The value of useless information

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Abstract

There are many situations in which individuals have a choice of whether or not to observe the eventual outcome. In these instances, individuals often prefer to *avoid* observing the outcome. The standard von Neumann-Morgenstern (vNM) Expected Utility model cannot accommodate these cases, since it does not distinguish between lotteries for which outcomes are observed by the agent and lotteries for which they are not. I develop a simple axiomatic model that admits preferences for observing the outcome or remaining in doubt. I then use this model to analyze the connection between the agent's attitude towards risk, doubt, and what I refer to as 'optimism'.

This framework accommodates a wide array of field and experimental observations that violate the vNM model, and that may not seem related, prima facie. For instance, this framework accommodates self-handicapping, in which an agent chooses to impair his own performance. It also admits a status quo bias, without having recourse to framing effects. In a political economy setting, a voter avoids free information if he believes other voters will do the same.

Keywords: Value of information, uncertainty, recursive utility, doubt, unobserved outcomes, unresolved lotteries.

Models of decision making under uncertainty usually assume that the agents expect to eventually observe the resolution of uncertainty. However, there are many situations in which individuals can choose to avoid finding out which outcome has occurred. In these cases, individuals often decide *not* to observe the resolution of uncertainty. Consider the classic example of genetic diseases. As Pinker (2007) discusses, "the children of parents with Huntington's disease [HD] usually refuse to take the test that would tell them whether they carry the gene for it." HD is a neurodegenerative disease with severe physical and cognitive symptoms. It reduces life expectancy significantly, and there is currently no known cure. A person can take a predictive test to determine whether he himself will develop HD. A prenatal test can also be done to determine whether his unborn child will have the disease as well.[1] In an experimental study, Adam et al. (1993) find low demand for prenatal testing for HD. This is supported by a number of other studies as well, and Simpson et al. (2002) find that the demand for prenatal testing is significantly lower than the demand for predictive tests. That is, individuals who are willing to know their own HD status are often unwilling to find out their unborn child's status. Observing the result is an important decision, since the prenatal test is done at a stage in which parents can still terminate the pregnancy. As for parents who do not consider pregnancy termination to be an option, the information could still impact the way they decide to raise their child. For example, if they know that their child will develop HD, they might choose to prepare him psychologically for the difficult choices he will have to make in the future.

It may seem puzzling that some parents prefer to avoid the test. It may appear particularly surprising that a person who prefers to be certain of his own HD status now rather than later would also choose not to find out whether his unborn child will develop the disease.[2] But note that the average age of onset for HD is high enough that the subjects who do not see the result of the prenatal test may *never* find out whether their children are affected. That is, while choosing the predictive test mostly reveals a preference for early resolution of uncertainty, choosing (or refusing) the prenatal test mainly reveals a preference for never observing the outcome of a lottery. It is precisely this type of preference on which this paper focuses.[3]

[1] An affected individual has a 50% chance of passing the disease to each child. The average age of onsets varies between ages 35 and 55. See Tyler et al. (1990) for details.

[2] The prenatal test is not costless, as the procedure does involve a small chance of miscarriage. However, this cost appears small compared to the severity and likelihood of the disease, considering that this procedure is routinely conducted to test for much less likely conditions.

[3] In particular, this paper does not consider other factors that are present in the HD example, such as parents' concern that their child will be treated differently if it is known that he has HD, as discussed in Simpson (2002).

The standard von Neumann-Morgenstern (vNM) Expected Utility model cannot accommodate preferences for remaining in doubt, since it does not make a distinction between lotteries for which the final outcomes are observed and lotteries for which they are not. Redefining the outcome space to include whether the prize is observed does not resolve the issue.[4] In this paper, I modify the basic axioms of the vNM framework to develop a model that admits strict preferences for remaining in doubt or for observing the outcome. The central aim is to demonstrate that this simple and natural extension of the vNM framework can accommodate a wide array of field and experimental observations that are considered incompatible with the vNM model.

Applications

I first use a simplified version of the model to accommodate seemingly unrelated behavioral patterns that have motivated frameworks that are significantly different from the standard vNM model. Two important examples are self-handicapping and the status quo bias. In this analysis, I assume throughout that the agent is doubt-prone, meaning that when given the choice between observing and not observing a lottery's resolution, they prefer *not* observing it.

Consider first self-handicapping, in which individuals choose to reduce their chances of succeeding at a task. As discussed in Benabou and Tirole (2002), people may "choose to remain ignorant about their own abilities, and [...] they sometimes deliberately impair their own performance or choose overambitious tasks in which they are sure to fail (self-handicapping)." This behavior has been studied extensively, and seems difficult to reconcile with the standard Expected Utility theory. For that reason, models that study self-handicapping make a substantial departure from the standard vNM assumptions. A number of models follow Akerlof and Dickens' (1982) approach of endowing agents with manipulable beliefs or selective memory. Alternatively, Carillo and Mariotti (2000) consider a model of temporal-inconsistency, in which a game is played between the selves, and Benabou and Tirole (2002) use both manipulable beliefs and time-inconsistent agents.[5]

[4] The term observation is defined as learning what the outcome is. This model does not take into account a possible disutility from the graphical nature of the observation itself. See appendix for a discussion on the problem with redefining the outcome space to include the observation.

[5] See also Compte and Postlewaite (2004), who focus on the positive welfare implications of having a degree of selective memory (assuming such technology exists) in the case where performance depends on emotions. Benabou (2008) and Benabou and Tirole (2006a, 2007) explore further implications of belief manipulation, particularly in political economy settings, in which multiple equilibria emerge. Brunnermeier and Parker (2005) treat a general-equilbrium model in which beliefs are essentially choice

The frameworks mentioned above capture a notion of self-deception, which involves either a hard-wired form of selective memory (or perhaps a rule of thumb), or some form of conflict between distinct selves. These models are often not axiomatized. In contrast, this model simply extends the vNM framework and does not allow agents to manipulate their beliefs or to have access to any other means for deceiving themselves.[6] Yet it still accommodates the decision to self-handicap, as is shown in section 2. Intuitively, a doubt-prone agent prefers doing worse in a task if this allows him to avoid information concerning his own ability. This is essentially a formalization of the colloquial 'fear of failure'; an agent exerts less effort so as to obtain a coarser signal.

This model can also accommodate a status quo bias. The status quo bias refers to the well-known tendency people have for preferring their current endowment to other alternatives. This phenomenon is often seen as a behavioral anomaly that cannot be explained using the vNM model. On the other hand, it can be accommodated using loss aversion, which refers to the agent being more averse to avoiding a loss than to making a gain (Kahneman, Knetch and Thaler (1991)). The status quo bias is therefore an immediate consequence of the agent taking the status quo to be the reference point for gains versus losses. The vNM model does not allow an agent to evaluate a bundle differently based on whether it is a gain or a loss, and hence cannot accommodate a status quo bias. Arguably, this is an important systematic violation of the vNM model, and is one of the reasons cited by Kahneman, Knetch and Thaler (1991) for suggesting "a revised version of preference theory that would assign a special role to the status quo."

This model does not make use of a notion of reference points or of relative gains and losses. In the cases where the choices also have an informational component on the agent's ability to perform a task well, a doubt-prone agent has incentive to choose the bundle that is less informative. This leads to a status quo bias when it is reasonable to assume that maintaining the status quo is a less informative indicator of the agent's ability than other actions. Since this model does not resort to reference points, there is no arbitrariness in defining what constitutes a gain and what constitutes a loss. The bias of a doubt-prone agent is always towards the least-informative signal of his ability.

variables in the first period; an agent manipulates his beliefs about the future to maximize his felicity, which depends on future utility flow. Caplin and Leahy (2001) present an axiomatic model where agents have 'anticipatory feelings' prior to resolution of uncertainty, which may lead to time inconsistency. Koszegi (2006) considers an application of Caplin and Leahy (2001). Wu (1999) presents a model of anxiety. See Berglass and Jones (1978) for the original experiment on self-handicapping.

[6]While the theoretical framework later introduces a notion of optimism, the agents are *not* allowed to be either optimistic or pessimistic in any of the applications considered, as it can perhaps be seen as a form of belief manipulation.

In instances where the status quo provides the most informative signal, the bias would be *against* the status quo. For example, an individual could have incentive to change activities frequently rather than obtaining a sharp signal of his ability in one particular field.

This framework admits other instances of seemingly paradoxical behavior. In one example, an individual pays a firm to invest for him even though he does not expect that firm to have superior expertise. In other words, the agent's utility not only depends on the outcome, but also on who makes the decision. This result is not due to a cost of effort, but rather to the amount of information acquired by the decision maker. This framework can also be used in a political economy setting, as there are many government decisions that are never observed by voters. As shown in section 2, voters may have strong incentives to remain ignorant over these issues, even if information is free. This is in line with the well-known observation that there has been a consistently high level of political ignorance amongst voters in the U.S. (see Bartels (1996) for details). This model suggests that if voters care more about policies that they may never observe, then they have *less* incentive to acquire information.

Lastly, recent experiments by Dana, Weber and Kuang (2007) are consistent with this framework. They consider a typical dictator game in which there is a clear 'altruistic' alternative and a 'selfish' alternative: in one box, the dictator receives \$6 and the recipient \$1, and in the other both receive \$5. As is expected, a high percentage of people choose the altruistic alternative. They then vary the experiment so that the dictator does not know what the recipient will receive in each box, as it depends on a coin toss. However, the dictator can observe the result of the coin toss *before* making his choice. He can do so at no cost, and in some cases he is even paid to observe the coin toss. Despite this, in this setting a high percentage of agents refuse to observe the coin toss, and choose the 'selfish' option. These results appear difficult to reconcile with either selfish behavior or altruistic behavior, but they are entirely in line with doubt-prone preferences.[7]

Framework

An agent has primitive preferences over general lotteries that lead either to outcomes that he observes or to lotteries that never resolve, from his frame of reference.[8] This is a

[7]A thorough discussion of these experiments is deferred to a second paper, which also considers an extension of this framework that accommodates behavior associated with anticipated regret, including preferences for smaller menus and the Allais paradox.

[8]Throughout this paper, probabilities are taken to be objective. With subjective probabilities, there are cases in which it may seem more natural to interpret the preferences as state-dependent. For

richer domain of lotteries than in the standard vNM case. If the agent receives a lottery that never resolves then he knows that he will not observe the outcome, and his terminal prize is the lottery itself. I apply the three standard vNM axioms on this expanded domain; that is, weak order, continuity and independence hold. I also assume that the agent is indifferent between observing a specific outcome and receiving an unresolved lottery that places probability one on that same outcome, since he is certain of the outcome's occurrence. The observation itself has no effect on the value of the outcome in this model. This property restricts the agent's allowable preferences over unresolved lotteries, as I demonstrate in section 3.

I obtain a representation theorem that separates the agent's risk-attitude over lotteries whose outcomes he observes from his risk attitude over unresolved lotteries. While this representation theorem suffices for most of the analysis, I also consider a second representation in a two-period setting in which the agent may learn 'early' or 'late' whether or not a lottery will resolve. His preferences over unresolved lotteries are allowed to change over time. In contrast, his preferences over lotteries that resolve do *not* change over time, as this model does not aim to capture a notion of anxiety.

Using the first (static) representation, I explore the connection between risk-aversion, doubt-proneness and a new notion of optimism over unresolved lotteries, which I formally define. Intuitively, an optimistic agent prefers more 'scrambled' information. I show that an agent who is both doubt-prone and risk-averse over the unresolved lotteries can be neither optimistic nor pessimistic . In addition, his utility function associated with unresolved lotteries must be more concave than his utility function associated with lotteries whose outcome he observes. If an agent exhibits optimism over unresolved lotteries has the same utility function for both lotteries that resolve and lotteries that do not, then he must be doubt-prone.

Restricting attention only to preferences over purely unresolved lotteries, this model does not assume that these preferences obey the independence axiom. Instead, I assume the Rank-Dependent Utility (RDU) axioms, for reasons discussed in section 3. As there exists an accepted notion of optimism (Quiggin (1982)) in an RDU setting, it is of interest to formally relate RDU optimism to this paper's definition of optimism. RDU optimism essentially corresponds to a notion of overweighing the probabilities over the better outcomes. I show that my definition of optimism is equivalent to RDU optimism, if it holds everywhere. In that sense, it serves as a new axiom for RDU optimism.

instance, if a person has an intrinsic preference over his ability but is unsure of his type, it is unclear whether ability is better viewed as a state of the world or a consequence.

Relation to the literature

The approach used in this paper is related to, but distinct from, the recursive expected Utility (REU) framework introduced by Kreps and Porteus (1978), and extended by Epstein and Zin (1989), Segal (1990) and Grant, Kajii and Polak (1998, 2000).[9] These earlier contribution address the issue of temporal resolution, in which an agent has a preference for knowing now versus knowing later. While the REU framework treats the issue of the timing of the resolution, this paper treats the case of *no* resolution. Simply adding a 'never' stage to the REU space does not yield an equivalent representation. To demonstrate this point, I place the agent in a two-stage model (section 5), but do not allow the agent to have preferences over temporal resolution. The agent may, however, change his preferences over unresolved lotteries over time. For instance, he may prefer to avoid information in the early stage, but be curious in the later stage. In addition to the formal differences between the two frameworks, there are also interpretational ones. The REU model captures a notion of 'anxiety' (wanting to know sooner or later) which is distinct from the notion of doubt-proneness (not wanting to know at all) addressed here.

This paper is structured as follows. Section 1 introduces a simplified version of the model, which is used in section 2 for the applications. Section 3 presents the model, and section 4 defines optimism and discusses the connection between doubt-proneness, optimism and risk-aversion. Section 5 presents a representation for a two-period setting, and analyzes the connection between this model and the Kreps-Porteus model. Section 6 concludes. All proofs are in the appendix.

1 Simplified Model

I begin with a simplified version of the model, which is sufficient for most applications of interest. The axiomatic treatment is deferred to section 3. The objects used throughout are as follows. Let $\mathbf{Z} = [\underline{z}, \bar{z}] \subset \Re$ be the outcome space, and let $\mathfrak{L}_0$ be the set of simple probability measures on $\mathbf{Z}$. For $f = (z_1, p_1; z_2, p_2; ...; z_m, p_m) \in \mathfrak{L}_0$, z_i occurs with probability p_i. I use the notation $f(z_i)$ to mean the probability p_i (in lottery f) that z_i occurs. Let $\mathfrak{L}_1$ be the set of simple lotteries over $\mathbf{Z} \cup \mathfrak{L}_0$. For $X \in \mathfrak{L}_1$, I use the notation $X = (z_1, q_1^I; ...; z_n, q_n^I;\ f_1, q_1^N; ...; f_m, q_m^N)$. Here, z_i occurs with probability q_i^I, and lottery

[9] Grant, Kajii and Polak (1998) focus on preferences for early resolution of uncertainty, and Dillenberger (2011) considers preferences for one-shot resolution of uncertainty. Selden's (1978) framework is also closely related to the REU model.

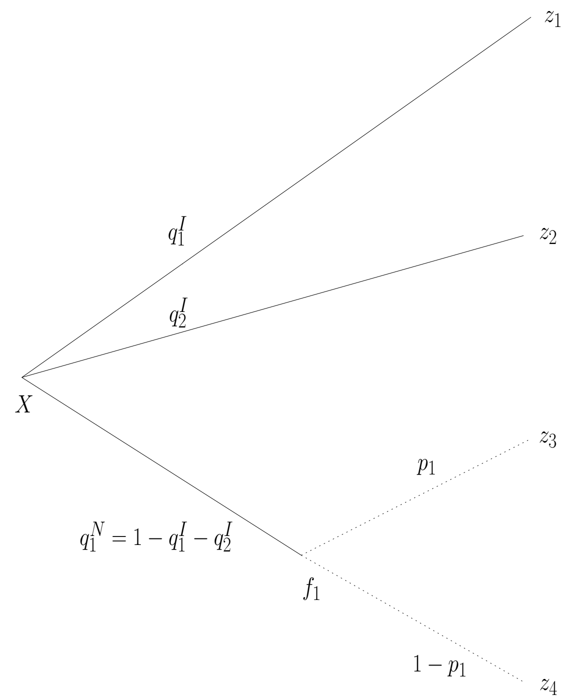

Figure 1: Lottery $X = (z_1, q_1^I; z_2, q_2^I; f_1, q_1^N)$, where $f_1 = (z_3, p_1; z_4, 1 - p_1)$

f_j occurs with probability q_j^N. Note that $\sum_{i=1}^{n} q_i^I + \sum_{i=1}^{m} q_i^N = 1$. The reason for using this notation, rather than the simpler enumeration $q_1, q_2, ..., q_n$ is explained shortly. Let $\succeq$ denotes the agent's preferences over $\mathfrak{L}_1$, and $\succ$, $\sim$ are defined in the usual manner. Assume the agent's preferences are monotone.

For any $X = (z_1, q_1^I; z_2, q_2^I; ...; z_n, q_n^I;\ f_1, q_1^N; f_2, q_2^N; ...; f_m, q_m^N)$, the agent expects to observe the outcome of the first-stage lottery. He knows, for instance, that with probability q_i^I, outcome z_i occurs, and furthermore he knows that he will observe it. Similarly, he knows that with probability q_i^N, lottery f_i occurs. But while he does observe that he is now faced with lottery f_i, he does *not* observe the outcome of f_i. I refer to lottery f_i as an 'unresolved' lottery. I also use the notation q_i^I and q_i^N to distinguish between probabilities that lead to prizes where the agent is informed of the outcome (since he directly observes which z occurs), and probabilities that lead to prizes where he is not (since he only observes the ensuing lottery). The superscript I in q_i^I stands for 'Informed', and N in q_i^N for 'Not informed' (see figure 1).

Denote the degenerate one-stage lottery that leads to $z_i \in \mathbf{Z}$ with certainty $\delta_{z_i} = (z_i, 1) \in \mathfrak{L}_0$. The degenerate lottery that leads to $f_i \in \mathfrak{L}_0$ with certainty is denoted $\delta_{f_i} = (f_i, 1) \in \mathfrak{L}_1$. Note that all lotteries of form $X = f$, where $f \in \mathfrak{L}_0$, are purely resolved (or 'informed') lotteries, in the sense that the agent expects to observe whatever outcome occurs. Similarly, all lotteries of form $X = \delta_f$, where $f \in \mathfrak{L}_o$, are purely unresolved lotteries. With slight abuse, the notation $f \succeq f'$ (or $\delta_f \succeq \delta_{f'}$) is used, where $f, f' \in \mathfrak{L}_o$. In addition, $f \succeq \delta_f$ (or $\delta_f \succeq f$) indicates that the agent prefers (not) to observe the outcome of lottery f than to remain in doubt. Under the simplest set of axioms, the representation collapses to the following:

Simple representation. $X \succ Y$ if and only if $W(X) > W(Y)$, where for all $X = ((z_1, q_1^I; ...; z_n, q_n^I;\ f_1, q_1^N; ...; f_m, q_m^N) \in \mathfrak{L}_1$,

$$W(X) = \sum_{i=1}^{n} q_i^I u(z_i) + \sum_{j=1}^{m} q_j^N u\left(v^{-1}\left(Ev(f_j)\right)\right)$$

The functions u and v are unique up to positive affine transformation.

In this simplified setting, the representation is essentially a Kreps-Porteus (KP) representation, albeit with a different interpretation. Section 3 considers more general axioms, and section 5 demonstrates that even this simplified model diverges from the KP representation if there is more than one period. But for most applications, this simple representation suffices.

The intuition behind this representation is straightforward. The first term is of the standard expected utility form for lotteries that are eventually observed, with utility functional u. As the aim of this model is to depart as little as possible from expected utility, when there are no unobserved lotteries the representation is indistinguishable from the standard EU form. As for when there are unobserved lotteries, they are treated in the same way as any other prize. Take an unobserved lottery f. The function v is used to find how this lottery f ranks with respect to other outcomes in the outcome space $\mathbf{Z}$. In other words, v is used to obtain the certainty equivalent $v^{-1}(Ev(f))$. Then, this representation uses standard expected utility analysis with function u, using the certainty equivalent $v^{-1}(Ev(f))$ in lieu of a final outcome z. I now define doubt-proneness, which refers to a preference for not observing a lottery, in the natural way.

Definition (Doubt-proneness)

- An agent is doubt-prone *somewhere* if there exists some f such that $\delta_f \succ f$.
- An agent is doubt-prone *everywhere* if: (i) there exists no $f \in \mathfrak{L}_o$ such that $f \succ \delta_f$ and (ii) there exists some f such that $\delta_f \succ f$.

An agent who prefers not to observe the resolution of some lottery than to observe it is doubt-prone somewhere. An agent who (weakly, and strictly for one lottery) prefers not to observe the outcome of *any* lottery is doubt-prone everywhere. Doubt-aversion is defined in a similar manner. Section 4 provides a thorough discussion on the relation between doubt-proneness and the functions of the more general model. The next section considers different settings in which agents are doubt-prone, so as to demonstrate that various behavioral anomalies can be accommodated naturally.

2 Applications

The aim of this section is to illustrate the scope of this simple extension of the vNM expected utility model. This section makes the same assumption over preferences throughout, namely that agents are doubt-prone (they would rather not observe the resolution of uncertainty). I consider two applications in this section. In the first, an agent's utility depends directly on his ability, since it is related to his self-image. He may never fully observe his ability, but his success at performing tasks provides him with an imperfect signal. How well he performs a task also depends on his effort. Performing a task better provides him with a reward, and so in the standard EU setting, he would always put in as much effort as he can if effort is costless. In this setting, however, there is a tradeoff between obtaining a better reward by putting in more effort and obtaining a coarser signal of ability by putting in less effort. The agent therefore has an incentive to self-handicap. This setup also accommodates other well known behavioral patterns. Under one version of this setup, an agent has an incentive to remain with the status quo. In another version of this setup, a risk-neutral agent prefers less risky bonds with a lower expected return to more risky stocks with a higher expected return. This agent is also willing to pay a firm to invest for him, even if he knows that the firm does not have superior expertise.

In the second application, voters all have the same preferences, but they do not know who the better candidate is. However, they can acquire this information at no cost. I demonstrate that there are equilibria in which they choose to remain ignorant, and the wrong candidate is as likely to win as the right candidate.

2.1 Preservation of self-image

I first introduce a general setup in which the agent has preferences over their self-image, before analyzing the implications of the results in different contexts. I assume that the agent places direct value on his ability, independently of the effect it has on his monetary reward. Arguably, individuals care about their self-image, and would rather think of themselves as being of higher ability than lower ability. Note that ability, or self-worth, is never directly observed by individuals, and so this framework applies. Instead, individuals' success at achieving their goals, given how much effort they put in, provides them with imperfect signals of their ability.

Suppose then that the agent is endowed with ability (or type) $t \in [\underline{t}, \overline{t}] \in \mathbb{R}$. He does not know what his ability is, but his prior probability of having ability t is $p(t)$. The agent chooses effort $e \in [\underline{e}, \overline{e}] \in \mathbb{R}$, to obtain a reward $m \in [\underline{m}, \overline{m}] \in \mathbb{R}$. Although the agent may never observe his ability, he does observe m. The reward depends on his ability, the effort he puts in, and an intrinsic uncertainty. Let $p(m|e,t)$ denote his probability of receiving reward m given his effort e and his ability t. Since he does not know what his ability is ex-ante, his prior probability of receiving m given effort e is $p(m|e) = \sum_{t \in [\underline{t}, \overline{t}]} p(m|e,t)p(t)$. Assume that the expected reward is higher if he puts in more effort for any given ability, and it is higher if he is of higher ability at any given effort level: $Em(e,t) > Em(e,t') \Leftrightarrow t > t'$, and $Em(e,t) > Em(e',t) \Leftrightarrow e > e'$.[10]

The agent's value function W depends on both his reward m and on his intrinsic ability t. Assume that his utility for m is linear; more precisely, his expected utility over m is $Em(e)$. In addition, it is linearly separable from his utility over t. He is weakly risk-averse over t (for both resolved and unresolved lotteries) as well as doubt-prone.[11] Recall that u is the utility associated with resolved lotteries, and v with unresolved lotteries. In this case, these lotteries are over his ability t.

In the standard case in which the agent expects to observe both his ability t and his reward m, then his value function is:

$$W(e) = Em(e) + Eu(t)$$

Since effort is costless, it is immediate that he should put in the highest level of effort, $e = \overline{e}$. But now suppose that he does *not* necessarily observe his ability ex-post. In this case, when he receives his monetary reward, he simply updates his probability on his ability, given m and his chosen effort level e. His value function is therefore:

$$W(e) = Em(e) + \sum_{m} p(m|e) u\left(v^{-1}(Ev(t|m,e))\right)$$

Depending on the functional form, the agent might not put in effort $e = \overline{e}$. His effort level also depends on his incentive to obtain the least information concerning his ability, since he is doubt-prone. In other words, he takes into account what the combination of his effort and the reward he obtains allow him to deduce about his ability. Suppose that

[10] All the probability distributions in this section have finite support.

[11] While the representation provided does not explicitly have a separate 'money' term, extending the model to include this term is trivial.

there is a unique effort level e_o (the 'ostrich' effort) that is entirely uninformative, i.e. $p(t|m, e_o) = p(t)$ for all $t \in [\underline{t}, \overline{t}]$ and for all $m \in [\underline{m}, \overline{m}]$. Note that e_o provides the agent with the highest expected utility over his ability. That is, define

$$C(e) \equiv u\left(v^{-1}(Ev(t))\right) - \sum_m p(m|e)u\left(v^{-1}(Ev(t|m,e))\right)$$

As shown in the appendix, it is always the case that $C(e) > 0$ (for $e \neq e_o$) for a doubt-prone agent, with $C(e_o) = 0$. Redefining the value function to be $\tilde{W}(e) = W(e) - u\left(v^{-1}(Ev(t))\right)$, the agent maximizes

$$\tilde{W}(e) = Em(e) - C(e)$$

Hence $C(e)$ is effectively the 'shadow' cost of effort due to acquiring information that he would rather ignore. The optimal effort level depends on the importance of the expected reward $Em(e)$ relative to the agent's disutility of acquiring information concerning his ability, as is captured by $C(e)$. Suppose now that $e_0 = \underline{e}$, and that the agent obtains a more informative signal (in the Blackwell sense) for a higher effort e. Then $C(\underline{e}) = 0$, and $C(e)$ is strictly increasing, so that the 'shadow' cost is increasing in effort level. The following simple example serves as an illustration.

Numerical Example
Let $\underline{e} = \underline{t} = 0$, $\overline{e} = \overline{t} = 1$, $p(t = 0) = \frac{1}{2}$ and $p(t = 1) = \frac{1}{2}$. The agent's reward m only takes value \$0 and \$100. The probability of obtaining reward $m = \$100$ given e and t are:

$$p(m = \$100|t = 1, e) = e$$
$$p(m = \$100|t = 0, e) = 0$$

and $p(m = \$0|t, e) = 1 - p(m = \$100|t, e)$. The utility functions are $u = a\sqrt{t}$ for some $a > 0$, and $v = t$.

Note that in this example, the completely uninformative effort e_o is equal to 0. At effort $e = 0$, he is sure to obtain \$0, and his posterior on his ability is the same as his prior. As he puts in more effort, he obtains a sharper signal of his ability. If he puts in maximum effort $e = 1$, then he will fully deduce his ability ex-post: if he obtains \$100 then he knows he has ability $t = 1$, and if he obtains \$0 then he knows he has ability $t = 0$. His value function is now:

$$\tilde{W}(e) = 50 - C(e)$$

where $C(e) = \frac{a}{2}(\sqrt{2} - e - \sqrt{2 - 3e + e^2})$.

The optimal level of effort e^* is in the full range $[0, 1]$, depending on a. As a increases, the monetary reward m becomes less significant, and effort level e^* decreases. As a decreases, the agent's utility over his ability becomes less significant, and effort e^* increases (see appendix for details).

Self-handicapping

The setup presented here can be applied to several different contexts, the most immediate of which is self-handicapping. There is strong anecdotal evidence that people are sometimes restrained by a 'fear of failure', and will not put in as much effort as they could. Berglas and Jones (1978) find in an experiment that individuals deliberately impede their own chances of success, and attribute this behavior to people's desire to protect the image of the self.[12] The amount of optimal self-handicapping depends on the doubt-proneness of the agent, and how good of a signal he expects to obtain. As discussed above, choosing a higher effort level leads to a tradeoff between the improved reward $Em(e)$ and the incurred cost $C(e)$ of learning more about one's actual ability. This model also confirms Berglas and Jones' intuition that those who are more likely to self-handicap are not the most successful or the least successful, but rather those who are uncertain about their own competence. Akerlof and Dickens' (1982) observation that people will remain ignorant so as to protect their ego is also in agreement with the implications of this framework. But notice that here, self-handicapping follows from the agent's doubt-proneness over his decision making ability, and not from an ability to lie to himself or to manipulate his beliefs in any way.

Status quo bias

The endowment effect and status quo bias are analyzed by Kahneman, Knetch and Thaler (1991), and are explained using framing effects and loss aversion. The agent's preference for avoiding a loss is taken to be stronger than his preference for making a gain, and the reference point for what constitutes a gain or a loss is assumed to be the status quo. However, Samuelson and Zeckhauser (1988) do not view the status quo bias to be solely a consequence of loss-aversion: "Our results show the presence of status quo bias even when there are no explicit gain/loss framing effects.... Thus, we conclude

[12] See Benabou and Tirole (2002) for an explanation that uses manipulable beliefs.

that status quo bias is a general experimental finding – consistent with, but not solely prompted by, loss aversion." The framework discussed here can be applied to some settings in which a status quo bias is present.

Suppose that e now represents a choice over different bundles rather than effort. For instance, suppose that the agent only places probability on $\underline{e}$ and $\overline{e}$, and that $\underline{e}$ corresponds to keeping the current allocation, while $\overline{e}$ corresponds to switching to another bundle. In addition, suppose that acquiring a bundle also carries information on the agent's decision making ability. In this case, rather than representing a cost of effort, $C(e)$ represents the cost of deviating from the bundle that is least informative of the agent's decision making ability. Suppose that $e_0 = \underline{e}$, so that keeping the same bundle is uninformative. Then the agent exhibits a status quo bias, since inaction (keeping the same bundle) has information cost $C(e_0) = 0$. If instead, however, keeping the status quo bundle were more informative than obtaining other bundles, then a doubt-prone agent would be biased *against* the status quo. An example would be an individual who skips from activity to activity rather than persevere with one, so as to avoid a sharper signal of his ability in that specific field.

The key difference between the model presented here and the standard vNM model is that this model allows for an asymmetry in the value of acquiring a bundle compared to losing that bundle. The bundle itself does not change value based on whether the agent is endowed with it or not, and in that sense there is no framing effect. Instead, acquiring a new bundle *in itself* has different informational implications than selling it. In the case where the unobserved prize is the agent's ability, then acquiring a new bundle may provide him with more information on his ability than keeping his current allocation.

Bonds, stocks and paternity

Consider the case in which e represents an investment decision rather than effort. A higher e represents a more risky investment, but in expectation it leads to a higher monetary reward. As before, t corresponds to a notion of ability. An individual who is of higher decision-making ability makes a wiser investment choice and therefore obtains a higher expected monetary reward, given the chosen risk level. For instance, $\underline{e}$ might be a portfolio consisting solely of bonds, while $\overline{e}$ consists solely of higher-risk stocks. Maintain the assumption that $e_o = \underline{e}$. In other words, the riskless option is also least informative concerning the agent's potential as an investor.

In this setting, although the agent is risk-neutral in money, his chosen bundle e^* may still consist of more bonds than it would if the reward were purely monetary, as there is a bias towards $\underline{e}$.[13] In addition, suppose that a firm exists which offers to invest the agent's money in his place. Even if the agent does not believe that the firm has superior expertise, he still agrees to pay. Since the optimal level of risk in this case is $\overline{e}$, he is willing to pay up to $Em(\overline{e}) - Em(e^*) + C(e^*)$. In fact, even if the firm were to choose the suboptimal level e^*, he would be willing to pay up to $C(e^*)$.

In the standard EU model, the agent's choice would only depend on the monetary reward he expects to obtain. In contrast, the framework presented here allows the agent's choice to depend not only on his expected reward, but also the decision making process. That is, the agent bases his choice on the *manner* in which he expects to obtain the monetary reward.

2.2 Political Ignorance

The high degree of political ignorance of voters has been thoroughly researched, particularly in the US (see Bartels (1996)). Given the length of electoral campaigns in American politics, the amount of media coverage and the accessibility of informational sources, it seems that the cost of acquiring information should not be prohibitive for voters. Note that there are political issues whose resolution the voters may never observe. For instance, the voters may choose not to observe the amount of foreign aid given, the degree of lobbying or nepotism, or the government stance on interrogation methods. For those issues, a doubt-prone agent may have incentive to ignore information even if it is free. In other words, making information more accessible would not necessarily have a strong impact on the individual's informativeness on these issues. Since voters affect the election result as a group, each individual's decision to acquire information has an externality on other voters and on *their* decision to acquire information. This section discusses a very simple example in which voters' information acquisition plays a dominant role on the other voters' decision to acquire information. Although voting is sincere, there is a strategic aspect to the decision to obtain information.

Consider an economy in which N citizens care about issue $\gamma \in [0, 1]$, which is determined by a politician that they vote for. They can choose not to observe what the politician does. Suppose that there are two candidates, A and B. One of the two will choose policy $\gamma = 0$ if elected, and the other will choose $\gamma = 1$. The voters do not know which one is which, and place probability 1/2 that A will choose $\gamma = 0$, and 1/2 that A will choose $\gamma = 1$ (and similarly for B). But they can acquire that information at no cost, if they choose to do so. Let p_i be the ex-post probability that the ith agent places

[13] A more careful study would of course be required to gauge the empirical significance.

on the winner being the candidate who implements $\gamma = 1$, where $i \in \{1, .., N\}$. The timing is as follows:

1) Each voter decides whether or not to observe where candidates A and B stand. A voter cannot force another voter to acquire information.

2) Each voter votes sincerely, i.e. he votes for the candidate on whom he places a higher probability of implementing policy γ that he prefers. If he is indifferent or if he places equal probability on either candidate implementing his preferred policy, then he tosses a fair coin and votes accordingly.

3) The candidate who obtains the majority wins the election. In case of a tie, a coin toss determines the winner. The winner then implements the policy he prefers, and there is no possibility of reelection.

Now suppose that every voter prefers γ to be higher. In addition, every voter is also strictly doubt-prone. Let his value function be W_i^I if he acquires information and W_i^N if he does not. Even though every voter prefers the candidate who implements $\gamma = 1$, and even though information is free, there is still an equilibrium in which no one acquires information, and the candidate who implements $\gamma = 0$ wins with probability $\frac{1}{2}$. This equilibrium is Pareto-dominated (in expectation) by the other equilibria, in which at least a strict majority of agents acquires information, and the candidate who implements $\gamma = 1$ wins with probability 1. This is briefly shown below.

Equilibrium in which no voter is informed. If no other voter is informed, then voter i does not acquire information either. Since $p_i \in (0, 1)$ if no one else is informed, it follows that $W_i^I < W_i^N$ (on his own he cannot force $p_i \in \{0, 1\}$). Unless agent i is certain that either the right candidate or the wrong candidate always wins the election, i.e. that $p_i = 1$ or that $p_i = 0$, he does not acquire information.
Note that there is no equilibrium in which a minority of voters acquires information, since each voter in the minority has incentive to deviate. Note also that the difference between W_i^I and W_i^N for a given $p_i \in (0, 1)$ is higher if the difference between the agent's utility of $\gamma = 1$ and $\gamma = 0$ is larger.

Equilibrium in which at least a strict majority is informed. If at least a strict majority is informed, then the right candidate wins with probability 1. Hence $p_i = 1$ for each agent i, and so he is indifferent, since $W_i^I = W_i^N$. Note, however, that this equilibrium does not survive if each voter i places an arbitrarily small probability $\delta > 0$ that each of the other voters does not acquire information.

The externality of information plays an excessive role in this simple example, however it may still have an impact in a more realistic model. In particular, this example suggests that it is precisely those who are most affected who may end up least informed: as the difference between the doubt-prone agent's utility of the good policy and his utility of the bad policy increases, he has *less* incentive to acquire information. Moreover, a Pareto gain would be achieved if enough voters were 'forced' to acquire information on the candidates' policies.

3 Model

I now present the general (static) model. Recall from the previous section that the outcome space is $\mathbf{Z} = [\underline{z}, \bar{z}] \subset \Re$, that $\mathfrak{L}_0$ be the set of simple probability measures on $\mathbf{Z}$, and that $\mathfrak{L}_1$ is the set of simple lotteries over $\mathbf{Z} \cup \mathfrak{L}_0$, with typical element $X \in \mathfrak{L}_1$. Depending on the lottery, the agent may or may not observes outcomes in $\mathbf{Z}$. For instance, the agent observes all the outcomes for lottery $X = f$, and does not observe any outcome for lottery $X' = \delta_{f_i}$.

3.1 General axioms

The following certainty axiom **A.1** is assumed throughout:

AXIOM A.1 (Certainty): Take any $z_i \in \mathbf{Z}$, and let $X = \delta_{z_i} = (z_i, 1)$ and $X' = (\delta_{z_i}, 1)$. Then $X \sim X'$.

The certainty axiom **A.1** concerns the case in which an agent is certain that an outcome z_i occurs. In that case, it makes no difference whether he is presented with a resolved lottery that leads to z_i for sure or an unresolved lottery that leads to z_i for sure. He is indifferent between the two lotteries. Hence axiom **A.1** does not allow the agent to have a preference for being informed of something that he already knows. This simple axiom provides a formal link between the agent's preferences over resolved lotteries and his preferences over unresolved lotteries. The following three axioms are standard.

AXIOM A.2 (Weak Order): $\succeq$ is complete and transitive.

AXIOM A.3 (Continuity): $\succeq$ is continuous in the weak convergence topology. That is, for each $X \in \mathfrak{L}_1$, the sets $\{X' \in \mathfrak{L}_1 : X' \succeq X\}$ and $\{X' \in \mathfrak{L}_1 : X \succeq X'\}$ are both closed in the weak convergence topology.

AXIOM A.4 (Independence): For all $X, Y, Z \in \mathfrak{L}_1$ and $\alpha \in (0, 1]$, $X \succ Y$ implies $\alpha X + (1-\alpha)Z \succ \alpha Y + (1-\alpha)Z$.

Focusing on axiom **A.4**, it is noteworthy that the agent's preferences $\succeq$ are on a richer space than in the standard framework. The independence axiom in the standard vNM model is taken on preferences over lotteries over outcomes, since all lotteries lead to outcomes that are eventually observed. In this paper, the agent's prize is not always an outcome z, and can instead be an unresolved lottery f. By assumption **A.4**, however, there is no axiomatic difference between receiving an outcome z as a prize and obtaining an unresolved lottery f as a prize. Under this approach, the rationale for using the independence axiom in the standard model holds in this case as well. Since the aim is to depart as little as possible from the vNM Expected Utility model, I assume the independence axiom **A.4** throughout.

Axioms **A.1** through **A.4** suffice for this model to subsume the standard vNM representation for preferences over outcomes that the agent eventually observes. That is, suppose we focus on lotteries of form $X = f$, i.e. lotteries that lead to outcomes. Then all the standard vNM axioms over these lotteries hold, and the EU representation follows directly. These axioms are not sufficient, however, to characterize the agent's preferences over lotteries that do not resolve. If, for instance, the agent receives a lottery $X = \delta_f$, it is unclear what his 'perception' of unresolved lottery f is. The next step, therefore, is to consider axioms that allow us to characterize the agent's preferences over these 'purely' unresolved lotteries of form $X = \delta_f$. As there is a natural isomorphism between lotteries of form $X = \delta_f \in \mathfrak{L}_1$ and one-stage lotteries in $\mathfrak{L}_0$, define the preference relation $\succeq_N$ in the following way:

Definition of $\succeq_N$ For any $f, f' \in \mathfrak{L}_0$, $f \succeq_N f'$ if $\delta_f \succeq \delta_{f'}$.

Define $\succ_N$ and $\sim_N$ in the usual way. I first make a continuity assumption:

AXIOM N.1 (Continuity): $\succeq_N$ is continuous in the weak convergence topology.

Assuming independence over the preference relation $\succeq_N$ would lead to the simplified representation introduced in section 1 and used in section 2 for all the applications considered. In this section, I do *not* assume independence over the preference relation $\succeq_N$, for the following reason. Suppose that an agent is given a choice between three lottery tickets. The first ticket consists of a lottery $f = (\$1000, 1/3; \$400, 1/3; \$0, 1/3)$. With probability 1/3, the ticket yields \$1000, with probability 1/3 it yields \$400, and it yields 0 otherwise. The second ticket consists of lottery $f' = (\$1000, 1/2; \$0, 1/2)$ and the third ticket consists of $f'' = (\$400, 1) = \delta_{400}$, which yields \$400 for certain. In addition, suppose that the agent does not purchase the ticket for himself, but for a charitable organization that he holds in high esteem.

It is plausible that a risk-averse agent prefers the safe lottery δ_{400} to lottery f', *if* he expects to observe the outcome of the lotteries (for instance, if the charity thanks him for his contribution of the quantity it receives). But it may also be the case that the same agent has different preferences and choose risky lottery f' over the safe lottery δ_{400} ($f' \succ_N \delta_{400}$), if he donates the unresolved ticket to the charity and does not expect to observe which outcome occurs. There is a 1/2 chance that the charity has received \$1000, and he does not expect to ever find out if it has received \$0. These preferences may be driven by a notion of 'optimism'.

Now compare lotteries f to f', still for the case in which the agent does not expect to observe the resolution of uncertainty. It is also plausible that the agent prefers lottery f to f' ($f \succ_N f'$): lottery f is less risky than lottery f', and at the same time he still does not find out whether the charity has received \$0:

$$(\$1000, 1/3; \$400, 1/3; \$0, 1/3) \succ_N (\$1000, 1/2; \$0, 1/2) \succ_N \delta_{400}.$$

These preferences appear reasonable, but they violate independence. In fact, they violate the stronger axiom of betweenness, and so do not fall in the Dekel (1986) class of preferences.[14]

This example illustrates that there are two distinct notions that play a role in the agent's preference over unresolved lotteries. The agent may be risk-averse over unresolved lotteries, and this risk-aversion manifests itself in his comparison between lottery f and the more risky lottery f'. At the same time, he may be 'optimistic' that the good outcome has occurred if he does not observe the lottery, which affects his assessment of lottery f', compared to the safe lottery δ_{400}. A single utility function v cannot capture

[14]Note that $f = \frac{2}{3}f' + \frac{1}{3}\delta_{400}$. This is a violation of independence (and betweenness) because the following does not hold: $f' \succ_N \frac{2}{3}f' + \frac{1}{3}\delta_{400} \succ_N \delta_{400}$. More specifically, this violates quasi-convexity.

both these notions, since risk-aversion and optimism over what is unobserved do not necessarily coincide. Since both risk-aversion and optimism are contributing factors to the agent's preferences to remain in doubt, this framework should allow the agent to express both dimensions of preference.

I now assume the Rank-Dependent Utility (RDU) axioms, which are general enough to allow the previous example. The RDU representation allows for two functions, v and w, the first that reweighs the outcomes (identically to the vNM model), and the second reweighs the probabilities. I show, in the following section, that an RDU representation captures a notion of risk and optimism that are suitable to this model, even though my formal definition of optimism will be different from the accepted RDU definition. I later consider conditions which force the function w to be linear, essentially reducing the representation of $\succeq_N$ to a vNM representation.[15]

3.2 RDU representation for $\succeq_N$.

The following notation is convenient for the RDU representation. For lottery $f = (z_1, p_1; ; ...; z_m, p_m) \in \mathfrak{L}_o$, the $z_i's$ are ordered from smallest to highest, i.e. $z_m > ... > z_1$. Recall that the agent's preferences are monotone, which implies that $\delta_{z_m} \succ_N ... \succ_N \delta_{z_1}$. In addition, p_i^* denotes the probability of reaching outcome z_i or an outcome that is weakly preferred to z_i. That is, $p_i^* = \sum_{j=i}^{m} p_j$. Note that for the least-preferred outcome z_1, $p_1^* = 1$. Probabilities p_i^* are referred to here as 'decumulative' probabilities. The RDU form, introduced by Quiggin (1982), is defined in the following manner:[16]

Definition (RDU) Rank-dependent utility (RDU) holds if there exists a strictly increasing continuous probability weighting function $w : [0, 1] \to [0, 1]$ with $w(0) = 0$ and $w(1) = 1$ and a strictly increasing utility function $v : \mathbf{Z} \to \Re$ such that for all $f, f' \in \mathfrak{L}_o$,

$$f \succ_N f' \text{ if and only if } V_{RDU}(f) > V_{RDU}(f')$$

where V_{RDU} is defined to be: for all $f = (z_1, p_1; z_2, p_2; ...; z_m, p_m)$,

$$V_{RDU}(f) = v(z_1) + \sum_{i=2}^{m} [v(z_i) - v(z_{i-1})] w(p_i^*)$$

[15] The notion of 'optimism' may seem at odds with the previous claim that an agent who is not allowed to manipulate his beliefs may still choose to 'self-handicapping'. That is, one interpretation of a rank-dependent utility representation is that the agent distorts the actual probability. For this reason, In the analysis of self-handicapping (section 2), I do *not* allow the agent to be either optimistic or pessimistic.

[16] See also Yaari (1987), and Diecidue and Wakker (2001) for a thorough discussion of RDU.

Moreover, v is unique up to positive affine transformation.

Note that if the weighting function w is linear, then V_{RDU} reduces to the standard EU form.[17] I now briefly discuss the axiomatic foundation of the RDU representation, in the context of this model. Suppose that

$$f_\alpha = (z_1, p_1; ...; \alpha, p_i; ...; z_m, p_m) \succeq_N (z_1', p_1; ...; \beta, p_i; ...; z_m', p_m) = f_\beta'$$
$$f_\kappa' = (z_1', p_1; ...; \kappa, p_i; ...; z_m', p_m) \succeq_N (z_1, p_1; ...; \gamma, p_i; ...; z_m, p_m) = f_\gamma$$

where $\alpha, \beta, \gamma, \kappa \in \mathbf{Z}$. Comparing lotteries f_α and f_γ, the only difference is in whether α or γ is reached with probability p_i. Since all the other outcomes are the same in both lotteries and are reached with the same probabilities, the difference is in the value of outcome α compared to the value of outcome γ (and similarly for f_β',f_κ' and β, κ). In the comparison of $f_\alpha \succeq_N f_\beta'$ and $f_\kappa' \succeq_N f_\gamma$, all the probabilities of reaching the (rank-preserved) outcomes are the same. For that reason, this model assumes that the switch in preference is due to a difference in the value of outcomes α and β relative to γ and κ, and not in the way the probabilities are aggregated. It is precisely this property that RDU provides: if $f_\alpha \succeq_N f_\beta'$ and $f_\kappa' \succeq_N, f_\gamma$, and if $\succeq_N$ is of the RDU form, then $v(\alpha) - v(\beta) \geq v(\gamma) - v(\kappa)$. Note that this does not depend on the choice of $z's$ and $p's$, and so the following axiom, adapted from Wakker (1994), must hold:

AXIOM N.RDU (Wakker tradeoff consistency for $\succeq_N$):
Let $f_\alpha = (z_1, p_1; ...; \alpha, p_i; ...; z_m, p_m)$, $f_\gamma = (z_1, p_1; ...; \gamma, p_i; ...; z_m, p_m)$, $f_\beta' = (z_1', p_1; ...; \beta, p_i; ...; z_m', p_m)$ and $f_\kappa' = (z_1', p_1; ...; \kappa, p_i; ...; z_m', p_m)$. If:

$$f_\alpha \succeq_N f_\beta'$$
$$f_\kappa' \succeq_N f_\gamma$$

then for any lotteries $g_\alpha = (\hat{z}_1, \hat{p}_1; ...; \alpha, \hat{p}_i; ...; \hat{z}_{\hat{m}}, \hat{p}_{\hat{m}})$, $g_\gamma = (\hat{z}_1, \hat{p}_1; ...; \gamma, \hat{p}_i; ...; \hat{z}_{\hat{m}}, \hat{p}_{\hat{m}})$, $g_\beta' = (\hat{z}_1', \hat{p}_1; ...; \beta, \hat{p}_i; ...; \hat{z}_{\hat{m}}', \hat{p}_{\hat{m}})$, $g_\kappa' = (\hat{z}_1', \hat{p}_1; ...; \kappa, p_i; ...; \hat{z}_{\hat{m}}', \hat{p}_{\hat{m}})$ such that $g_\gamma \succeq_N g_\kappa'$,
it must be that $g_\alpha \succeq_N g_\beta'$.

Under this axiom, only the values of α,β,γ and κ are relevant to the ordering of the

[17] This is not the most common form of RDU; this notation is taken from Abdellaoui (2002). Given the rank-ordering above, the typical form would be $V_{RDU} = \sum_{i=1}^{n-1}[w(p_i^*) - w(p_{i+1}^*)]v(z_i) + w(p_n^*)v(z_n)$. It is easy to check that the two representations are identical.

agent's preferences when all the probabilities of reaching all other outcomes are the same across the four lotteries. In fact, as shown in Wakker (1994), this axiom is sufficient, along with stochastic dominance and continuity, for the RDU representation to hold. Using this result, the general representation theorem for $\succeq$ is as follows:

Representation Theorem. *Suppose axioms* ***A.1*** *through* ***A.4*** *and axioms* ***N.1*** *and* ***N.RDU*** *hold. In addition, suppose stochastic dominance holds for $\succeq_N$. Then there exist strictly increasing, continuous and bounded functions $u : \mathbf{Z} \to \mathbb{R}$, $v : \mathbf{Z} \to \mathbb{R}$, $w : [0,1] \to [0,1]$ with $w(0) = 0$ and $w(1) = 1$, such that for all $X, Y \in \mathfrak{L}_1$,*

$$X \succ Y \text{ if and only if } W(X) > W(Y)$$

where W is defined to be: for all $X = ((z_1, q_1^I; ...; z_n, q_n^I;\ f_1, q_1^N; ...; f_m, q_m^N) \in \mathfrak{L}_1$,

$$W(X) = \sum_{i=1}^{n} q_i^I u(z_i) + \sum_{j=1}^{m} q_j^N u\left(v^{-1}\left(V_{RDU}(f_j^N)\right)\right)$$

and

$$V_{RDU}(f) = v(z_1) + \textstyle\sum_{h=2}^{m}[v(z_h) - v(z_{h-1})]w(p_i^*).$$

Moreover, u and v are unique up to positive affine transformation.

Note that u remains the utility function associated with the general lotteries (and final outcomes). In addition, v is the utility function associated with unresolved lotteries, and w is the probability weighting function associated with unresolved lotteries. It is not immediately clear from this representation what doubt-proneness implies, in terms of the shapes of the functions. The next section defines optimism, and formally relates it to the accepted notion of optimism in an RDU setting. I then connect doubt-proneness, risk-aversion, and this new notion of optimism.

4 Risk-aversion, doubt-proneness and optimism

In this section, I focus on the relationship between doubt-proneness and the shapes of the functions u, v and w. I first define formally what optimism means in this context. Returning to the charity example from the previous section, recall that lottery $f = (\$1000, 1/3; \$400, 1/3; \$0, 1/3)$, lottery $f' = (\$1000, 1/2; \$0, 1/2)$ and lottery $\delta_{400} = (\$400, 1)$. While $f' \succ_N \delta_{400}$, it is not the case that $f' \succ_N af' + (1+a)\delta_{400} \succ_N \delta_{400}$ for all a, which the independence axiom would imply. In this example, $f = \frac{2}{3}f' + \frac{1}{3}\delta_{400} \succ_N f'$.

The notion of optimism over unresolved lotteries I aim to capture allows the agent prefer more 'scrambled' information, since it essentially allows him to form a better assessment of these unresolved lotteries. Consider lottery δ_{400}, in which the agent is certain that the outcome is \$400. Now suppose that it is mixed with a lottery $\tilde{f}' = (\$400+\delta, 1/2; \$400-\epsilon)$, where $\tilde{f}'$ is chosen such that $\tilde{f}' \sim_N f'$, and ϵ is close to 0.[18] Specifically, consider the mixture $\tilde{f} = 2/3\ f' + 1/3\ \delta_{400} = (\$400+\delta, 1/3; \$400, 1/3; \$400-\epsilon, 1/3)$ (see figure 2). If the independence axiom over unresolved lotteries were to hold, then $f \sim \tilde{f}$. But I also allow $f \succ_N \tilde{f}$, with the reasoning that the optimist agent prefers knowing as little as as possible about the unresolved lottery. With lottery f, the optimist can form a more reassuring perception of the outcome, as it could be much higher (\$1000). With lottery $\tilde{f}$, however, as ϵ becomes smaller and smaller, it becomes less and less attractive to the optimist agent, as he is more and more certain of the vicinity of the outcome. In brief, an optimist has a preference for more 'scrambled' information. A pessimistic agent, on the other hand, prefers less scrambled information, since knowing less would lead him to form a more negative perception. I allow the agent to be optimist, pessimism or neutral (i.e. independence may hold), but I assume that his preferences are preserved, given a specific mixture a and specific probabilities. That is, if the agent prefers unresolved lottery f to $\tilde{f}$, as in the example above, then this preference is preserved as ϵ becomes smaller. I refer to this property, which I now generalize, as 'information scrambling consistency' (ISC).

Definition (ISC)
Let $f = (z_1, p_1; ...; z_i; p_i; z_{i+1}, p_{i+1}; ...; z_n, p_n)$, $f' = (z_1, p_1; ...; z'_i; p_i; z'_{i+1}, p_{i+1}; ...; z_n, p_n) \in \mathfrak{L}_o$ such that $f \sim_N f'$, and *case 1*: $(z'_i, z'_{i+1}) \subset (z_i, z_{i+1})$ (*case 2*: $(z_i, z_{i+1}) \subset (z'_i, z'_{i+1})$). If, for some $a \in (0,1)$ and some $z \in (z'_i, z'_{i+1})$:

$$af + (1-a)\delta_z \succeq_N af' + (1-a)\delta_z,$$

then $\succeq_N$ satisfies ISC if:

$$a\tilde{f} + (1-a)\delta_{\tilde{z}} \succeq_N a\tilde{f}' + (1-a)\delta_{\tilde{z}}$$

for any $\tilde{f} = (\tilde{z}_1, p_1; ...\tilde{z}_i; p_i; \tilde{z}_{i+1}, p_{i+1}; ...; \tilde{z}_n, p_n)$, $\tilde{f}' = (\tilde{z}_1, p_1; ...\tilde{z}'_i; p_i; \tilde{z}'_{i+1}, p_{i+1}; ...; \tilde{z}_n, p_n)$ and $\tilde{z}$ such that $\tilde{z} \in (\tilde{z}'_i, \tilde{z}'_{i+1}) \subset (\tilde{z}_i, \tilde{z}_{i+1})$ (*case 2*: $\tilde{z} \in (\tilde{z}_i, \tilde{z}_{i+1}) \subset (\tilde{z}'_i, \tilde{z}'_{i+1})$).

[18] For δ to also be close to 0, \$400 would have to be close to the certainty equivalent of the unresolved lottery $f' = (\$1000, 1/2; \$0, 1/2)$.

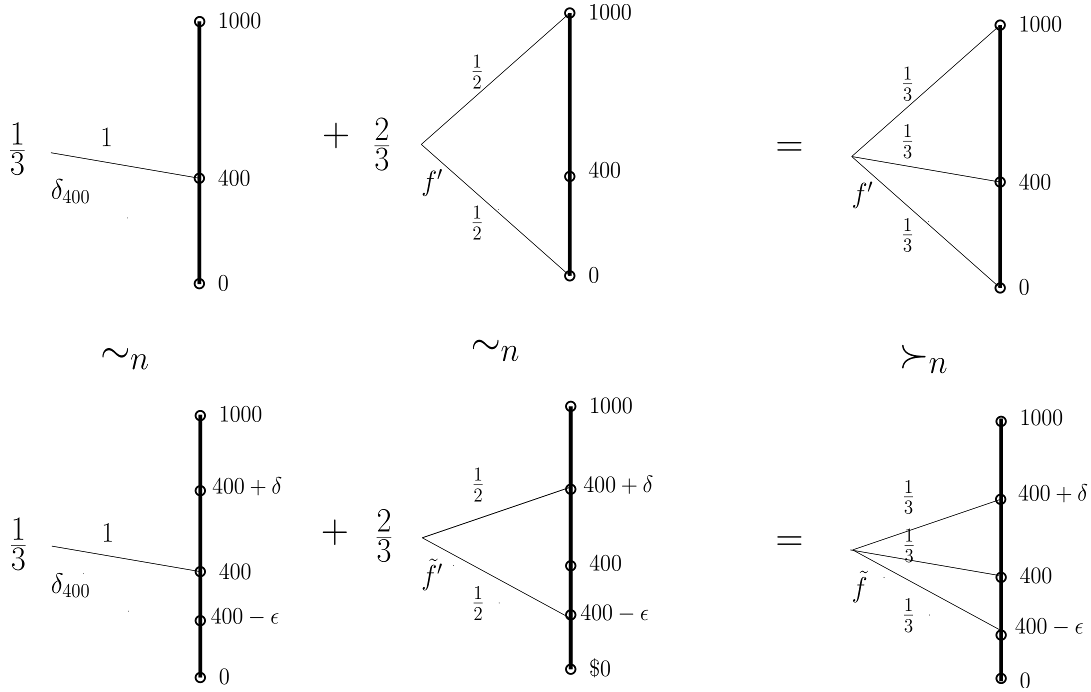

Figure 2: Optimism.

A preference for more scrambled information (optimism) corresponds to case 1, i.e. preferring $af + (1-a)\delta_z \succ af' + (1-a)\delta_z$ when $(z'_i, z'_{i+1}) \subset (z_i, z_{i+1})$. Similarly, a preference for less scrambled information (pessimism) corresponds to case 2. The appeal of the RDU representation is that it satisfies the ISC property:

Theorem 2. *Suppose that RDU holds for $\succeq_N$. Then $\succeq_N$ satisfies ISC.*

A local preference for more scrambled information, which I refer to as local optimism, does *not* correspond to the accepted RDU notion of optimism, analyzed by Wakker (1994). Focusing instead on a global preference for more scrambled information, which is denoted (global) optimism:

Definition (Optimism) The preference relation $\succeq_N$ exhibits optimism if and only if $\succeq_N$ always exhibits a preference for more scrambled information. That is, for any $f = (z_1, p_1; ...z_i; p_i; z_{i+1}, p_{i+1}; ...; z_n, p_n)$, $f' = (z_1, p_1; ...z'_i; p_i; z'_{i+1}, p_{i+1}; ...; z_n, p_n) \in \mathfrak{L}_\mathfrak{o}$ such that $f \sim_N f'$, and $(z'_i, z'_{i+1}) \subset (z_i, z_{i+1})$, and for all $a \in (0,1)$ and $z \in (z_i, z_{i+1})$,

$$af + (1-a)\delta_z \succeq_N af' + (1-a)\delta_z.$$

The next theorem states that an agent who exhibits *global* optimism according to this definition also exhibits optimism according to the standard RDU definition. In other words, an agent has a global preference for more scrambled information if and only if the weighting function w is concave, which corresponds to the accepted (Wakker) RDU definition of optimism.

Theorem 3. *Suppose that $\succeq_N$ satisfies RDU, and let w be the associated weighting function. Then w is concave (convex) if and only if $\succeq_N$ exhibits optimism.*

The following result connects doubt-proneness, the properties of the utility functions, and the properties of the probability weighting function $w(p)$. A similar result hold for doubt-aversion, and is deferred to the appendix.

Theorem 4. *Suppose that axioms **A.1** through **A.4** and the RDU axioms hold, and let u and v be the utility functions associated with the resolved and unresolved lotteries, respectively, and w be the decision weight associated with the unresolved lotteries. In addition, suppose that u and v are both differentiable. Then:*
(i) If there exists a $p \in (0,1)$ such that $p < w(p)$, then the agent is doubt-prone somewhere. Similarly, if there exists $p' \in (0,1)$ such that $p' > w(p')$, then the agent is doubt-averse somewhere.
(ii) If the agent is doubt-prone everywhere, then $p \leq w(p)$ for all $p \in (0,1)$. Moreover, if v exhibits stronger diminishing marginal utility than u, then $\succeq_N$ violates quasi-convexity (that is, there exists some $f', f'' \in \mathfrak{L}_o$, and $\alpha \in (0,1)$ such that $f' \succ f''$ and $\alpha f' + (1-\alpha)f'' \succ_N f'$).

The differentiability assumption, though common, may seem bothersome as it is not taken over the primitives. Alternatively, we could make an assumption over the primitives that guarantees (for instance) strict concavity of u and v, which would in fact be sufficient for the result.[19] Given the results above, an assumption or deduction over the agent's doubt-attitude has testable implications concerning his aggregation of probabilities (w) for unresolved lottery, and vice-versa. In addition, these implications can be disentangled from the agent's diminishing marginal utility. Since it is not necessary that w satisfies the same empirical properties as the typical case considered under rank-dependent utility, an experimental study would be useful for a better understanding of the shape of w. If, in addition to doubt-proneness, mean-preserving risk-aversion (in the standard sense) of $\succeq_N$ is assumed, then the RDU representation collapses to the recursive EU representation:

[19]For a discussion of the differentiability assumption, see Chew, Karni and Safra (1987).

Corollary 4.1. *Suppose that the conditions of theorem 4 all hold. Then the following two statements are equivalent:*

(i) Preference $\succeq$ displays doubt-proneness everywhere and $\succeq_N$ displays mean-preserving risk-aversion.

(ii) Function V_{RDU} is of the EU form (i.e. $w(p) = p$ for all $p \in [0,1]$), both u and v are concave, and $u = \lambda \circ v$ for some continuous, concave, and increasing λ.

This result further shows that attitude toward risk and attitude towards doubt constrain the probability weighting function, and can in fact completely characterize it.[20] But note that in an RDU setting, mean-preserving risk aversion is not identical to diminishing marginal utility. That is, the previous result does not imply that a doubt-prone agent who obeys risk aversion cannot have a concave utility function v. I now focus a counterexample for which doubt-proneness is entirely due to the weighting factor w, and *not* the difference in concavity between u and v.

Consider an agent for whom functions u and v are identical. It is already immediate from theorem 4 that for a doubt-prone agent, it is necessary that $p \leq w(p)$ for all p. In fact, this condition is sufficient.[21] The following result does not require differentiability.

Theorem 5. *Suppose that the conditions of theorem 4 all hold. Furthermore, suppose that $u(z) = v(z)$ for all $z \in \mathbf{Z}$ (or, more generally, $u = \lambda \circ v$ for some continuous, weakly concave, and increasing λ). Then the agent is doubt-prone everywhere if and only if $p \leq w(p)$ (with $p < w(p)$ for some $p \in (0,1)$ if $u(z) = v(z)$ for all $z \in \mathbf{Z}$).*

It follows that an optimistic agent for whom u is identical v (or for whom u is more concave than v) must be doubt-prone. These results therefore connect optimism, doubt-proneness, and risk-aversion (in the standard sense). Before concluding this section, note that extensive research has been conducted on the shape of w in the usual RDU setting, in which uncertainty eventually resolves.[22] As this a different setting, I have not made similar assumptions over the shape of w. Instead, I have shown that the induced preferences to remain in doubt have strong implications on the weighting function w.

[20]This last corollary is similar to a result in Grant, Kajii and Polak (2000) but with a notion of doubt-proneness that is weaker than the preference for late-resolution that would be required in the framework they use; the difference in assumptions is due to the difference in settings. It is also of note that under Grant, Kajii and Polak (2000)'s restriction, there is no need to assume differentiability, as it is in fact implied.

[21]It is clear that if $p = w(p)$ for all $p \in (0,1)$ and if $u(z) = v(z)$ for all $z \in \mathbf{Z}$, then the agent is doubt-neutral.

[22]See, for instance, Karni and Safra (1990), and Prelec (1998) for an axiomatic treatment of w.

Consider, for example, the common assumption that w is S-shaped (concave on the initial interval and convex beyond). In that case, it must be that the agent is doubt-prone for some lotteries and doubt-averse for others. But an empirical discussion of whether w is S-shaped in this setting is outside the scope of this paper.

5 Dynamic model and relation to the KP representation

In this section, I present a dynamic version of the model. This setup corresponds to the case in which an agent may not know now whether he will ever make an observation, but he may find out later. I do this in part because this type of scenario occurs frequently; it is often the case that an individual is not sure whether he can successfully avoid information in the next period. In the self-image application, for instance, the agent may take into consideration whether he might obtain information further in the future. Another reason for conducting this analysis is to illustrate the difference between this model and the KP representation (and, more generally, REU). I show that the models are formally distinct, even if independence axioms hold at every stage. This result may appear counterintuitive, since it may appear that a 'never' stage is formally equivalent to a 'much later' stage, but with a different interpretation. I discuss the reasons for the distinction between the two frameworks.

Suppose, for simplicity, that there are 2 stages of resolution (early and late) in a KP setup.[23] Assume, however, that the agent is indifferent between early and late resolution of uncertainty, so that there is a single utility function u associated with lotteries that resolve. It is clear that in this case, the KP representation is identical to an Expected Utility representation. But now, suppose that we include preferences over unresolved lotteries. That is, let $\mathfrak{L}_2$ is the set of simple lotteries over $\mathfrak{L}_1 \cup \mathfrak{L}_0$. For $\mathbf{X} \in \mathfrak{L}_2$, the notation $\mathbf{X} = (X_1, q^I_{1,e}; ...; X_{n_e}, q^I_{n_e,e};\ f_{1,e}, q^N_{1,e}; ...; f_{m_e,e}, q^N_{m_e,e}) \in \mathfrak{L}_2$, where $X_{i,e} \in \mathfrak{L}_1$, and $f_{j,e} \in \mathfrak{L}_0$. The subscript 'e' denotes the early stage. The agent's preferences $\succeq$ are now over $\mathfrak{L}_2$, rather than $\mathfrak{L}_1$ (see figure 3).

The timing is as follows. The agent first observes the outcome of the first stage lottery (the early stage). For instance, with probability $q^I_{i,e}$, he receives a second lottery $X_i \in \mathfrak{L}_1$. The superscript I ('Informed') denotes that the agent expects to observe the outcome of lottery X_i. With probability $q^N_{j,e}$, the agent receives a lottery $f^N_{j,e} \in \mathfrak{L}_0$, which does *not* resolve. Here, the superscript N ('Not informed' denotes that the agent never

[23]More stages of resolution can be added in the usual way.

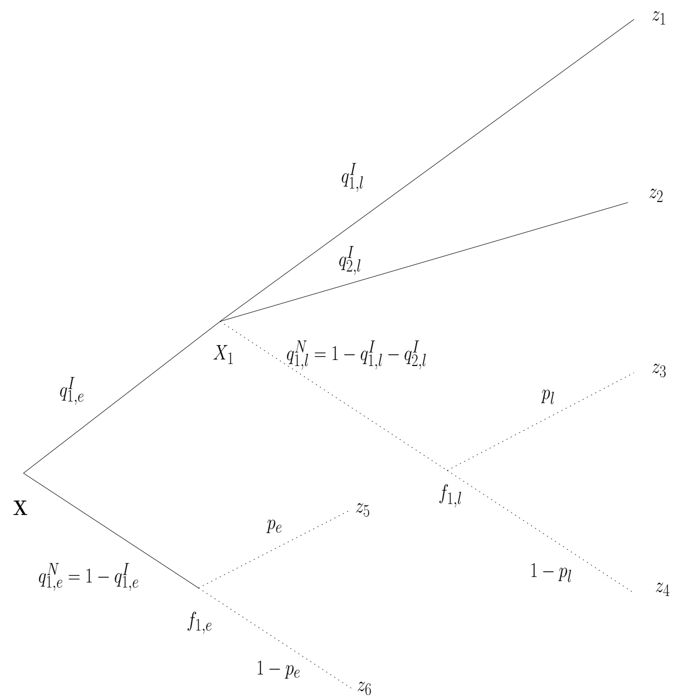

Figure 3: Lottery $\mathbf{X} = (X_1, q^I_{1,e}; f_{1,e}, q^N_{1,e} = 1 - q^I_{1,e}$, where $X_1 = (z_1, q^I_{1,l}; z_2, q^I_{2,l}; f_{1,l}, q^N_{1,l} = 1 - q^I_{1,l} - q^I_{2,l})$, $f_{1,e} = (z_5, p_e; z_6, 1 - p_e)$ and $f_{1,l} = (z_3, p_l; z_4, 1 - p_l)$.

observes the resolution of $f^N_{j,e}$. A lottery $f^N_{j,e}$ (henceforth 'early unresolved lottery') is a terminal node, in the sense that the agent does not expect it to lead to a second stage. Now suppose that the first (early) stage lottery leads to a second (late) stage lottery $X_i = (z_1, q^I_{1,l}; z_2, q^I_{2,l}; ...; z_n, q^I_{n_l,l};\ f_{1,l}, q^N_{1,l}; f_{2,l}, q^N_{2,l}; ...; f_{m,l}, q^N_{m,l})$. This second lottery always resolves. With probability $q^I_{h,l}$, the agent receives a final outcome $z^I_{h,l}$, which he observes. With probability $q^N_{k,l}$, he receives a lottery $f_{k,l} \in \mathfrak{L}_\mathfrak{o}$ which never resolves (henceforth 'late unresolved lottery'). The difference between a lottery f_e and a lottery f_l is that the agent knows after the early stage that he receives lottery f_e which does not resolve, while he doesn't find out until the *late* stage that he receives lottery f_l which does not resolve. As before, the q^I's and q^N's are used to distinguish between the probabilities that lead to prizes where he is fully informed of the outcome (since he directly observes which z occurs), and the probabilities that lead to prizes where he is *not* informed (since he only observes the ensuing lottery).

Suppose now that an continuity axiom and an independence axiom for unresolved lottery holds at every stage. That is, define $\succeq_{N,e}$ and $\succeq_{N,l}$ in the natural way, and let continuity and independence axioms hold for each of these preferences. In this case, there are unresolved utility functions v_e, v_l associated with $\succeq_{N,e}$ and $\succeq_{N,l}$, respectively:

$$\mathbf{W}(\mathbf{X}) = \sum q^I(z)u(z) + \sum q^I_{i,e}(z)\left(\sum q^N_{i,l}u\left(v_l^{-1}\left(Ev_l(z|f_{i,l})\right)\right)\right) + \sum q^N_{i,e}u\left(v_e^{-1}\left(Ev_e(z|f_{i,e})\right)\right)$$

Note that v_e and v_l need not be the same, since $\succeq_{N,e}$ and $\succeq_{N,l}$ are separate. Hence, there are three utility functions in this setting: utility u is associated with lotteries that eventually resolve, while functions v_e and v_l are associated with early and late unresolved lotteries. It is immediate, therefore, that having a KP model that accommodates unresolved lotteries is formally distinct from simply adding a 'never' stage, as this can only account for one additional utility function. The reason for this distinction is that the agent's perception of the unresolved lotteries need not be the same in the early stage as it is in the second stage.

There is another, and perhaps more fundamental, difference between temporal resolution and lack of resolution. While the early stage leads to the eventual occurrence of the late stage, there is no notion of sequence for unresolved lotteries. That is, the first unresolved lottery cannot lead to a second lottery; each unresolved lottery is a final prize, and hence a terminal node. For that reason, while the KP representation will have terms such as $u_e(u_l^{-1}(\cdot))$, there cannot be an equivalent unresolved term, $v_e(v_l^{-1}(\cdot))$. In this representation, both utility functions v_e and v_l are terminal, in the sense that the expectations are over outcomes, and not over any further lotteries. While the notation is cumbersome, this representation demonstrates that each unresolved lottery is essentially a final prize, and its value depends on whether it is obtained early or late. The agent's preferences over unresolved lotteries are allowed to vary in time, even when he has neutral preferences over the timing of resolution of uncertainty. The distinction between the KP representation and a representation that takes into account preferences for unresolved lotteries holds if the independence axioms over $\succeq_{N,e}$ and $\succeq_{N,l}$ are relaxed. In other words, this distinction carries through to more general REU representations.

6 Closing remarks

This paper provides a representation theorem for preferences over lotteries whose outcomes may never be observed. The agent's perception of the unobserved outcome, relative to his risk-aversion, induces his attitude towards doubt. This relation is captured by his resolved utility function u, his unresolved utility function v and his unresolved decision weighting function w. The model presented here is an extension of the vNM framework, and it does not entail a significant axiomatic departure. However, it can accommodate behavioral patterns that are inconsistent with expected utility, and that have motivated a wide array of different frameworks. For instance, doubt-prone individuals have an incentive to self-handicap, and this incentive is higher if they are less certain about their competence.[24] Doubt-prone individuals are also more likely to choose the status quo bundle, if making a decision is more informative than inaction. In addition, an agent who is risk-neutral may still favor less risky investments, and would pay a firm to invest for him, even if it does not have superior expertise. The agent's attempt to preserve his self-image implies that his utility depends not only on the outcome that results, but also on the action taken. In a political economy context, doubt-proneness encourages political ignorance. When individuals derive more utility from the policies that they are not required to observe, they have *less* incentive to acquire information. Moreover, agents have a greater disutility from acquiring information if they are more ignorant ex-ante.

Finally, note that experiments that address the impact of anticipated regret frequently allow for foregone outcomes that individuals do not observe (see Zeelenberg (1999)). Similarly, in experiments by Dana, Weber and Kuang (2007), subjects deliberately choose to ignore free information concerning the full consequences of their actions. These empirical findings would be useful in determining plausible degrees of doubt-proneness, although this is outside the scope of this paper.

[24] Recall that this model does not allow agents to be delusional, since they are unable to mislead themselves into having false beliefs.

Appendix

The appendix is structured as follows. Part 1 explains why the standard EU model is inappropriate when the agent does not expect to observe the resolution of uncertainty. Part 2 provides an example of the 'preservation of self-image' application. All the proofs are in part 3.

A.1 Limitations of the standard EU model

This example illustrates the problem with using the standard vNM EU model when there are outcomes that the agent never expects to observe. Consider the simple case of an agent who has performed a task and does not know how well he has done. There are no future decisions that depend on his performance. For example, as a simple adaptation of Savage's omelet, suppose that the agent does not know whether he has fed his guests a good omelet or a bad one. With probability p_t, he has done well ($\bar{t}$), and with probability $(1-p_t)$ he has done badly ($\underline{t}$). He prefers having done well to having done badly, although this will have no future repercussions. Given the choice between remaining forever in doubt (D) and perfectly resolving the uncertainty, (ND), it might appear that he compares:

$$U_D = p_t u(\bar{t}) + (1-p_t)u(\underline{t})$$

to

$$U_{ND} = p_t u(\bar{t}) + (1-p_t)u(\underline{t})$$

and that since $U_D = U_{ND}$, he is indifferent. But U_D is *not* necessarily the right function to use if he chooses to remain in doubt, because from his frame of reference the final outcome will not be $\bar{t}$ or $\underline{t}$. That is, he does not expect to 'obtain' ex-post utility $u(\bar{t})$ or $u(\underline{t})$ because he does not expect to observe either $\bar{t}$ or $\underline{t}$. As it is not clear what his perception of the consequence is if he does not expect the uncertainty to be resolved (from his viewpoint), his expected utility is undetermined. In its current form, the standard EU model does not offer a method for evaluating this choice. Using U_D effectively ignores that the relevant frame of reference is the agent's, not the modeler's.[25]

Redefining the outcome space to include the observation itself does not eliminate the problem. Suppose that the outcome space is taken to be $Z = \{\bar{t}_D, \underline{t}_D, \bar{t}_{ND}, \underline{t}_{ND}\}$ where $\bar{t}_D$ represents the outcome that he did well but doubts it, $\bar{t}_{ND}$ that he did well and does not doubt it, and so

[25]This issue is not resolved by starting with preferences over lotteries as primitives. In the standard framework, the agent has primitive preferences over lotteries over outcomes, and he is not allowed to choose between lotteries whose resolution he observes and lotteries whose resolution he does not observe. He is therefore not given the option to express those preferences.

forth. He therefore compares the following:

$$U_D = p_t u(\bar{t}_D) + (1 - p_t) u(\underline{t}_D)$$

to

$$U_{ND} = p_t u(\bar{t}_{ND}) + (1 - p_t) u(\underline{t}_{ND})$$

It is difficult to interpret the meaning of the consequence ‘did well, but doubts it’ from his frame of reference, since it is not clear what it means to be in doubt if he knows that he has done well. In addition, his preferences over $\bar{t}_D$ and $\underline{t}_D$ are completely pinned down. Consider the two extremes, $p_t = 1$ and $p_t = 0$. When $p_t = 1$, there is no intrinsic difference between U_D and U_{ND}, since he knows that he has done well. Hence, $u(\bar{t}_D) = u(\bar{t}_{ND})$. Similarly, when $p_t = 0$, he knows he has done badly, and so $u(\underline{t}_D) = u(\underline{t}_{ND})$. It then follows that $U_D = U_{ND}$ for any $p_t \in [0, 1]$. This definition of the outcome space is essentially the same as simply $Z = \{\bar{t}, \underline{t}\}$. His indifference between remaining in doubt and not remaining in doubt is a consequence of following this approach, it is not implicit from the standard EU model.
Redefining the outcome space so that his utility is constant if he remains in doubt is even more problematic. Suppose that $Z = \{\bar{t}_{ND}, \underline{t}_{ND}, D\}$, letting $\bar{t}_{ND}$ be the outcome ‘talented and he does not remain in doubt (he observes the outcome)’, $\underline{T}_{ND}$ be the outcome ‘untalented and he observes it’, and letting D mean that he does not observe the outcome, hence remaining in doubt. He now compares:

$$U_D = u(D)$$

to

$$U_{ND} = p_t u(\bar{t}_{ND}) + (1 - p_t) u(\underline{t}_{ND})$$

However, in the limit $p_t \to 1$, U_D should approach U_{ND}, which only occurs if $u(D) = u(\bar{t}_{ND})$. But in that case, as $p_t \to 0$, U_D does not approach U_{ND}, and so there is an unavoidable discontinuity.

A.2 Applications

Numerical Example (Preservation of Self-image)

The following is a more general version of the numerical example provided in the main body of the paper. Suppose he puts in effort $e \in [0, 1]$, and obtains reward $m \in [0, 100]$. He also has an unobserved talent $t \in [0, 1]$. The agent is doubt-prone and risk-averse for both resolved and unresolved lotteries on talent. Specifically, $u = at^{1/2}$ for some $a > 0$, and v = t. His expected utility of money is linearly separable from his utility of talent, and is equal to his expected reward Em. He therefore maximizes:

$$\tilde{W}(e) = Em(e) - C(e)$$

where $C(e) \equiv u\left(v^{-1}(Ev(t))\right) - \sum_m p(m|e)u \circ v^{-1}(Ev(t|m,e))$

The agent's prior is q that talent $t = 0$, and $1-q$ that talent $t = 1$. He can put in level $e \in [\underline{e}, \overline{e}]$. Given that he has talent $t = 1$ or $t = 0$ and puts in effort e, his respective probabilities of obtaining monetary reward $m = 100$ are $p(100|t = 1, e) = e$ and $p(100|t = 0, e) = be$, for $b \in [0, 1)$.

Note that the ostrich effort e_0 in this example is $e = 0$, since he is certain to obtain $m = 0$, independently of his talent. It follows from the probabilities given above that:

$$p(\$0|1, e) = 1 - e$$

$$p(\$0|0, e) = 1 - be$$

$$p(100|e) = e(q + b(1 - q)$$

$$p(\$0|e) = 1 - e(q + b(1 - q))$$

$$p(1|100, e) = \frac{q}{q + b(1 - q)}$$

Solving:

$W(e) = 100 * p(100|e) + a\left(p(0|e)p(\bar{t})p(0|\bar{t}, e)\right)^{1/2} + a\left(p(100|e)p(\bar{t})p(100|\bar{t}, e)\right)^{1/2}$

$= e(100\beta + a(\beta q)^{1/2}) + aq^{1/2}\left(1 - e(1 + \beta) + \beta e^2\right)^{1/2}$

where $\beta = q + b(1 - q)$. Let $\gamma = 100\beta + a(\beta q)^{1/2}$, and $D = \frac{4\gamma^2}{a^2 q}$. Then, from the first order conditions, we obtain:

$e^2(\beta C - 4\beta^2) + e(4\beta - C)(1 + \beta) + C - (1 + \beta)^2 = 0$

The example in the text corresponds to the case $b = 0$, $q = 1/2$, and so $\beta = 1/2$, $\gamma = 50 + \frac{a}{2}$, and $d = 2D = \left(\frac{200}{a} + 2\right)^2$.

A.3 Proofs

Representation Theorem. *Proof.* Let $X = (z_1, q_1^I; z_2, q_2^I; ...; z_n, q_n^I;\ f_1, q_1^N; f_2, q_2^N; ...; f_m, q_m^N)$. By continuity, there exists a function $H : \mathfrak{L}_0 \to \mathbf{Z}$ such that $\delta_{H(f)} \sim_N f$ (i.e. $\delta_{\delta_{H(f)}} \sim \delta_f$) for all $f \in \mathfrak{L}_0$. By the certainty axiom **A.3**, it follows that $\delta_{H(f)} \sim \delta_{\delta_{H(f)}}$. Hence $\delta_{H(f)} \sim \delta_f$ for any $f \in \mathfrak{L}_0$. By a well-known implication of the independence axiom **A.4**, $X \sim \tilde{X}$, where $\tilde{X} = (z_1, q_1^I; z_2, q_2^I; ...; z_n, q_n^I;\ H(f_1), q_1^N; H(f_2), q_2^N; ...; H(f_m), q_m^N)$, and so $X \sim \tilde{X}$. Defining $\tilde{Y}$ similarly, $Y \sim \tilde{Y}$. By transitivity, $X \succ Y \Rightarrow \tilde{X} \succ \tilde{Y}$. Note that all lotteries $\tilde{X}$ and $\tilde{Y}$ are one-stage lotteries, with final outcomes as prizes. Define the preference relation $\succ_I$ in the following way: $X \succ Y \Rightarrow \tilde{X} \succ_I \tilde{Y}$. All the EU axioms hold on $\succ_I$, and so $\tilde{X} \succ \tilde{Y}$ if and only if $W(\tilde{X}) > W(\tilde{Y})$, where

$$W(\tilde{X}) = \sum_{i=1}^{n} q_i^I u(z_i) + \sum_{i=1}^{m} q_i^N u\left(H(f_{z_i})\right)$$

and W is unique up to positive affine transformation. But since $X \succ Y \Rightarrow \tilde{X} \succ \tilde{Y}$, it follows that $X \succ Y$ if and only if $W(\tilde{X}) > W(\tilde{Y})$.

To obtain the representation of H: axioms **A.1-A.4** and axiom **N.1** imply that $\succeq_N$ is a weak order and that Jensen-continuity holds. The proof for the RDU representation of $\succeq_N$ then follows from Wakker (1994). Then, for any $f \in \mathfrak{L}_0$, we have shown that $\delta_{H(f)} \sim_N f$. Since $w(1) = 1$, it follows that $v(H(f)) = v^{-1}\left(V_{RDU}(f)\right)$, and hence $H(f) = v^{-1}\left(V_{RDU}(f)\right)$, which completes the proof.

Theorem 2. *Proof.* Case 1 is shown below, and case 2 can be proven in a similar way (by changing all the signs). Suppose RDU holds for $\succeq_N$.
There are two cases two consider:

(a) f, f' have more than 2 elements:
Let $f = (z_1, p_1; ...z_i; p_i; z_{i+1}, p_{i+1}; ...; z_n, p_n)$, $f' = (z_1, p_1; ...z_i'; p_i; z_{i+1}', p_{i+1}; ...; z_n, p_n) \in \mathfrak{L}_0$ such that $f \sim_N f'$, and $(z_i', z_{i+1}') \subset (z_i, z_{i+1})$. Suppose that, for some $a \in (0,1)$ and some $z \in (z_i', z_{i+1}')$,

$$af + (1-a)\delta_z \succeq_N af' + (1-a)\delta_z$$

Since RDU holds:

$$f \sim_N f' \Rightarrow V_{RDU}(f) = V_{RDU}(f')$$

$$\Rightarrow v(z_1) + \sum_{j=2}^{i-1} w(p_j^*)[v(z_j) - v(z_{j-1})] + w(p_i^*)[v(z_i) - v(z_{i-1})] + w(p_{i+1}^*)[v(z_{i+1}) - v(z_i)]$$

$$+w(p_{i+2}^*)[v(z_{i+2}) - v(z_{i+1})] + \sum_{j=i+3}^{n} w(p_j^*)[v(z_j) - v(z_{j-1})] \quad =$$

$$v(z_1) + \sum_{j=2}^{i-1} w(p_j^*)[v(z_j) - v(z_{j-1})] + w(p_i^*)[v(z_i') - v(z_{i-1})] + w(p_{i+1}^*)[v(z_{i+1}') - v(z_i')]$$

$$+w(p_{i+2}^*)[v(z_{i+2}) - v(z_{i+1}')] + \sum_{j=i+3}^{n} w(p_j^*)[v(z_j) - v(z_{j-1})]$$

$$\Rightarrow \frac{w(p_{i+1}^*) - w(p_{i+2}^*)}{w(p_i^*) - w(p_{i+1}^*)} = \frac{v(z_i') - v(z_i)}{v(z_{i+1}) - v(z_{i+1}')} \tag{1}$$

Note that $af+(1-a)\delta_z = (z_1, ap_1; ...z_i; ap_i; z, 1-a; z_{i+1}, ap_{i+1}; ...; z_n, ap_n)$, where the ranking of z is due to $z \in (z'_i, z'_{i+1}) \subset (z_i, z_{i+1})$. Similarly, $af'+(1-a)\delta_z = (z_1, ap_1; ...z'_i; ap_i; z, 1-a; z'_{i+1}, ap_{i+1}; ...; z_n, ap_n)$. Using the condition

$$af + (1-a)\delta_z \succeq_N af' + (1-a)\delta_z$$

it follows that

$$\begin{aligned}\Rightarrow v(z_1) + \sum_{j=2}^{i-1} w(ap_j^* + 1 - a)[v(z_j) - v(z_{j-1})] + w(ap_i^* + 1 - a)[v(z_i) - v(z_{i-1})] \\ +w(ap_{i+1}^* + 1 - a)[v(z) - v(z_i)] + w(ap_{i+1}^*)[v(z_{i+1}) - v(z)] \\ +w(ap_{i+2}^*)[v(z_{i+2}) - v(z_{i+1})] + \sum_{j=i+3}^{n} w(ap_j^*)[v(z_j) - v(z_{j-1})] \quad \geq \\ v(z_1) + \sum_{j=2}^{i-1} w(ap_j^* + 1 - a)[v(z_j) - v(z_{j-1})] + w(ap_i^* + 1 - a)[v(z'_i) - v(z_{i-1})] \\ +w(ap_{i+1}^* + 1 - a)[v(z) - v(z'_i)] + w(ap_{i+1}^*)[v(z'_{i+1}) - v(z)] \\ +w(ap_{i+2}^*)[v(z_{i+2}) - v(z'_{i+1})] + \sum_{j=i+3}^{n} w(ap_j^*)[v(z_j) - v(z_{j-1})]\end{aligned}$$

$$\Rightarrow \frac{w(ap_{i+1}^*) - w(ap_{i+2}^*)}{w(ap_i^* + 1 - a) - w(ap_{i+1}^* + 1 - a)} \geq \frac{v(z'_i) - v(z_i)}{v(z_{i+1}) - v(z'_{i+1})} \tag{2}$$

Combining (1) and (2), we obtain:

$$\frac{w(ap_{i+1}^*) - w(ap_{i+2}^*)}{w(ap_i^* + 1 - a) - w(ap_{i+1}^* + 1 - a)} \geq \frac{w(p_{i+1}^*) - w(p_{i+2}^*)}{w(p_i^*) - w(p_{i+1}^*)} \tag{3}$$

Note that this does not depend on the utility function v, but only on the weighting function w. Take any $\tilde{f} = (\tilde{z}_1, p_1; ...\tilde{z}_i; p_i; \tilde{z}_{i+1}, p_{i+1}; ...; \tilde{z}_n, p_n)$, $\tilde{f}' = (\tilde{z}_1, p_1; ...\tilde{z}'_i; p_i; \tilde{z}'_{i+1}, p_{i+1}; ...; \tilde{z}_n, p_n)$ and $\tilde{z}$ such that $\tilde{z} \in (\tilde{z}'_i, \tilde{z}'_{i+1}) \subset (\tilde{z}_i, \tilde{z}_{i+1})$. It must be that $a\tilde{f}+(1-a)\delta_{\tilde{z}} \succeq_N a\tilde{f}'+(1-a)\delta_{\tilde{z}}$. Suppose not, i.e. suppose that $a\tilde{f}' + (1-a)\delta_{\tilde{z}} \succ_N a\tilde{f} + (1-a)\delta_{\tilde{z}}$. Then, redoing a similar calculation to the one above, we obtain:

$$\frac{w(ap_{i+1}^*) - w(ap_{i+2}^*)}{w(ap_i^* + 1 - a) - w(ap_{i+1}^* + 1 - a)} < \frac{w(p_{i+1}^*) - w(p_{i+2}^*)}{w(p_i^*) - w(p_{i+1}^*)} \tag{4}$$

which contradicts (3). Hence ISC holds for this case.

(b) f, f' have exactly 2 elements:
Let $f = (z_1, 1-p; z_2, p)$, $f' = (z'_1, 1-p; z'_2, p) \in \mathfrak{L}_o$ such that $f \sim_N f'$, and $(z'_1, z'_2) \subset (z_1, z_2)$.

Suppose that, for some $a \in (0,1)$ and some $z \in (z'_1, z'_2)$. If $\succeq_N$ satisfies RDU, then:

$$f \sim_N f' \Rightarrow v(z_1) + w(p)[v(z_2) - v(z_1)] = v(z'_1) + w(p)[v(z'_2) - v(z'_1)]$$

$$\Rightarrow w(p) = \frac{v(z'_1) - v(z_1)}{[v(z'_1) - v(z_1)] + [v(z_2) - v(z'_2)]}$$

$$\Rightarrow \frac{w(p)}{1 - w(p)} = \frac{v(z'_1) - v(z_1)}{v(z_2) - v(z'_2)} \tag{5}$$

Since $af + (1-a)\delta_z = ((z_1, a(1-p); z, 1-a; z_2, ap)$ and $af' + (1-a)\delta_z = ((z'_1, a(1-p); z, 1-a; z'_2, ap)$, the condition $af + (1-a)\delta_z \succ_N af' + (1-a)\delta_z$ implies (using a similar calculation to the one used for obtaining (3)) that

$$\Rightarrow \frac{w(ap)}{1 - w(ap + 1 - a)} \geq \frac{v(z'_1) - v(z_1)}{v(z_2) - v(z'_2)} \tag{6}$$

and combining (4) and (5), it follows that

$$\Rightarrow \frac{w(ap)}{1 - w(ap + 1 - a)} \geq \frac{w(p)}{1 - w(p)} \tag{7}$$

As before, this does not depend on the $v's$, but only on the weighting function w. Take any $\tilde{f} = (\tilde{z}_1, 1-p; \tilde{z}_2, p)$, $\tilde{f}' = (\tilde{z}'_1, p_1; \tilde{z}'_2, p_2)$ and $\tilde{z}$ such that $\tilde{z} \in (\tilde{z}'_1, \tilde{z}'_2) \subset (\tilde{z}_1, \tilde{z}_2)$. It must be that $a\tilde{f} + (1-a)\delta_{\tilde{z}} \succeq_N a\tilde{f}' + (1-a)\delta_{\tilde{z}}$. Suppose not, i.e. suppose that $a\tilde{f}' + (1-a)\delta_{\tilde{z}} \succ_N a\tilde{f} + (1-a)\delta_{\tilde{z}}$. Then, redoing a similar calculation to the one above, we obtain:

$$\Rightarrow \frac{w(ap)}{1 - w(ap + 1 - a)} < \frac{w(p)}{1 - w(p)} \tag{8}$$

which contradicts (7). Hence ISC holds for this case as well, which completes the proof.

The following lemma is used in the proof of theorem 3:

Lemma 1. *Let $w : [0,1] \to [0,1]$. Take any $p, q, p', p' \in [\underline{p}, \overline{p}] \subseteq [0,1]$ such that $p > p' > q'$, $q > q'$. Then if w is concave on $[\underline{p}, \overline{p}]$:*

$$\frac{w(p) - w(q)}{p - q} \leq \frac{w(p') - w(q')}{p' - q'}$$

if w is convex on $[\underline{p}, \overline{p}]$:

$$\frac{w(p) - w(q)}{p - q} \geq \frac{w(p') - w(q')}{p' - q'}$$

Proof. The proof is only shown for a concave function w. We make use of the following well-known result that a function f is concave if and only if for any $\tilde{p} > \tilde{q} > \tilde{r}$,

$$\frac{f(\tilde{p}) - f(\tilde{q})}{\tilde{p} - \tilde{q}} \leq \frac{f(\tilde{p}) - f(\tilde{r})}{\tilde{p} - \tilde{r}} \leq \frac{f(\tilde{q}) - f(\tilde{r})}{\tilde{q} - \tilde{r}} \tag{9}$$

We now directly prove the claim for each of the three possible cases:

(i) $p > q > p' > q'$
Using (9) twice,

$$\frac{w(p) - w(q)}{p - q} \leq \frac{w(q) - w(p')}{q - p'} \leq \frac{w(p') - w(q')}{p' - q'}$$

(ii) $p > p' > q > q'$
Using (9) twice,

$$\frac{w(p) - w(q)}{p - q} \leq \frac{w(p') - w(q)}{p' - q} \leq \frac{w(p') - w(q')}{p' - q'}$$

(iii) $p > p' = q > q'$
In this case, the result follows immediately from (9):

$$\frac{w(p) - w(q)}{p - q} \leq \frac{w(q) - w(q')}{q - q'} = \frac{w(p') - w(q')}{p' - q'}$$

which completes the proof.

Theorem 3. *Proof.* Suppose that $\succeq_N$ satisfies RDU. We first show (A) that the weighting function w is concave implies that for any $f = (z_1, p_1; ...z_i; p_i; z_{i+1}, p_{i+1}; ...; z_n, p_n)$, $f' = (z_1, p_1; ...z'_i; p_i; z'_{i+1}, p_{i+1}; ...; z_n, p_n) \in \mathfrak{L}_0$ such that $f \sim_N f'$, and $(z'_i, z'_{i+1}) \subset (z_i, z_{i+1})$, and for all $a \in (0, 1)$ and $z \in (z_i, z_{i+1})$,

$$af + (1 - a)\delta_z \succeq_N af' + (1 - a)\delta_z$$

We then prove the converse (B).
Proof of (A) Suppose that the weighting function w is concave. We proceed by contradiction. There are two cases to consider:

(a) f, f' have more than two elements: Let $f = (z_1, p_1; ...z_i; p_i; z_{i+1}, p_{i+1}; ...; z_n, p_n)$, $f' = (z_1, p_1; ...z'_i; p_i; z'_{i+1}, p_{i+1}; ...; z_n, p_n) \in \mathfrak{L}_0$ such that $f \sim_N f'$, and $(z'_i, z'_{i+1}) \subset (z_i, z_{i+1})$. Suppose there exists some $a \in (0, 1)$ and some $z \in (z_i, z_{i+1})$ such that $af' + (1 - a)\delta_z \succ_N$

$af + (1-a)\delta_z$. Using the derivation of theorem 3, it follows that

$$\frac{w(ap^*_{i+1}) - w(ap^*_{i+2})}{w(ap^*_i + 1 - a) - w(ap^*_{i+1} + 1 - a)} < \frac{w(p^*_{i+1}) - w(p^*_{i+2})}{w(p^*_i) - w(p^*_{i+1})} \tag{10}$$

We now show:

(I) $w(ap^*_{i+1}) - w(ap^*_{i+2}) \geq a\left(w(p^*_{i+1}) - w(p^*_{i+2})\right)$
Note that $p^*_{i+1} > p^*_{i+2} > ap^*_{i+2}$, since $a \in (0,1)$, and using the definition of p^*. It is immediate that $ap^*_{i+1} > ap^*_{i+2}$. It follows, therefore, from lemma 1, that:

$$\frac{w(p^*_{i+1}) - w(p^*_{i+2})}{p^*_{i+1} - p^*_{i+2}} \leq \frac{w(ap^*_{i+1}) - w(ap^*_{i+2})}{ap^*_{i+1} - ap^*_{i+2}}$$

Rearranging, we obtain $w(ap^*_{i+1}) - w(ap^*_{i+2}) \geq a\left(w(p^*_{i+1}) - w(p^*_{i+2})\right)$.

(II) $w(ap^*_i + 1 - a) - w(ap^*_{i+1} + 1 - a) \leq a\left(w(p^*_i) - w(p^*_{i+1})\right)$
Note that $ap^*_i + 1 - a > p^*_i$, since $a, p^*_i \in (0,1)$ implies that $1 - a > p^*_i(1-a)$. Similarly, $ap^*_{i+1} + 1 - a > p^*_{i+1}$, and we know that $p^*_i > p^*_{i+1}$. Using lemma 1, it follows that:

$$\frac{w(ap^*_i + 1 - a) - w(ap^*_{i+1} + 1 - a)}{(ap^*_i + 1 - a) - \left(ap^*_{i+1} + 1 - a\right)} \leq \frac{w(p^*_i) - w(p^*_{i+1})}{p^*_i - p^*_{i+1}}$$

Rearranging, we obtain $w(ap^*_i + 1 - a) - w(ap^*_{i+1} + 1 - a) \leq a\left(w(p^*_i) - w(p^*_{i+1})\right)$

Combining (I) and (II) (noting that both sides of (II) are greater than zero), it follows that

$$\frac{w(ap^*_{i+1}) - w(ap^*_{i+2})}{w(ap^*_i + 1 - a) - w(ap^*_{i+1} + 1 - a)} \geq \frac{w(p^*_{i+1}) - w(p^*_{i+2})}{w(p^*_i) - w(p^*_{i+1})} \tag{11}$$

which is a contradiction of (10).

(b) f, f' have exactly 2 elements:
Let $f = (z_1, 1-p; z_2, p)$, $f' = (z'_1, 1-p; z'_2, p) \in \mathfrak{L}_o$ such that $f \sim_N f'$, and $(z'_1, z'_2) \subset (z_1, z_2)$. Suppose there exists some $a \in (0,1)$ and some $z \in (z_1, z_2)$ such that $af' + (1-a)\delta_z \succ_N af + (1-a)\delta_z$. Using the derivation of theorem 3, it follows that

$$\frac{w(ap)}{1 - w(ap + 1 - a)} < \frac{w(p)}{1 - w(p)} \tag{12}$$

We now show:

(I) $w(ap) \geq aw(p)$
$a \in (0,1)$ and so $p > ap > 0$. It follows from the well-known result (9) used in

proving lemma 1 that:

$$\frac{w(p)-w(0)}{p} \leq \frac{w(ap)-w(0)}{ap-0}$$

Using $w(0)=0$ and rearranging, we obtain $w(ap) \geq aw(p)$

(II) $1-w(ap+1-a) \leq a\,(1-w(p))$

Note that $1 > ap+1-a > p$, since it is immediate from $a, p \in (0,1)$ that $a > ap$ and $1-a > p(1-a)$.

Using (9) again,

$$\frac{w(1)-w(ap+1-a)}{1-(ap+1-a)} \leq \frac{w(1)-w(p)}{1-p}$$

Using $w(1)=1$ and rearranging, we obtain that $1-w(ap+1-a) \leq a\,(1-w(p))$.

Combining (I) and (II), we obtain

$$\frac{w(ap)}{1-w(ap+1-a)} \geq \frac{w(p)}{1-w(p)} \tag{13}$$

which contradicts (12).

Proof of (B) Suppose that for any $f = (z_1, p_1; ...z_i; p_i; z_{i+1}, p_{i+1}; ...; z_n, p_n)$, $f' = (z_1, p_1; ...z_i'; p_i; z_{i+1}', p_{i+1}; ...; z_n, p_n) \in \mathfrak{L}_0$ such that $f \sim_N f'$, and $(z_i', z_{i+1}') \subset (z_i, z_{i+1})$, and for all $a \in (0,1)$ and $z \in (z_i, z_{i+1})$,

$$af + (1-a)\delta_z \succeq_N af' + (1-a)\delta_z$$

We proceed as follows: (a) we first show that there is no interval $[\underline{p}, \overline{p}] \subseteq [0,1]$ on which w is strictly convex; (b) we then show that there is no interval $[\underline{p}, \overline{p}] \subseteq [0,1]$ such that for all $p \in [\underline{p}, \overline{p}]$, $w(p)$ is 'under the diagonal', i.e. $\frac{w(\overline{p})-w(p)}{\overline{p}-p} > \frac{w(\overline{p})-w(\underline{p})}{\overline{p}-\underline{p}} > \frac{w(p)-w(\underline{p})}{p-\underline{p}}$ (note that with stronger smoothness assumptions this would be sufficient for concavity); (c) we use results (a) and (b) to prove that w must be concave. We first note that it follows from the claim and from the derivation of theorem 3 that:

$$\frac{w(ap_1)-w(ap_2)}{w(ap_0+1-a)-w(ap_1+1-a)} \geq \frac{w(p_1)-w(p_2)}{w(p_0)-w(p_1)} \tag{14}$$

for all $0 \leq p_2 < p_1 < p_0 \leq 1$ and $a \in (0,1)$.

(a) We proceed by contradiction: suppose there does exist an interval $[\underline{p}, \overline{p}] \subseteq [0,1]$ on which w is strictly convex. Let $\underline{p} < p_2 < p_1 < p_0 < \overline{p}$, and let $\{\frac{\underline{p}}{p_2}, \frac{1-\overline{p}}{1-p_0}\} < a < 1$. It follows that $\underline{p} < ap_2 < ap_1 < ap_1 + 1 - a < ap_0 + 1 - a)\overline{p}$. Using lemma 1, it follows that:

$$\frac{w(p_1)-w(p_2)}{p_1-p_2} > \frac{w(ap_1)-w(ap_2)}{ap_1-ap_2} \tag{15}$$

$$\frac{w(ap_0+1-a)-w(ap_1+1-a)}{(ap_0+1-a)-(ap_1+1-a)} > \frac{w(p_0)-w(p_1)}{p_0-p_1} \tag{16}$$

Rearranging and combining (15) and (16), it follows that

$$\frac{w(ap_1)-w(ap_2)}{w(ap_0+1-a)-w(ap_1+1-a)} < \frac{w(p_1)-w(p_2)}{w(p_0)-w(p_1)}$$

which contradicts (14).

(b) We proceed again by contradiction: suppose that there does exist an interval $[\underline{p},\overline{p}] \subseteq [0,1]$ such that $\frac{w(\overline{p})-w(p)}{\overline{p}-p} > \frac{w(\overline{p})-w(\underline{p})}{\overline{p}-\underline{p}} > \frac{w(p)-w(\underline{p})}{p-\underline{p}}$ for all $p \in [\underline{p},\overline{p}]$.
Let $a = 1-(\overline{p}-\underline{p})+\epsilon$, for an arbitrarily small ϵ. Let $\tilde{p} = \underline{p}/a$. Using result (a), $[\tilde{p},\tilde{p}+\delta]$ cannot be strictly convex, for any $\delta \in (0,1-\tilde{p}]$. We can therefore find $\{p_0,p_1,p_2\} \in [\tilde{p},\tilde{p}+\delta]$ such that $p_2 < p_1 < p_0$ and

$$\frac{w(p_1)-w(p_2)}{p_1-p_2} \geq \frac{w(p_0)-w(p_1)}{p_0-p_1} \tag{17}$$

As δ,ϵ become arbitrarily small (and $a\delta \leq \epsilon$), $ap_2 \to \underline{p}$, $ap_0+1-a \to \overline{p}$ and $\{ap_2, ap_1, ap_1+1-a, ap_0+1-a\} \in [\underline{p},\overline{p}]$. We therefore have that for small enough δ,ϵ,

$$\frac{w(ap_0+1-a)-w(ap_1+1-a)}{(ap_0+1-a)-(ap_1+1-a)} > \frac{w(\overline{p})-w(\underline{p})}{\overline{p}-\underline{p}} \tag{18}$$

and

$$\frac{w(\overline{p})-w(\underline{p})}{\overline{p}-\underline{p}} > \frac{w(ap_1)-w(ap_2)}{a(p_1-p_2)} \tag{19}$$

Combining (18) and (19):

$$\frac{w(ap_1)-w(ap_2)}{w(ap_0+1-a)-w(ap_1+1-a)} < \frac{p_1-p_2}{p_0-p_1} \tag{20}$$

Combining (17) and (20), we obtain:

$$\frac{w(ap_1)-w(ap_2)}{w(ap_0+1-a)-w(ap_1+1-a)} < \frac{w(p_1)-w(p_2)}{w(p_0)-w(p_1)}$$

which contradicts (14).

(c) We now prove that w is concave. Suppose not, i.e. suppose there exist $0 \leq p < q < r < 1$ such that

$$\frac{w(r)-w(q)}{r-q} > \frac{w(q)-w(p)}{q-p} \tag{21}$$

Let $a = 1 - (r - q) + \epsilon$, for an arbitrarily small ϵ. Let $\tilde{p} = q/a$. Using result (a), $[\tilde{p} - \delta, \tilde{p}]$ cannot be strictly convex, for any $\delta \in (0, \tilde{p}]$. We can therefore find $\{p_0, p_1, p_2\} \in [\tilde{p} - \delta, \tilde{p}]$ such that $p_2 < p_1 < p_0$ and

$$\frac{w(p_1) - w(p_2)}{p_1 - p_2} \geq \frac{w(p_0) - w(p_1)}{p_0 - p_1} \tag{22}$$

As δ, ϵ become arbitrarily small (and $a\delta \leq \epsilon$), $ap_1 \to q$, $ap_0 + 1 - a \to r$, $\{ap_2, ap_1\} \in (p, q]$ and $\{ap_1 + 1 - a, ap_0 + 1 - a\} \in [q, r]$.
Using result (b), we have can find some (small enough) δ, ϵ such that

$$\frac{w(ap_1) - w(ap_2)}{a(p_1 - p_2)} \leq \frac{w(q) - w(p)}{q - p} \tag{23}$$

$$\frac{w(ap_0 + 1 - a) - w(ap_1 + 1 - a)}{(ap_0 + 1 - a) - (ap_1 + 1 - a)} \geq \frac{w(r) - w(q)}{r - q} \tag{24}$$

Combining (21, (23) and (24) we have

$$\frac{w(ap_1) - w(ap_2)}{w(ap_0 + 1 - a) - w(ap_1 + 1 - a)} < \frac{p_1 - p_2}{p_0 - p_1} \tag{25}$$

Combining (22) and (25), we have

$$\frac{w(ap_1) - w(ap_2)}{w(ap_0 + 1 - a) - w(ap_1 + 1 - a)} < \frac{w(p_1) - w(p_2)}{w(p_0) - w(p_1)}$$

which contradicts (14), and completes the proof.

Theorem 4. *Suppose that axioms **A.1** through **A.4** and the RDU axioms hold, and let u and v be the utility functions associated with the resolved and unresolved lotteries, respectively, and w be the decision weight associated with the unresolved lotteries. In addition, suppose that u, v are both differentiable. Then:*
(i) If there exists a $p \in (0, 1)$ such that $p < w(p)$, then the agent is doubt-prone somewhere. Similarly, if there exists $p' \in (0, 1)$ such that $p' > w(p')$, then the agent is doubt-averse somewhere.
(ii) If the agent is doubt-averse everywhere, then $p \geq w(p)$ for all $p \in (0, 1)$. Moreover, if u exhibits stronger diminishing marginal utility than v (i.e. $u = \lambda \circ v$ for some continuous, weakly concave, and increasing λ on $v([\underline{z}, \bar{z}])$), then $\succeq_N$ violates quasi-concavity. (that is, there exists some $f', f'' \in \mathfrak{L}_o$, and $\alpha \in (0, 1)$ such that $f' \succ f''$ and $f'' \succ_N \alpha f' + (1 - \alpha) f''$).
If the agent is doubt-prone everywhere, then $p \leq w(p)$ for all $p \in (0, 1)$. Moreover, if v exhibits

stronger diminishing marginal utility than u , then $\succeq_N$ violates quasi-convexity. (that is, there exists some $f', f'' \in \mathfrak{L}_o$, and $\alpha \in (0,1)$ such that $f' \succ f''$ and $\alpha f' + (1-\alpha) f'' \succ_N f'$).

Proof. (i) Suppose not, i.e. suppose that there exists $p \in (0,1)$ such that $p < w(p)$, and that $f \succeq \delta_f$ for all $f \in \mathfrak{L}_o$. Let $f_\epsilon = (z; 1-p; z+\epsilon, p)$ for some $z \in \mathbf{Z}$, $p \in \mathfrak{L}_o$, $0 < \epsilon < \bar{z} - z$. Since $f \succeq \delta_f$, by continuity (and using the certainty axiom), there exists a $\tilde{z}_\epsilon \in (z, z+\epsilon)$ such that $f \succeq \left[\delta_{\tilde{z}_\epsilon} \sim \delta_{\delta_{\tilde{z}_\epsilon}}\right] \succeq \delta_f$. Hence:

$$(1-p)u(z) + pu(z+\epsilon) \geq u(\tilde{z}_\epsilon)$$

$$w(p)\left(v(z+\epsilon) - v(z)\right) + v(z) \leq v(\tilde{z}_\epsilon)$$

Rearranging:

$$p \geq \frac{u(\tilde{z}_\epsilon) - u(z)}{u(z+\epsilon) - u(z)}$$

$$w(p) \leq \frac{v(\tilde{z}_\epsilon) - v(z)}{v(z+\epsilon) - v(z)}$$

Hence:

$$\frac{u(\tilde{z}_\epsilon) - u(z)}{u(z+\epsilon) - u(z)} - \frac{v(\tilde{z}_\epsilon) - v(z)}{v(z+\epsilon) - v(z)} \leq p - w(p)$$

But as $\epsilon \to 0$, $\frac{u(\tilde{z}_\epsilon)-u(z)}{u(z+\epsilon)-u(z)} \to \frac{u'(z)}{u'(z)}$, and $\frac{v(\tilde{z}_\epsilon)-v(z)}{v(z+\epsilon)-v(z)} \to \frac{v'(z)}{v'(z)}$, by differentiability. Since the left-hand-side goes to $1-1=0$ in the limit, while the right-hand-side does not change, it must be that $0 \leq p - w(p)$. But this is a contradiction, since $p < w(p)$.
The second part of the result can be proved in a similar manner, for the case $p' > w(p')$.
(ii) The result is only shown for doubt-aversion; a similar reasoning holds for doubt-proneness. By the contrapositive of (i), it is immediate that if $f \succeq \delta_f$ for all $f \in \mathfrak{L}_o$, then $w(p) \leq p$ for all $p \in (0,1)$. Now suppose that $f \succ \delta_f$ for some f, and that u is a (weakly) concave transformation of v. If w is not concave, then $\succeq_N$ cannot be quasi-concave, by Wakker (1994) theorem 25. Since $w(0) = 0$, $w(1) = 1$, $w(p) \geq p$ for a concave function. We have that $w(p) \leq p$, and so it suffices to show that $w(p) < p$ for some p. Suppose not. That is, $w(p) = p$ for all p. Since u is more concave than v, it must be that $u^{-1}(EU(f)) \leq v^{-1}(EV(f))$(that is, the certainty equivalent of f for the informed lotteries is not bigger than the certainty equivalent of f for the unresolved lotteries, by a well known result). However, since $f \succ \delta_f$, it must also be that $u^{-1}(EU(f)) > v^{-1}(EV(f))$, which is a contradiction.
Note that if $f \sim \delta_f$ for all $f \in \mathfrak{L}_o$, than trivially, u is a linear transformation of v, and $w(p) = p$.

Corollary 4.1. *Proof.* To prove (i) $\Rightarrow$ (ii):If $\succeq_N$ displays mean-preserving risk-aversion, then $w(p)$ is convex, by Chew, Epstein and Safra (1986) or Grant, Kajii and Polak (2000). Since $w(0) = 0$, $w(1) = 1$, it must be that $p \geq w(p)$. Since $\delta_f \succeq f$, it follows from result (ii) that

$p \leq w(p)$. Hence $w(p) = p$, implying that $\succeq_N$ satisfies expected utility.
Since $\delta_f \succeq f$ for all $f \in \mathfrak{L}_o$, and both u and v are of EU form, u must be a concave transformation of v. This is well-known, see for instance Kreps-Porteus (1978).
The other direction, (ii) $\Rightarrow$ (i), is trivial: if u and v are concave then they both display mean-preserving risk aversion by well known results, and if u is a concave transformation of v then $\delta_f \succeq f$ for all $f \in \mathfrak{L}_o$.

Theorem 5.

Proof. If $u(z) = v(z)$ for all $z \in \mathbf{Z}$, then $\delta_f \succeq f$ if and only if

$$u(z_1) + \sum_{i=2}^{m} [u(z_i) - u(z_{i-1})] w(p_i^*) \geq \sum_{i=1}^{m} u(z_i) p(z_i) \tag{26}$$

$$\Leftrightarrow u(z_1) + \sum_{i=2}^{m} [u(z_i) - u(z_{i-1})] w(p_i^*) \geq u(z_1) + \sum_{i=2}^{m} [u(z_i) - u(z_{i-1})] p_i^* \tag{27}$$

$$\Leftrightarrow \sum_{i=2}^{m} [u(z_i) - u(z_{i-1})](w(p_i^*) - p_i^*) \geq 0. \tag{28}$$

This expression is always true if and only if $w(p) \geq p$ for all $p \in [0, 1]$. For the agent to be doubt-prone, the inequality in (28) must be strict somewhere, hence $w(p) > p$ for some $p \in (0, 1)$. Now suppose $u = \lambda \circ v$ for some continuous, weakly concave and increasing λ. By theorem 4, the agent is doubt-prone everywhere only if $p \leq w(p)$. Now suppose that $w(p) > p$. Then using the same argument as above, we have:

$$v(z_1) + \sum_{i=2}^{m} [v(z_i) - v(z_{i-1})] w(p_i^*) \geq \sum_{i=1}^{m} v(z_i) p(z_i). \tag{29}$$

Hence:

$$u\left(v^{-1}\left(v(z_1) + \sum_{i=2}^{m} [v(z_i) - v(z_{i-1})] w(p_i^*)\right)\right) \geq u\left(v^{-1}\left(\sum_{i=1}^{m} v(z_i) p(z_i)\right)\right). \tag{30}$$

But by concavity of $u\left(v^{-1}(\cdot)\right)$, we know that

$$u\left(v^{-1}\left(\sum_{i=1}^{m} v(z_i) p(z_i)\right)\right) \geq \sum_{i=1}^{m} u(z_i) p(z_i), \tag{31}$$

with strict inequality somewhere, hence the agent is doubt-prone everywhere. This completes the proof.

Preservation of self-image. For an agent who is doubt-prone and risk-averse for both resolved and unresolved lotteries, the following holds:

$$C(e) \equiv u \circ v^{-1}(Ev(t)) - \sum_m p(m|e) u \circ v^{-1}(Ev(t|m,e)) \geq 0$$

Proof. Note that $u \circ v^{-1}(\cdot)$ is concave. Hence

$$\begin{aligned}
&\sum_m p(m|e) u \circ v^{-1}(Ev(t|m,e)) \leq u \circ v^{-1}\left(\sum_m p(m|e)(Ev(t|m,e))\right) \\
&\leq u \circ v^{-1}\left(\sum_m p(m|e) \sum_t \frac{p(m|t,e)p(t)}{p(m|e)} v(t)\right) \leq u \circ v^{-1}\left(\sum_m \sum_t p(m|t,e)p(t)v(t)\right) \\
&\leq u \circ v^{-1}\left(\sum_t \sum_m p(m|t,e)p(t)v(t)\right) \leq u \circ v^{-1}\left(\sum_t p(t)v(t)\right) = u \circ v^{-1}(Ev(t))
\end{aligned}$$

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