

# Loss Leading as an Exploitative Practice

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## Abstract

Large retailers often compete with smaller stores carrying a narrower range. We find that large retailers can exercise market power by pricing below cost some of the products also offered by smaller rivals, in order to discriminate multi-stop shoppers from one-stop shoppers. Loss leading then appears as an exploitative device rather than as an exclusionary instrument, although small rivals are hurt in the process; banning below-cost pricing increases consumer surplus, rivals' profits, and social welfare. Our insights extend to industries where established firms compete with entrants offering fewer products. They also apply to complementary products such as platforms and applications.

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# 1 Introduction

The last three decades have seen the emergence of large supermarkets offering a full range of groceries and other goods, to attract consumers through one-stop shopping. More recently, we have also witnessed an increase in concentration – due partly to zoning regulations which, by limiting their internal growth, have encouraged retail chains to merge and to acquire independent stores. As a result, at the local level these large supermarkets often have substantial market power,<sup>1</sup> and mainly compete with much smaller stores.<sup>2</sup>

While large retailers can exert their market power in various ways,<sup>3</sup> the recent literature on retail power has mostly focused on *buyer power* (against suppliers) rather than on *seller power* (against consumers and smaller rivals). Yet, as reflected by policy debates,<sup>4</sup> it is the large retailers' ability to distort competition that may bring the most profound market effects. This paper shows that large retailers can exercise their seller power by adopting a *loss leading* strategy, which consists of pricing below cost some of the competitive products (leader products) and charging higher prices for the other goods. This practice is indeed widely adopted by large retailers: in its groceries market investigation, the UK Competition Commission notes for example that most supermarket chains engage in loss leading, mainly for staples such as milk and dairy, alcohol, bread and bakery products that consumers purchase repeatedly and regularly – and which constitute the core product lines of small outlets such as hard-discount stores; it finds that loss leaders represent up to 6% of a retailer's total sales.<sup>5</sup>

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<sup>1</sup>In the UK, where the number of supermarket chains went from 7 to 4 within a few years, the Competition Commission (2008) reports that 27% of larger grocery stores are located in "highly-concentrated local markets" (three or fewer fasciae, and one having more than 60% of grocery sales within a 10-minute drive-time; see section 6). It also finds that the impact on a large retailer's profit from another large retailer is less than 4%, and that from small retailers is statistically insignificant (Appendix 4.4 at § 47).

<sup>2</sup>In France, where large retailers have stores exceeding 10,000 sq. mt. (up to 24,000+ sq. mt. for Carrefour), zoning regulations have limited the size of new entrants to 300 sq. mt. (now 1,000 sq. mt.).

<sup>3</sup>See Dobson and Waterson (1999) for a detailed discussion.

<sup>4</sup>See for example the reports of the US Federal Trade Commission (2001, 2003), the proceedings of the FTC conference held on May 24, 2007, available at <http://www.ftc.gov/be/grocery/index.shtm>, or the groceries market enquiries of the UK Competition Commission (2000, 2008) recommending the adoption of codes of practices. In France, two Acts adopted in 1996 aimed at curbing the expansion of large retailers as well as the exploitation of their market power.

<sup>5</sup>See Competition Commission (2008); Dobson (2002) also provides a detailed economic analysis of

Loss leading has stirred hot debates in antitrust circles. In the U.K., for example, in its first sector inquiry the Competition Commission expressed the concern that loss leading "may have a predatory impact on small and specialist retailers" and limit the growth of particular retailers such as hard discounters;<sup>6</sup> yet in its second inquiry it dismisses the concern and finds instead that loss leading "may represent effective competition between retailers and may benefit consumers by reducing the average price for a basket of products".<sup>7</sup> In Germany, in 2000 the Federal Cartel Office ordered dominant supermarket chains to stop selling below cost staples including milk and butter, arguing that this could impair competition and force smaller retailers to exit the market. By contrast, OECD (2007) argues that rules against loss leading are likely to protect inefficient competitors and harm consumers. A similar discrepancy appears in below-cost resale statutes.<sup>8</sup>

In the economic literature, loss leading has been viewed as an advertising strategy adopted to attract consumers who are imperfectly informed of prices;<sup>9</sup> however, this may be less relevant for routine grocery shopping, where consumers seem to be reasonably aware of prices.<sup>10</sup> Loss leading has also been interpreted as optimal cross-subsidization by a multi-product monopolist facing different demand elasticities across products;<sup>11</sup> in

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loss-leading pricing in UK grocery retailing, with particular emphasis on bakery retailers.

<sup>6</sup>Competition Commission (2000), p. 132.

<sup>7</sup>Competition Commission (2008), p. 9; it notes however that loss leading may have the "unintended effect" of disproportionately squeezing smaller rivals' margins and even forcing them to exit (see p. 96).

<sup>8</sup>In the US, 22 states are equipped with general sales-below-cost laws, and 16 additional states prohibit below-cost sales on motor fuel. In the EU, below-cost resale is banned in Belgium, France, Ireland, Luxembourg, Portugal, and Spain, and is restricted in other countries including Austria, Denmark, Germany, Greece, Italy, Sweden and Switzerland, whereas it is generally allowed in the Netherlands and the UK. See Skidmore *et al.* (2005) and Calvani (2001).

<sup>9</sup>Lal and Matutes (1994), for example, consider a situation where multi-product firms compete for consumers who are initially unaware of prices, and find that in equilibrium firms may indeed choose to advertise loss leaders in order to increase store traffic. Ellison (2005) develops the model to analyze add-on pricing, and shows that loss leading can be optimal when firms only advertise base goods.

<sup>10</sup>See e.g. Competition Commission (2008), stating at p. 97 that consumers' price comparisons depended "not only on the price of a selection of known-value items, but also on the basket price and many other factors", and concluding that loss leading was "unlikely to mislead consumers in relation to the overall cost of shopping at a particular grocery store".

<sup>11</sup>Bliss (1988) already views loss leading as a cross-subsidizing strategy, but does not formally establish existence conditions. Beard and Stern (2008) allow for continuous consumer demands and show that loss

practice, however, large retailers' choice of loss leaders appears driven by the competition from smaller stores on specific products.

By contrast, little attention has been devoted to the above-mentioned concerns, relating to the potential adverse effect on smaller rivals and consumers. And while it may be tempting to treat loss leading as predatory pricing,<sup>12</sup> the persistence of below-cost sales over time does not fit with a scenario in which the predator would seek to recoup the losses incurred during the predation phase by raising the prices afterwards, once rivals have been pushed out of the market. This begs several related questions: what is the rationale for loss leading if it is not predatory? What is then the impact on rivals, consumers and society?

This paper aims at filling this gap. We develop a model of asymmetric competition between large and small retailers, reflecting key features of these different formats.<sup>13</sup> Large stores offer a wide range of products, which gives them substantial flexibility in their pricing policies and allows consumers to fulfil their needs in one stop, whereas smaller stores focus instead on limited product selections on which they can offer better value. For instance, specialist retailers can offer higher quality or more services. Another successful example is provided by hard discounters – part of a chain themselves<sup>14</sup>. They focus on a limited range of basic goods for daily need in order to generate quick turnovers<sup>15</sup>

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leading can indeed arise although for rather specific demand functions. Ambrus and Weinstein (2008) study symmetric competition for one-stop shoppers, and show that loss leading can arise only when consumer demand is elastic and exhibits rather specific forms of complementarity. The scope for loss leading in these settings, as well as its impact on consumers and welfare, still needs to be assessed.

<sup>12</sup>See e.g., Bolton, Brodley and Riordan (2000) and Eckert and West (2003) for detailed discussions of predatory-pricing tests.

<sup>13</sup>Buyer power may reinforce seller power when suppliers, forced to offer better conditions to large retailers, are led to increase their prices to smaller retailers – see Dobson and Inderst (2007), and Inderst and Valletti (2008). However, as noted below, small stores often belong to large chains and thus also enjoy buyer power. We will thus assume away here any buyer power asymmetry, and focus specifically on how large retailers can use their *seller* power at the expense of consumers and smaller rivals.

<sup>14</sup>The leading discount chains, Aldi and Lidl, together account for more than 50% of discount sales in Europe (Cleeren *et al.* 2010); and in the U.S., where the hard discount format has emerged more recently, Aldi had already opened more than 1000 stores in 2009, either in the name of Trade Joe's or Aldi stores (Steenkamp and Kumar 2009). Another price-aggressive hard discounter, Dollar General, has already opened 9,000 local stores in the US.

<sup>15</sup>According to Competition Commission (2008), hard discounters typically stock less than 1500 store

which, together with the reliance on private labels, allows them to negotiate better supply conditions; they moreover adopt a no-frills approach so as to minimize their operating costs.<sup>16</sup> This low-cost business model allows hard discounters to offer prices that are up to 60% lower than those of leading name brands, and 40% lower than large retailers' own labels (Cleeren *et al.* 2010).

We abstract away from the above-mentioned efficiency justifications by assuming that consumers are perfectly informed of all prices and by allowing for homogeneous consumer valuations for the goods. Our key modelling feature is instead to account for the heterogeneity in consumers' shopping costs: some consumers face higher shopping costs, e.g., because of tighter time constraints or lower taste for shopping, and thus have a stronger preference for one-stop shopping, whereas others have lower shopping costs and can therefore benefit from multi-stop shopping.<sup>17</sup>

We first present the main insights in a stylized setting where consumers have homogeneous valuations over a range of products offered by a large retailer, who faces a competitive fringe of smaller but more efficient rivals on some of the goods (the competitive segment). Consumers with low shopping costs then buy the monopolized goods from the large retailer and the competitive goods from the small ones, as they offer better value on those goods. Consumers with higher shopping costs can choose between buying the full range from the large retailer, or buying only the competitive goods from the smaller rivals. We show that the larger retailer adopts a loss leading strategy whenever its broader range allows it to win the competition for these one-stop shoppers. The intuition is that, while the presence of the smaller rivals generates a competitive pressure, it also allows the large retailer to screen multi-stop shoppers from one-stop shoppers. Indeed, pricing the competitive goods below cost, and raising the price for the monopolized goods accordingly, does not affect the total price charged to one-stop shoppers but increases the margin earned on multi-stop shoppers in the monopolized segment. Loss leading thus appears as an effective exploitative device that allows the large retailer to increase its

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keeping units (SKUs), whereas supermarkets normally offer more than 150,000 SKUs.

<sup>16</sup>See for example Sachon (2009) for a discussion of the hard-discount business model.

<sup>17</sup>According to the marketing literature, patronizing multiple stores becomes an important pattern in the grocery retail business, see for instance Gijsbrechts *et al.* (2008). Stassen *et al.* (1999) concluded from the Progressive Grocer Reports (1990-1997) in the U.S. that roughly 75% of all grocery shoppers regularly shopping more than one store each week.

profit at the expense of consumers – when the comparative advantage conferred by its broader range is large enough, the large retailer can even obtain in this way more profit than in the absence of the smaller rivals.

We then extend the analysis to the case where the large retailer faces a strategic rival rather than a fringe in the competitive segment, in which case loss leading also hurts the rival by reducing its market share and squeezing its profit margin.<sup>18</sup> However, this margin squeeze appears here to be a by-product of exploitation rather than driven by exclusionary motives; indeed, it is the very presence of a rival, offering better terms on some products, that allows the large retailer to screen consumers according to their shopping costs. Yet, the lack of exclusionary intention, as well as the fact that the small retailers remain active, should not lead to the conclusion that loss leading is an innocuous strategy, since it hurts both consumers and rivals. We show that a ban on loss leading would discipline the large retailer and benefit consumers as well as the small rival, and would also increase social welfare by improving the distribution efficiency in the competitive segment.

We also show that loss leading still arises in more general settings with heterogeneous consumer valuations for the goods, product differentiation in the competitive segment, and (imperfect) competition among large retailers. The exploitative use of loss leading appears to be a robust feature in market environments where a few large retailers enjoy substantial market power over one-stop shoppers and compete with rivals who focus on narrower product lines, where they benefit from lower costs or better quality.

While this paper has been motivated by the use of loss leading in retail markets, its insights apply to a variety of situations where: (i) a firm enjoys substantial market power in one market and faces tough competition in other markets; (ii) dealing with a single supplier gives customers some benefits (e.g. due to scale economies, lower adoption or maintenance costs, etc.), which vary across customers. Pricing below cost in the competitive markets then allows the larger firm to screen customers more effectively and extract part of the benefits. This insight can shed a new light on antitrust cases such as the *IBM* and *Microsoft* cases;<sup>19</sup> while the debates have mainly focused on exclusionary

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<sup>18</sup>This is consistent with Dobson (2002) who, in a report for the Federation of Bakers, argues that the structure of the U.K. retail market, and the mix of different retail formats, is particularly conducive to the emergence of loss leading, as a form of competitive price discrimination which could lead to higher prices on other products, thus harming consumers as well as squeezing smaller rivals' profits.

<sup>19</sup>See e.g. *United States v. International Business Machines Corporation*, Docket number 69 Civ. DNE

purposes, our analysis suggests an alternative conceptual framework based instead on exploitative motives.

This paper can also be related to the literature on bundling and tying (particularly for IT goods which have negligible production costs, where both loss leading and bundling amount to giving the product for free). Part of this literature has focused on the use of (possibly mixed) bundling on price discrimination, in setups in which there is competition with (often symmetric) product differentiation in each segment; the main insight is that bundling tends to limit double marginalization problems and thus intensify competition.<sup>20</sup> Building on this literature, Armstrong and Vickers (2010) consider a symmetric duopoly in which consumers that "mix and match" incur an additional shopping cost; they show that, while prices remain above (or at) cost, mixed bundling tends to raise profit at the expense of consumers. Another part of the literature has focused instead on the use of tying as an entry deterrence device, e.g. by committing to fiercer competition in case of entry or by reducing the value of entering into a single market.<sup>21</sup> By contrast, loss leading has little impact here on the total price at which large firms offer their bundles, and does not intend to be exclusionary; instead, it primarily increases the price charged on the less competitive segment to those consumers who have lower shopping or adoption costs.

## 2 Loss leading as an exploitative device

We present here the main intuition in a stylized setting in which a large retailer ( $L$ ), supplying a broad range of products, competes with a fringe of smaller retailers ( $S$ ) that focus on a much narrower product line. For the sake of exposition, we simply assume that there are two markets (which can be interpreted as different goods or product lines):  $A$  is monopolized by  $L$ , whereas  $B$  can be supplied by  $L$  and  $S$ . Consumers desire at most one unit of  $A$  and one unit of  $B$ <sup>22</sup> and, to rule out cross-subsidy motives based on demand

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(S.D. NY) and *United States of America v. Microsoft Corporation*, Civil Action No. 98-1232 TPJ (D.C.).

<sup>20</sup>See e.g. Matutes and Regibeau (1988).

<sup>21</sup>See e.g. Whinston (1990), Nalebuff (2004), Carlton and Waldman (2002) and Choi and Stefanadis (2001); Rey and Tirole (2007) offers a review of this literature.

<sup>22</sup>The assumption of unit demands appears reasonable for groceries and other day-to-day consumer purchases. To be sure, price changes affect the composition of consumer baskets, but are less likely to have a large impact on the volume of purchases for staples.

elasticity differences such as in Bliss (1988), they have homogeneous valuations.

Consumers incur a shopping cost  $s$  for visiting a store, which reflects the opportunity cost of the time spent in traffic, parking, selecting products, checking out, and so forth; it may also account for the consumer's taste for shopping. Our key modelling feature, reflecting the fact that consumers may be more or less time-constrained, or value their shopping experience in different ways, is that the shopping cost  $s$  varies across consumers.

## 2.1 A simple example

A numerical example can illustrate the intuition. Suppose that  $L$  can supply  $A$  at no cost and  $B$  at unit cost  $c = 4$ , while consumers value  $A$  at  $u_A = 10$  and  $B$  at  $u_B = 6$ . Suppose further that half of the consumers face a high shopping cost  $s = 4$ , whereas the others can shop at no cost. If  $L$  were alone, it would supply all consumers at a total price (slightly below)  $p^m = u_A + u_B - s = 12$ , yielding a monopoly profit  $\Pi^m = 12 - 4 = 8$ .<sup>23</sup>  $L$  would thus extract all surplus from high-cost consumers but leave the others a surplus of 4, reflecting the difference in shopping costs.

Suppose now that  $B$  is also offered by a competitive fringe  $S$  at a price  $\hat{p} = 2$ .  $S$  cannot attract high-cost consumers, who would obtain  $u_B - \hat{p} - s = 0$ ;  $L$  can therefore still charge them a total price of  $p^m$ .  $L$  could for example price  $B$  at cost ( $p_B = 4$ ) and charge the rest on  $A$  ( $p_A = 8$ ):  $L$  would then sell  $A$  only to low-cost consumers (who become "multi-stop shoppers" and buy  $B$  from  $S$ ) and yet obtain the monopoly margin on both types of consumers:  $p^m - c = p_A = 8$ . However, the presence of the small rivals opens a door for screening consumers according to their shopping costs, and this is best achieved by selling  $B$  below cost; keeping the total price equal to  $p^m$ , lowering the price for  $B$  down to  $\hat{p} = 2$ , and increasing the price for  $A$  to  $\hat{p}_A = 10$ , does not affect the shopping behavior of high-cost consumers (who still face a total price of  $p^m$ ), but increases the margin earned on multi-stop shoppers (since  $\hat{p}_A > p_A$ ). This loss-leading strategy thus allows  $L$  to charge the monopoly price to one-stop shoppers, and actually extracts here the full value of  $A$  from multi-stop shoppers;<sup>24</sup> as a result, it earns a total profit  $\Pi = 9$ , which is greater than in the absence of  $S$ .

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<sup>23</sup>Selling only to low-cost consumers at a total price  $p = 16$  yields a lower profit  $(16 - 4)/2 = 6 < \Pi^m$ .

<sup>24</sup>This is the best  $L$  can achieve with low-cost shoppers, who are willing to pay at most  $\hat{p} < c$  for  $B$ .



## 2.2 Baseline model

We now consider more general supply and demand conditions, and in particular allow  $L$  and  $S$  to offer different varieties in the competitive market,  $B_L$  and  $B_S$ . We denote by  $u_A$ ,  $u_L$  and  $u_S$  the consumer valuations for  $A$ ,  $B_L$  and  $B_S$ , and by  $c_A$ ,  $c_L$  and  $c_S$  the (constant) unit costs. We are interested in the case where small retailers are more efficient in distributing  $B$  (otherwise,  $S$  would not sell anything, and multi-stop shopping would never arise):  $w_S \equiv u_S - c_S > w_L \equiv u_L - c_L (> 0)$ ; for instance,  $S$  can include chained, cost-cutting hard-discounters ( $c_S < c_L$ ) or specialist stores that offer better quality or more service ( $u_S > u_L$ ).  $L$  however benefits from its broader range ( $w_A \equiv u_A - c_A > 0$ ), and may overall offer a higher or lower value:  $w_{AL} \equiv w_A + w_L \gtrless w_S$ . Finally, we allow for continuous distributions of the shopping cost  $s$ , characterized by a cumulative distribution function  $F(\cdot)$  and a density function  $f(\cdot)$ ; to ensure the concavity of  $L$ 's profit, we assume that the inverse hazard rate,  $h(\cdot) \equiv F(\cdot)/f(\cdot)$ , is strictly increasing. Intuitively, consumers with a high  $s$  favor one-stop shopping, whereas those with a lower  $s$  can take advantage of multi-stop shopping and buy  $A$  from  $L$  but  $B$  from  $S$ ; the mix of multi-stop and one-stop shoppers is however endogenous and depends on  $L$ 's prices,  $p_A$  and  $p_L$ .

In the absence of  $S$ , consumers would buy both products from  $L$ <sup>25</sup> as long as  $s \leq v_{AL} \equiv u_A + u_L - p_A - p_L = w_{AL} - r_{AL}$ , where  $v_{AL}$  denotes the consumer value from purchasing  $A$  and  $B_L$ , and  $r_{AL} \equiv p_A - c_A + p_L - c_L$  denotes  $L$ 's total margin. The monopolist would thus face a demand  $F(v_{AL})$  and make a profit  $r_{AL}F(v_{AL}) = r_{AL}F(w_{AL} - r_{AL})$ . This profit function is quasi-concave in  $r_{AL}$ <sup>26</sup> and the monopoly outcome is thus characterized by the first-order condition:

$$r_{AL}^m = h(v_{AL}^m), \quad (1)$$

which, using  $r_{AL}^m = w_{AL} - v_{AL}^m$ , yields<sup>27</sup>

$$v_{AL}^m \equiv l^{-1}(w_{AL}), \quad (2)$$

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<sup>25</sup>The assumptions  $w_A, w_L > 0$  imply that it is more profitable to sell both products rather than one.

<sup>26</sup>Its derivative is of the form  $f(w_{AL} - r_{AL})\phi(r_{AL})$ , where  $\phi(r_{AL}) \equiv h(w_{AL} - r_{AL}) - r_{AL}$  is strictly decreasing. A similar reasoning applies below to the other profit functions of  $L$  and  $S$ .

<sup>27</sup>We implicitly assume away here any relevant upper bound on shopping costs. If  $s$  is instead distributed over a range  $[0, \bar{s}]$ , where  $\bar{s} \leq l^{-1}(w_{AL})$ , then the optimal (monopoly) value is  $v_{AL}^m = \bar{s}$  and the corresponding profit is  $w_{AL} - \bar{s}$ .

where  $l(s) \equiv s + h(s)$  is increasing in  $s$ ; the associated monopoly profit is  $\Pi_{AL}^m \equiv r_{AL}^m F(v_{AL}^m)$ .

Assume now that the fringe  $S$  supplies  $B_S$  at cost ( $p_S = c_S$ ), thus offering consumers a value  $v_S = w_S$ , and consider  $L$ 's response (we only sketch the reasoning here, and provide a complete analysis in Appendix A). If  $v_{AL} \geq w_S$ , one-stop shoppers prefer  $L$  to  $S$  ("regime  $L$ "), and they are indeed willing to patronize  $L$  as long as  $s \leq v_{AL}$ ; however, consumers favor multi-stop shopping if the additional cost of visiting  $S$  is lower than the extra value it offers, which we will denote by  $\tau$ :

$$s \leq \tau \equiv w_S - (w_L - r_L),$$

where  $r_L \equiv p_L - c_L$  denotes  $L$ 's margin on  $B_L$ . Thus, in regime  $L$  consumers visit  $L$  as long as  $s \leq v_{AL}$ , and visit both stores if  $s \leq \tau$  (see Figure 1).<sup>28</sup>

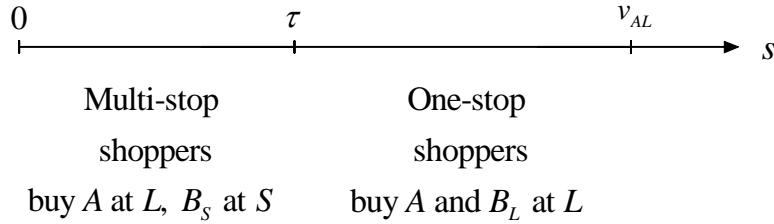


Figure 1: Regime  $L$

$L$  therefore attracts a demand  $F(v_{AL}) - F(\tau)$  for both products (from one-stop shoppers) and an additional demand  $F(\tau)$  for product  $A$  only (from multi-stop shoppers); it thus obtains a profit equal to:

$$r_{AL} (F(v_{AL}) - F(\tau)) + r_A F(\tau) = r_{AL} F(v_{AL}) - r_L F(\tau),$$

where  $r_A \equiv p_A - c_A = r_{AL} - r_L$  denotes  $L$ 's margin on  $A$ . Since  $v_{AL} = w_{AL} - r_{AL}$  and  $\tau = w_S - w_L + r_L$ , this profit expression is additively separable (and quasi-concave) in  $r_{AL}$  and  $r_L$ ; the optimal pricing policy in regime  $L$  thus consists in maximizing  $r_{AL} F(v_{AL}) = r_{AL} F(w_{AL} - r_{AL})$  w.r.t.  $r_{AL}$ , subject to  $v_{AL} = w_{AL} - r_{AL} \geq w_S$ , and minimizing  $r_L F(\tau) = r_L F(w_S - w_L + r_L)$  w.r.t.  $r_L$ . But the latter obviously leads to  $r_L < 0$ , that is, to selling  $B_L$  below cost.

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<sup>28</sup>Appendix A shows that prices leading to  $\tau < 0$  (resp.,  $\tau > v_{AL}$ ) are equivalent to prices yielding  $\tau = 0$  (resp.,  $\tau = v_{AL}$ ); therefore, without loss of generality, we can restrict attention to  $\tau \in [0, v_{AL}]$ .

We thus obtain our first insight: whenever  $L$  offers better value for one-stop shoppers (regime  $L$ ), it adopts a loss-leading strategy. The intuition is quite simple. Keeping the total margin  $r_{AL}$  constant, reducing  $r_L$  allows  $L$  to increase the margin  $r_A$  it charges on  $A$ ; this does not affect the overall margin on one-stop shoppers (who buy both  $A$  and  $B$ ), but enhances the margin on multi-stop shoppers (who only buy  $A$ ). However, this move also transforms some multi-stop shoppers (who initially buy  $B_S$  from  $S$ ) into one-stop shoppers (who now turn to  $B_L$ ); but this, too, benefits  $L$  as long as one-stop shoppers are more profitable, that is, as long as  $r_L > 0$ .  $L$  thus finds it optimal to keep reducing  $r_L$  until selling  $B_L$  below cost.

The optimal subsidy balances its favorable impact on  $r_A$  against its adverse effect on the mix of multi-stop shoppers (who become more profitable than one-stop shoppers when  $r_L < 0$ ), and is characterized by the first-order condition

$$r_L^* = -h(\tau^*) < 0, \quad (3)$$

where, using  $r_L^* = \tau^* - (w_S - w_L)$ , the optimal threshold  $\tau^*$  is given by:

$$\tau^* \equiv l^{-1}(w_S - w_L) > 0. \quad (4)$$

In the absence of any restriction on its total margin  $r_{AL}$ ,  $L$  would maximize the first term,  $r_{AL}F(v_{AL})$ , by charging the monopoly margin  $r_{AL}^m$ , thus offering one-stop shoppers a value  $v_{AL} = v_{AL}^m$ .

Conversely, this strategy does attract one-stop shoppers as long as  $v_{AL}^m \geq w_S$ , or  $w_{AL} \geq l(w_S) (> w_S)$ ; therefore, when  $L$  derives a sufficiently large comparative advantage from its broader product range, the optimal strategy consists of charging the monopoly margin  $r_{AL}^m$  for the bundle, and  $r_L^* = -h(\tau^*)$  for  $B_L$ . The loss-leading strategy then gives  $L$  a profit equal to:

$$\Pi_L^* = r_{AL}^m F(v_{AL}^m) - r_L^* F(\tau^*) = \Pi_{AL}^m + h(\tau^*) F(\tau^*),$$

which *exceeds* the monopolistic profit  $\Pi_{AL}^m$ .

When instead  $L$ 's comparative advantage is not large enough (namely,  $w_{AL} < l(w_S)$ ),  $L$  must improve its offer in order to keep attracting one-stop shoppers. It is then optimal for  $L$  to match the value offered by the competitive fringe:  $\tilde{v}_{AL}^* = w_S$ , or  $\tilde{r}_{AL}^* = w_{AL} -$

$w_S (< r_{AL}^m)$ .<sup>29</sup> The loss-leading strategy then gives  $L$  a profit equal to:

$$\tilde{\Pi}_L^* \equiv (w_{AL} - w_S) F(w_S) + h(\tau^*) F(\tau^*).$$

Alternatively,  $L$  can leave one-stop shoppers to the small retailers ("regime  $S$ ") and focus instead on multi-stop shoppers, who are willing to buy  $A$  from  $L$  as long as the added value  $v_A \equiv w_A - r_A$  exceeds the extra shopping cost  $s$ . In this way,  $L$  obtains:

$$\Pi_A^m \equiv r_A^m F(v_A^m) = \max_{r_A} r_A F(w_A - r_A).$$

The loss-leading strategy<sup>30</sup> is clearly preferable when  $v_{AL}^m \geq w_S$ , since it then gives  $L$  more profit than the monopolistic level  $\Pi_{AL}^m$  (which already exceeds  $\Pi_A^m$ ). As it turns out, it remains preferable as long as  $L$  enjoys a comparative advantage over  $S$ :

**Proposition 1** *Suppose the large retailer ( $L$ ) faces a competitive fringe of small retailers ( $S$ ). Then:*

- *Whenever  $L$  enjoys a comparative advantage over  $S$  (i.e.,  $w_{AL} > w_S$ ), its unique optimal pricing strategy involves loss leading:  $L$  sells the competitive product  $B_L$  below cost. Furthermore, when its comparative advantage is large (namely,  $v_{AL}^m \geq w_S$ ),  $L$  keeps the total margin for the two products at the monopoly level ( $r_{AL} = r_{AL}^m$ ) and earns a higher profit than in the absence of any rivals; otherwise  $L$  simply obtains a total margin reflecting its comparative advantage ( $r_{AL} = w_{AL} - w_S$ ).*
- *When instead  $L$  faces a comparative disadvantage (i.e.,  $w_{AL} < w_S$ ), its unique optimal pricing strategy consists of monopolizing the non-competitive product and leaving the market of the competitive product to the small retailers.*

**Proof.** See Appendix A. ■

*Illustration: Uniform density of shopping costs.* Suppose that the shopping cost is uniformly distributed:  $F(s) = s$ . The monopoly profit,  $r_{AL} v_{AL} = r_{AL} (w_{AL} - r_{AL})$ , is maximal for  $r_{AL}^m = v_{AL}^m = w_{AL}/2$ . Thus, as long as  $w_{AL} \geq 2w_S$ , offering the monopoly value suffices to attract one-stop shoppers ( $v_{AL}^m \geq w_S$ ) and  $L$ 's profit is given by:

$$r_{AL}^m v_{AL}^m - r_L \tau = \Pi_{AL}^m - r_L (w_S - w_L + r_L),$$

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<sup>29</sup>If needed,  $L$  can slightly enhance its offer to make sure that it attracts all one-stop shoppers.

<sup>30</sup>Throughout the paper, we refer to loss leading as actually *selling* a product below cost. In regime  $S$ ,  $L$  may keep *offering*  $B_L$  below cost when  $w_{AL} < w_S$  (e.g. by charging  $r_L = -r_A^m$ ), but only sells  $A$ .

which is maximal for:

$$r_L^* = -\tau^* = -\frac{w_S - w_L}{2} < 0.$$

In this way,  $L$  obtains more than the monopoly profit:

$$\Pi_L^* = \Pi_{AL}^m - r_L^* \tau^* = \frac{w_{AL}^2}{4} + \frac{(w_S - w_L)^2}{4}.$$

When  $2w_S > w_{AL}$ ,  $L$  maintains the subsidy  $r_L^*$  but can only charge  $\tilde{r}_{AL}^* = w_{AL} - w_S$  to one-stop shoppers; its profit reduces to:

$$\tilde{\Pi}_L^* = (w_{AL} - w_S) w_S + \frac{(w_S - w_L)^2}{4},$$

which coincides with

$$\Pi_A^m = \frac{(w_{AL} - w_L)^2}{4}$$

when  $w_{AL} = w_S$ . Finally, whenever  $w_{AL} < w_S$ ,  $L$  leaves the competitive segment to its smaller rivals and earns  $\Pi_A^m$  by exploiting its monopoly power on  $A$ .

*Remark: Bundled discounts.* In principle,  $L$  could offer three prices: one for  $A$ , one for  $B_L$  and one for the bundle. But as  $L$  sells  $A$  to all of its customers, only two prices matter here: the price  $p_A$  charged for  $A$  to multi-stop shoppers, and the total price  $p_{AL}$  charged for  $A$  and  $B_L$  to one-stop shoppers; since these prices can equivalently be implemented through stand-alone prices,  $p_A$  for  $A$  and  $p_L \equiv p_{AL} - p_A$  for  $B_L$ , offering an additional bundled discount cannot improve  $L$ 's profit here.

*Remark: Specialist stores versus hard discounters.* The baseline model covers two types of small retail formats who may benefit from a comparative advantage in market  $B$ : hard discount chains who focus on lower costs (but may offer a similar quality), and specialist stores, such as wine specialists, fruit and vegetable stores, and bakery stores, who offer higher quality (at possibly higher costs). In the latter case, however, consumers may differ in their preferences for quality, and those who do not value quality much may not be interested in multi-stop shopping even if they have low shopping costs. Yet, as shown in Appendix E, the above insight applies as long as some consumers favor quality over price.

*Remark: Asymmetric shopping costs.* In practice, a consumer may incur different costs when visiting  $L$  or  $S$  – visiting a larger store may for example be more time-consuming. Our analysis easily extends to such situations. Suppose, for example, that consumers bear a cost  $\alpha s$  when patronizing  $L$  (and  $s$ , as before, when visiting  $S$ ). The threshold  $\tau$  remains

unchanged, but one-stop shoppers are now willing to patronize  $L$  whenever  $s < v_{AL}/\alpha$ . Thus, as long as  $L$  attracts one-stop shoppers, its profit is now:

$$\Pi_L = r_{AL}F\left(\frac{v_{AL}}{\alpha}\right) - r_L F(\tau),$$

which leads  $L$  to adopt the same loss-leading strategy as before  $r_L^* = -h(\tau^*)$ .

### 3 Loss leading and margin squeeze

Focusing on the case where the small retailer is a competitive fringe allows us to highlight the pure exploitative effect of loss leading without considering its impact on the smaller rivals, since competition among them dissipates their margins anyway. Yet, in many antitrust cases, small retailers have complained that their profits were squeezed as a result of large retailers' loss-leading strategies. To analyze this margin-squeeze effect of loss leading, we consider here the case where  $L$  competes against a single smaller rival  $S$ , who can thus earn a positive margin  $r_S > 0$  on  $B_S$ .

The previous analysis of  $L$ 's pricing behavior still applies, replacing the competitive value  $w_S$  with the net value offered by  $S$ ,  $v_S = w_S - r_S$ . We will focus on the regime where  $L$  attracts one-stop shoppers by offering a better value than its rival ( $v_{AL} > v_S$ ).  $L$  then faces a demand  $F(v_{AL}) - F(\hat{\tau})$  on both products from one-stop shoppers, and an additional demand  $F(\hat{\tau})$  on product  $A$  from multi-stop shoppers, where the gain from multi-stop shopping,  $\hat{\tau}$ , is now given by:

$$\hat{\tau} \equiv v_S - v_L = w_S - w_L + r_L - r_S. \quad (5)$$

Maximizing its profit,  $\Pi_L = r_{AL}F(v_{AL}) - r_L F(\hat{\tau})$ , leads  $L$  to charge again the monopoly margin for the bundle ( $\hat{r}_{AL}^* = r_{AL}^m$ ) and to price the competitive good below cost, with a subsidy satisfying  $r_L = -h(\hat{\tau})$ .

Since  $S$  only attracts multi-stop shoppers, it obtains a profit  $\Pi_S = r_S F(\hat{\tau})$  and its best response to  $r_L$  is thus characterized by the first-order condition:

$$r_S = h(\hat{\tau}).$$

The equilibrium margin  $\hat{r}_L^*$  and  $\hat{r}_S^*$  and the resulting threshold  $\hat{\tau}^*$  thus satisfy:

$$\hat{\tau}^* = w_S - w_L + \hat{r}_L^* - \hat{r}_S^* = w_S - w_L - 2h(\hat{\tau}^*),$$

which yields

$$\hat{\tau}^* \equiv j^{-1}(w_S - w_L), \quad (6)$$

where  $j(x) \equiv x + 2h(x)$  is strictly increasing. In this candidate equilibrium,  $S$  earns a profit

$$\hat{\Pi}_S^* \equiv h(\hat{\tau}^*) F(\hat{\tau}^*),$$

while  $L$  obtains

$$\hat{\Pi}_L^* \equiv \Pi_{AL}^m + h(\hat{\tau}^*) F(\hat{\tau}^*).$$

Since  $\hat{\tau}^* = j^{-1}(w_S - w_L) < l^{-1}(w_S - w_L) = \tau^*$ ,  $L$ 's profit is lower than in the previous case, where it was facing a competitive fringe of small retailers. For these margins to form an equilibrium, two conditions must be satisfied: first,  $L$  must indeed attract one-stop shoppers; second, while  $L$  has no incentive to exclude its rival, since it earns more profit than a pure monopolist,  $S$  may want to attract one-stop shoppers by offering a higher value than  $v_{AL}^m$ . We show in Appendix B that these two conditions are satisfied when (and only when)  $L$  enjoys a significant comparative advantage:

**Proposition 2** *Suppose that  $L$  faces a strategic smaller rival  $S$ . Then loss leading arises in equilibrium if and only if  $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$ , where the threshold  $\hat{w}_{AL}(w_S, w_L)$  lies above  $w_S$  and increases with  $w_S$ ; conversely, in this range there is a unique Nash equilibrium, in which  $L$  sells the competitive product below-cost while keeping the total price for both products at the monopoly level, and earns a profit higher than in the absence of the rival.*

**Proof.** See Appendix B. ■

Loss leading thus constitutes a robust exploitative device, which allows  $L$  to discriminate multi-stop shoppers from one-stop shoppers even when competing with a strategic smaller rival. As before, adopting loss leading allows  $L$  to earn even more profit than a pure monopolist if its comparative advantage is large enough. Compared with the case of a competitive fringe, loss leading is now adopted in equilibrium only when it allows  $L$  to earn the full monopoly margin from one-stop shoppers, but it does so in a broader range of circumstances: it is shown in Appendix B that the equilibrium condition  $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$  is less stringent than the similar condition for the case of a competitive fringe ( $v_{AL}^m \geq w_S$ ).

Compared with the case of a competitive fringe of smaller retailers, whose profit is not affected by  $L$ 's behavior, the loss-leading strategy now reduces  $S$ 's profit, not only

by decreasing its market share, but also by squeezing its margin:  $S$ 's best response is  $r_S = h(\hat{\tau})$ , where  $\hat{\tau} = l^{-1}(w_S - w_L + r_L)$  increases with  $r_L$ . Yet, this appears here as a side effect of the exploitative motive rather than as the result of exclusionary motive. In particular, foreclosing the market through strategic tying or (pure) bundling would not be profitable here, since  $L$  could obtain at most the monopoly profit in the case of exclusion.

*Remark: Strategic margin squeeze.* Although margin squeeze appears here as a by-product of exploitation, the large retailer has an incentive to manipulate its rivals' prices: the lower  $S$ 's price for  $B_S$ , the more  $L$  can extract from multi-stop shoppers. Thus, if  $L$  could move first and act as a Stackelberg leader, it would *decrease* even further its price for  $B_L$ , so as to force  $S$  to respond by decreasing its own price (in contrast with the standard Stackelberg insight, where the leader usually benefits from higher rival prices) and in this way allow  $L$  to raise its price on  $A$  for multi-stop shoppers.

However, since  $L$  benefits from the presence of  $S$ , it may also want to tailor its loss-leading strategy in order to maintain that presence. Suppose for example that the entry of  $S$  is uncertain. It is then profitable for  $L$  to adopt a loss-leading strategy in case of entry, in order to extract additional rents from multi-stop shoppers; however, this can reduce the likelihood of entry, in which case  $L$  faces a trade-off between exploitation and entry accommodation. We develop a simple model along these lines in Appendix C, which yields the following insights:

**Proposition 3** *If  $L$  and  $S$  compete as Stackelberg leader and follower, then in the loss-leading equilibrium  $L$  prices further below-cost than in the absence of the first-mover advantage. However, if the entry of  $S$  depends on the realization of a random entry cost, then  $L$  limits instead the subsidy on  $B$  so as to increase the likelihood of entry.*

**Proof.** See Appendix C. ■

*Remark: complementary goods and adoption costs.* While we have focused here on the case where  $A$  and  $B$  are independent goods or partially substitutable, the analysis applies also – even more straightforwardly – to the case of complements. Suppose for example that  $A$  is a prerequisite for using  $B$  (as in the case of CD players and speakers): product  $B$  has no value on a stand-alone basis ( $u_L = u_S = 0$ ), and must be used together with product  $A$  (yielding utilities  $u_{AL}$  and  $u_{AS}$ , where  $w_{AS} = u_{AS} - c_A - c_S > w_{AL} = u_{AL} - c_A - c_L$ ). Interpreting  $w_L$  and  $w_S$  as  $w_{AL} - w_A$  and  $w_{AS} - w_A$ ,<sup>31</sup> the above analysis

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<sup>31</sup>The analysis applies irrespective of whether  $A$  generates or not a value on a stand-alone basis, as



goes through except that, since one-stop shoppers necessarily favor  $L$  (since there is no value in patronizing  $S$  only),  $L$  *always* engages in loss leading: it charges the monopoly margin  $r_{AL}^m$  for the bundle and an even greater margin  $r_{AL}^m + h(\tau^*)$  for  $A$  on a stand-alone basis (reflecting the "subsidy"  $r_L^* = -h(\tau^*)$  on  $B_L$ ).

Also, while we have focused so far on retail markets, the insights apply to industries in which the costs of adopting a technology, of learning how to use a product, of maintaining equipment, and so forth, play a role similar to the shopping costs that consumers incur to visit an additional store. Indeed, the same analysis goes through, interpreting  $s$  as the *additional* cost that customers must incur in order to use a rival application, rather than that provided by the incumbent. These insights can, therefore, shed a new light on famous antitrust cases such as the *Microsoft* saga, in which Microsoft has been accused of excluding rivals in adjacent markets – e.g., the markets for browsers or media players. While the arguments mainly focused there on the rationality of an exclusionary conduct, our analysis suggests an alternative motivation for subsidizing or otherwise encouraging customers to adopt the platform developer's own application, to the detriment of its rivals.

Similar insights also apply to industries in which procuring several categories of products from the same supplier allows a customer to save on operating costs. For example, in its decision on the proposed merger between Aerospatiale-Alenia and De Havilland,<sup>32</sup> the European Commission mentions that the new entity would benefit from being the only one to offer regional aircraft in all three relevant sizes, thus allowing "one-stop shopper" airlines to save on maintenance and spare parts as well as on pilot training and certification. To see how the analysis can be transposed in such industries, suppose for instance that  $L$  covers both segments  $A$  and  $B$  while  $S$  covers  $B$  only, and that procuring both products from the same supplier involves a maintenance cost  $f$ , while dealing with different suppliers increases the maintenance cost to  $f + s$ , where  $s$  is customer-specific. Then, whenever active customers prefer procuring both products (e.g., because the products are complements, or because airlines cannot be viable without operating aircraft in all relevant sizes), the same analysis as above applies, and  $L$  subsidizes again the competitive product (and charges, for example, the full value for the bundle if  $f$  is constant and the

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long as combining it with  $B$  generates a higher value.

<sup>32</sup>See the decision of the European Commission in case No. IV/M053 - Aerospatiale-Alenia/de Havilland (2 October 1991).

goods are complements, or mimics the pricing policy with asymmetric shopping costs if  $f$  is proportional to  $s$ ).

## 4 Banning loss leading

We now show that loss leading reduces consumer surplus and social welfare as well as smaller rivals' profits. For the sake of exposition, we consider here the scenario where  $L$  faces a strategic rival, and focus moreover on the regime in which  $L$  attracts one-stop shoppers and thus engages in loss leading (that is,  $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$ ).

Suppose  $L$  is not allowed to price below cost. We show in Appendix D that  $L$  then keeps attracting one-stop shoppers in equilibrium. Since  $L$ 's profit function is quasi-concave and separable in  $r_{AL}$  and  $r_L$ ,  $L$  maintains the total margin at the monopoly level ( $r_{AL}^m$ ) but now sells  $B_L$  at cost ( $r_L = 0$ ); consequently, its profit is reduced to  $\Pi_{AL}^m = r_{AL}^m F(v_{AL}^m)$ .

Since  $L$  no longer subsidizes the competitive segment,  $S$  faces more demand: the gain from multi-stop shopping increases from  $\tau = w_S - w_L + r_L^* - r_S$  to  $\tau = w_S - w_L - r_S$ . Maximizing its profit  $\Pi_S = r_S F(\tau)$  then leads  $S$  to charge a margin satisfying  $r_S = h(\tau) = h(w_S - w_L - r_S)$ , and the equilibrium threshold becomes:

$$\tau^* = l^{-1}(w_S - w_L) > j^{-1}(w_S - w_L) = \hat{\tau}^*.$$

That is,  $S$  increases its market share (from  $\hat{\tau}^*$  to  $\tau^*$ ) as well as its margin (from  $\hat{r}_S^* = h(\hat{\tau}^*)$  to  $\hat{r}_S^b \equiv h(\tau^*)$ ) and, consequently, increases its profit by

$$\Delta_{\Pi_S} = h(\tau^*) F(\tau^*) - h(\hat{\tau}^*) F(\hat{\tau}^*) > 0.$$

Banning loss leading does not affect the value of one-stop shopping, since  $L$  maintains the same total margin,  $r_{AL}^m$ . It, however, encourages consumers to take advantage of multi-stop shopping: banning loss leading forces  $L$  to compete "on the merits", which induces those consumers with a shopping cost lower than  $\tau^*$  to patronize both stores; in contrast, subsidizing  $B_L$  (and overcharging  $A$  by the same amount) discourages consumers with a shopping cost exceeding  $\hat{\tau}^*$  from visiting  $S$ . The ban on loss leading thus benefits consumers whose shopping cost lies between  $\hat{\tau}^*$  and  $\tau^*$ , since the resulting lower price for  $A$  allows them to save  $\tau^* - s$ . Using a revealed preference argument, it also benefits genuine multi-stop shoppers (those with a shopping cost  $s < \hat{\tau}^*$ ), by increasing the value

of multi-stop shopping from  $\hat{v}_{AS}^* \equiv v_{AL}^m + \hat{\tau}^*$  to  $v_{AS}^* \equiv v_{AL}^m + \tau^*$ . Overall, a ban on loss leading thus increases total consumer surplus by:

$$\Delta_{CS} = (\tau^* - \hat{\tau}^*) F(\hat{\tau}^*) + \int_{\hat{\tau}^*}^{\tau^*} (\tau^* - s) dF(s) > 0.$$

Finally, fostering multi-stop shopping also enhances efficiency, since more consumers benefit from a better distribution of  $B$ . The gain in social welfare is equal to:

$$\Delta_W = \int_{\hat{\tau}^*}^{\tau^*} (w_S - w_L - s) dF(s),$$

and is positive since  $\hat{\tau}^* < \tau^* < w_S - w_L$ . Therefore, we have:

**Proposition 4** *Assume that  $L$  faces a strategic rival and would engage in loss leading. Banning below-cost pricing then leads to an equilibrium where  $L$  maintains the same total margin but sells the competitive good at cost; as a result, the ban increases consumer surplus, the rival's profit, and social welfare.*

**Proof.** See Appendix D. ■

A similar analysis applies when  $L$  faces a competitive fringe. While loss leading no longer affects rivals' profit, it still reduces their market share and thus distorts distribution efficiency at the expense of consumers. Banning loss leading thus improves again consumer surplus and social welfare.

As noted in the introduction, competition authorities have been reluctant to treat loss leading as predatory pricing, and some countries have instead adopted below-cost pricing regulations. By showing that loss leading can be used as an exploitative device, to extract extra rents from multi-stop shoppers, rather than as an exclusionary or predatory practice, our analysis sheds a new light on the rationale of loss leading and can thus help placing the assessment of its anticompetitive effects on firmer ground.

## 5 Inter-format vs. intra-format competition

We have so far taken as given the market structure and focused on asymmetric competition between large and small retail formats. We now consider the implications of this analysis for retailers' format choices. When the founders of Aldi, the Albrecht brothers, took over their family's small neighborhood store in 1946, a retail cooperative was dominating

their local market. They took to selling a limited range of private label products, and the success of this innovative approach triggered the development of the hard-discount business model. Later on (in the 1990s), the large supermarket chains began imitating this business model and opened their own hard-discount chains to compete head-to-head with the existing hard discounters.<sup>33</sup>

To capture the key features of these developments, suppose that initially (period 0, say) a large retailer  $L$  enjoys a monopoly position for the distribution of two goods,  $A$  and  $B$ : potential entrants can open similar stores, in which case head-to-head competition drives prices down to cost for both goods. Then in period 1, one of the potential entrants,  $S$ , innovates and comes up with a new retail format which, by focusing on a limited product range (good  $B$ ), confers a comparative advantage in that product range ( $w_S > w_L$ ), although the product range is so limited that one-stop shoppers will never patronize it ( $\hat{w}_{AL}(w_S, w_L) < w_{AL}$ ). Finally, in period 2, the established large retailer and the other potential entrants can imitate the innovation and open a store with the new format.

Let  $f$  denote the cost of opening a new store. Clearly, no entrant will ever open another large store, since the resulting head-to-head competition would not allow recouping this set-up cost. By contrast, in period 1 the innovator will open a small store, even if it anticipates subsequent entry in period 2, as long as  $f < \hat{\Pi}_S^* = h(\hat{\tau}^*) F(\hat{\tau}^*)$ ; note that entry is not only profitable for the innovator, but it also increases  $L$ 's profit by  $h(\hat{\tau}^*) F(\hat{\tau}^*)$ . Consider now period 2. If the innovator already opened a small store in period 1, no other entrant will do so in period 2, since head-to-head competition would then eliminate the margin on  $B$ . However,  $L$  can benefit from opening its own small store: while this drives the profit of  $S$  down to zero, it allows  $L$  to extract even more surplus than before from multi-stop shoppers:  $h(\tau^*) F(\tau^*) > h(\hat{\tau}^*) F(\hat{\tau}^*)$ ; therefore,  $L$  will open its own small format store whenever  $f < h(\tau^*) F(\tau^*) - h(\hat{\tau}^*) F(\hat{\tau}^*)$ . The resulting competition has no impact on one-stop shoppers but fosters multi-stop shopping (the shopping cost threshold increases from  $\hat{\tau}^*$  to  $\tau^*$ ), and thus enhances consumer surplus as well as total welfare.

Alternatively, if mergers were allowed,  $L$  could acquire  $S$ , in which case  $L$  and  $S$  could

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<sup>33</sup>Carrefour, Casino and Rewer, for example, have already established their own discount chains (namely, ED, Leader Price and Penny); Auchan and Tesco are experimenting along the same lines, whereas Asda, one of the largest retailers in the UK, has acquired the hard discounter Netto.

together generate a total profit

$$\begin{aligned}\Pi_L + \Pi_S &= r_{AL} (F(v_{AL}) - F(\tau)) + (r_S + r_A) F(\tau) \\ &= r_{AL} F(v_{AL}) + (r_S - r_L) F(\tau),\end{aligned}$$

where the second line is derived by using  $r_A = r_{AL} - r_L$ . It is thus optimal to charge  $r_{AL} = r_{AL}^m$  and  $r_L - r_S = -h(\tau^*)$ , where  $\tau^* = l^{-1}(w_S - w_L)$ , and in this way  $L$  and  $S$  generate a joint profit equal to  $\Pi_L^* = \Pi_{AL}^m + h(\tau^*) F(\tau^*)$ . So, this scenario is equivalent to opening a new store for competing with  $S$ , but saves the cost  $f$  of opening another store. By construction, the profit achieved by the merged entity exceeds the joint profit of  $L$  and  $S$  in the other scenario; we thus have  $h(\tau^*) F(\tau^*) > 2h(\hat{\tau}^*) F(\hat{\tau}^*)$ , or  $h(\tau^*) F(\tau^*) - h(\hat{\tau}^*) F(\hat{\tau}^*) > h(\hat{\tau}^*) F(\hat{\tau}^*)$ . This discussion can be summarized as follows:

**Proposition 5** *Suppose  $0 < f < h(\hat{\tau}^*) F(\hat{\tau}^*)$ . Then: (i) no retailer ever opens another large store; (ii)  $S$  opens a small store when the innovation becomes available in period 1; and (iii) either  $L$  merges with  $S$ , if this is allowed, or it opens another small store when the innovation can be imitated in period 2.*

## 6 Robustness

So far, we have used a simple setting in which the large retailer ( $L$ ) competes only on specific product segments ("good  $B$ ") and enjoys a monopoly position in the others ("good  $A$ "); loss leading then allows it to better exploit its market power and charge higher prices to multi-stop shoppers in the monopolized segments. Furthermore, while we allowed for quite general distributions of consumers' shopping costs, we assumed that their valuations were homogenous. We describe here several extensions, showing that our insights apply more generally as long as one-stop shoppers favor the large retailer(s). We first introduce heterogeneity in consumer valuations, which makes the aggregate demand for goods " $A$ " or " $B$ " more sensitive to prices, but may also attenuate the intensity of competition in market  $B$  if  $L$  and  $S$  offer differentiated varieties. We then introduce (imperfect) competition in the monopolized markets.

Introducing heterogeneous valuations in the competitive market does not affect our analysis as long as most one-stop shoppers prefer patronizing  $L$ , and buy both goods from it, to patronizing  $S$ : keeping  $r_{AL}$  constant, reducing  $r_L$  and increasing  $r_A$  then still

does not affect one-stop shoppers, and can only transform some multi-stop shoppers into one-stop shoppers, which benefits  $L$  as long as  $r_L > 0$ ; therefore, in equilibrium  $L$  prices  $B_L$  below cost. To illustrate this, we present in Appendix E a simple setting in which  $L$  and  $S$  offer differentiated varieties and consumer relative preferences are distributed in such a way that some consumers may prefer  $B_L$  to  $B_S$ , while others have the reverse ranking, but all one-stop shoppers prefer the bundle  $A - B_L$  to consuming  $B_S$  only. This can for example be the case when  $S$  (a specialist store, say) offers a better quality, and consumers have heterogeneous values for quality; loss leading then arises whenever at least some consumers care about quality and engage in multi-stop shopping.

Introducing heterogeneous valuations for  $A$  makes its demand elastic, which limits  $L$ 's ability to raise prices in this segment; this may make loss leading less attractive, since the purpose of the exploitative device is precisely to earn more from multi-stop shoppers on this segment. Likewise, (imperfect) competition among large retailers curbs their capacity to charge high prices on  $A$  and may also discourage the use of loss leading as an exploitative device. To check the robustness of our analysis, we present in Appendix F a variant where consumers are distributed along a Hotelling line: specifically, a consumer located at  $x$  obtains a utility  $u_A - \frac{x}{\sigma} - p_A = w_A - r_A - \frac{x}{\sigma}$ , where  $\sigma$  represents the degree of consumer heterogeneity and  $x$  is distributed according to a cumulative distribution function  $G(\cdot)$ , with density  $g(\cdot)$ , so as to allow for a general elastic demand function. One-stop shoppers are thus willing to patronize  $L$  if  $s \leq v_{AL} - \frac{x}{\sigma}$  or, equivalently,  $x \leq x_{AL}(s) \equiv \sigma(v_{AL} - s)$ , and prefer this to patronizing  $S$  as long as  $x \leq \hat{x} \equiv \sigma(v_{AL} - v_S)$ . As before, consumers prefer multi-stop shopping to patronizing  $L$  as long as  $s \leq \tau$ ; however, they now prefer this to patronizing  $S$  only if the additional value from consuming  $A$  offsets the extra shopping cost:

$$s \leq v_A - \frac{x}{\sigma} \iff x \leq x_A(s) \equiv \sigma(v_A - s).$$

Therefore, as long as  $L$  attracts some one-stop shoppers ( $v_{AL} > v_S$ ) and  $S$  attracts some multi-stop shoppers ( $\tau > 0$ ), then (see Figure 2):

- consumers with  $s < \tau$  buy  $A$  from  $L$  and  $B_S$  from  $S$  if  $x < x_A(s)$  (region  $D_{AS}$ ), and only  $B_S$  otherwise (region  $D_S$ );

consumers with  $\tau < s < v_{AL}$  and  $x < x_{AL}(s)$  buy both  $A$  and  $B_L$  from  $L$  (region

$D_{AL}$ ), and otherwise buy either  $B_S$  only (if  $s \leq v_S$ ) or nothing (if  $s > v_S$ ).

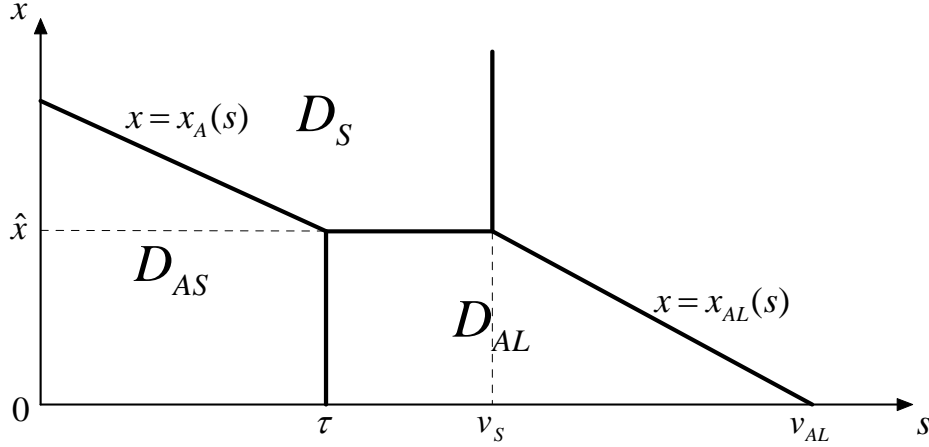


Figure 2: Heterogeneous valuations for  $A$

We also consider the possibility that  $L$  competes with another large retailer,  $L_2$ , located at the other end of the Hotelling line (thus giving consumers a utility  $w_A - r_{A_2} - \frac{1-x}{\sigma}$ ), in which case the large retailers may either compete for multi-stop shoppers only (Figure 3a), or for both types of consumers (Figure 3b).

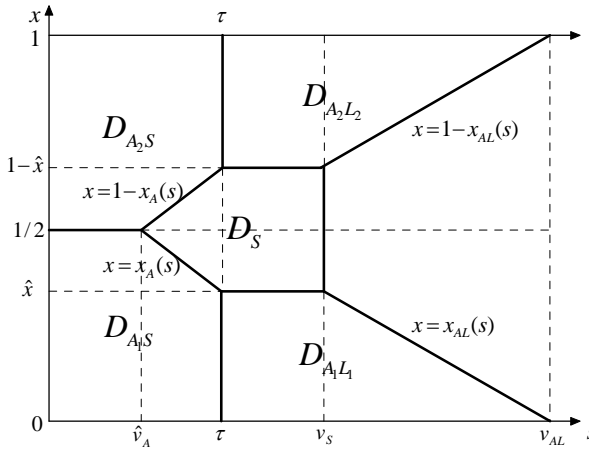


Figure 3a

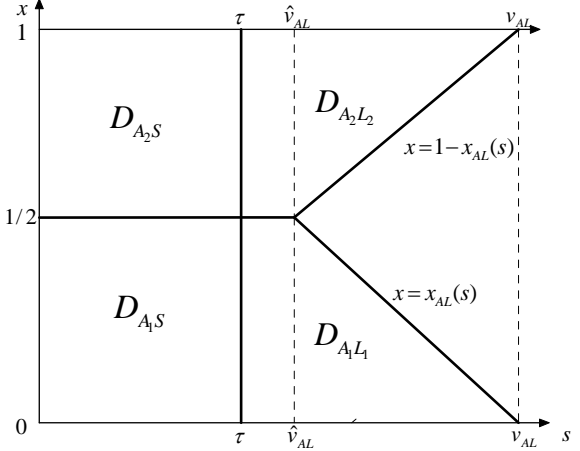


Figure 3b

In all these variants, we show that, whenever  $L$  attracts some one-stop shoppers, it adopts a loss-leading strategy. While competition limits  $L$ ' margins (on  $A$  as well as on the assortment  $AL$ ), loss leading still allows it to better discriminate consumers according to their shopping costs. As before, pricing  $B_L$  below cost, and increasing the price of  $A$  so as to maintain  $r_{AL}$  unchanged, does not affect one-stop shoppers but allows  $L$  to extract more surplus from multi-stop shoppers. While this strategy may now induce some multi-stop shoppers to stop buying  $A$  or switch to the other large retailer, the analysis

shows that, as long as the inverse hazard rate,  $k(\cdot) \equiv G(\cdot)/g(\cdot)$ , is increasing, multi-stop shoppers are actually less price-sensitive than one-stop shoppers; as a result,  $L$  aims again at charging greater margins on them, and the loss-leading strategy remains profitable:

**Proposition 6** *Suppose consumers have heterogeneous valuations for  $A$  and/or another large retailer competes à la Hotelling. Then, in any equilibrium in which  $L$  attracts some one-stop shoppers, it adopts a loss-leading pricing strategy to extract more surplus from multi-stop shoppers.*

*Proof.* See Appendix F. ■

## 7 Conclusion

This paper provides a new rationale for the adoption of loss leading and highlights its harmful impact on retail competition and consumers in the absence of efficiency justifications, thus giving support to small rivals' complaints and competition concerns.<sup>34</sup> It identifies two key drivers: asymmetry in the product range and heterogeneity in consumers' shopping patterns.<sup>35</sup> The analysis also supports the expressed doubts about the exclusionary motive of the practice, and stresses instead its role as an exploitative device. Yet, this exploitative use of loss leading harms consumers and society as well as the small rivals, which may provide a rationale for antitrust enforcement.<sup>36</sup>

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<sup>34</sup>Chambolle (2005) also studies asymmetric competition between a large retailer and a smaller one, in a different setting in which both retailers are equally efficient, but a majority of consumers is closer to the smaller store, and travel costs are too large for multi-stop shopping; the large retailer then never uses the competitive good as a loss leader, but can instead use in this way the monopolized good, in which case this can benefit consumers as well as society. This is in line with the observation that in practice, concerns are voiced when loss leaders are chosen among the staples offered by the smaller retailers.

<sup>35</sup>We have focused here on consumer shopping costs, which appear as a key factor for routine, repeated purchases. Other dimensions may be relevant for other types of purchases; for example, for less frequent, high value purchases, information and search costs may play a more important role – and customers with lower search costs are again likely to visit more stores. It would be interesting to study whether these alternative sources of underlying heterogeneity yield similar or distinct insights.

<sup>36</sup>Allain and Chambolle (2005) and Rey and Vergé (2010) note however that below-cost pricing regulations can allow manufacturers to impose price floors on their retailers, in which case they can be used to better exert market power or to reduce interbrand as well as intrabrand competition; banning loss leaders may then have a perverse effect on consumer welfare.



While the insights are quite robust to variations in cost and demand conditions, policy measures should however also take into account potential efficiency justifications, and empirical studies are needed to assess the resulting balance. We have furthermore restricted attention to individual unit demands, which appears reasonable for groceries and other day-to-day purchases, and also assumed away any correlation between consumers' valuations for the goods and their shopping costs; whether our insights apply to market environments where consumers' individual demands are elastic, or underlying characteristics (e.g., wealth) affect both shopping costs and willingness to pay, is left to future research. Likewise, our framework focuses on small retailers who have lower cost or offer better quality, such as hard discount stores or specialist stores, but does not account for other categories of small stores, such as convenience stores, who face higher cost (and charge higher prices) but allow consumers to save on shopping costs; we leave to future research the analysis of pricing strategies in such instances.

Finally, while the analysis focuses mainly on retail markets, our insights apply as well to industries where a firm, enjoying substantial market power in one segment, competes with more efficient rivals in other segments, and procuring these products from the same supplier generates customer-specific benefits. They also apply to complementary products, such as platforms and applications. While some of these industries have hosted heated antitrust cases focusing on predatory pricing or related conduct, our analysis provides an alternative rationale for below-cost pricing based on exploitation rather than exclusion.

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# Appendices

## Not for publication

### A Proof of Proposition 1

Suppose first that  $v_{AL} \geq w_S$ , that is,  $r_{AL} \leq w_{AL} - w_S$  ("regime  $L$ "). We first show that, without loss of generality, we can focus on prices such that  $\tau \in [0, v_{AL}]$ . If  $\tau > v_{AL}$  (i.e.,  $w_S - w_L + r_L > w_{AL} - r_{AL}$ , or  $r_L > r'_L \equiv (w_{AL} - r_A - (w_S - w_L))/2$ ), there are no one-stop shoppers: active consumers buy  $A$  from  $L$  and  $B_S$  from  $S$ , and do so as long as  $2s < v_A + w_S$ ; however, keeping  $r_A$  constant, decreasing  $r_L$  to  $r'_L$  such that  $\tau' = v'_{AL}$  does not affect the number of active consumers (since  $v_A$  does not change), who still visit both stores as before. If instead  $\tau < 0$  (i.e.,  $r_L < -w_S - w_L$ ), there are no multi-stop shoppers: active consumers only visit  $L$ , and do so as long as  $s < v_{AL} = w_{AL} - r_{AL}$ ; however, keeping  $r_{AL}$  constant, increasing  $r_L$  to  $r'_L = -(w_S - w_L)$  yields  $\tau' = 0$  without affecting consumer behavior. The condition  $\tau \geq 0$  moreover ensures that prospective multi-stop shoppers are indeed willing to buy  $A$  on a stand-alone basis:  $w_S \leq v_{AL} = w_{AL} - r_A - r_L$  implies  $r_A \leq w_{AL} - w_S - r_L = w_{AL} - w_L - \tau < w_A$ .

Thus, consumers whose shopping cost lies in  $[0, \tau]$  buy  $A$  from  $L$  (and  $B_S$  from  $S$ ), whereas those with a shopping cost in  $[\tau, v_{AL}]$  buy both  $A$  and  $B_L$  from  $L$ . Using  $v_{AL} = w_{AL} - r_{AL}$  and  $\tau = w_S - w_L + r_L$ ,  $L$ 's optimization program within regime  $L$  can thus be expressed as:

$$\begin{aligned} \max_{r_{AL}, r_L} \quad & \Pi_L(r_{AL}, r_L) = r_{AL}F(w_{AL} - r_{AL}) - r_LF(w_S - w_L + r_L), \\ \text{subject to} \quad & r_{AL} \leq w_{AL} - w_S \end{aligned} \tag{7}$$

where  $\Pi_L(r_{AL}, r_L)$  is additively separable and moreover strictly quasi-concave in  $r_{AL}$  and  $r_L$ .  $L$ 's optimization program can thus be decomposed into:

$$\begin{aligned} \max_{r_{AL}} \quad & r_{AL}F(w_{AL} - r_{AL}), \\ \text{s. t.} \quad & r_{AL} \leq w_{AL} - w_S \end{aligned}$$

which leads to  $r_{AL} = \min\{r_{AL}^m, w_{AL} - w_S\}$  and  $v_{AL} = \max\{v_{AL}^m, w_S\}$ , and

$$\min_{r_L} r_L F(w_S - w_L + r_L),$$

which yields the first-order condition:

$$r_L^* = -h(w_S - w_L + r_L^*) = -h(\tau^*) < 0. \tag{8}$$

Using  $r_L^* = \tau^* - (w_S - w_L) = -h(\tau^*)$ , the optimal threshold  $\tau^*$  is given by:

$$\tau^* \equiv l^{-1}(w_S - w_L) > 0. \quad (9)$$

Note that this threshold satisfies  $\tau^* < v_{AL}^m$ . To see this, take instead  $v_{AL}$  and  $\tau$  as control variables and rewrite  $L$ 's profit as  $\Pi_L(v_{AL}, \tau) = r_{AL}F(v_{AL}) - r_LF(\tau) = (w_{AL} - v_{AL})F(v_{AL}) + (w_S - w_L - \tau)F(\tau)$ . Then we have  $v_{AL}^m = \arg \max_v (w_{AL} - v)F(v) > \arg \max_v (w_S - w_L - v)F(v) = \tau^*$ , since  $w_{AL} \geq l(w_S) (> w_S \geq w_S - w_L)$ .

Suppose now that  $v_{AL} < w_S$ , that is,  $r_{AL} > w_{AL} - w_S$  ("regime  $S$ ").  $L$  then only attracts multi-stop shoppers, who buy  $A$  from it as long as  $s \leq v_A = w_A - r_A$ .  $L$  thus obtains:

$$\Pi_L = r_A F(v_A) = r_A F(w_A - r_A),$$

which is maximal for  $r_A^m$  and  $v_A^m = w_A - r_A^m$ , characterized by:

$$r_A^m = h(v_A^m), \quad v_A^m = l^{-1}(w_A).$$

$L$ 's profit in regime  $S$  is thus at most:

$$\Pi_A^m \equiv r_A^m F(v_A^m).$$

As already noted, regime  $L$  is clearly preferable when  $v_{AL}^m \geq w_S$ , since it then gives  $L$  more profit than the monopolistic level  $\Pi_{AL}^m$ , which itself is greater than  $\Pi_A^m$ :

$$\Pi_{AL}^m = \max_r r F(w_{AL} - r) > \max_r r F(w_A - r) = \Pi_A^m,$$

since  $w_{AL} > w_A$ . We now show that regime  $L$ , and the associated loss-leading strategy, remains profitable when  $w_{AL} \geq w_S > v_{AL}^m$ , where it involves  $r_L^* < 0$  and  $\tilde{r}_{AL}^* = w_{AL} - w_S$ . To see this, fixing  $\tilde{r}_{AL}^*$  and using  $r_A$  rather than  $r_L$  as the optimization variable, the margin on  $B_L$  and the shopping cost threshold can be expressed as:

$$r_L = \tilde{r}_{AL}^* - r_A = w_{AL} - w_S - r_A, \tau = w_S - w_L + r_L = w_{AL} - w_L - r_A = w_A - r_A.$$

Then, the maximum profit  $\tilde{\Pi}_L^*$  can then be written as:

$$\begin{aligned} \tilde{\Pi}_L^* &= \tilde{r}_{AL}^* (F(\tilde{v}_{AL}^*) - F(\tau^*)) + r_A^* F(\tau^*) \\ &= (w_{AL} - w_S) (F(w_S) - F(\tau^*)) + r_A^* F(\tau^*) \\ &= \max_{r_A} \{ (w_{AL} - w_S) (F(w_S) - F(w_A - r_A)) + r_A F(w_A - r_A) \} \\ &\geq (w_{AL} - w_S) (F(w_S) - F(w_A - r_A^m)) + r_A^m F(w_A - r_A^m) \\ &= (w_{AL} - w_S) (F(w_S) - F(v_A^m)) + \Pi_A^m. \end{aligned}$$

Since  $w_S > v_{AL}^m = l^{-1}(w_{AL}) > l^{-1}(w_A) = v_A^m$ , it follows that  $\tilde{\Pi}_L^* \geq \Pi_A^m$  whenever  $w_{AL} \geq w_S$ .

Conversely, when  $w_{AL} < w_S$ , then  $L$  can indeed achieve  $\Pi_A^m$  in regime  $S$  (it suffices to set  $r_L = 0$ , which together with  $r_A = r_A^m$ , satisfies  $r_{AL} = r_A^m > 0 > w_{AL} - w_S$ , and thus  $v_{AL} < w_S$ ), and we have:

$$\begin{aligned}\tilde{\Pi}_L^* &= (w_{AL} - w_S)(F(w_S) - F(w_A - \tilde{r}_A^*)) + \tilde{r}_A^* F(w_A - \tilde{r}_A^*) \\ &< \tilde{r}_A^* F(w_A - \tilde{r}_A^*) \\ &\leq \Pi_A^m,\end{aligned}$$

where the first inequality stems from  $w_S > w_{AL} (> w_A - \tilde{r}_A^*)$ .

Finally, in the limit case where  $w_{AL} = w_S$ , using  $B_L$  as a loss leader amounts to monopolizing product  $A$ . Notice that offering  $v_{AL} = w_S$  requires  $r_{AL} = w_{AL} - v_{AL} = 0$ , or  $r_A = -r_L$ , thus the margin on  $A$  reflects the subsidy on  $B_L$ . In this case, the optimal subsidy strategy maximizes  $-r_L F(\tau) = -r_L F(w_S - w_L + r_L) = r_A F(w_A - r_A)$ . Consumers are also indifferent between these two strategies: in both cases they face the same price for  $A$ . While the loss-leading strategy may yield a lower price for  $B_L$  (in the monopolization scenario,  $L$  may actually stop carrying  $B_L$ ), this does not affect multi-stop shoppers (who do not buy  $B_L$  from  $L$ ), whereas one-stop shoppers are indifferent between buying  $A$  and  $B_L$  from  $L$  or  $B_S$  only from  $S$ .

## B Proof of Proposition 2

We derive here the conditions under which the loss leading outcome ( $\hat{r}_{AL}^* = r_{AL}^m$  and  $\hat{r}_L^* = -\hat{r}_S^* = -h(\hat{\tau}^*)$ , where  $\hat{\tau}^* = j^{-1}(w_S - w_L)$ ) forms a Nash equilibrium, before checking the uniqueness of the equilibrium. To attract one-stop shoppers,  $L$  must offer a better value than  $S$ :<sup>37</sup>

$$v_{AL}^m \geq \hat{v}_S^* \equiv w_S - h(\hat{\tau}^*). \quad (10)$$

This condition implies  $v_{AL}^m \geq \hat{v}_S^* > \hat{v}_S^* - \hat{v}_L^* = \hat{\tau}^*$ , which in turn implies  $w_{AL} > w_S$ :

$$w_{AL} = l(v_{AL}^m) \geq l(\hat{v}_S^*) = \hat{v}_S^* + h(\hat{v}_S^*) = w_S - h(\hat{\tau}^*) + h(\hat{v}_S^*) > w_S.$$

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<sup>37</sup>As before, this is equivalent to  $w_{AL} - w_L - \hat{r}_A^* = v_{AL}^m - \hat{v}_L^* \geq \hat{v}_S^* - \hat{v}_L^* = \hat{\tau}^* (> 0)$ , which implies that multi-stop shoppers are indeed willing to buy  $A$  when visiting  $L$ . Moreover, this condition also implies  $v_{AL}^m > \hat{v}_S^* - \hat{v}_L^* = \hat{\tau}^* (> 0)$ .

Moreover, while  $L$  has no incentive to exclude its rival, since it earns more profit than a pure monopolist,  $S$  may want to attract one-stop shoppers by reducing  $r_S$  so as to offer  $v_S \geq v_{AL}^m$ . Such a deviation allows  $S$  to attract all consumers (one-stop or multi-stop shoppers) with shopping costs  $s \leq v_S$  and thus yields a profit  $\Pi_S^d(v_S) \equiv r_S F(v_S) = (w_S - v_S) F(v_S)$ . It is easy to check that the best deviation of this type is to offer  $v_S^d = v_{AL}^m$  (or slightly above  $v_{AL}^m$ , if one-stop shoppers are indifferent between two stores in this case). To see this, note that  $\Pi_S^d(v_S)$  is quasi-concave in  $v_S$  and let  $v_S^m$  denote the optimal value of  $v_S$ . Since the candidate equilibrium margin,  $\hat{v}_S^*$ , maximizes  $(w_S - w_L + \hat{r}_L^* - v_S) F(v_S)$ , where  $w_S - w_L + \hat{r}_L^* < w_S$ , a simple revealed argument yields  $v_S^m < \hat{v}_S^*$ . Thus, increasing  $v_S$  further above  $v_{AL}^m > \hat{v}_S^*$  would reduce  $S$ 's profit monotonically, and it is then optimal for  $S$  to offer precisely  $v_S^d = v_{AL}^m$ , which gives  $S$  a profit equal to  $\Pi_S^d(v_{AL}^m) = (w_S - v_{AL}^m) F(v_{AL}^m)$ . Thus, the loss-leading outcome is immune to such a deviation if and only if

$$\hat{\Pi}_S^* \equiv h(\hat{\tau}^*) F(\hat{\tau}^*) \geq \hat{\Pi}_S^d \equiv (w_S - v_{AL}^m) F(v_{AL}^m). \quad (11)$$

This condition can be further written as:

$$\Psi(w_{AL}; w_S) \equiv (w_S - v_{AL}^m) F(v_{AL}^m) \leq \hat{\Pi}_S^*, \quad (12)$$

where  $v_{AL}^m = l^{-1}(w_{AL})$  and thus satisfies  $v_{AL}^m + h(v_{AL}^m) = w_{AL}$ . Therefore:

$$\begin{aligned} \frac{\partial \Psi}{\partial w_{AL}}(w_{AL}; w_S) &= ((w_S - v_{AL}^m) f(v_{AL}^m) - F(v_{AL}^m)) \frac{dv_{AL}^m}{dw_{AL}} \\ &= (w_S - v_{AL}^m - h(v_{AL}^m)) \frac{f(v_{AL}^m)}{1 + h'(v_{AL}^m)} \\ &= (w_S - w_{AL}) \frac{f(v_{AL}^m)}{1 + h'(v_{AL}^m)}. \end{aligned}$$

It follows that, in the range  $w_{AL} \geq w_S$ ,  $\Psi(w_{AL}; w_S)$  decreases with  $w_{AL}$  (and strictly so for  $w_{AL} > w_S$ ). Thus, condition (11) amounts to  $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$ , where  $\hat{w}_{AL}(w_S, w_L)$  is the unique solution to  $\Psi(w_{AL}; w_S) = \hat{\Pi}_S^*$ . To show that this solution exists and lies above  $w_S$ , note first that  $\Psi$  becomes negative for  $w_{AL} > l(w_S)$  (since then  $v_{AL}^m = l^{-1}(w_{AL}) > w_S$ ), and that for  $w_{AL} = w_S$ ,  $\Psi(w_{AL}; w_S) = (w_{AL} - v_{AL}^m) F(v_{AL}^m) = \Pi_{AL}^m = \max_v (w_{AL} - v) F(v)$ ; since  $w_{AL} > w_S - w_L + \hat{r}_L^*$ , this exceeds  $\hat{\Pi}_S^* = \max_\tau (w_S - w_L + \hat{r}_L^* - \tau) F(\tau)$ .

Finally, in the range  $w_{AL} > w_S (> w_S - \hat{v}_L^*)$ , a simple revealed argument yields:

$$\hat{\tau}^* = \arg \max_v (w_S - \hat{v}_L^* - \tau) F(\tau) < v_{AL}^m = \arg \max_v (w_{AL} - v) F(v).$$



Therefore, (11), which is equivalent to:

$$v_{AL}^m \geq w_S - \frac{h(\hat{\tau}^*) F(\hat{\tau}^*)}{F(v_{AL}^m)}, \quad (13)$$

implies (10). The two conditions (10) and (11) thus boil down to  $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$ .

It remains to show that  $\hat{w}_{AL}(w_S, w_L)$  increases with  $w_S$ . Differentiating  $\hat{w}_{AL}(w_S, w_L)$  with respect to  $w_S$  yields:

$$\frac{\partial \hat{w}_{AL}}{\partial w_S} = \frac{\frac{\partial \Psi}{\partial w_S} - \frac{\partial \hat{\Pi}_S^*}{\partial w_S}}{-\frac{\partial \Psi}{\partial w_{AL}}},$$

where the denominator is positive in the relevant range, whereas the numerator is equal to:

$$\begin{aligned} \frac{\partial \Psi}{\partial w_S} - \frac{\partial \hat{\Pi}_S^*}{\partial w_S} &= F(v_{AL}^m) - \frac{d(h(\hat{\tau}^*) F(\hat{\tau}^*))}{d\hat{\tau}^*} \frac{\partial \hat{\tau}^*}{\partial w_S} \\ &= F(v_{AL}^m) - \frac{1 + h'(\hat{\tau}^*)}{1 + 2h'(\hat{\tau}^*)} F(\hat{\tau}^*), \end{aligned}$$

which is positive since  $v_{AL}^m > \hat{\tau}^*$ .

We now show that no other equilibrium exists when  $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$ . First, we turn to regime  $S$ , in which one-stop shoppers patronize  $S$  ( $v_{AL} < v_S$ ), and show that there is no such equilibrium when  $w_{AL} > w_S$ . In this regime,  $L$  faces only a demand  $F(v_A)$  for  $A$  from multi-stop shoppers, where  $v_A = w_A - r_A$ , and thus makes a profit equal to  $r_A F(v_A)$ .  $L$  could however deviate and attract one-stop shoppers by reducing  $r_L$  (keeping  $r_A$  and thus  $v_A$  constant) so as to offer  $v'_{AL} = v_S$  (or slightly above  $v_S$ ). Doing so would not change the number of multi-stop shoppers, since  $\tau' = v_S - v'_L = v'_{AL} - v'_L = v'_A = v_A$ , and  $L$  would obtain the same margin,  $r_A$ , from those consumers. But it would now attract one-stop shoppers (those for which  $v_A \leq s \leq v_{AL} = v_S$ ), from which  $L$  could earn a total margin  $r'_{AL} = w_{AL} - v'_{AL} = w_{AL} - v_S = w_{AL} - w_S + r_S$ . Since any candidate equilibrium requires  $r_S \geq 0$ , the deviation would be profitable when  $w_{AL} > w_S$ .

Second, consider the boundary between the two regimes, in which one-stop shoppers are indifferent between visiting  $L$  or  $S$  ( $v_{AL} = v_S$ ). Note that there must exist some active consumers, since either retailer can profitably attract consumers by charging a small positive margin; therefore, we must have  $v_{AL} = v_S > 0$ . Suppose that all active consumers are multi-stop shoppers (in which case  $L$  only sells  $A$  while  $S$  sells  $B_S$  to all consumers), which requires  $v_{AL} = v_S \leq \tau$ . Applying the same logic as in the beginning of Appendix B, we can without loss of generality focus on the case  $v_{AL} = v_S = \tau$ . It is

then profitable for  $L$  to transform some multi-stop shoppers into one-stop shoppers, by reducing its margin on  $B_L$  to  $r'_L = w_L - \varepsilon > 0$  and increasing  $r_A$  by  $\varepsilon$ , so as to keep  $v_{AL}$  constant: doing so does not affect the total number of active consumers, but transforms those whose shopping cost lies between  $\tau' = v_S - v'_L = \tau - \varepsilon$  and  $\tau$  into one-stop shoppers. While  $L$  obtains the same margin on them (since  $r'_{AL} = r_{AL}$ ), it now obtains a higher margin  $r'_A > r_A$  on the remaining multi-stop shoppers.

Therefore, some consumers must visit a single store, and by assumption must be indifferent between visiting either store ( $v_{AL} = v_S$ ). Suppose now some one-stop shoppers visit  $S$ . Since  $S$  can avoid making losses, we must then have  $r_S \geq 0$ . But then,  $v_{AL} = v_S$  implies  $r_{AL} = r_S + w_{AL} - w_S > 0$  and, thus, it would be profitable for  $L$  to reduce  $r_{AL}$  slightly, so as to attract all one-stop shoppers. Therefore, all one-stop shoppers must go to  $L$  if  $r_{AL} > 0$ . Conversely, we must have  $r_S \leq 0$ , otherwise  $S$  would benefit from slightly reducing its margin so as to attract all one-stop shoppers. Therefore, in any candidate equilibrium such that  $v_{AL} = v_S > 0$ , either:

- There are some multi-stop shoppers (i.e.  $\tau > 0$ ) and thus  $r_S = 0$ ; but then, slightly increasing  $r_S$  would allow  $S$  to keep attracting some multi-stop shoppers and obtain a positive profit, a contradiction.
- Or, all consumers buy both products from  $L$ , which requires  $r_L \leq r_S - (w_S - w_L) \leq -(w_S - w_L) < 0$ . But then, increasing  $r_L$  to  $r'_L = r_S - (w_S - w_L) + \varepsilon$  and reducing  $r_A$  by the same amount (so as to keep  $r_{AL}$  constant) would lead those consumers with  $s < \tau' = \varepsilon$  to buy  $B_S$  from  $S$ , allowing  $L$  to avoid granting them the subsidy  $r_L$ .

It follows that there is no equilibrium such that  $v_{AL} = v_S$ .

Finally, loss leading (in which  $L$  not only offers, but actually *sells* below cost) can only arise when  $L$  sells to one-stop shoppers, which thus requires  $v_{AL} \geq v_S$ . But this cannot be an equilibrium when  $w_{AL} < \hat{w}_{AL}(w_S, w_L)$ , since: (i) in the range  $v_{AL} > v_S$ , the only such candidate is the above described loss-leading outcome, which requires  $w_{AL} \geq \hat{w}_{AL}(w_S, w_L)$ ; and (ii) as just discussed, no equilibrium exists in the boundary case  $v_{AL} = v_S$ .

## C Proof of Proposition 3

*Stackelberg leadership.* Suppose that  $L$  benefits from a first-mover advantage: it sets its prices first, and then, having observed these prices,  $S$  sets its own price. Retail prices are often strategic complements, and it is indeed the case here for  $S$  in the  $B$  segment: as noted before,  $S$ 's best response,  $\hat{r}_S(r_L)$ , increases with  $r_L$ . Thus, in the case of "normal competition" in the  $B$  market,  $L$  would exploit its first-mover advantage by *increasing* its price for  $B_L$ , so as to encourage its rival to increase its own price and relax the competitive pressure. In contrast, here  $L$  has an incentive to *decrease*  $r_L$  even further. This leads  $S$  to decrease its own price, which allows  $L$  to raise the price for  $A$ . To see this, note that  $L$ 's Stackelberg profit from a loss-leading strategy can be written as:

$$\Pi_L^S(r_L) = \Pi_{AL}^m - r_L F(\hat{\tau}(r_L)) = \Pi_{AL}^m - r_L F(w_S - w_L + r_L - \hat{r}_S(r_L)).$$

Denoting by  $r_L^S$  the optimal Stackelberg margin and using  $\hat{r}_S(\hat{r}_L^*) = \hat{r}_S^*$ , where  $\hat{r}_L^*$  and  $\hat{r}_S^*$  are the equilibrium margins when  $L$  moves simultaneously with  $S$ , we have:

$$\begin{aligned} -r_L^S F(w_S - w_L + r_L^S - \hat{r}_S(r_L^S)) &\geq -\hat{r}_L^* F(w_S - w_L + \hat{r}_L^* - \hat{r}_S(\hat{r}_L^*)) \\ &\geq -r_L^S F(w_S - w_L + r_L^S - \hat{r}_S^*), \end{aligned}$$

where the second inequality stems from the fact that  $\hat{r}_L^*$  constitutes  $L$ 's best response to  $r_S^*$ . Since  $-r_L^S > 0$  and  $F(\cdot)$  and  $\hat{r}_S(\cdot)$  are both increasing, this in turn implies  $r_L^S \leq \hat{r}_L^*$ . This inequality is moreover strict, since (using  $\hat{\tau}(\hat{r}_L^*) = \hat{\tau}^*$ ):

$$(\Pi_L^S)'(\hat{r}_L^*) = -F(\hat{\tau}^*) - \hat{r}_L^* f(\hat{\tau}^*) (1 - \hat{r}_S'(\hat{r}_L^*)) = \hat{r}_L^* f(\hat{\tau}^*) \hat{r}_S'(\hat{r}_L^*) < 0.$$

Thus,  $L$  sells the competitive product  $B_L$  further below-cost, compared with what it would do in the absence of a first-mover advantage:  $r_L^S < \hat{r}_L^*$ .

*Entry accommodation.* Suppose now that the presence of  $S$  is uncertain. To capture this possibility, assume that  $S$  incurs a fixed cost for entering the market,  $\gamma$ , which is ex ante distributed according to a cumulative distribution function  $F_\gamma(\cdot)$ , and consider the following timing:

- In stage 1,  $L$  chooses its prices.
- In stage 2, the entry cost is realized, and  $S$  chooses whether to enter; if it enters, it then sets its own price.

If entry were certain, maximizing its Stackelberg profit would lead  $L$  to adopt  $r_L^S$ . But now,  $S$  enters only when its best response profit,  $\hat{\Pi}_S(r_L)$ , exceeds the realized cost  $\gamma$ , which occurs with probability  $\rho(r_L) \equiv F_\gamma(\hat{\Pi}_S(r_L))$ .  $L$ 's ex ante profit is therefore equal to

$$\hat{\Pi}_L^S(r_L) = \Pi_{AL}^m + \rho(r_L) \Pi_L^S(r_L).$$

The optimal margin,  $\hat{r}_L^S$ , thus satisfies

$$\rho(\hat{r}_L^S) \Pi_L^S(\hat{r}_L^S) \geq \rho(r_L^S) \Pi_L^S(r_L^S) \geq \rho(r_L^S) \Pi_L^S(\hat{r}_L^S),$$

which implies

$$\rho(\hat{r}_L^S) \geq \rho(r_L^S).$$

Since  $F_\gamma$  and  $\hat{\Pi}_S$  are both increasing in  $r_L$ , so is  $\rho$  and thus  $\hat{r}_L^S \geq r_L^S$ . This inequality is moreover strict, since

$$\left(\hat{\Pi}_L^S\right)'(r_L^S) = \rho'(r_L^S) \Pi_L^S(r_L^S) + \rho(r_L^S) (\Pi_L^S)'(r_L^S) = \rho'(r_L^S) \Pi_L^S(r_L^S) > 0.$$

Therefore, when  $L$ 's comparative advantage leads it to adopt a loss-leading strategy, it limits the subsidy on  $B$  so as to increase the likelihood of entry:  $\hat{r}_L^S > r_L^S$ .

## D Proof of Proposition 4

In the equilibrium where  $L$  attracts one-stop shoppers in the absence of a ban,  $L$  must offer a higher value than  $S$ :  $v_{AL} = v_{AL}^m > \hat{v}_S^* = w_S - \hat{r}_S^*$ , and  $S$  must moreover not be tempted to deviate and attract one-stop shoppers, which boils down to  $\hat{\Pi}_S^* = h(\hat{\tau}^*) F(\hat{\tau}^*) \geq \hat{\Pi}_S^d = (w_S - v_{AL}^m) F(v_{AL}^m)$ . If  $L$  keeps attracting one-stop shoppers (i.e.,  $v_{AL} > v_S$ ) when loss leading is banned, then the unique candidate equilibrium is  $r_{AL} = r_{AL}^m$ ,  $r_L = 0$  and  $\hat{r}_S^b = h(\tau^*)$ , where  $\tau^* = l^{-1}(w_S - w_L)$ .

We show now this candidate equilibrium prevails when loss-leading would arise if below-cost pricing were allowed. Note that, since  $S$  increases its price (i.e.,  $\hat{r}_S^b = h(\tau^*) > \hat{r}_S^* = h(\hat{\tau}^*)$ ), it offers less value ( $v_S = \hat{v}_S^b \equiv w_S - \hat{r}_S^b < \hat{v}_S^*$ ), and thus  $L$  indeed attracts one-stop shoppers:  $v_{AL} = v_{AL}^m > (\hat{v}_S^b >) \hat{v}_S^b$ . Furthermore, as  $S$  must again offer at least  $v_S = v_{AL}$  to attract one-stop shoppers, it still cannot obtain more than  $\hat{\Pi}_S^d$  by deviating in this way. Therefore, since  $S$  now obtains more profit ( $\Pi_S^* \equiv h(\tau^*) F(\tau^*) > \hat{\Pi}_S^* =$

$h(\hat{\tau}^*) F(\hat{\tau}^*)$ , it is less tempted to deviate:  $\Pi_S^* > (\hat{\Pi}_S^* >) \hat{\Pi}_S^d$ . It follows that the conditions for sustaining the above equilibrium are less stringent than that for the loss-leading equilibrium.

## E Product differentiation in the competitive market

We show that our main insights apply when consumers vary in their relative preferences over  $B_L$  and  $B_S$ . For example, suppose  $B_L$  is a "standard" variety generating a homogeneous utility  $u_L$ , whereas  $B_S$ , a better variety supplied by specialist stores, yields a utility  $u_S + \theta q$ ;  $\theta \in [0, 1]$  thus characterizes the consumer preference for quality and is distributed according to a c.d.f  $\Phi(\cdot)$  with density function  $\phi(\cdot)$ , whereas  $q$  measures the degree of consumer heterogeneity. For the sake of exposition, we consider here the case where  $B_S$  is supplied by a competitive fringe and assume that:

- $S$  provides better value for at least some quality-oriented consumers:  $w_S + q > w_L$ ; we allow however for  $w_L > w_S$ , in which case  $L$  offers higher value than  $S$  for less quality-oriented consumers.
- all one-stop shoppers favor  $L$ :  $v_{AL} \geq w_S + q$ .

As before, consumers are willing to patronize  $L$  if  $s \leq v_{AL}$ , and prefer multi-stop shopping to one-stop shopping if

$$s \leq w_S + \theta q - v_L = \tau + \theta q,$$

where  $\tau = w_S - w_L + r_L$ .  $L$  thus earns a profit

$$\Pi_L = r_{AL} D_{AL}(r_{AL}) - r_L D_{AS}(r_L)$$

where  $D_{AL}(r_{AL}) = F(v_{AL})$  and  $D_{AS}(r_L) = \int_0^1 F(\tau + \theta q) \phi(\theta) d\theta$ . The loss leading logic of the baseline model applies again here: since  $v_{AL} = w_{AL} - r_{AL}$  and  $\tau = w_S - w_L + r_L$ ,  $L$ 's profit is separable in  $r_{AL}$  and  $r_L$ , and still charges the price on  $B_L$  below-cost.

While we presented this example in terms of "vertical" quality differentiation, the same analysis applies to "horizontal" differentiation, where for example the utilities generated by  $B_L$  and  $B_S$  would be of the form  $u_L + (1 - \theta)q$  and  $u_S + \theta q$ ; the only difference is that,

since consumers have now heterogeneous valuations for  $B_L$  as well, the above demands become:

$$D_{AL}(r_{AL}) = \int_0^1 F(v_{AL} + (1 - \theta)q)\phi(\theta)d\theta, D_{AS}(r_L) = \int_0^1 F(\tau + (2\theta - 1)q)\phi(\theta)d\theta.$$

## F Proof of Proposition 6

### F.1 Local Monopolies with heterogeneous preferences on $A$

We show that introducing an elastic demand in market  $A$  does not preclude the large retailer from adopting a loss-leading strategy, so as to extract additional surplus from multi-stop shoppers. We focus on the large retailer's strategies, taking the strategies of the smaller retailer(s) as given; thus, whether the smaller rival is a strategic player or a competitive fringe does not matter here.

$L$ 's profit can be written as (see Figure 1):

$$\Pi_L = r_{AL}D_{AL} + r_AD_{AS} = r_{AL} \int_{\tau}^{v_{AL}} G(x_{AL}(s)) f(s) ds + r_A \int_0^{\tau} G(x_A(s)) f(s) ds.$$

To characterize the equilibrium values of  $r_L$  and  $r_{AL}$ , we now consider the impact of a small change on either variable.

Consider first a modification of  $r_A$  by  $dr$ , adjusting  $r_L$  by  $-dr$  so as to keep  $r_{AL}$  constant. Such a change does not affect the behavior of one-stop shoppers (it has no impact on  $v_{AL}$  and  $x_{AL}(s)$ ), but (see Figure 2):

- It affects multi-shop shoppers: for  $s < \tau$ , the marginal consumer indifferent between buying  $A$  from  $L$  or patronizing  $S$  only becomes  $x = x_A(s) - \sigma dr$ ; therefore,  $L$  loses  $\sigma g(x_A(s)) dr$  consumers, on which it no longer earns the margin  $r_A$ .  $L$  however increases its margin by  $dr$  on the mass  $G(x_A(s))$  of consumers that buy  $A$ . Thus, the overall impact of such an adjustment on multi-stop shoppers is equal to

$$\int_0^{\tau} [G(x_A(s)) - \sigma r_A g(x_A(s))] f(s) ds dr.$$

- In addition, it alters the choice between one-stop and multi-stop shopping: those consumers for which  $s \in [\tau - dr, \tau]$  and  $x \leq x_A(s)$  turn to one-stop shopping and now buy  $B$  as well as  $A$  from  $L_1$ , which (noting that  $x_A(\tau) = \hat{x}$ ) brings a gain  $r_L G(\hat{x}) f(\tau) dr$ .

These effects must cancel out in equilibrium, which yields

$$\int_0^\tau [\sigma r_A - k(x_A(s))] g(x_A(s)) f(s) ds = r_L G(\hat{x}) f(\tau).$$

Likewise, adjusting slightly  $r_{AL}$  by  $dr$ , keeping  $r_A$  constant (and thus changing  $r_L$  by  $dr$  as well) does not affect the behavior of multi-stop shoppers (it has no impact on  $v_{AS}$  and  $x_A(s)$ ), but:

- It affects one-stop shoppers: for  $s > \tau$ , the marginal shopper becomes  $x = x_{AL}(s) - \sigma dr$ , and the resulting change in profit is

$$\int_\tau^{v_{AL}} [G(x_{AL}(s)) - \sigma r_{AL} g(x_{AL}(s))] f(s) ds dr.$$

- In addition, those consumers for which  $s \in [\tau, \tau + dr]$  and  $x \leq x_{AL}(s)$  become multi-stop shoppers and stop buying  $B$  from  $L$ , which (noting that  $x_{AL}(\tau) = \hat{x}$ ) brings a net effect  $-r_L G(\hat{x}) f(\tau) dr$ .

In equilibrium, these effects must again cancel each other, which yields

$$\int_\tau^{v_{AL}} [\sigma r_{AL} - k(x_{AL}(s))] g(x_{AL}(s)) f(s) ds = -r_L G(\hat{x}) f(\tau).$$

Therefore, if in equilibrium  $r_L$  were non-negative, we would have

$$\int_0^\tau [\sigma r_A - k(x_A(s))] g(x_A(s)) f(s) ds \geq 0 \geq \int_\tau^{v_{AL}} [\sigma r_{AL} - k(x_{AL}(s))] g(x_{AL}(s)) f(s) ds,$$

that is,  $r_A$  would exceed a weighted average of  $k(x_A(s))/\sigma$  for  $s \in [0, \tau]$ , whereas  $r_{AL}$  would be lower than a weighted average of  $k(x_{AL}(s))/\sigma$  for  $s \in [\tau, v_{AL}]$ . But since  $k(x_A(s))$  and  $k(x_{AL}(s))$  decrease as  $s$  increases ( $k(\cdot)$  increases by assumption, and both  $x_A(s)$  and  $x_{AL}(s)$  decrease by construction), this would imply  $r_A > r_{AL}$ , a contradiction. Therefore, in equilibrium,  $r_L < 0$ .

If the shopping cost  $s$  is distributed over some interval  $[0, \bar{s}]$ , where  $\bar{s} > \tau$  to ensure that large retailers still attract some one-stop shoppers, the first-order conditions become:

$$\begin{aligned} \int_0^\tau [\sigma r_A - k(x_A(s))] g(x_A(s)) f(s) ds &= r_L G(\hat{x}) f(\tau), \\ \int_\tau^{\min\{v_{AL}, \bar{s}\}} [\sigma r_{AL} - k(x_{AL}(s))] g(x_{AL}(s)) f(s) ds &= -r_L G(\hat{x}) f(\tau); \end{aligned}$$

it thus suffices to replace  $v_{AL}$  with  $\min\{v_{AL}, \bar{s}\}$  in the above reasoning.

## F.2 Imperfect competition among large retailers

Suppose now that two large retailers are present,  $L_1$  and  $L_2$ , who incur the same costs in distributing  $A$  and  $B$ , and offer the same variety  $B_L$  but differentiated varieties  $A_1$  and  $A_2$ : a consumer with preference  $x$  then obtains a utility  $u_A - \frac{x}{\sigma} - p_{A_1} = w_A - r_{A_1} - \frac{x}{\sigma}$  from buying  $A_1$  and a utility  $w_A - r_{A_2} - \frac{1-x}{\sigma}$  from buying  $A_2$ . We will restrict attention to symmetric distributions (that is, the density  $g(\cdot)$  satisfies  $g(x) = g(1-x)$ ) and will focus on (symmetric) equilibria in which: (i) the large retailers compete against each other as well as against their smaller rivals; (ii) small retailers attract some multi-stop shoppers by offering a value  $v_S$  that exceeds the value  $v_L$  offered by large retailers on the  $B$  market; and (iii) large retailers attract some one-stop shoppers by offering them a value  $v_{AL}$  that exceeds  $v_S$ , as well as the value  $v_A$  that they offer on the  $A$  market alone.

Large retailers may compete against each other for one-stop and/or for multi-stop shoppers. In the former case, in a symmetric equilibrium (of the form  $r_{A_1 L_1} = r_{A_2 L_2} = r_{AL}$  and  $r_{L_1} = r_{L_2} = r_L$ ) some consumers (with  $x = 1/2$ ) are indifferent between buying both goods from either  $L_1$  or  $L_2$ , and prefer doing so to patronizing  $S$  only; this implies (using  $x = 1/2$ , and dropping the subscripts 1 and 2 for ease of exposition):

$$\hat{v}_{AL} \equiv v_{AL} - \frac{1}{2\sigma} \geq v_S,$$

which is equivalent to

$$\hat{v}_A \equiv v_A - \frac{1}{2\sigma} \geq \tau = v_S - v_L.$$

Therefore, consumers with preference  $x = 1/2$  and shopping cost  $s < \tau$ , who thus prefer multi-stop shopping (that is, buying  $B_S$  from  $S$  and  $A$  from either  $L_1$  or  $L_2$ ) to visiting  $L_1$  or  $L_2$  only, also prefer multi-stop shopping to patronizing  $S$  only (since  $s < \tau$  then implies  $s < \hat{v}_A$ ). In other words, if large retailers compete for one-stop shoppers, they will also compete for multi-stop shoppers. This observation allows us to classify the (symmetric) candidate equilibria into two types:

- Type  $M$ : large retailers compete only for *multi-stop* shoppers;
- Type  $O$ : large retailers compete for *one-stop* shoppers as well as for multi-stop shoppers.

In the first type of equilibria (which is illustrated in Figure 3a), for  $x = 1/2$  some consumers with low shopping costs are indifferent between assortments  $A_1 S$  and  $A_2 S$ , and



prefer those assortments to any other option, whereas consumers with higher shopping costs patronize  $S$  only; the relevant threshold for the shopping cost satisfies

$$\hat{v}_A + v_S - 2s = v_S - s,$$

that is,  $s = \hat{v}_A$ . Consumers with  $s < \hat{v}_A$  thus buy  $B$  from  $S$  and  $A$  from either  $L_1$  or  $L_2$  (depending on whether  $x$  is smaller or larger than  $1/2$ ). Conversely, consumers whose shopping costs exceed  $v_{AL}$  do not shop. As for consumers whose shopping costs lie between  $\hat{v}_A$  and  $v_{AL}$ :

- when  $s < \tau$ , consumers still buy  $B_S$  from  $S$ ; they also buy  $A$  from  $L_1$  if  $x < x_A(s) = \sigma(v_A - s)$ , or from  $L_2$  if  $x > 1 - x_A(s)$ ;
- when  $s > \tau$ :
  - if  $x < x_{AL}(s)$ , consumers buy both goods from  $L_1$ ;
  - if  $x > 1 - x_{AL}(s)$ , consumers buy both goods from  $L_2$ ;
  - if  $x_{AL}(s) < x < 1 - x_{AL}(s)$ , consumers patronize  $S$  if  $s < v_S$ , and buy nothing otherwise.

In the second type of equilibria (illustrated in Figure 3b), all consumers with a shopping cost  $s < \tau$  buy  $B_S$  from  $S$  and  $A$  from either  $L_1$  (if  $x < 1/2$ ) or  $L_2$  (if  $x > 1/2$ ), while consumers with  $s > v_{AL}$  buy nothing. For consumers with  $\tau < s < v_{AL}$ , then:

- if  $s < \hat{v}_{AL}$ , consumers will buy both goods from either  $L_1$  (if  $x < 1/2$ ) or  $L_2$  (if  $x > 1/2$ );
- if  $\hat{v}_{AL} < s < v_{AL}$ , consumers will buy both goods from  $L_1$  if  $x < x_{AL}(s)$  or from  $L_2$  if  $x > 1 - x_{AL}(s)$ , and buy nothing otherwise.

A similar description applies when the shopping cost  $s$  is bounded, truncating as necessary the interval for  $s$ .

We show now loss leading is still used as an exploitative device. Consider first (symmetric) equilibria of type  $M$ , in which large retailers compete only for multi-stop shoppers. In the absence of any bound on shopping costs, the demands for assortments  $A_1L_1$  and

$A_1S$  in such equilibrium, where  $r_{A_1L_1} = r_{A_2L_2} = r_{AL}$  and  $r_{L_1} = r_{L_2} = r_L$  (and thus  $r_{A_1} = r_{A_2} = r_A$ ), can be expressed as:

$$D_{AS} = \int_0^\tau G(\hat{x}_A(s)) f(s) ds \text{ and } D_{AL} = \int_\tau^{v_{AL}} G(x_{AL}(s)) f(s) ds,$$

where as before  $\tau = v_S - v_L$  and  $x_{AL}(s) = \sigma(v_{AL} - \max\{s, v_S\})$ , and  $\hat{x}_A(s) \equiv \sigma(v_A - \max\{s, \hat{v}_A\}) = \min\{1/2, x_A(s) = \sigma(v_A - s)\}$ .

Applying the same approach as above, starting from a candidate symmetric equilibrium, consider first a small change  $dr$  in  $r_{A_1}$ , adjusting  $r_{L_1}$  by  $-dr$  so as to keep  $r_{A_1L_1}$  constant:

- For  $s < \hat{v}_A$ , the marginal consumer who is indifferent between buying  $A$  from  $L_1$  or  $L_2$  is such that:

$$w_A - (r_A + dr) - \frac{x}{\sigma} = w_A - r_A - \frac{1-x}{\sigma},$$

or:

$$x = \frac{1}{2} - \frac{\sigma dr}{2}.$$

The overall impact on  $L_1$ 's profit is thus:

$$\int_0^{\hat{v}_A} [G(\hat{x}_A(s)) - \frac{\sigma}{2} r_{AG}(\hat{x}_A(s))] f(s) ds dr.$$

- For  $\hat{v}_A < s < \tau$ , the marginal consumer indifferent between buying  $A$  from  $L_1$  or patronizing  $S$  becomes  $x = x_A(s) - \sigma dr$ , and the resulting impact on profit is:

$$\int_{\hat{v}_A}^\tau [G(\hat{x}_A(s)) - \sigma r_{AG}(\hat{x}_A(s))] f(s) ds dr.$$

- In addition, those consumers for which  $s \in [\tau - dr, \tau]$  and  $x \leq \hat{x}_A(s)$  turn to one-stop shopping and now buy  $B$  as well as  $A$  from  $L_1$ , which brings an additional profit  $r_L G(\hat{x}) f(\tau) dr$ .

Therefore, in equilibrium, we must have:

$$\int_0^\tau [\sigma r_A - \eta_A(s)] \hat{g}(\hat{x}_A(s)) f(s) ds = r_L G(\hat{x}) f(\tau), \quad (14)$$

where (using  $\hat{x}_A(s) = 1/2$  for  $s \leq \hat{v}_A$ ):

$$\eta_A(s) \equiv \begin{cases} 2k(\hat{x}_A(s)) & \text{for } s < \hat{v}_A \\ k(\hat{x}_A(s)) & \text{for } s > \hat{v}_A \end{cases} \text{ and } \hat{g}(x) \equiv \begin{cases} \frac{g(1/2)}{2} & \text{for } x = \frac{1}{2} \\ g(x) & \text{for } x < \frac{1}{2} \end{cases}.$$

Consider now a small change  $dr$  in  $r_{A_1L_1}$ , keeping  $r_{A_1}$  constant (and thus adjusting  $r_{L_1}$  by  $dr$  as well):

- for  $s > \tau$ , the marginal (one-stop) shopper becomes  $x = x_{AL}(s) - \sigma dr$  and the impact on the profit is

$$\int_{\tau}^{v_{AL}} [G(x_{AL}(s)) - \sigma r_{AL} g(x_{AL}(s))] f(s) ds dr;$$

- in addition, those consumers for which  $s \in [\tau, \tau + dr]$  and  $x \leq x_{AL}(s)$  become multi-stop shoppers and stop buying  $B$  from  $L_1$ , which brings a net loss  $-r_L G(\hat{x}) f(\tau) dr$ .

In equilibrium, we must therefore have

$$\int_{\tau}^{v_{AL}} [\sigma r_{AL} - \eta_{AL}(s)] g(x_{AL}(s)) f(s) ds = -r_L G(\hat{x}) f(\tau), \quad (15)$$

where  $\eta_{AL}(s) \equiv k(x_{AL}(s))$ .

Thus, if  $r_L$  were non-negative, the two conditions (14) and (15) would imply

$$\int_0^{\tau} [\sigma r_A - \eta_A(s)] \hat{g}(\hat{x}_A(s)) f(s) ds \geq 0 \geq \int_{\tau}^{v_{AL}} [\sigma r_{AL} - \eta_{AL}(s)] g(x_{AL}(s)) f(s) ds,$$

where  $\eta_A$  and  $\eta_{AL}$  decrease as  $s$  increases, and coincide for  $s = \tau$ ; this, in turn, would imply  $r_A > r_{AL}$ , a contradiction. A similar argument applies when the shopping cost  $s$  is distributed over some interval  $[0, \bar{s}]$ .

The same approach can be used for (symmetric) equilibria of type  $O$ , in which large retailers compete as well for one-stop shoppers. In the absence of any bound on shopping costs, the demands for assortments  $A_1 L_1$  and  $A_1 S$  in such equilibrium can be expressed as

$$D_{AS} = \int_0^{\tau} G\left(\frac{1}{2}\right) f(s) ds \text{ and } D_{AL} = \int_{\tau}^{v_{AL}} G(\hat{x}_{AL}(s)) f(s) ds,$$

where  $\hat{x}_{AL}(s) \equiv \sigma(v_A - \max\{s, \hat{v}_{AL}\}) = \min\{1/2, x_{AL}(s) = \sigma(v_{AL} - s)\}$ .

Following a small change  $dr$  in  $r_{A_1}$ , adjusting  $r_{L_1}$  by  $-dr$  so as to keep  $r_{A_1 L_1}$  constant, we have:

- for  $s < \tau$ , the marginal consumer indifferent between buying  $A$  from  $L_1$  or  $L_2$  becomes  $1/2 - \sigma dr/2$ ;
- in addition, those consumers for which  $s \in [\tau - dr, \tau]$  and  $x \leq \hat{x}_A(s)$  become one-stop shoppers.

Therefore, in equilibrium we must have

$$\int_0^{\tau} [\sigma r_A - \hat{\eta}_A] \hat{g}\left(\frac{1}{2}\right) f(s) ds = r_L G\left(\frac{1}{2}\right) f(\tau),$$

where  $\hat{\eta}_A \equiv 2k(1/2)$  and  $\hat{g}(\frac{1}{2}) = g(1/2)/2$ .

Likewise, following a small change  $dr$  in  $r_{A_1 L_1}$ , keeping  $r_{A_1}$  constant (and thus changing  $r_{L_1}$  by  $dr$  as well), we have:

- for  $\tau < s < \hat{v}_{AL}$ , the marginal (one-stop) shopper becomes  $x = x_{AL}(s) - \sigma dr/2$ ;
- for  $\hat{v}_{AL} < s < v_{AL}$ , the marginal (one-stop) shopper becomes  $x = x_{AL}(s) - \sigma dr$ ;
- in addition, those consumers for which  $s \in [\tau, \tau + dr]$  and  $x \leq \hat{x}_{AL}(s)$  become multi-stop shoppers: they stop buying  $B$  from  $L_1$ .

We must therefore have

$$\int_{\tau}^{v_{AL}} [\sigma r_{AL} - \hat{\eta}_{AL}(s)] \hat{g}(\hat{x}_{AL}(s)) f(s) ds = -r_L G(\hat{x}) f(\tau),$$

where

$$\hat{\eta}_{AL}(s) \equiv \begin{cases} 2k(\hat{x}_{AL}(s)) & \text{for } s < \hat{v}_{AL} \\ k(\hat{x}_{AL}(s)) & \text{for } s > \hat{v}_{AL} \end{cases},$$

and  $\hat{g}(x)$  is defined above with  $\hat{x}_{AL}(s) = 1/2$  for  $\tau \leq s \leq \hat{v}_{AL}$ . Thus, if  $r_L$  were non-negative, the above two conditions would imply:

$$\int_0^{\tau} [\sigma r_A - \hat{\eta}_A] \hat{g}(\frac{1}{2}) f(s) ds \geq 0 \geq \int_{\tau}^{v_{AL}} [\sigma r_{AL} - \hat{\eta}_{AL}(s)] \hat{g}(\hat{x}_{AL}(s)) f(s) ds,$$

and a contradiction follows, since  $\hat{x}_{AL}(s) \leq 1/2$ , with a strict inequality for  $s > \hat{v}_{AL}$ , and thus  $\hat{\eta}_{AL}(s) \leq 2k(\hat{x}_{AL}(s)) \leq \hat{\eta}_A$ , with again a strict inequality for  $s > \hat{v}_{AL}$ . A similar argument applies again when the shopping cost  $s$  is distributed over some interval  $[0, \bar{s}]$ .

If instead  $\bar{s} < \hat{v}_{AL}$ , then all consumers buy both goods, in which case  $\hat{\eta}_{AL}(\cdot) = \hat{\eta}_A$  and  $\hat{g}(\hat{x}_{AL}(t)) = \hat{g}(\frac{1}{2})$ , and  $r_L = 0$ .