Climate Change and Clean Capital Accumulation: When To Invest?

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Abstract

Capturing CO_2 from fossil fuels burning using Carbon Capture and Storage (CCS) technology or producing energy based on low-carbon renewables require investments in specific forms of long-life clean capital. The present paper focuses on the optimal accumulation of clean capital when the carbon-emitting resource, a fossil fuel, is exhaustible and the regulation takes the form of a cap over the CO_2 concentration. By impacting differently the marginal value dynamics of CCS systems and renewables power plants, the exhaustibility of the carbon-emitting resource leads to different optimal investments paths. Because CCS systems are useless in the long run, it may be profitable to deploy them before the carbon cap is reached if their depreciation is slow or the carbon-emitting resource is scarce enough. On the contrary, it is never optimal to invest in renewables power plants before the CO_2 concentration reaches the carbon cap, with constant investment costs. Investments in renewables power plants start to maintain the consumption flow at a level determined by their characteristics of cost and duration and by the energy demand. When the CO_2 concentration is at the ceiling, energy may be provided at the ceiling by an energy mix based on fossil fuels with CCS and renewables. Introducing a pool of CCS technologies and renewables, differentiated only by their depreciation rate and their constant capital investment cost, we find that only one kind of renewables is used along the optimal path, whereas several CCS technologies may be used with long-life technologies in first place.

Keywords: Dynamic Models, Climate change, Externalities, Nonrenewable Resources, Carbon Capture, Renewables, Investments, Energy Markets, Capital Accumulation.

JEL Classification: Q31, Q38, Q41, Q54, Q55

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1 Introduction

In 2009, CO_2 from energy production represents 65% of greenhouse gas emissions and fossil fuels account for 81% of the world energy supply (IEA [2011]). Two energy options are expected to play an important role in the mitigation of climate change: low-carbon alternative energy sources that are substitutes to fossil fuels (solar energy, wind energy, hydropower, biomass, biofuels, nuclear power etc.) or fossil fuels used in association with carbon capture (biological, oceanic, geological) options[1]. To stabilize in 2030 the greenhouse gas concentration to 450 ppm CO2-eq, renewables and nuclear should respectively account for 37% and 18% of electricity production; by contrast, in the IEA-reference scenario, fossil fuels (coal, oil and gas) will account for two-thirds (IEA [2009b]). Among capture options, Carbon capture and storage technology (CCS) is one of particular interest. The rationale of Carbon Capture and Storage is to enable the use of fossil fuels without drastically increasing anthropic carbon dioxide emissions by filtering CO_2 form the emission flow and storing it under the ground. It takes an important role in the range of low-cost mitigation options since without CCS, the overall costs of reducing CO_2 emissions to 2005 levels by 2050 are expected to increase by 70% (IEA [2009a]).

Deploying renewables and carbon capture requires investments in specific power plants (hydropower plants, nuclear power plants, geothermy systems for very long life types of capital) and capture and storage systems, that are expected to last over several decades. Capital investment costs drive the most important part of the costs of renewables: the marginal cost of producing energy is relatively low as long as installed power capacities are not saturated. Current investments costs estimates for large hydropower station, offshore wind, biomass combustion for power (solid fuels) and solar photovoltaic are respectively 1000-5500, 2200-3000, 2000-3000 and 5000-6500 USD/kW, while production costs estimates range between 0.3-1.2, 0.8-1.2, 0.6-1.9 and 2-8 USD/kWh (IEA [2009b]). Carbon capture involves a costly system of filters, pipelines and safe storage reservoirs with large uncertainty over the costs of commercial plants. CCS technology is already implemented in large-scale commercial or pilot plants in Algeria (In Salah), Australia (Otway, Munmorah), Canada (Weyburn-Midale, Fort Nelson CCS project) China (Jilin, Gaobeidian, Shindongkou), Germnay (Schwarze Pumpe), France (Lacq, Le Havre), Italy (Brindisi), Netherlands (K12-B, Willem Alexander), Norway (Snohvit, Sleipner, Technology Centre Mongstad), Spain (Puertollano), Sweden (Karlshamn Field Pilot), UK (Don Valley, SCCS), the USA (Plant Barry, LaBarge, Koch Nitrogen Plant, Century Gas Processing, Val Verde CO2 pipeline, Mountaineer Pilot).

The previous Hotelling-based papers with a carbon cap regulation (Chakravorty et al. [2006], Lafforgue et al. [2007]) assume that CCS and renewables can be used without prior investments, and so the exhaustibility of the carbon-emitting resource does not play a significant role to determine when mitigation should start i.e *before* or *when* the CO_2 concentration hits the carbon ceiling. As long as solar energy and carbon capture can be used at a constant cost without constraint, these options are never used before the CO_2 concentration reaches the carbon cap. Late abatement efforts are preferred in order to benefit from the natural atmospheric dilution of carbon and from the decreasing discounted costs of renewables and CCS.

This theoretical paper focuses on the optimal timing of investments in CCS systems and renewables plants and the role played by the scarcity of the exhaustible carbon-emitting resource. To get tractable results and compare

[1] Energy efficiency improvements i.e reducing the energy needs and so the carbon emissions for a given level of utility, are not considered in that paper.

them with those found in previous literature, the analysis is based on a standard Hotelling-like model, in which utility comes from two sources of energy: a carbon-emitting exhaustible resource (a fossil fuel) and a carbon-free renewable resource (hereafter solar energy). The regulation takes the form of a carbon cap over the atmospheric carbon stock. This threshold can be considered as an exogenous constraint, for instance stemming from a Kyoto-like Protocol, or as the first-best carbon policy. The second interpretation of the ceiling is correct if the damage function can be approximated by a binary-convex damage function with nil marginal damage when the CO_2 concentration is kept under or equal to the threshold and infinite otherwise. Dealing with the pollution problem requires to properly distinguish three main sorts of capital: dirty productive capital, clean productive capital (renewables) and non-productive capital that allows to clean the environment (CCS systems). CCS systems and solar plants are built trough time by investing in specific forms of capital at a constant unitary cost regardless of the amount of investment, thus capacities constraints are not exogenous but result from investments decisions that drive capital accumulation. Solar plants are assumed to be more expensive than CCS systems. To simplify, we exclude the dirty capital (extraction and fossil fuels burning plants) from the analysis. It follows that we do not make any distinction between CCS retrofitting of the existing coal fleet and new power plants including CCS systems.

Even in the simple framework of our model, optimal paths of fossil fuels extraction and investments in CCS systems and solar plants are complex and various. The long duration of "clean capital" (CCS systems or solar plants) in the economy is not a sufficient condition for early mitigation effort. The key result is that the exhaustibility of the polluting resource may lead to drastically different optimal investments rules for clean productive capital and clean unproductive capital. With constant investment costs, investments in solar plants should not start before the carbon ceiling is reached regardless of their depreciation rate. On the contrary, investments in CCS systems should take place before the carbon ceiling if they depreciate slowly enough or if the carbon-emitting resource is scarce enough or the ceiling high enough; in spite of the natural dilution of CO_2 in the atmosphere and the decreasing discounted investment costs, that tend to postpone mitigation efforts. The optimal date of starting investments depends on the strength of these contradictory forces: early CCS investments allow to capture more emissions whose costs are globally increasing then decreasing, but a part of emissions would have been naturally absorbed and the unitary cost of investment is decreasing trough time. Whereas the investment costs dynamics of both options is similar, their marginal benefits dynamics differ as a consequence of the exhaustibility of the polluting resource. The marginal value of solar plants is increasing due to the overall decrease of the energy demand driven by the exhaustibility of the fossil fuel. In the short term, the value of CCS systems may increase due to the increasing cost of pollution they allow to neutralize. However, this value is necessarily decreasing in the long term since CCS systems would become useless when the fossil fuel is exhausted and may start to decrease early if the quantity of captured emissions is strongly decreasing trough time (low-depreciation rate). Investments in solar plants (after the fossil fuel exhaustion or at the ceiling if solar plants are cheap enough) help to maintain the consumption flow to a specific level, determined by the characteristics of the solar energy supply and the energy demand. Both CCS and solar energy can be used at the same time when the CO_2 concentration is kept equal to the ceiling before the fossil fuel gets exhausted. Introducing increasing investments costs in CCS systems and solar plants tends to dilute investment efforts trough time to avoid costly massive investments. It may lead to invest in solar plants before the ceiling if their price is low enough, and necessarily leads to deploy solar plants before the fossil fuel is exhausted. The carbon tax exhibits a complex pattern. It increases before the CO_2 concentration reaches the carbon ceiling, then the carbon tax is set to keep non captured emissions just equal to natural absorption at the ceiling. As a consequence, at the ceiling the carbon tax may be decreasing only or constant and then inverse-U-shaped. We do not exclude that the carbon tax may exhibit a U-shape through time at the ceiling. With a pool of CCS technologies and renewables, differentiated only by their depreciation rate and their capital cost (still constant), only one renewable is used along the optimal path. By contrast, several types of CCS technologies might be used along the optimal path, long-life technologies in first place.

One could argue that fossil fuels are abundant enough (especially those, like coal, used for power generation) to make insignificant the effect of their exhaustibility on the dynamics of the long term benefit of CCS systems; as a consequence, our conclusions would be purely theoretical. A direct answer would be that a resource becomes *economically* exhausted before being *physically* exhausted in real life. The use of a resource stops when it becomes definitely more costly than its benefit or than an other resource for the same benefit. Considering local pollution due to coal extraction and coal burning, or increasing extraction costs supports the non physical exhaustion of coal.

The remainder of this paper is organized as follows. Section 2 contains a review of the previous related literature, Section 3 shows the social planner model we use and the first order conditions of optimality. Section 4 presents the optimal paths of extraction and investments in CCS systems and solar plants. Section 4 presents concluding remarks.

2 Review of Literature

This paper is at the intersection of the natural resources literature that examines the transition from exhaustible resources towards renewables and the literature on the optimal timing of carbon mitigation. To my knowledge, no paper has tried to define the optimal time-path of the energy use and the carbon-mitigation policy, taking account of the specific forms of capitals incorporated in low-carbon energy options and the exhaustibility of the carbon-emitting resource.

Extensions of the Hotelling model (Hotelling [1931]) include different features (increasing extraction costs, R&D in the backstop, capacities constraint etc.) to well describe the time-path of extraction of different resources. Studying the capital accumulation to extract an exhaustible mineral from a finite stock and extraction capacities that do no depreciate, Campbell [1980] finds that investments in extraction capacities should be done at the beginning of the time period due to the globally decreasing pattern of extraction. Holland [2003] finds that when extraction capacity is limited, the "least-cost first" extraction rule (Herfindahl [1967]), no more holds. Expensive resource deposit may be used first and may be exhausted before exhausting the low-cost deposit. Tsur and Zemel [2003] study optimal R&D in the backstop and transition to the backstop from exhaustible resources. Amigues et al. [2011] introduce adjustment costs in the transition to the backstop resource without extraction constraint. In this paper, the accumulation of capital for the backstop start whereas the exhaustible resource is not yet exhausted. The transition towards the backstop technology is only driven by the exhaustibility of the cheapest energy resource. If the exhaustible resource is abundant enough, the equilibrium path is composed of a first path where only the exhaustible resource is used, followed by a joint-use phase until the depletion of the exhaustible resource and finally a phase where only the renewable is used.

The other aspect of the paper concerns issues arisen by the carbon mitigation. The common argument for late mitigation efforts is that a delay in abatement would only slightly affect climate change damages that depend on accumulated emissions and mitigation costs could be decreasing trough time due to the joint effect of the natural atmospheric CO_2 absorption, discounting and technical progress in the low-carbon technologies. However, strong decreasing returns to scale in low-carbon energy options favor early and smooth mitigation to avoid costly drastic change towards low-carbon options. Capital accumulation with pollution problem have also been examined. Introducing the pollution problem requires to properly distinguish three main sorts of capital: dirty productive capital, clean productive capital and non-productive capital that allows to clean the environment. In Ploeg and Withagen [1991] pollution comes from production and investments allow to clean the environment: investments in clean productive capital or exhaustible resources are simply evoked and not connected together in their framework. show that long-life forms of "dirty" capital favor early and smooth mitigation to avoid costly drastic change towards low-carbon options and shutdowns of still productive "dirty" installations. Using an Hotelling model (Hotelling [1931]), some papers introduce the exhaustibility of the polluting resource in the carbon mitigation problem, but without considering the accumulation of specific forms of capitals incorporated in CCS and solar energy use. Chakravorty et al. [2006] examine CO_2 abatement from the stock in a Hotelling model with a carbon ceiling, considering stationary and non-stationary energy demands, with rare or abundant solar energy. In a similar frame, Lafforgue et al. [2007] focus on CCS technology and determine the optimal extraction and CO_2 capture with constrained and unconstrained storage capacities. If all emissions are capturable and no CCS systems have to be installed, it is never optimal to abate or to capture CO_2 before the ceiling binds regardless of the level of the costs and the size of storage reservoirs. Abating later is preferred in order to benefit from the natural free decay of the pollution stock and from the decreasing unitary discounted cost of capture. But as shown byAmigues et al. [2010] and Coulomb and Henriet [2010], when the flow of capturable emissions is constrained, starting CO_2 capture before the ceiling to slack the environmental constraint may be optimal. Without natural dilution, Farzin [1996] finds that the abatement option must be used while the ceiling does not bind. In all these papers, the exhaustibility of the polluting resource impacts the dynamics of extraction but does not interact with the optimal mitigation timing, since the exhaustibility does not modify the direct cost of a specific low-carbon option (solar or CCS). Without pollution problem, in a constant costs framework, the backstop is used only once the fossil fuel is exhausted. With a carbon ceiling, Chakravorty et al. [2006] show that solar energy is used only once the fossil fuel is exhausted or can be used when the CO_2 concentration is at the ceiling, if its price is low enough. With learning-by-doing in the backstop technology, solar energy can be used before the ceiling binds (Chakravorty et al. [2009]). Kama et al. [2009] propose a growth model with CCS with storage costs dedicated to avoid carbon leakage but without considering duration of clean capitals.

Our paper combines the different features of previous models: the capital accumulation and the exhaustibility of the polluting resource. By considering two types of clean capitals (productive capital like renewables power plants and non-productive clean capital like CCS systems) that depreciate trough time, we point out the role played by the scarcity of the exhaustible resource in the carbon mitigation. Capacities constraints are not exogenous but result from the investments decision that drive capital accumulation. Constraints over the solar energy flow or the carbon capture flow can be relaxed by investments. Investments are done at a constant current unitary cost regardless of the amount of investment.

3 The Social Planner Model

3.1 Assumptions and notations

By assumption, there is no improvement in energy efficiency, so energy can be used as a proxy for goods entering the utility function. Utility comes from energy consumption based on two perfectly substitutable primary energy sources: a carbon-free renewable energy source (solar energy), K_s and a polluting exhaustible energy source (a fossil fuel like coal), x: $u(x, K_s) = u(x + K_s)$. Problems of imperfect substitution between resources across sectors, intermittency of renewables, R&D etc. are not considered here. The utility function satisfies the standard regularity conditions ($u \epsilon C^2$, $u' > 0$, $u'' < 0$) and Inada conditions ($\lim_{g \to \infty} u'(g) = 0$ and $\lim_{g \to 0} u'(g) = +\infty$). Energy demand is stationary and writes $D(p_t)$, where p_t is the energy price. r stands for the constant social discount rate.

CCS (ccs) and solar energy (s) require specific forms of capitals to be used. For $i \epsilon \{ccs; s\}$, installed capital of option i available at time t is written $K_i(t)$. The unitary costs of capital i is constant and equals FC_i. For convenience, one unit of CCS systems captures instantaneously one unit of CO_2, and one unit of solar plants provides instantaneously one unit of solar energy. For the sake of simplicity and without any loss of generality, marginal costs of use of CCS and solar plants are assumed to be nil in the rest of the paper (see appendices for a rewriting of the problem with non-nil constant cost of use). With nil cost of use, installed solar plants are fully used, as well as installed CCS systems (as long as the flow of non-captured emissions is positive). Capacities are assumed to be perfectly divisible and depreciate at rate δ_i, $\delta_i < 1$; the extreme case where capital fully depreciate, $\delta_{ccs} = \delta_s = 1$, is studied in Chakravorty et al. [2006]. Writing $I_i(t)$, the investments in capital i for $i \epsilon \{ccs; s\}$, the capital i accumulation follows $\dot{K}_i(t) = -\delta_i K_i(t) + I_i(t)$. Initially, there is no CCS systems or solar plants ($K_i(0) = 0$). CCS systems are assumed to be cheap enough to be used.

Geological reservoirs where CO_2 is stored are assumed to be large enough, so that the cumulative amount of captured emissions along the optimal path is not constrained by their size. Looking at facts and figures may help to support this assumption since the theoretical storage capacity ranges from 8 090 to 15 500 GtCO2 (IEA [2009a]) whereas economically recoverable conventional and unconventional fossil fuels range from 3865 to 5509 GtCO2. Risks coming from CO_2 storage are not considered here; for an interesting overview of these risks, one can refer to IPCC [2005].

Q_t ($Q_0 > 0$) and $x(t)$ represent the stock and the extracted flow of the exhaustible resource at time t. Its coefficient of pollution equals one: burning one unit of fossil fuel emits one unit of CO_2. The constant unitary extraction cost is written c_x. In real life, to extract fossil fuel and to produce energy from fossil fuels burning, specific "dirty" capacities (for extraction or power generation) are needed. We exclude this feature to simplify calculation and presentation of optimal paths. The necessity to invest in "dirty" capacities rise an important issue concerning the use of CCS retrofitting vs. CCS implementation in new power plants, and issues over shutdowns cost of still-productive dirty installations[2].

The variation of carbon concentration equals non captured anthropic emissions minus natural dilution. CCS can be used only to capture CO_2 from the flow of emissions and not directly from the atmospheric CO_2 stock, so $x(t) \geq K_{ccs}(t)$: capture is limited by the (endogenous) size of gross emissions, and this constraint is called "the capture from the flow" constraint. If along the optimal path that constraint would never never bind, the optimal CCS use could be seen as well as the optimal capture from the stock if both capture costs were equal. Assuming that the natural absorption is proportional to the CO_2 concentration and writing α, the instantaneous rate of absorption of CO_2, the law of motion of the CO_2 concentration, Z_t, writes:

$$\dot{Z}_t = -\alpha Z_t + x(t) - K_{ccs}(t) \tag{1}$$

3.2 The social planner decision problem

The social planner seeks to find the extraction and the investment paths $\{x(t), I_{ccs}(t), I_s(t)\}$ which maximize the net discounted social surplus under the environmental constraint:

$$\int_0^\infty e^{-rt}\Big(u\big(x(t)+K_s(t)\big) - c_x x(t) - FC_{ccs}.I_{ccs}(t) - FC_s.I_s(t)\Big)dt$$

subject to $\forall t, \forall i \epsilon \{ccs; s\}$,

$$\begin{aligned}
\dot{Q}_t &= -x(t)\\
\dot{Z}_t &= -\alpha Z_t + x(t) - K_{ccs}(t)\\
Z_t &\leq \overline{Z}\\
\dot{K}_i(t) &= -\delta_i K_i(t) + I_i(t)\\
x(t) &\geq K_{ccs}(t)\\
Q_t, x(t), I_i(t) &\geq 0
\end{aligned}$$

Transversality conditions are given by:

$$\lim_{t\to\infty} \lambda_t e^{-rt} Q_t = 0 \tag{2}$$

$$\lim_{t\to\infty} \mu_t e^{-rt} Z_t = 0 \tag{3}$$

[2]With "dirty" capacities that depreciate slowly enough, we expect a similar result than in Campbell [1980]: investments in "dirty" capacities should be done at the beginning of the time period, because the "dirty" capacities constraint binds only at the beginning of the period due to the decreasing pattern of extraction.

Equation 2 simply states that the fossil fuel must be exhausted in the long run, otherwise the scarcity rent must equal 0.

3.3 Optimal energy price and optimal carbon price

The Hamiltonian in current value writes:

$$
\begin{aligned}
\mathbb{H} \;=\;& u(x(t)+K_s(t)) - c_x x(t) - FC_{ccs}.I_{ccs}(t) - FC_s.I_s(t) \\
& -\lambda_t x(t) - \mu_t(-\alpha Z_t + x(t) - K_{ccs}(t)) \\
& +\eta_{ccs,t}(-\delta_{ccs}K_{ccs}(t) + I_{ccs}(t)) \\
& +\eta_{s,t}(-\delta_s K_s(t) + I_s(t))
\end{aligned}
$$

With the following slackness conditions, $\forall t, \forall i \epsilon \{ccs; s\}$:

$$\nu_t \geq 0, \quad \text{and} \quad \nu_t(\overline{Z} - Z_t) = 0 \tag{4}$$
$$b_t \geq 0 \quad \text{and} \quad b_t Q_t = 0 \tag{5}$$
$$a_t \geq 0, \quad \text{and} \quad a_t x(t) = 0 \tag{6}$$
$$\gamma_{i,t} \geq 0, \quad \text{and} \quad \gamma_{i,t} I_i(t) = 0 \tag{7}$$
$$\sigma_t \geq 0 \quad \text{and} \quad \sigma_t(x(t) - K_{ccs}(t)) = 0 \tag{8}$$

Along the optimal paths of extraction and investments, the dynamics of the co-state variables, λ_t, μ_t, $\eta_{ccs,t}$ and $\eta_{s,t}$ are determined by:

$$\dot{\lambda}_t = r\lambda_t - \frac{\partial H(t)}{\partial Q_t} \Longleftrightarrow \dot{\lambda}_t = r\lambda_t \tag{9}$$
$$\dot{\mu}_t = r\mu_t - \frac{\partial H(t)}{\partial Z_t} \Longleftrightarrow \dot{\mu}_t = (r+\alpha)\mu_t + \nu_t \tag{10}$$
$$\dot{\eta}_{ccs,t} = r\eta_{ccs,t} - \frac{\partial H(t)}{\partial K_{ccs,t}} \Longleftrightarrow \dot{\eta}_{ccs,t} = (r+\delta_{ccs})\eta_{ccs,t} - \mu_t + \sigma_t \tag{11}$$
$$\dot{\eta}_{s,t} = r\eta_{s,t} - \frac{\partial H(t)}{\partial K_{s,t}} \Longleftrightarrow \dot{\eta}_{s,t} = (r+\delta_s)\eta_{s,t} - u_s'(x(t)+K_s(t)) \tag{12}$$

The First Order Conditions are:

$$\frac{\partial H(t)}{\partial x(t)} = 0 \quad \Longleftrightarrow \quad p_x(t) = c_x + \lambda_t + \mu_t - \sigma_t \tag{13}$$
$$\frac{\partial H(t)}{\partial I_i(t)} = 0 \quad \Longleftrightarrow \quad FC_i = \eta_{i,t} + \gamma_{i,t} \tag{14}$$

The co-state variable λ_t represents the current value of the scarcity rent of the exhaustible resource. As shown by Hotelling [1931], it increases at rate r: the discounted net marginal surplus of extraction must be constant. In other words,extracting a supplementary unit must be equivalent to saving it for a latter use along the optimal path.

The co-state variable μ_t represents the current value of the shadow cost of marginal pollution. It exhibits a familiar pattern driven by the ceiling-shaped carbon regulation and the modelization of the natural dilution of CO_2 in the atmosphere. In a decentralized economy with perfect competition, optimal taxation requires to tax non-captured emissions of time t with an unitary tax equaling this marginal cost μ_t; the whole time path of the carbon tax must be initially credibly announced. Before the ceiling binds, the carbon tax increases at the rate of the sum of the discount rate and the absorption rate ($\nu_t = 0$; $\mu_t = \mu_0 e^{(r+\alpha)t}$). At the ceiling, the carbon tax is set to make non-captured emissions just equal to natural absorption, taking account of available CCS systems. Because of the duration and depreciation of CCS capacities, the pollution cost may be decreasing or increasing over some intervals of time. Once the ceiling no more binds, the pollution cost is nil. So, the pollution cost writes:

$$\begin{cases} \mu_t = \mu_0 e^{(r+\alpha)t} & for Z < \overline{Z} if Z = \overline{Z} in the future \\ \mu_t = D^{-1}(\alpha\overline{Z} + K_{ccs}(t)) - \lambda_t & for Z = \overline{Z} \\ \mu_t = 0 & otherwise \end{cases}$$

By equation 13, the price of fossil fuels writes $p_x(t) = c_x + \lambda_t + \mu_t$, if the installed capacities do not exceed gross emissions ($x(t) \geq K_{ccs}(t)$ and $\sigma_t = 0$). It writes $p_x(t) = c_x + \lambda_t + \mu_t - \sigma_t$ if the installed capacities exceed gross emissions ($x(t) < K_{ccs}(t)$ and $\sigma_t > 0$); in that case, the marginal pollution would be costless since it could be captured without extending the capacities, so the fossil fuel price would equal $p_x(t) = c_x + \lambda_t$. Since there is no incentive to build capacities that are not immediately used, just after any investment capacities cannot exceed the gross emissions flow ($x(t) \geq K_{ccs}(t)$ and $p_x(t) = c_x + \lambda_t + \mu_t$). Let assume that after a while $x(t) < K_{ccs}(t)$ and $p_x(t) = c_x + \lambda_t$. If this switch occurs before the ceiling, the consumption would be put upward in a discrete way while $K_{ccs}(t)$ continuously depreciates: so, having $x(t) \geq K_{ccs}(t)$ then $x(t) < K_{ccs}(t)$ is not possible. At the ceiling, net emissions are strictly positive. Finally, along the optimal path, capacities are fully used until the ceiling stops to bind so $p_x(t) = c_x + \lambda_t + \mu_t$. By setting the tax equal to μ_t, in a competitive world, the social planner forces, the CCS providers to give access to capture facilities at a unitary cost of μ_t. Since investments costs are constant, the social planner must in addition give quantities targets for the construction of CCS systems and solar plants.

The extraction $x(t)$ is globally decreasing. This is obviously the case before the ceiling or after the ceiling. A decrease of the fossil fuel price at the ceiling would imply that μ_t is decreasing and CCS capacities increase to keep the ceiling just binding, so investments must be done and μ_t is constant, and so the extraction is decreasing. Contradiction. So the fossil fuel extraction is decreasing over the optimal path of extraction.

Co-state variable $\eta_{ccs,t}$ represents the marginal value of CCS systems at time t. It comes that $\eta_{ccs,t} = \int_t^\infty e^{-(r+\delta_{ccs})(j-t)} \mu_j dj$ (CCS systems are always fully used when $\mu_j > 0$ as shown above). The marginal benefit of a CCS capacity equals the sum of the discounted pollution costs that would be waved by this capacity. Co-state variable $\eta_{s,t}$ represents the marginal value of solar plants at time t. We get that $\eta_{s,t} = \int_t^\infty e^{-(r+\delta_s)(j-t)} u'(x(j)+K_s(j))dj$.

The marginal benefit of a solar capacity equals the sum of the discounted flows of marginal utility trough time. Because of the natural dilution and the exhaustibility of the fossil fuel, in the long run, the ceiling will no more bind at a date $\bar{t}$ and CCS systems will be no more useful after that date, contrary to solar plants. Investments occur as long as the cost of investment is lower than the marginal value of capital; so, $FC_i = \eta_{i,t}$ when we invest in capital i by equation 14 and the slackness condition 7. With nil cost of use, installed solar plants are fully used, as well as installed CCS systems (as long as the flow of non-captured emissions is positive). There is no incentive to extend CCS capacities withing the period of time the ceiling binds since it would imply a discontinuity in the energy consumption at the ceiling. It is not possible to invest at the same time in solar plants and in CCS systems. If solar is used, price must be constant and so the shadow cost of pollution must decrease to compensate the increase of the scarcity rent, and so it is not possible to invest in CCS systems when μ_t is decreasing.

In a competitive decentralized economy where investments in CCS systems and solar plants come from private firms, firms face negative returns on investments in the early period of capital exploitation; investments costs are asymptotically balanced in the long run for solar plants, and only once CCS systems stop to be used for CCS systems. Exhaustible carbon-emitting resources and long-duration of clean capital rise important issues concerning the credibility of the carbon tax time path. Implementing the social best policy requires a credible announce of the future path of the carbon tax since both the fossil fuels owners, clean energy providers and CCS systems providers take their decision over an intertemporal horizon. Both the cost of pollution and the cost of fossil fuels including the pollution cost are important to give the good incentives for deploying CCS and renewables.

4 Optimal Paths of Extraction and Investments in CCS Systems and Solar Plants.

4.1 Properties of optimal investments paths in CCS systems and solar plants

From First order conditions and Transversality conditions, we derive the key features of the different types of optimal paths of extraction and clean capital accumulation. See Appendices for proofs.

Proposition 4.1. *If the depreciation rate of CCS systems is low enough or the resource is scarce enough or the carbon ceiling is high enough, it is optimal to build some CCS systems before the CO_2 concentration reaches the carbon ceiling. Otherwise, investments start when the CO_2 concentration reaches the carbon cap, are decreasing and must stop before the CO_2 concentration falls under the carbon cap.*

Claim. If $u''' \leq 0$, the ceiling binds only once. For some parameters and demand function presenting strong local convexity, the ceiling may bind over several separate interval of time if CCS is used. WORK IN PROGRESS

Remark. Due to investments in CCS before the ceiling, the CO_2 concentration may decreases after the investment shot before increasing to reach the ceiling, despite the decreasing gross emissions flow.

Proposition 4.2. *It is never optimal to invest in CCS systems at two different dates before the CO_2 concentration reaches the carbon ceiling if the captured emissions flow is never constrained by the size of the gross emissions flow between these two dates.*

Claim. If $u''' \leq 0$, there is no stop-and go pattern in CCS investments. However for some parameters and demand function presenting strong local convexity, optimal investments may exhibit a stop-and-go pattern. WORK IN PROGRESS

Proposition 4.3. *Investments in solar plants never stop and help to maintain the energy consumption at a level, determined by the energy demand and the solar energy supply.*

Corollary. *Since the extraction of fossil fuel is decreasing, the overall energy consumption is decreasing and then constant once solar energy start to be used.*

Proposition 4.4. *It is never optimal to invest in solar plants before the CO_2 concentration reaches the ceiling.*

Proposition 4.5. *If there is a pool of renewables, with different constant investments costs and different depreciation rate, only one type of renewables is used along the optimal path.*

Proposition 4.6. *If there is a pool of CCS technologies, with different constant investments costs and different depreciation rates, several types of CCS technologies can be used along the optimal path.*

Claim. If two types of CCS technologies are used before the ceiling or if $u''' \leq 0$, the longest-life type of technologies being necessarily used in first place.

Proposition 4.7. *If current investment costs are strictly increasing with the amount of current investments, investments are diluted trough time to avoid expensive investment costs: investments in CCS systems necessarily start before the ceiling and investments in solar plants may start before the ceiling if their cost is low enough.*

4.2 Description of the Different Optimal Paths of Investments and Extraction

Following figures describe different optimal paths: CCS investments occur before the ceiling (Figure 4.1), CCS investments start at the beginning of the ceiling (Figure 4.2) and CCS is never used (Figure 4.3). For each figure, the upper graphs describe the clean capital accumulation, the middle graphs present the CO_2 stock and the energy mix, and finally the lower graphs show the energy price, the scarcity rent and the shadow cost of pollution. All figures present on the left side (respectively on the right-side) the case where solar energy is used before (when) the fossil fuel is (gets) exhausted. We do not present the unusual paths where the ceiling binds twice, where CCS investments exhibit a stop-and-go pattern, or where the gross emissions flow constraints carbon capture before the ceiling. Also, we do not focus on the alternative settings with increasing investment costs or with several types of renewables or CCS.

4.2.1 Solar power plants are cheap enough to be used at the ceiling ($E_{LR} > \alpha\overline{Z} + min\{K_{ccs}\}$). (See the left-side graph in each figure).

Depending on their price, solar plants can be used at the ceiling or after the ceiling when the fossil fuel gets exhausted. In other words, if the long run energy flow E_{LR} ($\frac{u'(E_{LR})}{r+\delta_s} = FC_s$) is higher than the minimum of the consumption of fossil fuel at the ceiling ($\alpha\overline{Z} + min\{K_{ccs}\}$), solar plants are used at the ceiling to maintain the level of consumption equal to E_{LR}.

If CCS systems are cheap enough to be installed and their depreciation rate is low, investments in CCS start before the ceiling binds (See figure 4.1)

- Phase 1 $[0; t_{ccs})$: Fossil fuel energy source phase without CCS.
  The CO_2 concentration is under the ceiling and increases. The energy demand decreases. The marginal benefit of an additional unit of CCS, $\eta_{ccs,t}$, is increasing. K_{ccs} CCS systems are built at time t_{ccs} such that $\eta_{ccs,t_{ccs}} = FC_{ccs}$ and $\dot{\eta}_{ccs,t_{ccs}} = 0$, to slow down the accumulation of CO_2 in the atmosphere.

- Phase 2 $[t_{ccs}; \underline{t})$: Fossil fuel energy source phase with CCS.
  All the CCS systems are fully used hereafter. The quantity of CCS systems is decreasing at rate δ_{ccs} hereafter. Captured emissions equal $K_t = K_{t_{ccs}} e^{-\delta(t-t_{ccs})}$ at time t. The CO_2 concentration may decrease first before being increasing to reach the carbon ceiling. The dynamics of the shadow cost of pollution and the dynamics of the energy price is unchanged. The energy demand is decreasing. This phase lasts until $\underline{t}$ when the ceiling constraint starts to bind.

- Phase 3 $[\underline{t}; \bar{t})$: The ceiling is binding, fossil fuel with CCS.
  The amount of non-captured emissions equals $\alpha\overline{Z}$ to keep the ceiling just binding. CCS capacities are decreasing. The fossil fuel price is set such that the ceiling is just binding ($p_t = D^{-1}(\alpha\overline{Z} + K_{t_{ccs}} e^{-\delta(t-t_{ccs})})$, so it is increasing and the energy demand is decreasing. This phase lasts until the energy consumption sufficiently low to make investment in solar plants profitable (exactly when the energy demand equals E_{LR} such that $\frac{u'(E_{LR})}{r+\delta_s} = FC_s$).

- Phase 4 $[\bar{t}; T)$: The ceiling is binding, energy mix with fossil fuel burning with CCS and solar energy.
  The share of fossil fuel in the energy mix is decreasing. Solar plants are deployed to maintain the flow of consumption equal to E_{LR} and the energy price is constant. This phase lasts until the fossil fuel is exhausted and the ceiling stops to bind.

- Phase 5 $[T; \infty)$: Carbon-free energy source.
  The fossil fuel is exhausted. Power plants are continuously built up to compensate their depreciation in order to maintain the level of energy consumption at E_{LR}. The CO_2 concentration continuously decreases at rate α.

If CCS systems are cheap enough to be installed and their depreciation rate is high enough, CCS is used at the beginning of the ceiling (See figure 4.2). Compared to the previous path, phase 2 is replaced by a phase with investments in CCS systems at the ceiling. The quantity of installed CCS systems is decreasing trough time and still help to keep the ceiling just binding. During that phase, the shadow price of pollution must be set equal to $(r + \alpha)FC_{ccs}$. When the resource is scarce enough, the accumulated future costs of pollution becomes relatively low, and thus there is a date ($t_{ccs,stop}$) at which the marginal benefit of CCS systems falls under FC_{ccs}. The carbon tax must be constant and equal to $FC_{ccs}(r + \delta_{ccs})$ when investments occur, then increasing and decreasing. Solar investments start while the carbon tax is decreasing since the energy price must be constant (thus $\dot{\lambda}_t = -\dot{\mu}_t$). The rest of the path is similar.

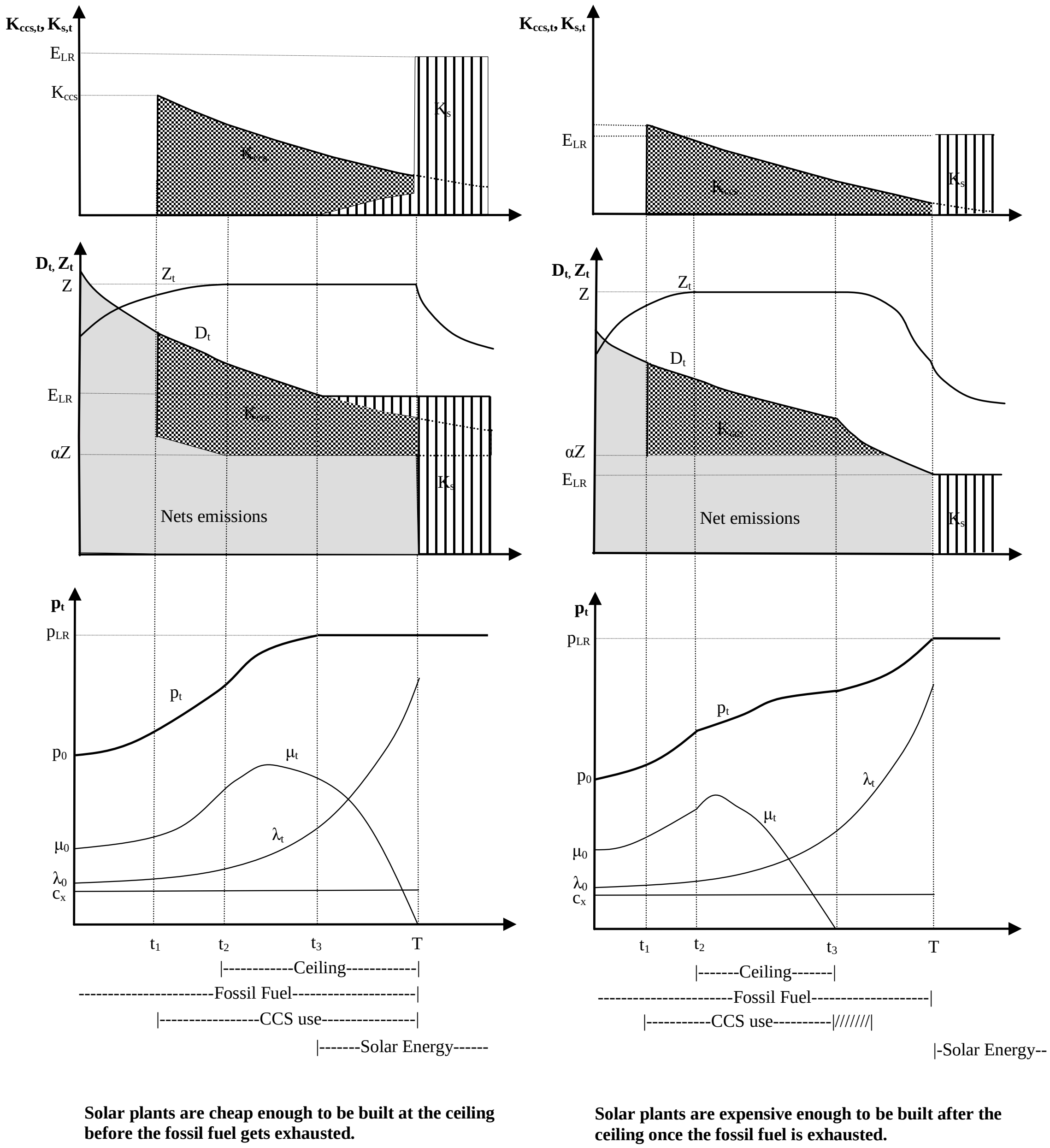


Figure 4.1: Clean capital accumulation, energy mix and energy price when CCS investments occur before the ceiling.

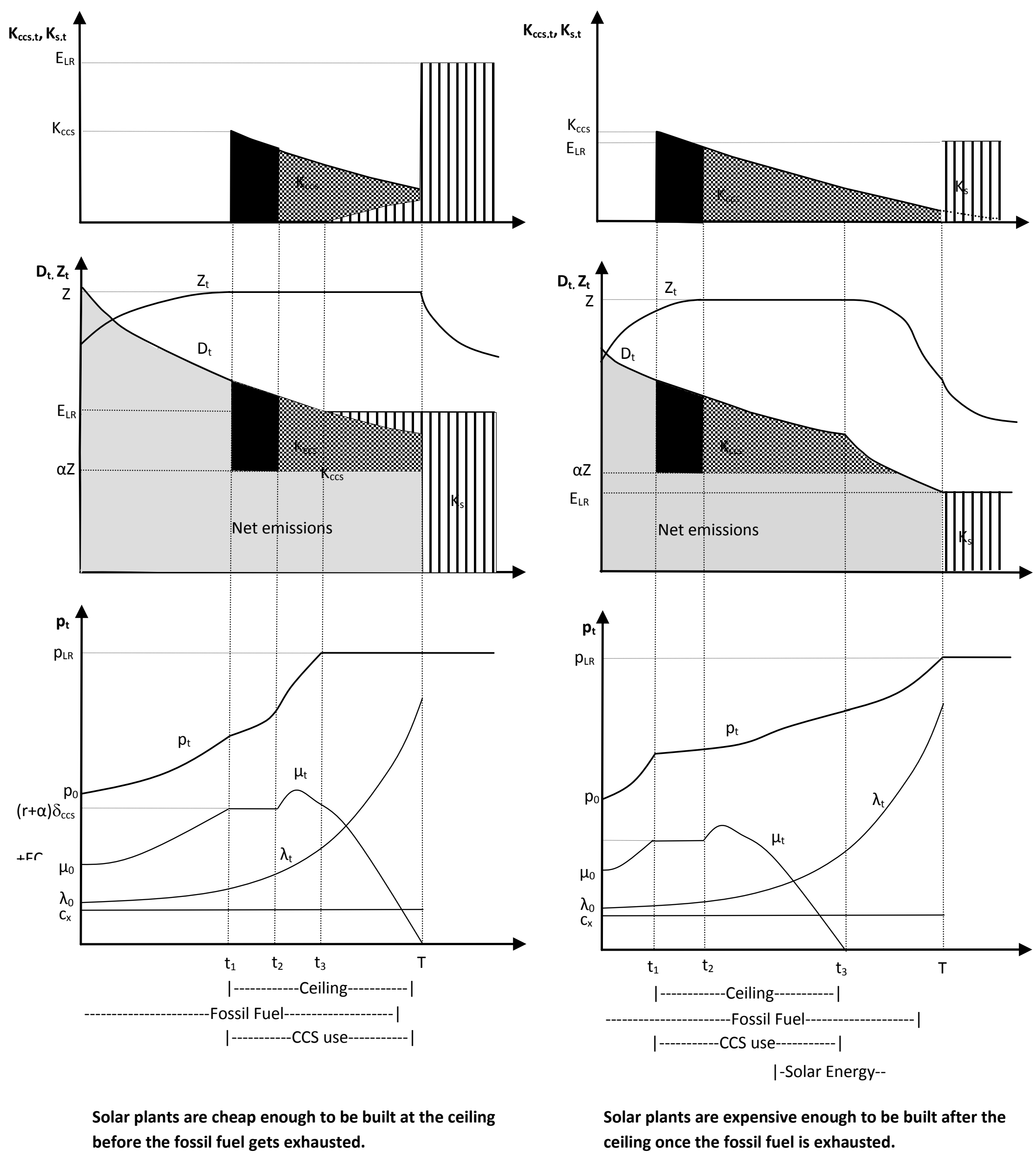


Figure 4.2: Clean capital accumulation, energy mix and energy price when CCS investments start at the ceiling.

If CCS systems are too expensive, CCS is not used at all (See figure 4.3). The fossil fuel price is first increasing before the ceiling and then constant to keep emissions just equal to natural absorption. At the ceiling and after the ceiling, the energy consumption is kept at the level E_{LR}, so solar plants provide a flow of energy of $E_{LR} - \alpha\overline{Z}$ at the ceiling and then E_{LR} after the ceiling, when the fossil fuel is exhausted. If solar energy is used at the ceiling, it is used at the beginning of the ceiling. The end of the path is similar to previously described paths.

4.2.2 Solar power plants are expensive enough to be installed only once the fossil fuel is exhausted. $E_{LR} < \alpha\overline{Z} + min\{K_{ccs}\}$.

If at the end of the ceiling, the energy consumption is higher than E_{LR}, there is no incentive to install solar plants before the fossil fuel is exhausted. The fossil fuel is not exhausted when the ceiling stops to bind. After the ceiling, extraction follows a pure Hotelling path until exhaustion. After the fossil fuel is exhausted, solar plants are built up, the energy price equals p_{LR} and the energy consumption is kept equal to E_{LR}. As in previous sections, CCS can be used before the ceiling, at the ceiling, or not at all depending on parameters (See the right-side graph in each figure).

4.2.3 Determination of the endogenous variables.

The initial scarcity rent, the initial carbon tax and the dates defined above $\{\lambda_0, \mu_0, t_{ccs}, \underline{t}, \overline{t}, T, K_{ccs}(t)\}$ satisfy the following conditions:

- The continuity of energy prices between phases:

$$\lambda_0 e^{r\underline{t}} + \mu_0 e^{(\alpha+r)\underline{t}} = D^{-1}(\alpha\bar{Z} + K_{\underline{t}}) \tag{15}$$

$$\lambda_0 e^{r\bar{t}} = D^{-1}(\alpha\bar{Z} + K_{\bar{t}}) \tag{16}$$

$$\lambda_0 e^{rt_{solar}} + \mu_{t_{solar}} = FC_s(r+\delta_s) = p_{LR} \tag{17}$$

  - If solar energy is used before the fossil fuel is exhausted thus, $t_{solar} < T$ and $T = \bar{t}$. Otherwise, $t_{solar} = T$ and equation 17 becomes $\lambda_0 e^{rT} = FC_s(r+\delta_s)$ since $\mu_t = 0$ for $t > \bar{t}$,

- The carbon concentration reaches $\bar{Z}$ at time $\underline{t}$:

$$e^{-\alpha\underline{t}} Z_0 + \int_0^{\underline{t}} e^{-\alpha(\underline{t}-t)} D(\lambda_0 e^{rt} + \mu_0 e^{(r+\alpha)t}) dt \quad - \int_{t_{ccs}}^{\underline{t}} e^{-\alpha(\underline{t}-t)} K_t dt = \quad \bar{Z} \tag{18}$$

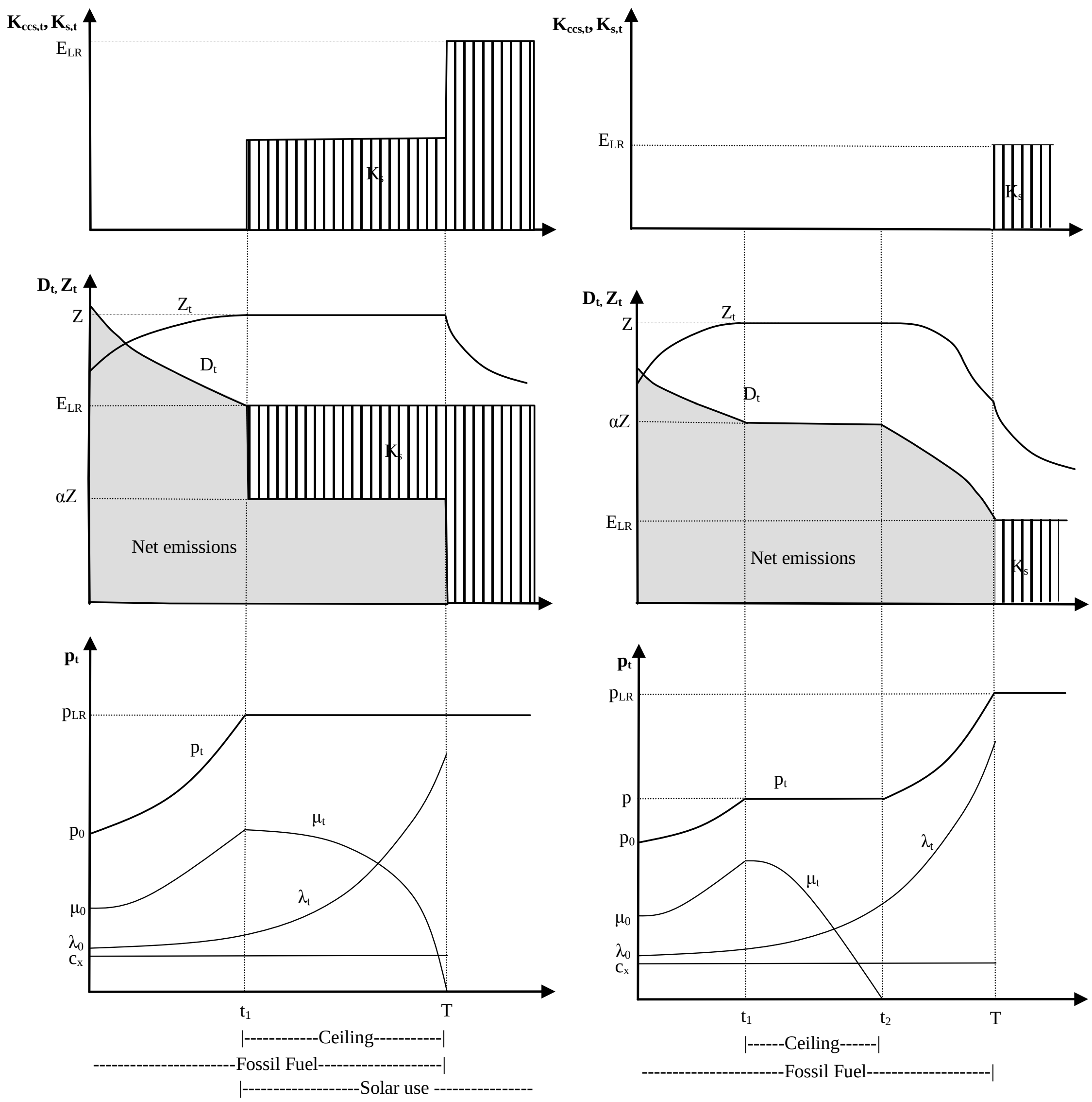
- The non renewable resource is finally exhausted at time T:

$$\int_0^{\underline{t}} D(\lambda_0 e^{rt} + \mu_0 e^{(r+\alpha)t}) dt + \int_{\underline{t}}^{\bar{t}} \left(\alpha\bar{Z} + K_t\right) dt + \int_{\bar{t}}^{T} D(\lambda_0 e^{rt}) dt = Q_0 \tag{19}$$

- Investments start at time t_{ccs} and stop at time $t_{ccs,stop}$:

$$\mu_0 e^{(\alpha+r)t_{ccs}} = (r+\delta)FC_{ccs} \tag{20}$$

$$\eta_{t_{ccs}} = \int_{t_{ccs}}^{\underline{t}} \mu_0 e^{(r+\alpha)t} e^{-(r+\delta)(t-t_{ccs})} dt + \int_{\underline{t}}^{\bar{t}} \left[D^{-1}(\alpha\bar{Z} + K_t) - \lambda_0 e^{rt}\right] e^{-(r+\delta)(t-t_{ccs})} dt = FC_{ccs} \tag{21}$$



Solar plants are cheap enough to be built at the ceiling before the fossil fuel gets exhausted.

Solar plants are expensive enough to be built after the ceiling once the fossil fuel is exhausted.

Figure 4.3: Clean capital accumulation, energy mix and energy price without CCS investments.

- If CCS is not used, $\forall t \; K_t = 0$, equations 20 and 21 are no more relevant.
- If investments occur before the ceiling $t_{ccs} < \underline{t}$, thus for $t \geq t_{ccs}$, $K_t = K_{t_{ccs}} e^{-\delta(t-t_{ccs})}$ and $t_{ccs} = t_{ccs,stop}$.
- If investments start at the beginning of the ceiling, $t_{ccs} = \underline{t}$ and last until $t_{ccs,stop}$ when $\eta_{ccs,t_{ccs,stop}} = \int_{t_{ccs}}^{t_{ccs,stop}} \mu_0 e^{(r+\alpha)t} e^{-(r+\delta)(t-t_{ccs})} dt + \int_{t_{ccs,stop}}^{\bar{t}} \left[D^{-1}(\alpha \bar{Z} + K_t) - \lambda_0 e^{rt}\right] e^{-(r+\delta)(t-t_{ccs})} dt < FC_{ccs}$. During the investments period $[t_{ccs}; t_{ccs,stop}]$ at the ceiling, $\mu_t = (r+\delta) FC_{ccs}$ thus for a given scarcity rent, the quantity of CCS sytems needed at each period of time is well defined.

5 Conclusions

This theoretical paper focuses on the optimal timing of investments in CCS systems and solar plants and the role played by the scarcity of the exhaustible carbon-emitting resource. To get tractable results and compare them with those found in previous literature, the analysis is based on a standard Hotelling-like model, in which utility comes from two sources of energy: a carbon-emitting exhaustible resource (a fossil fuel) and a carbon-free renewable resource (solar energy). The regulation takes the form of a carbon cap over the atmospheric carbon stock. By impacting differently the marginal value dynamics of CCS systems and low-carbon renewables power plants, the exhaustibility of the carbon-emitting resource leads to drastically different optimal investments paths. Investing in CCS systems before the atmospheric carbon stock reaches the carbon cap is optimal if their depreciation is slow and the carbon-emitting resource is scarce enough, whereas it is never optimal to invest in solar plants before the ceiling, with constant investment costs. Investments in renewables power plants start to maintain the consumption flow at a level determined by their characteristics (cost, duration) and the energy demand. Energy may be provided at the ceiling by an energy mix from fossil fuels burning, whose emissions are partly captured, and renewables. Introducing a pool of CCS technologies and renewables, differentiated only by their depreciation rate and their constant capital investment cost, we find that only one kind of renewables is used along the optimal path, whereas several CCS technologies may be used with long-life technologies in first place.

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Appendices

The social planner decision problem with cost of use of existing capacities

We assume that costs of use of solar plants and CCS systems are constant but strictly positive. The direct consequence is that the quantity of capital used at time t, written $K_i^u(t)$, may be different that the available quantity of capital at that date. Writing MC_i, the marginal cost of use of K_i, the instantaneous marginal benefit of capturing CO_2 and using solar energy in current value become respectively $\mu_t - MC_{ccs}$ and $u'(x(t)+K_s^u(t)) - MC_s$. It leads to slight modifications of previous writings:

The social planner objective becomes:

$$\int_0^{\infty} e^{-rt}\Big(u\big(x(t)+K_s^u(t)\big) - c_x x(t) - FC_{ccs}.I_{ccs}(t) - MC_{ccs}.K_{ccs}^u(t) - FC_s.I_s(t) - MC_s.K_s^u(t)\Big)dt$$

The Hamiltonian in current value becomes:

$$\begin{aligned}
\mathbb{H} \;=\; & u(x(t)+K_s^u(t)) - c_x x(t) - FC_{ccs}.I_{ccs}(t) - MC_{ccs}.K_{ccs}^u(t) - FC_s.I_s(t) - MC_s.K_s^u(t) \\
& -\lambda_t x(t) - \mu_t(-\alpha Z_t + x(t) - K_{ccs}^u(t)) \\
& +\eta_{ccs,t}(-\delta_{ccs}K_{ccs}(t) + I_{ccs}(t)) \\
& +\eta_{s,t}(-\delta_s K_s(t) + I_s(t))
\end{aligned}$$

With this additional slackness condition, $\forall t,\forall i \epsilon \{ccs;s\}$:

$$\theta_{i,t} \geq 0, \quad \text{and} \quad \theta_{i,t}(K_i(t) - K_i^u(t)) = 0 \tag{22}$$

The dynamics of the co-state variables,$\eta_{ccs,t}$ and $\eta_{s,t}$ become:

$$\dot{\eta}_{ccs,t} = r\eta_{ccs,t} - \frac{\partial H(t)}{\partial K_{ccs,t}} \Longleftrightarrow \dot{\eta}_{ccs,t} = (r+\delta_{ccs})\eta_{ccs,t} - (\mu_t - MC_{ccs}) + \theta_{ccs,t} + \sigma_t \tag{23}$$

$$\dot{\eta}_{s,t} = r\eta_{s,t} - \frac{\partial H(t)}{\partial K_{s,t}} \Longleftrightarrow \dot{\eta}_{s,t} = (r+\delta_s)\eta_{s,t} - (u_s'(x(t)+K_s^u(t)) - MC_s) + \theta_{s,t} \tag{24}$$

The First Order Conditions for the use of capital are:

$$\frac{\partial H(t)}{\partial K_{ccs}^u(t)} = 0 \quad \Longleftrightarrow \quad MC_{ccs} = \mu_t + \theta_{ccs,t} + \sigma_t \tag{25}$$

$$\frac{\partial H(t)}{\partial K_s^u(t)} = 0 \quad \Longleftrightarrow \quad MC_s = u'(x(t)+K_s^u(t)) + \theta_{s,t} \tag{26}$$

By equation 25, CCS systems are used as long as the marginal cost of use is under the marginal cost of pollution and the flow of non-captured emissions is positive; so, CCS systems stop to be used before the ceiling stops to binds i.e before $\mu_t = 0$. By equation 26, solar plants are used as long as the marginal cost of use is under the marginal utility of energy.

Co-state variables $\eta_{ccs,t}$ and $\eta_{s,t}$ represent the marginal value of CCS systems and solar plants at time t. It comes that $\eta_{ccs,t} = \int_t^{\infty} e^{-(r+\delta_{ccs})(j-t)}(\mu_j - MC_{ccs})dj$ (capacities are always fully used when $\mu_j > 0$ as shown above) and $\eta_{s,t} = \int_t^{\infty} e^{-(r+\delta_s)(j-t)}(u'(x(j) + K_s(j)) - MC_{ccs})dj$. The marginal benefit of a CCS capacity equals the sum of the discounted pollution costs that would be waved by this capacity, minus the cost of use. The marginal benefit of a solar capacity equals the sum of the discounted flows of marginal utility trough time, minus the cost of use. Because of the natural dilution and the exhaustibility of the fossil fuel, in the long run, the ceiling will no more bind at date $\bar{t}$ and CCS systems will be no more useful after that date, contrary to solar plants.

Previous results are not modified, except that CCS capacities stop to be used before the ceiling stops to bind. A stop-and-go pattern in the use of CCS systems is not optimal. Concerning solar plants, a stop-and-go pattern in the use or in investments in solar plants is not optimal.

Proofs of propositions

Proposition 5.1. *If the depreciation rate of CCS systems is low enough and the resource is scarce enough, it is optimal to build some CCS systems before the CO_2 concentration reaches the carbon ceiling; otherwise investments start when the CO_2 concentration reaches the carbon cap, are decreasing and must stop before the CO_2 concentration falls under the carbon cap.*

Proof. The extreme case of a full instantaneous depreciation is analyzed in previous literature (see for instance Lafforgue et al. [2007]), in that case, it is never optimal to use CCS before the ceiling binds if costs are constant. For a given quantity of fossil fuel, for δ_{ccs} high enough, it is optima to invest only at the ceiling. For any $\delta < 1$, if the polluting resource is abundant enough, it is optimal to invest only at the ceiling. In the extreme case of a non exhaustible polluting resource, its price simply equals the shadow cost of pollution plus the cost of extraction ($p_t = c_x + \mu_t$). The optimal trajectory implies to stabilize the CO_2 concentration at the ceiling until the end. Fossil fuel with CCS becomes a perfect competitor of solar energy, CCS and solar energy cannot be used along the same optimal path: CCS can only be used at the ceiling to maintain the energy flow to a specific value, $\alpha\overline{Z} + \overline{K}_{ccs}$, such that $\frac{u'\left(\alpha\overline{Z}+\overline{K}_{ccs}\right)}{r+\delta_{ccs}} = FC_{ccs}$ (See proposition 5.3). If there is no CCS investments before the ceiling, investment start at the beginning of the ceiling and never before. Indeed, otherwise from the beginning of the ceiling to the investment date, μ_t must decrease to keep the fossil fuel consumption just equal to $\alpha\overline{Z}$, and after that date, μ_t is decreasing, is lower than this value of μ_t, since otherwise the price will be higher and so, taking account of existing CCS capacities, the ceiling will not bind anymore. So if there is no CCS capacities at the beginning of the ceiling, it is not optimal to invest at the ceiling. If there are investments at the ceiling over an interval of time, μ_t must be constant over this interval by equation 14, so the fossil fuel price is increasing and the fossil-fuel-based energy consumption is decreasing, so emissions are decreasing and CCS capacities decrease to keep the ceiling just binding. After investments stop, μ_t cannot be only decreasing. Indeed, otherwise it would imply that when investments in CCS systems are done, $\eta_{ccs,t} = \int_t^{\infty} e^{-(r+\delta_{ccs})(j-t)}(r + \delta_{ccs})FC_{ccs}dj < \eta_{ccs,t}$. So, μ_t is increasing after being constant and then decreasing. μ_t increasing at the ceiling requires that $u'(x(t) + K_s(t)) - \lambda_t - c_x$ is increasing and so that $x(t) + K_s(t)$ is decreasing, so there cannot be any investments in solar plants at that time since the energy consumption, $x(t) + K_s(t)$ would be constant by proposition 5.3 . The increasing tax at the ceiling, when investments stop, reflects that consuming oil leads to a strong negative externality since the opportunity of capture is constrained by the decreasing stock of CCS plants. the carbon tax is finally decreasing since in the ling run, the ceiling will stop to bind.

Let us assume that it is never optimal to invest in CCS systems before the CO_2 concentration reaches the carbon ceiling. As shown supra, investments necessarily start at the beginning of the ceiling at a date we call t_1. Let us call $\bar{t}$, the date when the ceiling would stop to bind and t', the date when investments in CCS systems would stop, $t' < \bar{t} < \infty$. Such dates exist since due to the exhaustibility of the carbon-emitting resource, the ceiling will no more bind in the long-run ($\bar{t} < \infty$), and by continuity of the energy price at the ceiling and after, μ_t must be decreasing towards zero when approaching the end of the ceiling phase, thus there exists a date at which investing in CCS capacities is no more profitable while the ceiling still binds ($t' < \bar{t}$). If the last unit of CCS system at t' is not installed, the CO_2 concentration would go slightly over the ceiling at that date. The cheapest way to keep the ceiling just binding at time t' is to invest at time $t^*$when the cost of reducing by one unit the CO_2 concentration of time t' is minimized. For $\delta_{ccs} < 1$, investing in one unit at time t lowers the carbon stock of time t' by $\int_t^{t'} e^{-\delta_{ccs}(j-t)}e^{-\alpha(t'-j)}dj$. Thus, the cost of reducing the CO_2 concentration of time t' by one unit by investing at time t in value of the initial period writes, $f_{t,t'} = \frac{e^{-rt}FC_{ccs}}{\int_t^{t'} e^{-\delta_{ccs}(j-t)}e^{-\alpha(t'-j)}dj}$. $\dot{f}_{t,t'}$ has the sign of $\frac{1}{(\alpha-\delta_{ccs})}\left[(\alpha+r)e^{(\alpha-\delta_{ccs})t} - (\delta_{ccs}+r)e^{(\alpha-\delta_{ccs})t'}\right]$ and $\ddot{f}_{t,t'} > 0$. For t close enough to t', $\dot{f}_{t,t'}$ is positive, $\dot{f}_{t,t'}$ may change its sign in the segment $[0, t']$ from negative to positive at time t^* ($t^* = \frac{ln(\frac{\delta_{ccs}+r}{\alpha+r})}{\alpha-\delta_{ccs}} + t'$), or can be only positive depending on parameters, in that case t_0 is the date of investments; the optimal date of investments writes $Max\{t^* = \frac{ln(\frac{\delta_{ccs}+r}{\alpha+r})}{\alpha-\delta_{ccs}} + t'; t_0\}$; $\frac{dt^*}{d\alpha} > 0$, $\frac{dt^*}{d\delta_{ccs}} > 0$ and $\frac{dt^*}{dr} > 0$: the lower the depreciation or the absorption rate or the discount rate is, the earlier the mitigation should start. If the resource is scarce enough, the interval $[t', \bar{t};]$ is short. $t^* < t_1$: it is optimal to invest at time t^* before the ceiling rather than at time t', so, the ceiling will be reached later at time t_2, slightly after t_1, and so we still have $t^* < t_2$.(See figure 5.1) So, starting investments at the ceiling is not optimal. Even without natural dilution ($\alpha = 0$), this result still holds. □

Remark. If optimality requires drastic investments in CCS systems before the ceiling, the CO_2 concentration decreases after the investments before increasing to reach the ceiling, despite the decreasing gross emissions flow.

Claim. If $u''' < 0$, the ceiling binds only once. For some parameters and demand function presenting strong local convexity, the ceiling may bind over several separate interval of time if CCS is used. WORK IN PROGRESS

Proof. See the proof of the following claim. □

Proposition 5.2. *It is never optimal to invest in CCS capacities at two different dates before the CO_2 concentration reaches the carbon ceiling if the captured emissions flow is never constrained by the size of the gross emissions flow between these two dates.*

Proof. From equation 14, if CCS capacities are built up over an interval of time while $x(t) - K_{ccs}(t) > 0$ ($\sigma_t = 0$), by equation 11, $\mu_t = FC_{ccs}(r + \delta_{ccs})$ over that interval of time. Investing at two separate dates implies that $\eta_{ccs,t}$ is decreasing and then increasing, so deriving equation 11 trough time, if $x(t) - K_{ccs}(t) > 0$ between these two dates, it implies that a date $\hat{t}$ exists, such that $\ddot{\eta}_{ccs,\hat{t}} > 0$ and $\dot{\eta}_{ccs,\hat{t}} = 0$, so $\dot{\mu}_{\hat{t}} < 0$. Both cases are impossible since μ_t is strictly increasing before the ceiling by equation 10.
If investments occur before the ceiling over an interval of time, the constraint "capture from the flow must bind" over that interval of time. In they case, the CO_2 concentration must decrease, before increasing to reach the

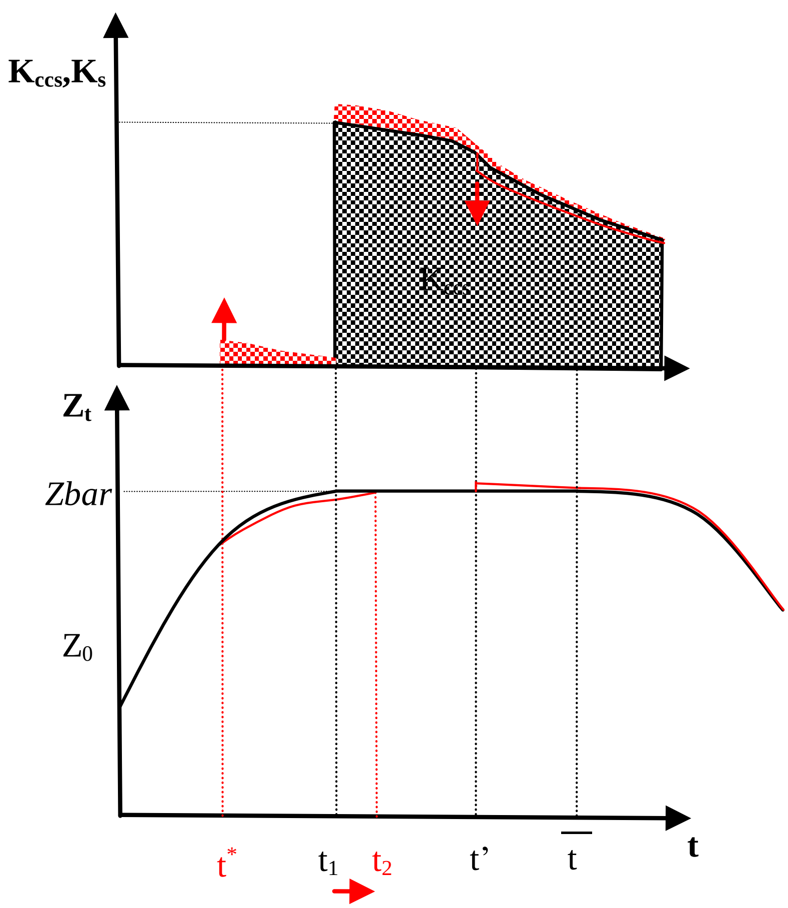

Figure 5.1: Figure Proposition 1

ceiling. The "Capture from the flow constraint" may bind only before the ceiling, since at the ceiling, non-captured emissions are strictly positive and equal $\alpha\overline{Z}$. If this constraint binds, it is still optimal to invest before the ceiling for some parameters, but the unique investment shot at time t^* when $\eta_{ccs,t^*} = \int_{t^*}^{\bar{t}} e^{-(r+\delta_{ccs})(j-t^*)}\mu_j dj$, is no more the solution. Investments occur at several dates rather than at a unique date. Between two dates of investments, the "capture from the flow" constraint must bind at least once (indeed, otherwise $\eta_{ccs,t}$ cannot decrease and then increase to reach FC_{ccs}). As indicated supra, there is no excess of CCS systems before $\mu_t = 0$ i.e before the ceiling stops to bind. Along the optimal path, t^* stands for the optimal date to deploy capture systems if CO_2 could be captured directly from the atmospheric stock. First units of CCS systems must be invested at that date, if there are investments before or after t^*, as closest as possible to t^* i.e when the constraint does not bind to reduce the overall mitigation cost. The constraint cannot be binding over an interval of time without investments. Indeed, if there is no investment over an interval when the constraint binds, $x(t) = K_{ccs}(t)$ and $\dot{K}_{ccs}(t) = -\delta_{ccs}K_{ccs}(t)$ so $\frac{\dot{x}(t)}{x(t)} = -\delta_{ccs}$, thus $x(t) = x(t_a)e^{-\delta_{ccs}(t-t_a)}$ with t_a the consumption level when investments start. Extraction writes $x(t) = u'^{-1}(p(t)) = u'^{-1}(c_x + \lambda_0 e^{(r+\alpha)t} + \mu_0 e^{(r+\alpha)t})$. Finally, it follows that $u'^{-1}(c_x + \lambda_0 e^{(r+\alpha)t} + \mu_0 e^{(r+\alpha)t}) = x(t_a)e^{-\delta_{ccs}(t-t_a)}$, which is not possible over a non-nil interval. It follows that investments before the ceiling occur over an interval of time if the constraint "capture from the flow" binds and exactly when the constraint binds. In any case, if investments start before the ceiling, they stop before the ceiling. Indeed, if the CO_2 concentration increases, installed capacities must be lower than the gross emissions flow. □

Claim. If $u''' \leq 0$, it is not possible to get a stop-and go pattern for investments in CCS. However for some parameters and demand function presenting strong local convexity, optimal investments may exhibit a stop-and-go pattern. WORK IN PROGRESS

Proof. If $u''' \leq 0$, μ_t cannot be decreasing and then increasing at the ceiling, so neither does η_t, and it is not possible to get a stop-and go pattern for investments in CCS. Indeed if μ_t is decreasing until t^* then increasing, thus $\dot{p}_{t^{*+}} > \dot{p}_{t^{*-}}$ due the increase of the scarcity rent, equivalent to $u''(x_{t^{*+}})\dot{x}_{t^{*+}} > u''(x_{t^{*-}})\dot{x}_{t^{*-}}$ i.e $u''(\alpha\overline{Z}+K_{t^{*+}})\dot{K}_{t^{*+}} > u''(\alpha\overline{Z}+K_{t^{*-}})\dot{K}_{t^{*-}}$; it implies that $u''(\alpha\overline{Z}+K_{t^{*+}}) < u''(\alpha\overline{Z}+K_{t^{*-}}) < 0$, since $0 > \dot{K}_{t^{*+}} > \dot{K}_{t^{*-}}$, thus u'' must be strictly increasing. A U-shape in μ_t dynamics trough time implies that $f(K) = -u''(\alpha\overline{Z}+K_{t^{*+}})\delta K_t$ is decreasing and thus $f'(K) = -u'''(\alpha\overline{Z}+K_t)K_t - u''(\alpha\overline{Z}+K_t) < 0$; $u''' > 0$ is clearly not a sufficient condition to get $f(K)$ decreasing. In addition, $\dot{p}_{t^{*+}} > \dot{p}_{t^{*-}}$ does not imply that μ_t is decreasing until t^* then increasing that does not imply that η_t exhibits a U-shape; thus this pattern may be not common.
A stop-and-go pattern for CCS investments would imply that the pollution cost decreases and then increases i.e the ceiling constraint, in spite of the increase of the scarcity rent, becomes more and more difficult to satisfy and due to the depreciation of CCS systems.The energy demand must decrease quickly enough in a first time and then decrease relatively slowly. We describe this specific case in Figure AA. Let us assume that $Z_0 = \overline{Z}$. We consider the following continuous demand function : for $p \leq \overline{p}_1$, $D(p) = \overline{D}$, for $\overline{p}_1 \leq p \leq \overline{p}_2$, $D(p) = \overline{D} - b(p - \overline{p}_1)$ and for $\overline{p}_2 \leq \overline{p}_3$, $D(p) = \underline{D}$, and for $\overline{p}_3 \leq p$, $D(p) = \underline{D} - h(p - \overline{p}_3)$ we assume that $\overline{D} > \alpha\overline{Z}$. If $p_0 \leq \overline{p}_1$ (condition 1), thus there are investments in a first period to allow for consumption $\overline{D}$, the quantity of installed capital is $\overline{K}$, such that $\overline{D} < \alpha\overline{Z} + \overline{K}$. By continuity of the energy price and due to the depreciation of CCS systems, investment cannot stop before $t_1$when $p_t = \overline{p}_1$ since the carbon stock would go over the ceiling. Over an interval of investments, as shown above, μ_t is constant and such that $\overline{\mu} = \mu_t = (r + \delta_{ccs})FC_{ccs}$, thus investments must stop at time $t_1$if $-D'(\lambda_{t_1} + \overline{\mu})\dot{\lambda}_{t_1} < -\delta_{ccs}\overline{K} < 0$ (condition 2) at time t_1 the ceiling stops to bind because the demand falls under $\alpha\overline{Z} + \overline{K}$ and since the price si continuous, μ_t cannot drop suddenly from $\overline{\mu}$ to 0 thus the shadow price cannot be

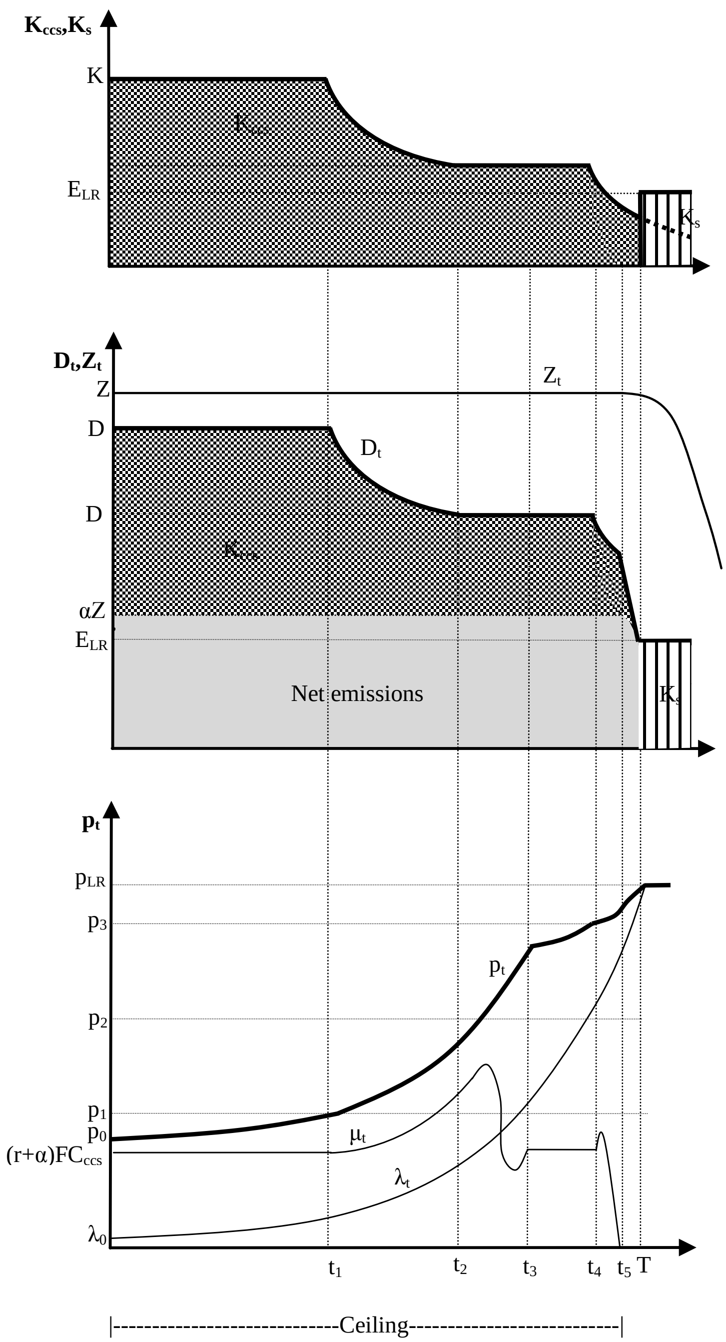

Figure 5.2: Figure Stop-and-go pattern in CCS investments

nil while the carbon stock is not at the ceiling and thus the ceiling constraint must bind in the future. μ_t must be increasing when the ceiling does not bind. Because CCS systems depreciate, there exist date t_3 at which the capacities just equal $\underline{D}$, $\overline{K}e^{-\delta_{ccs}(t_3-t_1)}$. From date t_3, defined above, the carbon stock restarts to increase until reaching the ceiling (at time t_4). The fossil fuel is still used at that time if $t_4 < T$ with $\int_0^T D(p_t)dt = Q_0$ and $\lambda_T = q$, or equivalently if $p_{t_4} < q$. The fossil fuel is not exhausted at time t_4 if $\int_0^{t_4} D(p_t)dt < Q_0$ since demand is globally decreasing, if $\int_0^{t_4} D(p_0)dt < Q_0$ i.e $t_4\overline{D} < Q_0$. If the fossil fuel is not yet exhausted, thus CCS investments must restart to maintain the consumption to the $\underline{D}$ level at time t_4. When the ceiling binds, μ_t must be decreasing at the ceiling and then increasing to reach $\overline{\mu}$ at time t_4.

- $-br\lambda_0 e^{rt_1} < -\delta_{ccs}\overline{K}$
- $\lambda_0 + \overline{\mu} < \overline{p}_1$
- $t_1\overline{D} + \frac{\overline{K}}{\delta}(1 - e^{-\delta_{ccs}(t_3-t_1)}) < Q_0$

□

We could check that for Q_0 high enough, and b high enough, $\overline{D}$ close enough to $\underline{D}$, the three conditions are satisfied and thus the ceiling binds twice and CCS investments exhibit a stop-and-go pattern.
In the case we describe, the ceiling binds twice which is another new feature of optimal path when CCS systems last and depreciate trough time. Assuming that the demand is no fixed for price higher than $\overline{p}_3$, would allow to get the ceiling binding while CCS investments will not necessarily restart.

Note that in CCS investment stop while the ceiling keeps on being binding, investments must restart date $t_3$described above if the fossil fuel is not yet exhausted at time t_3 and the demand equals $\underline{D}$ at that date i.e if $\int_0^{t_1} \overline{D}dt + \int_{t_1}^{t_3} \overline{K}e^{-\delta_{ccs}(t-t_1)}dt < Q_0$ i.e $t_1\overline{D} + \frac{\overline{K}}{\delta}(1 - e^{-\delta_{ccs}(t_3-t_1)}) < Q_0$.

To simplify the proof, we could consider the case where CCS systems cost nothing and thus the first best policy consist in extracting as in the pure Hotelling model without pollution cost, and show that there exist parameters such that CCS capacities are not fully used. See Figure 5.1.

Remark. In any case, there is no incentive to extend CCS capacities withing the period of time the ceiling binds since it would imply a discontinuity in the energy consumption at the ceiling.

Proposition 5.3. *Investments in solar plants never stop and help to maintain the energy consumption at a level, determined by the energy demand and the solar energy supply.*

Proof. Due to the exhaustibility of fossil fuels, energy is provided by solar plants in the long run. From equation 12, $\ddot{\eta}_{s,t} = (r+\delta_s)\dot{\eta}_t - u''(x(t)+K_s(t)).(\dot{x}(t_1)+\dot{K}_s(t_1))$, if investments in solar plants exhibit a stop-and-go pattern, there must exist a date t_1 at which $\ddot{\eta}_{s,t_1} > 0$, and $\dot{\eta}_{s,t_1} = 0$, so $\dot{x}(t_1)+\dot{K}_s(t_1) > 0$. Since $\dot{K}_s(t_1) < 0$, thus $\dot{x}(t_1) > 0$ thus $\dot{p}(t_1) < 0$ it is impossible both before the ceiling (see equation 13) and at the ceiling (an increase of $x(t)$ would imply an extension of CCS capacities i.e the price would decrease while the scarcity rent is going up and the pollution cost is increasing which is impossible as indicated supra). By equation 12, investments start to stabilize the energy level to E_{LR} such that $\frac{u'(E_{LR})}{r+\delta_s} = FC_s$. This long-run energy flow does not depend on the features of the exhaustible resource or on the stringency of the carbon policy, but only on the characteristics of solar plants and the energy demand. □

Corollary. *Since the extraction of fossil fuel is decreasing, the overall energy consumption is decreasing and then constant once solar energy start to be used.*

Proposition 5.4. *It is never optimal to invest in solar plants before the CO_2 concentration reaches the ceiling.*

Proof. Let us assume that it is optimal to invest in solar plants at date t_1 before the ceiling. By proposition 5.3, investments in solar plants never stop and help to maintain the energy consumption to E_{LR}. It follows that after t_1 and before the ceiling, energy production would be based on fossil fuels and solar energy whose costs must be equal. At time t, the marginal (increasing) cost of fossil fuel consumption is $p_t = c_x + \lambda_t + \mu_t$, while the marginal (constant) cost of solar energy all things given equal writes $FC_{ccs}(1 + \delta_{ccs}) - \frac{FC_{ccs}}{1+r}$ in current value (substituting investments at time t to later investments with the same amount of power plants after t). Obviously, we cannot have $p_t = FC_{ccs}(1 + \delta_{ccs}) - \frac{FC_{ccs}}{1+r}$ over an interval of time.
It is optimal to start investments in solar plants before the ceiling iff. $r = 0$ and $\delta_s = 0$. In that case, the discounted cost of investments is current trough time, and early investments allow to provide more energy for the same overall cost. In that case, it is optimal to invest as early as possible i.e at the beginning of the time period. Contrary to the case of CCS systems, the benefit of a delay in the building up of solar plants is not slacked by a bounded horizon of time since the solar plants provide energy on the whole path hereafter.
Our results are obviously driven by the assumption of constant investments costs for solar capacities. □

Proposition 5.5. *If there is a pool of renewables, with different constant investments costs and different depreciation rate, only one type of renewables is used along the optimal path.*

Proof. Let us assume two renewables$\{1; 2\}$ such that $\delta^1_R < \delta^2_R$; $FC^1_R > FC^2_R$. By equations 14 and 24, using both options along the same interval implies that $FC^1_R(r+\delta^1_R) = FC^2_R(r+\delta^2_R)$. Otherwise, only the cheapest option i such that $FC^i_R(r+\delta^i_R) < FC^{-i}_R(r+\delta^{-i}_R)$ is used[3]. Indeed, if we assume that option $-i$ is used at a date due to equation 10 and $FC^i_R(r+\delta^i_R) < FC^{-i}_R(r+\delta^{-i}_R)$, option i is not used at that date. Option i will be necessarily used again indeed, otherwise the energy level would converge to $FC^{-i}_R(r + \delta^{-i}_R) = u'(E^{-i}_{LR})$ and it would be possible to increase the energy level by investing in option i since $FC^i_R(r+\delta^i_R) < u'(E^{-i}_{LR})$. So, option i must be necessarily used at a latter date, thus its benefit must increase: we note t^* the date when the dynamics of $\dot{\eta}_R$ changes from negative to positive, at that time capacities of types $-i$ are increasing, thus investments in option $-i$ take place and so $\dot{\eta}^i = \dot{\eta}^{-i} = 0$ and using equation 24 we get: $FC^{-i}_R(r + \delta^{-i}_R) = u'(X_t + K^i_R + K^{-i}_R)$ and $\eta^i(r + \delta^i_R) < (FC^i_R(r + \delta^i_R)$ since option i is not used at date t^* and so $\dot{\eta}^i < (FC^i_R(r + \delta^i_R) - u'(X_t + K^i_R + K^{-i}_R) < FC^{-i}_R(r + \delta^{-i}_R) - u'(X_t + K^i_R + K^{-i}_R) = 0$ that contradicts the initial assumption. □

Claim 5.6. If $u''' < 0$, the ceiling binds only once. For some parameters and demand function presenting strong local convexity, the ceiling may bind over several separate interval of time if CCS is used. WORK IN PROGRESS

Proof. If CCS is not used, the carbon stock is increasing until reaching the carbon ceiling, then it is kept at the carbon ceiling level and then decreases toward zero. In any case, the ceiling constraint is only binding over an interval of time, thus when the carbon stock falls under the carbon ceiling, it will stay definitively under the carbon ceiling. Indeed, if the carbon constraints binds twice, it implies that the carbon stock decreases and the increases again whereas the energy demand based on fossil fuels decreases since the fossil fuel price increases when the ceiling does not bind.
Considering now the case where CCS is used, it comes that when $u''' < 0$ the ceiling binds only once. Indeed, if $u''' \leq 0$ thus $\dot{D}$ is negative and decreasing when the price increases, whereas by assumption $\dot{K}$ is negative and

[3] In real life, the use of a pool of renewables is driven by the heterogeneity of costs of renewables trough space, the intermittency of power generation and the instantaneous increasing cost (of investment or use) of some renewables. Heterogeneous technical change can be an additional reason.

increasing, thus when once $0 > \dot{K} > \dot{D}$, hereafter $0 > \dot{K} > \dot{D}$, and thus $D(pt) < \alpha\overline{Z} + K_{ccs,t}$ and the carbon stock keeps on decreasing and thus the ceiling will no more bind. If the ceiling binds twice, the demand must be such that given the existing CCS capacities, the energy demand is relatively low in a first time, and in a second time given the depreciation of the CCS capacities, the energy demand is relatively strong to make the carbon stock increases to reach the ceiling again. We leave that point for further examination.
As shown in proof of claim 5 for some specific demand function exhibiting string local convexity, it is possible that the carbon stock binds twice. $\square$

Proposition 5.7. *If there is a pool of CCS technologies, with different constant investments costs and different depreciation rates, several types of CCS technologies can be used along the optimal path.*

Proof. If there are two CCS technologies $\{1; 2\}$, such that $\delta^1_{ccs} < \delta^2_{ccs}$ and $FC^1_{ccs} > FC^2_{ccs}$, both can be used along the same path but not at the same time. For instance, we can show that there exist parameters such that using CCS only at the ceiling is preferable than using CCS before the ceiling for any CCS type (Condition 1) and using only one CCS type leads to using CCS of option 1 (Condition 2), and that there is still an incentive to change of type of CCS over some period of time (condition 3). To simplify, we assume that the energy demand equals $\overline{D}$ thus we directly get the date of exhaustion T, $T = \frac{Q_0}{\overline{D}}$ and we assume that $\delta^2_{ccs} = 1$, thus option 2 cannot be used before the ceiling.
From the constant demand assumption, it comes that investing before the ceiling implies to hit the ceiling constraint only at date T when the fossil fuel is exhausted. Investing before the ceiling at the cheapest cost requires to choose the date that minimizes $f_{t,t'} = \frac{e^{-rt}FC_{ccs}}{\int_t^{t'} e^{-\delta^1_{ccs}(j-t)}dj}$ as indicated supra. $\dot{f}_{t,t'}$ has the sign of $-re^{(-\delta^1_{ccs})t} + (\delta^1_{ccs}+r)e^{(-\delta^1_{ccs})t'}$ and $\ddot{f}_{t,t'} > 0$. ($\dot{f}_{t^*,t'} = 0$ and $t^* = \frac{ln(\frac{\delta^1_{ccs}}{r})}{-\delta^1_{ccs}} + T$). The quantity of invested capital at time t_{ccs}, K^1_{ccs} must satisfy this condition: $Z_0 + \int_0^{t_{ccs}} \overline{D}ds + \int_{t_{ccs}}^T (\overline{D} - K^1_{ccs}e^{-\delta^1_{ccs}(s-t_{ccs})})ds = \overline{Z}$. We get $K^1_{ccs} = \frac{\delta(\overline{Z}-Z_0-Q_0)}{e^{-\delta^1_{ccs}[\frac{\overline{D}}{Q_0}-t_{ccs}]}-1}$. The cost of investing before the ceiling finally writes $C_1 = FC^1_{ccs} * K^1_{ccs}e^{-rt_{ccs}} = FC^1_{ccs}\frac{\delta^1_{ccs}(\overline{Z}-Z_0-Q_0)}{e^{-\delta^1_{ccs}[\frac{\overline{D}}{Q_0}-(\frac{ln(\frac{\delta^1_{ccs}+r}{r})}{-\delta^1_{ccs}}+T)]}-1}e^{-r(\frac{ln(\frac{\delta^1_{ccs}+r}{r})}{-\delta^1_{ccs}}+T)}$, when this action is feasible.
With no investments before the ceiling, the ceiling is reached at time $\underline{t}$ such that $Z_0 + \int_0^{\underline{t}} \overline{D}ds = \overline{Z}$, so $\underline{t} = \frac{\overline{Z}-Z_0}{\overline{D}}$.
From $\underline{t}$ until date T, investments enable to consume $\overline{D}$, the cost of investing at the ceiling writes $C_2 = FC^1_{ccs}\overline{D}e^{-r\underline{t}} + \int_{\underline{t}}^T FC^1_{ccs}\delta^1_{ccs}\overline{D}e^{-rt}ds = FC^1_{ccs}\overline{D}[\frac{r+\delta^1_{ccs}}{r}e^{-r\frac{\overline{Z}-Z_0}{\overline{D}}} - \frac{\delta^1_{ccs}}{r}e^{r\frac{Q_0}{\overline{D}}}]$.
Condition 1 is equivalent to $C_2 < C_1$. For Q_0, high enough, it is too costly to invest before the ceiling rather than at the ceiling, and for $Q_0 > Q_{max}$ it becomes clearly impossible to satisfy the demand and the ceiling constraint by investing before the ceiling only, and so for Q_0 high enough, investing at the ceiling is a better option than investing before the ceiling. We take Q_0 sufficient high to satisfy condition 1.
Condition 2 is equivalent to $C_1 < FC^2_{ccs}\overline{D}[\frac{r+1}{r}e^{-r\frac{\overline{Z}-Z_0}{\overline{D}}} - \frac{1}{r}e^{r\frac{Q_0}{\overline{D}}}]$. It is always possible to find parameters such that Condition 2 and condition 1 are simultaneously satisfied with $FC^2_{ccs} < FC^1_{ccs}$.

Finally, considering the last unit of capital invested at time T to keep the ceiling just binding, it would be cheaper to invest this last unit in technology 2. Indeed the difference between investing in technology 2 and technology 1 at the ceiling writes: $g_{t^*} = \int_{t^*}^T \overline{D}FC^2_{ccs}e^{-rs}ds + [FC^2_{ccs}\delta^1_{ccs} - FC^1_{ccs}\delta^1_{ccs}]\overline{D}e^{-rt^*} - \int_{t^*}^T \delta^1_{ccs}\overline{D}FC^1_{ccs}e^{-rs}ds$. $g_{t^*}$is negative for t^* close enough to T.

In conclusion, there exist parameters such that using both types of CCS technologies along the extraction path is optimal in spite of constant investments costs. $\square$

Claim. If two types of CCS technologies are used before the ceiling or if $u''' \leq 0$, the longest-life type of technologies being necessarily used in first place.

Proof. A necessary condition to use the long life option 1 is that $(r+\delta_1)FC^1_{ccs} < FC^2_{ccs}(r+1)$. Indeed, the cost of replacing a unit of capital of type 1 at time t^* by capital of type 2 all things given equal should be positive: $C = (FC^2_{ccs} - FC^1_{ccs})e^{-rt^*} + FC^2_{ccs}\int_{t^*}^{\bar{t}}(1-\delta_1)e^{-rs}e^{-\delta^1_{ccs}(s-t^*)}ds < -(r+\delta^1_{ccs})FC^1_{ccs} + FC^2_{ccs}(r+\delta^1_{ccs}) + FC^2_{ccs} - \delta^1_{ccs}FC_2$ that implies $FC^1_{ccs}(r+\delta^1_{ccs}) < FC^2_{ccs}(r+1)$. Assuming that $(r+\delta^1_{ccs})FC^1_{ccs} < FC^2_{ccs}(r+1)$ and option 2 is used before option 1 at time t_2, it comes $FC^2_{ccs}(r+1) = \mu_{t_2}$ and $\eta^1_{t_2} < FC^1_{ccs}$; if $u''' \leq 0$, η^1_t cannot decrease and then increase so η^1_t is increasing at time t_2, $(r+\delta^1_{ccs})\eta^1_{t_2} > \mu_{t_2}$that implies $(r+\delta^1_{ccs})FC^1_{ccs} > FC^2_{ccs}(r+1)$, that contradicts previous assumption. For any demand function, η^1_t cannot decrease before the ceiling and so it is not possible to use option 2 and then option 1 before the ceiling. $\square$

Proposition 5.8. *If current investment costs are strictly increasing with the amount of current investments, investments are diluted trough time to avoid expensive investment costs: investments in CCS systems necessarily start before the ceiling and investments in solar plants may start before the ceiling if their cost is low enough.*

Proof. The costs of investments in CCS or solar capacities are increasing with the quantity of investments at a date i.e the production function of low-carbon capacities exhibits decreasing returns to scale. Writing $FC(I_t)$ the cost of investing I_t at time t, with $FC'(I_t) > 0$ and $FC''(I_t) > 0$, equation 14 becomes $FC_i(I^*_t) = \eta_{i,t} + \gamma_{i,t}$. Let us assume that investments exhibit a discrete jump from 0 to $I^* > 0$ at time t^*; the marginal cost of investments is $FC'(0)$ on the left of $t^*$and $FC'(I_{t*})$ on the right of t^*while the benefit function of investment is continuous. Similarly, there cannot be a discrete stop in the investments in low-carbon options.

It follows that investments in CCS necessarily start before the ceiling: if the resource abundant enough, CCS investments start before the ceiling and last over an interval of time at the ceiling too, otherwise they last over an interval of time strictly before the ceiling. The marginal gross benefit is increasing then decreasing or simply decreasing if investments start at the beginning of the time period, so, investments are increasing then decreasing trough time or only decreasing if they start from the beginning.

For solar plants, drastic investments are no more optimal: if they are cheap enough, solar investment may start before the ceiling; in any case, they start before the exhaustion of the fossil fuel. The energy demand tends to a limit, $K_{s,LR}$, such that the cost of investments equals the marginal benefit of solar plants ($\frac{u'(K_{s,LR})}{r+\delta} = FC'(K_{s,LR})$). Without pollution concerns i.e once the ceiling no more binds in our framework, the transition from the exhaustible resource to the backstop is as described in Amigues et al. [2011]. $\square$