

# Regulation, Diversity, and Arbitrage in Market Models Possesing a Stochastic Number of Assets

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## Abstract

We discuss financial market models in which the number of assets is a finite but unbounded stochastic process, and study the existence of various forms of arbitrage in these models. The framework permits modeling of equity markets where companies may enter, leave, merge, and split. The asset price process is taken to be a  $\cup_{n=1}^{\infty} \mathbb{R}^n$ -valued piecewise semimartingale, defined herein. Stochastic integration with respect to  $\cup_{n=1}^{\infty} \mathbb{R}^n$ -valued piecewise semimartingales is extended from the  $\mathbb{R}^n$  case by use of localization and partitioning. The “No free lunch with vanishing risk” equivalence to the existence of an equivalent supermartingale measure for the class of nonnegative wealth processes, and the “No arbitrage of the first kind” equivalence to the existence of an equivalent local martingale deflator for the price process are extended to this setting. In the case where the price process is a  $\cup_{n=1}^{\infty} \mathbb{R}^n$ -valued piecewise Itô process, straightforward functionally generated relative arbitrage is found to be less readily available than in  $n$ -dimensional Itô process models.

In 1999 Robert Fernholz observed an inconsistency between the normative assumption of existence of an equivalent martingale measure (EMM) and the empirical reality of diversity in equity markets. We explore a method of imposing diversity on market models by a type of antitrust regulation that is compatible with EMMs. The regulatory procedure breaks up companies that become too large, preventing any from becoming monopolies.

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