Systemic banks and the lender of last resort*

Jorge Ponce† Marc Rennert‡

March 21, 2012

First version: October 2011

Preliminary and incomplete

Abstract

This paper proposes a model where systemic banks coexist with non-systemic banks. Troubles in a systemic bank may hurt non-systemic banks but not vice versa. We analyze the decision of the central banker and the deposit insurer to provide emergency liquidity assistance to illiquid banks whose solvency conditions are only observed through supervision. We find that the existence of systemic banks provides a rationale for the central bank to act as lender of last resort in a larger range of banks' liquidity shortfalls than when all banks are non-systemic. We discuss policy implications that inform current reform efforts to the architecture of banking regulation and supervision.

1 Introduction

The recent financial crisis has shown that one source for the fragility of the financial system is the concentration of banking activities on systemically important institutions. The existence of systemic banks imposes special challenges to policymaking because a failure of one of those creates significant negative externalities to other banks, the financial system and the economy as a whole. After an intensive political and academic debate the Basel Committee on Banking Supervision and the Financial Stability Board announced

*We thank Dominik Grafenhofer, David Heilmann und Markus Reisinger. All errors remain our own. The views expressed herein are those of the authors and do not necessarily represent the views of the institutions to which they are affiliated.

†Banco Central del Uruguay

‡Deutsche Bundesbank and IAE Toulouse

on the 4^{th} of November 2011[1] policy measures to address systemically important financial institution: higher capital requirements, recovery and resolution plans as well as more efficient supervision. However, the implication of considering the existence of systemically important banks on the design of the lender of last resort policy has not received much attention. Although it is a very important issue for policy makers.

We offer a study of the implications of the existence of systemically important banks for the design of the lender of last resort policy. In our model a systemic and a non-systemic bank coexist. Both banks invest their deposits into risky illiquid assets. A failure of the systemic bank may hurt the return of the non-systemic bank but not vice versa. A collapse might occur because banks are exposed to a verifiable liquidity shock. Only an emergency liquidity assistance from a lender of last resort can ensure the continuation of the bank in trouble. It is social optimal to provide emergency liquidity assistance if the bank's asset quality is high. The quality of the banks' assets is not verifiable and, for this reason, not contractible. Self-interested supervisors (i.e. a central banker and a deposit insurer) observe the quality of the banks' assets and decide whether or not to support the illiquid bank with an emergency loan. The deposit insurer has to compensate the depositors of a failed bank, but it can liquidate the illiquid bank at the interim state and realize the liquidation value. For this reason it is biased towards liquidation. The central bank's expected losses increase with the size of the emergency loan. Hence, it provides emergency liquidity assistance if the quality of the banks' risky assets is high enough. Additionally, the policy maker has the unconditional bailout rule at his disposal. This policy tool implies that the bank in trouble receives an emergency loan independent of the quality of its assets.

Our main result is that the existence of systemic banks provide a rationale for the central bank to act as lender of last resort in a larger range of banks' liquidity shortfalls than when all banks are non-systemic. The second best optimal allocation of the lender of last resort responsibilities for the systemic as well as for the non-systemic bank consists of two intervals. For small liquidity shocks the central bank should be in charge of the lender of last resort responsibility. For larger liquidity shortfalls the unconditional bailout rule should be applied. Comparing the allocation with the framework of only non-systemic banks the central bank is in charge for a larger range of liquidity shocks.

To explain this result we have to distinguish between the non-systemic and the systemic

[1]The press release of the Basel Committee on Banking Supervision is available following *http://www.bis.org/press/p111104.htm*. The press release of the Financial Stability Board can be found following *http://www.financialstabilityboard.org/press/pr_111104cc.pdf*. A list of 29 global systemically important banks was published by the Financial Stability Board at the 4^{th} of November 2011 (*http://www.financialstabilityboard.org/publications/r_111104bb.pdf*)

bank. First of all, note that the central bank's lending decision gets tougher with increasing liquidity shocks, because its expected losses increase with the emergency loan provided to the bank in trouble. In a framework consisting only of non-systemic banks the central bank is too soft for small liquidity shock and too tough for large liquidity shocks compared to the first best lending decision. From these observation, the following can be shown for the non-systemic bank. Given that the central bank's expected losses are unaffected by negative externalities of the systemic bank on the non-systemic bank the lending decision does not change. The first best lending decision for the non-systemic bank in contrast gets tougher if the systemic bank fails because the expected return of the non-systemic bank's asset is lower. As a consequence the central bank is assigned as the lender of last resort for a larger interval of liquidity shocks at the expense of the unconditional bailout rule because the central bank's lending decision is now closer to the first best lending decision than the unconditional bailout rule.

Concerning the systemic bank there are two effects. On the one hand the benchmark the first best allocation internalize the negative externalities of the systemic bank on the non-systemic bank and is for this reason more forbearing. Everything else constant this implies that the central bank should act as the lender of last resort on a smaller interval of liquidity shocks. The central bank on the other hand does internalize the effect of a liquidation of the systemic bank on the central bank's expected losses from the non-systemic bank. This makes the central bank also more forbearing. Under some conditions the central bank is so much forbearing such that it is optimal to assign it with the lender of last resort responsibilities for a larger interval of liquidity shortfalls.

Our model is inspired by Repullo (2000) and Espinosa-Vega, Kahn, Matta, and Sole (2011). Repullo (2000) first addressed the question of optimal institutional allocation of lender of last resort responsibilities. Espinosa-Vega, Kahn, Matta, and Sole (2011) extended Repullo's (2000) framework by introducing a systemic bank in an incomplete contract framework. Focusing on the question whether a unified regulatory architectures where the lender of last resort is combined with the deposit insurance in a single regulator dominates an architecture with separate agencies in a framework with systemic and non-systemic banks. They find that a unified regulatory is on the one hand more forbearance towards the systemic institution but on the other hand under some conditions a unified regulator can reduce systemic risk.

Our model differs in several points from Espinosa-Vega, Kahn, Matta, and Sole (2011). First, we are interested in the optimal institutional allocation of lender of last resort responsibilities between separate agencies. Second, we are modeling the impact of a systemic bank in a different way. Espinosa-Vega, Kahn, Matta, and Sole (2011) assume that a failure of the systemic bank reduces the probability of success for the non-systemic bank. In our approach the failure of the systemic bank reduces the return of the non-systemic bank. From our point of view a collapse of the systemic bank causes in the first place losses for the non-systemic bank but does not influence the probabilities of success of the risky asset. It rather increase the cost of operation for the non-systemic bank because risk management efforts have to be increased like the replacements of hedging contracts. Third, Espinosa-Vega, Kahn, Matta, and Sole (2011) assume like in Kahn and Santos (2005) that the political cost for the regulators exceed the social cost of a failure. They claim that supervisors are biased towards forbearance. However assuming as we do that the policy cost for the regulator are inferior to the social cost the supervisor may exert forbearance and be tougher than optimal depending on the solvency of the bank. The last difference is that in Espinosa-Vega, Kahn, Matta, and Sole (2011) the maturity of the risky asset differs between the systemic and the non-systemic bank. In our model both banks invest into identical assets.

2 Literature

To be defined.

3 The model

We propose a model inspired by Espinosa-Vega, Kahn, Matta, and Sole (2011) and Repullo (2000) with two banks: a systemic bank (S) and a non-systemic one (N). To fund their activities both banks offer demand deposit contracts. At the beginning of their operations each bank raises one unit of deposits. We assume that deposits are fully insured by the deposit insurance and that they can be withdrawn either after the first or the second period of operation.

Each bank has access to an illiquid risky asset which yields after two periods for each unit invested a random return $\tilde{R}$. The asset can either succeed, $\tilde{R} = R$, or fail, $\tilde{R} = 0$. The asset is ex ante profitable: $E(\tilde{R}) > 1$, but it can not be sold at an intermediate date. However, the bank can be liquidated at this date. The liquidation value is $L \in (0, 1)$. For simplification we assume that L is equal for both banks.

Both banks are exposed to two type of shocks: a liquidity shock v_i and a solvency shock

u_i with $i \in S, N$. After the first period of the asset's duration a fraction $v_i \in (0,1)$ of the bank's deposits are withdrawn. The liquidity shock v_i is publicly verified and correspond to the realization of a random variable $\tilde{v}_i$ with a cumulative distribution G. It has the support in [0,1]. If $v_i > 0$ the bank faces liquidation unless the lender of last resort provides emergency liquidity assistance. A liquidation causes social cost of c.

Simultaneously with the liquidity shock v_i a signal u_i about the profitability of the bank's asset at maturity is privately observed by the lender of last resort. The signal u_i is the realization of a random variable $\tilde{u}_i$ with a cumulative distribution F and the support in [0,1].

Departing from Espinosa-Vega, Kahn, Matta, and Sole (2011) and Repullo (2000) the banks differ in several points. While the systemic bank S raises deposits and invests them into the asset at date 1 the starting date of operation for the non-systemic bank N is delayed to date 2. This is in contrast to Espinosa-Vega, Kahn, Matta, and Sole (2011) where both banks start at the same date but the duration of the investment project differs.

Our model also differs from Espinosa-Vega, Kahn, Matta, and Sole (2011) in the modeling of the systemic risk. Espinosa-Vega, Kahn, Matta, and Sole (2011) assume that a collapse of the systemic bank reduces the probability of the successful state of the non-systemic bank's asset. In our model the failure or the liquidation of the systemic bank both reduces the return of the non-systemic bank's asset in the successful state to $\tilde{R} = R - \gamma$ but not vice versa. From our point of view the failure of the systemic bank reduces in the first place the profitability of the non-systemic bank, because the non-systemic bank has to replace all links to the systemic bank with new contracts but with different counterparties. This will increase the cost of operation for the non-systemic bank and thus reduce the return on the asset.

As in Repullo (2000) there are two agencies available to carry out the role of the lender of last resort (LLR) for illiquid banks: the central bank and the deposit insurance. While the central bank is a natural source of liquidity due to its monetary policy tools the deposit insurance is funded through premiums paid by banks. As in Ponce (2010) we assume that deposit insurance premiums are normalized to zero. The agency assign with the LLR responsibilities is provided with the authority to accumulate all necessary information form the illiquid bank in order to fulfill its mandate.

The agencies differ in their incentives to support the illiquid bank. Both agencies care about their expected final wealth net of the incurred political cost from a bank's failure. The central bank's cost to provide liquidity correspond to the amount of the emergency loan in case the bank in trouble fails. The deposit insurance on the other hand only takes into account the cost of reimbursing the depositors of the collapsed bank. Only the deposit insurance can liquidate the illiquid bank and realized the liquidation value L. Additionally, both agencies incur the political cost of a failure, but only when they had been in charge of the LLR responsibilities. The political cost are αc for the central bank respectively βc for the deposit insurance with $\alpha < 1$ and $\beta < 1$. Espinosa-Vega, Kahn, Matta, and Sole (2011) in contrast assume that the political cost exceeds the social cost.

As in Ponce (2010) apart from allocating the responsibility to one of the two agencies the policy maker can implement an unconditional bailout. In this case the central bank is instructed to provide liquidity to the troubled bank without any negative effect on her final wealth in case of default. The deposit insurance has to compensate the remaining depositors in case the supported bank fails. The liquidity injection of the policy maker through the central bank are not protected by the deposit insurance. Neither does the deposit insurance nor the central bank incur any political cost from a failure when the unconditional bailout rule is applied.

3.1 Timing

Date 0.

- Policy maker announces lender of last resort policy for the systemic bank S and the non-systemic bank N.

- Bank S raises 1 unit of deposits and invests it into a risky asset.

Date 1.

- Bank S's solvency signal u_S is privately observed (Distribution F_S with support in [0,1]).

- Bank S's liquidity shortfall v_S is publicly verified (Distribution G_S with support in [0,1]).

- LLR policy for the systemic bank S is applied. Either bank S receives an emergency loan and continues to operate or bank S is closed. In the latter case the deposit insurance liquidates bank S and repays the depositors.

- Bank N raises 1 unit of deposits and invests it into a risky asset.

Date 2.

- Bank N's solvency signal u_N is privately observed (Distribution F_N with support in [0,1]).

- Bank N's liquidity shock v_N is publicly verified (Distribution G_N with support in [0,1]).

- The regulatory agency in charge applies the lender of last resort policy. Bank N is either closed or it remains open if the responsible agency provides an emergency loan. Otherwise the deposit insurance conducts the liquidation of the institute and repays depositors.

- Return of bank S's risky asset is realized in case bank S was not liquidated before. If risky assets yield a zero return deposit insurance reimburses depositors.

Date 3.

- In case bank N is still operating at date 3 the return of bank N's risky asset is realized. If risky assets is not successful deposit insurance reimburses depositors.

For simplification we assume $F_S = F_N = F$, $G_S = G_N = G$. The timing of the model is summarized in figure 1 and the payoff structure in figure 2.

[Figure 1 about here.]

[Figure 2 about here.]

4 Allocation of lender of last resort responsibilities

In order to find the second best allocation of LLR responsibilities for the systemic bank we solve the model backwards beginning with the non-systemic bank N followed by the systemic bank S.

We define the following indicator variables with a value equal to one in case the below-mentioned conditions hold:

- $\mathbb{1}_S^{SS} = 1$ if systemic bank S succeeds at date 2.

- $\mathbb{1}_S^{SF} = 1$ if systemic bank S fails at date 2 or was closed at date 1.

- $\mathbb{1}_S = 1$ if LLR loan is provides to systemic bank S.

- $\mathbb{1}_N^{SS} = 1$ if LLR loan is provided to non-systemic bank N given systemic bank S succeeded.

- $\mathbb{1}_N^{SF} = 1$ if LLR loan is provided to non-systemic bank N given systemic bank S failed at date 2 or was closed at date 1.

4.1 Non-systemic bank

4.1.1 First-best

For the determination of the socially optimal allocation of the LLR responsibilities we assume that the liquidity shock v_S and the solvency signal u_S are both verifiable. The expected social welfare from bank N is

$$\begin{aligned} W_N =& E\{\mathbb{1}_S^{SS}[\{\mathbb{1}_N^{SS}(u_N R - (1-u_N)c) + (1-\mathbb{1}_N^{SS})(L-c)] \\ &+ \mathbb{1}_S^{SF}[\mathbb{1}_N^{SF}(u_N(R-\gamma) - (1-u_N)c) + (1-\mathbb{1}_N^{SF})(L-c)]\}, \end{aligned} \tag{1}$$

where the first part of this expression is the expected social welfare given a successful bank S (case SS). In this situation bank N if supported with an emergency loan succeeds with probability u_N and yields a return R. A failure occurs with a probability $(1 - u_N)$ and causes social cost c. If bank N is not supported she will be liquidated with a liquidation value L. The closure causes social cost of c.

The second part of the expression is the expected social welfare in case bank S was liquidated or its risky asset failed (case SF). If bank N receives an emergency loan its risky asset will succeed with probability u_N and yield a return $R-\gamma$. Bank N's risky asset fails with probability $(1 - u_N)$ which causes social costs of c. If the emergency loan is rejected bank N is liquidated with a liquidation value L. A liquidation causes social cost of c.

To determine the thresholds on the solvency signal u_N for the liquidity provided both states of bank S are considered. First, the case of a successful bank S is analyzed. It is optimal to provide an emergency loan to bank N if the expected social value from bank N's continuation exceeds the social value of bank N's liquidation. The social optimal lending decision to bank N in case SS is

$$\begin{aligned} u_N R - (1-u_N)c &\geq L - c, \\ u_N \geq u_N^{SS} &\equiv \frac{L}{R+c}. \end{aligned} \tag{2}$$

In case the systemic bank S failed it is optimal to provide emergency liquidity assistance to bank N if

$$\begin{aligned} u_N(R-\gamma)-(1-u_N)c &\geq L-c, \\ u_N \geq u_N^{SF} &\equiv \frac{L}{R+c-\gamma}. \end{aligned} \tag{3}$$

4.1.2 Central bank as the LLR

The central bank will only support the non-systemic bank N if the expected cost from providing the emergency loan is lower than the cost of closing the non-systemic bank N immediately. The expected cost for the central bank of an emergency loan to bank N is the amount of the liquidity injection v_N and the political cost of bank N's failure αc. Not supporting bank N results in a closure and causes political cost αc for the central bank.

In both cases the central bank's cost are independent from the state of bank S. Thus the central bank's benefit is

$$\begin{aligned} B_N =& \mathbb{1}_S^{SS}[\{\mathbb{1}_N^{SS}(-(1-u_N)(\alpha c+v_N))+(1-\mathbb{1}_N^{SS})(-\alpha c)] \\ &+\mathbb{1}_S^{SF}[\mathbb{1}_N^{SF}(-(1-u_N)(\alpha c+v_N))+(1-\mathbb{1}_N^{SF})(-\alpha c)]. \end{aligned} \tag{4}$$

From (4) it is obvious that the central bank will provide the emergency liquidity if

$$\begin{aligned} u_N(\alpha c+v_N) &\geq v_N, \\ u_N \geq u_N^{CB} &\equiv \frac{v_N}{v_N+\alpha c}. \end{aligned} \tag{5}$$

4.1.3 Deposit insurance as the LLR

As the central bank the deposit insurance will only support the non-systemic bank N if the expected cost from providing liquidity is lower than the cost of closing the non-systemic bank N at date 2. The expected cost of the emergency loan for the deposit insurance are the compensation of the depositors and the political cost βc weighted with the probability $(1-u_N)$. The cost of closing bank N immediately are equivalent to the political cost βc. The deposit insurance still has to compensate the depositors, but it can liquidate the closed bank and realize the liquidation value L.

The benefit for the deposit insurance is

$$\begin{aligned} D_N = & \mathbb{1}_S^{SS}[\{\mathbb{1}_N^{SS}(-(1-u_N)(1+\beta c)) + (1-\mathbb{1}_N^{SS})(L-1-\beta c)] \\ & + \mathbb{1}_S^{SF}[\mathbb{1}_N^{SF}(-(1-u_N)(1+\beta c)) + (1-\mathbb{1}_N^{SF})(L-1-\beta c)]. \end{aligned} \tag{6}$$

Given (6) the deposit insurance will provide an emergency loan to bank N if

$$\begin{aligned} u_N(1+\beta c) \geq L, \\ u_N \geq u_N^{DI} \equiv \frac{L}{1+\beta c}. \end{aligned} \tag{7}$$

4.1.4 The unconditional bailout rule

Under the unconditional bailout rule the bank receives an emergency loan regardless of its solvency signal u_N,

$$u_N \geq 0 \equiv u_N^{UBR}. \tag{8}$$

Figure 3 summaries the thresholds for the liquidity provision as defined above.

[Figure 3 about here.]

4.1.5 Optimal allocation

The thresholds for the provision of the emergency loan to the non-systematic bank N of the central bank and deposit insurance are independent of bank S's state. The first best threshold however differs between the two cases SS and SF. Thus we will study the optimal allocation of LLR responsibilities for both cases separately and therefore define $\nu \in \{SS, SF\}$.

On the basis of case SS we illustrate our approach to define the optimal second best allocation of LLR responsibilities. The expected social welfare in (1) given the socially optimal threshold to provide emergency liquidity $u_N^{SS} \equiv \frac{L}{R+c}$ when bank S was successful can be expressed as

$$W_N^{SS} = E[\mathbb{1}_N^{SS}(u_N - u_N^{SS})](R+c) + (L-c). \tag{9}$$

As in Ponce (2010) to maximize (9) it is sufficient to maximize the normalized social welfare

$$w_N^{SS} = E[\mathbb{1}_N^{SS}(u_N - u_N^{SS})]. \tag{10}$$

As the approach for $\nu = SF$ is analogous it follows for $\nu \in \{SF, SS\}$ that the normalized social welfare for the central bank, the deposit insurance and the unconditional bailout rule is

$$w_N^{CB,\nu}(v_N) = \int_{u_N^{CB}(v_N)}^{1} (u_N - u_N^{\nu})\, \mathrm{d}F(u), \tag{11}$$

$$w_N^{DI,\nu} = \int_{u_N^{DI}}^{1} (u_N - u_N^{\nu})\, \mathrm{d}F(u), \tag{12}$$

$$w_N^{UBR,\nu} = \int_{0}^{1} (u_N - u_N^{\nu})\, \mathrm{d}F(u). \tag{13}$$

Lemma 1 follows Ponce's (2010) results adapted to the model studied here.

Lemma 1. *Assume that $E\left(\widetilde{u_N} \mid u_N \leq u_N^{DI}\right) > u_N^{SF}$. Then, (1) if the systemic bank succeeded, $\nu = SS$ (respectively failed, $\nu = SF$), then (i) $w_N^{CB,SS}(v_N)$ is increasing in v_N if $v_N < v_N^A \equiv \frac{\alpha c L}{R-L+c}$ (respectively $v_N < v_N^C \equiv \frac{\alpha c L}{R-L+c-\gamma}$), (ii) decreasing if $v_N > v_N^A$ (respectively $v_N > v_N^C$), and (iii) has a global maximum at $v_N = v_N^A$ (respectively at $v_N = v_N^C$); (2) $w_N^{CB,\nu}(0) = w_N^{UBR,\nu} > w_N^{DI,\nu}$; (3) If $v_N < v_N^B \equiv \frac{\alpha c L}{1-L+\beta c}$, then $w_N^{DI,\nu} < w_N^{CB,\nu}(v_N)$, otherwise $w_N^{DI,\nu} \geq w_N^{CB,\nu}(v_N)$; (4) $w_N^{DI,\nu} > w_N^{CB,\nu}(1) > 0 \;\forall\; \nu \in \{SS, SF\}$.*

Proof. (1) The first derivative of $w_N^{CB,\nu}(v_N)$ is: $\dot{w}_N^{CB,\nu}(v_N) = -\dot{u}_N^{CB}(v_N)\left[u_N^{CB}(v_N) - u_N^{\nu}\right] f(u)$, where f is the density function of the random variable $\widetilde{u_N}$. Since $\dot{u_N^{CB}}(v_N)$ and $f(u)$ are positive for all v_N and u_N, $w_N^{CB,\nu}(v_N)$ is increasing in v_N if $u_N^{CB}(v_N) < u_N^{\nu}$, decreasing if $u_N^{CB}(v_N) > u_N^{\nu}$, and has a global maximum for $u_N^{CB}(v_N) = u_N^{\nu}$. Since $\dot{u_N^{CB}}(v_N) > 0$ and v_N^A (respectively v_N^C) is such that $u_N^{CB}\left(v_N^A\right) = u_N^{SS}$ (respectively $u_N^{CB}\left(v_N^C\right) = u_N^{SF}$) (see Figure 3), the result follows.

(2) (a) Since $u_{CB,}(0) = 0 = u_{UBR}$, then $w_N^{CB,\nu}(0) = w_N^{UBR,\nu}$. (b) Assume $w_N^{UBR,\nu} - w_N^{DI,\nu} \leq 0$. Then $\int_0^1 (u_N - u_N^{\nu})\, dF(u) - \int_{u^{DI}}^1 (u_N - u_N^{\nu})\, dF(u) \leq 0$,
$\int_0^{u_N^{DI}} (u_N - u_N^{\nu})\, dF(u) \leq 0$, $\left[E\left(\widetilde{u_N} \mid u_N \leq u_N^{DI}\right) - u_N^{\nu}\right] F\left(u_N^{DI}\right) \leq 0$, and
$E\left(\widetilde{u_N} \mid u_N \leq u_N^{DI}\right) \leq u_N^{\nu}$. A contradiction.

(3) Since v_N^B is such that $u_N^{CB}\left(v_N^B\right) = u_N^{DI}$, then $w_N^{CB,\nu}\left(v_N^B\right) = w_N^{DI,\nu}$. Properties (1) and (2)(a) imply that $w_N^{DI,\nu} < w_N^{UBR,\nu} \leq w_N^{CB,\nu}(v_N)$ for $v_N < v_N^B$ and that $w_N^{DI,\nu} \geq w_N^{CB,\nu}(v_N)$ for $v_N \geq v_N^B$.

(4) Since $v_N^B < 1$, Property (3) implies that $w_N^{DI,\nu} > w_N^{CB,\nu}(1)$. $w_N^{CB}(1) = \int_{u_N^{CB,\nu}(1)}^{1} (u_N - u_N^\nu)\, dF(u) = \left[E\left(\widetilde{u_N} \mid u_N > u_N^{CB}(1)\right) - u_N^\nu\right]\left[1 - F\left(u_N^{CB,\nu}(1)\right)\right]$. Since $u_N^\nu < \frac{1}{1+\alpha c} = u_N^{CB}(1) < 1$ both factors are positive, then $w_N^{CB}(1) > 0$.

□

The policy maker will allocate the lender of last resort responsibilities according to the size of the liquidity shock v_N, because only the liquidity shock v_N is verifiable. To maximize the expected social welfare lemma 1 implies the following optimal second best allocation:

Proposition 1. *Assume that $E\left(\widetilde{u_N} \mid u_N \leq u_N^{DI}\right) > u_N^{SF}$. If the systemic bank succeeded, $\nu = SS$ (respectively failed, $\nu = SF$), there exists a threshold for the liquidity shortfall of the non-systemic bank $v_N^{SS} \in (v_N^A, v_N^B)$ (respectively $v_N^{SF} \in (v_N^C, v_N^B)$) such that it is optimal to allocate the lender of last resort responsibilities for the non-systemic bank to the central bank for liquidity shortfalls below the threshold and to apply the unconditional bailout rule for liquidity shortfalls above it.*

Proposition 1 can be explained as followed. Given condition $E\left(\widetilde{u_N} \mid u_N \leq u_N^{DI}\right) > u_N^{SF}$ it is more likely that a random non-systemic bank's asset is of average quality (i.e. $u \in [u_N^{SF}, u_N^{DI}]$) than of low quality (i.e. $u \in [0, u_N^{SF}]$). It is more likely that a non-systemic bank for which liquidity support is social optimal does not receive an emergency loan from the deposit insurance than a non-systemic bank with low quality assets is bailed out unconditionally. Therefore, the policy maker chooses to apply the unconditional bail out rule instead of assigning the deposit insurance with the LLR responsibility.

It is not always optimal to support illiquid banks. For small liquidity shock the central bank's threshold are closer to the social optimal one than the unconditional bailout rule. Therefore, the LLR responsibilities for small liquidity shock is allocated to the central bank. If the liquidity shock is large the central bank's lending decision is too restrictive. The unconditional bailout rule maximizes the expected social welfare.

Proposition 2. *The central bank should act as a lender of last resort in a larger range of liquidity shortfalls of the non-systemic bank when the systemic bank failed than when it succeeded (i.e. $v_N^{SS} < v_N^{SF}$).*

Proof. Given equation (11) and (13) (a) $w_N^{UBR,SS} - w_N^{UBR,SF} = u_N^{SF} - u_N^{SS}$, (b) $w_N^{CB,SS}(v_N) - w_N^{CB,SF}(v_N) = (u_N^{SF} - u_N^{SS})[1 - F(u_N^{CB}(v_N))]$, (c) $w_N^{CB,SS}(v_N) - w_N^{CB,SF}(v_N)$ is non-increasing in v_N, (d) To the right of v_N^C both $w_N^{CB,SS}(v_N)$ and $w_N^{CB,SF}(v_N)$ are decreasing. It follows that $v_N^{SS} < v_N^{SF}$ □

The intuition for this result is the following. With a failure of the systemic bank the expected return of the non-systemic bank falls, such that for all liquidity shocks the social optimal threshold for the provision of the emergency loan increases. Since the central bank become less forbearing with increasing liquidity shocks the social optimal lending decision is closer to the central bank's one for a larger interval of liquidity shocks. However, for very large liquidity shocks the central bank is still too tough, such that unconditional bail out rule maximizes in this interval the expected social welfare.

[Figure 4 about here.]

4.2 Systemic bank (incomplete)

4.2.1 First-best

As for the non-systemic bank the first best provision of emergency liquidity assistance is determined by maximizing the expected social welfare given that the liquidity shock as well as the solvency shock are both verifiable. Thus the expected social welfare is

$$\begin{aligned} W_S =& E\{\mathbb{1}_S[u_S R - (1-u_S)c + W_N^{SC}] + (1-\mathbb{1}_S)[L - c + W_N^{SF}]\}, \\ W_S =& E\{\mathbb{1}_S[u_S(R+c) - L + W_N^{SC} - W_N^{SF}] + L - c + W_N^{SF}\}, \end{aligned} \tag{14}$$

where the first part of the expression is the social welfare when the systemic bank receives emergency liquidity assistance. In this case the social welfare has two components. The first component is the systemic bank's expected continuation value at date 2 net of social cost: $u_S R - (1-u_S)c$. The second component is the expected social welfare of the non-systemic bank W_N^{SC} which has to be considered because the state of the systemic bank has an impact on the provision of emergency liquidity for the non-systemic bank and therefore on the overall social welfare from the banking sector. The second part of the expression is the social welfare in case of a liquidated systemic bank. The liquidation value of the systemic bank net of social cost of a failure at date 1 is $L-c$. As before the expected social welfare from the non-systemic bank given a liquidated systemic bank has to be considered.

The expected social welfare from the non-systemic bank if the systemic bank continues to operate is

$$\begin{aligned} W_N^{SC} =& u_S E\{\mathbb{1}_N^{SS}(u_N R - (1-u_N)c) + (1-\mathbb{1}_N^{SS})(L-c)\} \\ &+ (1-u_S)E\{\mathbb{1}_N^{SF}(u_N(R-\gamma) - (1-u_N)c) + (1-\mathbb{1}_N^{SF})(L-c)\}, \end{aligned} \tag{15}$$

which consists of the expected social welfare from the non-systemic bank in case the systemic bank is successful or fails at date 2. In the first case the non-systemic bank's expected continuation value net of the social cost of the bank's failure is $u_N R - (1 - u_N)c$. Is the emergency liquidity assistance not provided the liquidation value of the non-systemic bank net of the social cost of a failure at date 2 is $L - c$. In case the systemic bank failed at date 2 the non-systemic bank's expected continuation value net of the expected social cost of the bank's failure is $u_N(R - \gamma) - (1 - u_N)c$ and the liquidation value of the non-systemic bank net of the social cost of a failure is $L - c$.

The expected social welfare from the non-systemic bank in case the systemic bank was liquidated at date 1 is

$$W_N^{SF} = E\{\mathbb{1}_N^{SF}(u_N(R - \gamma) - (1 - u_N)c) + (1 - \mathbb{1}_N^{SF})(L - c)\}, \tag{16}$$

where the non-systemic bank's expected continuation value net of the social cost of a failure is $u_N(R - \gamma) - (1 - u_N)c$ and the liquidation value net of the social cost of a failure at date 2 is $L - c$.

We define

$$W_N^{\Delta} = E\{(\mathbb{1}_N^{SS} - \mathbb{1}_N^{SF})(u_N(R + c) - L) + \mathbb{1}_N^{SF} u_N \gamma\} > 0, \tag{17}$$

such that

$$W_N^{SC} - W_N^{SF} = u_S W_N^{\Delta}. \tag{18}$$

Given (14) and (18) it is social optimal to provide the emergency loan to the systemic bank if

$$\begin{aligned} u_S(R + c + W_N^{\Delta}) &\geq L, \\ u_S \geq u_S^* &\equiv \frac{L}{R + c + W_N^{\Delta}}. \end{aligned} \tag{19}$$

4.2.2 Central bank as the LLR

As for the non-systemic bank the central bank cares about its income and therefore has the objective to minimize financial and political cost when acting as the LLR. In case the systemic bank is supported but fails the central bank's cost are the liquidity injection v_S, the political costs αc and the utility from the non-systemic bank N given the systemic bank continues to operate B_N^{SC}. Not supporting the systemic bank results in the closure of bank S. Since the central did not provide any liquidity to the troubled bank the central bank's cost only consists of the political cost αc. As above the utility from the non-systemic bank N given the closure of the systemic bank B_N^{SF} enters into the central bank's utility, because the state of the systemic bank influence the expected profitability of the bank N. Thus the central bank's expected utility from the systemic bank S is

$$\begin{aligned} B_S =& E\{\mathbb{1}_S[-(1-u_S)(\alpha c + v_S) + B_N^{SC}] + (1-\mathbb{1}_S)[-\alpha c + B_N^{SF}]\}, \\ B_S =& E\{\mathbb{1}_S[u_S(v_S + \alpha c) - v_S + B_N^{SC} - B_N^{SF}] - \alpha c + B_N^{SF}\}. \end{aligned} \tag{20}$$

The central bank's utility from the non-systemic bank N given the systemic bank continues to operate is

$$\begin{aligned} B_N^{SC} =& u_S \int_0^{v_N^{SS}} \left[\int_0^{u_N^{CB}(v_N)} -(\alpha c) dF(u) + \int_{u_N^{CB}(v_N)}^1 -(1-u_N)(\alpha c + v_N) dF(u) \right] dG(v_N) \\ &+ (1-u_S) \int_0^{v_N^{SF}} \left[\int_0^{u_N^{CB}(v_N)} -(\alpha c) dF(u) + \int_{u_N^{CB}(v_N)}^1 -(1-u_N)(\alpha c + v_N) dF(u) \right] dG(v_N), \end{aligned} \tag{21}$$

where both states of the systemic bank are considered. v_N^{SS} and v_N^{SF} are the optimal second best liquidity shock thresholds below which the central bank is the lender of last resort for the non-systemic bank as defined in proposition 1. When the central bank is responsible for the provision of the emergency loan, the central bank will only support the bank in trouble if the solvency signal is above its threshold. In case the non-systemic bank fails while being supported the central bank will lose the emergency loan v_N and bear the political cost of bank failure αc. Below the solvency threshold the central bank will never support bank N and bear the political cost αc.

The utility from the non-systemic bank N given the closure of the systemic bank S is

$$B_N^{SF} = \int_0^{v_N^{SF}} \left[\int_0^{u_N^{CB}(v_N)} -(\alpha c) dF(u) + \int_{u_N^{CB}(v_N)}^1 -(1-u_N)(\alpha c + v_N) dF(u) \right] dG(v_N), \tag{22}$$

where v_N^{SF} is the non-systemic's bank liquidity shock below which the central bank is the lender of last resort. Is the solvency signal for the non-systemic bank below $u_N^{CB}(v_N)$ the central bank will refuse the emergency loan and bear the political cost αc. With a solvency signal above the threshold the central bank supports bank N but might lose the emergency loan v_N and bear the political cost αc when the non-systemic bank fails.

We define

$$B_N^{\Delta} = \int_{v_N^{SS}}^{v_N^{SF}} \left[\int_0^{u_N^{CB}(v_N)} (\alpha c) dF(u) + \int_{u_N^{CB}(v_N)}^{1} (1 - u_N)(\alpha c + v_N) dF(u) \right] dG(v_N) > 0,$$

such that

$$B_N^{SC} - B_N^{SF} = u_S B_N^{\Delta}. \tag{23}$$

Given (20) and (23) the central bank lends to bank S if

$$\begin{aligned} u_S(v_S + \alpha c + B_N^{\Delta}) &\geq v_S, \\ u_S \geq u_S^{CB} &\equiv \frac{v_S}{v_S + \alpha c + B_N^{\Delta}}. \end{aligned} \tag{24}$$

4.2.3 Deposit insurance as the LLR

The deposit insurance's objective is to reduce the financial and political cost of the emergency liquidity provision. If the deposit insurance supports the systemic bank with probability $(1 - u_S)$ bank S might fail. As a result the deposit insurance loses the provided liquidity and has to compensate the remaining depositors. Additionally the deposit insurance has to bear the political cost βc. Independent of bank S's success the decision to support the systemic bank influences the bank N's profitability and consequently also the utility for the deposit insurance from the systemic bank. As before the deposit insurance's benefit from the non-systemic bank given the systemic bank continues to operate enters into the expected utility of the deposit insurance. In case the systemic bank is not supported the expected cost are $L - 1 - \beta c + D_N^{SF}$ where L is the liquidation value and D_N^{SF} is the deposit insurance's benefit from the non-systemic bank in case the systemic bank is closed. The deposit insurance's expected utility is given by

$$\begin{aligned} D_S =& E\{\mathbb{1}_S[-(1 - u_S)(1 + \beta c) + D_N^{SC}] + (1 - \mathbb{1}_S)[L - 1 - \beta c + D_N^{SF}]\}, \\ D_S =& E\{\mathbb{1}_S[u_S(1 + \beta c) - L + (D_N^{SC} - D_N^{SF})] + L - 1 - \beta c + D_N^{SF}\}. \end{aligned} \tag{25}$$

According to proposition 1 the deposit insurance is not responsible for the provision of emergency loans to the non-systemic bank N. For this reason it does not bear any political cost if bank N fails or is closed. The deposit insurance still has to compensate bank N's depositors in case of distress. When the central bank is the lender of last resort and she does not support bank N the deposit insurance has to compensate all depositors net of the liquidation value $(1 - L)$. If the central bank supports bank N and the non-systemic bank fails the deposit insurance only has to compensate the remaining depositors. Thus the expected cost for the deposit insurance are $(1 - u_N)(1 - v_N)$. If the unconditional bailout rule is applied the deposit insurance has to compensate all depositors. Therefore, the expected cost are $(1 - u_N)$. This said the utility for the deposit insurance from the non-systemic bank B if the systemic bank S is not liquidated correspond to

$$\begin{aligned} D_N^{SC} =& u_S \left[\int_0^{v_N^{SS}} \left[\int_0^{u_N^{CB}(v_N)} -(1-L)dF(u) + \int_{u_N^{CB}(v_N)}^1 -(1-u_N)(1-v_N)dF(u) \right] dG(v_N) \right. \\ & \left. + \int_{v_N^{SS}}^1 \int_0^1 -(1-u_N)(1-v_N)dF(u_N)dG(v_N) \right] + (1-u_S) \\ & \left[\int_0^{v_N^{SF}} \left[\int_0^{u_N^{CB}(v_N)} -(1-L)dF(u) + \int_{u_N^{CB}(v_N)}^1 -(1-u_N)(1-v_N)(1-v_N)dF(u) \right] dG(v_N) \right. \\ & \left. + \int_{v_N^{SF}}^1 \int_0^1 -(1-u_N)dF(u)dG(v_N) \right], \end{aligned} \tag{26}$$

where v_N^{SS} and v_N^{SS} are the second best threshold as defined in proposition 1.

The deposit insurance's utility from bank N if bank S is liquidated is

$$\begin{aligned} D_N^{SF} =& \int_0^{v_N^{SF}} \left[\int_0^{u_N^{CB}(v_N)} -(1-L)dF(u) + \int_{u_N^{CB}(v_N)}^1 -(1-u_N)(1-v_N)dF(u) \right] dG(v_N) \\ & + \int_{v_N^{SF}}^1 \int_0^1 -(1-u_N)(1-v_N)dF(u)dG(v_N), \end{aligned} \tag{27}$$

which follows the same reasoning as above. In case the central bank does not support bank N the deposit insurance liquidates the bank and compensates the depositors. If the central bank provided the emergency loan but bank N fails the deposit insurance has to compensate only the remaining depositors. If the unconditional bail out rule is applied and bank N fails the deposit insurance compensates all depositors.

We define

$$D_N^{\Delta} = \int_{v_N^{SS}}^{v_N^{SF}} \left[\int_0^{u_N^{CB}(v_N)} (1-L) - (1-u_N)(1-v_N) dF(u) \right] dG(v_N),$$

such that

$$D_N^{SC} - D_N^{SF} = u_S D_N^{\Delta}. \tag{28}$$

Given (25) and (28) the deposit insurance lends to bank S if

$$\begin{aligned} u_S(1 + \beta c + D_N^{\Delta}) \geq L, \\ u_S \geq u_S^{DI} \equiv \frac{L}{1 + \beta c + D_N^{\Delta}}. \end{aligned} \tag{29}$$

4.2.4 Unconditional bailout rule

Under the unconditional bailout rule the bank receives an emergency loan regardless of its solvency signal u_S.

$$u_S \geq 0 \equiv u_S^{UBR}. \tag{30}$$

4.2.5 Optimal allocation (incomplete)

[Figure 5 about here.]

As above the normalized welfare for the systemic bank S is derived from (14) given $u_S^* = \frac{L}{R+c+W_N^{\Delta}}$.

$$W_S = E[\mathbb{1}_S(u_N - u_S^*)](R + c) + (L - c + W_N^{SF}). \tag{31}$$

The normalized social welfare for the central bank, the deposit insurance and the unconditional bailout rule is

$$w_S^{CB}(v_N) = \int_{u_S^{CB}(v_S)}^{1} (u_S - u_S^*) \, \mathrm{d}F(u), \tag{32}$$

$$w_S^{DI} = \int_{u_S^{DI}}^{1} (u_S - u_S^*) \, \mathrm{d}F(u), \tag{33}$$

$$w_S^{UBR} = \int_{0}^{1} (u_S - u_S^*) \, \mathrm{d}F(u). \tag{34}$$

Given the first order derivation of $w_S^{CB}(v_S)$ we know that this function has a maximum at $u_S^{CB} = u_S^*$. Thus I can show that the maximum of the central bank's normalized welfare is reached at $v_S^A = \frac{\alpha cL + LB_N^{\Delta}}{R - L + c + W_N^{\Delta}}$.

Next step would be to compare $W_N^{SC} - W_N^{SF} > 0$ and $B_N^{SC} - B_N^{SF} > 0$ in order to find out if v_S^A is left or right of v_N^B.

5 Extensions (incomplete)

In the previous sections we assumed that the systemic effect γ of bank S on bank N's return is public information. In this section studies the incentives of the agencies to share information if it is costly to verify the systemic effect.

We assume that the systemic effect γ of bank S on bank N is private information. Both banks are informed about their connectivness, because they have profound knowledge about their portfolio structure and the whole set of business activities they carry out.

The regulators can collect information about the systemic impact by requesting the banks to report the systemic effect γ of bank A on bank B. Every inquiry is costly and therefore the regulator has to decide to ask one bank or both. The banks can report truthfully or not. Their report will influence the emergency liquidity provision thresholds.

We want to answer the following questions: What are the incentives of banks to report a manipulated systemic effect? Will the bank over- or understate the systemic effect? Do both banks report differently? Should regulator ask only one bank or both? Is there a difference between the central bank and the deposit insurance?

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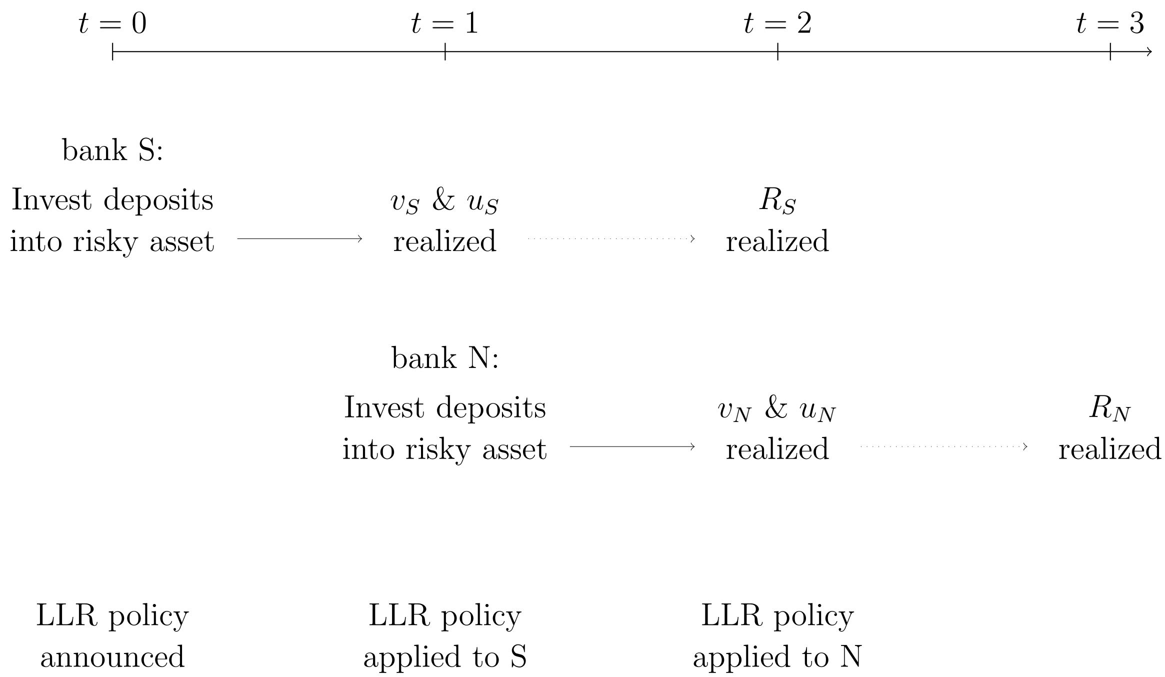

Figure 1: Timing of the model.

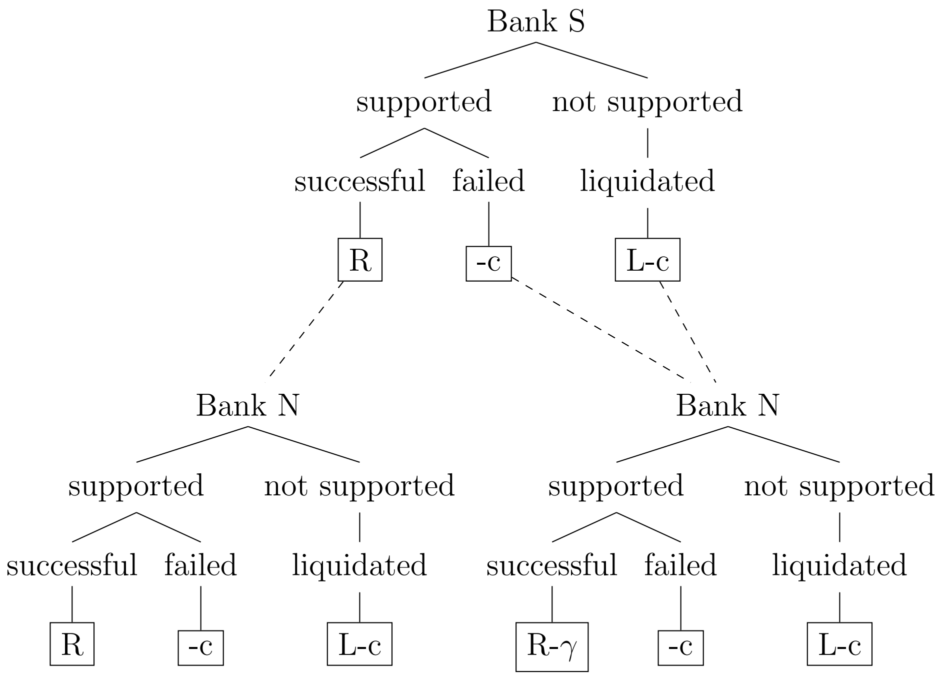

Figure 2: Payoff structure.

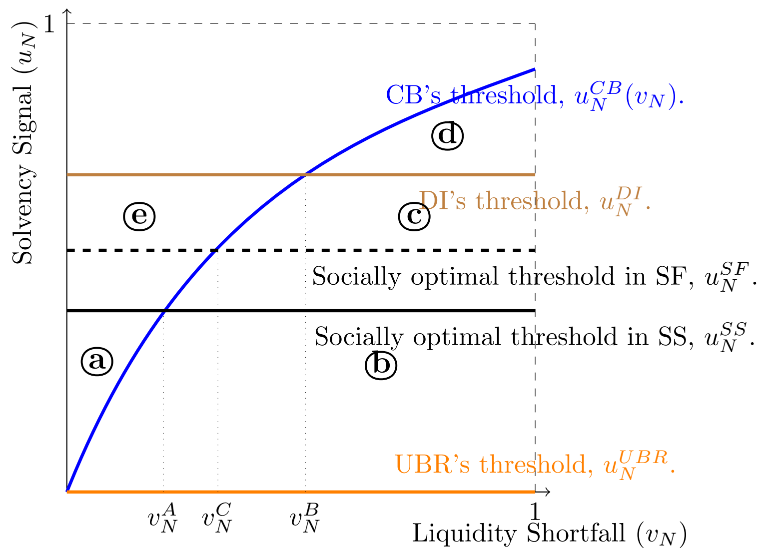

Figure 3: Lending Decisions for the non-systemic bank. It is socially optimal to lend to non-systemic banks with solvency signals above u_N^i for $i \in \{SS, SF\}$. In region ⓐ the central banker (CB) provides socially non-desirable emergency loans; in regions ⓒ and ⓓ she does not provide socially desirable emergency loans. In regions ⓒ and ⓔ the deposit insurer (DI) does not provide socially desirable emergency loans. In regions ⓐ and ⓑ, socially non-desirable emergency loans are provided by following the unconditional bailout rule (UBR).
Let ${}^{A}_{N} \equiv \frac{\alpha c L}{R-L+c}$ be the value for v_N such that $u_N^{CB}(v_N) = u_N^{SS}$ $v_N^C \equiv \frac{\alpha c L}{R-L+c-\gamma}$ be the value for v_N such that $u_N^{CB}(v_N) = u_N^{SF}$ and $v_N^B \equiv \frac{\alpha c L}{1-L+\beta c}$ the value for v_N such that $u_N^{CB}(v_N) = u_N^{DI}$. It is immediate that $0 < v_N^A < v_N^C < v_N^B$. Moreover, $c < \frac{1-L}{L}$ implies that $v_N^B < 1$.

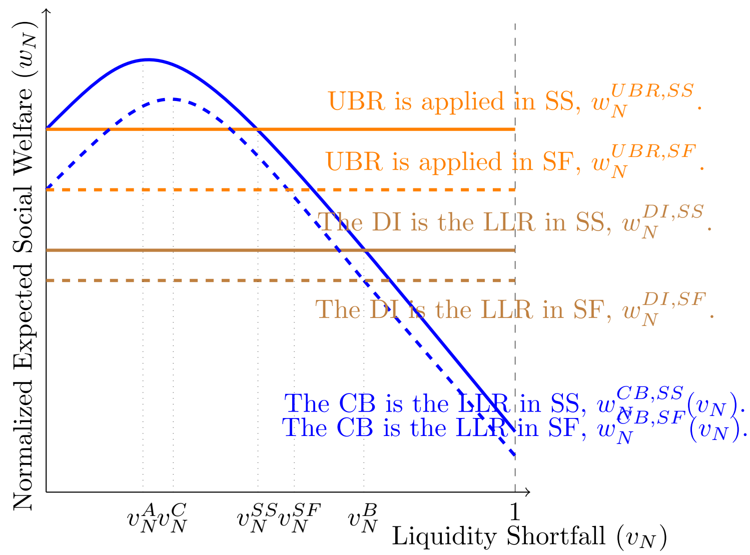

Figure 4: Normalized Expected Social Welfare for the non-systemic bank. The optimal allocation of the lender of last resort activity for the non-systemic bank follows the upper envelope of solid functions in case the systemic bank survives and is successful. Otherwise it follows the upper envelope of the dashed functions: for $v_N < v_N^i$ the central banker's decision maximizes w_N; for $v_N \geq v_N^i$ the unconditional bailout rule maximizes w_N for $i \in \{SS, SF\}$.

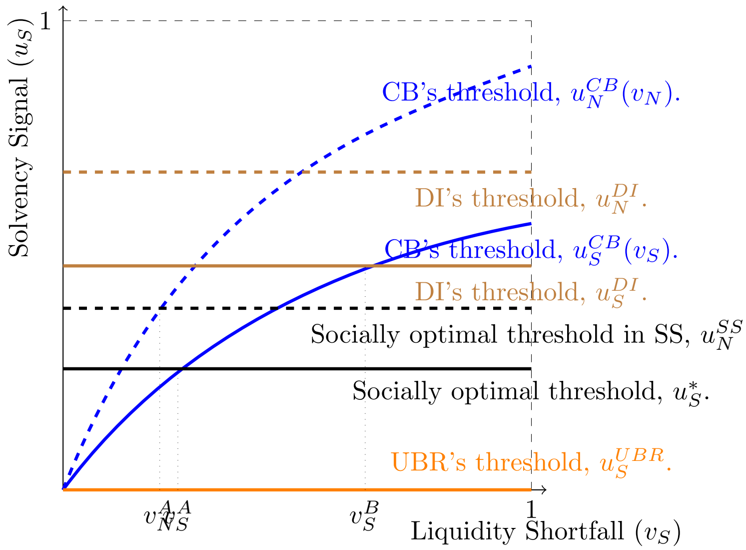

Figure 5: Lending Decisions for the systemic bank. ....