DEPARTMENT OF ECONOMICS
OxCarre (Oxford Centre for the Analysis of Resource Rich Economies)

Manor Road Building, Manor Road, Oxford OX1 3UQ
Tel: +44(0)1865 281281 Fax: +44(0)1865 281163
reception@economics.ox.ac.uk www.economics.ox.ac.uk

UNIVERSITY OF OXFORD

OxCarre Research Paper 35

Is There Really a Green Paradox?

Revised, 19 September 2010

Frederick van der Ploeg

OxCarre

and

Cees Withagen

VU University Amsterdam & Tinbergen Institute

Direct tel: +44(0) 1865 281281 E-mail: celia.kingham@economics.ox.ac.uk

IS THERE REALLY A GREEN PARADOX?[1]

Frederick van der Ploeg[2], University of Oxford

Cees Withagen[3], VU University Amsterdam and Tinbergen Institute

Abstract

The Green Paradox states that, in the absence of an appropriate tax on CO2 emissions, subsidizing a renewable backstop such as solar or wind energy brings forward the date at which fossil fuels become exhausted and consequently global warming is aggravated. We shed light on this issue by solving a model of depletion of non-renewable fossil fuels followed by a switch to a clean renewable backstop, paying attention to timing of the switch and the amount of fossil fuels remaining unexploited. We show that the Green Paradox occurs if the backstop is relatively expensive and full exhaustion of fossil fuels is optimal, but does not occur if the backstop is sufficiently cheap relative to the cost of extracting the last drop of fossil fuels plus marginal global warming damages as then it is attractive to leave fossil fuels unexploited and thus limit CO2 emissions. We show that, without a carbon tax, subsidizing (taxing) the backstop might enhance social welfare if fossil fuel reserves are not fully (fully) exhausted. We also discuss the potential for limit pricing when the non-renewable resource is owned by a monopolist. Finally, we show that if backstop are already used and there is a new sequence of backstops becoming economically viable as the price of fossil fuels rises, a lower cost of the backstop will either postpone fossil fuel exhaustion or leave more fossil fuel in situ, thus boosting green welfare. However, if a market economy does not internalize global warming externalities and renewables have not kicked in yet, full exhaustion of fossil fuels will occur in finite time and a backstop subsidy always curbs green welfare.

Keywords: Green Paradox, Hotelling rule, non-renewable resource, renewable backstop, simultaneous use, global warming, carbon tax, monopoly, limit pricing

JEL codes: Q30, Q42, Q54

Revised 19 September 2010

---

[1] We are grateful to Rob Aalbers, Alex Bowen, Christa Brunnschweiler, Reyer Gerlagh, Dieter Helm, Cameron Hepburn, Erik Verhoef and Amos Zemel for helpful suggestions on the costs and CO2 intensity of various energy sources, Reyer Gerlagh, Michael Hoel, Ngo Van Long, Stephen Salant, Hans-Werner Sinn, Tony Venables and participants in seminars at OxCarre, Groningen, CORE, Louvain, CPB, The Hague and CenDEF at the University of Amsterdam, and at the conferences SURED in Ascona and WCERE 2010 in Montreal for helpful comments on earlier versions of the paper, and Vincent van Goes for preparing figure 3.
[2] Manor Road Building, Oxford OX 1 3 UQ, England. Email: rick.vanderploeg@economics.ox.ac.uk. Also affiliated with the University of Amsterdam and the Tinbergen Institute.
[3] Corresponding author. Department of Economics, VU University Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands. Email: cwithagen@feweb.vu.nl.

1. Introduction

The accumulation of $CO2$ due to the extraction and use of fossil fuels is the main cause of climate change. In a somewhat different context d'Arge and Kogiku (1973) argue".. the 'pure' mining problem must be coupled with the 'pure' pollution problem and questions like these become relevant: which should we run out first, air to breathe or fossil fuels to pollute the air we breathe?". In the design of optimal climate policy one could neglect the exhaustibility of fossil fuels by arguing that they are abundant until the far future, as is the case for coal or oil from tar sands. However, this may lead to the failure of climate policy. In the absence of renewable resources such as solar or wind energy, some fossil fuels such as oil and gas are essentially available in limited amounts and their optimal intertemporal use needs to be determined in conjunction with any adverse effects this may have on global warming. The optimal policy of extracting such fossil fuels and combating climate change should take into account the order in which the fuels are to be extracted. In doing so, differences in extraction costs for the various sources of energy as well as differences in the contributions the resources make to climate change play a role. With the availability of renewable backstops these problems persist. In addition, the timing, order and speed of extraction in conjunction with the introduction of the backstop are crucial for future welfare. Our aim is to present a dynamic welfare analysis in a world where climate change poses a serious negative externality. We explicitly consider exhaustibility of some fossil fuels, but also look at renewable backstops[4]. Backstops are defined as renewable resources that are perfect substitutes for fossil fuel and not constrained by exhaustibility. Our special interest in the role of backstops is motivated by the argument that subsidizing backstops may have negative detrimental climate effects (Sinn, 2008ab).[5] This is Sinn's 'Green Paradox' that has received a lot of attention in the press, but recently also has been scrutinized more rigorously (e.g., Hoel, 2008; Gerlagh, 2009; Grafton, Kompas and Long, 2010). We aim to critically review this argument and analyze its consequences in a model that is closely related to the one employed by Sinn. We emphasize the following features.

In the first place, we study in detail the situation where marginal extraction costs of the non-renewable resource depend on the existing stock. It follows that lowering the cost of supplying the renewable backstop may lead to a positive remaining stock of fossil fuel reserves in case the backstop price is lower than the marginal extraction costs at low resource stocks.

---

[4] Papers addressing externalities and exhaustibility, but abstracting from a backstop, include Krautkraemer (1985), who is mainly interested in preservation in view of amenity values, Withagen (1994), who shows that initial use of the exhaustible resource is smaller than without the externality, Ulph and Ulph (1994), who deal with optimal (dynamic) taxation of fossil fuels and their detrimental effect on the environment, and Sinclair (1994), who argues that with endogenous growth optimal fossil fuel taxes may fall rather than rise over time.

[5] This builds on earlier contributions (Sinn, 1981, 1982).

Secondly, we focus on backstops which do not cause CO_2 emissions such as solar or wind energy. [6] In first instance we consider the case where the unit production costs of the backstop are constant. Whether a backstop technology is cheap relative to fossil fuel depends on its own production cost and the total cost associated with fossil fuel, consisting of the stock dependent extraction costs and the damage caused by the accumulation of CO_2 emissions.[7] Therefore, the relative cheapness of backstops is changing over time with a decreasing stock of fossil fuel and an increasing stock of CO_2.

We pay special attention to the case where these backstops are still relatively expensive, possibly not when it comes to the marginal production costs once capacity is installed, but surely it is expensive to increase capacity, there are costs to do with intermittence and especially offshore wind mills are very costly to repair. Wind energy is estimated to be at least three times as expensive as 'grey' electricity (Wikipedia). However, a recent study suggests that, as far as the electricity industry is concerned, the costs of renewable sources of energy have fallen quite a bit: solar energy is currently 50% more expensive than conventional electricity, wind energy has the same cost and is (apart from the problem of intermittence) competitive, and biomass, CCS coal/gas and advanced natural gas combined cycle have mark-ups of 10%, 60% and 20%, respectively (Paltsev et al., 2009). These mark-ups for renewable energy sources are measured from a very low base and may not be so impressive when they account for a much larger market share. We will show that the Green Paradox prevails if the backstop is becoming cheaper provided that the backstop remains expensive. Another energy source that does not emit CO_2 is nuclear energy, which is deemed to be rather competitive already, possibly due to the neglect of the cost to be incurred after the plants become obsolete including the cost of disposing of nuclear waste. We show that with this cost configuration the Green Paradox no longer holds. In as far as advanced nuclear is much more expensive than conventional energy as suggested by a mark-up of 70% (Paltsev et al., 2009), our arguments suggest that the Green Paradox may not hold. In a sense carbon sequestration of electricity-generating industries may be viewed as an expensive backstop compared to conventional oil or gas but with lower CO_2 emissions.

[6] We thus abstract from heavily polluting and expensive backstops (van der Ploeg and Withagen, 2010). An example of this is the tar sands, because their reserves are much larger than conventional oil and gas reserves. Although burning oil from tar sands yields same emissions as burning conventional oil, a lot of energy is used in producing oil from tar sands and therefore CO_2 emissions are at least 50% higher and in some cases perhaps even 3 to 5 times higher than those of conventional oil. They also adversely affect the livelihood of indigenous communities via large-scale leakage of toxic waste in groundwater and destruction of ancient forests larger than the size of England. We also abstract from coal which is heavily polluting (electricity from coal-fired plants are 30% higher than oil-fired plants), but cheap to exploit (depending on location and soil characteristics). Also, the process of making coal liquid so that it can be a substitute for oil in transportation takes a lot of energy.

[7] Useful studies on the costs of producing various renewable and non-renewable forms of energy are Shihab Eldin (2002, European Commission (2003), Neuhoff (2004) and Paltsev et al. (2009).

Thirdly, our policy recommendations follow straightforwardly from dynamic optimization. Optimal taxes correspond to the shadow prices that are generated in the social optimum. We will characterize them, also in a dynamic setting. However, on a worldwide scale these optimal taxes are difficult to implement. In the long run new technologies are indispensable and one could therefore advocate subsidizing the development of clean backstop technologies. In addition to earlier analyses we add a global social welfare perspective and show that a lower cost of supplying the backstop may be beneficial, albeit not for green welfare. We give the conditions under which a subsidy enhances social welfare when there is no carbon tax. However, unless the reduction is realized in a costless way, the policy will in general not be first best.

Sinn (2008a) discusses a neoclassical optimal growth model with a non-renewable resource and a global warming externality, but abstracts from explicitly analyzing backstop alternatives. He argues that in as far they are imperfect substitutes[8], they are already incorporated in the demand function for oil/gas, and perfect substitutes such as bio-fuels will require too much of scarce resources – land – and will meet political opposition. This is why Sinn (2008a) concentrates on policies that limit the speed of extraction, not on policies that limit the total amount of extraction of fossil fuels, and focuses only at green rather than total welfare. Our main objective is to model backstops explicitly and to analyze the effects of subsidies and taxes on the use of the backstop on total as well as green welfare paying attention to situations where it is and where it is not optimal to leave some of the fossil fuel reserves in situ. Following Sinn (2008ab), we analyze what happens if a Hotelling ramp for taxes on C02 emissions is politically infeasible. If the government then resorts to subsidizing solar or wind energy, as is done on a large scale in Germany, depletion of fossil fuels may occur more rapidly and discounting then implies that climate change damages increase. If the atmosphere is already polluted with lots of CO2 emissions, we show that it is socially optimal to postpone depletion of fossil fuels to combat global warming.

Fourthly, some argue that the Green Paradox is the result of rational speculative behaviour of resource-owners under perfect competition (Sinn, 2008ab) or resource-owning monopolists (Gerlagh, 2009), we analyze the implications of imperfect competition more formally. When it comes to the theory of non-renewable resources, this is the case studied most extensively elsewhere in the literature. The reason is that neither the oil market dominated by OPEC nor the gas market dominated by Russia, Iran, Qatar and Venezuela can be characterized as competitive. It goes beyond the scope of the present paper to extend the cartel-fringe model (e.g., Groot et al., 2003) by allowing for a renewable backstop and the interactions with climate change. However, we do pay attention to the case of a resource-owning monopolist, in order

[8] In Section 5, introduced in point 5 below, we implicitly allow for backstops varying in production costs.

to incorporate the phenomenon of limit pricing (cf., Salant, 1977; Hoel, 1978). We show that depletion under a monopolist will be slower and, if the backstop is relatively cheap, a Green Paradox need not arise.

Finally, we investigate what happens if there is a continuum of backstops coming on stream as the price of fossil fuels gradually rises over time. In this case, the analysis becomes more complicated as it will be optimal to have a phase where fossil fuels and the renewable backstop are used simultaneously if global warming externalities are properly internalized. We then show that even when it is optimal to fully exhaust fossil fuel reserves, the Green Paradox no longer holds provided renewables are already being used alongside fossil fuels as lowering the backstop cost will either leave more fossil fuel in situ or will postpone exhaustion of fossil fuels. However, if there is an initial phase where only fossil fuel is used, subsidizing the backstop may lead to the Green Paradox, especially if global warming externalities are not properly internalized and exhaustion takes place in finite time.

The outline of the paper is as follows. Section 2 analyzes the socially optimal transition from conventional oil and gas to a clean renewable backstop, in the face of climate externalities and stock-dependent extraction costs.[9] Damage from CO2 emissions can be modelled through a negative externality in production (cf., Heal, 1985; Sinn, 2008ab), but we follow the mainstream approach where damage adversely affects social welfare. We abstract from capital accumulation.[10] Section 3 studies the outcome in a decentralized market economy and shows how the social optimum can be sustained with a rising CO2 tax. It also shows that in the second-best situation where a rising CO2 tax is infeasible, subsidizing the backstop need not lead to a Green Paradox if this encourages private resource owners to leave more fossil fuels in the soil and to switch more quickly to the clean backstop. Section 4 offers some ideas on the Green Paradox and imperfect competition. Section 5 investigates the implications of a sequence of backstops becoming economically viable as the social price of fossil fuel rises due to an upwards sloping supply of renewables. It also contrasts the social optimum with the market outcome and investigates the effects of subsidizing the renewable backstop. Section 6 concludes and discusses policy implications.

2. Switching from dirty fossil fuels to a clean backstop: social optimum

We study the optimal extraction of non-renewable resources (fossil fuels) with a renewable backstop kicking in once fossil fuels become too expensive. The backstop is a perfect substitute for the non-

---

[9] Many papers address transitions from non-renewables to backstops (e.g., Heal, 1976; Tsur and Zemel, 2003, 2005), but they do not explicitly analyze environmental externalities.

[10] Golosov et al. (2009) also study a general equilibrium model of fossil taxes and a backstop fuel, but focus on capital accumulation and ignore exhaustibility of fossil reserves. They show that optimal ad-valorem taxes on oil consumption decline over time.

renewable resource and its supply is infinitely elastic. To assess the Sinn (2008a and 2008b) arguments properly, we add climate change externalities as part of social welfare. The easiest way is to introduce a convex function in past CO2 emissions in the felicity function to capture the damage done by the CO2 emissions into the atmosphere from burning oil. A widely used description of the accumulation of CO2 in the atmosphere reads $\dot{E}(t) = q(t) - \nu E(t).$ However, in order to get tractable solutions we set $\nu = 0$.[11] With quasi-linear preferences, the social planner's problem then reads:

(1) $$\max \int_0^\infty \exp(-\rho t)[U(q(t)+x(t)) - G(S(t))q(t) - bx(t) - D(E(t))]dt$$

subject to $\dot{E}(t) = q(t),\ E(0) = E_0,$ where E_0 indicates the CO2 emissions that have taken place up to time zero, the non-negativity condition $x(t) \geq 0$, and the depletion equation

(2) $$\dot{S}(t) = -q(t),\ q(t) \geq 0,\ S(t) \geq 0,\ S(0) = S_0, \text{ given.}$$

Here ρ is the constant rate of time preference, q denotes the extraction rate, G the per unit extraction costs, x the rate of use of the backstop and b the unit cost of supplying the backstop energy source. The instantaneous utility function is U with $U' > 0$ and we suppose that at the optimum $U'' < 0$. The per unit extraction costs of the non-renewable resource are a decreasing function of the *in situ* stock, $G' < 0$. Hence, as reserves diminish, unit cost of extraction rises. The set of constraints (2) implies that total current and future depletion cannot exceed reserves, $\int_0^\infty q(t)\mathrm{d}t \leq S_0$. Note that $E(t) = E_0 + S_0 - S(t)$.

The current-value Hamiltonian is defined by

(3) $$H(q,x,S,\lambda,\mu) \equiv U(q+x) - G(S)q - bx - D(E) - \lambda q - \mu q,$$

where λ is the shadow price of the non-renewable resource and $\mu \geq 0$ the shadow cost of the CO2 stock. The necessary conditions for a social optimum are:

(3a) $$\dot{\lambda}(t) = \rho\lambda(t) + G'(S(t))q(t), \quad \dot{\mu}(t) = \rho\mu(t) - D'(E(t)),$$

(3b) $$\lim_{t\to\infty} \exp(-\rho t)\left[\lambda(t)S(t) - \mu(t)E(t)\right] = 0,$$

[11] Hoel and Kverndokk (1996) do consider decay, but do not study sensitivity with respect to the cost of supplying the backstop which is much easier by assuming away decay.

(3c) $$U'(q(t)+x(t))-\lambda(t)-\mu(t)-G(S(t))\leq 0,\, q(t)\geq 0, \quad \text{c.s.},$$

(3d) $$U'(q(t)+x(t))-b\leq 0,\, x(t)\geq 0,\ c.s.,$$

where c.s. refers to complimentary slackness. Define η as the social value of the non-renewable resource consisting of the sum of the value of the stock of fossil fuels and the environmental value of keeping the stock in the ground, $\eta \equiv \lambda + \mu.$ The necessary conditions can be interpreted as follows. The modified Hotelling rule implied by (3a) says that the rate of increase in the total scarcity rent of the non-renewable resource ($\dot{\eta}/\eta$) equals the rate of time preference ρ plus the sensitivity of the marginal cost of extraction to the stock of remaining reserves $G'(S)q$ minus marginal global warming damages $D'(E),$ both normalized by the social value U'. With global warming damages it is socially optimal to deplete the stock of fossil fuels more slowly. A rapidly increasing unit cost of extraction also slows down the depletion of fossil fuel. The transversality condition (3b) states that the sum of the present value of the remaining stock of fossil fuel and of the social cost of the pollution stock vanishes as time goes to infinity. Equation (3c) says that no resource extraction takes place if the marginal utility of the resource is below marginal extraction costs plus the social cost of the resource. Equation (3d) says that the backstop is used unless the marginal utility of energy falls short of the supply price b.

In order to avoid the necessity of distinguishing between too many cases we assume that $U'(x)=b$ has a positive solution $\bar{x}$. This means that it is profitable from a social welfare perspective to employ the backstop technology after extraction of fossil fuel has come to an end. Moreover, we assume that $U'(0)>G(0)+D'(E_0+S_0)/\rho.$ This means that in the absence of the backstop, extraction of fossil fuel will continue till full exhaustion. A weaker assumption is $U'(0)>G(S_0)+D'(E_0)/\rho$, so that in the absence of the backstop it will initially be profitable to extract fossil fuel. But then we have to deal with the possibility of the use of fossil fuel coming to an end due to lack of contribution to social welfare rather than due to its cost becoming too high relative to the cost of the backstop.

We now characterize the social optimum.

Proposition 1: The social optimum is characterized by an initial phase where only fossil fuel is used. After finite time *T* the backstop takes over indefinitely. The use of the backstop, the use and stock of fossil fuel, and the atmospheric CO2 concentration from time *T* on are given by:

(4) $$x(t)=\bar{x},\ q(t)=0,\ S(t)=S(T),\ \text{and } E(t)=E_0+S_0-S(T), \quad \text{for all } t\geq T.$$

At the switch from fossil fuel to the backstop, we have:

$$\lambda(T)+\frac{D'\left(E_0+S_0-S(T)\right)}{\rho}=b-G\left(S(T)\right). \tag{5}$$

Proof: Assume that there is simultaneous use of fossil fuel and the backstop. Equations (3c) and (3d) give $U'(q(t)+x(t))=b=\eta(t)+G(S(t))$. Therefore equation (3a) implies that $\dot{\eta}(t)=-G'(S(t))\dot{S}(t)=$ $\rho\left[b-G(S(t))\right]+G'(S(t))q(t)-D'(E_0+S_0-S(t))$ or $\eta(t)=D'(E_0+S_0-S(t))/\rho>0$. Hence $\dot{\eta}(t)=D''(E(t))q(t)/\rho>0.$ But we also have $\dot{\eta}(t)=G'(S(t))q(t)<0.$ This is a contradiction, so at any instant of time t, $q(t)$ and $x(t)$ cannot both be positive . Furthermore, note that a transition from the backstop to the fossil fuel cannot take place. Once the backstop is in use, the state of the system no longer changes and there is no reason to fall back on fossil fuel. Also, note that as long as extraction takes place we have from equation (3c) $q(t)\geq\overline{x}$. Hence, as long as there is resource extraction, it is bounded from below by a positive constant, implying from the limited availability of the resource that extraction will come to an end within finite time, say at *T*. The above establishes equation (4) as well. We have from the solution to the first and second differential equation of (3a) that $\lambda(t)=\lambda(T)\exp(\rho(t-T))$ and $\mu(t)=D'(E(T))/\rho+(\mu(T)-D'(E(T))/\rho)\exp(\rho(t-T)),$ respectively, for $t\geq T$. Hence, substituting these solutions and $S(t) = S(T)$ for all $t \geq T$, into the transversality condition (3b), we obtain:

$$\lim_{t\to\infty}\exp(-\rho t)\left[\lambda(T)\exp(\rho(t-T))S(T)-\left[\frac{D'(E(T))}{\rho}+\left(\mu(T)-\frac{D'(E(T))}{\rho}\right)\exp(\rho(t-T))\right]E(t)\right]=$$

$$\lim_{t\to\infty}\left[\lambda(T)S(T)-\left[\left(\mu(T)-\frac{D'(E(T))}{\rho}\right)\right]E(T)\right]\exp(-\rho T)=0$$

$\lambda(T)S(T)=\left(\mu(T)-D'(E(T))/\rho\right)E(T).$ and therefore $\lambda(T)S(T)=\left(\mu(T)-D'(E(T))/\rho\right)E(T).$ Since $\lambda(T)S(T)=0$ must hold, this implies that $\mu(T)=D'(E(T))/\rho.$ Hence, taking into account that $q(T)=0,\ x(T)=\overline{x},$ we obtain equation (5) from equations (3c) and (3d). Q.E.D.

Equation (4) of proposition 1 indicates that a higher cost of the backstop implies less use of the backstop in the post-oil era. Furthermore, given our assumption of no natural decay in the atmospheric CO2 concentration, the stock of CO2 in the atmosphere remains what is at the time of the switch to the clean backstop and is higher if the amount of oil burnt in the oil era (i.e., $S_0 - S(T)$) has been more substantial. The condition given in (5) says that the marginal value of fossil fuel left in situ at the time of the switch to the backstop plus the present value of the global warming damages of the carbon stock in the atmosphere must equal the cost advantage of supplying the renewable backstop (i.e., the cost of the backstop minus the cost of extracting fossil fuel given the amount of stock that is left in situ at the time of the switch).

2.1. How much fossil fuel to leave in the soil?

Three possibilities arise: full exhaustion $(S(T)=0)$, partial exhaustion $(S_0 > S(T) > 0)$ and zero extraction of fossil fuel $(S(T)=0)$. Consider the second case with some fossil fuel reserves left in situ at the time of the switch to the backstop. Then the shadow price of fossil fuel at the switch to the backstop is zero (i.e., $\lambda(T) = 0$), so that equation (5) implies that the present value of marginal future global warming damages of remaining fossil fuel reserves (*FMD*) equals the marginal benefit of extracting fossil fuel rather than using the backstop (*MB*) at the time of the switch to the backstop:

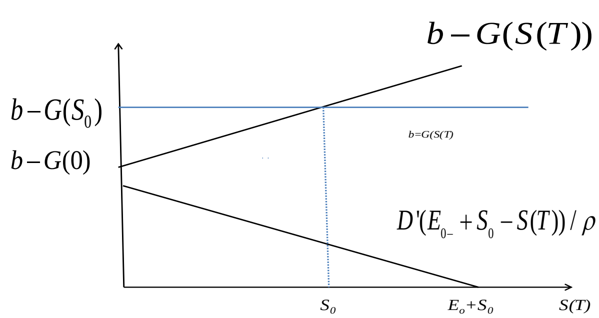
$$(5') \qquad FMD = D'\left(E_0 + S_0 - \bar{S}\right)/\rho = b - G\left(\bar{S}\right) \equiv MB \quad \Rightarrow \quad S(t) = \bar{S} \text{ provided } \bar{S} > 0.$$

So the cost of supplying the renewable backstop must be higher than the marginal cost of extracting the final drop of fossil fuel to cover the present value of the marginal social costs of global warming.

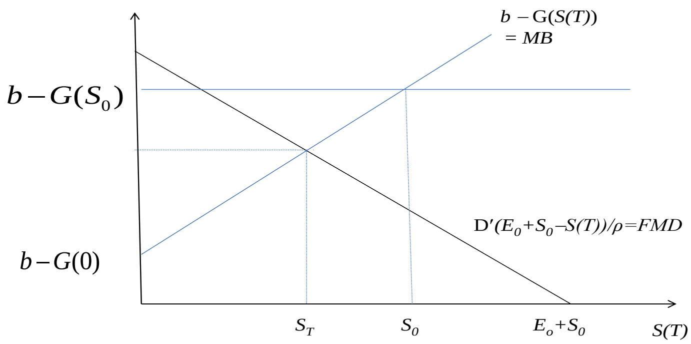
It could be that the solution $\bar{S}$ of equation (5′) is negative, e.g., if $b > G(0)$ and the aversion to global warming is small (i.e., preferences for a low CO2 stock are low or the rate used to discount marginal global warming is very high). In such cases the stock of fossil fuel will be fully exhausted at the moment the backstop energy source takes over, so that $S(T)=0$. This is the first case. The third case arises if the backstop is very cheap and the climate challenge is acute. In that case, $\lambda(T)$ is strictly negative and equation (5) implies that $b - G(S_0) < D'(E_0)/\rho$, so that it is optimal to never use fossil fuel and start using the backstop straight away. The three possible outcomes are depicted in fig. 1.

Figure 1: Marginal global warming damages and marginal benefits of oil extraction

(a) **Full exhaustion:** expensive backstop and modest climate challenge $b - G(0) > D'(E_0 + S_0)/\rho$



(b) **Partial exhaustion:** cheap backstop and acute climate challenge $b - G(0) < D'(E_0 + S_0)/\rho$



Note that *FMD* is strictly decreasing, whereas *MB* is strictly increasing in the final stock of fossil fuel left in situ. Panel (a) depicts the case where marginal global warming damages (after full exhaustion) are not very important and/or we have a high value of the rate of discount ρ and where the cost of supplying fossil fuel, even at low levels of the stock, is low relative to the cost of supplying the backstop. Then the *FMD* locus hits the vertical axis before it intersects the *MB* locus. In that case it is optimal to fully extract conventional fossil fuel reserves before switching to the backstop energy source. However, if marginal global warming damages are believed to be important and if prudent discounting is used (as advocated by the Stern Review (2007)), the *FMD* locus crosses the *MB* locus yielding a non-zero stock of fossil fuel left in situ ($S_0 > S(T) = \bar{S} > 0$) as portrayed in panel (b) of fig. 1. If the backstop is very cheap, the marginal cost of global warming is high and the rate used to discount global warming damages is very low, and the climate challenge is very acute, i.e., $b - G(S_0) < D'(E_0)/\rho,$ the intersection point lies to the right of S_0. The above is summarized in the following proposition.

Proposition 2: Full, partial and no exhaustion of fossil fuels occurs under the following conditions:

$$\text{if } b > G(0) + D'(E_0 + S_0)/\rho \text{ then } T > 0, S(T) = 0,$$

(6) $$\text{if } G(S_0) + D'(E_0)/\rho < b < G(0) + D'(E_0 + S_0)/\rho \text{ then } T > 0, S(T) = \bar{S} > 0,$$

$$\text{and if } G(S_0) + D'(E_0)/\rho > b \text{ then } T = 0, \; S(T) = S_0.$$

As technical progress reduces the cost of the backstop or a lower discount rate is used, one moves from a regime of full exhaustion to partial exhaustion and eventually to zero exhaustion of oil and gas reserves. In the sequel we rule out that fossil fuel is never extracted, so we suppose $G(S_0) + D'(E_0)/\rho < b.$ Note that the final stock of fossil fuel does not depend on preferences regarding energy consumption.

2.2. When to switch from fossil fuel to the renewable backstop?

The question we address next is how the time at which extraction of fossil fuel stops, depends on several crucial parameters of the model. Most of our results are derived for general functional forms. However, here we solve the model explicitly by making use of the following functional forms for utility of fuel consumption, per-unit cost of extracting oil and global warming damages:

(7) $$U(y) = \alpha y - \beta y^2/2, \quad G(S) = \gamma - \delta S, \quad D(E) = \kappa E^2/2.$$

Therefore, $U'(\overline{x}) = b$ implies $\overline{x} = (\alpha - b)/\beta.$ Moreover, the condition $U'(0) > G(0) + D'(E_0 + S_0)/\rho$ now says that the choke price of fuel exceeds the social cost of extracting the last drop of fossil fuel, $\alpha > \gamma + \kappa(E_0 + S_0)/\rho.$ This ensures that the choke price is high enough, $\alpha > \gamma - \delta S_0 + \kappa E_0/\rho,$ to make it optimal to start using fossil fuel from the outset ($T > 0$), $\alpha > \gamma - \delta S_0 + \kappa E_0/\rho.$ It thus follows from equations (5′) and (6) with the functional forms given by (7) that

(8)
$$S(T) = 0 \text{ if } b > \gamma + \kappa(E_0 + S_0)/\rho$$
$$S(T) = \overline{S} \equiv \frac{\gamma + \kappa(E_0 + S_0)/\rho - b}{\delta + \kappa/\rho} \text{ if } b < \gamma + \kappa(E_0 + S_0)/\rho.$$

So if the cost of the backstop exceeds the social cost of extracting the last drop of fossil fuel, not all reserves will be fully exhausted. We obtain the following proposition.

Proposition 3: With the specific functional forms introduced above, the time of the switch from fossil fuel to the backstop *T* is increasing in the initial stocks of fossil fuel and pollution, in the marginal production costs of the backstop, in the maximal marginal extraction costs (γ) and decreasing in maximal marginal utility (α). It is also decreasing in $S(T) = \overline{S}$ for positive $S(T)$.

Proof: From the first-order conditions (3a)-(3d), we get for $t \in [0, T)$ the following differential equation:

$$\rho\left[U'(-\dot{S}(t)) - G(S(t))\right] + U''(-\dot{S}(t))\ddot{S}(t) = D'(E_0 + S_0 - S(t)),$$

where the boundary conditions are $S(0) = S_0$ and $q(T) = -\dot{S}(T) = \bar{x}$. With our specific functional forms the equation becomes $\beta\ddot{S} - \rho\beta\dot{S} - (\rho\delta + \kappa)S = \rho(\alpha - \gamma) - \kappa(E_0 + S_0)$. The solution for $t \le T$ is given by $S(t) = K_1 \exp(s_1 t) + K_2 \exp(s_2 t) - \Gamma$, where $\Gamma \equiv \dfrac{\alpha - \gamma - \kappa(E_0 + S_0)/\rho}{\delta + \kappa/\rho} > 0$. The characteristic equation $\beta s^2 - \rho\beta s - (\rho\delta + \kappa) = 0$ gives the roots $s_1 = \frac{1}{2}\rho + \frac{1}{2}\sqrt{\rho^2 + 4(\rho\delta + \kappa)/\beta} > \rho$ and $s_2 = \rho - s_1 < 0$. Using the boundary conditions $K_1 + K_2 - \Gamma = S_0$ and $K_1 \exp(s_1 T) + K_2 \exp(s_2 T) - \Gamma = S(T)$, we solve for K_1 and K_2 and obtain

$$\begin{aligned} S(t) = {} & \left[\frac{\left(S(T) + \Gamma\right)\exp(-s_1 T) - (S_0 + \Gamma)\exp\left((s_2 - s_1)T\right)}{1 - \exp\left[(s_2 - s_1)T\right]}\right]\exp(s_1 t) \\ & + \left[\frac{S_0 + \Gamma - \left(S(T) + \Gamma\right)\exp(-s_1 T)}{1 - \exp\left[(s_2 - s_1)T\right]}\right]\exp(s_2 t) - \Gamma, \quad t \le T, \end{aligned} \tag{9}$$

Continuity of the Hamiltonian, $q(T) = -\dot{S}(T) = -s_1 K_1 \exp(s_1 T) - s_2 K_2 \exp(s_2 T) = \bar{x}$, gives:

$$\begin{aligned} \bar{x} = \frac{\alpha - b}{\beta} & = \frac{(S_0 + \Gamma)(s_1 - s_2)\exp(s_2 T) + (S(T) + \Gamma)\left[s_2 \exp((s_2 - s_1)T) - s_1\right]}{1 - \exp((s_2 - s_1)T)} \\ & \equiv \Phi\left(\overset{-}{T}, \overset{+}{S_0}, \overset{-}{S(T)}, \overset{-}{\Gamma}, s_1, s_2\right) \quad \Rightarrow \quad T = \mathrm{T}\left(\overset{+}{S_0}, \overset{-}{S(T)}, \overset{-}{\Gamma}, s_1, s_2, \overset{-}{\frac{\alpha - b}{\beta}}\right). \end{aligned} \tag{10}$$

With $s_1 > \rho > 0$ and $s_2 < 0$, the denominator on the right-hand side of equation (10) is strictly positive and increasing in T; given that we suppose that $\Gamma > 0$ and that $S(T) < S_0$, the numerator of equation (10) is decreasing in T, $\dfrac{\partial Num}{\partial T} = \underbrace{s_2(s_2 - s_1)\exp(s_2 T)}_{+}\left[\underbrace{(S(T)\exp(-s_1 T) - S_0}_{-} - \Gamma\underbrace{\left[1 - \exp(-s_1 T)\right]}_{+}\right] < 0.$ Hence, $\Phi_T < 0$. The signs of the derivatives of $\Phi(.)$ with respect to S_0 and $S(T)$ follow immediately, where we note that $S(T)$ itself is endogenous and depends on various parameters of interest. The derivative of $\Phi(.)$ with respect to Γ is given by $\dfrac{(s_1 - s_2)\exp(s_2 T) + s_2 \exp((s_2 - s_1)T) - s_1}{1 - \exp\left((s_2 - s_1)T\right)}$, which tends to $-s_1 < 0$ as $T \to \infty$. Since the numerator decreases and the denominator increases as T increases, the derivative of $\Phi(.)$ with respect to Γ is negative.

For $S(T) = 0$ we can now unambiguously determine the effects of those parameters that appear in Γ but do not appear in s_1 or s_2, namely S_0, $\bar{S}$, b, α and γ.[12] We thus obtain that the effect of α on $\Phi(.)$ via Γ is negative, hence applying the implicit function theorem to (10) we find that the total effect of α on T (via Γ and via $\bar{x}$) is negative. We also find that the effect of γ on T (operating via Γ) is positive. Since the effect of S_0 via Γ on T is positive, application of the implicit function theorem to (10) implies that the total effect of S_0 on T in case of full exhaustion is unambiguously positive. Furthermore, the effect of E_0 on T is easily seen to be positive as well. Applying the implicit function theorem to (10), we also see that b has a positive effect on T.

For $b \ge \gamma + \kappa(E_0 + S_0)/\rho$, we have partial exhaustion with $S(T) = \bar{S}$ given in (8). Applying the implicit function theorem to equation (10) shows that there is a negative effect of $S(T)$ on T. Substituting the expression for $S(T)$ into T(.), we obtain the T(.) for the case that $b \ge \gamma + \kappa(E_0 + S_0)/\rho$. Q.E.D.

[12] The signs of the partial derivatives with respect to ρ, δ, β and κ are more difficult to determine, since they operate both via the eigenvalues s_1 and s_2 and via Γ.

More substantial global warming externalities (higher κ) imply that the socially optimal outcome gives rise to slower depletion of the stock of non-renewable resources. We have also established that the transition from fossil fuel to the backstop occurs more quickly with a smaller initial stock of fossil fuel reserves (lower S_0), a lower initial atmospheric concentration of CO2 (lower E_0), lower marginal cost of extracting the last drop of fossil fuel (lower γ) and a higher choke price of fossil fuels (higher α). Furthermore, a lower cost of the renewable backstop (lower b) induces a quicker switch from fossil fuel to the backstop as well. These comparative statics effects occur both in case fossil fuels are fully and partially exhausted at the time of the switch to the backstop. However, if part of the reserves of fossil fuels remains in situ, there are some additional effects operating via the negative effect of the final stock of fossil fuels $\overline{S}$ on the transition date T. Most importantly, a lower cost of the backstop (lower b) implies that it is more attractive from a social perspective to keep more fossil fuels in situ at the time of transition and this in itself reinforces the quickening of the transition from fossil fuels to the renewable backstop. We will show in proposition 4 that this core result also holds for general functional forms. We also find that with partial exhaustion a lower marginal cost of extracting the last drop of fossil fuel (lower γ) makes it optimal to hold less fossil fuel reserves in situ at the time of the switch (lower $\overline{S}$), and this postpones the date of transition to the backstop. Hence, with partial exhaustion the speeding up of the transition to the backstop is somewhat offset and may even be reversed. We also see that a lower initial stock of fossil fuel reserves and a lower initial atmospheric concentration of CO2 make it optimal to keep less fossil fuels in situ at the time of the switch, especially if marginal global warming damages rise rapidly with emissions, and this also offsets and possibly reverses the speeding up of the transition from fossil fuels to renewables.

2.3. Green welfare and the cost of supplying the backstop energy source

If the backstop is relatively expensive compared to extracting the last drop of oil and gas plus the present value of marginal global warming damages, proposition 2 indicates that fossil fuel reserves are fully exhausted. In that case we know for the specific functional forms (7) used in proposition 3 that lowering the price of the renewable backstop leads to a more rapid switch to the backstop, but we will show in proposition below that this also holds for general functional forms. Due to discounting and convexity of global warming damages, one may argue that faster depletion of fossil fuel reserves must lead to worsening of green welfare. However, this conjecture is not valid if it is not optimal to fully exhaust fossil fuel reserves, i.e., if the backstop is relatively cheap compared with the cost of extracting the final drop of oil and gas plus the present value of marginal global warming damages. We show this in proposition 4 below regarding the effect of the cost of the backstop on green welfare, where green welfare is defined as:

(11) $$\Lambda \equiv -\int_0^\infty e^{-\rho t} D(E(t))dt = -\int_0^T e^{-\rho t} D(E(t))dt - \frac{D(E(T))e^{-\rho T}}{\rho}.$$

Proposition 4: If $b > G(0) + D'(E_0 + S_0)/\rho$, then with general functional forms for $U(.)$, $G(.)$ and $D(.)$ a marginal decrease in the cost of the backstop leads to earlier exhaustion of oil and gas reserves and a reduction in green welfare, thus giving a Green Paradox. Otherwise, there is only partial exhaustion of fossil fuel reserves while a decrease in the cost of the backstop costs slows down carbon extraction and boosts green welfare, so that there is no Green Paradox.

Proof: We obtain the marginal effect of the cost of the backstop on green welfare:

(12) $$\frac{\partial \Lambda}{\partial b} = \int_0^T \exp(-\rho t) D'(E(t) \frac{\partial S(t)}{\partial b} dt + \frac{D'(E(T))\exp(-\rho T)}{\rho} \frac{\partial S(T)}{\partial b}$$

Based on proposition 2 we distinguish two cases:

(a) If $b > G(0) + D'(E_0 + S_0)/\rho$, fossil fuel reserves get fully exhausted, $S(T) = 0$. This means that the second term in (11) vanishes. We next show that the first term is positive. Take some b^* satisfying $b > b^* > G(0) + D'(E_0 + S_0)/\rho$ and denote the corresponding socially optimal values by asterisks. We have $S^*(T^*) = 0$. Suppose there exists $0 < t^* < T^*$ such that $q^*(t) < q(t)$ for all $t \in [0, t^*)$ and $q^*(t^*) = q(t^*)$. The latter equality must hold because in both cases the reserves get fully exhausted. We have $S^*(t^*) > S(t^*)$, $G(S^*(t^*)) < G(S(t^*))$, $D'(E^*(t^*)) < D'(E(t^*))$. It follows from $U'(q^*(t^*)) = \eta^*(t^*) + G(S^*(t^*)) = U'(q(t^*)) = \eta(t^*) + G(S(t^*))$ that $\eta^*(t^*) > \eta(t^*)$. But then $\dot{q}^*(t^*) = \frac{\rho\eta^*(t^*) - D'(E^*(t^*))}{U''(q^*(t^*)} < \frac{\rho\eta(t^*) - D'(E(t^*))}{U''(q(t^*)} = \dot{q}(t^*)$. But, obviously we need $\dot{q}^*(t^*) > \dot{q}(t^*)$. Hence, initially we have higher extraction at b^*. And this will be the case over the entire program, because the argument used above can be repeated to show that we cannot have $q^*(t) = q(t) > 0$ for any t. So, the lower backstop price leads to a uniformly higher rate of extraction, a uniformly lower stock and earlier exhaustion. The conclusion is that $\partial S(t)/\partial b > 0$, implying that green welfare decreases as b falls.

(b) If $b < G(0) + D'(E_0 + S_0)/\rho$, reserves do not get fully, $S(T) > 0$. It has already been shown (see fig. 1b) that a lower backstop price increases the final stock. Hence, the second term in (12) must be negative. We now show that the first term is negative as well by showing that a lower b increases the resource stock at all instants of time. Suppose there exists an interval of time $[0, t^*]$ such that $q^*(t) > q(t)$ for all $t \in [0, t^*)$ and $q^*(t^*) = q(t^*)$. The latter equality must hold, because $S^*(T^*) > S(T)$. We have $S^*(t^*) < S(t^*)$, $G(S^*(t^*)) > G(S(t^*))$, $D'(E^*(t^*)) > D'(E(t^*))$. It then follows from $U'(q^*(t^*)) = \eta^*(t^*) + G(S^*(t^*)) = U'(q(t^*)) = \eta(t^*) + G(S(t^*))$ that $\eta^*(t^*) < \eta(t^*)$. But then $\dot{q}^*(t^*) = \frac{\rho\eta^*(t^*) - D'(E^*(t^*))}{U''(q^*(t^*)} > \frac{\rho\eta(t^*) - D'(E(t^*))}{U''(q(t^*)} = \dot{q}(t^*)$. But, obviously we need $\dot{q}^*(t^*) < \dot{q}(t^*)$. Hence, initially we have lower extraction at the lower b. And this will be the case over the entire program. The conclusion is that $\partial S(t)/\partial b < 0$, so that Λ increases as b falls. Q.E.D.

With full exhaustion of conventional oil and gas reserves, a lower cost of the backstop brings forward the date of exhaustion and switch to the backstop. Furthermore, it curbs green welfare. However, if the backstop is cheap and the global warming challenge acute, it is not optimal to fully exhaust fossil fuel reserves. We then recall from equation (10) in proposition 3 that a cheaper backstop brings forward the switch to the backstop even more, that is $\mathrm{T}_b - \left(\dfrac{\rho}{\kappa + \rho\delta}\right)\mathrm{T}_{\bar{S}} > \mathrm{T}_b > 0$ where T_b and $\mathrm{T}_{\bar{S}}$ denote the partial derivatives of the function T(.) with respect to *b* and $\bar{S}$, respectively. Furthermore, equation (8) indicates that it is optimal to leave more oil and gas in situ at the time of the switch. A cheaper backstop cuts extraction of oil and gas as a greater proportion of reserves are kept in situ. In that case, climate damages will be less and there will be no Green Paradox.

Proposition 4 establishes whether a cheaper backstop (e.g., due to technical progress or a subsidy) lowers or increases *green* welfare. Note that technical progress always boosts total *social* welfare whereas a backstop subsidy financed by lump-sum taxes always lowers *social* welfare if the socially optimal CO2 tax is in place.

3. Climate policy in the competitive decentralized market outcome

To assess whether the social optimum can be sustained in a competitive market economy, we consider behaviour of households and resource owners. Households maximize $U(q+x)+C$ subject to the budget constraint, $C + p(q+x) + A - T,$ where *C*, *p*, *A* and *T* denote consumer expenditures on all other commodities than oil, the market price of oil, endowment of households and lump-sum taxes, respectively. Households thus set $U'(q+x) = p,$ so the demand for fuel is a decreasing function of the market price of fuel (as $U'' < 0$). We assume that the mining company has access to the backstop. This is equivalent to having a separate mining company and another company supplying the backstop in competition with each other. Taking the time paths of the price of oil *p*, the carbon tax τ and the backstop subsidy σ as given, the resource-owning firms maximize profits

$$(13) \qquad \int_0^\infty \left\{ p(t)(q(t)+x(t)) - (G(S(t)) + \tau(t))q(t) - (b - \sigma(t))x(t) \right\} \exp(-\rho t)\mathrm{d}t,$$

subject to the depletion equation (2). This yields the first-order conditions:

$$(3a') \qquad \dot{\omega}(t) = \rho\omega(t) + G'(S(t))q(t),$$

(3b′) $$\lim_{t\to\infty} \exp(-\rho t)\omega(t)S(t) = 0,$$

(3c′) $$p(t) - G(S(t)) - \tau(t) - \omega(t) \le 0,\ q(t) \ge 0, \quad c.s.,$$

(3d′) $$p(t) - b + \sigma(t) \le 0,\ x(t) \ge 0,\ \text{c.s.},$$

where ω is the private marginal value of the fossil fuel stock. Equation (3a′) is the Hotelling rule, which states that the rate of increase in the scarcity rent of fossil fuel equals the rate of time preference ρ plus the sensitivity of the marginal cost of extraction to the stock of remaining reserves $G'(S)q$, normalized by the shadow price of fossil fuel. Comparing this with equation (3a), we see that the shadow price rises more quickly than in the social optimum so that depletion occurs too fast in a market economy unless the CO2 tax corrects for this externality. Equation (3c′) says that fossil fuel extraction does not take place if the fuel price is below marginal extraction costs plus the CO2 tax. Equation (3d′) says that the backstop is used unless the fuel price falls short of the supply price, net of the subsidy $b-\sigma$.

3.1. Sustaining the first-best outcome

We first characterize how the socially optimal outcome can be achieved in a market economy.

Proposition 5: The social optimum can be sustained in a market economy by a CO2 tax ramp given by $\dot{\tau}/\tau = \rho - D'(E)/\tau < \rho$ and $\sigma(t) = 0$.

Proof: Comparing the optimality conditions for the market economy, (3a′)-(3d′), with those of the social optimum, (3a)-(3d), and using $U'(q+x) = p$, we see that to replicate the social optimum the CO2 tax at time *t* must equal $\tau(t) = \eta(t) - \omega(t)$ and $\sigma(t) = 0$ with revenue rebated in lump-sum fashion. Using this in equation (3a′), we get $\dot{\eta} - \dot{\tau} = \rho(\eta - \tau) + G'(S)q$. Substituting equation (3a), we obtain $\dot{\tau}/\tau = \rho - D'(E)/\tau < \rho$. Q.E.D.

The optimal rate of change in the carbon tax thus consists of a Hotelling term equal to the rate of time preference minus a term depending on marginal global warming damages.[13] It is thus socially optimal to have the CO2 tax rate growing at a slower rate than the discount rate.

3.2. Second-best outcome if carbon tax infeasible

Sinn (2008a and 2008b), however, argues that a (rapidly) rising CO2 tax may be tough to sell to the people. Instead, governments may resort to the second-best policy of a constant subsidy or tax on the renewable backstop and financing this with lump-sum taxes whilst ruling out a CO2 tax. We also rule out a time-varying backstop subsidy or tax even though a constant backstop subsidy or tax may not be

[13] Similar results have been obtained earlier (e.g., Hoel and Kverndokk, 1996).

second-best optimal. Still, it is a fair first approximation and, moreover, we show that, contrary to what is suggested in the literature, a subsidy on the backstop can be beneficial from a social welfare perspective rather than from the narrower perspective or green welfare.

Before we do that, it is useful to briefly compare the decentralized market outcome without a carbon tax with the socially optimal outcome, but with a backstop subsidy.

Proposition 6: The decentralized market outcome has an initial phase where only fossil fuel is used and after finite time T the economy switches to only using the backstop. Use of the backstop is a decreasing function of the cost of the backstop (net of subsidy), that is $x(t) = \bar{x}(b-\sigma).$ Full exhaustion of fossil fuel reserves occurs if $b-\sigma > G(0),$ partial exhaustion if $G(S_0) < b-\sigma < G(0)$ with $S(T) > 0,$ and no exhaustion at all if $G(S_0) > b-\sigma.$ With the functional forms given in (7) the instant of time where extraction of fossil fuel is abandoned, occurs later if the initial fossil fuel stock S_0 is high, the cost of the backstop net of the subsidy $b-\sigma$ is high, the maximal marginal extraction costs γ is high and the choke price for fuel α is low. In case of partial exhaustion, the larger the amount left in the ground, the earlier the switch to the backstop takes place.

Proof: Replace b by $b-\sigma$ and set $\kappa = 0$ in propositions 1, 2 and 3. Q.E.D.

To illustrate the proposition, reconsider fig. 1. We note that the *FMD* line for the decentralized market outcome now excludes marginal global warming damages and thus corresponds to the horizontal axis. As fig. 1 is drawn for the case $b > \gamma$, reserves will be fully exhausted. For the case that $b < \gamma$, the *MB* line crosses the negative part of the vertical axis and the stock of fossil fuel reserves left in situ will equal $S(T) = (\gamma-b)/\delta$ provided this is less than S_0 (otherwise, fossil fuel reserves will never be extracted). Of course, less oil and gas will be left in situ in the competitive economy than in the social outcome which internalizes global warming damages. Introduction of a subsidy on the backstop in the decentralized market economy brings forward the date of the switch to the backstop, makes it more likely that some of the fossil fuel reserves will be left in situ, and will lead to a bigger stock of unexploited fossil fuel reserves in case of partial exhaustion. The following proposition characterizes the welfare consequences of introducing a subsidy for the renewable backstop.

Proposition 7: Assume a CO2 tax is not feasible. If the renewable backstop is always more expensive than carbon fuels (i.e., $b > G(0)$) and the atmospheric CO2 concentration is severely damaging in the margin, introducing a tax on the backstop enhances welfare. If the backstop is at some point cheaper than carbon fuels (e.g., $b < G(0)$) and the stock of CO2 is severely damaging, subsidizing the backstop enhances welfare**.**

Proof: We fix all parameters except *b*. In the competitive economy the time paths of *q*, *S*, *x*, *E* and *T* depend on *b*, and we henceforth write these endogenous variables as a function of *b* and the parameter κ, indicating the severity of damage. Let us first consider a positive backstop subsidy, and thus suppose that $G(0) > b > b^* = b - \sigma$. We decompose social welfare in the competitive economy into three parts: the private component of social welfare

$$V(b) \equiv \int_0^\infty \exp(-\rho t)\left[U(q(t;b)+x(t;b)) - G(S(t;b))q(t;b) - bx(t;b)\right]dt$$

green welfare

$$\Lambda(b) = -\int_0^\infty \exp(-\rho t)\kappa D(E(t;b))dt,$$

and subsidies

$$R(b-b^*) \equiv \int_{T(b')}^\infty \exp(-\rho t)(b-b^*)\overline{x}^* dt = \frac{(b-b^*)\overline{x}^*}{\rho}\exp[-\rho T(b^*)] \text{ with } U'(\overline{x}^*) = b^*.$$

Denote the welfare difference by $\Delta(b, b^*, \kappa)$. Clearly, $\Delta(b, b^*, 0) < 0$ because without global warming externalities the decentralized economy is socially optimal. We also observe that, for any given *b* and *b**, $\Delta(b, b^*, \kappa)$ is monotonic in κ. In the case at hand, $\Lambda(b^*) > \Lambda(b)$ because a lower backstop price will slow down extraction and leave more oil in situ. Hence, there exists a critical $\hat{\kappa}(b, b^*)$ such that for $\kappa < \hat{\kappa}(b, b^*)$ no subsidy should be given and for $\kappa > \hat{\kappa}(b, b^*)$ a subsidy enhances social welfare. Now assume $b > G(0)$, implying full exhaustion. If the backstop is subsidized in such a way that $b - \sigma > G(0)$ then, according to proposition 6, exhaustion will take place earlier, which is bad for social welfare if κ is large enough. Thus one should tax the backstop implying later exhaustion. Q.E.D.

Two remarks are in order. First, once fossil fuel reserves are fully depleted, it becomes socially optimal to abolish the tax on the renewable backstop. This may lead to a credibility problem. Second, it could be that the high global warming damage parameter needed in the proof to warrant a subsidy or a tax is such that a regime switch occurs.[14] Related to this is the observation that in case of an expensive backstop, an alternative policy is to subsidize the backstop to such an extent that it becomes cheaper than oil (i.e., *b* > *G*(0) > *b**). Then the policy is non-marginal, which might work for a very negative global warming externality, as can be illustrated in an example using the functional forms of section 2.2. We take $\alpha = 100, \beta = 1, \gamma = 30, \delta = 0.5, \rho = 0.2, b = 30, S_0 = 9, E_0 = 1$. In contrast to the *marginal* effects of a backstop subsidy or tax on welfare presented in proposition 7, fig. 2 plots the *non-marginal* effects of introducing a backstop subsidy and a backstop tax of varying orders of magnitude on welfare (net of the

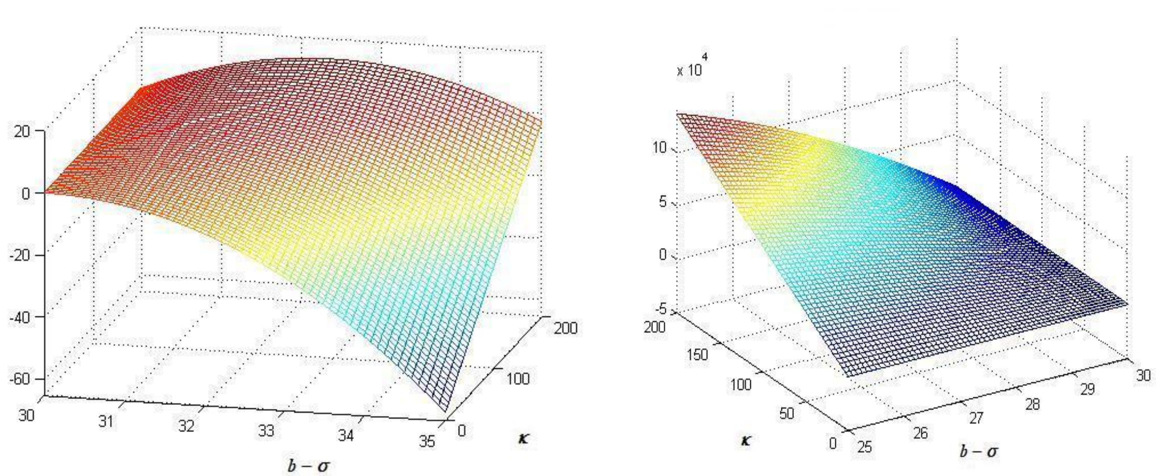
[14] For example, starting in the unlikely socially optimal regime with full exhaustion, the damage parameter needed to justify a tax might be such that we arrive in a regime where no exhaustion of fossil fuel takes place at all.

lump-sum taxes needed to finance the subsidy or the lump-sum transfers made possible by the tax on the backstop)[15] for different values of the damage parameter κ.

In panel (a) of fig. 2 we introduce a tax on the backstop, leading to the backstop cost being larger than γ . In the competitive economy fossil fuel reserves will then always be fully exhausted. The higher the tax and the cost of the backstop, the smaller is the initial rate of fossil fuel extraction and the later exhaustion of fossil fuel reserves takes place. This curbs emissions and implies an initial positive effect on green welfare. However, the private component of social welfare falls. With no or little concern about global warming (small κ), taxing the backstop always harms total welfare. Only if society cares a lot about global warming damages and the tax is not too high, is the welfare effect positive. Hence, if the increase in green welfare is large enough, it outweighs the fall in the private component of social welfare. However, too large backstop taxes lower social welfare even if society cares a lot about CO2 damages (high κ). For $\kappa = 200$, welfare is maximized if the backstop tax equals 2.

Figure 2: Backstop subsidies can boost second-best social welfare

(a) Taxing the backstop (full exhaustion) **(b) Subsidizing the backstop (partial exhaustion)**



Panel (b) of fig. 2 deals with the case of a subsidy which leads to a lower backstop cost and therefore to partial exhaustion of reserves with a positive final stock of fossil fuels. With no concern about global warming ($\kappa = 0$) introducing a backstop subsidy affects social welfare negatively, since the competitive outcome is socially efficient. However, it turns out that for these parameter values, the net effect of introducing a backstop subsidy is rather small. We also see that even for relatively little concern about

[15] Since use of the backstop is given by $x(t) = (\alpha - b + \sigma) / \beta > 0$ for all $t \geq T$, we have to subtract $\exp(-\rho T)\sigma(\alpha - b + \sigma) / (\beta\rho)$ from social welfare at time zero.

global warming, there is a substantial[16] welfare gain from introducing the backstop subsidy. This suggests that with $b > \gamma$ it is better from a welfare perspective to subsidize the backstop so that the effective cost is reduced below 30, rather than taxing the backstop.

Summing up, given the availability of a clean backstop, the appropriate way of realizing the first best is for the government to implement a C02 tax ramp, not a backstop subsidy. If a carbon tax is infeasible, subsidizing the backstop runs into the Green Paradox if the backstop is initially relative expensive. Green welfare will fall as oil and gas reserves are more quickly exhausted, but overall welfare may increase. However, if the backstop is (made) cheap enough compared with current extraction costs of oil and gas, it is optimal to keep some fossil fuel reserves unexploited which benefits the environment. Subsidizing the renewable backstop then means that the switch away from oil and gas to the clean backstop occurs more rapidly; and also that a bigger fraction of fossil fuel reserves remains in situ. CO2 emissions are less, so that the Green Paradox is avoided. An alternative is to compensate the owner of non-renewable resources for keeping some of its reserves unexploited. Interestingly, Ecuador demanded at the 2009 United Nations Climate Change Conference in Copenhagen $4.5 billion as compensation to keep oil in the soil and thus preserve the Amazon rain forest and curb CO2 emissions by 410 million tons. In practice, mining companies also attempt to bribe indigenous people to accept their resources being plundered.

4. Monopolistic supply of fossil fuels

So far, we have discussed socially optimal outcomes and outcomes that would prevail in a competitive market economy. Clearly on the markets for non-renewables imperfect competition prevails. Therefore, it is relevant to study the Green Paradox under the assumption of imperfect competition. As a first step we consider the case of a monopoly. It is well known that with monopolies in natural resource markets, limit pricing may occur (Salant, 1977; Hoel, 1978). This means that in the presence of a backstop technology with price *b* and constant marginal extraction costs of the non-renewable resource smaller than *b*, there is an initial phase until some T_1 where the monopolist keeps the market price of oil or gas below the cost of supplying the backstop price, and subsequently a final phase $(T_1, T_2]$ where the backstop price is undercut by an infinitely small margin. The instants of time (T_1, T_2) are determined endogenously by maximizing over the two parts of the trajectory. With stock-dependent extraction costs matters are more complicated. However, limit pricing may still occur. To see this, and to investigate its consequences, we consider a

[16] Note that in panel (b) the vertical axis is several orders of magnitude larger than in panel (a).

monopolist facing a linear inverse demand function $p(t) = \alpha - \beta q(t)$ and having extraction costs $G(S) = \gamma - \delta S.$ The cost of supplying the renewable backstop is *b*. The monopolist's problem is then

(14) $$\max_{q,T} \int_0^T \exp(-\rho t)\left[\alpha - \beta q(t) - \gamma + \delta S(t)\right] q(t)\mathrm{d}t$$

subject to the depletion equation (2) and the inverse demand function $p(t) = \alpha - \beta q(t) \le b.$ Note that the maximization also takes place with respect to the date *T* at which extraction definitely stops.

Proposition 8: Suppose that the owner of the non-renewable resource is a monopolist who is faced with a renewable backstop fuel over which it has no control. If the backstop price is high compared to the marginal cost of extracting fossil fuel ($b > \gamma$), lowering the cost of the backstop implies that it takes a shorter time to exhaust fossil fuels and in this sense the Green Paradox prevails. If the backstop price is relatively low ($b < \gamma$), then initial extraction of fossil fuels is excessive, but it lasts shorter than before, the stock of remaining fossil fuel at the time of the switch is $S(T) = (\gamma - b)/\delta > 0,$ and the Green Paradox need not necessarily arise. In both cases there is a phase of limit pricing, where fossil fuels are priced marginally below the cost of the backstop.

Proof: The current value Hamiltonian reads $\left[\alpha - \beta q - \gamma + \delta S\right] q - \lambda q.$ A necessary condition for optimality is $\alpha - 2\beta q(t) - \gamma + \delta S(t) = \lambda(t)$ as long as $q(t) > 0$ and $\alpha - \beta q(t) < b,$ where $\dot{\lambda}(t) = \rho\lambda(t) - \delta q(t).$ Moreover, at the time *T* when extraction definitely finishes, the Hamiltonian vanishes:

$$H(T) = \left[\alpha - \beta q(T) - \gamma + \delta S(T)\right] q(T) - \lambda(T) q(T) = 0.$$

We consider first the case where $b > \gamma$. Fossil fuels are then fully exhausted. Hence, at some instant of time *T* we have $S(T) = 0$ and $p(T) = \alpha - \beta\, q(T) = b$. Moreover, from $H(T) = 0$ we then have $\lambda(T) = b - \gamma > 0$. If the solution would be interior ($q(t) > 0$ and $\alpha - \beta\, q(t) < b$) until exhaustion, meaning no limit pricing, we have $\lim_{t\to T} \lambda(t) = \lim_{t\to T} \alpha - 2\beta q(t) - \gamma + \delta S(t) = 2b - \alpha - \gamma.$ However, this contradicts $\lambda(T) = b - \gamma.$ Therefore, there must be a phase with limit pricing. Hence, there exist $0 < T_1 < T_2$ such that for $0 \le t \le T_1$ we have $p(t) < b,$ and for $T_1 \le t \le T_2$ we have $p(t) = b.$ A marginal decrease of the backstop price results in a smaller shadow price λ. Indeed a smaller backstop price reduces the constraint set of the monopolist and thereby the shadow price of the non-renewable resource. Consequently, extraction increases during the first phase as well as in the second phase. Therefore it takes a shorter period of time to exhaust the resource. This result obtains a fortiori if limit pricing occurs from the outset.

Now consider the second case where $b < \gamma$. To have an interesting problem, suppose $\gamma - \delta S_0 < b < \gamma.$ Otherwise, extraction would never take place. At the time where the monopolist leaves the market (*T*), the price must equal the backstop price. Hence, $q(T) = (\alpha - b) / \beta.$ Moreover, it should not be profitable to extract anymore, $\alpha - \beta(\alpha - b) / \beta - \gamma + \delta S(T) = 0.$ Hence, $S(T) = (\gamma - b) / \delta.$ It follows, as before, that there is a phase with limit pricing. Regarding the Green Paradox two countervailing effects are at work. On the one hand, a smaller backstop price increases the remaining stock of fossil fuels kept in situ, which runs counter to the Green Paradox. On the other hand, it increases the final extraction rate and thereby all extraction rates during the regime of limit pricing and of the extraction rates before limit pricing starts. This can only happen if non-renewables are taken out of exploitation earlier than before. Hence, initially extraction becomes larger, but lasts shorter than before. Q.E.D.

It is interesting to see whether the Green Paradox is more prominent under monopoly than under perfect competition. Indeed, it is sometimes said that "the monopolist is the conservationist's best friend" (Dasgupta and Heal, 1979, p. 323) [17] and the question is whether this conjecture also holds when it comes to climate change and backstop technologies. With linear demand and zero marginal extraction costs, the initial market price will be higher under monopoly than under perfect competition. Moreover, it will take the monopolist longer to exhaust fossil fuel reserves which will be reinforced once account is taken of constant marginal extraction costs. It is easily established that also with a backstop and stock-dependent extraction costs, the price path under monopoly will initially be higher than under perfect competition. [18] So also with a backstop and stock-dependent extraction costs the monopolist will exhaust fossil fuel reserves at a later instant of time. However, for the total amount of fossil fuel left in situ, the market structure is irrelevant. Whether the exhaustion dates under monopoly and perfect competition come closer as the cost of the backstop is reduced, is left for further research.

5. Convex backstop production costs

We now consider non-constant production costs of the backstop. The problem is then given by:

$$(15) \qquad \max \int_0^\infty \exp(-\rho t)[U(q(t) + x(t)) - G(S(t))q(t) - B(x(t)) - D(E(t))]dt$$

subject to $\dot{E}(t) = q(t),\ E(0) = E_0$ and (2), where $B(x)$ with B'>0, B''>0 is the convex production cost function of the backstop. The current-value Hamiltonian is now defined as

[17] In a general equilibrium model with capital accumulation but without exhaustibility of oil and in the presence of a backstop Hassler et al. (2009) also conclude that an oil monopoly is good for the environment.

[18] If the perfect competition price would cross the monopoly price from above at some instant of time T, then it follows from the necessary conditions that $(\alpha - \gamma) / 2 > \delta(S^{pc}(T) - S^{mon}(T)).$ But the expression on the right-hand side is positive since in perfect competition more is left in the ground until T.

(16) $$H(q,x,S,\lambda,\mu) \equiv U(q+x) - G(S)q - B(x) - D(E) - \lambda q - \mu q,$$

so that the necessary conditions for a social optimum become:

(3a″) $$\dot{\lambda}(t) = \rho\lambda(t) + G'(S(t))q(t), \quad \dot{\mu}(t) = \rho\mu(t) - D'(E(t)),$$

(3b″) $$\lim_{t\to\infty} \exp(-\rho t)\left[\lambda(t)S(t) - \mu(t)E(t)\right] = 0,$$

(3c″) $$U'(q(t)+x(t)) - \lambda(t) - \mu(t) - G(S(t)) \le 0,\ q(t) \ge 0,\ \text{c.s.},$$

(3d″) $$U'(q(t)+x(t)) - B'(x) \le 0,\ x(t) \ge 0,\ c.s.$$

Before we characterize the optimal order of use of the two types of energy, it is useful to define the optimal use of the backstop when fossil fuel is not used, namely $x = \bar{x}$ such that the marginal utility of the backstop (price of the backstop fuel) equals the marginal cost of the backstop, $U'(\bar{x}) = B'(\bar{x}) \equiv b > 0$. We suppose that $\bar{x} > 0$.

Proposition 9: With convex backstop production costs, the optimal sequence is to first have a phase $t \in [0, T_1)$ where only fossil fuel is used, then a phase $t \in [T_1, T_2)$ of simultaneous use of fossil fuel and the backstop, and finally a phase $t \in [T_2, \infty)$ where only the backstop is used. The first phase or the third phase may be degenerate (so that $T_1 < 0$ or $T_2 \to \infty$, respectively). If the cost of using only the backstop b exceeds or equals the social cost of using the last drop of fossil fuel $G(0) + D'(E_0 + S_0)/\rho$, fossil fuels will be fully exhausted: $S(T_2) = 0$. Otherwise, there will be partial exhaustion of fossil fuels: $S(T_2) > 0$.

Proof: Clearly, once only the backstop is used, this will remain so. Moreover, then it follows from (3d″) that $x(t) = \bar{x}$. A transition from simultaneous use to use of only fossil fuel is ruled out by the following argument. Along an interval of simultaneous use we have from (3a″) and (3c″) that $(\dot{q}+\dot{x})U'' = \rho\lambda + \rho\mu - D'(E)$. The right-hand side of this expression is positive since $\lambda \ge 0$ and $\rho\mu - D'(E) \ge 0$ since otherwise μ becomes negative eventually, which is not allowed. Hence $q + x$ is decreasing. It then follows from (3d″) that q is decreasing and x increasing. A transition to only fossil fuel use then requires a downward jump in the use of the backstop and an upward jump in the use of oil. But (3c″) implies continuity of $q+x$ whereas (3d″) requires an upward jump in $q+x$. Finally, there will never be a transition from only fossil fuel to only the backstop. To see this, suppose that a transition takes place at some instant of time t_1. Right before the transition we have $U'(q) \le B'(0)$ and right after $U'(x) = B'(x)$. Again, continuity is violated. So, a generic sequence reads: $q > 0 \to (q > 0, x > 0) \to x > 0$ with transition dates T_1 and T_2 respectively. We could have $T_1 = 0$ and $T_2 = \infty$. But $x(T_1) = 0$ if $T_1 > 0$ and $q(T_2) = 0$ if $T_2 < \infty$, from continuity. If $b > G(0) + D'(E_0 + S_0)/\rho$ then $S(T_2) = 0$. Otherwise, it would pay to continue using fossil fuel and reduce the use of the backstop marginally. If $b < G(0) + D'(E_0 + S_0)/\rho$ then $S(T_2) > 0$. Q.E.D.

Hence, the conditions for partial or full depletion are quite similar to those that we had before. However, we will show that the implications of the backstop becoming cheaper for the Green Paradox are not robust. We therefore turn to the specific functional forms used in section 2.2 and furthermore suppose a quadratic cost function for the backstop,

(17) $$B(x) = \psi x + \pi x^2 / 2, \quad \psi, \pi > 0 \text{ implying } \bar{x} = \frac{\alpha - \psi}{\beta + \pi} > 0 \text{ and } b = \frac{\alpha\pi + \beta\psi}{\beta + \pi} > 0.$$

We suppose that the choke price of fuel α exceeds the cost of the first unit of the backstop ψ. We also define the following parameters:

(18) $$\bar{S} = \frac{\gamma + \kappa(E_0 + S_0)/\rho - b}{\delta + \kappa/\rho}, \quad \hat{s}_1 \equiv \rho - \hat{s}_2 > 0, \quad \hat{s}_2 \equiv \frac{1}{2}\rho - \frac{1}{2}\sqrt{\rho^2 + 4(\rho\delta + \kappa)/\hat{\beta}} < 0,$$

with $\hat{\beta} \equiv \beta\pi / (\beta + \pi)$.

Proposition 10: Simultaneous use from the outset

I. *Asymptotic partial exhaustion:* Suppose $-(\alpha - \psi)/(\hat{s}_2\beta) + \bar{S} > S_0 > \bar{S} > 0.$ Then it is optimal to have simultaneous use from the outset forever ($T_1 = 0, T_2 = \infty$). The path for the fossil fuel resource stock is then given by $S(t) = (S_0 - \bar{S})\exp(\hat{s}_2 t) + \bar{S}$ and the stock approaches $\bar{S}$ asymptotically. Fossil fuel use is given by $q(t) = -(S_0 - \bar{S})\hat{s}_2 \exp(\hat{s}_2 t)$.

II. *Full exhaustion in time:* Let $(\hat{K}_1, \hat{K}_2, T_2)$ solve $\bar{S} + \hat{K}_1 + \hat{K}_2 = S_0$, $-\hat{s}_1 e^{\hat{s}_1 T_2}\hat{K}_1 - \hat{s}_2 e^{\hat{s}_2 T_2}\hat{K}_2 = 0,$ $e^{\hat{s}_1 T_2}\hat{K}_1 + e^{\hat{s}_2 T_2}\hat{K}_2 = 0.$ Suppose $-\hat{s}_1\hat{K}_1 - \hat{s}_2\hat{K}_2 \le (\alpha - \psi)/\beta.$ Then it is optimal to have simultaneous use from the outset with the backstop taking over and fossil fuels being fully exhausted in finite time T_2.

Backstop use follows in both cases from $x(t)=\frac{\alpha-\psi}{\beta+\pi}-\frac{\beta}{\beta+\pi}q(t)>0.$ It is never optimal to start with simultaneous use and then the backstop taking over fully within finite time at a positive level of fossil fuel reserves in situ.

Proof: Along an interval of time with simultaneous use we have $\alpha-\beta(q(t)+x(t))=\psi+\pi x(t).$ We get the second-order differential equation: $\hat{\beta}\ddot{S}-\rho\hat{\beta}\dot{S}-(\delta\rho+\kappa)S=\rho(b-\gamma)-\kappa(E_0+S_0).$ The solution is given by

(19) $S(t)=\hat{K}_1\exp(\hat{s}_1 t)+\hat{K}_2\exp(\hat{s}_2 t)+\bar{S}$ and $q(t)=-\hat{s}_1 e^{\hat{s}_1 t}\hat{K}_1-\hat{s}_2 e^{\hat{s}_2 t}\hat{K}_2.$

where $\hat{K}_1$ and $\hat{K}_2$ are constants to be determined. Under the conditions mentioned under case I, we have a solution satisfying all the necessary conditions. Indeed, we can take $\hat{K}_1=0$. Then $S(t)\to\bar{S}>0$ as $t\to\infty$. We must then have $\hat{K}_2=S_0-\bar{S}$. Moreover, $q(0)=-\hat{s}_2\hat{K}_2\leq(\alpha-\psi)/\beta$ and therefore $x(0)>0.$ Hence, this is an optimum. The same holds for the conditions mentioned under case II. It cannot be optimal to start with simultaneous use and then the backstop taking over completely within finite time at a positive stock level. If this would occur, then at the transition we should have $b=\gamma-\delta\bar{S}+\kappa(E_0+S_0-\bar{S})/\rho.$ Therefore, the solution of the differential equation gives $\bar{S}=\hat{K}_1\exp(\hat{s}_1T_2)+\hat{K}_2\exp(\hat{s}_2T_2)+\bar{S}$ but this is at variance with $q(T_2)=-\hat{s}_1\hat{K}_1\exp(\hat{s}_1T_2)-\hat{s}_2\hat{K}_2\exp(\hat{s}_2T_2)$ unless $T_2\to\infty$. Q.E.D.

We now characterize two further cases for when it is optimal to start with a phase using only fossil fuel and analyze the effects of a lower cost of the backstop technology using propositions 9 and 10. In our discussion we make use of the result that introducing a backstop subsidy, σ, in a decentralized market economy which does not internalize global warming externalities ($\kappa=0$) is analytically equivalent to lowering the marginal cost of the backstop, ψ, in the socially optimal outcome. When we speak of the Green Paradox, we take a narrow interpretation in the sense of a backstop subsidy reducing green welfare.

I. Backstop kicks in immediately and asymptotic partial exhaustion of fossil fuel

The conditions of case I of proposition 10 are relevant if the social cost of extracting the last drop of fossil fuel exceeds the cost of using only the backstop. A decrease in ψ, which is the marginal cost of the backstop at zero production, lowers b and has no impact on $\hat{s}_1$ or $\hat{s}_2$. The expression $(\alpha-\psi)/\rho$ gets larger, so that we remain in the regime with simultaneous use throughout. Hence, upon a decrease in the marginal cost of the backstop, the asymptotic stock of fossil fuel gets higher ($\bar{S}$ increases). More fossil fuel is left in the ground and there is no incentive to extract fossil fuel faster, so green welfare is boosted.

II. Backstop kicks in immediately and full exhaustion of fossil fuel in finite time

The conditions of case II of proposition 10 are relevant if the social cost of extracting the last drop of fossil fuel is less than the cost of using only the backstop. From equation (19) in the proof of proposition 10 we find that given the conditions $q(T_2) = S(T_2) = 0$ we can solve for $\hat{K}_1$ and $\hat{K}_2$ and obtain the paths of fossil fuel reserves and fossil fuel use:

$$S(t) = \bar{S} - \bar{S}\left(\frac{\hat{s}_1 \exp\left(\hat{s}_2 (t-T_2)\right) - \hat{s}_2 \exp\left(\hat{s}_1 (t-T_2)\right)}{\hat{s}_1 - \hat{s}_2} \right) \text{ and}$$

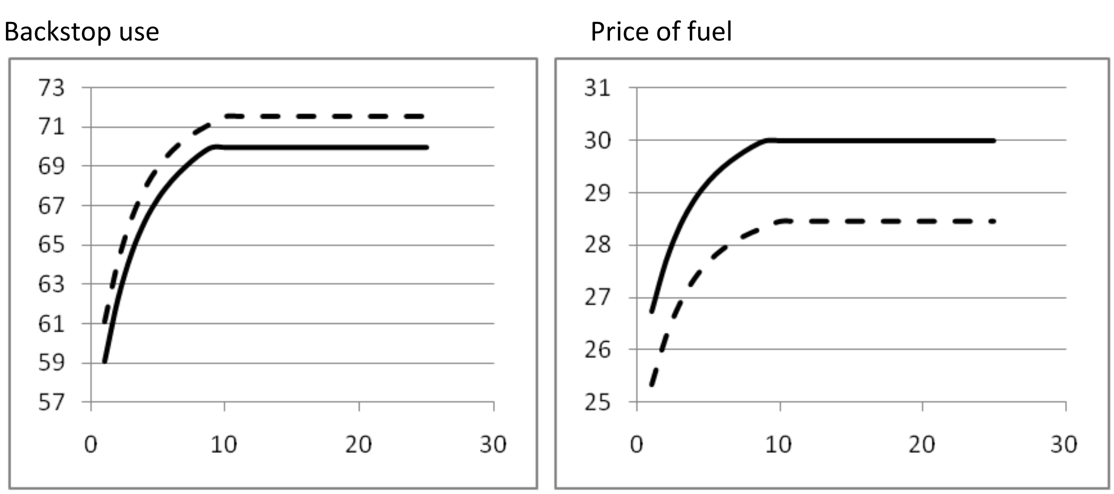
(19′) and

$$q(t) = \left(\frac{\hat{s}_1 \hat{s}_2}{\hat{s}_1 - \hat{s}_2} \right) \bar{S} \left[\exp\left(\hat{s}_2 (t-T_2)\right) - \exp\left(\hat{s}_1 (t-T_2)\right) \right] > 0 \text{ for } t \geq 0.$$

The time at which fossil fuel is fully exhausted T_2 follows from the condition

$S_0 = \bar{S} - \bar{S}\left(\frac{\hat{s}_1 \exp\left(-\hat{s}_2 T_2\right) - \hat{s}_2 \exp\left(-\hat{s}_1 T_2\right)}{\hat{s}_1 - \hat{s}_2} \right)$ or $\frac{\hat{s}_1 \exp(-\hat{s}_2 T_2) - \hat{s}_2 \exp(-\hat{s}_1 T_2)}{\hat{s}_1 - \hat{s}_2} = \frac{b - \gamma + \delta S_0 - \kappa E_0 / \rho}{b - \gamma - \kappa (E_0 + S_0) / \rho}$. The left-hand side of this equation is increasing in T_2. The right-hand side is increasing in the initial oil stock, in the initial atmospheric CO2 concentration, in the marginal extraction cost of the last drop of fossil fuel, and decreasing in b. The latter implies that in the case at hand lower production costs of the backstop (e.g., due to lower ψ or π) give rise to *later* exhaustion, contrary to the case of constant marginal cost of the backstop. The reason is that now the cheaper backstop already substitutes for fossil fuel during the phase of simultaneous use, which boosts green welfare as on the one hand a non-polluting backstop is introduced more quickly and on the other hand the depletion of fossil fuel occurs more gradually. We thus obtain the insight that with convex backstop production costs the Green Paradox does not hold even if there is full exhaustion of fossil fuel reserves. However, this case requires the backstop to be used from the outset and thus the initial resource stock should be small.

Figure 3: Full exhaustion of fossil reserves in finite time and backstop kicking in at time zero



These insights are confirmed by the illustrative solution paths for use of the backstop and the social price of fossil fuel (i.e., the market price plus the optimal Pigouvian CO2 tax) plotted in fig. 3 with parameter values α = 100, β = 1, γ = 1, δ = 0.5, ρ = 0.2, S_0 = 9, E_0 = 1, κ = 0.5 and ψ = 9 or 7, so that b = 30 or 28.5 and $\overline{S}$ = -1.3 or -0.8. The horizontal axes display time as 4t. Introducing a backstop subsidy (i.e., lowering ψ from 9 to 7; see dashed lines) thus postpones the date of exhaustion from 7.76 to 9. Backstop use is boosted while fuel prices are lower everywhere. So in contrast to the case where the backstop is constant (as in sections 2 and 3), the regime of full exhaustion of fossil fuel reserves now does not yield a Green Paradox either. This stark rebuttal of the Green Paradox arises from the fastened gradual phasing in of more and more expensive backstops from time zero.

III. Backstop kicks in later than fossil fuels and asymptotic partial exhaustion of fossil fuel

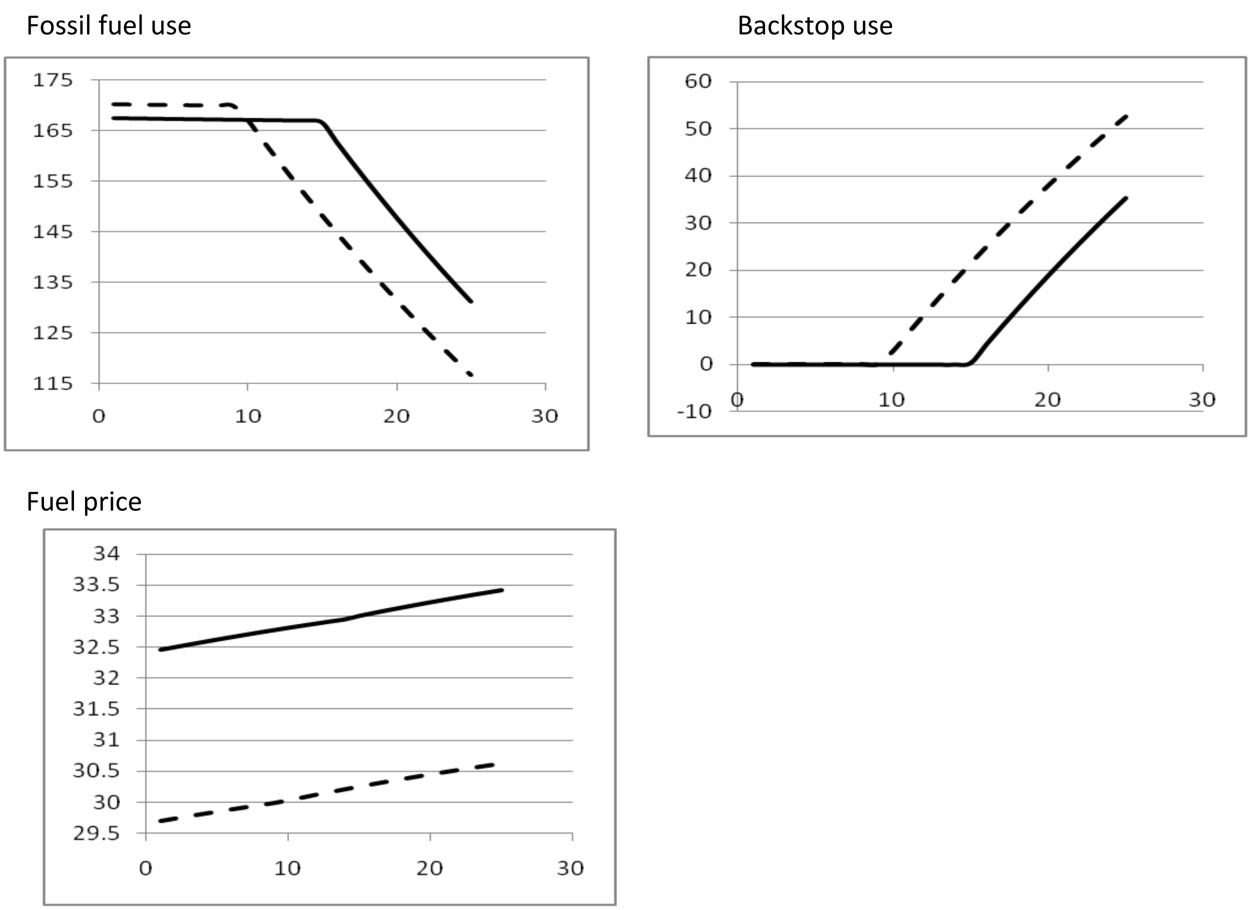
Now a positive stock of fossil fuels is left in situ, but the initial stock is too large to warrant immediate simultaneous use. So, suppose we have an initial interval of time with only fossil fuel use until T_1. From this point on, we have simultaneous use forever. The backstop thus kicks in at $t = T_1$ with $x(0) = 0$ after a phase of using only fossil fuel. It then rises asymptotically to $\overline{x}$. Fossil fuel use starts with $q(0) > (\alpha-\psi)/\beta$, then declines gradually reaching $q(T_1) = (\alpha-\psi)/\beta$ at $t = T_1$ and asymptotically approaching zero as $t \rightarrow \infty$. Fossil fuel stocks fall asymptotically from S_0 at time zero to $\overline{S}$. The differential equation for $t \geq T_1$ is $\hat{\beta}\ddot{S} - \rho\hat{\beta}\dot{S} - (\delta\rho + \kappa)S = \rho(b-\gamma) - \kappa(E_{T_1} + S_{T_1}) = \rho(b-\gamma) - \kappa(E_0 + S_0)$. Hence, with $\overline{S} > 0$, its solution is $S(t) = \hat{K}_2 e^{\hat{s}_2(t-T_1)} + \overline{S}\ (t \geq T_1)$ with $\hat{K}_2 = -(\alpha-\psi)/\hat{s}_2\beta.$ Hence $S(T_1) = \hat{K}_2 + \overline{S}$. Now suppose we are in a regime with $T_1 > 0$ and that ψ falls. Hence, both $\overline{S}$ and $\hat{K}_2$ increase. Ultimately more fossil fuel is left in situ, and at the instant of time where simultaneous use starts, fossil fuel extraction gets higher. Hence, before the new T_1 extraction is increased. These two effects are compatible only if the new T_1 is smaller than the old one and the economy phases in renewables more quickly. Hence, initially we have more extraction, for a shorter period of time. Fig. 4 gives some illustrative solution trajectories with parameter values α = 200, β = 1, γ = 100, δ = 1, ρ = 0.526, S_0 = 100, E_0 = 0, κ = 0.263, ψ = 33 or 30, and π = 0.0118.[19] The horizontal axes display time as 400 t. These parameter values imply $\overline{x} = 165$ or 168, b = 34.9 or 32.0, $\overline{S}$ = 76.7 or 78.7, Γ = 33.3, s_2 = –0.66 or $\hat{s}_2 = -9.56.$ For these parameter values $q(0)$ is

---

[19] With parameters as above except $\kappa = \rho/2$ and S_0 = 97, we have $S_0 > \overline{S} = 79 > 0$ and $\gamma - \delta S > 0.$ We have chosen ρ so that $q(0) = (\alpha-\psi)/\beta = 172 = -\hat{s}_2\hat{K}_2 = -18\hat{s}_2$ holds. So the economy is exactly on the boundary between an initial phase of using only fossil fuel and the backstop kicking in immediately. A higher value of S_0, say, 100 ensures that the former regime (case III) prevails. A lower value of S_0 ensures that case I prevails.

167 or 170, both of which exceed $(\alpha - \beta)/\psi$, so that there will be an initial phase where only fossil fuel is used. The main insights are confirmed, so introducing a backstop subsidy (lowering ψ from 33 to 30; see dashed lines) brings forward the date at which the backstop is phased in and also leaves more fossil fuel in situ in the long run. As a result of $-s_2$ being much smaller than $-\hat{s}_2$, fossil fuel use hardly diminishes with time during the first phase but it must fall much more rapidly as soon as the backstop is introduced and phased in more and more. This ensures that the fuel price follows a smooth path during the two phases. Interestingly, the backstop subsidy pulls down the whole trajectory of fuel prices, including those that prevail in the first phase when the backstop has not been phased in. This is the Hotelling intertemporal arbitrage logic in action. Since the clean backstop is introduced more quickly and more aggressively and more fossil fuel is left in situ, green welfare increases. But also more fossil fuel is extracted in the initial phase, which reduces green welfare. Hence, there may be a Green Paradox. The numerical exercises performed thus far suggest that the Green Paradox does not appear.

Figure 4: Asymptotic partial exhaustion of reserves with backstop kicking in at time $T_1 > 0$



IV. Backstop kicks in later than fossil fuels and full exhaustion of fossil fuels in finite time

The final case is where $\bar{S} < 0$ and the initial resource stock is too large to warrant the backstop kicking in immediately. From the proof of proposition 10 we find that given the conditions $q(T_2) = S(T_2) = 0$ we can solve for $\hat{K}_1$ and $\hat{K}_2$ and obtain the paths of fossil fuel reserves and fossil fuel use:

(19″)
$$S(t) = \bar{S} - \bar{S}\left(\frac{\hat{s}_1 \exp\left(\hat{s}_2(t-T_2)\right) - \hat{s}_2 \exp\left(\hat{s}_1(t-T_2)\right)}{\hat{s}_1 - \hat{s}_2}\right) \text{ and}$$
$$q(t) = \left(\frac{\hat{s}_1\hat{s}_2}{\hat{s}_1 - \hat{s}_2}\right)\bar{S}\left[\exp\left(\hat{s}_2(t-T_2)\right) - \exp\left(\hat{s}_1(t-T_2)\right)\right] > 0 \text{ for } t \in [T_1, T_2].$$

Making use of the expressions for b in (17) and $\bar{S}, \hat{s}_1$ and $\hat{s}_2$ in (18) and of the fact that $q(T_1)$ given below is a monotonic decreasing function of $T_2 - T_1$, the boundary condition at the moment that renewables are phased in can be used to see how $T_2 - T_1$ depends on the various parameters:

(20)
$$q(T_1) = \left(\frac{\hat{s}_1\hat{s}_2}{\hat{s}_1 - \hat{s}_2}\right)\bar{S}\left[\exp\left(-\hat{s}_2(T_2-T_1)\right) - \exp\left(-\hat{s}_1(T_2-T_1)\right)\right] = \frac{\alpha-\psi}{\beta} > 0$$
$$\Rightarrow \quad T_2 - T_1 = \Upsilon(\overset{+}{\kappa(E_0+S_0)}, \overset{+}{\gamma}, \overset{+}{\delta}, \overset{-}{\psi}, \overset{-}{\pi}, \overset{+/-}{\alpha}, \overset{-}{\beta}, \overset{+/-}{\rho}).$$

Hence, the duration of the phase of simultaneous use of fossil fuel and the backstop is high if fossil fuel extraction is expensive and becomes rapidly more expensive as reserves run out (γ and δ are high), the marginal cost of global warming is high (i.e., if the initial stock of fossil fuel, S_0, and the initial atmospheric CO2 concentration, E_0, are high and κ is high), the backstop is cheap but becomes rapidly more expensive at the margin as more of it is used (low ψ and π), and the demand for fuel is not very price sensitive (low β). Differentiation of equation (7) with respect to time gives the path for fossil fuel reserves for the phase $t \in [0, T_1)$ and thus the boundary condition

$$q(T_1) = -\dot{S}(T_1) = (s_1 - s_2)\left[\frac{\left(S_0+\Gamma\right)\exp(s_2T_1)}{1-\exp\left[(s_2-s_1)T_1\right]}\right] - \left[\frac{\left(s_1 + s_2\exp\left[\left(s_2-s_1\right)T_1\right]\right)\left(S_{T_1}+\Gamma\right)}{1-\exp\left[\left(s_2-s_1\right)T_1\right]}\right] = \frac{\alpha-\psi}{\beta}.$$ This

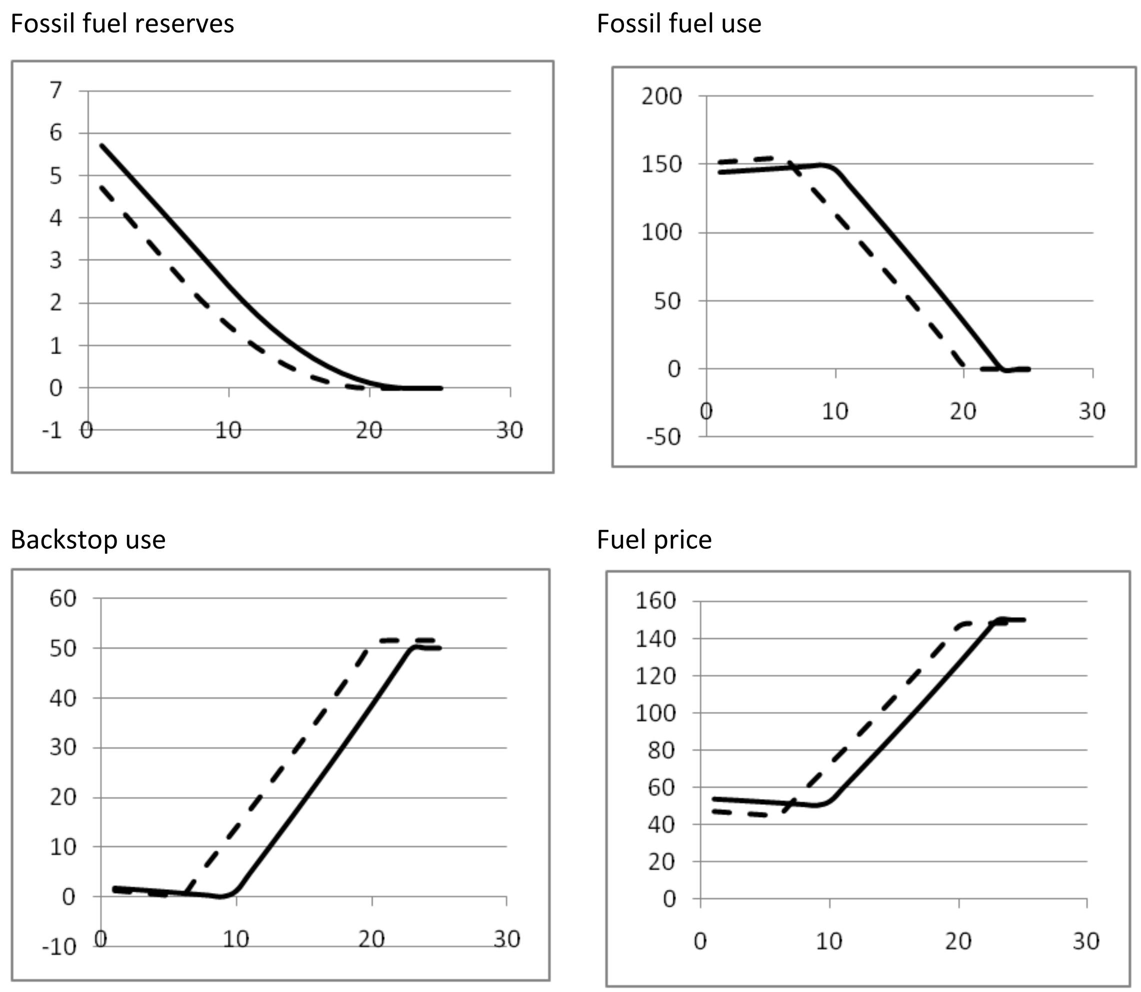
condition together with the final condition $S_{T_1} = \bar{S} - \bar{S}\left(\dfrac{\hat{s}_1\exp\left(-\hat{s}_2(T_2-T_1)\right) - \hat{s}_2\exp\left(-\hat{s}_1(T_2-T_1)\right)}{\hat{s}_1-\hat{s}_2}\right)$

allows one to calculate the values of S_{T_1} and T_1 and thus obtain the full solution for the three phases.

In order to get better intuition of the market outcome, we suppose that private agents do not internalize global warming externalities and therefore set $\kappa = 0$. Setting the other parameters to $\alpha = 200$, $\beta = 1$, $\gamma =$

100, $\delta = 1$, $\rho = 6$, $\psi = 50$ or 45, $\pi = 2$, $E_0 = 0$ and $S_0 = 100$, we get $b = 150$ or 148.3and $\bar{S} = -50$ or -48.3.[20] Fig. 5 then gives the solution paths with the horizontal axes giving time as $0.66 + t/400$.

Figure 5: Full exhaustion with three phases of fuel, fuel/backstop and backstop use



We observe that a backstop subsidy (lower ψ) induces a higher value of fossil fuel use at the end of the first phase, but during the second phase fossil fuel is phased out more quickly. There is a quicker and more aggressive phasing in of renewables (at date 4.95 rather than 8.63). The second phase of simultaneous use of fossil fuel and renewables lasts longer (14.2 rather than 13.2 periods), which boosts green welfare. The date of exhaustion of fossil fuel reserves is brought forward (from date 21.88 to 19.18), which is bad for green welfare. However, as can be seen from the first panel of fig. 5, fossil fuel reserves are lower throughout the whole trajectory and therefore the second effect dominates the effect of

[20] Again, we chose ρ such that for $S_0 = 97$ the economy is exactly on the boundary where $T_1 = 0$. Raising S_0 a little puts the economy into regime IV whilst lowering it a little puts it into regime II.

a quicker and more aggressive phasing in of renewables. This result holds whenever the cost of using only the backstop, b, is bigger than the cost of extracting the drop of fossil fuel, γ. Fuel prices are lower during the initial phase where only fossil fuel is used, but higher during the phase of simultaneous use.

The characterization of the four regimes and the effects of reducing the cost of the backstop in each of these regimes are summarized in Table 1. We conclude that the Green Paradox can be ruled out in regimes I and II, but in regime III it may occur and in regime IV where full exhaustion takes place in finite time and there is an initial phase where only fossil fuel is used it will definitely occur. Sadly, this fourth regime is the one which is most likely to prevail in a decentralized market economies where the costs of global warming are not internalized by an appropriate carbon tax.

Table 1: Effects of lower marginal cost of renewable backstop

	Asymptotic partial exhaustion: $(b < \gamma + \kappa(E_0 + S_0)/\rho, \bar{S} > 0)$	Full exhaustion in finite time: $(b > \gamma + \kappa(E_0 + S_0)/\rho, \bar{S} < 0)$
Backstop kicks in immediately: low initial fossil fuel reserves	I. More fossil fuel is left in situ. Extraction speed unaffected. No Green Paradox.	II. Postpones exhaustion of fossil fuel. More aggressive phasing on renewable. No Green Paradox.
First initial phase with only fossil fuel: high initial fossil fuel reserves	III. Initially more rapid fossil fuel extraction, but in the long run more fossil fuel is left in situ. Renewables are phased in more quickly (and more aggressively) at which point fossil fuel extraction speeds up. Green Paradox may arise.	IV. Initial more fossil fuel is left in the ground. Backstop is introduced more quickly and the simultaneous phase lasts longer. Fossil fuels are exhausted more quickly. Green Paradox will occur in a market economy.

6. Conclusions

We show that a smaller initial stock of fossil fuel reserves, a positive shock to demand for energy fuels, and a lower cost of extracting fossil fuels, mean that fossil fuels are more rapidly exhausted in a first-best economy with a clean backstop, having constant marginal cost. We also show that, if the atmosphere has already been polluted with a lot of CO2 emissions, it is socially optimal to postpone depletion of oil and gas in order to combat global warming. Sinn's Green Paradox arises if the backstop (e.g., solar or wind energy) is currently economically less attractive than oil or gas, but more attractive from an environmental point of view as C02 emissions are insignificant. If, following Sinn, we suppose that a Hotelling ramp for taxes on C02 emissions is politically infeasible, then the government might resort to subsidizing solar or wind energy, as is done on a large scale in Germany. In that case, depletion of oil and gas might occur more rapidly and climate change damages increase. This phenomenon is called the Green Paradox. It occurs if the backstop price is high relative to the extraction costs. We also show that total welfare might decrease. If the concern for the environment is substantial, it would be better to tax the clean backstop in order to postpone exhaustion. However, if a substantial subsidy renders the clean backstop cheaper than fossil fuel, total welfare will be enhanced if the concern for the environment is large enough.

However, Sheik Ahmed Zaki Yamani, the colourful former Saudi oil minister, has been quoted in the New York Times as saying “The Stone Age came to an end not for a lack of stones and the Oil Age will end, but not for lack of oil”. It is this insight which lies at the heart of our critique of the Green Paradox. Fossil fuel will not and should perhaps not last forever if cheap and clean alternatives are available or become available in the future. We show that if the backstop is relatively cheap and low on C02 emissions compared to oil and gas, subsidizing the backstop leads to a bigger final in situ stock of oil and gas reserves and to a higher rate of extraction of oil and gas at the time that the economy switches to using the backstop. Subsidizing the backstop leads to less extraction so that not all oil and gas reserves will be extracted from the earth. Climate damages will now be less and there is no Green Paradox.

If the non-renewable resource is owned by a monopolist, limit pricing will occur. Moreover, due to our assumption of linear demand, initial monopolistic extraction is larger than under perfect competition. The Green Paradox prevails if the backstop price is relatively high compared to the initial marginal cost of extracting oil or gas. If the backstop price is relatively low, a larger stock of oil and gas reserves is left in situ. Interestingly, the Green Paradox need not necessarily occur yet this is the situation that is closest to what Sinn (2008ab) had in mind.

We offer also insights with increasing marginal cost of supplying the backstop. In that case, the optimal path is characterized by a first phase of only fossil fuels, a second phase with simultaneous use of fossil fuels and the backstop, and a third phase with only use of the backstop (where the first and third phases may be degenerate). Even if it is optimal to fully exhaust fossil fuel reserves, the Green Paradox need no longer hold provided renewable are already being used alongside fossil fuels as lowering the backstop cost will either postpone exhaustion of fossil fuels or lead to more fossil fuels to be left in situ. If renewables are not being used yet, a backstop subsidy will bring in the backstop more quickly alongside fossil fuels and for a longer period; but during the phase that only fossil fuels are being used, fossil fuel extraction will be higher. If some fossil fuel reserves are left in situ, we can therefore not say whether green welfare will fall or rise. But in the more likely market outcome where global warming externalities are not internalized and all fossil fuel reserves are fully exhausted in finite time, a backstop subsidy definitely reduces green welfare in line with the Green Paradox.

It may be worthwhile to extend our analysis in the following directions. First, it may be of interest to allow for imperfect substitution in the demand for the non-renewable and the backstop energy source. This may arise from concerns with security of energy supplies, diversification and/or intermittence of backstops such as wind and solar energy and will lead to the simultaneous use of both the non-renewable and the backstop. Second, it is important to investigate what happens if there are various types of backstop available at the same time. If it is possible to rank them, e.g., clean but competitive (nuclear), clean and expensive (wind, solar, advanced nuclear) and dirty and expensive (tar sands), it is best to go for the cleanest and cheapest backstop. However, with dirty and cheap backstops, matters are more complicated especially if we allow for upward-sloping supply schedules of the backstop. Third, given that once non-renewables are exhausted, it becomes attractive to abolish the tax on the backstop and therefore it is of interest to investigate credibility aspects of optimal climate change policies. Fourth, the analysis could be extended to an international context by analyzing issues of carbon leakage and ways to sustain international cooperation (see Hoel, 2008; Eichner and Pethig, 2009, 2010). Fifth, one could investigate the issues we addressed in this paper within the context of a Ramsey model with capital formation and pollution. Finally, one could use the analysis to empirically investigate the various policies that can be used to combat global warming.

References

D'Arge, R. and K. Kogiku (1973). Economic growth and the environment, *Review of Economic Studies*, 40, 61-77.

Dasgupta, P. and G. Heal (1979), *Economic Theory and Exhaustible Resources*, Cambridge University Press, Cambridge.

Edenhofer, O. and M. Kalkuhl (2009). Das Grünen Paradox – Menetekel oder Prognose, to appear.

Eichner, T. and R. Pethig (2009). Carbon leakage, the green paradox and perfect future markets, Working Paper 2542, CESifo, Munich.

Eichner, T. and R. Pethig (2010). International carbon emissions trading and strategic incentives to subsidize green energy, Working Paper No. 3083, CESifo, Munich.

European Commission (2003). World Energy, Technology and Climate Policy Outlook 2030 – WETO, Directorate General for Research- Energy, European Commission, Brussels.

Gerlach, R. (2009). Too much oil, Keynote Lecture, CESifo Conference, Munich University.

Golosov, M., J. Hassler, P. Krusell and A. Tsyvinski (2009). Optimal taxes on fossil fuel in general equilibrium, mimeo.

Grafton, R.Q., T. Kompas and N.V. Long (2010). Biofuels subsidies and the Green Paradox, Working Paper No. 2960, CESifo, Munich.

Groot, F., A.J. de Zeeuw and C. Withagen (2003). Strong time-consistency in the cartel-versus-fringe model, *Journal of Economic Dynamics and Control*. 28, 2, 287-306.

Hassler, J., P. Krusell and C. Olovsson (2009). Oil monopoly and the climate, American Economic Review, Papers and Proceedings, to appear.

Heal, G. (1976). The relationship between price and extraction cost for a resource with a backstop technology, *Bell Journal of Economics*, *7*, 371-378.

Heal, G. (1985). Interaction between economy and climate: A framework for policy design under uncertainty, in V. Smith and A. White (eds.), *Advances in Applied Microeconomics*, JAI Press, pp. 151-168.

Hoel, M. (1978). Resource extraction, substitute production, and monopoly, *Journal of Economic Theory*, 19, 28-37.

Hoel, M. (1983). Monopoly resource extractions under the presence of predetermined substitute production, *Journal of Economic Theory*, 30, 201-212.

Hoel, M. (2008), Bush meets Hotelling: effects of improved renewable energy technology on greenhouse gas emissions, Working Paper 2492, CESifo, Munich.

Hoel, M. and S. Kverndokk (1996). Depletion of fossil fuels and the impacts of global warming, *Resource and Energy Economics*, 18, 115-136.

Krautkraemer, J. (1985). Optimal growth, resource amenities and the preservation of natural environments, *Review of Economic Studies*, 51, 153-170

Neuhoff, K. (2004). Large scale deployment of renewables for electricity generation, Working Paper 59, The Cambridge-MIT Institute.

Paltsev, S., J.M. Reilly, H.D. Jacoby and J.F. Morris (2009). The cost of climate policy in the United States, Report No. 173, MIT Joint Program on the Science and Policy of Global Change, MIT, Cambridge, Mass.

Ploeg, F. van der and C. Withagen (2010). Coal and global warming: dirty backstops and the Green Paradox, OxCarre, Research Paper, University of Oxford.

Salant, S.W. (1977). Staving off the backstop: dynamic limit-pricing with a kinked demand curve, International Finance Discussion Papers110, Board of Governors of the Federal Reserve System, U.S.

Shihab-Eldin, A. (2002). New energy technologies: trends in the development of clean and energy efficient energy technologies, Organization of the Petroleum Exporting Countries,

Sinclair, P.J.N. (1994). On the optimum trend of fossil fuel taxation, *Oxford Economic Papers*, 46, 869-877.

Sinn, H.W. (1981). The theory of exhaustible resources, *Zeitschrift für Nationalökonomie*, 41, 1-2, 183-192.

Sinn, H.W. (1982). Absatzsteuern, Ölförderung und das Almendeproblem, in H. Siebert (ed.), *Reaktionen Auf Energiepreissteigerungen*, Lang, Frankfurt and Bern.

Sinn, H.W. (2008a). Public policies against global warming, *International Tax and Public Finance*, 15, 4, 360-394.

Sinn, H.W. (2008b). *Das Grüne Paradoxon. Plädoyer für eine Illusionsfreie Klimapolitik*, Econ, Berlin.

Stern, N. H. (2007). *The Economics of Climate Change: The Stern Review*, Cambridge University Press, Cambridge, U.K.

Tsur, Y. and A. Zemel (2003). Optimal transition to backstop substitutes for nonrenewable resources, *Journal of Economic Dynamics and Control*, 27, 551-572.

Tsur, Y. and A. Zemel (2005). Scarcity, growth and R&D, *Journal of Environmental Economics and Management*, 49, 484-499.

Ulph A. and D. Ulph (1994). The optimal time path of a carbon tax, *Oxford Economics Papers*, 46, 857-868.

Withagen, C. (1994). Pollution and exhaustibility of fossil fuels, *Resource and Energy Economics*, 16, 235-242.