Upfront Payment, Renegotiation and (Mis)coordination in Multilateral Vertical Contracting*

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Abstract

The paper analyzes the competitive effects of vertical contracts in a situation where competition exists both upstream and downstream, and both sides have balanced and differentiated bargaining power. It develops the framework of sequential bilateral negotiations between two rival manufacturers and two competing retailers with only one manufacturer negotiating with both retailers and conversely only one retailer negotiating with both manufacturers. It finds that when the supply contracts consist of three-part tariffs (i.e., upfront payments and quantity discounts) and can be renegotiated (from scratch) at any time before retail competition takes place, firms fail to maximize their total profits. The paper also shows that, while the manufacturer dealing with both retailers may use upfront fees as a tool to dampen intrabrand competition, the other dealing with one retailer only may use it as a means to compensate for the negative impact of the sales of its product on the total profits from selling its rival's one. The results contrast with those obtained when competition exists at one level only: in a similar contracting environment firms could sustain monopoly prices and, if only a single, common retailer were available, neither manufacturer would need to pay upfront.

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1 Introduction

The retailing sector, which until recently has been viewed as fully competitive, nowadays is strikingly concentrated. A notable example is the grocery sector. As reported in Dobson (2005), *"while in 1960 the UK grocery supermarkets accounted 20% of total sales, in 2002 their share increased to 89% with the top 5 – stores controlling 67% of all sales."* There is a broad consensus that, accompanying this trend towards higher concentration, retailers are gaining more market

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power over manufacturers who face less alternatives for distributing their products. Specifically, there is a large amount of evidence showing that not only retailers no longer passively accept the offers made by manufacturers but in fact are capable to negotiate more favorable contractual arrangements which include a variety of discounts as well as a number of add-on fees not related to the volume of their purchases.[1] In particular, the upfront fees have attracted much of attention and triggered heated debates both in the US and Europe. Such payments (usually per unit of time) include slotting fees to get access to (often limited) retailers' shelf space, fees for having new products introduced in a single store, or fees for staying in retailers' lists of potential suppliers. Although common in supermarket industry, they are also prevalent in other retail sectors, such as drug stores, bookstores, record stores, software stores and electronic commerce on the Internet. These payments can hardly be disregarded since in aggregate they can amount to millions of dollars annually. According to the FTC (2003) report on retail grocery industry, *"a national rollout to 85% of the stores, where 85% of these stores receive a slotting allowance, would cost $1.55 million for bread, $2.20 million for hot dogs, $1.98 million for ice cream, $0.80 million for pasta, and $1.17 million for salad dressing."*

The economic literature on slotting allowances and more generally on vertical contracting has mostly concentrated on situations where competition exists either upstream or downstream.[2] Although there are exceptions where a single manufacturer faces no close competitors or a single retailer acts as a 'gatekeeper' in a market for final consumers, it seems more reasonable to assume that every single retailer carries the products of many rival manufacturers and every single manufacturer in turn distributes its product through many (possibly competing) retailers. Furthermore, given that the business of a single manufacturer and a single retailer typically represents only a proportion of the business for each of the parties involved in relationship, it is reasonable to assume that the market power is distributed somewhat equally between them rather than concentrated on one side. This is particularly relevant for the case when manufacturers of some must-stock brands or multinational companies seek to distribute their products through large supermarket chains. Since both parties have strong market positions, they are more likely to negotiate their contracts rather than accept that one of them makes take-it-or-leave-it offers. For example, in referring to the Safeway's comments on the relationship with its main suppliers, the UK Competition Commission (2000) states:

> "Safeway said that it negotiated with suppliers in a variety of ways. At the simplest level a basic cost price was agreed. Many suppliers would then initiate or participate in volume-related discounts, which had the effect of sharing the risk. In addition, many

[1] Although it is common in the literature to relate upfront fees to retailers' increasing market power, there is a contrary view that they result from a dramatic change in manufacturers' pattern of advertising with the emphasis being now placed on trade promotions. The commentators also note that, by paying retailers upfront, manufacturers are, in effect, using them to test-market new products instead of paying for test market research. For more on these issues, see Klein and Wright (2007).

[2] A notable exception is Shaffer (1991) who studies the setting of perfectly competitive manufacturers making offers to two differentiated retailers. Restricting attention to two-part tariffs but allowing for negative fixed fees, Shaffer shows that there exists an equilibrium in which manufacturers offer wholesale prices above marginal cost and pay slotting allowances.

suppliers wanted to run or participate in promotions, and sought contributions from retailers. In other cases, Safeway might offer to give greater prominence to some products, asking in return from suppliers a contribution which reflected the benefit they would receive from the additional sales."

The present paper analyzes the competitive effects of upfront fees in a richer environment where competition exists both upstream and downstream, and both manufacturers and retailers have balanced and differentiated bargaining power. It develops the framework of bilateral negotiations taking place sequentially between two rival manufacturers and two competing retailers. It is assumed that only one manufacturer can distribute its product through both retailers, and conversely only one retailer can carry the products of both manufacturers.[3] This can occur because, say, negotiations between one manufacturer and one retailer have previously ended in a breakdown[4] (and the parties can no longer renegotiate), or one retailer have decided to delist one manufacturer's product and instead launch its own-label imitation of it, or the entry of one retailer has been initiated by the other manufacturer provided that it will not carry the product of its rival.[5] In this framework, negotiations start off between the retailer representing both manufacturers and the manufacturer supplying only this retailer. After that negotiations take place between the common retailer and the manufacturer supplying both retailers, who then negotiates with the retailer representing only this manufacturer. Building on Stole and Zwiebel (1996), it is assumed that any time a manufacturer-retailer pair fails to reach an agreement, it cannot renegotiate at another time while all the contracts signed earlier are renegotiated from scratch, preserving the same order of negotiations. This approach captures the idea that firms are free to renegotiate their contracts at any time before retail competition takes place.[6] In practice, contracts are likely to be renegotiated in case of a material change of circumstances. Letting firms rewrite their contracts according to changed market conditions is a way

[3] Relaxing this assumption, i.e., allowing each manufacturer to supply both retailers is the subject of future research.

[4] A conflict between MTS (one of the big 3 telecom operators in Russia) and Euroset (a dominant cellular retail chain in Russia) that broke out in April 2009 and caused the parties to cancel their dealership agreement is an example. Although a dispute legally arose over mutual debts, commentators say that the real issue may have been caused by the MTS's suspicion that Euroset was trying to reduce sales of MTS network connection contracts. More precisely, MTS suspected that Euroset's sales managers were recommending to the majority of potential subscribers to connect to its main rival – VimpelCom – who at that time had bought a 49.9%-stake in Euroset. Although in November 2009 the companies signed a settlement agreement whereby they offset their mutual obligations, it was not until July 2010 that MTS restarted selling its contracts via Euroset.

[5] Alternatively, it could be that a dominant retailer has enforced one or some manufacturers to sign an exclusive dealing agreement with it. For example, in the landmark case Toy R Us Inc. v. FTC (1996) it was found that Toys was in essence forcing its main suppliers not to sell to its main competitors - warehouse clubs. In particular, the agreement between Toys and the suppliers imposed that exclusive and clearance products had to be offered to Toys while only 'discontinued' products could be offered to warehouses. Toys enforced the agreement by threatening to stop buying from any producer that was reported to have 'cheated.'

[6] *"Full written agreements between the main parties and their suppliers are unusual. Day-to-day negotiations (particularly on price and quantity) are usually conducted orally . . . With computerized sales-based ordering and EDI, the relationship with suppliers was about a constant series of interactions, with volumes and delivery dates being confirmed electronically."* The UK's Competition Commission's report on supermarkets (2000).

to capture such "reactive renegotiation". By focusing on three-part tariffs,[7] the present paper aims to determine: *(i)* the price levels when the retailers carry the products of all their respective suppliers and *(ii)* whether all the trade links are established in equilibrium. The main finding of the paper is that because of competition at both levels firms can no longer maintain monopoly prices. Moreover, it can be that in equilibrium some links are necessarily missing.[8]

These results contrast sharply with those obtained when competition exists at only one level. Specifically, in a situation where a monopolistic manufacturer negotiates sequentially with two competing retailers, Bedre (2009) shows that three-part tariffs allow the firms to eliminate intrabrand rivalry and achieve the fully integrated monopoly outcome.[9] In the other polar case of a single retailer negotiating sequentially with two upstream competitors, even two-part tariffs may suffice to obtain a perfectly cooperative outcome.[10]

To gain the intuition of why in a similar contracting environment the firms fail to maximize the total profits when competition exists at both levels, start with the case where two manufacturers rely on a single, common retailer to distribute their products. In that case, neither manufacturer-retailer pair has incentives to deviate from marginal cost pricing (provided that firms are allowed to renegotiate)[11] which suffices to maintain retail prices at monopoly levels. Allowing the second manufacturer to distribute its product through another (second) retailer gives rise not only to intrabrand competition but also to interbrand competition between different retailers. The first effect gives it the incentives to behave opportunistically with respect to the common retailer while the second effect gives it the incentives to behave opportunistically with respect to its rival. As in Bedre (2009), three-part tariffs and renegotiation allow the second manufacturer to suppress intrabrand competition and coordinate the decisions of the three firms so as to maximize their joint trilateral profits. By doing so, they however fail to account for the impact of their decisions on the first manufacturer's profit and thus have incentives to free-ride on its product's margin. Since the first manufacturer can affect the incentives of only one retailer to accept a discount while its rival is tempting to undercut it in both stores, it is not able to fully eliminate the scope for free-riding on its margin. As a result, the monopoly outcome fails to exist.[12]

The paper also highlights the role of upfront fees in successive oligopolies

[7] That is, the contracts that consist of an upfront fee and a variable payment related to the volume of the trade.

[8] Note that exclusion of some firm (i.e., the situation when it fails to establish trade relationship with any partner) is also possible.

[9] Miklos-Thal et al. (2010) show that these results also hold when retailers have all the bargaining power and offer contingent three-part tariffs.

[10] Most importantly, for this result to hold in a framework of sequential contracting is that contracts are not binding and are subject to renegotiation (from scratch). Otherwise, as Marx and Shaffer (1999) and more recently Caprice and Schlippenbach (2010) show, the firms fail to sustain monopoly prices. See section 3 for more details.

[11] Renegotiation eliminates the externalities (resulting from sequential contracting) which prevent the firms from obtaining the fully efficient outcome. Although the second manufacturer and the retailer do not deviate from the marginal cost, the first manufacturer and the retailer may have incentives to do so. In particular, assuming that contracts are binding, Marx and Shaffer (1999) show that they set the wholesale price below marginal cost while, taking into account consumer shopping costs, Caprice and Schlippenbach (2010) find that the wholesale price in first negotiation is upward distorted.

[12] That is the outcome that would emerge if, taking the pattern of trade as given, all the firms coordinated their decisions on maximizing their joint profits.

characterized by “interlocking relationships”. In particular, in a situation where two manufacturers contract sequentially with a single, common retailer, it shows that there is no need for upfront payments provided that renegotiation is allowed.[13] In contrast, when the second manufacturer can distribute its product through another retailer, both manufacturers may need to pay upfront to the common retailer. While the second manufacturer may use an upfront payment (combined with quantity discounts) as a tool to suppress intrabrand competition, the first manufacturer may use it as a means to compensate for the negative impact of the sales of its product on the sales of the second manufacturer’s product (in both retail outlets). Intuitively, anticipating that its rival and the common retailer will seek to maximize the total profits on the sales of its rival’s product, the first manufacturer seeks to convince this retailer to carry its product anyway. Moreover, it seeks to induce its rival to charge a higher wholesale price to the second retailer (since this would reduce the competitive pressure on its product) and a lower wholesale price to the common retailer (since this would allow it to lower its own wholesale price which in turn would lead to more interbrand coordination). By setting fixed fee equal to zero (or negative if it has a weak bargaining power), the first manufacturer secures the sales of its product through the common retailer. When the degree of interbrand rivalry between the retailers is high, this reduces demand for its rival’s product sold through the second retailer. As a result, its rival gains less from giving variable discounts to the second retailer which in turn reduces its incentives to behave opportunistically (with respect to the common one). Consequently, it is led to negotiates a lower wholesale price with the common retailer and a higher wholesale price with the second one.

The paper also contributes to the literature on vertical contracting in bilaterally oligopolistic industries where bargaining power is distributed somewhat evenly between vertically related firms. To my knowledge, few papers address this issue.[14] Among them, Bjornerstedt and Stennek (2006) develop the framework of simultaneous bilateral negotiations between sellers and buyers of intermediate goods. Assuming that contracts consist of a quantity and a price, the authors show that in equilibrium the volume of trade is socially efficient, despite the presence of externalities[15] and the market power concentrated at both sides. In a similar framework of competing buyers and sellers, de Fontenay and Gans (2007) instead assume that bilateral negotiations take place sequentially. In their framework, the breakdown of any negotiations is public (in which case all the agreements previously reached are renegotiated from scratch) while the terms of contract (which consist of a quantity and a price) agreed upon in a given buyer-seller pair is private. In contrast to Bjornerstedt and Stennek (2006), de Fontenay and Gans find that the equilibrium profile of quantities fails to max-

[13]Throughout the paper, it is assumed that there is no competition for retailers’ shelf space since this would a priori give rise to the emergence of slotting fees.

[14]Also, there are few papers that explore a setting where two rival manufacturers distribute their products through two competing retailers and the bargaining power is entirely upstream. Assuming linear wholesale prices, Dobson and Waterson (2001) find that it is possible to have a situation where each retailer carries only one brand. Allowing instead for two-part tariffs, Rey and Verge (2004) show that there does not always exist an equilibrium in which each retailer carries the products of both manufacturers, despite the fact that consumers are willing to buy each of them.

[15]In their framework it is implicitly assumed that the quantity agreed upon between a buyer and a seller affects the payoffs of all other agents in a network.

imize the total surplus. Although the present paper uses a similar framework of sequential bilateral negotiations, it differs from the aforementioned papers in that it treats the case when the contracts are public and more sophisticated than quantity fixing ones.

The rest of the paper is organized as follows. Section 2 outlines the framework for the analysis. Section 3 derives all equilibrium continuations following a break-down of negotiations in one or two manufacturer-retailer pair(s). In particular, it shows that there can be a situation where a single retailer is left to negotiate with two manufacturers, in which case two-part tariffs may suffice to maximize the joint profits of all the active firms. Using the results of section 3, section 4 characterizes the equilibria in which all manufacturer-retailer pairs reach an agreement and the retailers carry the product(s) of all their respective supplier(s). Finally, section 5 discusses some policy implications of the findings and concludes.

2 Framework

Consider an environment where manufacturer A (hereafter M_A) seeks to distribute its product through retailer 1 while its rival - manufacturer B (hereafter M_B) - seeks to distribute its product through retailers 1 and 2 (hereafter R_1 and R_2). Each manufacturer $k = A, B$ incurs marginal cost $c_k \geq 0$ and zero fixed cost of production while the retailers incur no distribution cost. The retailers differ in their locations or services provided, so if each of them carries the product(s) of all its respective supplier(s), there are three imperfectly substitute "products" on the market: two of them - $A1$ and $B1$ - are sold in retail outlet 1 and one - $B2$ - is sold in retail outlet 2.

Denote by $\mathbf{q} \equiv (q_{A1}, q_{B1}, q_{A2})$ the quantity vector and by $R_{ki}(\mathbf{q})$ the revenue from selling q_{ki} units of product $ki = \{A1, B1, B2\}$ in outlet $i = 1, 2$, given that q_{hj} units of product $hj \neq ki$ are sold in outlet j. The function $R_{ki}(.)$ equals zero for $q_{ki} = 0$, strictly concave in q_{ki} and twice continuously differentiable. Furthermore, selling each extra unit of any product $hj \neq ki$ is assumed to reduce both the revenue and the marginal revenue from selling product ki.

Assumption A1. $\dfrac{\partial R_{ki}}{\partial q_{hj}} < 0$ *for any* $hj \neq ki = \{A1, B1, B2\}$.

Assumption A2. $\dfrac{\partial^2 R_{ki}}{\partial q_{hj} \partial q_{ki}} < 0$ *for any* $hj \neq ki = \{A1, B1, B2\}$.

These assumptions are satisfied in many standard oligopoly models and imply that all the products are imperfect substitutes.

Denote by Π^m the maximal industry profit that could be obtained for a given configuration of trade links (i.e., provided that R_2 does not carry product A):

$$\Pi^m \equiv \max_{\mathbf{q}} \sum_{k=A,B} \left(R_{k1}(\mathbf{q}) - c_k q_{k1}\right) + \left(R_{B2}(\mathbf{q}) - c_B q_{B2}\right).$$

Throughout the paper, Π^m will be referred to as the industry wide monopoly profit. If R_2 were inactive, the maximal industry profit would be given by:

$$\Pi^m_{A1B1} \equiv \max_{q_{A1}, q_{B1}} \sum_{k=A,B} \left(R_{k1}(q_{A1}, q_{B1}, 0) - c_k q_{k1}\right).$$

Distributing only product B in both retail outlets would generate the total profits:

$$\Pi^m_{B1B2} \equiv \max_{q_{B1},q_{B2}} \sum_{i=1,2} \left(R_{Bi}\left(0, q_{B1}, q_{B2}\right) - c_B q_{Bi}\right),$$

while distributing only product ki would generate the total profit:

$$\Pi^m_{ki} \equiv \max_{q_{ki}} R_{ki}\left(q_{ki}, 0, 0\right) - c_k q_{ki}.$$

The relationship between M_k and R_i is governed by a supply contract $\mathcal{C}_{ki} \equiv (w_{ki}, F_{ki}, S_{ki})$ which specifies the following transfer payment:

$$T_{ki}\left(q_{ki}|\mathcal{C}_{ki}\right) = \begin{cases} w_{ki}q_{ki} + F_{ki} + S_{ki}, & \text{for } q_{ki} > 0 \\ S_{ki}, & \text{for } q_{ki} = 0 \end{cases},$$

where q_{ki} is the quantity of product k purchased by R_i from M_k, w_k is the price that R_i pays for each unit of product k, F_{ki} is a conditional fixed fee which R_i pays only if it purchases positive quantity of product k and S_{ki} is an up-front fee which R_i pays regardless of whether it will purchase any quantity of product k afterwards. More precisely, S_{ki} is paid when $\mathcal{C}_{ki}$ is just signed while F_{ki} is paid when R_i makes a decision on q_{ki}.

The terms of each contract $\mathcal{C}_{ki}$ are assumed to be determined through negotiations between M_k and R_i which are modeled as the alternating-offers bargaining game introduced by Binmore *et al.* (1986).[16] As the authors show, the equilibrium of such a game is defined as the solution to the generalized Nash bargaining problem which implies that each party obtains its disagreement payoff plus a share of the gains from trade in proportion to its bargaining power. Throughout the paper, it will be assumed that each party possesses some bargaining power, so that in the event of agreement between M_k and R_i, R_i gets the share λ_{ki} (correspondingly, M_k gets the share $1 - \lambda_{ki}$) of the gains where $\lambda_{ki} \in (0, 1)$.

Each firm's disagreement payoffs is defined as the payoff that it would receive if the current negotiations ended in a breakdown and all the earlier signed contracts were renegotiated "from scratch". This approach, originally introduced by Stole and Zwiebel (1996) and recently followed by De Fontenay and Gans (2005, 2007) and Bedre (2009), captures the idea that firms can renegotiate any contract before retail competition takes place. In a setting of sequential contracting this implies that the earlier signed contracts cannot influence the contracts signed later on. This is because, if the later contracts are not signed, the earlier contracts will be renegotiated anyway.

Any time a contract is signed it is assumed to be public. This assumption is made to avoid the technical difficulties related to the proper specification of

[16] In this game in each period of time one of the parties makes an offer to its counterpart. If the offer is accepted, the game ends and the parties obtain their payoffs according to the contract signed. If the offer is rejected, then there is an exogenous risk of breakdown of negotiations in which case each party obtains its disagreement payoff; otherwise the rejecting party makes a counter offer. Alternating offers continues until some of the parties accepts an offer or negotiations break down.

beliefs (about all the contracts signed earlier). Moreover, if the contracts were private, the equilibrium outcome would *a priori* be inefficient.[17]

The contract negotiations between the manufacturers and the retailers are modeled as a sequential game G of bilateral negotiations introduced by Stole and Zwiebel (1996). Formally, the game unfolds as follows: at first R_1 and M_A negotiate $\mathcal{C}_{A1}$, then R_1 and M_B negotiate $\mathcal{C}_{B1}$ and, finally, R_2 and M_B negotiate $\mathcal{C}_{B2}$. Building on Stole and Zwiebel (1996), it is assumed that any time a manufacturer-retailer pair reaches an agreement in any round of negotiations, the next manufacturer-retailer pair in the ordering proceeds with negotiations. If instead a negotiation round ends in a breakdown, the corresponding pair will never renegotiate at any time, negotiations start over from the beginning with the pair that has previously reached an agreement, following the same order over all the remaining pairs. All negotiations stop when the last pair in the ordering reaches an agreement. The retailers then decide on the quantities to purchase from each manufacturer with whom they have signed the contracts. Retail competition takes place and the firms obtain the payoffs according to the contracts signed.

The game is solved in a recursive manner using subgame-perfect Nash equilibrium as a solution concept. As a starting point of the analysis, the outcome of retail competition is characterized. As it will be clear later, it is not necessarily to specify whether the retailers compete in prices or quantities; the analysis applies to both cases. Instead, it will prove to be sufficient to make the following assumption.

Assumption A3. *(i) For any vector of wholesale prices* $\mathbf{w} \equiv (w_{A1}, w_{B1}, w_{B2})$ *there exists a unique retail equilibrium characterized by the vector of equilibrium demand functions* $\mathbf{q}(\mathbf{w}) \equiv (q_{A1}(\mathbf{w}), q_{B1}(\mathbf{w}), q_{B2}(\mathbf{w}))$; *(ii) an increase in the wholesale price for product ki decreases the demand for that product and increases the demand for all other products, i.e.,*

$$\frac{\partial q_{ki}(\mathbf{w})}{\partial w_{ki}} < 0 < \frac{\partial q_{hj}(\mathbf{w})}{\partial w_{ki}},$$

for any $hj \neq ki = \{A1, B1, B2\}$*; and (iii) for any pair of wholesale prices* $(w_{hj}, w_{h'j'})$ *where* $hj \neq h'j' \neq ki$ *there exists a threshold such that for all* w_{ki} *above this threshold* $q_{ki}(\mathbf{w}) = 0$, *and for all* w_{ki} *below this threshold* $q_{ki}(\mathbf{w}) > 0$.

These conditions are satisfied when quantities are strategic substitutes. Note also that this formulation allows for the definition of equilibrium demand functions even if some product(s) is not available which can be viewed as the limit of setting the corresponding wholesale price(s) to infinity. For example, if R_1 chooses not to carry product B then the equilibrium demand functions for products $A1$ and $B2$ are given by $q_{A1}(w_{A1}, \infty, w_{B2})$ and $q_{B2}(w_{A1}, \infty, w_{B2})$, respectively.

Denote the R_i's equilibrium flow profits from selling product k by:

$$\pi_k^{R_i}(\mathbf{w}) = R_{ki}(\mathbf{q}(\mathbf{w})) - w_{ki} q_{ki}(\mathbf{w}).$$

The function $\pi_k^{R_i}(\mathbf{w})$ is assumed to display the following properties.

[17]When contracts are private, each manufacturer negotiating with competing retailers has incentives to behave opportunistically. As shown by Hart and Tirole (1990), O'Brien and Shaffer (1992), McAfee and Schwartz (1994) and, more recently, Rey and Verge (2004), this prevents firms from implementing the fully integrated monopoly outcome.

Assumption A4. *For each $k \neq h = A, B$ and each $i \neq j = 1, 2$:*
(i) $\pi_k^{R_i}(\mathbf{w}) > 0$ *if* $q_{ki}(\mathbf{w}) > 0$ *and* $\pi_k^{R_i}(\mathbf{w}) = 0$ *otherwise;*
(ii) $\dfrac{\partial \pi_A^{R_1}(\mathbf{w})}{\partial w_{k1}} + \dfrac{\partial \pi_B^{R_1}(\mathbf{w})}{\partial w_{k1}} < 0$ *if* $\pi_k^{R_1}(\mathbf{w}) > 0$*;*
$\dfrac{\partial \pi_B^{R_2}(\mathbf{w})}{\partial w_{B2}} < 0$ *if* $\pi_B^{R_2}(\mathbf{w}) > 0$ *and* $\dfrac{\partial \pi_B^{R_2}(\mathbf{w})}{\partial w_{B2}} = 0$ *otherwise;*
(iii) $\dfrac{\partial \pi_k^{R_i}(\mathbf{w})}{\partial w_{hj}} > 0$ *if* $\pi_k^{R_i}(\mathbf{w}) > 0$ *and* $\dfrac{\partial \pi_k^{R_i}(\mathbf{w})}{\partial w_{hj}} = 0$ *otherwise;*
(iv) $\dfrac{\partial \pi_A^{R_1}(\mathbf{w})}{\partial w_{k1} \partial w_{B2}} + \dfrac{\partial \pi_B^{R_1}(\mathbf{w})}{\partial w_{k1} \partial w_{B2}} < 0$ *if* $\pi_A^{R_1}(\mathbf{w}) > 0$ *and* $\pi_B^{R_1}(\mathbf{w}) > 0$.

Condition A4(ii) is standard in the literature and implies that the (total) profit from selling product ki is lower, the larger is the unit cost of ki. To gain the intuition of condition A4(iii), consider an increase, say, in w_{A1}. By assumption A3(ii), this will lead to a decrease in sales of $A1$ and to an increase sales of both $B1$ and $B2$. Since the profit, say, $\pi_B^{R_1}$ increases in the amount of sales of $B1$ but decreases in the amount of sales of $B2$, the ultimate effect on $\pi_B^{R_1}$ may be ambiguous. A4(iii) implies that the first effect dominates the second one.

Finally, condition A4(iv) implies that R_1 benefits less from a decrease in the marginal cost of any of the products that it carries, the lower is the marginal cost of the product carried by its rival. [18] Intuitively, R_1 gains from a decrease in w_{k1} in two ways: first, because its marginal cost is lower (this is a direct effect) and, second, because, by increasing sales of $k1$, it induces R_2 to contract its sales of $B2$ (this is a strategic effect). The gain from the direct effect is proportional to the size of sales of $k1$ and thus is lower, the lower is the marginal cost of $B2$. The gain from the strategic effect depends on R_2's reduction in sales of $B2$ in response to a decrease in w_{k1}. Whether the size of the reduction is lower or larger, the lower is the marginal cost of $B2$, in general, is uncertain which makes the impact of the strategic effect somewhat ambiguous. Condition A4(iv) implies that whatever its sign, the overall effect of a decrease in the marginal cost of $B2$ on the R_1's marginal gain from a decrease in its own marginal price is negative.[19] All the conditions of A4 are for example satisfied in the linear demand function model.

Denote the M_k's equilibrium flow profits from selling its product in outlet i by:

$$\pi_i^{M_k}(\mathbf{w}) = (w_{ki} - c_k)\, q_{ki}(\mathbf{w}).$$

[18] A similar assumption but in a slightly different context is made in the literature. For example, in modeling the setting in which a monopolistic supplier faces n firms which compete in the downstream market and use the supplier's input to produce substitute products, McAfee and Schwartz (1994) assume that "a decrease in a firm's marginal cost is less valuable to it the lower a rival's marginal cost" (p. 216). The same approch is followed by Marx and Shaffer (2004).

[19] For the sake of illustration, suppose that the retailers compete *a la* Cournot. Using the first order conditions which define the equilibrium outcomes, one then gets:

$$\frac{\partial \pi_{A1}^{R}(\mathbf{w})}{\partial w_{k1} \partial w_{B2}} + \frac{\partial \pi_{B1}^{R}(\mathbf{w})}{\partial w_{k1} \partial w_{B2}} = -\frac{\partial q_{k1}}{\partial w_{B2}} + \frac{\partial}{\partial w_{B2}} \left(\left(\frac{\partial R_{A1}}{\partial q_{B2}} + \frac{\partial R_{B1}}{\partial q_{B2}} \right) \frac{\partial q_{B2}}{\partial w_{k1}} \right),$$

for each $k = A, B$. By assumption A3(ii), the first term in the above expression (which captures the direct effect) is negative while the sign of the second term (which captures the strategic effect) is ambiguous.

Assuming that all firms are active, their total industry profits are equal to:

$$\Pi(\mathbf{w}) = \sum_{k=A,B} \left(\pi_k^{R_1}(\mathbf{w}) + \pi_1^{M_k}(\mathbf{w})\right) + \left(\pi_B^{R_2}(\mathbf{w}) + \pi_2^{M_B}(\mathbf{w})\right). \qquad (1)$$

If all firms could coordinate their decisions, they would set the wholesale prices so as to maximize $\Pi_{A1B1B2}(\mathbf{w})$. For the sake of exposition, from now on the following assumption will be made:

Assumption A5. *The function* $\Pi(\mathbf{w})$ *is quasi-concave in* $\mathbf{w}$ *and there exists a unique vector of wholesale prices* $\mathbf{w}^m \equiv (w_{A1}^m, w_{B1}^m, w_{B2}^m)$ *that generates the industry wide monopoly profit, i.e.,* $\Pi(\mathbf{w}^m) = \Pi^m$.

Because of the impact of retail competition marginal cost pricing cannot implement the monopoly outcome; since quantities are strategic substitutes, each retailer would have incentives to sell more than a monopolist controlling the sales in all retail outlets. Thus, wholesale prices above the unit costs are needed to offset the impact of retail competition and, therefore, $w_{ki}^m > c_k$ for each $k = A, B$ and each $i = 1, 2$.

It will prove useful to define the wholesale price $w_{A1}^{BR}(w_{B1}, w_{B2})$ which maximizes $\Pi(w_{A1}, w_{B1}, w_{B2})$ for a given pair (w_{B1}, w_{B2}):

$$w_{A1}^{BR}(w_{B1}, w_{B2}) = \arg\max_{w_{A1}} \Pi(w_{A1}, w_{B1}, w_{B2}).$$

Assumption A6. $\frac{\partial w_{A1}^{BR}}{\partial w_{B1}} > 0$ *and* $\frac{\partial w_{A1}^{BR}}{\partial w_{B2}} < 0$.

This assumption is for example satisfied in the linear demand function model. It implies that while maximizing the total industry profit w_{A1} and w_{B1} can be viewed as strategic complements while w_{A1} and w_{B2} can be viewed as strategic substitutes. Intuitively, by acting as a common agent, R_1 internalizes all the externalities arising between competing brands. Furthermore, since the total profit is larger when it carries both brands rather than only one of them, A and B tend to complement each other when they are sold at the same store. In contrast, when A and B are sold at different stores, the (negative) externalities are still present which tends to make them substitutes: following an increase in sales, say, of product A, a monopolist controlling the sales in all retail outlets would optimally choose to contract the sales of product B in order to maintain retail prices at a higher level.

Taking the assumptions about the retail equilibrium as given, the task boils down to determining the outcome of negotiations in each manufacturer-retailer pair. Since, while bargaining over the terms of the contract, each manufacturer-retailer pair takes into account what each party would obtain if the current negotiations broke down, it proves to be convenient to begin the analysis by characterizing the equilibrium continuations for all cases of break-down of negotiations before solving for the equilibrium of the whole game.

3 Break-down of negotiations

To begin, suppose that negotiations in two manufacturer-retailer pairs have broken down. The equilibrium continuation then implies that the remaining pair, say, $M_k - R_i$, negotiates the following contract: *(i)* $w_{ki} = c_k$ so that the joint bilateral profits are maximal and equal to Π_{ki}^m, and *(ii)* F_{ki} and S_{ki} are set so that these profits are divided according to each party's bargaining power, i.e., M_k gets $(1-\lambda_{ki})\,\Pi_{ki}^m$ while R_i gets $\lambda_{ki}\Pi_{ki}^m$.

Suppose now that negotiations in just one manufacturer-retailer pair have broken down. Three cases need then to be distinguished.

3.1 Break-down of negotiations between M_A and R_1

Since M_A and R_1 can no longer negotiate, the continuation play is the sequential game of bilateral negotiations taking place, first, between M_B and R_1 and then between M_B and R_2. This game is analyzed in Bedre (2010) whose result can be stated as follows.

Proposition 1 *Suppose that negotiations between M_A and R_1 have broken down. Then, all continuation equilibria of the game G imply (i) the wholesale prices are set at the levels that generate the monopoly profit Π_{B1B2}^m; (ii) R_1 gives all of its variable profit as a conditional fee while R_2's conditional fee is set just to ensure that it is active; (iii) the unconditional fees are set so that the gains from trade in each manufacturer-retailer pair are shared according to each party's bargaining power. In all such equilibria M_A obtains zero while M_B, R_1 and R_2 obtain the following payoffs:*

$$\begin{aligned}
\widehat{u}^{M_B} &= (1-\lambda_{B1})\left[\Pi_{B1}^m + (1-\lambda_{B2})\,\widehat{\Delta}\right], \\
\widehat{u}^{R_1} &= \Pi_{B1B2}^m - (1-\lambda_{B1})\left[\Pi_{B1}^m + \widehat{\Delta}\right], \\
\widehat{u}^{R_2} &= (1-\lambda_{B1})\,\lambda_{B2}\widehat{\Delta},
\end{aligned}$$

where

$$\widehat{\Delta} \equiv \max\left\{0, \Pi_{B1B2}^m - \frac{1-\lambda_{B2}+\lambda_{B1}\lambda_{B2}}{1-\lambda_{B2}}\Pi_{B1}^m + \frac{\lambda_{B1}}{1-\lambda_{B1}}\Pi_{B2}^m\right\}.$$

Proof. See Bedre (2009). ■

Both three-part tariffs and renegotiation are important for the equilibrium outcome to be efficient. Intuitively, since M_B contracts sequentially with the retailers, it has incentives to free-ride on its contract with R_1 while signing a contract with R_2. To protect itself against such an opportunistic move, R_1 agrees to give all its variable profit as a conditional payment. This deters M_B from giving variable discounts to R_2 since otherwise R_1 would prefer to "opt out" and avoid the payment larger than its variable profit. On the other hand, R_1 is willing to give all its variable profit because M_B pays it upfront, so that, eventually, it gets its share of the gains from trade.

Three-part tariffs alone are however not sufficient to achieve the efficiency. Without renegotiation M_B could use its contract with R_1 as a tool to influence its contract with R_2 and thus distort the wholesale prices generating the monopoly profit. Renegotiation eliminates these contractual externalities and aligns the bilateral incentives of M_B and R_1 with maximizing the joint profits of M_B, R_1 and R_2.

3.2 Break-down of negotiations between M_B and R_1

Suppose that negotiations between M_A and R_1 have ended in an agreement while negotiations between M_B and R_1 have broken down. The subgame continuation then implies that M_A and R_1 renegotiate their contract from scratch; after that negotiations take place between M_B and R_2. This implies competition between two vertical structures $M_A - R_1$ and $M_B - R_2$. In equilibrium each manufacturer-retailer pair maximizes its joint bilateral profit which is then shared according to each party's bargaining power.

Formally, the break down of negotiations between M_B and R_1 implies that R_1 no longer carries product B. By assumption A3, the equilibrium demand functions for products $A1$ and $B2$ are then defined by $q_{A1}(w_{A1}, \infty, w_{B2})$ and $q_{B2}(w_{A1}, \infty, w_{B2})$ respectively. Denote by $\overline{\pi}_{A1}(w_{A1}, w_{B2})$ and $\overline{\pi}_{B2}(w_{A1}, w_{B2})$ the joint bilateral profits of the pairs $M_A - R_1$ and $M_B - R_2$, i.e.,

$$
\begin{aligned}
\overline{\pi}_{A1}(w_{A1}, w_{B2}) &\equiv \pi_A^{R_1}(w_{A1}, \infty, w_{B2}) + (w_{A1} - c_A)\, q_{A1}(w_{A1}, \infty, w_{B2}), \\
\overline{\pi}_{B2}(w_{A1}, w_{B2}) &\equiv \pi_B^{R_2}(w_{A1}, \infty, w_{B2}) + (w_{B2} - c_B)\, q_{B2}(w_{A1}, \infty, w_{B2}),
\end{aligned}
$$

respectively.

Since M_B and R_2 negotiate their contract while observing the contract signed by M_A and R_1, it is convenient to define their best response $\overline{w}_{B2}^{BR}(w_{A1})$ to the wholesale price w_{A1}:

$$\overline{w}_{B2}^{BR}(w_{A1}) = \arg\max_{w_{B2}} \overline{\pi}_{B2}(w_{A1}, w_{B2}).$$

Anticipating the effect of their negotiations on the subsequent play of the game, M_A and R_1 set the wholesale price $\overline{w}_{A1}$ which maximizes their joint bilateral profits:

$$\overline{w}_{A1} = \arg\max_{w_{A1}} \overline{\pi}_{A1}(w_{A1}, \overline{w}_{B2}^{BR}(w_{A1})).$$

M_B and R_2 then optimally respond to $\overline{w}_{A1}$ by setting the wholesale price $\overline{w}_{B2} \equiv \overline{w}_{B2}^{BR}(\overline{w}_{A1})$. The following proposition summarizes the discussion.

Proposition 2 *Suppose that negotiations between M_B and R_1 have broken down. Then, all continuation equilibria of the game G imply (i) M_A and R_1 set $w_{A1} = \overline{w}_{A1}$ while M_B and R_2 set $w_{B2} = \overline{w}_{B2}$; (ii) the conditional fees are set to ensure that both retailers are active, i.e., $F_{A1} \leq \pi_A^{R_1}(\overline{w}_{A1}, \infty, \overline{w}_{B2})$ and $F_{B2} \leq \pi_B^{R_2}(\overline{w}_{A1}, \infty, \overline{w}_{B2})$; (iii) the unconditional fees are set so that the gains from trade in each manufacturer-retailer pair are shared according to each party's bargaining power. In all such equilibria the firms obtain the following payoffs:*

$$
\begin{aligned}
\overline{u}^{M_A} &= (1 - \lambda_{A1})\, \overline{\pi}_{A1}(\overline{w}_{A1}, \overline{w}_{B2}), \\
\overline{u}^{M_B} &= (1 - \lambda_{B2})\, \overline{\pi}_{B2}(\overline{w}_{A1}, \overline{w}_{B2}), \\
\overline{u}^{R_1} &= \lambda_{A1} \overline{\pi}_{A1}(\overline{w}_{A1}, \overline{w}_{B2}) \\
\overline{u}^{R_2} &= \lambda_{B2} \overline{\pi}_{B2}(\overline{w}_{A1}, \overline{w}_{B2}).
\end{aligned}
$$

3.3 Break-down of negotiations between M_B and R_2

Suppose now that both pairs $M_A - R_1$ and $M_B - R_1$ have succeeded in their negotiations while negotiations between M_B and R_2 have broken down. Since all the contracts signed earlier are then renegotiated from scratch, from then onwards the firms play the game in which R_1 negotiates sequentially with M_A and M_B. Such a game was first studied by Marx and Shaffer (1999) and recently by Caprice and Schlippenbach (2010), however, under the assumption that the supply contracts are restricted to two-part tariffs and non-renegotiable. Both papers show that the equilibrium outcome is generally inefficient (from the firms' point of view): while the second pair $M_B - R_1$ sets the wholesale price equal to the marginal cost, the first pair $M_A - R_1$ has incentives to distort the marginal cost pricing. The distortion occurs because, by signing its contract with M_A, R_1 affects its disagreement payoff in its bargaining with M_B. Thus, even though R_1 acting as a common agent internalizes any impact of the sales of one product on the sales of the other, the contractual externalities stemming from sequential contracting do not allow the firms to achieve the monopoly outcome.

Although renegotiation eliminates the contractual externalities, it does not always restore the efficiency when the contracts consist of two-part tariffs. The following proposition states formally this result.

Proposition 3 *Suppose that in the game G negotiations between M_B and R_2 are not allowed and the supply contracts are restricted to two-part tariffs. Then, the firms can implement the monopoly outcome as a common agency equilibrium only if:*

$$\begin{aligned} & \frac{(1-\lambda_{A1}+\lambda_{A1}\lambda_{B1})\,\Pi^m_{A1}-\lambda_{B1}\Pi^m_{A1B1}}{1-\lambda_{B1}} \\ < \quad & \min\left\{(1-\lambda_{A1})\Delta,\ \Pi^m_{A1B1}-\max\{\Pi^m_{B1},\lambda_{A1}\Pi^m_{A1}\}\right\}. \end{aligned} \tag{2}$$

where $\Delta \equiv \Pi^m_{A1B1} - \left(\Pi^m_{B1} - \frac{(1-\lambda_{B1})\lambda_{A1}}{\lambda_{B1}}\Pi^m_{A1}\right)$.

Proof. Available upon request. ■

The intuition is as follows. On the one hand, in any common agency equilibrium neither manufacturer can demand a conditional payment larger than the incremental value of its product. On the other hand, each manufacturer-retailer pair uses the conditional fee as a tool to share the bilateral gains from trade according to each party's bargaining power. Condition (2) defines the range of parameter values when, under marginal cost pricing, such a sharing rule does not destroy the R_1's incentives to carry both products.

This is however not the case when (2) is violated which is possible for λ_{A1} and λ_{B1} sufficiently small.[20] Intuitively, when R_1 has little bargaining power vis-a-vis M_B it may happen that the M_B's share of the gains from trade exceeds the incremental value of its product. In which case, M_B must be allocated a

[20] To see this, let $\lambda_{A1} = k\lambda_{B1}$ where $k > 0$ and choose λ_{B1} such that $k\lambda_{B1}\Pi^m_{A1} \leq \Pi^m_{B1}$ and $\Pi^m_{A1B1} - \Pi^m_{B1} \leq (1-k\lambda_{B1})\left(\Pi^m_{A1B1} - \Pi^m_{B1} + k(1-\lambda_{B1})\Pi^m_{A1}\right)$. In which case (2) boils down to $\Pi^m_{A1} + \Pi^m_{B1} - \Pi^m_{A1B1} < \lambda_{B1}\left(\Pi^m_{B1} + k(1-\lambda_{B1})\Pi^m_{A1}\right)$. Since the products are imperfect substitutes, then $\Pi^m_{A1} + \Pi^m_{B1} - \Pi^m_{A1B1} > 0$ and, therefore, there always exists λ_{B1} sufficiently small which violates this condition.

smaller part of these gains for the common agency equilibrium to be preserved. This is however no longer optimal from the point of view of maximizing the total industry profits. In particular, M_A and R_1 are then lead to maximize only a part of the industry profit which implies that they set the wholesale price below the marginal cost. As a result, common agency equilibria either yield an inefficient outcome or do not exist.

If instead *three*-part tariffs are allowed, securing R_1's incentives to carry both brands is no longer a constraint in dividing bilateral gains from trade. As a result, there always exist equilibria in which both manufacturers are active and in all such equilibria the firms achieve the fully integrated monopoly outcome. Formally, we have:

Proposition 4 *Suppose that negotiations between M_B and R_2 have broken down. Then, there always exist common agency continuation equilibria of the game G in which (i) the wholesale prices are set at marginal costs, i.e., $w_{k1} = c_k$ for each $k = \{A, B\}$; (ii) the conditional fees are set so as to ensure that R_1 accepts to carry both products, i.e., $F_{k1} \leq \Pi^m_{A1B1} - \Pi^m_{h1}$ for each $k \neq h = \{A, B\}$; (iii) the unconditional fees are set so that the gains from trade in each manufacturer-retailer pair are shared according to each party's bargaining power. In all such equilibria R_2 obtains zero while R_1, M_A and M_B obtain the following payoffs:*

$$\begin{aligned}
\widetilde{u}^{R_1} &= \lambda_{A1}\left(\Pi^m_{A1} + \lambda_{B1}\widetilde{\Delta}\right), \\
\widetilde{u}^{M_A} &= \Pi^m_{A1B1} - \lambda_{A1}\left(\Pi^m_{A1} + \widetilde{\Delta}\right), \\
\widetilde{u}^{M_B} &= \lambda_{A1}\left(1 - \lambda_{B1}\right)\widetilde{\Delta},
\end{aligned}$$

where

$$\widetilde{\Delta} \equiv \max\left\{0, \Pi^m_{A1B1} - \frac{1 - \lambda_{A1} + \lambda_{A1}\lambda_{B1}}{\lambda_{B1}}\Pi^m_{A1} + \frac{1 - \lambda_{A1}}{\lambda_{A1}}\Pi^m_{B1}\right\}.$$

Moreover, if $\lambda_{A1}\widetilde{\Delta} \geq \Pi^m_{A1B1} - \Pi^m_{A1}$, there also exist continuation equilibria in which only R_1 and M_A are active. In all such equilibria R_1 obtains the payoff $\widetilde{u}^{R_1} = \lambda_{A1}\Pi^m_{A1}$, M_A obtains the payoff $\widetilde{u}^{M_A} = (1 - \lambda_{A1})\Pi^m_{A1}$ while R_2 and M_B obtain zero.

Proof. See Appendix *A*. ■

The proposition, in particular, states that different types of equilibria are possible. Intuitively, besides inducing continuation equilibrium in which R_1 carries both brands, M_A and R_1 can instead induce a continuation equilibrium in which R_1 carries only brand A.[21] The condition $\lambda_{A1}\widetilde{\Delta} \geq \Pi^m_{A1B1} - \Pi^m_{A1} (> 0)$ implies that in that case M_A would obtain the payoff larger than the one it obtains in any common agency continuation equilibrium. In contrast, R_1 always prefers continuation equilibria in which it carries both brands since $\lambda_{A1}(\Pi^m_{A1} +$

[21] R_1 and M_A can do so, by setting the unconditional fee sufficiently large, so that R_1 and M_B could never obtain non-negative gains from trade. Formally, this would induce the break down of negotiations between R_1 and M_B and, concequently, trigger renegotiations between R_1 and M_A (from scratch).

$\lambda_{B1}\widetilde{\Delta}) \geq \lambda_{A1}\Pi_{A1}^m$. The divergence of preferences about the continuation play gives rise to multiple types of equilibria.

Propositions 3 and 4 imply that when a single, common retailer contracts sequentially with rival manufacturers, three-part tariffs may not be needed to obtain efficiency provided that renegotiation is allowed. In contrast, as it will be shown below, even three-part tariffs do not suffice to maintain monopoly prices when competition exists both upstream and downstream.

4 Equilibrium of the game G

This section derives (subgame perfect) equilibria in which the retailers carry the products of all their respective suppliers. As before, the game is solved by using the algorithm of backward induction: at any time a manufacturer and a retailer negotiate a contract, they take all the earlier signed contracts as given and anticipate the effect of the outcome of their negotiations on the subsequent play of the game.

4.1 Retail competition

To begin, suppose that all the contracts have been signed, i.e., all the retailer-manufacturer pairs have succeeded in their negotiations, and consider the retail competition stage. Given that R_1 accepts to carry both products, R_2 will accept to carry product B only if, by doing so, it earns non-negative profit, i.e., only if:

$$\pi_B^{R_2}(w_{A1}, w_{B1}, w_{B2}) - F_{B2} \geq 0. \qquad \mathrm{PC}_2^{AB}$$

On the other hand, given that R_2 accepts to carry product B, R_1 will accept to carry both products only if, by doing so, it earns the profits that are not only non-negative, but also higher than the profit it could earn from selling only one of them. Since, by selling only product B, it could earn the profit $\pi_B^{R_1}(\infty, w_{B1}, w_{B2}) - F_{B1}$ while, by selling only product A, it could earn the profit $\pi_A^{R_1}(w_{A1}, \infty, w_{B2}) - F_{A1}$, in any equilibrium under study the following constraints must be satisfied:

$$\begin{aligned} \sum_{k=A,B} \pi_k^{R_1}(w_{A1}, w_{B1}, w_{B2}) - F_{k1} &\geq 0, && \mathrm{PC}_1^{AB} \\ &\geq \pi_B^{R_1}(\infty, w_{B1}, w_{B2}) - F_{B1}, && \mathrm{IC}_{A1}^{AB} \\ &\geq \pi_A^{R_1}(w_{A1}, \infty, w_{B2}) - F_{A1}. && \mathrm{IC}_{B1}^{AB} \end{aligned}$$

Since products A and B are imperfect substitute, removing one of them increases the profit from selling the other. This allows R_1 to behave opportunistically and guarantee itself positive profits.

Lemma 1 *In any equilibrium in which the retailers carry the products of all their respective suppliers R_1 earns positive profits.*

Proof. Suppose not, i.e., R_1 earns zero. Summing up IC_{A1}^{AB} and IC_{B1}^{AB} and, using the fact that PC_1 binds, yields:

$$\begin{aligned} 0 \quad \geq \quad & \left(\pi_A^{R_1}(w_{A1},\infty,w_{B2}) - \pi_A^{R_1}(w_{A1},w_{B1},w_{B2})\right) \\ & + \left(\pi_B^{R_1}(\infty,w_{B1},w_{B2}) - \pi_B^{R_1}(w_{A1},w_{B1},w_{B2})\right). \end{aligned}$$

By assumption A5, the function $\pi_k^{R_1}(w_{k1},w_{h1},w_{B2})$ increases in w_{h1} for $k \neq h = \{A,B\}$. This implies that the right hand side of the above condition is strictly positive which is a contradiction. ■

The implication of lemma 1 is that in any equilibrium under study PC_1^{AB} is not binding and thus can be omitted in the subsequent analysis.

Denote by $\mathfrak{C} \equiv (\mathcal{C}_{A1},\mathcal{C}_{B1},\mathcal{C}_{B2})$ the triple of contracts signed between M_A and R_1, M_B and R_1, M_B and R_2 respectively. For a given $\mathfrak{C}$ denote by $u^{R_i}(\mathfrak{C})$ the R_i's overall payoff for each $i = 1,2$ and by $u^{M_k}(\mathfrak{C})$ the M_k's overall payoff for each $k = A,B$. If all the contracts satisfy IP_i^{AB} for each $i = 1,2$ and IC_{k1}^{AB} for each $k = A,B$, then:

$$u^{R_1}(\mathfrak{C}) = \sum_{k=A,B}\left(\pi_k^{R_1}(\mathbf{w}) - (F_{k1}+S_{k1})\right), \tag{3}$$

$$u^{R_2}(\mathfrak{C}) = \pi_B^{R_2}(\mathbf{w}) - (F_{B2}+S_{B2}), \tag{4}$$

$$u^{M_A}(\mathfrak{C}) = \pi_1^{M_A}(\mathbf{w}) + (F_{A1}+S_{A1}), \tag{5}$$

$$u^{M_B}(\mathfrak{C}) = \sum_{i=1,2}\left(\pi_i^{M_B}(\mathbf{w}) + (F_{Bi}+S_{Bi})\right). \tag{6}$$

4.2 Negotiations between M_B and R_2

Suppose that $\mathcal{C}_{A1}$ and $\mathcal{C}_{B1}$ have been signed. Taking $\mathcal{C}_{A1}$ and $\mathcal{C}_{B1}$ as given and anticipating the outcome of retail competition, M_B and R_2 negotiate the contract $\mathcal{C}_{B2}^*$ which solves the generalized Nash bargaining problem provided that the M_B's disagreement payoff is $\widetilde{u}^{M_B}$ (this is what M_B would get while renegotiating with R_1 if negotiations with R_2 failed) while the R_2's disagreement payoff is zero (once negotiations between M_B and R_2 break down, the parties cannot renegotiate at another time). Furthermore, in order to obtain the continuation equilibrium in which all three products are available on the market, it is necessary that IC_{A1}^{AB}, IC_{B1}^{AB} and PC_2^{AB} are satisfied which implies that $\mathcal{C}_{B2}^*$ must be a solution to the following problem:

$$\max_{\mathcal{C}_{B2}} \left(u^{R_2}(\mathfrak{C})\right)^{\lambda_{B2}} \left(u^{M_B}(\mathfrak{C}) - \widetilde{u}^{M_B}\right)^{1-\lambda_{B2}}, \tag{$\mathcal{P}_1$}$$

$s.t.$ IC_{A1}^{AB}, IC_{B1}^{AB} and PC_2^{AB} hold.

Denote by $G_{B2}(\mathfrak{C})$ the incremental gains from trade between M_B and R_2, i.e.,

$$G_{B2}(\mathfrak{C}) \equiv u^{R_2}(\mathfrak{C}) + u^{M_B}(\mathfrak{C}) - \widetilde{u}^{M_B} = \widehat{\Pi}(\mathbf{w}) + (F_{B1}+S_{B1}) - \widetilde{u}^{M_B}, \tag{7}$$

where

$$\widehat{\Pi}(\mathbf{w}) \equiv \pi_B^{R_2}(\mathbf{w}) + \left(\pi_1^{M_B}(\mathbf{w}) + \pi_2^{M_B}(\mathbf{w})\right).$$

Denote by w_{B2}^* the wholesale price which maximizes the joint bilateral profits of M_B and R_2 subject to the constraint that the R_1's incentives to carry both products are preserved, i.e.,

$$\begin{aligned} w_{B2}^*(\mathbf{w}_1, F_{A1}, F_{B1}) &= \arg\max_{w_{B2}} \widehat{\Pi}(\mathbf{w}_1, w_{B2}) \qquad \mathcal{P}_2 \\ & \quad s.t.\ \mathrm{IC}_{A1}^{AB} \text{ and } \mathrm{IC}_{B1}^{AB} \text{ hold} \end{aligned}$$

where $\mathbf{w}_1 \equiv (w_{A1}, w_{B1})$.

Define the gains from trade G_{B2}^* between M_B and R_2 as the difference between the joint bilateral profits when they trade and the joint bilateral profits when they do not. Provided that these gains are non-negative for $w_{B2} = w_{B2}^*$,[22] i.e.,

$$G_{B2}^*(\mathcal{C}_{A1}, \mathcal{C}_{B1}) \equiv \widehat{\Pi}(\mathbf{w}_1, w_{B2}^*) + (F_{B1} + S_{B1}) - \widetilde{u}^{M_B} \geq 0, \qquad \mathrm{GT}_{B2}$$

the solution $\mathcal{C}_{B2}^*$ to $\mathcal{P}_1$ implies that *(i)* the wholesale price is set equal to w_{B2}^*; *(ii)* the conditional fee F_{B2}^* is set so that PC_2^{AB} is satisfied for $w_{B2} = w_{B2}^*$, i.e., $F_{B2}^* \leq \pi_B^{R_2}(\mathbf{w}_1, w_{B2}^*)$; and *(iii)* the unconditional fee S_{B2}^* is set so that M_B and R_2 divide their gains from trade according to each party's bargaining power, i.e.,

$$F_{B2}^* + S_{B2}^* = \pi_B^{R_2}(\mathbf{w}_1, w_{B2}^*) - \lambda_{B2} G_{B2}^*(\mathcal{C}_{A1}, \mathcal{C}_{B1}),$$

which implies that R_2 gets the payoff:

$$u^{R_2*}(\mathcal{C}_{A1}, \mathcal{C}_{B1}) \equiv u^{R_2}(\mathcal{C}_{A1}, \mathcal{C}_{B1}, \mathcal{C}_{B2}^*) = \lambda_{B2} G_{B2}^*(\mathcal{C}_{A1}, \mathcal{C}_{B1}), \tag{8}$$

while M_B gets the payoff:

$$u^{M_B*}(\mathcal{C}_{A1}, \mathcal{C}_{B1}) \equiv u^{M_B}(\mathcal{C}_{A1}, \mathcal{C}_{B1}, \mathcal{C}_{B2}^*) = \widetilde{u}^{M_B} + (1 - \lambda_{B2}) G_{B2}^*(\mathcal{C}_{A1}, \mathcal{C}_{B1}). \tag{9}$$

Consider now the solution to $\mathcal{P}_2$ which defines the wholesale price set by M_B and R_2 in equilibrium under study. For the sake of exposition, the following assumption will be made.

Assumption A7. *The function* $\widehat{\Pi}(\mathbf{w}_1, w_{B2})$ *is quasi-concave in* w_{B2} *for any vector* $\mathbf{w}_1$ *and achieves its maximum for* $w_{B2} = \widehat{w}_{B2}(\mathbf{w}_1)$ *where*

$$\widehat{w}_{B2}(\mathbf{w}_1) = \arg\max_{w_{B2}} \widehat{\Pi}(\mathbf{w}_1, w_{B2}).$$

The wholesale price $\widehat{w}_{B2}(\mathbf{w}_1)$ is the best reply of M_B and R_2 to the wholesale prices w_{A1} and w_{B1}. In particular, M_B and R_2 would set this price, if R_1 had to absorb any impact on its profit due to an increase in sales of R_2.

Using that $\pi_A^{R_1}(\infty, w_{B1}, w_{B2}) = \pi_B^{R_1}(\infty, w_{A1}, w_{B2}) = 0$, write the IC_{k1}^{AB} constraint for each $k = \{A, B\}$ as follows:

[22] Otherwise negotiations between M_B and R_2 are assumed to break down.

$$\left(\pi_A^{R_1}(w_{A1},w_{B1},w_{B2})+\pi_B^{R_1}(w_{A1},w_{B1},w_{B2})\right)$$
$$-\left(\pi_A^{R_1}(\infty,w_{h1},w_{B2})+\pi_B^{R_1}(\infty,w_{h1},w_{B2})\right)\geq F_{k1},$$

for $h\neq k$. Differentiating the left hand side of the above condition w.r.t. w_{B2} and using assumption A6 yields:

$$\begin{aligned}
&\left(\frac{\partial\pi_A^{R_1}(w_{A1},w_{B1},w_{B2})}{\partial w_{B2}}+\frac{\partial\pi_B^{R_1}(w_{A1},w_{B1},w_{B2})}{\partial w_{B2}}\right)\\
&-\left(\frac{\partial\pi_A^{R_1}(\infty,w_{h1},w_{B2})}{\partial w_{B2}}+\frac{\partial\pi_B^{R_1}(\infty,w_{h1},w_{B2})}{\partial w_{B2}}\right)\\
=\;&-\int_{w_{k1}}^{\infty}\left(\frac{\partial\pi_A^{R_1}(w'_{k1},w_{h1},w_{B2})}{\partial w'_{k1}\partial w_{B2}}+\frac{\partial\pi_B^{R_1}(w'_{k1},w_{h1},w_{B2})}{\partial w'_{k1}\partial w_{B2}}\right)dw'_{k1}>0,
\end{aligned}$$

Thus, for a given pair $(\mathbf{w}_1,F_{k1})$ the set of wholesale prices w_{B2} satisfying IC_{k1}^{AB} is the set $\{w_{B2}:w_{B2}\geq\mathfrak{w}_{B2}^k(\mathbf{w}_1,F_{k1})\}$ where $\mathfrak{w}_{B2}^k(\mathbf{w}_1,F_{k1})$ is the wholesale price for which IC_{k1}^{AB} binds. Taken with assumption A6, this implies that the solution to $\mathcal{P}_2$ can be written as:

$$w_{B2}^*(\mathbf{w}_1,F_{A1},F_{B1})=\max\left\{\widehat{w}_{B2}(\mathbf{w}_1),\mathfrak{w}_{B2}^A(\mathbf{w}_1,F_{A1}),\mathfrak{w}_{B2}^B(\mathbf{w}_1,F_{B1})\right\}.$$

In what follows, the focus of the analysis will be on the equilibrium in which $w_{Bi}>c_B$ for each $i=1,2$.[23] Keeping this in mind, consider pair-wise deviations in which M_B and R_2 set w_{B2} below w_{B2}^*. When R_1 carries two products, by doing so, M_B and R_2 may gain in two respect. First, as in Bedre (2009), decreasing w_{B2} allows them to free-ride on the contract $\mathcal{C}_{B1}$ signed earlier between M_B and R_1 and, second, it allows them to exclude M_A. The later, in particular, implies that in any equilibrium it cannot be that $w_{B2}^*=\mathfrak{w}_{B2}^A>\max\{\widehat{w}_{B2},\mathfrak{w}_{B2}^B\}$, since, by setting a wholesale price (slightly) below $\mathfrak{w}_{B2}^A$, M_B and R_2 could then induce the continuation equilibrium in which R_1 removes brand A (while still carrying brand B).[24] The next lemma states formally this result.[25]

Lemma 2 *Any equilibrium in which $w_{Bi}>c_B$ for each $i=1,2$ implies that*

$$\max\left\{\widehat{w}_{B2}(\mathbf{w}_1),\mathfrak{w}_{B2}^B(\mathbf{w}_1,F_{B1})\right\}\geq\mathfrak{w}_{B2}^A(\mathbf{w}_1,F_{A1}).$$

Proof. See Appendix *B*. ■

Thus, although both IC_{A1}^{AB} and IC_{B1}^{AB} must be satisfied in equilibrium, they serve different roles in determining the equilibrium contracts. Whereas R_1 can use F_{B1} as a tool to influence the outcome of negotiations between M_B and R_2,

[23] As it will be shown below, in the most preferred continuation equilibrium R_1, R_2 and M_B seek to maximize the total profits from selling product B and thus set the wholesale prices above costs.

[24] Assumption A4(iv) ensures the existance of such a continuation equilibrium.

[25] Hereafter it will be assumed that in equilibrium $w_{Bi}^c>c_B$ for each $i=1,2$. The exact condition that guarantees this will be stated below.

M_A is deemed to "accommodate", i.e., it sets F_{A1} so that to render its exclusion unprofitable.

As it will be shown below, while signing their contract $\mathcal{C}_{B1}$, M_B and R_1 seek to induce M_B and R_2 to set a wholesale price above $\widehat{w}_{B2}$ (in order to prevent them from free-riding on $\mathcal{C}_{B1}$). Thus, from now on one will restrict attention to the case when $\mathfrak{w}_{B2}^B \geq \widehat{w}_{B2}$[26] which implies that (using also lemma 2),

$$w_{B2}^* (\mathbf{w}_1, F_{B1}) = \mathfrak{w}_{B2}^B (\mathbf{w}_1, F_{B1}) \geq \max \left\{ \widehat{w}_{B2} (\mathbf{w}_1), \mathfrak{w}_{B2}^A (\mathbf{w}_1, F_{A1}) \right\}.$$

Assuming that w_{B2}^* is defined as above, denote by Π_{B2}^d the maximal bilateral profits that M_B and R_2 could obtain by setting their wholesale price below $\mathfrak{w}_{B2}^B$. By doing so, they would destroy the incentives of R_1 to carry both products (since IC_{B1}^{AB} is no longer satisfied) and instead induce it to carry either product A or none of the products.[27] As it will be shown below, what matters for the analysis is the bounds on Π_{B2}^d rather than its individual values.

First of all, Π_{B2}^d cannot exceed Π_{B2}^m which is the maximal bilateral profit that M_B and R_2 could obtain if, in response to their setting $w_{B2} = c_B$, R_1 would choose to carry none of the products. On the other hand, inducing R_1 to remove product A may not be possible. In particular, it is not possible if $F_{A1} = 0$. This is because the incremental contribution of product A to the total profits earned in outlet 1 is always non-negative[28] and R_1 would then carry brand A anyway. In which case the maximum bilateral profits that M_B and R_2 could jointly obtain from decreasing w_{B2} is given by:

$$\Pi_{B2} (w_{A1}) = \max_{w_{B2}} \pi_B^{R_2} (w_{A1}, \infty, w_{B2}) + \pi_2^{M_B} (w_{A1}, \infty, w_{B2}) < \Pi_{B2}^m, \tag{10}$$

where the inequality is implied by imperfect substitutability of the products. It then follows that $\Pi_{B2}^d \in [\Pi_{B2} (w_{A1}), \Pi_{B2}^m]$.

The condition that any equilibrium must be immune to all possible deviations, in particular, implies that M_B and R_2 cannot derive any gain from setting w_{B2} below $\mathfrak{w}_{B2}^B$. Since in any such deviation R_1 removes product B and thus waives its fixed payment to M_B, it would not be jointly profitable for M_B and R_2 only if:

[26] This rules out the off-equilibrium continuations which cannot affect the firms' actions taken along the equilibrium path.

[27] By applying a similar reasoning as in the proof of lemma 2, it can be shown that if M_B and R_2 set a wholesale price $w_{B2} \in (\underline{\mathfrak{w}}_{B2}^A, \mathfrak{w}_{B2}^B)$ where $\underline{\mathfrak{w}}_{B2}^A < \mathfrak{w}_{B2}^A$, the (unique) retail equilibrium implies that R_1 carries only product A. On the other hand, when w_{B2} is sufficiently small, R_1 may find it profitable to opt out and carry none of the products. In general, there is no systematic reason to rule out the situation that for some w_{B2} lying in between there exists a retail equilibrium in which R_1 carries only product B, even though it is not possible to show this without specifying demand functions. Taking this into consideration would somewhat complicate the analysis and, moreover, could make the equilibrium under study impossible to sustain (since M_B and R_2 might always profitably deviate). Hence, from now on it will be assumed that setting w_{B2} below $\mathfrak{w}_{B2}^B$ induces retail equilibria in which R_1 carries either product A or none of the products only.

[28] To see this, write IC_{A1}^{AB} taken for $F_{A1} = 0$ as:

$$\pi_A^{R_1} (w_{A1}, w_{B1}, w_{B2}) + \pi_B^{R_1} (w_{A1}, w_{B1}, w_{B2}) - \pi_B^{R_1} (\infty, w_{B1}, w_{B2}) \geq 0.$$

The left hand side of the above condition evaluated for $w_{A1} = \infty$ equals to zero while, by assumption A4(ii), it decreases in w_{A1}. Hence, this condition is satisfied for any (w_{A1}, w_{B1}, w_{B2}).

$$\widehat{\Pi}\left(\mathbf{w}_1, \mathfrak{w}_{B2}^B\right) + F_{B1} \geq \Pi_{B2}^d.$$

Using that $\mathfrak{w}_{B2}^B$ is the wholesale price for which IC_{B1}^{AB} binds, the above condition can be written as:

$$\begin{aligned} & \sum_{i=1,2}\left(\pi_B^{R_i}\left(w_{A1}, w_{B1}, \mathfrak{w}_{B2}^B\right) + \pi_i^{M_B}\left(w_{A1}, w_{B1}, \mathfrak{w}_{B2}^B\right)\right) - \Pi_{B2}^d \ \mathrm{ND}_{B2} \\ \geq \ & \pi_A^{R_1}(w_{A1}, \infty, \mathfrak{w}_{B2}^B) - \pi_A^{R_1}\left(w_{A1}, w_{B1}, \mathfrak{w}_{B2}^B\right), \end{aligned}$$

which implies that in equilibrium the increase in the *total* profits from selling product B in two rather than one retail outlet must more than offset the reduction in the profit from selling product A (because it is then sold together with product B rather than alone).[29]

4.3 Negotiations between M_B and R_1

Taking $\mathcal{C}_{A1}$ as given and anticipating the outcome of negotiations between M_B and R_1, M_B and R_1 negotiate the contract $\mathcal{C}_{B1}^{**}$ which solves the generalized Nash bargaining problem provided that the M_B's disagreement payoff is $\overline{u}^{M_B}$ (this is what M_B would get while renegotiating with R_2 if negotiations with R_1 failed) while the R_1's disagreement payoff is $\overline{u}^{R_1}$ (this is what R_1 would get while renegotiating with M_A if negotiations with M_B failed). Furthermore, if M_B and R_1 wish to induce an equilibrium continuation in which M_B and R_2 do not fail their negotiations and the retailers carry the products of all their respective suppliers, $\mathcal{C}_{B1}^{**}$ must also ensure that *(i)* M_B and R_2 obtain non-negative gains from trade and *(ii)* they cannot gain by cutting their wholesale price to below $w_{B2}^* = \mathfrak{w}_{B2}^B$. Thus, in any such equilibrium $\mathcal{C}_{B1}^{**}$ must be a solution to the following problem:

$$\begin{aligned} & \max_{\mathcal{C}_{B1}} \left(u^{R_1 *}\left(\mathcal{C}_{A1}, \mathcal{C}_{B1}\right) - \overline{u}^{R_1}\right)^{\lambda_{B1}} \left(u^{M_B *}\left(\mathcal{C}_{A1}, \mathcal{C}_{B1}\right) - \overline{u}^{M_B}\right)^{1-\lambda_{B1}}, \quad \mathcal{P}_3 \\ & s.t.\ \mathrm{GT}_{B2} \text{ and } \mathrm{ND}_{B2} \text{ hold} \end{aligned}$$

where $u^{M_B *}\left(\mathcal{C}_{A1}, \mathcal{C}_{B1}\right)$ is given by (9) while $u_1^{R*}\left(\mathcal{C}_{A1}, \mathcal{C}_{B1}\right) \equiv u_1^R\left(\mathcal{C}_{A1}, \mathcal{C}_{B1}, \mathcal{C}_{B2}^*\right)$ is given by (using (3)):

$$u^{R_1 *}\left(\mathcal{C}_{A1}, \mathcal{C}_{B1}\right) = \sum_{k=A,B}\left(\pi_k^{R_1}(\mathbf{w}_1, w_{B2}^*) - (F_{k1} + S_{k1})\right). \qquad (11)$$

Denote by $\widetilde{\Pi}(\mathbf{w})$ the joint *variable* profits of R_1, R_2 and M_B, i.e.,

$$\begin{aligned} \widetilde{\Pi}(\mathbf{w}) & \equiv \left(\pi_A^{R_1}(\mathbf{w}) + \pi_B^{R_1}(\mathbf{w})\right) + \widehat{\Pi}(\mathbf{w}) \qquad (12) \\ & = \pi_A^{R_1}(\mathbf{w}) + \sum_{i=1,2}\left(\pi_B^{R_i}(\mathbf{w}) + \pi_i^{M_B}(\mathbf{w})\right). \end{aligned}$$

[29] Note that for $w_{A1} = \infty$ the ND_{B2} costraint boils down to the one obtained in the case when R_1 carries only brand B.

which is the sum of the total profits from selling product B and the R_1's flow profit from selling product A.

For given w_{A1} and Π^d_{B2}, denote by $\mathbf{w}_B^{**} \equiv (w_{B1}^{**}, w_{B2}^{**})$ the pair of wholesale prices which maximizes $\widetilde{\Pi}(w_{A1}, \mathbf{w}_B)$ subject to the constraint that M_B and R_2 cannot gain from undercutting w_{B2}^{**}, i.e.,

$$\begin{aligned} \mathbf{w}_B^{**}\left(w_{A1}, \Pi^d_{B2}\right) &= \arg\max_{\mathbf{w}_B} \widetilde{\Pi}(w_{A1}, \mathbf{w}_B), \qquad \mathcal{P}_4 \\ & \quad s.t.\ \mathrm{ND}_{B2} \text{ holds for } \mathfrak{w}^B_{B2} = w_{B2}. \end{aligned}$$

Define the gains from trade $G^*_{B1}(\mathcal{C}_{A1}, \mathcal{C}_{B1})$ between M_B and R_1 as follows:

$$\begin{aligned} G^*_{B1}(\mathcal{C}_{A1}, \mathcal{C}_{B1}) &\equiv \frac{u^{M_B*}(\mathcal{C}_{A1}, \mathcal{C}_{B1}) - \widetilde{u}^{M_B}}{1-\lambda_{B2}} + u^{R_1*}(\mathcal{C}_{A1}, \mathcal{C}_{B1}) - \left(\overline{u}^{R_1} + \frac{\overline{u}^{M_B} - \widetilde{u}^{M_B}}{1-\lambda_{B2}}\right) \\ &= \widetilde{\Pi}(w_{A1}, \mathbf{w}_B) - (F_{A1} + S_{A1}) - \left(\overline{u}^{R_1} + \widetilde{u}^{M_B} + \frac{\overline{u}^{M_B} - \widetilde{u}^{M_B}}{1-\lambda_{B2}}\right). \end{aligned}$$

Intuitively, this is because condition (9) implies that in equilibrium M_B obtains at least the payoff $\widetilde{u}^{M_B}$ and, moreover, a unit increase in the variable part of its payoff due to its trade with R_1 increases its overall payoff by just $(1-\lambda_{B2})$. Hence, M_B can gain from trade with R_1 only if the variable part exceeds the value $\frac{\overline{u}^{M_B} - \widetilde{u}^{M_B}}{1-\lambda_{B2}}$ which can be regarded as its 'effective' disagreement payoff in bargaining with R_1. Denote by $G^{**}_{B1}(\mathcal{C}_{A1})$ the value of $G^*_{B1}(\mathcal{C}_{A1}, \mathcal{C}_{B1})$ taken for $\mathbf{w}_B = \mathbf{w}_B^{**}$, i.e.,[30]

$$G^{**}_{B1}(\mathcal{C}_{A1}) \equiv \widetilde{\Pi}(w_{A1}, \mathbf{w}_B^{**}) - (F_{A1} + S_{A1}) - \left(\overline{u}^{R_1} + \widetilde{u}^{M_B} + \frac{\overline{u}^{M_B} - \widetilde{u}^{M_B}}{1-\lambda_{B2}}\right). \quad (13)$$

Next lemma characterizes the solution to $\mathcal{P}_3$.

Lemma 3 *The solution to $\mathcal{P}_3$ exists if and only if:*

$$G^{**}_{B1}(\mathcal{C}_{A1}) \geq \max\left\{0, \frac{\widetilde{u}^{M_B} - \overline{u}^{M_B}}{1-\lambda_{B2}}\right\}, \qquad \mathrm{GT}_{B1}$$

in which case $w_{B1} = w^{**}_{B1}$ and $F_{B1} = F^{**}_{B1}$ where F^{**}_{B1} is equal to the incremental contribution of product B to the total profits earned in outlet 1 and evaluated for $\mathbf{w}_B = \mathbf{w}_B^{**}$, i.e.,

$$F^{**}_{B1} = \sum_{k=A,B} \pi_k^{R_1}(w_{A1}, \mathbf{w}_B^{**}) - \pi_A^{R_1}(w_{A1}, \infty, w^{**}_{B2}).$$

If $G^{**}_{B1}(\mathcal{C}_{A1})$ satisfies the condition,

$$G^{**}_{B1}(\mathcal{C}_{A1}) \geq \max\left\{0, \frac{\widetilde{u}^{M_B} - \overline{u}^{M_B}}{(1-\lambda_{B1})(1-\lambda_{B2})}\right\},$$

[30] To shortcut the notation, the argument $\left(w_{A1}, \Pi^d_{B2}\right)$ will be suppressed where it does not lead to any confusion.

then S_{B1}^{**} *is set so that* R_1 *and* M_B *share their gains from trade according to each party's bargaining power and obtain the following payoffs:*

$$u^{R_1**}(\mathcal{C}_{A1}) = \overline{u}^{R_1} + \lambda_{B1} G_{B1}^{**}(\mathcal{C}_{A1}), \tag{14}$$

$$u^{M_B**}(\mathcal{C}_{A1}) = \overline{u}^{M_B} + (1-\lambda_{B1})(1-\lambda_{B2}) G_{B1}^{**}(\mathcal{C}_{A1}). \tag{15}$$

If $\widetilde{u}^{M_B} > \overline{u}^{M_B}$ *and* $G_{B1}^{**}(\mathcal{C}_{A1})$ *satisfies the condition,*

$$\frac{\widetilde{u}^{M_B} - \overline{u}^{M_B}}{1-\lambda_{B2}} \leq G_{B1}^{**}(\mathcal{C}_{A1}) < \frac{\widetilde{u}^{M_B} - \overline{u}^{M_B}}{(1-\lambda_{B1})(1-\lambda_{B2})},$$

then S_{B1}^{**} *is set so that* R_2 *and* M_B *obtain zero gains from trade while* R_1 *and* M_B *obtain the following payoffs:*

$$u^{R_1**}(\mathcal{C}_{A1}) = \overline{u}^{R_1} + \left(G_{B1}^{**}(\mathcal{C}_{A1}) - \frac{\widetilde{u}^{M_B} - \overline{u}^{M_B}}{1-\lambda_{B2}}\right), \tag{16}$$

$$u^{M_B**}(\mathcal{C}_{A1}) = \widetilde{u}^{M_B}. \tag{17}$$

Proof. See Appendix C. ■

As is known in the literature, when M_B contracts sequentially with R_1 and R_2, there is always a scope for opportunistic behavior: while signing their contract, M_B and R_2 have incentives to free-ride on the contract $\mathcal{C}_{B1}$ signed earlier between M_B and R_1. Allowing R_1 to carry the products of both M_A and M_B however alters these incentives in two respects.[31] First, since products A and B are imperfect substitutes, the sales of product A reduce the total profits from selling product B, i.e.,

$$\max_{\mathbf{w}_B} \sum_{i=1,2} \left(\pi_B^{R_i}(w_{A1}, \mathbf{w}_B) + \pi_i^{M_B}(w_{A1}, \mathbf{w}_B)\right) < \Pi_{B1B2}^m.$$

This *strengthens* the bilateral incentives of M_B and R_2 to free-ride on $\mathcal{C}_{B1}$. In particular, these incentives are stronger, the stronger is the substitution effect and the larger are the joint bilateral profits Π_{B2}^d that R_2 and M_B would obtain from cutting their wholesale price to bellow $\mathfrak{w}_{B2}^B$.

Second, as long as R_1 removes product B, it seeks to recoup the losses by increasing the sales of product A (in which case it gains $\pi_A^{R_1}(w_{A1}, \infty, w_{B1}) - \pi_A^{R_1}(w_{A1}, \mathbf{w}_B)$). This *weakens* the bilateral incentives of M_B and R_2 to free-ride on $\mathcal{C}_{B1}$, since an increase in the sales of product A (through R_1) reduces the sales of product B (through R_2) and thus decreases the profit Π_{B2}^d. The ND_{B2} constraint thus determines the extent to which R_1 and M_A are able to influence the wholesale prices w_{B1} and w_{B2} through the impact of their contract on the incentives of R_2 and M_B to free-ride on $\mathcal{C}_{B1}$.

To see this formally, denote by $\widetilde{\mathbf{w}}_B(w_{A1}) \equiv (\widetilde{w}_{B1}(w_{A1}), \widetilde{w}_{B2}(w_{A1}))$ the pair of wholesale prices that R_1, R_2 and M_B would set in response to w_{A1} if the three firms acted as a single entity and coordinated their decisions on the sales of product B in both retail outlets:

[31] In the absence of M_A problem $\mathcal{P}_4$ boils down to maximizing the total profits on sales of the M_B's product in both retail outlets. In which case w_{B1}^{**} and w_{B2}^{**} would be set so as to obtain the monopoly profit Π_{B1B2}^m while ND_{B2} would per se be irrelevant since, by lowering their wholesale price, the maximum what R_2 and M_B could jointly obtain is $\Pi_{B2}^m < \Pi_{B1B2}^m$.

$$\widetilde{\mathbf{w}}_B(w_{A1}) = \arg\max_{\mathbf{w}_B} \widetilde{\Pi}\left(w_{A1}, \mathbf{w}_B\right).$$

For the sake of exposition, from now on it will be assumed that the function $\widetilde{\Pi}\left(\cdot\right)$ satisfies the following regularity conditions:

Assumption A8. *(i) The function $\widetilde{\Pi}\left(w_{A1}, \mathbf{w}_B\right)$ is quasi-concave in $\mathbf{w}_B$ for any w_{A1} and achieves its maximum for $\mathbf{w}_B = \widetilde{\mathbf{w}}_B(w_{A1})$; (ii) $\widetilde{w}_{Bi}(w_{A1}) > c_B$ for each $i = 1, 2$ and any w_{A1}, i.e., despite the sales of product A the wholesale prices above the unit costs are needed to maximize the total profits on sales of product B; (iii) $\frac{\partial \widetilde{\Pi}}{\partial w_{B1} \partial w_{B2}} < 0$ for any w_{A1}, i.e., the impact on the profit $\widetilde{\Pi}$ of a small increase in w_{Bi} is lower, the lower is $w_{Bi'}$ for $i \neq i' = 1, 2$.*

This assumption is satisfied in many standard oligopolistic models. Using assumption A4(iii), one can verify that, in response to the wholesale price $\widetilde{w}_{B1}$ set by M_B and R_1, M_B and R_2 would set the wholesale price $\widehat{w}_{B2}\left(w_{A1}, \widetilde{w}_{B1}\right) < \widetilde{w}_{B2}$ (for any given w_{A1}), i.e., free-ride on the margin of product B sold through R_1. Similar to Bedre (2009), by setting the conditional fee equal to the incremental contribution of product B to the total profits on sales in outlet 1, M_B and R_1 seek to make undercutting $\widetilde{w}_{B2}$ unprofitable for M_B and R_2. Thus, if the ND_{B2} constraint were satisfied for $\mathbf{w}_B = \widetilde{\mathbf{w}}_B(w_{A1})$, this would be an equilibrium continuation. Otherwise M_B and R_1 distort the wholesale prices $\widetilde{w}_{B1}$ and $\widetilde{w}_{B2}$ in order to reduce the incentives of M_B and R_2 to engage in opportunistic behavior.

Using (12) and rearranging the terms, the ND_{B2} constraint (taken for $\mathfrak{w}_{B2}^B = w_{B2}$) can be written as:

$$\widetilde{\Pi}\left(w_{A1}, \mathbf{w}_B\right) - \pi_A^{R_1}\left(w_{A1}, \infty, w_{B2}\right) \geq \Pi_{B2}^d. \tag{18}$$

Thus, in order for problem $\mathcal{P}_4$ to be convex, the following assumption will be made.

Assumption A9. *The function $\widetilde{\Pi}\left(w_{A1}, \mathbf{w}_B\right) - \pi_A^{R_1}\left(w_{A1}, \infty, w_{B2}\right)$ is quasi-concave in $\mathbf{w}_B$ for any $w_{A1} \geq c_A$ and achieves its maximum for $\mathbf{w}_B = \widetilde{\mathbf{w}}'_B(w_{A1})$ given by:*

$$\widetilde{\mathbf{w}}'_B(w_{A1}) = \arg\max_{\mathbf{w}_B} \widetilde{\Pi}\left(w_{A1}, \mathbf{w}_B\right) - \pi_A^{R_1}\left(w_{A1}, \infty, w_{B2}\right).$$

Moreover, the solution to the above program is such that $\widetilde{w}'_{B2}(w_{A1}) \geq c_B$ and $\widetilde{w}'_{B2}(w_{A1}) \geq \widehat{w}_{B2}(w_{A1}, \widetilde{w}'_{B1}(w_{A1}))$ for any $w_{A1} \geq c_A$.

Note that the condition $\frac{\partial \widetilde{\Pi}}{\partial w_{B1} \partial w_{B2}} < 0$ implies that $\widetilde{w}_{B1}\left(w_{A1}\right) < \widetilde{w}'_{B1}\left(w_{A1}\right)$ and $\widetilde{w}'_{B2}\left(w_{A1}\right) < \widetilde{w}_{B2}\left(w_{A1}\right)$. Solving $\mathcal{P}_4$ while taking into account that the ND_{B2} constraint can be binding leads to the following result.

Lemma 4 *The solution to $\mathcal{P}_4$ implies that $\widetilde{w}_{B1}\left(w_{A1}\right) \leq w_{B1}^{**}\left(w_{A1}, \Pi_{B2}^d\right) \leq \widetilde{w}'_{B1}\left(w_{A1}\right)$ and $\widetilde{w}'_{B2}\left(w_{A1}\right) \leq w_{B2}^{**}\left(w_{A1}, \Pi_{B2}^d\right) \leq \widetilde{w}_{B2}\left(w_{A1}\right)$ for any $\left(w_{A1}, \Pi_{B2}^d\right)$. Moreover, $w_{B1}^{**}\left(w_{A1}, \Pi_{B2}^d\right)$ is non-decreasing while $w_{B2}^{**}\left(w_{A1}, \Pi_{B2}^d\right)$ is non-increasing in Π_{B2}^d.*

Proof. See Appendix D. ■

Lemma 4 thus determines the impact of the incentives of M_B and R_2 to free-ride on their rivals' margins on the the wholesale prices $\widetilde{w}_{B1}$ and $\widetilde{w}_{B2}$ maximizing the joint profits of R_1, R_2 and M_B in the presence of sales of product A.

Intuitively, R_2 and M_B have stronger incentives to so, the more they can gain from undercutting w_{B2}^{**} (i.e., the larger the value of Π_{B2}^d is). This calls for a reduction in w_{B2}^{**} (i.e., setting $w_{B2}^{**} < \widetilde{w}_{B2}$) and an increase in w_{B1}^{**} (i.e., setting $w_{B1}^{**} > \widetilde{w}_{B1}$).

Assumptions A8 and A9 guarantee that any solution $(w_{B1}^{**}, w_{B2}^{**})$ to $\mathcal{P}_4$ satisfies the condition $w_{Bi}^{**} > c_B$ for each $i = 1, 2$. It then immediately follows:

Corollary *In any candidate equilibrium in which the retailers carry the products of all their respective suppliers the wholesale prices w_{B1}^{**} and w_{B2}^{**} must maximize the joint profits of R_1, R_2 and M_B subject to the constraint that M_B and R_2 cannot gain by cutting their wholesale price to below w_{B2}^{**}.*

Since $\widetilde{\Pi}(\mathbf{w}) = \Pi(\mathbf{w}) - \pi_1^{M_A}(\mathbf{w})$, then, using assumption A3(ii) and a simple revealed preference argument, one can verify that the best reply to w_{A1}^m of R_1, R_2 and M_B acting as a single entity would be their setting $\widetilde{w}_{Bi}(w_{A1}^m) < w_{Bi}^m$ for each $i = 1, 2$. Taken with lemma 4, this leads to:

Proposition 5 *In all subgame perfect equilibria of the game G the fully integrated monopoly outcome cannot be implemented, i.e., $w_{ki} \neq w_{ki}^m$ at least for some $ki = \{A1, B1, B2\}$.*

Intuitively, the fact that the supply contracts are three-part tariffs and non-binding allows M_B to suppress intrabrand competition and thus coordinate the decisions of M_B, R_1 and R_2 so as to maximize their joint trilateral profits from selling product B.[32] Since in doing so the three firms fail to account for the impact of their decisions on the M_A's profit, they have incentives to free-ride on its product's margin. Furthermore, given that product A competes with product B in *both* retail outlets, M_B is tempting to convince both R_1 and R_2 to undercut its rival. Since in its turn M_A can reduce the incentives of *only* one retailer to accept a discount, it is not able to fully prevent M_B from free-riding on its margin. As a result, the fully integrated monopoly outcome fails to exist.

Consider now (pair-wise) deviations of R_1 and M_B. First, they may break down their negotiations and get their disagreement payoffs. As long as R_1 and M_B obtain at least non-negative gains from trade (which is necessary to sustain the equilibrium under study), they could never gain from such a deviation.

Second, R_1 and M_B may induce break-down negotiations between M_B and R_2.[33] This would trigger renegotiation (from scratch) of all the contracts signed earlier. Since in that case M_B and R_2 can no longer negotiate, the equilibrium continuation is characterized by proposition 3 which implies that R_1 would obtain the payoff $\widetilde{u}^{R_1}$ while M_B would obtain the payoff $\widetilde{u}^{M_B}$. Thus, neither R_1 nor M_B could gain from such a deviation only if the following two conditions hold:

$$
\begin{aligned}
u^{R_1**}(\mathcal{C}_{A1}) &\geq \widetilde{u}^{R_1}, && (19)\\
u^{M_B**}(\mathcal{C}_{A1}) &\geq \widetilde{u}^{M_B}. && (20)
\end{aligned}
$$

[32] As in Bedre (2010), the conditional fee F_{B1}^{**} serves to protect R_1 from the opportunistic behavior of M_B while renegotiation allows the firms to eliminates contractual externalities (due to sequential contracting).

[33] Since the gains from trade between M_B and R_2 are given by $\max\{\widehat{\Pi} + F_{B1}, \Pi_{B2}^d\} + S_{B1} - \widetilde{u}^{M_B}$, they are always negative for some $S_{B1} < 0$.

Note that condition (9) implies that M_B obtains the payoff which is at least as large as $\widetilde{u}^{M_B}$ (since in any equilibrium M_B and R_2 must obtain non-negative gains from trade). As a result, in equilibrium condition (20) is always satisfied and thus can be omitted in the subsequent analysis.

Finally, R_1 and M_B may deviate by inducing the retail equilibrium in which R_1 does not carry product A while both retailers carry product B. Note that such a deviation may not always be possible as long as the incremental contribution of product A is non-negative.[34] As before, it is not possible if M_A demands no conditional payment from R_1, i.e., sets $F_{A1} = 0$. Denote by $\Pi^d_{B1}(w_{A1}, F_{A1})$ the maximal profit that M_B, R_1 and R_2 could jointly obtain by excluding M_A and let $\Pi^d_{B1}(w_{A1}, F_{A1}) = \underline{\Pi}^d_{B1} \leq 0$[35] in case they could not do so.[36] Since exclusion of M_A allows the three firms to save on the fixed payment F_{A1}, they would not gain from such a deviation only if:

$$\widetilde{\Pi}(w_{A1}, \mathbf{w}^{**}_B) - \Pi^d_{B1}(w_{A1}, F_{A1}) \geq F_{A1}. \qquad \mathrm{ND}_{B1}$$

This condition is analogous to the one obtained in case when M_B is restricted to distribute its product only through R_1. In particular, it implies that the conditional payment to M_A must not exceed the incremental contribution of its product to the joint trilateral profits of M_B, R_1 and R_2.

4.4 Negotiations between M_A and R_1

Anticipating all the contracts that will be signed afterwards, M_A and R_1 negotiate the contract $\mathcal{C}^{***}_{A1}$ which solves the generalized Nash bargaining problem provided that the M_A's disagreement payoff is zero (once negotiations between M_A and R_1 break down, the parties cannot renegotiate at another time) while the R_1's disagreement payoff is $\widehat{u}^{R_1}$ (this is what R_1 would get while renegotiating with M_B if negotiations with M_A failed). The analysis so far implies that if M_A and R_1 wish to induce the continuation equilibrium in which neither R_1 and M_B nor R_2 and M_B break down their negotiations, $\mathcal{C}^{***}_{A1}$ must also satisfy the following four conditions: *(i)* R_1, R_2 and M_B cannot exclude M_A by setting w_{B2} just below $w^{**}_{B2}(w_{A1}, \Pi^d_{B2})$, i.e., it must be that,

$$\mathfrak{w}^A_{B2}\left(w_{A1}, w^{**}_{B1}\left(w_{A1}, \Pi^d_{B2}\right), F_{A1}\right) \leq w^{**}_{B2}\left(w_{A1}, \Pi^d_{B2}\right), \qquad \mathrm{NE}_A$$

where Π^d_{B2} is a function of (w_{A1}, F_{A1}), *(ii)* M_B and R_1 obtain at least non-negative gains from trade, i.e., GT_{B1} must be satisfied, *(iii)* while negotiating with M_B, R_1 have no incentives to opt for break-down of negotiations between M_B and R_2, i.e., (19) must be satisfied, and *(iv)* M_B and R_1 cannot gain from excluding M_A, i.e., ND_{B1} must be satisfied.

Lemma 3 implies that depending on the values of $\widetilde{u}^{M_B}$ and $\overline{u}^{M_B}$, different continuation equilibria are possible. Thus, for the sake of exposition only, the following assumption will be made.[37]

[34] See footnote ?

[35] Setting $\underline{\Pi}^d_{B1} \leq 0$ is meant to satisfy the ND_{B1} constraint whenever exclusion of M_A is not possible.

[36] Since the contracts are three-part tariffs and non-binding, then, even by deviating to excusion of M_A, R_1 and M_B will seek to maximize the joint profit of M_B, R_1 and R_2.

[37] It can be shown that the analysis of the opposite case where $\widetilde{u}^M_{B1} > \overline{u}^M_{B2}$ will result simply in a different division of the gains from trade between M_A and R_1.

Assumption A10. $\overline{u}^{M_B} \geq \widetilde{u}^{M_B}$.

Under this assumption, GT_{B1} boils down to $G^{**}_{B1}(\mathcal{C}_{A1}) \geq 0$. Furthermore, A10 implies that in all continuation equilibria $u_1^{R**}(\mathcal{C}_{A1})$ is given by (14) which after substituting into (19) and rearranging the terms yields:

$$G^{**}_{B1}(\mathcal{C}_{A1}) \geq \frac{\widetilde{u}^{R_1} - \overline{u}^{R_1}}{\lambda_{B1}}. \qquad \mathrm{GT}^{**}_{B1}$$

Propositions 2 and 3 and the fact that $\Pi^m_{A1} > \overline{\pi}_{A1}(\overline{w}_{A1}, \overline{w}_{B2})$ imply that $\widetilde{u}^{R_1} > \overline{u}^{R_1}$ meaning that GT^{**}_{B1} is stronger than GT_{B1}. Taken all together, this implies that in any equilibrium under study $\mathcal{C}^{***}_{A1}$ must be a solution to the following problem:

$$\begin{aligned}
&\max_{\mathcal{C}_{A1}} \left(u^{R_1**}(\mathcal{C}_{A1}) - \widehat{u}^{R_1}\right)^{\lambda_{A1}} \left(u^{M_A**}(\mathcal{C}_{A1})\right)^{1-\lambda_{A1}}, \qquad \mathcal{P}_5\\
&s.t.\ \mathrm{GT}^{**}_{B1},\ \mathrm{NE}_A \text{ and } \mathrm{ND}_{B1} \text{ hold}
\end{aligned}$$

where $u^{R_1**}(\mathcal{C}_{A1})$ is given by (14) while $u^{M_A**}(\mathcal{C}_{A1})$ is given by:

$$u^{M_A**}(\mathcal{C}_{A1}) = \pi_1^{M_A}(w_{A1}, \mathbf{w}^{**}_B) + (F_{A1} + S_{A1}). \qquad (21)$$

Denote by $(w^{***}_{A1}, F^{***}_{A1})$ the conditional payment which maximizes the total industry profit subject to the constraint that in the subsequent play of the game R_1, R_2 and M_B acting as a single entity cannot jointly gain from excluding M_A, i.e.,[38]

$$\begin{aligned}
(w^{***}_{A1}, F^{***}_{A1}) &= \arg\max_{w_{A1}, F_{A1}} \Pi(w_{A1}, \mathbf{w}^{**}_B(w_{A1}, \Pi^d_{B2})) \qquad \mathcal{P}_6\\
&\quad s.t.\ \mathrm{NE}_A \text{ and } \mathrm{ND}_{B1} \text{ hold.}
\end{aligned}$$

Note that condition (14) implies that in equilibrium R_1 obtains at least the payoff $\overline{u}^{R_1}$ and, moreover, a unit increase in the variable part of its payoff due to the trade with M_A increases its overall payoff by just λ_{B1}. This implies that R_1 can gain from trade with M_A only if the variable part exceeds the value $\frac{\widehat{u}^{R_1} - \overline{u}^{R_1}}{\lambda_{B1}}$. Hence, as before, the gains from trade $G^{**}_{A1}(\mathcal{C}_{A1})$ between M_A and R_1 can be defined as follows (using (13) and $\Pi(\mathbf{w}) = \widetilde{\Pi}(\mathbf{w}) + \pi_1^{M_A}(\mathbf{w})$):

$$\begin{aligned}
G^{**}_{A1}(\mathcal{C}_{A1}) &\equiv \frac{u^{R_1**}(\mathcal{C}_{A1}) - \overline{u}^{R_1}}{\lambda_{B1}} + u^{M_A**}(\mathcal{C}_{A1}) - \frac{\widehat{u}^{R_1} - \overline{u}^{R_1}}{\lambda_{B1}}\\
&= \Pi(w_{A1}, \mathbf{w}^{**}_B) - d_{A1},
\end{aligned}$$

where

$$d_{A1} \equiv \left(\overline{u}^{R_1} + \frac{\widehat{u}^{R_1} - \overline{u}^{R_1}}{\lambda_{B1}}\right) + \left(\widetilde{u}^{M_B} + \frac{\overline{u}^{M_B} - \widetilde{u}^{M_B}}{1 - \lambda_{B2}}\right), \qquad (22)$$

which is the sum of the disagreement payoffs of all the parties involved in a series of pair-wise negotiations. Denote by G^{***}_{A1} the value of $G^{**}_{A1}(\mathcal{C}_{A1})$ taken for $(w_{A1}, F_{A1}) = (w^{***}_{A1}, F^{***}_{A1})$, i.e., $G^{***}_{A1} \equiv \Pi^{***} - d_{A1}$.

[38] Here, it is implied that Π^d_{B2} is a function of (w_{A1}, F_{A1}).

Next lemma characterizes the solution to $\mathcal{P}_5$.

Lemma 5 *The solution to $\mathcal{P}_5$ exists if and only if:*

$$G_{A1}^{***} \geq \max\left\{0, \frac{\widetilde{u}^{R_1} - \widehat{u}^{R_1}}{\lambda_{B1}}\right\}, \qquad \mathrm{GT}_{A1}$$

in which case $(w_{A1}, F_{A1}) = (w_{A1}^{***}, F_{A1}^{***})$. If GT_{A1}^{***} satisfies the condition,

$$G_{A1}^{***} \geq \max\left\{0, \frac{\widetilde{u}^{R_1} - \widehat{u}^{R_1}}{\lambda_{A1}\lambda_{B1}}\right\}, \tag{23}$$

then S_{A1}^{***} is set so that R_1 and M_A share their gains from trade according to each party's bargaining power; the equilibrium continuation then implies that M_A, R_1, M_B and R_2 obtain the payoffs,

$$u^{M_A} = (1 - \lambda_{A1})\, G_{A1}^{***}, \tag{24}$$
$$u^{R_1} = \widehat{u}^{R_1} + \lambda_{A1}\lambda_{B1}G_{A1}^{***}, \tag{25}$$
$$u^{M_B} = \overline{u}^{M_B} + (1 - \lambda_{B1})(1 - \lambda_{B2}) \left[\frac{\widehat{u}^{R_1} - \overline{u}^{R_1}}{\lambda_{B1}} + \lambda_{A1}G_{A1}^{***}\right], \tag{26}$$
$$u^{R_2} = (1 - \lambda_{B1})\lambda_{B2} \left[\frac{\overline{u}^{M_B} - \widetilde{u}^{M_B}}{(1 - \lambda_{B1})(1 - \lambda_{B2})} + \frac{\widehat{u}^{R_1} - \overline{u}^{R_1}}{\lambda_{B1}} + \lambda_{A1}G_{A1}^{***}\right] \tag{27}$$

If $\widetilde{u}^{R_1} > \widehat{u}^{R_1}$ and G_{A1}^{***} satisfies the condition,

$$\frac{\widetilde{u}^{R_1} - \widehat{u}^{R_1}}{\lambda_{B1}} \leq G_{A1}^{***} < \frac{\widetilde{u}^{R_1} - \widehat{u}^{R_1}}{\lambda_{A1}\lambda_{B1}}, \tag{28}$$

then S_{A1}^{***} is set so that the GT_{B1}^{**} constraint is binding; the equilibrium continuation then implies that R_1 obtains the payoff $u^{R_1} = \widetilde{u}^{R_1}$ while M_A, M_B and R_2 obtain the payoffs,

$$u^{M_A} = G_{A1}^{***} - \frac{\widetilde{u}^{R_1} - \widehat{u}^{R_1}}{\lambda_{B1}}, \tag{29}$$
$$u^{M_B} = \overline{u}^{M_B} + (1 - \lambda_{B1})(1 - \lambda_{B2}) \frac{\widetilde{u}^{R_1} - \overline{u}^{R_1}}{\lambda_{B1}}, \tag{30}$$
$$u^{R_2} = (1 - \lambda_{B1})\lambda_{B2} \left[\frac{\overline{u}^{M_B} - \widetilde{u}^{M_B}}{(1 - \lambda_{B1})(1 - \lambda_{B2})} + \frac{\widetilde{u}^{R_1} - \overline{u}^{R_1}}{\lambda_{B1}}\right]. \tag{31}$$

Proof. See Appendix *E*. ■

An immediate implication of lemma 5 is that in any equilibrium under study the GT_{A1} constraint must necessarily hold. Since in general there is no systematic reason for this to be the case, we have:[39]

[39] As an example, choose λ_{A1} so as to satisfy the condition $\Pi_{B1}^m + \lambda_{A1}\overline{\pi}_{A1}(\overline{w}_{A1}, \overline{w}_{B2}) < \Pi_{B1B2}^m$ (such λ_{A1} always exists since $\Pi_{B1B2}^m > \Pi_{B1}^m$). For such λ_{A1} choose λ'_{B1} so as to satisfy the condition $\widetilde{\Delta} = 0$ (this is always possible since $\widetilde{\Delta}$ is non-decreasing in λ_{B1} and equals to zero for λ_{B1} sufficiently small). Set $\widehat{\Delta} = 0$. Using this and the definitions of $\overline{u}^{M_B}$, $\widetilde{u}^{M_B}$, $\widehat{u}^{R_1}$ and $\overline{u}^{R_1}$ (see propositions 1, 2 and 4), it can be verified that,

Corollary *In a framework of sequential contracting with competition being present both upstream and downstream there do not always exist equilibria in which the retailers carry the products of all their respective suppliers.*

Note that this result holds regardless of whether the equilibrium outcome is efficient or not and is due entirely to multilateral contracting. More precisely, this is because any party involved in negotiations with two (or more) counterparties cannot fully appropriate the benefits of individual trade with each of them. This effectively increases that party's outside option of failing some negotiation(s) which in turn makes it somewhat difficult to sustain the equilibrium in which all the trade links are active.[40],[41]

Another implication of lemma 5 is that the variable part (w_{A1}, F_{A1}) of the equilibrium contract between M_A and R_1 must be a solution to $\mathcal{P}_6$. Since it is somewhat difficult to solve $\mathcal{P}_6$ for general demand functions, in what follows two polar cases will be considered.

Suppose first that the ND_{B2} constraint is never binding, i.e., it is satisfied for any vector $(w_{A1}, \widetilde{\mathbf{w}}_B(w_{A1}))$ and any Π^d_{B2}. This is likely to occur when the degree of interbrand rivalry is low, i.e., when the sales of product B (through both retail outlets) have a small impact on the sales of product A. Anticipating that, in response to w_{A1}, R_1, R_2 and M_B will set the wholesale prices $\mathbf{w}^{**}_B = \widetilde{\mathbf{w}}_B(w_{A1})$, R_1 and M_A then set the wholesale price,

$$\widetilde{w}_{A1} = \arg\max_{w_{A1}} \Pi(w_{A1}, \widetilde{\mathbf{w}}_B(w_{A1})),$$

and choose the conditional payment $\widetilde{F}_{A1}$ so as to satisfy the NE_A and ND_{B1} constraints taken for $w_{A1} = \widetilde{w}_{A1}$ and $\mathbf{w}_B = \widetilde{\mathbf{w}}_B(\widetilde{w}_{A1})$. This is similar to the case when the manufacturers can distribute their products only through R_1. In both cases M_A cannot alter the decision(s) of its rival (through the conditional fee that it can extract from R_1) to set the wholesale price(s) which are not the best response(s) to w_{A1}. Instead it chooses F_{A1} so as to render its exclusion unprofitable for M_B.

Suppose now that the ND_{B2} constraint is always binding which, in particular, implies that R_1, R_2 and M_B set $\mathbf{w}_B = \mathbf{w}^{**}_B\left(w_{A1}, \Pi^d_{B2}\right)$ where $\mathbf{w}^{**}_B\left(w_{A1}, \Pi^d_{B2}\right)$ is a solution to $\mathcal{P}_4$. Consider now a marginal change in the total profits $d\Pi$ following a small increase in Π^d_{B2} while keeping w_{A1} constant. Since $\mathbf{w}^{**}_B\left(w_{A1}, \Pi^d_{B2}\right)$ is smooth differentiable in Π^d_{B2}, we have:

$$d_{A1} = \overline{\pi}_{B2}(\overline{w}_{A1}, \overline{w}_{B2}) + \frac{\Pi^m_{B1B2} - (1-\lambda_{B1})\left(\Pi^m_{B1} + \lambda_{A1}\overline{\pi}_{A1}(\overline{w}_{A1}, \overline{w}_{B2})\right)}{\lambda_{B1}}.$$

It then follows that for any $\Pi^{***} > 0$ there always exists λ''_{B1} (sufficiently small) such that $d_{A1} > \Pi^{***}$. Take $\lambda_{B1} \le \min\{\lambda'_{B1}, \lambda''_{B1}\}$. To complete, for such λ_{B1} choose λ_{B2} so as to satisfy the condition $\widehat{\Delta} = 0$ (which is always possible since $\widehat{\Delta}$ is non-increasing in λ_{B2} and equals to zero for λ_{B2} sufficiently close to one).

[40]For example, in any equilibrium in which R_1 purchases from both manufacturers, it receives just $\lambda_{A1}\lambda_{B1}$ of every unit increase in the gains from trade with M_A while the rest $\lambda_{A1}(1-\lambda_{B1})$ benefits M_B. Consequently, the value of its outside option rises by $1/\lambda_{B1}$ in its bargaining with M_A.

[41]A similar effect is also present in situations where competition exists at only one level (e.g., as in Bedre (2009)), but there it is offset by an increase in total profits when all firms stay active.

$$d\Pi(w_{A1}, \mathbf{w}_B^{**}) = \left(\frac{\partial \Pi(w_{A1}, \mathbf{w}_B^{**})}{\partial w_{B1}} \frac{\partial w_{B1}^{**}}{\partial \Pi_{B2}^d} + \frac{\partial \Pi(w_{A1}, \mathbf{w}_B^{**})}{\partial w_{B2}} \frac{\partial w_{B2}^{**}}{\partial \Pi_{B2}^d} \right) d\Pi_{B2}^d.$$

As it is shown in the proof of lemma 4, the solution to $\mathcal{P}_4$ must necessarily satisfy the condition $\frac{\partial \widetilde{\Pi}(w_{A1}, \mathbf{w}_B^{**})}{\partial w_{B1}} = 0$. Differentiating this condition w.r.t. Π_{B2}^d and rearranging the terms yields:

$$\frac{\partial w_{B1}^{**}}{\partial \Pi_{B2}^d} = -\frac{\dfrac{\partial^2 \widetilde{\Pi}(w_{A1}, \mathbf{w}_B^{**})}{\partial w_{B1} \partial w_{B2}}}{\dfrac{\partial^2 \widetilde{\Pi}(w_{A1}, \mathbf{w}_B^{**})}{\partial w_{B1}^2}} \frac{\partial w_{B2}^{**}}{\partial \Pi_{B2}^d}.$$

Lemma 4 implies that $\frac{\partial w_{B2}^{**}}{\partial \Pi_{B2}^d} < 0$ while assumption A8 implies that $\frac{\partial^2 \widetilde{\Pi}}{\partial w_{B1}^2} < 0$. It then follows that $d\Pi(w_{A1}, \mathbf{w}_B^{**}) < 0$ if and only if:

$$\left(\frac{\partial^2 \widetilde{\Pi}(\mathbf{w})}{\partial w_{B1} \partial w_{B2}} \frac{\partial \Pi(\mathbf{w})}{\partial w_{B1}} - \frac{\partial^2 \widetilde{\Pi}(\mathbf{w})}{\partial w_{B1}^2} \frac{\partial \Pi(\mathbf{w})}{\partial w_{B2}} \right)_{\mathbf{w}=(w_{A1}, \mathbf{w}_B^{**})} > 0. \tag{32}$$

Using the first order conditions to problem $\mathcal{P}_4$[42] and $\Pi(\mathbf{w}) = \widetilde{\Pi}(\mathbf{w}) + \pi_1^{M_A}(\mathbf{w})$, one obtains:

$$\begin{aligned} \frac{\partial \Pi(w_{A1}, \mathbf{w}_B^{**})}{\partial w_{B1}} &= \frac{\partial \pi_1^{M_A}(w_{A1}, \mathbf{w}_B^{**})}{\partial w_{B1}}, \\ \frac{\partial \Pi(w_{A1}, \mathbf{w}_B^{**})}{\partial w_{B2}} &= \frac{\partial \pi_1^{M_A}(w_{A1}, \mathbf{w}_B^{**})}{\partial w_{B2}} + \frac{\psi}{1+\psi} \frac{\partial \pi_A^{R_1}(w_{A1}, \infty, w_{B2}^{**})}{\partial w_{B2}}, \end{aligned}$$

where $\psi \geq 0$ is the Lagrange multiplier to the ND_{B2} constraint in $\mathcal{P}_4$. Plugging the above conditions into (32) and using that $\frac{\partial \pi_A^{R_1}(w_{A1}, \infty, w_{B2})}{\partial w_{B2}} > 0$ (which is implied by assumption A4(iii)) yields:

$$\begin{aligned} & \left(\frac{\partial^2 \widetilde{\Pi}(\mathbf{w})}{\partial w_{B1} \partial w_{B2}} \frac{\partial \Pi(\mathbf{w})}{\partial w_{B1}} - \frac{\partial^2 \widetilde{\Pi}(\mathbf{w})}{\partial w_{B1}^2} \frac{\partial \Pi(\mathbf{w})}{\partial w_{B2}} \right)_{\mathbf{w}=(w_{A1}, \mathbf{w}_B^{**})} \\ \geq & \left(\frac{\partial^2 \widetilde{\Pi}(\mathbf{w})}{\partial w_{B1} \partial w_{B2}} \frac{\partial \pi_1^{M_A}(\mathbf{w})}{\partial w_{B1}} - \frac{\partial^2 \widetilde{\Pi}(\mathbf{w})}{\partial w_{B1}^2} \frac{\partial \pi_1^{M_A}(\mathbf{w})}{\partial w_{B2}} \right)_{\mathbf{w}=(w_{A1}, \mathbf{w}_B^{**})}. \end{aligned}$$

Since $\pi_1^{M_A}(\mathbf{w}) = (w_{A1} - c_A)\, q_{A1}(\mathbf{w})$, then, provided that $w_{A1} > c_A$, the condition,

$$\frac{\partial^2 \widetilde{\Pi}(\mathbf{w})}{\partial w_{B1} \partial w_{B2}} \frac{\partial q_{A1}(\mathbf{w})}{\partial w_{B1}} - \frac{\partial^2 \widetilde{\Pi}(\mathbf{w})}{\partial w_{B1}^2} \frac{\partial q_{A1}(\mathbf{w})}{\partial w_{B2}} > 0, \tag{33}$$

ensures that $d\Pi(w_{A1}, \mathbf{w}_B^{**}) < 0$.

In the linear demand function model there always exist parameter values for which condition (33) is satisfied. In particular, it is satisfied when the degree of

[42] See Appendix *D* for more details.

interbrand rivalry between the retailers is high, i.e., when the sales of product B through R_2 exert competitive pressure on the sales of product A through R_1.[43]

Define the wholesale prices $\underline{\mathbf{w}}_B^{**}(w_{A1})$ that would be set by M_B, R_1 and R_2 acting as a single entity in a situation where M_B and R_2 have minimal incentives to deviate, i.e., $\underline{\mathbf{w}}_B^{**}(w_{A1})$ solves $\mathcal{P}_4$ for $\Pi_{B2}^d = \Pi_{B2}(w_{A1})$ where $\Pi_{B2}(w_{A1})$ is given by (10). Denote by $\underline{w}_{A1}$ the wholesale price which maximizes the total profits provided that M_B, R_1 and R_2 set $\mathbf{w}_B = \underline{\mathbf{w}}_B^{**}(w_{A1})$, i.e.,

$$\underline{w}_{A1} = \arg\max_{w_{A1}} \Pi(w_{A1}, \underline{\mathbf{w}}_B^{**}(w_{A1})).$$

Assumption A11. $\underline{w}_{A1} > c_A$.

Note that the condition $d\Pi(w_{A1}, \mathbf{w}_B^{**}) < 0$ implies that it is optimal to set Π_{B2}^d at its lowest level. Since Π_{B2}^d is minimal for $F_{A1} = 0$ (for any given w_{A1}) and the NE_A and ND_{B1} constraints are satisfied for $F_{A1} = 0$, we can state:

Proposition 6 *Suppose that condition* (33) *is satisfied and assumption A11 holds. Then, any equilibrium in which both retailers carry the products of all their respective suppliers implies that M_A demands no conditional payment from R_1, i.e., it sets $F_{A1} = 0$. Moreover, the equilibrium wholesale prices are given by $w_{A1} = \underline{w}_{A1}$ and $w_{Bi} = \underline{w}_{Bi}^{**}(\underline{w}_{A1})$ for each $i = 1, 2$.*

Thus, when the intensity of interbrand rivalry between the retailers is strong, M_A and R_1 seek to minimize the incentives of M_B and R_2 to decrease their wholesale price. Intuitively, since w_{A1} and w_{B1} are typically strategic complements while w_{A1} and w_{B2} are typically strategic substitutes,[44] M_A and R_1 would jointly prefer M_B and R_2 to set a higher wholesale price (and thus contract the sales of product B at the second store). More precisely, M_A would prefer them to do so because this would lessen the competitive pressure on its own product and, moreover, allow for more coordination of the sales through R_1.[45] In its turn, R_1 would prefer M_B to have less incentives to free-ride on their contract because this would make it more tractable while negotiating price concessions. By setting its conditional fee equal to zero, M_A ensures that M_B will not be able to exclude it anyway. As long as the two brands compete vigorously while being sold through different retailers, this decreases demand for the M_B's product at the second store which in turn leads it to gain less from giving variable discounts to R_2. As a result, by securing the sales of product A, R_1 and M_A per se reduce the benefits that M_B could derive from distributing its product

[43] For example, in a linear model the inverse demand functions for products $A1$, $B1$ and $B2$ are given by $p_{A1} = 1 - q_{A1} - \beta q_{B1} - \gamma q_{B2}$, $p_{B1} = 1 - q_{B1} - \beta q_{A1} - \alpha q_{B2}$ and $p_{B2} = 1 - q_{B2} - \alpha q_{B1} - \gamma q_{A1}$, respectively, where $\alpha, \beta, \gamma \in (0, 1)$. The parameters α and β measure the degree of intrabrand and interbrand rivalry, respectively, while γ reflects interbrand rivalry between R_1 and R_2, i.e., the demand effect of the rival brand sold at the rival retailer. Assuming quantity competition between the retailers, it can be verified that (33) then boils down to:

$$\begin{aligned} & (\gamma - \alpha\beta)\left[\left(4 - \gamma^2\right)\left(4(1 - \beta^2) - \left(\alpha^2 - 2\alpha\beta\gamma + \gamma^2\right)\right) - 8(\alpha - \gamma\beta)^2\right] \\ > \; & 2\left(4\beta - \alpha\gamma\right)\left(\alpha - \beta\gamma\right)\left(\alpha^2 - 2\beta\alpha\gamma + \gamma^2\right), \end{aligned}$$

which is satisfied for γ sufficiently large.

[44] See Assumption A6.

[45] This is because, in response to an increase in w_{B2}, M_A and M_B would lower their wholesale prices for R_1.

through R_2 and thus induce it to accept a lower wholesale price in negotiations with R_1 and set a higher wholesale price in negotiations with R_2.

Moreover, as it is shown in the proof of lemma 5, the unconditional payment by R_1 is then equal to (using that $F_{A1} = 0$):

$$S_{A1}^{***} = \begin{cases} (1 - \lambda_{A1})\, G_{A1}^{***} - \pi_{A1}^{M***}, & \text{if (23) holds} \\ (G_{A1}^{***} - \frac{\widetilde{u}^{R_1} - \widehat{u}^{R_1}}{\lambda_{B1}}) - \pi_{A1}^{M***}, & \text{if (28) holds} \end{cases}.$$

The above condition implies that it is possible to have $S_{A1}^{***} < 0$. For example, this may happen when M_A has a weak bargaining power, i.e., when λ_{A1} is sufficiently large. Alternatively, this may happen when the incremental contribution of the M_A's product to the joint profits of M_B, R_1 and R_2 is sufficiently small, i.e.,

$$\left(G_{A1}^{***} - \frac{\widetilde{u}^{R_1} - \widehat{u}^{R_1}}{\lambda_{B1}} \right) - \pi_{A1}^{M***} = \widetilde{\Pi}^{***} - \left(d_{A1} + \frac{\widetilde{u}^{R_1} - \widehat{u}^{R_1}}{\lambda_{B1}} \right) < 0.$$

In either case M_A does not only demand any fixed payment from R_1 but itself pays to R_1 for distributing its product. The intuition is that M_B, R_1 and R_2 may obtain a higher value of their joint trilateral profits by selling only product B in both retail outlets. Hence, M_A may be willing to pay upfront in order to compensate the three firms for the negative impact of the sales of its product on the sales of product B.

5 Conclusion

The paper highlights the role of vertical contracts in a situation where competition exists between both upstream and downstream firms, and both sides wield some bargaining power. As mentioned earlier, much of the literature on vertical contracting assumes that one of the markets is effectively monopolized. Yet, there is a plenty of evidence showing that competition is present in an increasing number of markets, though it may take place between a small number of strategic players. Moreover, many industries are better characterized as successive oligopolies with "interlocking relationships". This raises the issue of whether the results obtained in the literature will carry over to a more sophisticated environment in which upstream firms have alternatives in distributing their products and conversely downstream firms have alternatives in choosing their sources of supply. The present paper makes a start in this direction.

More precisely, it develops the framework of sequential bilateral negotiations between two rival manufacturers and two competing retailers with only one manufacturer negotiating with both retailers and only one retailer purchasing from both manufacturers. The main focus of the analysis is on the contracts that include an upfront payment and a quantity discount (i.e., two-part tariff), since they are common in many industries characterized by vertical relationships. In contrast to the extant literature, the paper shows that even if such contracts are renegotiable, they are no longer sufficient to maintain retail prices at monopoly levels. Moreover, the market outcome does not necessarily involve that all trade links are active, i.e., it can be that in all equilibria retailers carry the brands of some manufacturers only. The paper also sheds some light on the role of slotting allowances in multilateral vertical contracting. More precisely, it extends the result obtained in the literature that a manufacturer may use them as a means to dampen intrabrand rivalry to a more general setting. On the other hand, it provides a new result that a manufacturer having a smaller distribution network may use slotting fees as a means of compensation for the negative impact of the sales of its product on the total profits from selling the product of its rival having a larger distribution network. Thereby, it formalizes the idea that slotting allowances may be paid for having a manufacturer's product to remain on the retailer's shelves.

Although the analysis has been performed for a given order of negotiations, the qualitative insights seem robust to alternative specifications of the order of negotiations. My conjecture is that even in those cases the firms will still fail to maintain prices at monopoly levels (provided that the three links are active).[46] This is because three-part tariffs do not suffice to eliminate interbrand rivalry, in particular, between different retailers which provides each manufacturer with the incentives to free-ride on the margin(s) of its rival. Likewise, it will not always be the case that all links are active in equilibrium. Because of contractual externalities a firm negotiating with two (or more) counterparties cannot fully appropriate the benefits of individual trade with each of them. Consequently, regardless of the order of negotiations it may have incentives to fail some of them.

The results have important implications for competition authorities. In particular, they suggest that the impact of slotting allowances may be less anti-competitive when competition exists both upstream and downstream. This is more relevant for situations where the intensity of interbrand rivalry between retailers is strong. The point is that even if each manufacturer can suppress rivalry between retailers carrying its own brand (by means of three-part tariffs) and in its turn each retailer (acting as a common agent) can lessen competition between competing brands, there still remains competition between brands sold at different retail outlets which exerts downward pressure on prices. Finally, the analysis suggests that slotting fees may be used to ensure that a retailer does not remove a manufacturer's product from its store.

Appendix

A. Proof of Proposition 2

When R_1 acts as the sole distributor for both manufacturers, its demand for products A and B is defined as the limit case of retail competition when R_2 faces the distribution cost $w_{B2} = \infty$. Specifically, denote by $\widetilde{q}_{k1}(w_{A1}, w_{B1}) \equiv q_{k1}(w_{A1}, w_{B1}, \infty)$ the R_1's demand function for each product $k = \{A, B\}$ and by $\widetilde{\pi}_k^{R_1}(w_{A1}, w_{B1}) \equiv \pi_k^{R_1}(w_{A1}, w_{B1}, \infty)$ its flow profit from selling product k:

$$\pi_k^{R_1}(w_{A1}, w_{B1}, \infty) = \widetilde{R}_{k1}(\widetilde{q}_{A1}(w_{A1}, w_{B1}), \widetilde{q}_{B1}(w_{A1}, w_{B1})) - w_{k1}\widetilde{q}_{k1}(w_{A1}, w_{B1}),$$

where $\widetilde{R}_{k1}(q_{A1}, q_{B1}) \equiv R_{k1}(q_{A1}, q_{B1}, 0)$. Likewise, the M_k's flow profit is given by:

[46] Although the exact level of prices depends on the order of negotiations.

$$\widetilde{\pi}^{M_k}\left(w_{A1}, w_{B1}\right) \equiv \left(w_{k1} - c_k\right) \widetilde{q}_{k1}\left(w_{A1}, w_{B1}\right).$$

If R_1 purchases positive quantities from both M_A and M_B, it gets the overall profit:

$$\widetilde{u}^{R_1}\left(\mathcal{C}_{A1}, \mathcal{C}_{B1}\right) = \sum_{k=A,B} \widetilde{\pi}_k^{R_1}\left(w_{A1}, w_{B1}\right) - \left(F_{k1} + S_{k1}\right), \tag{34}$$

while each M_k gets the overall profit:

$$\widetilde{u}^{M_k}\left(\mathcal{C}_{A1}, \mathcal{C}_{B1}\right) = \widetilde{\pi}^{M_k}\left(w_{A1}, w_{B1}\right) + \left(F_{k1} + S_{k1}\right). \tag{35}$$

If instead R_1 chooses to carry only brand k (alternatively, it has signed a contract only with M_k), its overall profit is:

$$\widetilde{u}_k^{R_1}\left(\mathcal{C}_{k1}\right) = \widetilde{\pi}_k^{R_1}\left(w_{k1}, \infty\right) - \left(F_{k1} + S_{k1}\right),$$

while the M_k's overall profit is:

$$\widetilde{u}_k^{M_k}\left(\mathcal{C}_{k1}\right) = \widetilde{\pi}^{M_k}\left(w_{k1}, \infty\right) + \left(F_{k1} + S_{k1}\right).$$

If all the active firms could fully coordinate their decisions, they would seek to maximize their joint profits given by:

$$\Pi_{A1B1}\left(w_{A1}, w_{B1}\right) = \sum_{k=A,B} \widetilde{R}_{k1}\left(\widetilde{q}_{A1}\left(w_{A1}, w_{B1}\right), \widetilde{q}_{B1}\left(w_{A1}, w_{B1}\right)\right) - c_k \widetilde{q}_{k1}\left(w_{A1}, w_{B1}\right).$$

In what follows, it will be assumed that the wholesale prices equal to marginal costs suffice to achieve the monopoly profit, i.e., $\Pi_{A1B1}\left(c_A, c_B\right) = \Pi_{A1B1}^m$.

Common agency equilibria

In any such equilibrium R_1 will find it profitable to carry brands A and B if, by doing so, it earns the profit that is not only non-negative, but also higher than the profit it could earn by carrying only one of them. This implies that contracts $\mathcal{C}_{A1}$ and $\mathcal{C}_{B1}$ must satisfy the following three conditions:

$$\begin{aligned}
\sum_{k=A,B} \widetilde{\pi}_k^{R_1}\left(w_{A1}, w_{B1}\right) - F_{k1} &\geq 0, && \mathrm{IP}^{AB} \\
&\geq \widetilde{\pi}_A^{R_1}\left(w_{A1}, \infty\right) - F_{A1}, && \mathrm{IC}_B^{AB} \\
&\geq \widetilde{\pi}_B^{R_1}\left(w_{B1}, \infty\right) - F_{B1}. && \mathrm{IC}_A^{AB}
\end{aligned}$$

Since products A and B are imperfect substitutes, removing one of them increases the profit from selling the other. This allows R_1 to behave opportunistically and guarantee itself positive profits.

Lemma 1 *In any common agency equilibrium R_1 earns positive profits.*

Proof. Suppose instead that R_1 earns zero, i.e.,

$$\sum_{k=A,B} \widetilde{\pi}_k^{R_1}\left(w_{A1}, w_{B1}\right) - F_{k1} = 0. \tag{36}$$

Summing up IC_A^{AB} and IC_B^{AB} and using (36) yields:

$$\widetilde{\pi}_A^{R_1}(w_{A1},\infty)+\widetilde{\pi}_B^{R_1}(w_{B1},\infty)\leq\widetilde{\pi}_A^{R_1}(w_{A1},w_{B1})+\widetilde{\pi}_B^{R_1}(w_{A1},w_{B1}). \tag{37}$$

The assumption of imperfect substitutability of A and B, in particular, implies:

$$\widetilde{R}_{k1}\left(\widetilde{q}_{A1}(w_{A1},w_{B1}),\widetilde{q}_{B1}(w_{A1},w_{B1})\right)<\widetilde{R}_{k1}\left(\widetilde{q}_{k1}(w_{A1},w_{B1}),0\right), \tag{38}$$

for each $k=\{A,B\}$. Using (38) and the fact that $\widetilde{\pi}_k^{R_1}(w_{k1},\infty)$ is maximal for $\widetilde{q}_{k1}(w_{k1},\infty)$, one obtains:

$$\begin{aligned}\widetilde{\pi}_k^{R_1}(w_{A1},w_{B1}) &= \widetilde{R}_{k1}\left(\widetilde{q}_{A1}(w_{A1},w_{B1}),\widetilde{q}_{B1}(w_{A1},w_{B1})\right)-w_{k1}\widetilde{q}_{k1}(w_{A1},w_{B1})\\ &< \widetilde{R}_{k1}\left(\widetilde{q}_{k1}(w_{A1},w_{B1}),0\right)-w_{k1}\widetilde{q}_{k1}(w_{A1},w_{B1})\\ &\leq \widetilde{R}_{k1}\left(\widetilde{q}_{k1}(w_{k1},\infty),0\right)-w_{k1}\widetilde{q}_{k1}(w_{k1},\infty)\\ &= \widetilde{\pi}_k^{R_1}(w_{k1},\infty),\end{aligned}$$

for each $k=\{A,B\}$. This is a contradiction to (37). ■

Lemma A1 thus implies that in any common agency equilibrium the IP^{AB} constraint is not binding and thus can be omitted in the subsequent analysis.

Suppose now that M_A and R_1 have signed a contract $\mathcal{C}_{A1}$. Taken $\mathcal{C}_{A1}$ as given, M_B and R_1 sign the contract $\mathcal{C}_{B1}^*$ which is a solution to the generalized Nash bargaining problem provided that the R_1's disagreement payoff is $\lambda_{A1}\Pi_{A1}^m$ (this is what R_1 would get while renegotiating with M_A if negotiations with M_B failed) while the M_B's disagreement payoff is zero.[47] Since R_1 will accept to carry both products only if IC_A^{AB} and IC_B^{AB} are satisfied, then in any common agency equilibrium $\mathcal{C}_{B1}^*$ must solve the following problem:

$$\max_{\mathcal{C}_{B1}}\left(\widetilde{u}^{R_1}(\mathcal{C}_{A1},\mathcal{C}_{B1})-\lambda_{A1}\Pi_{A1}^m\right)^{\lambda_{B1}}\left(\widetilde{u}^{M_B}(\mathcal{C}_{A1},\mathcal{C}_{B1})\right)^{1-\lambda_{B1}} \qquad \mathcal{P}_1^A$$
$$s.t.\ \mathrm{IC}_A^{AB} \text{ and } \mathrm{IC}_B^{AB} \text{ hold.}$$

Denote by $\widetilde{G}_{B1}(\mathcal{C}_{A1},w_{B1})$ the gains from trade between M_B and R_1, i.e.,

$$\widetilde{G}_{B1}(\mathcal{C}_{A1},w_{B1})\equiv\widetilde{\Pi}_{B1}(w_{A1},w_{B1})-(F_{A1}+S_{A1})-\lambda_{A1}\Pi_{A1}^m,$$

where $\widetilde{\Pi}_{B1}(w_{A1},w_{B1})$ is the sum of the flow profits of M_B and R_1:

$$\widetilde{\Pi}_{B1}(w_{A1},w_{B1})=\widetilde{\pi}_A^{R_1}(w_{A1},w_{B1})+\widetilde{\pi}_B^{R_1}(w_{A1},w_{B1})+\widetilde{\pi}^{M_B}(w_{A1},w_{B1}).$$

Denote by w_{B1}^* the wholesale price which maximizes $\widetilde{\Pi}_{B1}(w_{A1},w_{B1})$ subject to the IC_A^{AB} constraint, i.e.,

$$w_{B1}^* = \arg\max_{w_{B1}}\widetilde{\Pi}_{B1}(w_{A1},w_{B1}), \qquad \mathcal{P}_2^A$$
$$s.t.\ \mathrm{IC}_A^{AB} \text{ holds}$$

[47] Since, by assumption, M_B has already failed to reach an agreement with R_2, then the failure to reach an agreement with R_1 leaves M_B with no channels for distributing its product.

Provided that $\widetilde{G}_{B1}(\mathcal{C}_{A1}, w^*_{B1}) \geq 0$, the solution $\mathcal{C}^*_{B1}$ implies *(i)* the wholesale price is equal to w^*_{B1}; *(ii)* the conditional fee F^*_{B1} is set so as to satisfy IC^{AB}_B for $w_{B1} = w^*_{B1}$, i.e.,

$$F^*_{B1} \leq \widetilde{\pi}^{R_1}_A (w_{A1}, w^*_{B1}) + \widetilde{\pi}^{R_1}_B (w_{A1}, w^*_{B1}) - \widetilde{\pi}^{R_1}_A (w_{A1}, \infty),$$

and *(iii)* the unconditional fee S^*_{B1} is set so that R_1 and M_B divide their gains from trade according to each party's bargaining power, i.e.,

$$F^*_{B1} + S^*_{B1} = (1 - \lambda_{B1})\widetilde{G}_{B1}(\mathcal{C}_{A1}, w^*_{B1}) - \widetilde{\pi}^{M_B} (w_{A1}, w^*_{B1}),$$

and obtain the following payoffs:

$$\begin{aligned} \widetilde{u}^{R_1} (\mathcal{C}_{A1}, \mathcal{C}^*_{B1}) &= \lambda_{A1}\Pi^m_{A1} + \lambda_{B1}\widetilde{G}_{B1}(\mathcal{C}_{A1}, w^*_{B1}), \\ \widetilde{u}^{M_B} (\mathcal{C}_{A1}, \mathcal{C}^*_{B1}) &= (1 - \lambda_{B1})\widetilde{G}_{B1}(\mathcal{C}_{A1}, w^*_{B1}), \end{aligned}$$

respectively.

Consider now problem $\mathcal{P}^A_2$. Differentiating $\widetilde{\Pi}_{B1} (w_{A1}, w_{B1})$ w.r.t. w_{B1} and using the envelop theorem yields:[48]

$$\begin{aligned} \frac{\partial \widetilde{\Pi}_{B1} (w_{A1}, w_{B1})}{\partial w_{B1}} &= \frac{\partial}{\partial w_{B1}} \left(\widetilde{\pi}^{R_1}_A (w_{A1}, w_{B1}) + \widetilde{\pi}^{R_1}_B (w_{A1}, w_{B1}) + \widetilde{\pi}^{M_B} (w_{A1}, w_{B1}) \right) \\ &= (w_{B1} - c_B) \frac{\partial \widetilde{q}_{B1} (w_{A1}, w_{B1})}{\partial w_{B1}}. \end{aligned}$$

By assumption A3, $\frac{\partial \widetilde{q}_{B1}(w_{A1}, w_{B1})}{\partial w_{B1}} < 0$ and, therefore, $\widetilde{\Pi}_{B1} (w_{A1}, w_{B1})$ is maximal for $w_{B1} = c_B$.

Write now the IC^{AB}_A constraint as follows:

$$\widetilde{\pi}^{R_1}_A (w_{A1}, w_{B1}) + \widetilde{\pi}^{R_1}_B (w_{A1}, w_{B1}) - \widetilde{\pi}^{R_1}_B (w_{B1}, \infty) \geq F_{A1}. \tag{39}$$

Differentiating the left hand side of (39) and again using the envelop theorem leads to:

$$\begin{aligned} & \frac{\partial}{\partial w_{B1}} \left(\widetilde{\pi}^{R_1}_A (w_{A1}, w_{B1}) + \widetilde{\pi}^{R_1}_B (w_{A1}, w_{B1}) - \widetilde{\pi}^{R_1}_B (w_{B1}, \infty) \right) \\ = \;& -\widetilde{q}_{B1} (w_{A1}, w_{B1}) + \widetilde{q}_{B1} (w_{B1}, \infty). \end{aligned}$$

Since products A and B are imperfect substitutes, then $\widetilde{q}_{B1} (\infty, w_{B1}) > \widetilde{q}_{B1} (w_{A1}, w_{B1})$ and, therefore, the left hand side of (39) is increasing in w_{B1}. Thus, the set of wholesale prices satisfying IC^{AB}_A is the set $\{w_{B1} : w_{B1} \geq \widetilde{w}_{B1}\}$ where $\widetilde{w}_{B1}$ is the wholesale price for which IC^{AB}_A binds. Given that $\widetilde{\Pi}_{B1} (w_{A1}, w_{B1})$ is maximal for $w_{B1} = c_B$, the solution to $\mathcal{P}^A_2$ can be written as:

$$w^*_{B1} = \max\{c_B, \widetilde{w}_{B1}\}.$$

The above condition implies that in any common agency equilibrium M_B and R_1 never set the wholesale price below the marginal cost, while if they set it above c_B then IC^{AB}_A must necessarily be binding.

[48] Since the R_1's profits are maximal for $\widetilde{q}_{A1} (w_{A1}, w_{B1})$ and $\widetilde{q}_{B1} (w_{A1}, w_{B1})$, then $\frac{\partial}{\partial w_{k1}} \left(\widetilde{\pi}^{R_1}_A (w_{A1}, w_{B1}) + \widetilde{\pi}^{R_1}_B (w_{A1}, w_{B1}) \right) = -\widetilde{q}_{k1} (w_{A1}, w_{B1})$.

Instead of inducing a continuation equilibrium in which R_1 carries both products (by setting $w_{B1} = w^*_{B1}$), M_B and R_1 might deviate by setting w_{B1} below w^*_{B1} and thus induce a continuation equilibrium in which R_1 carries only product B. In particular, if it were $w^*_{B1} > c_B$ then such a pair-wise deviation would always be profitable since, by doing so, M_B and R_1 could not only increase their joint variable profits but also economize on the fixed payment to M_A. Given that the joint profits of M_B and R_1 are maximal for $w_{B1} = c_B$, this is the only candidate for equilibrium. The following lemma confirms this intuition.

Lemma 2 *In any common agency equilibrium $w^*_{B1} = c_B$.*

Proof. Suppose instead that $w^*_{B1} = \widetilde{w}_{B1} > c_B$ which implies that IC^{AB}_A is binding, i.e.,

$$F_{A1} = \widetilde{\pi}^{R_1}_A (w_{A1}, w^*_{B1}) + \widetilde{\pi}^{R_1}_B (w_{A1}, w^*_{B1}) - \widetilde{\pi}^{R_1}_B (w^*_{B1}, \infty). \tag{40}$$

By setting $w^*_{B1} = \widetilde{w}_{B1}$, M_B and R_1 obtain the joint profits (using (40)):

$$\widetilde{u}^{R_1} (\mathcal{C}_{A1}, \mathcal{C}^*_{B1}) + \widetilde{u}^{M_B} (\mathcal{C}_{A1}, \mathcal{C}^*_{B1}) = \widetilde{\pi}^{R_1}_B (w^*_{B1}, \infty) + \widetilde{\pi}^{M_B} (w_{A1}, w^*_{B1}) - S_{A1}.$$

Since $w^*_{B1} > c_B$ and $\frac{\partial \widetilde{q}_{B1}(w_{A1}, w_{B1})}{\partial w_{A1}} > 0$, the function $\widetilde{\pi}^{M_B} (w_{A1}, w^*_{B1}) = (w^*_{B1} - c_B)\widetilde{q}_{B1} (w_{A1}, w^*_{B1})$ is increasing in w_{A1} and, therefore:

$$\begin{aligned} \widetilde{u}^{R_1} (\mathcal{C}_{A1}, \mathcal{C}^*_{B1}) + \widetilde{u}^{M_B} (\mathcal{C}_{A1}, \mathcal{C}^*_{B1}) &< \widetilde{\pi}^{R_1}_B (w^*_{B1}, \infty) + \widetilde{\pi}^{M_B} (w^*_{B1}, \infty) - S_{A1} \\ &< \Pi^m_{B1} - S_{A1}. \end{aligned}$$

If instead M_B and R_1 wish to induce a continuation equilibrium in which R_1 carries only brand B, they must sign a contract $\mathcal{C}_{B1}$ which satisfies the following constraints:

$$\begin{aligned} \widetilde{\pi}^{R_1}_B (w_{B1}, \infty) - F_{B1} &\geq 0, && \mathrm{IP}^B \\ &\geq \widetilde{\pi}^{R_1}_A (w_{A1}, \infty) - F_{A1}, && \mathrm{IC}^B_A \\ &\geq \sum_{k=A,B} \widetilde{\pi}^{R_1}_k (w_{A1}, w_{B1}) - F_{k1}. && \mathrm{IC}^B_{AB} \end{aligned}$$

Note that IC^B_{AB} is satisfied as long as IC^{AB}_A is violated. In particular, it is satisfied for $w_{B1} = c_B < w^*_{B1}$. Note also that IP^B and IC^B_A can be satisfied by an appropriate choice of F_{B1} while S_{B1} can be chosen so as to divide the gains from trade according to the parties' relative bargaining power. This implies that in the most profitable deviation M_B and R_1 jointly obtain the profit $\Pi^m_{B1} - S_{A1}$ which in turn implies that a wholesale price $w^*_{B1} > c_B$ can never be an equilibrium.

It remains to check that $w^*_{B1} = c_B$ constitutes a continuation equilibrium. In that case the most profitable deviation of M_B and R_1 involve their cutting the wholesale price to below $\widetilde{w}_{B1}(\leq c_B)$. By applying a similar reasoning as before, it can be verified that, by doing so, M_B and R_1 can jointly obtain at most the profit $\widetilde{\pi}^{R_1}_B(\widetilde{w}_{B1}, \infty) + \widetilde{\pi}^{M_B}(\widetilde{w}_{B1}, \infty) - S_{A1}$. On the other hand, IC^{AB}_A taken for $w^*_{B1} = c_B$ implies:

$$F_{A1} \leq \widetilde{\pi}^{R_1}_A (w_{A1}, c_B) + \widetilde{\pi}^{R_1}_B (w_{A1}, c_B) - \widetilde{\pi}^{R_1}_B (c_B, \infty).$$

Using the above condition and $\widetilde{\pi}^{M_B}(w_{A1}, c_B) = 0$ leads to:

$$\begin{aligned}\widetilde{u}^{R_1}(\mathcal{C}_{A1}, \mathcal{C}_{B1}^*) + \widetilde{u}^{M_B}(\mathcal{C}_{A1}, \mathcal{C}_{B1}^*) &= \widetilde{\pi}_A^{R_1}(w_{A1}, c_B) + \widetilde{\pi}_B^{R_1}(w_{A1}, c_B) - (F_{A1} + S_{A1}) \\ &\geq \widetilde{\pi}_B^{R_1}(c_B, \infty) - S_{A1}.\end{aligned}$$

Since $\widetilde{\pi}_B^{R_1}(c_B, \infty) = \Pi_{B1}^m \geq \widetilde{\pi}_B^{R_1}(\widetilde{w}_{B1}, \infty) + \widetilde{\pi}^{M_B}(\widetilde{w}_{B1}, \infty)$, the above condition implies that M_B and R_1 cannot gain from such a deviation. ■

Anticipating the contract $\mathcal{C}_{B1}^*$ that will be signed by M_B and R_1 afterwards, M_A and R_1 sign the contract $\mathcal{C}_{A1}^*$ which solves the generalized Nash bargaining problem provided that the R_1's disagreement payoff is $\lambda_{B1}\Pi_{B1}^m$ (this is what R_1 would get while renegotiating with M_B if negotiations with M_A failed) while the M_A's disagreement payoff is zero (this is because M_A and R_1 are not allowed to renegotiate at any time). Furthermore, if M_A and R_1 wish to induce a continuation equilibrium in which R_1 carries the products of both manufacturers, $\mathcal{C}_{A1}$ must also satisfy the following two conditions: *(i)* it must secure the R_1's incentives to carry brand A, i.e., it must satisfy IC_A^{AB} taken for $w_{B1} = c_B$ which, using that $\widetilde{\Pi}_{B1}(w_{A1}, c_B) = \widetilde{\pi}_{A1}^R(w_{A1}, c_B) + \widetilde{\pi}_{B1}^R(w_{A1}, c_B)$ and $\widetilde{\pi}_B^{R_1}(c_B, \infty) = \Pi_{B1}^m$, boils down to:

$$\widetilde{\Pi}_{B1}(w_{A1}, c_B) - F_{A1} - \Pi_{B1}^m \geq 0, \qquad \mathrm{IC}_A^{AB}(c_B)$$

and *(ii)* it must ensure that M_B and R_1 obtain non-negative gains from trade evaluated for $w_{B1}^* = c_B$:

$$\widetilde{G}_{B1}(\mathcal{C}_{A1}, c_B) = \widetilde{\Pi}_{B1}(w_{A1}, c_B) - (F_{A1} + S_{A1}) - \lambda_{A1}\Pi_{A1}^m \geq 0. \qquad \mathrm{GT}_{B1}(c_B)$$

Taken together, this implies that in any common agency equilibrium $\mathcal{C}_{A1}^*$ must be a solution to the following problem:

$$\max_{\mathcal{C}_{A1}} \left(\widetilde{u}^{R_1}(\mathcal{C}_{A1}, \mathcal{C}_{B1}^*) - \lambda_{B1}\Pi_{B1}^m\right)^{\lambda_{A1}} \left(\widetilde{u}^{M_A}(\mathcal{C}_{A1}, \mathcal{C}_{B1}^*)\right)^{1-\lambda_{A1}} \qquad \mathcal{P}_3^A$$
$$s.t.\ \mathrm{IC}_A^{AB}(c_B) \text{ and } \mathrm{GT}_{B1}(c_B) \text{ hold}$$

where $\widetilde{u}^{M_A}(\mathcal{C}_{A1}, \mathcal{C}_{B1}^*)$ is given by:

$$\widetilde{u}^{M_A}(\mathcal{C}_{A1}, \mathcal{C}_{B1}^*) = \widetilde{\pi}^{M_A}(w_{A1}, c_B) + (F_{A1} + S_{A1}), \tag{41}$$

while $\widetilde{u}^{R_1}(\mathcal{C}_{A1}, \mathcal{C}_{B1}^*)$ is given by:

$$\begin{aligned}\widetilde{u}^{R_1}(\mathcal{C}_{A1}, \mathcal{C}_{B1}^*) &= \lambda_{A1}\Pi_{A1}^m + \lambda_{B1}\widetilde{G}_{B1}(\mathcal{C}_{A1}, c_B) \qquad (42)\\ &= \lambda_{B1}\Pi_{B1}^m + \lambda_{B1}\left(\widetilde{\Pi}_{B1}(w_{A1}, c_B) - (F_{A1} + S_{A1}) - \widetilde{d}_{A1}\right),\end{aligned}$$

where

$$\widetilde{d}_{A1} \equiv \Pi_{B1}^m - \frac{(1-\lambda_{B1})\lambda_{A1}}{\lambda_{B1}}\Pi_{A1}^m.$$

Note that what matters for the solution to $\mathcal{P}_3^A$ is the sum $F_{A1} + S_{A1}$ rather than the individual values of F_{A1} and S_{A1}. This, in particular, implies that F_{A1}

can always be chosen so as to satisfy $\mathrm{IC}_A^{AB}(c_B)$ while S_{A1} can then be adjusted so as to achieve the optimal value of $F_{A1}+S_{A1}$. As a consequence, $\mathrm{IC}_A^{AB}(c_B)$ can be omitted while solving $\mathcal{P}_3^A$.

Using (41) and (42) , the Lagrange function $\mathcal{L}$ for $\mathcal{P}_3^A$ writes as (denoting by $s_{A1}\equiv F_{A1}+S_{A1}$):

$$\begin{aligned}\mathcal{L} &= \left(\widetilde{\Pi}_{B1}(w_{A1},c_B)-s_{A1}-\widetilde{d}_{A1}\right)^{\lambda_{A1}}\left(\widetilde{\pi}^{M_A}(w_{A1},c_B)+s_{A1}\right)^{1-\lambda_{A1}}\\ &\quad +\psi\left(\widetilde{\Pi}_{B1}(w_{A1},c_B)-s_{A1}-\lambda_{A1}\Pi_{A1}^m\right),\end{aligned}$$

where $\psi\geq 0$ is the Lagrange multiplier to $\mathrm{GT}_{B1}(c_B)$. Differentiating $\mathcal{L}$ w.r.t. w_{A1} and s_{A1} yields the following first order conditions:

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial w_{A1}} &= \left[\lambda_{A1}\left(\frac{\widetilde{\pi}^{M_A}(w_{A1},c_B)+s_{A1}}{\widetilde{\Pi}_{B1}(w_{A1},c_B)-s_{A1}-\widetilde{d}_{A1}}\right)^{1-\lambda_{A1}}+\psi\right]\frac{\partial\widetilde{\Pi}_{B1}(w_{A1},c_B)}{\partial w_{A1}} \qquad (43)\\ &\quad +(1-\lambda_{A1})\left(\frac{\widetilde{\Pi}_{B1}(w_{A1},c_B)-s_{A1}-\widetilde{d}_{A1}}{\widetilde{\pi}^{M_A}(w_{A1},c_B)+s_{A1}}\right)^{\lambda_{A1}}\frac{\partial\widetilde{\pi}^{M_A}(w_{A1},c_B)}{\partial w_{A1}}=0,\end{aligned}$$

and

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial s_{A1}} &= -\psi-\lambda_{A1}\left(\frac{\widetilde{\pi}^{M_A}(w_{A1},c_B)+s_{A1}}{\widetilde{\Pi}_{B1}(w_{A1},c_B)-s_{A1}-\widetilde{d}_{A1}}\right)^{1-\lambda_{A1}} \qquad (44)\\ &\quad +(1-\lambda_{A1})\left(\frac{\widetilde{\Pi}_{B1}(w_{A1},c_B)-s_{A1}-\widetilde{d}_{A1}}{\widetilde{\pi}^{M_A}(w_{A1},c_B)+s_{A1}}\right)^{\lambda_{A1}}=0.\end{aligned}$$

Plugging (44) into (43) yields (by omitting the term $1-\lambda_{A1}>0$):

$$\left(\frac{\widetilde{\Pi}_{B1}(w_{A1},c_B)-s_{A1}-\widetilde{d}_{A1}}{\widetilde{\pi}^{M_A}(w_{A1},c_B)+s_{A1}}\right)^{\lambda_{A1}}\frac{\partial}{\partial w_{A1}}\left(\widetilde{\Pi}_{B1}(w_{A1},c_B)+\widetilde{\pi}^{M_A}(w_{A1},c_B)\right)=0.$$

Since the first term in brackets in the above expression is strictly positive, we have (using the envelope theorem):

$$\begin{aligned}\frac{\partial}{\partial w_{A1}}\left(\widetilde{\Pi}_{B1}(w_{A1},c_B)+\widetilde{\pi}^{M_A}(w_{A1},c_B)\right) &= \frac{\partial}{\partial w_{B1}}\left(\widetilde{\pi}_A^{R_1}(w_{A1},c_B)+\widetilde{\pi}_B^{R_1}(w_{A1},c_B)\right.\\ \left.+\,\widetilde{\pi}_1^{M_A}(w_{A1},c_B)\right) &= (w_{A1}-c_A)\frac{\partial\widetilde{q}_{A1}(w_{A1},c_B)}{\partial w_{A1}}=0,\end{aligned}$$

which, given that $\frac{\partial\widetilde{q}_{A1}(w_{A1},w_{B1})}{\partial w_{A1}}<0$, implies that $w_{A1}^*=c_A$. In other words, M_A and R_1 set the wholesale price equal to the marginal cost regardless of whether $\mathrm{GT}_{B1}(c_B)$ is binding or not. This, in particular, implies that $\widetilde{\pi}^{M_A}(c_A,c_B)=0$ and

$$\widetilde{\Pi}_{B1}(c_A,c_B)=\widetilde{\pi}_{A1}^R(w_{A1},c_B)+\widetilde{\pi}_{B1}^R(w_{A1},c_B)=\Pi_{A1B1}^m.$$

***Case* 1** $\mathrm{GT}_{B1}(c_B)$ is slack and, therefore, $\psi=0$. Plugging $\psi=0$ into (44) and rearranging the terms yields (since $\widetilde{\pi}^{M_A}(c_A,c_B)=0$ and $\widetilde{\Pi}_{B1}(c_A,c_B)=\Pi_{A1B1}^m$):

$$s_{A1}^*=(1-\lambda_{A1})\left(\Pi_{A1B1}^m-\widetilde{d}_{A1}\right),$$

which implies that R_1, M_A and M_B respectively obtain the following payoffs:

$$\begin{aligned}\widetilde{u}^{R_1} &= \lambda_{A1}\Pi^m_{A1} + \lambda_{B1}\widetilde{G}_{B1}(c_A, c_B),\\ \widetilde{u}^{M_A} &= \Pi^m_{A1B1} - \lambda_{A1}\Pi^m_{A1} - \widetilde{G}_{B1}(c_A, c_B),\\ \widetilde{u}^{M_B} &= (1-\lambda_{B1})\,\widetilde{G}_{B1}(c_A, c_B),\end{aligned}$$

where

$$\begin{aligned}\widetilde{G}_{B1}(c_A, c_B) &= \widetilde{\Pi}_{B1}(c_A, c_B) - s^*_{A1} - \lambda_{A1}\Pi^m_{A1}\\ &= \lambda_{A1}\left(\Pi^m_{A1B1} - \frac{1-\lambda_{A1}+\lambda_{A1}\lambda_{B1}}{\lambda_{B1}}\Pi^m_{A1}\right) + (1-\lambda_{A1})\,\Pi^m_{B1} > 0.\end{aligned}$$

***Case* 2** $\mathrm{GT}_{B1}(c_B)$ binds which implies that $\widetilde{\Pi}_{B1}(c_A, c_B) - s^*_{A1} - \lambda_{A1}\Pi^m_{A1} = 0$ while ψ is then given by:

$$\psi = \frac{(1-\lambda_{A1})\left(\Pi^m_{A1B1} - \widetilde{d}_{A1}\right) - s^*_{A1}}{(s^*_{A1})^{\lambda_{A1}}\left(\Pi^m_{A1B1} - s^*_{A1} - \widetilde{d}_{A1}\right)^{1-\lambda_{A1}}}.$$

Using that $s^*_{A1} = \Pi^m_{A1B1} - \lambda_{A1}\Pi^m_{A1}$, the condition $\psi \geq 0$ implies that

$$\begin{aligned}(1-\lambda_{A1})\left(\Pi^m_{A1B1} - \widetilde{d}_{A1}\right) - (\Pi^m_{A1B1} - \lambda_{A1}\Pi^m_{A1}) &\geq 0 \iff\\ \widetilde{G}_{B1}(c_A, c_B) &\leq 0.\end{aligned}$$

One can verify that in that case R_1, M_A and M_B obtain the payoffs $\widetilde{u}^{R_1} = \lambda_{A1}\Pi^m_{A1}$, $\widetilde{u}^{M_A} = \Pi^m_{A1B1} - \lambda_{A1}\Pi^m_{A1}$ and $\widetilde{u}^{M_B} = 0$ respectively.

Besides inducing common agency continuation equilibrium (hereafter alternative 1) M_A and R_1 have other alternatives: they can fail their negotiations in which case R_1 gets the payoff $\lambda_{B1}\Pi^m_{B1}$ while M_A gets zero (hereafter alternative 2); they can induce a breakdown of negotiations between M_B and R_1[49] in which case R_1 gets the payoff $\lambda_{A1}\Pi^m_{A1}$ while M_A gets the payoff $(1-\lambda_{A1})\,\Pi^m_{A1}$ (hereafter alternative 3).

Consider first the preferred ranking of the alternatives from the point of view of R_1. If $\widetilde{G}_{B1}(c_A, c_B) > 0$, then alternative 1 gives R_1 the payoff:

$$\widetilde{u}^{R_1} = \lambda_{B1}\Pi^m_{B1} + \lambda_{A1}\lambda_{B1}\left(\Pi^m_{A1B1} - \widetilde{d}_{A1}\right) > \lambda_{B1}\Pi^m_{B1}.$$

Otherwise, it obtains the payoff $\widetilde{u}^{R_1} = \lambda_{A1}\Pi^m_{A1} > \lambda_{B1}\Pi^m_{B1}$.[50] Hence, R_1 always prefers alternative 1 to alternative 2. Furthermore, since $\widetilde{u}^{R_1} \geq \lambda_{A1}\Pi^m_{A1}$, then alternative 1 is the most preferred for R_1.

[49] It suffices to set $(F_{A1} + S_{A1})$ sufficiently large so that M_B and R_1 can never obtain non-negative gains from trade.

[50] Since the products are imperfect substitutes, then $\Pi^m_{A1B1} > \Pi^m_{k1}$ for $k = \{A, B\}$ which taken with the condition $\widetilde{G}_{B1}(c_A, c_B) \leq 0$ yields:

$$\Pi^m_{k1} < \left(1 + \frac{(1-\lambda_{A1})(1-\lambda_{B1})}{\lambda_{B1}}\right)\Pi^m_{A1} - \frac{1-\lambda_{A1}}{\lambda_{A1}}\Pi^m_{B1},$$

for $k = \{A, B\}$. Taken for $k = A$, this condition implies that $\lambda_{B1}\Pi^m_{B1} < (1-\lambda_{B1})\,\lambda_{A1}\Pi^m_{A1} < \lambda_{A1}\Pi^m_{A1}$ while, taken for $k = B$, it implies that $\lambda_{B1}\Pi^m_{B1} < (1-\lambda_{A1}+\lambda_{A1}\lambda_{B1})\,\lambda_{A1}\Pi^m_{A1} < \lambda_{A1}\Pi^m_{A1}$.

Let us now turn to M_A. Note that alternative 2 is its least preferred since any other alternatives gives it a larger payoff. Furthermore, if $\widetilde{G}_{B1}(c_A, c_B) \leq 0$, then M_A is always better off with alternative 1 rather than with alternative 3. In contrast, if $\widetilde{G}_{B1}(c_A, c_B) > 0$, it prefers alternative 3 to alternative 1 whenever the following condition is satisfied:

$$(1 - \lambda_{A1}) \Pi^m_{A1} > \Pi^m_{A1B1} - \lambda_{A1}\Pi^m_{A1} - \widetilde{G}_{B1}(c_A, c_B)$$

or

$$\lambda_{A1}\left(\Pi^m_{A1B1} - \frac{1-\lambda_{A1}+\lambda_{A1}\lambda_{B1}}{\lambda_{B1}}\Pi^m_{A1} + \frac{1-\lambda_{A1}}{\lambda_{A1}}\Pi^m_{B1}\right) > \Pi^m_{A1B1} - \Pi^m_{A1}.$$

Thus, both common agency and exclusive equilibria are possible in that case.

B. Proof of Lemma 2

Denote by w^c_{ki} and F^c_{ki} the wholesale price and the conditional payment, respectively, for each product $ki = \{A1, B1, B2\}$ in a candidate equilibrium in which $w^c_{Bi} > c_B$ for each $i = 1, 2$ and let $\mathfrak{w}^{Bc}_{B2} \equiv \mathfrak{w}^B_{B2}(\mathbf{w}^c_1, F^c_{B1})$ and $\mathfrak{w}^{Ac}_{B2} \equiv \mathfrak{w}^A_{B2}(\mathbf{w}^c_1, F^c_{A1})$. Note that the condition $\mathfrak{w}^{Ac}_{B2} > \max\{\widehat{w}_{B2}, \mathfrak{w}^{Bc}_{B2}\}$, in particular, implies that $\mathfrak{w}^{Ac}_{B2} > \mathfrak{w}^{Bc}_{B2}$. Consider now a pair-wise deviation in which M_B and R_2 set w_{B2} slightly below $\mathfrak{w}^{Ac}_{B2}$. As long as $w^c_{Bi} > c_B$ for $i = 1, 2$, such a deviation would always be profitable provided that, as a response to it, R_1 removed just product A. Indeed, in that case M_B and R_2 would jointly obtain the profits:

$$\begin{aligned}\widehat{\Pi}\left(\infty, w^c_{B1}, \mathfrak{w}^{Ac}_{B2}\right) + (F^c_{B1} + S^c_{B1}) \quad &= \quad \sum_{i=1,2} \pi^{M_B}_i\left(\infty, w^c_{B1}, \mathfrak{w}^{Ac}_{B2}\right) + \pi^{R_2}_B\left(\infty, w^c_{B1}, \mathfrak{w}^{Ac}_{B2}\right) \\ &\quad\quad + (F^c_{B1} + S^c_{B1}).\end{aligned}$$

By assumption A4(ii), $\frac{\partial \pi^{R_2}_B}{\partial w_{A1}} > 0$. Furthermore, assumption A3(ii) implies that $\frac{\partial q_{Bi}}{\partial w_{A1}} > 0$ which, taken with the fact that $w^c_{Bi} > c_B$, yields:

$$\frac{\partial \pi^{M_B}_i}{\partial w_{A1}} = (w^c_{Bi} - c_B)\frac{\partial q_{Bi}}{\partial w_{A1}} > 0,$$

for each $i = 1, 2$. It then immediately follows that $\widehat{\Pi}\left(\infty, w^c_{B1}, \mathfrak{w}^{Ac}_{B2}\right) > \widehat{\Pi}\left(w^c_{A1}, w^c_{B1}, \mathfrak{w}^{Ac}_{B2}\right)$.

It remains to verify that, by setting w_{B2} below $\mathfrak{w}^{Ac}_{B2}$, M_B and R_2 indeed induce a continuation equilibrium in which R_1 carries only product B. Since the R_2's participation is always secured, this would be the case only if the following three constraints were satisfied:

$$\begin{aligned}\pi^{R_1}_B(\infty, w^c_{B1}, w_{B2}) - F^c_{B1} \quad &> \quad 0, & \mathrm{PC}^B_1 \\ &> \quad \pi^{R_1}_A(w^c_{A1}, \infty, w_{B2}) - F^c_{A1}, & \mathrm{IC}^B_{A1} \\ &> \quad \sum_{k=A,B} \pi^{R_1}_k\left(w^c_{A1}, w^c_{B1}, w_{B2}\right) - F^c_{k1}. & \mathrm{IC}^B_{AB1}\end{aligned}$$

IC_{AB1}^{B} implies that R_1 earns a higher profit by selling product B alone rather than by selling it together with product A while IC_{A}^{B} implies that doing so is more profitable than selling only product A. Finally, PC^{B} implies that selling product B is also more profitable than selling none of the products.

Note that IC_{AB1}^{B} is the reverse of IC_{A1}^{AB} and, therefore, it is always satisfied for $w_{B2} < \mathfrak{w}_{B2}^{Ac}$. Hence, it remains to check that PC_{1}^{B} and IC_{A1}^{B} are also satisfied for $w_{B2} < \mathfrak{w}_{B2}^{Ac}$.

Check of PC_{1}^{B}

Using the definition of $\mathfrak{w}_{B2}^{Bc}$, we have:

$$F_{B1}^{c} = \pi_{A}^{R_1}(w_{A1}^{c}, w_{B1}^{c}, \mathfrak{w}_{B2}^{Bc}) + \pi_{B}^{R_1}(w_{A1}^{c}, w_{B1}^{c}, \mathfrak{w}_{B2}^{Bc}) - \pi_{A}^{R_1}(w_{A1}^{c}, \infty, \mathfrak{w}_{B2}^{Bc}). \quad (45)$$

Plugging the above condition into the left hand side of PC_{1}^{B} yields:

$$\begin{aligned}\pi_{B}^{R_1}(\infty, w_{B1}^{c}, w_{B2}) - F_{B1}^{c} \quad = \quad & \left(\pi_{A}^{R_1}(w_{A1}^{c}, \infty, \mathfrak{w}_{B2}^{Bc}) - \pi_{A}^{R_1}(w_{A1}^{c}, w_{B1}^{c}, \mathfrak{w}_{B2}^{Bc})\right) \\ & + \left(\pi_{B}^{R_1}(\infty, w_{B1}^{c}, w_{B2}) - \pi_{B}^{R_1}(w_{A1}^{c}, w_{B1}^{c}, \mathfrak{w}_{B2}^{Bc})\right).\end{aligned}$$

Note that assumption A5 taken for $hj = A1$ and $ki = B1$ implies that

$$\pi_{A}^{R_1}(w_{A1}^{c}, \infty, \mathfrak{w}_{B2}^{Bc}) - \pi_{A}^{R_1}(w_{A1}^{c}, w_{B1}^{c}, \mathfrak{w}_{B2}^{Bc}) > 0. \quad (46)$$

Define now the function $\phi(w_{B2})$ as follows:

$$\phi(w_{B2}) \equiv \pi_{B}^{R_1}(\infty, w_{B1}^{c}, w_{B2}) - \pi_{B}^{R_1}(w_{A1}^{c}, w_{B1}^{c}, \mathfrak{w}_{B2}^{Bc}).$$

Assumption A5, when it is taken for $hj = B1$ and $ki = A1$, implies that

$$\phi(\mathfrak{w}_{B2}^{Bc}) = \pi_{B}^{R_1}(\infty, w_{B1}^{c}, \mathfrak{w}_{B2}^{Bc}) - \pi_{B}^{R_1}(w_{A1}^{c}, w_{B1}^{c}, \mathfrak{w}_{B2}^{Bc}) > 0, \quad (47)$$

and, when it is taken for $hj = B1$ and $ki = B2$, implies that

$$\frac{\partial \phi(w_{B2})}{\partial w_{B2}} = \frac{\partial \pi_{B}^{R_1}(\infty, w_{B1}^{c}, w_{B2})}{\partial w_{B2}} > 0. \quad (48)$$

Using (47) and (48), one obtains that $\phi(w_{B2}) > 0$ for any $w_{B2} \geq \mathfrak{w}_{B2}^{Bc}$ which, taken with (46), implies that PC_{1}^{B} is satisfied for any $w_{B2} \in [\mathfrak{w}_{B2}^{Bc}, \mathfrak{w}_{B2}^{Ac})$.

Check of IC_{A1}^{B}

Write IC_{A1}^{B} as follows:

$$\Lambda(w_{B2}) \equiv \pi_{B}^{R_1}(\infty, w_{B1}^{c}, w_{B2}) - \pi_{A}^{R_1}(w_{A1}^{c}, \infty, w_{B2}) + F_{A1}^{c} - F_{B1}^{c} > 0. \quad (49)$$

Using the definition of $\mathfrak{w}_{B2}^{Ac}$, we have:

$$F_{A1}^{c} = \pi_{A}^{R_1}(w_{A1}^{c}, w_{B1}^{c}, \mathfrak{w}_{B2}^{Ac}) + \pi_{B}^{R_1}(w_{A1}^{c}, w_{B1}^{c}, \mathfrak{w}_{B2}^{Ac}) - \pi_{B}^{R_1}(\infty, w_{B1}^{c}, \mathfrak{w}_{B2}^{Ac}). \quad (50)$$

Plugging (45) and (50) into (49) and rearranging the terms yields:

$$\Lambda\left(w_{B2}\right)=\Lambda_{A}\left(w_{B2}\right)+\Lambda_{B}\left(w_{B2}\right), \tag{51}$$

where

$$\begin{aligned}\Lambda_{A}\left(w_{B2}\right) &\equiv \left(\pi_{A}^{R_1}(w_{A1}^{c},w_{B1}^{c},\mathfrak{w}_{B2}^{Ac})-\pi_{A}^{R_1}(w_{A1}^{c},w_{B1}^{c},\mathfrak{w}_{B2}^{Bc})\right)\\ &\quad-\left(\pi_{A}^{R_1}(w_{A1}^{c},\infty,w_{B2})-\pi_{A}^{R_1}(w_{A1}^{c},\infty,\mathfrak{w}_{B2}^{Bc})\right),\end{aligned}$$

and

$$\begin{aligned}\Lambda_{B}\left(w_{B2}\right) &\equiv \left(\pi_{B}^{R_1}(w_{A1}^{c},w_{B1}^{c},\mathfrak{w}_{B2}^{Ac})-\pi_{B}^{R_1}(w_{A1}^{c},w_{B1}^{c},\mathfrak{w}_{B2}^{Bc})\right)\\ &\quad-\left(\pi_{B}^{R_1}(\infty,w_{B1}^{c},\mathfrak{w}_{B2}^{Ac})-\pi_{B}^{R_1}(\infty,w_{B1}^{c},w_{B2})\right).\end{aligned}$$

Using that $\pi_{A}^{R_1}(\infty,w_{B1}^{c},\mathfrak{w}_{B2}^{Ac})=\pi_{A}^{R_1}(\infty,w_{B1}^{c},w_{B2})=0$, $\Lambda_{A}\left(w_{B2}\right)$ can be written as:

$$\begin{aligned}\Lambda_{A}\left(w_{B2}\right) &= \Big[\left(\pi_{A}^{R_1}(w_{A1}^{c},w_{B1}^{c},w_{B2})-\pi_{A}^{R_1}(w_{A1}^{c},w_{B1}^{c},\mathfrak{w}_{B2}^{Bc})\right)\\ &\quad-\left(\pi_{A}^{R_1}(w_{A1}^{c},\infty,w_{B2})-\pi_{A}^{R_1}(w_{A1}^{c},\infty,\mathfrak{w}_{B2}^{Bc})\right)\Big]\\ &\quad+\Big[\left(\pi_{A}^{R_1}(w_{A1}^{c},w_{B1}^{c},\mathfrak{w}_{B2}^{Ac})-\pi_{A}^{R_1}(w_{A1}^{c},w_{B1}^{c},w_{B2})\right)\\ &\quad-\left(\pi_{A}^{R_1}(\infty,w_{B1}^{c},\mathfrak{w}_{B2}^{Ac})-\pi_{A}^{R_1}(\infty,w_{B1}^{c},w_{B2})\right)\Big].\end{aligned}$$

Routing calculations yield:

$$\begin{aligned}&\left(\pi_{A}^{R_1}(w_{A1}^{c},w_{B1}^{c},w_{B2})-\pi_{A}^{R_1}(w_{A1}^{c},w_{B1}^{c},\mathfrak{w}_{B2}^{Bc})\right)\\ &-\left(\pi_{A}^{R_1}(w_{A1}^{c},\infty,w_{B2})-\pi_{A}^{R_1}(w_{A1}^{c},\infty,\mathfrak{w}_{B2}^{Bc})\right)\\ =\;&-\int_{w_{B1}^{c}}^{\infty}dw_{B1}\int_{\mathfrak{w}_{B2}^{Bc}}^{w_{B2}}dw_{B2}'\frac{\partial\pi_{A}^{R_1}(w_{A1}^{c},w_{B1},w_{B2}')}{\partial w_{B1}\partial w_{B2}'},\end{aligned}$$

and

$$\begin{aligned}&\left(\pi_{A}^{R_1}(w_{A1}^{c},w_{B1}^{c},\mathfrak{w}_{B2}^{Ac})-\pi_{A}^{R_1}(w_{A1}^{c},w_{B1}^{c},w_{B2})\right)\\ &-\left(\pi_{A}^{R_1}(\infty,w_{B1}^{c},\mathfrak{w}_{B2}^{Ac})-\pi_{A}^{R_1}(\infty,w_{B1}^{c},w_{B2})\right)\\ =\;&-\int_{w_{A1}^{c}}^{\infty}dw_{A1}\int_{w_{B2}}^{\mathfrak{w}_{B2}^{Ac}}dw_{B2}'\frac{\partial\pi_{A}^{R_1}(w_{A1},w_{B1}^{c},w_{B2}')}{\partial w_{A1}\partial w_{B2}'}.\end{aligned}$$

Hence,

$$\begin{aligned}\Lambda_{A}\left(w_{B2}\right) &= -\int_{w_{B1}^{c}}^{\infty}dw_{B1}\int_{\mathfrak{w}_{B2}^{Bc}}^{w_{B2}}dw_{B2}'\frac{\partial\pi_{A}^{R_1}(w_{A1}^{c},w_{B1},w_{B2}')}{\partial w_{B1}\partial w_{B2}'}\\ &\quad-\int_{w_{A1}^{c}}^{\infty}dw_{A1}\int_{w_{B2}}^{\mathfrak{w}_{B2}^{Ac}}dw_{B2}'\frac{\partial\pi_{A}^{R_1}(w_{A1},w_{B1}^{c},w_{B2}')}{\partial w_{A1}\partial w_{B2}'}.\end{aligned} \tag{52}$$

Using that $\pi_B^{R_1}(w_{A1}^c, \infty, w_{B2}) = \pi_B^{R_1}(w_{A1}^c, \infty, \mathfrak{w}_{B2}^{Bc}) = 0$, $\Lambda_{B1}(w_{B2})$ can be written as:

$$\begin{aligned}\Lambda_B(w_{B2}) \equiv & \Big[\Big(\pi_B^{R_1}(w_{A1}^c, w_{B1}^c, \mathfrak{w}_{B2}^{Ac}) - \pi_B^{R_1}(w_{A1}^c, w_{B1}^c, w_{B2})\Big) \\ & - \Big(\pi_{B1}^{R}(\infty, w_{B1}^c, \mathfrak{w}_{B2}^{Ac}) - \pi_B^{R_1}(\infty, w_{B1}^c, w_{B2})\Big)\Big] \\ & + \Big[\Big(\pi_B^{R_1}(w_{A1}^c, w_{B1}^c, w_{B2}) - \pi_B^{R_1}(w_{A1}^c, w_{B1}^c, \mathfrak{w}_{B2}^{Bc})\Big) \\ & - \Big(\pi_B^{R_1}(w_{A1}^c, \infty, w_{B2}) - \pi_B^{R_1}(w_{A1}^c, \infty, \mathfrak{w}_{B2}^{Bc})\Big)\Big].\end{aligned}$$

By performing similar calculations as in case for $\Lambda_A(w_{B2})$, it can be checked that

$$\begin{aligned}\Lambda_B(w_{B2}) = & -\int_{w_{A1}^c}^{\infty} dw_{A1} \int_{w_{B2}}^{\mathfrak{w}_{B2}^{Ac}} dw'_{B2} \frac{\partial \pi_B^{R_1}(w_{A1}, w_{B1}^c, w'_{B2})}{\partial w_{A1} \partial w'_{B2}} \\ & - \int_{w_{B1}^c}^{\infty} dw_{B1} \int_{\mathfrak{w}_{B2}^{Bc}}^{w_{B2}} dw'_{B2} \frac{\partial \pi_B^{R_1}(w_{A1}^c, w_{B1}, w'_{B2})}{\partial w_{B1} \partial w'_{B2}}.\end{aligned} \tag{53}$$

Plugging (52) and (53) into (51) and rearranging the terms yields:

$$\begin{aligned}\Lambda(w_{B2}) = & -\int_{w_{A1}^c}^{\infty} dw_{A1} \int_{w_{B2}}^{\mathfrak{w}_{B2}^{Ac}} dw'_{B2} \left(\frac{\partial \pi_A^{R_1}(w_{A1}, w_{B1}^c, w'_{B2})}{\partial w_{A1} \partial w'_{B2}} + \frac{\partial \pi_B^{R_1}(w_{A1}, w_{B1}^c, w'_{B2})}{\partial w_{A1} \partial w'_{B2}} \right) \\ & - \int_{w_{B1}^c}^{\infty} dw_{B1} \int_{\mathfrak{w}_{B2}^{Bc}}^{w_{B2}} dw'_{B2} \left(\frac{\partial \pi_A^{R_1}(w_{A1}^c, w_{B1}, w'_{B2})}{\partial w_{B1} \partial w'_{B2}} + \frac{\partial \pi_B^{R_1}(w_{A1}^c, w_{B1}, w'_{B2})}{\partial w_{B1} \partial w'_{B2}} \right).\end{aligned}$$

By assumption A4(iv), both terms in brackets in the above condition are strictly negative which implies that $\Lambda(w_{B2}) > 0$ for any $w_{B2} \in [\mathfrak{w}_{B2}^{Bc}, \mathfrak{w}_{B2}^{Ac})$.

C. Proof of lemma 3

Note first that as long as a pair of wholesale prices $\mathbf{w}_B = (w_{B1}, w_{B2})$ satisfies the ND$_{B2}$ constraint then in the subsequent stage of the game R_2 and M_B will not deviate by decreasing their wholesale price below w_{B2}. In which case setting F_{B1} equal to:

$$F_{B1} = \sum_{k=A,B} \pi_{k1}^{R}(w_{A1}, w_{B1}, w_{B2}) - \pi_{A1}^{R}(w_{A1}, \infty, w_{B2}), \tag{54}$$

also ensures that ND$_{B2}$ is satisfied for such (w_{B1}, w_{B2}).

Note next that, as long as $w_{B2}^* = w_{B2}$ is given, the maximand in $\mathcal{P}_3$ as well as the GT$_{B2}$ constraint depends only on the sum $s_{B1} \equiv F_{B1} + S_{B1}$ rather than on the individual values of S_{B1} and F_{B1}. Hence, instead of optimizing with respect to (w_{B1}, F_{B1}, S_{B1}) one can optimize with respect to (w_{B1}, w_{B2}, s_{B1}) and then recover F_{B1} by using (54) and S_{B1} by setting $S_{B1} = s_{B1} - F_{B1}$.

Using (9) and (7), it can be verified that,

$$u^{M_B*}(\mathcal{C}_{A1},\mathcal{C}_{B1}) - \overline{u}^{M_B} = (1-\lambda_{B2})\left[\widehat{\Pi}(\mathbf{w}_1, w_{B2}^{*}) + (F_{B1}+S_{B1}) - d_{B1}\right], \quad (55)$$

where

$$d_{B1} \equiv \widetilde{u}^{M_B} + \frac{\overline{u}^{M_B} - \widetilde{u}^{M_B}}{1-\lambda_{B2}}. \quad (56)$$

Likewise, using (11), (12) and (56), it can be verified that,

$$u^{R_1*}(\mathcal{C}_{A1},\mathcal{C}_{B1}) - \overline{u}^{R_1} = G_{B1}^{*}(\mathcal{C}_{A1},\mathbf{w}_B) - \widehat{\Pi}(w_{A1},\mathbf{w}_B) - (F_{B1}+S_{B1}) + d_{B1}, \quad (57)$$

where

$$G_{B1}^{*}(\mathcal{C}_{A1},\mathbf{w}_B) \equiv \widetilde{\Pi}(w_{A1},\mathbf{w}_B) - (F_{A1}+S_{A1}) - \left(\overline{u}^{R_1} + \widetilde{u}^{M_B} + \frac{\overline{u}^{M_B} - \widetilde{u}^{M_B}}{1-\lambda_{B2}}\right).$$

Substituting (55) and (57) into $\mathcal{P}_3$ gives rise to the following problem (using that $s_{B1} \equiv F_{B1} + S_{B1}$):

$$\begin{aligned}
&\max_{\mathbf{w}_B, s_{B1}} \left(G_{B1}^{*}(\mathcal{C}_{A1},\mathbf{w}_B) - \widehat{\Pi}(w_{A1},\mathbf{w}_B) - s_{B1} + d_{B1}\right)^{\lambda_{B1}} \qquad \mathcal{P}^C \\
&\times \left(\widehat{\Pi}(w_{A1},\mathbf{w}_B) + s_{B1} - d_{B1}\right)^{1-\lambda_{B1}}, \\
&s.t.\ \mathrm{GT}_{B2} \text{ and } \mathrm{ND}_{B2} \text{ hold for } \mathfrak{w}_{B2}^{B} = w_{B2}
\end{aligned}$$

Since s_{B1} enters into GT_{B2}, two cases need to be distinguished.

***Case* 1** GT_{B2} is not binding

This implies that GT_{B2} can be omitted. Unconstrained maximization w.r.t. s_{B1} then yields:

$$s_{B1} = (1-\lambda_{B1})\, G_{B1}^{*}(\mathcal{C}_{A1},\mathbf{w}_B) - \left(\widehat{\Pi}(w_{A1},\mathbf{w}_B) - d_{B1}\right) \quad (58)$$

Plugging the above condition into problem $\mathcal{P}^C$ gives rise to problem $\mathcal{P}_4$. It is left for the reader to verify that when s_{A1} is given by (58) and $\mathbf{w}_B = \mathbf{w}_B^{**}$ where $\mathbf{w}_B^{**}$ is a solution to $\mathcal{P}_4$, the payoffs of R_1 and M_B are given by (14) and (15), respectively.

The solution is valid only if the following two conditions are satisfied: *(i)* R_1 and M_B obtain non-negative gains from trade, i.e., $G_{B1}^{**}(\mathcal{C}_{A1}) \equiv G_{B1}^{*}(\mathcal{C}_{A1}, \mathbf{w}_B^{**}) \geq 0$, and *(ii)* R_2 and M_B obtain non-negative gains from trade (i.e., ND_{B2} is satisfied). Plugging (58) into (7) and evaluating it for $\mathbf{w}_B = \mathbf{w}_B^{**}$ gives rise to:

$$\begin{aligned}
G_{B2}^{**}(\mathcal{C}_{A1}) &= \widehat{\Pi}(w_{A1},\mathbf{w}_B^{**}) + s_{B1}(\mathcal{C}_{A1},\mathbf{w}_B^{**}) - \widetilde{u}^{M_B} \\
&= \frac{\overline{u}^{M_B} - \widetilde{u}^{M_B}}{1-\lambda_{B2}} + (1-\lambda_{B1})\, G_{B1}^{**}(\mathcal{C}_{A1}) \geq 0,
\end{aligned}$$

which yields:

$$G^{**}_{B1}(\mathcal{C}_{A1}) \geq \frac{\widetilde{u}^{M_B} - \overline{u}^{M_B}}{(1-\lambda_{B1})(1-\lambda_{B2})}. \tag{59}$$

Taken with $G^{**}_{B1}(\mathcal{C}_{A1}) \geq 0$, the above condition implies that in optimum GT_{B2} is not binding if and only if:

$$G^{**}_{B1}(\mathcal{C}_{A1}) \geq \max\left\{0, \frac{\widetilde{u}^{M_B} - \overline{u}^{M_B}}{(1-\lambda_{B1})(1-\lambda_{B2})}\right\}.$$

***Case* 2** GT_{B2} is binding

This implies that the following condition must be satisfied (using that $s_{B1} \equiv F_{B1} + S_{B1}$),

$$\widehat{\Pi}(w_{A1}, \mathbf{w}_B) + s_{B1} - \widetilde{u}^{M_B} = 0,$$

which yields:

$$s_{B1} = -\left(\widehat{\Pi}(w_{A1}, \mathbf{w}_B) - \widetilde{u}^{M_B}\right). \tag{60}$$

Plugging (60) into (55) yields $u^{M_B*}(\mathcal{C}_{A1}, \mathcal{C}_{B1}) = \widetilde{u}^{M_B}$ while plugging it into (57) yields:

$$u^{R_1*}(\mathcal{C}_{A1}, \mathcal{C}_{B1}) = G^{**}_{B1}(\mathcal{C}_{A1}, \mathbf{w}_B) - \frac{\widetilde{u}^{M_B} - \overline{u}^{M_B}}{1-\lambda_{B2}}.$$

Since $u^{M_B*}(\mathcal{C}_{A1}, \mathcal{C}_{B1}) = \widetilde{u}^{M_B}$ while $u^{R_1*}(\mathcal{C}_{A1}, \mathcal{C}_{B1})$ is given by the above condition then problem $\mathcal{P}^C$ effectively boils down to problem $\mathcal{P}_4$.

Condition (60) implies that the firms do not share their gains from trade according to the relative bargaining power. As a result, it does not need to be the case that both firms will benefit from trade even if their bilateral gains from trade are non-negative, i.e., $G^{**}_{B1}(\mathcal{C}_{A1}) \geq 0$. Thus, for the trade to take place, it must be that each firm derives *individual* benefits from it, i.e., it must be that $u^{M_B*}(\mathcal{C}_{A1}, \mathcal{C}_{B1}) = \widetilde{u}^{M_B} > \overline{u}^{M_B}$ and,

$$G^{**}_{B1}(\mathcal{C}_{A1}) \geq \frac{\widetilde{u}^{M_B} - \overline{u}^{M_B}}{1-\lambda_{B2}}.$$

Since in case when $G^{**}_{B1}(\mathcal{C}_{A1})$ satisfies (58), there exists a solution to unconstrained maximization (w.r.t. s_{B1}) and the maximand is a quasi-concave function of s_{B1}, it follows that in optimum GT_{B2} is binding if and only if:

$$\frac{\widetilde{u}^{M_B} - \overline{u}^{M_B}}{1-\lambda_{B2}} \leq G^{**}_{B1}(\mathcal{C}_{A1}) < \frac{\widetilde{u}^{M_B} - \overline{u}^{M_B}}{(1-\lambda_{B1})(1-\lambda_{B2})}.$$

Taken all together, this implies that the solution to $\mathcal{P}_3$ exists if and only if GT_{B1} is satisfied.

D. Proof of lemma 4

Using (18), the Lagrange function for $\mathcal{P}_4$ writes as follows:

$$\mathcal{L} = \widetilde{\Pi}(w_{A1}, \mathbf{w}_B) + \psi\left(\widetilde{\Pi}(w_{A1}, \mathbf{w}_B) - \pi^{R_1}_A(w_{A1}, \infty, w_{B2}) - \Pi^d_{B2}\right).$$

where $\psi \geq 0$ is the Lagrange multiplier to the ND_{B2} constraint in $\mathcal{P}_4$. Differentiating $\mathcal{L}$ w.r.t. w_{B1} and w_{B2} yields the following first order conditions:

$$
\begin{aligned}
(1+\psi)\frac{\partial\widetilde{\Pi}(w_{A1},\mathbf{w}_B)}{\partial w_{B1}} &= 0, &(61)\\
(1+\psi)\frac{\partial\widetilde{\Pi}(w_{A1},\mathbf{w}_B)}{\partial w_{B2}} - \psi\frac{\partial\pi_A^{R_1}(w_{A1},\infty,w_{B2})}{\partial w_{B2}} &= 0. &(62)
\end{aligned}
$$

Consider first the case when ND_{B2} is not binding. This implies that $\psi = 0$ and, therefore, the solution $(\widetilde{w}_{B1},\widetilde{w}_{B2})$ to unconstrained maximization of $\widetilde{\Pi}(\cdot)$ and the solution $(w_{B1}^{**},w_{B2}^{**})$ to constrained maximization of $\widetilde{\Pi}(\cdot)$ coincide, i.e., $(w_{B1}^{**},w_{B2}^{**}) = (\widetilde{w}_{B1},\widetilde{w}_{B2})$.

Consider now the case when ND_{B2} is binding and, therefore, $\psi > 0$. Define the functions $W_{1B}(w_{A1},w_{2B})$ and $W_{2B}(w_{A1},w_{1B})$ such that $w_{1B} = W_{1B}(w_{A1},w_{2B})$ solves (61) for any (w_{A1},w_{2B}) while $w_{2B} = W_{2B}(w_{A1},w_{1B},\psi)$ solves (62) for any (w_{A1},w_{1B},ψ). Note that W_{1B} cannot depend on ψ which implies that the solutions $(\widetilde{w}_{B1},\widetilde{w}_{B2})$ and $(w_{B1}^{**},w_{B2}^{**})$ must lie on the same "reaction curve", i.e., it must be that $\widetilde{w}_{B1} = W_{1B}(w_{A1},\widetilde{w}_{B2})$ and $w_{B1}^{**} = W_{1B}(w_{A1},w_{B2}^{**})$.

Differentiating (61) w.r.t. w_{B2} yields:

$$
\frac{\partial^2\widetilde{\Pi}(w_{A1},\mathbf{w}_B)}{\partial w_{B1}^2}\frac{\partial W_{1B}(w_{A1},w_{2B})}{\partial w_{B2}} + \frac{\partial\widetilde{\Pi}(w_{A1},\mathbf{w}_B)}{\partial w_{B1}\partial w_{B2}} = 0.
$$

The assumptions that $\widetilde{\Pi}(\cdot)$ is quasi-concave in $\mathbf{w}_B$ and that $\frac{\partial\widetilde{\Pi}(w_{A1},\mathbf{w}_B)}{\partial w_{B1}\partial w_{B2}} < 0$ imply that $\frac{\partial W_{1B}(w_{A1},w_{2B})}{\partial w_{B2}} < 0$. Thus, only two cases need to be considered: *(i)* $w_{B1}^{**} < \widetilde{w}_{B1}$ and $w_{B2}^{**} > \widetilde{w}_{B2}$ and *(ii)* $w_{B1}^{**} > \widetilde{w}_{B1}$ and $w_{B2}^{**} < \widetilde{w}_{B2}$.

It suffices to show that the first case is impossible. Indeed, any solution $(w_{B1}^{**},w_{B2}^{**}) \neq (\widetilde{w}_{B1},\widetilde{w}_{B2})$ and such that ND_{B2} is binding implies that the following two conditions are satisfied:

$$
\begin{aligned}
\widetilde{\Pi}(w_{A1},\mathbf{w}_B^{**}) - \pi_{A1}^R(w_{A1},\infty,w_{B2}^{**}) &= \Pi_{B2}^d,\\
\widetilde{\Pi}(w_{A1},\widetilde{\mathbf{w}}_B) - \pi_{A1}^R(w_{A1},\infty,\widetilde{w}_{B2}) &< \Pi_{B2}^d.
\end{aligned}
$$

Since the function $\widetilde{\Pi}(\cdot)$ is maximal for $(\widetilde{w}_{B1},\widetilde{w}_{B2})$, then $\widetilde{\Pi}(w_{A1},\mathbf{w}_B^{**}) \leq \widetilde{\Pi}(w_{A1},\widetilde{\mathbf{w}}_B)$. By assumption A4(iii), $\pi_{A1}^R(w_{A1},\infty,w_{B2})$ increases in w_{B2} and, therefore, $\pi_{A1}^R(w_{A1},\infty,w_{B2}^{**}) > \pi_{A1}^R(w_{A1},\infty,\widetilde{w}_{B2})$ for $w_{B2}^{**} > \widetilde{w}_{B2}$. Taken together this implies that,

$$
\begin{aligned}
\Pi_{B2}^d &= \widetilde{\Pi}(w_{A1},\mathbf{w}_B^{**}) - \pi_{A1}^R(w_{A1},\infty,w_{B2}^{**})\\
&< \widetilde{\Pi}(w_{A1},\widetilde{\mathbf{w}}_B) - \pi_{A1}^R(w_{A1},\infty,\widetilde{w}_{B2}) < \Pi_{B2}^d,
\end{aligned}
$$

which is a contradiction.

Assuming that in optimum ND_{B2} is binding, denote by $(w_{B1}^{**},w_{B2}^{**})$ and $(w_{B1}^{**\prime},w_{B2}^{**\prime})$ the solutions to $\mathcal{P}_4$ for Π_{B2}^d and $\Pi_{B2}^{d\prime} > \Pi_{B2}^d$, respectively. Note that $(w_{B1}^{**\prime},w_{B2}^{**\prime})$ must necessarily be such $w_{B1}^{**\prime} = W_{1B}(w_{A1},w_{B2}^{**\prime})$. Since $W_{1B}(w_{A1},w_{B2})$ is strictly decreasing in w_{B2}, only two cases need to be considered: *(i)* $w_{B1}^{**\prime} < w_{B1}^{**}$ and $w_{B2}^{**\prime} > w_{B2}^{**}$ and *(ii)* $w_{B1}^{**\prime} > w_{B1}^{**}$ and $w_{B2}^{**\prime} < w_{B2}^{**}$.

As before, it suffices to show that the first case is impossible. Note first that the following condition must be satisfied:

$$\widetilde{\Pi}(w_{A1}, \mathbf{w}_B^{**\prime}) - \pi_{A1}^R(w_{A1}, \infty, w_{B2}^{**\prime}) = \Pi_{B2}^{d\prime} > \Pi_{B2}^d.$$

Then, the fact that $(w_{B1}^{**}, w_{B2}^{**})$ solves $\mathcal{P}_4$ for Π_{B2}^d implies that $\widetilde{\Pi}(w_{A1}, \mathbf{w}_B^{**}) \geq \widetilde{\Pi}(w_{A1}, \mathbf{w}_B^{**\prime})$. This is because while solving $\mathcal{P}_4$ for Π_{B2}^d, the point $(w_{B1}^{**\prime}, w_{B2}^{**\prime})$ is available while the point $(w_{B1}^{**}, w_{B2}^{**})$ is optimal.

Using assumption A4(iii) that $\pi_{A1}^R(w_{A1}, \infty, w_{B2})$ increases in w_{B2}, one obtains (for $w_{B2}^{**\prime} > w_{B2}^{**}$):

$$\begin{aligned} \Pi_{B2}^d &= \widetilde{\Pi}(w_{A1}, \mathbf{w}_B^{**}) - \pi_{A1}^R(w_{A1}, \infty, w_{B2}^{**}) \\ &> \widetilde{\Pi}(w_{A1}, \mathbf{w}_B^{**\prime}) - \pi_{A1}^R(w_{A1}, \infty, w_{B2}^{**\prime}) = \Pi_{B2}^{d\prime}, \end{aligned}$$

which is a contradiction.

E. Proof of lemma 5

Using (14) and (13), it can be verified that,

$$u^{R_1**}(\mathcal{C}_{A1}) - \widehat{u}^{R_1} = \lambda_{B1}\left(\widetilde{\Pi}(w_{A1}, \mathbf{w}_B^{**}) - (F_{A1} + S_{A1}) - d_{A1}\right), \tag{63}$$

where d_{A1} is given by (22). Plugging (63) and (21) into $\mathcal{P}_5$ leads to the following problem:

$$\begin{aligned} &\max_{w_{A1}, F_{A1}, S_{A1}} \left(\widetilde{\Pi}(w_{A1}, \mathbf{w}_B^{**}) - (F_{A1} + S_{A1}) - d_{A1}\right)^{\lambda_{A1}} \\ &\times \left(\pi_1^{M_A}(w_{A1}, \mathbf{w}_B^{**}) + (F_{A1} + S_{A1})\right)^{1-\lambda_{A1}}, \\ &s.t.\ \mathrm{GT}_{B1}^{**},\ \mathrm{NE}_A \text{ and } \mathrm{ND}_{B1} \text{ hold} \end{aligned} \tag{$\mathcal{P}^E$}$$

Note that what matters for the optimum of $\mathcal{P}^E$ is the sum $s_{A1} \equiv F_{A1} + S_{A1}$ rather than the individual value of S_{A1}. This is because both the maximand and the GT_{B1}^{**} constraint depend on the sum $F_{A1} + S_{A1}$ while neither the NE_A nor ND_{B1} constraint depends on S_{A1}. Hence, instead of maximizing with respect to S_{A1} one can maximize with respect to s_{A1} and then recover S_{A1} by setting $S_{A1} = s_{A1} - F_{A1}$. Two cases need to be distinguished.

***Case* 1** GT_{B1}^{**} is not binding

This implies that GT_{B1}^{**} can be omitted. Unconstrained maximization w.r.t. s_{A1} then yields:

$$s_{A1} = (1 - \lambda_{A1})\left(\widetilde{\Pi}(w_{A1}, \mathbf{w}_B^{**}) - d_{A1}\right) - \lambda_{A1}\pi_1^{M_A}(w_{A1}, \mathbf{w}_B^{**}). \tag{64}$$

Substituting the above condition into problem $\mathcal{P}^E$ gives rise to problem $\mathcal{P}_6$. It is left for the reader to verify that when s_{A1} is given by (64) and $(w_{A1}, F_{A1}) = (w_{A1}^{***}, F_{A1}^{***})$ where $(w_{A1}^{***}, F_{A1}^{***})$ is a solution to $\mathcal{P}_6$, the payoffs of the four firms are given by (24) − (27).

The solution is valid only if the following two conditions are satisfied: *(i)* R_1 and M_A obtain non-negative gains from trade, i.e., it must be that $G^{***}_{A1} \geq 0$,and *(ii)* GT^{**}_{B1} must be satisfied. Plugging (64) into (13) and rearranging the terms yields:

$$\begin{aligned} G^{***}_{B1} &= \frac{\widehat{u}^{R_1} - \overline{u}^{R_1}}{\lambda_{B1}} + \lambda_{A1} G^{***}_{A1} \geq 0 \iff \\ G^{***}_{A1} &\geq \frac{\widetilde{u}^{R_1} - \widehat{u}^{R_1}}{\lambda_{A1}\lambda_{B1}}. \end{aligned}$$

Taken together with $G^{***}_{A1} \geq 0$, the above condition implies that in optimum GT^{**}_{B1} is not binding if and only if (23) is satisfied.

***Case* 2** GT^{**}_{B1} is binding

This implies that the following condition must be satisfied (using (13)),

$$\widetilde{\Pi}\left(w_{A1}, \mathbf{w}^{**}_B\right) - s_{A1} - \left(\overline{u}^{R_1} + \widetilde{u}^{M_B} + \frac{\overline{u}^{M_B} - \widetilde{u}^{M_B}}{1 - \lambda_{B2}}\right) = \frac{\widetilde{u}^{R_1} - \overline{u}^{R_1}}{\lambda_{B1}},$$

which yields (using (22)) :

$$s_{A1} = \widetilde{\Pi}\left(w_{A1}, \mathbf{w}^{**}_B\right) - \left(d_{A1} + \frac{\widetilde{u}^{R_1} - \widehat{u}^{R_1}}{\lambda_{B1}}\right). \tag{65}$$

Plugging (65) into (63) yields $u^{R_1**}\left(\mathcal{C}_{A1}\right) = \widetilde{u}^{R_1}$ while plugging it into (21) yields:

$$u^{M_A**}\left(\mathcal{C}_{A1}\right) = G^{**}_{A1}\left(\mathcal{C}_{A1}\right) - \frac{\widetilde{u}^{R_1} - \widehat{u}^{R_1}}{\lambda_{B1}},$$

where $G^{**}_{A1}\left(\mathcal{C}_{A1}\right) \equiv \Pi(w_{A1}, \mathbf{w}^{**}_B) - d_{A1}$. Since $u^{R_1**}\left(\mathcal{C}_{A1}\right) = \widetilde{u}^{R_1}$ while $u^{M_A**}\left(\mathcal{C}_{A1}\right)$ is given by the above condition then problem $\mathcal{P}^E$ effectively boils down to problem $\mathcal{P}_6$.

In contrast to (64), condition (65) implies that M_A and R_1 do not share their gains from trade according to their relative bargaining power. As a result, it does not need to be the case that both firms benefit from trade even if their bilateral gains from trade are non-negative. More precisely, (**??**) implies that R_1 obtains positive gains from trade only if $\widetilde{u}^{R_1} > \widehat{u}^{R_1}$ while (**??**) implies that M_A obtains non-negative gains from trade only if:

$$G^{***}_{A1} \geq \frac{\widetilde{u}^{R_1} - \widehat{u}^{R_1}}{\lambda_{B1}}.$$

Since in case when G^{***}_{A1} satisfies (23), there exists a solution to unconstrained maximization (w.r.t. s_{A1}) and the maximand is a quasi-concave function of s_{A1}, it follows that in optimum GT^{**}_{B1} is binding if and only if:

$$\frac{\widetilde{u}^{R_1} - \widehat{u}^{R_1}}{\lambda_{B1}} \leq G^{***}_{A1} < \frac{\widetilde{u}^{R_1} - \widehat{u}^{R_1}}{\lambda_{A1}\lambda_{B1}}.$$

Taken all together, this implies that the solution to $\mathcal{P}_5$ exists if and only if GT_{A1} is satisfied.

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