

INTRAHOUSEHOLD RISK SHARING, INCENTIVES AND NUTRITION: EVIDENCE FROM THE PHILIPPINES

PIERRE DUBOIS AND ETHAN LIGON

ABSTRACT. Using data on individual consumption and earnings from the Philippines, we test for efficient or constrained-efficient risk sharing within the household by constructing a dynamic model of intrahousehold food allocation under uncertainty distinguishing between the nutrients supplied by a particular food bundle and the quality of that bundle. We characterize constrained-efficient outcomes when there is private information on household members activities. We show that when nutrition affects only contemporaneous utility then under rather general conditions food bundles of identical quality to all members of the household are optimal even when household members have private information about their actions. In this case the quantity of food may provide incentives, but quality remains common within the household even if individuals are imperfectly insured in "quantity" against income shocks because of the trade-off with incentives. However, if nutrition affects future productivity, then, when the household invests more in the nutrition of one member it will simultaneously reduce the quality of her food bundle. We find evidence that individual consumptions respond to individual earnings shocks both because of nutritional investment and incentives.

PIERRE DUBOIS, TOULOUSE SCHOOL OF ECONOMICS

ETHAN LIGON, UNIVERSITY OF CALIFORNIA, BERKELEY

E-mail addresses: pierre.dubois@tse-fr.eu, ligon@berkeley.edu.

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1. INTRODUCTION

In recent years, a variety of authors have sought to test the hypothesis that intra-household allocations are efficient. Often these have been construed as tests of the “collective household” model. Special cases of this model are associated with Samuelson (1956) and Becker (1974); more recent formulations are associated with work by McElroy, Chiappori, and others (e.g. McElroy, 1990; Browning and Chiappori, 1998; Chiappori, 1992; Mazzocco, 2007; Bourguignon et al., 2009). Simultaneously, following Mace (1991), Cochrane (1991) and Townsend (1994), the hypothesis of complete markets has been tested extensively on household consumption data from developing countries as well as developed economies. Most tests of full risk sharing usually address the question of risk sharing across households and reject the hypothesis. Asymmetric information (imperfect or incomplete) and limited commitment have been considered as the potential sources of incompleteness of markets, by Ligon (1998), Ligon et al. (2002), Dubois et al. (2008), Bold (2009), Kinnan (2010) among others. However, as “markets” may be incomplete *within* the household, leading to rejections of full risk sharing across households. Thus, a natural extension is to test whether full risk sharing holds at least within the household. Dercon and Krishnan (2000) test the hypothesis of full intra-household risk-sharing in Ethiopia by looking at the response of individual nutritional status to illness shocks. They assume that utility depends on food consumption only via anthropometric status and reject intra-household efficient risk sharing, at least for poorer households. We can relax this assumption and use data on individual level food allocation and individual earnings to better understand the process which determines the allocation of food within households in the rural Philippines. In Dubois and Ligon (2011) we already extend the tests of full risk sharing across households to tests within the households and find that an individual’s allocation of household food expenditures respond to individual-level variations in earnings. This means that we can reject at least the full risk sharing implications of static efficiency within the household since idiosyncratic shocks to earnings would be insured away, and individual level consumption shares would remain constant. Full intra-household efficiency implies both *productive* efficiency, as well as *allocational* efficiency. Most previous literature has tested one or another of these (though see Rangel and Thomas, 2006, for tests of both). Udry (1996), for example, focuses on productive efficiency, while a much larger number of authors have focused on allocational efficiency (e.g., Thomas, 1990; Lundberg et al., 1997; Browning and Chiappori, 1998). One important difficulty involved in testing intra-household allocational efficiency is that intra-household allocations are seldom observed and most data would consist of

household-level consumption. In this paper we exploit a carefully collected dataset which records expenditures for each individual within a household, and thus are able to conduct a first direct test of intra-household efficient or full risk sharing. In Dubois and Ligon (2011), assuming utility is additively separable across broad categories of consumption goods, we test allocational efficiency which implies that the marginal rate of substitution between any two commodities will be equated across household members. Contrary to Dubois and Ligon (2011), we do not use marginal rates of substitutions between commodities but allow the consumption from utility of individuals to depend on two endogenous attributes: the quantity and the quality of consumption. Using this more general utility model, we extend the full risk sharing test allowing to take into account the individual marginal returns to nutritional investment as well as asymmetric information among households members.

In this paper we make an attempt to sort out two different hypotheses which could explain the empirical evidence of sensitivity of individual consumptions to individual earnings, offering an alternative to the simpler full risk sharing hypothesis with no investment value for consumption. The first one is what we will call the hypothesis of nutritional investment: the household allocates more food to some individuals because the returns to this kind of nutritional investment are higher than for other individuals. If, for example, one family member has the opportunity to earn more by ploughing others' fields (a task which requires both strength and energy) then he may receive both more calories and protein than other family members engaged in less strenuous and remunerative tasks. The second hypothesis is based on asymmetric information within the household and states that food is used to provide incentives because there are hidden actions taken by family members. For example, when one goes to work in another's field his efforts may be unobserved by the other household members. Under this hypothesis, a family member who brings home higher than usual earnings may be rewarded with more food. These two ideas were already introduced under the terms of efficiency wages, either with a moral hazard argument (Shapiro and Stiglitz (1984)) or a nutritional investment argument on future productivity (Bliss and Stern (Bliss and Stern); Deollikar (1988)) in agrarian poor developing countries contexts. Our approach to distinguishing between these two hypotheses involves developing a model in which food is characterized both by its quantity and its quality providing some "efficient" consumption units. The consumption quantity will be the only consumption dimension which affects future nutritional status and thus will correspond to the amount of calories or proteins consumed because we assume that what matters for future productivity is only nutrition through calories or proteins

and not whether these calories or proteins come from vegetables, meat or rice. As for the quality of consumption, we will assume that the unit cost of the consumption quantity will be an increasing function of quality. One surprising result is that under quite general conditions food quality will vary across individuals in the household only when nutritional investment is important. Even when there are hidden actions and food provides incentives, these incentives will involve only varying the quantity and not the quality of food unless nutritional investments matter.

Using the same dataset that we do, Foster and Rosenzweig (1994) asks whether or not individual anthropometric measures depend on the nature of the contract governing compensation for off-farm work, interpreting this as a test for the importance of shirking with respect to off farm employers. As in Dercon and Krishnan (2000), Foster and Rosenzweig assume that food only influences utility to the extent that it influences measures of weight for height, but find that indeed incentives provided in the workplace influence consumption and physical status. In contrast to Foster and Rosenzweig, our focus is on the allocation of goods *within* the household, and on the role that food consumption may play in providing incentives above and beyond the role that food consumption may play as a sort of nutritional investment.

We proceed as follows. First, we provide an extended description of the data in Section 2. We describe some patterns observed in the consumption allocations of these rural Philippine households, including expected levels of consumption, and both individual and household-level measures of risk in both consumption and income.

Second, in Section 3 we formulate several models, each corresponding to a dynamic program which characterizes allocations in different environments. In the first dynamic model, utility and future productivity depends on consumption, but there is no asymmetric information among household members. In particular, while food consumption produces direct utility (which depends on the quantity and quality of different kinds of foodstuffs), it also represents a human capital investment which influences labor productivity but this investment depends only on the quantity and nutritional content of foodstuffs, and *not* on food quality. This model of nutritional investments reproduces some of the features of models formulated by, e.g., Pitt et al. (1990) or Pitt and Rosenzweig (1985). We then characterize allocations in the ‘complete markets’ hypothesis. We do so by assuming that a household head (or social planner) allocates state contingent consumption goods, makes investment decisions, and assigns activities to other household members in order to maximize a household welfare objective. We then study how to characterize the Pareto efficient allocations. In this model there is no private information and hence no need to provide incentives, but

the optimal allocation of food depends on the effect that consumption has on both utility and productivity. This model implies a set of restrictions on household members' intertemporal marginal rates of substitution which depend on whether consumption affects future productivity. A key testable prediction of the presence of nutritional investment model is that if there's an increase in the returns to nutritional investment for household member i , then nutritional consumption of this member will increase at the same time that the quality of food consumed by i decreases.

Then, we extend the model with nutritional investment so that the off-farm labor effort of other household members is private information¹. Foster and Rosenzweig (1994) find evidence which suggests that workers labor effort is difficult to monitor by farm employers, so that geographically remote household members can't observe this effort either. Accordingly, the intra-household sharing rule must be incentive compatible. Household members must be provided with appropriate incentives to induce them to take the recommended actions. We show that in this model of efficient intra-household incentives food quality should respond to unpredicted individual earnings shocks. Section 6 concludes.

2. THE DATA

The main data used in this paper are drawn from a survey conducted by the International Food Policy Research Institute and the Research Institute for Mindanao Culture in the Southern region of the Bukidnon Province of Mindanao Island in the Philippines during 1984–1985. These data are described in greater detail by Bouis and Haddad (1990) and in the references contained therein. Additional data on weather used in this paper were collected by us at the weather station of Malay-Balay in Bukidnon.

2.1. Survey Design. Bukidnon is a poor rural and mainly agricultural area of the Philippines. Early in 1984, a random sample of 2039 households was drawn from 18 villages in the area of interest. A preliminary survey was administered to each household to elicit information used to develop criteria for a stratified random sample later selected for more detailed study. The preliminary survey indicated that farms larger than 15 hectares amounted to less than 3 per cent of all households, a figure corresponding closely to the 1980 agricultural census. Only households farming less than 15 hectares and having at least one child under five years old were eligible for selection. Based on this preliminary survey, a stratified random sample of 510 households from ten villages was chosen. Some attrition (mostly because of

¹“Off-farm labor” in this context means agricultural work on land operated by some other household.

out-migration) occurred during the study—a total of 448 households from ten villages finally participated in the four surveys conducted at four month intervals beginning in July 1984. The total number of persons in the survey is 3294.

2.2. Food Expenditures and Nutrition. The nutritional component of the survey involved interviewing respondents to elicit recall of individual food intake over the previous twenty-four hours. In addition, there were monthly interviews to measure household-level food expenditures, and every four month interviews to measure household level non-food expenditures. The measurement of food intake involved collecting data on quantity purchased (along with prices) at the household level, and the quantity consumed at the individual level. Information on both individual- and household-level food consumption was highly disaggregated. For individual level consumption, data was collected on over eighty different items or dishes. For each dish, there was a corresponding recipe mapping ingredients into quantities of the dish. One can then back out the quantities of all ingredients implied by the data collected on dishes. For almost all of these, there is a corresponding entry in a food conversion table which translates quantities of each ingredient into a basket of nutrients. Individual food expenditures are computed using information on prices from the household expenditure survey, multiplied by the quantities consumed by different individuals. Appropriate adjustments are made to account for food consumed out of own-production or in-kind transactions.

Later in the paper, estimable equations will lead us to examine changes in individuals' consumption, and not *levels* of consumption. However, some of these differences are interesting, and so some information on levels of individual expenditures along with caloric and protein intakes are given in Table 1. Turning first to the final columns of the table, note that the average individual in our sample is not terribly well-fed. Comparing the figures in Table 1 to standard guidelines for energy-protein requirements (WHO, 1985) reveals that even the average person in our sample faces something of an energy deficit.

When we consider the average consumption of different age-sex groups, it becomes clear that some groups are particularly malnourished. Also, these figures show clearly that the relationship between consumptions and age consistently follows an inverse “U”-shaped pattern. Since the energy and protein needs of the very old and the very young are less than that of prime age adults, the appearance of this presumed pattern in the data is reassuring.

The picture of inequality drawn by our attention to energy and protein intakes is, if anything, exacerbated by closer attention to the sources of

	Expend.	Rice	Corn	Staples	Meat	Veg.	Snacks	Calories	Protein
Sample	6.38	0.80	1.30	0.32	1.98	0.53	0.90	1972.15	58.94
Male	7.06	0.89	1.39	0.31	2.12	0.55	1.25	2100.17	62.56
Female	5.66	0.69	1.21	0.32	1.83	0.51	0.54	1837.44	55.13
≤ 5 years	3.80	0.40	0.73	0.23	1.41	0.24	0.39	1137.08	34.89
6–10 years	4.84	0.62	1.05	0.28	1.64	0.37	0.43	1618.38	48.50
11–15 years	6.88	0.87	1.51	0.37	2.24	0.62	0.63	2232.41	66.06
16–25 years	7.93	1.06	1.61	0.36	2.27	0.76	1.24	2412.47	71.97
26–50 years	8.88	1.01	1.61	0.36	2.49	0.69	2.07	2416.09	72.50
> 50 years	7.12	0.88	1.44	0.25	1.88	0.57	1.36	2136.46	61.65
≤ 5 years (Male)	3.72	0.42	0.73	0.23	1.41	0.22	0.29	1166.96	35.48
6–10 years (Male)	5.01	0.69	1.04	0.28	1.73	0.37	0.46	1653.66	49.33
11–15 years (Male)	7.11	0.92	1.67	0.38	2.28	0.58	0.72	2360.03	69.04
16–25 years (Male)	9.46	1.21	1.83	0.35	2.62	0.84	1.96	2688.41	80.57
26–50 years (Male)	10.41	1.18	1.77	0.35	2.69	0.74	3.01	2653.38	79.36
> 50 years (Male)	7.96	1.04	1.50	0.23	1.94	0.63	1.73	2300.73	66.91
≤ 5 years (Female)	3.90	0.37	0.72	0.22	1.41	0.27	0.50	1101.53	34.18
6–10 years (Female)	4.66	0.55	1.06	0.28	1.54	0.36	0.40	1582.22	47.65
11–15 years (Female)	6.66	0.82	1.36	0.36	2.20	0.66	0.55	2109.17	63.18
16–25 years (Female)	6.61	0.93	1.42	0.37	1.97	0.69	0.62	2176.37	64.62
26–50 years (Female)	6.86	0.78	1.41	0.37	2.23	0.63	0.84	2102.99	63.43
> 50 years (Female)	4.57	0.39	1.27	0.30	1.67	0.39	0.25	1639.46	45.76

TABLE 1. Mean Daily Food Consumption. First column reports total expenditures per person (in constant Philippine pesos). The next six columns report means for particular sorts of food (differences between total food expenditures and the sum of its constituents is accounted for by “other non-staple” foods). The final two columns report individual Calories and grams of protein obtained from individual-level food consumption.

nutrition. While all of the foods considered here are sources of calories and protein, it also seems likely that food consumption is valued not *only* for its nutritive content, but that individuals also derive some direct utility from certain kinds of consumption. This point receives some striking support from Table 1. Consider, for example, average daily expenditures by males aged 26–50, compared with the same category of expenditures by women of the same age. The value of expenditures on male consumption of all staples is 51 per cent greater than that of females of the same age. This difference could be attributed to differences in activity or metabolic rate. However, comparing expenditures on what are presumably superior goods, we find that expenditures on male consumption of meat (and fish),

vegetables, and snacks (including fruit) is 72 per cent greater than the corresponding expenditures by women in the same age group 26-50. We can also see from Table 1 that such difference comes partially from differences in quantities but not only and that differences in average price also explain the larger expenditures for males for such categories. Since nothing like a difference of this size shows up in calories or protein, this seems like very strong evidence that intra-household allocation mechanisms are designed to put a particularly high weight on the *utility* of prime-age males relative to other household members, quite independent of those prime-age males' greater energy-protein requirements. Note that although these differences in consumption seem to point to an inequalitarian allocation, these differences provide no evidence to suggest that household allocations are inefficient.

2.3. Weather. Weather data coming from the weather station of Malay Balay which is at the center of the Bukidnon province were collected on the period 1961-1994, which includes the period of the survey. These data are monthly data about the number of cloudy days during the month, the number of rainy days, the maximum daily rain quantity, the average rate of humidity, the minimum daily temperature, the maximum daily temperature during the month. Using these data, we first estimate a VAR model for these 6 variables. Computing likelihood ratio tests as well as other information criteria like the Akaike Information criterium or the final prediction error, we choose to estimate a VAR model of order 3 with a 12 month lag included. The results of the VAR model are in Table 2. The fit of each equation varies from an R^2 of 21% for maximum rain to an R^2 of 62% for maximum temperature. This shows that there is a substantial variation in weather which is not easily predicted and that weather shocks are relatively important in that environment. For example, the number of days of rain per month varies substantially. On average it is 17.9 days per month but with a standard deviation of 6.37. Moreover, more than half of such variance cannot be explained by our VAR model. Weather shocks are thus quite important in the Bukidnon context, since rain and temperature vary substantially and that a large part of this variation is not simply due to predicted seasonal variations.

2.4. Agricultural Labor Income. Households in these data are seen to derive income from a variety of sources. All of the households in the sample are agriculturalists, and cultivate some land. Typically this sort of cultivation will involve labor from several members of the household, so that it's not possible to reliably attribute income or production from cultivation to a particular individual.

We are interested in the question of whether individuals may be somehow compensated for their contributions to the pooled resources of the

Variables	cloud _t	maxrain _t	rainydays _t	humid _t	mintemp _t	maxtemp _t
cloud _{t-1}	0.0283 (0.0737)	-5.879*** (1.871)	-0.885** (0.357)	0.0257 (0.226)	-0.0318 (0.0433)	0.0935 (0.0625)
cloud _{t-2}	0.340*** (0.0713)	-0.448 (1.810)	0.284 (0.346)	0.0444 (0.219)	-0.00328 (0.0419)	0.0986 (0.0605)
cloud _{t-3}	0.131* (0.0736)	0.160 (1.868)	0.209 (0.357)	-0.110 (0.226)	0.0324 (0.0432)	0.0642 (0.0624)
cloud _{t-12}	-0.180*** (0.0671)	-1.950 (1.705)	-0.934*** (0.325)	-0.570*** (0.206)	-0.0780** (0.0395)	0.144** (0.0570)
mxrain _{t-1}	0.000558 (0.00247)	-0.0409 (0.0627)	0.00785 (0.0120)	-0.00779 (0.00758)	-0.00106 (0.00145)	0.00450** (0.00209)
mxrain _{t-2}	-3.68e-05 (0.00251)	0.0133 (0.0636)	-0.0158 (0.0121)	0.00727 (0.00770)	-0.000920 (0.00147)	0.000393 (0.00213)
mxrain _{t-3}	0.00252 (0.00249)	0.0100 (0.0632)	0.0155 (0.0121)	0.00917 (0.00764)	0.00233 (0.00146)	-0.00372* (0.00211)
mxrain _{t-12}	0.00166 (0.00256)	0.0327 (0.0649)	-0.00239 (0.0124)	-0.00382 (0.00785)	0.00203 (0.00150)	0.00189 (0.00217)
rainydays _{t-1}	0.00513 (0.0180)	0.715 (0.457)	0.303*** (0.0872)	0.0245 (0.0552)	-0.000978 (0.0106)	-0.0200 (0.0153)
rainydays _{t-2}	0.0189 (0.0183)	0.0177 (0.465)	0.0993 (0.0888)	-0.00737 (0.0563)	0.000411 (0.0108)	-0.0210 (0.0155)
rainydays _{t-3}	-0.00357 (0.0169)	0.969** (0.430)	0.134 (0.0822)	0.0320 (0.0520)	-0.0165* (0.00996)	-0.0164 (0.0144)
rainydays _{t-12}	0.0167 (0.0168)	0.931** (0.426)	0.348*** (0.0813)	0.0941* (0.0515)	0.0208** (0.00985)	-0.00800 (0.0142)
humid _{t-1}	-0.0313 (0.0267)	0.759 (0.679)	-0.148 (0.130)	0.547*** (0.0821)	-0.00158 (0.0157)	-0.0148 (0.0227)
humid _{t-2}	-0.0281 (0.0302)	-0.0540 (0.767)	-0.0707 (0.147)	0.102 (0.0928)	-0.00934 (0.0178)	0.0280 (0.0256)
humid _{t-3}	0.000389 (0.0261)	0.0913 (0.662)	-0.0445 (0.126)	-0.0969 (0.0800)	-0.0208 (0.0153)	0.00951 (0.0221)
humid _{t-12}	0.00144 (0.0202)	-0.452 (0.513)	-0.0744 (0.0979)	0.112* (0.0620)	-0.00599 (0.0119)	0.0217 (0.0171)
mintemp _{t-1}	0.234** (0.106)	7.109*** (2.685)	1.876*** (0.513)	0.413 (0.325)	0.560*** (0.0621)	-0.247*** (0.0897)
mintemp _{t-2}	-0.0925 (0.120)	-2.158 (3.055)	0.239 (0.583)	0.174 (0.370)	0.102 (0.0707)	-0.165 (0.102)
mintemp _{t-3}	-0.217** (0.106)	-6.073** (2.695)	-1.405*** (0.515)	-0.345 (0.326)	-0.0275 (0.0624)	0.182** (0.0901)
mintemp _{t-12}	0.297*** (0.0904)	6.200*** (2.294)	1.132*** (0.438)	0.345 (0.277)	0.213*** (0.0531)	-0.0465 (0.0767)
maxtemp _{t-1}	-0.215** (0.0946)	2.626 (2.402)	0.195 (0.459)	0.706** (0.291)	0.136** (0.0556)	0.300*** (0.0803)
maxtemp _{t-2}	0.126 (0.0994)	-1.840 (2.523)	-0.428 (0.482)	0.280 (0.305)	-0.0631 (0.0584)	0.0234 (0.0843)
maxtemp _{t-3}	0.122 (0.0845)	2.193 (2.146)	0.808** (0.410)	0.0622 (0.260)	-0.0836* (0.0497)	0.0211 (0.0717)
maxtemp _{t-12}	0.0478 (0.0758)	-0.885 (1.924)	0.117 (0.367)	-0.589** (0.233)	0.0834* (0.0445)	0.491*** (0.0643)
Constant	1.665 (3.682)	-132.6 (93.48)	-15.94 (17.85)	4.821 (11.31)	4.196* (2.163)	4.774 (3.124)
R ²	0.34	0.21	0.45	0.51	0.58	0.62

TABLE 2. VAR results for monthly weather statistics (1961-1994). Standard Errors are in parenthesis. *, **, ***, indicate significance at the 10, 5, 1 percent levels respectively. cloud is the number of cloudy days per month, maxrain is the maximum rain quantity per day during the month, rainydays is the number of rainy days during the month, humid is average rate of air humidity during the month, mintemp and maxtemp are the monthly average of the minimum and maximum daily temperatures.

household, and for this we need some measure of individual contribution. One component of total household income can be ascribed to individuals; that's earnings from off-farm agricultural labor. By "agricultural labor" note that we mean labor on *other* farms, compensated either in cash or in kind. Twenty-three percent of the individuals in the sample obtain some income from such labor in at least one of the four rounds, though in any given round only around thirteen percent of individuals are so engaged.

As agricultural productivity is usually importantly affected by the quantity of rain, it is not surprising that off farm labor earnings may vary substantially according to weather shocks and in particular according to the number of rainy days.

In order to separate the predictable and seasonal variations of weather from the unpredictable weather shocks, we use the results of the VAR model presented before to construct a predicted and unpredicted measure of each weather variable w_{kt} for measure k at period t . We denote by w_{kt}^p the predicted measure of current weather variable.

Then, we use these measures to disentangle the predicted and unpredicted wage earnings shocks by survey round. However, as Foster and Rosenzweig (1996) show, agricultural earnings in this region are related to the physical productivity of workers and thus to their anthropometric characteristics in addition to education and age. We thus take into account these factors in explaining wage earnings in addition to the part of earnings that can be related to predictable weather variations. We do this by regressing the log earnings $\ln y_{it}$ of each individual i at period t on a set of individual characteristics like gender, education level, age, age square, height and height squared, weight and weight squared plus predicted weather variations interacted with village dummy variables and gender dummies (to allow weather variation to affect earnings of males and females differently). The total R^2 of this regression is 23 per cent. The predicted log earnings is then denoted $\ln y_{it}^p$ while the residual is the unpredicted log earnings denoted $\ln y_{it}^u$.

A joint F test that all predicted weather variables do not affect earnings rejects strongly the null ($F(43, 1680) = 7.57$). Concerning individual characteristics, education and age are the main determinants of earnings.

3. MODELING INTRA-HOUSEHOLD ALLOCATIONS

In this section we present two dynamic models of intra-household allocation. The first of these assumes that there's full information within the household and features fully Pareto efficient allocations, while the second

assumes that household members' actions may be private, and delivers allocations which are constrained efficient.

We begin in Section 3.1 by describing basic elements of the economic environment common to both models. In Section 3.2 we present a dynamic model of intra-household allocation in which the household head assigns investments, labor effort, and consumption allocations so as to maximize a weighted sum of discounted expected utilities. A special case of this model occurs when food consumption produces utility, but has no effect on productivity. We derive restrictions on the intra-household allocation of both nutrients and food expenditures conditional on the absence of nutritional investment being correct. In Section 3.3 we extend this model to allow for the possibility that household members' actions may not be observed by the household head. Such hidden actions result in allocations not being fully Pareto efficient, and we give a partial characterization of the incentive-constrained allocations.

3.1. Economic Environment. Here we describe the stochastic environment involving both public shocks and individual-specific stochastic production in Section 3.1.2. We then describe the manner in which nutritional investments can influence certain forms of human capital in Section 3.1.3. Next, we discuss the connection between food quality and food prices in Section 3.1.4.

3.1.1. Preliminaries. Consider a household having n members, indexed by $i = 1, 2, \dots, n$, where an index of 1 is understood to refer to the household head. Time is indexed by $t = 0, 1, 2, \dots$. During each period, member i consumes a *quantity* of nutrients c_{it} . These nutrients have a corresponding *quality* ϕ_{it} , assumed to be in the interval $[0, \bar{\phi}]$. At the same time, i supplies some labor (or takes some action) a_{it} .

Household member i derives direct utility from both the quantity and quality of his consumption and disutility from his labor. Further, at time t person i possesses a set of characteristics (e.g., sex, health, age) which we denote by the L -vector b_{it} . These characteristics may have an influence on the utility he derives from both consumption and activities. Thus, we write his momentary utility at t as some $U(\phi_{it}, c_{it}, b_{it}) + Z_i(a_{it}, b_{it})$, where the function U is assumed to be increasing, concave, and continuously differentiable in both the quantity and quality of nutrients, and where Z_i is a function describing the disutility of labor a_{it} for an individual with characteristics b_{it} . Future utility is discounted via a common discount factor $\beta \in (0, 1)$.

3.1.2. Stochastic Structure. Households face two basic sources of uncertainty. First, in each period t some public shock $\theta_t \in \Theta = \{1, 2, \dots, S\}$ is

realized. One should think of θ_t as an index of the publicly observable state of nature. The realization of θ_t determines weather shocks, the prices faced by the household, the health of different people, and so on. Note that though publicly observed, this shock need not be common. For example, in a particular state $\hat{\theta}$ it may be that it rains on everyone, but person one is healthy while person two is sick. The probability of some particular θ_t occurring at time t may depend on the previous period's realization θ_{t-1} via a collection of Markovian transition probabilities $\Pr(\theta_t|\theta_{t-1})$, while the cumulative probability that the realization of θ_t will be less than or equal to some value θ' is written $G(\theta'|\theta_{t-1}) = \sum_{r=1}^{\theta'} \Pr(r|\theta_{t-1})$.

The second source of uncertainty faced by households is production or earnings uncertainty. Each person $i = 1, \dots, n$ begins each period t with some productive inputs z_{it} purchased by the household the previous period. Each household member then supplies labor actions a_{it} . Conditional on the inputs z_{it} provided and labor actions a_{it} being taken, the public shock θ_t is realized. Investments, actions, and public shocks together influence the distribution of person i 's production; the realization of person i 's production at t is denoted $y_{it} \in \mathbb{R}$, and is measured in units of the numeraire good. Each y_{it} is drawn from a continuous distribution $F^i(y|a_{it}, z_{it}, \theta_t)$ which depends on the labor supplied by person i a_{it} , the investment (of the numeraire good) z_{it} made by the household in the previous period, and on the public shock θ_t . We assume that for every (a, z, θ') the corresponding density $f^i(y|a, z, \theta')$ exists, and that the support of f_i doesn't vary with a . We further assume that f^i is a continuously differentiable function of a , and denote such derivatives by $f_a^i(y|a, z, \theta')$.

As the notation is meant to suggest, output y_{it} is assumed to be conditionally independent of y_{js} for all $(i, t) \neq (j, s)$. Accordingly, using bold characters to denote vectors of variables for each family member, let $y = (y_1, y_2, \dots, y_n)$ denote the vector of outputs for each member of the family, with joint distribution $F(y|a, z, \theta') = \prod_{i=1}^n F^i(y_i|a_i, z_i, \theta')$ and joint density $f(y|a, z, \theta')$. Denote by ω the collection of random elements which affect household i at the end of a period, so that $\omega = (y, \theta')$; for the sake of brevity, let $H(\omega|a, z, \theta) = F(y|a, z, \theta')G(\theta'|\theta)$.

3.1.3. Nutritional Investments. The individual characteristics of an individual are allowed to evolve over time. Let M_i be a law of motion for the vector of characteristics of individual i , with $b_{it+1} = M_i(b_{it}, c_{it}, \theta_t)$, and let $M_i^\ell(b_{it}, c_{it}, \theta_t)$ denote the law of motion for the ℓ th characteristic. Similarly, let \mathbf{b}_t denote the list of vectors of individual characteristics at t , with \mathbf{M} the law of motion for characteristics for all n family members. For example, a person's strength (and thus labor productivity) in period $t+1$ may depend

on his strength in the previous period as well as on the quantity of his consumption c_{it} . Note that the evolution of b_{it} is assumed *not* to depend on the quality of consumption, but only on its quantity where the consumption quantity c is nutrients (e.g., calories or grams of protein). A variety of different diets could plausibly provide the same quantity of nutrients c ; however, not all of these diets will provide the same level of utility because of different quality levels φ .

3.1.4. Prices and Qualities. The implicit price of nutrients may depend on quality and the public shock θ' ; the price of a unit of nutrients having quality φ is $p(\varphi, \theta')$. We will denote the partial derivative of the function p with respect to quality φ by $p'(\varphi, \theta')$, and the second partial derivative of p with respect to φ by $p''(\varphi, \theta')$.

Assumption 1 (Price of Quality). *Let $\Phi = [0, \bar{\varphi}] \subset \mathbb{R}$. We make the following assumptions regarding the manner in which prices vary with quality:*

- (1) $p : \Phi \times \Theta$ is a twice continuously differentiable function of quality.
- (2) $p(0, \theta') > 0$, $p'(\varphi, \theta') > 0$, $p''(\varphi, \theta') > 0$ for all $\varphi \in \Phi$ and $\theta' \in \Theta$.
- (3) $\lim_{\varphi \rightarrow \infty} \varphi p'(\varphi, \theta') - p''(\varphi, \theta') > 1$

The first part of this assumption implies that the prices of even low quality nutrients must always be positive, and higher quality nutrients must cost more. The assumption that the price function is convex guarantees that the quality elasticity of the price function denoted $\eta(\cdot, \theta') (\equiv \varphi p'(\varphi, \theta') / p(\varphi, \theta'))$ is strictly increasing. It will appear as a sufficient condition to insure that the optimal quality choice of food is uniquely determined. The final part of Assumption 1 is critical to guaranteeing that an interior solution to the quality choice problem exists. Indeed, it implies that whatever the public shock, there exists a quality level of food such that the price elasticity is equal to one: for any $\theta' \in \Theta$ there exists some $\hat{\varphi} \in \Phi$ such that $\eta(\hat{\varphi}, \theta') \equiv \hat{\varphi} p'(\hat{\varphi}, \theta') / p(\hat{\varphi}, \theta') = 1$ because the previous assumptions imply that $\xi(\varphi; (\omega), \theta') \equiv p(\varphi; (\omega), \theta') - \varphi_i(\omega) p'(\varphi_i(\omega), \theta')$ is strictly increasing and negative in zero. A simple example of a price function which satisfies Assumption 1 is $p(\varphi, \theta') = g(\theta')(1 + \varphi)^\rho$, with $g(\theta')$ positive, $\rho > 1$, and $\bar{\varphi} > 1/(\rho - 1)$.

3.2. The Household Head's Problem with Full Information. The household head decides the labor each household member should supply, how much the household should collectively save or invest, how to allocate the remaining household resources across household members, and what resources should be promised to individual family members in the future.² We

²There is a small literature devoted to the subject of division of responsibilities between adult males and females in Philippine households (e.g., Illo, 1995; Eder, 2006). Though

formulate the problem as a dynamic program, adopting an approach similar to e.g., Spear and Srivastava (1987) in which future ‘utility promises’ are made by the head to individual family members and appear as state variables in the head’s dynamic programming problem. In this recursive formulation of the problem we can drop the time subscripts from variables: all variables may be assumed to be dated t unless adorned by the notation $'$, in which case these are dated $t + 1$.

The head chooses the allocation of consumption only after observing the realization of ω , so that consumptions are assigned after individual outputs are determined and the public shock θ' is observed. Allocating consumption involves choosing both nutrients $c_i(\omega)$ to award to person i in state ω as well as a corresponding quality $\varphi_i(\omega)$.

For any time t realization of ω_t , the head must satisfy the budget constraint

$$(1) \quad \sum_{i=1}^n p(\varphi_{it}(\omega_t), \theta_t) c_{it}(\omega_t) \leq \sum_{i=1}^n y_{it} - \sum_{i=1}^n z_{it}(\omega_{t-1}).$$

At the same time that the head allocates contemporaneous consumption, she also makes promises to other household members about their future levels of (discounted, expected) utility. Past promises must also be honored. Thus, if the head promised future utils w_{it} to person i at date $t - 1$ (when ω_{t-1} is realized), then honoring that promise requires that

$$(2) \quad \int [U(\varphi_{it}(\omega_t), c_{it}(\omega_t), b_{it}) + Z_i(a_{it}, b_{it}) + \beta w_{it+1}(\omega_t)] dH(\omega_t | a_t, \theta_t) = w_{it}.$$

3.2.1. Allocations Under Full Information with Nutritional Investments.

We formulate the problem facing the head recursively. At the beginning of a period, the head takes as given an n -vector reflecting her current utility promises to other household members (w), investments from the previous period (z), a list of the characteristics of household members (b), and the previous period’s public shock θ . Given her preferences, the head then assigns labor, decides how much to save/invest for subsequent periods, and makes another set of utility promises, all subject to the constraints implied by these prices and resources. In particular, let $V(w, z, b, \theta)$ denote the discounted, expected utility of the head given the current state, and let this function satisfy

unfortunately we can’t offer independent evidence from our dataset, this list of tasks seems to be consistent with the kinds of responsibilities typically undertaken by adult females, rather than males. Accordingly, we assume below that the senior female in the household is the household head unless noted otherwise.

Program 1.

(3)

$$V(w, z, b, \theta) = \max_{\{a_i, z'_i, \{\varphi_i(\omega), c_i(\omega), w'_i(\omega)\}_{\omega \in \Omega}\}_{i=1}^n} \int [U(\varphi_1(\omega), c_1(\omega), b_1) + Z_1(a_1, b_1) + \beta V(w'(\omega), z', M(b, c(\omega), \theta'), \theta')] dH(\omega|a, z, \theta)$$

subject to the household budget constraint

$$(4) \quad \sum_{i=1}^n p(\varphi_i(\omega), \theta') c_i(\omega) \leq \sum_{i=1}^n y_i - \sum_{i=1}^n z'_i$$

for all $\omega = (y, \theta') \in \Omega$, and subject also to a set of promise-keeping constraints

$$(5) \quad \int [U(\varphi_i(\omega), c_i(\omega), b_i) + Z_i(a_i, b_i) + \beta w'_i(\omega)] dH(\omega|a, z, \theta) = w_i,$$

for all $i = 1, \dots, n$; and subject finally to the requirement $\varphi_i(\omega) \in \Phi$ for all $i = 1, \dots, n$ and all states $\omega \in \Omega$.

We begin our analysis of this problem by giving a statement of a simple result: in the full-information environment of Program 1, the marginal cost to the head of promising utility w_{it} to person i will be constant across time and different states.

Lemma 1. *Let $\alpha_{it} = \frac{\partial V}{\partial w_i}(w_t, z_t, b_t, \theta_t)$ in Program 1. If $U(\varphi, c, b)$ is a concave function of (φ, c) for all b , then $\alpha_{it} = \alpha_{it+1}$ for all dates t .*

Proof. The envelope condition with respect to w_i in Program 1 implies that $\frac{\partial V}{\partial w_i}(w_t, z_t, b_t, \theta_t) \equiv \alpha_{it}$ is equal to the Lagrange multiplier on the promise-keeping constraint (5). This fact along with the first order condition for the head's problem with respect to $w_i(\omega_{t+1})$ then imply that $\alpha_{it} = \alpha_{it+1}$. The concavity of U is a sufficient condition for this first order condition to characterize the solution to the head's problem. \square

Since individual utilities affect the head's value function in a linear way, the lemma simply reflects the point that in environments such as that described here, optimal allocations of consumption will keep the ratio of marginal utilities of different household members constant across both dates and states (as in, e.g., Wilson, 1968); this is basically a consequence of risk aversion and our assumption of time separable expected utility.

We want to consider the outcomes predicted by this simple model under a variety of different assumptions regarding preferences and other aspects of the environment facing the household. We begin by adopting some restrictions on the utility function U :

Assumption 2 (Utility Functions). *We place the following restrictions on the form of the utility function for person i :*

- (1) $U(\phi, c, b) = h(b)u_i(\phi c)$;
- (2) $u_i : \mathbb{R} \rightarrow \mathbb{R}$ is strictly increasing concave; and
- (3) $\lim_{x \searrow 0} u'_i(x) = -\infty$.

Point (1) of this assumption implies that the utility function satisfies a form of multiplicative separability between characteristics b and the quality and quantity of consumption. It implies that characteristics affect the marginal utility of consumption only multiplicatively in a common form across different people. This specification choice is however commonly used in the literature on risk sharing and in consumption studies (Dercon and Krishnan (2000), Blundell et al. (1993), Attanasio and Weber (1994), Attanasio et al. (1999), Mace (1991), Cochrane (1991) and Townsend (1994)) and all the relevance and robustness of this restriction will hinge on the degree of heterogeneity introduced in characteristics b . This point (1) of Assumption 2 also implies that the utility depends on "efficient" units of consumption measured by the product of quantity and quality of nutrients ³. Remark that usual specifications (Mace (1991), Cochrane (1991), Townsend (1994), Ligon (1998), Dubois et al. (2008), Dubois and Ligon (2011)) would not take into account the quality ϕ and implicitly restrict even more utility to depend only on the consumption quantity. Assumptions Assumption 1 and Assumption 2 regarding preferences and the dependence of price on quality imply that the individual indifference curve between quality and quantity will cross the non linear budget constraint only once and at a point where the price elasticity of quality is equal to one (the budget constraint being linear in quantity but non linear in quality, the budget lines in the quantity-quality plan will not be linear). Point (2) of Assumption 2 is entirely standard, while point (3) is one of the 'Inada' conditions which allows to avoid the possibility of zero consumption as being assigned to any household member.

Throughout the paper, Assumption 1 and Assumption 2 will be maintained so that all propositions and other results are derived under these two assumptions.

We now characterize the relationship between consumption, expenditures, and quality for various household members relative to the household head. With full information and nutritional investment, denoting Δ the operator differencing across time periods, we have the following proposition:

³It is possible to relax this assumption considerably by assuming, for example, that utility depends on an individual-specific Cobb-Douglas functions of quantity and quality. The addition of this additional idiosyncratic element of individual preferences requires only modest changes to our results, which are available from the authors on request.

Proposition 1. *Under full information and nutritional investment consumption allocations which solve Program 1 will satisfy*

$$(6) \quad \Delta \ln \frac{u'_i(\varphi_{it}c_{it})}{u'_1(\varphi_{1t}c_{1t})} + \Delta \ln \frac{h(b_{it})}{h(b_{1t})} = \Delta \ln \frac{p'(\varphi_{it}, \theta_{t+1})}{p'(\varphi_{1t}, \theta_{t+1})}$$

for all $i = 1, \dots, n$, and all dates t .

Proof. For any $i = 2, \dots, n$, let α_i denote the Lagrange multiplier associated with the head of household's corresponding promise-keeping constraint (5). We adopt the convention that $\alpha_1 \equiv 1$.

Then the first order conditions from the head's problem for φ_i can be rearranged to yield the following expression for person i 's consumption quantity in state ω :

$$(7) \quad c_i(\omega) = \frac{\alpha_i}{\mu(\omega)} \frac{\partial U(\varphi_i(\omega), c_i(\omega), b_i)}{\partial \varphi} \frac{1}{p'(\varphi_i(\omega), \theta')}$$

where $\mu(\omega)$ is the Lagrange multiplier of the collective household budget constraint when the state of nature is ω , and recalling that $p'(\cdot, \theta')$ is the partial derivative of the price function with respect to quality φ .

Dividing the expression given by (7) for the consumption quantity of person $i > 1$ by the corresponding expression for $i = 1$ yields

$$\frac{c_i(\omega)}{c_1(\omega)} = \alpha_i \frac{\partial U(\varphi_i(\omega), c_i(\omega), b_i) / \partial \varphi}{\partial U(\varphi_1(\omega), c_1(\omega), b_1) / \partial \varphi} \frac{p'(\varphi_1(\omega), \theta')}{p'(\varphi_i(\omega), \theta')}.$$

Fix a particular period t ; Lemma 1 implies that α_i is invariant over time. Then taking logarithms and differences across adjacent time periods gives us

$$(8) \quad \Delta \ln \frac{c_{it}}{c_{1t}} = \Delta \ln \frac{\partial U(\varphi_{it}, c_{it}, b_{it}) / \partial \varphi}{\partial U(\varphi_{1t}, c_{1t}, b_{1t}) / \partial \varphi} + \Delta \ln \frac{p'(\varphi_{1t}, \theta_{t+1})}{p'(\varphi_{it}, \theta_{t+1})}.$$

Using Assumption 2, the first term on the right-hand side of this expression is equal to

$$\Delta \ln \frac{h(b_{it})}{h(b_{1t})} + \Delta \ln \frac{c_{it}}{c_{1t}} + \Delta \ln \frac{u'_i(\varphi_{it}c_{it})}{u'_1(\varphi_{1t}c_{1t})}.$$

Substituting this into (8) yields the result. \square

In words, Equation (6) can be thought of as governing the allocation of food quality, conditional on the allocation of nutrients. At an optimum the household head will equate the (change in) the marginal rate of substitution between quality for the head and quality for person i . This marginal rate of substitution is just the left-hand-side of (6), and is equated to the (change in the) marginal rate of transformation, which depends on the ratio of marginal prices $p'(\varphi_{1t}, \theta_{t+1}) / p'(\varphi_{it}, \theta_{t+1})$. The household head can increase

the quality of her consumption at the expense of the quality of i 's consumption, but she'll only do this to the point at which it becomes less expensive to satisfy utility promises to i by giving i a greater *quantity* of consumption.

The return to giving i additional nutrition depends partly on the additional utility i receives from having more food, but may also depend on the effects that this additional nutrition may have on productivity. When nutrients influence productivity, one can think of consumption as a sort of nutritional investment. The returns to this investment depend both on the marginal product of characteristics b and on the marginal impact that nutrition has on these characteristics.

In any state ω the marginal return to a nutritional investment in person i will generally depend on the future utilities promised to all family members $w'(\omega)$, other investments made in future production z' , current characteristics b , i 's level of consumption c_i , the current public shock θ' (which plays a role in determining the price of food given to i) and the household head's marginal utility of income in the current state ω , which we denote by $\mu(\omega)$. Then given the L individual characteristics b_i , the marginal return (in utils) in state ω to additional nutritional investment c_i in person i is

$$R_i(c(\omega), w'(\omega), z', b, \theta') = \frac{1}{\mu(\omega)} \sum_{\ell=1}^L \frac{\partial V}{\partial b_i^\ell}(w'(\omega), z', M(b, c(\omega), \theta'), \theta') \frac{\partial M_i^\ell}{\partial c_i}(b, c(\omega), \theta').$$

Note that $\mu(\omega)$ can also be interpreted as the Lagrange multiplier associated with the budget constraint (4) in the present state ω , and is thus related to the cost of diverting resources from investments z' , or from consumption assigned to others.

3.2.2. Allocations Under Full Information with No Nutritional Investments. In general, (6) will not suffice to determine the allocation of either quantities or qualities—the latter depend on the allocation of the former, and the assignment of quantities will generally depend on the returns these nutritional investments have in terms of increased production.

To establish a simple benchmark, consider instead the case in which there is no return to nutritional investment (food consumption does not influence future productive characteristics like anthropometrics). We express this case using the following assumption:

Assumption 3 (No Nutritional Investment). *The law of motion for characteristics satisfies $M(b, c, \theta') = M(b, \hat{c}, \theta')$ for all (b, θ') and all $c, \hat{c} \in \mathbb{R}$.*

Note that when Assumption 3 is satisfied, the returns to nutritional investment (R_i) will be zero.

Proposition 2. *If there is no nutritional investment (Assumption 3), then the allocated quality of nutrients which solves Program 1 will not vary across*

individuals in the household, so that $\varphi_{it}(\omega) = \varphi_{1t}(\omega)$ for all $i = 1, \dots, n$, $t = 1, 2, \dots$, and all $\omega \in \Omega$.

Proof. We show that $\varphi_i = \varphi_1$. The first order condition of Program 1 with respect to $c_i(\omega)$ is

$$\alpha_i \frac{\partial U}{\partial c}(\varphi_i(\omega), c_i(\omega), b_i) = \mu(\omega) [p(\varphi_i(\omega), \theta') - \beta R_i(c(\omega), w(\omega), z', b, \theta')].$$

However, the assumption of no nutritional investment (Assumption 3) implies that the term involving R_i is equal to zero. Using this fact along with the preferences restrictions (Assumption 2) and re-arranging yields

$$(9) \quad \alpha_i h(b_i) \varphi_i(\omega) u'_i(\varphi_i(\omega) c_i(\omega)) = \mu(\omega) p(\varphi_i(\omega), \theta').$$

The first order condition with respect to $\varphi_i(\omega)$ is equation (7) which can be similarly re-arranged to yield

$$(10) \quad \alpha_i h(b_i) u'_i(\varphi_i(\omega) c_i(\omega)) = \mu(\omega) p'(\varphi_i(\omega), \theta').$$

As we assumed that the price $p(\varphi, \theta')$ and its partial derivative with respect to φ will always be strictly positive (Assumption 1), dividing (9) by (10) and rearranging yields the result that

$$\frac{p'(\varphi_i(\omega), \theta')}{p(\varphi_i(\omega), \theta')} \varphi_i(\omega) = 1$$

for $i = 1, \dots, n$ and all $\omega \in \Omega$.

If there is a unique solution to this equation, then quality will be constant across household members. One way to insure that this is the case is to maintain the sufficient assumption that the price function is increasing convex. Indeed, the last two parts of the maintained assumption on prices (Assumption 1) guarantee that this equation has a unique solution. Then since the price function is common to all household members, the solution must be the same for all household members. \square

If quality is the same across household members, then we can use this fact to make a stronger claim about the allocation of nutrients. When qualities are identical within the household, the ratio $p'(\varphi_i, \theta')/p'(\varphi_1, \theta')$ is equal to one. Using this fact along with (6) from Proposition 1 yields

$$(11) \quad \Delta \ln \frac{u'_i(\varphi_{1t} c_{it})}{u'_1(\varphi_{1t} c_{1t})} = \Delta \ln \frac{h(b_{1t})}{h(b_{it})}$$

for all $i = 1, \dots, n$, and all dates t . This expression is similar in spirit to the restrictions on the evolution of marginal utilities familiar from tests of inter-household risk sharing, and follows immediately from the observation that with full risk-sharing individuals' marginal utilities of consumption will be perfectly correlated (e.g., Mace, 1991)—the restriction here differs only because we allow marginal utilities to depend on individual characteristics.

3.3. The Household Allocation with Hidden Actions. We next turn our attention to characterizing within-household consumption allocations when the labor of members can't be directly observed by the household head, in which case a situation of moral hazard arises.

3.3.1. Allocations Under Hidden Actions with Nutritional Investment. In this situation, the maximization problem facing the head can be expressed as follows:

Program 2.

$$(12) \quad V(w, z, b, \theta) = \max_{\{a_i, z'_i, \{\varphi_i(\omega), c_i(\omega), w'_i(\omega)\}_{\omega \in \Omega}\}_{i=1}^n} \int [U(\varphi_1(\omega), c_1(\omega), b_1) + Z_1(a_1, b_1) + \beta V(w'(\omega), z', M(b, c(\omega), \theta'), \theta')] dH(\omega|a, z, \theta)$$

subject to the household budget constraint (4) for all $\omega = (y, \theta') \in \Omega$; and subject also to the set of promise-keeping constraints (5). In addition, the household head must choose allocations and labor assignments which satisfy a set of incentive compatibility constraints

$$(13) \quad a_i \in \arg \max_a \int [U(\varphi_i(\omega), c_i(\omega), b_i) + Z_i(a, b_i) + \beta w_i(\omega)] \cdot d \left(F^i(y_i|a, z_i, \theta') \prod_{j \neq i} F^j(y_j|a_j, z_j, \theta') G(\theta'|\theta) \right)$$

for all $i = 1, \dots, n$; and subject finally to a requirement that $\varphi_i(\omega) \in \Phi$ for all $i = 1, \dots, n$; and all states $\omega \in \Omega$.

Remark that Program 2 is identical to Program 1 save for the addition of the incentive compatibility constraint (13), which requires that person i will have no incentive to deviate from the action a_i recommended by the head. Unfortunately, it's difficult to characterize the effects of this kind of hidden action when the requirement of incentive compatibility is expressed in the form of (13). The following program provides an alternative 'relaxed' optimization problem, which will have the same solutions as Program 2 so long as the so-called 'first order approach' is valid (Ábrahám and Pavoni, 2008).

Program 3.

$$(14) \quad V(w, z, b, \theta) = \max_{\{a_i, z'_i, \{\varphi_i(\omega), c_i(\omega), w'_i(\omega)\}_{\omega \in \Omega}\}_{i=1}^n} \int [U(\varphi_1(\omega), c_1(\omega), b_1) + Z_1(a_1, b_1) + \beta V(w'(\omega), z', M(b, c(\omega), \theta'), \theta')] dH(\omega|a, z, \theta)$$

subject to the household budget constraint (4) for all $\omega = (y, \theta') \in \Omega$; and subject also to the set of promise-keeping constraints (5). In addition, the household head must choose allocations and labor assignments which satisfy the first-order conditions from the agent's choice of labor effort,

$$(15) \quad \int [U(\varphi_i(\omega), c_i(\omega), b_i) + Z_i(a, b_i) + \beta w'_i(\omega)] \frac{f_a^i(y_i | a_i, z_i, \theta')}{f^i(y_i | a_i, z_i, \theta')} \cdot d(F(y | a, z, \theta') G(\theta' | \theta)) = - \frac{\partial Z_i(a_i, b_i)}{\partial a_i}$$

for all $i = 1, \dots, n$; and subject finally to a requirement that $\varphi_i(\omega) \in \Phi$ for all $i = 1, \dots, n$; and all states $\omega \in \Omega$.

Then, we will maintain the assumption of validity of the first order approach, which we clearly state in the following assumption:

Assumption 4 (The First Order Approach is Valid). *Any solution to the 'relaxed' Program 3 is also a solution to Program 2.*

We can then show the following lemma:

Lemma 2. *Let $\alpha_{it} = \frac{\partial V}{\partial w_i}(w_t, z_t, b_t, \theta_t)$ in Program 3, and let v_{it} be the Lagrange multiplier associated with (15). If Assumption 4 is satisfied, then $\alpha_{it+1} = \alpha_{it} + v_{it} \frac{f_a^i(y_{it} | a_{it}, z_{it}, \theta_{t+1})}{f^i(y_{it} | a_{it}, z_{it}, \theta_{t+1})}$ for all dates t .*

Proof. The envelope condition with respect to w_i in Program 1 implies that $\frac{\partial V}{\partial w_i}(w_t, z_t, b_t, \theta_t) \equiv \alpha_{it}$ is equal to the Lagrange multiplier on the promise-keeping constraint (5). This fact along with the first order condition for the head's problem with respect to $w'_i(\omega_{t+1})$ then imply that $\alpha_{it+1} = \alpha_{it} + v_{it} \frac{f_a^i(y_{it} | a_{it}, z_{it}, \theta_{t+1})}{f^i(y_{it} | a_{it}, z_{it}, \theta_{t+1})}$. By Assumption 4, these first order conditions characterize the solution to the head's problem in Program 2. \square

Remark that Lemma 2 boils down to Lemma 1 if there is no asymmetric information within the household in which case the incentive constraint is not binding ($v_{it} = 0$).

Proposition 3. *If the First Order Approach (Assumption 4) is satisfied, then the consumption allocation for person i at time t solving Program 2 will satisfy*

$$(16) \quad \Delta \ln \frac{u'_1(\varphi_{1t} c_{1t})}{u'_i(\varphi_{it} c_{it})} = \Delta \ln \frac{h(b_{it})}{h(b_{1t})} - \Delta \ln \frac{p'(\varphi_{it}, \theta_{t+1})}{p'(\varphi_{1t}, \theta_{t+1})} + \ln \left[1 + \frac{v_{it}}{\alpha_{it}} \frac{f_a^i(y_{it} | a_{it}, z_{it}, \theta_{t+1})}{f^i(y_{it} | a_{it}, z_{it}, \theta_{t+1})} \right]$$

for some non-negative scalar v_{it} .

Proof. Let α_{it} denote the Lagrange multiplier associated with the promise-keeping constraint (5) for person i in period t . Then the first order conditions from the head's problem for φ_i and φ_1 imply that

$$(17) \quad \frac{\partial U(\varphi_{it}, c_{it}, b_{it}) / \partial \varphi}{\partial U(\varphi_{1t}, c_{1t}, b_{1t}) / \partial \varphi} \left[\alpha_{it} + v_{it} \frac{f_a^i(y_{it} | a_{it}, z_{it}, \theta_{t+1})}{f^i(y_{it} | a_{it}, z_{it}, \theta_{t+1})} \right] = \frac{p'(\varphi_{it}, \theta_{t+1})}{p'(\varphi_{1t}, \theta_{t+1})} \frac{c_{it}}{c_{1t}},$$

where v_{it} is the Lagrange multiplier associated with the incentive compatibility constraint (15). Lemma 2 implies that the expression in brackets on the left-hand side of (17) is equal to α_{it+1} ; using this fact, we can write

$$(18) \quad \alpha_{it+1} = \frac{\partial U(\varphi_{1t}, c_{1t}, b_{1t}) / \partial \varphi}{\partial U(\varphi_{it}, c_{it}, b_{it}) / \partial \varphi} \frac{p'(\varphi_{it}, \theta_{t+1})}{p'(\varphi_{1t}, \theta_{t+1})} \frac{c_{it}}{c_{1t}}.$$

Using Assumption 2, this becomes

$$\alpha_{it+1} = \frac{u'_1(\varphi_{1t} c_{1t})}{u'_i(\varphi_{it} c_{it})} \frac{p'(\varphi_{it}, \theta_{t+1})}{p'(\varphi_{1t}, \theta_{t+1})} \frac{h(b_{1t})}{h(b_{it})}.$$

Combining this with Lemma 2 it follows that

$$\frac{u'_1(\varphi_{1t} c_{1t})}{u'_i(\varphi_{it} c_{it})} \frac{p'(\varphi_{it}, \theta_{t+1})}{p'(\varphi_{1t}, \theta_{t+1})} \frac{h(b_{1t})}{h(b_{it})} = \alpha_{it} + v_{it} \frac{f_a^i(y_{it} | a_{it}, z_{it}, \theta_{t+1})}{f^i(y_{it} | a_{it}, z_{it}, \theta_{t+1})}.$$

Dividing this expression on both sides by α_{it} , taking logarithms, and re-arranging yields the result. \square

We also have a corresponding characterization governing the assignment of quality, given by the following proposition.

Proposition 4. *The quality assignment which solves Program 3 will satisfy*

$$(19) \quad \eta(\varphi_i(\omega), \theta') = 1 - \frac{\beta}{p(\varphi_i(\omega), \theta')} R_i(c(\omega), w(\omega), z', b, \theta'),$$

where η is the quality elasticity of price. The quantity assignment will be such that

$$(20) \quad h(b_i) u'_i(\varphi_i(\omega) c_i(\omega)) = \frac{\mu(\omega)}{\alpha_i + v_i \frac{f_a^i(y_i | a_i, z_i, \theta')}{f^i(y_i | a_i, z_i, \theta')}} p'(\varphi_i(\omega), \theta')$$

Moreover, increases in R_i will result in a decrease in food quality $\varphi_i(\omega)$ and an increase in food quantity $c_i(\omega)$, ceteris paribus.

Proof. The first order condition of Program 3 with respect to $c_i(\omega)$ is

$$\begin{aligned} \frac{\partial U}{\partial c}(\varphi_i(\omega), c_i(\omega), b_i) \left[\alpha_i + v_i \frac{f_a^i(y_i | a_i, z_i, \theta')}{f^i(y_i | a_i, z_i, \theta')} \right] - \mu(\omega) p(\varphi_i(\omega), \theta') \\ + \beta \mu(\omega) R_i(c(\omega), w(\omega), z', b, \theta') = 0. \end{aligned}$$

Using Assumption 2 and rearranging, this yields

$$(21) \quad h(b_i)\varphi_i(\omega)\alpha_i u'_i(\varphi_i(\omega)c_i(\omega)) \left[1 + \frac{v_i f_a^i(y_i|a_i, z_i, \theta')}{\alpha_i f^i(y_i|a_i, z_i, \theta')} \right] \\ = \mu(\omega)p(\varphi_i(\omega), \theta') - \beta\mu(\omega)R_i(c(\omega), w(\omega), z', b, \theta').$$

The first order condition with respect to $\varphi_i(\omega)$ is

$$\frac{\partial U}{\partial \varphi}(\varphi_i(\omega), c_i(\omega), b_i) \left[\alpha_i + v_i \frac{f_a^i(y_i|a_i, z_i, \theta')}{f^i(y_i|a_i, z_i, \theta')} \right] - \mu(\omega)p'(\varphi_i(\omega), \theta')c_i(\omega) = 0,$$

which (because $c_i(\omega) > 0$), can be similarly rearranged to yield

$$(22) \quad h(b_i)\alpha_i u'_i(\varphi_i(\omega)c_i(\omega)) \left[1 + \frac{v_i f_a^i(y_i|a_i, z_i, \theta')}{\alpha_i f^i(y_i|a_i, z_i, \theta')} \right] = \mu(\omega)p'(\varphi_i(\omega), \theta').$$

Dividing (21) by (22) then yields (19).

Let $\xi(\varphi_i(\omega), \theta') = p(\varphi_i(\omega), \theta') - \varphi_i(\omega)p'(\varphi_i(\omega), \theta')$. Expressed in terms of ξ instead of the elasticity η , (19) becomes $\xi(\varphi_i(\omega), \theta') = \beta R_i(c(\omega), w(\omega), z', b, \theta')$.

Assumption 1 implies that the function ξ is decreasing in $\varphi_i(\omega)$, as $\frac{\partial \xi(\varphi_i(\omega), \theta')}{\partial \varphi_i(\omega)} = -\varphi_i(\omega)p''(\varphi_i(\omega), \theta') < 0$, and hence each state contingent $\varphi_i(\omega)$ is a decreasing function of R_i . We directly see that the Lagrange multiplier v_i with respect to the incentive constraint does not affect the quality assignment conditional on the returns to nutritional investment. That's why this equation is also satisfied for Program 1 even if the returns to investment are different in this case. Using (21) and (19) we obtain (20) and as p' is increasing in quality, a decrease in quality $\varphi_i(\omega)$, everything else equal, decreases $u'_i(\varphi_i(\omega)c_i(\omega))$ meaning that $\varphi_i(\omega)c_i(\omega)$ increases and thus that $c_i(\omega)$ increases. \square

Quality assignment for Program 1 will also satisfy the same equation (19) even if under Program 1, where there is no incentive problem, the consumption quantities $c_i(\omega)$ in (19) will be different than under Program 3 with both nutritional investment and moral hazard (remember that with no moral hazard (no incentive problem), (20) is modified by the fact that $v_i = 0$).

Proposition 4 allows us to show that if there is no moral hazard, given total household resources ($\mu(\omega)$) and given food quality assignment $\varphi_i(\omega)$, individual earnings shocks y_i should not affect the quantity of nutrients $c_i(\omega)$. Indeed, when there is no asymmetric information, $v_i = 0$ in (20) and we obtain:

$$(23) \quad h(b_i)u'_i(\varphi_i(\omega)c_i(\omega)) = \frac{\mu(\omega)}{\alpha_i}p'(\varphi_i(\omega), \theta')$$

Furthermore, we assume that the Monotone Likelihood Ratio Property is satisfied for the probability distribution function of any individual earnings :

Assumption 5 (Monotone Likelihood Ratio Property). *The ratio $\frac{f_a^i(y_i|a_i, z_i, \theta')}{f^i(y_i|a_i, z_i, \theta')}$ increasing in y_i .*

This assumption is commonly used in moral hazard settings (Rogerson, 1985) and means that the log-likelihood of a higher effort is increasing in the outcome y . This is a usual condition allowing to insure that a higher output is informative of a higher effort and prevents optimal contracts to incur some bunching. Then, given the assumption of Monotone Likelihood Ratio Property, Proposition 4 implies that unexpected individual earnings shocks should be positively correlated with individual consumption of nutrient (quantity $c_i(\omega)$), everything else equal and in particular given household resources and given the quality of nutrients $\varphi_i(\omega)$.

3.3.2. Allocations Under Hidden Actions with No Nutritional Investment. Proposition 4 allows us to characterize the assignment of quality within the household when there is both nutritional investment and also hidden actions. We now characterize quality assignment if there are hidden actions but no nutritional investment.

Corollary 1. *If there is no nutritional investment (Assumption 3) and the first order approach is valid (Assumption 4), then assigned quality will not vary across members of the household, and the provision of incentives will be achieved only via changes in the quantity of nutrients, not their quality.*

Proof. Assumption 3 implies that the function R_i is identically zero for all $i = 1, \dots, n$, so that $\eta(\varphi_i(\omega), \theta') = 1$, which in turn implies that $\varphi_i = \varphi_1$ for all $i = 1, \dots, n$. \square

If quality doesn't vary across household members in the absence of nutritional investment, then it immediately follows that when there's no nutritional investment (and the other assumptions of the corollary are satisfied), by using (16), we can express the ratio of marginal utilities of consumption between the head-of-household and person i as

$$(24) \quad \Delta \ln \frac{u'_1(\varphi_{1t} c_{1t})}{u'_i(\varphi_{1t} c_{it})} = \Delta \ln \frac{h(b_{it})}{h(b_{1t})} + \ln \left[1 + \frac{v_{it}}{\alpha_{it}} \frac{f_a^i(y_{it}|a_{it}, z_{it}, \theta_{t+1})}{f^i(y_{it}|a_{it}, z_{it}, \theta_{t+1})} \right],$$

where v_{it} is non-negative, as before. We know that in the absence of nutritional investment the provision of incentives will take place entirely via changes in quantities, and (24) tells us how. This equation implicitly expresses changes in the sharing rule for quality-adjusted consumption between the head and person i as a function of the common quality φ_{it} , of

individual characteristics, of past utility-promises (captured by the α 's) and as a function of the informational value of earnings shocks (which in turn depends upon the likelihood ratio which appears in the final term).

4. ECONOMETRIC IDENTIFICATION AND EMPIRICAL SPECIFICATIONS

4.1. Additional Assumptions for Testing. Before taking the predictions of the models described in Section 3 to the data, we first (i) make some specification choices regarding individuals' utility functions; and (ii) explicitly consider measurement errors for the quantities and prices of consumption.

4.1.1. Individual utility functions. Recall that we assumed that each individual i had a momentary utility function of the form $U(\varphi, c, b) + Z_i(a, b)$ with $U(\varphi, c, b) = h(b)u_i(\varphi c)$ (Assumption 2). For the need of empirical estimation, we adopt the following specification choices: assumptions on both h and u_i .

Assumption 6. *We place the following restrictions on the form of the utility function of consumption for person i :*

- (1) *The utility function $u_i(\varphi c) = \frac{(\varphi c)^{1-\gamma_i}}{1-\gamma_i}$;*
- (2) *$\ln h(b)$ is a linear function of b ;*

Point (1) means that we specify the utility function to exhibit Constant Relative Risk Aversion (CRRA), which is widely used in the literature. Point (2) restricts $\ln h(b)$ to be a single linear index. Then, we partition the vector of characteristics b_{it} into a pair (v_i, x_{it}) where we introduce two different notations for fixed and time-varying individual characteristics. We let v_i denote time invariant characteristics of person i (such as sex or metabolic rate), which may or may not be observed by the econometrician, and we let x_{it} denote observed time-varying characteristics of the same person (such as age and health). Then, without additional loss of generality, let (ι_h, δ_h) be a pair of vectors which select and weight characteristics which influence the utility of consumption of nutrients such that $\ln h(b_{it}) = \iota_h v_i + \delta_h x_{it}$.

4.1.2. Measurement Errors. Considering seriously measurement errors on consumption, expenditures and prices is necessary. Here, individual consumption is only measured on a full but single day during each four-month period, and so even if consumption within this single day was measured without any error whatever, it is still presumably a noisy signal of what consumption was over the entire period. Accordingly, as unit prices are computed from measured expenditures and measured consumption quantities, the measurement errors affecting consumption quantities and expenditures determine the measurement error on prices.

Thus, we assume that the true value of consumption quantity (calories or

proteins) for individual i during period t , c_{it} , is measured with a measurement error $\tilde{\epsilon}_{it}$ such that observed consumption is $\tilde{c}_{it} = c_{it}e^{\tilde{\epsilon}_{it}}$. As measurement errors are likely to exist also on expenditures, we assume that the measured expenditure variable denoted $\widetilde{p_{it}c_{it}}$ is measured with a multiplicative error $e^{\tilde{\xi}_{it}}$ having an unknown mean, such that $\widetilde{p_{it}c_{it}} = p_{it}c_{it}e^{\tilde{\xi}_{it}}$.

Then, the price variable being obtained from the ratio of expenditures and quantities of consumption (nutrients such as calories or proteins), the measured prices \tilde{p}_{it} (e.g., expenditures per calorie) are affected by a multiplicative error since $\tilde{p}_{it} = \frac{\widetilde{p_{it}c_{it}}}{\tilde{c}_{it}} = p_{it}e^{\tilde{\xi}_{it}-\tilde{\epsilon}_{it}}$.

The distribution and expected value of ϵ_{it} and ξ_{it} can vary across individuals, and we don't impose that first differences be mean independent be independent of every other variable but allow them to depend on observable characteristics b_{it} , assuming:

Assumption 7. *Measurement errors $\tilde{\epsilon}_{it}$ and $\tilde{\xi}_{it}$ on consumption quantities and individual expenditures are such that $\tilde{c}_{it} = c_{it}e^{\tilde{\epsilon}_{it}}$ and $\widetilde{p_{it}c_{it}} = p_{it}c_{it}e^{\tilde{\xi}_{it}}$. They satisfy $\tilde{\epsilon}_{it} = \mathbf{v}_c\mathbf{v}_i + \delta_c x_{it} + \epsilon_{it}$ and $\tilde{\xi}_{it} = \mathbf{v}_p\mathbf{v}_i + \delta_p x_{it} + \xi_{it}$, where ϵ_{it} and ξ_{it} are independent across periods and such that $\Delta\epsilon_{it}$ and $\Delta\xi_{it}$ are independent and mean independent of observable variables.*

The parameter vectors (\mathbf{v}_c, δ_c) and (\mathbf{v}_p, δ_p) select and weight characteristics which influence the measurement error in consumption quantities and expenditures.

4.2. Identification and Testing under the Different Regimes. From Proposition 4 it follows that

$$\varphi_{it} = \frac{p_{it} - \beta R_{it}}{p'_{it}}.$$

Quality φ_{it} is not directly observable, but using this along with the chosen specification (Assumption 6) allows us to re-write (16) as

$$(25) \quad \gamma_i \Delta \ln c_{it} - \gamma_1 \Delta \ln c_{1t} = \delta_h (\Delta x_{it} - \Delta x_{1t}) \\ - \Delta \left[(\ln p'_{it} - \ln p'_{1t}) + \gamma_i \ln(\varphi_{it}) - \gamma_1 \ln(\varphi_{1t}) \right] \\ + \ln \left[1 + \frac{\mathbf{v}_{it} f_a^i(y_{it} | a_{it}, z_{it}, \theta_t)}{\alpha_{it} f^i(y_{it} | a_{it}, z_{it}, \theta_t)} \right].$$

Equation (25) pins down the relationship between quantities and individual characteristics, shocks to earnings, returns to nutritional investments, and prices and qualities. However, there are several problems which one must address before using this to construct an estimator. The first of these is that while prices $p_{it} = p(\varphi_{it}, \theta_t)$ can be observed (though perhaps with error), we do not observe the qualities φ_{it} and marginal prices of quality

$p'_{it} = p'(\varphi_{it}, \theta_t)$. We'll propose different approaches to dealing with this problem.

4.2.1. Full information and No Nutritional Investment. If there is full information and no nutritional investment (i.e., there are no hidden actions and Assumption 3 is satisfied), then current consumption contributes only to utility, and doesn't influence productivity or other characteristics. This setting is analogous to that envisioned in most research on *inter*-household risk-sharing, such as Townsend (1994) or the few tests of risk sharing within the household (Dercon and Krishnan (2000), Dubois and Ligon (2011)). Then, idiosyncratic earnings shocks should be efficiently shared, with the consequence that these shocks will have no influence on person i 's consumption relative to the consumption of the household head (per Proposition 1). Further, since in this regime consumption has no influence on productivity or other characteristics of household members, then the assigned quality of nutrients will be the same across all members of the household.

To be more precise, when consumption doesn't affect future productive characteristics (Assumption 3), the prices of food delivered within the household do not vary across individuals and quality is at the level where the elasticity of price with respect to quality is one, per Proposition 2. Then, quantities and qualities within the household will satisfy (11). Combining (11) with assumptions regarding the measurement error process for c_{it} (Assumption 7), or imposing $v_{it} = 0$ and returns to nutritional investment $R_i = R_1 = 0$ in (25), it follows that

$$(26) \quad \Delta \ln \tilde{c}_{it} = \frac{\gamma_1}{\gamma_i} \Delta \ln \tilde{c}_{1t} + \left(\frac{\delta_h}{\gamma_i} + \delta_c \right) \Delta x_{it} - \left(\frac{\delta_h}{\gamma_i} + \frac{\gamma_1}{\gamma_i} \delta_c \right) \Delta x_{1t} \\ + \left(\frac{\gamma_1}{\gamma_i} - 1 \right) \Delta \ln \varphi_{1t} + \Delta \varepsilon_{it} - \frac{\gamma_1}{\gamma_i} \Delta \varepsilon_{1t}.$$

because $\frac{p(\varphi_{1t}, \theta_t)}{p'(\varphi_{1t}, \theta_t)} = \varphi_{1t} = \frac{p(\varphi_{it}, \theta_t)}{p'(\varphi_{it}, \theta_t)} = \varphi_{it}$ and that individual expenditures will follow:

$$(27) \quad \Delta \ln(\widetilde{p_{it} c_{it}}) = \frac{\gamma_1}{\gamma_i} \Delta \ln(\widetilde{p_{1t} c_{1t}}) + \left(\frac{\delta_h}{\gamma_i} + \delta_p \right) \Delta x_{it} - \left(\frac{\delta_h}{\gamma_i} + \frac{\gamma_1}{\gamma_i} \delta_p \right) \Delta x_{1t} \\ - \left(\frac{\gamma_1}{\gamma_i} - 1 \right) \Delta \ln p'(\varphi_{1t}, \theta_t) + \Delta \xi_{it} - \frac{\gamma_1}{\gamma_i} \Delta \xi_{1t}.$$

In words, changes in the quantity of consumption for person i will be proportional to changes in the quantity of consumption for the head, adjusted for the effect of differences in time-varying observable characteristics x_{it} such as age and health and changes in the consumption (common to the

household in this case because $\phi_{it} = \phi_{1t}$), and the effect of possibly unobservable but fixed characteristics such as sex or metabolic rate.

Several remarks are in order: Equation (26) or equation (27) differ from a straightforward extension within the household of the well-known full risk sharing tests of Townsend (1994) in two ways. First, unlike most risk sharing tests, we allow for heterogeneity of preferences and in particular in risk aversion, allowing it to depend on observables as in Dubois (2000) or Dubois (2009). This introduces the changes in household head consumption (or expenditure) on the right hand side variables with a coefficient equal to the ratio of relative risk aversion parameters of each household member with respect to the household head. Second, the fact that preferences depend on both quantity and quality of food (even without dynamic effect introduced by nutritional investment) also introduces another important change seen in equation (26) where changes in the log of quality of food also appears on the right hand side variables. This term disappears only if one assumes homogeneity of preferences with respect to risk ($\gamma_i = \gamma_1$). Otherwise, it shows that changes in the quality of food (reflected in the price for nutrients) at the household level will be negatively correlated with the change in quantity of food of more (than the household head) risk averse members, or that less risk averse household members ($\gamma_i < \gamma_1$) will have a change in the quantity of nutrients that is negatively correlated with the change of quality of the head or the household (since quality will be the same within the household). On expenditures, we see that the change in log expenditures with depend on the change in log marginal prices of quality which by Assumption 1 are increasing functions of quality. Thus, changes in expenditures (relative to the head) will be positively correlated with changes in food quality for household members more risk averse than the household head ($\gamma_i > \gamma_1$) and negatively for others ($\gamma_i < \gamma_1$).

Equations (26) or (27) could very nearly serve as a basis for estimating and testing the hypothesis, but we need to be careful for two measurement issues. First, as the head's observed consumption and observed expenditures may be contaminated by measurement errors, we have an endogeneity problem of the head's consumption quantity in (26) and of the head's expenditures in (27) because

$$E \left(\Delta \epsilon_{it} - \frac{\gamma_1}{\gamma_i} \Delta \epsilon_{1t} \mid \Delta \ln \tilde{c}_{1t}, \Delta x_{it}, \Delta x_{1t} \right) \neq 0,$$

and

$$E \left(\Delta \xi_{it} - \frac{\gamma_1}{\gamma_i} \Delta \xi_{1t} \mid \Delta \ln \widetilde{p_{1t} c_{1t}}, \Delta x_{it}, \Delta x_{1t} \right) \neq 0,$$

since $\Delta \ln \tilde{c}_{1t}$ and $\Delta \ln \widetilde{p_{1t} c_{1t}}$ are correlated with $\Delta \epsilon_{1t}$

To deal with these problems of measurement errors in the head's consumption, we use household level measures of consumption and food expenditures. As these are not based on the individual level measures of caloric intakes, they are good candidate instrumental variables; since measurement errors in household-level measures of consumption or expenditures should be independent of measurement errors in the individual-level measures. Data on household-level consumption and expenditures are collected using an entirely different survey instrument than are data on individual food expenditures, and the period covered by the household-level data is a month, instead of a twenty-four hour period.

Second, though we observe measures of the prices $p(\varphi_{1t}, \theta_t)$, we do not observe $p'(\varphi_{1t}, \theta_t)$ or φ_{1t} which appear on the right hand side of (26) and (27). Omitting the term $(\gamma_1/\gamma_i - 1)\Delta \ln p'(\varphi_{1t}, \theta_t)$ or $(\gamma_1/\gamma_i - 1)\Delta \ln \varphi_{1t}$ would also contaminate the disturbance term with an unobservable which is correlated with other observable right-hand-side variables, again introducing an endogeneity bias. This problem of unobservable quality is fundamental in our proposed generalization of tests of risk sharing, even in the first simple case of no nutritional investment and no moral hazard. If preferences depend on food quality which is an important endogenous choice in consumption, usual tests will be biased by the omitted control for food quality even if it is optimal to choose a common household level quality, just because quality vary across periods and households. The only case where one does not need to control for household level unobserved quality is when preferences are homogenous ($\gamma_i = \gamma_1$) in which case consumption quantity or expenditures growths do not depend on the time varying allocated quality of food.

We thus propose several approaches to dealing with the problem that $p'(\varphi_{1t}, \theta_t)$ and φ_{1t} are unobserved. First, one could assume additional re-

strictions on the function p and for example the assumption that the price function is 'factorizable', so that:

Assumption 8. *[Prices are factorizable] There exist functions (p^1, p^2) such that $p(\varphi, \theta) = p^1(\varphi)p^2(\theta)$.*

In this case, unobserved quality φ does not depend on θ and is constant across periods and states of nature because $\frac{p(\varphi_{it}, \theta_t)}{p'(\varphi_{it}, \theta_t)} = \varphi_{it}$ without nutritional investment and the ratio p/p' does not depend on θ . Thus $\ln \varphi$ falls out of (27) after differencing. Moreover, with (Assumption 8), as φ is constant across states of nature, $\Delta \ln p'(\varphi, \theta) = \Delta \ln p^2(\theta)$ which is equivalent to simple time specific effects (which are not explicitly in equations below just for brevity but that will be introduced in estimations). Thus, with factorizable

price (Assumption 8), (26) and (27) become:

$$(28) \quad \Delta \ln \tilde{c}_{it} = \frac{\gamma_1}{\gamma_i} \Delta \ln \tilde{c}_{1t} + \left(\frac{\delta_h}{\gamma_i} + \delta_c \right) \Delta x_{it} - \left(\frac{\delta_h}{\gamma_i} + \frac{\gamma_1}{\gamma_i} \delta_c \right) \Delta x_{1t} \\ + \Delta \varepsilon_{it} - \frac{\gamma_1}{\gamma_i} \Delta \varepsilon_{1t}.$$

and

$$(29) \quad \Delta \ln(\widetilde{p_{it}c_{it}}) = \frac{\gamma_1}{\gamma_i} \Delta \ln(\widetilde{p_{1t}c_{1t}}) + \left(\frac{\delta_h}{\gamma_i} + \delta_p \right) \Delta x_{it} - \left(\frac{\delta_h}{\gamma_i} + \frac{\gamma_1}{\gamma_i} \delta_p \right) \Delta x_{1t} \\ + \Delta \xi_{it} - \frac{\gamma_1}{\gamma_i} \Delta \xi_{1t}.$$

Second, one could assume that the price function takes a known flexible but parametric form as follows:

Assumption 9. [*Parametric Price Function*]

$$(30) \quad p(\varphi, \theta) = g(\theta) (f(\theta) + \varphi)^\rho$$

for some real-valued positive functions f and g , and parameter $\rho > 1$.

Note that this price function is *not* generally factorizable, and so quality will respond to changes in the state of nature θ . However, under Assumption 9, we have $\frac{p'(\varphi, \theta)}{p(\varphi, \theta)} = \rho \left(\frac{p(\varphi, \theta)}{g(\theta)} \right)^{-1/\rho}$ and thus this particular functional form implies that the logarithm of the ratio p/p' is a linear function of p , so that $\Delta \ln \varphi_{1t} = \Delta \ln \frac{p(\varphi_{1t}, \theta_t)}{p'(\varphi_{1t}, \theta_t)}$ is proportional to $\Delta \ln p(\varphi_{1t}, \theta_t)$.

Then the entire expression involving unobserved quality in (26) and elsewhere is just a linear function of the change in the logarithm of prices, and similarly for (27). Indeed in this case, with Assumption 9, and using the assumption on measurement errors on prices, (26) and (27) become:

$$(31) \quad \Delta \ln \tilde{c}_{it} = \frac{\gamma_1}{\gamma_i} \Delta \ln \tilde{c}_{1t} + \left(\frac{\delta_h}{\gamma_i} + \delta_c \right) \Delta x_{it} - \left(\frac{\delta_h}{\gamma_i} + \frac{\gamma_1}{\gamma_i} \delta_c + \left(\frac{\gamma_1}{\gamma_i} - 1 \right) \frac{1}{\rho} (\delta_c - \delta_p) \right) \Delta x_{1t} \\ + \left(\frac{\gamma_1}{\gamma_i} - 1 \right) \frac{1}{\rho} \Delta \ln \tilde{p}_{1t} + \Delta \varepsilon_{it} - \frac{\gamma_1}{\gamma_i} \Delta \varepsilon_{1t} - \left(\frac{\gamma_1}{\gamma_i} - 1 \right) \frac{1}{\rho} \Delta \xi_{1t}$$

and

$$\begin{aligned}
(32) \quad \Delta \ln(\widetilde{p_{it}c_{it}}) &= \frac{\gamma_1}{\gamma_i} \Delta \ln(\widetilde{p_{1t}c_{1t}}) - \left(\frac{\gamma_1}{\gamma_i} - 1 \right) \frac{\rho - 1}{\rho} \Delta \ln \tilde{p}_{1t} \\
&+ \left(\frac{\delta_h}{\gamma_i} + \delta_p \right) \Delta x_{it} - \left(\frac{\delta_h}{\gamma_i} + \frac{\gamma_1}{\gamma_i} \delta_p + \left(\frac{\gamma_1}{\gamma_i} - 1 \right) \frac{\rho - 1}{\rho} (\delta_c - \delta_p) \right) \Delta x_{1t} \\
&+ \Delta \xi_{it} - \left(\frac{\gamma_1}{\gamma_i} - 1 \right) \frac{\rho - 1}{\rho} \Delta \varepsilon_{1t} + \left(\frac{\gamma_i - \gamma_1}{\rho \gamma_i} - 1 \right) \Delta \xi_{1t}
\end{aligned}$$

Remark that in both previous equations, we omitted the constant and time specific dummies for brevity of equations but these must and will be included in the estimations. Remark also that because of the introduction of the price of the head on the right hand side, the measurement errors of prices also appears in this equation.

Finally, the third way to estimate (26) and (27) is to use the fact that $p(\varphi, \theta)$ being increasing in φ , it is invertible. We can then approximate the inverse quality function of price $\varphi^{-1}(p, \theta)$ to identify parameters of equation with a semi-parametric approach. Using the fact that $p'(\varphi, \theta)$ is increasing in φ and thus invertible, we can approximate the following increasing function $p'^{-1}(\varphi(p, \theta), \theta)$ with a non parametric approach, denoting $p'^{-1}(\varphi(p, \theta), \theta) \equiv \psi(p, \theta)$.

As $\ln \varphi^{-1}(\cdot, \theta)$ and $\ln \psi(\cdot, \theta)$ are unknown, we will approximate them by spline functions. The case where we approximate them with a linear function corresponds to the exact case described before but we will in some sense generalize this approach by introducing some non linear flexible function of price on the right hand side to control for unobserved quality.

Of course, as prices are endogenous, one will need to account also for the endogeneity of these price functions. We will do that by introducing as additional controls for endogeneity the residuals of the first stage price regression (denoted u_{1t-1} and u_{1t}) as suggested by Blundell and Powell (2003) and used for example in a semi-parametric estimation with splines in limited commitment model by Dubois et al. (2008).

Before turning to the explanation of this spline method, remark that we will add an additional restriction mostly for empirical tractability of this non parametric method by assuming that the functions $\ln \varphi^{-1}(p, \theta)$ and $\ln \psi(p, \theta)$ are separable between p and θ . The function of p will be estimated as H_φ for the consumption quantity equation and $H_{p'}$ for the expenditure equation. Then, we use penalized a spline regression following Yu and Ruppert (2002). We choose function $H(\cdot)$ in the following set:

$$(33) \quad H(u; \alpha) = \alpha_1 u + \dots + \alpha_p u^p + \sum_{k=1}^{K-1} \alpha_{p+k} S_p(u - \kappa_k)$$

where

$$S_p(u - \kappa_k) = \begin{cases} (u - \kappa_k)^p \mathbf{1}\{u > \kappa_k\} & \text{for } k > p/2 \\ (\kappa_k - u)^p \mathbf{1}\{u < \kappa_k\} & \text{for } k \leq p/2 \end{cases}$$

where p is the degree of the spline (in practice we will use $p = 3$), $K - 1$ is a fixed number of knots (in practice we will use $K = 10$), κ_k are the $1/K$ -quantiles of u ($\mathbf{1}\{A\}$ is the indicator function of A).

Parameters to estimate are the vector $\alpha = (\alpha_1, \dots, \alpha_{p+K-1})$ of parameters of the spline. The number of knots $K - 1$ is fixed (see Ruppert (2002), for a procedure of selection) and the locations of the knots κ_k are supposed to be given by the $1/K$ -quantiles of u .

Then, (26) will be

$$(34) \quad \Delta \ln \tilde{c}_{it} = \frac{\gamma_1}{\gamma_i} \Delta \ln \tilde{c}_{1t} + \left(\frac{\delta_h}{\gamma_i} + \delta_c \right) \Delta x_{it} - \left(\frac{\delta_h}{\gamma_i} + \frac{\gamma_1}{\gamma_i} \delta_c \right) \Delta x_{1t} \\ + \left(\frac{\gamma_1}{\gamma_i} - 1 \right) \Delta H_\varphi(p_{1t}) + \Delta \varepsilon_{it} - \frac{\gamma_1}{\gamma_i} \Delta \varepsilon_{1t}.$$

Then, (27) will be

$$(35) \quad \Delta \ln(\widetilde{p_{it}c_{it}}) = \frac{\gamma_1}{\gamma_i} \Delta \ln(\widetilde{p_{1t}c_{1t}}) - \left(\frac{\gamma_1}{\gamma_i} - 1 \right) \Delta H_{p'}(p_{1t}) \\ + \left(\frac{\delta_h}{\gamma_i} + \delta_p \right) \Delta x_{it} - \left(\frac{\delta_h}{\gamma_i} + \frac{\gamma_1}{\gamma_i} \delta_p \right) \Delta x_{1t} + \Delta \xi_{it} - \frac{\gamma_1}{\gamma_i} \Delta \xi_{1t}.$$

Denoting $\lambda_{1i} = \frac{\gamma_1}{\gamma_i}$, $\lambda_{2i} = \left(\frac{\delta_h}{\gamma_i} + \delta_c \right)$, $\lambda_{3i} = \left(\frac{\delta_h}{\gamma_i} + \frac{\gamma_1}{\gamma_i} \delta_c \right)$, the estimation method of (34) consists of finding the global minimum of:

(36)

$$Q_c(\lambda_{1i}, \lambda_{2i}, \lambda_{3i}, \lambda_4, \lambda_5, \alpha) = \sum_{i,t} (\Delta \ln \tilde{c}_{it} - \lambda_{1i} \Delta \ln \tilde{c}_{1t} - \lambda_{2i} \Delta x_{it} + \lambda_{3i} \Delta x_{1t} \\ - (\lambda_{1i} - 1) \Delta H_\varphi(p_{1t}) - \lambda_4 u_{1t} - \lambda_5 u_{1t-1})^2 + v \sum_{k=1}^{K-1} \alpha_{p+k}^2$$

where v is a penalty weight penalizing equally the elements α_{p+1} to α_{p+K-1} (so that we do not impose any shape to function $H(\cdot)$). The model then predicts that functions H_φ and $H_{p'}$ should be increasing functions.

Remark that contrary to the application in Dubois et al. (2008) the spline function appears in first difference, we thus are not able to identify its constant term (which is here normalized to zero since $H(0) = 0$) and we define knots as quantiles of the distribution of the price p_{1t} over the T periods of data. The reader is referred to Yu and Ruppert (2002) for all proofs

of asymptotic properties including the sandwich formula for the variance-covariance matrix of estimates.⁴

The same method is applied to the estimation of (35).

Then, one can test for over-identifying restrictions by testing that individual income shocks do not affect these changes in marginal utilities and are thus insured against within the household.

4.2.2. Testing for incentives in the absence of nutritional investment. In the regime with hidden actions but no nutritional investment, current consumption contributes only to utility, and doesn't influence productivity or individual characteristics. As we have seen, perhaps surprisingly, the quality of nutrients is then constant across household members, as in the full information model without nutritional investment—per Corollary 1 it's optimal to provide incentives only by varying the *quantity* of nutrients, not their quality.

However, we don't observe quality directly. Instead, we observe expenditures and quantities, each of which may be measured with error. Prices may of course vary *across* households, either because households demand different levels of quality, or because of (e.g., spatial) variation in the prices households face.

Starting with the fact that $p(\varphi_{it}, \theta_t) = p(\varphi_{1t}, \theta_t)$, adding our assumptions regarding measurement error in prices, and then taking logarithms and differences of the resulting expression gives the estimating equation for observed prices \tilde{p}_{it} :

$$(37) \quad \Delta \ln \tilde{p}_{it} - \Delta \ln \tilde{p}_{1t} = (\delta_p - \delta_c)(\Delta x_{it} - \Delta x_{1t}) + e_{it},$$

where the disturbance term is $e_{it} = \Delta \xi_{it} - \Delta \xi_{1t} + \Delta \epsilon_{1t} - \Delta \epsilon_{it}$. Then the null hypothesis of no nutritional investment implies the testable restriction that the disturbance term e_{it} is mean-independent of (possibly changing) individual characteristics b_{it} , of the public shock θ_t , of individual earnings realizations y_{it} and that the parameter β_p is equal to one in

$$(38) \quad \Delta \ln \tilde{p}_{it} = \beta_p \Delta \ln \tilde{p}_{1t} + (\delta_p - \delta_c)(\Delta x_{it} - \Delta x_{1t}) + e_{it},$$

Of course, because the price for the household head p_{1t} is assumed to be observed only with error, it doesn't follow that $E(e_{it} | \Delta \ln \tilde{p}_{1t}) = 0$. However, we can use our data on household level expenditures and on household level quantities to construct an instrument to deal with this problem. Denoting

⁴Other details concerning the algorithm, the choice of the smoothing parameter v by generalized cross-validation and the choice of other regularization parameters are available upon request.

this instrument for prices by \bar{p}_t , we can then use the conditional moment restriction

$$(39) \quad E(e_{it} | \Delta \ln(\bar{p}_t), \Delta x_{it} - \Delta x_{1t}, \Delta \ln y_{1t}, \Delta \ln y_{it}) = 0$$

both to estimate the parameters (β_p, δ_p) and to test the hypothesis that individual prices are mean independent of individual earnings.

One of the implications of the absence of nutritional investment is that individual prices will be equal within each household, and this can be tested by using (39) as the basis for estimating and testing (37). This test is valid whatever the asymmetry of information within the household because we have shown that this implication is true under the model with moral hazard or not.

However, when actions are hidden, the household head may award additional consumption quantity to members with unexpectedly high earnings as a way of preventing shirking or reducing consumption for household members with earnings which are less than expected—thus, the restrictions implied by (26) should *not* be expected to hold in this environment because of an additional term due to the necessary provision of incentives. Instead, using Proposition 3 and Corollary 1, with our preference specification (Assumption 6) and the measurement errors introduction, we obtain

$$(40) \quad \Delta \ln \tilde{c}_{it} = \frac{\gamma_1}{\gamma_i} \Delta \ln \tilde{c}_{1t} + \left(\frac{\delta_h}{\gamma_i} + \delta_c \right) \Delta x_{it} - \left(\frac{\delta_h}{\gamma_i} + \frac{\gamma_1}{\gamma_i} \delta_c \right) \Delta x_{1t} \\ + \left(\frac{\gamma_1}{\gamma_i} - 1 \right) \Delta \ln \varphi_{1t} + \left(\Delta \varepsilon_{it} - \frac{\gamma_1}{\gamma_i} \Delta \varepsilon_{1t} \right) \\ + \frac{1}{\gamma_i} \ln \left[1 + \frac{v_{it}}{\alpha_{it}} \frac{f_a^i(y_{it} | a_{it}, z_{it}, \theta_t)}{f^i(y_{it} | a_{it}, z_{it}, \theta_t)} \right].$$

When the first order approach is valid (Assumption 4)⁵, we should expect unexpected changes in earnings y_{it}^u to be correlated with the final term. Indeed, the final term implies that quantities of consumption will be related to earnings shocks. In contrast to the case with full information, in which any such shocks are insured away, here consumption varies with earnings in order to provide incentives.

As before, we need to be concerned about the effects of measurement error in the head's consumption, and as before we need to deal with the fact that the quantity φ_{1t} in (40) is unobserved. As in the case in which there

⁵Standard sufficient conditions for the validity of the first order approach in static principal-agent models either include (Rogerson, 1985) or imply (Jewitt, 1988; Conlon, 2009) an assumption that there's a monotone relationship between y_{it}^u and the likelihood ratio $f_a^i(y_{it} | a_{it}, z_{it}, \theta_t) / f^i(y_{it} | a_{it}, z_{it}, \theta_t)$.

was full information, we deal with this latter problem by adopting one of several different assumptions regarding the price function.

As seen before in Section 4.2.1, we have several ways to deal with these unobserved prices that we can apply again here. First, we can assume that prices are factorizable (Assumption 8), then the term involving quality drops out of (40). The second case consists in assuming that besides prices may be "non-factorizable", they take a parametric form described in Assumption 9, then we have

$$\Delta \ln \varphi_{1t} = \Delta \ln \frac{p(\varphi_{1t}, \theta_t)}{p'(\varphi_{1t}, \theta_t)} = \frac{1}{\rho} [\Delta \ln p(\varphi_{1t}, \theta_t) - \Delta \ln g(\theta_t)].$$

In this case, the unobserved quality term in (40) will be replaced by a linear function of changes in log prices. The third case consists in a control function approach, already described in the previous section and which amounts to approximate the inverse quality function of prices to account for $\Delta \ln \varphi_{1t}$. The approximation will in practice be done with polynomials of $\Delta \ln p_{1t}$.

4.2.3. Testing for nutritional investment under full information. The third regime, *nutritional investment* without incentives problems, is one in which the allocation of nutrients affects not only the utility of different household members, but also the production possibility set of the household. In this model, the allocation of energy and protein in the household may respond to changes in the expected marginal productivity of actions for a particular household member (but not to unexpected productivity shocks). The most obvious example might have to do with the additional energy required by some household members during different seasons: household members who engage in heavy agricultural labor may be assigned a disproportionate share of calories during the harvest season, for example, or these same people may receive a greater share of protein in advance of a period of hard labor. In this case, the consumption quantity obeys the following equation (corresponding to (25) with $v_{it} = 0$ and using Proposition 4):

$$\begin{aligned} (41) \quad \Delta \ln \tilde{c}_{it} = & \frac{\gamma_1}{\gamma_i} \Delta \ln \tilde{c}_{1t} + \left(\frac{\delta_h}{\gamma_i} + \delta_c \right) \Delta x_{it} - \left(\frac{\delta_h}{\gamma_i} + \frac{\gamma_1}{\gamma_i} \delta_c \right) \Delta x_{1t} \\ & + \frac{1}{\gamma_i} [\Delta \ln p'_{1t} - \Delta \ln p'_{it}] + \frac{\gamma_1}{\gamma_i} \Delta \ln(\varphi_{1t}) - \Delta \ln(\varphi_{it}) \\ & + (\Delta \varepsilon_{it} - \frac{\gamma_1}{\gamma_i} \Delta \varepsilon_{1t}). \end{aligned}$$

Remark that there are equivalent ways of rewriting this equation using Proposition 4 because $\varphi_{it} = \frac{p_{it} - \beta R_{it}}{p'_{it}}$.

Contrary to the case where there is no nutritional investment, unobserved quality will not be common within the household and thus $\ln \phi_{it}$ and $\ln p'_{it}$ will not disappear when we add the restriction of factorizable price functions (Assumption 8). Assuming the parametric function for price (Assumption 9) also does not help a lot in this case.

We only know that $\ln \phi_{it}$ and $\ln p'_{it}$ are an unknown increasing functions of price p_{it} .

By imposing the price function to be both factorizable (Assumption 8) and parametric (Assumption 9) which amounts to assume the following restriction on prices, we can however obtain a know linear formulation for (41).

Assumption 10. *The price function is such that $p(\phi, \theta) = g(\theta)\phi^\rho$.*

Then, this particular case of a factorizable price function has the advantage of being log linear in quality such that we can replace $\ln \phi_{it}$ and $\ln p'_{it}$ by $\frac{1}{\rho} \ln p_{it}$ and $\frac{\rho-1}{\rho} \ln p_{it}$ respectively, up to time fixed effects. Then, when we assume the parametric factorizable function for price (Assumption 10), and including measurement errors on prices, (41) becomes:

$$\begin{aligned}
 (42) \quad \Delta \ln \tilde{c}_{it} = & \frac{\gamma_1}{\gamma_i} \Delta \ln \tilde{c}_{1t} + \left(\frac{\delta_h}{\gamma_i} + \delta_c - \left(\frac{\gamma_i + \rho - 1}{\rho \gamma_i} \right) (\delta_c - \delta_p) \right) \Delta x_{it} \\
 & - \left(\frac{\delta_h}{\gamma_i} + \frac{\gamma_1}{\gamma_i} \delta_c - \left(\frac{\gamma_1 + \rho - 1}{\rho \gamma_i} \right) (\delta_c - \delta_p) \right) \Delta x_{1t} \\
 & + \left(\frac{\gamma_1 + \rho - 1}{\rho \gamma_i} \right) \Delta \ln \tilde{p}_{1t} - \left(\frac{\gamma_i + \rho - 1}{\rho \gamma_i} \right) \Delta \ln \tilde{p}_{it} \\
 & + \left(\frac{\gamma_1 + \rho - 1}{\rho \gamma_i} \right) (\Delta \varepsilon_{1t} - \Delta \xi_{1t}) - \left(\frac{\gamma_i + \rho - 1}{\rho \gamma_i} \right) (\Delta \varepsilon_{it} - \Delta \xi_{it}) + \left(\Delta \varepsilon_{it} - \frac{\gamma_1}{\gamma_i} \Delta \varepsilon_{1t} \right)
 \end{aligned}$$

In this case, we can estimate all model parameters thanks to the observation of prices for individual i and the household head. We see that, contrary to the case where there is no nutritional investment, household members will be allocated different food qualities.

Of course, one will need to instrument $\Delta \ln p_{it}$ and $\Delta \ln p_{1t}$. In order to instrument for these prices, one can use household level prices but also exogenous shifters of prices and thus of unobserved quality. As given by the unobserved quality equation (19), we know that shifters of returns to investment should be correlated with quality. Assuming that the body mass index (BMI) of individuals affects returns to investment (Foster and Rosenzweig (1994) show it is correlated with wage earnings, and our earnings equation confirms this fact), we can also use the BMI of each individuals as an instrumental variable for prices and thus changes in BMI over periods as instrumental variables for changes in prices.

In the case where we don't want to impose the parametric assumption (Assumption 9), we can replace $\ln p'_{it} + \gamma_i \ln \phi_{it}$ and $\ln p'_{1t} + \gamma_1 \ln \phi_{1t}$ by non parametric functions of observed prices p_{it} and p_{1t} . Indeed, we assumed that the price function is increasing convex, implying that both are increasing functions of unobserved qualities.

Then, we can estimate (41) with polynomials of prices p_{1t} and p_{it} on the right hand side, allows to identify model parameters γ and δ_h under the pure nutritional investment model but also to test the over-identifying restrictions as for all i :

$$\begin{aligned} E(\Delta \varepsilon_{it} - \Delta \xi_{it} | \Delta \ln \tilde{c}_{1t}, \Delta x_{it}, \Delta x_{1t}, y_{it}^u) &= 0 \\ E(\Delta \varepsilon_{it} - (\gamma_1 / \gamma_i) \Delta \varepsilon_{1t} | \Delta \ln \tilde{c}_{1t}, \Delta x_{it}, \Delta x_{1t}, y_{it}^u) &= 0 \end{aligned}$$

which will be rejected in particular if there is moral hazard.

As shown in Section 4.2.1, we can instead use penalized spline functions for $\ln p'_{it} + \gamma_i \ln \phi_{it}$ which we know is an increasing function of price p_{it} .

Using spline functions $H_{p'}$ and H_ϕ as define in (33), denoting

$\lambda_{1i} = \frac{\gamma_1}{\gamma_i}$, $\lambda_{2i} = \left(\frac{\delta_h}{\gamma_i} + \delta_c \right)$, $\lambda_{3i} = \left(\frac{\delta_h}{\gamma_i} + \frac{\gamma_1}{\gamma_i} \delta_c \right)$, $\lambda_{4i} = \frac{1}{\gamma_i}$, the estimation method of (41) consists of finding the global minimum of:

(43)

$$\begin{aligned} Q_c(\lambda_{1i}, \lambda_{2i}, \lambda_{3i}, \lambda_{4i}, \lambda_5, \lambda_6, \alpha) &= \sum_{i,t} (\Delta \ln \tilde{c}_{it} - \lambda_{1i} \Delta \ln \tilde{c}_{1t} - \lambda_{2i} \Delta x_{it} + \lambda_{3i} \Delta x_{1t} \\ &\quad - \lambda_{4i} (\Delta H_{p'}(p_{1t}) - \Delta H_{p'}(p_{it})) - \lambda_{1i} \Delta H_\phi(p_{1t}) + \Delta H_\phi(p_{it}) - \lambda_5 u_{1t} - \lambda_6 u_{1t-1})^2 + v \sum_{k=1}^{K-1} \alpha_{p+k}^2 \end{aligned}$$

where v is a penalty weight. The model then predicts that functions $H_{p'}$ and H_ϕ should be increasing functions.

Moreover, this equation is also useful to get some qualitative predictions. Actually, per Proposition 4, the model predicts that in this regime the *quality* of nutrients ϕ_{it} will be *negatively* correlated with positive shocks to the returns to investment R_{it} . However, given resources and quality of food, the quantity of nutrients should not depend on unexpected individual earnings shocks y_{it}^u . The basic idea is that since the household member is being given additional *quantities* of food (which increases not only productivity but also utility), the household head can reduce the *quality* of food while still making good on *ex ante* utility promises. Thus, in the nutritional investment regime our model predicts that the quantity of nutrients consumed will be positively correlated with predicted earnings if they are positively correlated with returns to investment but not with unpredicted earnings shocks.

4.2.4. Testing for incentives and investment. The fourth regime combines the attributes of the nutritional investment regime and the incentives regime,

and so we title it *incentives and investment*. In this regime consumption is allocated *both* as an investment and to provide incentives for household members to exert effort. In this case, consumption quantity is allocated according to

$$\begin{aligned}
 (44) \quad \Delta \ln \tilde{c}_{it} = & \frac{\gamma_1}{\gamma_i} \Delta \ln \tilde{c}_{1t} + \left(\frac{\delta_h}{\gamma_i} + \delta_c \right) \Delta x_{it} - \left(\frac{\delta_h}{\gamma_i} + \frac{\gamma_1}{\gamma_i} \delta_c \right) \Delta x_{1t} \\
 & + \frac{1}{\gamma_i} [\Delta \ln p'_{1t} - \Delta \ln p'_{it}] + \frac{\gamma_1}{\gamma_i} \Delta \ln(\varphi_{1t}) - \Delta \ln(\varphi_{it}) \\
 & + \frac{1}{\gamma_i} \ln \left[1 + \frac{v_{it}}{\alpha_{it}} \frac{f_a^i(y_{it}|a_{it}, z_{it}, \theta_t)}{f^i(y_{it}|a_{it}, z_{it}, \theta_t)} \right] + (\Delta \varepsilon_{it} - \frac{\gamma_1}{\gamma_i} \Delta \varepsilon_{1t}).
 \end{aligned}$$

As before, this equation includes some unobservable terms that are increasing functions of prices. Again assuming a parametric factorizable function for price (Assumption 10), and including measurement errors on prices, (44) becomes:

$$\begin{aligned}
 (45) \quad \Delta \ln \tilde{c}_{it} = & \frac{\gamma_1}{\gamma_i} \Delta \ln \tilde{c}_{1t} + \left(\frac{\delta_h}{\gamma_i} + \delta_c - \left(\frac{\gamma_i + \rho - 1}{\rho \gamma_i} \right) (\delta_c - \delta_p) \right) \Delta x_{it} \\
 & - \left(\frac{\delta_h}{\gamma_i} + \frac{\gamma_1}{\gamma_i} \delta_c - \left(\frac{\gamma_1 + \rho - 1}{\rho \gamma_i} \right) (\delta_c - \delta_p) \right) \Delta x_{1t} \\
 & + \left(\frac{\gamma_1 + \rho - 1}{\rho \gamma_i} \right) \Delta \ln \tilde{p}_{1t} - \left(\frac{\gamma_i + \rho - 1}{\rho \gamma_i} \right) \Delta \ln \tilde{p}_{it} \\
 & + \left(\frac{\gamma_1 + \rho - 1}{\rho \gamma_i} \right) (\Delta \varepsilon_{1t} - \Delta \xi_{1t}) - \left(\frac{\gamma_i + \rho - 1}{\rho \gamma_i} \right) (\Delta \varepsilon_{it} - \Delta \xi_{it}) \\
 & + \frac{1}{\gamma_i} \ln \left[1 + \frac{v_{it}}{\alpha_{it}} \frac{f_a^i(y_{it}|a_{it}, z_{it}, \theta_t)}{f^i(y_{it}|a_{it}, z_{it}, \theta_t)} \right] + (\Delta \varepsilon_{it} - \frac{\gamma_1}{\gamma_i} \Delta \varepsilon_{1t})
 \end{aligned}$$

Because of the last line of this equation, our model predicts that unpredictable earnings shocks will be positively correlated with quantities of nutrients. Further, when one considers the sign of the correlation between *predictable* earnings shocks (returns to nutritional investment) and quality, the prediction of the model in the investment and incentives regime is also that predictable earnings shocks should be *negatively* correlated with quality.

Though in the investment and incentive regime predictable earnings shocks will be negatively correlated with consumption quality, the direction of the effect of *unpredictable* earnings shocks on quality is ambiguous. Increased quantities of nutrition assigned to a worker for investment purposes will lead the household to reduce the quality assigned, but the efficient provision of incentives in this regime calls for an increase in quality, so the ultimate

sign of the correlation between unexpected earnings shocks and quality depends on which of the investment or incentives effects dominate.

In the pure nutritional investment regime there are no unobserved actions, and hence no need to provide incentives. In this case (19) implies that quality will vary negatively with the expected marginal benefit of nutritional investments. However, a rejection of the hypothesis that quality is perfectly correlated in the household is also consistent with the investment and incentives regime. As we cannot perform a non structural estimation of this model because of the unobserved likelihood ratios, we will proceed as explained in the previous section, by testing the model with over-identifying tests based on the inclusion of individual earnings in (42) which should be significant according to (45). Moreover, (45) and previous propositions predict that unexpected earnings shocks should have a positive coefficient while expected earnings shocks should have a negative coefficient.

Finally, one can also do the same overidentifying test in the case of the semi-parametric estimation of (34) by adding overidentifying individual earnings variables.

4.3. Parameters Identification. In all previous models, we can show easily that $\frac{\gamma_l}{\gamma_i}$, $\frac{\delta_h}{\gamma_i}$, δ_c , δ_p , ρ (when relevant) are identified.

Indeed, $\frac{\gamma_l}{\gamma_i}$ is identified by the coefficient on $\Delta \ln \tilde{c}_{1t}$. Summing the coefficients of Δx_{1t} and Δx_{it} allows to identify $\left(1 - \frac{\gamma_l}{\gamma_i}\right) \delta_c$ and thus δ_c . The identification of δ_c then allows to identify $\frac{\delta_h}{\gamma_i}$ using the coefficient of Δx_{it} . The price equation in the case of no nutritional investment allows identifying $\delta_c - \delta_p$ and thus δ_p is also identified. Then, when relevant the addition of the coefficients of $\Delta \ln \tilde{p}_{1t}$ and $\Delta \ln \tilde{p}_{it}$ identifies $\left(\frac{\gamma_l}{\gamma_i} - 1\right) \frac{1}{\rho}$ and thus ρ .

4.4. Summary of Testable Predictions. The models we have described above give rise to different possible regimes, depending on whether or not consumption influences subsequent productivity (Assumption 3) and on whether or not household members take actions which are hidden from the household head.

Summarizing the results of the different propositions, Table 3 describes the pattern of correlations between both predicted and unpredicted changes in earnings with both the quality and quantity of nutrients under different assumptions of the model regarding asymmetric information and nutritional investment. Under the general encompassing case where information about efforts can be private and consumption is changing future productivity, the

estimation of model parameters is not possible unless additional assumptions are done. Testable predictions summarized in Table 3 give the possibility to reject some hypothesis within the most general model considered here.

Information	Nutritional Investment	Predicted Earnings		Unpredicted Earnings	
		Quantity	Quality	Quantity	Quality
Full	No	0	0	0	0
Private	No	—	0	+	0
Full	Yes	+	—	+	—
Private	Yes	+	—	+	+/-

TABLE 3. Correlations between predicted or unpredicted changes in earnings and the quantity and quality of nutrients in different regimes.

It's ultimately differences in the pattern of these correlations predicted which allows us to advance a claim regarding the regime which actually prevails in the real-world environment which generates the data described in Section 2.

The restrictions on the relationship between earnings and the quantity and quality of consumption derive from Proposition 3 (for quantity) and Proposition 4 (for quality).

5. EMPIRICAL RESULTS

5.1. Tests of Full Risk Sharing. When nutrition is supposed not to influence future characteristics, we can identify all parameters in (28) and (29) using the assumption on price factorization (Assumption 8), or in (31) and (32) using the parametric assumption (Assumption 9), or in (34) and (35) using the control function approach. Of course, measurement errors in prices need also to be taken into account such that coefficients depend on the correlations between measurement errors in prices with the x through coefficients δ_p .

In order to reduce the dimension of parameters to estimate, and given that we use a short panel, we specify the risk aversion parameters as functions of observable characteristics, with $\gamma_i = \gamma'v_i$. As described previously, we use household level expenditure data to instrument the right hand side household head consumption changes and take into account the structure of heteroskedasticity due to measurement errors in the estimation (see Section B in appendix). We estimate a system of three equations, corresponding to

	Expend.	Calories	Protein	F(<i>p</i> -value)
$\frac{\gamma v_1}{\gamma v_i}$: Male	1.6038* (0.0306)	1.4944* (0.0408)	1.6179* (0.0296)	1428.5493 (0.0000)
$\frac{\gamma v_1}{\gamma v_i}$: Female	1.0152* (0.0258)	1.1512* (0.0391)	1.0625* (0.0246)	781.9572 (0.0000)
$\frac{\delta_h}{\gamma v_i}$: Age male	0.4148* (0.1346)	0.3330 (0.1819)	0.4452* (0.1446)	4.8577 (0.0022)
$\frac{\delta_h}{\gamma v_i}$: Age female	0.1814 (0.1559)	0.0398 (0.2108)	0.0670 (0.1675)	0.6086 (0.6093)
$\frac{\delta_h}{\gamma v_i}$: Days sick, male	0.0048 (0.0032)	0.0078 (0.0043)	0.0052 (0.0034)	1.2738 (0.2814)
$\frac{\delta_h}{\gamma v_i}$: Days sick, female	-0.0038 (0.0040)	-0.0058 (0.0054)	-0.0096* (0.0043)	3.6258 (0.0124)
$\frac{\delta_h}{\gamma v_i}$: Pregnant	-0.1676* (0.0835)	-0.2411* (0.1130)	-0.1847* (0.0898)	1.7809 (0.1484)
$\frac{\delta_h}{\gamma v_i}$: Nursing	-0.0098 (0.0102)	-0.0132 (0.0138)	-0.0092 (0.0109)	0.4182 (0.7399)
Second quarter	0.0749* (0.0208)	0.1409* (0.0279)	0.1071* (0.0221)	8.7307 (0.0000)
Third quarter	0.0128 (0.0209)	0.1056* (0.0283)	0.1086* (0.0225)	14.0427 (0.0000)
Fourth quarter	0.0836* (0.0213)	0.0125 (0.0281)	-0.0162 (0.0224)	15.6535 (0.0000)

TABLE 4. Parameter estimates under efficient risk sharing within the household and no nutritional investment. Standard Errors are in parenthesis.

individual food expenditures, individual calorie intake and individual protein intake. For time-varying individual characteristics x_{it} , we have used the logarithm of age, a set of time effects, the number of days sick in the most recent period, an indicator of pregnancy, and a measure of lactation. For fixed individual characteristics, we have simply used sex interacted with whether or not the individual is the head of household. To deal with the issue of the endogeneity of the household head consumption quantity \tilde{c}_{1t} , we use total household expenditures measured from a different module as an instrument for heads' consumption quantity of calories (or proteins). Results are in Table 4. The last column of this Table presents the F tests of the joint significance of coefficients across the three equations.

The ratios of risk aversion parameters γ s can also be interpreted as the elasticity of i 's consumption growth with respect to the growth in the head's consumption. e.g., a one per cent increase in the head's consumption expenditures will deliver a 1.6 percent increase in the expenditures for male household members. Elasticities for other females are around one. These parameter estimates show that males are less risk averse than females and are sheltering females from shocks which buffet total household expenditures; alternatively it shows that males consume a larger share of the surplus in good states.

Unsurprisingly, males tend to receive a larger share of household resources as their age increases; e.g., a 3-year old male with a 23 year old mother will have a consumption expenditure share eleven per cent larger (relative to his mother) at the end of the survey. We also find that sick women consume less protein, but that nutrient intake is not significantly affected. Women who become pregnant have *decreases* in consumption expenditures, calories, and protein intake. Consistent with evidence on low-birth weight from Bouis and Haddad (1990).

These estimates shed light on the intra-household allocation of consumption given the validity of our specification of preferences and given that in fact intra-household allocations are Pareto optimal. If the full information efficient household model is correct, then consumption allocation decisions ought to be orthogonal to individual earnings shocks. In particular, the vector of estimated disturbances ought to be orthogonal to unpredicted individual earning realizations. To implement this overidentifying test, we use the results of the off-farm earnings equation whose estimation is described in Section 2.4. Using weather variation, these earnings equations allow in particular to decompose individual earnings into a predicted and unpredicted part denoted y_{it+1}^p and y_{it+1}^u for any individual i at period t . Actually, as said in section Section 2.4, y_{it+1}^p is obtained from a linear regression of log earnings $\ln y_{it}$ of each individual i at period t on a set of individual characteristics like gender, education level, age, age square, height and height squared, weight and weight squared plus predicted weather variations interacted with village dummy variables and gender dummies (to allow weather variation to affect earnings of males and females differently).

Table 5 shows that we can then reject full risk-sharing within the household because individual income shocks as well as household head income shocks are jointly significant. In particular, unpredicted earnings shocks are significantly affecting consumption changes which constitutes the usual testing strategy of full risk sharing (usually across households). Remark that in our formulation, we control for household aggregate shocks referring to the household head consumption. Under the assumption of complete

	Expend.	Calories	Protein	$F(p)$
y_{1t+1}^p	0.1632 (0.2757)	0.0619 (0.3709)	-0.2186 (0.2940)	2.4632 (0.0605)
y_{1t+1}^u	-0.0089 (0.0260)	-0.0140 (0.0352)	0.0309 (0.0279)	4.9312 (0.0020)
y_{it+1}^p	-0.7757* (0.2654)	-0.6427 (0.3563)	-0.2094 (0.2823)	6.9830 (0.0001)
y_{it+1}^u	0.0404 (0.0297)	-0.0640 (0.0402)	-0.0340 (0.0319)	3.9608 (0.0078)

TABLE 5. Overidentifying Tests of Full Risk Sharing. Standard errors are in parenthesis for coefficients and p-values for the F-tests of the last column.

markets within the household (no private information) and the assumption that there is no nutritional investment, this testing strategy is equivalent to the usual one used for risk sharing test across households where aggregate shocks at the household level would be controlled by average household consumption. In such case, allowing for heterogeneity of preferences with respect to risk involves that the coefficient of household consumption change on the right hand side would be the ratio of the individual relative risk aversion to the household average relative risk aversion. In order to test the robustness of our rejection of full risk sharing, we implement such alternative test which is reported in Table 8 in appendix Section D. We find then an even stronger rejection of full risk sharing.

If prices are not factorizable (Assumption 8 is not valid), unobserved food quality may bias those estimates. Thus we implement such tests allowing for unobserved quality to affect consumption changes as in (??)

5.2. Testing for Nutritional Investment Effects. The interpretation of the previous tests can be misleading as nutritional investment effects can generate a correlation between consumption and individual earnings, just as in the efficiency wages theory based on the nutrition-health-productivity relationship. We thus now test for the presence of nutritional investment using (38) which can be rewritten as:

$$(46) \quad \Delta \ln \tilde{p}_{it} = \beta_p \Delta \ln \tilde{p}_{1t} + (\delta_p + \delta_c)(\Delta x_{it} - \Delta x_{1t}) + e_{it},$$

Our model predicts that all incentives will be provided via variations in quantity of nutrients, not quality, even if moral hazard is present and imposes a constrained efficient allocation of consumption, incentives being given only through the quantity of nutrients. Here, for tests involving the

	Calorie Price	Protein Price	F(<i>p</i> -value)
β_p : Male	3.8164* (0.1087)	1.8719* (0.0365)	1615.2035 (0.0000)
β_p : Female	1.5882* (0.0985)	1.0857* (0.0381)	470.3805 (0.0000)
δ_p : Age male	0.0036* (0.0017)	-0.0183 (0.0257)	10.1705 (0.0000)
δ_p : Age female	0.0000 (0.0020)	0.0140 (0.0297)	0.2853 (0.7518)
Days sick, male	0.0001* (0.0000)	0.0009 (0.0006)	5.4003 (0.0045)
Days sick, female	0.0000 (0.0001)	0.0010 (0.0008)	1.1505 (0.3165)
Pregnant	0.0003 (0.0011)	-0.0016 (0.0159)	0.1688 (0.8447)
Nursing	-0.0000 (0.0001)	-0.0007 (0.0019)	0.1451 (0.8649)
Second quarter	0.0004 (0.0003)	0.0048 (0.0039)	1.1122 (0.3289)
Third quarter	-0.0007* (0.0003)	-0.0112* (0.0040)	4.0745 (0.0170)
Fourth quarter	0.0011* (0.0003)	0.0147* (0.0041)	8.0336 (0.0003)

TABLE 6. Tests of Nutritional Investment. Standard errors are in parenthesis.

relationship between the individual characteristics b_{jt} and quality prices, we use a set of time-varying individual characteristics $\Delta \ln x_{it}$, which include a set of (quarterly) time effects; interactions between sex and the logarithm of age in years, and between sex and the number of days sick in the most recent period; an indicator with the value of one if person i is in the second or third trimester of pregnancy; and a measure of lactation (the number of minutes spent nursing per day). For each of these measures we compute the difference between the value of the measure for person i and the value of the same measure for the household head. We also use gender as fixed individual characteristic.

Table 6 shows that $\beta_p = 1$ can be rejected, which means that the quality of food varies across household members, rejecting the absence of nutritional investment.

Either of the regimes without nutritional investment implies that the coefficient associated with the quality (price) of the head's consumption should be one. In Table 6 we interact the (change in the logarithm of) the head's consumption quality with the sex of person i , allowing us to test whether changes in the allocation of quality within the household depend on sex. Variables used to estimate (37) are all transformed in such a way that the estimated coefficients can be interpreted as elasticities.

The first two rows of Table 6 provide a dramatic rejection of the hypothesis that nutritional investment doesn't matter. For every one per cent increase in the cost per calorie for the head, we estimate that males in the household will receive an increase of 3.8 per cent, while other females in the household will receive an increase of 1.5 per cent. Each of these estimated coefficients is highly significant, and significantly different from one. Estimated elasticities associated with the cost per gram of protein are less dramatic, but also significantly different from one. Further, the coefficients associated with the costs of both calories and protein are jointly significant (the final column of Table 6 reports the F -statistics associated with the joint test of the hypothesis that both coefficients are zero, with p -values in parenthesis).

The results show that males see significantly more variation in quality than female household members. This leads us to reject the hypothesis that private information alone can account for rejection of full risk sharing. It proves that nutritional investment must be present.

5.3. Tests of Nutrition and Incentives. Now, some individual characteristics of an individual that affect preferences and productivity are allowed to evolve endogenously over time through the consumption quantity. The idea here is that e.g., person i 's weight at $t + 1$ may depend on his weight in the previous period as well as on his consumption. As already said, the evolution of b_{it} is assumed *not* to depend on the quality of consumption ϕ_{it} ; but only on quantities c_{it} .

The central interpretation of our model predictions is that if a household member is given more food as an *investment*, we should see an increase in *quantity* and earnings, but can also expect a *decrease* in quality.

Table 7 reports results of projecting changes in different measures of prices (using both the changes in the unit cost per calorie and in the unit cost per gram of protein).

As Table 7 shows, because quality goes up in response to unexpected earnings shocks, it means that incentives play a role in determining allocations. Quality goes down in response to *expected* earnings.

	Calorie Price	Protein Price	F(<i>p</i> -value)
y_{1t+1}^p	0.0033 (0.0035)	0.0577 (0.0524)	0.6111 (0.5428)
y_{1t+1}^u	-0.0002 (0.0003)	-0.0098* (0.0050)	3.4886 (0.0306)
y_{it+1}^p	-0.0055 (0.0034)	-0.1107* (0.0504)	2.4266 (0.0884)
y_{it+1}^u	0.0002 (0.0004)	0.0044 (0.0057)	0.3244 (0.7230)

TABLE 7. Tests of Nutritional Investment. Standard errors are in parenthesis.

We infer that the assignment of food within households in our dataset depends on nutritional investments, and assert that neither the full information without nutritional investment regime nor the pure incentive regime describes the mechanism used to assign food quality in the Philippine setting which generated our data.

We thus have two distinct explanations for the failure of the standard full risk sharing hypothesis. Predictable changes in earnings help explain consumption and prices and thus quality of individual food consumption is varying across household members, which leads to consider a model with nutritional investments. A nutrition based efficiency wage effect seems at work. Moreover, as off-farm labor can't be observed, food plays an incentive role and augmenting the private information model to allow nutritional investments helps to reconcile the model with the data. In particular, unpredictable shocks in earnings lead to increases in quantity of nutrients. Though high earners receive more calories and protein, *in addition* they receive higher quality (more expensive) food. This is consistent with the provision of incentives.

6. CONCLUSION

In this paper we have constructed a direct test of the hypothesis that food is efficiently allocated within households in some part of the rural Philippines using individual observation of food consumption. Conditional on our specification of preferences, we are able to reject this hypothesis, as the allocation of food expenditures, calories, and protein is significantly related to the realization of each individual's off-farm earnings. Our tests also allow household members to have different risk preferences so that the rejection cannot be attributed to a differential tolerance with respect to uncertain consumption by men versus women (for example).

We then turn to two alternative explanations of this feature of the data. We first consider a model in which the off-farm efforts of individual family members cannot be observed, so that the allocation of food is designed to provide incentives to these workers. Second, we consider a model in which food consumption produces not only utils but also functions as a form of nutritional investment, which may be used to directly influence the marginal productivity of workers. Of these two motives (investment and incentives), we are able to reject the hypothesis that changes in the allocation of food are used *solely* to provide incentives, and are similarly able to reject the hypothesis that changes in the allocation of food are used solely as a form of nutritional investment. We are left with evidence that households in this setting allocate food both to provide incentives and as a form of nutritional investment. These two mechanisms are reminiscent of efficiency wages theories both in its nutrition-health-productivity and incentives provision versions.

APPENDIX A. DATA

Details on the consumption data:

- (1) Data on individual food intake comes from 24 hour recall interviews. Some eighty different sorts of foodstuffs are found in the data, but only 49 appear with sufficient frequency to be usefully categorized as anything but “other.” These include corn (boiled/grits/meal), soft drinks, alcoholic drinks, rice and rice products, corn products, bread products, kamote, potatoes, cassava and cassava products, other root crops, sugar, cooking oil, mantika, fresh fish, dried fish, shrimps and other shellfish, cooked meat, organ meat, processed meat, chicken, bagoong, patis, buro, sardines, pork (lean), beef (lean), carabeef and goat meat, eggs (all types), milk (all types), mongo, soybeans, other dried beans, kamote tops, kangkong, malonggay, other leafy greens, squashes, tomatoes, mangoes and papayas, bananas, other fruit, and other vegetables.
- (2) Calorie and protein individual intakes are computed using equivalence tables of these quantitative food intakes on each of the 49 food categories.
- (3) Individual expenditures of food are computed using these food intakes valued at a household-specific price.

APPENDIX B. VARIANCE

For individual i and household j and period t , ϵ_{it}^j is the measurement error on consumption for individual i in household j .

Our assumptions on measurement errors imply that

$$\begin{aligned} V\left(\Delta \epsilon_{it}^j\right) &= 2\sigma_{\epsilon}^2 \forall i, j, t \\ E\left(\Delta \epsilon_{it}^j \Delta \epsilon_{1t}^{j'}\right) &= 0 \forall i, j, j', t \end{aligned}$$

Then, we obtain that

$$E\left[\left(\Delta \epsilon_{it}^j - \frac{\gamma_1}{\gamma_i} \Delta \epsilon_{1t}^j\right) \left(\Delta \epsilon_{1t'}^{j'} - \frac{\gamma_1}{\gamma_{i'}} \Delta \epsilon_{1t'}^{j'}\right)\right] = 0 \text{ if } j \neq j'$$

and for all $j = 1, \dots, J$:

$$\begin{aligned}
 E \left[\left(\Delta \varepsilon_{it}^j - \frac{\gamma_1}{\gamma_i} \Delta \varepsilon_{1t}^j \right) \left(\Delta \varepsilon_{i't'}^j - \frac{\gamma_1}{\gamma_{i'}} \Delta \varepsilon_{1t'}^j \right) \right] &= 2 \left(1 + \left(\frac{\gamma_1}{\gamma_i} \right)^2 \right) \sigma_\varepsilon^2 \text{ if } i = i' \text{ and } t = t' \\
 &= 2 \frac{\gamma_1^2}{\gamma_i \gamma_{i'}} \sigma_\varepsilon^2 \text{ if } i \neq i' \text{ and } t = t' \\
 &= - \left(1 + \left(\frac{\gamma_1}{\gamma_i} \right)^2 \right) \sigma_\varepsilon^2 \text{ if } i = i' \text{ and } |t - t'| = 1 \\
 &= - \frac{\gamma_1^2}{\gamma_i \gamma_{i'}} \sigma_\varepsilon^2 \text{ if } i \neq i' \text{ and } |t - t'| = 1
 \end{aligned}$$

We denote Λ_j the following matrix

$$\begin{aligned}
 \Lambda_j &= I_{N_j-1} + \left[\frac{\gamma_1^2}{\gamma_i \gamma_{i'}} \right]_{i=2, \dots, N_j, i'=2, \dots, N_j} \\
 &= \begin{bmatrix} 1 + \left(\frac{\gamma_1}{\gamma_2} \right)^2 & \frac{\gamma_1^2}{\gamma_3 \gamma_2} & \cdot & \frac{\gamma_1^2}{\gamma_{N_j} \gamma_2} \\ \frac{\gamma_1^2}{\gamma_2 \gamma_3} & 1 + \left(\frac{\gamma_1}{\gamma_3} \right)^2 & \cdot & \frac{\gamma_1^2}{\gamma_{N_j} \gamma_3} \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\gamma_1^2}{\gamma_2 \gamma_{N_j}} & \frac{\gamma_1^2}{\gamma_3 \gamma_{N_j}} & \cdot & 1 + \left(\frac{\gamma_1}{\gamma_{N_j}} \right)^2 \end{bmatrix} \quad \dim(N_j - 1, N_j - 1)
 \end{aligned}$$

The variance covariance matrix of household j ,

$$\Omega_j = \left[E \left(\Delta \varepsilon_{it}^j - \frac{\gamma_1}{\gamma_i} \Delta \varepsilon_{1t}^j \right) \left(\Delta \varepsilon_{i't'}^j - \frac{\gamma_1}{\gamma_{i'}} \Delta \varepsilon_{1t'}^j \right) \right]_{i=1, \dots, N_j, t=1, \dots, T} \quad (47)$$

is such that

$$\Omega_j = \sigma_\varepsilon^2 \begin{bmatrix} 2\Lambda_j & -\Lambda_j & 0 & 0 \\ -\Lambda_j & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & -\Lambda_j \\ 0 & 0 & -\Lambda_j & 2\Lambda_j \end{bmatrix} \quad \dim((N_j - 1) \times T, (N_j - 1) \times T)$$

The variance covariance matrix Ω of $\left(\Delta \epsilon_{it}^j - \frac{\gamma_1}{\gamma_t} \Delta \epsilon_{1t}^j\right)_{i,j,t}$ for the whole sample is then

$$\Omega = \begin{bmatrix} \Omega_1 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \Omega_J \end{bmatrix} \quad \dim \left(\sum_{j=1}^J (N_j - 1) \times T, \sum_{j=1}^J (N_j - 1) \times T \right)$$

APPENDIX C. ESTIMATOR

Here we devise an estimator with which to estimate the system of equations.

Notations:

$$\begin{aligned} \xi &= \begin{bmatrix} \xi_{11} \\ \vdots \\ \xi_{it} \\ \vdots \\ \xi_{NT} \end{bmatrix}, X^{k'} = \begin{bmatrix} X_{11} \\ \vdots \\ X_{it} \\ \vdots \\ X_{NT} \end{bmatrix} = \begin{bmatrix} (X_{11}(1), \dots, X_{11}(p)) \\ \vdots \\ (X_{it}(1), \dots, X_{it}(p)) \\ \vdots \\ (X_{NT}(1), \dots, X_{NT}(p)) \end{bmatrix} \quad (\dim : NT * p), \\ Z &= \begin{bmatrix} Z_{11} \\ \vdots \\ Z_{it} \\ \vdots \\ Z_{NT} \end{bmatrix} = \begin{bmatrix} (Z_{11}(1), \dots, Z_{11}(k)) \\ \vdots \\ (Z_{it}(1), \dots, Z_{it}(k)) \\ \vdots \\ (Z_{NT}(1), \dots, Z_{NT}(k)) \end{bmatrix} \quad (\dim : NT * k), Y = \begin{bmatrix} Y_{11} \\ \vdots \\ Y_{it} \\ \vdots \\ Y_{NT} \end{bmatrix} \quad (\dim : 3NT * 1) \\ \xi &= \begin{pmatrix} \xi^1 \\ \xi^2 \\ \xi^3 \end{pmatrix} \quad (\dim : 3NT * 1), X = \begin{pmatrix} X^1 & 0 & 0 \\ 0 & X^2 & 0 \\ 0 & 0 & X^3 \end{pmatrix} \quad (\dim : 3p * 3NT), \beta = \\ &\begin{pmatrix} \beta^1 \\ \beta^2 \\ \beta^3 \end{pmatrix} \quad (\dim : 3p * 1) \\ Z &= \begin{pmatrix} Z^1 \\ Z^2 \\ Z^3 \end{pmatrix} \quad (\dim : 3NT * k), Y = \begin{pmatrix} Y^1 \\ Y^2 \\ Y^3 \end{pmatrix} \quad (\dim : 3NT * 1) \end{aligned}$$

Then

$$\begin{aligned} Y &= X'\beta + \xi \\ \dim &: (3NT * 1) = (3NT * 3p) * (3p * 1) + (3NT * 1) \end{aligned}$$

Denoting

$$\begin{aligned} P_Z &= Z(Z'Z)^{-1}Z' \\ \dim &: (3NT * 3NT) \end{aligned}$$

we have

$$\begin{aligned} \beta_{OLS} &= (X'X)^{-1}X'Y \\ \beta_{IV} &= (X'P_ZX)^{-1}X'P_ZY \\ \beta_{GLS} &= (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y \\ \beta_{IVGLS} &= (X'P_Z\Omega^{-1}P_ZX)^{-1}X'P_Z\Omega^{-1}Y \end{aligned}$$

$$\begin{aligned} \text{var}(\beta_{OLS}) &= (X'X)^{-1} \\ \text{var}(\beta_{IV}) &= (X'P_ZX)^{-1} \\ \text{var}(\beta_{GLS}) &= (X'\Omega^{-1}X)^{-1} \\ \text{var}(\beta_{IVGLS}) &= (P_ZX\Omega^{-1}X'P_Z)^{-1} \end{aligned}$$

where $\Omega = E(\xi\xi')$

$$\Omega = E(\xi\xi') = E \begin{pmatrix} \xi^1 \\ \xi^2 \\ \xi^3 \end{pmatrix} \begin{pmatrix} \xi^1 \\ \xi^2 \\ \xi^3 \end{pmatrix}' = \begin{pmatrix} E\xi^1\xi^{1'} & E\xi^2\xi^1 & E\xi^1\xi^{3'} \\ E\xi^2\xi^{1'} & E\xi^2\xi^{2'} & E\xi^2\xi^{3'} \\ E\xi^3\xi^{1'} & E\xi^3\xi^{2'} & E\xi^3\xi^{3'} \end{pmatrix} \quad (\dim : 3NT * 3NT)$$

Using the matrix notations:

- When

$$\begin{aligned} \text{cov}(\xi_{it}, \xi_{i't}) &= \sigma_{kk'} \text{ if } i = i' \\ &= 0 \text{ if } i \neq i' \end{aligned}$$

then

$$\Omega = \Sigma_3 \otimes I_{NT}$$

with

$$\Sigma_3 = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix}$$

$$\Omega^{-1} = \Sigma_3^{-1} \otimes I_{NT}$$

Thus

$$\begin{aligned} \beta_{GLS} &= (X'(\Sigma_3^{-1} \otimes I_{NT})X)^{-1}X'(\Sigma_3^{-1} \otimes I_{NT})Y \\ \beta_{IVGLS} &= (X'P_Z(\Sigma_3^{-1} \otimes I_{NT})P_ZX)^{-1}X'P_Z(\Sigma_3^{-1} \otimes I_{NT})Y \end{aligned}$$

- When

$$cov(\xi_{it}, \xi_{i't'}) = \sigma_{kk'} \lambda(z_{it} - z_{i't'})$$

then

$$\Omega = \Sigma_3 \otimes \Sigma(z)$$

with

$$\begin{aligned} \Sigma(z) &= I_{NT} + \lambda \left[(z * J_{1,NT}) - (z * J_{1,NT})' \right] \\ \Omega^{-1} &= \Sigma_3^{-1} \otimes \Sigma(z)^{-1} \end{aligned}$$

- When

$$\begin{aligned} cov(\xi_{it}, \xi_{i't'}) &= \sigma_{kk'} \lambda(z_{it} - z_{i't'}) \\ cov\left(\frac{\xi_{it}}{\gamma'_k v_i}, \frac{\xi_{i't'}}{\gamma'_{k'} v_{i'}}\right) &= \sigma_{kk'} \frac{\lambda(z_{it} - z_{i't'})}{(\gamma'_k v_i) (\gamma'_{k'} v_{i'})} \end{aligned}$$

then

$$\Omega = \Sigma_3 \otimes \Sigma(z)$$

with

$$\Sigma(z) = (I_{NT} + \lambda \left[(z * J_{1,NT}) - (z * J_{1,NT})' \right]) \cdot / (A)$$

where $\cdot /$ is for the element by element division and

$$\begin{aligned} A[it, i't'] &= (\gamma'_k v_i) (\gamma'_{k'} v_{i'}) \\ \Omega^{-1} &= \Sigma_3^{-1} \otimes \Sigma(z)^{-1} \end{aligned}$$

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APPENDIX D. ROBUSTNESS CHECKS

	Food Exp.	Calories	Protein	F (p -value)	Food Exp.	Calories	Protein	F (p -value)
$\frac{\gamma'v_{1i}}{\gamma'v_i}$: Male	0.4702* (0.0735)	0.0341 (0.1268)	0.1847 (0.1065)	12.9939 (0.0000)	0.4403* (0.0726)	0.0317 (0.1128)	0.1684 (0.0999)	11.6410 (0.0000)
$\frac{\gamma'v_{1i}}{\gamma'v_i}$: Female	0.6032* (0.0700)	0.3862* (0.0927)	0.3739* (0.0823)	16.1549 (0.0000)	0.5875* (0.0686)	0.3746* (0.0865)	0.3654* (0.0794)	15.5333 (0.0000)
$\frac{\delta}{\gamma'v_i}$: Age male	0.2151* (0.1029)	0.1902 (0.1384)	0.3819* (0.1016)	6.9259 (0.0001)	0.1946 (0.1040)	0.2052 (0.1283)	0.3830* (0.0988)	7.4874 (0.0001)
$\frac{\delta}{\gamma'v_i}$: Age female	0.0897 (0.1151)	0.2634* (0.1287)	0.2378* (0.1078)	2.0944 (0.0987)	0.0899 (0.1138)	0.2706* (0.1281)	0.2432* (0.1075)	2.1150 (0.0960)
Days sick, male	-0.0165* (0.0028)	-0.0037 (0.0031)	-0.0087* (0.0027)	15.4364 (0.0000)	-0.0165* (0.0028)	-0.0036 (0.0031)	-0.0087* (0.0026)	15.8961 (0.0000)
Days sick, female	-0.0067* (0.0025)	-0.0002 (0.0028)	-0.0033 (0.0023)	4.2902 (0.0049)	-0.0068* (0.0025)	-0.0003 (0.0027)	-0.0033 (0.0023)	4.3165 (0.0048)
Pregnant	-0.0862 (0.0726)	-0.1526 (0.0852)	-0.1290 (0.0684)	1.2427 (0.2924)	-0.0846 (0.0718)	-0.1512 (0.0839)	-0.1280 (0.0680)	1.2350 (0.2952)
Nursing	0.0048 (0.0099)	0.0074 (0.0107)	0.0066 (0.0091)	0.1856 (0.9063)	0.0057 (0.0098)	0.0079 (0.0106)	0.0073 (0.0091)	0.2219 (0.8812)
Second quarter	0.0234 (0.0236)	0.0059 (0.0259)	-0.0100 (0.0222)	1.1556 (0.3251)	0.0048 (0.0229)	-0.0044 (0.0225)	-0.0225 (0.0203)	1.0862 (0.3535)
Third quarter	0.0941* (0.0166)	-0.0159 (0.0219)	-0.0335 (0.0173)	12.2146 (0.0000)	0.1161* (0.0172)	-0.0158 (0.0234)	-0.0282 (0.0185)	15.2183 (0.0000)
Fourth quarter	-0.0993* (0.0194)	0.0250 (0.0197)	0.0834* (0.0170)	17.4917 (0.0000)	-0.1302* (0.0215)	0.0284 (0.0234)	0.0792* (0.0198)	15.8459 (0.0000)
y_{1t+1}^p	— —	— —	— —	— —	-0.4855* (0.1369)	-0.0374 (0.1840)	-0.1675 (0.1429)	4.0752 (0.0067)
y_{1t+1}^u	— —	— —	— —	— —	0.0040 (0.0139)	0.0106 (0.0157)	0.0131 (0.0135)	0.4097 (0.7461)
y_{it+1}^p	— —	— —	— —	— —	0.3682 (0.2635)	-0.2278 (0.3319)	-0.0619 (0.2684)	0.8312 (0.4765)
y_{it+1}^u	— —	— —	— —	— —	0.0554* (0.0251)	0.0475 (0.0300)	0.0377 (0.0246)	1.6861 (0.1676)

TABLE 8. Full Risk Sharing using Average Household Consumption for Aggregate Shock. Standard Errors are in parenthesis.