Agency, Firm Growth, and Managerial Turnover*

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Abstract

We study the relation between firm growth and optimal managerial contracting under moral hazard when a long-lived firm is operated by a sequence of managers. In our model, firms replace their managers not only upon poor performance to provide incentives, but also when outside managers are at a comparative advantage to lead the firm through a new growth phase. Firms with better investment prospects have higher managerial turnover and rely on more front-loaded compensation schemes. Realized firm growth depends jointly on the exogenous arrival of growth opportunities and the severity of the moral hazard problem. Whenever agency constitutes an obstacle to firm growth, excessive managerial retention adds to agency costs due to a contractual externality affecting future managers.

1 Introduction

Firms extract value not only from operating their existing assets, but also from the expected future profits of their growth opportunities. The latter source of value creation typically involves implementing major changes of strategy, exploring new markets, developing new products, adopting innovative production techniques or changing the organization of labor within the firm. However incumbent managers, for a variety of reasons, may lack the vision or the skills that are necessary to lead the firm through a new growth phase. Firms often find that major management changes are needed to pursue their growth opportunities successfully.

This paper explores how growth-induced management turnover interacts with the provision of managerial incentives in a dynamic moral hazard model. We consider a firm with assets in place and growth opportunities, which is run by a sequence of managers throughout its life-cycle. As in previous studies on optimal long term contracts

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with limited liability, firms can use the threat of early termination to discipline their incumbent managers, i.e., firms often fire their managers after periods of poor performance. But in contrast with previous studies, our paper stresses that firms may also fire their managers despite good performance if a change of management is the best or only option to seize valuable growth opportunities.

In our model, a risk-neutral manager is hired by a risk neutral, long-lived firm to run its existing assets. Cash flows are only observable by the manager, who can then underreport and divert them for his own private benefit. The firm can fire its incumbent manager at any point in time, and replace him at a cost. Growth opportunities are stochastic and may arrive in any period. We assume that growth is efficient under first-best and growth opportunities are contractible. In our baseline model, the firm needs to replace its current manager in order to pursue a growth opportunity. We later show that growth-driven turnover arises endogenously when it is more profitable for a firm to grow with a new manager. Upon taking up a growth opportunity, the firm pays the costs of investing and replacing the manager, and the scale of its business increases.

We solve for the optimal long-term contract signed between the firm and each of its successive managers at the time they are hired. As in other papers in the literature on dynamic moral hazard, a manager's expected discounted payoff under the optimal contract, or *continuation value*, evolves over time and its sensitivity to cashflows is related to the severity of the agency problem. A key feature in our analysis is that the continuation value of the firm upon replacing an incumbent manager is endogenous (equal to the value of the firm under the newly hired manager), and contingent on the current availability of a growth opportunity. This contrasts with most of the existing dynamic contracting models where, upon firing the manager, the firm obtains an exogenously given liquidation value.

Our results in the baseline model are as follows. First, the realized growth of firms depends both on the technological features of the growth process and on the severity of moral hazard. This implies that a firm's corporate governance can be a key determinant of corporate growth. In our model, two firms with similar growth opportunities may end up having very different realized growth profiles just because they differ in the severity of the agency problem they face. A firm plagued with more severe agency problem may forego a growth opportunity and decide instead to retain its incumbent management, when the growth opportunity arises after a period of good performance. Throughout the paper, we therefore distinguish between two (endogenous) types of firms: *low growth* firms that may or may not undertake growth opportunities depending upon the past performance of the incumbent manager, and *high growth* firms that undertake all growth opportunities when they arise. In the former type of firms, underinvestment adds to the usual inefficiency that, for the sake of *ex ante* incentive provision, managers can be fired upon poor past performance in the absence of a growth opportunity.

Second, the probability of replacing an incumbent manager in our model depends not only on past and current performance, as summarized by the manager's continuation value, but also on the availability of a growth opportunity. In all firms, the conditional probability of managerial replacement is higher in states of the world where a growth opportunity is available. In low growth firms, the performance threshold being used to determine replacement decision is set at a higher level in these states, making replacement more likely. In high growth firm, the incumbent management is systematically replaced when a growth opportunity arises.

Third, we characterize the optimal compensation scheme of incumbent managers, and determine how the availability of growth opportunities affects managerial compensation. We find that the optimal managerial contract is readily implementable by a system of deferred compensation credit and bonuses. Deferred compensation is used, along with the threat of inefficient replacement, in order to provide incentives in the best possible way. We show that the degree to which firms rely on back-loading of compensation is affected by their growth prospects. Namely, the extent of back-loading decreases with the quality of firms' growth opportunities.

We also find that severance is not required under the optimal contract. The reason is that it is more efficient for the firm, in order to save on agency costs, to give zero severance and instead increase the manager's promise contingent on him being retained. The important assumption that drives our result is that the arrival of a growth opportunity is exogenous and contractible. The zero-severance result in our model therefore contributes to our understanding of why firms make severance payments to their managers. We conjecture that growth-induced turnover would go along with positive severance if the principal had to incentivize the agent to ensure growth, or to truthfully report the arrival of a growth opportunity.

Lastly, we identify a new component of agency costs that arises in our framework, which is due to a form of contractual externality. When a firm offers a contract to a newly hired manager, it fails to take into account the spillover effect upon the expected amount of time before hiring future managers and thus the present value of compensation received by all future managers. The agency cost induced by this externality is naturally larger for *low growth* firms, where the arrival of a growth opportunity does not always result in managerial turnover. This externality of the current binding contracts of the firm on its future binding contracts does not arise in earlier papers in the literature, in which firms are liquidated at an exogenous value upon termination of the incumbent, and only, manager of the firm.

We also consider an extension of the baseline model where firms can grow with their incumbent managers, possibly at a different cost than when they grow with a new manager. Whenever it is sufficiently costly to grow with the incumbent manager, e.g., because realizing a growth opportunity would require paying an army of external consultants to help the firm reinvent itself, all the results of the baseline model survive. However, when the costs of growing with the incumbent manager are reasonably low, a new set of predictions emerges. In that case, we find that a firm always grows with its incumbent manager upon good past performance. On the other hand, if poor performance leads to dismissal of the incumbent at a time a growth opportunity is available, the firm grows with its new manager if the costs of doing so are not prohibitive. We illustrate how our results on optimal history-contingent compensation and turnover policies extend in this new environment by way of a simple numerical example.

Our notion that the growth of a firm may require replacing the incumbent manager is found in many early contributions to the managerial theory of the firm. Penrose (1959) discusses why firms may operate successfully under competent managers but may still fail to take full advantage of their opportunities of expansion. Williamson (1966) elaborates on how management constraints affect the realized growth of firms. More recently, Roberts (2004) echoes Penrose by emphasizing the need for different organizational capabilities in the *exploration* and *exploitation* of firms' investment projects. He discusses a number of business cases where this effect if prominent. In their empirical study of U.S. firms, Murphy and Zimmerman (1993) study a variety of measures of firm performance in the years preceding and following CEO turnover. They report a decline in capital expenditures in the year of CEO replacement followed by a sharp increase subsequently. In their theoretical analysis using a repeated moral hazard framework but without optimal contracting, Anderson and Nyborg (2011) show the link between managerial replacement and firm growth is affected by the firm's choice of debt or equity financing. Working in a general contracting framework but without growth opportunities, Spear and Wang (2005) study a model where a firm can fire the incumbent manager and hire a new one from an external labor market.

Our paper relates directly to several strands in the finance and economics literature on dynamic contracting and the theory of the firm. The works by Quadrini (2004), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007a), Biais et al. (2010), DeMarzo et al. (2011), and Philippon and Sannikov (2011) explore, as we do, the link between dynamic moral hazard and contractible investment opportunities.[1] Our framework differs from these papers in several dimensions. The key difference is that in our framework growth may entail replacing the current manager; whereas, all of the papers mentioned assume that a firm retains the same manager over its entire life-cycle. Furthermore, in contrast with Quadrini (2004), Clementi and Hopenhayn (2006), and DeMarzo and Fishman (2007a), we endogenize the liquidation value of the firm, and focus on managerial turnover rather than firm survival. In contrast with DeMarzo and Fishman (2007a) and DeMarzo et al. (2011), we consider growth opportunities which arrive stochastically.

Our also paper relates to the empirical literature that highlights how managerial turnover and incentives relate to realized growth. In the context of venture capital, Kaplan, Sensoy and Stromberg (2009) find that the management team of firms in their early stages of growth undergoes high turnover before the IPO. This is consistent with the prediction in our model that firms with high realized growth have high managerial turnover. The testable implications of our model on managerial turnover and growth also relate to the recent study by Jenter and Lewellen (2011) on CEO turnover and acquisitions. As in this paper, acquisitions are major investments in which target CEOs are either fired or forced to retire early; Jenter and Lewellen (2011) then show that all else equal takeovers are more likely when incumbent CEOs reach their retirement age and hence it is cheaper to replace them. There are links as well to other empirical literature including Mikkelson and Partch (1997), Kaplan and Minton (2008), and Murphy (1985) (2001) which we discuss after having derived our principal results.

Finally, there has been recent theoretical work on managerial turnover which echoes

[1] He (2008) considers an environment where growth is affected by non-observable effort.

our motivation that firms may need different managers at different times, but does so in a context without growth. Eisfeldt and Kuhnen (2012) consider a competitive matching framework without agency conflicts, and explore the role of industry conditions in determining managerial turnover, managerial compensation and the type of CEOs being hired. Another related paper by Garrett and Pavan (2012) considers a matching model with optimal dynamic contracting and predict excessive managerial retention; their result is similar in some ways to the contractual externality due to managerial turnover which we find in our paper.

The paper proceeds as follows. Section 2 describes the model. Section 3 derives the optimal long-term contract, and provides an informal discussion of its main features. Section 4 provides an illustration in the stationary limit of the model. Section 5 employs numerical simulations to further analyze the implications of our model and quantify the effects. Section 6 extends our results to a more general environment where the firm can grow with its current manager. Section 7 concludes, and a mathematical appendix includes proofs of some key results.

2 The model

2.1 Setup

We consider a project/firm that generates a stream of risky cashflows $\{Y_1, Y_2, ..., Y_T\}$ over T periods (we later consider the stationary limit as T goes to infinity). The project is run by an agent (the *manager*) who can underreport cashflows and divert them for his own private benefit. The agent gets $\lambda \leq 1$ for each unit of diverted cash, so that λ captures the severity of moral hazard. In any period, an incumbent agent can be fired (at some cost) and replaced by a new agent. For simplicity, we normalize the value of an agent's best outside option upon being fired to zero. Agents and principal are risk-neutral with discount rates ρ and $r < \rho$, respectively.

The firm cashflow in period t is $Y_t = \Phi_t y_t$, where Φ_t is the size of the firm at the beginning of period t and y_t is independently and identically distributed with support $\mathcal{Y}$, $\mathbb{E}(y_t) = \mu$ and $\min(\mathcal{Y}) = 0$. In each period, with probability q and independently from current cashflow realization, the firm has an opportunity to grow. The state variable $\theta_t \in \{G, N\}$ describes whether a growth opportunity is available ($\theta_t = G$) or not ($\theta_t = N$) in period t. Taking up a growth opportunity involves paying some investment cost and hiring a new manager. Specifically, if a growth opportunity realizes in period t, given an initial size Φ_t, the firm can grow to a size $(1 + \gamma)\Phi_t$ in period $t+1$ at a cost of $(\chi + \kappa)\Phi_t$, where χ and κ denote the proportional costs of scaling-up and replacing the manager, respectively.[2] If there is no growth opportunity or if an available growth opportunity is not taken up, the size of the firm remains constant. Figure 1 summarizes the timing within each period.

Our baseline assumption (relaxed later in Section 6) that growth necessarily entails replacing the incumbent manager is quite natural in circumstances where firm growth

[2]When considering the stationary limit of the model as $T \to \infty$, we impose that $q\gamma < e^r - 1$ to ensure finite valuation.

requires a new skill set and/or a change in corporate culture. The incumbent manager, whose human capital has to some degree become specific to the firm in its current form during his tenure, will have lost the flexibility to adapt his skills to new requirements. While we have in mind drastic changes of the firm, as a modeling convenience we capture this as a discrete change in firm size, which scales up the distribution of cashflows. Note however that growth in our model may not involve an increase in physical capital. Instead it could simply be the result of finding better management causing a permanent increase in firm productivity.

We focus our analysis on situations where it is first-best efficient to replace management to take up an available growth opportunity, which in the infinite horizon limit of the model amounts to the following parameter restriction

$$\frac{\gamma\mu}{e^r - 1} > \kappa + \chi. \tag{1}$$

Absent a growth opportunity, a manager would never be fired under first best when $\kappa > 0$. As a benchmark, we can define $V_t(\Phi)$, the first-best value of the firm in period t given size Φ, ex-cashflow and before the growth opportunity realization. The sequence of first-best value functions is given recursively by

$$V_t(\Phi) = q\left[-(\kappa+\chi)\Phi + e^{-r}\left\{(1+\gamma)\Phi\mu + V_{t+1}[(1+\gamma)\Phi]\right\}\right] + (1-q)e^{-r}\{\Phi\mu + V_{t+1}(\Phi)\},$$

where the recursion starts at $V_T(\Phi) = 0$, for all Φ. In the infinite horizon stationary limit, the homogenous nature of the model allows us to write $V(\Phi) = v^*\Phi$, where

$$v^* = \frac{-q(\kappa+\chi) + e^{-r}(1+q\gamma)\mu}{1 - e^{-r}(1+q\gamma)}. \tag{2}$$

2.2 Contracting

We now consider optimal second-best contracting under asymmetric information about cashflows. A contract is established between the firm and the manager at the outset of his tenure. When the latter is replaced, the contract is terminated and a new contract is established with a new manager. A contract specifies as a function of history (i.e., the sequence of payments received by the principal, and the history of growth opportunity realizations), circumstances under which an agent is fired (i.e., history-contingent firing probabilities), investment and growth, and non-negative cash compensation from principal to agents. Agents have limited liability, and the principal has deep pockets implying that he will not pass up growth opportunities because he is cash constrained. For simplicity, we assume a contractual environment with full-commitment (no renegotiation) and we rule out private savings by the agent.[3] The amount of diversion is the only decision over which the agent has control. In searching for an optimal contract, we restrict our attention to contracts that induce truthful reporting (since $\lambda \leq 1$ diversion is at least weakly inefficient). An optimal contract is

[3]DeMarzo and Fishman (2007b), Section 2.1 and Corollary 1, show that if the rate of return available to the agent is less than or equal to r (i.e., private saving is weakly inefficient), even if allowed to do so, the agent would have no incentive to use private savings under the derived optimal contract.

one that gives maximum payoff to the principal subject to providing a certain payoff to the agent, while satisfying incentive compatibility and limited liability constraints. We assume that the contract is designed so as to give an expected discounted value of Φw_0 to a manager hired to run the firm at size Φ.

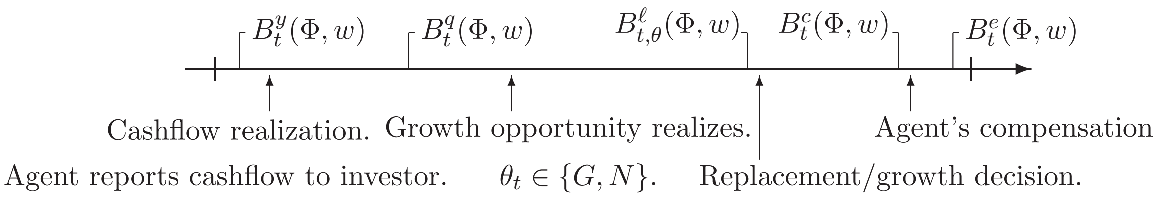

Figure 1: Intra-period timing

3 The optimal contract

In this section, we characterize managerial compensation, managerial turnover, and realized firm growth under the optimal contract. Our derivation of the optimal contract follows the approach of DeMarzo and Fishman (2007b).[4] In our context, history can be summarized by two state variables: the current size of the firm Φ, and the agent's size-adjusted continuation value w. Given this simplified state space, the optimal contracting problem can be solved by dynamic programming. To this end, it is useful to introduce a number of value functions to keep track of the principal's discounted expected payoff at different points within a period (as shown in Figure 1). We let $B_t^y(\Phi, w)$ denote the principal's value under the optimal contract at the beginning of period t, before cashflow realization, given current size Φ and (size-adjusted) continuation value w to be delivered to the agent; $B_t^q(\Phi, w)$ denotes the principal's value in period t, after cashflow realization, but before the growth opportunity is realized; $B_{t,\theta}^\ell(\Phi, w)$ denotes the principal's value conditional on a growth opportunity being available or not, before replacement and growth decisions; $B_t^c(\Phi, w)$ denotes the principal's value before compensation to the agent, conditional on the firm entering period $t+1$ with size Φ; and $B_t^e(\Phi, w)$ denotes the principal's value at the end of period t, conditional on the firm entering period $t+1$ with size Φ and with (size-adjusted) continuation value $e^\rho w$ to be delivered to the manager as of the beginning of period $t+1$. Our assumptions that firm cashflows and costs are all proportional to size guarantee that these value functions are all homogenous in current firm size.

Lemma 1. *All value functions satisfy the following homogeneity property*

$$B_t^i(\Phi, w) = \Phi B_t^i(1, w) \equiv \Phi b_t^i(w), \qquad i \in \{y, q, \ell, c, e\}. \tag{3}$$

[4]See Green (1987) and Spear and Srivastava (1987) for early applications of recursive techniques in the context of dynamic moral hazard. See DeMarzo and Fishman (2007a) and Biais et al. (2010) for applications involving time-varying firm size.

The analysis can therefore be simplified by applying dynamic programming directly onto the size-adjusted value functions. In the end, an optimal contract is entirely characterized via a set of rules specifying the evolution of the state variable w, and a set of policy functions specifying the agent's compensation and the optimal replacement and growth policies as a function of the current value of w and of whether a growth opportunity is currently available or not.

3.1 Properties of the optimal contract

We now solve for the size-adjusted value functions, the law of motion for the agent's continuation value w, along with the optimal compensation, replacement and growth policies by backward induction. We provide an informal discussion of the main features of the optimal contract and its implementation in Section 3.2.
The recursion starts in the final period with $b^{\ell}_{T,\theta}(w) = -w$ for $\theta = G, N$. Now consider the construction of $b^q_{t+1}(w)$ for $t \leq T-1$ along with the determination of w_G and w_N, the continuation promises contingent upon the availability or not of a growth opportunity, taking $b^{\ell}_{t+1,G}(w)$ and $b^{\ell}_{t+1,N}(w)$ as given. We have

$$b^q_{t+1}(w) = \max_{w_G, w_N} q b^{\ell}_{t+1,G}(w_G) + (1-q) b^{\ell}_{t+1,N}(w_N), \tag{4}$$

subject to the promise-keeping condition $qw_G + (1-q)w_N = w$ and limited liability $w_\theta \geq 0$ for $\theta = G, N$. We describe the solution to this problem below in Proposition 2 after having characterized the continuation value functions $b^{\ell}_{t+1,\theta}$.

The beginning-of-period value function is obtained as

$$b^y_{t+1}(w) = \max_{\{w^q(y)\}_{y\in\mathcal{Y}}} \mu + \mathbb{E}\{b^q_{t+1}[w^q(y)]\}, \tag{5}$$

where the expectation is taken over the distribution of y, subject to the promise-keeping condition $\mathbb{E}[w^q(y)] = w$, limited liability $w^q(y) \geq 0$, and incentive compatibility

$$w^q(y) \geq w^q(\tilde{y}) + \lambda(y - \tilde{y}), \quad \forall y \in \mathcal{Y},\ \forall \tilde{y} \in [0, y]. \tag{6}$$

The following lemma further characterizes the beginning-of-period value function, as well as the cashflow sensitivity of the agent's updated continuation value.

Lemma 2. *In any period t, b^y_t is only defined for $w \geq \lambda\mu$. Moreover,*

$$w^q(y, w) = w + \lambda(y - \mu), \qquad w \geq \lambda\mu. \tag{7}$$

The intuition behind Eq. (7) is that in order to induce the agent not to divert, his continuation value must have a sensitivity λ to his payment to the principal. Hence the incentive-compatibility condition gives the slope of w^q with respect to y, while the promise-keeping condition gives the level of the schedule. The fact that b^y_t is only defined for $w \geq \lambda\mu$ comes from the limited liability constraint: indeed, w needs to be high enough to guarantee that even for the lowest possible cashflow realization, the continuation value $w^q(y)$ consistent with incentive-compatibility and promise-keeping constraints remains non-negative.[5] Given b^y_{t+1}, the end-of-period value function in period t is simply given by

$$b^e_t(w) = e^{-r} b^y_{t+1}(e^{\rho} w), \qquad w \geq e^{-\rho}\lambda\mu, \tag{8}$$

where the domain of b^e_t follows directly from that of b^y_{t+1}.

Lemma 3. *For $t < T-1$, b^e_t is concave in w.*

In a Modigliani-Miller world, increasing the agent's value would merely amount to redistributing total firm value, and the principal's value would simply be linearly decreasing in the agent's value with a slope of -1. In the presence of moral hazard and costly replacement, a change in w also affects the principal's value via its impact on the likelihood of inefficient firing. Under the contract, the investor is committed to firing the agent following a string of bad cash flow realizations even though this may be costly (i.e., *ex post* inefficient) for the investor. When the manager's current promise is low, this *ex post* bad outcome for the investor is relatively likely. Increasing the agent's promise by some given amount hurts the investor by sacrificing some portion of future cash flows, but this is mitigated by the fact that it reduces the prospect of a costly turnover. When the manager's current promise is relatively high, the prospect of turnover is slight and the benefit derived from reducing it is also slight.[6] In the mathematical appendix, we provide a proof to Lemma 3. The key property that drives the result is that at the next but last period before the end of the firm (period $T-1$), in order to be able to properly discipline the agent in the last period, there will be circumstances that lead to the inefficient liquidation of the firm. Then one can show recursively that if the payoff function to the principal at one stage of the firm is concave, the construction of the optimal contract guarantees that the payoff to the principal at earlier stages is also concave.

3.1.1 Cash compensation

The value function b^c_t captures the principal's value contingent on the incumbent manager being retained. The problem at this stage is to find the best possible way to compensate the agent over time, by employing the optimal mix of present versus future compensation. Formally, for $w \geq e^{-\rho}\lambda\mu$

$$b^c_t(w) = \max_{c, w^e} \; -c + b^e_t(w^e) \tag{9}$$

subject to the promise keeping condition $c + w^e = w$, the limited liability condition $c \geq 0$ and $w^e \geq e^{-\rho}\lambda\mu$.

Lemma 4. *Let $\overline{w}_t$ such that $b^{e\,\prime}_t(\overline{w}_t) = -1$. The optimal compensation policy is*

$$c_t(w) = \begin{cases} 0, & w \leq \overline{w}_t, \\ w - \overline{w}_t, & w > \overline{w}_t. \end{cases} \tag{10}$$

Therefore, $b^c_t(w) = b^e_t(w)$ for $w \leq \overline{w}_t$ and $b^c_t(w) = b^e_t(\overline{w}_t) - (w - \overline{w}_t)$ for $w > \overline{w}_t$.

[5] Recall that $\min(\mathcal{Y}) = 0$. More generally, the lower bound of the domain of b^y is $\lambda(\mu - \min(\mathcal{Y}))$.

[6] This reasoning ignores the impact of a change in w on the growth prospects of the firm, as implied by Proposition 1.

Lemma 4 states that it is optimal to defer an agent's compensation until his continuation value has reached the threshold $\overline{w}_t$. The optimal compensation threshold is determined by a basic tradeoff: delayed compensation is preferable because it keeps the agent's promise from falling closer to the inefficient termination threshold, while early compensation is preferable because the agent is more impatient than the principal. Formally, the compensation threshold $\overline{w}_t$ is determined by comparing the marginal cost for the principal of present versus deferred compensation. By compensating the agent with Δc in period t, the principal's value is $-\Delta c + b_t^e(w - \Delta c)$. For a small Δc, this can be approximated by $b_t^e(w) + \Delta c(-1 - b_t^{e\,\prime}(w))$, which shows that non-zero compensation is optimal if and only if $b_t^{e\,\prime}(w) < -1$.

3.1.2 Replacement and growth

We can now proceed with the construction of $b_{t,\theta}^\ell$ for $\theta = G, N$. At this stage, given the realization of θ and the manager's continuation value w, the contract specifies the firing probability $p_{t,\theta}(w)$, the updated continuation value $w_{t,\theta}^c(w)$ that the incumbent manager gets upon being retained, and a possible severance pay $s_{t,\theta}(w)$ awarded if he is not. Note that there is no growth opportunity available such that $\theta = N$, the principal's continuation value (adjusted by current size) upon replacing the incumbent manager is:

$$\ell_{t,N} = e^{-r} b_{t+1}^y(w_0) - \kappa. \tag{11}$$

If instead a growth opportunity is available in period t such that $\theta = G$, the principal's continuation value upon hiring a new manager depends on whether the opportunity is taken up or not. We restrict our attention to situations where the cost of growth (captured by χ) is sufficiently small relative to the benefit of growth (captured by γ), so as to rule out the uninteresting case where the firm would never grow under second best. Hence

$$\ell_{t,G} = e^{-r}(1+\gamma) b_{t+1}^y(w_0) - (\kappa + \chi) > \ell_{t,N}, \tag{12}$$

and $p_{t,G}(w)$ can also be interpreted as the probability of growing conditional on a growth opportunity being available.

The optimal severance and replacement/growth policies are obtained by considering the following constrained maximization problem, separately for $\theta = G$ and $\theta = N$:

$$b_{t,\theta}^\ell(w) = \max_{p,s,w^c} \; p(\ell_{t,\theta} - s) + (1-p) b_t^c(w^c) \tag{13}$$

subject to the promise keeping condition $ps + (1-p)w^c = w$, the limited liability condition $s \geq 0$, $w^c \geq e^{-\rho}\lambda\mu$, and $p \in [0,1]$. To analyze this problem, it is useful to introduce for $\theta \in \{G, N\}$,

$$\delta_{t,\theta} = \sup\left\{ \frac{b_t^c(w) - \ell_{t,\theta}}{w} : w \geq e^{-\rho}\lambda\mu \right\}, \tag{14}$$

and

$$\underline{w}_{t,\theta} = \begin{cases} \inf\{w \geq e^{-\rho}\lambda\mu : b_t^{c\,\prime}(w) \leq \delta_{t,\theta}\}, & \text{if } \delta_{t,\theta} > -1, \\ \infty, & \text{otherwise.} \end{cases} \tag{15}$$

Graphically, $\delta_{t,\theta}$ and $\underline{w}_{t,\theta}$ are determined by finding the line of maximum slope relating the termination point $(0, \ell_{t,\theta})$ to the curve representing $b_t^c(w)$.[7] The slope of this line gives $\delta_{t,\theta}$, while $\underline{w}_{t,\theta}$ is defined as the value of w at the intersection/tangency point if $\delta_{t,\theta} > -1$ and $\underline{w}_{t,\theta} = \infty$ otherwise.

Proposition 1. *For any realization of $\theta \in \{G, N\}$, the optimal replacement policy can be described as follows:*

(i) *if $\delta_{t,\theta} > -1$, the probability of the incumbent agent being replaced is*

$$p_{t,\theta}(w) = \begin{cases} 1 - w/\underline{w}_{t,\theta}, & 0 \leq w < \underline{w}_{t,\theta}, \\ 0, & w \geq \underline{w}_{t,\theta}. \end{cases} \tag{16}$$

The agent receives no severance pay upon being fired, $s_{t,\theta}(w) = 0$, $\forall w < \underline{w}_{t,\theta}$, and his continuation value upon being retained is

$$w_{t,\theta}^c(w) = \begin{cases} \underline{w}_{t,\theta}, & 0 < w < \underline{w}_{t,\theta}, \\ w, & w \geq \underline{w}_{t,\theta}, \end{cases} \tag{17}$$

Hence

$$b_{t,\theta}^\ell(w) = \begin{cases} \ell_{t,\theta} + \delta_{t,\theta} w, & 0 \leq w \leq \underline{w}_{t,\theta}, \\ b_t^c(w), & w \geq \underline{w}_{t,\theta}. \end{cases} \tag{18}$$

(ii) *if $\delta_{t,\theta} \leq -1$, the incumbent manager is replaced with probability one independently of the agent's promised value, $p_{t,\theta}(w) = 1$ for all $w \geq 0$. Upon being replaced, the manager receives $s_{t,\theta}(w) = w$, and*

$$b_{t,\theta}^\ell(w) = \ell_{t,\theta} - w, \quad \forall w \geq 0. \tag{19}$$

We will proceed under the assumption that replacement in the absence of growth is always ex-post inefficient, hence $\delta_{t,N} > -1$ and part *(i)* of Proposition 1 applies in the absence of a growth opportunity. As further discussed in Section 3.2, whether $\delta_{t,G}$ is greater or lower than -1 essentially depends on the quality of growth opportunities relative to the cost of pursuing them (captured by the parameters γ and χ, respectively).[8]

3.1.3 Contractual response to the arrival of a growth opportunity

We now close the derivation of the optimal contract by characterizing how the agent's continuation payoff is affected by the realization of a growth opportunity. This involves solving the optimization problem entering in the definition of b_t^q, as stated in (4).

Proposition 2. *For a given promise w, the contingent continuation payoffs (w_G, w_N) in period t are characterized as follows:*

(a) *If $\delta_{t,G} > -1$*

[7] See Figures 2 and 3.

[8] Note that (14) along with $\ell_{t,G} > \ell_{t,N}$ implies $\delta_{t,G} < \delta_{t,N}$.

(i) *if* $w < (1-q)\underline{w}_{t,G}$, $w_G = 0$ *and* $w_N = w/(1-q)$*;*

(ii) *if* $(1-q)\underline{w}_{t,G} \leq w < \underline{w}_{t,G}$, $w_G = (w - (1-q)\underline{w}_{t,G})/q$ *and* $w_N = \underline{w}_{t,G}$*;*

(iii) *if* $\underline{w}_{t,G} \leq w \leq \overline{w}_t$, $w_G = w_N = w$*;*

(iv) *if* $w > \overline{w}_t$*, any combination of* w_G *and* w_N *such that* $w_G \geq \overline{w}_t$, $w_N \geq \overline{w}_t$*, and* $qw_G + (1-q)w_N = w$ *can be chosen.*

(b) *If* $\delta_{t,G} \leq -1$

(i) *if* $w < (1-q)\overline{w}_t$, $w_G = 0$ *and* $w_N = w/(1-q)$*;*

(ii) *if* $w > (1-q)\overline{w}_t$*, any combination of* $w_G \geq 0$ *and* w_N *such that* $w_N \geq \overline{w}_t$ *and* $qw_G + (1-q)w_N = w$ *can be chosen.*

Combining Proposition 1 and Proposition 2, it immediately follows that

Corollary 1. *There always exists an optimal contract under which the agent receives no severance pay upon being replaced.*

Corollary 1 establishes that severance pay plays no material role in the optimal dynamic contract. Positive severance pay can never arise in the absence of a growth opportunity, or even upon realization of such an opportunity as long as $\delta_{t,G} > -1$. Indeed in both circumstances, part *(i)* of Proposition 1 applies. The only circumstance, though somewhat artificial, where severance pay could arise under an optimal contract is if $\delta_{t,G} \leq -1$, and the firm has had good recent performance so that the agent's promise after cashflow realization is above $(1-q)\overline{w}_t$. In that case, combining part *(ii)* of Proposition 1 and case *(b-ii)* of Proposition 2, it appears that an agent could indifferently be given a non-zero severance pay contingent on $\theta_t = G$, or zero severance and a higher continuation payoff contingent on $\theta_t = N$.

3.2 Discussion of the optimal contract

Having formally derived the optimal contract in our setting, it is useful to summarize it informally and to discuss how it can be implemented in practice. The optimal contract between the firm and its manager sets out the conditions under which the manager will be compensated during his tenure at the firm and also those which will lead to his leaving the firm. These terms and conditions are chosen to maximize the value of payoffs to the firm's owners subject to incentivizing the manager to truthfully report realized cashflows. Payments and retention/replacement decisions are made over time as a function of the value of promised deferred payments, w_t, which evolves under the influence of the firm's operating performance and growth opportunity realizations. The contractual features in force at time t are summarized in the threshold values $\underline{w}_{t,G}$, $\underline{w}_{t,N}$, and $\overline{w}_t$. The manager receives qualitatively different treatment depending upon whether w_t is above or below these thresholds.

The threshold values $\underline{w}_{t,G}$ and $\underline{w}_{t,N}$ may be thought of as *replacement thresholds*. As the replacement decision is made after the availability of a growth opportunity (or lack thereof) has been observed, these thresholds are conditioned on such opportunity being available or not. $\underline{w}_{t,N}$ is the dismissal threshold when there is no growth opportunity available. If the manager's current promise lies above this threshold, $w_t > \underline{w}_{t,N}$, then he knows that he will be retained. If rather the operating performance has been so poor that the manager's promise is below the threshold, $w_t < \underline{w}_{t,N}$, then he is at risk of being fired. In effect, he is given a lottery whereby with some probability he will be dismissed and will receive no further payments from the firm. If he survives this, he stays with the firm and is awarded a continuing promise that is increased to the dismissal threshold amount, $w_t = \underline{w}_{t,N}$. The intuition for why there is zero severance pay is that by reducing the payment upon dismissal to zero the principal is able to increase the promise to the agent if he survives the dismissal threat, thus reducing the agency problem faced by the firm subsequently. The probability of dismissal is chosen so that the lottery is fair, i.e., its expected value equals the agent's promise w_t.

The logic of the dismissal decision when the growth opportunity is available is similar to the above; however, it is made by comparing the agent's promise to the growth dismissal threshold $\underline{w}_{t,G}$ which is higher than that without growth ($\underline{w}_{t,G} > \underline{w}_{t,N}$). That is, risk of dismissal weakly increases if a growth opportunity arises. If the manager's promise is above the threshold $\underline{w}_{t,G}$ he knows he is safe. If he is below this threshold he is given a fair lottery in which, if he is dismissed, he leaves the firm with no further compensation, and if he survives, he is given a continuing promise which is increased to $\underline{w}_{t,G}$.[9]

The threshold value $\overline{w}_t$ can be thought of as the *bonus threshold.* In any period, if the agent has survived the replacement phase, he may be entitled to cash compensation. If the adjusted promise of a surviving agent lies above the bonus threshold such that $w_t > \overline{w}_t$, a bonus is awarded in that period equal to the excess $w_t - \overline{w}_t$, and the agent's continuing promise is reduced to the threshold amount $\overline{w}_t$. Otherwise, if $w_t \leq \overline{w}_t$, the agent receives no compensation in that period and continues with his promise w_t, which is adjusted to $e^{\rho} w_t$ at the beginning of the next period as a fair compensation to the agent for his payoff being delayed.

The promise that the agent takes into a period undergoes two adjustments prior to the replacement and compensation phases. First, upon the report of the cash flow for the period, the agent's promise is adjusted linearly as described in equation (7), the cash flow sensitivity being set so as to provide the right incentives for the agent not to divert. Then upon the realization of θ the promise is further adjusted as described in Proposition 2. The logic is to deliver a given promise w in the form of contingent continuation payoffs (w_G, w_N), taking into account that the reduction in agency costs induced by a marginal increase in the agent's promise depends on whether a growth opportunity is available or not. If the agent's post-cashflow promise is low — i.e., cases *(a-i)* and *(b-i)*, it is optimal to deliver his promise entirely in form of a higher continuation payoff contingent on no growth opportunity becoming available, so as to reduce the likelihood of inefficient replacement. His continuation payoff contingent on a growth opportunity becoming available is set to zero, implying that the agent will be dismissed for sure if a growth opportunity arises, with no severance pay. For higher

[9] As further discussed below, when the benefit of growing is great enough ($\delta_{t,G} \leq 1$), the incumbent manager is systematically replaced when a growth opportunity is available, independently of past performance ($\underline{w}_{t,G} = \infty$). In that case, there exists an optimal contract featuring no severance, though as already mentioned, the contract could equivalently be designed so that the leaving manager would receive positive severance if past performance has been sufficiently good (see Corollary 1).

levels of w, the same comparison between the marginal costs of an increased promise across states of the world (i.e., realizations of θ_t) operates.[10]

The optimal contract calls for zero severance pay to a dismissed manager under most circumstances (in particular if a manager is not dismissed upon growth), and positive severance is always at least weakly dominated by no severance (Corollary 1). The reason is that it is more economical for the firm (again in terms of saving on agency costs) to give zero severance and instead increase the manager's promise contingent on him being retained. Our zero-severance result relies crucially on the assumption that growth opportunities are both exogenous and contractible. We conjecture that growth-induced turnover would go along with positive severance if the principal had to incentivize the agent to ensure growth, or to truthfully report the arrival of a growth opportunity.

The optimal contract we have just described can be implemented fairly directly using standard employment contracts, and there is some evidence that features of our optimal contracts are used in practice. The bonus calculation in this contract is very much like the typical contract that was found by Murphy (2001) in his study of the bonus contracts of large U.S. firms in 1997. The key parameters he identifies are the performance target, the pay-performance-sensitivity (pps), and the bonus threshold. In our contracts, these are μ, λ, and $\overline{w}_t$ respectively.

Our contract specifies an indefinite term with both the manager and the firm having the right to terminate at will.[11] Actual employment contracts are often written in this way.[12] In practice, it is not unheard of that following a period of poor performance when the manager was thought to be under threat of dismissal, the firm instead retains the manager and gives him an improved compensation package as a vote of confidence. This is analogous to the award of deferred compensation of $\underline{w}_\theta - w$ when the manager survives a dismissal threat.

Our analysis implies that it is useful to distinguish two categories of firms depending upon the quality of their growth prospects, both in terms of the frequency of arrival of growth opportunities and of their attractiveness when they become available. The tenure of an incumbent manager will be heavily dependent upon the type of firm he is running. A *high growth* firm is one that will undertake growth any time it has an opportunity independently of the firm's past operating performance, thus generating a lot of growth-induced turnover. Other firms, which for simplicity we call *low growth*

[10] Case *(a-ii)* of Proposition 2, applies to a situation where the agent will survive for sure if there is no growth opportunity and has a positive probability of surviving when a growth opportunity is available. In this case the contingent promises are set so that any surviving agent will face the same incentives, i.e., will carry a promise of $\underline{w}_{t,G}$ into the compensation phase of the period. A similar logic applies in case *(a-iii)*, where the agent will be retained for sure independently of whether or not the growth opportunity is available. For very high values of w, the prospects of inefficient liquidation are so distant that the marginal benefit, in terms of reduced agency costs, of an increased promise is equal to zero across both realizations of θ_t. Obviously the marginal benefit of an increased promise contingent on $\theta_t = G$ is also zero if the agent is replaced for sure upon realization of a growth opportunity.

[11] Our setup could easily be extended to incorporate a positive reservation value for the agent. With zero reservation value and limited liability, inducing the agent to remain in the contract is never an issue.

[12] Of course, some employment laws may constrain this, e.g., by imposing a mandatory notice period which may vary with the tenure.

firms even though in practice they may grow quite fast, do not always take up an available growth opportunity. Instead, if past performance has been good enough and the manager has accumulated a high promised compensation, they will retain the current manager and keep operating assets at the current scale.[13] Proposition 1 shows that the distinction between high and low growth firms depends crucially on $\delta_{t,G}$ defined by Eq. (14). *Low growth* firms are characterized by $\delta_{t,G} > -1$, whereas *High growth* firms satisfy $\delta_{t,G} \leq -1$.[14] *High growth* firms and *low growth* firms behave in dramatically different ways. While *high growth* firms always seize an opportunity to invest and grow, fully realizing their growth potential, *low growth* firms do not systematically take up available growth opportunities, thus wasting part of their growth potential. Hence, for the latter firms, an important source of agency cost is under-investment. For *low growth* firms, the probability of taking a growth opportunity, $p_{t,G}(w)$, is decreasing in w. That is, the better has been the operating performance recently, the less likely that the firm will take up a growth opportunity. These firms do not take up growth opportunities for high w because the overall cost of taking up the growth opportunity is too high.[15]

4 Optimal stationary contract

We now consider our model in the stationary limit where $T \to \infty$. This is a useful simplification because the key features of the optimal contract, adjusting for changes of scale as the firm grows, will be constant over the life of the firm. This allows us to better understand the relationship between these contract features and the deep underlying characteristics of the firm, in particular, the severity of managerial moral hazard and the frequency of growth opportunities.

To do this, we solve numerically for the value functions and associated replacement, growth, severance and compensation policies by iterating backward until convergence for a large value of T. When considering the stationary limit of the optimal contract, we drop all time subscripts. We assume size-adjusted cashflows are independently, identically and uniformly distributed on $\{0, 1, 2, ..., 20\}$, with mean $\mu = 10$. The moral hazard parameter is $\lambda = 0.9$. Discount rates for the principal and the agent are such that $e^r - 1 = 6.5\%$ and $e^\rho - 1 = 7\%$. The cost of firing and replacing a manager is equal to 2% of annual mean cashflow ($\kappa = 0.2$), while the investment cost required for the firm to scale up is set to 20% of annual mean cashflow ($\chi = 2$). We set the scale adjusted reservation compensation for a new manager at $w_0 = 14$. Other parameter values to be specified are q and γ, capturing the likelihood and the magnitude of growth opportunities, respectively.

[13]Note that in both types of firms, the probability of an agent being dismissed conditional on $\theta = N$ weakly increases with poor (past and current) performance.

[14]In Section 4.3, we provide a mapping of *high growth* vs. *low growth* firms in the parameter space in the stationary limit of the model.

[15]This result contrasts with DeMarzo and Fishman (2007a) who find that investment is increasing in the agent's promise because the return on investment is high then.

4.1 Two baseline cases

Our analysis in Section 3.1 shows that the optimal stationary contract is entirely summarized by three threshold values $\underline{w}_N$, $\underline{w}_G$ and $\overline{w}$. Consider first the case where $q = 0.1$ and $\gamma = 0.25$. In this case, the optimal stationary thresholds are $\underline{w}_N = 8.42$, $\underline{w}_G = \infty$ and $\overline{w} = 26.06$. The fact that $\underline{w}_G = \infty$ indicates that it is optimal to grow and replace the agent with probability 1 whenever a growth opportunity is available. That is, this is a *high growth* firm. Figure 2 represents the corresponding stationary value functions. Note that, $b_G^\ell(w)$ decreases linearly with slope -1 and lies above $b^c(w)$ for all w indicates graphically that this is a case of high growth. The agent's compensation threshold $\overline{w} = 26.06$ means that an agent who enters the job with an expected discounted payoff of $w_0 = 14$ must experience a sustained run of good cashflow realizations before receiving any cash compensation.

Suppose instead $\gamma = 0.1$, while all other parameters are kept the same. The optimal stationary thresholds become $\underline{w}_N = 8.42$, $\underline{w}_G = 18.06$ and $\overline{w} = 33.29$. Having reduced the rate at which the firm is allowed to grow upon arrival of a growth opportunity, we now have a firm which does not take up efficient growth opportunities systematically when available, but only if w is below the threshold $\underline{w}_G = 18.06$. This is a *low growth* firm. Figure 3 shows the stationary value functions in this case. Note that, in this case $b_G^\ell(w)$ initially decreases linearly with slope greater than -1 and is tangent to $b^c(w)$ at $\underline{w}_G = 18.06$. Note that in the bonus threshold in the low growth benchmark firm is higher than in the high growth benchmark (33.29 versus 26.06). Later when we simulate the model we will see that on average compensation will arrive much later for the agent in this lower growth case.

4.2 Sensitivity of contract terms

The realized earnings and growth performance of firms are the result of managers' and owners' responses to cashflow shocks and to the arrival of growth opportunities, and these reactions will be shaped by the terms of the contract as set out in the pay-performance sensitivity and in the thresholds, $\underline{w}_N$, $\underline{w}_G$ and $\overline{w}$. Thus understanding how these thresholds are affected by changes in the deep parameters of the model is an important step toward understanding how the earnings and growth experience of firms is determined.

Figure 4 depicts the three thresholds as functions of the severity of moral hazard, λ, and the arrival growth opportunity frequency, q, for a firm with a finite $\underline{w}_G$, that is, for a low growth firm. The understanding of $\underline{w}_N$, the dismissal threshold in the absence of growth opportunities, is quite straightforward because here we have an analytical formula: $\underline{w}_N = e^{-\rho}\lambda\mu$. That is, the non-growth dismissal threshold is linearly increasing in λ and independent of q. Intuitively, in the face of increased moral hazard, the principal will increase the dismissal threshold, thereby increasing the risk of disciplinary dismissal.

Next consider the impact of λ on the bonus threshold, $\overline{w}$. It is increasing in λ reflecting an increased benefit of deferred compensation. This is because the inefficient termination threshold is higher and the pay-performance sensitivity increases, implying that it takes a shorter run of poor performance for the no-growth dismissal threat to be active.

To understand the effect of increasing λ on $\underline{w}_G$, recall that an increase in this threshold means the agent's promise is more likely to be below it, which in turn means that the probability that the firm will take a growth opportunity and fire the manager increases. That is, there is a positive relationship between $\underline{w}_G$ and conditional probability of growth. In light of this, a higher λ results in a higher $\underline{w}_G$ because this has two benefits. There is a higher probability that the firm will undertake the attractive growth opportunity. And if no growth opportunity arrives, agent continues with a higher promise, $w = \underline{w}_G$, which makes subsequent inefficient liquidation less likely.

We turn next to the impact of q on $\overline{w}$ and $\underline{w}_G$, again for low growth firm. A higher q causes a fall in the bonus threshold, $\overline{w}$, implying that cash payouts will be made following a shorter run of good performance. This follows because, a higher q implies higher unconditional probability of early termination, with no severance pay. Thus in order to deliver the reservation value, w_0, ex ante, the cash compensation needs to be paid earlier. Furthermore, for the same reason, in order to increase the probability of getting to the bonus threshold the growth dismissal threshold, $\underline{w}_G$, decreases because this decreases the probability of dismissal, conditional on $\theta = G$.

Finally, for high-growth firms, by definition $\underline{w}_G = \infty$. The sensitivities of $\underline{w}_N$ and $\overline{w}$ are similar to those in the the low-growth case and for similar reasons. Again, in our framework, $\underline{w}_N = e^{-\rho}\lambda\mu$. The bonus threshold $\overline{w}$ is increasing in λ and decreasing in q, as is the case for low-growth firms. $\overline{w}$ falls with an increase in q because the marginal cost of earlier bonus payments decreases as q increases. This is because as q increases it is more likely that a growth opportunity will arrive soon, in which case it will be taken up for sure. Therefore the likelihood of inefficient replacement is reduced and the marginal benefit of deferred compensation is reduced.

4.3 What makes a firm grow fast?

Our baseline examples in Section 4.1 show that two firms that differ only in the size of the growth opportunity will have very different contracts for top management. These differences translate into very different policies toward growth opportunities with high-growth firms undertaking all opportunities that present themselves and low-growth firms undertaking opportunities only if incumbent management is not performing well.

It is also the case that differences only in agency costs may result in very different growth experiences. To see this, consider an example of two firms that have the same size of their growth opportunities ($\gamma = .125$), the same probability of having a stochastic growth opportunity $q = 10\%$, and only differ in the degree of moral hazard λ. All other parameters are as in our baseline cases. In this example, our model predicts that the firm with $\lambda = 0.5$ grows at an average rate of 1.25%. This is because it is a high-growth firm that undertakes all the growth opportunities that arise. Meanwhile, the firm with $\lambda = 1.0$ grows at an average rate of around 0.41%.[16] Stated otherwise, suppose the two firms start out life with identical scale of operations. Fifty years on, $t = 50$, the

[16]The latter statement is based on simulations.

expectation is that the firm with low agency problems will have a scale (measured by the mean cashflow rate) that is 52% larger than the high agency cost firm.

This holds for other parameters as well. That is, we may have two firms that differ only slightly in their deep parameters, with one a high-growth firm and the other a low-growth firm. Figure 5 depicts regions of the parameter space corresponding to high-growth firms and low-growth firms. All parameters are set as in the second baseline case (low-growth firm) of Section 4 except for the two parameters depicted in the diagram.

To summarize, small differences in parameters can result in dramatically different growth and turnover behavior. Growing firms need a good flow (high q) of good growth opportunities (high γ)for expanding markets and improving technology. They need to manage transitions well (low κ, low χ). And they need to keep agency problems under control, for example, through increased monitoring (low λ).

5 Turnover, compensation timing, agency costs

5.1 Simulating the model

We now simulate the model to understand its implications for management turnover and the relative importance of deferred compensation. Simulations also allow us to assess the importance of the agency costs due to the contracting imperfections present in this framework.

Specifically we draw repeatedly a sequence of cashflows and growth opportunity realizations, keeping track of compensation, growth and termination decisions commanded by the optimal contract. We then characterize these histories using a variety of summary statistics. We focus on three statistics that are of particular interest. First we calculate the average longevity or 'tenure' of managers, which is inversely related to the replacement frequency. Second we calculate the unconditional probabilities of efficient termination (i.e. fire the agent to undertake growth) and inefficient termination (i.e. fire the agent without growing) as the corresponding realized sample frequencies. Third, to measure the extent to which the optimal contract relies on deferred compensation, we calculate the average duration of the agent's compensation conditional on the agent receiving non-zero compensation during his tenure in the firm. This is calculated as the weighted average tenure years of the agent's realized payments with weights calculated as the ratio of discounted cash flow to the sum of discounted cash flows.

For example, consider the results for the benchmark cases given in Section 4.1. For the *high growth* firm with $\gamma = 0.25$, average tenure of an agent is 8.6 years. The average probability of efficient termination is 10% per year, reflecting the fact that for a *high growth* firm any available growth opportunity is undertaken. The probability of inefficient termination is about 1.57% per year. And the average duration of compensation is 7.1 years.

In contrast for the *low growth* firm with $\gamma = 0.1$, the average tenure is 109 years. The probability of inefficient termination is 0.25% which is lower than the probability of efficient termination (0.66%). The average duration of compensation is 20.4 years. Comparing results for the two cases, we see that *high growth* firms receive compensation earlier than on average do agents in *low growth* firms.

5.2 Comparative statics

In this section, we further explore predictions from our model in terms of its comparative statics with respect to some key parameters. Specifically, we solve our model for alternative values of these parameters and then simulate the model assuming the same realizations for underlying cashflow shocks and growth opportunities. We record the histories of management turnover, whether turnover takes place for growth or for disciplinary reasons, and the compensation histories for each of the firm's managers. The parameters we vary are q, the probability of having a stochastic growth opportunity, and λ, the severity of agency problems. The default values of these parameters take on when the other parameter is varied are $q = 0.1$ and $\lambda = 0.9$. Other parameters are as in our baseline cases of Section 4.1.

5.2.1 Management turnover

In our model managers are replaced either to facilitate growth or because a history of poor operating results leads to dismissal. The exact conditions under which managers are replaced are sensitive to both the growth prospects of the firm and to the severity of agency problems faced by the firm.

Representing the quality of the growth prospects by the frequency of arrival of growth opportunities, q, we show the sensitivity to this parameter of average manager tenure. This is depicted in the left panel of Figure 6 for a high growth firm with $\gamma = 0.25$. From the figure we see that as the probability of growth opportunity in a year rises from 5% to 25% the average tenure of the agent declines from 15 years to something under 4 years. A similar negative sensitivity to increases in q holds for low growth firms (e.g., with $\gamma < 0.1$), with the difference that, for a given q, the average tenure is much higher.

Thus tenure falls and turnover rises for firms with better growth prospects. To our knowledge this hypothesis has not been submitted to direct empirical testing. However, there is some indirect evidence which is supportive of the hypothesis. Specifically, Mikkelson and Partch (1997) compare top management turnover intensity in two successive five-year periods with very different mergers and acquisitions activity. They find that in the active take-over period of 1984-1988, 33% of firms in the sample underwent complete management changes (i.e., replaced all of the president, CEO and Chairman); whereas this intensity was only 17% in the subsequent period 1989-1993 when take-over activity was low. Interestingly their notion of complete management corresponds better to our model which associates turnover and major changes of direction than does most of the literature which has focused exclusively on CEO turnover. While they do not specifically make a link of management turnover and firm growth, the two periods they cover coincide with very different experiences of firm growth and investment. Specifically, in the 1984-88 period U.S. annual non-residential investment spending increased 28%; whereas, between 1989 and 1993 it increased only 12.5%.[17]

In the right panel of Figure 6 we see the consequences of increasing the severity of managerial moral hazard. As the rent extraction efficiency (λ) of the agent rises the average longevity declines. This is a reflection of the fact that the optimal contract relies more heavily on the threat of termination in the face of more severe moral hazard. Again, a similar pattern is found for low growth firms as well.

5.2.2 Efficient and inefficient replacement probabilities

As already noted, turnover may occur for growth or for discipline. These two kinds of managerial turnover are affected differently by changes in the firm's underlying characteristics. To distinguish these effects, we calculate the average frequency of these two types of turnover in the simulated histories and plot these as functions of q and λ in Figure 7. The top row pertains to the high growth case, with $\gamma = 0.25$ as above. In high growth firms the unconditional probability of replacement for reasons of growth are higher than the probability of disciplinary replacement. Since all growth opportunities are taken up in these firms, this frequency increases linearly in q.

The effect of more severe agency problems on dismissal frequencies in high growth firms is given in the upper right panel of Figure 7. Since all growth opportunities are taken up, changes in λ have no effect on the efficient dismissal probability. The probability of inefficient dismissal is slightly increasing in λ. This reflects an increased reliance on the termination threat when moral hazard is more severe.

The sensitivities of dismissal probabilities for low growth firms are given in the bottom row of Figure 7. As for high growth firms, efficient dismissal probability is increasing in q. Recalling that in low growth firms, growth opportunities are taken only when incumbent managers have been performing poorly, we see that more such managers are eliminated through growth when growth arrives more frequently (i.e., as q increases). In the right panel, the probability of inefficient replacement increases with increasing λ reflecting greater reliance on the dismissal threat (increased $\underline{w}_N$). Thus more managers are replaced before any growth opportunity arrives, implying a decline in the unconditional efficient dismissal probability, as seen in the figure.

5.2.3 Compensation duration

To assess the consequence of changing parameters for the reliance on front loading of compensation, we have calculated the realized duration of compensation from bonuses during agents' tenure. These sensitivities are given in Figure 8. From the top row we see that for both high and low growth firms an increase in q reduces the duration of compensation. That is, when growth opportunities arrive more frequently, firms optimally rely on more front-loading of compensation. The effect works through the lower bonus threshold for high-growth firms.

The second row of Figure 8 shows the effect of increasing λ. For both high growth firms and low growth firms the average duration of compensation rises as λ rises. The

[17]Based on annual U.S. National Income Statistics.

reason for this is that a higher λ increases bonus threshold, $\overline{w}$. Managers receive compensation only after a sustained run of good performance.

Again, to our knowledge, there are no empirical studies that directly test whether these effects on the timing of compensation hold. However, recently Kaplan and Minton (2008) have studied the evolution of top CEO turnover since 1990, a period that saw very rapid increases in the amount of top management compensation. They find evidence of more rapid turnover, especially after 2000. They argue that the observed increases in CEO pay are compensation for shorter tenure. This is consistent with our theory in which high growth will be associated with shorter tenure and more front-loading of compensation.

5.3 Agency costs

In this section we assess the loss of value caused by the non-contractibility of cashflows. In our framework with repeated growth options, the first-best value of the firm is the expected present discounted value of all cashflows net of dismissal and investment costs when the firm undertakes all growth opportunities that present themselves but does not dismiss any manager in the absence of growth. Under the optimal contract in the face of non-contractible cashflow, the firm will fall short of this value for several distinct reasons. First, as in previous studies of agency in a dynamic setting, under the optimal contract the firm will dismiss managers for disciplinary reasons following a series of poor cashflow realizations even though this is ex post inefficient. Second, there is an inefficiency due to the reliance on deferred compensation when mangers are more impatient than investors, $\rho > r$. Third, under the optimal contract the firm will sometimes retain an incumbent manager and pass-up growth opportunities even though growth is ex post efficient. Finally, there is a more subtle form of agency costs which we have not emphasized in our discussion until now. This is due to the fact that at the time of agreeing a contract with an incoming manager the firm does not take into account the spill-over effect on the timing of future managers' hiring. As noted in the Introduction, this effect is absent in the previous literature.

Specifically, the second best value of the firm is the expected present value of all cashflows that accrue to the principal and to all managers who successively run the firm under optimal contracts as set out in Proposition 1. Two subtleties should be noted in calculating this second best value. First, cash flows to agents are discounted at the agents' discount rate, ρ; whereas, investor cash flows are discounted at rate r. Since $\rho > r$, the promise to an agent is worth less to the agent than it costs the firm. Second, the calculation of agent cash flows includes payments to *all* agents, both current and future. Thus in the stationary case we can write the size-adjusted, beginning-of-period second-best value of the firm as

$$v(w) = b^y(w) + w + f(w), \tag{20}$$

where $f(w)$ denotes the expected discounted value of payoffs to future agents as a function of the current agent's promised value, w.[18] To assess the extent of agency

[18] The last term, $f(w)$, does not appear in the earlier contributions to the literature on optimal long-term

costs, the total value of the firm under the optimal contract $v(w)$ can be compared to the beginning-of-period, first-best value of the firm, $\mu + v^*$, for v^* defined in (2).

Figure 9 depicts values under the second best optimal contact for the *high growth* ($\gamma = 0.25$ in the top panel) and *low growth* firms ($\gamma = 0.1$ in the bottom panel) as set out in Section 4.1. The left panel gives the value for the principal and the incumbent agent, $b(w) + w$. The middle panel give the present value of compensation to future agents who are not party to the current contract but who are affected by the current contract and the current promise to the incumbent agent, $f(w)$. The right panel gives the sum of all these components, that is, the second best value of the firm defined above, $v(w) = b(w) + w + f(w)$. These can be compared to the corresponding first best values ($\mu + v^*$) of 260.39 and 189.37, respectively. The second-best value function $v(w)$ shows only little sensitivity to the current agent's promise w. Agency costs amount to roughly 5% of first-best value for the high growth case and about 13% in the low growth case. That is, agency costs represent about fifteen months of expected cashflows for the high-growth firm and about thirty-four months of expected cashflows for the low-growth firm. The principal reason why agency costs are less for the high-growth firm is because it undertakes all investment opportunities, even under the second-best contract, whereas a low-growth firm suffers from under-investment.

In the left panels of Figure 9 we see that for both high and low growth firms the combined value to the principal and the incumbent manager is increasing in the promise to this manager. This reflects the relaxation of agency problems affecting the two parties to the current contract, and this is an effect already seen in previous dynamic agency models. Interestingly, the second-best firm value, taking into account the effect on future managers, is not increasing and concave in w. This is seen in the right panel of Figure 9 where, for both high-growth and low-growth firms, $v(w)$ becomes decreasing beyond a certain point.

Why? The answer is that the second best contract is designed so as to maximize investor value subject to the incentive compatibility condition (6) *vis à vis* the incumbent agent. This condition does not take into consideration the consequences for *future* agents. Thus incentivizing the current agent with a higher promise may come at the cost of reducing payoffs to future agents. Specifically, if the current agent will be succeeded by future agents at stochastic stopping times τ_i, $i = 1, 2, 3, ...$, the expected present values of the amounts they will receive, $\mathbb{E}[e^{-\rho\tau_i}\Phi_{\tau_i} w_0]$, are both missing and affected by the current w since this affects the distribution of stopping times.

As can be seen from the central panel of Figure 9, the present value of payoffs to future agents, $f(w)$, is decreasing in the current promise. In the case of low growth firms there are two separate effects. A higher promise w tends to decrease the probability that the incumbent will be replaced for disciplinary reasons. It is also reduces the probability of replacing the agent in order to undertake growth. In the case of high growth firms, by definition, growth opportunities are undertaken whenever they appear,

contracts where there is a single agent and the "liquidation" value of the firm is exogenous. For instance, the liquidation value of the firm is set equal to zero in Biais et al. (2007). In their welfare analysis DeMarzo and Fishman (2007b) take the liquidation value to be equal to an exogenous fraction of the first-best value. Garrett and Pavan (2012) do identify a tendency toward excessive retention of managers which implies a loss of welfare somewhat akin to what we capture in $f(w)$.

independently of w. Thus only the first effect is present. This is the reason that the value $f(w)$ is less sensitive to changes in w in the high growth case than in the low growth case. Note that as w increases from 10 to 30, $f(w)$ declines by about 5 for the high-growth firm and by about 9 for the low-growth firm.

6 Extension: who grows the firm?

The maintained assumption in our analysis so far was that in order to pursue an opportunity to grow, the incumbent manager had to be replaced. We now consider a more general environment where upon the arrival of a growth opportunity, the firm can decide to grow either with a new manager or with the incumbent manager. Endogenizing the choice of managerial replacement upon growth makes the analysis of the model more complex. However, the economic forces we have highlighted so far remain at play, and the analysis will help clarify under which circumstances our conclusions from earlier sections still hold, and how they need to be modified in other cases.

We now let χ^i denote the (size-adjusted) cost of taking the growth opportunity with the incumbent manager, and χ^n the cost of growing with a new manager.[19] The derivation of the optimal contract follows the same logic as in Section 3.1, except for the construction of b_G^ℓ.[20] The key novel feature of the optimal contract in the extended environment is that, when faced with a growth opportunity, the firm needs to decide whether the incumbent manager, if retained, would keep running the firm at the same size or at an expanded size. Formally, we define

$$\bar{b}_G^\ell(w) = \max_{p,s,w^c} p(\ell_G - s) + (1-p)b^c(w^c) \tag{21}$$

subject to the promise keeping condition $ps + (1-p)w^c = w$ the limited liability condition $s \geq 0$, $w^c \geq e^{-\rho}\lambda\mu$, and $p \in [0,1]$. We also define

$$\hat{b}_G^\ell(w) = \max_{p,s,w^c} p(\ell_G - s) + (1-p)[(1+\gamma)b^c(w^c) - \chi^i] \tag{22}$$

subject to the alternative promise keeping condition $ps + (1-p)(1+\gamma)w^c = w$. The value function $\bar{b}_G^\ell$ corresponds to the case where upon retaining its incumbent manager the firm does not take up the growth opportunity. The value function $\hat{b}_G^\ell$ corresponds to the alternative case where, if retained, the incumbent manager does implement the growth opportunity.[21] Note that ℓ_G, the continuation value upon replacement contingent on $\theta = G$, is generally defined as

$$\ell_G = \max\{e^{-r}(1+\gamma)b^y(w_0) - \kappa - \chi^n;\ e^{-r}b^y(w_0) - \kappa\}. \tag{23}$$

Whenever the cost of growing with a new manager χ^n is sufficiently small (relative to γ), if a new manager is hired at a time a growth opportunity is available, growth

[19]We assume that $\gamma\mu/(e^r - 1) > \min(\chi^i, \chi^n + \kappa)$, so that the first-best policy in steady state involves taking all growth opportunities. Under first best, the firm grows with new managers if and only if $\chi^n + \kappa < \chi^i$.

[20]For notational convenience, we drop all time subscripts in this section.

[21]Note that in that case, the probability of managerial replacement $p_G(w)$, which appears as p in (22), no longer coincides with the probability of growing conditional on $\theta = G$.

is implemented ($\ell_G > \ell_N$). For high values of χ^n, the firm never grows with a new manager ($\ell_G = \ell_N$).

6.1 When the incumbent never grows the firm

We start our analysis of our extended model by showing that under some circumstances the firm will never grow with incumbent manager and that in this case our analysis of sections 3, 4 and 5 goes through. The firm chooses optimally whether to grow or not upon retaining an incumbent manager at times a growth opportunity is available. In particular, if it is prohibitively costly to do so (χ^i very large), a firm would never grow with an incumbent manager and would only ever grow with new managers (as long as the costs of doing so, captured by χ^n, are reasonably low). Our analysis of the baseline model directly applies to such configurations.

Proposition 3. *When χ^i is large, $b^\ell_G = \bar{b}^\ell_G$, and a firm never grows with an incumbent manager. If moreover χ^n is relatively small, all the results of Section 3 apply.*

In the remaining of this section, we turn our attention to alternative configurations where χ^i is low relative to the gains from growth (implying $\hat{b}^\ell_G > \bar{b}^\ell_G$), so that it is optimal for the firm to sometimes grow with an incumbent manager.[22]

6.2 When the incumbent may grow the firm

Our next proposition describes the construction of b^ℓ_G and the associated replacement and severance policies (conditional on $\theta = G$) in circumstances when the firm may grow under incumbent management. In our extending setting this will depend crucially on χ^i being low enough relative to the other parameters of the firm. Note that b^ℓ_N along with $p_N(w)$, $s_N(w)$ and $w^c_N(w)$ are still obtained along the lines of Proposition 1.

Proposition 4. *When χ^i is low, $b^\ell_G = \hat{b}^\ell_G$, where the construction of $\hat{b}^\ell_G$ given b^c proceeds as follows. Let $\underline{w}_G = (1+\gamma)e^{-\rho}\lambda\mu$ and*

$$\hat{b}^c(w) = (1+\gamma)b^c\left(\frac{w}{1+\gamma}\right) - \chi^i, \qquad w \geq \underline{w}_G \tag{24}$$

and define $\delta_G = \frac{\hat{b}^c(\underline{w}_G)-\ell_G}{\underline{w}_G}$. Then

$$\hat{b}^\ell_G(w) = \begin{cases} \ell_G + \delta_G w, & 0 \leq w \leq \underline{w}_G, \\ \hat{b}^c(w), & w \geq \underline{w}_G. \end{cases} \tag{25}$$

The managerial replacement probability conditional on $\theta = G$ is

$$p_G(w) = \begin{cases} 1 - (w/\underline{w}_G), & 0 \leq w < \underline{w}_G, \\ 0, & w \geq \underline{w}_G. \end{cases} \tag{26}$$

[22] We ignore situations where $\hat{b}^\ell_G(w) > \bar{b}^\ell_G(w)$ if and only if w is above some threshold.

Severance pay conditional on $\theta = G$ *is* $s_G(w) = 0$, $\forall w$, *and the continuation value upon being retained (adjusted by end-of-the-period size) is*

$$w_G^c(w) = \begin{cases} \hat{\underline{w}}_G/(1+\gamma), & 0 < w < \underline{w}_G, \\ w/(1+\gamma), & w \geq \underline{w}_G, \end{cases} \tag{27}$$

Our next proposition characterizes the adjustment of an agent's expected payoff to the arrival of a growth opportunity. For low values of w, the adjustment of an agent's promise contingent on θ crucially depends on the relative magnitude of δ_G vs δ_N, i.e., on the relative ex-post inefficiency of replacement across realizations of θ. For instance, if the inefficiency is greater in the no-growth state of the world ($\delta_N > \delta_G$) and the agent's promise post-cashflow is sufficiently small, then it is optimal to deliver all the promise in the form of a higher continuation payoff contingent on the realization of that state so as to reduce the prospect of the most inefficient type of turnover.

Proposition 5. *For a given promise* w, *the contingent continuation payoffs* (w_G, w_N) *in period* t *are characterized as follows.*

(a) *if* $\delta_N > \delta_G$ *[for low* χ^n*]*

(i) *if* $w < (1-q)\underline{w}_N$, $w_G = 0$ *and* $w_N = \frac{w}{1-q}$;

(ii) *if* $w \in [(1-q)\underline{w}_N, q\underline{w}_G + (1-q)\underline{w}_N]$, $w_G = \frac{w-(1-q)\underline{w}_N}{q}$ *and* $w_N = \underline{w}_N$;

(iii) *if* $q\underline{w}_G + (1-q)\underline{w}_N \leq w \leq (1+\gamma q)\overline{w}$, $w_G = \frac{1+\gamma}{1+\gamma q}w$ *and* $w_N = \frac{1}{1+\gamma q}w$;

(iv) *if* $w > (1+\gamma q)\overline{w}$, *any pair* (w_G, w_N) *such that* $w_G \geq (1+\gamma)\overline{w}$, $w_N \geq \overline{w}$, *and* $qw_G + (1-q)w_N = w$ *is optimal.*

(b) *if* $\delta_N < \delta_G$ *[for high* χ^n*]*

(i) *if* $w < q\underline{w}_G$, $w_G = w/q$ *and* $w_N = 0$;

(ii) *if* $w \in [(q\underline{w}_G, q\underline{w}_G + (1-q)\underline{w}_N]$, $w_G = \underline{w}_G$ *and* $w_N = \frac{w-q\underline{w}_G}{1-q}$;

and (iii) *and* (iv) *of case* (a) *apply for higher values of* w.

Figure 10 depicts stationary value functions b_G^ℓ and b_N^ℓ for parameter values such that Proposition 4 applies and the firm sometimes grows with the incumbent manager.[23] Threshold values are $\underline{w}_N = 8.41$, $\underline{w}_G = 9.25$, and $\overline{w} = 44.93$. In that example $\ell_G > \ell_N$, i.e., if turnover occurs at times a growth opportunity is available, the firm will grow with its new manager. Moreover, $\delta_N = 0.77$ and $\delta_G = 0.69$, i.e., replacement is more inefficient ex-post in the absence of a growth opportunity, and part *(a)* of Proposition 5 applies.

To conclude this section, we illustrate the optimal history-contingent compensation and turnover policies by way of a simple numerical example, for the parameter values used in Figure 10. The goal of the example is twofold. First, we illustrate under which circumstances the firm finds it optimal to grow with the incumbent manager. Second, we illustrate how managerial turnover is affected by past and current cashflow realizations and the availability of a growth opportunity.

[23]Here we assume $\chi^i = \chi^n = 2$, $\kappa = 7.5$, $\gamma = 0.1$, $q = 0.2$, and the other parameters are as in the benchmark case of Section 4.1, i.e., $\lambda = 0.9$, $e^r - 1 = 6.5\%$, $e^\rho - 1 = 7\%$, $\mu = 10$ and $w_0 = 14$.

Table 1: An illustration of the optimal contract for low χ_i

Period t	6	7	8	...	14	15
Size Φ_t	1	1	1.1	—	1.1	1.1
Promise w_t^y	30	36.55	41.58	—	12	9.00
Cashflow y_t	15	13	17	—	4	9
Promise w_t^q	34.50	39.25	47.88	—	6.60	8.10
Growth option θ_t	N	G	N	—	N	G
Promise $w_{t,\theta}^{\ell}$	34.16	42.75	47.41	—	7.33	5.30
Replacement proba p	0	0	0	—	0.13	0.43
Promise w^c	34.16	38.86	47.41	—	8.41	
Cash compensation c	0	0	2.48	—	0	
Promise w^e	34.16	38.86	44.93	—	8.41	

Table 1 presents the evolution of the contractual promise to the incumbent manager for a particular path of scale adjusted cash flows and growth opportunity realizations. At the beginning of the episode we consider, the manager is still running the firm at its initial size, and has accumulated a high promise as a consequence of sustained good performance. His promise w_N^{ℓ} is much higher than the dismissal threshold, but not high enough to warrant a bonus. Thus he continues into period $t = 7$ carrying a promise that has been augmented from previous period to take into account the manager's rate of time preference, ρ. A growth opportunity presents itself and given its high promise level, the manager is retained and is allowed to grow the firm. Notice that his scale-adjusted promise is reduced (from $w_G^{\ell} = 42.75$ to $w^c = 38.86$) to reflect that in the future he will be running a larger firm and therefore will be facing a high expected cash flow implying higher compensation (i.e., his expected payoff is kept at the same level). Subsequently, the firm is operated at a scale of 1.1 and following another good cashflow in period $t = 8$ the agent has accumulated a sufficiently high promise to be awarded a bonus.

After period $t = 8$, the firm goes through several periods of sustained poor performance, and the manager starts period $t = 14$ with an expected discounted payoff $w^y = 12$. After another poor cashflow realization, his promise falls at a low point point of $w^q = 6.60$. Inefficient termination is looming and case *(a-i)* of Proposition 5 applies. If a growth opportunity arrived, the agent would be dismissed with certainty with zero severance; on the other hand, with a contingent continuation promise w_N^{ℓ} raised to 7.33 the manager has a higher chance of surviving the dismissal stage in case no growth opportunity arises, i.e., the most inefficient form of turnover is made less likely. In our example, no growth opportunity arises in period $t = 14$, but the agent's promise is still below the dismissal threshold $\underline{w}_N = 8.39$, and therefore he is at risk of being fired with no severance (with 13% chance). The challenged manager is fortunate enough to be retained with an increased promise, set in such a way that he starts the following period with a promise high enough ($w^y = 9$) to guarantee that limited liability does not go into the way of pay-performance sensitivity to provide appropriate incentives not to divert cash. The firm performance in that period is not good enough for the manager to be sure to keep his job ($w^q < q\underline{w}_G + (1-q)\underline{w}_N = 8.49$). Case *(a-ii)* of Proposition 5 now applies since $w^q > (1-q)\underline{w}_N = 7.57$. If no growth opportunity had materialized in that period, the manager would have been safe ($w_N^\ell = \underline{w}_N$). However, given the realization of a second growth opportunity, he is again at risk of being fired (with 43% chance). In our example, the manager is dismissed and his tenure ends after 15 periods. A new manager is hired to run the firm at a size of 1.21 (indeed χ^n is small enough to guarantee that $\ell_G > \ell_N$, i.e., it is more beneficial for the firm to take up growth with its new manager than not).

7 Conclusion

In this paper we explore the relationship between managerial compensation and growth in a dynamic agency framework. In contrast with previous studies, we consider a long-lived firm with growth prospects that can hire a sequence of managers over time. In this setting management replacement may occur not only to discipline management but also possibly to facilitate growth. This framework produces new insights on managerial compensation and turnover. We find that the firm's growth trajectory depends on the severity of agency problems as well as the quality of its growth opportunities. We show how optimal contracts in firms with growth opportunities can be implemented with a system of deferred compensation credit and bonuses that are similar to that found in practice. We find that firms with very good growth prospects tend to rely less on back-loading of compensation than firms with poor growth prospects. We also identify a new component of agency costs which relates exclusively to managerial turnover. This new component of agency costs is due to the spillover effect of the length of an existing managerial contract onto the present value of all future contracts signed by the firm.

Our study suggests a number of open issues concerning the relation between growth and incentive provision. In our framework, the growth event is modeled very simply. Within a single time period, a growth opportunity appears, and the firm decides whether or not to take it up and whether or not to replace the incumbent with a new manager. In reality, many growth opportunities may require or at least benefit from a prolonged transition during which outgoing and incoming management need to cooperate. Extending our model in this direction might yield new predictions on optimal managerial contracts.

In a different vein, it would be interesting to explore the determinants of the growth opportunity arrival process which here we have treated as exogenous. In particular, current management may need to allocate their efforts between producing cash flows from assets in place and developing new opportunities for growth. There may be a trade-off between two activities in that they may both require top management time but also because they use different management skills.

A Appendix

Proof of Lemma 1: Non scale-adjusted value functions are defined recursively as follows. Given $B^{\ell}_{t+1,G}(\Phi, w)$ and $B^{\ell}_{t+1,N}(\Phi, w)$, we have

$$B^q_{t+1}(\Phi, w) = \max_{w_G, w_N \geq 0} qB^{\ell}_{t+1,G}(\Phi, w_G) + (1-q)B^{\ell}_{t+1,N}(\Phi, w_N), \tag{28}$$

subject to $qw_G + (1-q)w_N = w$. Then

$$B^y_{t+1}(\Phi, w) = \max_{\{w^q(y)\}_{y \in \mathcal{Y}}} \Phi\mu + \mathbb{E}_y\{B^q_{t+1}[\Phi, w^q(y)]\} \tag{29}$$

subject to promise-keeping condition $\mathbb{E}_y[w^q(y)] = w$, limited liability $w^q(y) \geq 0$, and incentive-compatibility constraint

$$w^q(y) \geq w^q(\tilde{y}) + \lambda(y - \tilde{y}), \quad \forall y \in \mathcal{Y}, \forall \tilde{y} \in [0, y]. \tag{30}$$

Note that the limited liability and incentive-compatibility constraints imply that B^y_{t+1} is only defined for $w \geq \lambda\mu$. Now, given B^y_{t+1}, we can define

$$B^e_t(\Phi, w) = e^{-r}B^y_{t+1}(\Phi, e^{\rho}w), \qquad w \geq e^{-\rho}\lambda\mu \tag{31}$$

Next

$$B^c_t(\Phi, w) = \max_{C, w^e \geq 0} -C + B^e_t(\Phi, w^e) \tag{32}$$

subject to $C + \Phi w^e = \Phi w$. Note that the first argument in functions B^c and B^e is the beginning-of-next-period size, which has already been determined, and cash compensation C is not size-adjusted. We can also define

$$L_{t,N}(\Phi) = e^{-r}B^y_{t+1}(\Phi, w_0) - \kappa\Phi, \tag{33}$$

$$L_{t,G}(\Phi) = e^{-r}B^y_{t+1}((1+\gamma)\Phi, w_0) - (\kappa + \chi)\Phi, \tag{34}$$

and

$$B^{\ell}_{t,\theta}(\Phi, a) = \max_{p, S, w^c} p(L_{t,\theta}(\Phi) - S) + (1-p)B^c_t(\Phi, w^c) \tag{35}$$

subject to $pS + (1-p)\Phi w^c = \Phi w$, $S \geq 0$, $p \in [0, 1]$, and $w^c \geq e^{-\rho}\lambda\mu$. The homogeneity result and the definition of the scale-adjusted value functions as they appear in Section 3.1 follows directly from the observation that in the last period $B^q_T(\Phi, w) = -\Phi w$. Then given the homogeneity of B^q_T, the homogeneity of B^y_T follows, and homogeneity of earlier value functions obtains recursively. ∎

Proof of Lemma 3: Our goal here is to show how the concavity of b^e_t arises for $t < T-1$. For that purpose, we need to go through the detailed construction of the value functions within period $T-1$. Our starting point is that in the last period $b^y_T(w) = \mu - w$, for $w \geq \lambda\mu$, which in turn implies $b^e_{T-1}(w) = e^{-r}\mu - e^{\rho - r}w$, for $w \geq e^{-\rho}\lambda\mu$. Since the slope of b^e_{T-1} is strictly below -1, the solution of the constrained maximization problem in (9) involves setting $w^e = e^{-\rho}\lambda\mu$ and $c = w - e^{-\rho}\lambda\mu$. Therefore,

$$b^c_{T-1}(w) = e^{-\rho}\lambda\mu + (1-\lambda)e^{-r}\mu - w, \qquad w \geq e^{-\rho}\lambda\mu. \tag{36}$$

We can now analyze $b^\ell_{T-1,N}$. The relevant continuation value upon replacement is

$$\ell_{T-1,N} = e^{-r}b^y(w_0) - \kappa = e^{-r}\mu - (e^{-r}w_0 + \kappa). \tag{37}$$

Note that $w_0 \geq \lambda\mu$ implies that $\ell_{T-1,N} < e^{-\rho}\lambda\mu + (1-\lambda)e^{-r}\mu$, which in turn implies that $\delta_{T-1,N} > -1$ and $b^\ell_{T-1,N}$ is piecewise linear and globally concave, with a kink at $\underline{w}_{T-1,N} = e^{-\rho}\lambda\mu$. The same characterization applies to $b^\ell_{T-1,G}$ if $\delta_{T-1,G} > -1$; otherwise $b^\ell_{T-1,G}$ is simply linearly decreasing with slope -1. Furthermore, note that $\ell_{G,T-1} > \ell_{N,T-1}$ implies $\delta_{T-1,G} < \delta_{T-1,N}$. Consider now the constrained optimization problem in (4). Given our previous characterization of $b^\ell_{T-1,N}$ and $b^\ell_{T-1,G}$, we know the maximum is reached (though not necessarily uniquely) by setting $w_G = 0$ and $w_N = w/(1-q)$. Therefore we can write

$$b^q_{T-1}(w) = q\ell_{G,T-1} + (1-q)b^\ell_{T-1,N}\left(\frac{w}{1-q}\right). \tag{38}$$

This further implies that b^q_{T-1} is piecewise linear and globally concave, with slope $\delta_{T-1,N} > -1$ for $w < (1-q)\underline{w}_{T-1,N}$ and slope -1 for $w > (1-q)\underline{w}_{T-1,N}$, with a kink at $(1-q)\underline{w}_{T-1,N}$. We now turn to the function b^y_{T-1} as defined in 5. Using Lemma 2, we can write

$$b^y_{T-1}(w) = \mu + \int b^q_{T-1}(w + \lambda(y-\mu))dF(y), \tag{39}$$

where F denotes the cumulative probability distribution of size-adjusted cashflows. Consider two promises w_A and w_B greater or equal to $\lambda\mu$, and for $\alpha \in (0,1)$, define $w_C = \alpha w_A + (1-\alpha)w_B$. Note that

$$\begin{aligned}
& \alpha \int b^q_{T-1}(w_A + \lambda(y-\mu))dF(y) + (1-\alpha)\int b^q_{T-1}(w_B + \lambda(y-\mu))dF(y) \\
= & \int [\alpha b^q_{T-1}(w_A + \lambda(y-\mu)) + (1-\alpha)b^q_{T-1}(w_B + \lambda(y-\mu))]dF(y) \\
\leq & \int b^q_{T-1}[\alpha(w_A + \lambda(y-\mu)) + (1-\alpha)(w_B + \lambda(y-\mu))]dF(y) \\
= & \int b^q_{T-1}[(\alpha w_A + (1-\alpha)w_B) + \lambda(y-\mu)]dF(y).
\end{aligned}$$

Therefore $\alpha b^y_{T-1}(w_A) + (1-\alpha)b^y_{T-1}(w_B) \leq b^y_{T-1}(w_C)$, and b^y_{T-1} is concave. Further inspection shows that b^y_{T-1} is strictly concave for $w < (1-q)\underline{w}_{T-1,N} + \lambda\mu$, and decreases linearly with slope -1 above that threshold. The concavity of b^e_{T-2} follows directly. That concavity is preserved in earlier periods can be established using similar arguments. ∎

Proof of Proposition 2: We drop time subscripts for notational convenience and define the function $V_w(w_G) = qb^\ell_G(w_G) + (1-q)b^\ell_N[w_N(w_G, w)]$, where

$$w_N(w_G, w) = \frac{1}{1-q}(w - qw_G). \tag{40}$$

For any $w \geq 0$, we consider the problem

$$\max_{w_G \in \left[0, \frac{w}{q}\right]} V_w(w_G). \tag{41}$$

Note that $V'_w(w_G)$ has the sign of $b^\ell_G{}'(w_G) - b^\ell_N{}'[w_N(w_G, w)]$. Consider first the case where $\delta_G > -1$ and $\underline{w}_G < \infty$, as depicted in Figure 3. For $w < (1-q)\underline{w}_G$, $V'_w(0) < 0$; indeed $w_N(0, w) = w/(1-q) < \underline{w}_G$ and therefore $b^\ell_N{}'(w_N(0, w)) > \delta_G$. Hence we have the corner solution $w_G = 0$ and $w_N = w/(1-q)$. For $w \geq (1-q)\underline{w}_G$, the first-order optimality condition $V'_w(w_G) = 0$ is satisfied at $w_G = w$. Indeed $w_N(w, w) = w$, and $b^\ell_G{}'(w) = b^\ell_N{}'(w)$ since b^ℓ_G and b^ℓ_N both coincide with b^c in that range. Setting $w_G = w_N = w$ is the unique solution when $w \in [(1-q)\underline{w}_G, \overline{w}]$ since V_w is strictly concave over that range. However for $w > \overline{w}$, the maximum of V_w is reached at any $w_G \geq \overline{w}$ such that $w_N(w_G, w) \geq \overline{w}$. This comes from the fact that b^c is linear over that region. Consider now the case where $\underline{w}_G = \infty$ and b^ℓ_G decreases linearly with slope -1. This case is as depicted in Figure 2. For $w < (1-q)\overline{w}$, $V'_w(0) < 0$; indeed $w_N(0, w) < \overline{w}$ and therefore $b^\ell_N{}'(w_N(0, w)) > -1$. Hence we have the corner solution $w_G = 0$ and $w_N = w/(1-q)$. However for $w > (1-q)\overline{w}$, the maximum of V_w is reached at any $w_G \geq 0$ such that $w_N(w_G, w) \geq \overline{w}$. ∎

Proof of Proposition 4: Consider the constrained optimization problem in (22). For given $w > (1+\gamma)e^{-\rho}\lambda\mu$, the objective function evaluated at the candidate solution $p = 0$, $s = 0$ and $w^c = w/(1+\gamma)$ is equal to $\hat{b}^c(w)$, where $\hat{b}^c$ is defined in (24). Note that the lower bound of the domain of $\hat{b}^c$ follows directly from the lower bound of the domain of b^c. All achievable payoffs are within the convex hull of $(0, \ell_G)$ and the payoff frontier $\hat{b}^c$. ∎

Proof of Proposition 5: The argument of the proof relies crucially on the slopes of the value functions b^ℓ_G and b^ℓ_N. When χ^i is low and $b^\ell_G = \hat{b}^\ell_G$, then for $w > \underline{w}_G$, $b^\ell_G{}'(w) = \hat{b}^{c\prime}(w) = b^{c\prime}(w/(1+\gamma))$. Then we apply the same logic as in the proof of Proposition 2 ∎

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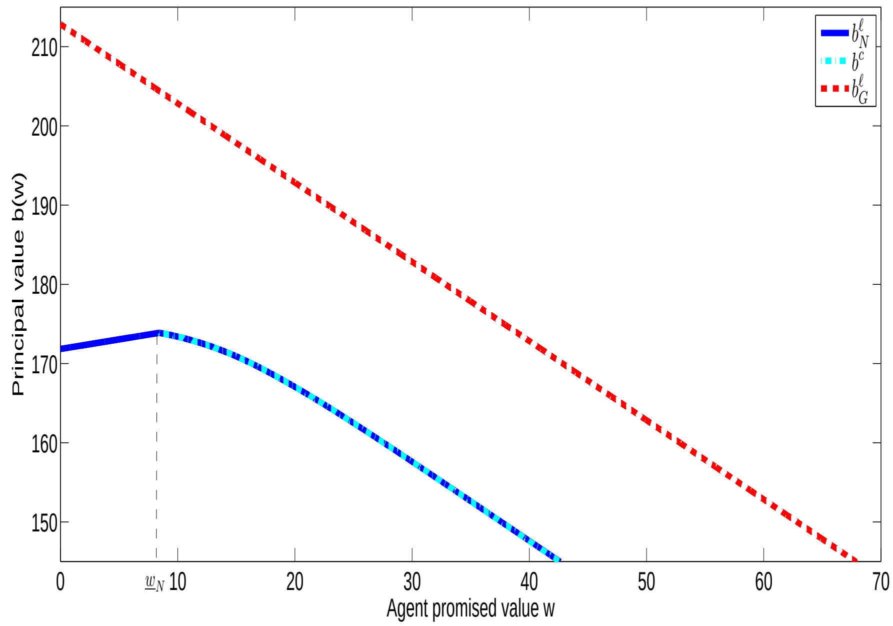

Figure 2: Value functions for high growth firm ($\gamma = 0.25$)

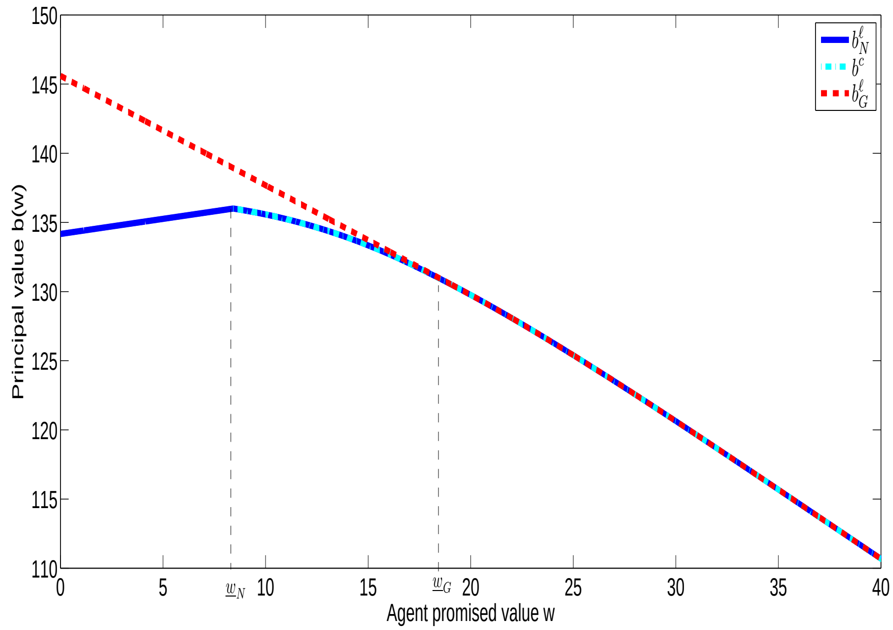

Figure 3: Value functions for low growth firms ($\gamma = 0.1$)

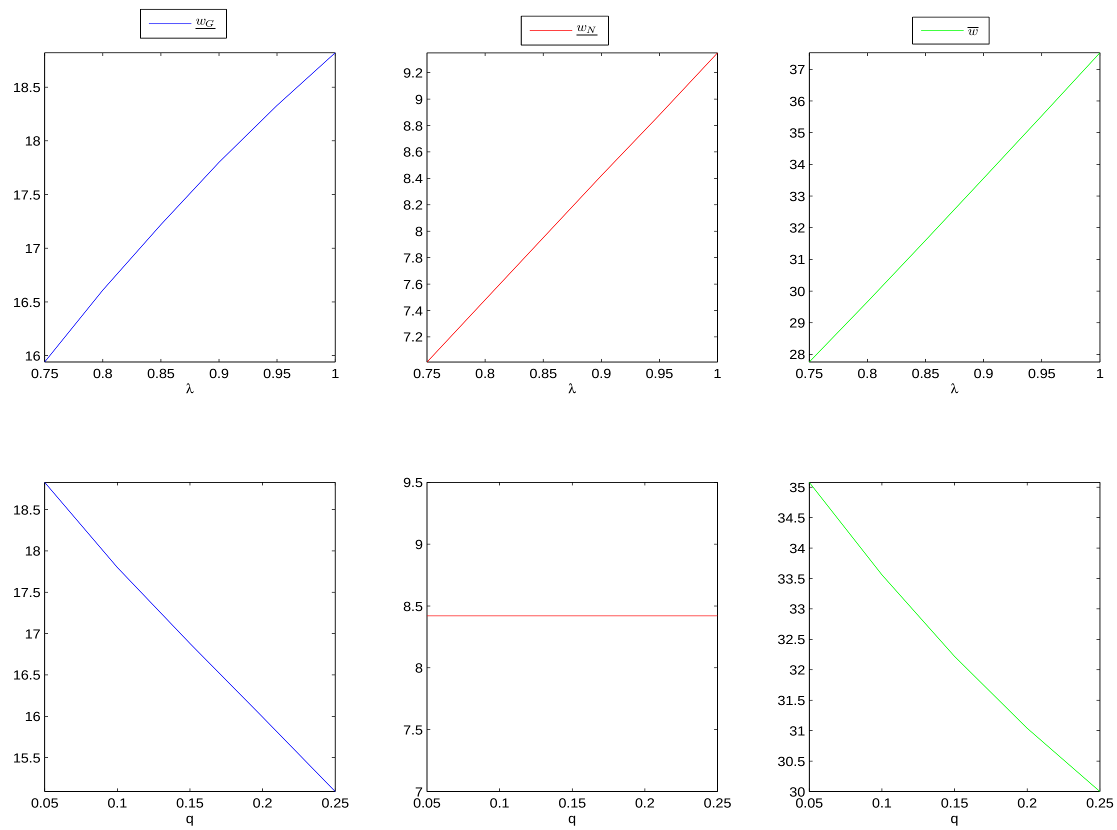

Figure 4: Threshold sensitivities: low growth firm

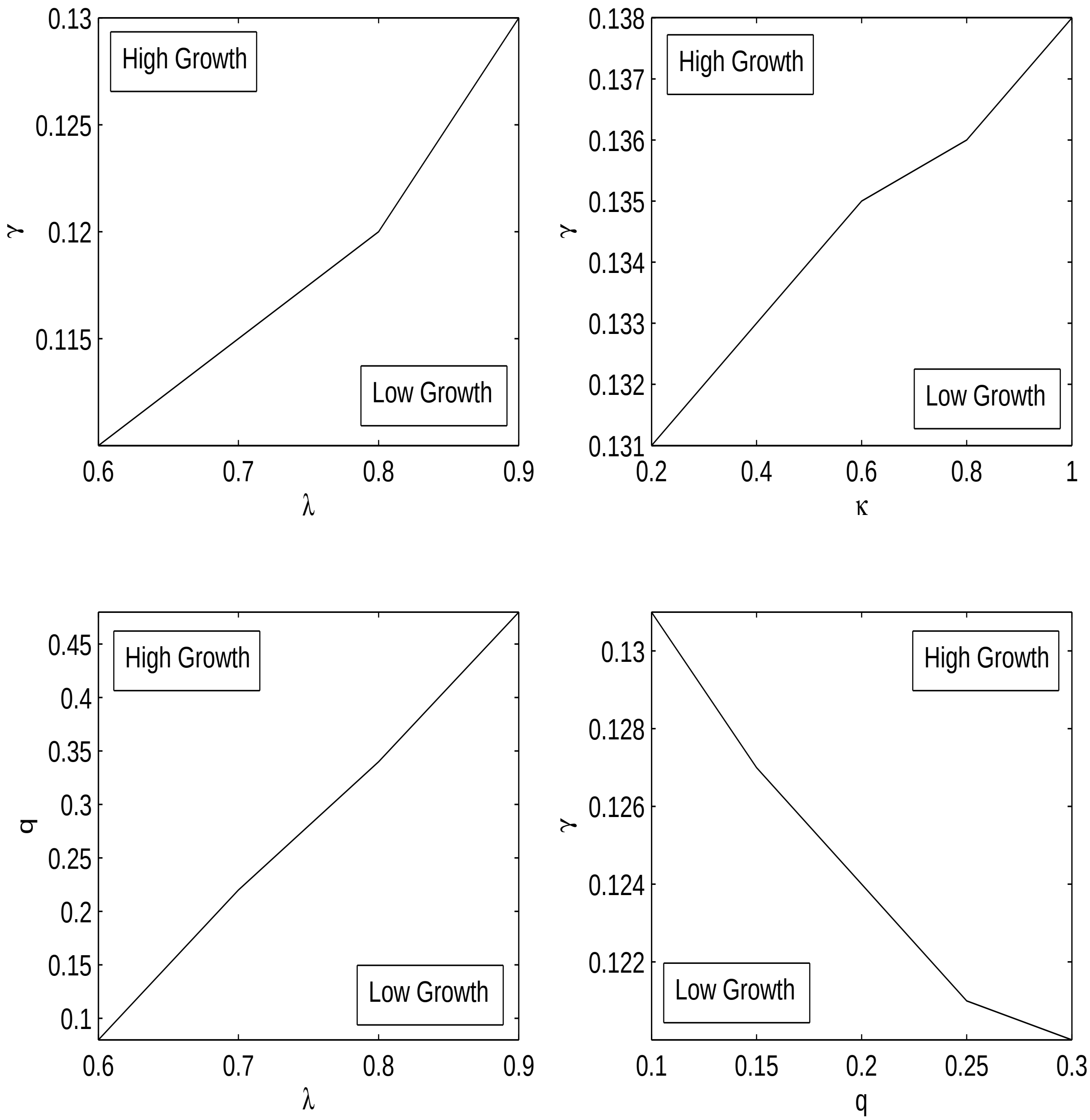

Figure 5: High growth and low growth regions of parameters space

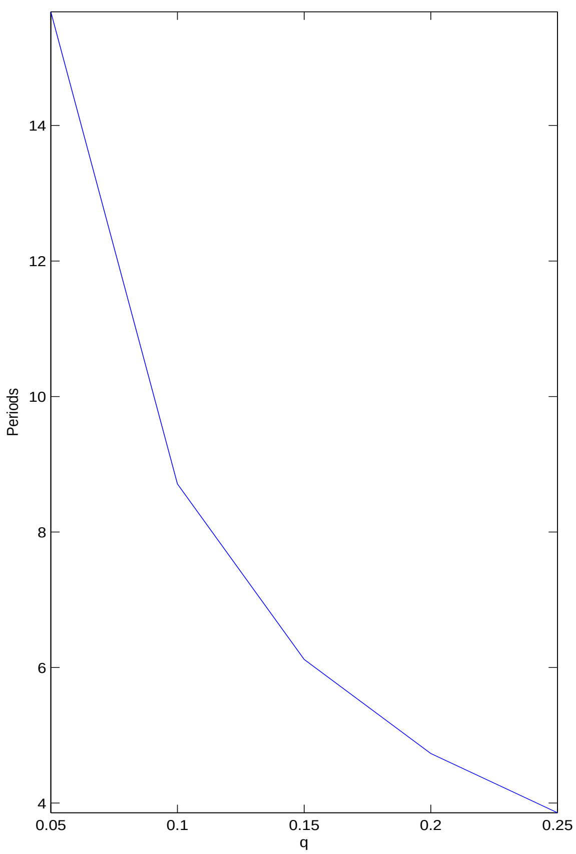

Figure 6: Average tenure in high-growth firms

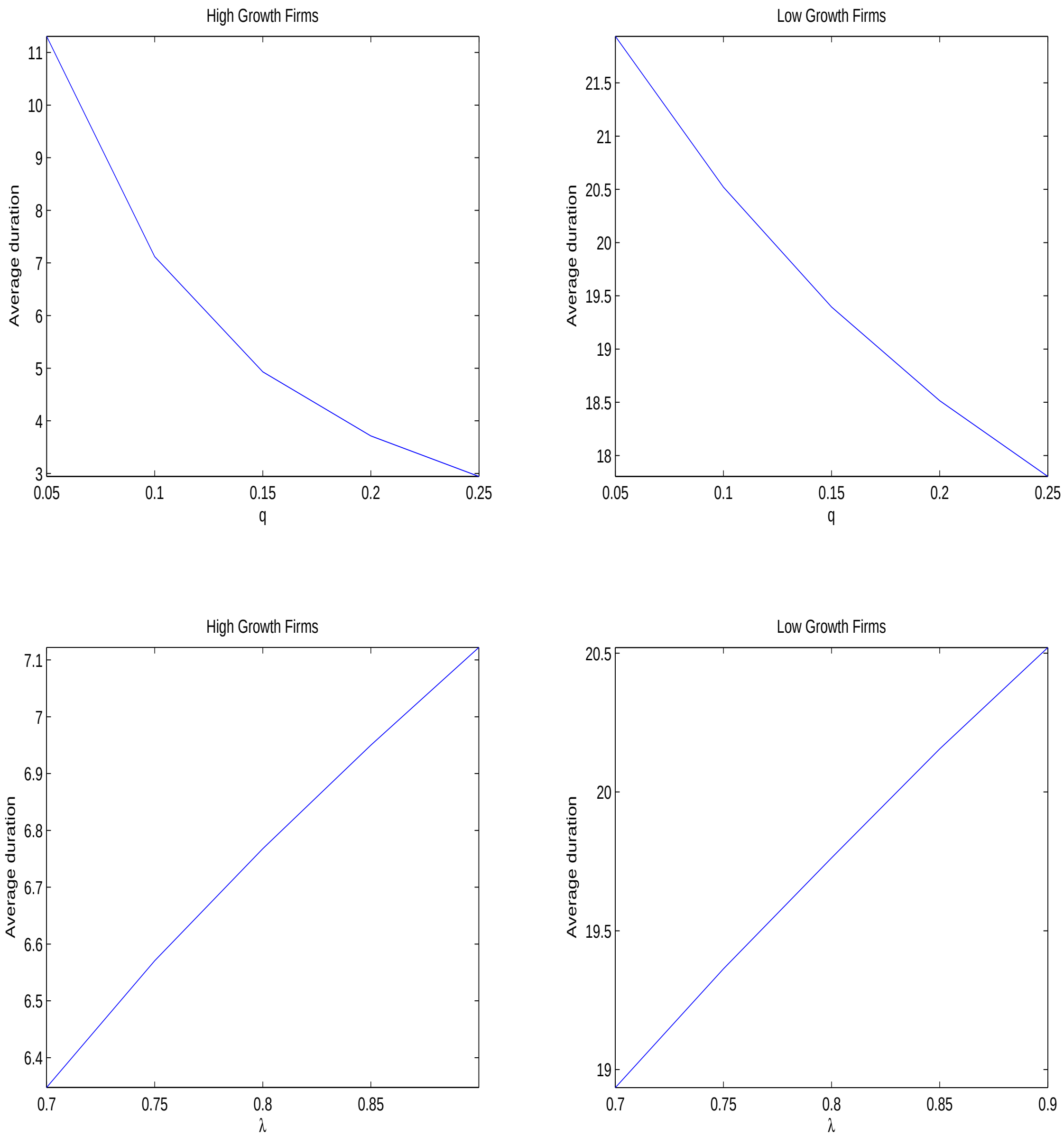

Figure 7: Average dismissal rates

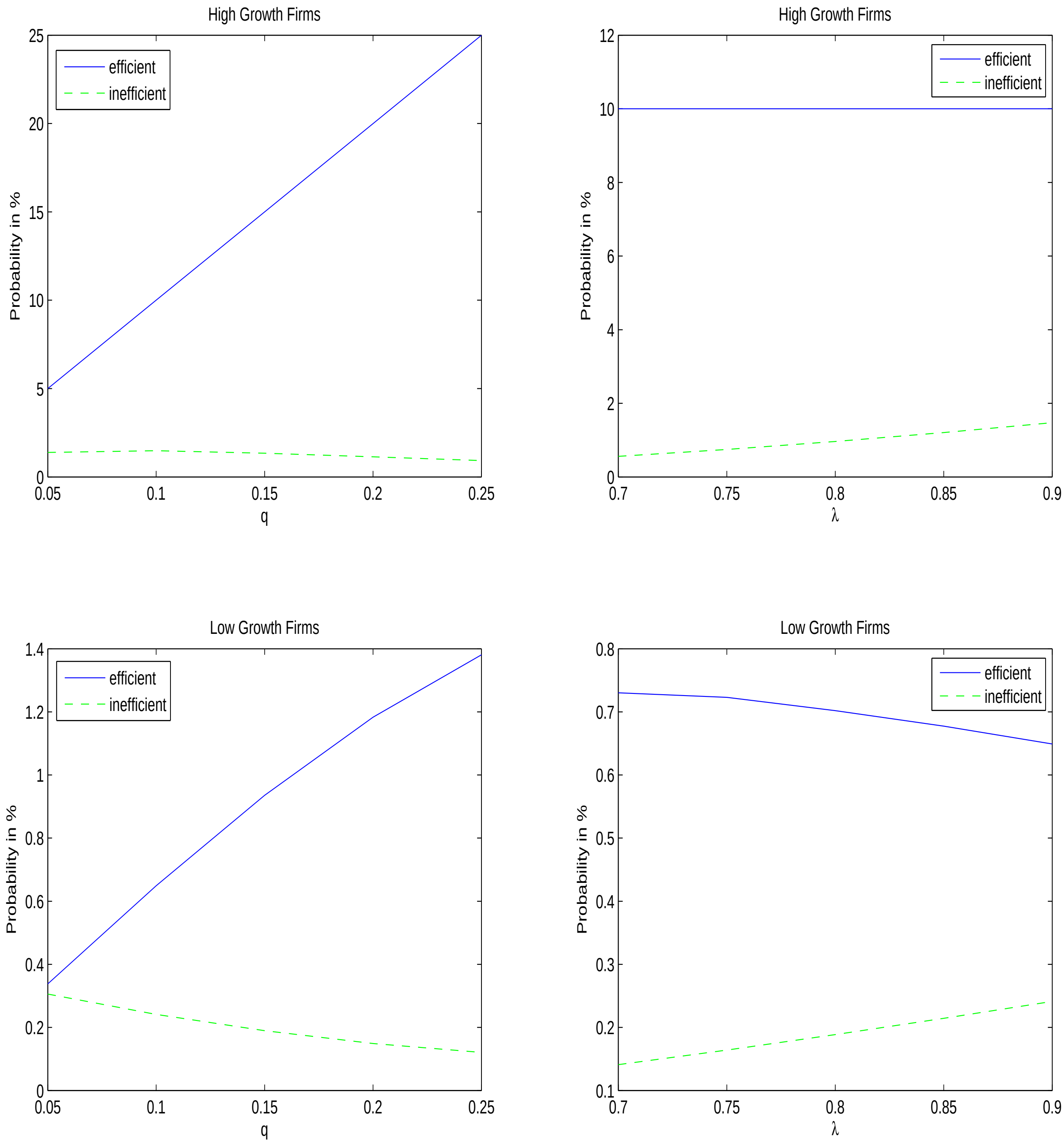

Figure 8: Compensation duration

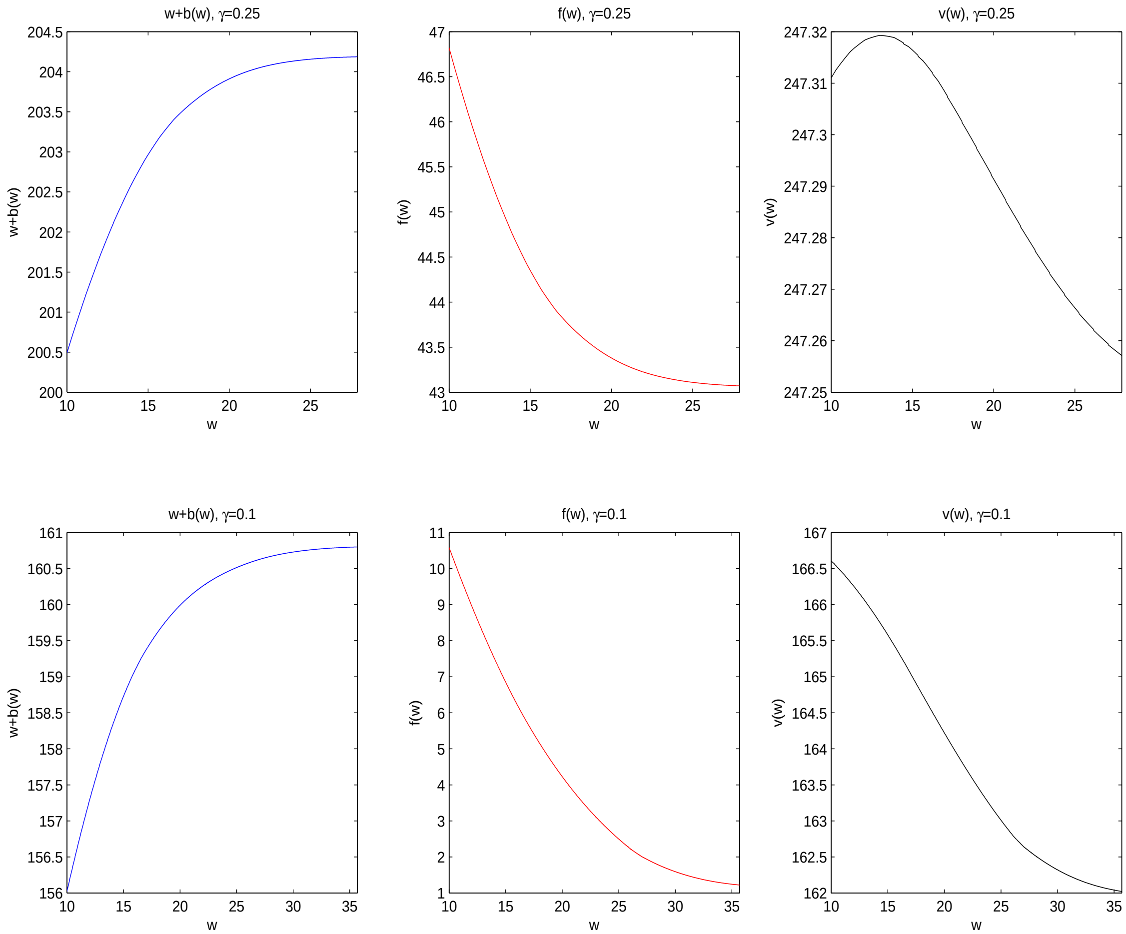

Figure 9: Second Best Values for high growth firms (top) and low growth firms (bottom)

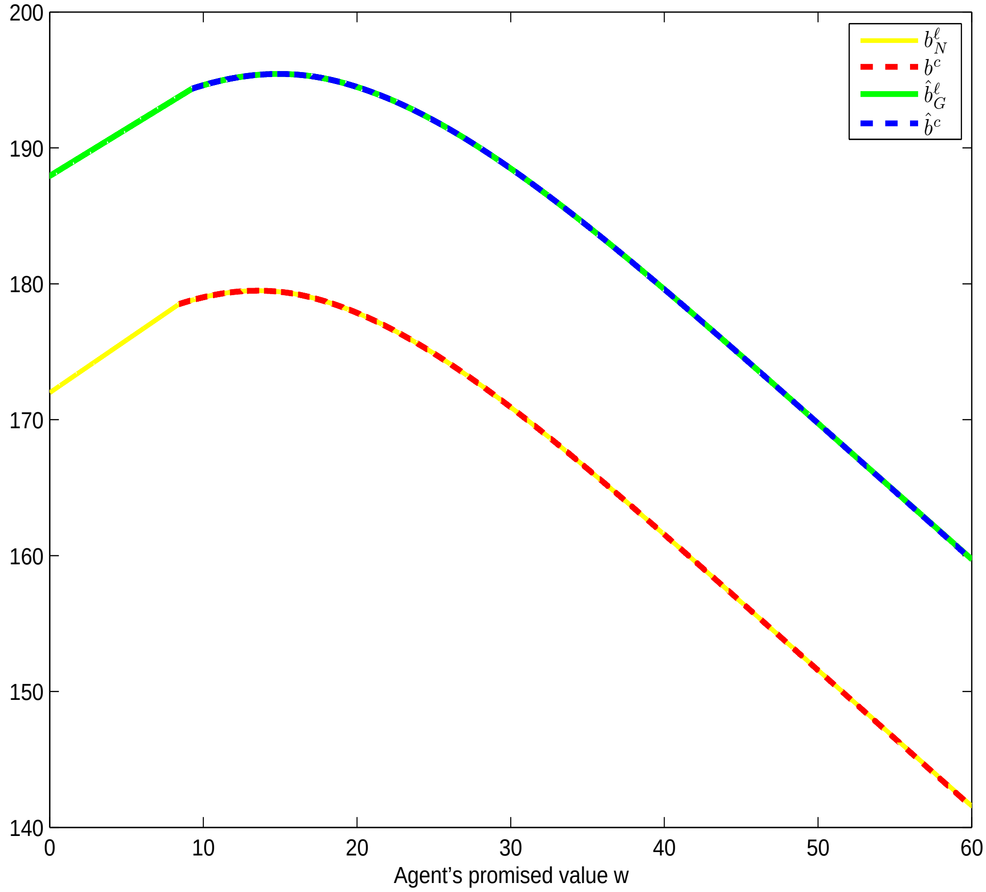

Figure 10: Extension: Low χ^i