# Functional linear instrumental regression

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We consider the problem of estimating the slope function $\beta$ in functional linear instrumental regression, where in the presence of a functional instrument $W$ the dependence of a scalar response $Y$ on the variation of an endogenous explanatory random function $X$ is modeled by $Y = \int_0^1 \beta(t)X(t)dt + U$ for some error term $U$ uncorrelated with $W$. Given an iid. $n$-sample of $(Y, X, W)$ we show a lower bound of the maximal mean integrated squared error for any estimator of $\beta$ over a certain ellipsoid of slope functions. This bound is essentially determined by the mapping properties of the cross-covariance operator associated to $X$ and the best linear predictor $W_{BLP}$ of $X$ given the instrument $W$. Assuming first that $W_{BLP}$ is known we propose a least squares estimator of $\beta$ which is based on a dimension reduction technique and additional thresholding. It is shown that this estimator can attain the lower bound up to a constant under mild additional moment conditions. However, the best linear predictor of $X$ given the instrument $W$ is in general not known. We provide sufficient conditions to ensure the minimax-optimality of the least squares estimator where $W_{BLP}$ is replaced by the instrument $W$. Finally, we show that theses conditions can be weakened if the slope function is estimated by a two stage least squares approach where in a first step $W_{BLP}$ is estimated by a linear regression. In the second step the estimator of $W_{BLP}$ is used to construct the least squares estimator of the slope function. We illustrate these results by considering Sobolev ellipsoids and finitely or infinitely smoothing cross-covariance operators.

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## References

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