DIRECTED TECHNOLOGICAL CHANGE: IT'S ALL ABOUT KNOWLEDGE

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ABSTRACT. Directed technological change (DTC) hinges on how stocks of factor-augmenting knowledge evolve relative to each other, which depends in turn on the nature of the links between them. We analyse these links in an analytical model focusing explicitly on the relative knowledge stock: in the process we generalize and systematize a number of results from the literature on DTC, and generate some novel results. Finally, we discuss future research directions in the light of existing theory, empirical evidence, and policy problems concerning environment and resources.

Please note that this is work in progress: it may suffer from errors, and undoubtedly suffers from omissions.

1. INTRODUCTION

The direction of technological change has occupied economists at least since Hicks (1932). However, the overall rate of change has received far greater attention, as it has become clear (since for instance Solow (1956)) that this rate is what determines the overall growth rate in the economy; interest exploded following the methodological and conceptual breakthroughs in understanding endogenous technological change exemplified by Romer (1990) and Aghion and Howitt (1992).

In the case of endogenous directed technological change — henceforth *DTC* — the methodological breakthroughs came even more recently (indeed they built on the earlier work), and are exemplified by the work of Acemoglu (e.g. 1998; 2002) where the focus is on returns to skilled contra unskilled labour. The case presents a puzzle since, during the 20th century, rises in the quantity of skilled labour (assumed exogenous) were followed, after a lag, by rises in relative returns to skilled labour; thus the long-run demand curve for skilled labour over the period slopes upwards, and the factor share of skilled labour has risen beyond recognition since 1900. This puzzle is interesting, but does not give rise to an obviously important policy question to match the question of how to raise the overall growth rate in an economy. This may explain why interest in the field has been relatively lukewarm compared to interest in the determinants of the overall growth rate.

An area of research where the importance of DTC is obvious is long-run links between the economy, the environment, and natural resources. We know that physical flows (for instance flows of resource inputs, or polluting by-products) must be bounded in the long run. The traditional approach when facing this fact has been to focus on the degree of substitutability between input factors given constant technology. This was justified explicitly by Solow in his Richard T. Ely lecture (Solow, 1974, p.11):

> ...there is virtue in analyzing the zero-technical-progress case because it is easy to see how technical progress can relieve and perhaps eliminate the drag on economic welfare exercised by natural-resource scarcity.

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However, we know (and Solow knew) that it is technological change — the creation and application of ideas — which drives the process of long-run growth, i.e. the process by which an increasing flow of value is produced using given inputs of labour. This follows because there must be decreasing returns to increasing the quantity of physical capital, i.e. the sheer number of machines of a given type. By the same logic, technological change must also lie behind the process by which an increasing flow of value is produced using given inputs of resources, a process which must occur if economic growth is to continue while resource flows are bounded. Therefore, to study growth and resource flows simultaneously we must study technological change with respect to the productivity of both labour inputs and resource inputs: DTC.

Consider for instance the relatively new and very important policy question of how best to reduce the use of fossil fuels in the economy and hence reduce CO_2 emissions to the atmosphere. Since fossil fuels are factors of production, it is essential to understand how markets for fossil fuels (the supply side) and technology (such as fuel-augmenting or labour-augmenting technologies) may respond to measures by individual countries, or coalitions of countries, to reduce their fuel use. Or indeed how markets are likely to respond in case global supply considerations push up the fuel price. That is, models of DTC are required.

Acemoglu's framework is a powerful one, and it is already being applied to energy and resource markets; see for instance Smulders and de Nooij (2003) and Acemoglu et al. (2009). Nevertheless, we argue that the existing literature is overly narrow in the range of modelling approaches, and that many fundamental questions remain to be answered about the driving forces of DTC. Our aim is thus to deepen the understanding of directed technological change, with a particular focus on the nature of knowledge (technology) spillovers, generalizing existing results and generating new results.

We now return to the literature, aiming to establish the two claims made above, i.e. that the literature is overly narrow and that important questions remain unanswered. Concerning the first claim, the modern approach flows to a great extent from the pathbreaking work of Acemoglu (e.g. 1998; 2002), although Kiley (1999) should also be noted. Acemoglu's approach is essentially to build two Romer-type sectors, each using one input and a sector-specific range of intermediate goods; the wider the range, the greater the productivity in that sector. In our notation we have inputs A and B, quantities q_a, q_b and productivity levels k_a, k_b; so effective input quantities are $k_a q_a$ and $k_b q_b$. The two aggregate goods compete with one another on the consumption side (they are imperfect substitutes). The long-run development of the economy depends on rates of increase in the respective ranges of intermediate goods. The large majority of papers incorporating DTC have used Acemoglu's model, and no alternative framework has been fleshed out.[1]

Concerning unanswered questions, note first that in a series of papers the result emerges that long-run factor shares are fixed despite changes in the quantities of factors. However, nowhere are conditions derived under which this result holds. This is of course a very important lacuna, since the result — fixed factor shares — is a beautiful one, making it a trivial matter to predict the market response to changes in factor quantities.

An early paper with the fixed-shares result is Kennedy (1964). Kennedy considers, in effect, a single product manufactured by a single firm using two factors which are complements; each factor has its own (independent) stock of factor-augmenting knowledge,

[1]Note that Hart (2009) uses an alternative approach, but it is not a central part of the model and not fully developed.

and the two stocks grow at constant relative rates given constant relative investments. Under these circumstances optimal investments will be increasing functions of the relevant factor share, and if the share of an input rises, investments in factor-augmenting technology will rise, pushing the share back down again. Acemoglu (2002), in his seminal paper on DTC, notes that the fixed-shares result emerges from his model in one particular specification, with 'extreme state dependence', but does not comment further. Smulders and de Nooij (2003), in a model of the factor share of energy, use an adapted version of the Acemoglu model. They also generate the fixed-share result, but unlike in the work of Acemoglu, it follows from their baseline model. Finally, Hart (2009) models supply and demand for non-renewable resources, and generates the fixed-share result *inter alia* in a novel model of DTC.

A result which is a very explicit focus for Acemoglu is the possibility of upward-sloping demand, that is that if the relative quantity of an input increases, its price may increase. Acemoglu (2002) derives two conditions for this result which apply respective specifications of his model, and he further investigates the more general of them in Acemoglu (2007), but it is not clear from this work how general the condition is, particularly since a clear intuitive interpretation of the condition is lacking.[2]

Acemoglu (2002) also mentions conditions for stability of the internal solutions; the alternative is corner solutions in which one or the other of the inputs captures all the returns. The latter case is highlighted in Acemoglu et al. (2009), where Acemoglu's previous modelling of DTC is applied to the problem of growth with environmental constraints. In the basic model there are two sectors, one 'dirty' and one 'clean', and environmental damages are directly proportional to aggregate output from the dirty sector Y_d.[3] When the goods (and hence also the inputs) are substitutes, innovation favours the more advanced sector, which thus tends to take over; we have a corner solution. There is of course a 'clean corner' and a 'dirty corner', and assuming (with good reason) that the dirty sector dominates initially, and (for simplicity) that factor quantities are constant, then the economy will always end up in the dirty corner. A striking policy conclusion follows; if a regulator can tip the balance in favour of innovation in the clean sector by (for instance) taxing the dirty good, then only a limited period of taxation will be necessary before the clean sector advances ahead of the dirty sector and is thus favoured by innovators irrespective of any regulations.

Finally, Smulders and de Nooij (2003) discuss energy, where an increase in quantity (relative to labour) has been accompanied by a steep decline in the relative price, so steep that the factor share of energy has actually fallen. Smulders and de Nooij derive such a decline in a model with fixed long-run factor shares, assuming that the economy is on a transition path from an initial position with an above-equilibrium factor share of energy.

To sum up, many theoretical papers produce a 'fixed shares' result, but there is no general theory concerning the circumstances under which this result applies. Furthermore, empirical observations show fundamental differences between different cases; a rise in

[2]The results are all based either on perfect markets, a completely centralized economy, or an economy with the same underlying structure as in Acemoglu (1998, 2002). Since we are interested in decentralized but imperfect markets, and wish to generalize from the above-mentioned structure, Acemoglu (2007) is of little use to us.

[3]Note that the latter assumption drastically weakens the relevance of their policy simulations in the case of climate change, since the non-renewable resource is an input into the production of Y_d, and hence only indirectly linked to it. On the other hand, we know that fossil-fuel consumption is *directly* linked to CO_2 emissions and consequent damage.

factor abundance may be followed by anything from a fall in factor returns so steep that the factor share falls, to a rise in factor returns (upward sloping demand curve). The overall growth literature strongly suggests that the modelling of decentralized markets with knowledge spillovers is likely to be crucial in understanding these observations, but the range of such models in the DTC literature is severely limited and the models are very specific.

We begin by setting up a general model of factor-augmenting DTC with investment by decentralized firms and knowledge spillovers between them, and using it to establish some theoretical results. Firstly, we establish conditions under which the fixed-shares result — i.e. that the factor shares on an interior b.g.p. in the economy are independent of the physical quantities of the factors employed — holds. We show that the result applies very generally in economies with two factors and factor-augmenting technologies, if the elasticity of the respective levels of factor-augmenting technologies to investment is independent of the relative levels of the technologies.

To generate further straightforward results, we restrict the model in two ways which limit its ability to describe short-run dynamic adjustment processes, but do not affect its ability to describe long-run processes.We know that the fixed-shares result holds whether the factors are substitutes or complements. However, in the restricted model we show that when they are substitutes the b.g.p. is unstable, and the economy approaches a corner solution in which one factor or the other dominates completely in terms of returns; if factor quantities are endogenous then the quantity of the factor earning zero returns will typically also fall to zero. This result is closely related to the results of Acemoglu et al. (2009). We go on to show that when the levels of factor-augmenting knowledge are linked (knowledge associated with one factor tends to spill over and help in the accumulation of knowledge augmenting the other factor) then these conclusions change, but substitutability/complementarity remains crucial. Specifically, when the factors are substitutes the factor share of a more abundant factor rises, whereas when they are complements the factor share of such a factor falls; these cases correspond to skilled/unskilled labour, and energy/labour, respectively. Here we generalize and reinterpret many of the results of Acemoglu (2002). In the final part of the paper we propose a more natural concept of knowledge spillovers, and show that it leads to more complex results with multiple equilibria, path dependence, and thresholds.

In Section 2 we describe the basic framework, before deriving results for independent knowledge stocks in Section 3, and then for linked knowledge stock in Section 4. In Section 5) we discuss future research directions in the light of existing theory, empirical evidence, and policy problems concerning environment and resources, and in Section 6 we conclude.

2. THE BASIC MODEL

In this section we set up a very general model of a decentralized economy with factor-augmenting technological change, knowledge spillovers between firms, and the potential for long-run balanced growth given constant factor quantities.

First, the production function. We have a continuum of firms indexed by i, and production from firm i is y_i. There are two physical inputs, A and B, which are used in quantities q_{ai} and q_{bi} by firm i, at prices p_a and p_b (the firms are price takers w.r.t. inputs). Furthermore, firm i owns capital (human capital, machines, etc.) with embodied technology which augments the two inputs by factors k_{ai} and k_{bi} respectively. We abstract from the physical quantity of capital and treat k_{ai} and k_{bi} as the levels of factor-augmenting technology. Now denote the *effective* quantities of inputs used by firm i as $\mathcal{A}_i$ and $\mathcal{B}_i$, where $\mathcal{A}_i = k_{ai}q_{ai}$, $\mathcal{B}_i = k_{bi}q_{bi}$. Then production y_i is given by

$$y_i = f(\mathcal{A}_i, \mathcal{B}_i). \tag{1}$$

Holding technology constant, the effect of changes in factor quantities on the factor shares depends on the elasticity of substitution in the production function. However, we are interested in the long-run factor shares when technology adapts; changes in quantities of factors will change relative prices and thus lead to *directed technological change*. To model this, we work in discrete time, but all the results go through in the corresponding continuous-time framework.

Each firm has its own levels of factor-augmenting knowledge, and knowledge levels in period $t+1$ are a function of investments I_a and I_b, and the vectors of knowledge levels of all firms in period t $\mathbf{k}_{at}$ and $\mathbf{k}_{bt}$, as follows:

$$k_{a,t+1} = g(\mathbf{k}_{at}, \mathbf{k}_{bt})\frac{I^{\phi}_{a,t+1}}{r_a}; \tag{2}$$

$$k_{b,t+1} = h(\mathbf{k}_{bt}, \mathbf{k}_{at})\frac{I^{\phi}_{b,t+1}}{r_b}. \tag{3}$$

Here r_a and r_b are positive parameters, ϕ is a positive parameter less than unity, and the functions g and h are homogeneous of degree 1. We thus assume that firms can raise the quality of their labour and machines through investment, and that the elasticity of next-period quality w.r.t. firm investment is constant. (Alternatively, if quality is supplied by another firm, the price set by that firm has the same correspondence to quality.) Furthermore, next-period quality is also a function of current quality within the firm, and also the quality (knowledge) of other firms, as is standard for growth models. Note that r_a and r_b affect the cost of new technology; the higher they are, the lower the cost of the corresponding technology. In effect, firms can purchase increasingly high-quality technology, but at an increasing price. Since g and h are homogeneous of degree one we can also write

$$k_{a,t+1}/k_{at} = g[(1/k_{at})\mathbf{k}_{at}, (1/k_{at})\mathbf{k}_{bt}]\frac{I^{\phi}_{a,t+1}}{r_a}; \tag{4}$$

$$k_{b,t+1}/k_{bt} = h[(1/k_{bt})\mathbf{k}_{bt}, (1/k_{bt})\mathbf{k}_{at}]\frac{I^{\phi}_{b,t+1}}{r_b}. \tag{5}$$

Note that we do not specify what the investment good is; we do not need to, since although we introduce the price of the investment good below (equation 6), it quickly drops out again, since we are only interested in the direction of technological change, not its rate. However, it is important to note that it is the same investment good used in both sectors.

Now assume that the discount factor per period is constant, ρ.[4] Furthermore, the price of the final good produced by the firm is p_{yt}, which may be a function of y_t (that is, we do not rule out market power). Then we can write the following Lagrangian for a single

[4]This simplification makes sense because we are interested in the direction of technological change and not its rate.

firm:

$$\mathcal{L} = \sum_{t=0}^{\infty} \rho^t \{ p_{yt} f(k_{at}q_{at}, k_{bt}q_{bt}) - p_{at}q_{at} - p_{bt}q_{bt} - p_{It}(I_{at}+I_{bt}) \\ - \lambda_{at}\left[k_{at} - g(\mathbf{k}_{a,t-1}, \mathbf{k}_{b,t-1})\frac{I_{at}^{\phi}}{r_a}\right] \\ - \lambda_{bt}\left[k_{bt} - h(\mathbf{k}_{b,t-1}, \mathbf{k}_{a,t-1})\frac{I_{bt}^{\phi}}{r_b}\right] \}. \tag{6}$$

The first-order conditions in q_a and q_b then yield

$$p_{at}q_{at} = (p'_{yt}y_t + p_{yt})\mathcal{A}_t f'_{\mathcal{A}t}; \quad p_b q_b = (p'_{yt}y_t + p_{yt})\mathcal{B}_t f'_{\mathcal{B}t}, \tag{7}$$

and the relative factor shares are

$$\frac{p_a q_a}{p_b q_b} = \frac{\mathcal{A} f'_{\mathcal{A}}}{\mathcal{B} f'_{\mathcal{B}}}. \tag{8}$$

The first-order conditions in I_a and k_a yield

$$p_{It} = \phi \lambda_{at} k_{at} / I_{at} \tag{9}$$

$$\lambda_{at} = q_{at}(p'_{yt}y_t + p_{yt})f'_{\mathcal{A}t} + \rho\lambda_{a,t+1}g'_{kat}(\mathbf{k}_{at}, \mathbf{k}_{bt})I_{a,t+1}^{\phi}/r_a \\ + \rho\lambda_{b,t+1}h'_{kat}(\mathbf{k}_{bt}, \mathbf{k}_{at})I_{b,t+1}^{\phi}/r_b. \tag{10}$$

The first condition states that the marginal cost of investment (p_{It}) must equal its marginal returns, and the second states that the value of knowledge is the sum of its immediate value in boosting production and its value in contributing to next-period knowledge within the same firm.

3. Independent knowledge stocks

In this section we investigate the properties of the model economy when knowledge stocks are independent, the key result being *fixed factor shares*, that factor shares on a b.g.p. are independent of factor quantities.

3.1. The balanced growth path: fixed factor shares.

First we define *independent knowledge stocks*, a *balanced growth path (b.g.p.)*, and *states* 0 and 1, and go on to derive Proposition 1 about when factor shares are fixed.

Definition 1. Knowledge stocks are *independent* if and only if we can write equations 4 and 5 as

$$k_{a,t+1}/k_{at} = g[(1/k_{at})\mathbf{k}_{at}]I_{a,t+1}^{\phi}/r_a; \tag{11}$$

$$k_{b,t+1}/k_{bt} = h[(1/k_{bt})\mathbf{k}_{bt}]I_{b,t+1}^{\phi}/r_b. \tag{12}$$

That is, an increase in a-augmenting technology does not make it easier for firms to generate b-augmenting technology; there are no spillovers between the technology types. If stocks are not independent then they are *linked*.

Definition 2. When the economy is on a b.g.p. then knowledge levels in all sectors grow at the same rate, by a factor θ per period. Then the vectors $(1/k_{at})\mathbf{k}_{at}$ and $(1/k_{bt})\mathbf{k}_{bt}$ are constant, and can be written $\tilde{\mathbf{k}}_a$ and $\tilde{\mathbf{k}}_b$, and $g(\tilde{\mathbf{k}}_a)I_{a,t+1}^{\phi}/r_a = h(\tilde{\mathbf{k}}_b)I_{b,t+1}^{\phi}/r_b = \theta$. Furthermore, factor quantities are constant and prices rise by θ per period, whereas the prices of final goods and the shadow price of knowledge are constant.

Definition 3. In state 0 total factor quantities are q_{ao} and q_{bo} and the knowledge levels of firms are $\mathbf{k}_{a0}$ and $\mathbf{k}_{b0}$; in state 1 the corresponding quantities are q_{a1}, q_{b1}, $\mathbf{k}_{a1}$ and $\mathbf{k}_{b1}$, where $q_{a1} = sq_{ao}$, $q_{b1} = tq_{bo}$, $\mathbf{k}_{a1} = (1/s)\mathbf{k}_{a0}$, $\mathbf{k}_{b1} = (1/t)\mathbf{k}_{b0}$, and s and t are positive scalars.

Proposition 1. Fixed factor shares. *If an economy with independent knowledge stocks is on a b.g.p. in state* 0*, then it is also on a b.g.p. in state* 1*. Furthermore, the relative factor shares in each case are equal.*

For a given firm, the relative factor shares are given by

$$\frac{p_a q_a}{p_b q_b} = \left(\frac{r_a}{r_b}\right)^{1/\phi} \left\{\frac{h[(1/k_{bt})\mathbf{k}_{bt}]}{g[(1/k_{at})\mathbf{k}_{at}]}\right\}^{1/\phi} \frac{1-\rho\theta g'_{kat}(\mathbf{k}_{at},\mathbf{k}_{bt})/g(\tilde{\mathbf{k}}_a)}{1-\rho\theta h'_{kbt}(\mathbf{k}_{bt},\mathbf{k}_{at})/h(\tilde{\mathbf{k}}_b)}. \tag{13}$$

Proof. The condition for balanced growth when knowledge stocks are independent follows from equations 11 and 12, and is that

$$\frac{I_{a,t+1}}{I_{b,t+1}} = \left\{\frac{h[(1/k_{bt})\mathbf{k}_{bt}]}{g[(1/k_{at})\mathbf{k}_{at}]}\frac{r_a}{r_b}\right\}^{1/\phi} \tag{14}$$

for all firms in the economy; that is, investment in each firm is such that the growth rates of knowledge stocks within each firm are equal. We proceed by demonstrating that if this holds in state 0 then it also holds in state 1.

Use equations 9 and 10, the corresponding equations for q_b, and the assumptions of independent knowledge and balanced growth, to show that the optimal investment ratio chosen by a given firm is given by

$$\frac{I_{at}}{I_{bt}} = \frac{f'_{\mathcal{A}t}\mathcal{A}_t}{f'_{\mathcal{B}t}\mathcal{B}_t} \cdot \frac{1-\rho\theta h'_{kbt}(\mathbf{k}_{bt})/h(\tilde{\mathbf{k}}_b)}{1-\rho\theta g'_{kat}(\mathbf{k}_{at})/g(\tilde{\mathbf{k}}_a)}. \tag{15}$$

But $\tilde{k}_{a0} = \tilde{k}_{a1}$ and $\tilde{k}_{b0} = \tilde{k}_{b1}$, from the definitions above. Furthermore, since functions g and h are homogeneous of degree one their first derivatives are homogeneous of degree zero, implying that $g'_{ka0} = g'_{ka1}$ and $h'_{kb0} = h'_{kb1}$. Thus from equation 15 the investment ratios are identical in states 0 and 1 in all firms. By assumption, the economy is on a b.g.p. in state 0. The equality of investment in the two states, and equation 14, then show that the economy must also be on a b.g.p. in state 1. Since the investment ratio is constant, we can use equations 14, 15, and 8 to derive equation 13. □

The intuition behind the proposition is straightforward, and follows Kennedy (1964); assuming a single firm, the assumptions about how knowledge grows imply that a situation with factor-augmenting knowledge k_{a0} and factor quantity q_a is indistinguishable from a situation with knowledge k_{a0}/s and quantity sq_{ao}. Furthermore, factor shares reflect not the physical quantity of factors, but the ease with which factor-augmenting knowledge can be generated; when such knowledge is relatively expensive to generate then the factor share is high. Assuming a single firm (or symmetric firms) within which knowledge carries forward from period t to $t+1$ in the same way in both sectors (q_a and q_b) then we simply have that the relative factor shares are equal to $(r_a/r_b)^{1/\phi}$. If firms are heterogeneous and the sectors differ w.r.t. knowledge spillovers and depreciation then the situation will be more complex; for instance, if knowledge k_a spreads more easily between firms than knowledge k_b then this will reduce the relative factor share of q_a.

Finally, note that the result (Proposition 1) is quite general: it applies to any economy, however complex (for instance with multiple goods and multiple sectors) where growth is achieved through augmenting fixed factor inputs, and knowledge stocks are independent as defined.

3.2. **Stability of the b.g.p.: complements and substitutes.** We now make more restrictive assumptions—defining a *simple economy*—in order to derive an explicit solution to the model and thus straightforwardly demonstrate the stability properties of balanced growth paths in the economy. In further work it would be useful to extend these results to more general economies, but here our focus is on the intuition behind the results and therefore the more restrictive scenario is sufficient.

Definition 4. Define a *simple* economy as one in which there is a continuum of symmetric firms, and investment occurs periodically in such a way that private knowledge depreciates completely from one period to the next, that is $g'_{kat}(\mathbf{k}_{at}, \mathbf{k}_{bt}) = g'_{kbt}(\mathbf{k}_{at}, \mathbf{k}_{bt}) = 0$, and similarly in sector B. Furthermore, the production function of each firm is C.E.S., so we have (for firm i)

$$y_i = f(\mathcal{A}_i, \mathcal{B}_i) = [(k_{at}q_{at})^{\varepsilon} + (k_{bt}q_{bt})^{\varepsilon}]^{1/\varepsilon}, \tag{16}$$

where $\varepsilon \in (-\infty, 1)$, so when $\varepsilon > 0$ the inputs are substitutes.

Note that we can express the development of a simple economy using a single state equation, derived from equations 4 and 5, where $F = g/h$, and reflects spillovers between the sectors:

$$\frac{(k_a/k_b)_{t+1}}{(k_a/k_b)_t} = F\left[(k_a/k_b)_t\right]\left(\frac{I_a}{I_b}\right)^{\phi}_{t+1}\frac{r_b}{r_a}. \tag{17}$$

When knowledge stocks are independent (Definition 1), $F = 1$; when stocks are *linked*, F is decreasing in its argument because the higher is k_a/k_b, the less sector A benefits from spillovers from sector B. Note also that in a simple economy the first-order conditions 9 and 10 yield directly (not just on a b.g.p.) that

$$\left(\frac{I_a}{I_b}\right)_t = \frac{f'_{\mathcal{A}t}\mathcal{A}_t}{f'_{\mathcal{B}t}\mathcal{B}_t} = \left(\frac{\mathcal{A}}{\mathcal{B}}\right)^{\varepsilon}_t. \tag{18}$$

Now, for convenience, define

$$\begin{aligned} &K_t = (k_a/k_b)_t;\ \hat{K}_t = K_{t+1}/K_t; \\ &I_t = (I_a/I_b)_t;\ r = (r_a/r_b);\ P_t = (p_a/p_b)_t;\ Q_t = (q_a/q_b)_t \text{ and } S_t = P_tQ_t. \end{aligned} \tag{19}$$

Substitute equation 18 into 17 to obtain $\hat{K}_t = Q^{\phi\varepsilon}_{t+1}(r_b/r_a)F(K_t)K^{\phi\varepsilon}_{t+1}$, and then use the definition of $\hat{K}_t$ to simplify:

$$\hat{K}_t = \left[Q^{\phi\varepsilon}_{t+1}(r_b/r_a)F(K_t)K^{\phi\varepsilon}_t\right]^{1/(1-\phi\varepsilon)}. \tag{20}$$

Now use equations 8 and 18 to obtain

$$S_t = P_tQ_t = (K_tQ_t)^{\varepsilon}. \tag{21}$$

Recall that Q evolves exogenously, and K is the state variable. These two equations thus describe the evolution of the system from a given starting point K_t, and we have the following proposition.

Proposition 2. *In a simple economy with independent knowledge stocks there is a unique interior b.g.p. at which $S = r^{1/\phi}$. The b.g.p. is globally stable when the elasticity of substitution between the effective factor inputs $\mathcal{A}$ and $\mathcal{B}$ is less than unity, and unstable when it is greater than unity. In the latter case the economy approaches a corner solution in which one or the other factor earns all the returns.*

Proof. Set $F = 1$ and $\hat{K} = 1$ to derive $S = r^{1/\phi}$ from 20 and 21. Return to equation 20, and note that the elasticity of $\hat{K}$ w.r.t. K is simply $\phi\varepsilon$ when $F = 1$, which is positive if $\varepsilon > 0$, and negative if $\varepsilon < 0$. Furthermore, as the economy approaches the corner solutions, $k_a/k_b \to 0$ and $k_a/k_b \to \infty$ the growth factor approaches 0 and ∞ respectively in the former case, and vice versa in the latter case. Hence if $\varepsilon > 0$ we have instability, whereas if $\varepsilon < 0$ we have stability. □

The intuition is straightforward. When the factors are complements, the relative factor share of $\mathcal{A}$, PQ, falls when its quantity rises, dampening investment in factor-augmenting knowledge k_a and hence reversing the increase in $\mathcal{A}$. On the other hand, when factors are substitutes relative cost share of $\mathcal{A}$ rises when its quantity rises, stimulating investment in factor-augmenting knowledge k_a and hence accelerating the rise in $\mathcal{A}$. Hence instead of returning to the b.g.p., the economy approaches a corner at which $\mathcal{A}$ captures all the returns. Finally, the factor share on the b.g.p. depends on the relative ease of developing factor-augmenting technology; a factor which is easily augmented earns a low share.

Propositions 1 and 2 tally with Acemoglu's results in the knowledge-based-R&D model with 'extreme state dependence', which is Acemoglu's term for what we call independent knowledge stocks. In Acemoglu (1998, 2002) the factors are substitutes in the production function, but may nevertheless be 'gross' complements (Acemoglu's terminology) due to complementarity of the two final goods. Acemoglu (2002) notes on p.795 that—given independent knowledge stocks—b.g.p. factor shares are fixed irrespective of factor quantities (Proposition 1), and on p.794 that 'the system is stable only [...] when the two factors are gross complements' (Proposition 2). Acemoglu does not highlight the implication of instability, which is that we have a corner solution in which either skilled or unskilled labour dominate completely.

4. Linked knowledge stocks

We now turn to the situation where knowledge stocks are linked, as defined in Definition 1. Linkage implies that knowledge spills over from sector A to sector B and vice versa, which is of course to be expected in general. For instance, when firms learn a lot about how to use labour efficiently to produce final goods, we would expect some of that knowledge to spill over into how to use fossil fuels efficiently. Thus if labour-augmenting knowledge moves ahead of fuel-augmenting knowledge, pushing fuel use up, it should (technologically) become easier and easier to raise the level of fuel-augmenting knowledge by borrowing from the large stock of labour-augmenting knowledge. We work with two specifications of linkage, the first—denoted 'constant elasticity dependence'—following Acemoglu (2002), the second based on what we argue to be a more natural concept of spillovers, and thus simply denoted 'knowledge spillovers'.

4.1. **Constant elasticity dependence.**

Recall that function $F = g/h$ (equation 17) determines the nature of links between knowledge stocks. When $F = 1$ there are no links; when F is a decreasing function of K (recall that $K = k_a/k_b$) there are links. We analyse two functions F, the first in this section and the second in the following section.

Definition 5. There is *constant elasticity knowledge dependence* when

$$F(K_t) = K_t^{-\sigma_c},$$

where $\sigma_c \in (0,1]$.

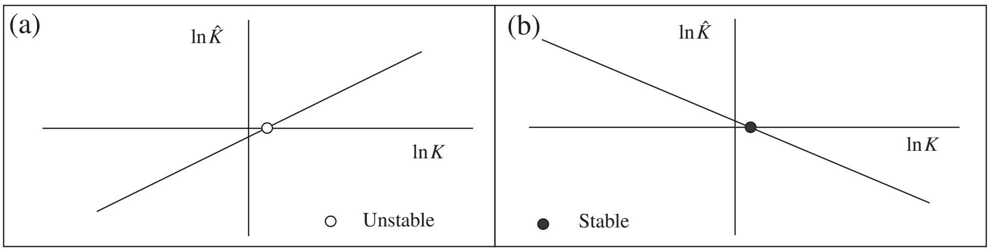

FIGURE 1. The growth factor $\hat{K}_t$ as a function of knowledge K_t. Recall that $\hat{K}$ is monotonically increasing in Q. In (a) we have constant elasticity knowledge dependence and $\phi\varepsilon > \sigma_c$, whereas in (b) we have $\phi\varepsilon < \sigma_c$. The ring indicates an unstable b.g.p., and the dot indicates a stable b.g.p.

Here σ_c measures the strength of knowledge dependence; when σ_c approaches zero there is no dependence, and when $\sigma_c = 1$ knowledge levels are effectively tied together. We then have the following proposition.

Proposition 3. *In a simple economy with constant elasticity knowledge dependence there is a unique interior b.g.p. which is globally stable when $\sigma_c > \phi\varepsilon$, and unstable given the opposite. In the latter case the economy approaches a corner solution in which one or the other factor earns all the returns.*

Proof. Substitute for F in equation 20 using Definition 5 to obtain

$$\hat{K}_t = \left[Q_{t+1}^{\phi\varepsilon}(r_b/r_a)K_t^{\phi\varepsilon-\sigma_c}\right]^{1/(1-\phi\varepsilon)}. \tag{22}$$

Since $\phi\varepsilon < 1$ the elasticity of $\hat{K}_t$ w.r.t. K_t is everywhere positive if $\phi\varepsilon > \sigma_c$, and negative if $\sigma_c > \phi\varepsilon$. Furthermore, as the economy approaches the corner solutions ($K \to 0$ and $K \to \infty$), $\hat{K}$ approaches 0 and ∞ respectively in the former case (i.e. when $\phi\varepsilon > \sigma_c$), and vice versa in the latter case. □

The intuition here is that links between the knowledge stocks encourage accumulation of k_b when k_a/k_b (i.e. K) rises, and this favours stability. The higher is σ_c, the stronger are the links. On the other hand, the higher are ε and ϕ the stronger are the countervailing forces. Because elasticities are everywhere constant, we have a unique interior b.g.p. The two alternatives are illustrated in Figure 1.

We now turn our attention to analysis of comparative statics of factor shares and factor returns on the b.g.p., when relative factor quantities q_a/q_b change, and hence Q changes. Recall Proposition 1, that when knowledge stocks are independent then factor shares on a b.g.p. are not affected by changes in factor quantities. What is the effect of such changes in a simple economy with constant elasticity knowledge dependence?

Proposition 4. *In a simple economy with constant elasticity knowledge dependence on a globally stable b.g.p., an increase in Q (corresponding to an increase in the ratio q_a/q_b) leads to a fall in the relative factor share of q_a, PQ, on the new b.g.p. if the factors are complements ($\varepsilon < 0$), and a rise in the relative factor share if they are substitutes ($\varepsilon > 0$). In the latter case, the relative returns to q_a on that b.g.p., P, rise if $\phi\varepsilon/(1-\varepsilon) > \sigma_c$.*

Proof. Put $\hat{K} = 1$ into equation 22 to yield, in long-run equilibrium,

$$KQ = (K^{\sigma_c}R)^{1/(\phi\varepsilon)}.$$

Now use equation 21 ($S = PQ = (KQ)^\varepsilon$) to obtain

$$S = (Q^{\sigma_c}/R)^{\varepsilon/(\sigma_c - \phi\varepsilon)},$$

and

$$P = \left[R^{-\varepsilon/(1-\varepsilon)} Q^{\phi\varepsilon/(1-\varepsilon)-\sigma_c}\right]^{(1-\varepsilon)/(\sigma_c-\phi\varepsilon)}. \tag{23}$$

Since we know that $\varepsilon < 1$ and $\sigma_c > \phi\varepsilon$ (we assumed stability), it follows by inspection that (i) the elasticity of relative factor shares S w.r.t. relative quantities Q is positive if the factors are substitutes ($\varepsilon > 0$) and negative if they are complements, and (ii) the elasticity of relative factor returns P w.r.t. Q is positive if $\phi\varepsilon/(1-\varepsilon) > \sigma_c$. □

The intuition here is as follows. Assume the economy starts on a b.g.p. When $\varepsilon < 0$ the factors are complements and when Q rises, relative factor share S falls, and hence K falls, which tends to restore factor shares. When $\sigma_c = 0$ there is nothing to brake this process, and S returns to its original level: However, when $\sigma_c > 0$ then when K falls, production of k_a becomes relatively easier, which brakes the fall in K and leads to a new b.g.p. with higher KQ, and hence a lower factor share for A. Given a lower share, it follows trivially that relative factor returns P (recall that $P = p_a/p_b$) are also lower.

Conversely, when $\varepsilon > 0$, the factors are substitutes, and when Q rises, S rises, and hence K rises, which tends to further raise S. When $\sigma_c = 0$ there is nothing to brake this process, and S goes to infinity (a corner solution in which returns to B are zero). However, when $\sigma_c > 0$ then when K rises, production of k_a becomes relatively harder, which brakes the rise in K. When $\sigma_c > \phi\varepsilon$ this effect is strong enough to stop the increase in K before the economy goes to a corner, and if $\sigma_c > \phi\varepsilon/(1-\varepsilon)$ then the effect is strong enough to stop the increase in K before relative factor returns have risen.

Note that all of these results can be seen as generalizations of the special cases to be found in Acemoglu (2002). To see this, start by noting the following:

(1) follow Acemoglu by denoting the elasticity of substitution between the input factors as σ, and use equation 16 to show that $\sigma = 1/(1-\varepsilon)$
(2) ϕ — the elasticity of knowledge to investment — is unity throughout in Acemoglu (2002) (see equations 19 and 24 for the cases of lab equipment and knowledge-based R&D);
(3) in the lab equipment model, if one sector advances relative to the other by x percent, the relative productivity of investment inputs (lab equipment) in that sector falls by x percent.[5] This corresponds to $\sigma_c = 1$ in our model;
(4) In the knowledge-based R&D model Acemoglu introduces a parameter δ (equation 24), and comparison to Definition 5 shows that our σ_c corresponds to $1-\delta$.

Now consider our stability condition from Proposition 3, $\sigma_c > \phi\varepsilon$. Using the points above, this becomes $\sigma < 1$ for the lab equipment model, and $\sigma < 1/\delta$ for the knowledge-based R&D model. Furthermore, the condition for upward-sloping demand from Proposition 4 becomes $\sigma > 2$ for the lab equipment model, and $\sigma > 2-\delta$ for the knowledge-based R&D model.

[5] In Acemoglu's notation, if the ratio of investment efforts R_L/R_Z is held constant then the ratio of the growth rates of productivity in the sectors is proportional to the inverse of the ratio of productivity levels:

$$\frac{\dot{N}_L/N_L}{\dot{N}_Z/N_Z} = \frac{\eta_L}{\eta_Z}\frac{R_L}{R_Z}\frac{N_Z}{N_L}.$$

4.2. **Knowledge spillovers between sectors.** Constant elasticity knowledge dependence (previous section) is not very attractive intuitively as it implies that knowledge in sector A is essential for knowledge accumulation in sector B; thus if there is zero knowledge in sector A then knowledge accumulation in sector B is impossible. This is not consistent with the concept of spillovers, which implies that knowledge in sector A can help sector B, but is not essential. Hence the following definition.

Definition 6. There are *knowledge spillovers between sectors* when

$$F(K_t) = \frac{1}{K_t}\frac{K_t+\sigma_s}{1+\sigma_s K_t}. \tag{24}$$

where $\sigma_s \in (0,1]$.

This function gives the properties intuitive for spillovers: there is baseline knowledge growth building on knowledge within a given sector, and knowledge spillovers from outside the sector can also help. The parameter σ_s measures the size of the spillover effect; if $\sigma_s = 1$ then a unit of sector-A knowledge is equally useful as a unit of sector-A knowledge in boosting accumulation in sector B. Note that at the limits, constant elasticity dependence and knowledge spillovers are identical: when $\sigma = 0$ knowledge stocks grow independently, and when $\sigma = 1$ then $F = 1/K$.

Knowledge spillovers are more complex than constant elasticity dependence, and give rise to multiple equilibria and thresholds, as described in the following two propositions.

Proposition 5. *In a simple economy with* knowledge spillovers between sectors *there is always at least one interior b.g.p. When $\phi\varepsilon < 2\sigma_s/(1+\sigma_s)$ the b.g.p. is unique and stable, and when $\phi\varepsilon > 2\sigma_s/(1+\sigma_s)$ there is either a unique stable b.g.p., or three balanced growth paths, of which two are stable and one (with an intermediate value of the state variable) is unstable. For sufficiently low or high values of Q there is a unique stable b.g.p., whereas for an intermediate range of values there are three b.g.p.s.*

Proof. Substitute for F in equation 20 using Definition 6 to obtain an expression corresponding to equation 22:

$$\hat{K}_t = \left[Q_{t+1}^{\phi\varepsilon}(r_b/r_a)K_t^{\phi\varepsilon}\frac{1+\sigma_s/K_t}{1+\sigma_s K_t}\right]^{1/(1-\phi\varepsilon)}. \tag{25}$$

Differentiate to obtain the elasticity of the $\hat{K}$ w.r.t. the K, ζ:

$$\zeta = \frac{1}{1-\phi\varepsilon}\left[\phi\varepsilon - \frac{\sigma_s/K_t}{1+\sigma_s/K_t} - \frac{\sigma_s K_t}{1+\sigma_s K_t}\right]. \tag{26}$$

At the limits ($K \to 0$ and $K \to \infty$) the elasticity is -1. Between the limits it has a single turning point—a maximum—at $K = 1$, at which point $\zeta = \phi\varepsilon - 2\sigma_s/(1+\sigma_s)$. If ζ is negative at this turning point then $\hat{K}$ must be monotonically decreasing, whereas if it is positive at this turning point $\hat{K}$ must have two turning points, first a local minimum, then a local maximum. Since $\hat{K}$ is positive and increasing in Q with constant elasticity, if Q is sufficiently large then $\hat{K} > 1$ at the local minimum, and the locus of $\hat{K}$ only crosses $\hat{K} = 1$ once, with negative slope; similarly, if Q is sufficiently small then $\hat{K} < 1$ at the local maximum, and again the locus of $\hat{K}$ only crosses $\hat{K} = 1$ once, with negative slope; in these cases there is a unique stable b.g.p. In between there must be a range of values of Q such that $\hat{K} < 1$ at the minimum and $\hat{K} > 1$ at the maximum, and the locus of $\hat{K}$ crosses $\hat{K} = 1$ three times and there are three b.g.p.s, of which the outer two are stable since the elasticity is negative. □

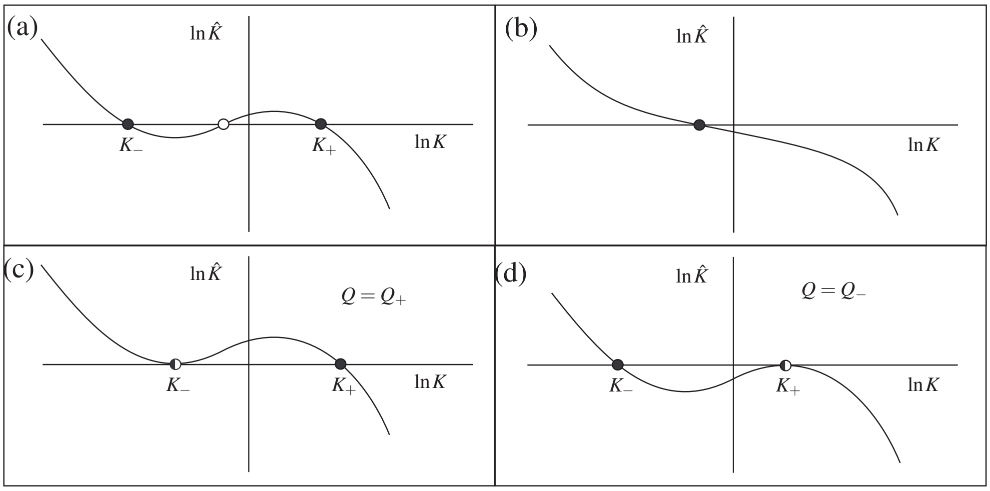

FIGURE 2. The growth factor $\hat{K}_t$ as a function of knowledge K_t. Recall that $\hat{K}$ is monotonically increasing in Q. In (a) we have knowledge spillovers and $\phi\varepsilon > 2\sigma_s/(1+\sigma_s)$, whereas in (b) we have $\phi\varepsilon < 2\sigma_s/(1+\sigma_s)$. In (c) and (d) we have the the critical levels of Q (Q_+ and Q_- respectively) such that a marginal increase (decrease) in Q leads to a discontinuity in b.g.p. factor returns and shares when the economy starts at K_- (K_+). The rings indicate unstable b.g.p.s, the dots indicate stable b.g.p.s, and the half-filled rings are saddle points. K_- and K_+ indicate the b.g.p. knowledge levels.

The implications of Proposition 5 are powerful, given that the case with knowledge spillovers between sectors is the most general. First, note that σ_s is a measure of the directionality technological change; when $\sigma_s \to 1$ spillovers are so strong that technology levels in the two sectors are essentially tied together, whereas when $\sigma_s = 0$ there are no spillovers at all; the technologies develop separately and technological change is fully directional. Now, when the factors are complements ($\varepsilon < 0$) there is always a unique, stable b.g.p. (as in Figure 2(b)). However, when the factors are substitutes (as with skilled and unskilled labour, or wind power and fossil fuels) then if technological change is sufficiently directional (σ_s sufficiently low) then there will be two stable b.g.p.s for given quantities of the factors; on one of these paths factor A dominates, whereas on the other factor B dominates, and history will determine the b.g.p. to which the economy converges (Figure 2(a)); if K starts high then the economy converges to K_+, and (given that the goods are substitutes) PQ is high, i.e. factor A dominates.

Proposition 6.

6.1 *In a simple economy with knowledge spillovers between sectors on a locally stable b.g.p., a marginal increase in Q (corresponding to an increase in the ratio q_a/q_b) leads to a marginal increase in the b.g.p. factor share if the factors are substitutes, and a decrease if they are complements.*

6.2 *If*

$$\frac{2\sigma}{1+\sigma} > \phi\varepsilon > \frac{\phi\frac{2\sigma}{1+\sigma}}{\phi + \frac{2\sigma}{1+\sigma}}$$

(implying inter alia *that the factors are substitutes) then there is a range of values of Q for which a marginal increase in Q leads to a marginal increase in b.g.p. factor returns P.*

6.3 *If $\phi\varepsilon > 2\sigma_s/(1+\sigma_s)$ (again implying substitutes) then as Q increases from zero, at some critical value Q_+ there is a discontinuity such that a marginal increase in Q leads to a discrete increase in $\widetilde{K}$ from $\widetilde{K}_-$ to $\widetilde{K}_+$, and there are discrete increases in both the b.g.p. factor share $Q\widetilde{P}$ and b.g.p. factor returns $\widetilde{P}$. Symmetrically, when Q falls from $+\infty$ there is a lower value of Q, denoted Q_-, at which there is a discontinuity such that there is a discrete fall in both $Q\widetilde{P}$ and $\widetilde{P}$. See Figures 2(c) and 2(d).*

Proof. 6.1. Set $\hat{K} = 1$ in equation 25 to obtain, in long-run equilibrium,

$$KQ = \left(\frac{r_a}{r_b}\frac{1+\sigma_s K}{1+\sigma_s/K}\right)^{1/(\phi\varepsilon)}, \tag{27}$$

and substitute in equation 28 to obtain

$$PQ = \left(\frac{r_a}{r_b}\frac{1+\sigma_s K}{1+\sigma_s/K}\right)^{1/\phi}. \tag{28}$$

Local stability implies that $\partial K/\partial Q$—where K and Q without time subscripts denote long-run equilibrium values—must be positive if the factors are substitutes, and negative if they are complements. Since it is straightforward to show that $\partial PQ/\partial K$ is positive, this proves that $\partial PQ/\partial Q$ is positive given substitutes, and negative given complements.

6.2. Since $2\sigma/(1+\sigma) > \phi\varepsilon$ we know from Proposition 5 that there is a unique b.g.p. for given Q. Use equations 27 and 28 to show that

$$\psi = \frac{Q}{P}\frac{\partial P}{\partial Q} = -1 + \frac{1/\phi}{1/(\phi\varepsilon) - 1/\xi}, \tag{29}$$

where

$$\xi = \sigma\left(\frac{K}{1+\sigma K} + \frac{1}{K+\sigma}\right).$$

Thus ψ is decreasing in ξ, and ξ is positive and has a single turning point, a minimum, at $K = 1$. Thus if ψ is positive at $K = 1$ then there is a range of values of K, and associated values of Q, at which a marginal increase in Q leads to a marginal increase in $\widetilde{P}$. The condition for $\psi > 0$ when $K = 1$ (and hence $\xi = 2\sigma/(1+\sigma)$) is (from equation 29) $\varepsilon > 2\sigma/(1+\sigma)/[\phi + 2\sigma/(1+\sigma)]$.

6.3. From Proposition 5 we know that given $\phi\varepsilon > 2\sigma_s/(1+\sigma_s)$ then there is a range of values of Q for which there exist two stable b.g.p.s and one unstable. Denote the levels of K at the stable b.g.p.s as K_- and K_+, as illustrated in Figure 2. Since $\hat{K}$ is increasing in Q, if we consider the values of K_- for successively higher values of Q, at some Q, denoted Q_-, K_- must be at a local minimum in $\hat{K}(K)$, and a marginal increase in K causes the economy to move discontinuously to a new b.g.p. at K_+, at which both the factor share and factor returns are (discontinuously) higher than at K_-. □

Intuitively, if one factor has dominated historically (fossil energy, or unskilled labour) then rises in the availability of a substitute factor may not be sufficient to shift the economy into a new equilibrium in which the substitute factor dominates, even when such an equilibrium would be stable; the reason is the accumulated knowledge augmenting the historically dominant factor. However, if the substitute factor rises sufficiently in availability then a transition to use of that factor will always occur.

Note that in the proofs quantity is exogenous and price endogenous. If (as is more realistic) prices and quantities of factor inputs are linked, more complex results will emerge. We mention two cases. Firstly, if there is a downward-sloping supply curve for an input such as skilled labour, then an exogenous shift outwards in the supply curve (leading to a short-run decrease in price and increase in quantity) could trigger a long-run price increase, leading to further increases in supply and (in turn) further increases in price. The process would stop when the skilled factor had taken over the market and the elasticity of price w.r.t. quantity had again become negative. Secondly, assume that there is a floor on the price at which an input can be produced (consider for example the minimum extraction cost of coal as an example), and there is a shift in the supply curve of a substitute energy source (say a safe, clean and cheap nuclear technology is developed). Then the expansion of the alternative sector (nuclear) may not stop at reducing the share of the original factor (coal); if the price of coal falls through the floor, extraction will cease and the factor share falls to zero.

5. Discussion

In this section we assess the significance of the results in the context of environmental and resource economics.

5.1. Fixed factor shares and independent knowledge. First we assess the case of fixed factor shares and independent knowledge. Recall that this case is mentioned in Acemoglu (2002), and used in Smulders and de Nooij (2003), Hart (2009), and Acemoglu et al. (2009). In Section 3 we showed that if — given some level of factor quantities q_a and q_b — there is a b.g.p. with relative factor shares $\overline{S}$, then there is a corresponding b.g.p. for any fixed factor quantities, on which the relative factor shares are also $\overline{S}$. Furthermore, if the factors are substitutes any b.g.p. will be unstable, and if they are everywhere substitutes we will have a corner solution with all returns going to one factor.

This result is analytically pleasing, but is it empirically relevant? If it is, the implications are huge. Consider first when augmented factors are complements in the production function. The result then implies that there are unbounded possibilities for *physical* input factors to substitute for one another nevertheless. (Perhaps this is what Solow was thinking about in the quotation in the introduction above.) Assume, for instance, a balanced growth path with a high rate of consumption of some resource. If that resource becomes scarce such that flows must be cut drastically, then a new growth path can be reached — after a period of investment in technology augmenting that resource — which is identical to the old in all other respects (growth rate, investment rates, factor shares, etc.). Consider now when the augmented factors are substitutes. If knowledge stocks are independent then one will always dominate in the long run, and which one dominates is a function of history, not primarily the properties of the factors.

Superficially, there seems to be some support for the result in the data on resource consumption rates and prices. For instance, Smulders and de Nooij (2003) (citing Jones (2002)) note that the factor share of energy has only declined slowly over a long period, while quantity has risen. Furthermore, Hart (2009) shows the same result for two — more-or-less randomly chosen — minerals. Finally, a more systematic study by Heidrich (2010) shows the same result for the consumption rate and factor share of all metals aggregated together. We reproduce her data, adapted, in Figure 3. Note that quantity rises more slowly than global product, while price is constant overall, hence the factor share

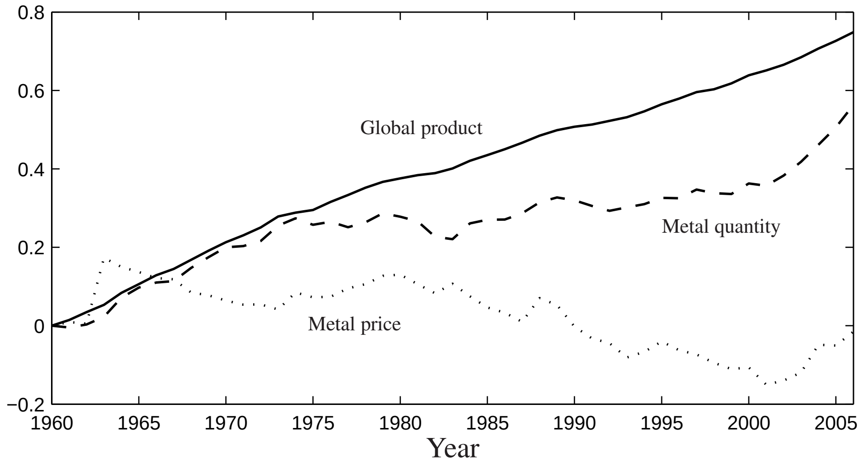

FIGURE 3. Trends in the price and quantity of aggregate metals, compared to global product. Vertical axis: logarithms of values normalized by base year. Adapted from Heidrich (2010).

declines over the period. The rate of decline is 1.0 percent per year, which is similar to the decline in energy share noted by Jones and Smulders and de Nooij.

If we consider factor inputs as metals and labour (or indeed energy and labour) then clearly they are complements. Hence, if there levels of factor-augmenting knowledge grew independently we would expect stable balanced growth with constant factor shares, even when the relative (physical) factor quantities change. On the other hand, if knowledge stocks are linked then when labour-augmenting knowledge grows faster than metal-augmenting knowledge it should to some extent pull metal-augmenting knowledge along with it, hence an increase in the physical quantity of metals relative to labour will be accompanied by an increase in augmented inputs of metals in the long run, hence (since the factors are complements) the share of metals will decline.

It is tempting to find support for the idea that links between knowledge stocks are weak, since the decline in factor share is slight (1.0 percent per year) despite the large increase in metal quantity (2.9 percent per year). However, this would be to ignore the fact that labour inputs have also risen during the period, by at least 1 percent per year over the period, hence the ratio of metal inputs to labour has grown by at most 2 percent per year, more likely just 1 percent per year. Now the effect of changes in relative quantities on factor shares looks large, suggesting strong links between knowledge stocks.

5.2. **The nature of links between knowledge stocks.** If we accept that knowledge stocks are in general linked, the natural question then is what is the nature of these links? Is the simple constant-elasticity model sufficient, or is a more sophisticated model — such as the one we develop above and denote ‘knowledge spillovers’ — required? We illustrate the latter model in Figure 4, where we show long-run demand curves for substitute resources produced from a full model (not presented here) of knowledge spillovers as defined above. By contrast, the long-run demand curve resulting from a model with constant-elasticity knowledge dependence is simply linear. Given knowledge spillovers, when one input dominates in terms of physical quantity, it also does so in terms of factor price. When the quantity of the second input rises, it remains peripheral and its price may even fall. However, as the quantity increases further the share of the second input rises, and investment in knowledge augmenting this input becomes more attractive. At some point a threshold is passed, and much more is invested into augmenting the second good. The data presented by Acemoglu (2002, Figure 1) concerning returns to skilled and unskilled labour are suggestive of such a threshold being passed when the relative supply of college skills exceeds 0.4, but much more research would be required to support (or reject) the idea of knowledge spillovers as set out above.

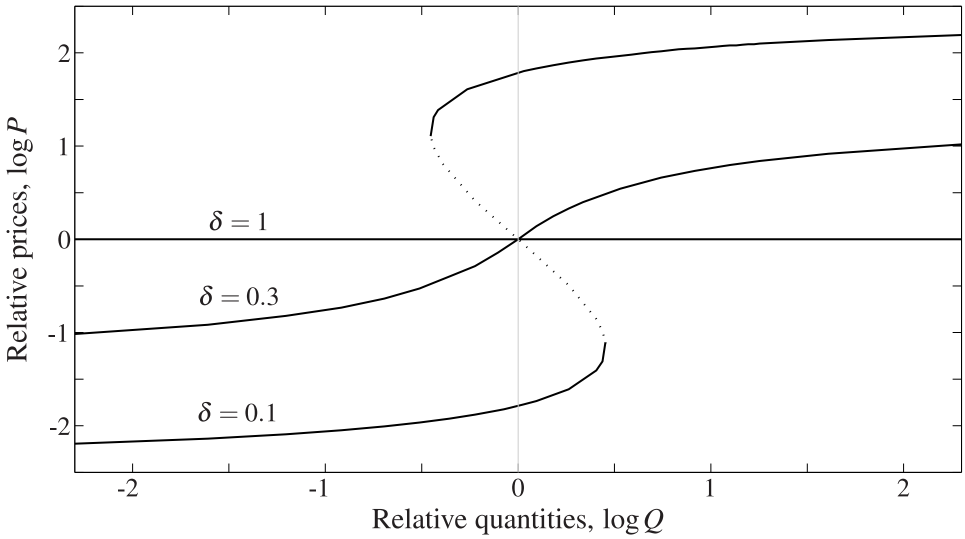

FIGURE 4. Simulation of long-run demand curves from a full model (not presented here) of knowledge spillovers as defined above; the bigger is δ, the more tightly are knowledge levels tied together.

The presence of such thresholds would be good news for advocates of active environmental policy, since they imply that there is an interval of factor quantities over which history determines that the 'old' dirty factor dominates, even though the 'new' clean factor has the potential to take over and be a cheaper long-run input to production. Of course, this is not as strong a case for intervention as that presented by Acemoglu et al. (2009), but on the other hand, their conclusion rests on the economy not having moved 'too far' into the corner. Since we have had a fossil-fuel based economy for well over 100 years, the logic of their approach would suggest that levels of knowledge augmenting the clean input are irretrievably low.

6. CONCLUSIONS

The above models are intended to clarify the drivers of directed technological change, establish some baseline results, and act as a basis for further — more applied — modelling. A first step is of course to integrate the insights about knowledge growth into an overall model of growth and DTC. There is a huge need for further research both in building models, and testing them against data. For instance, to model fossil-fuel demand and supply satisfactorily we need a nested model including both overall demand for energy (a complement to other inputs) and the production of energy where fossil fuels are substitutes for other inputs such as renewables and nuclear power. Furthermore, final goods should differ in their intrinsic energy intensity in order to reflect the observation that different sectors have different energy factor shares, and these sectors can change in size relative to one another.

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