F^a_{hh} and F^a_{ll}, and prices collateral using the time-invariant pricing function of the DEF $q^{DEF}(b,\varepsilon,\kappa)$.[11] Hence, conditional efficiency for this planner means that it can implement the same set of feasible credit positions as private agents in the DEF.

The two planners' optimization problems in standard intertemporal form can be summarized as follows:

$$E_0^i\left[\sum_{t=0}^{\infty}\beta^t\frac{c_t^{1-\sigma}}{1-\sigma}\right] \qquad \text{for} \quad i \;=\; SP1, SP2 \tag{13}$$

$$s.t. \qquad c_t+\frac{b_{t+1}}{R_t} \;=\; b_t+\varepsilon_t Y(1) \tag{14}$$

$$-\frac{b_{t+1}}{R_t} \;\leq\; \kappa_t q_t^i \tag{15}$$

with $q_t^{SP1}=q_t^{DEL}$ and $q_t^{SP2}=q^{DEF}$. Note that in SP1, the planner solves a similar Bayesian learning problem as private agents observing the same history of credit regimes κ^T. This planner's initial priors are denoted p_0^{ij} for $i,j=h,l$. If $p_0^{ij}=n_0^{ij}$, which will be our baseline scenario, so that both SP1 and private agents have identical beliefs at all times. Later in sensitivity analysis we

[11] These pricing functions are time invariant because they correspond to the solutions of a standard recursive rational expectations equilibrium. The resulting planning problem is analogous to the one solved in Bianchi and Mendoza (2010).

examine the implications of relaxing this assumption. In SP2, the planner uses the true transition probabilities F^a_{hh} and F^a_{ll}.

We solve the problem of each planner in recursive form, and to simplify the exposition we represent the two AU problems in recursive form.[12] For each planner $i = SP1, SP2$ the solution to the date-t AUOP is given by policy functions $(b'_t(b,\varepsilon,\kappa), c_t(b,\varepsilon,\kappa), \mu_t(b,\varepsilon,\kappa))$ that satisfy the following recursive equilibrium conditions:

$$u'\left(c_t(b,\varepsilon,\kappa)\right) - \mu_t(b,\varepsilon,\kappa) = \tag{16}$$

$$\beta R \left[\sum_{\varepsilon' \in E} \sum_{\kappa' \in \{\kappa^h, \kappa^l\}} E^i_t[\kappa'|\kappa]\pi(\varepsilon'|\varepsilon) \left[u'(c_t(b',\varepsilon',\kappa')) + \kappa' \mu_t(b',\varepsilon',\kappa') \frac{\partial q^i_t(b',\varepsilon',\kappa')}{\partial b'} \right] \right]$$

$$c_t(b,\varepsilon,\kappa) + \frac{b'_t(b,\varepsilon,\kappa)}{R} = \varepsilon Y(1) + b \tag{17}$$

$$\frac{b'_t(b,\varepsilon,\kappa)}{R} \geq -\kappa q^i_t(b,\varepsilon,\kappa) 1 \tag{18}$$

where the pricing functions for each planner are $q^{SP1}_t(b,\varepsilon,\kappa) = q^{DEL}_t(b,\varepsilon,\kappa)$ and $q^{SP2}_t(b,\varepsilon,\kappa) = q^{DEF}(b,\varepsilon,\kappa)$. Moreover, expectations in each planner's date-t AUOP are taken using $E^{SP1}_t[\kappa'|\kappa] \equiv \begin{bmatrix} E_t[F^g_{hh}] & 1 - E_t[F^g_{hh}] \\ 1 - E_t[F^g_{ll}] & E_t[F^g_{ll}] \end{bmatrix}$ and $E^i_t[\kappa'|\kappa] \equiv \begin{bmatrix} F^a_{hh} & 1 - F^a_{hh} \\ 1 - F^a_{ll} & F^a_{ll} \end{bmatrix}$ for $i = SP2$.[13] Note also that in these problems the time subindexes of expectations operators, decision rules and pricing functions represent the date of the AUOP to which they pertain, and not the indexing of time within each AUOP. That is, in the date-t AUOP the planner creates expectations of the prices and allocations of all future periods using the date-t recursive decision rules and pricing functions (e.g. in the date-t AUOP, consumption projected for $t+1$ is given by the expectation of $c_t(b',\varepsilon',\kappa')$). Moreover, for SP2, since the planner has full information and can implement the credit feasibility set of the DEF, the decision rules are actually time-invariant at equilibrium (all date-t AUOP's for SP2 are identical because they use the true Markov process of κ and the DEF time-invariant pricing functions).

We can now define the two recursive social planner problems for a given history of realizations κ^T:

[12] This is redundant for SP2 because this planner solves a standard full-information rational expectations recursive equilibrium with time-invariant decision rules and pricing functions.

[13] By analogy with the results in (6), the posterior means of the government's learning dynamics satisfy: $E_t[F^g_{hh}] = p^{hh}_t/(p^{hh}_t + p^{hl}_t)$, $E_t[F^g_{ll}] = p^{ll}_t/(p^{ll}_t + p^{lh}_t)$. Note that, since both the private sector and the government observe the same κ sequence, these counters can differ from those of private agents only because of differences in date-0 priors.

SP1 Equilibrium Given the DEL time-varying asset pricing functions $[q_t^{DEL}(b, \varepsilon, \kappa)]_{t=1}^T$, a recursive AU equilibrium for the SP1 planner is given by a sequence of decision rules $[b_t'(b, \varepsilon, \kappa), c_t(b, \varepsilon, \kappa), \mu_t(b, \varepsilon, \kappa)]_{t=1}^T$ such that: (a) the decision rules for date t solve SP1's date-t AUOP conditional on $E_t^g[\kappa'|\kappa]$; and (b) the elements of $E_t^g[\kappa'|\kappa]$ are the posterior means produced by the date-t posterior densities of F_{hh}^g and F_{ll}^g determined by the Bayesian learning process.

SP2 Equilibrium Given the DEF time-invariant asset pricing function $q^{DEF}(b, \varepsilon, \kappa)$, a recursive AU equilibrium for the SP2 planner is given by time-invariant decision rules $[b'(b, \varepsilon, \kappa), c(b, \varepsilon, \kappa), \mu(b, \varepsilon, \kappa)]$ such that the decision rules solve SP2's date-t AUOP conditional on $E^a[\kappa'|\kappa]$ for all t.

2.6 Pecuniary Externality and Decentralization of Planners' Allocations

The key difference between the first-order conditions of the social planners and those obtained in the private agents' DEL is the pecuniary externality reflected in the right-hand-side of the planners' Euler equation for bonds (eq. (16)): The planners internalize how, in states in which the collateral constraint is expected to bind next period (i.e. $\mu_t(b', \varepsilon', \kappa') > 0$ for at least some states), the choice of debt made in the current period, b', will alter the tightness of the constraint by affecting prices in the next period ($\frac{\partial q_t^i(b', \varepsilon', \kappa')}{\partial b'}$). This derivative represents the response of the land price tomorrow to changes in the debt chosen today, which can be a very steep function when the collateral constraint binds because of the Fisherian deflation mechanism.

While the two planning problems consider the above price derivative, they differ sharply in how they do it. Consider again the period of optimism produced by the effect on the private agents' beliefs of the initial spell of κ^h realizations after financial innovation starts. Since in the baseline case SP1 has the same initial priors as private agents (because the baseline assumes $p_0^{ij} = n_0^{ij}$), its beliefs are always identical to those of private agents. Thus, SP1 shares in the agent's optimism both directly, in terms of beliefs about transition probabilities of κ, and indirectly, in terms of facing the feasible set of credit positions implied by optimistic collateral prices in the DEL pricing function. This planner still wants to use macro-prudential policy to dampen credit growth because it internalizes the slope of the asset pricing function when the collateral constraint on debt is expected to bind, but this planner's expectations are as optimistic as the private agents' and hence it assigns very low probability to a financial crash (i.e. a transition from κ^h to κ^l), and it internalizes a pricing function inflated by optimism. Our quantitative findings show that, if optimism builds quickly (i.e. $E_t[F_{hh}^s]$ approaches 1) and the collateral constraint binds tightly in the early stages of financial innovation, these limitations can result in SP1 attaining equilibrium debt and land prices close to those of the DEL, thus reducing the effectiveness of macro-prudential policy. But if optimism builds gradually and/or the collateral constraint is not tightly binding, SP1 attains lower debt positions than private agents in the DEL, and this causes the crash to be significantly less severe when financial conditions reverse.

SP2 differs sharply because it does not share the private agents' optimistic beliefs and thus assign higher probability to the likelihood of observing a κ^h-to-κ^l transition than in the DEL, which therefore strengthens SP2's incentive to build precautionary savings and borrow less. SP2 is also more cautious than SP1, because it assigns higher probability to transitions from states with optimistic prices to those with pessimistic crash prices. Again depending on whether the constraint binds and how optimistic beliefs are, SP2 acquires less debt and experiences lower land price booms than both SP1 and DEL, and for the same reason its use of macro prudential policy is more intensive.

Given the model's pecuniary externality, the most natural choice to model the implementation of macro-prudential policies are Pigouvian taxes. In particular, using taxes on debt ($\tau^i_{b,t}$) and land dividends ($\tau^i_{l,t}$) we can fully implement the planner problems' allocations (for $i = SP1, SP2$). With these taxes, the budget constraint of private agents becomes:

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R_t(1+\tau^i_{b,t})} = q_t k_t + b_t + \varepsilon_t Y(k_t)(1-\tau^i_{l,t}) + T^i_t. \tag{19}$$

T^i_t represents lump-sum transfers by which the government rebates to private agents all its tax revenue (or a lump-sum tax in case the tax rates are negative, which is not ruled out).

The Euler equations of the competitive equilibrium with the macro-prudential policy in place are:

$$u'(t) = \beta R(1+\tau^i_{b,t}) E^s_t\left[u'(t+1)\right] + \mu_t \tag{20}$$

$$q_t(u'(t) - \mu_t\kappa) = \beta E^s_t\left[u'(t+1)\left(\varepsilon_{t+1}Y_k(k_{t+1})(1-\tau^i_{l,t}) + q_{t+1}\right)\right]. \tag{21}$$

We compute the state-contingent, time-varying schedules of these taxes by replacing each planner's allocations in these optimality conditions and then solving for the corresponding tax rates, so that the DEL *with* macro-prudential policy supports both the same allocations of each planner's problem and the corresponding asset pricing functions that each planner uses to value collateral.

The debt tax is needed to replicate the planner's debt choices, and the dividends tax is needed to support the pricing functions that the planner used to value collateral as the competitive equilibrium asset pricing functions.[14] The tax schedules in recursive form are denoted $\tau^i_{b,t}(b, \varepsilon, \kappa)$ and $\tau^i_{l,t}(b, \varepsilon, \kappa)$. There will be one of these schedules for each date-t AUOP solved by private agents in the DEL with Pigouvian taxes.

It is important to note that when the collateral constraint is binding at t, one can construct multiple representations of the tax schedules that implement the allocations of the planning problems. This is because when $\mu_t > 0$ the value of b_{t+1} is determined by the collateral constraint and not by the Euler equation for bonds. In particular, if we plug a given planner's consumption and debt plans and collateral pricing function in conditions (20) and (21), there are different schedules of debt and dividend taxes depending on a chosen schedule for the shadow value μ_t in the DEL with taxes. For simplicity, we chose the tax schedules such that $\tau^i_{b,t} = 0$ when the collateral constraint binds. Hence, when $\mu_t > 0$, the shadow value of the constraint is set at $\mu_t = u'(t) - \beta R E^s_t \left[u'(t+1)\right]$, and given this the corresponding tax on dividends when the constraint binds follows from condition (21).

The debt tax can be decomposed into three terms that are useful for interpreting how macroprudential policy responds to the effects of imperfect information, the pecuniary externality and the interaction of these two. In particular, combining (20) and (16) and rearranging terms, the debt tax for each planner can be expressed as follows:

$$\tau^i_{b,t} = \underbrace{\frac{E^i_t[u'(t+1)]}{E^s_t[u'(t+1)]} - 1}_{\text{information}} + \underbrace{\frac{E^i_t\left[\kappa_{t+1}\mu_t(t+1)\frac{\partial q^i_t(t+1)}{\partial b'}\right] - E^s_t\left[\kappa_{t+1}\mu_t(t+1)\frac{\partial q^i_t(t+1)}{\partial b'}\right]}{E^s_t[u'(t+1)]}}_{\text{interaction}} + \underbrace{\frac{E^s_t\left[\kappa_{t+1}\mu_t(t+1)\frac{\partial q^i_t(t+1)}{\partial b'}\right]}{E^s_t[u'(t+1)]}}_{\text{externality}} \tag{22}$$

where all variables are evaluated at the corresponding planner's problem. The first term in the right-hand-side of this expression is labeled "information" because it reflects the contribution to the debt tax that arises from deviations in the one-period-ahead expected marginal utilities of private agents and planner i, which arise because of the beliefs formed with their different information sets. If the two information sets, and hence beliefs, are identical, as they are in our baseline SP1, this term vanishes, but for SP2 it does not vanish. The second term, labeled "interaction", reflects differences in the expected value of the externality when evaluated using the beliefs of each planner

[14] The tax on debt is also equivalent to tightening margin requirements, i.e., reducing κ when the credit constraint is slack (see Bianchi (2011)).

v. the private agents' beliefs. This term is zero when either the information sets are the same or the DEL is far from the region where the constraint binds, and hence the externality term is zero for all possible states in $t+1$. Thus, the label "interaction" reflects the fact that both the informational difference and the externality need to be present for this term to be nonzero. Finally, the third term labeled "externality" is simply the value of the externality evaluated using the beliefs of private agents.

3 Quantitative Analysis

This Section explores the quantitative implications of the model. We discuss first the baseline calibration of parameter values, and then compare the DEL with the two planning problems. We also quantify the macro-prudential tax schedules that decentralize the planners' allocations and decompose them into their three components.

3.1 Baseline Calibration

We borrow the baseline calibration from Boz and Mendoza (2010), so we keep the description here short. In contrast with their work, however, our aim here is to study how the interaction between the learning friction and the pecuniary externality affect the design of macro-prudential policy in the aftermath of financial innovation. This implies that the uncertainty surrounding the values of the parameters driving the learning process and the collateral constraint takes particular relevance, so we view this initial calibration more as a baseline to begin the quantitative analysis than as a calibration intended to judge the model's ability to match the data.

The model is calibrated to U.S. quarterly data at annualized rates and assuming a learning period of length T in which $\kappa = \kappa^h$ from $t = 1, ..., J$ (the optimistic phase) and $\kappa = \kappa^l$ from $J + 1$ to T (the pessimistic phase). The parameter values are listed in Table 1.

As in Boz and Mendoza (2010), we set the start of the learning dynamics and the dates T and J based on observations from a timeline of the financial innovation process and events leading to the 2008 U.S. financial crisis. As explained earlier, we define financial innovation as a structural change that creates a new environment with stochastic switches between κ^h and κ^l. Before financial innovation, we assume the financial environment was characterized by a regime with a single time-invariant collateral coefficient κ^l. We set the date of the structural change as of 1997Q1 to be consistent with two important facts. First, 1997 was the year of the first publicly-available

Table 1: Baseline Parameter Values

β	Discount factor (annualized)	0.91
σ	Risk aversion coefficient	2.0
c	Consumption-GDP ratio	0.670
A	Lump-sum absorption	0.321
r	Interest rate (annualized)	2.660
ρ	Persistence of endowment shocks	0.869
σ_e	Standard deviation of TFP shocks	0.008
α	Factor share of land in production	0.025
κ^h	Value of κ in the high securitization regime	0.926
κ^l	Value of κ in the low securitization regime	0.642
F^a_{hh}	True persistence of κ^h	0.95
F^a_{ll}	True persistence of κ^l	0.95
n^{hh}_0,n^{hl}_0	Priors	0.0205

securitization of mortgages under the New Community Reinvestment Act and the first issuance of corporate CDS's by JPMorgan.[15] Second, this was also the year in which the net credit assets-GDP ratio of U.S. households started a protracted decline that lasted until the end of 2008, while prior to 1997 this ratio was quite stable at about -30 percent. We date the start of the financial crisis at 2007Q1, consistent with the initial nation-wide decline in home prices and the early signs of difficulties in the subprime mortgage market. The experiment ends two years later. These assumptions imply setting $T = 48$ and $J = 40$ (i.e. 40 consecutive quarters of κ^h realizations followed by 8 consecutive quarters of κ^l).

The model's parameters are calibrated as follows: First, the values of $(\sigma, R, \rho, \sigma_e, \kappa^l, \kappa^h, F^a_{hh}, F^a_{ll})$ are calculated directly from the data or set to standard values from the quantitative DSGE literature. Second, the values of (α, β) are calibrated such that the model's pre-financial innovation stochastic stationary state is consistent with various averages from U.S. data from the pre-financial-innovation period (i.e. pre-1997), assuming that in that period the financial constraint with κ^l was binding on average. Finally, the initial priors are calibrated assuming that they are symmetric, with a common value n_0 for all transitions targeted to match an estimate of observed excess land returns, as described later in this Section.

[15]Several other major financial reforms were also introduced in the late 1990s, including the Commodity Futures Modernization Act, which moved over-the-counter derivatives beyond the reach of regulators, and the Gramm-Leach-Bliley act, which removed legal barriers separating bank and non-bank financial intermediaries set in 1933 with the Glass-Steagall act.

We set the real interest rate to the average ex-post real interest rate on U.S. three-month T-bills during the period 1980Q1-1996Q4, which is 2.66 percent annually. The value of σ is set to $\sigma = 2.0$, the standard value in DSGE models of the U.S. economy.

To pin down κ^l and κ^h, we use the data on net credit market assets of U.S. households and non-profit organizations from the *Flow of Funds* of the Federal Reserve Board as a proxy for b in the model. The proxy for ql is obtained from the estimates of the value of residential land provided by Davis and Heathcote (2007). On average over the 1980Q1-1996Q4 period, the ratios of the value of residential land and net credit market assets relative to GDP were stable around 0.477 and -0.313, respectively. Next, we construct a macro estimate of the household leverage ratio, or the loan-to-value ratio, by dividing net credit market assets by the value of residential land. We set the value of κ^l by combining the 1980Q1-1996Q4 average of this ratio with the calibrated value of R which yields $\kappa^l = 0.659/1.0266 = 0.642$. Following a similar idea, we set κ^h to the 2006Q4 value of the estimated leverage ratio, hence $\kappa^h = 0.926$.

The value of F^a_{hh} is set based on Mendoza and Terrones (2008)'s finding that the mean duration of credit booms in industrial economies is 7 years. To match this mean duration, we set $F^a_{hh} = 0.95$. We assume a symmetric process by setting $F^a_{ll} = 0.95$. Notice that the true transition probability matrix across financial regimes is not needed to solve the DEL, but is necessary for solving SP2 and DEF.[16]

We assume a standard Cobb-Douglas production function: $Y(k_t) = k_t^\alpha$. Using the 1980Q1-1996Q4 average of the value of residential land to GDP, the value of R, and the condition that arbitrages the returns on land and bonds, which follows from the optimality conditions (4)-(5), the implied value for α is $\alpha = 0.0251$ [17].

The stochastic process for ε is set to approximate an AR(1) process ($\ln(\varepsilon_t) = \rho \ln(\varepsilon_{t-1}) + e_t$) fitted to HP-filtered real U.S. GDP per capita using data for the period 1965Q1-1996Q4. The parameter estimates of this process are $\rho = 0.869$ and $\sigma_e = 0.00833$, which imply a standard deviation of TFP of $\sigma_\varepsilon = 1.68$ percent.

[16] Notice also that while knowing F^a_{hh} and F^a_{ll} is not necessary for solving the DEL over the assumed sequence of 48 realizations of κ, solving the DEL over an infinite horizon does require the true probabilities, because the counters must satisfy the condition that the ratios n^{hl}/n^{hh} and n^{lh}/n^{ll} need to converge to F^a_{hl}/F^a_{hh} and F^a_{lh}/F^a_{ll} respectively as the counters go to infinity.

[17] Since the model with a single financial regime set at κ^l (i.e., the pre-financial-innovation regime) yields a collateral constraint that is almost always binding and a negligible excess return on land, we use the approximation $E[R^q] \approx R$, and then conditions (4) and (5) imply: $\alpha = (ql/l^\alpha)[R - 1 + \beta^{-1}(1 - \beta R)(1 - \kappa^l)]$

The value of β is set so that in the pre-financial-innovation stochastic steady state the model matches the observed standard deviation of consumption relative to output over the 1980Q1-1996Q4 period, which is 0.8. This yields $\beta = 0.91$.

We introduce an exogenous, time-invariant amount of autonomous spending in order to make the model's average consumption-output ratio and average resource constraint consistent with the data. As noted earlier, the *Flow of Funds* data show that the observed average ratio of net credit assets to GDP in the 1980Q1-1996Q4 period was very stable at $\bar{b} = -0.313$. In the case of the consumption-GDP ratio, the data show a slight trend, so we use the last observation of the pre-financial-innovation regime (1996Q4), which implies $\bar{c} = 0.670$.[18] To make these ratios consistent with the model's resource constraint in the average of the stochastic stationary state for that same financial regime, we introduce autonomous spending by the share A of GDP, so that the long-run average of the resource constraint is given by $1 = \bar{c} + A - \bar{b}(R-1)/R$. Given the values for $\bar{b}$, $\bar{c}$ and R, A is calculated as a residual $A = 1 - \bar{c} + \bar{b}(R-1)/R = 0.321$.[19] This adjustment represents the averages of investment and government expenditures, which are not explicitly modeled.

The remaining parameters are the counters of the beta-binomial distributions that determine the initial priors. Because we assume symmetric priors, $n_0 = n_0^{hl} = n_0^{hh} = n_0^{ll} = n_0^{lh}$, so that there is only one parameter to calibrate. We set n_0 so that in the DEL the implied expected excess return on land one period ahead of $t = 0$ matches the annualized 1997Q2 spread on the Fannie Mae residential MBS with 30-year maturity over the T-bill rate. This excess return was equal to 47.6 basis points and the model matches it with $n_0 = 0.0205$.

3.2 Baseline Results

The main quantitative experiment compares the dynamics triggered by financial innovation over the learning period ($t = 1, ..., 48$) in the DEL with those of the two planning problems. These dynamics are computed by solving the sequence of AUOPs for each date t that define each equilibrium, and constructing simulations that chain together the decision rules of each date-t AUOP as described in Boz and Mendoza (2010).[20] These simulations keep TFP unchanged at its mean value ($\varepsilon = 1$) and start from the initial condition $b_0 = -0.345$, which corresponds to the net credit market assets-GDP

[18] Consumption and GDP data were obtained from the *International Financial Statistics* of the IMF.

[19] Note that, since land is in fixed unit supply and the unconditional mean of ε equal to 1, the mean of output in the model is also 1.

[20] Recall that, as explained in Section 2, the decision rules of DEL and SP1 change every period as their beliefs evolve, and hence the dynamics shown for these scenarios result from chaining together the corresponding period's bond decision rules and equilibrium prices.

ratio of U.S. households observed in the data in 1996Q4. Figure 1 plots the dynamics of bonds and land prices (Panels (a) and (b)), the shadow value of collateral μ_t (Panel (c)), the agents' beliefs (Panel (d)), and the pecuniary externality, defined as $E_t[\kappa_{t+1}\mu_t(t+1)\frac{\partial q_t^i(t+1))}{\partial b'}]$ for $i = SP1, SP2$ (Panel (e)).

The evolution of beliefs in Panel (d) shows the large and rapid buildup of optimism that follows the arrival of financial innovation. $E_t[F_{hh}^s]$ rises from 0.980 to 0.999 from $t = 1$ to $t = 40$, as agents observe the long spell of κ^hs. Since there are no observations of κ^l, the beliefs about κ^l do not change during this time (recall Equation (6). At date 41, when the economy switches to κ^l for the first time, $E_{41}[F_{hh}^s]$ falls to 0.975, and more importantly $E_{41}[F_{ll}^s]$ rises sharply from 0.5 to 0.98. Hence, beliefs turn pessimistic very quickly after the first realization of κ^l. Panel (d) also shows the time-invariant true transition probabilities F_{hh}^a and F_{ll}^a, which are the same because we assumed a symmetric process for κ. The excess of $E_t[F_{hh}^s]$ over F_{hh}^a (F_{ll}^a over $E_t[F_{ll}^s]$) measures the degree of optimism (pessimism) built during the optimistic (pessimistic) phase.

The increase in $E_t[F_{hh}^s]$ from date 1 to 40 may appear small (from 0.98 to 0.999) and the difference relative to F_{hh}^a (which is set at 0.95) may also seem small. However, even these small differences have important implications for the perception of riskiness of the financial environment, particularly for the expected mean duration of the κ^h regime and the perceived variability of the κ process. The expected mean duration of κ^h rises from 50 quarters with $E_1[F_{hh}^s] = 0.98$ to 1,000 quarters with $E_{40}[F_{hh}^s] = 0.999$ at the peak of the optimistic phase, and the coefficient of variation of κ based on date-40 beliefs is about 1/4 of that based on date-1 beliefs. Thus, agents' expectations of the riskiness of the new financial environment drop dramatically as the optimistic phase progresses. This is also true relative to $F_{hh}^a = 0.95$, which implies that in the true regime-switching Markov process the κ^h regime has a significantly shorter mean duration of 28 periods. This is about half of what the agents that are learning perceive already at $t = 1$ of the optimistic phase, and a negligible fraction of the mean duration they expect by $t = 40$.

The difference between $E_t[F_{ll}^s]$ as learning progresses and F_{ll}^a has a similar implication. During the optimistic phase, in which $E_t[F_{ll}^s]$ remains constant at 1/2, and since $F_{ll}^a = 0.95$, agents' beliefs imply a projected mean duration for the κ^l regime of only 2 periods, whereas the true mean duration is 28 periods.

These sharp differences in projected mean durations of both κ regimes play a key role in driving the much stronger incentives for precautionary savings of SP2. This planner anticipates that κ^h (κ^l) will arrive less (more) often and that sequences of κ^h (κ^l) are likely to be of much shorter (longer) duration than what agents in the DEL and SP1 believe. Thus, DEL and SP1 perceive much less riskiness in the new financial environment than SP2.

In line with the above description of the evolution of beliefs, Panel (a) of Figure 1 shows that in DEL there is a large and sustained increase in debt (a decline in bonds) for the first 40 periods and a very sharp correction at date 41. This increase in debt accounts for about 2/3rds of the observed rise in net credit liabilities of U.S. households. Panel (b) shows that the surge in debt in the DEL is accompanied by a sharp increase in the price of the risky asset, which is about 44 percent the observed rise in U.S. residential land prices. These two results are reassuring, because they show that the model's baseline DEL is a reasonable laboratory in which to conduct macro-prudential policy experiments inasmuch as the Fisherian interaction of the financial friction and financial innovation produce sizable, sustained booms in debt and land prices. Moreover, as Boz and Mendoza (2010) showed, these booms are twice as large as what the model would predict by either removing the debt-deflation amplification mechanism or the informational friction.

Panel (a) also shows that the two social planners choose lower debt positions than the DEL during the optimistic phase, but the size of the adjustment differs across the two planners. SP1 chooses only slightly smaller debt (higher bonds) than DEL, while SP2 chooses debt levels that are much smaller than those of SP1 and DEL.

SP1 borrows slightly less than DEL in the early periods after financial innovation is introduced, but then bond holdings and asset prices become nearly identical in the two equilibria starting at $t = 7$. This may seem puzzling, because in principle SP1 still has the incentive to use macro-prudential policy, as reflected in the positive externality terms for SP1 displayed in Panel (e). In fact, as we show later in the sensitivity analysis, the nearly identical bond and price dynamics in the baseline DEL and SP1 is *not* a general result. It is the outcome for the baseline calibration because the borrowing constraint binds tightly as a result of the rapid surge in optimism soon after financial innovation starts (see Panel (d)). Thus, households' willingness to borrow induces them to face a high shadow value from relaxing the collateral constraint, and hence, since SP1 also considers the high value of current consumption attributed by households, it also decides to borrow up to the limit. Notice that although date-t borrowing decisions for DEL and SP1 coincide, the fact that the collateral constraint is expected to bind one period ahead still generates an externality for SP1 (see Panel (e)), but this is not strong enough to offset the high value assigned to date-t borrowing, which pushes both private agents and SP1 to borrow up to the limit.

Panel (e) also shows that SP1's externality becomes weaker over time as it approaches the end of the optimistic phase at $t = 40$. The weak externality at this date is easier to interpret by examining Figure 2, which plots the bond decision rules and pricing functions for $t = 40$ in the two κ regimes (for $\varepsilon = 1$). The externality term is given by $E_{40}[\kappa' \mu_{40}(b', \varepsilon', \kappa') \frac{\partial q_{40}^{DEL}(b', \varepsilon', \kappa')}{\partial b'}]$. As Panel (b) of Figure 2 shows, the pricing function $q_{40}^{DEL}(b, 1, \kappa^h)$ is relatively flat, which means that land prices do not differ much for different choices of b'. Thus, in the κ^h state that SP1 believes most likely to continue, the price derivative driving the externality, $\frac{\partial q_{40}^{DEL}(b', \varepsilon', \kappa')}{\partial b'}$, is small. In the other financial regime, κ^l, the pricing function $q_t^{DEL}(b, 1, \kappa^l)$ is very steep (see Panel (d)), but this carries a very small weight because SP1 assigns a negligible probability to switching from κ^h to κ^l. A similar dynamic is at play as the externality weakens from $t = 6$ to $t = 40$. The externality weakens because optimistic beliefs imply that, conditional on having observed $\kappa_t = \kappa^h$ at each date of the optimistic pase, SP1's perceived probability of a switch to κ^l is very low (i.e. $E_t[F_{hl}^s]$ is close to zero). Hence, as the optimistic phase progresses, SP1 evaluates the externality assigning a large and increasing weight to the one-period-ahead state with the small derivative $\frac{\partial q_t^{DEL}(b', 1, \kappa^h)}{\partial b'}$ and nearly zero weight to the state with the large derivative $\frac{\partial q_t^{DEL}(b', 1, \kappa^l)}{\partial b'}$. Notice that there is another effect that goes in the opposite direction. At higher levels of debt along the transition path, the slope of the pricing functions becomes relatively steeper, which would make the externality term larger. But the previous effect on the increasing weight on κ^h regime still dominates.

Using the true regime-switching transition probabilities across the κ regimes, SP2 perceives higher risk in the new financial environment (both in terms of the likelihood of switching to κ^l one period ahead of each date $t = 1, ..., 40$ and in terms of the long-run perceived mean duration of the κ^h regime and the volatility of the κ process). Thus, SP2 has significantly stronger precautionary savings motives, and chooses much lower debt levels than SP1 and DEL during the optimistic phase (see Panel (a) of Figure 1). In fact, on average SP2 avoids hitting the borrowing constraint during the entire optimistic phase, and thus obtains $\mu_t = 0$ for $t = 1, ..., 40$ (see Panel (e)).

The equilibrium prices for SP2 are lower than SP1, because SP1 faces the DEF pricing function, $q^{DEF}(b', 1, \kappa^h)$. Moreover, in the dynamics, land prices actually fall slightly for SP2 (see Panel (b) of Figure (1), because in the DEF the new regime with switching $\kappa's$ allows for more debt on average than the pre-financial-innovation regime with a constant κ^l, but it also entails more risk because of the variability of κ. Under full information the latter effect dominates, thus causing land prices to fall slightly. The $q^{DEF}(b', 1, \kappa^h)$ pricing function supports lower land prices because it is unaffected by the optimistic beliefs and underpricing of risk present in the DEL, although it retains the property that prices are relatively flat for κ^h and very steep for κ^l when the collateral constraint binds (see Panels (b) and (d) of Figure 2). Hence, SP2 chooses lower debt levels because of precautionary reasons, and these credit positions support lower land prices because this planner can attain credit positions that undo the effect of optimistic expectations on prices.

The dynamics of consumption are easy to infer from the debt and price dynamics. During the early periods of the optimistic phase, consumption in DEL and SP1 exceeds that of SP2, in line with the larger debt buildup in those equilibria.

Consider now the model dynamics for the DEL and the two planners when the first switch to κ^l arrives at $t = 41$, which we define as a "crisis episode." To illustrate the crisis dynamics clearly, Figure 4 shows event windows for seven quarters before and after the crisis. As shown in Panel (a) of this figure, SP2, who chose the lowest levels of debt in the optimistic phase, experiences the smallest debt correction. This is consistent with the macro-prudential behavior that led SP2 to take precautionary action and choose lower debt levels, because SP2 can correct the optimism of private beliefs and their effect on the set of feasible credit positions (i.e. it can support collateral values consistent with those of the DEF). With both sources of overborrowing shut down, the smaller correction in debt at $t = 41$ is in response to the exogenous tightening of the constraint due to the lower realization of κ. This exogenous debt correction cannot be avoided even with full information about the transition probability matrix across financial regimes.

The realization of κ^l in period 41 leads to a change in the beliefs of SP1 and DEL about the persistence of the κ^h regime, making the debt correction they experience more pronounced. Since in the baseline calibration SP1 can do almost nothing to undo the overborrowing effect of optimistic beliefs, it cannot avoid arriving at date 40 with the same debt level as in the DEL, at which the economy is vulnerable to a large correction in case of a transition to κ^l. In addition, once the credit regime shift occurs, beliefs turn pessimistic increasing the perceived riskiness of the financial environment and strengthening the feedback process of the Fisherian deflation mechanism as described in the previous Section. As a result, the change in debt that SP1 experiences in date 41 is more than twice as large as that of SP2.

The ranking of the price declines in SP2, SP1 and DEL (with SP2 smaller and DEL and SP1 larger) follows the ranking of the debt correction. Consistent with the sharp change in beliefs at date 41, having built a larger debt than SP2 and facing the same set of feasible credit positions as DEL, SP1 cannot avoid falling on the relatively steep portion of the pricing function, as plotted in panel (d) of Figure 3. In the region where SP1 chose debt levels, prices vary significantly across debt positions in the κ^l regime, leading to a large in decline in the asset price for SP1, as shown in Panel (b) of Figure 4. Notice also how the differences across the different equilibria shrink for all the macroeconomic series plotted in this figure towards the end of the time-series experiment, as the beliefs get closer to rational expectations.

Figure 5 shows the taxes on debt, τ_b, and dividends, τ_l, necessary to support each planners' allocations as DELs with taxes. In line with the above results showing that SP1's debt and land prices deviate only slightly from the outcomes of the DEL without taxes, SP1 makes limited use of these taxes. In the seven periods after financial innovation starts, it uses a debt tax of about 2-3 percent and a subsidy on dividends of up to 3 percent. After that, as the collateral constraint becomes binding for SP1, the debt tax drops to zero and the dividends rises to about 2 percent. In contrast, SP2 uses macro-prudential taxes more actively. SP2 increases debt taxes gradually from about 4 percent to 8 percent in the optimistic phase, and then cuts it to zero as the financial crisis erupts. Moreover, SP2 increases the subsidy on land dividends in the early stages of the optimistic phase, and then keeps it constant at about 5 percent until the crisis occurs, at which time the subsidy falls to almost zero.

Figure 6 decomposes the above tax policy dynamics in terms of the information, interaction, and externality terms. Since SP1 goes through the same learning process as private agents in the DEL without taxes, the information and interaction terms of the debt taxes for this planner are always zero. The externality term (which captures the expected value of the pecuniary externality in units of marginal utility using the DEL's beliefs) accounts for the full amount of SP1's debt taxes shown in Figure 5. This terms rises up to a maximum of about 3 percent, before vanishing after the seventh period. Again, the results that the externality tax term and the total debt tax itself are small are consistent with the finding that SP1's debt and land prices deviate slightly from those obtained in the DEL without taxes.

The debt taxes of SP2 are significantly higher than those of SP1, but not because of the externality component. In fact, the externality component remains relatively small for SP2.[21] In the early stages after financial innovation starts, this component is even smaller for SP2 than for SP1, but in contrast with SP1, the externality component remains slightly positive throughout the optimistic phase. The debt taxes of SP2 are higher because of large information and interaction components, which rise gradually during the early stages of the optimistic phase to stabilize at

[21] Interestingly, the size of the externality tax component is comparable in magnitude to the debt taxes estimated by Bianchi and Mendoza (2010) in a model with a similar collateral constraint but with a constant κ, production with labor and working capital financing of wages, and rational expectations formed with full information.

about 3.3 and 4.2 percent respectively. In contrast, both of these components are zero for SP1, as explained above.

3.3 Welfare Analysis

The welfare effects of the informational friction, the pecuniary externality and the use of macro-prudential policy can be quantified by computing compensating variations in terms of constant consumption levels that yield the same lifetime utility as the consumption allocations of DEL, SP1 and SP2. This is similar to the standard welfare analysis used in DSGE models, but we make modifications to take into account the time-varying nature of the value functions and decision rules pertaining to the AU optimization problems at each date $t = 0, ..., 48$. In particular, since the solution to each AU problem represents the "perceived" full solution of an infinite-horizon dynamic programming problem with the given set of beliefs, we construct a perceived lifetime welfare measure at each date t as follows:

$$V_t(b, \varepsilon, \kappa) = u(c_t(b, \varepsilon, \kappa)) + \beta E^i[V(b_t'(b, \varepsilon, \kappa), \varepsilon', \kappa')] \quad i = s, a$$

where $c_t(b, \varepsilon, \kappa)$ and $b_t'(b, \varepsilon, \kappa)$ are the decision rules for consumption and bonds for the date-t optimization problem (with a pair of these decision rules for DEL, SP1 and SP2). The expectation in the right-hand-side can be computed using either the true transition probabilities across financial regimes or the subjective beliefs. This procedure yields a welfare number $V_t(b, \varepsilon, \kappa)$ at each date t of the simulation for each triple in the state space (b, ε, κ) and for each of the three model economies. We then convert each welfare number into a constant consumption level that is equivalent in terms of lifetime utility (i.e. the value $\bar{c}$ that solves $V_t(b, \varepsilon, \kappa) = \sum_{t=0}^{\infty} \beta^t \frac{\bar{c}^{1-\sigma}}{1-\sigma}$).

Table 2 reports welfare effects as the percentage change in the welfare-equivalent consumption levels across the different economies for $t = 1$ and 40, computed using both the true transition probabilities and the subjective beliefs. Since we have a value of $\bar{c}$ for each triple in the state space in each economy, we report average welfare effects based on the perceived ergodic distribution of (b, ε, κ) at each date t in the DEL.[22] For comparison, we also report welfare effects fixing $b = b_1^{DEL}$ or b_{40}^{DEL} (i.e. the values of b along the simulated time series path of the DEL in Figure (1)) and

[22] Figure (7) in Boz and Mendoza (2010) plots the evolution of these perceived ergodic distributions over the 48 periods of the DEL simulation. As optimism increases, the ergodic distribution shifts sharply to the left, supporting large levels of debt at high probabilities, which otherwise would have zero probability in the DEF ergodic distribution.

Table 2: Welfare gains (in percentage)

	Average		$(b_t^{DEL}, \kappa_t, \varepsilon_t)$	
	$t = 1$	$t = 40$	$t = 1$	$t = 40$
True probabilities				
(1) SP2 versus DEF	0.052	0.05	0.06	0.07
(2) SP2 versus DEL	0.37	7.4	0.30	7.39
(3) SP1 versus DEL	0.17	0.03	0.17	0.03
Subjective beliefs				
(4) SP1 versus DEL	0.025	0.0	0.025	0.0
(5) DEL versus SP2	-0.39	-2.7	-0.27	-2.73

$\varepsilon_t = E[\varepsilon] = 1$ and $\kappa_t = \kappa^h$. The results are similar, suggesting that the aggregation using the DEL ergodic distributions is not biasing the analysis.

The largest welfare gains are obtained when comparing SP2 v. DEL using the true transition probabilities across financial regimes. In this case, the planner is internalizing the pecuniary externality and avoiding all the effects of the misperception of risk implied by optimistic beliefs. The latter include both the effects on the equilibrium allocations and the effects from underestimating the transition to κ^l in the expectation taken in the right-hand-side of $V_t(b, \varepsilon, \kappa)$. At date 1 this translates into an average welfare gain of about 1/3rd of a percent, and at the peak of optimism just before the crisis ($t = 40$), a gain of 7.4 percent. This is a very large gain relative to existing estimates in the DSGE literature on the cost of business cycles, the benefits of faster growth, or the benefits of fully eliminating tax distortions. Note also that since SP1 and DEL display nearly identical dynamics as they approach the peak of optimism, the welfare gains of SP2 v. SP1 at $t = 40$ would be the same as in the comparison SP2 v. DEL. This indicates that in the baseline experiment the large welfare gains obtained by SP2 are largely the result of removing the informational friction and the associated financial amplification mechanism (which is what the comparison of SP2 v. SP1 isolates, since both planners internalize the pecuniary externality).

The welfare gain of SP2 v. DEL is smaller if instead of computing welfare effects using the true probabilities in the expression for $V_t(b, \varepsilon, \kappa)$ above we use subjective beliefs, and take the measure as the ratio of DEL v. SP2. In this case, the losses for DEL are about -0.4 percent at $t = 1$ and -2.7 percent at $t = 40$. The welfare losses that result from comparing DEL against SP2 under subjective beliefs arise because in this case the probabilities used to compute expected utility under DEL are biased towards more positive outcomes. Notice that in absolute value, the $t = 1$ loss is about the same as the gain of SP2 over DEL based on true probabilities, but for $t = 40$ the loss based on subjective beliefs is about 1/3rd the size of the gain based on true probabilities. This is further indication of the large social cost of the informational friction and its financial implications, because it shows that keeping allocations the same and isolating only the effect of assigning the correct transition probabilities across $\kappa' s$, instead of underestimating significantly the likelihood of a κ^h-to-κ^l transition, results in a welfare gain of SP2 v. DEL that is 4.7 percentage points larger (7.4 gain under SP2 v. DEL with true probabilities, relative to the absolute value of the -2.7 percent loss of DEL v. SP2 with subjective beliefs). In qualitative terms, a similar message follows from comparing SP1 v. DEL using true probabilities against SP1 v. DEL using subjective beliefs. Again the welfare gains are larger when true probabilities are used to assess the risk of financial regime switches.

Finally, the welfare effects comparing SP2 v. DEF using the true transition probabilities, and SP1 v. DEL using subjective beliefs, isolate the benefits of internalizing the pecuniary externality alone, leaving the informational friction either absent (SP2 v. DEF) or with identical beliefs across private agents an the social planner (SP1 v. DEL). As reported in Bianchi and Mendoza (2010), this results in positive but modest welfare gains of up 0.052 percent.

3.4 Sensitivity Analysis

This sub-section conducts a sensitivity analysis to study how the parameterization of the initial priors affect baseline results. This is important because there is obviously a lot of uncertainty about the values of the initial priors, and as we indicated earlier, in this regard the baseline parameterization is more a benchmark to start the quantitative analysis than a calibration backed by robust empirical estimates. We will show in particular that the baseline result indicating that debt and land price dynamics of SP1 and DEL are similar, and hence that SP1's macro-prudential policy makes little difference during the optimistic phase, is not robust to alternative specifications of the priors.

We conduct two sets of sensitivity experiments altering the initial priors. In one we change the initial priors in the DEL and SP1 to induce a gradual buildup of optimism, and in the other we introduce heterogeneous priors between private agents and the social planner. In all of these experiments, the optimization problem of the social planner is analogous to that solved by SP1 in problem (13). Table 3 reports the values of the initial counters that characterize the initial

Table 3: Summary of Priors

	n_0^{hh}	n_0^{hl}	n_0^{ll}	n_0^{lh}
DEL & SP1				
Baseline	0.02	0.02	0.02	0.02
Gradual Optimism	7.6	0.4	0.38	0.02
SP2 & SP3	$\rightarrow\infty$	$\frac{n_0^{hh}(1-F_{hh}^a)}{F_{hh}^a}$	$\rightarrow\infty$	$\frac{n_0^{ll}(1-F_{ll}^a)}{F_{ll}^a}$
SP4	0.2	0.2	0.2	0.2

priors for these experiments, and Figure 7 plots the evolution of beliefs that corresponds to each experiment.

(a) Gradual Optimism

In this experiment, initial priors are still the same for the government and the private sector, but they are constructed so that optimism builds more gradually in the early stages of financial innovation than in the baseline. This is accomplished by setting the initial priors so that the date-0 posterior means are equal to the true transition probabilities and the initial counters are asymmetric across the four transitions. In particular, we assume that $n_0^{hh} = 7.6$, higher than in the baseline, but then set $n_0^{hl} = 0.4$ so that $E_0[F_{hh}^s] = F_{hh}^a = 0.95$, and we keep $n_0^{hl} = 0.02$ as in the baseline, but then set $n_0^{ll} = 0.38$ so that $E_0[F_{ll}^s] = F_{ll}^a = 0.95$. In this experiment, as Figure 7 shows, learning starts from $E_0[F_{hh}^s] = 0.95$ and rises gradually towards 1, while in the baseline it starts at $E_0[F_{hh}^s] = 0.5$ and jumps to 0.98 with just the first observation of κ^h (by contrast, the gradual optimism scenario reaches 0.98 after 12 observations of κ^h). Keep in mind also that while beliefs start at the true transition probabilities, neither private agents nor SP1 know that, and hence their beliefs shift away from the true probabilities as realizations of κ arrive, until they converge back to the true values in the long run.

Gradual optimism yields noticeably larger differences between the outcomes attained by DEL and SP1 (see Figure 8). In the run-up to the crisis, debt levels in DEL reach about 4 percentage points of GDP more than in SP1. Moreover, during the crash, asset prices are 16 percent higher for SP1 due to the lower leverage at the time of the crisis. Clearly, SP1 accumulates less debt during the transition phase which leads to a smaller crash at $t = 41$, and hence in this scenario macro-prudential policy is more effective even when both private agents and the government face the same learning problem and the same collateral pricing functions.

The key reason for the different results under baseline and gradual optimism is that in the baseline the combination of the rapid surge in optimism and the households' impatience leads them to borrow up to the limit, and attain a high shadow value from relaxing the collateral constraint. Since the benevolent planner also considers the high value assigned to current consumption by the households, it also decides to borrow up to the limit. In line with this reasoning, Figure 8 shows that the differences in bond positions between DEL and SP1 narrow as the optimistic phase progresses and the shadow value of relaxing the collateral constraint increases.

In general, we find that the more gradual is the build-up of optimism, the more the collateral constraint is likely to remain slack or be marginally binding during the optimistic phase, and the more effective is macro-prudential policy, even if the planner is as uninformed as private agents. The planner that has full information continues to be significantly more cautious, however, and hence implements more active macro-prudential policies.

The prices under the DEL and SP1 continue to be very similar because the DEL pricing function in the κ^h regime continues to be relatively flat (see Figure 9). The externality itself, however, is actually larger, because the pricing function is again very steep for the κ^l regime and the gradual buildup of optimism means that SP1 assigns higher probability to switching to this regime than it did in the baseline. Hence, SP1 levies larger debt taxes in this scenario than in the baseline (see Figure 10). In contrast, SP2 charges slightly lower debt taxes.

In terms of the components of the debt tax (Figure 11), gradual optimism reduces the information component for SP2 (recall it is always zero for SP1). The interaction term is also now smaller than baseline for SP2. By contrast, the externality component of the taxes rises sharply for both planners under the gradual optimism scenario, relative to the baseline. This is in line with the previous findings indicating that the gradual buildup of optimism enlarges the externality and creates more room for macro-prudential policy.

(b) Heterogeneous Priors between Government and Private Agents

In the experiments we study next the initial priors of the government and the private sector differ. This is interesting because heterogenous beliefs can be used to construct variants of the SP1 planner in which the government can be more or less optimistic than private agents about the new financial regime, and thus will have incentives to respond more or less forcefully with macro-prudential policy. From a technical standpoint, this is similar to applying the Hansen-Sargent risk-sensitivity operator to bias the date-0 priors of the government.[23]

We study two experiments with heterogenous priors. In both experiments we keep the initial priors of private agents as in the baseline DEL. In the first experiment, we modify SP1 to construct an extreme case in which the planner (now labeled SP3) has initial priors such that effectively the planner's beliefs have converged to the true transition probabilities. This occurs as the initial counters that represent the planner's priors go to infinity under the conditions that $\frac{p_0^{hl}}{p_0^{hh}} = \frac{F_{hl}^a}{F_{hh}^a}$ and $\frac{p_0^{lh}}{p_0^{ll}} = \frac{F_{lh}^a}{F_{ll}^a}$. The second experiment represents a social planner, SP4, who has initial priors given by $p_0^{hh} = p_0^{ll} = 0.2$. Here the priors remain symmetric, so that $E_0^p[F_{hh}] = E_0^p[F_{ll}] = 0.5$ as plotted in Figure 7. Recall also that both SP3 and SP4, like SP1, still have to value collateral using the land pricing functions of the DEL, which are influenced by the private agents' beliefs.

Consider first the results for SP3. SP3 is similar to the fully informed SP2 in that it assesses the correct probabilities of landing and remaining in states with good and bad credit regimes, so its incentives to build precautionary savings are stronger than SP1, as suggested by a comparison of Figures 1 and 12. Hence, SP3 chooses lower debt levels than SP1 and DEL during the optimistic phase (see Panel (a) of Figure 12). SP3 cannot, however, correct the agent's mispricing of collateral under the DEL's beliefs, and hence still allows larger debt positions than SP2.

Despite SP3 choosing significantly less debt than SP1, the land prices of the two are similar because both SP1 and SP3 use the same collateral pricing functions $q_t^{DEL}(b, 1, \kappa)$, *and* because these pricing functions have a flat slope in the κ^h state. In particular, since $\frac{\partial q_t^{DEL}(b', 1, \kappa^h)}{\partial b'}$ is small for SP1 and SP3 for $t = 1, ..., 40$, their different choice of bonds translates into small differences in land prices. If the pricing functions were steeper at the optimal debt choices, the equilibrium dynamics of land prices would be different even though both SP1 and SP3 use the same pricing functions, because the lower debt levels chosen by SP3 would imply different date-t prices picked from the same pricing function (i.e. the same $q_t^{DEL}(b', 1, \kappa^h)$ would return different prices for each planner because of the different choices of b').

SP3 actively uses macro-prudential taxes as shown in the top panel of Figure 13. Taxes on debt increase gradually from about 4 percent to close to 9 percent in the optimistic phase, and then drop to zero as the financial crisis erupts and the collateral constraint binds. Comparing Figures 6 and 13, SP3 and SP2's dividends tax policies are qualitatively similar, with subsidies that increase

[23] Cogley and Sargent (2008a) used this approach to bias downard beliefs about consumption growth at the end of the Great Depression so as to support large equity premia in their learning asset pricing model.

gradually during the optimistic phase, but quantitatively SP2 uses smaller subsidies, because SP2 aims to support the DEF asset pricing functions, which are uniformly lower than the DEL pricing functions supported by SP3. Thus, SP2 taxes debt just as much as SP3 to weaken the incentives of private agents to borrow, but subsidizes land dividends less to deflate the effect of optimistic beliefs on land prices.

The dynamics of the tax components show that the externality component remains small for SP3, but the information and interaction components are large and rise rise gradually during the optimistic phase. Interestingly, SP3 displays a lower information component than SP2, along with a higher interaction component. The higher information term for SP2 is due to the fact that consumption booms less under this planner than under SP3, which results in higher expected marginal utility. Given that the externality components for SP2 and SP3 are similar, it follows from Equation (22) that the higher interaction term for SP3 is due to the fact that this planner has higher expected externality terms ($E_t^{SP2}\left[\kappa' \mu(b', \varepsilon', \kappa') \frac{\partial q_t^{DEL}(.)}{\partial b'}\right] > E_t^{SP3}\left[\kappa' \mu(b', \varepsilon', \kappa') \frac{\partial q_t^{DEF}(.)}{\partial b'}\right]$), which in turn result from the steeper DEL collateral pricing functions in the κ^l regime than in the DEF (see Figure 2) and the fact that both SP2 and SP3 assign more weight to κ^l using the true Markov-switching probabilities across financial regimes.

Now we turn to the experiment for SP4. SP4 has higher initial counters for the persistence of each regime than DEL or SP1, which alters significantly the perception of the riskiness of the new financial environment. For instance, at date $t = 1$ after the first realization of κ^h is observed, SP4 expects the mean duration of the κ^h regime to be about 6 quarters while the private agents and SP1 expect a mean duration of 50 quarters under the baseline calibration of $n_0^{hh} = 0.0205$. Hence, SP4 perceives more riskiness in the financial environment inasmuch as it believes the mean duration of the good credit regime will be significantly shorter. Moreover, this scenario has a feature similar to the gradual optimism experiment because optimism builds more gradually for SP4 than for the DEL and SP1. The evolution of the mean beliefs under SP4 and the DEL are plotted in Panels (f) and (d) of Figure 12, respectively.

Panel (a) of Figure 12 reveals that introducing asymmetric beliefs to make SP4 perceive more risk once again results in an outcome in which a planner subject to learning and facing the same collateral prices of the DEL chooses lower debt positions than in the DEL. In fact, SP4's debt levels are also lower than those of SP1 plotted in Figure 1. [24] This is because of the combination

[24] The comparison of SP4 with SP1 is very relevant because both of them go through a learning process and face the same collateral pricing function.

of the higher perception of risk, the more gradual buildup of optimism, and the fact that under the influence of these forces the collateral constraint is not binding for SP4 under the entire optimistic phase. Given a uniformly higher externality term, as plotted in Panel (e), and uniformly less optimistic beliefs than SP1, SP4 chooses lower debt levels, and not up to the point where the constraint binds. In fact, SP4 only hits the borrowing limit when the economy switches to the κ^l state. This is evident in the shadow price being almost always zero in Panel (c) of Figure 12 except in period 41 and the last few periods of the experiment.

It is also interesting to note in Panel (e) of Figures 12 and 1 that the dynamics of the externality term have similar shapes for SP4 and SP1. The main difference is in that the levels are uniformly higher for SP4. Similar forces are at play for these planners, initially the fast buildup of debt relative to the buildup of optimism increases the probability assigned to the constraint becoming binding at date $t+1$. After about period 10, the buildup of debt slows down and this effect is dominated by the beliefs becoming more optimistic over time leading to a weakening of the externality as these planners assign smaller probabilities to a switch to κ^l, where the derivative of the pricing function, $\frac{\partial q_t^{DEL}(b',1,\kappa^l)}{\partial b'}$, is large. The externality term is uniformly larger for SP4 than SP1 because its beliefs are uniformly less optimistic. SP4 always assigns a higher weight to the κ^l regime where the derivative of the pricing function, $\frac{\partial q_t^{DEL}(b',1,\kappa^l)}{\partial b'}$ is large.

The price dynamics of SP4 are very similar to those of DEL. The lower debt choices of this planner do not translate into large differences in the price given the flatness of the pricing function.[25] In fact, SP4 prices are slightly above those of DEL since lower debt positions are associated with higher land prices.

Consistent with the externality being large, SP4 levies taxes on debt that are higher than in the baseline and also higher than in the gradual optimism scenario. Moreover, for SP4 the interaction component of the debt tax is the largest almost throughout the entire experiment, as was the case for SP2 and SP3 in the baseline. Hence, our finding that the interaction of financial and information frictions play a key role in the design of macro-prudential policy remains robust to considering a social planner with beliefs different from those of private agents or from the true rational expectations, and this was also the case in the gradual optimism scenario.

[25]Since DEL prices do not change when we change the priors of the planner, we do not re-plot them here.

4 Conclusion

This paper provides a quantitative dynamic stochastic general equilibrium framework for studying macro-prudential policy that incorporates two key elements of the financial amplification mechanism: Imperfect information about the true riskiness of new financial regimes and a credit constraint that limits the debt of agents to a fraction of the market value of their assets. The fraction of the value of assets that can be pledged as collateral increases with financial innovation, but risk also increases because this collateral coefficient also becomes stochastic, and the persistence of regimes with high and low ability to borrow needs to be learned over time. As learning progresses, agents go through waves of optimism and pessimism which distort their debt decisions and hence equilibrium asset prices. In addition, the credit constraint introduces a pecuniary externality whereby individual agents do not internalize the effect of their borrowing decisions on equilibrium prices. The interaction of waves of optimistic and pessimistic beliefs with this pecuniary externality produces a powerful amplification mechanism that can yield large increases in debt and asset prices in a decentralized competitive equilibrium.

We study the effects of macroprudential policies in the form of Pigouvian taxes on debt and dividends in this environment, considering two conditionally efficient social planner problems that face different information sets and feasible credit positions. The first planner faces a similar learning problem as private agents and faces the collateral pricing function of the decentralized competitive equilibrium with learning (i.e., the same set of feasible credit positions as private agents). The second planner has full information and in addition it faces the collateral pricing function of a rational expectations equilibrium with full information (i.e. the set of feasible credit positions of private agents in this equilibrium).

In a baseline calibration to U.S. data, the second social planner supports debt positions and land prices that are much lower than those in the decentralized competitive equilibrium with learning, and hence faces smaller corrections in debt, consumption and land prices when financial crises hit. In contrast, the first planner supports allocations and prices that deviate only slightly from those of the DEL. Thus, in the baseline parameterization, macro-prudential policy is significantly more effective when the planner has full information and can support collateral values free from the effect of optimistic beliefs, and has negligible effects when the planner is subject to the same subjective beliefs and collateral pricing conditions of the DEL. Sensitivity analysis shows, however, that by varying the initial priors of the learning setup, particularly by modifying them so as to induce a gradual buildup of optimism in the early stages of financial innovation or to introduce heterogeneous priors between the social planner and private agents, it is possible even for the first planner to use macro-prudential policy to attain different equilibrium debt than DEL, and thus improve the performance of the economy during financial crises.

Under our baseline parameterization, we find large welfare losses from the undervaluation of risk during credit expansions and the resulting collapse during the reversal of financial conditions. This suggests that there is a key interaction between perception of risk, and the externality introduced by the systemic feedback loop between asset prices and collateral constraints. The welfare gains from correcting purely the pecuniary externality are much smaller.

These results highlight the importance of considering the information set of policymakers in the design of macro-prudential policies. If regulators operate with the same incomplete information set as the private agents, the effects of these policies are more limited and can even be negligible. This is particularly important in a boom-bust cycle in credit largely driven by financial innovation, about which the regulators are likely to be just as uninformed as the private agents. If on the other hand, in a credit boom episode where the private agents operate under incomplete or misleading information while the regulators can acquire better information, say by looking at similar previous episodes in the history of the country or other countries in similar situations, then macro-prudential policy has good potential to contain the amplitude of the boom-bust cycle.

One important aspect that we have not considered in our analysis is belief heterogeneity within the private sector. As proposed by Geanakoplos (2010) and more recently investigated in Cao (2011) and Simsek (2010), this can generate credit cycles with important effects over asset price volatility and investment volatility. The connection between financial innovation and Fisherian deflation in such a framework can shed further light on the effectiveness of macro-prudential policy.

5 Appendix: Recursive Optimization Problems

We assume that agents make decisions according to the anticipated utility approach. Accordingly, the recursive optimization problem can be written as

$$
\begin{aligned}
V_t(b, k, B, \varepsilon) &= \max_{b', k', c} \frac{c^{1-\sigma}}{1-\sigma} + \beta E_t^s \left[V_{t+1}(b', k', B', \varepsilon') \right] \qquad (23) \\
s.t. \quad q(B, \varepsilon) k' + c + \frac{b'}{R} &= q(B, \varepsilon) k + b + \varepsilon F(k) \\
B' &= \Gamma(B, \varepsilon) \\
\frac{b'}{R} &\leq \kappa q(B, \varepsilon) k'
\end{aligned}
$$

Notice that the value function is indexed by t because beliefs are changing over time. In rational expectations, instead, the value function would be a time-invariant function of the individual and aggregate state variables.

Definition: The *(AU) recursive competitive equilibrium* is defined by a subjective conditional-expectation operator E_t^s, an asset pricing function $q_t(B, \varepsilon)$, a perceived law of motion for aggregate bond holdings $\Gamma_t(B, \varepsilon)$, and a set of decision rules $\left\{ \hat{b}_t'(b, k, B, \varepsilon), \hat{k}_t'(b, k, B, \varepsilon), \hat{c}_t(b, k, B, \varepsilon) \right\}$ with associated value function $V_t(b, k, B, \varepsilon)$ such that:

1. $\left\{ \hat{b}_t'(b, k, B, \varepsilon), \hat{k}_t'(b, k, B, \varepsilon), \hat{c}_t(b, k, B, \varepsilon)) \right\}$ *and* $V_t(b, k, B, \varepsilon)$ *solve (23), taking as given* $q_t(B, \varepsilon)$, $\Gamma_t(B, \varepsilon)$.

2. *The perceived law of motion for aggregate bonds is consistent with the actual law of motion:* $\Gamma_t(B, \varepsilon) = \hat{b}_t'(B, \bar{K}, B, \varepsilon)$.

3. Land prices satisfy $q(B, \varepsilon) = E_{\varepsilon'|\varepsilon} \left\{ \frac{\beta u'\left(\hat{c}(\Gamma(B,\varepsilon), \bar{K}, \Gamma(B,\varepsilon), \varepsilon')\right) \left[\varepsilon' F_k(\bar{K}, \varepsilon') + q(\Gamma_t(B,\varepsilon), \varepsilon')\right]}{u'\left(\hat{c}(B, \bar{K}, B, \varepsilon)\right) - \kappa \max\left[0, u'\left(\hat{c}(B, \bar{K}, B, \varepsilon)\right) - \beta R E_{\varepsilon'|\varepsilon} u'(\hat{c}(\Gamma(B,\varepsilon), \bar{K}, \Gamma(B,\varepsilon), \varepsilon')\right]} \right\}$

4. Goods and asset *markets clear:* $\frac{\hat{b}'(B, \bar{K}, B, \varepsilon)}{R} + c(B, \bar{K}, B, \varepsilon) = \varepsilon f(\bar{K}) + B_t$ and $\hat{k}(B, \bar{K}, B, \varepsilon) = \bar{K}$

References

Aiyagari, R., and M. Gertler (1999): "Overreaction of Asset Prices in General Equilibrium," *Review of Economic Dynamics*, 2(1), 3–35.

Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E. Young (2010): "Financial Crises and Macro-Prudential Policy," Mimeo, University of Virginia.

Bernanke, B., M. Gertler, and S. Gilchrist (1999): "The financial accelerator in a quantitative business cycle model," in *Handbook of Macroeconomics*, ed. by J. Taylor, and M. Woodford, vol. 1C. by North-Holland.

Bianchi, J. (2011): "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, 101(7), 3400–3426.

Bianchi, J., and E. G. Mendoza (2010): "Overborrowing, Financial Crises and 'Macroprudential' Policy," NBER Working Paper No. 16091.

Boz, E., and E. G. Mendoza (2010): "Financial Innovation, the Discovery of Risk, and the U.S. Credit Crisis," NBER Working Paper 16020.

Cao, D. (2011): "Collateral Shortages, Asset Price and Investment Volatility with Heterogeneous Beliefs," mimeo, Georgetown University.

Cogley, T., and T. Sargent (2008a): "The market price of risk and the equity premium: A legacy of the Great Depression?," *Journal of Monetary Economics*, 55(3), 454–476.

Cogley, T., and T. J. Sargent (2008b): "Anticipated Utility and Rational Expectations as Approximations of Bayesian Decision Making," *International Economic Review*, 49, 185–221.

Davis, M., and J. Heathcote (2007): "The price and quantity of residential land in the United States," *Journal of Monetary Economics*, 54(8), 2595–2620.

Fisher, I. (1933): "The debt-deflation theory of great depressions," *Econometrica*, 1(4), 337–357.

Geanakoplos, J. (2010): "The leverage cycle," *NBER Macro-economics Annual 2009*, University of Chicago Press, Chicago, 2010, D. Acemoglu, K. Rogoff, and M. Woodford, eds.

Gennaioli, N., and A. Shleifer (2010): "What comes to mind," *The Quarterly Journal of Economics*, 125(4), 1399.

Gennaioli, N., A. Shleifer, and R. Vishny (2010): "Neglected risks, financial innovation, and financial fragility," Journal of Financial Economics.

Jeanne, O., and A. Korinek (2010): "Managing Credit Booms and Busts: A Pigouvian Taxation Approach," Discussion paper, NBER Working Paper.

Kehoe, T. J., and D. Levine (1993): "Debt-Constrained Asset Markets," *Review of Economic Studies*, 60, 865–888.

Kiyotaki, N., and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105, 211–248.

Korinek, A. (2010): "Systemic Risk-Taking: Accelerator Effects, Externalities, and Regulatory," Mimeo, University of Maryland.

Lorenzoni, G. (2008): "Inefficient Credit Booms," *Review of Economic Studies*, 75, 809–833.

Lustig, H. (2000): "Secured lending and asset prices," mimeo, Stanford University.

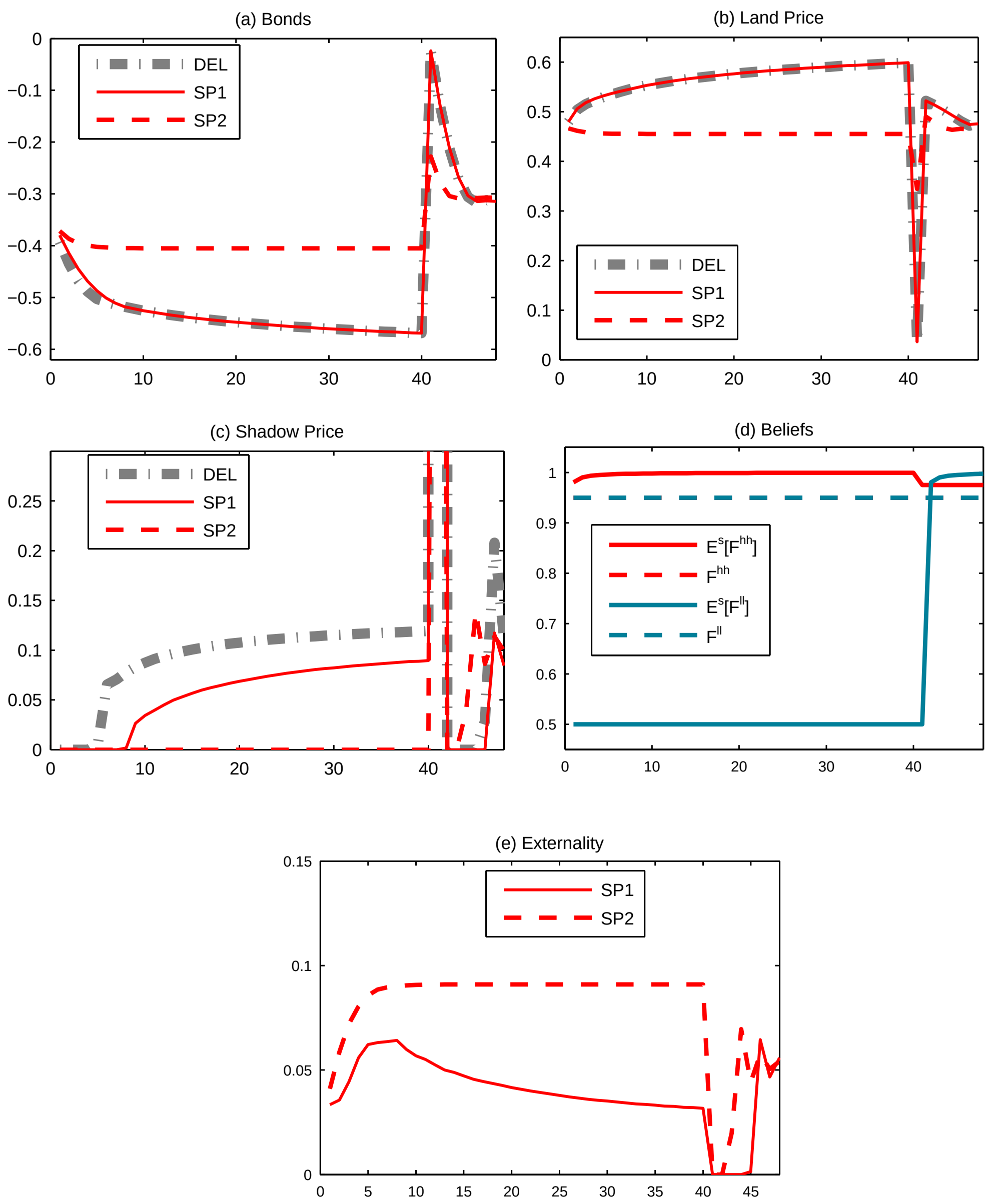
Mendoza, E. G., and M. E. Terrones (2008): "An Anatomy Of Credit Booms: Evidence From Macro Aggregates And Micro Data," NBER Working Paper 14049.

Minsky, H. (1992): "The financial instability hypothesis," *The Jerome Levy Economics Institute Working Paper No. 74*.

Simsek, A. (2010): "Belief disagreements and collateral constraints," Mimeo, Harvard University.

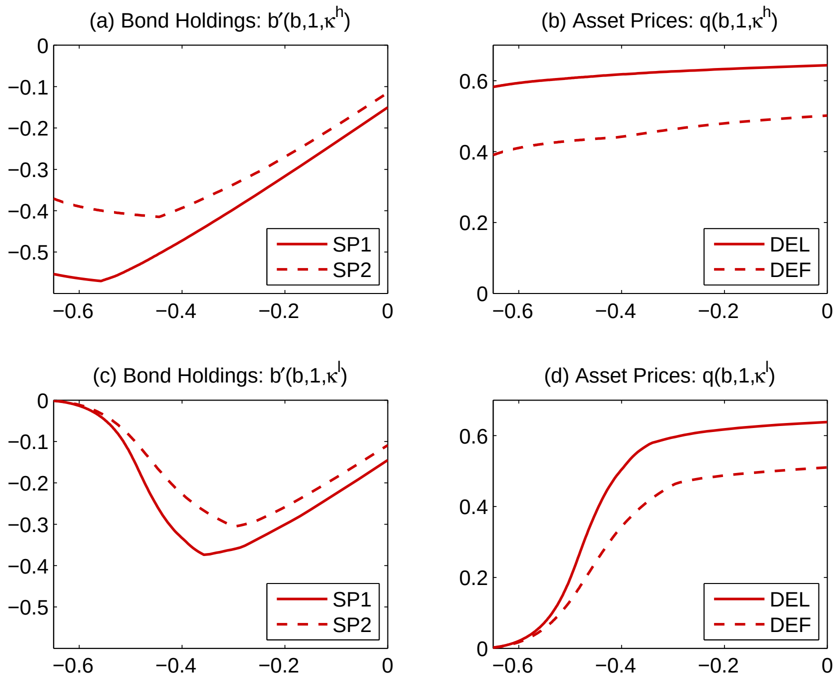
Stein, J. (2011): "Monetary policy as financial-stability regulation," Discussion paper, NBER Working Paper.

Figure 1: Dynamics in the Baseline Calibration



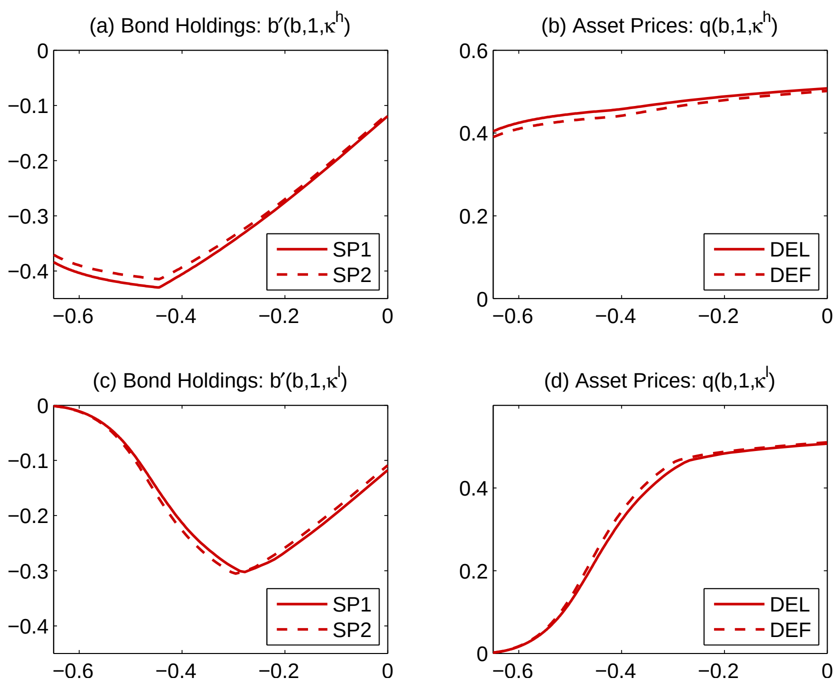
Notes: DEL: Imperfect information decentralized equilibrium, SP1: Social planner with imperfect information implementing the set of feasible credit positions of DEL, SP2: Social planner with full information implementing the set of feasible credit positions of DEF.

Figure 2: Period 40 Bond Holdings and Asset Prices



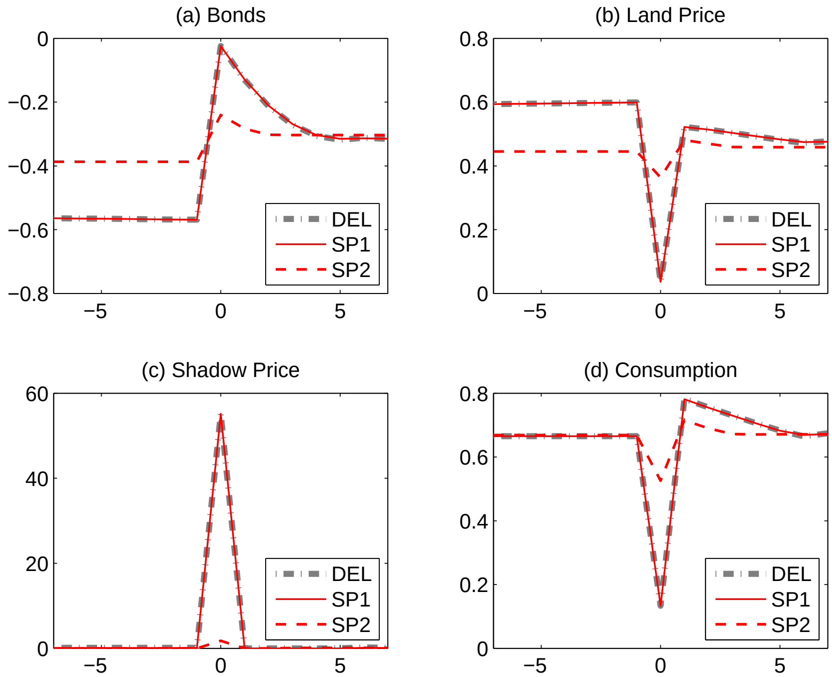
Notes: SP1: Social planner with imperfect information implementing the set of feasible credit positions of imperfect information decentralized equilibrium, SP2: Social planner with full information implementing the set of feasible credit positions of full information decentralized equilibrium.

Figure 3: Period 41 Bond Holdings and Asset Prices



Notes: SP1: Social planner with imperfect information implementing the set of feasible credit positions of imperfect information decentralized equilibrium, SP2: Social planner with full information implementing the set of feasible credit positions of full information decentralized equilibrium.

Figure 4: Crisis Episode

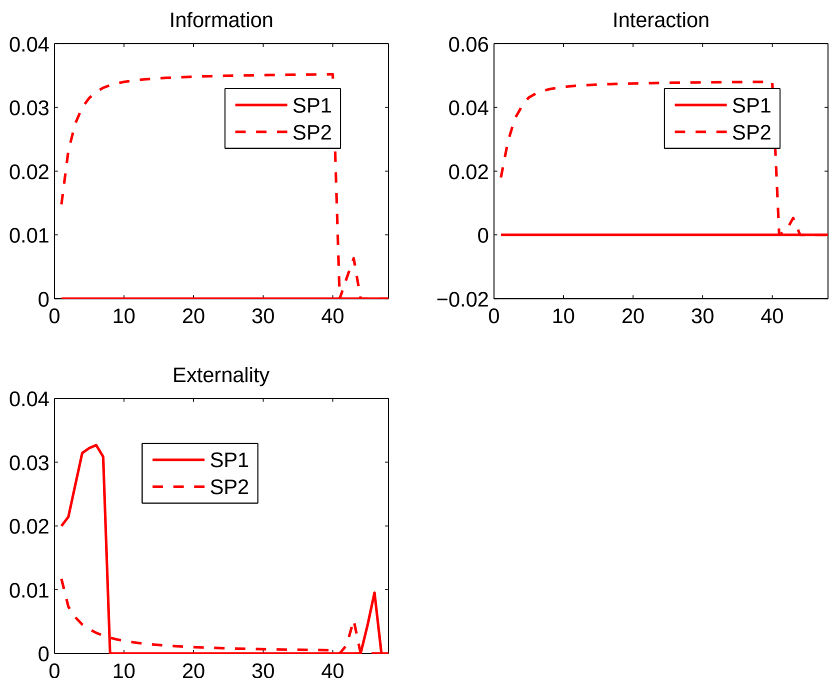


Notes: This figure plots the time series dynamics in periods 41 ± 7. DEL: Imperfect information decentralized equilibrium, SP1: Social planner with imperfect information implementing the set of feasible credit positions of DEL, SP2: Social planner with full information implementing the set of feasible credit positions of DEF.

Figure 5: Taxes on Debt and Land Dividends

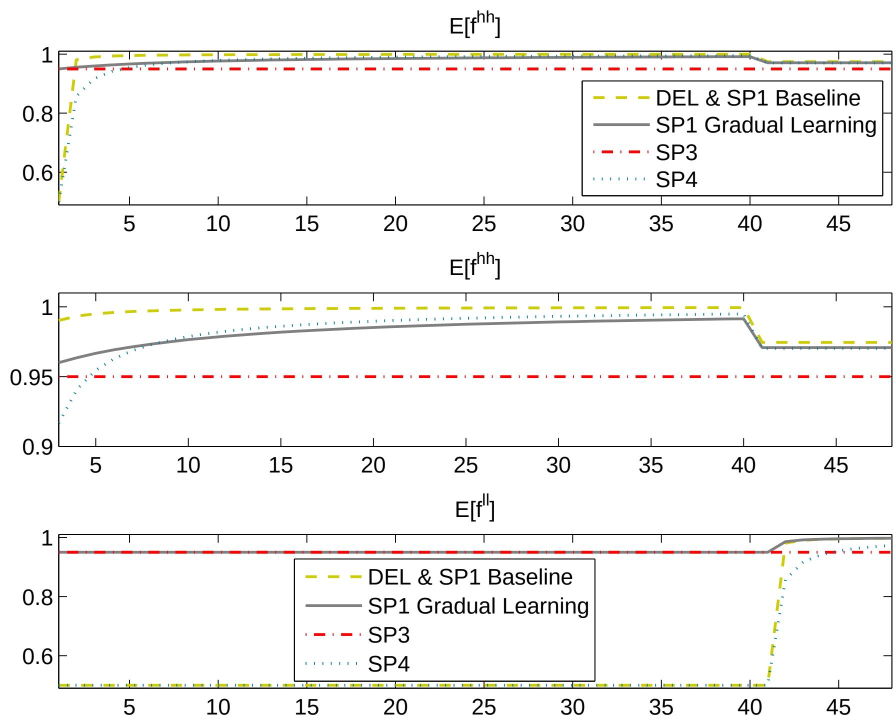
Notes: This figure plots the taxes on debt and on land dividends that support the corresponding planners allocations as competitive equilibrium. SP1: Social planner with imperfect information implementing the set of feasible credit positions of DEL, SP2: Social planner with full information implementing the set of feasible credit positions of DEF.

Figure 6: Decomposition of Taxes on Debt



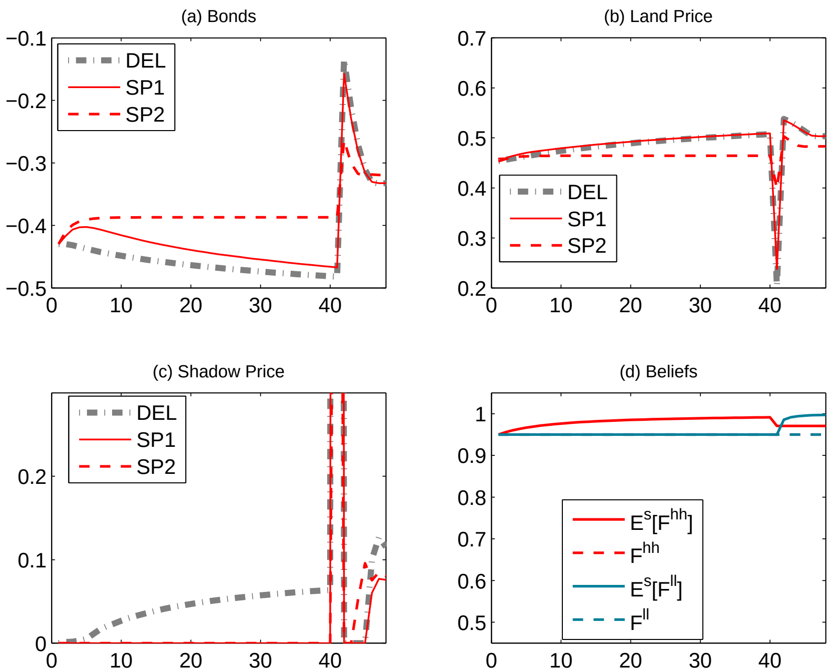
Notes: This figure plots the decomposition of taxes on debt to three distinct parts: 'information' arises due to the differences in the expectation of one period ahead consumption between private agents and the social planner, 'externality' captures the pecuniary externality, 'interaction' is due to the differences in the expectation of the one period ahead externality between private agents and the social planner. SP1: Social planner with imperfect information implementing the set of feasible credit positions of DEL, SP2: Social planner with full information implementing the set of feasible credit positions of DEF.

Figure 7: Priors



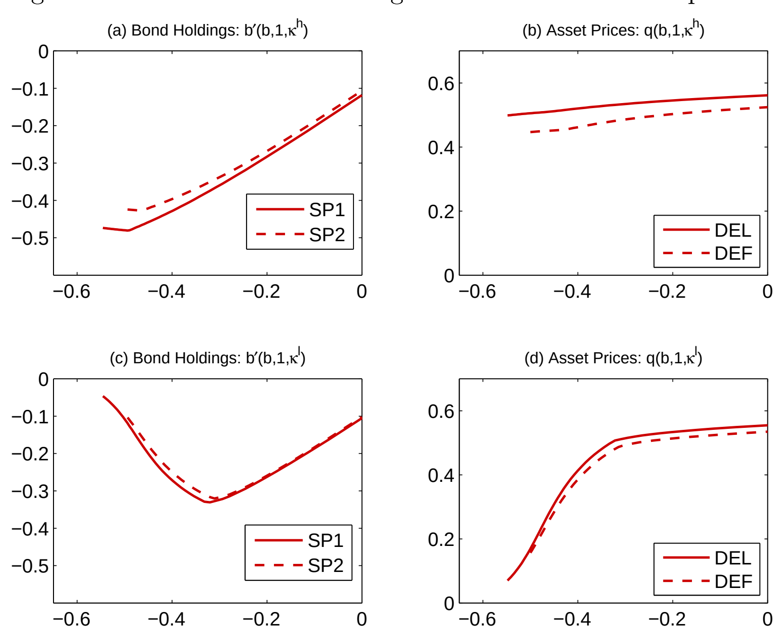
Notes: DEL: Imperfect information decentralized equilibrium, SP1: Social planner with imperfect information implementing the set of feasible credit positions of DEL, Gradual Learning: Scenario with priors such that optimism builds more gradually in the early stages of financial innovation than in the baseline. SP3: Social planner with full information implementing the set of feasible credit positions of DEF. SP4: Social planner with imperfect information and different priors than private agents implementing the set of feasible credit positions of DEL.

Figure 8: Dynamics in Gradual Optimism Calibration



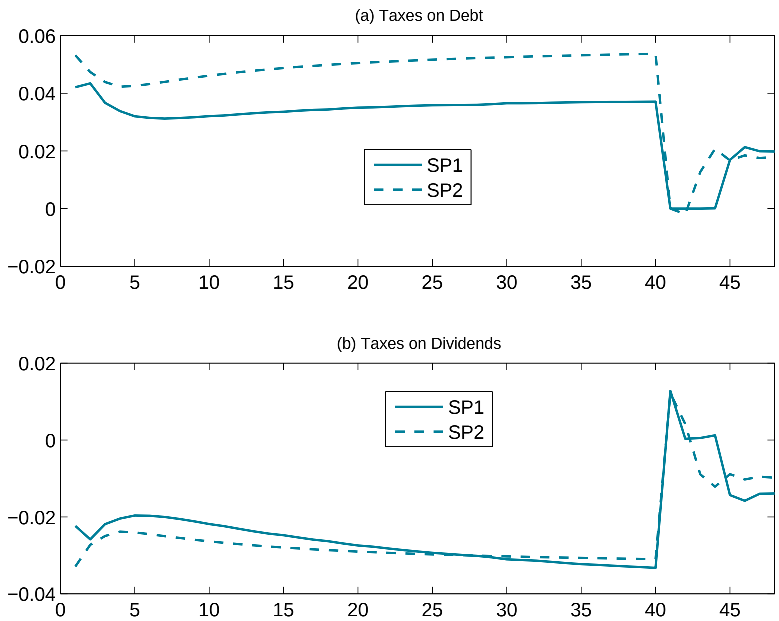
Notes: DEL: Imperfect information decentralized equilibrium, SP1: Social planner with imperfect information implementing the set of feasible credit positions of DEL, SP2: Social planner with full information implementing the set of feasible credit positions of DEF.

Figure 9: Period 40 Bond Holdings and Prices: Gradual Optimism



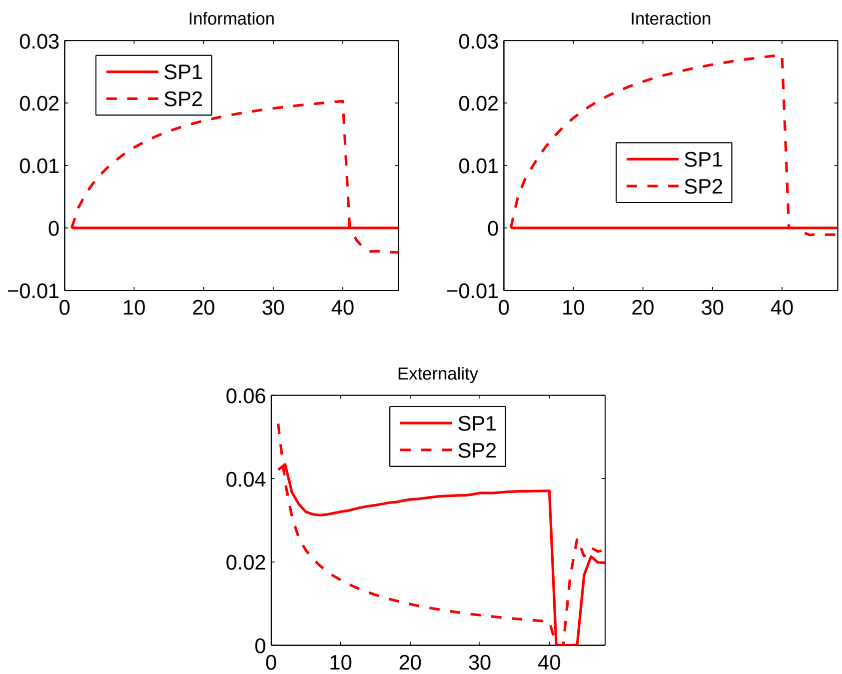
Notes: SP1: Social planner with imperfect information implementing the set of feasible credit positions of imperfect information decentralized equilibrium, SP2: Social planner with full information implementing the set of feasible credit positions of full information decentralized equilibrium.

Figure 10: Taxes on Debt and Land Dividends: Gradual Optimism



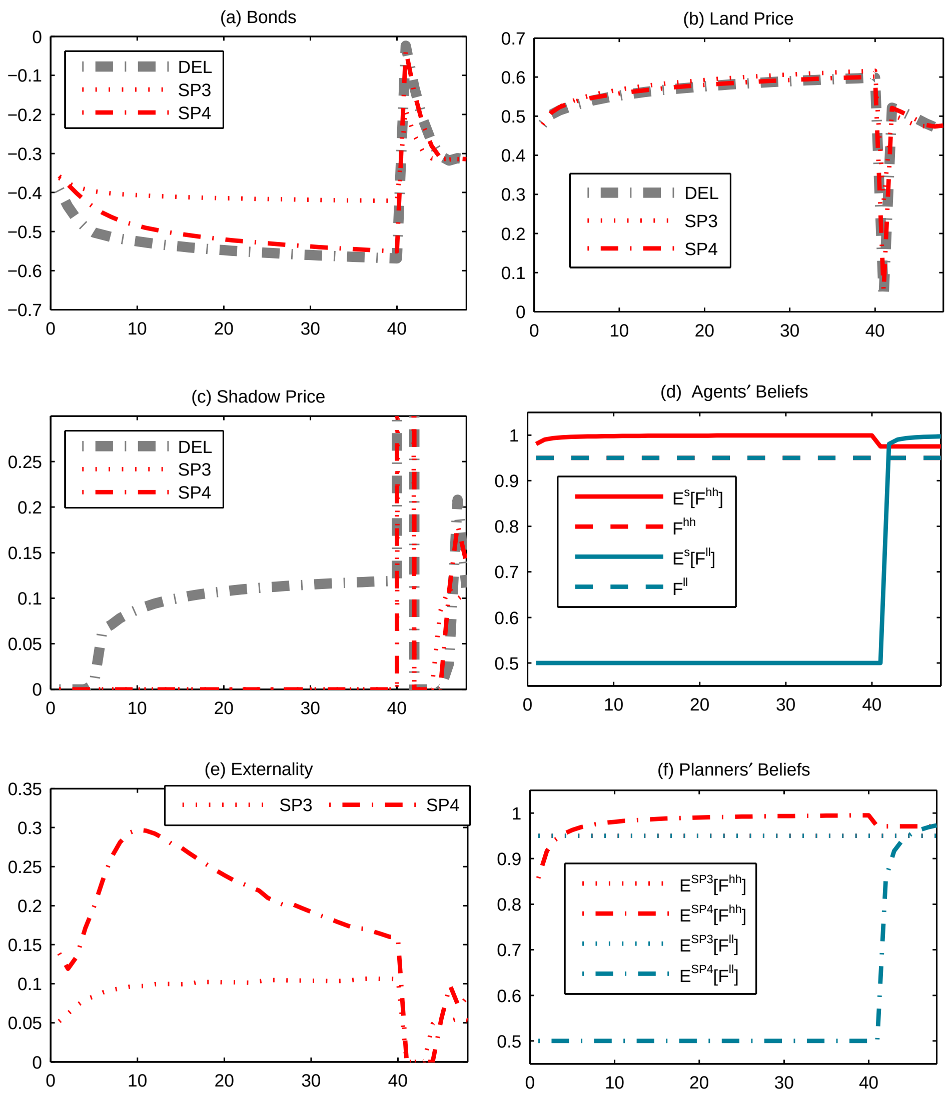
Notes: This figure plots the taxes on debt and on land dividends that support the corresponding planners allocations as competitive equilibrium. SP1: Social planner with imperfect information implementing the set of feasible credit positions of DEL, SP2: Social planner with full information implementing the set of feasible credit positions of DEF.

Figure 11: Decomposition of Taxes on Debt: Gradual Optimism



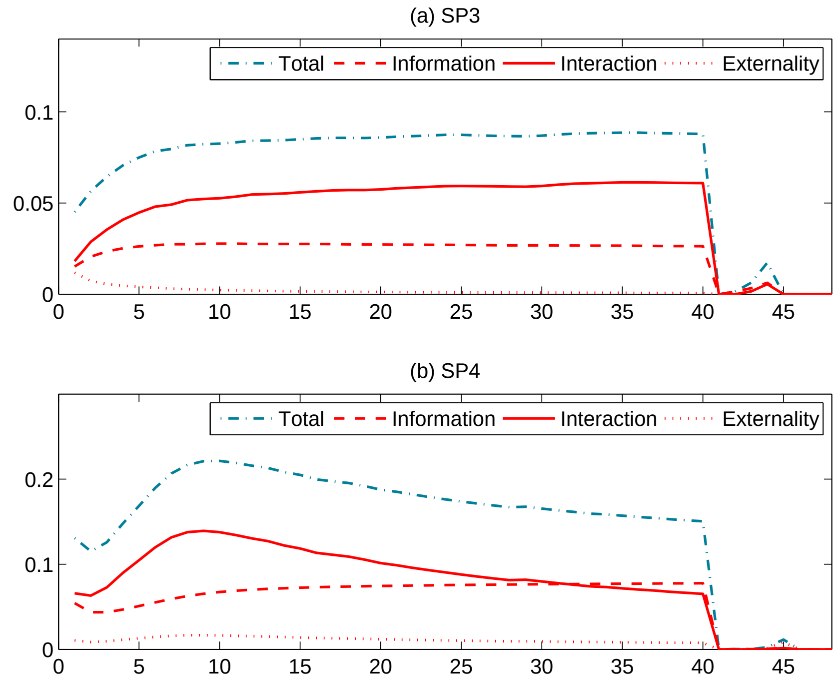
Notes: This figure plots the decomposition of taxes on debt to three distinct parts: 'information' arises due to the differences in the expectation of one period ahead consumption between private agents and the social planner, 'externality' captures the pecuniary externality, 'interaction' is due to the differences in the expectation of the one period ahead externality between private agents and the social planner. SP1: Social planner with imperfect information implementing the set of feasible credit positions of DEL, SP2: Social planner with full information implementing the set of feasible credit positions of DEF.

Figure 12: Dynamics in Asymmetric Priors Calibration



Notes: DEL: Imperfect information decentralized equilibrium, SP3: Social planner with full information implementing the set of feasible credit positions of DEL, SP4: Social planner with imperfect information and different priors than private agents implementing the set of feasible credit positions of DEL.

Figure 13: Taxes on Debt: Asymmetric Priors



Notes: This figure plots the taxes on debt that support the corresponding planners allocations as competitive equilibrium for SP3 and SP4. (SP3: Social planner with full information implementing the set of feasible credit positions of DEL, SP4: Social planner with imperfect information and different priors than private agents implementing the set of feasible credit positions of DEL.) Taxes are decomposed into three distinct parts: 'information' arises due to the differences in the expectation of one period ahead consumption between private agents and the social planner, 'externality' captures the pecuniary externality, 'interaction' is due to the differences in the expectation of the one period ahead externality between private agents and the social planner.