Bargaining with intertemporal maximin payoffs

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Abstract

We present a new class of dynamic bargaining problems, called "bargaining problems with intertemporal maximin payoffs," that may reflect sustainability problems having to encompass conflicting issues in the long-run. Each bargainer (or stake-holder) has a representative indicator, namely a function of the state and decisions, and aims at maximizing its minimal value over time. Bargaining on sustainability issues consists in defining the vector of stake-holder's payoffs. We are interested in defining the set of feasible outcomes of such problems. This set is interpreted as a support for a social choice of sustainability objectives. We introduce a `MONDAI` condition – Monotonicity of Dynamics And Indicators – consistent with many economic problems and, in particular, "environmental economic" sustainability issues. We characterize the set of feasible outcomes for problems satisfying these monotonicity properties, and the bargaining solutions under the axioms of *Pareto efficiency* and *Independence of Irrelevant Alternatives.* We also provide a "satisficing" common decision rule to achieve any given solution. We then examine the time-consistency of the solution under the axioms of *Veto Power* and *Individual Rationality.*

Keywords: bargaining theory, dynamics, maximin, monotonicity, feasibility set, sustainability

1 Introduction

We present a new class of dynamic bargaining problems inspired by sustainable development issues. A group of stake-holders can agree on a common sequence of actions (intertemporal path of consumption or investment for instance), as in Fershtman [1983]. These actions modify the state of the economy (capital stocks) according to a dynamics representing production possibilities. Stake-holders get an intertemporal payoff which depend on the resulting economic trajectory. If they do not agree, they get a reference payoff given by the status-quo, or business-as-usual economic trajectory (this is the equivalent of the disagreement point of static bargaining problems). In Fershtman [1983], the intertemporal payoffs were given by the discounted utility criterion. The novelty of the present paper is that we consider intertemporal maximin payoff functions [Solow, 1974, Cairns and Long, 2006, Long, 2006]. The payoff of each stake-holder is defined as the minimal level over time of an individual indicator (representing a sustainability issue). The payoffs are not directly transferable between stake-holders as the indicators can be of different nature, with different units. At each time, the level of the indicators depend on the state of the economy and the commonly agreed decision at that time.

The set of feasible outcomes represents all the achievable stake-holders payoffs (minimal level of the indicators over time). This set depends on the economic state and dynamics, which makes the problem environment-dependent [Roemer, 1986, 1988, Chen and Maskin, 1999], and is not "comprehensive."[1] This makes its characterization difficult, which is an obstacle

[1] The set of feasible outcomes in not comprehensive when having a given payoff vector

to the resolution of the bargaining problem.

Considering the classical axioms of Pareto efficiency and Independence of irrelevant alternatives, we show that the solution of the bargaining problem is the same as the solution of a more tractable problem based on a comprehensive set containing the set of feasible outcomes of the original problem. In particular, the Pareto frontiers of both set coincide. We characterize this auxiliary comprehensive set under some monotonicity properties – called $\texttt{MONDAI}_k$ (Monotonicity of the dynamics and k indicators). Roughly speaking, these monotonicity properties correspond to a requirement that capital stocks are "productive" (in the sense that having more capital stocks makes it possible to produce more, and does not reduce the payoff of any stake-holder), and that some (not all) indicators depend in a monotonic way on the action (i.e., decreasing), which makes the stake-holders having payoffs depending on these indicators belong to some "interest group." When there is such an interest group, the Pareto frontier of the set of feasible outcomes is shown to be of a lower dimension than the number of stake-holders.

We then provide an interpretation of the bargaining problem in terms of social choice, and describe the corresponding social planner problem (Social Welfare Function). We also characterize the evolution of the bargaining solution over time, showing that it exhibits "time-monotonicity" (the payoff of all stake-holders is non-decreasing over time) if stake-holders are "individually rational" and have a "veto power."

in the set does not mean that lower payoffs are achievable [Kalai, 1977, Zhou, 1996]. This can occur in particular when payoffs are not transferable between agents. We give a formal definition in Section 3.

To our knowledge, this class of problems has never been studied. Our paper is therefore an original contribution to the bargaining literature. The closer analyses are that of Fershtman [1983], who studied the same kind of dynamic bargaining on a sequence of actions, but with payoffs defined by discounted utility, and that of Long [2006], who considered a dynamic game with intertemporal maximin preferences. This latter problem differs from ours as each player has his own decision, and the solution depends on the strategic interactions between players, relating his paper to the field of dynamic games while we adopt the bargaining theory approach.

The paper is organized as follows. In the next section, we present the motivation for studying such dynamic bargaining problems, and their interpretation in terms of sustainable development. In Section 3, we present the general dynamic bargaining problem, the axioms, and the equivalence of its solution with that of another, more tractable problem. All results are obtained without any specification on the functional forms. In Section 4, we introduce the $\mathtt{MONDAI}_k$ monotonicity properties, and characterize the set of outcomes of the auxiliary problem, along with a "satisficing" feedback decision rule making it possible to achieve any Pareto solution. We then provide, in Section 5 the Social Choice problem equivalent to our bargaining problem, and examine the time-consistency of solutions. We conclude in Section 6 on future research avenues. Proofs and examples are in the appendix.

2 Motivation and Settings

The new class of dynamic bargaining problems introduced in this paper is inspired by sustainable development issues. This section first describes the general issue at stake, and then relates this issue and the proposed problem to the existing literature on bargaining.

Sustainable development issues are dynamics and encompass several (potentially conflicting) dimensions, such as environmental and economic issues. In practice, multicriteria approaches based on sustainability indicators and thresholds are used.[2] Sustainability indicators, depending on the state and decisions of the economy, follow the dynamic evolution of quantities representing the various issues (e.g., the per capita GDP, the employment rate, the greenhouse gases (GHG) atmospheric concentration, the spawning stock biomass of a targeted species in a fishery). Thresholds represent constraints which should not be overshot (e.g., minimal per capita income, minimal em-

[2]For example, the climate change issue is addressed by defining a limit thresholds for GHG atmospheric concentration [UN, 1998]. Regarding biodiversity, a somehow similar approach is applied worldwide, with the creation of reserves to protect natural habitat of species [**?**]. These reserves are constraints on the development of land-use for alternative economic use such as agriculture or urban development. Other examples include minimal stock size for fisheries [FAO, 2005], or thresholds for pollution of air and water. These approaches are based on quantities (indicators and thresholds) rather than on prices. From an economic point of view, damage or benefit functions could be defined and used for cost-benefit analyses. However, as sustainability concerns are often related to non-market goods which value is difficult to assess, and involve future generations which are not present to state their preferences, such an approach is not always possible and physical indicators are used. As there is no "easy" common currency between the various issues at stake, each issue is tackled on its own.

ployment, maximal GHG concentration, minimal stock for a fishery). These thresholds are chosen socially.[3] "Any social decision is the ultimate outcome of some kind of collective bargaining process" [Kalai et al., 1976, p.233]. There are necessary trade-offs between these environmental issues and other issues, e.g., economic development. We are interested in representing these trade-offs and the underlying bargaining process. We argue that the definition of thresholds that sustainability indicators should not overshoot can be seen as a bargaining problem, the thresholds being interpreted as the payoffs of some representative stake-holders (being they real, or virtual as is public opinion).

Bargaining problems have been widely studied since their formulation by Nash [1950, 1953]. Traditional bargaining theory assumes that p stakeholders may agree on an allocation within a set of feasible outcomes, or end up at a disagreement point. The usual assumption in static bargaining theory is to consider that the set of feasible outcomes is a compact and convex (and usually comprehensive) subset of $\mathbb{R}^p$. In static problems, defining such a set is not an issue. In particular, if the cooperative solution is implemented and monetary transfers between players are possible, the Pareto frontier of the feasibility set is defined by a "budget" line. For some problems however, this set may me nonconvex, which makes the definition of solutions more difficult [Zhou, 1996, Mariotti, 2000, Denicolò and Mariotti, 2000]. There

[3] These thresholds often have scientific basis (as the IPCC advices for the climate change issue), but also account for economic and social issues. A clear argument showing that sustainability thresholds are socially chosen is that they differ between countries, and in particular with the level of development. Environmental standards are higher in developed countries with high income than in developing countries [Dasgupta et al., 2001].

are several solutions to the bargaining problem, depending on the definition of preferences over feasible outcomes [Border and Segal, 1997]. Pareto efficiency implies that solutions are located on the frontier of the set of feasible outcomes.

Bargaining problems on a dynamical system have received less attention than static problems.[4] Fershtman [1983] introduced "dynamic bargaining problems" in which stakeholders have to agree on a time path of actions, i.e., a common set of decision parameters for the system, and have intertemporal payoffs depending on the economic path. In such dynamic bargaining problems, the set of feasible outcomes is not straightforward to characterize. In particular, it depends on the dynamics of the system, which will strongly influence the results. The usual assumption on the existence of a convex set of feasible outcomes is thus stronger than in the static case and, in the dynamic case, the description of this set is an issue in and of itself. This is particularly true when no (intertemporal) transfers between players are possible, as in the sustainability issue described above.

Following Fershtman [1983], we consider a dynamic bargaining problem in which stake-holders have to agree on a time path of actions. Fershtman [1983] considered discounted utility payoff functions. In the sustainability issue, the discounted utility criterion has been criticized and qualified as a "dictatorship of the present" [Chichilnisky, 1996]. An alternative criterion proposed to address the sustainability issue is the maximin [Solow, 1974], which treats all

[4] In the dynamic framework, attention has been devoted either to repeated or iterative static bargaining, such as price negotiation, or to dynamic games. In dynamic games, each player has a decision parameter [Jørgensen and Zaccour, 2007], which is not the case when stake-holders bargain over a set of decision parameters for a dynamic system.

generations with anonymity. In an economy using sustainability indicators, what is sustained (i.e., supported from below, literally) is the minimal level of the indicators over time. The usual formulation of a maximin problem is in utilitarian terms, but the maximin criterion can also be applied to the sustainment of environmental indicators [Cairns and Long, 2006]. We thus assume that stake-holders have maximin intertemporal payoffs depending on the minimal level over time of an indicator.[5] Such formalization results as a new class of dynamic bargaining problems in which payoffs are not transferable between stake-holders [Kalai and Samet, 1985]. We are interested in bargaining problems involving a dynamic system and several stake-holders whose intertemporal payoff are represented by maximin functions. Given the initial state of the system, we aim at defining the *set of feasible outcomes* of our dynamic problem, and the solutions of a bargaining problem satisfying the axioms of *Pareto efficiency* and *Independence to irrelevant alternatives.*

The reader would note that this problem is slightly different from the issue of defining thresholds that sustainability indicators should not overshoot, which was the problem described at the beginning of this section. However, we shall prove that the two problems are in fact very interrelated. They have the same solution, and their Pareto frontiers coincide. In terms of economic interpretations, this means that addressing sustainability using indicators and thresholds is equivalent of considering (virtual) stake-holders with intertemporal maximin payoff.

[5] Without loss of generality, it is always possible to take the negative level of an indicator representing a "bad," such as pollution, to be able to consider that the payoff is the minimal level over time of the indicator.

How the society makes its final choice among the set of feasible outcomes is beyond the scope of this paper. We however emphasize the correspondence between intertemporal dynamic multi-objective problems and dynamic bargaining, and its implications on the definition of sustainability thresholds. We refer to the corresponding social criterion for sustainable development and interpret the solution of the bargaining problem in terms of the underlying social choice rule. The kind of problem described here can be used to *i)* define the set of negotiation between stake-holders having intertemporal maximin utility functions, *ii)* define the distributional possibilities of a policy maker between such stake-holders, and then describe the necessary trade-offs between sustainability issues to be satisfied over time, and *iii)* define the feedback decision rule to be implemented to achieve a given Pareto efficient outcome.

3 Bargaining problem with intertemporal maximin payoff

3.1 Overview of the bargaining problem

Suppose that $p \geq 2$ stake-holders identified by $i = 1, \dots, p$ have to agree on a common decision which will have consequences on their individual payoff. In an intertemporal context where time t is discrete and runs as $t = t_0, t_0+1, \dots$, the bargaining is upon a sequence of actions $a(\cdot) := \Big(a(t_0), a(t_0+1), \dots\Big)$, where each action $a(t)$ is taken in a set $\mathbb{A}$. Stake-holders' payoff will depend on that sequence.

Now, consider that the sequence of decisions and the payoffs are related by the evolution of the economic state, in a dynamic framework $x(t+1) = g\Big(x(t), a(t)\Big)$. Past decisions influence the dynamic state of the economy, resulting in an economic trajectory $x(\cdot) := \Big(x(t_0), x(t_0+1), \ldots\Big)$ where at each period, $x(t)$ belongs to the state space $\mathbb{X}$.

For $i = 1, \ldots, p$, an indicator $I_i : \mathbb{X} \times \mathbb{A} \mapsto \mathbb{R}$, depending on the economic state $x(t)$ and decision $a(t)$, represents the measurement of the i^{th} bargainer interest at each time t. Each indicator has its own unit, which does not allow direct transfers between bargainers.

In this paper, we consider that bargainers have intertemporal payoffs $\inf_{t=t_0,t_0+1,\ldots} I_i\Big(x(t), a(t)\Big)$, equal to the minimal level over time of their indicator. The stake-holders bargain on a sequence of actions $a(\cdot)$ which, given the initial economic state x_0, defines an outcome within a set of feasible outcomes. Note that the bargained sequence of actions $a(\cdot)$ can be given by an open-loop decision, or by a state-dependent (Markovian) decision rule, namely a mapping $\mathfrak{a} : \mathbb{X} \mapsto \mathbb{A}$ giving each decision as a function of the state by $a(t) = \mathfrak{a}\Big(x(t)\Big)$.

3.2 The dynamic model

The evolution of the system is described by a nonlinear discrete-time dynamical control system through the difference equation

$$\begin{cases} x(t+1) = g\Big(x(t), a(t)\Big), \quad t = t_0, t_0+1, \ldots \\ x(t_0) = x_0 \quad \text{given}, \end{cases} \tag{1}$$

where the *state variable* $x(t)$ belongs to the finite dimensional state space $\mathbb{X} \subset \mathbb{R}^{n_{\mathbb{X}}}$, the *decision variable* $a(t)$ is an element of the *decision set* $\mathbb{A} \subset \mathbb{R}^{n_{\mathbb{A}}}$

while the *dynamics* g maps $\mathbb{X} \times \mathbb{A}$ into $\mathbb{X}$ (for the sake of simplicity, we consider the time-autonomous case).

When the sequence of actions is defined by a Markovian feedback rule, i.e., a mapping $\mathfrak{a} : \mathbb{X} \to \mathbb{A}$ giving each decision as a function of the state by $a(t) = \mathfrak{a}\Big(x(t)\Big)$, one gets the closed-loop dynamics

$$\begin{cases} x(t_0) = x_0 \\ a(t) = \mathfrak{a}\Big(x(t)\Big) \\ x(t+1) = g\Big(x(t), a(t)\Big) . \end{cases} \tag{2}$$

Given a bargained sequence of action $a(\cdot)$, the intertemporal payoff of the i^{th} stake-holder is defined according to

$$J_i^a(x_0) = \inf_{t=t_0, t_0+1, \ldots} I_i\Big(x(t), a(t)\Big) \qquad i = 1, \ldots, p \tag{3}$$

where $I_i : \mathbb{X} \times \mathbb{A} \mapsto \mathbb{R}$ is an instantaneous indicator (i.e., a measurement of interest for the i^{th} stake-holder). Bargainer i aims at maximizing his intertemporal payoff, which leads to consider a maximin problem.

3.3 The bargaining problem

A bargaining problem is characterized by a set of feasible outcomes and a disagreement point. A bargaining solution consists of choosing a particular element of the set, under some axioms.

In our context, both the set of feasible outcomes and the disagreement point depend on the initial state of the economy and on the dynamics of the system. They thus depend on the economic environment.[6] Moreover, a

[6]Classical bargaining theory is based only on the shape of the set of feasible outcomes.

bargaining solution, which is a vector of payoffs receiving the agreement of all players, is only defined by an associated sequence of actions to achieve it.

The set of feasible outcomes. The set $\mathcal{F}(x_0)$ of all *feasible outcomes* starting from x_0 is the collection of achievable intertemporal payoffs (3):

$$\mathcal{F}(x_0) := \left\{ \theta = (\theta_1, \ldots, \theta_p) \in \mathbb{R}^p \;\middle|\; \begin{array}{l} \exists\ (a(t_0), a(t_0+1), \ldots) \text{ and} \\ (x(t_0), x(t_0+1), \ldots) \\ \text{satisfying } x(t_0) = x_0 \\ x(t+1) = g\Big(x(t), a(t)\Big) \\ \forall\ t = t_0, t_0+1, \ldots \text{ and} \\ \inf_{t=t_0,t_0+1,\ldots} I_i\Big(x(t), a(t)\Big) = \theta_i \\ \forall\ i = 1, \ldots, p. \end{array} \right\} . \quad (4)$$

Note that this set is not necessarily convex[7] or comprehensive (i.e., a lower set[8]) in the space $\mathbb{R}^p$.

Disagreement point. If there is no agreement, the economy stays on the business as usual trajectory (BAU). This defines a disagreement outcome vector $(\theta_1^{BAU}, \ldots, \theta_p^{BAU})$.

Roemer [1986, 1988] criticized this approach as it does not account for the economic environment and the nature of the goods to be shared. Chen and Maskin [1999] enriched the economic context of the bargaining problem by considering the possibility of production. We go one step further by considering the whole economic dynamics, in an intertemporal framework.

[7]An example of such a problem with non-convex set is given in the appendix.

[8]If a point $(\theta_1, \ldots, \theta_p)$ is in $\mathcal{F}(x_0)$, a point $(\theta'_1, \ldots, \theta'_p) \leq (\theta_1, \ldots, \theta_p)$ (component-wise) may not be in $\mathcal{F}(x_0)$, i.e., may not be feasible.

The axioms We assume that the bargaining solution satisfies the axioms of *Pareto efficiency* and *Independence of Irrelevant Alternatives* defined as follows.

Definition 1 (Weak Pareto Efficiency)
Let $\mathcal{A} \subset \mathbb{R}^p$ be given. A vector of outcomes $\theta = (\theta_1, \ldots, \theta_p) \in \mathcal{A}$ is said to be weakly Pareto efficient *in $\mathcal{A}$ if, for any $\sigma = (\sigma_1, \ldots \sigma_p)$ such that $\sigma_i > \theta_i$, for all $i = 1, \ldots, p$, one has $\sigma \notin \mathcal{A}$.*

All weakly Pareto efficient points are on the boundary of $\mathcal{A}$. We shall denote by $\mathcal{P}^w{}_{\mathcal{A}}$ the set of all weak Pareto boundary points of $\mathcal{A}$.

Definition 2 ((Strong) Pareto Efficiency)
A vector of outcomes $\theta = (\theta_1, \ldots, \theta_p) \in \mathcal{A} \subset \mathbb{R}^p$ is (strongly) Pareto efficient *– Pareto efficient for short – in $\mathcal{A}$ if, for any $\sigma = (\sigma_1, \ldots \sigma_p)$ such that $\sigma_i \geq \theta_i$ for all $i = 1, \ldots, p$ and $\sigma_i > \theta_i$ for some i, one has $\sigma \notin \mathcal{A}$.*

We shall denote by $\mathcal{P}_{\mathcal{A}}$ the set of all Pareto boundary points of $\mathcal{A}$. Note that $\mathcal{P}_{\mathcal{A}} \subset \mathcal{P}^w{}_{\mathcal{A}}$. An outcome $(\theta_1, \ldots, \theta_p) \in \mathcal{P}^w{}_{\mathcal{A}} \backslash \mathcal{P}_{\mathcal{A}}$ is dominated in the sense that one can increase the payoff of at least one stake-holder without decreasing that of the others.

Definition 3 (Independence of Irrelevant Alternatives)
Consider two sets $\mathcal{A}$ and $\mathcal{A}'$ such that $\mathcal{A}' \subset \mathcal{A}$. A bargaining solution satisfies the property of independence of irrelevant alternatives *if, whenever a solution a of the bargaining problem on $\mathcal{A}$ satisfies $a \in \mathcal{A}'$, then a is also the solution of the bargaining problem on $\mathcal{A}'$.*

This axiom means that, if the solution of a bargaining problem belongs to a subset of the set of feasible outcomes, the solution of the bargaining problem on this subset is the same as the solution on the whole set. Reducing the set feasible outcomes by suppression "irrelevant alternatives" (i.e., elements of the set which were not solution of the bargaining problem) does not change the solution.

3.4 An auxiliary bargaining problem

As we consider Pareto efficient solutions, we are interested in the Pareto frontier of the set $\mathcal{F}(x_0)$. As this set is not comprehensive, we rather turn toward the following more practical set of *satisficing* outcomes. We shall see that this set defines an alternative problem which has the same (strong) Pareto solutions as our bargaining problem.

Another bargaining problem: satisficing outcomes. We define the set of *satisficing outcomes*[9] $\mathcal{S}(x_0)$ starting from x_0 as follows

$$\mathcal{S}(x_0) := \left\{ \theta = (\theta_1, \ldots, \theta_p) \in \mathbb{R}^p \;\middle|\; \begin{array}{l} \exists \; (a(t_0), a(t_0+1), \ldots) \text{ and} \\ (x(t_0), x(t_0+1), \ldots) \\ \text{satisfying } \; x(t_0) = x_0 \\ x(t+1) = g\Big(x(t), a(t)\Big) \\ \forall \; t = t_0, t_0+1, \ldots \text{ and} \\ I_i\Big(x(t), a(t)\Big) \geq \theta_i \quad \forall \; i = 1, \ldots, p \end{array} \right\} . \tag{5}$$

The only difference between the above satisficing outcomes set (5) and the feasible outcomes set (4) is an equality replaced by an inequality in the final lines of their definitions. Notice that if $\theta \in \mathcal{S}(x_0)$ then, $\theta' \leq \theta$ (with the componentwise order) also belongs to $\mathcal{S}(x_0)$ and therefore, this set is comprehensive (lower set). The following proposition states that this set encompasses the set of feasible outcomes of our bargaining problem with intertemporal maximin payoffs.

Proposition 1

$$\mathcal{F}(x_0) \subset \mathcal{S}(x_0)$$

This result is obvious.

[9] Satisficing means that these outcomes are guaranteed in the sense actual payoff is greater than or equal to these levels. Replaced in our initial motivation of defining the necessary trade-offs in the definition of sustainability thresholds, this set corresponds the the set of achievable sustainability thresholds.

Equivalence of solutions The following Proposition states that the Pareto solutions of the original bargaining problem (i.e., part of frontier of $\mathcal{F}(x_0)$) correspond to the Pareto frontier of the set $\mathcal{S}(x_0)$.

Proposition 2

The Pareto frontiers of $\mathcal{F}(x_0)$ and $\mathcal{S}(x_0)$ are the same. That is

$$\mathcal{P}_{\mathcal{F}(x_0)} = \mathcal{P}_{\mathcal{S}(x_0)} \ .$$

The interpretation of this proposition is quite simple. When thresholds $(\theta_1, \ldots, \theta_p) \in \mathcal{S}(x_0)$ are Pareto efficient, these thresholds are actually achieved in the sense that the constraint is eventually binding. They are thus feasible outcomes for the dynamic bargaining problem with intertemporal maximin payoffs. In fact, the following proposition states that the solution of a bargaining on $\mathcal{S}(x_0)$ is the same the solution of a bargaining problem on $\mathcal{F}(x_0)$.

Proposition 3

Under the axioms of Pareto efficiency *and* Independence of Irrelevant Alternatives*, and given Propositions 1 and 2 the solution of the bargaining problem on $\mathcal{S}(x_0)$ is also solution of the bargaining problem on $\mathcal{F}(x_0)$.*

This result means that this is equivalent, from the solution point of view, to bargain on minimal values of an indicator or on a threshold which should not be overshot. As we stated in Section 2, there is a strong link between the two problems. An interesting remark is that addressing the sustainability issue by defining thresholds which should be overshot by sustainability indicators is equivalent of considering (virtual) stake-holders having intertemporal maximin payoffs.

The general setting of the bargaining problem does not allow us to discuss its solution in more detail without specifying some general properties of the functions under consideration. In particular, one needs to know the set of feasible outcomes. In the next section, we show that, under some monotonicity properties, it is possible to compute $\mathcal{S}(x_0)$. According to proposition 3, the solution of the bargaining problem on this set is the solution of the bargaining problem on $\mathcal{F}(x_0)$.

4 The monotonic case

In this section, we study the dynamic bargaining problem with intertemporal maximin payoff under some monotonicity assumptions for the dynamics and the indicators. We name these assumptions $\mathtt{MONDAI}_k$ (MONotonicity of Dynamics And k Indicators). These assumptions have significant economic interpretations. In particular, they can represent environmental economic problems, related to sustainability issues.

4.1 Monotonicity assumptions

Monotonic dynamics. Some dynamic models have the following qualitative properties (*ceteris paribus*): (i) the higher the state vector at a period is, the higher it is at the following period; (ii) the higher the decision at a period is, the lower the state vector is at the following period. This is the case for many economic problems in which capital stocks are productive[10] and consumption comes from foregone investment. As we put a particular focus on environmental issues, let us emphasize that these properties are satisfied, for instance, for problems of air quality dynamics and pollutant emissions[11], or natural resource stocks (renewable or not) and extraction/harvesting.[12]

Monotonic indicators and interest groups. With respect to the indicators, we can also exhibit such monotonicity properties. If all capital stocks are defined as "goods," indicators will usually increase with the state, i.e., the larger the state vector, the higher the indicators.[13] Some indicators may also be monotonically responding to the decisions. This is the case for environmental indicators which are monotonically decreasing with the decisions

[10] It requires that the various components of the capital vector have no negative effect one on the others.

[11] The better the air quality at one period, the better at the following period (*ceteris paribus*). And the higher the pollutant emission at one period, the worse the air quality at the following period. This works for the climate change issue and greenhouse gases emissions, taking the negative level of CO_2 atmospheric concentration as a state.

[12] The larger the resource stock at one period, the larger at the following. The larger the extraction or harvesting, the lower the resource stock at the following period. Note that these assumptions are not satisfied for multispecies ecological models when there is a prey-predator relationship, as a larger predator stock may reduce the next period prey stock.

[13] This is true for economic indicators, which may depend for instance on capital stocks, knowledge / human capital, or infrastructures. This is also true for ecological indicators as long as the capital stocks are properly defined, by accounting for "bads" (pollution for instance) by their negative level.

such as pollutant emissions or resource extraction.[14]

In order to represent the mentioned above behaviors, we supply the state space $\mathbb{X} \subset \mathbb{R}^{n_{\mathbb{X}}}$ and the decision space $\mathbb{A} \subset \mathbb{R}^{n_{\mathbb{A}}}$ with the componentwise order: $y' \geq y$ if and only if each component of $y' = (y'_1, \ldots, y'_d)$ is greater or equal than to the corresponding component of $y = (y_1, \ldots, y_d)$. We say that a mapping $f : \mathbb{X} \times \mathbb{A} \longrightarrow \mathbb{R}^d$, defined for state and decision variables, with values in $\mathbb{R}^d$ (we will use $d = n_{\mathbb{X}}$ for the dynamics case, and $d = 1$ for the indicator case), is increasing with respect to the state variable if it satisfies $\forall\ (x, x', a) \in \mathbb{X} \times \mathbb{X} \times \mathbb{A},\ x' \geq x \Rightarrow f(x', a) \geq f(x, a)$, and is *decreasing with respect to the decision* if $\forall\ (x, a, a') \in \mathbb{X} \times \mathbb{A} \times \mathbb{A},\ a' \geq a \Rightarrow f(x, a') \leq f(x, a)$. Obviously, according to the previous definition, if a function does not depend on the state or the decision, it will be both increasing and decreasing with respect to such variable.

$\texttt{MONDAI}_k$ property: Monotonicity of the dynamics and k indicators.

Definition 4 ($\texttt{MONDAI}_k$)

Let $k \in \{1, \ldots, p-1\}$. A dynamic bargaining problem with intertemporal maximin payoffs is $\texttt{MONDAI}_k$ if:

- *the dynamics $g : \mathbb{X} \times \mathbb{A} \longrightarrow \mathbb{X}$ is increasing in the state variable and decreasing in the decision;*
- *all the indicators $I_i : \mathbb{X} \times \mathbb{A} \longrightarrow \mathbb{R}$ are continuous, and are increasing*

[14] Note that economic indicators may be monotonically increasing with the decisions, but not necessarily. For example, fishermen may favor an increase of fishing effort as long as it increases their profit, but no more when the associated cost is higher than the benefit from fishing.

in the state variable;

- *the first k indicators $I_1, \dots, I_k$ are decreasing in the decision variable.*

In the previous definition, all the indicators are increasing with the state. All capital stocks are valuable (or at least not damageable). The stake-holders of the first group $\{1, \dots, k\}$ have a particular interest in having the decision always as small as possible (e.g., GHG emissions, deforestation, fishing effort), which is interpreted as a "pro-environmental group" in our environmental issue context. Their payoff is always decreasing when the decision variables increase. On the contrary, the second group of indicators does not depend on the decision in a particular way (or some of the indicators may be increasing with the decision, in opposition to the indicators of the first group). The stake-holders $k+1, \dots, p$ are called *outsiders* of the interest group $\{1, \dots, k\}$ as they have no systematic "monotonic" interest in the decision level.[15]

4.2 Satisficing decision rule

In what follows, we will consider a scalar decision, i.e., $\mathbb{A} \subset \mathbb{R}$ and, assuming they exist, we denote by $a_\flat$, $a_\sharp \in \mathbb{A}$ the lower and upper bounds of the set $\mathbb{A}$, i.e., $a_\flat \leq a \leq a_\sharp$ for all $a \in \mathbb{A}$.

For a vector of satisficing outcomes $\theta = (\theta_1, \dots, \theta_p) \in \mathcal{S}(x_0)$ – under the monotony assumptions $\texttt{MONDAI}_k$ – we shall describe a common feedback

[15]The particular $\texttt{MONDAI}_{p-1}$ case (all indicators are environmental except one, interpreted as an economic instantaneous payoff such as utility or consumption) has an interesting economic interpretation. This case actually corresponds to a well-known problem in economics, namely a maximin under environmental constraints [Cairns and Long, 2006].

decision rule that ensures to obtain at least these thresholds. This rule is parametrized by guaranteed payoffs of the outsiders of the interest group.

Proposition 4

Assume that the bargaining problem is $\texttt{MONDAI}_k$*, for some* $k \in \{1, \ldots, p-1\}$*. Consider* $p-k$ *thresholds* $\theta_{k+1:p} = (\theta_{k+1}, \ldots, \theta_p) \in \mathbb{R}^{p-k}$ *and define the decision rule* $\mathfrak{a}^{\star}_{\theta_{k+1:p}}$ *by*[16]

$$\mathfrak{a}^{\star}_{\theta_{k+1:p}}(x) := \inf\{a \in \mathbb{A} \mid I_i(x, a) \geq \theta_i \ , \quad i = k+1, \ldots, p\} \ . \tag{6}$$

Then, for any $\theta_{1:k} \in \mathbb{R}^k$*, the vector of thresholds* $\theta = (\theta_{1:k}, \theta_{k+1:p})$ *belongs to* $\mathcal{S}(x_0)$ *if and only if the feedback decision* $\mathfrak{a}^{\star}_{\theta_{k+1:p}}$ *is a common decision rule that allows to obtain at least* θ*, starting from* x_0*.*

The interest of the previous result is twofold. On the one hand, if $(\theta_1, \ldots, \theta_p) \in \mathcal{S}(x_0)$, the trajectory starting from the initial state x_0 and defined by the feedback rule $\mathfrak{a}^{\star}_{\theta_{k+1:p}}$, that is

$$\begin{cases} x(t_0) = x_0 \\ x(t+1) = g\left(x(t), \mathfrak{a}^{\star}_{\theta_{k+1:p}}(x(t))\right) \quad t = t_0, t_0+1, \ldots, \end{cases} \tag{7}$$

guarantee the given outcomes. On the other hand, given a partial set of outcomes $\theta_{k+1:p}$, if the economic trajectory (7) defined by $\mathfrak{a}^{\star}_{\theta_{k+1:p}}$ does not achieve a given complementary set of outcomes $\tilde{\theta}_{1:k}$, no other rule will. It means that outcomes $(\tilde{\theta}_{1:k}, \theta_{k+1:p})$ cannot be guaranteed, namely $(\tilde{\theta}_{1:k}, \theta_{k+1:p}) \notin \mathcal{S}(x_0)$.

[16] Notice that $\mathfrak{a}^{\star}_{\theta_{k+1:p}}(x)$ is not defined for thoses states x such that $\{a \in \mathbb{A} \mid I_i(x, a) \geq \theta_i \ , \quad i = k+1, \ldots, p\} = \emptyset$.

4.3 Interest group and low-dimensional Pareto frontier

Thanks to the result of Proposition 4, we shall provide a way to describe the set of satisficing outcomes $\mathcal{S}(x_0)$.

PROPOSITION 5

If the dynamics g and the indicators $I_1, \ldots, I_p$ are $\texttt{MONDAI}_k$ *for some* $k \in \{1, \ldots, p-1\}$ *then,*

$$\mathcal{S}(x_0) = \{\theta = (\theta_{1:k}, \theta_{k+1:p}) \in \mathbb{R}^p \ \mid \theta_{1:k} \leq \Theta_{1:k}(\theta_{k+1:p}, x_0)\} \tag{8}$$

where the components of $\Theta_{1:k}(\theta_{k+1:p}, x_0) = (\Theta_1(\theta_{k+1:p}, x_0), \ldots, \Theta_k(\theta_{k+1:p}, x_0))$ *are defined by*

$$\Theta_i(\theta_{k+1:p}, x_0) = \inf_{t=t_0, t_0+1, \ldots} I_i\Big(x(t), \mathfrak{a}^{\star}_{\theta_{k+1:p}}(x(t))\Big) \qquad i = 1, \ldots, k \ . \tag{9}$$

Here above, the decision rule $\mathfrak{a}^{\star}_{\theta_{k+1:p}}$ *is given by* (6) *and the state by the closed loop dynamics* (2).

Equality (8) establishes that the set of satisficing thresholds is parameterized by the $p-k$ outcomes of the outsiders. Indeed, the outcomes $\theta = (\Theta_{1:k}(\theta_{k+1:p}, x_0), \theta_{k+1:p})$, when $\theta_{k+1:p}$ covers different values on $\mathbb{R}^{p-k}$, allow to compute the set $\mathcal{S}(x_0)$ by the relation (deduced from Proposition 5)

$$\mathcal{S}(x_0) = \widehat{\mathcal{S}}(x_0) + \mathbb{R}^p_-$$

where

$$\widehat{\mathcal{S}}(x_0) = \Big\{(\Theta_{1:k}(\theta_{k+1:p}, x_0), \theta_{k+1:p}) \ \mid \ \theta_{k+1:p} \in \mathbb{R}^{p-k}\Big\} \ , \tag{10}$$

and $\mathbb{R}^p_-$ is the p dimensional negative octant $\mathbb{R}^p_- = \{(\sigma_1, \ldots, \sigma_p) \ \mid \ \sigma_i \leq 0\,, \quad i = 1, \ldots, p\}$. Thus, the set of satisficing outcomes $\mathcal{S}(x_0)$ is obtained by means of $\widehat{\mathcal{S}}(x_0)$ which is more tractable to compute. Figure 1 illustrates how to compute $\widehat{\mathcal{S}}(x_0)$ and therefore $\mathcal{S}(x_0)$. The figure corresponds to a case with $p = 3$ and $k = 2$. Taking $\mathfrak{a}^{\star}_{\theta_3}(x) = \inf\{a | I(x,a) \geq \theta_3\}$ and computing Θ_1 and Θ_2 for all θ_3.

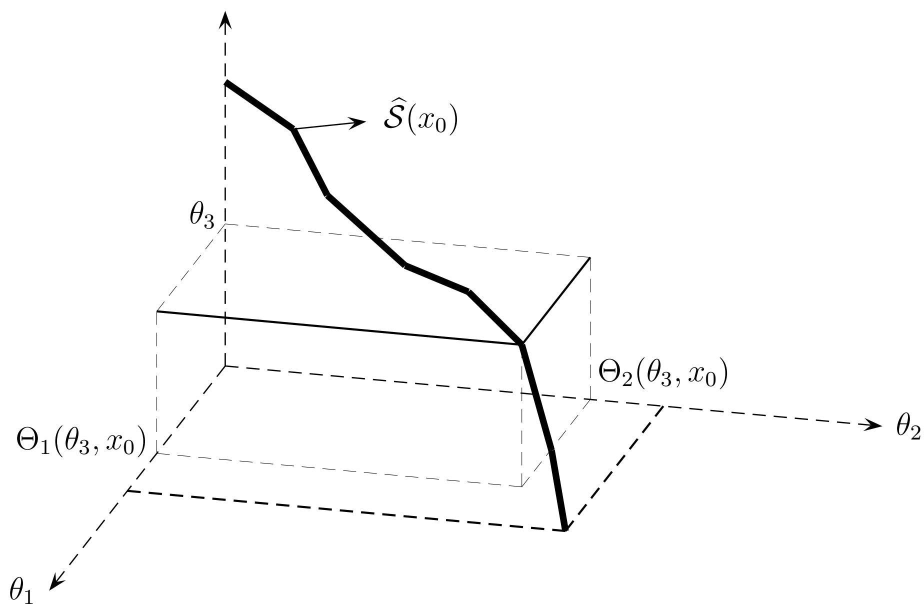

Figure 1: Satisficing outcomes parameterized by threshold θ_3.

Moreover, as we will establish in the next section, the set $\widehat{\mathcal{S}}(x_0)$ is strongly related to the Pareto frontier of $\mathcal{S}(x_0)$ and then (from Proposition 2), with the Pareto frontier of feasible outcomes $\mathcal{F}(x_0)$ of the bargaining problem.

4.4 Pareto bargaining solutions

Proposition 5 implies that the outcomes $\theta = (\Theta_{1:k}(\theta_{k+1:p}, x_0), \theta_{k+1:p})$, when the outsiders's thresholds $\theta_{k+1:p}$ covers different values on $\mathbb{R}^{p-k}$, are related to the Pareto vectors (weak and strong: see definitions 1 and 2) of $\mathcal{S}(x_0)$ in the $\texttt{MONDAI}$ framework as it is established in the following result.[17]

PROPOSITION 6

If the bargaining problem is $\texttt{MONDAI}_k$*, for some* $k \in \{1, \dots, p-1\}$*, then*

$$\mathcal{P}_{\mathcal{S}(x_0)} \subset \widehat{\mathcal{S}}(x_0) \subset \mathcal{P}^w{}_{\mathcal{S}(x_0)},$$

where $\widehat{\mathcal{S}}(x_0)$ *is given by* (10)*.*

As the set $\widehat{\mathcal{S}}(x_0)$ is fully characterized, it is now possible to give the solution of the dynamic bargaining problem with intertemporal maximin payoff under the $\texttt{MONDAI}_k$ assumption.

PROPOSITION 7

For $\texttt{MONDAI}_k$ *dynamic bargaining problems with intertemporal maximin payoffs, and under the axioms of* Pareto efficiency *and* Independence of Irrelevant Alternatives*, given Propositions 2 and 6, the solution of the bargaining problem on* $\mathcal{F}(x_0)$ *is the solution of the bargaining problem on* $\widehat{\mathcal{S}}(x_0)$*.*

Our description of the Pareto optimal solutions encompasses the information on the economic context and economic dynamics, as the set of feasible outcomes $\mathcal{F}(x_0)$ accounts for the dynamics, state and decisions [Roemer, 1988]. In particular, we have shown that, with an interest group of k stakeholders ($k \in \{1, \dots, p-1\}$), the dimension of the Pareto frontier of our dynamic bargaining problem is lower than or equal to $p - k + 1$.

[17] Note that, if the set of feasible outcomes is smooth and strictly convex, both weak and strong Pareto frontier coincide, and are fully characterized by $\widehat{\mathcal{S}}(x_0)$.

5 Discussion

The problem presented in this paper raises several new theoretical issues. We would like to put emphasis on two of them: the interpretation of the bargaining solution as a social choice issue, and the time-consistency of a particular solution of the bargaining problem.

5.1 Social choice: equivalent sustainability criterion

How the society makes its final choice among the Pareto efficient solutions $\mathcal{P}_{\mathcal{F}(x_0)}$ is beyond the scope of this paper. It is however worthwhile to note that there is an underlying criterion representing the choice of sustainability thresholds corresponding to the solution of the bargaining problem described in this paper. Given the equivalence of Pareto solutions of problem (4) and (5) (Proposition 2), the bargained solution corresponds to a social choice of sustainability thresholds. Any Pareto solution of our bargaining problem can be the solution of a "social welfare ordering" represented by a strictly increasing real-valued function which obeys the Pareto principle [Denicolò and Mariotti, 2000, Mariotti, 2000]. A criterion introduced by Martinet [2011] describes the choice of sustainability thresholds. It is based on a "social welfare function" $W(\theta_1, \ldots, \theta_p)$ ranking all alternative thresholds, where W is increasing in all its arguments. The criterion reads

$$\begin{aligned}
& \max W(\theta_1, \ldots, \theta_p) \\
s.t. \quad & x(t_0) = x_0\,, \quad x(t+1) = g\Big(x(t), a(t)\Big) \\
& I_i\Big(x(t), a(t)\Big) \geq \theta_i\,, \quad \forall i = 1, \ldots, p,\ \forall t = t_0, t_0+1, \ldots
\end{aligned}$$

which is equivalent to

$$\max W(\theta_1, \ldots, \theta_p) \qquad (11)$$
$$s.t. \quad (\theta_1, \ldots, \theta_p) \in \mathcal{S}(x_0).$$

This criterion is interpreted as a generalized maximin criterion. It is the sustainability criterion which corresponds to the dynamic bargaining problem described in this paper. The solution belongs to $\mathcal{P}_{\mathcal{S}(x_0)}$ and thus to $\mathcal{P}_{\mathcal{F}(x_0)}$.

Any solution of the bargaining problem (i.e., any bargaining mechanism) has an equivalent in the social choice problem (11), for some welfare function. For instance, the preference over the sustainability issue (thresholds) may correspond to the weight associated to the outcome of each stake-holder (i.e., the importance of the sustainability issue is related to some bargaining power).[18]

[18] In the sustainability context, the final outcome cannot be independent of the issue at stake. Sustainability raises equity issues, both between generations and between concerns of different dimension. Roemer [1986, 1988] argued that bargaining theory is not sufficient to address distributive justice, mainly because it is, in its original formulation, "context free" and neglects preferences and needs. In our approach, sustainability thresholds can be interpreted as minimal rights to be guaranteed to all generations, which may be defined according to some basic needs. Note also that we fully consider the economic environment (the set of feasible outcomes depends on the state $x(t)$, and economic dynamics). Roemer [1986, 1988] showed that the welfare egalitarian mechanism is the only mechanism to satisfy the set of axioms he proposed. This corresponds to a Rawlsian conception of justice. Our problem corresponds to some generalized maximin approach in an intertemporal framework.

5.2 Evolution of the feasibility set and improvement of outcomes over time

Contrary to static bargaining problems, breaking the agreement does not bring the players back to the initial situation as the state of the economy evolves over time (and thus the set of feasible outcomes). Fershtman [1983] is concerned with the stability of the bargaining solution for dynamic bargaining problems. In his model, where stake-holders have intertemporal payoffs given by discounted utility, one player may consider breaking the agreement at some time $t > t_0$ if he has received most of his planned payoff at that time. In our model, the stability issue also rises: stake-holders may want to bargain again after some time.

Consider a bargained sequence of actions $a(.)$. Each player's payoff is non-decreasing over time along the resulting trajectory. Indeed, define

$$(J_1^a(t), \ldots, J_p^a(t)) := \left(\inf_{s \geq t} I_1\left(x(s), a(s)\right), \ldots, \inf_{s \geq t} I_p\left(x(s), a(s)\right) \right) .$$

Then, the following condition is always satisfied[19]

$$(J_1^a(t), \ldots, J_p^a(t)) \quad \geq \quad (J_1^a(t_0), \ldots, J_p^a(t_0))$$

Definition 5 (Veto Power)

The bargaining problem is constrained by "veto power" if, at any time period $t \geq 0$ *along the business-as-usual trajectory generated by a decision rule* $a(x(t))$*, any of the* p *stake-holder can veto a change in the decision rule.*

Under the axiom of *Veto Power*, any stake-holder can dismiss an alternative path of action.

[19]The payoffs $J_i(t)$ being defined by eq. (3) computed along the trajectory defined by the initial state x_0, the dynamics (1), and the bargained decision path $\mathfrak{a}^\star(.)$.

Definition 6 (Individual Rationality)
A bargaining solution satisfies the property of Individual Rationality *if, the payoff of all stake-holders is greater than or equal to that of the disagreement point.*

Under *Individual Rationality*, no stake-holder should accept an alternative decision rule reducing his intertemporal payoff.

Along any given trajectory, the outcomes can only increase. The vector $(J_1^a(t), \ldots, J_p^a(t))$ defines a new disagreement point if bargaining takes place again at time t. This disagreement point corresponds to the actual payoffs along the initial bargained trajectory.[20] The set of feasible outcomes is defined by $\mathcal{F}(x(t))$, according to (4). The stake-holders then face a new problem at time t. Bargaining should take place again if the (dynamic) disagreement point is not on the Pareto frontier of (dynamic) feasibility set $\mathcal{F}(x(t))$.

We now consider the case of such a re-bargaining. If stake-holders have a veto power and under the axiom of Individual Rationality, they can rule out any path of action which reduces their outcome. Under these assumptions, we can prove that the solution of the dynamic bargaining is monotonic, in the sense that it is not decreasing over time.

Proposition 8 (Time monotonicity)
Assuming Veto power *and* Individual Rationality, *when time passes and the economic state evolves, neither stake-holder's payoff falls when bargaining again.*

[20]Note that, as this vector increases over time, some stake-holder may have interest to delay the bargaining process in order to have a higher disagreement outcome. Considering such temporal strategies is behind the scope of this paper.

The proof of this proposition is immediate. It shows that, if bargaining takes place again at some time $t > t_0$, the outcomes can only increase, as stake-holders can remain on the initial path by vetoing any alternative path. Implementing time-monotonic solution is then required when Individual Rationality and veto are considered. This gives us an equivalent of the conclusion by Kalai and Samet [1985], but in an intertemporal framework.

It is interesting to note that, if we do not assume the property of Veto Power, as it can be the case if there is no actual stake-holder standing for a sustainability issue, or if we consider the equivalent Social Choice Problem (in which the decision-maker can reduce the outcome of some stake-holders to increase that of some others if this decision increases the Social Welfare), the solution does not necessarily satisfy time-monotonicity. In the sustainability context, it would mean that some environmental standards or objectives may be reduced when the set of feasible outcome evolves. In a sense, it may not be an issue, as society's choice may change, and new trade-off may be made, when the economic context and associated opportunities change.[21]

6 Conclusion

We introduced a new class of dynamic bargaining problems, in which stake-holders have to agree on a time path of actions which influence the dynamic state of the economy. The payoff of stake-holders is defined as the minimal

[21]For instance, the ceiling constraint on greenhouse gases atmospheric concentrations may change in several decades.

level over time of some indicators depending on the evolution of the economic state and decisions (intertemporal maximin criterion). We showed that, in this kind of bargaining problems, the set of feasible outcomes is not easy to define, in particular because it is not a comprehensive set. Looking for Pareto efficient solutions, we show that the solution of the bargaining problem can be obtained by studying an auxiliary problem (involving a comprehensive set).

In dynamic bargaining problems, the shape of outcome possibilities depends on the dynamics of the system. In particular, we show that the set of Pareto efficient outcomes can be determined under some monotonicity properties, and we exhibit a common decision rule to achieve any solution. Regarding the sustainability issue that motivated us, this decision rule is intuitively interpreted as a conservative approach to protect the environment given non-environmental outcomes. When there are "interest groups," the Pareto frontier of the set is of a lower dimension than the initial problem, allowing an easier computation of the Pareto solutions.

We describe the corresponding social choice problem, and sustainability criterion. This is the generalized maximin criterion introduced by Martinet [2011]. We also show that, under "individual rationality" axiom and "veto power", the bargaining solution is "time-monotonic," i.e., none player has his payoff decreasing over time. This is not the case when the veto condition is dropped, or if one considers the "social welfare function" maximization. In the sustainability debate, it means that environmental objectives can be reduced over time.

Future research could focus on the analysis of time-consistent solutions when there is no Veto Power. Another interesting issue would be to consider intertemporal bargaining problems with stake-holders having different forms of intertemporal payoff (e.g., some have discounted utility payoffs and some others have maximin).

A Proofs

Proof of Proposition 2.

For proving $\mathcal{P}_{\mathcal{F}(x_0)} = \mathcal{P}_{\mathcal{S}(x_0)}$ let us start showing the inclusion $\mathcal{P}_{\mathcal{S}(x_0)} \subset \mathcal{P}_{\mathcal{F}(x_0)}$.

First, we claim that $\mathcal{P}_{\mathcal{S}(x_0)} \subset \mathcal{F}(x_0)$. For $(\theta_1, \ldots, \theta_p) \in \mathcal{P}_{\mathcal{S}(x_0)} \subset \mathcal{S}(x_0)$ there exists a trajectory of decisions $a(\cdot)$ and states $x(\cdot)$, starting from x_0, such that for all $i = 1, \ldots, p$ and $t \geq t_0$ one has $I_i\Big(x(t), a(t)\Big) \geq \theta_i$.

Assuming $(\theta_1, \ldots, \theta_p) \notin \mathcal{F}(x_0)$ would mean that there is no trajectory starting from x_0 such that $\inf_{t=t_0,t_0+1,\ldots} I_i\Big(x(t), a(t)\Big) = \theta_i$ for all $i = 1, \ldots, p$. That implies that along the trajectories previously considered, at least one constraint is not binding, and therefore there exists $i \in \{1, \ldots, p\}$ such that $\inf_{t=t_0,t_0+1,\ldots} I_i\Big(x(t), a(t)\Big) = \theta_i' > \theta_i$. Hence, $(\theta_1, , \ldots, \theta_i' \ldots, \theta_p) \in \mathcal{S}(x_0)$, which is in contradiction with $(\theta_1, \ldots, \theta_i \ldots, \theta_p) \in \mathcal{P}_{\mathcal{S}(x_0)}$, concluding then $(\theta_1, \ldots, \theta_p) \in \mathcal{F}(x_0)$.

Now, for $(\theta_1, \ldots, \theta_p) \in \mathcal{P}_{\mathcal{S}(x_0)} \subset \mathcal{F}(x_0)$, if we assume $(\theta_1, \ldots, \theta_p) \notin \mathcal{P}_{\mathcal{F}(x_0)}$, there exists some $i \in \{1, \ldots, p\}$ and $\theta_i' > \theta_i$ such that $(\theta_1, \ldots, \theta_i' \ldots, \theta_p) \in \mathcal{F}(x_0)$. Thus, there is a trajectory $(x(\cdot), a(\cdot))$ starting from x_0 such that $\inf_{t=t_0,t_0+1,\ldots} I_i\Big(x(t), a(t)\Big) = \theta_i'$. This would mean that $(\theta_1, \ldots, \theta_i' \ldots, \theta_p) \in \mathcal{S}(x_0)$, which is a contradiction with $(\theta_1, \ldots, \theta_p) \in \mathcal{P}_{\mathcal{S}(x_0)}$, concluding then

$(\theta_1, \ldots, \theta_p) \in \mathcal{P}_{\mathcal{F}(x_0)}$.

For proving the reverse inclusion $\mathcal{P}_{\mathcal{F}(x_0)} \subset \mathcal{P}_{\mathcal{S}(x_0)}$, consider $(\theta_1, \ldots, \theta_p) \in \mathcal{P}_{\mathcal{F}(x_0)}$. Since $\mathcal{P}_{\mathcal{F}(x_0)} \subset \mathcal{F}(x_0) \subset \mathcal{S}(x_0)$ we get that $(\theta_1, \ldots, \theta_p) \in \mathcal{S}(x_0)$. If $(\theta_1, \ldots, \theta_p) \notin \mathcal{P}_{\mathcal{S}(x_0)}$, there exist some $i \in \{1, \ldots, p\}$ and $\theta'_i > \theta_i$ such that $(\theta_1, \ldots, \theta'_i \ldots, \theta_p) \in \mathcal{S}(x_0)$, and therefore there is a trajectory of decisions $a(\cdot)$ and states $x(\cdot)$, starting from x_0, such that $\inf_{t=t_0,t_0+1,\ldots} I_i\Big(x(t), a(t)\Big) \geq \theta'_i$. Along this trajectory, let us define $\tilde{\theta}_i = \inf_{t=t_0,t_0+1,\ldots} I_i\Big(x(t), a(t)\Big) \geq \theta'_i > \theta_i$. This would mean that $(\theta_1, \ldots, \tilde{\theta}_i, \ldots, \theta_p) \in \mathcal{F}(x_0)$, which is a contradiction with $(\theta_1, \ldots, \theta_i, \ldots, \theta_p) \in \mathcal{P}_{\mathcal{F}(x_0)}$ concluding then $(\theta_1, \ldots, \theta_p) \in \mathcal{P}_{\mathcal{S}(x_0)}$.

□

Proof of Proposition 3. Under the axiom of Pareto efficiency, a solution $\theta \in \mathcal{S}(x_0)$ belongs to $\mathcal{P}_{\mathcal{S}(x_0)}$. According to Proposition 2, this means that $\theta \in \mathcal{P}_{\mathcal{F}(x_0)} \in \mathcal{F}(x_0)$. Given the axiom of Independence of irrelevant alternatives and Proposition 1, θ is also the solution of the dynamic bargaining problem on $\mathcal{F}(x_0)$. □

Proof of Proposition 4.

We have to show that for an initial state x_0, if the vector of thresholds $\theta = (\theta_{1:k}, \theta_{k+1:p})$ belong to $\mathcal{S}(x_0)$ then, $a^\star(t) = \mathfrak{a}^\star_{\theta_{k+1:p}}(x(t))$ defined in (6) is a decision rule that allows to obtain at least θ.

Take $\theta = (\theta_{1:k}, \theta_{k+1:p}) \in \mathcal{S}(x_0)$ and a sequence of decisions $a(t_0), a(t_0 + 1) \ldots$ that allows to guarante these thresholds. Since $\theta \in \mathcal{S}(x_0)$, the decision $\mathfrak{a}^\star_{\theta_{k+1:p}}(x_0)$ is well defined (the infimum is taken over an nonempty set) and (from the definition of $\mathfrak{a}^\star_{\theta_{k+1:p}}(\cdot)$ in (6)), we have that $a(t_0) \geq \mathfrak{a}^\star_{\theta_{k+1:p}}(x(t_0))$

and therefore, due to g is decreasing in the decision variable, we obtain that

$$x^\star(t_0+1) = g(x(t_0), \mathfrak{a}^\star_{\theta_{k+1:p}}(x(t_0))) \geq g(x(t_0), a(t_0)) = x(t_0+1) \ .$$

As above, in the following we will denote by $x^\star(\cdot)$ and $x(\cdot)$ the trajectories of the states generated by feedback decisions $\mathfrak{a}^\star_{\theta_{k+1:p}}$ and decisions $a(\cdot)$ respectively.

Since indicators I_i, $i = k+1, \ldots, p$, are increasing with the state x, we can see in (6) that $\mathfrak{a}^\star_{\theta_{k+1:p}}(x)$ is decreasing with the state x. Hence

$$\begin{aligned} a(t_0+1) \ &\geq \ \mathfrak{a}^\star_{\theta_{k+1:p}}(x(t_0+1)) \\ &\qquad \text{by definition of } \mathfrak{a}^\star_{\theta_{k+1:p}} \text{ because} \\ &\qquad I_i(x(t_0+1), a(t_0+1)) \geq \theta_i \ , \quad i = k+1, \ldots, p \\ &\geq \ \mathfrak{a}^\star_{\theta_{k+1:p}}(x^\star(t_0+1)) \\ &\qquad \text{because } \mathfrak{a}^\star_{\theta_{k+1:p}}(\cdot) \text{ is decreasing in the state variable .} \end{aligned}$$

We thus obtain that

$$\begin{aligned} x^\star(t_0+2) \ &= \ g(x^\star(t_0+1), \mathfrak{a}^\star_{\theta_{k+1:p}}(x^\star(t_0+1))) \\ &\geq \ g(x^\star(t_0+1), a(t_0+1)) \\ &\qquad \text{because the dynamics } g \text{ is decreasing in the control variable} \\ &\geq \ g(x(t_0+1), a(t_0+1)) \\ &\qquad \text{because the dynamics } g \text{ is increasing in the state variable} \\ &= \ x(t_0+2) \ . \end{aligned}$$

Recursively we can conclude that $x^\star(t) \geq x(t)$ and $\mathfrak{a}^\star_{\theta_{k+1:p}}(x^\star(t)) \leq a(t)$ for all $t \geq t_0$.

On the other hand, by assumption, the indicators $I_1, \dots, I_k$ are increasing in the state and decreasing in the decision variable. We deduce then that for $i = 1, \dots, k$,

$$I_i\Big(x^\star(t), \mathfrak{a}^\star_{\theta_{k+1:p}}(x^\star(t))\Big) \geq I_i\Big(x(t), a(t)\Big) \geq \theta_i \ .$$

For $i = k+1, \dots, p$, notice that $I_i\Big(x^\star(t), \mathfrak{a}^\star_{\theta_{k+1:p}}(x^\star(t))\Big) \geq \theta_i$ by definition of $\mathfrak{a}^\star_{\theta_{k+1:p}}$ which allows to conclude the desired result.

Finally, if $\mathfrak{a}^\star_{\theta_{k+1:p}}$ is a common decision rule that allow to obtain at least $\theta = (\theta_{1:k}, \theta_{k+1:p})$, obviously $\theta \in \mathcal{S}(x_0)$. □

Proof of Proposition 5.

For $\theta = (\theta_1, \dots, \theta_p) = (\theta_{1:k}, \theta_{k+1:p})$ in $\mathcal{S}(x_0)$ we first prove that the inequalities $\theta_i \leq \Theta_i(\theta_{k+1:p}, x_0)$ for $i = 1, \dots, k$ hold. From the definition of $\mathcal{S}(x_0)$, there exists a sequence of controls $a(t_0), a(t_0+1), \dots$ such that the trajectory given by

$$\begin{cases} \tilde{x}(t+1) = g(\tilde{x}(t), a(t)), \quad t = t_0, t_0+1, \dots \\ \tilde{x}(t_0) = x_0 \end{cases}$$

satisfies

$$I_i(\tilde{x}(t), a(t)) \geq \theta_i \qquad i = 1, 2, \dots, p \qquad t = t_0, t_0+1, \dots . \tag{12}$$

Since $I_i(x_0, a(t_0)) \geq \theta_i$, for $i = k+1, \dots, p$, from the definition of $\mathfrak{a}^\star_{\theta_{k+1:p}}$ one has $a(t_0) \geq \mathfrak{a}^\star_{\theta_{k+1:p}}(x_0)$ which, from (12) (for $t = t_0$) and monotonicity properties of indicators $I_1, \dots, I_k$, implies

$$I_i\Big(x_0, \mathfrak{a}^\star_{\theta_{k+1:p}}(x_0)\Big) \geq \theta_i \qquad i = 1, \dots, k \ .$$

If we consider the trajectory

$$\begin{cases} x(t+1) = g\Big(x(t), \mathfrak{a}^{\star}_{\theta_{k+1:p}}(x(t))\Big), \quad t = t_0, t_0+1, \dots \\ x(t_0) = x_0 \end{cases} \tag{13}$$

inductively we can prove that $\mathfrak{a}^{\star}_{\theta_{k+1:p}}(x(t)) \leq a(t)$ and $x(t) \geq \tilde{x}(t)$ for all $t = t_0, t_0+1, \dots$. Therefore

$$I_i\Big(x(t), \mathfrak{a}^{\star}_{\theta_{k+1:p}}(x(t))\Big) \geq I_i\Big(\tilde{x}(t), a(t)\Big) \geq \theta_i \qquad i = 1, \dots, k, \ \ t = t_0, t_0+1, \dots$$

implying $\theta_i \leq \Theta_i(\theta_{k+1:p}, x_0)$ for $i = 1, \dots, k$.

For the reverse inclusion in (8), take $\theta = (\theta_{1:k}, \theta_{k+1:p}) \in \mathbb{R}^p$. If $\theta_{1:k} \leq \Theta_{1:k}(\theta_{k+1:p}, x_0)$, from he definition of $\Theta_i(\theta_{k+1:p}, x_0)$ in (9), we have that the trajectory defined in (13) satisfies

$$I_i\Big(x(t), \mathfrak{a}^{\star}_{\theta_{k+1:p}}(x(t))\Big) \geq \Theta_i(\theta_{k+1:p}, x_0) \geq \theta_i \qquad i = 1, \dots, k, \ \ t \geq t_0$$

and, from definition of $\mathfrak{a}^{\star}_{\theta_{k+1:p}}(\cdot)$, one has

$$I_i\Big(x(t), \mathfrak{a}^{\star}_{\theta_{k+1:p}}(x(t))\Big) \geq \theta_i \qquad i = k+1, \dots, p, \ t \geq t_0$$

concluding that $\theta = (\theta_{1:k}, \theta_{k+1:p}) \in \mathcal{S}(x_0)$, because the common decision rule $\mathfrak{a}^{\star}_{\theta_{k+1:p}}$ is admissible for θ. $\square$

Proof of Proposition 6

Let us start proving the inclusion

$$\widehat{\mathcal{S}}(x_0) = \Big\{\Big(\widehat{\theta}_{1:k}, \theta_{k+1:p}(\theta_{1:k}, x_0)\Big) \in \mathbb{R}^p \ \ | \ \ \theta_{k+1:p} \in \mathbb{R}^{p-k}\Big\} \subset \mathcal{P}^w{}_{\mathcal{S}(x_0)}$$

In order to do that, we need the following Lemma.

Lemma 1

Assume that the dynamics g and the indicators $I_1, \dots, I_p$ are $\texttt{MONDAI}_k$ for some $k \in \{1, \dots, p-1\}$. Let $\theta_{k+1:p}$ and $\sigma_{k+1:p}$ be two vectors of partial thresholds such that $\theta_{k+1:p} \leq \sigma_{k+1:p}$. Then,

$$\widehat{\sigma}_{1:k}(\sigma_{k+1:p}, x_0) \leq \Theta_{1:k}(\theta_{k+1:p}, x_0), \tag{14}$$

where, for a fixed set of thresholds $\theta_{k+1:p} = (\theta_{k+1}, \dots, \theta_p)$ (resp. $\sigma_{k+1:p} = (\sigma_{k+1}, \dots, \sigma_p)$), the vector $\Theta_{1:k}(\theta_{k+1:p}, x_0)$ (resp. $\widehat{\sigma}_{1:k}(\sigma_{k+1:p}, x_0)$) is given by (9).

Proof of Lemma 1

From Proposition 5, we have that the vectors $\Big(\Theta_{1:k}(\theta_{k+1:p}, x_0), \theta_{k+1:p}\Big)$ and $\Big(\widehat{\sigma}_{1:k}(\sigma_{k+1:p}, x_0), \sigma_{k+1:p}\Big)$ are in $\mathcal{S}(x_0)$. For $\theta_{k+1:p}$ and $\sigma_{k+1:p}$, we consider the associated decision rules $\mathfrak{a}^\star_{\theta_{k+1:p}}$ and $\mathfrak{a}^\star_{\sigma_{k+1:p}}$ (defined in (6)) and the generated trajectories

$$\begin{cases} x(t_0) = x_0 \\ x(t+1) = g\Big(x(t), \mathfrak{a}^\star_{\theta_{k+1:p}}(x(t))\Big) \quad t = t_0, t_0+1, \dots, \end{cases}$$

$$\begin{cases} \tilde{x}(t_0) = x_0 \\ \tilde{x}(t+1) = g\Big(\tilde{x}(t), \mathfrak{a}^\star_{\sigma_{k+1:p}}(\tilde{x}(t))\Big) \quad t = t_0, t_0+1, \dots, \end{cases}$$

From the hypothesis $\theta_{k+1:p} \leq \sigma_{k+1:p}$ and the definition of $\mathfrak{a}^\star_{\sigma_{k+1:p}}$, we have

$$I_i\Big(x_0, \mathfrak{a}^\star_{\sigma_{k+1:p}}(x(t_0))\Big) \geq \sigma_i \geq \theta_i \qquad i = k+1, \dots, p$$

and, from the definition of $\mathfrak{a}^\star_{\theta_{k+1:p}}$ one has $\mathfrak{a}^\star_{\sigma_{k+1:p}}(\tilde{x}(t_0)) \geq \mathfrak{a}^\star_{\theta_{k+1:p}}(x(t_0))$. Therefore,

$$x(t_0+1) = g\Big(x_0, \mathfrak{a}^\star_{\theta_{k+1:p}}(x(t_0))\Big) \geq g\Big(x_0, \mathfrak{a}^\star_{\sigma_{k+1:p}}(\tilde{x}(t_0))\Big) = \tilde{x}(t_0+1).$$

Inductively we can prove that

$$x(t) \geq \tilde{x}(t) \quad \text{and} \quad \mathfrak{a}^{\star}_{\sigma_{k+1:p}}(\tilde{x}(t)) \geq \mathfrak{a}^{\star}_{\theta_{k+1:p}}(x(t)) \qquad \forall\, t \geq t_0,$$

implying

$$I_i\Big(\tilde{x}(t), \mathfrak{a}^{\star}_{\sigma_{k+1:p}}(\tilde{x}(t))\Big) \leq I_i\Big(x(t), \mathfrak{a}^{\star}_{\theta_{k+1:p}}(x(t))\Big) \qquad i = 1, \ldots, k \quad t \geq t_0.$$

Taking minimum on $t = t_0, t_0 + 1, \ldots$, we prove the inequality (14).

□

Consider now $\Big(\Theta_{1:k}(\theta_{k+1:p}, x_0), \theta_{k+1:p}\Big) \in \widehat{\mathcal{S}}(x_0)$ and $\sigma = (\sigma_{1:k}, \sigma_{k+1:p})$ such that

$$\sigma = (\sigma_{1:k}, \sigma_{k+1:p}) > \Big(\Theta_{1:k}(\theta_{k+1:p}, x_0), \theta_{k+1:p}\Big). \tag{15}$$

From Lemma 1, we have

$$\widehat{\sigma}_{1:k}(\sigma_{k+1:p}, x_0) \leq \Theta_{1:k}(\theta_{k+1:p}, x_0). \tag{16}$$

If $\sigma \in \mathcal{S}(x_0)$, Proposition 5 allows us to conclude

$$\sigma_{1:k} \leq \widehat{\sigma}_{1:k}(\sigma_{k+1:p}, x_0).$$

The above inequality, together with (16), is a contradiction with (15), concluding thus that $\sigma \notin \mathcal{S}(x_0)$ and hence $\Big(\Theta_{1:k}(\theta_{k+1:p}, x_0), \theta_{k+1:p}\Big) \in {\mathcal{P}^w}_{\mathcal{S}(x_0)}$.

For the inclusion

$$\mathcal{P}_{\mathcal{S}(x_0)} \subset \widehat{\mathcal{S}}(x_0),$$

take $\theta = (\theta_{1:k}, \theta_{k+1:p}) \in \mathcal{P}_{\mathcal{S}(x_0)} \subset \mathcal{S}(x_0)$. From Proposition 5 we get

$$\theta_{1:k} \leq \Theta_{1:k}(\theta_{k+1:p}, x_0).$$

Since $(\theta_{k+1:p}, \Theta_{1:k}(\theta_{k+1:p}, x_0)) \in \mathcal{S}(x_0)$ and $\theta = (\theta_{1:k}, \theta_{k+1:p}) \in \mathcal{P}_{\mathcal{S}(x_0)}$ we conclude that $\theta_{1:k} = \Theta_{1:k}(\theta_{k+1:p}, x_0)$, and then

$$\theta = (\theta_{1:k}, \theta_{k+1:p}) = \Big(\Theta_{1:k}(\theta_{k+1:p}, x_0), \theta_{k+1:p}\Big) \in \widehat{\mathcal{S}}(x_0).$$

□

Proof of Proposition 7. The proof is similar (*mutatis mutandis*) to that of Proposition 3. □

B Example

Consider the dynamical system

$$\begin{cases} x(t+1) = g\Big(x(t), a(t)\Big) = \frac{x(t)}{a(t)}, \quad t = t_0, t_0+1, \ldots \\ x(t_0) = x_0 \quad \text{given,} \end{cases}$$

where $0 < a_\flat \leq a(t) \leq a_\sharp \leq 1$.

Given the following indicators:

$$\begin{aligned} I_1(x, a) &= h(a) \\ I_2(x, a) &= x\ a \end{aligned}$$

where $h(\cdot)$ is a decreasing function, we observe that the dynamics g and the indicators are $\texttt{MONDAI}_1$.

For a threshold $\theta_2 \geq 0$ one has

$$\begin{aligned} \mathfrak{a}^\star_{\theta_2}(x) &= \inf \{a \in [a_\flat, a_\sharp] \mid I_2(x, a) = x\ a \geq \theta_2\ \} \\ &= \begin{cases} \max\Big\{a_\flat, \frac{\theta_2}{x}\Big\} & \text{if } \theta_2 \leq a_\sharp\ x \\ +\infty & \text{if } \theta_2 > a_\sharp\ x \end{cases} \end{aligned}$$

Since $a_\sharp \leq 1$ one has that $\theta_2 \leq a_\sharp\ x$ implies $\theta_2 \leq a_\sharp\ g(x, \mathfrak{a}^\star_{\theta_2}(x))$.

Consider now the dynamic associated with the feedback control $\mathfrak{a}^\star_{\theta_2}(\cdot)$. First, it is straightforward to check that thresholds $\theta_2 > a_\sharp\ x_0$ are not satisficing. If $\theta_2 \in [0, a_\flat\ x_0]$ we deduce that for all $t \geq t_0$, one has $\mathfrak{a}^\star_{\theta_2}(x(t)) = a_\flat$ and $x(t)$ is an increasing sequence tending to $+\infty$. Therefore

$$\Theta_1(\theta_2, x_0) = \inf_{t=t_0,t_0+1,\ldots} I_1\Big(x(t), \mathfrak{a}^\star_{\theta_2}(x(t))\Big) = h(a_\flat)\ .$$

Finally, when $\theta_2 \in [a_\flat\ x_0, a_\sharp\ x_0]$ we can deduce that the trajectory $x(t)$ is increasing and converging to $+\infty$ and the sequence of controls $\mathfrak{a}^\star_{\theta_2}(x(t))$ is decreasing. Even more, we can deduce that there exists t^* such that $\mathfrak{a}^\star_{\theta_2}(x(t)) = a_\flat$ for all $t \geq t^*$. Hence

$$\Theta_1(\theta_2, x_0) = \inf_{t=t_0,t_0+1,\ldots} I_1\Big(x(t), \mathfrak{a}^\star_{\theta_2}(x(t))\Big) = h(a(t_0)) = h\left(\frac{\theta_2}{x_0}\right).$$

Figure 2 shows the set $\mathcal{S}(x_0)$ in this example.

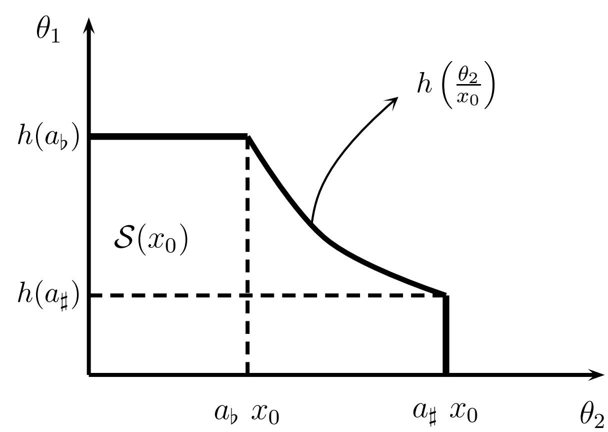

Figure 2: Set $\mathcal{S}(x_0)$ for the bargaining problem ??????????

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