A Theory of Tacit Collusion*

Joseph E. Harrington, Jr.
Department of Economics
Johns Hopkins University
Baltimore, MD 21218
410-516-7615, -7600 (Fax)
joe.harrington@jhu.edu
www.econ.jhu.edu/People/Harrington

November 2011

Abstract

A theory of tacit collusion is developed based on coordination through price leadership and less than full mutual understanding of strategies. It is common knowledge that price increases are to be at least matched but who should lead and at what price is not common knowledge. The steady-state price is characterized and it falls short of the best collusive equilibrium price. That coordination is through tacit means and not express communication is then shown to limit the extent of the price rise from collusion.

*I appreciate the comments of George Mailath, seminar participants at Universitat Pompeu Fabra, Universitat de València, Penn-Wharton, and EARIE 2011, and the research assistance of Wei Zhao.

1 Introduction

The economic theory of collusion focuses on what outcomes are sustainable and the strategy profiles that sustain them. What prices and market allocations can be supported? What are the most effective strategies for monitoring compliance? What are the most severe punishments that can be imposed in response to evidence of non-compliance? The literature is rich in taking account of the determinants of the set of collusive outcomes including market traits such as product differentiation and demand volatility, firm traits such as capacity, cost, and time preference, and the amount of public and private information available to firms.

In comparison, the primary focus of antitrust law is not on the outcome nor the strategies that sustain an outcome but rather the means by which a collusive arrangement is achieved. The illegality comes from firms having an agreement to coordinate their behavior.

> [A]ntitrust law clarified that the idea of an agreement describes a process that firms engage in, not merely the outcome that they reach. Not every parallel pricing outcome constitutes an agreement because not every such outcome was reached through the process to which the law objects: a negotiation that concludes when the firms convey mutual assurances that the understanding they reached will be carried out.[1]

To establish the presence of an agreement - and thereby a violation of Section 1 of the Sherman Act - it must be shown that firms "had a conscious commitment to a common scheme designed to achieve an unlawful objective,"[2] that they had a "unity of purpose or a common design and understanding, or a meeting of minds."[3] Thus, the law focuses on what mutual understanding exists among firms and how that mutual understanding was achieved.[4]

From this perspective, U.S. antitrust law has identified three types of collusion. *Explicit collusion* is when supracompetitive prices are achieved via express communication about an agreement; there has been a direct exchange of assurances regarding the coordination of their conduct. Mutual understanding is significant and is acquired through express communication. Explicit collusion is illegal. *Conscious parallelism* is when supracompetitive prices are achieved without express communication. A common example is two adjacent gasoline stations in which one station raises its price to a supracompetitive level and the other station matches the price hike. While there may be mutual understanding regarding the underlying mechanism that stabilizes those supracompetitive prices (for example, any price undercutting results in a return to competitive prices), this understanding was not reached through express communi-

[1] Baker (1993), p. 179.

[2] *Monsanto Co. v. Spray-Rite Serv. Corp.*, 465 U.S. 752 (1984); 753.

[3] *American Tobacco Co. v. United States*, 328 U.S. 781 (1946); 810.

[4] The distinction between the economic and legal approaches to collusion is presented in Kaplow and Shapiro (2007); also, see Kaplow (2011a, 2011b, 2011c).

cation. Conscious parallelism is legal.[5] *Concerted action* resides between these two extremes and refers to when supracompetitive prices are achieved with direct communication - such as about intentions - but firms do not expressly propose and reach an agreement (Page 2007).[6] For example, concerted practices may involve a firm's public announcement of a proposed pricing policy which, without the express affirmative response from its rivals, is followed by the common adoption of that policy with a subsequent rise in price. The extent of mutual understanding is more than conscious parallelism but does not reach the level of explicit collusion. Concerted action lies in the gray area of what is legal and what is not. Conscious parallelism and concerted action are both forms of tacit collusion in that a substantive part of the collusive arrangement is achieved without express communication.

While the distinction between explicit and tacit collusion exists in practice and in the law, it is a distinction that is largely absent from economic theory.[7] The economic theory of collusion - based on equilibrium analysis - presumes mutual understanding is complete (that is, the strategy profile is common knowledge) and does not deal with how mutual understanding is achieved, nor the extent of coordinated behavior that can result when there are gaps in mutual understanding. Furthermore, there is good reason for firms to try to collude without express communication, and thus find themselves dealing with less than full mutual understanding. Given that explicit collusion is illegal and tacit collusion is generally legal, if firms can achieve a collusive outcome through tacit means then they will presumably do so and thereby avoid the possibility of financial penalties and jail time. This then leads one to ask: What types of markets are conducive to tacit collusion? What types of public announcements are able to generate sufficient mutual understanding to produce collusion? In markets for which both explicit and tacit collusion are feasible, when is collusion through explicit means significantly more profitable? To address those questions requires developing distinct theories of explicit collusion and tacit collusion. Of course, the

[5] "Conscious parallelism is parallel behavior that typically appears in markets with small number of sellers. It is not the result of an explicit agreement [and] refers to a form of tacit collusion in which each firm in an oligopoly realizes that it is within the interests of the entire group of firms to maintain a high price or to avoid vigorous price competition, and the firms act in accordance with this realization." (Hylton, 2003, p. 73)

[6] From *Interstate Circuit, Inc. v. United States*, 306 U.S. 226 (1939): "It was enough that, knowing that concerted action was contemplated or invited, the distributors gave their adherence to the scheme and participated in it. ... [A]cceptance by competitors, without previous agreement, of an invitation to participate in a plan, the necessary consequence of which, if carried out, is restraint of interstate commerce, is sufficient to establish an unlawful conspiracy under the Sherman Act."

[7] Rightfully and frequently, lawyers remind economists of our inadequacy in this regard:

> While properly applied economic science may allow an economist to reach conclusions about "collusion," the term as used by economists may include both tacit and overt collusion among competitors ... and it is unclear whether economists have any special expertise to distinguish between the kinds of "agreement." [Milne and Pace (2003), p. 36]
>
> On the ultimate issue of whether behavior is the result of a contract, combination, or conspiracy, however, courts routinely prevent economists from offering an opinion, because economics has surprisingly little to say about this issue. [Page (2007), p. 424]

primary challenge to modelling tacit collusion is dispensing with the assumption of equilibrium and allowing for less than full mutual understanding among firms.[8]

The contribution of this paper is in developing a theory of tacit collusion. Two essential elements of a model of tacit collusion are: 1) a transparent mechanism for coordinating on a collusive outcome; and 2) a plausible amount of mutual understanding among firms. The coordination mechanism considered here is price leadership, which is a commonly observed method of tacit collusion.[9] In terms of mutual understanding, it is assumed that it is common knowledge among firms that price increases will be at least matched and that failure to do so results in reversion to the pre-collusive outcome.[10] What is not common knowledge is leadership protocol. Which firm will lead by raising price? What price will it set? Is another firm expected to lead the next round of price hikes? In other words, there is mutual understanding among firms about the general mechanism of price leadership and price matching, but firms may lack common beliefs regarding the specific sequence of prices. Another way to view this assumption on mutual understanding is that, rather than suppose a strategy profile is common knowledge as is done with an equilibrium analysis, it is instead assumed to be common knowledge that firms' strategies lie in a subset of the strategy space. I will argue that this assumption on mutual understanding is plausibly achieved without express communication of the variety that would be a Section 1 violation.

Without the equilibrium assumption, two questions are of particular interest. First, can we characterize firms' prices when they lack mutual understanding as to their strategies? How much mutual understanding is required to say something precise? Second, assuming we can say something precise, what is the cost to firms from not having full mutual understanding? Is price lower under tacit collusion than if they were to engage in express communication and achieve the mutual understanding of strategies implicit in equilibrium?

In answer to the first question, I show that a precise statement can be made as to the steady-state price, though the transition path eludes characterization. As regards the second question, the lack of full mutual understanding does indeed constrain the

[8] One should not be misled to believe that the theoretical industrial organization literature is replete with theories of tacit collusion by virtue of these theories being called "tacit collusion," as exemplified by the excellent survey "The Economics of Tacit Collusion" (Ivaldi et al, 2003). These theories characterize collusive behavior assuming full mutual understanding of strategies (that is, equilibrium) and are agnostic regarding how mutual understanding is reached. There is, however, some research that is most naturally considered explicit collusion because it assumes firms expressly communicate within the context of an equilibrium. Cheap talk messages about firms' private information on cost are exchanged in Athey and Bagwell (2001, 2008), on demand in Aoyagi (2002), Hanazono and Yang (2007), and Gerlach (2009), and on sales in Harrington and Skrzypacz (2011). There is also a body of work on bidding rings in auctions where participation in the auction is preceded by a mechanism among the ring members that involves the exchange of reported valuations; see, for example, Graham and Marshall (1987) and Krishna (2010).

[9] See Markham (1951) for an early discussion of price leadership and collusion, and Scherer (1980, Chapter 6) for several examples. In the equilibrium setting, some relevant papers exploring price leadership as a collusive device include Rotemberg and Saloner (1990) and Mouraviev and Rey (2010).

[10] The role of price-matching here is to coordinate on a collusive outcome. It has also been explored as a form of punishment; see Lu and Wright (2010) and Garrod (2011).

extent of collusion; the steady-state price is strictly below the highest sustainable equilibrium price. In other words, if firms could expressly communicate, they would sustain a price in excess of that which is achieved under tacit collusion.

To my knowledge, the only other theory of tacit collusion is MacLeod (1985), whose approach is very different. To begin, it is based on firms announcing proposed price changes rather than making actual price changes. Axioms specify how firms respond to a price announcement, and these axioms are common knowledge. A firm's price response is allowed to depend on the existing price vector and the announced price change, and it is assumed the firm which announces the price change will implement it. If it is assumed that the price response is continuous with respect to the announcement, invariant to scale changes, and independent of firm identity then the response function must entail matching the announced price change.[11] When firms are symmetric, the theory predicts that the joint profit maximum is achieved. To the contrary, the theory developed here predicts price is always below the highest equilibrium price.

Of some relevance to the current paper is the literature on the rational learning of strategies in a repeated game; see, for example, Kalai and Lehrer (1993) and Nachbar (2005). The main result of Kalai and Lehrer (1993) is that if players are rational and each starts with a set of beliefs on other players' strategies that are compatible with the strategies actually chosen then play must converge in finite time to an $\varepsilon-$Nash equilibrium of the repeated game, for arbitrarily small ε. Assumptions are very weak in that a player need not know other players' payoffs or whether they are rational. In contrast, it is assumed here that rationality and payoff functions are common knowledge. While both that literature and the current paper explore behavior in a repeated game setting when strategies are not common knowledge, their objectives are very different. The rational learning literature seeks to determine how weak one can make the assumptions on beliefs in an infinitely repeated setting and still achieve convergence on an equilibrium. The current paper's goal is to develop a theory of tacit collusion; that is, making predictions on price based on plausible assumptions on mutual understanding. Given these distinct goals, the amount of structure placed on prior beliefs is very different. The rational learning literature only requires that a player's prior beliefs on the other players' strategies include their actual strategies in the support. The paper here draws from the context of tacit collusion in a market to place a substantive, though plausible, amount of structure on prior beliefs, and then derive its implications for prices. The results are more precise but then the assumptions are stronger.

In Section 2, standard assumptions are made regarding cost, demand, and firm objectives. In Section 3, the assumption of equilibrium is replaced with alternative assumptions on the behavior and beliefs of firms. The main result is in Section 4, and the case of linear demand and cost functions is considered in Section 5 to further explore the price effect of firms coordinating their behavior through tacit, rather than

[11] If it was not assumed to be common knowledge that the firm announcing the price change would implement it then another price response function which satisfies the axioms is one which has a zero price response. In fact, it should be stated as a fourth axiom that the price change of the firm announcing the price change equals that announcement.

explicit, means.

2 Assumptions on the Market

Consider a symmetric differentiated products price game with n firms. $\pi(p_i, \mathbf{p}_{-i})$ is a firm 's profit when it prices at p_i and its rivals price at $\mathbf{p}_{-i} = (p_1, ..., p_{i-1}, p_{i+1}, ..., p_n)$. Assume $\pi(p_i, \mathbf{p}_{-i})$ is bounded, twice continuously differentiable, increasing in a rival's price p_j $(j \neq i)$, and strictly concave in own price p_i. A firm's best reply function then exists:

$$\psi(\mathbf{p}_{-i}) = \arg\max_{p_i} \pi(p_i, \mathbf{p}_{-i}).$$

Further assume

$$\frac{\partial^2 \pi}{\partial p_i \partial p_j} > 0, \ \forall j \neq i$$

from which it follows that $\psi(\mathbf{p}_{-i})$ is increasing in p_j, $j \neq i$. A symmetric Nash equilibrium price, p^N, exists and is assumed to be unique, which implies:

$$\psi(p, .., p) \gtreqless p \text{ as } p \lesseqgtr p^N$$

and let

$$\pi^N \equiv \pi\left(p^N, ..., p^N\right) > 0.$$

Assuming $\pi(p, ..., p)$ is strictly concave in p, there exists a unique joint profit maximum p^M,

$$\sum_{j=1}^{n} \frac{\partial \pi(p, ..., p)}{\partial p_j} \gtreqless 0 \text{ as } p \lesseqgtr p^M,$$

and $p^M > p^N$.

Firms interact in an infinitely repeated price game with perfect monitoring. A collusive price $p' > p^N$ is sustainable with the grim trigger strategy if and only if:[12]

$$\left(\frac{1}{1-\delta_i}\right) \pi\left(p', ..., p'\right) \geq \max_{p_i < p} \pi\left(p_i, p', ..., p'\right) + \left(\frac{\delta_i}{1-\delta_i}\right) \pi^N, \ \forall i = 1, ..., n. \quad (1)$$

δ_i is the discount factor of firm i and firms are allowed to be heterogeneous in that respect:

$$0 < \delta_n \leq \delta_{n-1} \leq \cdots \leq \delta_1 < 1.$$

Define $\widetilde{p}$ as the best price sustainable using the grim trigger strategy:

$$\widetilde{p} \equiv \max\left\{p \in \left[p^N, p^M\right] : \left(\frac{1}{1-\delta_i}\right) \pi(p, ..., p) \geq \max_{p_i < p} \pi(p_i, p, ..., p) + \left(\frac{\delta_i}{1-\delta_i}\right) \pi^N, \forall i = 1, ..., n\right\}.$$

[12] The grim trigger strategy has any deviation from the collusive price p' result in a price of p^N forever.

Assume $\widetilde{p} > p^N$ and if $\widetilde{p} \in \left(p^N, p^M\right)$ then

$$\left(\frac{1}{1-\delta_n}\right)\pi\left(p,...,p\right) \gtreqless \pi\left(\psi\left(p,...,p\right),p,...,p\right)+\left(\frac{\delta_n}{1-\delta_n}\right)\pi^N \text{ as } p \lesseqgtr \widetilde{p} \text{ for } p \in \left[p^N, p^M\right]. \tag{2}$$

Note that the preceding condition is for firm n because, given it has the (weakly) lowest discount factor, its incentive compatibility constraint is the first to bind. $\widetilde{p}$ will prove to be a useful benchmark.[13]

For the later analysis, consider the "price matching" objective function for firm i:

$$W_i\left(p_i, \mathbf{p}_{-i}\right) \equiv \pi\left(p_i, \mathbf{p}_{-i}\right) + \left(\frac{\delta_i}{1-\delta_i}\right)\pi\left(p_i,...,p_i\right).$$

Given its rivals price at $\mathbf{p}_{-i}$ in the current period, $W_i\left(p_i, \mathbf{p}_{-i}\right)$ is firm i's payoff from pricing at p_i in the current period and all firms matching that price in all ensuing periods. Note that

$$\frac{\partial W_i\left(p_i, \mathbf{p}_{-i}\right)}{\partial p_i} = \frac{\partial \pi\left(p_i, \mathbf{p}_{-i}\right)}{\partial p_i} + \left(\frac{\delta_i}{1-\delta_i}\right)\sum_{j=1}^{n}\frac{\partial \pi\left(p_i,...,p_i\right)}{\partial p_j}.$$

If $p_i < p^M$ then the second term is positive; by raising its current price, a firm increases the future profit stream under the assumption that its price increase will be matched by its rivals. If $p_i > \psi\left(p_{-i}\right)$ then the first term is negative. Evaluate $\frac{\partial W_i\left(p_i, p_{-i}\right)}{\partial p_i}$ when firms price at a common level p:

$$\begin{aligned}\frac{\partial W_i\left(p,...,p\right)}{\partial p_i} &= \frac{\partial \pi\left(p,...,p\right)}{\partial p_i} + \left(\frac{\delta_i}{1-\delta_i}\right)\sum_{j=1}^{n}\frac{\partial \pi\left(p,...,p\right)}{\partial p_j} \\ &= \left(\frac{1}{1-\delta_i}\right)\left(\frac{\partial \pi\left(p,...,p\right)}{\partial p_i} + \delta\sum_{j\neq i}^{n}\frac{\partial \pi\left(p,...,p\right)}{\partial p_j}\right)\end{aligned}$$

Thus, when $p \in \left(p^N, p^M\right)$, raising price lowering current profit, $\frac{\partial \pi(p,...,p)}{\partial p_i} < 0$, and increases future profit, $\sum_{j=1}^{n}\frac{\partial \pi(p,...,p)}{\partial p_j} > 0$. By the preceding assumptions, $W_i\left(p_i, \mathbf{p}_{-i}\right)$ is strictly concave in p_i since it is the weighted sum of two strictly concave functions. Hence, a unique optimal price exists,

$$\phi_i\left(\mathbf{p}_{-i}\right) = \max_{p_i} W_i\left(p_i, \mathbf{p}_{-i}\right). \tag{3}$$

$\phi_i\left(\mathbf{p}_{-i}\right)$ is referred to as the price matching best reply function for firm i. By the preceding assumptions, $\phi_i\left(\mathbf{p}_{-i}\right)$ is increasing in a rival's price as

$$\frac{\partial \phi_i\left(\mathbf{p}_{-i}\right)}{\partial p_j} = -\frac{\partial^2 W\left(p_i, \mathbf{p}_{-i}\right)/\partial p_i \partial p_j}{\partial^2 W\left(p_i, \mathbf{p}_{-i}\right)/\partial p_i^2} = -\frac{\frac{\partial^2 \pi(p_i, \mathbf{p}_{-i})}{\partial p_i \partial p_j}}{\frac{\partial^2 \pi(p_i, \mathbf{p}_{-i})}{\partial p_i^2} + \left(\frac{\delta_i}{1-\delta_i}\right)\left(\frac{\partial^2 \pi(p,...,p)}{\partial p^2}\right)} > 0.$$

[13] Firms are not allowed to coordinate on a collusive outcome with unequal market shares which is one way to improve collusion when firms have different discount factors; see Harrington (1989). This restriction would seem reasonable given that firms are tacitly colluding in which case it isn't clear how they would achieve mutual understanding regarding a market allocation without engaging in express communication.

As there is a benefit in terms of future profit from raising price (as long as it does not exceed the joint profit maximum) then the price matching best reply function results in a higher price than the standard best reply function. To show this result, consider

$$\begin{aligned}\frac{\partial W_i\left(\psi\left(\mathbf{p}_{-i}\right),\mathbf{p}_{-i}\right)}{\partial p_i} &= \frac{\partial \pi\left(\psi\left(\mathbf{p}_{-i}\right),\mathbf{p}_{-i}\right)}{\partial p_i}+\left(\frac{\delta_i}{1-\delta_i}\right)\sum_{j=1}^{n}\frac{\partial \pi\left(\psi\left(\mathbf{p}_{-i}\right),...,\psi\left(\mathbf{p}_{-i}\right)\right)}{\partial p_j}\\ &= \left(\frac{\delta_i}{1-\delta_i}\right)\sum_{j=1}^{n}\frac{\partial \pi\left(\psi\left(\mathbf{p}_{-i}\right),...,\psi\left(\mathbf{p}_{-i}\right)\right)}{\partial p_j}>0,\end{aligned}$$

which is positive because $\mathbf{p}_{-i}\leq\left(p^M,...,p^M\right)$ implies $\psi\left(\mathbf{p}_{-i}\right)<p^M$.[14] By the strict concavity of W_i, $\phi_i\left(\mathbf{p}_{-i}\right)>\psi\left(\mathbf{p}_{-i}\right)$.

ϕ_i has a fixed point p_i^* because it is continuous, $\phi_i\left(p^N,...,p^N\right)>p^N$, and

$$\frac{\partial W_i\left(p^M,...,p^M\right)}{\partial p_i}=\frac{\partial \pi\left(p^M,...,p^M\right)}{\partial p_i}<0\Rightarrow\phi_i\left(p^M,...,p^M\right)<p^M.$$

Further assume the fixed point is unique:

$$\phi_i\left(p,...,p\right)\gtreqless p\text{ as }p\lesseqgtr p_i^*.$$

Thus, if rival firms price at p_i^*, firm i prefers to price at p_i^* rather than price differently under the assumption that its price will be matched.

To characterize the relationship between a firm's discount factor and p_i^*, first note that

$$\frac{\partial^2 W_i\left(p,...,p\right)}{\partial p_i\partial\delta}=\left(\frac{1}{\left(1-\delta_i\right)^2}\right)\sum_{j=1}^{n}\frac{\partial\pi\left(p,...,p\right)}{\partial p_j}>0,\text{ if }p<p^M. \tag{4}$$

p_{i+1}^* is defined by:

$$\frac{\partial W_{i+1}\left(p_{i+1}^*,...,p_{i+1}^*\right)}{\partial p_{i+1}}=\frac{\partial\pi\left(p_{i+1}^*,...,p_{i+1}^*\right)}{\partial p_{i+1}}+\left(\frac{\delta_{i+1}}{1-\delta_{i+1}}\right)\sum_{j=1}^{n}\frac{\partial\pi\left(p_{i+1}^*,...,p_{i+1}^*\right)}{\partial p_j}=0. \tag{5}$$

Substitute δ_i for δ_{i+1} in (5) and then using $\delta_i\geq\delta_{i+1}$ and (4), it follows that

$$\frac{\partial W_i\left(p_{i+1}^*,...,p_{i+1}^*\right)}{\partial p_i}=\frac{\partial\pi\left(p_{i+1}^*,...,p_{i+1}^*\right)}{\partial p_i}+\left(\frac{\delta_i}{1-\delta_i}\right)\sum_{j=1}^{n}\frac{\partial\pi\left(p_{i+1}^*,...,p_{i+1}^*\right)}{\partial p_j}\geq 0.$$

The concavity of W_i then implies $p_i^*\geq p_{i+1}^*$. Hence,

$$p^N<p_n^*\leq p_{n-1}^*\leq\cdots\leq p_1^*<p^M.$$

p_1^* will be an important benchmark for the analysis.

The example in Section 5 of linear demand and cost satisfies all of the assumptions made here.

[14] Since $\psi\left(p,..,p\right)\gtreqless p$ as $p\lesseqgtr p^N$ then $\psi\left(p^M,..,p^M\right)<p^M$. Given that ψ is increasing then $\mathbf{p}_{-i}\leq\left(p^M,...,p^M\right)$ implies $\psi\left(\mathbf{p}_{-i}\right)<\psi\left(p^M,..,p^M\right)<p^M$.

3 Assumptions on Beliefs and Behavior

The standard equilibrium approach to characterizing firm pricing entails making assumptions on behavior - each firm acts to maximize its payoff given the conjectured strategies of the other firms - and beliefs - each firm's conjectures are accurate. As the focus here is on tacit collusion - in which case firms do not engage in express communication - it is problematic that they would have accurate beliefs as to collusive strategies, especially in light of the abundance of collusive equilibria. Thus, rather than start with a particular strategy profile that is presumed to be common knowledge, a class of strategy profiles will be constructed through a series of assumptions on firms' beliefs. It will still involve some mutual understanding among firms though much less than is presumed with an equilibrium analysis and, as will be argued, the extent of mutual understanding is plausibly achieved without express communication.

In thinking about tacit collusion through price leadership, it is natural, and will prove useful, to break it down into three elements: 1) leadership - who leads? when? at what price?; 2) followership - how is a firm supposed to respond to another firm leading?; and 3) default - what does a firm do when behavior is inconsistent with commonly held beliefs? But before making assumptions to deal with each of those elements, let us first make a standard assumption about firm objectives and their beliefs on other firms' objectives.

Assumption 1: Each firm is rational in the sense of maximizing the present value of its expected profit stream, and rationality is common knowledge.

A key premise is that firms strive to at least match price increases to the greatest extent feasible and desirable, and this is common knowledge. Towards implementing that idea, I begin by defining when a strategy has the price matching plus (PMP) property.

Definition: The strategy of firm i satisfies the *price matching plus property* if: $p_j^\tau \geq \min\left\{\max\left\{p_1^{\tau-1},...,p_n^{\tau-1}\right\},\overline{p}\right\}\ \forall j, \forall \tau \leq t-1$ implies

$$p_i^t \begin{cases} \geq \max\left\{p_1^{t-1},...,p_n^{t-1}\right\} & \text{if } \max\left\{p_1^{t-1},...,p_n^{t-1}\right\} < \overline{p} \\ = \overline{p} & \text{if } \max\left\{p_1^{t-1},...,p_n^{t-1}\right\} \geq \overline{p} \end{cases}$$

By the PMP property, a firm will set price at least as high as the highest price in the previous period though not exceeding a price of $\overline{p}$, as long as no firm has veered from such behavior in the past. A key implication is that a firm will (at least) match price increases up to a price of $\overline{p}$. $\overline{p}$ is specified later in a manner consistent with the premise that firms seek to follow price increases as long as doing so is feasible and desirable.

Assumption 2: It is common knowledge that firms' strategies satisfy the PMP property.

By Assumption 2, there is a "meeting of minds" among firms that price increases will (at least) be matched, as long as price is not too high and past price increases have always been (at least) matched. How could this mutual understanding be achieved without express communications of the type associated with explicit collusion? First, it could occur through unilateral public announcements whereby one firm's manager declares elements of a strategy that encompasses price leadership and price matching. In the one-way truck rental market, the FTC claimed that, in a public announcement regarding earnings, the CEO of U-Haul repeatedly emphasized that U-Haul was demonstrating "price leadership" and was "trying to force prices."[15] While this was a case in which there were legal proceedings associated with Section 5 of the FTC Act (though not regarding Section 1 of the Sherman Act), it is not difficult to imagine such proclamations being made that would avoid prosecution. Second, mutual understanding could be achieved by the adoption of actions that served to communicate an expectation among firms that they will engage in coordinated pricing. It is argued in Harrington (2011) that, under certain market conditions, the mutual adoption of the posted price format signals that firms expect to collude. In the case of the turbine generator market, General Electric and Westinghouse mutually adopted the posted price format and subsequently engaged in tacit collusion through price leadership and price matching.[16] Thus, by taking certain costly actions that would only be optimal if firms did engage in tacit collusion, mutual understanding regarding price matching could be achieved. Third, mutual understanding of price matching could be acquired by way of example. One firm could raise its price and if rivals subsequently matched that price then firms may then have mutual understanding regarding price matching; from that point onward, Assumption 2 could well hold. This is a view that has been expressed by Richard Posner, first as a scholar and then as a judge in High Fructose Corn Syrup (2002):

> [O]ne seller communicates his "offer" by restricting output, and the offer is "accepted" by the actions of this rivals in restricting their outputs as well. It may therefore be appropriate in some cases to instruct a jury to find an agreement to fix prices if it is satisfied that there was a tacit meeting of the minds of the defendants on maintaining a noncompetitive pricing policy.[17]
>
> Section 1 of the Sherman Act ... is broad enough ... to encompass a purely tacit agreement to fix prices, that is, an agreement made without any actual communication among the parties to the agreement. If a firm raises price in the expectation that its competitors will do likewise, and they do, the firm's behavior can be conceptualized as the offer of a unilateral contract that the offerees accept by raising their prices.[18].

[15] *Matter of U-Haul Int'l Inc. and AMERCO* (FTC File No. 081-0157, July 10, 2010).

[16] The posted price format has firms publicly announce a non-negotiable price. For details on the turbine generator case, see Scherer (1980, p. 182) and Hay (2000).

[17] Posner (2001), pp. 94-95.

[18] *In Re High Fructose Corn Syrup Antitrust Litigation Appeal of A & W Bottling Inc et al*, United States Court of Appeals, 295 F3d 651, (7th Cir., 2002); 652.

In sum, Assumption 2 would seem, at a minimum, to be plausible even without the direct forms of communication associated with explicit collusion.

The next matter to consider is what firms will do if, contrary to common expectations, a firm violates the PMP property. Here, I will draw on Lewis (1969) to argue that the incongruity between observed behavior and expectations causes firms to focus on an outcome that is salient. Lewis (1969) defines a salient outcome as "one that stands out from the rest by its uniqueness in some conspicuous respect"[19] and that precedence is one source of saliency: "We may tend to repeat the action that succeeded before if we have no strong reason to do otherwise."[20] Cubitt and Sugden (2003) stress the latter qualifier and note that "precedent allows the individual to make inductive inferences in which she has *some* confidence, but which are overridden whenever deductive analysis points clearly in a different direction."[21]

With this perspective in mind, the movement from competition to tacit collusion can be seen as a shift from inductive to deductive reasoning. Firms had been competing and would be expected to continue to do so by induction. However, either through price signalling or public announcement of strategies or some other coordinating event, firms supplant inductive inferences with deductive reasoning so that a common expectation of competition is replaced with one of price leadership and price matching (as described in Assumption 2). With this as a backdrop, my claim is that, in response to a subsequent departure in behavior from Assumption 2 and thus a breakdown in the efficacy of deductive reasoning, firms revert to the original inductive analysis which then results in a competitive solution. Here I am appealing to the view that firms will "tend to pick the salient as a last resort."[22] The saliency of the competitive solution emanates from it being the most recent outcome (prior to the current episode of tacit collusion) that was common knowledge to firms.[23]

Assumption 3: It is common knowledge that behavior inconsistent with the PMP property results in pricing at p^N forever.

Before moving on, there are two implicit assumptions in the rationale for Assumption 3 that warrant discussion. First, the saliency of the competitive solution relies on it prevailing prior to this episode of tacit collusion, however that is not essential for the paper's main result. If some other behavior described the pre-collusion setting then that behavior can be inserted into Assumption 3. What *is* critical is that how firms respond to the departure from the PMP property is common knowledge and the associated continuation payoff is lower than if firms had abided by the PMP property. A second assumption, which figures prominently in discussions of saliency

[19] Lewis, (1969), p. 35.

[20] Lewis, (1969), p. 37.

[21] Cubitt and Sugden (2003), p. 196. Also see Sugden (2011).

[22] Lewis, (1969), p. 35.

[23] If two people are physically separated and in search of each other, a salient place for them to meet is the last place that they were together. That place is common knowledge to them - as they both witnessed each other there - and it is singular in being the most recent place visited that is common knowledge. Analogously, if there is inconsistency in firm behavior, firms may return to the most recent strategy profile that was common knowledge.

(such as in Lewis, 1969), is that the current post-collusion situation is sufficiently similar to the pre-collusion situation so that induction on the latter is compelling. It is well-recognized that

> no two interactions are exactly alike. Any two real-world interactions will differ in matters of detail, quite apart from the inescapable fact that "previous" and "current" interactions occur at different points in time. Thus, the idea of "repeating what was done in previous instances of the game" is not well-defined. Precedent has to depend on analogy: to follow precedent in the presence instance is to behave in a way that is *analogous with* behaviour in past instances. ... Inductive inference is possible only because a very small subset of the set of possible patterns is privileged.[24]

The post-collusion scenario most notably differs from the pre-collusion scenario in that the former was preceded by an episode of collusion, while the latter was (probably) not. Though this difference could disrupt the saliency of the pre-collusion outcome when it comes to responding to a departure from the PMP property, I believe it is plausible that its saliency remains intact.

Thus far, it is assumed to be common knowledge that firms are rational, firms will (at least) match price increases up to some maximum price $\overline{p}$, and departures from that price matching behavior results in a return to the competitive solution. The next step is to specify $\overline{p}$. My working assumption has been that firms will match price increases to the greatest extent feasible and desirable. The feasibility constraint is determined by which price increases are sustainable. Define p^U as the highest price satisfying (1); note that it can exceed p^M. Now suppose $\overline{p} > p^U$. If $\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\} = \overline{p}$ then, by Assumption 2, $p_j^\tau = \overline{p}\ \forall j, \forall \tau \geq t$. However, this pricing behavior contradicts Assumption 1 as it follows from $\overline{p} > p^U$ that

$$\left(\frac{1}{1-\delta_n}\right)\pi\left(\overline{p}, ..., \overline{p}\right) < \pi\left(\psi\left(\overline{p}, ..., \overline{p}\right), \overline{p}, ..., \overline{p}\right) + \left(\frac{\delta_n}{1-\delta_n}\right)\pi^N,$$

which implies firm n does better by pricing at $\psi\left(\overline{p}, ..., \overline{p}\right)$ and earning π^N thereafter (which ensues by virtue of Assumption 3). Hence, if $\overline{p} > p^U$ then Assumptions 1-3 are incompatible and, therefore, matching price increases up to $\overline{p}$ are not feasible. Turning to the desirability constraint, firms earn higher profit by matching higher price increases as long as the price being matched does not exceed p^M. It is then assumed that price increases up to p^M will at least be matched as long as they are sustainable. This discussion motivates Assumption 4.[25]

Assumption 4: $\overline{p} = \widetilde{p}$.

A strategy satisfying the properties in Assumptions 2-4 is referred to as "PMP-compatible."

[24] Cubitt and Sugden (2003), pp. 196-7.

[25] As argued in the text, A1-A3 imply $\overline{p} \leq \widetilde{p}$. Thus, the additional structure imposed by A4 is that $\overline{p}$ is not strictly less than $\widetilde{p}$.

Assumptions 2 and 4 describe followership behavior, while Assumption 3 describes default behavior in response to a departure from common expectations. We have yet to specify leadership behavior, which is the most problematic from a common knowledge perspective. In some markets, a particular firm may be the salient leader by virtue of its size or access to information (what is referred to as barometric price leadership; see, for example, Cooper, 1997). However, keep in mind that leadership is costly in that a firm that leads will lose demand prior to its price being matched.[26] As each firm would prefer another firm to take the lead by raising price, this could cause a lack of common knowledge as to who will lead, as well as the price to be set. In light of this discussion, it would seem problematic to assume the identity of the price leader and the pattern of price increases to be common knowledge, at least in the absence of express communication. For that reason, a very weak assumption concerning price leadership will be made.

Define $h^t(p', t')$ to be a period t history such that $(p_1^\tau, ..., p_n^\tau) = (p', ..., p'), \forall \tau = t', ..., t-1$. That is, all firms have priced at p' in the preceding $t' - t + 1$ periods. Next define $\rho_i\left(\mathbf{p}_{-i}^t \,\middle|\, h^t\right)$ as firm i's beliefs on other firms' period t prices, conditional on the history.

Assumption 5: $\lim_{t\to\infty} \rho_i\left(\mathbf{p}_{-i}^t = (p', ..., p') \,\middle|\, h^t(p', t')\right) = 1.$

By Assumption 5, if prices have remained at the same level for a long time then a firm assigns high probability to other firms maintaining that price. Of particular relevance for the ensuing analysis, a firm assigns low probability to one of its rivals taking the lead and raising price. Such beliefs are consistent with common learning rules such as fictitious play. For tacit collusion through price leadership to be assured of having any success in raising price, Assumption 5 is essential. Otherwise, each firm could always expect a rival to imminently raise price and, since a firm would prefer to match a rival's price increase then to take the lead itself, this would result in each firm not raising price *ad infinitum*.

Let me summarize the assumptions on behavior and beliefs. In terms of behavior, it is assumed that a firm is rational, a firm will (at least) match a rival's price as long as price does not exceed the highest sustainable price, and a firm will respond with competitive pricing if any firm should depart from this price matching behavior. As regards beliefs, the previous description of behavior is common knowledge and, in addition, each firm believes it is unlikely a rival will change its price when prices have not changed for a long time.

[26] Wang (2009) provides indirect evidence of the costliness of price leadership. In a retail gasoline market in Perth, Australia, Shell was the price leader over 85% of the time until a new law increased the cost of price leadership, after which the three large firms - BP, Caltex, and Shell - much more evenly shared the role of price leader. The law specified that every gasoline station was to notify the government by 2pm of its next day's retail prices, and to post prices on its price board at the start of the next day *for a duration of at least 24 hours*. Hence, a firm which led in price could not expect its rivals to match its price until the subsequent day. The difference between price being matched in an hour and in a day is actually quite significant given the high elasticity of firm demand in the retail gasoline market. For the Quebec City gasoline market, Clark and Houde (2011, p. 20) find that "a station that posts a price more than 2 cents above the minimum price in the city loses between 35% and 50% of its daily volume."

Consistent with tacit collusion, I believe this is a plausible amount of mutual understanding that could reasonably be achieved without the express communication associated with explicit collusion. Furthermore, there remains significant residual uncertainty among firms about the strategies of their rivals. It is not common knowledge as to who will lead a price increase, when it will occur, what price a leader will set, and whether price increases will just be matched or instead exceeded. In sum, it is common knowledge that the strategy profile lies in a subset of the strategy space (defined by those strategies satisfying A2-A4), but that subset is still quite large and encompasses strategy profiles that would produce no change in price, a single firm acting as a price leader, rotating price leadership, a short or long series of price increases, and many other possibilities. As shown in the next section, what will allow us to say something precise about the resultant price path will be the assumption that it is common knowledge that each firm acts to maximize the present value of its expected profit stream.

4 Steady-State Price under Tacit Collusion

The main result is shown when the price set is finite.[27] Assume the price set is $\Delta_\varepsilon \equiv \{0, \varepsilon, 2\varepsilon, ..., \}$, where $\varepsilon > 0$ and is presumed to be small. For convenience, suppose $p^N, p^*, \widetilde{p} \in \Delta_\varepsilon$, where p^N, p^*, and $\widetilde{p}$ are defined for when the price set is $\Re_+$.[28] As the discreteness of the price set could generate multiple optima, define the best reply correspondence for the price matching objective function:

$$\overline{\phi}_i(\mathbf{p}_{-i}) \equiv \arg\max_{p_i \in \Delta_\varepsilon} \pi(p_i, \mathbf{p}_{-i}) + \left(\frac{\delta_i}{1-\delta_i}\right)\pi(p_i, ..., p_i).$$

The property of the best reply correspondence required for proving the main result is:[29]

$$\overline{\phi}_i(\mathbf{p}_{-i}) \begin{cases} \subseteq \{p' + \varepsilon, ..., p_i^*\} & \text{if } \mathbf{p}_{-i} = (p', ..., p') \text{ where } p' < p_i^* - \varepsilon \\ = \{p_i^*\} & \text{if } \mathbf{p}_{-i} = (p_i^*, ..., p_i^*) \\ \subseteq \{p_i^*, ..., p' - \varepsilon\} & \text{if } \mathbf{p}_{-i} = (p', ..., p') \text{ where } p' > p_i^* + \varepsilon \end{cases} \tag{6}$$

Recall that p_i^* is a fixed point for $\phi_i(\mathbf{p}_{-i})$ and also for $\overline{\phi}_i(\mathbf{p}_{-i})$. By (6), if all rival firms price at p' then firm i's best reply has it price above p' when $p' < p_i^* - \varepsilon$. Analogously, if $p' > p_i^* + \varepsilon$ then firm i's best reply has it price below p'. Note that an implication of (6) is that the set of symmetric fixed points is, at most, $\{p_i^* - \varepsilon, p_i^*, p_i^* + \varepsilon\}$.[30]

Our main result is that the steady-state price is (close to) p_1^* when $p_1^* < \widetilde{p}$ and otherwise is $\widetilde{p}$.

[27] A discussion of the case of an infinite price set is provided at the end of this section.

[28] Note that if $\widetilde{p} \in \Delta_\varepsilon$ and $\psi(\widetilde{p}, ..., \widetilde{p}) \in \Delta_\varepsilon$, which is true if ε is sufficiently small, then $\widetilde{p}$ is still the best price sustainable using the grim punishment.

[29] General conditions for which (6) holds are provided in Appendix B. For example, it holds when demand and cost functions are linear.

[30] The main conclusion would still hold even if the set of fixed points includes other points since our interest lies in when ε is small in which case all fixed points are close to p_i^* when ε is small.

Theorem 1 *Assume A1-A5. If* $\left(p_1^0, ..., p_n^0\right) \in \left\{p^N, ..., \min\left\{p_1^* - \varepsilon, \widetilde{p}\right\}\right\}^n$ *then there exists finite* T *such that* $p_1^t = \cdots = p_n^t = \widehat{p}$ *for all* $t \geq T$ *where*

$$\widehat{p} \begin{cases} \in \{p_1^* - \varepsilon, p_1^*, p_1^* + \varepsilon\} & \text{if } p_1^* + \varepsilon \leq \widetilde{p} \\ \in \{p_1^* - \varepsilon, p_1^*\} & \text{if } \widetilde{p} = p_1^* \\ = \widetilde{p} & \text{if } \widetilde{p} \leq p_1^* - \varepsilon \end{cases}.$$

Proof: Given a finite price set and the boundedness and monotonicity of prices (Assumption 2), prices converge in finite time and, by Assumption 2, firms have identical prices. Hence, there exists $\widehat{p} \in \left\{p^N, ..., \widetilde{p}\right\}$ and finite T such that $p_1^t = \cdots = p_n^t = \widehat{p}$ for all $t \geq T$.

Let us first argue that: 1a) if $p_1^* - \varepsilon < \widetilde{p}$ then $\widehat{p} \in \{p_1^* - \varepsilon, ..., \widetilde{p}\}$; and 2a) if $p_1^* - \varepsilon \geq \widetilde{p}$ then $\widehat{p} = \widetilde{p}$. Since $\widehat{p} \leq \widetilde{p}$ then, for (1a) and (2a) not to be true, either: 1b) $p_1^* - \varepsilon < \widetilde{p}$ and $\widehat{p} < p_1^* - \varepsilon$; and/or 2b) $p_1^* - \varepsilon \geq \widetilde{p}$ and $\widehat{p} < \widetilde{p}$. In both cases, $\widehat{p} < p_1^* - \varepsilon$ and $\widehat{p} < \widetilde{p}$. Hence, a necessary condition for (1a) and (2a) not to be true is that $\widehat{p} < \min\{p_1^* - \varepsilon, \widetilde{p}\}$. Let us show that if $\widehat{p} < \min\{p_1^* - \varepsilon, \widetilde{p}\}$ then $\widehat{p}$ is inconsistent with Assumptions 1-5. Thus, suppose $\widehat{p} < \min\{p_1^* - \varepsilon, \widetilde{p}\}$. For $t > T$, Assumption 5 implies firm 1's expected payoff from pricing at $\widehat{p}$ converges to $\pi(\widehat{p}, ..., \widehat{p}) / (1 - \delta_1)$. To derive a lower bound on firm 1's payoff from pricing instead at $\widehat{p} + \varepsilon$, I use the property that a lower bound on a firm's period t continuation payoff is that associated with all firms pricing at $\min\left\{\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right\}$ in all periods. The proof of this lemma is in Appendix A.

Lemma 1: Let V_i denote firm i's maximal payoff given the other firms' strategies are PMP-compatible (and all firms have acted in a manner consistent with the PMP property in past periods). Then

$$V_i \geq \frac{\pi\left(\min\left\{\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right\}, ..., \min\left\{\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right\}\right)}{1 - \delta_i}.$$

Intuitively, if all rivals to firm i are using PMP-compatible strategies then they will price at least as high as $\min\left\{\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right\}$ in all ensuing periods (as long as firm i does not violate the PMP property and induce a shift to p^N) which means that firm i can at least earn the profit from all firms (including i) pricing at $\min\left\{\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right\}$. With this property, $\widehat{p} < \widetilde{p}$ implies that a lower bound on firm 1's payoff from pricing at $\widehat{p} + \varepsilon$ is

$$\pi(\widehat{p} + \varepsilon, \widehat{p}, ..., \widehat{p}) + \left(\frac{\delta_1}{1 - \delta_1}\right) \pi(\widehat{p} + \varepsilon, ..., \widehat{p} + \varepsilon). \tag{7}$$

Since $\widehat{p} < p_1^* - \varepsilon$, (6) implies $p_1 > \widehat{p}\ \forall p_1 \in \overline{\phi}_1(\widehat{p}, ..., \widehat{p})$, from which it follows that

$$\pi(\widehat{p} + \varepsilon, \widehat{p}, ..., \widehat{p}) + \left(\frac{\delta_1}{1 - \delta_1}\right) \pi(\widehat{p} + \varepsilon, ..., \widehat{p} + \varepsilon) > \frac{\pi(\widehat{p}, ..., \widehat{p})}{1 - \delta_1}, \tag{8}$$

where we use the strict concavity of W_1. Thus, if the steady-state price is $\widehat{p}$ and $\widehat{p} < \min\{p_1^* - \varepsilon, \widetilde{p}\}$ then firm 1 would eventually prefer to raise price to $\widehat{p} + \varepsilon$ rather than keep it at $\widehat{p}$ because the payoff to the former has a lower bound of (7) and, by (8), it exceeds the payoff from pricing at $\widehat{p}$. This fact contradicts price converging to $\widehat{p}$. We have then proven (1a) and (2a).

Consistent with Theorem 1, (1a)-(2a) imply:

$$\widehat{p}\begin{cases} = \widetilde{p} & \text{if } \widetilde{p} \leq p_1^* - \varepsilon \\ \in \{p_1^* - \varepsilon, p_1^*\} & \text{if } \widetilde{p} = p_1^* \\ \in \{p_1^* - \varepsilon, p_1^*, p_1^* + \varepsilon\} & \text{if } \widetilde{p} = p_1^* + \varepsilon \end{cases}.$$

To complete the proof, it needs to be shown:

$$\text{if } \widetilde{p} > p_1^* + \varepsilon \text{ then } \widehat{p} \in \{p_1^* - \varepsilon, p_1^*, p_1^* + \varepsilon\}.$$

Hence, from hereon assume $\widetilde{p} > p_1^* + \varepsilon$. Recall that $p_1^* \geq p_2^* \geq \cdots \geq p_n^*$ and, therefore, $\widetilde{p} > p_1^* + \varepsilon$ implies $\widetilde{p} > p_i^* + \varepsilon$ for all $i = 1, ..., n$.

Having proven property (1a), we already know that $\widehat{p} \geq p_1^* - \varepsilon$ when $\widetilde{p} > p_1^* + \varepsilon$. Proving that $\widehat{p} \leq p_1^* + \varepsilon$ is more subtle. Before embarking on the proof, an overview is provided. First it is shown that if a firm is rational and believes the other firms use PMP-compatible strategies then a firm will not price at $\widetilde{p}$. The reason is that firm i would find it optimal to price above $p_i^* + \varepsilon$ only if it induced at least one of its rivals to enact further price increases (and not just match the firm's price). However, if a firm believes its rivals will not price above $\widetilde{p}$ (which follows from believing its rivals use PMP-compatible strategies) then it is not optimal for a firm to raise price to $\widetilde{p}$ because it will only expect its rivals to match a price of $\widetilde{p}$, not exceed it. This argument works as well to show that each of the other firms will not raise price to $\widetilde{p}$. Hence, there is an upper bound on price of $\widetilde{p} - \varepsilon$. The proof is completed by induction using the common knowledge in Assumptions 1-4. If a firm believes its rivals will not price above p' then it can be shown that a firm will find it optimal not to price above $p' - \varepsilon$. This argument works only when $p' \geq p_1^* + 2\varepsilon$ which implies that an upper bound on price is $p_1^* + \varepsilon$, which is the desired result.

The key property of $\overline{\phi}_i$ which we'll need is:

$$\text{if } \mathbf{p}_{-i} \leq \left(p', ..., p'\right) \text{ and } p' > p_i^* + \varepsilon \text{ then } \overline{\phi}_i^U\left(\mathbf{p}_{-i}\right) \leq p' - \varepsilon, \tag{9}$$

where $\overline{\phi}_i^U\left(\mathbf{p}_{-i}\right)$ is the maximal element of $\overline{\phi}_i\left(\mathbf{p}_{-i}\right)$. By (6), $p' > p_i^* + \varepsilon$ implies $\overline{\phi}_i^U\left(p', ..., p'\right) \leq p' - \varepsilon$, and (9) follows from the fact that $\overline{\phi}_i^U\left(\mathbf{p}_{-i}\right)$ is non-decreasing in $\mathbf{p}_{-i}$.[31] From the strict concavity of the price matching objective function, it follows:

$$\text{if } p'' > p' \geq \overline{\phi}_i^U\left(\mathbf{p}_{-i}\right) \text{ then} \tag{10}$$

[31] By the definition of $\overline{\phi}^U\left(\mathbf{p}_{-i}\right)$, we know that:

$$W\left(\overline{\phi}^U\left(\mathbf{p}'_{-i}\right), \mathbf{p}'_{-i}\right) - W\left(p_i, \mathbf{p}'_{-i}\right) > 0, \ \forall p_i \in A \equiv \left\{p \in \Delta_\varepsilon : p > \overline{\phi}^U\left(\mathbf{p}'_{-i}\right)\right\}.$$

Since $\frac{\partial^2 W\left(p_i, \mathbf{p}_{-i}\right)}{\partial p_i \partial p_j} = \frac{\partial^2 \pi\left(p_i, \mathbf{p}_{-i}\right)}{\partial p_i \partial p_j} > 0$ then $\mathbf{p}''_{-i} \leq \mathbf{p}'_{-i}$ implies

$$W\left(p_i, \mathbf{p}'_{-i}\right) - W\left(p_i, \mathbf{p}''_{-i}\right) \geq W\left(\overline{\phi}^U\left(\mathbf{p}'_{-i}\right), \mathbf{p}'_{-i}\right) - W\left(\overline{\phi}^U\left(\mathbf{p}'_{-i}\right), \mathbf{p}''_{-i}\right), \ \forall p_i \in A,$$

$$\pi\left(p',\mathbf{p}_{-i}\right)+\left(\frac{\delta_i}{1-\delta_i}\right)\pi\left(p',...,p'\right)>\pi\left(p'',\mathbf{p}_{-i}\right)+\left(\frac{\delta_i}{1-\delta_i}\right)\pi\left(p'',...,p''\right).$$

Let us show that if firm i believes the other firms' strategies are PMP-compatible then a price of $\widetilde{p}-\varepsilon$ is strictly preferred to $\widetilde{p}$. Firm i's beliefs on $\mathbf{p}_{-i}^t$ have support $\left[\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}\right]^{n-1}$ because it believes the other firms' strategies are PMP-compatible. For any $\mathbf{p}_{-i}^t\in\left[\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}\right]^{n-1}$, Lemma 1 implies that a lower bound on its payoff from $p_i^t=\widetilde{p}-\varepsilon$ is

$$\pi\left(\widetilde{p}-\varepsilon,\mathbf{p}_{-i}^t\right)+\left(\frac{\delta_i}{1-\delta_i}\right)\pi\left(\widetilde{p}-\varepsilon,...,\widetilde{p}-\varepsilon\right). \tag{11}$$

For any $\mathbf{p}_{-i}^t\in\left[\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}\right]^{n-1}$, it follows from all firms using PMP-compatible strategies that firm i's payoff from $p_i^t=\widetilde{p}$ is

$$\pi\left(\widetilde{p},\mathbf{p}_{-i}^t\right)+\left(\frac{\delta_i}{1-\delta_i}\right)\pi\left(\widetilde{p},...,\widetilde{p}\right). \tag{12}$$

Given that $\widetilde{p}>p_i^*+\varepsilon$ then $\overline{\phi}_i^U\left(\mathbf{p}_{-i}^t\right)\leq\widetilde{p}-\varepsilon$ for all $\mathbf{p}_{-i}^t\leq\left(\widetilde{p},...,\widetilde{p}\right)$ by (9). It then follows from (10) that (11) strictly exceeds (12). Therefore, for any beliefs of firm i with support $\left[\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}\right]^{n-1}$, a price of $\widetilde{p}-\varepsilon$ is strictly preferred to $\widetilde{p}$. It follows that if a firm is rational and believes the other firms use PMP-compatible strategies then its optimal price does not exceed $\widetilde{p}-\varepsilon$.

Given the common knowledge from Assumptions 1-4, it is also the case that firm i believes firm $j\ (\neq i)$ is rational and that firm j believes firm h (for all $h\neq j$) uses a PMP-compatible strategy. Hence, applying the preceding argument to firm j, firm i believes firm j will not price above $\widetilde{p}-\varepsilon$. Firm i's beliefs on $\mathbf{p}_{-i}^t$ then have support $\left[\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}-\varepsilon\right]^{n-1}$. If $\widetilde{p}-\varepsilon>p_i^*+\varepsilon$ then, by (9), $\overline{\phi}_i^U\left(\mathbf{p}_{-i}^t\right)\leq\widetilde{p}-2\varepsilon$ for all $\mathbf{p}_{-i}^t\leq\left(\widetilde{p}-\varepsilon,...,\widetilde{p}-\varepsilon\right)$.[32] By the same logic as above, a lower bound on firm i's payoff from $p_i^t=\widetilde{p}-2\varepsilon$ is

$$\pi\left(\widetilde{p}-2\varepsilon,\mathbf{p}_{-i}^t\right)+\left(\frac{\delta_i}{1-\delta_i}\right)\pi\left(\widetilde{p}-2\varepsilon,...,\widetilde{p}-2\varepsilon\right), \tag{13}$$

while its payoff from $p_i^t=\widetilde{p}-\varepsilon$ is

$$\pi\left(\widetilde{p}-\varepsilon,\mathbf{p}_{-i}^t\right)+\left(\frac{\delta_i}{1-\delta_i}\right)\pi\left(\widetilde{p}-\varepsilon,...,\widetilde{p}-\varepsilon\right). \tag{14}$$

---

and, re-arranging, we have

$$W\left(\phi^U\left(\mathbf{p}'_{-i}\right),\mathbf{p}''_{-i}\right)-W\left(p_i,\mathbf{p}''_{-i}\right)\geq W\left(\overline{\phi}^U\left(\mathbf{p}'_{-i}\right),\mathbf{p}'_{-i}\right)-W\left(p_i,\mathbf{p}'_{-i}\right),\ \forall p_i\in A.$$

Therefore,

$$W\left(\overline{\phi}^U\left(\mathbf{p}'_{-i}\right),\mathbf{p}''_{-i}\right)-W\left(p_i,\mathbf{p}''_{-i}\right)>0,\ \forall p_i\in A.$$

Hence, $\mathbf{p}''_{-i}\leq\mathbf{p}'_{-i}$ implies $\overline{\phi}^U\left(\mathbf{p}''_{-i}\right)\leq\overline{\phi}^U\left(\mathbf{p}'_{-i}\right)$, so $\overline{\phi}_i^U\left(\mathbf{p}_{-i}\right)$ is non-decreasing.

[32] If instead $\widetilde{p}-\varepsilon\leq p_i^*+\varepsilon$ then, given that it has already been shown $\widetilde{p}-\varepsilon$ is an upper bound on the limit price, it follows that $p_i^*+\varepsilon$ is an upper bound and we're done.

With (14), we used the fact that firms will not price above $\widetilde{p}-\varepsilon$, which was derived in the first step. Again using (10), it is concluded that (13) strictly exceeds (14). Therefore, for any beliefs of firm i over $\mathbf{p}_{-i}^{t}$ with support $\left[\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}-\varepsilon\right]^{n-1}$, a price of $\widetilde{p}-2\varepsilon$ is strictly preferred to $\widetilde{p}-\varepsilon$. It follows that if a firm is rational and a firm believes other firms' strategies are PMP-compatible, believes the other firms are rational, and believes each of the other firms believes its rivals use PMP-compatible strategies then a firm's optimal price does not exceed $\widetilde{p}-2\varepsilon$. Hence, all firms will not price above $\widetilde{p}-2\varepsilon$.

The proof is completed by induction. Suppose we have shown that firm i believes that the other firms will not price above p' so firm i's beliefs on $\mathbf{p}_{-i}^{t}$ have support $\left[\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},p'\right]^{n-1}$. (That we can get to the point that firms have those beliefs relies on rationality and that firms use PMP-compatible strategies are both common knowledge.) If $p'>p^*+\varepsilon$ then $\overline{\phi}_i^U\left(\mathbf{p}_{-i}^{t}\right)\leq p'-\varepsilon$ for all $\mathbf{p}_{-i}^{t}\leq(p',...,p')$. A lower bound on firm i's payoff from $p_i^t=p'-\varepsilon$ is

$$\pi\left(p'-\varepsilon,\mathbf{p}_{-i}^{t}\right)+\left(\frac{\delta_i}{1-\delta_i}\right)\pi\left(p'-\varepsilon,...,p'-\varepsilon\right), \tag{15}$$

while its payoff from $p_i^t=p'$ is

$$\pi\left(p',\mathbf{p}_{-i}^{t}\right)+\left(\frac{\delta_i}{1-\delta_i}\right)\pi\left(p',...,p'\right), \tag{16}$$

since all firms have an upper bound of p' on their prices. Using (10), it is concluded that (15) strictly exceeds (16). Therefore, for any beliefs of firm i over $\mathbf{p}_{-i}^{t}$ with support $\left[\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},p'\right]^{n-1}$, a price of $p'-\varepsilon$ is strictly preferred to p'. It follows that firms' prices are bounded above by $p'-\varepsilon$. The preceding argument is correct as long as $p'>p_i^*+\varepsilon\ \forall i$ or, equivalently, $p'\geq p_1^*+2\varepsilon$. Thus, if $p'=p_1^*+2\varepsilon$ then $p'-\varepsilon=p_1^*+\varepsilon$ and, therefore, price is bounded above by $p_1^*+\varepsilon$. This completes the proof of: if $\widetilde{p}>p_1^*+\varepsilon$ then the limit price lies in $\{p_1^*-\varepsilon,p_1^*,p_1^*+\varepsilon\}$. ■

In explaining the basis for Theorem 1, suppose $p_1^*+\varepsilon\leq\widetilde{p}$ so that the steady-state price lies in $\{p_1^*-\varepsilon,p_1^*,p_1^*+\varepsilon\}$ and thus is close to p_1^*. Recall that p_i^* is defined by the condition whereby firm i is indifferent between keeping price at p_i^* and marginally raising it and having the higher price matched in all subsequent periods. In the proof of Theorem 1, Assumption 5 is used to avoid price converging on some level below (approximately) p_1^*. If each firm always expected a rival to take the lead in raising price then it could be optimal for each firm to keep price fixed, while anticipating it'll match its rival's price increase in the subsequent period. Compared to raising price in the current period, this yields higher current profit by being a lower-priced firm, and the same profit tomorrow. It would seem unnatural, however, for a firm to continually believe a rival is just about to raise price when its rival has kept its price fixed for many periods. By Assumption 5, firm i will eventually come to believe that its rivals will not raise price which would then induce firm i to take the lead by raising price. This serves to eventually result in price climbing up to (approximately) p_1^* since $p_1^*\geq\cdots\geq p_n^*$. Assumption 5 serves only this role in the proof.

The less obvious part of the proof is showing that price cannot converge above (approximately) p_1^* (where we are still considering the case when $p_1^* + \varepsilon \leq \widetilde{p}$). If firm i expected that its price increase would only be met - and never exceeded by its rival - then it would not want to raise price above p_i^*. However, it might be willing to raise price above p_i^* *if* it led one of its rivals to further increase price; that is, it induced future price leadership by another firm. Next note that no firm will raise price beyond $\widetilde{p}$, which is the minimum of the highest sustainable price and the joint profit maximum. This means that firm i would never want to raise price to $\widetilde{p}$ since such a price increase would only induce its rivals to match that price. Thus, if firm i is rational and firm i believes the other firms use PMP-compatible strategies then firm i would never raise price to $\widetilde{p}$. This argument applies to all firms which puts an upper bound on price of $\widetilde{p} - \varepsilon$. Given the common knowledge of rationality and that firms use PMP-compatible strategies, the same argument can be used to prove that firms will not raise price to $\widetilde{p} - \varepsilon$; therefore, $\widetilde{p} - 2\varepsilon$ is an upper bound. By induction, it is then established that no firm would ever raise price to a level exceeding $p_1^* + \varepsilon$. Hence, the limit price is approximately p_1^*.

If instead $\widetilde{p} \leq p_1^* - \varepsilon$ then the steady-state price is $\widetilde{p}$. Firm 1 is willing to raise price to a level beyond $\widetilde{p}$ (though not beyond p_1^*) if other firms would follow, but firms will not follow because prices in excess of $\widetilde{p}$ are not sustainable given that $\widetilde{p}$ is the highest steady-state price supportable when a deviation results in stage game Nash equilibrium pricing forever. Price leadership then ends at a price of $\widetilde{p}$.

In deriving the steady-state outcome, the punishment for deviation is reversion to a stage game Nash equilibrium. Such a punishment could, in principle, sustain a price as high as $\widetilde{p}$. To what extent can tacit collusion achieve that potential? The next result helps address that question. Recall that firm n has the (weakly) lowest discount factor. The proof is in Appendix A.

Theorem 2 $p_n^* \in \left(p^N, \widetilde{p}\right)$.

In explaining Theorem 2, recall that p_i^* is the price at which the reduction in current profit from a marginal increase in firm i's price is exactly equal in magnitude to the rise in the present value of the future profit stream when that higher price is matched by all firms for the infinite future. Equivalently stated, p_i^* is the price for firm i at which the increase in current profit from a marginal *decrease* in price to $p_i^* - \varepsilon$ is exactly equal in magnitude to the *fall* in the present value of the future profit stream *when the firm's rivals lower price to* $p_i^* - \varepsilon$ (where suppose ε is small). In comparison, define $\widetilde{p}_i$ as the price for firm i at which the increase in current profit from a marginal decrease in price is exactly equal in magnitude to the fall in the present value of the future profit stream *when the firm's rivals lower price to* p^N.[33] Given that the punishment is more severe in the latter case, it follows that the maximal sustainable price is higher: $\widetilde{p}_i > p_i^*$. Theorem 2 pertains to firm n because, as the firm with the (weakly) lowest discount factor, its incentive compatibility constraint defines $\widetilde{p}$ (that is, $\widetilde{p} = \widetilde{p}_n$).

[33] That is, $\widetilde{p}_i$ is the highest price for which firm i's incentive compatibility constraint, (1), holds. For this discussion, suppose $\widetilde{p}_i < p^M$.

It follows from $\delta_1 \geq \cdots \geq \delta_n$ that $p_1^* \geq \cdots \geq p_n^*$. By Theorem 1, if $\widetilde{p} \leq p_1^*$ then the tacitly collusive price is $\widetilde{p}$, so tacit collusion is able to achieve the maximum sustainable price. However, if $p_1^* < \widetilde{p}$ then, under tacit collusion, the steady-state price is (approximately) p_1^* even though higher prices are sustainable. Furthermore, note that if firms are symmetric then, by Theorem 2, $p_n^* = \cdots = p_1^* = p^* < \widetilde{p}$ and thus tacit collusion always falls short of what can be sustained. In other words, if firms started at a price of $\widetilde{p}$ then such a price would persist, as would any price in $[p^*, \widetilde{p}]$. But if firms start with prices below p^*, such as at the non-collusive price p^N, then prices will eventually end up around p^*, even though higher prices are sustainable. The problem is that it is not in the interests of any firm to lead a price change beyond p^*.

Corollary 3 *Assume A1-A5 and $\delta_1 = \cdots = \delta_n$. If $\left(p_1^0, ..., p_n^0\right) \in \left\{p^N, ..., \min\left\{p^* - \varepsilon, \widetilde{p}\right\}\right\}^n$ then there exists finite T such that $p_1^t = \cdots = p_n^t = \widehat{p}$ for all $t \geq T$ where $\widehat{p} \in \{p^* - \varepsilon, p^*, p^* + \varepsilon\}$. p^* is defined by*

$$\frac{\partial \pi\left(p^*, ..., p^*\right)}{\partial p_i} + \delta \sum_{j \neq i}^{n} \frac{\partial \pi\left(p^*, ..., p^*\right)}{\partial p_j} = 0,$$

and $p^* < \widetilde{p}$.

When firms are not too asymmetric - so that $p_1^* \simeq p_n^*$ and thus (by Theorem 2) $p_1^* < \widetilde{p}$ - the constraint on the steady-state price is that no firm wants to be a price leader once the price reaches p_1^*. When firms are sufficiently asymmetric - so that $p_n^* < \widetilde{p} < p_1^*$ - the constraint on the steady-state price is that prices higher than $\widetilde{p}$ are not sustainable. While the more patient firms - such as firm 1 - would be willing to raise price beyond $\widetilde{p}$ if it was matched, the more impatient firms - such as firm n - would prefer to undercut such a price. Hence, price only rises up to $\widetilde{p}$. In this case, tacit collusion is not limiting the price that can be achieved since price leadership results in a steady-state price of $\widetilde{p}$.[34]

Note that Theorems 1 and 2 are robust to the form of the punishment. p_i^* is independent of the punishment and, given another punishment, $\widetilde{p}$ would just be the highest sustainable price for that punishment. In particular, if the punishment is at least as severe as the grim punishment then Corollary 3 is true.

In concluding, let me discuss the role of the finiteness of the price set. Assume identical discount factors so that the steady-state price is (approximately) p^* and $p^* < \widetilde{p}$. These simplifications are not important for the explanation but make for an easier discussion. p^* is the highest price to which a firm will raise price if it can only anticipate that other firms will match its price. Thus, a firm is willing to take the lead and price above p^* only if, by doing so, it induces a rival to enact further price increases. Since no firm will price above $\widetilde{p}$ then raising price to $\widetilde{p}$ cannot induce rivals to lead future price increases. Thus, a firm will not raise price to a

[34] In another sense, tacit collusion is still limiting the extent of collusion because firms are not able to allocate the market. A higher price than $\widetilde{p}$ is sustainable if the more impatient firms were given a bigger share of the market.

level beyond $\widetilde{p}-\varepsilon$, which means $\widetilde{p}-\varepsilon$ is an upper bound on price. This argument works iteratively to ultimately conclude that p^* is (approximately) an upper bound on price. The finiteness of price is critical in this proof strategy for without it $\widetilde{p}-\varepsilon$ is not well-defined. However, even with an infinite price set, it is still the case that a necessary condition for a firm to lead and raise price above p^* is that it will induce a rival to enact further price increases. As that must always be true then, if the limit price exceeds p^*, price cannot converge in finite time. But since it is still the case that $\widetilde{p}$ is an upper bound on price, the price increases must then get arbitrarily small; eventually, each successive price increase must bring forth a smaller future price increase by a rival. I am not arguing that this argument will prevent Theorem 1 from extending to the infinite price set but rather that it is the only argument that could possibly do so. Either Theorem 1 extends to when the price set is infinite or, if it does not, then it implies a not very credible price path with never-ending price increases that eventually become arbitrarily small. The oddity of such a price path would seem an artifact of assuming an infinite set of prices when, in fact, the set of prices available to firms is finite.

5 Linear Example: Explicit vs. Tacit Collusion

In comparing explicit and tacit collusion, I will assume that firms, if they could expressly communicate, would agree to simultaneously raise price to the best equilibrium price of $\widetilde{p}$. Focusing on the case of identical discount factors, the steady-state price differential between explicit and tacit collusion is then measured by $\widetilde{p}-p^*$. This does presume that the same punishment is deployed with explicit collusion as with tacit collusion which is likely to result in an underestimate of the price under explicit collusion since presumably more punishments are available to firms if they can coordinate through express communication. It is then best to think of this comparison as isolating the effect of the method of coordination - price leadership versus express communication - while controlling for the mechanism that sustains the collusive outcome.

Assuming linear demand and cost functions, a firm's profit function is

$$\pi\left(p_i,\mathbf{p}_{-i}\right)=\left(a-bp_i+d\left(\frac{1}{n-1}\right)\sum_{j\neq i}p_j\right)\left(p_i-c\right),\text{where } a>bc>0, b>d>0.$$

The non-collusive stage game Nash equilibrium price and the joint profit-maximizing price are, respectively,

$$p^N=\frac{a+bc}{2b-d},\ p^M=\frac{a+(b-d)\,c}{2\,(b-d)}.$$

The price matching best reply function is

$$\phi\left(\mathbf{p}_{-i}\right)=\frac{a+(b-\delta d)\,c}{2\,(b-\delta d)}+\left(\frac{(1-\delta)\,d}{2\,(b-\delta d)}\right)\left(\frac{1}{n-1}\right)\sum_{j\neq i}p_j,$$

from which we can derive p^*:

$$p^* = \phi\left(p^*, ..., p^*\right) = \frac{a + (b - \delta d)\, c}{2\,(b - \delta d)} + \left(\frac{(1-\delta)\, d}{2\,(b - \delta d)}\right) p^* \Rightarrow$$

$$p^* = \frac{a + (b - \delta d)\, c}{2b - (1+\delta)\, d}$$

p^* is an increasing convex function of the discount factor:

$$\frac{\partial p^*}{\partial \delta} = \frac{d\,(a - (b-d)\, c)}{(2b - (1+\delta)\, d)^2} > 0,\ \frac{\partial^2 p^*}{\partial \delta^2} = \frac{d^2\,(a - (b-d)\, c)}{(2b - (1+\delta)\, d)^3} > 0.$$

It is straightforward to derive price under explicit collusion by solving (2):

$$\begin{aligned}\widetilde{p} \quad &= \quad \min\left\{\frac{4ab^2 + ad^2 + 4b^3c + bcd^2 - 4b^2cd - ad^2\delta - 4abd + 4abd\delta + 3bcd^2\delta - 4b^2cd\delta}{6bd^2 - 12b^2d + d^3\delta + 8b^3 - d^3 - 2bd^2\delta},\right.\\ &\left.\frac{a + (b-d)\, c}{2\,(b-d)}\right\}.\end{aligned}$$

If $\widetilde{p} < p^M$ then $\widetilde{p}$ is also an increasing convex function of the discount factor:

$$\frac{\partial \widetilde{p}}{\partial \delta} = \frac{4bd\,(a - bc + cd)\,(2b - d)}{\left(4b\,(b-d) + d^2\,(1-\delta)\right)^2} > 0,\ \frac{\partial^2 \widetilde{p}}{\partial \delta^2} = \frac{8bd^3\,(a - (b-d)\, c)\,(2b - d)}{\left(4b\,(b-d) + d^2\,(1-\delta)\right)^3} > 0.$$

Define $\delta^* \in (0,1)$ by: if $\delta < (>)\, \delta^*$ then $\widetilde{p}\,(\delta) < (=)\, p^M$.

The next result shows that $\widetilde{p} - p^*$ is increasing in δ when δ is low - so that a higher discount factor exacerbates the cost from coordinating through price leadership - but is decreasing in δ when δ is high. The proof is in Appendix A.

Theorem 4 *Assume linear demand and cost functions. Then* $\frac{\partial(\widetilde{p} - p^*)}{\partial \delta} > (<)\, 0$ *as* $\delta < (>)\, \delta^*$.

Illustrating this result for $a = 1, b = 1, d = .9, c = 0$, Figures 1 and 2 compare price under explicit collusion and tacit collusion, and how this comparison varies with the discount factor. To begin, the forces determining the steady-state price varies between when coordination is through tacit means and explicit means. With tacit collusion and price leadership, a firm that leads on price trades off lower current profit - as its demand falls by raising its price - and higher future profit - as rivals subsequently match that price. With a current loss and a future gain, a firm is more willing to engage in price leadership when its discount factor is higher; hence p^* is increasing in δ. It is then the profitability of leading that determines the steady-state price under tacit collusion. By comparison, explicit collusion allows firms to simultaneously raise price so there is no price leader and thus no current loss incurred; what constrains the collusive price is sustainability and, by the usual argument, $\widetilde{p}$ is increasing in δ (when $\widetilde{p} < p^M$). In sum, the steady-state price under explicit collusion is determined by the profitability of not undercutting that price as opposed to the profitability of leading a price increase which is what derives the steady-state price under tacit collusion.

When the discount factor is low, price under explicit collusion is near the competitive price because only prices close to the competitive price are sustainable. Price under tacit collusion is also near the competitive price because only for small price increases above the competitive price is the current loss exceeded by the future gain, and that is because the current loss is near zero when all firms price at p^N. Hence, when the discount factor is low, the type of coordination mechanism makes little difference. When the discount factor is high, the collusive price is near the joint profit maximum under either explicit or tacit collusion. Given firms' long-run view, high prices are sustainable and firms are strongly inclined to lead price increases. It is when the discount factor is moderate that the coordination mechanism makes the biggest difference. Firms are able to sustain high prices but no firm is willing to act as a price leader to achieve them. For the numerical example in Figure 1 with $\delta = .7$, the competitive price is .91 and explicit collusion results in a price of 4.47 which is close to the joint profit maximum of 5.00; however, tacit collusion with price leadership results in a price of only 2.13. It is when firms are moderately patient that the means of coordination has a significant impact on the steady-state price.

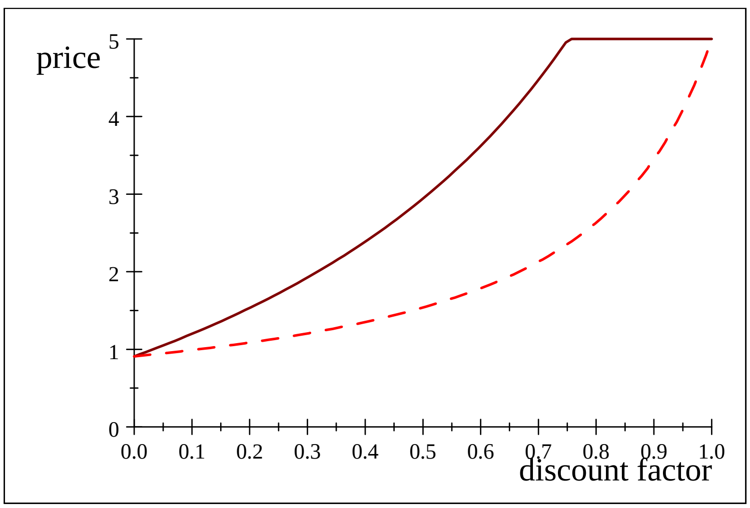

Figure 1: Price under explicit collusion (solid line) and tacit collusion (dashed line)

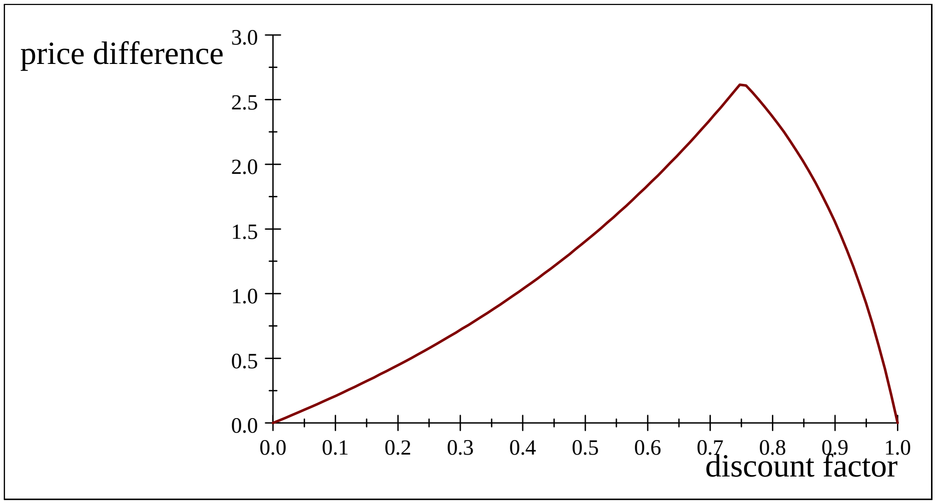

Figure 2: Price difference between explicit and tacit collusion, $\widetilde{p} - p^*$

This result may also have implications for when cartel formation (that is, explicit collusion) is most likely. When the discount factor is sufficiently low, cartel formation is not likely because the rise in price is small (whether firms, in the absence of cartel formation, would compete or tacitly collude). When the discount factor is sufficiently high, cartel formation is not likely either if the alternative is tacit collusion because tacit collusion does nearly as well.[35] It is when the discount factor is moderate that cartel formation is most attractive because it results in a much higher price than if firms either competed or tacitly colluded. While the attractiveness of tacit collusion (compared to competition) is always greater when the discount factor is higher, that is not the case with the attractiveness of explicit collusion (compared to tacit collusion). What are facilitating conditions for collusion can then depend on whether collusion is explicit or tacit.

6 Concluding Remarks

In his classic examination of imperfect competition, Chamberlain (1948) originally argued that collusion would naturally emerge because each firm would recognize the incentive to maintain a collusive price, rather than undercut its rivals' prices and bring forth retaliation. We now know that it is a non-trivial matter for firms to coordinate on a collusive solution because there are so many collusive equilibria. These equilibria differ in terms of the mechanism that sustains collusion as well as the particular outcome that is sustained. Modern oligopoly theory has generally ignored the question of how a collusive arrangement is achieved and instead focused on what can be sustained if firms are able to reach mutual understanding; in other words, the properties of equilibrium outcomes. While such mutual understanding can be

[35] This results is at best suggestive because it comes with at least two serious caveats. First, if more severe punishments can be coordinated upon under explicit collusion then price will be higher. Second, the comparison focuses on steady-state profit and ignores how the transition path might differ between tacit and explicit collusion.

acquired through express communication, this leaves unaddressed non-explicit forms of collusion, which are accepted by economists and the courts to occur in practice and are well-documented by experimental evidence.[36] This lack of theoretical attention to the distinction between explicit and tacit collusion has prevented advances in our understanding of how the means of coordination impacts the form and extent of collusion and, as a consequence, limited the role of economic theory in defining the contours of what is legal and illegal according to antitrust law.

The primary contribution of this paper is to characterize what collusive pricing looks like when firms deploy tacit means of coordination, specifically, price leadership. A model of tacit collusion requires jettisoning the assumption of equilibrium and instead imposing plausible assumptions on what firms commonly believe about their behavior. With mutual understanding about the method of tacit collusion - price leadership with price increases that are at least matched - but not about the specifics regarding the sequence of prices, it proved possible to characterize the steady-state price. If firms are not too asymmetric, the steady-state price under tacit collusion is strictly less than the maximal equilibrium price and, therefore, less than the price that could be achieved with explicit collusion. While tacit coordination avoids the possibility of legal action, it produces a lower price than if firms were to expressly communicate. Thus, if the threat of penalties due to antitrust enforcement deters firms from engaging in explicit collusion, there is a welfare gain even if firms manage to tacitly collude.

The importance of understanding the distinction between explicit and tacit collusion is especially relevant when it comes to policy. If the objective is to detect and prosecute cartels then explicit collusion is relevant in which case we need theories of explicit collusion to produce patterns to look for in the data. If the objective is to prevent horizontal mergers with coordinated effects then tacit collusion is most relevant in which case we need to know for what market structures tacit collusion is more likely to occur and lead to significant price increases. It is hoped that the progress that has been made here in developing a theory of tacit collusion will spur more research on modelling the distinction between explicit and tacit collusion, and thereby serve to close the gap between theory and practice on the matter of collusion.

[36] Some recent work showing the emergence of tacit collusion in an experimental setting includes Fonseca and Normann (2011) - who investigate when express means of coordination are especially valuable relative to tacit means - and Rojas (2011) - who shows that tacit collusion in the lab can be quite sophisticated in that the degree of collusion can vary with the current state of demand. For general references on tacit collusion in experiments, see Huck, Normann, and Oechssler (2004) and Engel (2007).

7 Appendix A

Proof of Lemma 1. Define $P \equiv \min\left\{\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right\}$. Wlog, the analysis will be conducted from the perspective of period 1 (and suppose 0 was a period of collusion). Consider firm i pricing at P in the current period and then, in all ensuing periods, matching the maximum price of the other firms' in the previous period:

$$p_i^1 = P;\ p_i^t = \max\left\{\mathbf{p}_{-i}^{t-1}\right\} \text{ for } t = 2, ...$$

where

$$\max\left\{\mathbf{p}_{-i}^{t-1}\right\} \equiv \max\left\{p_1^{t-1}, ..., p_{i-1}^{t-1}, p_{i+1}^{t-1}, ..., p_n^{t-1}\right\}.$$

Given this strategy for firm i and that the other firms' strategies are PMP-compatible, there will never be a violation of the PMP property. Hence, firm i's payoff is

$$\pi\left(P, \mathbf{p}_{-i}^1\right) + \sum_{t=2}^{\infty} \delta_i^{t-1} \pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\}, \mathbf{p}_{-i}^t\right).$$

Since $P \leq \max\left\{\mathbf{p}_{-i}^{t-1}\right\}$ (as all firms are pricing according to Assumption 2) and $\max\left\{\mathbf{p}_{-i}^{t-1}\right\} \leq p_j^t\ \forall j \neq i$, $\forall t \geq 2$, it follows from firm i's profit being increasing in the other firms' prices that

$$\begin{aligned} & \pi\left(P, \mathbf{p}_{-i}^1\right) + \sum_{t=2}^{\infty} \delta_i^{t-1} \pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\}, \mathbf{p}_{-i}\right) \\ \geq\ & \pi(P, ..., P) + \sum_{t=2}^{\infty} \delta_i^{t-1} \pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\}, ..., \max\left\{\mathbf{p}_{-i}^{t-1}\right\}\right). \end{aligned} \tag{17}$$

Next note that $P \leq \max\left\{\mathbf{p}_{-i}^{t-1}\right\} \leq \widetilde{p} \leq p^M$ which implies

$$\pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\}, ..., \max\left\{\mathbf{p}_{-i}^{t-1}\right\}\right) \geq \pi(P, ..., P).$$

Using this fact on the RHS of (17),

$$\begin{aligned} & \pi(P, ..., P) + \sum_{t=2}^{\infty} \delta_i^{t-1} \pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\}, ..., \max\left\{\mathbf{p}_{-i}^{t-1}\right\}\right) \\ \geq\ & \pi(P, ..., P) + \sum_{t=2}^{\infty} \delta_i^{t-1} \pi(P, ..., P) = \frac{\pi(P, ..., P)}{1 - \delta_i}. \end{aligned} \tag{18}$$

(17) and (18) imply

$$\pi\left(P, \mathbf{p}_{-i}^1\right) + \sum_{t=2}^{\infty} \delta_i^{t-1} \pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\}, \mathbf{p}_{-i}^t\right) \geq \frac{\pi(P, ..., P)}{1 - \delta_i},$$

from which we conclude $V_i \geq \pi(P, ..., P) / 1 - \delta_i$. ■

Proof of Theorem 2. p_i^* is defined by

$$\frac{\partial W_i\left(p_i^*, ..., p_i^*\right)}{\partial p_i} = \frac{\partial \pi\left(p_i^*, ..., p_i^*\right)}{\partial p_i} + \left(\frac{\delta_i}{1-\delta_i}\right) \sum_{j=1}^{n} \frac{\partial \pi\left(p_i^*, ..., p_i^*\right)}{\partial p_j} = 0$$

or

$$\frac{\partial \pi\left(p_i^*, ..., p_i^*\right)}{\partial p_i} + \delta_i \sum_{j \neq i}^{n} \frac{\partial \pi\left(p_i^*, ..., p_i^*\right)}{\partial p_j} = 0.$$

For all $p \geq p^M$,

$$\frac{\partial \pi\left(p, ..., p\right)}{\partial p_i} < 0 \text{ and } \sum_{j=1}^{n} \frac{\partial \pi\left(p, ..., p\right)}{\partial p_j} \leq 0,$$

which implies $p_i^* < p^M$. To show $p_i^* > p^N$, note that $\phi_i\left(p, ..., p\right) > \psi\left(p, ..., p\right)$ and $\psi\left(p, ..., p\right) \geq p\ \forall p \leq p^N$ implies $\phi_i\left(p, ..., p\right) > p\ \forall p \leq p^N$. Since $\phi_i\left(p, ..., p\right) \gtreqqless p$ as $p \lesseqqgtr p_i^*$ then $p_i^* > p^N$. We have then shown $p_i^* \in \left(p^N, p^M\right)\ \forall i$.

If $\widetilde{p} = p^M$ then $p_i^* \in \left(p^N, \widetilde{p}\right)\ \forall i$ and we are done. From hereon, suppose $\widetilde{p} < p^M$ in which case the incentive compatibility constraint (ICC) binds for firm n:[37]

$$\frac{\pi\left(\widetilde{p}, ..., \widetilde{p}\right)}{1-\delta_n} = \pi\left(\psi\left(\widetilde{p}, ..., \widetilde{p}\right), \widetilde{p}, ..., \widetilde{p}\right) + \left(\frac{\delta_n}{1-\delta_n}\right) \pi\left(p^N, ..., p^N\right)\ . \tag{19}$$

As $p \in \left(p^N, p_n^*\right]$ implies $\psi\left(p, ..., p\right) < p \leq \phi_n\left(p, ..., p\right)$ then, by strict concavity of W_n,

$$W_n\left(p, ..., p\right) > W_n\left(\psi\left(p\right), p, ..., p\right),$$

which is equivalently expressed as

$$\frac{\pi\left(p, ..., p\right)}{1-\delta_n} > \pi\left(\psi\left(p\right), p, ..., p\right) + \left(\frac{\delta_n}{1-\delta_n}\right) \pi\left(\psi\left(p\right), ..., \psi\left(p\right)\right). \tag{20}$$

$p > p^N$ implies $\psi\left(p, ..., p\right) \in \left(p^N, p\right)$. Next note $\psi\left(p, ..., p\right) < p \leq p_n^* < p^M$ implies $\psi\left(p, ..., p\right) < p^M$. It then follows from $\psi\left(p, ..., p\right) \in \left(p^N, p^M\right)$ that $\pi\left(\psi\left(p\right), ..., \psi\left(p\right)\right) > \pi\left(p^N, ..., p^N\right)$. Using this property in (20), we have

$$\frac{\pi\left(p, ..., p\right)}{1-\delta_n} > \pi\left(\psi\left(p\right), p, ..., p\right) + \left(\frac{\delta_n}{1-\delta_n}\right) \pi\left(p^N, ..., p^N\right),\ \forall p \in \left(p^N, p_n^*\right]. \tag{21}$$

Therefore, $p \in \left(p^N, p_n^*\right]$ is sustainable with the grim trigger strategy. Given (19) - where the ICC binds for $p = \widetilde{p}$ - and evaluating (21) at $p = p_n^*$ - so the ICC does not bind - it follows from (2) that $\widetilde{p} > p_n^*$. ∎

Proof of Theorem 3. Given that $\frac{\partial p^*}{\partial \delta} > 0$ and $\frac{\partial \widetilde{p}}{\partial \delta} = 0$ for $\delta > \delta^*$ then: if $\delta > \delta^*$ then $\frac{\partial(\widetilde{p}-p^*)}{\partial \delta} < 0$. The remainder of the proof focuses on showing: if $\delta < \delta^*$ then $\frac{\partial(\widetilde{p}-p^*)}{\partial \delta} > 0$.

[37] Recall that $\delta_1 \geq \cdots \geq \delta_n$.

If $\delta < \delta^*$ then

$$\widetilde{p}-p^* = \frac{4ab^2+ad^2+4b^3c+bcd^2-4b^2cd-ad^2\delta-4abd+4abd\delta+3bcd^2\delta-4b^2cd\delta}{6bd^2-12b^2d+d^3\delta+8b^3-d^3-2bd^2\delta} - \frac{a+(b-\delta d)c}{2b-(1+\delta)d}$$

$$\begin{aligned} & \frac{\partial\left(\widetilde{p}-p^*\right)}{\partial\delta} \\ = & -\left[\frac{d(a-bc+dc)}{\left(8b^3-4b^2d\delta-12b^2d+2bd^2\delta+6bd^2+d^3\delta^2-d^3\right)^2}\right]\times \\ & \left(-16b^4+32b^3d\delta+16b^3d-8b^2d^2\delta^2-40b^2d^2\delta+4bd^3\delta^2+16bd^3\delta-4bd^3+d^4\delta^2-2d^4\delta+d^4\right). \end{aligned}$$

As the term in [ ] is positive then

$$sign\left\{\frac{\partial\left(\widetilde{p}-p^*\right)}{\partial\delta}\right\} = sign\left\{\Psi(\delta)\right\}$$

where

$$\Psi(\delta) \equiv -\left(-16b^4+32b^3d\delta+16b^3d-8b^2d^2\delta^2-40b^2d^2\delta+4bd^3\delta^2+16bd^3\delta-4bd^3+d^4\delta^2-2d^4\delta+d^4\right).$$

In evaluating the sign of $\Psi(\delta)$, first note it is positive at the extreme values of δ:

$$\Psi(0) = 16b^4-16b^3d+4bd^3-d^4 = 16b^3(b-d)+d^3(4b-d) > 0$$

$$\begin{aligned} \Psi(1) &= 16b^4-48b^3d+48b^2d^2-16bd^3 = 16b\left(b^3-3b^2d+3bd^2-d^3\right) \\ &= 16b\left(b^2(b-d)-2bd(b-d)+d^2(b-d)\right) = 16b(b-d)\left(b^2-2bd+d^2\right) \\ &= 16b(b-d)(b-d)^2 > 0, \end{aligned}$$

which follow from $b > d > 0$. Given $\Psi(0), \Psi(1) > 0$, if $\Psi(\delta)$ is weakly monotonic then $\Psi(\delta) > 0 \forall\delta \in [0,1]$ and thus $\Psi(\delta) > 0 \forall\delta \in [0,\delta^*)$. Let us show $\Psi'(\delta) < 0$. Consider:

$$\Psi'(\delta) = 40b^2d^2-32b^3d-2d^4\delta-16bd^3+2d^4+16b^2d^2\delta-8bd^3\delta.$$

Since

$$\Psi''(\delta) = -2d^4+16b^2d^2-8bd^3 = 2d^2\left(8b^2-4bd-d^2\right) > 0,$$

$\Psi'(1) < 0$ is a sufficient condition to establish that $\Psi'(\delta) < 0 \forall\delta \in [0,1]$. Given that

$$\Psi'(1) = -8bd(b-d)(4b-3d) < 0,$$

we are done. ■

8 Appendix B

For when the price set is Δ_ε and $p_i^* \in \Delta_\varepsilon$, let us show that the property in (6) holds, which is reproduced here:

$$\overline{\phi}_i(\mathbf{p}_{-i}) \begin{cases} \subseteq \{p' + \varepsilon, ..., p_i^*\} & \text{if } \mathbf{p}_{-i} \leq (p', ..., p') \text{ where } p' < p_i^* - \varepsilon \\ = \{p_i^*\} & \text{if } \mathbf{p}_{-i} = (p_i^*, ..., p_i^*) \\ \subseteq \{p_i^*, ..., p' - \varepsilon\} & \text{if } \mathbf{p}_{-i} \geq (p', ..., p') \text{ where } p' > p_i^* + \varepsilon \end{cases}$$

To show that this holds for $p' < p_i^* - \varepsilon$, it is sufficient to establish that a lower bound on $\overline{\phi}_i(p_i^* - \eta\varepsilon, ..., p_i^* - \eta\varepsilon)$ is $p_i^* - \eta\varepsilon + \varepsilon$ when $\eta \in \{2, 3, ...\}$. If the unconstrained optimum is at least $p_i^* - \eta\varepsilon + \varepsilon$ then that is indeed the case.

Define $\widehat{\phi}_i(p) \equiv \phi_i(p, ..., p)$ as the best reply function when all other firms price at p, and $\widehat{\phi}_i : [0, \infty) \to [0, \infty)$. We want to show: if $\eta \in \{2, 3, ...\}$ then $\widehat{\phi}_i(p_i^* - \eta\varepsilon) \geq p_i^* - (\eta - 1)\varepsilon$. It will be shown that a sufficient condition for this result is $\widehat{\phi}_i'(p) \leq 1/2$. Note that:

$$\widehat{\phi}_i'(p) = -\frac{\frac{\partial^2 \pi}{\partial p_i \partial p_{-i}}}{\frac{\partial^2 \pi}{\partial p_i^2} + \left(\frac{\delta_i}{1-\delta_i}\right)\left(\frac{d^2\pi}{dp^2}\right)} \leq -\frac{\frac{\partial^2 \pi}{\partial p_i \partial p_{-i}}}{\frac{\partial^2 \pi}{\partial p_i^2}},$$

so $\widehat{\phi}_i'(p) \leq 1/2$ holds when

$$-\frac{\partial^2 \pi}{\partial p_i^2} \geq 2\frac{\partial^2 \pi}{\partial p_i \partial p_{-i}}.$$

Using the functional forms in Section 5, $\widehat{\phi}_i'(p) < 1/2$ holds for the case of linear demand and cost:

$$\widehat{\phi}_i'(p) = \frac{(1-\delta_i)\, d}{2(b - \delta_i d)} < \frac{d}{2b} < \frac{1}{2}, \ \forall \delta_i \in (0,1).$$

First note:

$$\widehat{\phi}_i(p_i^*) = p_i^*$$

and

$$\widehat{\phi}_i(p_i^*) = \widehat{\phi}_i(p_i^* - \varepsilon) + \int_{p_i^*-\varepsilon}^{p_i^*} \widehat{\phi}_i'(p)\, dp.$$

Given $\widehat{\phi}_i' \leq 1/2$, it follows from the previous equality:

$$\begin{aligned} \widehat{\phi}_i(p_i^*) &\leq \widehat{\phi}_i(p_i^* - \varepsilon) + \frac{\varepsilon}{2} \\ \widehat{\phi}_i(p_i^* - \varepsilon) &\geq \widehat{\phi}_i(p_i^*) - \frac{\varepsilon}{2} \Rightarrow \widehat{\phi}_i(p_i^* - \varepsilon) \geq p_i^* - \frac{\varepsilon}{2} \end{aligned}$$

Next consider:

$$\begin{aligned} \widehat{\phi}_i(p_i^* - \varepsilon) &= \widehat{\phi}_i(p_i^* - 2\varepsilon) + \int_{p_i^*-2\varepsilon}^{p_i^*-\varepsilon} \widehat{\phi}_i'(p)\, dp \\ \widehat{\phi}_i(p_i^* - \varepsilon) &\leq \widehat{\phi}_i(p_i^* - 2\varepsilon) + \frac{\varepsilon}{2} \Rightarrow \widehat{\phi}_i(p_i^* - 2\varepsilon) \geq \widehat{\phi}_i(p_i^* - \varepsilon) - \frac{\varepsilon}{2} \end{aligned}$$

Using $\widehat{\phi}_i\left(p_i^* - \varepsilon\right) \geq p_i^* - \frac{\varepsilon}{2}$, the previous inequality implies:

$$\widehat{\phi}_i\left(p_i^* - 2\varepsilon\right) \geq p_i^* - \frac{\varepsilon}{2} - \frac{\varepsilon}{2} \Rightarrow \widehat{\phi}_i\left(p_i^* - 2\varepsilon\right) \geq p_i^* - \varepsilon$$

which is the desired result for the case of $\eta = 2$. The proof is completed by induction. Suppose for $\eta \geq 2$, it is true that:

$$\widehat{\phi}_i\left(p_i^* - \eta\varepsilon\right) \geq p_i^* - (\eta - 1)\,\varepsilon.$$

Consider:

$$\begin{aligned}
\widehat{\phi}_i\left(p_i^* - \eta\varepsilon\right) &= \widehat{\phi}_i\left(p_i^* - (\eta+1)\,\varepsilon\right) + \int_{p_i^*-(\eta+1)\varepsilon}^{p_i^*-\eta\varepsilon} \widehat{\phi}_i'(p)\,dp \\
\widehat{\phi}_i\left(p_i^* - \eta\varepsilon\right) &\leq \widehat{\phi}_i\left(p_i^* - (\eta+1)\,\varepsilon\right) + \frac{\varepsilon}{2} \\
\widehat{\phi}_i\left(p_i^* - (\eta+1)\,\varepsilon\right) &\geq \widehat{\phi}_i\left(p_i^* - \eta\varepsilon\right) - \frac{\varepsilon}{2}
\end{aligned}$$

Using $\widehat{\phi}_i\left(p_i^* - \eta\varepsilon\right) \geq p_i^* - (\eta - 1)\,\varepsilon$ in the preceding inequality,

$$\begin{aligned}
\widehat{\phi}_i\left(p_i^* - (\eta+1)\,\varepsilon\right) &\geq p_i^* - (\eta-1)\,\varepsilon - \frac{\varepsilon}{2} \\
\widehat{\phi}_i\left(p_i^* - (\eta+1)\,\varepsilon\right) &\geq p_i^* - \eta\varepsilon + \frac{\varepsilon}{2} > p_i^* - \eta\varepsilon,
\end{aligned}$$

which proves the result. The proof when $p' > p_i^* - \varepsilon$ is analogous.

References

[1] Aoyagi, Masaki, “Collusion in Dynamic Bertrand Oligopoly with Correlated Private Signals and Communication.” *Journal of Economic Theory*, 102 (2002), 229-248.

[2] Athey, Susan and Kyle Bagwell, “Optimal Collusion with Private Information.” *RAND Journal of Economics*, 32 (2001), 428-465.

[3] Athey, Susan and Kyle Bagwell, “Collusion with Persistent Cost Shocks.” *Econometrica*, 76 (2008), 493-540.

[4] Baker, Jonathan B., “Two Sherman Act Section 1 Dilemmas: Parallel Pricing, the Oligopoly Problem, and Contemporary Economic Theory,” *Antitrust Bulletin*, 38 (1993), 143-219.

[5] Chamberlain, Edward, *The Theory of Monopolistic Competition*, Cambridge: Harvard University Press, 1938.

[6] Clark, Robert and Jean-François Houde, “Collusion with Asymmetric Retailers: Evidence from a Gasoline Price-Fixing Case,” HEC Montréal, February 2011.

[7] Cooper, David J., “Barometric Price Leadership,” *International Journal of Industrial Organization*, 15 (1997), 301-325.

[8] Cubitt, Robin P. and Robert Sugden, “Common Knowledge, Salience and Convention: A Reconstruction of David Lewis’ Game Theory,” *Economics and Philosophy*, 19 (2003), 175-210.

[9] Engel, Christoph, “How Much Collusion? A Meta-Analysis of Oligopoly Experiments,” *Journal of Competition Law and Economics*, 3 (2007), 491-549.

[10] Fonseca, Miguel A. and Hans-Theo Normann, “Explicit vs. Tacit Collusion - The Impact of Communication in Oligopoly Experiments,” Duesseldorf Institute for Competition Economics, January 2011.

[11] Garrod, Luke, “Collusive Price Rigidity under Price-Matching Punishments,” Loughborough University, September 2011.

[12] Gerlach, Heiko, “Stochastic Market Sharing, Partial Communication and Collusion,” *International Journal of Industrial Organization*, 27 (2009), 656-666.

[13] Graham, Daniel A. and Robert C. Marshall, “Collusive Behavior at Single-Object Second-Price and English Auctions,” *Journal of Political Economy*, 95 (1987), 1217-1239.

[14] Hanazono, Makato and Huanxing Yang, “Collusion, Fluctuating Demand, and Price Rigidity,” *International Economic Review*, 48 (2007), 483-515.

[15] Harrington, Joseph E. Jr., “Collusion Among Asymmetric Firms: The Case of Different Discount Factors,” *International Journal of Industrial Organization*, 7 (1989), 289-307.

[16] Harrington, Joseph E. Jr., “Posted Pricing as a Plus Factor,” *Journal of Competition Law and Economics*, 7 (2011), 1-35.

[17] Harrington, Joseph E. Jr. and Andrzej Skrzypacz, “Private Monitoring and Communication in Cartels: Explaining Recent Collusive Practices,” *American Economic Review*, 101 (2011), 2425-2449.

[18] Hay, George A., “The Meaning of ‘Agreement’ under the Sherman Act: Thoughts from the ‘Facilitating Practices’ Experience,” *Review of Industrial Organization*, 16 (2000), 113-129.

[19] Huck, Steffen, Hans-Theo Normann, and Jörg Oechssler, “Two are Few and Four are Many: Number Effects in Experimental Oligopolies,” *Journal of Economic Behavior and Organization*, 53 (2004), 435-446.

[20] Hylton, Keith N., *Antitrust Law: Economic Theory and Common Law Evolution*, Cambridge: Cambridge University Press, 2003.

[21] Ivaldi, Marc, Bruno Jullien, Patrick Rey, Paul Seabright, and Jean Tirole, “The Economics of Tacit Collusion,” IDEI Toulouse, Final Report for DG Competition, European Commission, March 2003.

[22] Kalai, Ehud and Ehud Lehrer, “Rational Learning Leads to Nash Equilibrium,” *Econometrica*, 561 (1993), 1019-1045.

[23] Kaplow, Louis and Carl Shapiro, “Antitrust,” in *Handbook of Law and Economics, Volume 2*, A. Mitchell Polinsky and Steven Shavell, eds., Amsterdam: Elsevier B.V., 2007.

[24] Kaplow, Louis, "On the Meaning of Horizontal Agreements in Competition Law," *California Law Review*, 99 (2011a), 683-818.

[25] Kaplow, Louis, "An Economic Approach to Price Fixing," *Antitrust Law Journal,* 77 (2011b), 343-449.

[26] Kaplow, Louis, "Direct Versus Commnications-Based Prohibitions on Price Fixing," John M. Olin Center for Law, Economics, and Business, Harvard Law School, Discussion Paper. 703, July 2011c (forthcoming, *Journal of Legal Analysis*).

[27] Krishna, Vijay, *Auction Theory*, Second Edition, Amsterdam: Academic Press, 2010.

[28] Lewis, David, *Convention: A Philosophical Study*, Cambridge: Harvard University Press, 1969.

[29] Lu, Yuanzhu and Julian Wright, "Tacit Collusion with Price-Matching Punishments," *International Journal of Industrial Organization*, 28 (2010), 298-306.

[30] MacLeod, W. Bentley, "A Theory of Conscious Parallelism," *European Economic Review*, 27 (1985), 25-44.

[31] Markham, Jesse W., "The Nature and Significance of Price Leadership," *American Economic Review*, 41 (1951), 891-905.

[32] Milne, Robert A. and Jack E. Pace, III, "The Scope of Expert Testimony on the Subject of Conspiracy in a Sherman Act Case," *Antitrust*, 17 (2003), 36-43

[33] Mouraviev, Igor and Patrick Rey, "Collusion and Leadership," Toulouse School of Economics, December 2010 (*International Journal of Industrial Organization*, forthcoming).

[34] Nachbar, John H., "Beliefs in Repeated Games," *Econometrica*, 73 (2005), 459-480.

[35] Page, William H., "Communication and Concerted Action," *Loyola University Chicago Law Journal*, 38 (2007), 405-460.

[36] Posner, Richard A., *Antitrust Law*, Second Edition, Chicago: University of Chicago Press, 2001.

[37] Rojas, Christian, "The Role of Information and Monitoring on Collusion," U. of Massachusetts-Amherst, August 2011.

[38] Rotemberg, Julio J. and Garth Saloner, "Price Leadership," *Journal of Industrial Economics,* 39 (1990), 93-111.

[39] Scherer, F. M., *Industrial Market Structure and Economic Performance*, 2nd Edition, Boston: Houghton Mifflin, 1980.

[40] Wang, Zhongmin, "(Mixed) Strategy in Oligopoly Pricing: Evidence from Gasoline Pricing Cycles Before and Under a Timing Regulation," *Journal of Political Economy*, 117 (2009), 987-1030.