Optimal extraction of a polluting non-renewable resource with R&D toward a clean backstop technology

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July 14, 2011

Abstract

We study the optimal extraction of a polluting non-renewable resource within the following framework: environmental regulation is imposed in the form of a ceiling on the stock of pollution and a clean unlimited backstop technology can be developed by research and development. More specifically, the time taken to develop a new technology depends on the amount spent on R&D. A surprising result is that the stringency of the ceiling and the size of the initial stock of the polluting non-renewable resource have a bearing on whether environmental regulation speeds up the optimal arrival date of this new technology. Compared to a scenario with no environmental externalities, stringent environmental regulation drives up the optimal R&D investment and advances the optimal backstop arrival date only in the case of a large initial resource stock. Otherwise, if the initial resource stock is small, regulation reduces optimal R&D and postpones the optimal backstop arrival date. These results are explained by the two roles played by the backstop technology. Firstly, the backstop serves to replace oil once it has been exhausted. As extraction is slowed down by regulation, the exhaustion of the non-renewable resource is postponed and the gains of innovation are lowered. Secondly, environmental regulation raises the gains of innovation by increasing the cost of consuming just oil.

Key words: Climate change; Non-renewable resources; Hotelling; Environmental Regulation, Innovation.

JEL Classification: Q30, Q42, Q54, Q55

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1 Introduction

New energy technologies, such as wind, solar or hydro energy, are likely to be needed to combat climate change and public R&D is being done in order to improve these technologies and lower their costs. The date when they will be broadly available for consumption should have an impact on oil supply and it is not straightforward to determine which combination of instruments (carbon tax and R&D toward clean technology) should be used in order to mitigate Greenhouse Gases (GHG) emissions in an efficient manner. This paper combines two strands of the literature to determine what effect a ceiling on the stock of pollution might have on the optimal extraction path of a polluting non renewable resource and on the optimal R&D toward an alternative backstop technology.

The first strand of the literature uses the Hotelling textbook model of exhaustible resources (Hotelling (1931)) to determine the optimal path of extraction of a polluting exhaustible resource, and thus the optimal time path for a carbon tax. The fact that polluting resources are non renewable is critical for the design of the optimal time path of a carbon tax. Choosing a wrong price path could exacerbate climate change by giving non-renewable resource owners the incentive to accelerate extraction in anticipation of future losses, thereby generating a green paradox (see Sinn (2008), Strand (2010)). Chakravorty, Magne & Moreaux (2006) characterize the effect of environmental regulation, in the form of a ceiling on the stock of pollution, on the dynamics of the transition from an exhaustible resource to an existing clean backstop technology. We adopt the same modeling but, rather than assuming that a clean renewable resource is already available at a given cost, we assume that R&D investment at date 0 determines the date at which a new technology is available, based on a deterministic innovation process as in Dasgupta et al. (1982).

The second strand of the literature that we refer to, in particular Dasgupta et al. (1982), deals with the optimal timing of *R&D* toward a backstop technology aiming to replace a non renewable resource. We compare the optimal backstop technology arrival date when there is a ceiling on the stock of pollution, with the optimal backstop technology arrival date without any pollution constraint, as determined by Dasgupta et al. (1982). This paper addresses the following questions: Does the optimal R&D investment toward an alternative backstop technology increase or decrease with environmental regulation ? The underlying intuition is that environmental regulation has two effects. The first is that it postpones the date of oil exhaustion, lengthening the time for which a potential backstop competes with oil. From this point of view, R&D toward an alternative technology and carbon taxation are substitutes: the energy used by future generations can be oil that has been saved due to the carbon tax or an alternative backstop. The second effect is that environmental regulation increases the marginal cost of consuming oil and gives incentives to develop the backstop.

In the first section, we introduce the model, combination of Chakravorty, Magne & Moreaux (2006) and Dasgupta et al. (1982).

2 Model with pollution and endogenous innovation

2.1 Assumptions and notations

We consider the energy commodity market. The corresponding gross surplus associated with its consumption is given by a twice continuously differentiable function u, strictly increasing and strictly concave. We denote the corresponding demand $(u')^{-1}(p)$ by $D(p)$.

Energy can be generated from a non-renewable resource such as oil. The non-renewable resource is extracted from a stock Q_0 known with certainty from the outset. Let x_t the rate of extraction of the non-renewable resource. The residual stock Q_t at date t is hence:

$$Q_t = Q_0 - \int_0^t x_\tau d\tau \tag{1}$$

The cost of extraction of this non-renewable resource is zero[1]. The burning of this resource increases the stock of CO_2, denoted Z_t, in the atmosphere. We assume natural decay at a constant rate α such that:

$$\dot{Z}_t = x_t - \alpha Z_t \tag{2}$$

The stock of pollution at date t is thus (Z_0 given):

$$Z_t = e^{-\alpha t}\left(Z_0 + \int_0^t e^{\alpha u} x_u du\right)$$

The environmental constraint is that the stock of pollution should not exceed a limit $\overline{Z}$: $\forall t,\ Z_t \leq \bar{Z}$. This ceiling $\bar{Z}$ may be exogenously fixed by a regulatory institution, or a bang-bang damage function. Once the stock of pollution has reached the ceiling, the flow of the non-renewable resource that can be burnt cannot exceed the natural decay, *i.e.* $x_t \leq \alpha\overline{Z} \equiv \overline{x}$. The corresponding energy minimal price is given by $\bar{p} = u^{'}(\bar{x})$.

A backstop technology can be developed by R&D. The supply of the energy from the backstop technology is inelastic at price q once it has been developed. We call y_t the consumption of backstop energy at date t. The total energy consumed at date t is hence $x_t + y_t$. The date at which this innovation occurs depends on the amount spent on R&D. As in Dasgupta et al. (1982), we assume that the necessary investment is reduced (in date 0 value) by postponing the date of innovation. Let T be the date of invention; the *R&D* cost function is $c(T)$, with $c'(T) \leq 0$. We make the following assumptions.

1. Assumption 1 : The marginal benefit, expressed at date T, of delaying innovation $-c^{'}(T)e^{rT}$ is increasing with T.
2. Assumption 2 : $\lim_{T\to 0} -e^{rT}c^{'}(T) = +\infty$, so that for all $Q_0 > 0$ and backstop cost $q > 0$ it is never optimal to innovate at date 0
3. Assumption 3 : $\lim_{T\to\infty} -e^{rT}c^{'}(T) < u(D(q)) - qD(q)$, otherwise performing no *R&D* at all would be optimal, irrespective of Q_0.
4. Assumption 4 : $q < \bar{p}$. At the ceiling, it is cheaper to use the backstop technology if any, than the non-renewable resource alone.

2.2 The social planner problem

The social planner maximizes the net surplus by choosing extraction rate x_t, innovation date T and rate of use of the backstop y_t for $t \geq T$, which maximize:

$$\int_0^T u(x_t)e^{-rt}dt + \int_T^{+\infty} \big(u(x_t + y_t) - qy_t\big)e^{-rt}dt - c(T)$$

[1]Choosing a low constant extraction cost would not change the results

subject to:

$$\begin{cases} \dot{Q}_t & = & -x_t & \left(\lambda_t^T\right) \\ \dot{Z}_t & = & -\alpha Z_t + x_t & \left(\mu_t^T\right) \\ Z_t & \leq & \overline{Z} & \left(\upsilon_t^T\right) \\ x_t, y_t & \geq & 0 & \left(a_t^T \geq 0, b_t^T \geq 0\right) \\ Q_0, Z_0 & \text{given} & & \\ \lim_{t\to+\infty} Q_t \geq 0 & & & \end{cases}$$

In the above program we have denoted the Lagrange multipliers in brackets.

2.2.1 Solving the planning problem

For sake of clarity we first derive the solution for T given. We will, in a second step, determine the optimal date of innovation. Denote by $H(s)$, the indicator function of $\mathbb{R}^+$. The first order conditions are:

$$u^{'}(x_t + H(T-t)y_t) - \lambda_t^T + \mu_t^T + a_t^T = 0$$
$$u^{'}(x_t + H(t-T)y_t) - q + b_t^T = 0$$

Together with the complementary slackness conditions:

$$\begin{aligned} \nu_t^T \geq 0 \quad & \text{and} \quad \nu_t^T \left(\overline{Z} - Z_t\right) = 0 \\ a_t^T \geq 0 \quad & \text{and} \quad a_t^T x_t = 0 \\ b_t^T \geq 0 \quad & \text{and} \quad b_t^T y_t = 0 \end{aligned}$$

The dynamics of the costate variables are determined by:

$$\dot{\lambda}_t^T = r\lambda_t^T \Longleftrightarrow \lambda_t^T = \lambda_0^T e^{rt}$$
$$\dot{\mu}_t^T = (r+\alpha)\mu_t^T + \nu_t^T$$

Remark that the costate variable μ_t^T is non positive. If $Z_t < \bar{Z}$ over some time interval, then $\nu_t^T = 0$ over that interval and $\mu_t^T = \mu_0^T e^{(r+\alpha)t}$ over that interval. The transversality conditions at infinity are moreover given by:

$$\lim_{t\to+\infty} e^{-rt}\lambda_t^T Q_t = \lambda_0^T \left(\lim_{t\to+\infty} Q_t\right) = 0$$

and

$$\lim_{t\to+\infty} e^{-rt}\mu_t^T Z_t = 0$$

The solution is denoted by starred letters. As long as the non-renewable resource is used (a_t^{*T}), its full marginal cost (equal to the marginal utility of consumption) is given by $p_t^{*T} = \lambda_t^{*T} - \mu_t^{*T}$. This is the sum of the scarcity rent $\lambda_t^{*T} = \lambda_0^{*T}e^{rt}$ and the shadow cost of pollution $-\mu_t^{*T}$. At the ceiling, the price of the resource is $p_t^{*T} = \bar{p}$ if the backstop has not yet arrived (*i.e.* if $t < T$). In this case $x_t^{*T} = \alpha\bar{Z}$ and $y_t^{*T} = 0$. If the ceiling is binding and the backstop technology has arrived (*i.e.* if $t \geq T$), then $p_t^{*T} = q$, $x_t^{*T} = \alpha\bar{Z}$, and $u^{'}(x_t^{*T} + y_t^{*T}) = q$, so that $y_t^{*T} = D(q) - \alpha\bar{Z}$. If the scarcity rent is above $\bar{p}$, the ceiling constraint is not binding and the shadow price follows a Hotelling path : $-\mu_t^{*T} = 0$ and $u'\left(x_t^{*T}\right) = \lambda_0^{*T}e^{rt}$.

We can write the following proposition characterizing the solution (with T fixed).

Proposition 1. *With T fixed, calling T_e^* the date of oil exhaustion, the solution $(x_t^{*T}, y_t^{*T}, \lambda_0^{*T}, \mu_0^{*T}, \mu_t^{*T}, T_e^*)$ of the optimization problem is given by*[2] *:*

1. $u'\left(x_t^{*T} + y_t^{*T}\right) = \left(\lambda_0^{*T} e^{rt} - \mu_t^{*T}\right) H(T-t) + \min\left(q, \lambda_0^{*T} e^{rt} - \mu_t^{*T}\right) H(t-T)$
2. $-\mu_t^{*T} = \max\left[\min\left(\bar{p}H(T-t) + qH(t-T) - \lambda_0^{*T} e^{rt}, -\mu_0^{*T} e^{(r+\alpha)t}\right), 0\right]$
3. $y_t^{*T} = \left((D(q) - \bar{x})H(T_e^* - t) + D(q)H(t - T_e^*)\right)H(t-T)$
4. $\max_t \left(e^{-\alpha t}\left(Z_0 + \int_0^t e^{\alpha u} x_u^{*T} du\right)\right) = \overline{Z}$
5. $T_e^* = \max\left(T, \frac{1}{r}\ln(\frac{q}{\lambda_0^{*T}})\right)$
6. $\int_0^{T_e^*} x_u^{*T} du = Q_0$

The problem is very similar to that studied by Chakravorty, Magne & Moreaux (2006). However, unlike them, the backstop technology cannot be used before arrival date T. The solution exposed in Chakravorty, Magne & Moreaux (2006) would be the solution to the problem presented here if the *R&D* cost was zero for all T.

In Proposition 1, the second item shows that several configurations are possible, depending on arrival date T. For instance if the backstop arrives after the binding date of the ceiling, while the ceiling is binding and before exhaustion, then the shadow price $u'(x_t^{*T} + y_t^{*T} H(t-T))$ falls from $\bar{p}$ to q at the very date of innovation. If the binding date coincides with T then before innovation date the shadow price is equal to the sum of the scarcity rent $\lambda_0^{*T} e^{rt}$ and the shadow price of pollution $-\mu_t^{*T} = -\mu_0^{*T} e^{(r+\alpha)t}$, with $\lambda_0^{*T} e^{rT} - \mu_0^{*T} e^{(r+\alpha)T} \leq \bar{p}$, and falls to q after. In the sequel we shall see all the possible configurations of the optimal shadow prices paths.

At this point of the presentation one can already do several important remarks. The scarcity rent $\lambda_0^{*T} e^{rt}$ is indeed the price that would be set by oil owners in perfect competition if there was a carbon tax $-\mu_t^{*T}$, and if the backstop was planned to arrive at date T. We therefore refer to $-\mu_t^{*T}$ as the "carbon tax" even though there are several possible carbon taxes. Indeed, in this model, any carbon tax of the form: $\lambda e^{rt} - \mu_t^{*T}$ with $\lambda \leq \lambda_0^{*T}$ would lead to the same optimal extraction path, it would only impact the sharing of the rent between the tax levier and the oil owners. It is important for the time path of the carbon tax to depend on the arrival date of the backstop. For instance, if the arrival date of the backstop was exogenously advanced after the carbon tax had been implemented (or if the backstop was subsidized, as in Van der Ploeg & Withagen (2010)), it would result in accelerated extraction and more pollution, thereby generating a "green paradox".

The second step consists in finding the optimal innovation arrival date, which maximizes $V(Q_0, Z_0, \bar{Z}, T) - c(T)$.

In the following, we compute the optimal extraction rate and optimal backstop arrival date. We compute the optimal price path for each possible innovation date. We derive from the optimal price path for arrival date T the marginal cost of delaying innovation at this arrival date. We

[2] If it exists, otherwise the ceiling is not binding and the solution is in Dasgupta et al. (1982).

first recap (see Dasgupta et al. (1982)), in Section 3.1, the solution to this planning program when there is no constraint on the stock of pollution. Then, in Section 3.2, we move on to the case of a low ceiling on the stock of pollution. In order to gain intuition, we first consider the case where the initial stock of pollution is already at the (stringent) ceiling in subsection 3.2.1. Then we study the general case with a stringent ceiling in subsection 3.2.2.

3 Optimal innovation date and extraction path

3.1 Unconstrained case: no externality

This case is a special case of the problem presented above, when $\bar{Z} = +\infty$. This case is studied in Dasgupta et al. (1982). It is never useful to develop the backstop technology before the oil is exhausted. The backstop's only role is to replace the non-renewable resource when there is no oil left. We denote with a ˜ on top the solution of the problem without externality. Let denote $\tilde{T}_0$ the date at which the oil is exhausted in a Hotelling model with backstop available at price q from the outset.

Price path for any arrival date T : For any arrival date before $\tilde{T}_0$, the backstop remains unused before $\tilde{T}_0$ and the non-renewable resource's price path follows a Hotelling price path $\tilde{p}_t^{*T} = \tilde{\lambda}_0 e^{rt}$. It is exhausted at date $\tilde{T}_0$, and the backstop is used from this date on, at price q. For any arrival date greater than $\tilde{T}_0$, the oil is exhausted exactly at the innovation date. Prior to invention, the non-renewable resource's price path follows a Hotelling price path :

$$u^{'}(\tilde{x}_t^{*T}) = \tilde{p}_t^{*T} = \tilde{\lambda}_0^{*T} e^{rt}$$

where $\tilde{\lambda}_0^{*T}$ is the initial scarcity rent for arrival date T, satisfying that $\int_0^T \tilde{x}(\tilde{\lambda}_0^{*T} e^{rt})dt = Q_0$. The final scarcity rent $\tilde{\lambda}_0^{*T} e^{rT}$ is above q. At arrival date, oil is exhausted and the energy price falls from $\tilde{\lambda}_0^{*T} e^{rT}$ to q. After the invention, energy price is equal to the marginal cost of producing the substitute q.

Optimal arrival date: The marginal benefit from delaying innovation due to saved R&D, $-c^{'}(T)e^{rT}$, is equal, at the optimal arrival date, to the marginal cost of delaying innovation. Because the backstop remains unused prior to date $\tilde{T}_0$, the marginal cost of delaying innovation is zero for all $T \leq \tilde{T}_0$. For any arrival date after $\tilde{T}_0$, this marginal cost is $(u(D(q)) - qD(q)) - (u(\tilde{x}_T^{*T}) - \tilde{p}_T^{*T}\tilde{x}_T^{*T})$. It is the sum of the welfare loss from consuming the oil at date T instead of consuming the backstop $u(\tilde{x}_T^{*T}) - (u(D(q) - qD(q))$ and the welfare loss from reduced oil consumption at all dates before T: consumption is reduced during the consumption path ending at $T - dT$ by exactly $\tilde{x}_T^{*T}$. It costs the discounted marginal utility of consumption, which is constant along the Pareto efficiency path, times the amount of consumption transferred to be consumed at date T, $-\tilde{p}_T^{*T}\tilde{x}_T^{*T}$

The optimal price path is represented in Figure 1.

Proposition 2. *As shown in Dasgupta et al. (1982), at the optimal unconstrained innovation date $\tilde{T}$:*

$$-c^{'}(\tilde{T})e^{r\tilde{T}} = (u(D(q)) - qD(q)) - (u(\tilde{x}_{\tilde{T}}^{*\tilde{T}}) - \tilde{p}_{\tilde{T}}^{*\tilde{T}}\tilde{x}_{\tilde{T}}^{*\tilde{T}}) \tag{3}$$

The main results are :

- ***Result 1:*** *The non-renewable resource is exhausted exactly at the date of innovation.*
- ***Result 2:*** *The price of the non-renewable resource at optimal arrival date $\tilde{T}$ is higher than the exogenous price of the renewable resource $\tilde{p}_{\tilde{T}}^{*\tilde{T}} > q$.*

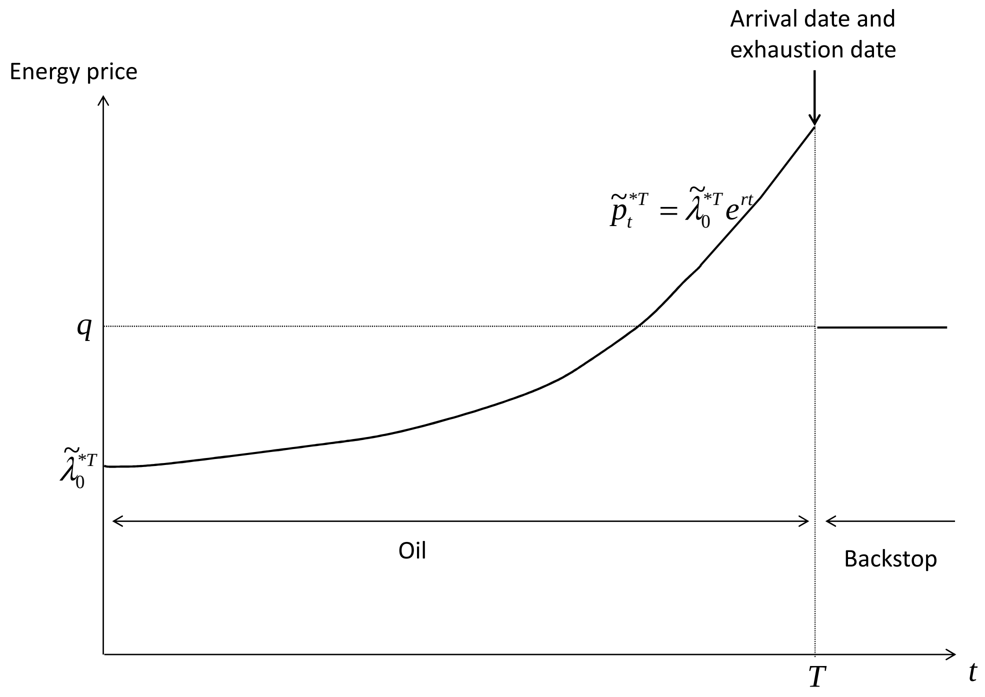

Figure 1: Price path for endogenous arrival date, without pollution ($\bar{Z} = +\infty$).

- ***Result 3:*** *The optimal introduction date for the backstop technology is increasing with the initial stock of the non-renewable resource: $\tilde{T}(Q_0)$ is an increasing function of Q_0.*
- ***Result 4:*** *The final price of the non renewable resource decreases with the initial stock of the non renewable resource.*

At the optimal arrival date, Eq.3 holds, so that the larger Q_0, the later $\tilde{T}$, and the lower $\tilde{p}_{\tilde{T}}^{*\tilde{T}}$, and then the lower the initial scarcity rent. Therefore, the extraction rate is increasing with the initial stock of the non-renewable resource, as is the stock of pollution in the atmosphere. For a given ceiling constraint on the stock of pollution $\bar{Z}$, there exists a threshold level for the non-renewable resource such that, if the initial stock of the non-renewable resource is over this threshold, the unconstrained stock of pollution becomes greater than the constrained stock of pollution: $\forall \bar{Z}, \exists Q_{\min}(\bar{Z})$ s.t. $Q > Q_{\min}(\bar{Z}) \Rightarrow Z_t > \bar{Z}$ for some t. (Formal proof in Appendix A.)

The same modeling approach is taken in the next section, but the externality caused by the burning of fossil fuels is taken into account. All the notations corresponding to the case with no externalities are indicated with a tilde $\tilde{}$ on top. In the following, we denote $\tilde{N}(T) = (u(D(q)) - qD(q)) - (u(\tilde{x}_T^{*T}) - \tilde{p}_T^{*T}\tilde{x}_T^{*T})$.

3.2 Externality case with a stringent ceiling on the stock of pollution: $\alpha\bar{Z} < D(q)$

If $\alpha\bar{Z} < D(q)$, the maximum quantity of CO_2 that can be released into the atmosphere at the ceiling, $\alpha\bar{Z}$, is less than the demand for energy at price q, $D(q)$. This is the case if $q < \bar{p}$, with q being the price for the backstop once it has been developed. When the ceiling is binding, in order to remain at the ceiling, either oil can be consumed at price $\bar{p}$ or oil and the backstop can be consumed together at price q if the backstop has been developed. With a low ceiling, this second option is preferable, because $q < \bar{p}$. The backstop is not only useful to replace oil when it is exhausted, but also to lower energy cost when the ceiling is binding.

3.2.1 Starting from the ceiling $Z_0 = \bar{Z}$

In order to build an intuition and understand how environmental regulation affects innovation and extraction, we assume first that the initial stock of pollution is at the ceiling $Z_0 = \bar{Z}$, this implies that, for all t, $x_t \leq \alpha \bar{Z}$.

Optimal price path for a given arrival date. Let denote $t_0(Q_0)$ the date when oil would be exhausted if $x_t = \alpha \bar{Z}$ for all t, *i.e.* $t_0(Q_0) \equiv \frac{Q_0}{\alpha \bar{Z}}$.

- If arrival date T is prior to $t_0(Q_0)$, extraction is limited to $\alpha \bar{Z}$ at each date until exhaustion at date $t_0(Q_0)$. Before arrival, the ceiling is binding, then $\nu_t^{*T} > 0$ and $p_t^{*T} = \bar{p}$. Up to the innovation date, the price is equal to $\bar{p}$. At the arrival date T, oil is not exhausted and the price falls from $\bar{p}$ to the backstop price q. As μ_t^{*T} is non positive, item 2 of Proposition 1 implies that the scarcity rent at arrival date $\lambda_0^{*T} e^{rT}$ is thus lower than q and the shadow cost of pollution falls from $-\mu_{T^-}^{*T} = \bar{p} - \lambda_0^{*T} e^{rT}$ to $-\mu_{T^+}^{*T} = q - \lambda_0^{*T} e^{rT}$. Then, as long as there is oil left, the non-renewable resource and the backstop are used jointly at price q. At date $t_0(Q_0)$, the scarcity rent reaches the backstop price $\lambda_0^{*T} e^{rt_0(Q_0)} = q$ and the oil is exhausted. From this date $t_0(Q_0)$, only energy supplied by the backstop is consumed, at price q. The oil extraction path, as well as the scarcity rent, is exactly the same as it would have been if the backstop had been available from the outset: for all $T \leq t_0(Q_0)$, $\lambda_0^{*T} = \lambda_0^0$. The shadow cost of pollution, however, depends on the arrival date T. The price path is illustrated on Fig.2.

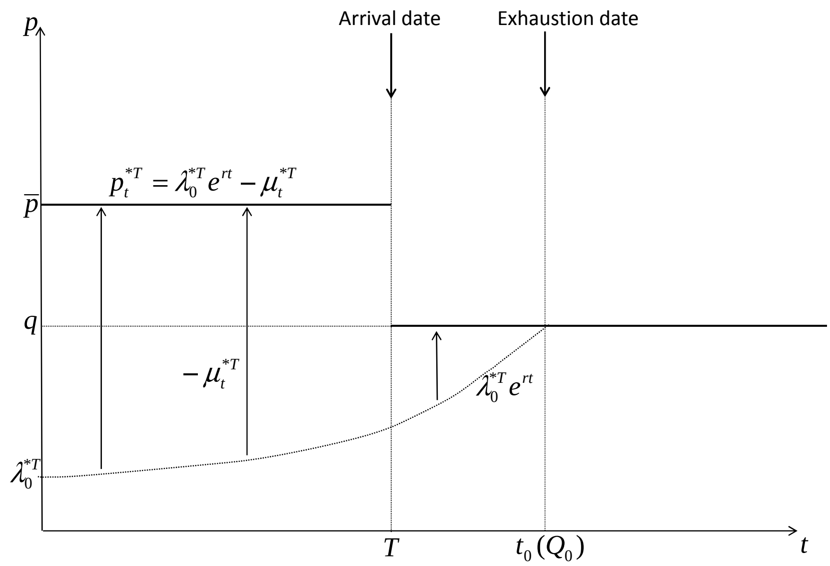

Figure 2: Price path (bold solid line) for exogenous arrival date before $t_0(Q_0)$ in the constrained case.

- If innovation arrives at date T later than $t_0(Q_0)$, initially as the ceiling is binding, then $\nu_t^{*T} > 0$ and $p_t^{*T} = \bar{p}$. The scarcity rent $\lambda_0^{*T} e^{rt}$ reaches $\bar{p}$ before $t_0(Q_0)$. From this date, as $x_t < \alpha\bar{Z}$ the ceiling ceases to be binding, the shadow price of the stock of carbon $-\mu_t^{*T}$ is zero and resource use follows a Hotelling path through to exhaustion. During this phase, what determines oil price is the constraint on the stock of oil (scarcity) and not the constraint on the stock of pollution (ceiling). The resource price and the scarcity rent are equal. The final price of the oil is over $\bar{p}$. The price path is illustrated on Fig.3

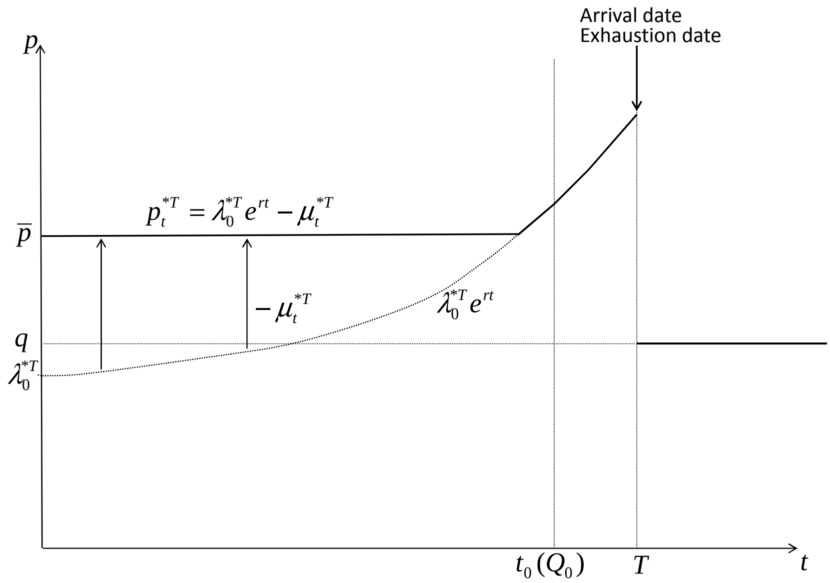

Figure 3: Price path (bold solid line) for exogenous arrival date after $t_0(Q_0)$ in the constrained (solid line).

Optimal arrival date. • The value of innovating at date $T < t_0(Q_0)$ can be written:

$$\begin{aligned} V(Q_0, \bar{Z}, Z_0, T) &= \int_0^T u(\bar{x})e^{-rt}dt + \int_T^{t_0(Q_0)} (u(D(q)) - q(D(q) - \bar{x}))e^{-rt}dt \\ &\quad + \int_{t_0(Q_0)}^{\infty} (u(D(q)) - qD(q))e^{-rt}dt \end{aligned}$$

So that the marginal cost $N(T)$ of delaying innovation expressed at date T, writes, for all $T < t_0(Q_0)$:

$$N(T) = e^{rT}\frac{\partial V}{\partial T} = (u(D(q)) - qD(q)) - (u(\bar{x}) - q\bar{x})$$

The marginal cost of delaying innovation can be broken down into two terms.

$$N(T) = \underbrace{\Big(\big(u(D(q)) - qD(q)\big) - \big(u(\bar{x}) - \bar{p}\bar{x}\big)\Big)}_{\text{constrained oil price increases N}} - \underbrace{\bar{x}\Big(\bar{p} - q\Big)}_{\text{oil left at arrival date decreases N}}$$

The first term in the marginal cost of delaying arrival looks like the marginal cost in the unconstrained case, $\tilde{N}(T) = \big(u(D(q)) - qD(q)\big) - \big(u(\tilde{x}_T^{*T}) - \tilde{p}_T^{*T}\tilde{x}_T^{*T}\big)$. As

$T < t_0(Q_0)$, $\bar{p}$ is necessarily greater than final oil price of the unconstrained case $\tilde{p}_T^{*T}$, so that this first term is greater than in the unconstrained case. Therefore, the incentive to innovate to lower the energy price is greater in the constrained case. However, This is not the only driver of innovation. Indeed the second term reduces the marginal cost of delaying innovation. Indeed, because the oil is not exhausted at arrival date T, delaying arrival does not reduce the quantity of oil consumed before T, contrary to the unconstrained case with final price $\bar{p}$. Delaying innovation then reduces welfare before T more in the unconstrained case than in the constrained case[3] for the same energy price drop from $\bar{p}$ to q.

- The value of innovating at $T > t_0(Q)$ writes, if we denote $h(T)$ the end of the ceiling period:

$$\begin{aligned} V(Q_0, \bar{Z}, Z_0, T) &= \int_0^{h(T)} u(\bar{x})e^{-rt}dt + \int_{h(T)}^{T} u(x_t^{*T})e^{-rt}dt \\ &+ \int_T^{\infty} (u(D(q)) - qD(q))e^{-rt}dt \end{aligned}$$

As in the unconstrained case, the marginal cost of delaying innovation can be written $N(T) = (u(D(q)) - qD(q)) - (u(x_T^{*T}) - p_T^{*T} x_T^{*T})$, because delaying innovation entails a loss in welfare from consuming, at date T, oil at price p_T^{*T} instead of energy from the backstop at price q $((u(D(q)) - qD(q)) - u(x_T^{*T}))$ and it entails a loss in welfare from decreased oil consumption, by x_T^{*T}, before T, valued at price p_T^{*T}. This expression is the same as the marginal cost of delaying innovation for the unconstrained case. At the arrival date, oil is exhausted. Regulation slows down oil extraction. The final price p_T^{*T} is consequently less than the final price in the unregulated case for the same arrival date $\tilde{p}_T^{*T}$.

The following proposition holds:

Proposition 3. *(Preliminary results)*

1. *The optimal arrival date T^* is given by[4] :*

$$-c'(T^*)e^{rT^*} = N(T^*)$$

With, starting from the ceiling:

$$N(T) = \begin{cases} (u(D(q)) - qD(q)) - (u(\bar{x}) - q\bar{x}) & \text{if } T < t_0(Q_0) \\ (u(D(q)) - qD(q)) - (u(x_T^{*T}) - p_T^{*T} x_T^{*T}) & T \geq t_0(Q_0) \end{cases}$$

2. *There exists an initial stock of oil Q^l such that:*

   - *For all $Q_0 \leq Q^l$, innovation is postponed by regulation compared to the case with no externality: $T^* > \tilde{T}$*

[3] Delaying innovation reduces welfare before T more, by $\bar{p}\bar{x}$, in the unconstrained case than in the constrained case. However, for the same reason, expenses saved due to a delayed arrival, and hence extracting oil at no cost instead of purchasing energy from the backstop, is $q(D(q) - \bar{x})$ in the constrained case whereas it is $qD(q)$ in the unconstrained case with final price $\bar{p}$. The marginal cost of delaying arrival is consequently $(\bar{p} - q)\bar{x}$ lower, with a ceiling constraint on the stock of pollution, than the unconstrained case with final price $\bar{p}$ and exhaustion.

[4] $V(Q_0, Z_0, \bar{Z}, T)$ is continuous but not derivable in $t_0(Q_0)$. However $\lim_{T\to T^{*-}} \left(e^{rT}\frac{\partial V((Q_0,Z_0,\bar{Z},T)}{\partial T}\right)$ increases with T^*, so that $V(Q_0, Z_0, \bar{Z}, T) - c(T)$ has a unique maximum. For ease of notation, we write that, at optimal arrival date $-e^{rT^*}c'(T^*) = e^{rT^*}\frac{\partial V(Q_0,T)}{\partial T}$ also if $\lim_{T\to T^{*-}} \left(e^{rT}\frac{\partial V((Q_0,Z_0,\bar{Z},T)}{\partial T}\right) \leq -e^{rT^*}c'(T^*)$ and $\lim_{T\to T^{*+}} \left(e^{rT}\frac{\partial V((Q_0,Z_0,\bar{Z},T)}{\partial T}\right) \geq -e^{rT^*}c'(T^*)$

- *For all $Q_0 > Q^l$, innovation is advanced by regulation compared to the case with no externality: $T^* \leq \tilde{T}$*

If $\lim_{T\to+\infty} -e^{rT}c^{'}(T) \leq u(D(q)) - qD(q) - (u(\bar{x}) - q\bar{x})$, *then* $Q^l < +\infty$

$N(T)$ is increasing and $-e^{rT}c^{'}(T)$ is decreasing so that there is a single maximum.

Proof . Consider the limit price p^l, satisfying

$$u(x(p^l)) - p^l x(p^l) = u(\bar{x}) - q\bar{x}$$

Then $\bar{p} > p^l > q$. If the unconstrained optimal final price is below p^l, then the entire price path is below $\bar{p}$, so that date $t_0(Q_0)$ is after the optimal innovation date $\tilde{T}$. The constrained marginal cost of delaying innovation at date $\tilde{T}$ satisfies $N(\tilde{T}) = (u(D(q)) - qD(q)) - (u(\bar{x}) - q\bar{x}) = (u(D(q)) - qD(q)) - (u(x(p^l)) - p^l x(p^l)) \geq \big(u(D(q)) - qD(q)) - (u(\tilde{x}_{\tilde{T}}) - \tilde{p}_{\tilde{T}}\tilde{x}_{\tilde{T}}\big) = \tilde{N}(\tilde{T})$: regulation drives up optimal innovation if final unconstrained case is below p^l (see the upper figure of Fig.4). Similarly, if final unconstrained optimal final price is above p^l, regulation postpones optimal arrival date, compared to the unconstrained case. Let T^l be such that[5]:

$$(u(D(q)) - qD(q)) - (u(\bar{x}) - q\bar{x}) = e^{-rT^l}c^{'}(T^l) \tag{4}$$

and (Q^l, λ^l) be the solution of :

$$\begin{cases} Q^l = \int_0^{T^l} D(\lambda^l e^{rt})dt \\ \lambda^l e^{rT^l} = p^l \end{cases} \tag{5}$$

As $p^l < \bar{p}$, then $T^l < t_0(Q^l)$ and T^l is the optimal date of innovation if $Q_0 = Q^l$ for both the no externality case and the externality case. For all $Q_0 > Q^l$, as $d\tilde{p}_t^{*T}/dQ_0 < 0$, the final unconstrained price satisfies $\tilde{p}_{\tilde{T}}^{*\tilde{T}} < p^l$ and the optimal unconstrained arrival date $\tilde{T}$ satisfies $\tilde{T} > T^l$. As $dt_0(Q_0)/dQ_0 > 0$, then $t_0(Q_0) > T^l$, the optimal date of innovation in the constrained case is thus $T = T^l$. Optimal arrival date is advanced by the constraint on the stock of pollution. For Q_0 such that $Q_{min} < Q_0 < Q^l$, the optimal date of innovation in the unconstrained case is before T^l, as the optimal arrival date increases with the initial stock of oil in the unconstrained case. If $t_0(Q_0) \geq T^l$, the optimal date of innovation is $T = T^l$ in the externality case (see the lower figure of Fig.4), so that innovation occurs later in the externality case than in the no externality case. If $t_0(Q_0) < T^l$, then at date $\tilde{T}$, the price path is in a Hotelling phase in both constrained and unconstrained case, with $p_{\tilde{T}}^{*\tilde{T}} \leq \tilde{p}_{\tilde{T}}^{*\tilde{T}}$, so that $N(\tilde{T}) < \tilde{N}(\tilde{T})$, so that innovation occurs later in the externality case than in the no externality case.

The comparison between the marginal cost of delaying the arrival date in the constrained case and in the unconstrained case depends on the relative magnitude of two diverging effects: a higher oil price due to regulation and less arrival utility because there is oil left. The larger the initial stock of non-renewable resource Q_0, the lower the oil price at optimal arrival date in the unconstrained case. If Q_0 is large, the gains from reducing the constrained oil price, from $\bar{p}$ to q, are sufficiently high compared to the gain from reducing the unconstrained oil price to offset the fact that the backstop is made less useful in the long run to replace the oil once it is exhausted (because regulation postpones oil exhaustion). On the other hand, if Q_0 is small enough so that the final price in the unconstrained case is not much below $\bar{p}$, the difference between oil prices at the unconstrained optimal arrival date in the unconstrained and constrained case does

[5]if $T \to -e^{rT}c^{'}(T)$ is always above $(u(D(q)) - qD(q)) - (u(\bar{x}) - q\bar{x})$, then Q^l defined hereafter is equal to $+\infty$

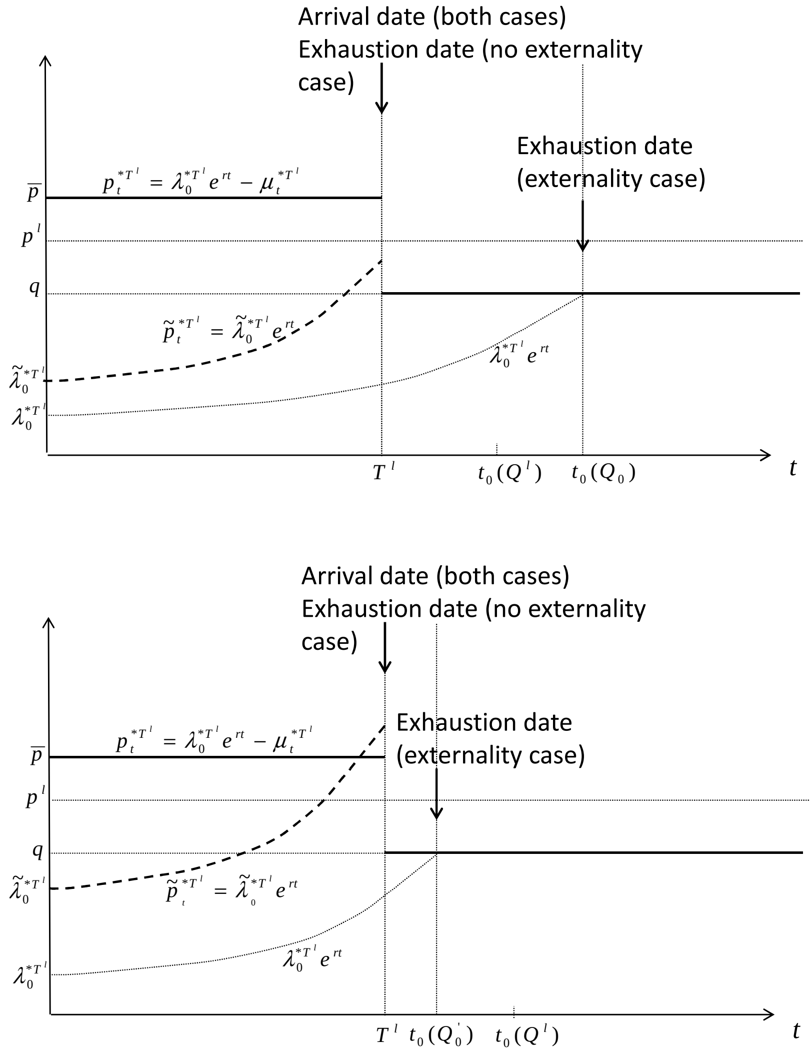

Figure 4: Price path for optimal constrained arrival date T^l, for $Q_0 > Q^l$ (upper figure) and for $Q_0^{'} < Q^l$ (lower figure), in the constrained (solid line) and unconstrained case (dotted line).

not offset the second term of the marginal cost of delaying innovation in the constrained case $(\bar{p} - q)\bar{x}$. If Q_0 is small enough so that the final price in the unconstrained case is above $\bar{p}$, then final unconstrained oil price is above final constrained price, so that there is only one of the two effects: regulation postpones extraction and thus postpones optimal arrival date.

3.2.2 Starting from below the ceiling: $Z_0 < \bar{Z}$

Starting from below the ceiling, it is never useful to innovate before the ceiling has been reached. It would result in a dormant technology, which has been developed but is not used because cheap oil can be used as long as the ceiling is not binding. This result echoes the Chakravorty, Magne & Moreaux (2006) finding. In their case, the backstop is never used before the ceiling is reached. We restrict our attention, in the following, to the case when extraction is slowed down by regulation, for arrival at $\tilde{T}$ (assumption 5). This seems a reasonable assumption, which is obviously satisfied if, for a given arrival date, regulation initially increases oil price. For all $(Z_0, \bar{Z})$, a sufficient condition is that $\bar{Z}$ is not too large compared to Z_0 (Then $\forall Q_0$, at the optimal unconstrained date $\tilde{T}$, either oil is not exhausted in the constrained case or $p^{*\tilde{T}}_{\tilde{T}} \leq \tilde{p}^{*\tilde{T}}_{\tilde{T}}$, see Appendix C)[6]. In this case there are two diverging effect of the externality on optimal backstop arrival date: Firstly, the externality makes innovation useful, as it allows for the consumption of cheap energy despite the externality. Secondly, regulation postpones the exhaustion of the oil. In the future, energy from the backstop will be in competition with oil that has been saved by regulation.

Optimal price path for a given innovation date. It is first necessary to characterize the optimal price path for any arrival date to compute the marginal cost of delaying innovation at each date. The expression of the marginal cost of delaying innovation depends on the arrival date and this expression changes at some pivotal dates that we define below.

- The date t_1 is the start of the ceiling period when there is backstop available at price q from the outset. If arrival occurs before t_1, *i.e.* while the ceiling constraint is non-binding, the price path is the same as if the backstop was available from the outset (it is described in Chakravorty, Magne & Moreaux (2006)). The oil price $p^{*T}_t = \lambda^{*T}_0 e^{rt} - \mu^{*T}_0 e^{(r+\alpha)t}$ reaches, along the optimal path, price q at the start of ceiling period t_1. From this date on, the backstop and the oil are used jointly at price q until oil exhaustion at date θ_1. Date t_1 is such that $(\lambda^{*t_1}_0, \mu^{*t_1}_0, t_1, \theta_1)$ satisfy:

$$\begin{cases} Z_{t_1} & = \bar{Z} \\ \lambda^{*t_1}_0 e^{rt_1} - \mu^{*t_1}_0 e^{(r+\alpha)t_1} & = q \\ \lambda^{*t_1}_0 e^{r\theta_1} & = q \\ \int_0^{\theta_1} x^{*t_1}_t dt + (\theta_1 - t_1)\alpha\bar{Z} & = Q \end{cases}$$

  The price path and extraction path are represented in Fig.5.

- If the arrival date is after t_1, then the resource price at the arrival date is greater than q. We define date t_2, such that, when the arrival date is between t_1 and t_2, the ceiling is reached exactly at T, at price p^{*T}_T between q and $\bar{p}$. Before T, the ceiling is non-binding and the shadow price of pollution rises at rate $r + \alpha$, for $t \leq T$, $p^{*T}_t = \lambda^{*T}_0 e^{rt} - \mu^{*T}_0 e^{(r+\alpha)t}$. At arrival date, the price falls from p^{*T}_T to q. After the arrival date, the backstop and the non-renewable resource are used at price q until exhaustion at date $h(T)$. $(\lambda^{*T}_0, \mu^{*T}_0, h(T))$ satisfy:

$$\begin{cases} Z_T & = \bar{Z} \\ \int_0^{h(T)} x^{*T}_t dt & = Q \\ \lambda^{*T}_0 e^{rh(T)} & = q \end{cases}$$

- If the arrival date is equal to t_2 or happens after t_2, the oil price remains equal to $\bar{p}$ from date t_2 until the arrival date or until the scarcity rent reaches $\bar{p}$. Date t_3 is such

[6] The fact that regulation actually slows down oil extraction is always true for large Q, but, because we have assumed that natural dilution is proportional to the stock of pollution, accelerating extraction at the beginning, *i.e.* before the ceiling is reached, increases the maximum quantity of oil that can be burnt before a given date for a given pollution constraint. If $\bar{Z} - Z_0$ is large, it might be the case that the proportional pollution dilution makes it profitable to accelerate extraction (and not to postpone it) so that the final price in the unconstrained case is below the final price in the constrained case. We do not consider this case here, but it could be studied in further work. It is interesting to note, however, that if instead of assuming proportional natural decay, we had assumed that α decreases inversely with the stock of pollution in the atmosphere Z_t, then this assumption would always be satisfied, and all the results hereafter would be true, irrespective of Z_0. Stock dependent decreasing natural decay would be more accurate, according to Joos et al. (1996). See a calibration in appendix C.

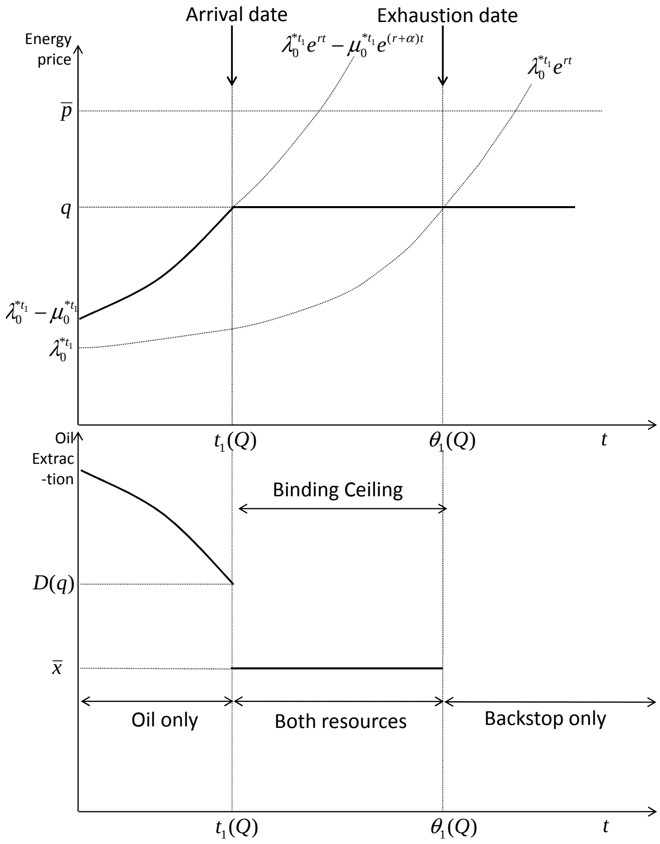

Figure 5: Price and extraction paths for arrival date $t_1(Q)$

that, for any arrival date T between t_2 and t_3, extraction follows the (same) following path: the ceiling is non-binding in the first phase ending at date t_2 at price $p_{t_2}^{*T} = \bar{p}$, during this first phase $p_t^{*T} = \lambda_0^{*T} e^{rt} - \mu_0^{*T} e^{(r+\alpha)t}$; then, from date t_2 until date t_3, the oil is used in quantity $\bar{x}$ at each date until oil exhaustion at date t_3, when the scarcity rent reaches q, $\lambda_0^{*T} e^{rt_3} = q$. Between arrival date T and exhaustion t_3, both the backstop and oil are used, at price q, exactly $\bar{x}$ of oil is consumed at each date. For arrival date between t_2 and t_3, $\lambda_0^{*T} = \lambda_0^{*t_2}$, $\mu_0^{*T} = \mu_0^{*t_2}$ and $x_t^{*T} = x_t^{*t_2}$. Dates t_2 and t_3 are such that $(\lambda_0^{*T}, \mu_0^{*T}, t_2, t_3)$ solve:

$$\begin{cases} Z_{t_2} & = \bar{Z} \\ \lambda_0^{*T} e^{rt_2} - \mu_0^T e^{(r+\alpha)t_2} & = \bar{p} \\ \lambda_0^{*T} e^{rt_3} & = q \\ \int_0^{t_2} x_t^{*T} dt + (t_3 - t_2)\alpha \bar{Z} & = Q \end{cases}$$

The price path and extraction path for arrival date t_2 are represented in Fig.6. The oil is exhausted at date t_3. For any arrival date between t_2 and t_3, the extraction path is the same as in Fig.6.

- If the arrival date is after t_3, the scarcity rent is higher than q at the arrival date and the oil is exhausted precisely at the arrival date. We define t_4 such that for any arrival date between t_3 and t_4, the scarcity rent at the innovation date is between

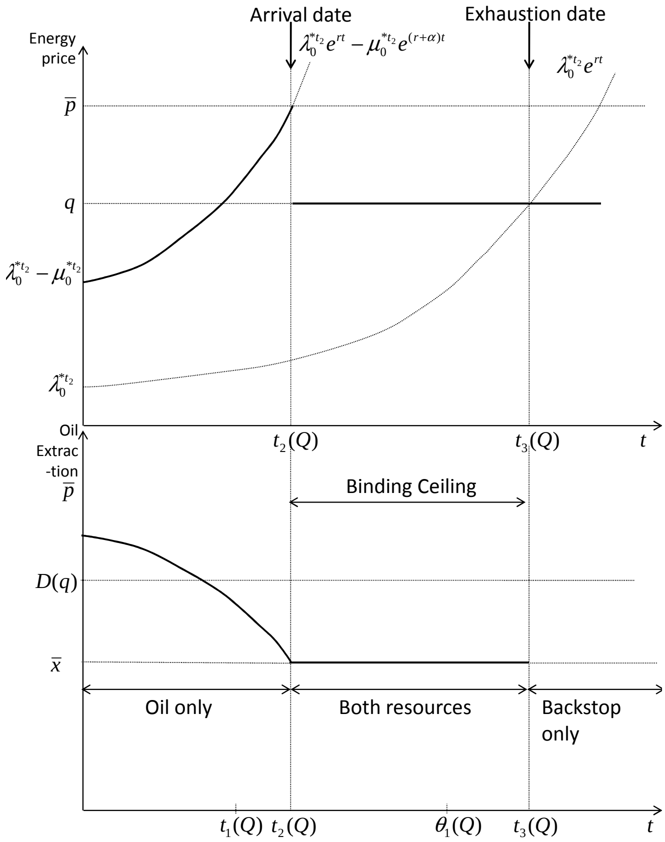

Figure 6: Price path for arrival date t_2. The oil is exhausted at date t_3 for any arrival date between t_2 and t_3.

q and $\bar{p}$. At the arrival date between t_3 and t_4, the ceiling constraint is tight and the price falls from $\bar{p}$ to q, and the oil is exhausted. Denoting $g(T)$ as the start of the ceiling period, for $t \le g(T)$, $p_t^{*T} = \lambda_0^{*T} e^{rt} - \mu_0^{*T} e^{(r+\alpha)t}$ then, for $g(T) \le t \le T$, $p_t^{*T} = \bar{p}$. With $(\lambda_0^{*T}, \mu_0^{*T}, g(T))$ solving:

$$\begin{cases} Z_{g(T)} & = \bar{Z} \\ \int_0^T x_t^{*T} dt & = Q \\ \lambda_0^{*T} e^{rg(T)} - \mu_0^{*T} e^{(r+\alpha)g(T)} & = \bar{p} \end{cases}$$

- For innovation date t_4, the ceiling is reached at date θ_3. Before that date, $p_t^{*T} = \lambda_0^{*T} e^{rt} - \mu_0^{*T} e^{(r+\alpha)t}$, with $p_{\theta_3}^{*T} = \bar{p}$. From θ_3 to t_4, price equals $\bar{p}$. Oil is exhausted at t_4, with final scarcity rent $\bar{p}$. By definition, $(\lambda_0^{*t_4}, \mu_0^{*t_4}, \theta_3, t_4)$ is the solution of:

$$\begin{cases} Z_{\theta_3} & = \bar{Z} \\ \lambda_0^{*t_4} e^{r\theta_3} - \mu_0^{*t_4} e^{(r+\alpha)\theta_3} & = \bar{p} \\ \lambda_0^{*t_4} e^{rt_4} & = \bar{p} \\ \int_0^{\theta_3} x_t^{*t_4} dt + (t_4 - \theta_3)\alpha \bar{Z} & = Q \end{cases}$$

The price path and extraction path for arrival date t_4 are represented in Fig.7.

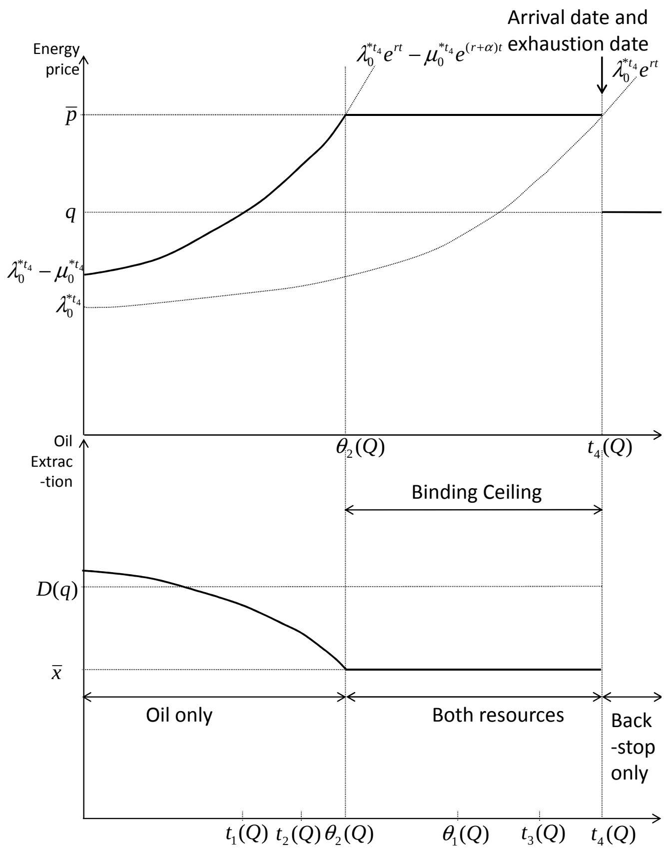

Figure 7: Price path for arrival date $t_4(Q)$. The oil is exhausted at the arrival date.

- If the arrival date is after t_4, the scarcity rent is higher than $\bar{p}$ at the arrival date, the constraint is non-binding at the arrival date and the price follows a "Hotelling path"; the oil is exhausted precisely at this date. Calling $f_1(T)$ the start of the ceiling period and $f_2(T)$ the end of the ceiling period, for $t \leq f_1(T)$, $p_t^{*T} = \lambda_0^{*T} e^{rt} - \mu_0^{*T} e^{(r+\alpha)t}$; then, for $f_1(T) \leq t \leq f_2(T)$, $p_t = \bar{p}$ and finally, for $t \geq f_2(T)$, $p_t^{*T} = \lambda_0^{*T} e^{rt}$. With $(\lambda_0^T, \mu_0^T, f_1(T), f_1(T))$ solving:

$$\begin{cases} Z_{f_1(T)} & = & \bar{Z} \\ \int_0^T x_t^{*T} dt & = & Q \\ \lambda_0^{*T} e^{r f_1(T)} - \mu_0^{*T} e^{(r+\alpha) f_1(T)} & = & \bar{p} \\ \lambda_0^{*T} e^{r f_2(T)} & = & \bar{p} \end{cases}$$

Optimal arrival date . All dates (t_1, t_2, t_3, t_4) depend on Q and $\bar{Z}$.

Proposition 4. *The optimal arrival date T^* is characterized by:*

$$e^{-rT^*} c^{'}(T^*) = N(T^*)$$

With:

$$N(T) = \begin{cases} 0 & T \leq t_1 \\ (u(D(q)) - qD(q)) - (u(x_T^{*T}) - x_T^{*T} p_T^{*T}) - \bar{x}(p_T^{*T} - q) & t_1 < T \leq t_2 \\ (u(D(q)) - qD(q)) - (u(\bar{x}) - q\bar{x}) & t_2 < T \leq t_3 \\ (u(D(q)) - qD(q)) - (u(\bar{x}) - \lambda_0^{*T} e^{rt} \bar{x}) & t_3 < T \leq t_4 \\ (u(D(q)) - qD(q)) - (u(x_T^{*T}) - p_T^{*T} x_T^{*T}) & T > t_4 \end{cases}$$

N(T) is computed in appendix D.

The oil price p_T^{*T} at arrival date between t_1 and t_2 increases with T (see appendix B.1), and by definition $p_{t_1}^{*t_1} = q$ and $p_{t_2}^{*t_2} = \bar{p}$; the final scarcity rent increases with T between t_3 and t_4 (see appendix B.2) and by definition, $\lambda_0^{*t_3} e^{rt_3} = q$ and $\lambda_0^{*t_4} e^{rt_4} = \bar{p}$. The marginal cost of delaying innovation is thus increasing with T and continuous at dates t_1, t_2, t_3 and t_4. So that there is a single solution to the maximization problem. The exhaustion date is increasing with the innovation arrival date. For any arrival date, the energy price is first increasing then decreasing at the arrival date. This result contrasts with Chakravorty et al. (2009). They find that, with learning by doing in the clean substitute, regulation may lead to cyclical energy price behavior. They assume that the unit cost of the renewable resource increases with the quantity supplied in each period, such that solar energy may be used before the ceiling is binding, but decreases between two periods with cumulative use. The process studied here is very different, as we assume a fixed cost for energy supplied by the backstop. If we assumed learning by doing after arrival, or positive spillovers in the clean sector (as in Acemoglu et al. (2009)), regulation might further speed up the optimal arrival of the clean substitute. Assuming an increasing unit cost of clean energy in each period may lead to more complicated patterns for the price paths in our case as well.

The marginal cost of delaying innovation expressed at arrival date, $N(T)$, must be positive at the arrival date: $p_{T^*}^{*T^*} > q$, so that **Result 2** remains true with environmental regulation. Moreover, **Result 3** continues to hold:

Proposition 5. *The optimal arrival date increases with the initial stock of the non-renewable. However, if* $\lim_{T\to\infty} -e^{rT} c^{'}(T) < u(D(q)) - qD(q) - (u(\bar{x}) - q\bar{x})$*, there is a cut-off date* $T^{\lim}$ *such that arrival is never after* $T^{\lim}$*.*

The proof is presented in Appendix E. Increasing the stock of the non-renewable resource reduces the incentive to innovate. However, if Q_0 is sufficiently high, it is as if the supply of Q_0 were infinite. In the unconstrained case, if the stock of oil were infinite, innovation would be useless, whereas in the constrained case, because regulation increases the price of consuming oil, it is useful to innovate at some date $T^{\lim}$ even when the stock of oil is infinite.

The following proposition holds:

Proposition 6. *There exists a stock* $Q^l(Z_0, \bar{Z})$ *of the non-renewable resource, such that:*

- $\forall Q \geq Q^l$*, regulation advances the optimal date of innovation compared to a case with no externality.*

- *$\forall Q < Q^l$, regulation postpones the optimal innovation date compared to the case with no externality.*

This result expands on the intuition in the previous section (proof in Appendix F). The intuition is the following: for Q_0 large enough, optimal unconstrained final price is low enough such that the gain from reducing marginal consumption cost in the constrained case is much higher than the gain from reducing the marginal consumption cost in the unconstrained case. This effect is sufficiently large to offset the fact that, because of regulation, the backstop is in competition with oil until oil exhaustion. If the oil stock is sufficiently large, regulation advances the optimal innovation date.

Result 1 no longer holds. The oil might not be exhausted at the optimal arrival date. This result parallels Chakravorty, Magne & Moreaux (2006): they show that both the non-renewable resource and the renewable resource can be used jointly. However, whether both resources are used simultaneously depends on the initial stock of the non-renewable resource and not only on the price of energy supplied with the new technology. When the stock of the non-renewable resource is small, it is optimal for the backstop to arrive after the ceiling period: For small Q_0, for optimal unconstrained arrival date $\tilde{T}$, constrained price path is also in a Hotelling phase, but oil is less scarce at $\tilde{T}$ in the constrained case than in the unconstrained case. The fact that regulation postpones exhaustion makes R&D less useful.

As the ceiling can be expected to become tighter over time, it is interesting to see how the optimal R&D investment varies with the stringency of the ceiling[7].

Proposition 7. *If, for initial oil stock Q_0 and ceiling $\bar{Z}^l$, regulation strictly advances innovation, then increasing the stringency of the ceiling from $\bar{Z}^l$ to $\bar{Z} < \bar{Z}^l$ also increases optimal R&D effort compared to the unconstrained case. (Proof in Appendix G).*

Corollary: The smallest initial amount of oil such that regulation advances innovation arrival, $Q^l(Z_0, \bar{Z})$ is decreasing with the stringency of the ceiling, *i.e* is increasing with $\bar{Z}$.

The central planner should thus increase R&D if the ceiling is lowered. In a decentralized economy, the market outcome of lowering the ceiling depends on the structure of the R&D sector. For a private sector engaging in R&D, there would be also two effects of carbon regulation: the date at which the backstop is used alone is postponed, so that possible full monopoly profits are postponed by regulation, but on the other hand, the backstop can be used earlier, in combination with oil.

3.3 Insights on possible extensions

Additional R&D to decrease the backstop cost q . If the social planner can also conduct additional R&D to decrease the backstop price, the problem becomes more complicated. The optimal cost q of the backstop, as well as its arrival date, depends on the characteristics of the cost function. If, for instance, this cost is separable in its two arguments, q and T,

[7]For values of the ceiling satisfying assumption 5

then the aim of the central planner is to find (q, T), which maximizes:

$$\max_{q,T} \Big(V(q,T) - C(q) - C(T) \Big)$$

This maximization problem is rewritten:

$$\max_{q} \Big((\max_{T} \big(V(q,T) - C(T) \big) - C(q) \Big)$$

The optimal arrival date is $T^*(q)$, *i.e.* the solution of: $\max_T \big(V(q,T) - C(T) \big)$. If we assume, as before, that regulation postpones extraction for all possible values of q, we have the following result:

Proposition 8. *If the R&D cost function $c(T, q)$ is separable in its two arguments, optimal innovation arrival date $T^*(q)$ is increasing with the cost q of energy supplied by the backstop. (Proof in Appendix H)*

The marginal cost of delaying the arrival date decreases with the cost of energy supplied by the backstop. If the backstop cost is high, the drop in energy price for a given arrival date is lower and innovation is less desirable. Thus, the social planner makes a trade-off between developing a cheap backstop early or an expensive backstop later, as in a case without externality. More precise demand and cost function assumptions may be necessary before a complete characterization of the solution can be made, and for a comparison between the externality and the no-externality cases, which is beyond the scope of this paper.

Positive extraction costs. With positive extraction costs c_e greater than the backstop price q, oil would not be used after innovation so that regulation would only increase the marginal cost of consuming oil, but would not lengthen the period during which the backstop competes with oil. Starting from the ceiling, the limit price p^l_c is such that: $u(x(p^l_c)) - p^l_c x(p^l_c) = u(\bar{x}) - c_e \bar{x} \leq u(\bar{x}) - q\bar{x}$, so that $p^l_c > p^l$. The corresponding initial oil stock such that regulation leads to advance innovation is thus smaller: $Q^l_c < Q^l$. Regulation would advance optimal arrival date in this case, even for Q_0 smaller than Q^l defined in section 3.2.1.

3.4 Policy implications

If the non-renewable resource owners are in perfect competition and if the central planner chooses the R&D strategy and can put a carbon tax in place, then the optimal carbon tax is equal to the shadow cost of pollution of the planned economy. It is first increasing then decreasing, as in Chakravorty, Magne & Moreaux (2006).

Moreover, if the initial stock of oil is large, placing a stringent ceiling on the stock of pollution should drive up the effort toward an alternative backstop technology, and not to push back the optimal arrival date, contrary to the intuition of the green paradox. On the other hand, imposing a high ceiling on the stock of pollution would reduce research into an alternative backstop technology compared to the unconstrained case.

These results do not imply, however, when research is conducted privately, that there should be subsidies to green innovation if the initial oil stock is large. Indeed, it would depend on the market structure associated with R&D. Taking as a starting point the Chakravorty, Magne &

Moreaux (2006) framework, where there is a backstop available at price $\bar{q} > \bar{p}$ from the outset, the potential for endogenous R&D toward a backstop technology consisting in a drop in the backstop cost from $\bar{q}$ to q increases the initial optimal environmental tax $-\mu_0$. Endogenous innovation lowers the scarcity rent due to the Sinn (2008) green paradox: policies that become greener over time make the oil producers' price fall. So it becomes necessary to increase the carbon tax.

4 Conclusion

We consider the effect of a cap on the stock of pollution on optimal R&D toward a clean backstop technology when the effect of R&D is to speed up the arrival of the backstop. We show that if the price of the backstop technology is such that the environmental cap can be met at a relatively low cost, innovation arrives earlier compared to the case with no externalities when the stock of the non-renewable resource is large. If the stock is small, on the other hand, the optimal arrival date of the backstop is postponed. This comes from two contradictory effects. The backstop fulfills two roles: it is used to meet the constraint on the stock of pollution, without limiting energy consumption too much, and it is also used to replace the non-renewable resource once it is exhausted. When the price of the backstop is relatively high (or the constraint on the stock of pollution is slack), only the second role is played by the backstop and regulation postpones the optimal innovation date for the backstop.

We have examined a case with zero extraction costs, implicitly assuming exogenous technological progress in extraction costs. We have not take into account learning by doing in the clean substitute. One avenue for research would be to consider this kind of technological progress.

Another avenue for future research would be to allow for imperfect substitution between the non-renewable resource and the backstop. Also, we could discard the hypothesis of the deterministic feature of $R\&D$, the intuitions would remain the same. Optimal environmental regulation has two effect on the desirability of increasing the probability of arrival of the backstop: firstly it postpones exhaustion, so that the backstop is less desirable ; secondly, it increases the price of oil, making the development of a cheap substitute more desirable. However, introducing probabilistic $R\&D$ makes the computation much harder and the optimal environmental regulation would not be as simple as it is in the deterministic model.

More importantly, it would be interesting to study this problem in a second-best setting, where R&D is conducted privately. Yet the solution of this problem cannot easily be decentralized if R&D is conducted by the private sector (for instance, with a monopoly engaging in R&D at date 0) without binding contracts. The reason for this is that ex-ante incentives (lowering the backstop price in order to accelerate exhaustion) then enter into conflict with ex-post incentives (setting the monopoly price once oil is exhausted). The monopoly price itself depends on the quantity of oil left at the arrival date. How the transition to a clean substitute would be affected by regulation would depend on the market structure associated with R&D.

A For Q_0 large enough, $Z_t \geq \bar{Z}$ for some t on the unconstrained price path

If there exists $Q_{\min}$ such that the stock of pollution in the atmosphere is above $\bar{Z}$ at some date in the unconstrained case with endogenous innovation, then, as the optimal initial scarcity rent decreases with Q_0, for all $Q_0 \geq Q_{\min}$, the stock of pollution also exceeds the ceiling at some date. Let find one value of Q_0 such that the stock of pollution in the atmosphere exceeds the ceiling on the unconstrained price path. Consider (λ, T_0) such that $p_t = \lambda e^{rt}$ and the ceiling is reached when the p_t reaches q:

$$\begin{aligned} e^{-\alpha T_0}(Z_0 + \int_0^{T_0} e^{\alpha t} x_t dt) &= \bar{Z} \\ \lambda e^{rT_0} &= q \end{aligned}$$

For this λ, consider the price path $p_t = \lambda e^{rt}$. There exists a date T, such that:

$$-e^{rT} c^{'}(T) + u(x(p_T)) - p_T x(p_T) - (u(D(q) - qD(q)) = 0$$

Indeed the function $T \rightarrow -e^{rT} c^{'}(T) + u(x(p_t)) - p_t x(p_t) - (u(D(q) - qD(q))$ is continuous (as a composition of continuous functions) and is positive, equal to $-e^{rT_0} c^{'}(T_0)$ at date T_0, and negative for T sufficiently high such that $-e^{rT} c^{'}(T) + u(x(p_t)) - p_t x(p_t) - (u(D(q) - qD(q)) \rightarrow_{t \rightarrow +\infty} -\alpha < 0$. Take $T_{\min}$ such that $-e^{rT_{\min}} c^{'}(T_{\min}) + u(x(p_t)) - p_t x(p_t) - (u(D(q) - qD(q)) = 0$ and λ defined above, take the stock of pollution Q_0, such that if the price is equal to λe^{rt}, Q_0 is exhausted at date $T_{\min}$. For this Q_0, by construction, the ceiling is reached at some date on the unconstrained price path.

B Intermediate results

B.1 Intermediate result 1: Constrained price at date T, p_T^T, increases with T between t_1 and t_2

If arrival date is T between t_1 and t_2, then:

$$\begin{cases} e^{-\alpha T}(Z_0 + \int_0^T e^{\alpha t} x_t^{*T} dt) &= \bar{Z} \\ \int_0^{h(T)} x_t^{*T} dt &= Q \\ \lambda_0^{*T} e^{rh(T)} &= q \end{cases}$$

We differentiate with respect to T and we get:

$$d\lambda_0^{*T} = \frac{-\left(\int_0^T \frac{\partial x_t^{*T}}{\partial p_t^{*T}} e^{(r+\alpha)t} dt\right)(x_T^{*T} - \bar{x}) rq dT}{\bar{x} e^{rh(T)} \int_0^T \frac{\partial x_t^{*T}}{\partial p_t^{*T}} e^{(r+2\alpha)t} + \left(\left(\int_0^T \frac{\partial x_t^{*T}}{\partial p_t^{*T}} e^{(r+\alpha)t} dt\right)^2 - \left(\int_0^T \frac{\partial x_t^{*T}}{\partial p_t^{*T}} e^{rt} dt\right)\left(\int_0^T \frac{\partial x_t^{*T}}{\partial p_t^{*T}} e^{(r+2\alpha)t} dt\right)\right)} \leq 0$$

$$d\mu_0^{*T} = \frac{\left(\left(\int_0^T \frac{\partial x_t^{*T}}{\partial p_t^{*T}} e^{rt} dt\right) rq - \bar{x} e^{rh(T)}\right)(x_T^{*T} - \bar{x}) dT}{\bar{x} e^{rh(T)} \int_0^T \frac{\partial x_t^{*T}}{\partial p_t^{*T}} e^{(r+2\alpha)t} + \left(\left(\int_0^T \frac{\partial x_t^{*T}}{\partial p_t^{*T}} e^{(r+\alpha)t} dt\right)^2 - \left(\int_0^T \frac{\partial x_t^{*T}}{\partial p_t^{*T}} e^{rt} dt\right)\left(\int_0^T \frac{\partial x_t^{*T}}{\partial p_t^{*T}} e^{(r+2\alpha)t} dt\right)\right)} \geq 0$$

As $d\lambda_0^{*T} \leq 0$ and $d\mu_0^{*T} \geq 0$, it must be the case that $d\lambda_T^{*T} + d\mu_{*T}^T \geq 0$, otherwise the price between 0 and T would be decreased at all dates when T increases and the stock of pollution would exceed the ceiling at some date.

B.2 Intermediate result 2: final scarcity rent $\lambda_0^{*T} e^{rT}$ increases with T between t_3 and t_4

If arrival date is T between t_3 and t_4, then:

$$\begin{cases} e^{-\alpha g(T)}(Z_0 + \int_0^{g(T)} e^{\alpha t} x_t^{*T} dt) &= \bar{Z} \\ \int_0^T x_t^{*T} dt &= Q \\ \lambda_0^{*T} e^{rg(T)} - \mu_0^{*T} e^{(r+\alpha)g(T)} &= \bar{p} \end{cases}$$

We differentiate with respect to T and we get:

$$d\lambda_0^{*T} = \frac{\left(\int_0^T \frac{\partial x_t^{*T}}{\partial p_t^{*T}} e^{(r+2\alpha)t} dt\right) \bar{x} dT}{\left(\left(\int_0^T \frac{\partial x_t^{*T}}{\partial p_t^{*T}} e^{(r+\alpha)t} dt\right)^2 - \left(\int_0^T \frac{\partial x_t^{*T}}{\partial p_t^{*T}} e^{rt} dt\right)\left(\int_0^T \frac{\partial x_t^{*T}}{\partial p_t^{*T}} e^{(r+2\alpha)t} dt\right)\right)} \geq 0$$

So that $d(\lambda_0^{*T} e^{rT}) > 0$

B.3 Intermediate result 3: date $t_1(Q)$ decreases with Q

Consider two stocks $Q^{'} > Q$. We call λ (resp. $\lambda^{'}$) and $-\mu$ (resp. $-\mu^{'}$) the scarcity rent and carbon tax associated with Q(resp. $Q^{'}$) when there is a backstop from the outset. It is first straightforward that $\lambda > \lambda^{'} \Leftrightarrow -\mu < -\mu^{'}$. Otherwise, the pollution stock would exceed the ceiling (or remain strictly below) for one of the stocks. Assume moreover that $\lambda > \lambda^{'}$. We denote x_t (resp. $x_t^{'}$) the extraction at date t. from equation, it must be that :

$$e^{-\alpha t_1^{'}}(Z_0 + \int_0^{t_1^{'}} e^{\alpha t} x_t dt) = \bar{Z}$$

$$e^{-\alpha t_1^{'}}(Z_0 + \int_0^{t_1^{'}} e^{\alpha t} x^{'}(t) dt) = \bar{Z}$$

If $\lambda < \lambda^{'}$, then $x_t - x_t^{'}$ is decreasing with t, It is first positive and then negative. We call θ_1 the date at which price paths cross. It follows that :

$$\int_0^{\theta_1} e^{\alpha t}(x_t - x_t^{'})dt = \int_{\theta_1}^{t_1^{'}} e^{\alpha t}(x_t^{'} - x_t)dt > 0$$

Then,

$$\int_0^{\theta_1} e^{\alpha t}(x_t - x_t^{'})dt < \int_0^{\theta_1} e^{\alpha \theta_1}(x_t - x_t^{'})$$

and $\int_{\theta_1}^{t_1^{'}} e^{\alpha t}(x_t^{'} - x_t)dt > \int_0^{\theta_1} e^{\alpha \theta_1}(x_t^{'} - x_t)$, and as a result

$$\int_0^{t_1^{'}} (x_t - x_t^{'})dt > 0$$

More oil is consumed between 0 and $t_1^{'}$ with Q than $Q^{'}$. But as $\lambda < \lambda^{'}$, the date of oil exhaustion is also later with Q than $Q^{'}$, so that more oil is consumed, which is in contradiction with $Q < Q^{'}$. So that $\lambda > \lambda^{'}$ and $-\mu < -\mu^{'}$, implying that $t_1(Q) > t_1(Q^{'})$

B.4 Intermediate result 4: $t_2(Q)$ decreases with Q and $t_3(Q)$ increases with Q

Date t_2 and t_3 are defined by

$$\begin{aligned}
e^{-\alpha t_2}(Z_0 + \int_0^{t_2} e^{\alpha t} x_t^{*t_2} dt) &= \bar{Z} \\
\lambda_0^{*t_2} e^{r t_2} - \mu_0^{*t_2} e^{(r+\alpha)t_2} &= \bar{p} \\
\lambda_0^{*t_2} e^{r t_3} &= q \\
\int_0^{t_2} x_t^{*t_2} dt + (t_3 - t_2)\alpha \bar{Z} &= Q
\end{aligned}$$

Differentiating wrt Q, we get:

$$(\int_0^{t_2} e^{(\alpha+r)t} x^{'}(p_t)dt) d\lambda_0^{*t_2} - (\int_0^{t_2} e^{(2\alpha+r)t} x^{'}(p_t^{*t_2})) d\mu_0^{*t_2} = 0 \tag{6}$$

$$d\lambda_0^{*t_2} e^{r t_2} - d\mu_0^{*t_2} e^{(r+\alpha)t_2} + (r\lambda_0^{*t_2} e^{r t_2} + (r+\alpha)e^{(r+\alpha)t_2})dt_2 = 0 \tag{7}$$

$$d\lambda_0^{*t_2} e^{r t_3} + r\lambda_0^{*t_2} e^{r t_3} dt_3 = 0 \tag{8}$$

$$(\int_0^{t_2} e^{rt} x_t^{*t_2} dt) d\lambda - (\int_0^{t_2} e^{(r+\alpha)t} x_t^{*t_2} dt) d\mu_0^{*t_2} + dt_3 \bar{x} = dQ \tag{9}$$

Combining Eq7, **??** and 9, we get:

$$\begin{aligned}
&\left((\int_0^{t_2} e^{(\alpha+r)t} x^{'}(p_t^{*t_2})dt)^2 - \int_0^{t_2} e^{(2\alpha+r)t} x^{'}(p_t^{*t_2})dt \int_0^{t_2} e^{(r)t} x^{'}(p_t^{*t_2})dt \right) r\lambda_0^{*t_2} d\lambda_0^{*t_2} \\
&+\bar{x}(\int_0^{t_2} e^{(2\alpha+r)t} x^{'}(p_t^{*t_2})dt) d\lambda_0^{*t_2} = -r\lambda_0^{*t_2} \int_0^{t_2} e^{(\alpha+r)t} x^{'}(p_t^{*t_2}) dt dQ
\end{aligned}$$

Using the Cauchy Schwartz inequality, it appears that $d\lambda_0^{*t_2} < 0$, the scarcity rent decreases with Q. It implies that

$$dt_3 > 0$$

From 7, we have that

$$-d\mu_0^{*t_2}e^{(r+\alpha)t_2} = -\frac{\int_0^{t_2} e^{\alpha t_2}e^{(\alpha+r)t}x^{'}(p_t^{*t_2})dt)}{(\int_0^{t_2} e^{(2\alpha+r)t}x^{'}(p_t^{*t_2}))}d\lambda_0^{*t_2}e^{rt_2} > -d\lambda_0^{*t_2}e^{rt_2}$$

From 8, we then have that :

$$dt_2 < 0$$

B.5 Intermediate result 5 : for a date of innovation T given, with that $t_1(Q) < T < t_2(Q)$ in the regulated case, the energy price increases with Q

We take $Q^{'} > Q$. Assume that $\lambda^{'} > \lambda$, then $-\mu^{'} < -\mu$ (for ease of notation, we write $\lambda \equiv \lambda_0^{*T}$ etc), and the prices must cross at some date (otherwise, pollution would exceed the ceiling) .
As $\lambda < \lambda^{'}$, then $x_t - x_t^{'}$ is decreasing with t, It is first positive until date θ_1 and then negative. It follows that :

$$\int_0^{\theta_1} e^{\alpha t}(x_t - x_t^{'})dt = \int_{\theta_1}^{T} e^{\alpha t}(x_t^{'} - x_t)dt > 0$$

But, because $x_t - x_t^{'}$ is decreasing with t, It is first positive until date θ_1 and then negative,

$$\int_0^{\theta_1} e^{\alpha t}(x_t - x_t^{'})dt < \int_0^{\theta_1} e^{\alpha\theta_1}(x_t - x_t^{'})dt$$

and

$$\int_{\theta_1}^{T} e^{\alpha t}(x_t^{'} - x_t)dt > \int_{\theta_1}^{T} e^{\alpha\theta_1}(x_t^{'} - x_t)dt$$

, and as a result

$$\int_0^{T} (x_t - x_t^{'})dt > 0$$

But as $\lambda < \lambda^{'}$, more oil is consumed after innovation when the stock is Q than when the stock is $Q^{'}$, which is a contradiction (more oil is consumed during all the price path). So that $Q^{'} > Q$ implies that $\lambda > \lambda^{'}$ and $-\mu < -\mu^{'}$, so that, at the date of innovation T, $p_T^{'} > p_T$

B.6 Intermediate result 6: the final scarcity rent increases with Q between $t_3(Q)$ and $t_4(Q)$

For arrival date T between $t_3(Q)$ and $t_4(Q)$, calling t_2 the date at which the ceiling is reached, $(t_2, \lambda_0^{*T}, \mu_0^{*T})$ satisfy the set of equation (for ease of notation, we write $\lambda \equiv \lambda_0^{*T}$ etc):

$$\begin{aligned} e^{-\alpha t_2}(Z_0 + \int_0^{t_2} e^{\alpha t}x_t dt) &= \bar{Z} \\ \lambda e^{rt_2} - \mu e^{(r+\alpha)t_2} &= \bar{p} \\ \int_0^{t_2} x_t dt + (T - t_2)\alpha\bar{Z} &= Q \end{aligned}$$

With $x_t = D(p_t)$ and $p_t = (\lambda e^{rt} - \mu e^{(r+\alpha)t})$ for $t < t_2$ and $p_t = \bar{p}$ for $t_2 < t < T$.
If T is also between $t_3(Q + dQ)$ and $t_4(Q + dQ)$, differentiating wrt Q, we get:

$$\begin{aligned} (\int_0^{t_2} e^{(\alpha+r)t}\frac{\partial x_t}{\partial p_t}dt)d\lambda - (\int_0^{t_2} e^{(2\alpha+r)t}\frac{\partial x_t}{\partial p_t}dt)d\mu &= 0 \\ d\lambda e^{rt_2} + (r\lambda e^{rt_2} + (r+\alpha)e^{(r+\alpha)t_2})dt_2 - d\mu e^{(r+\alpha)t_2} &= 0 \\ d\lambda(\int_0^{t_2} e^{rt}\frac{\partial x_t}{\partial p_t}dt) - d\mu(\int_0^{t_2} e^{(r+\alpha)t}\frac{\partial x_t}{\partial p_t}dt) &= dQ \end{aligned}$$

From the first equation:

$$d\lambda = +\frac{\int_0^{t_2} e^{(2\alpha+r)t}\frac{\partial x_t}{\partial p_t}dt}{\int_0^{t_2} e^{(\alpha+r)t}\frac{\partial x_t}{\partial p_t}dt}d\mu$$

From the fourth:

$$d\mu\frac{(\int_0^{t_2} e^{(\alpha+r)t}\frac{\partial x_t}{\partial p_t}dt)^2 - \int_0^{t_2} e^{(2\alpha+r)t}\frac{\partial x_t}{\partial p_t}dt\int_0^{t_2} e^{(r)t}\frac{\partial x_t}{\partial p_t}dt}{\int_0^{t_2} e^{(\alpha+r)t}\frac{\partial x_t}{\partial p_t}dt} = dQ$$

So that, using the Cauchy Schwartz inequality, $-d\mu > 0$ if $dQ > 0$ and $d\lambda < 0$.

C Regulation postpones extraction if, for all $(\bar{Z}, Z_0)$, $\bar{Z}$ is not too large compared to Z_0.

1. We first show that for any $\bar{Z}$, there is a $\bar{Z}_0$, with $\bar{Z}_0 < \bar{Z}$, such that if $Z_0 > \bar{Z}_0$, regulation slows extraction down.

   - Assume that $Q_0 = \infty$ and there is no backstop. The constrained price path is $p_t = -\mu_\infty e^{(r+\alpha)t}$ until p_t reaches $\bar{p}$ at date T^Z_∞ when the stock of pollution reaches $\bar{Z}$. From this date T^Z_∞, the price equals $\bar{p}$, we call this price path the infinite price path. Define Q^Z_∞ the total amount of the non renewable resource consumed in this case between dates 0 and date T^Z_∞. This date $T^Z_\infty(Z_0)$ is decreasing with Z_0 and $Q^Z_\infty(Z_0)$ is decreasing in Z_0. Indeed (μ_∞, T^Z_∞) are implicitly defined by:

$$e^{-\alpha T^Z_\infty}(Z_0 + \int_0^{T^Z_\infty} e^{\alpha t} x(p_t)dt) = \bar{Z} \tag{10}$$

$$-\mu e^{(r+\alpha)T^Z_\infty} = \bar{p} \tag{11}$$

     Differentiating wrt Z_0, we get that[8] : $dT^Z_\infty < 0$. Then[9] $dQ^Z_\infty < 0$ and[10]$dQ^Z_\infty - dT^Z_\infty \bar{x} < 0$

   - Consider t_1 and λ solutions of

$$\lambda e^{rt_1} = \bar{p} \tag{12}$$

$$\int_0^{t_1} x(\lambda e^{rt})dt = Q^Z_\infty + (t_1 - T^Z_\infty)\bar{x} \tag{13}$$

     $\int_0^{t_1} x(\bar{p}e^{-r(t_1-t)})dt - t_1\bar{x}$ is continuous and increasing with t_1, it is equal to zero at $t_1 = 0$ and tends to $+\infty$ when t_1 tends to $+\infty$. So that we can find t_1 that solves the equation. It cannot[11] be the case that $t_1 \leq T^Z_\infty$. If t_1 and λ are solutions of the above system, the quantity consumed on the infinite price path between 0 and t_1 is the same as the quantity consumed on the Hotelling price path ending at date t_1 at price $\bar{p}$. If $Z_0 = \bar{Z}$, this quantity is zero.

   - The date t_1 decreases with Z_0. Indeed $\int_0^{t_1} x(\bar{p}e^{-r(t_1-t)})dt - t_1\bar{x}$ is increasing with t_1, whereas $Q^Z_\infty - T^Z_\infty\bar{x}$ is decreasing with Z_0, so that t_1 is continuous and decreasing with Z_0. If $Z_0 = \bar{Z}$, then $t_1 = 0$. For Z_0 high enough, it is then the case that $-e^{rt_1}c'(t_1) \geq (u(D(q)) - qD(q)) - (u(\bar{x}) - \bar{p}\bar{x})$. If this is satisfied, then for $Q^* = Q^Z_\infty + (t_1 - T^Z_\infty)\bar{x}$, Q^* is sufficiently small so that the final optimal unconstrained price for Q^* is above $\bar{p}$. . If it is the case, then we can show that for all Q such that the regulation is binding, regulation postpones extraction.

   - If final price in the unconstrained case for arrival date $\tilde{T}$ is above $\bar{p}$ then, as the scarcity rent is lowered by the constraint, the final constrained price is less than the final unconstrained price for this arrival date. If final price in the unconstrained case for arrival date $\tilde{T}$ is below or equal to $\bar{p}$, then $Q > Q^*$. We first show that innovation arrives necessarily after date t_1 if oil is exhausted in the constrained case. Assume that innovation arrives at date t_1 defined above at final constrained price less or equal to $\bar{p}$, we show that it is not possible to exhaust more than Q^* under the constraint at date t_1. Indeed, if $\lambda > 0$ then $-\mu \leq -\mu_\infty$. But the infinite price path and the t_1 price path must cross once before date T^Z_∞ (otherwise if the price path remains below the infinite price path before T^Z_∞ the ceiling constraint is not satisfied, and if it is above always, then less than Q^* is consumed). Between 0 and t_1, it is the case that $\int_0^{t_1} e^{\alpha t}x_t dt \leq \int_0^{t_1} e^{\alpha t}x^\infty_t dt = \bar{Z}$. As prices cross once, there exists θ, such that for $t < \theta$, the infinite price is below and for $t > \theta_1$, infinite price is above. So that $\int_0^{\theta} e^{\alpha t}(x^\infty_t - x_t)dt \geq \int_\theta^{t_1} e^{\alpha t}(x_t - x^\infty_t)dt \geq 0$, so that $e^{\alpha\theta}\int_0^{\theta}(x^\infty_t - x_t)dt > \int_0^{\theta} e^{\alpha t}(x_t - x^\infty_t)dt \geq \int_\theta^{t_1} e^{\alpha t}(x_t - x^\infty_t)dt > e^{\alpha\theta}\int_\theta^{t_1}(x_t - x^\infty_t)dt$. So that more is consumed on the infinite price path between 0 and T^Z_∞. As a result at date t_1 oil cannot be exhausted for $Q \geq Q^*$, so that $T > t_1$ necessarily. If $T > t_1$, at date t_1, as $Q > Q^*$ and oil is exhausted at price below $\bar{p}$, then unconstrained price is

[8] We use that $dT^Z_\infty = \frac{e^{-\alpha T^Z_\infty}}{-\mu(r+\alpha)\int_0^{T^Z_\infty} e^{(r+2\alpha)t}x'(p_t)dt} dZ_0$

[9] As $\int_0^{T^{Z\infty}} x(p_t)dt = Q^Z_\infty$, if Z_0 increases the shadow cost increases everywhere.

[10] $dQ^Z_\infty - dT^Z_\infty\bar{x} = -d\mu\int_0^{T^Z_\infty} x'(p_t)e^{(r+\alpha)t}dt = -\frac{\int_0^{T^Z_\infty} x'(p_t)e^{(r+\alpha)t}dt}{\int_0^{T^Z_\infty} x'(p_t)e^{(r+2\alpha)t}dt} e^{-\alpha T^Z_\infty}dZ_0 < 0.$

[11] Assume that it is the case, then $\lambda > -\mu_\infty$ and the quantity consumed between 0 and t_1 is less than the quantity consumed between these two dates on the infinite price path. But as the price on the infinite price path is below $\bar{p}$ between t_1 and T^Z_∞, and that the total quantity consumed on this price path ending at date T^Z_∞ is Q^Z_∞, then the quantity consumed between 0 and t_1 on the infinite price path is strictly less than $Q^Z_\infty - (T^Z_\infty - t_1)\bar{x} = Q^Z_\infty + (t_1 - T^Z_\infty)\bar{x}$. So that the quantity consumed between 0 and t_1 on the λe^{rt} price path is also strictly less than $Q^Z_\infty + (t_1 - T^Z_\infty)\bar{x}$, which contradicts the fact that λ, t_1 solve the above system of equation.

below $\bar{p}$. Then the quantity consumed between 0 and t_1 is more than Q^*. On the other hand, on the constrained price path, we have seen that the quantity consumed increase with Q, so that between 0 and t_1, less is consumed on the constrained price path than if Q is infinite, *i.e*, less than Q^* is consumed. But if the constrained price at t_1 is $\bar{p}$, then less is consumed also on the constrained path between t_1 and $\tilde{T}$ than on the unconstrained price path, which contradicts the hypothesis that oil is exhausted at $\tilde{T}$ in both cases.

2. Similarly, we show that for any initial stock of pollution Z_0, there is a $\bar{Z}^* > Z_0$ such that if $\bar{Z} \leq \bar{Z}^*$, regulation slows extraction down. Take $(Q_\infty, T_\infty, \lambda, t_1)$ defined in Eqs. 12, 13, 10 and 11. It is straightforward that λ decreases with $\bar{Z}$. Call $t_2(\bar{Z})$ such that $\lambda e^{rt_2} = \bar{p}_0$. For $\bar{Z} = Z_0$, $t_2 = t_1 = 0$. The date t_2 is continuous and increasing with $\bar{Z}$. We can find t_2^* such that, for all $T \leq t_2^*$, $-e^{rT} c^{'}(T) > u(D(q)) - qD(q) - (u(\bar{x}_0) - \bar{p}_0 \bar{x}_0)$. Denote $\bar{Z}^*$ the value of the ceiling corresponding to t_2^*. Then $\bar{Z}^* < Z_0$ as $t_2^* > 0$. For all value of the ceiling such that $\bar{Z}^* < \bar{Z} < Z_0$, the optimal unconstrained final price for $Q_0 = Q_\infty + (t_1 - T_\infty)\alpha \bar{Z}$ is above $\bar{p}$. The end of the demonstration is the same as above.

3. Remark: On can find a backstop cost q^* such that assumption 4 is always true for all $(Z_0, \bar{Z})$ satisfying $D(q) > \alpha \bar{Z} > \alpha \bar{Z}_0$, as long as $q > q^*$.

4. Calibration: Using data on price elasticity in Chakravorty, Magné & Moreaux (2006), data on sectoral energy demands and CO_2 content of fossil fuel in Coulomb & Henriet (2010). We compute Q^* defined above. We find that if the backstop was to arrive at price below $\bar{p}$ for this Q^*, it would be necessary that the backstop arrives before 7.8 years, which is highly unlikely.

D Marginal cost of delaying innovation with a low ceiling

For $T \leq t_1$, the marginal change in surplus is zero. If innovation arrives at date T between date t_1 and date t_2, the price path is such that the ceiling is reached exactly at date T, the scarcity rent λ (for ease of notation, we denote $\lambda \equiv \lambda_0^{*T}$ etc) the shadow cost of pollution $-\mu$, exhaustion date $h(T)$ satisfy:

$$e^{-\alpha T}(Z_0 + \int_0^T e^{\alpha t} x_t dt) = \bar{Z} \tag{14}$$

$$\int_0^T x_t dt + (h(T) - T)\alpha \bar{Z} = Q \tag{15}$$

$$\lambda e^{rh(T)} = q \tag{16}$$

And the surplus can be written:

$$\int_0^T u(x_t)e^{-rt}dt + \int_T^{h(T)} (u(D(q)) - q(D(q) - \bar{x})e^{-rt}dt + \int_{h(T)}^{\infty} (u(D(q)) - qD(q))e^{-rt}dt$$

The marginal cost of delaying innovation, expressed at date T, is:

$$N(T) = -e^{rT}\frac{\partial}{\partial T}\Big(\int_0^T u(x_t)e^{-rt}dt + \int_T^{h(T)} (u(D(q)) - q(D(q) - \bar{x})e^{-rt}dt + \int_{h(T)}^{\infty} (u(D(q)) - qD(q))e^{-rt}dt\Big) \tag{17}$$

Developing equation 17:

$$\begin{aligned} & e^{rT}\Big(- u(x_T)e^{-rT} - \int_0^T p_t \frac{\partial x_t}{\partial T} e^{-rt}dt - \Big(\frac{\partial h(T)}{\partial T} e^{-rh(T)} + e^{-rT}\Big)(u(D(q)) - q(D(q) - \bar{x})) \\ & + \frac{\partial h(T)}{\partial T} e^{-rh(T)}(u(D(q)) - qD(q))\Big) \\ = \; & -u(x_T) - e^{rT}\int_0^T p_t \frac{\partial x_t}{\partial T} e^{-rt}dt - \frac{\partial h(T)}{\partial T} e^{r(T-h(T))} q\bar{x} + (u(D(q)) - q(D(q) - \bar{x})) \end{aligned}$$

And we know that between the dates 0 et T, $p_t = (\lambda - \mu e^{\alpha t})e^{rt}$,then

$$\int_0^T p_t \frac{\partial x_t}{\partial T} e^{-rt}dt = \lambda \int_0^T \frac{\partial x_t}{\partial T} - \int_0^T \mu \frac{\partial x_t}{\partial T} e^{\alpha t}dt \tag{18}$$

Differentiating Eq.14, we obtain :

$$-\alpha \bar{Z} + x_T + e^{-\alpha T}\int_0^T e^{\alpha t}\frac{\partial x_t}{\partial T} = 0$$

so that :

$$\int_0^T e^{\alpha t}\frac{\partial x_t}{\partial T} = (\alpha\bar{Z} - x_T)e^{\alpha T}$$

Differentiating Eq.15, we obtain :

$$\int_0^T \frac{\partial x_t}{\partial T} = (\alpha\bar{Z} - x_T) - \frac{\partial h(T)}{\partial T}\alpha\bar{Z}$$

so that Eq.18 can be rewritten:

$$\int_0^T p_t\frac{\partial x_t}{\partial T}e^{-rt}dt = \lambda(\alpha\bar{Z} - x_T) - \mu(\alpha\bar{Z} - x_T)e^{\alpha T} - \lambda\frac{\partial h(T)}{\partial T}\bar{x} \tag{19}$$

Moreover, using Eq.16 and the fact that $p_T = \lambda e^{rT} - \mu e^{(r+\alpha)T}$, and $\bar{x} = \alpha\bar{Z}$:

$$\int_0^T p_t\frac{\partial x_t}{\partial T}e^{-rt}dt = (\bar{x} - x_T)p_T e^{-rT} - qe^{-rh(T)}\frac{\partial h(T)}{\partial T}\bar{x} \tag{20}$$

Delaying the arrival date of innovation induces a marginal loss in welfare, at the arrival date :

$$N(T) = -\Big(u(x_T) - x_T)p_T\Big) - \bar{x}(p_T - q) + (u(D(q)) - qD(q)) \tag{21}$$

and we define

$$N(T) = (u(D(q)) - qD(q)) - \Big(u(x_T) - x_T p_T\Big) - \bar{x}(p_T - q) \tag{22}$$

The greater is T, the greater the price at the innovation date, and the greater the surplus loss.
If innovation occurs between t_2 et t_3, the price is $p_t = \lambda e^{rt} - \mu e^{(r+\alpha)t}$ until date t_2, $\bar{p}$ between t_2 and T, oil and the backstop are used jointly at price q until date t_3, when oil is exhausted, the backstop is used alone at price q. During this period, λ, μ, the date at which the ceiling is reached t_2 and exhaustion date t_3 do not depend on T. The surplus writes:

$$\int_0^{t_2} u(x_t)e^{-rt}dt + \int_{t_2}^T (u(\bar{x})e^{-rt}dt + \int_T^{t_3}(u(D(q)) - q(D(q) - \bar{x})e^{-rt}dt + \int_{t_3}^\infty (u(D(q)) - qD(q))e^{-rt}dt \tag{23}$$

Deriving Eq.23, we find that delaying the innovation date induces cost, expressed at date T :

$$N(T) = -(u(\bar{x}) - q\bar{x}) + (u(D(q)) - q(D(q)) \tag{24}$$

If innovation occurs between t_3 and t_4, then the price path is first $p_t = \lambda e^{rt} - \mu e^{(r+\alpha)t}$ until the ceiling is reached at price $\bar{p}$, then during the ceiling period, price equals $\bar{p}$ until exhaustion. At exhaustion date, the scarcity rent is above q and below $\bar{p}$, it is increasing with T. The scarcity rent λ, the shadow cost $-\mu$, and the date $g(T)$ when the ceiling is reached satisfy:

$$e^{-\alpha g(T)}(Z_0 + \int_0^{g(T)} e^{\alpha t}x_t dt) = \bar{Z} \tag{25}$$

$$\int_0^{g(T)} x_t dt + (T - g(T))\alpha\bar{Z} = Q \tag{26}$$

$$\lambda e^{rT} - \mu e^{(r+\alpha)T} = \bar{p} \tag{27}$$

The surplus writes:

$$\int_0^{g(T)} u(x_t)e^{-rt}dt + \int_{g(T)}^T u(\bar{x})e^{-rt}dt + \int_T^\infty (u(D(q)) - qD(q))e^{-rt}dt$$

The marginal cost of delaying innovation, expressed at date T, writes:

$$\begin{aligned} N(T) &= -e^{rT}\frac{\partial}{\partial T}\Big(\int_0^{g(T)} u(x_t)e^{-rt}dt + \int_{g(T)}^T u(\bar{x})e^{-rt}dt + \int_T^\infty (u(D(q)) - qD(q))e^{-rt}dt\Big) \\ &= u(D(q)) - qD(q) - e^{rT}\int_0^{g(T)} p_t\frac{\partial x_t}{\partial p_t}e^{-rt}dt - u(\bar{x}) \end{aligned}$$

Using that $p_t = \lambda e^{rt} - \mu e^{(r+\alpha)t}$

$$\int_0^{g(T)} p_t\frac{\partial x_t}{\partial p_t}e^{-rt}dt = \lambda\int_0^{g(T)}\frac{\partial x_t}{\partial p_t}dt - \mu\int_0^{g(T)}\frac{\partial x_t}{\partial p_t}e^{\alpha t}dt$$

Deriving Eq.26:

$$\lambda \int_0^{g(T)} \frac{\partial x_t}{\partial p_t} dt = -\lambda \bar{x}$$

Deriving Eq.25:

$$\lambda \int_0^{g(T)} e^{\alpha t} \frac{\partial x_t}{\partial p_t} dt = 0$$

So that:

$$N(T) \quad = \quad u(D(q)) - qD(q) - (u(\bar{x}) - \lambda e^{rT} \bar{x})$$

If innovation occurs after t_4, oil price follows a Hotelling path at arrival date, and oil is exhausted at this date. The surplus can be written:

$$\int_0^T u(x_t) e^{-rt} + \int_T^\infty (u(D(q)) - qD(q)) e^{-rt} dt$$

So that the marginal cost of delaying innovation, at arrival date, writes:

$$N(T) = (u(D(q)) - qD(q)) - e^{rT} u(x_T) e^{-rT} - e^{rT} \int_0^T p_t \frac{\partial x_t}{\partial T} dt$$

using that $p_t = \lambda e^{rt}$ after date T:

$$N(T) = (u(D(q)) - qD(q)) - u(x_T) - \lambda e^{rT} \int_0^T \frac{\partial x_t}{\partial T} dt$$

Oil is exhausted at date T, so that $\int_0^T x_t = Q$, implying that $\int_0^T \frac{\partial x_t}{\partial T} dt = -x_T$. The marginal cost of delaying innovation, expressed at the arrival date, writes:

$$N(T) = \Big(u(D(q))) - qD(q) \Big) - \Big(u(x_T) - p_T x_T \Big)$$

E Proof of Proposition 5

In order to demonstrate Proposition 5, we adopt a recursive reasoning. We show first that for all Q_0^1 such that innovation arrives at date T_1 after $t_2(Q_0^1)$, then for any $Q_0^2 > Q_0^1$ innovation arrives at date $T_2 > T_1$, satisfying $T_2 > t_2(Q_0^2)$. Denote $N_1(T)$ the marginal cost of delaying innovation for $Q_0 = Q_0^1$ and $N_2(T)$ the marginal cost of delaying innovation for $Q_0 = Q_0^2$. Denote $\lambda_0^T(Q_0^i)$ the initial scarcity rent for arrival date T and $Q_0 = Q_0^i$. Notice first that $T_1 > t_2(Q_0^2)$ as $t_2(Q_0)$ is decreasing with Q_0 (see appendix B.4).

- If T_1 satisfies $t_2(Q_0^1) \le T_1 \le t_3(Q_0^1)$, then $t_2(Q_0^2) \le T_1 \le t_3(Q_0^2)$, so $N_1(T_1) = N_2(T_1)$, as $t_3(Q_0)$ increases with Q_0 (see appendix B.4).
- If T_1 satisfies $t_3(Q_0^1) < T_1 \le \min(t_3(Q_0^2), t_4(Q_0^1))$, then $N_1(T_1) = u(D(q)) - qD(q) - (u(\bar{x}) - \lambda_0^{T_1}(Q_0^1) e^{rT_1} \bar{x}) \ge u(D(q)) - qD(q) - (u(\bar{x}) - q\bar{x}) = N_2(T_1)$. If $t_3(Q_0^2) < t_4(Q_0^1)$, and innovation arrives between $t_3(Q_0^2)$ and $t_4(Q_0^1)$, as the final scarcity rent λ_T^T decreases with Q_0 (see appendix B.6), then it is the case that $N_1(T_1) = u(D(q)) - qD(q) - (u(\bar{x}) - \lambda_0^{T_1}(Q_1) e^{rT_1} \bar{x}) \ge u(D(q)) - qD(q) - (u(\bar{x}) - \lambda_0^{T_1}(Q_2)(T_1) e^{rT_1} e^{rT_1} \bar{x}) = N_2(T_1)$. If $t_3(Q_0^2) > t_4(Q_0^1)$, and innovation arrives between $t_4(Q_0^1)$ and $t_3(Q_0^2)$,$N_1(T_1) \ge u(D(q)) - qD(q) - (u(\bar{x}) - \bar{p}\bar{x}) \ge u(D(q)) - qD(q) - (u(\bar{x}) - q\bar{x}) = N_2(T_1)$.
- Between $\max(t_3(Q_0^2), t_4(Q_0^1))$ and $t_4(Q_0^1)$, $N_1(T_1) \ge u(D(q)) - qD(q) - (u(\bar{x}) - \bar{p}\bar{x}) \ge u(D(q)) - qD(q) - (u(\bar{x}) - \lambda_0^{T_1}(Q_2)(T_1) e^{rT_1} \bar{x}) = N_2(T_1)$.
- After $t_4(Q_0^1)$, the bigger Q_0 the lower the scarcity rent and so $N_1(T_1) \ge N_2(T_2)$. So that, at T_1, $-e^{rT_1} C'(T_1) \ge N_2(T_1)$, so that optimal arrival for Q_0^2 is after T_1. We find that $T_2 > T_1 > t_2(Q_0^2)$.

We show now that for $Q_{\min}$, optimal arrival is after $t_2(Q_{\min})$. We know that final price $p_{\min}$ at optimal arrival date for $Q_0 = Q_{\min}$ satisfies $p_{\min} > \bar{p}$ (see Appendix C). We call θ_2 the date at which price $\bar{p}$ is reached along the optimal price path for $Q_0 = Q_{\min}$. As for $t > \theta_2$ $p_t > \bar{p}$, the stock of pollution in the atmosphere declines, for $t > \theta_2$ whereas while $t \le \theta_2$, the stock of pollution in the atmosphere increases, the ceiling is reached exactly at date θ_2. By definition, $t_2(Q_{\min})$ is the lowest possible date such that the ceiling is reached, when $Q_0 = Q_{\min}$, at price $\bar{p}$. So that, $\theta_2 < t_2(Q_{\min})$ and as final price is above $p_{\min} > \bar{p}$, optimal arrival date is at $T_{\min} \ge \theta_2 > t_2(Q)$. So that for $Q_0 = Q_{\min}$, optimal arrival date is after $t_2(Q_{\min})$, so that by recursive reasoning, optimal innovation for increases with Q_0.

Corollary: Arrival date is always after $t_2(Q_0)$.

F Proof of proposition 6

We first show that, if there exists a Q^l such that regulation advances innovation, then for all $Q > Q^l$, regulation also advances innovation. We assume that there is a Q^l such that at optimal unconstrained innovation date T^l, $N(Q^l, T^l) > \tilde{N}(Q^l, T^l)$. Notice first that, if $N(Q^l, T^l) > \tilde{N}(Q^l, T^l)$ then oil is not exhausted at this date. If it was the case, then it would be the case that $\tilde{p}^{*T^l}_{T^l} > p^{*T^l}_{T^l}$, which is not possible if $N(Q^l, T^l) > \tilde{N}(Q^l, T^l)$. Indeed, one can verify that $N(Q^l, T^l) \leq (u(D(q)) - qD(q)) - (u(x^{*T^l}_{T^l}) - p^{*T^l}_{T^l} x^{*T^l}_{T^l})$, so that if $\tilde{p}^{*T^l}_{T^l} > p^{*T^l}_{T^l}$, then $N(Q^l, T^l) \leq \tilde{N}(Q^l, T^l)$. So: if innovation is advanced for a given Q^l, it must be the case that oil is not exhausted at the optimal unconstrained innovation date in the constrained case, and as a result, that $T^l < t_3(Q^l)$. We have shown that $t_3(Q_0)$ increases with Q_0 (see B.4), so that $\forall Q_0 > Q^l, T^l < t_3(Q_0)$. let denote T^* the optimal constrained innovation date for $Q_0 > Q^l$. We know that, for $Q_0 > Q^l$, $t_1(Q_0) < t_1(Q^l)$(see B.3), then if the T^* is between $t_1(Q_0)$ and $t_1(Q^l)$, then $N(T^*, Q_0) > N(T^*, Q^l) = 0$. So that $T^* < T^l$. We differentiate $u(x^{*T}_{T^*}) - p^{*T^*}_{T^*} x^{*T^*}_{T^*} + \bar{x}(p^{*T^*}_{T^*} - q)$ wrt Q_0, we obtain $(\bar{x} - x^{*T^*}_{T^*})\frac{\partial p^{*T^*}_{T}}{\partial Q_0}$. But we know that $\frac{\partial p^{*T^*}_{T^*}}{\partial Q_0} > 0$ (see appendix B.5), so that if T^* is between dates $t_1(Q^l)$ and $t_2(Q_0)$ ($t_2(Q_0) < t_2(Q^l)$ see B.3), then $N(T^*, Q_0) > N(T^*, Q^l)$, so that $T^* < T^l$. Between dates $t_2(Q_0)$ and $t_2(Q^l)$, it is straightforward that $N(T^*, Q_0) > N(T^*, Q^l)$ as, $N(T^*, Q_0) \geq N(t_2(Q_0), Q_0) = N(t_2(Q^l), Q^l) > N(T^*, Q^l)$, so that $T^* < T^l$. If T^* is between $t_2(Q^l)$ and $t_3(Q^l)$, then $N(T^*, Q_0) = N(T^*, Q^l)$, so that $T^* = T^l$. So that it is always the case $T^* \leq T^l$. On the other hand, if $Q_0 > Q^l$, we know that $\tilde{T} > T^l$, so that for $Q_0 > Q^l$, we have that $T^* < \tilde{T}$
So that the implication :

$$\left\{\begin{array}{ccc} Q & > & Q^l \\ N(Q^l, T^l) & > & \tilde{N}(Q^l, T^l) \\ \tilde{N}(Q^l, T^l) & = & -e^{rT^l} c^{'}(T^l) \\ \tilde{N}(Q, \tilde{T}) & = & -e^{rT} c^{'}(\tilde{T}) \end{array}\right. \quad \Rightarrow \quad N(Q, \tilde{T}) > \tilde{N}(Q, \tilde{T})$$

is true.

We show now that there exists an initial stock Q^l of the non renewable resource such that, at the optimum, innovation arrives earlier in the constrained case than in the unconstrained. Consider the date T^l satisfying $(u(D(q)) - qD(q)) - (u(\bar{x}) - q\bar{x}) = -e^{rT^l} c^{'}(T^l)$. Take Q_1 such that innovation arrives strictly after $t_2(Q_{\min})$ in the unconstrained case, and such that $\tilde{N_1}(T^l) < -e^{rT^l} c^{'}(T^l)$ (($N_1(T^l)$ decreases with Q_0 and tends to 0). Then, as $t_2(Q_0)$ decreases with Q_0, optimal unconstrained arrival date $\tilde{T} > t_2(Q_1)$. We have then that $N_1(\tilde{T}) \geq (u(D(q)) - qD(q)) - (u(\bar{x}) - q\bar{x}) = -e^{rT^*} c^{'}(T^l) \geq \tilde{N_1}(T^l) \geq \tilde{N_1}(\tilde{T})$.

G Proof of Proposition 7

Take $\bar{Z}_1 < \bar{Z}^*$. According to appendix E, arrival date for $\bar{Z}^*$ is after $t_2(Q, \bar{Z}^*)$. Moreover, as we have assumed that regulation postpones innovation for $\bar{Z}^*$, then it must be the case that optimal arrival date for $\bar{Z}^*$ is prior to $t_3(Q, \bar{Z}^*)$. We can show now that t_3 increases when $\bar{Z}$ decreases. Indeed, using the set of equation defining t_3, we get that:

$$dt_3 = \frac{-d\bar{Z} e^{rt_3} \left(\int_0^{t_2} x^{'}_t e^{(r+\alpha)t} dt + \int_0^{t_2} x^{'}_t e^{(r+2\alpha)t} dt (t_3 - t_2)\alpha \right)}{\bar{x} e^{rt_3} int_0^{t_2} x'(t) e^{(r+2\alpha)t} dt + r\lambda e^{rt_3} \left(\left(\int_0^{t_2} x^{'}_t e^{(r+\alpha)t} dt \right)^2 - \int_0^{t_2} x^{'}_t e^{(r+2\alpha)t} dt \int_0^{t_2} x^{'}_t e^{r} dt \right)}$$

So that we $d\bar{Z} < 0$, we get $dt_3 > 0$. If $dt_2 < 0$, as $u(\bar{x}^*) - q\bar{x}^* > u(\bar{x}) - q\bar{x}$, then it is straightforward that more stringent ceiling advances innovation. If $dt_2 > 0$, we find that $d\lambda < 0$ (cf $dt_3 > 0$) and then necessarily $-d\mu > 0$. As a result, the price at arrival date $t_2(\bar{Z}^*)$ in the case where the ceiling is $\bar{Z}_1$ is greater then $\bar{p}^*$. Then: $u(x_{t_2(\bar{Z}^*)}) - p_{t_2(\bar{Z}^*)} x_{t_2(\bar{Z}^*)} + \bar{x}_1(p_{t_2(\bar{Z}^*)} - q) = u(x_{t_2(\bar{Z}^*)}) - qx_{t_2(\bar{Z}^*)} - (p_{t_2(\bar{Z}^*)} - q)(x_{t_2(\bar{Z}^*)} - \bar{x}_1) \leq u(x_{t_2(\bar{Z}^*)}) - qx_{t_2(\bar{Z}^*)} \leq u(\bar{x}^*) - q\bar{x}^*$ As a result, if arrival date for $\bar{Z}^*$ is between t_2^* and $t_2(\bar{Z}_1)$, innovation is also advanced by a more stringent ceiling.

H Proof of Proposition 8

We first show that $t_2(Q)$ increases with q. Indeed, taking equations defining $t_2(Q)$ and differentiating wrt q, we get:

$$dt_2 = \frac{\bar{x}(e^{rt_2} \int_0^{t_2} e^{(r+\alpha)t} x^{'}(p_t)(e^{\alpha t_1} - e^{\alpha t}) dt dq}{(r\lambda e^{rt_2} + (r+\alpha)\mu e^{(r+\alpha)t_2}) \left(\bar{x} e^{r\theta_2} \int_0^{t_2} x'(e^{(r+2\alpha)t} dt + rq \left((\int_0^{t_2} x'(e^{(r+\alpha)t} dt))^2 - \int_0^{t_2} x'(e^{(r+2\alpha)t} dt \int_0^{t_2} x' e^{rt} dt \right) \right)}$$

Using the Cauchy Schwartz inequality, we get that $dt_2 > 0$. Innovation always arrives after t_2 (see corollary in E). We can show that $dt_3 > 0$, indeed, we get:

$$dt_3 = \frac{\bar{x}(\int_0^{t_2} e^{(r+\alpha)t} x^{'}(p_t)dt)dq}{(r\lambda e^{rt_2} + (r+\alpha)\mu e^{(r+\alpha)t_2})\left(\bar{x}e^{r\theta_2}\int_0^{t_2} x'(e^{(r+2\alpha)t}dt + rq\left((\int_0^{t_2} x'(e^{(r+\alpha)t}dt))^2 - \int_0^{t_2} x'(e^{(r+2\alpha)t}dt\int_0^{t_2} x' e^{rt}dt\right)\right)}$$

It is clear that $dt_4 = 0$. Consider $q_1 < q_2$, at date $t_2(q_2)$, $N(t_2(q_2), q_1) = u(D(q_1)) - q_1 D(q_1) - (u(\bar{x}) - q_1\bar{x}) > u(D(q_2)) - q_2 D(q_2) - (u(\bar{x}) - q_2\bar{x}) = N(t_2(q_2), q_2)$. So that $N(t_2(q_2), q_1) > N(t_2(q_2), q_2)$. For all T satisfying $t_3(q_2) > T > t_2(q_2)$, $N(T, q_1) = N(t_2(q_2), q_1) > N(t_2(q_2), q_2) = N(T, q_2)$. Between $t_3(q_1)$ and $t_3(q_2)$, $N(T, q_1) > u(D(q_1)) - q_1 D(q_1) - (u(\bar{x}) - q_1\bar{x}) > u(D(q_2)) - q_2 D(q_2) - (u(\bar{x}) - q_2\bar{x}) = N(t_2(q_2), q_2)$. After $t_3(q_2)$, the constrained price path is the same for both cases but as $u(D(q_1)) - q_1 D(q_1) > u(D(q_2)) - q_2 D(q_2)$, $N(T, q_1) > N(t_2(q_2), q_2)$. So that the higher q (below $\bar{p}$), the later innovation.

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