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Leniency programs for multimarket firms: The effect of Amnesty Plus on cartel formation

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Abstract

We examine the effect of the Amnesty Plus policy on the incentives of firms to engage in cartel activities. Amnesty Plus is aimed at attracting amnesty applications by encouraging firms, convicted in one market, to report their collusive agreements in other markets. It has been vigorously advertised that Amnesty Plus weakens cartel stability. We show to the contrary that Amnesty Plus may not have this desirable effect, and, if improperly designed, may even stabilize a cartel. We suggest a simple discount-setting rule to avoid this anticompetitive effect.

Keywords: Amnesty Plus, Leniency program, multimarket contact, antitrust policy.

JEL Classification: K21, K42, L41

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1 Introduction

Experience garnered over many years has taught antitrust authorities in the United States (US) and the European Union (EU) that companies which have been colluding in one specific product market are more likely to have engaged in cartel activities in adjacent markets.

Due to the high diversity of businesses in multinational firms, cartel activities bear all the marks of contagion within companies. The probably most well-known example for such a cross-linked collusive pattern is the vitamin conspiracy. The striking feature of this complex of infringements is the central role played by Hoffmann-La Roche (HLR) and BASF, the two main vitamin producers, over the course of ten years in virtually every cartel affecting the whole extent of bulk vitamin production.¹ HLR, BASF and Rhône-Poulenc instigated the first group of cartels which consisted of price fixing agreements in the markets for vitamins A and E. The initial success of these arrangements inspired their replication in other vitamin markets. Accordingly, the European Commission (EC) stated that "the simultaneous existence of the collusive arrangements in the various vitamins was not a spontaneous or haphazard development, but was conceived and directed by the same persons at the most senior levels of the companies concerned".² Rhône-Poulenc's disclosure of evidence on collusion in the markets for vitamins A and E led to the opening of an investigation. However, only the comprehensive collaboration of BASF with the US Department of Justice (DoJ) under the Amnesty Plus program led to the successful prosecution of all participants. When Rhône-Poulenc plead guilty to price fixing in the vitamins A and E, it did however not provide any information on its participation in the vitamin D3 infringement and even pursued cartel activities in other product markets such as the markets for methionine and methylglucamine.³

In 1999, the DoJ implemented the Amnesty Plus program as part of its Corporate Leniency Policy in response to the increasing number of parallel offenders. According to Hammond, "The Division's Amnesty Plus program creates an attractive inducement for encouraging companies who are *already under investigation* to report the full extent of their antitrust crimes [...]" (Hammond, 2004, p.16).

Leniency programs cancel the fine against the first cartel member that brings decisive evidence to the antitrust authority. Amnesty Plus aims at attracting amnesty applications by encouraging subjects of ongoing investigations to consider whether they qualify for amnesty in other than the currently inspected markets. In particular, Amnesty Plus offers a firm, which plea-bargains an agreement for participation in one cartel, where it cannot obtain amnesty, complete immunity in a second cartel affecting another market. Provided that the firm agrees

¹Concerned were the markets for vitamins A, E, B1, B2, B3 (niacin), B4 (choline chloride), B5, B6, B9 (folic acid), B12, C, D3, H (biotin), beta carotene, carotenoids and premixes.

 $^{^{2}}$ EC IP/01/1625 November 2001, p.2.

 $^{^3\}mathrm{EC}$ IP/01/1625 and OJ L 6 of 10.1.2003, p.1-89; EC IP/02/976 and OJ L 255, 08.10.2003, p.1-32; EC IP/02/1746 and OJ L 38, 10.2.2004, p.18-46.

to fully cooperate in the investigation of the conspiracy of which the DoJ was previously not aware, it is automatically granted amnesty for this second offense. Moreover, the company benefits from a "substantial additional discount" (Hammond, 2006, p.10), i.e. the Plus, in the calculation of its fine in any plea agreement for the initial matter under investigation.

Under the current EC Leniency Notice, Amnesty Plus does not exist. Although the Organization for Economic Co-operation and Development (OECD) recommended the inclusion of Amnesty Plus as part of the 2002 reforms of the EU Leniency Program, the EC did not seize the opportunity to follow the US example by adopting a similar policy. Also in 2006, the EC failed to incorporate Amnesty Plus in the reform package.

The present paper studies how the Amnesty Plus policy affects firms' incentives to form a cartel.⁴ Following a conviction of one cartel, Amnesty Plus may encourage firms to report another cartel by granting the first firm which applies for this program a discount on the fine already imposed. Ex ante, however, the opportunity to benefit from Amnesty Plus may decrease the expected fine in one market and make the formation of a cartel - not in this but in another market, more attractive.

We study two markets in which two identical firms play an infinitely repeated game of collusion. In each period, the firms can choose to form a cartel before interacting in the product market. Collusion generates incriminating evidence which the antitrust authority can discover with some probability. Each firm can also bring this evidence to the authority. When a cartel is detected, each cartel member, except the first reporting firm, pays a fine. Amnesty Plus sets in when the firms decide whether to report the second cartel after having been convicted in the first market.

Our main result is that Amnesty Plus can increase the extent of collusion if the discount on the fine imposed for the initial infringement exceeds the fine the Amnesty Plus applicant would have incurred in the second market. To avoid this adverse effect, the design of the Amnesty Plus policy must respect a discount-setting rule that fixes the discount in the first market equal or below the fine in the second market. A leniency policy with an Amnesty Plus program that sticks to this rule always performs weakly better, in terms of cartel deterrence, than a standard leniency policy without Amnesty Plus. The reason is that Amnesty Plus may induce the reporting of the second cartel after a first detection. Increased desistance from cartel activities in the reporting stage reduces the value of joint collusion provided that the fine discount does not increase the expected collusive value at this stage.

Recent theoretical contributions such as Harrington (2008), Chen and Rey (2007), Aubert et al. (2006), Spagnolo (2004) and Motta and Polo (2003) study the trade-off leniency generates between less cartel stability through encouraged reporting and more cartel stability through lower expected fines. The overall conclusion is that leniency programs, if properly

 $^{^{4}}$ In a previous study, Roux and Von Ungern-Sternberg (2007) examine how Amnesty Plus affects the companies' incentives to reveal their collusive conduct when they are engaged in two cartels simultaneously. The authors use a static model and focus on the effect of Amnesty Plus on cartel stability ex post.

designed, make collusion more difficult.⁵ Several studies suggest that positive rewards may further strengthen the deterrence power of leniency programs (Aubert et al., 2006; Spagnolo, 2004).

Amnesty Plus is equivalent to a leniency program, coupled with a positive reward for the first informant, that is available in one market - say market 2 - only if the cartel in market 1 is discovered. Amnesty Plus, unlike a standard leniency program, therefore strategically links two markets. The reward can stabilize cartel 2 if cartel 1 is formed and thereby increase the extent of collusion. This market linkage also has an important implication for the procompetitive potential of Amnesty Plus. Contrary to a standard leniency program, Amnesty Plus may destabilize a cartel even if the probability of detection in that market is zero. Detection must just be likely enough in the other market. Amnesty Plus may thus be particularly useful when probabilities of detection differ across markets.

Another strand of literature closely linked to our analysis studies the role of multimarket contact between firms in sustaining collusion when there is no antitrust enforcement. In their seminal paper, Bernheim and Whinston (1990) give theoretical support to the informal argument, first raised by Edwards (1955), that multimarket contact may enhance collusion. They show that the firms can pool the incentive constraints of the different markets where they operate in order to transfer slack from a more to a less collusive market. At worst, with identical firms and markets, multimarket contact does not affect the opportunities for cooperation. At best, it facilitates collusion.⁶

In a recent paper, Choi and Gerlach (2009b) examine the sustainability of collusion in two markets linked by demand relationships. They find that successful prosecution in one market may destabilize collusion in the adjacent market if products are substitutes, whereas in the case of complements, successful prosecution in one market may increase cartel stability in the adjacent market. Choi and Gerlach (2009a) focus on substitutes and show that, if there is one local authority per market, free-rider problems, that arise due to positive prosecution externalities in each market, can only be solved by coordinating enforcement efforts across jurisdictions. Although both studies combine multimarket contact with antitrust enforcement, they do not analyze the strategic effects generated by Amnesty Plus or even leniency programs in this context.

The remainder of this paper is organized as follows. Section 2 sets up the model. Sections 3 and 4 analyze cartel formation. Section 5 extends our analysis to the case of heterogenous detection probabilities, partial collusion and more than 2 firms and 2 markets. Section 6 concludes.

⁵See also Miller (2009), Goeree and Helland (2009) and Brenner (2005) for empirical studies and Bigoni et al. (2009), Hinloopen and Soetevent (2008) and Apesteguia et al. (2007) for experiments.

⁶Multimarket contact can also lower firms' payoffs if it is combined with imperfect monitoring. See Thomas and Willig (2006).

2 The Model

2.1 Set-up

We consider two markets, 1 and 2, in which two identical firms play an infinitely repeated game where, in each period, they can choose to form a cartel before interacting in the product market. Communication is necessary for collusion and generates hard evidence which makes it possible to establish the antitrust offense.⁷ Markets 1 and 2 differ in profitability. In particular, market 1 is more profitable than market 2. Firms discount future payoffs by a common discount factor $\delta \in [0, 1[$. We compare the firms' decisions to form cartels under the EU and the US antitrust legislations whose sole difference here is that the latter comprises an Amnesty Plus program.

Throughout the analysis we use the following notation: We refer to variables of a specific market by using the indices 1 and 2. When considering any of the two markets, we use the index k, and we refer to the other market by using the index -k.

In each period, the fully collusive joint profit in market k is $2\pi_k > 0$, and thus, each firm makes a cartel profit equal to π_k .⁸ If the firms compete, they make zero profits. In case one firm unilaterally deviates from the collusive agreement while the other continues to collude, the deviating firm earns the whole short-term cartel profit $2\pi_k$ alone, whereas the other firm gets nothing. The firms use (grim) trigger strategies. The punishment they agreed upon starts the period following the deviation and lasts forever after.

At the time the firms decide whether to enter a collusive agreement, they observe the exogenous per-period conviction probability q > 0 with which the Antitrust Authority (AA) detects a cartel and convicts the colluding firms. Detection is independent across markets and over time.⁹ Each convicted firm pays a strictly positive, market specific fine F_k which is reduced under Amnesty Plus to $F_k - R_k$ in return for the disclosure of the second cartel. $R_k \in [0, F_k]$ represents the fine reduction granted to the first informant. The higher R_k the more generous the Amnesty Plus policy. The successful applicant receives amnesty in the second infringement because it is the first company reporting in that market. If both firms simultaneously apply for Amnesty Plus, each is first with probability $\frac{1}{2}$.

The fines are such that $F_1 \ge F_2$.¹⁰ We assume that the fine-profit ratio is higher for

⁷For collusion to be illegal, there must be evidence of an explicit agreement between the firms (McCutcheon, 1997). The view that collusion is self-enforcing but requires communication is common in the literature on leniency programs. See Aubert et al. (2006).

 $^{^{8}}$ We focus on full collusion in the main analysis and allow for partial collusion in extension 5.4.

⁹Detection in one market may increase as well as decrease the probability of detection in the other market. Rollover investigations make a second conviction more likely whereas limited resources of the AA and increased efforts by the firms to conceal the remaining conspiracy make it less likely. Assuming independence across markets is equivalent to saying that both effects are equally strong. By the independence over time assumption we impose stationarity.

¹⁰We believe that, in the light of $\pi_1 > \pi_2$, the assumption $F_1 \ge F_2$ is plausible. In practice, fines are set according to judicial principles which link them to the gravity of the infringement, and thus, to the nature and

market 2 than for market 1, i.e. $\frac{F_2}{\pi_2} > \frac{F_1}{\pi_1}$. This reflects the idea that the fine rises less than proportionally with the cartel profit. Legislative provisions and fine records support this assumption.¹¹ Heterogenous fine-profit ratios create heterogenous cartel formation incentives across markets and thereby generate a parameter range where, in the absence of Amnesty Plus, the firms form only one of the cartels. It is in this range where Amnesty Plus can produce a negative effect by inducing the formation of the second cartel.¹²

We assume that the evidence of collusion lasts for one period. Thus, after a firm has deviated from a collusive agreement it is held liable for its cartel behavior and can be fined until the end of the period in which the deviation occurred.¹³ Each cartel member has the possibility to bring the incriminating evidence to the AA. The first informant receives immunity from fines under a standard leniency program. Again, if both firms simultaneously apply for leniency, each is first with probability $\frac{1}{2}$.

Following a cartel conviction, we assume that the AA closely monitors the previously collusive industry and thus, firms compete, and they never return back to collusion in the same market.

2.2 Timing

The timing of the game is a version of the time structure used in Chen and Rey (2007), adapted to multimarket contact. In each period, the structure is as follows:

Stage 0: Each firm decides whether to enter a collusive agreement in the market(s) where no cartel has been previously convicted. If at least one firm decides not to collude in market k, competition takes place in this market. If this happens in both markets, the firms compete, and the game ends for that period. If both firms choose to collude in market k, their communication leaves some hard evidence.

importance of the anticompetitive behavior. The latter relates, at least indirectly, to the collusive overcharge which is, with zero competitive profits, equivalent to cartel profits.

¹¹The fine-profit ratio decreases with market size if small fines are inflated compared to high fines. The EU fine setting guidelines suggest that this is the case: First, the basic amount of the fine can be increased to ensure a sufficient deterrent effect of the fine. As a fine of a big absolute size is more likely to act as a deterrent (e.g. because of high media coverage), the deterrent uplift for a small fine seems to be relatively bigger than for a high fine. Second, the legal maximum, i.e. 10% of the firm's total turnover in the preceding business year, imposes a cap on large fines. Hence, the fines for large cartels are more likely to be capped than the fines for smaller cartels. Third, the "multiplier" increases the final amount of the fine if the Commissioner judges that the turnover of the convicted market is too small, and thus, the fine too low, relative to the company's entire turnover. The fine is multiplied by a number, historically between 2 and 5, to increase the financial impact of the penalty. There is also empirical evidence supporting our hypothesis. In particular, see Combe and Monnier (2009) and Connor (2005).

¹²Instead of heterogenous fine-profit ratios, we could use anything else that creates an asymmetry between the incentive compatibility constraints.

 $^{^{13}}$ The limitation period of the liability for antitrust offenses is generally a positive number of years. Article 25 of the EC Council Regulation 1/2003 states that the Commission can sue for Administrative Action until five years from the date of the infringement. Moreover, "[...] in the case of continuing or repeated infringements, time shall begin to run on the day on which the infringement ceases".

Stage 1: Each firm decides whether to stick to, or to deviate from, the collusive agreement(s). Its rival does not observe this decision until the end of stage 2.

Stage 2: Each firm decides whether to report the evidence it holds in each cartel to the AA. A cartel is convicted (with certainty) if at least one firm self-reports. The first informant gets complete immunity from fines in this market, whereas the other firm has to pay the full fine. If each cartel formed in stage 0 is reported in this stage, the game ends for this period; otherwise:

Stage 3: Each cartel formed in stage 0 and not reported in stage 2 is detected with probability q. If the AA does not detect any cartel, the game ends for that period. If the AA however detects the cartel(s) formed in stage 0 and not reported in stage 2, the colluding firms pay the corresponding fines, and the game ends for that period. If the firms have formed both cartels in stage 0 and not reported them in stage 2, and the AA has detected only one of them, then:

Stage 4: Each firm chooses whether to report the remaining cartel.

If Amnesty Plus exists, it is relevant only if the game reaches stage 4. This stage forms the reporting subgame where, after the detection of cartel k, the firms decide whether to report the remaining cartel -k. Amnesty Plus can alter the equilibria of this subgame and thereby affect the equilibria of the entire game.

Under each leniency policy, we define a set of strategies corresponding to three regimes: collusion in one market only, sequential collusion and joint collusion. We then determine the best collusive (subgame-perfect) symmetric equilibrium of the game without Amnesty Plus, constituted by these strategies, and compare it to its counterpart in the game with Amnesty Plus.

3 Collusion Under the EU Leniency Program

A strategy is denoted s over a single period and S over all periods. In particular, we denote s_0 (S₀) the strategy that consists of competing over one period (all periods).

3.1 Collusion in One Market

To analyze collusion in only one market, we consider the following strategies: s_k : collude in market k only, neither deviate from the collusive agreement nor report. S_k : play s_k in t = 0 and in any subsequent period as long as there is neither deviation from the collusive agreement nor reporting nor detection; otherwise play s_0 for the remaining periods. The cartel in market k is *individually stable*, i.e. the strategy pair (S_k, S_k) is an equilibrium, if and only if the gain from any unilateral deviation does not exceed the present discounted expected payoff $V_k(\delta)$ when both firms play S_k . This payoff is recursively defined as

$$V_k(\delta) = q(\pi_k - F_k) + (1 - q)(\pi_k + \delta V_k(\delta))$$

which we rewrite as

$$V_k(\delta) = \frac{\pi_k - qF_k}{1 - \delta(1 - q)}$$

In the presence of a leniency policy where the first informant pays no fine, the optimal unilateral deviation is to deviate from and to immediately report the collusive agreement. This deviation yields a payoff equal to $2\pi_k$. Both deviating without reporting and reporting without deviating yield lower payoffs, namely $2\pi_k - qF_k$ and 0. Thus, (S_k, S_k) is an equilibrium if and only if the following incentive compatibility constraint holds:

$$V_k(\delta) \ge 2\pi_k$$

We rewrite this constraint as

$$\delta \geq \widetilde{\delta}_k \equiv \frac{1 + q \frac{F_k}{\pi_k}}{2(1 - q)}$$

The individual stability threshold $\tilde{\delta}_k$ is increasing in q and $\frac{F_k}{\pi_k}$. Intuitively, the higher the probability of conviction and the higher the fine-profit ratio, the more firms have to value future flows of collusive profits, and thus, the higher the discount factor needed to individually sustain the cartel. Our assumption $\frac{F_2}{\pi_2} > \frac{F_1}{\pi_1}$ implies that $\tilde{\delta}_2 > \tilde{\delta}_1$, i.e. the cartel in market 2 is harder to sustain that the cartel in the more profitable market 1. Finally, we assume that $q < \frac{\pi_2}{2\pi_2 + F_2}$. Otherwise, cartel 2 would be individually unstable for any value $\delta \in [0, 1]$.¹⁴

3.2 Sequential Collusion

Sequential collusion refers to a situation in which the firms collude in only one market as long as they go undetected. After a detection in this market, they switch to collusion in the other market. We consider the following strategy:

 $S_{k\to-k}$: play s_k in t = 0 and in any subsequent period as long as there is neither deviation from the collusive agreement nor reporting nor detection; if there is detection but no deviation from the collusive agreement in t, play s_{-k} in t + 1 and in any subsequent period as long as

¹⁴The probability of detection seems to be quite low also in reality. Bryant and Eckard (1991) estimate the maximum probability of getting caught by the US authorities in any given year at 13% to 17%. Combe et al. (2008) find around 13% for a European sample.

there is neither deviation from the collusive agreement nor reporting nor detection; in all other cases, play s_0 for the remaining periods.

We focus on the sequential strategy $S_{1\to 2}$.¹⁵ The cartels are sequentially stable, i.e. the strategy pair $(S_{1\to 2}, S_{1\to 2})$ is an equilibrium, if and only if no firm has an incentive to deviate both when collusion occurs in market 1 and when it occurs in market 2. Each firm's present discounted expected payoff $V_{1\to 2}(\delta)$ when both firms play $S_{1\to 2}$ is recursively defined as

$$V_{1\to 2}(\delta) = q (\pi_1 - F_1 + \delta V_2(\delta)) + (1 - q) (\pi_1 + \delta V_{1\to 2}(\delta))$$

which can be rewritten as

$$V_{1\to 2}(\delta) = V_1(\delta) + q \frac{\delta}{1 - \delta(1 - q)} V_2(\delta)$$

The strategy pair $(S_{1\to 2}, S_{1\to 2})$ is an equilibrium if and only if the following incentive compatibility constraints hold:

$$V_{1\to 2}(\delta) \ge 2\pi_1$$

 $V_2(\delta) \ge 2\pi_2$

It is straightforward that the latter constraint implies the former. Thus, $(S_{1\to 2}, S_{1\to 2})$ is an equilibrium if and only if cartel 2 is individually stable, i.e. $\delta \geq \tilde{\delta}_2$.

3.3 Joint Collusion

To study simultaneous collusion in both markets, we consider the following strategies:

 s_{12} : collude in both markets, neither deviate from any of the collusive agreements nor report. S_{12} : play s_{12} in t = 0 and in any subsequent period as long as there is neither deviation from any collusive agreement nor reporting nor detection; if there is detection of cartel k but neither deviation from any collusive agreement nor reporting in t, play s_{-k} in t + 1, and in any subsequent period as long as there is neither deviation from the collusive agreement in market -k nor reporting nor detection, if cartel -k is individually stable; in all other cases, play s_0 for the remaining periods.

The two cartels are *jointly stable* under the EU policy, i.e. the strategy pair (S_{12}, S_{12}) is an equilibrium, if and only if the gain from any unilateral deviation does not exceed the present discounted expected value $V_{12}(\delta)$ when both firms play S_{12} . We denote $V_{12}(\delta)$ the 'value of

¹⁵In appendix A we show that there exists $\tilde{q} > 0$ such that $(S_{2\to 1}, S_{2\to 1})$ can never be the best collusive equilibrium if $q < \tilde{q}$. Throughout the paper we assume that the latter condition holds. This assumption has no qualitative implications for the analysis of the effect of Amnesty Plus but eases the exposition by eliminating $(S_{2\to 1}, S_{2\to 1})$ as a possible candidate for the best collusive equilibrium.

joint collusion' for the EU. The strategy S_{12} involves multimarket punishment. If one firm unilaterally deviates from the collusive agreement in one of the markets, the co-conspirator reverts to competition in *both* markets. The optimal unilateral deviation is then to deviate from the collusive agreements in both markets simultaneously and report both cartels. This deviation ensures a payoff equal to $2\pi_1 + 2\pi_2$. Thus, (S_{12}, S_{12}) is an equilibrium if and only if the following incentive compatibility constraint holds:

$$V_{12}(\delta) \ge 2\pi_1 + 2\pi_2$$

The value of joint collusion depends on whether the cartels are individually stable. There are three cases:

a-If cartel 1 is individually stable while cartel 2 is not, i.e. $\tilde{\delta}_1 \leq \delta < \tilde{\delta}_2$, the value of joint collusion is recursively defined as

$$V_{12}(\delta) = q^2(\pi_1 + \pi_2 - F_1 - F_2) + q(1-q)(\pi_1 + \pi_2 - F_1) + q(1-q)(\pi_1 + \pi_2 - F_2 + \delta V_1(\delta)) + (1-q)^2(\pi_1 + \pi_2 + \delta V_{12}(\delta))$$

From the independence assumption on the AA's detection technology it follows that the probability of detecting both cartels during a specific period is q^2 , only cartel 1 (cartel 2) is q(1-q), and none of the cartels $(1-q)^2$. If the AA detects cartel 1, the firms stop forming the individually unstable cartel 2. We rewrite the value of joint collusion as

$$V_{12}(\delta) = \frac{\pi_1 - qF_1}{1 - \delta(1 - q)} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)^2} \tag{1}$$

b- If both cartels are individually unstable, i.e. $\delta < \tilde{\delta}_1$, the value of joint collusion is

$$V_{12}(\delta) = \frac{\pi_1 - qF_1}{1 - \delta(1 - q)^2} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)^2}$$
(2)

c-If both cartels are individually stable, i.e. $\tilde{\delta}_2 \leq \delta < 1$, the value of joint collusion is

$$V_{12}(\delta) = \frac{\pi_1 - qF_1}{1 - \delta(1 - q)} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)}$$
(3)

3.4 Best Collusive Equilibrium

Proposition 1 characterizes the Pareto dominant equilibrium under the EU antitrust policy.

Proposition 1 There exists a joint stability threshold $\tilde{\delta}_{12} \in [\tilde{\delta}_1, \tilde{\delta}_2]$ such that: - If $\delta < \tilde{\delta}_1$, the competitive equilibrium (S_0, S_0) is the only equilibrium. - If $\tilde{\delta}_1 \leq \delta < \tilde{\delta}_{12}$, the individual collusion equilibrium (S_1, S_1) is the best collusive equilibrium. - If $\tilde{\delta}_{12} \leq \delta < 1$, the joint collusion equilibrium (S_{12}, S_{12}) is the best collusive equilibrium. **Proof.** See appendix C.

Over the interval $[\tilde{\delta}_{12}, 1]$, the expected lifespan of cartel 2 depends on the size of δ . For $\delta \in [\tilde{\delta}_{12}, \tilde{\delta}_2]$, cartel 2 is sustained only as long as the AA does not detect any of the cartels, whereas for $\delta \in [\tilde{\delta}_2, 1]$, cartel 2 is sustained up to its own detection.

By linking the punishment across markets, the firms can potentially transfer slack enforcement power from market 1 to market 2 and sustain collusion in both markets for values of $\delta < \tilde{\delta}_2$, i.e. even when cartel 2 is individually unstable. Multimarket contact has this procollusive effect if and only if $\tilde{\delta}_{12} < \tilde{\delta}_2$. In appendix B, we provide a necessary and sufficient condition for the latter inequality to hold and discuss how the possibility of cartel detection affects the procollusive potential of multimarket contact.

4 Collusion Under the US Leniency Program

We now introduce Amnesty Plus and examine its effect, first, on the equilibrium of the reporting subgame in stage 4 and, second, on the best collusive equilibrium of the entire game.

4.1 Reporting Subgame

In the absence of Amnesty Plus, the subgame exhibits two possible equilibria if cartel k is detected in the previous stage: Both firms reporting and both firms not reporting the remaining cartel. The Pareto dominant equilibrium is to not report the remaining cartel -k because each firm's expected payoff is $-\frac{1}{2}F_{-k}$ if both firms report compared to zero (when cartel -k is individually unstable) and the continuation value δV_{-k} (when cartel -k is individually stable) if none reports. As the firms do not report in the Pareto dominant equilibrium, they only desist from cartel -k if it is individually unstable. Figures 1 and 2 show the payoff matrices of this subgame.

F1, F2	R	NR	F1	, F2	R	NR
R	$-\frac{1}{2}F_k, -\frac{1}{2}F_k$	$0, -F_k$	R		$-\frac{1}{2}F_k, -\frac{1}{2}F_k$	$0, -F_k$
NR	$-F_k,0$	0,0	NI	R	$-F_k,0$	$\delta V_k, \delta V_k$

Figure 1: Cartel k unstable

Figure 2: Cartel k stable

Amnesty Plus may alter the firms' reporting decisions by creating a prisoners' dilemma where reporting cartel -k forms an equilibrium in dominant strategies. If a firm anticipates that its partner reports, it always prefers reporting. If a firm anticipates that its co-conspirator does not report, it prefers to report for any fine reduction R_k if cartel -k is individually unstable (Figure 3) because it gets a strictly positive R_k from reporting versus zero from not reporting. If cartel -k is individually stable, a firm, which anticipates that its partner does not report, finds it (strictly) optimal to report if and only if $R_k > \delta V_{-k}(\delta)$ (Figure 4). Not reporting would imply the renewed formation of the cartel in the next period and is therefore dominated by reporting only if the fine reduction exceeds the present discounted expected payoff a firm gets from this cartel. It is in these two cases where the problem of Amnesty Plus becomes apparent: While Amnesty Plus induces the firms to make the desired reporting decision, it may increase each firm's equilibrium payoff $X = \frac{1}{2}(R_k - F_{-k})$ above the equilibrium payoff in the subgame under the EU policy. Amnesty Plus may therefore raise the value of collusion over the entire game.

F1, F2	R	NR	\mathbf{F}^{1}	1, F2	R	NR
R	X,X	$R_{-k}, -F_k$	R		X,X	$R_{-k}, -F_k$
NR	$-F_k, R_{-k}$	0,0	N	R	$-F_k, R_{-k}$	$\delta V_k, \delta V_k$

Figure 3: Cartel k unstable

Figure 4: Cartel k stable

Amnesty Plus does not affect the firms' decisions to not report an individually stable cartel if $R_k \leq \delta V_{-k}(\delta)$. The subgame exhibits again two possible equilibria, but not reporting Pareto dominates reporting because $\frac{1}{2}(R_k - F_{-k}) < R_k \leq \delta V_{-k}(\delta)$. Notice that we can rewrite $R_k \leq \delta V_{-k}(\delta)$ as

$$\delta \ge \widehat{\delta}_{-k}(R_k) \equiv \frac{R_k}{\pi_{-k} - qF_{-k} + (1-q)R_k}$$

where $\hat{\delta}_{-k}(R_k)$ defines a robustness threshold for an individually stable cartel -k such that, above this threshold, it is *robust* to, and thus, survives the detection of cartel k.

4.2 Joint Collusion

Amnesty Plus cannot alter strategy profiles that do not involve simultaneous collusive interaction in the two markets. Hence, the strategies s_0 , s_k , S_0 , S_k and $S_{1\to 2}$ are identical with and without Amnesty Plus. The strategy profile for joint collusion is now given by:

 s_{12}^{AP} : collude in both markets, neither deviate from any of the collusive agreements nor report; if there is detection of one cartel, do not report the remaining cartel under Amnesty Plus if it is individually stable and robust, otherwise report.

 S_{12}^{AP} : play s_{12}^{AP} in t = 0 and in any subsequent period as long as there is neither deviation from any of the collusive agreements nor reporting nor detection; if there is detection of cartel k but neither deviation from any collusive agreement nor reporting in t, play s_{-k} in t+1, and in any subsequent period as long as there is neither deviation from the collusive agreement in market -k nor reporting nor detection, if cartel -k is individually stable and robust; in all other cases, play s_0 for the remaining periods.

The two cartels are *jointly stable* under the US policy, i.e. the strategy pair $(S_{12}^{AP}, S_{12}^{AP})$ is an equilibrium, if and only if the gain from any unilateral deviation does not exceed the present discounted expected value $V_{12}^{AP}(\delta, R_1, R_2)$ when both firms play S_{12}^{AP} . We denote $V_{12}(\delta)$ the 'value of joint collusion' for the US. Here again, the optimal unilateral deviation is to deviate from the collusive agreements in both markets and to report both cartels. Thus, $(S_{12}^{AP}, S_{12}^{AP})$ is an equilibrium if and only if the following incentive compatibility constraint holds:

$$V_{12}^{AP}(\delta, R_1, R_2) \ge 2\pi_1 + 2\pi_2$$

The value of joint collusion depends on the outcome of the reporting subgame in stage 4. There are four different cases:

a-If cartel 1 is stable and robust while cartel 2 is either unstable or stable but not robust, i.e. $\max(\tilde{\delta}_1, \hat{\delta}_1(R_2)) \leq \delta < \max(\tilde{\delta}_2, \hat{\delta}_2(R_1))$, both firms report cartel 2 but not cartel 1 in the reporting subgame. The value of joint collusion is thus recursively defined as

$$V_{12}^{AP}(\delta, R_1, R_2) = q^2(\pi_1 + \pi_2 - F_1 - F_2) + q(1-q)\left(\pi_1 + \pi_2 - F_1 + \frac{1}{2}(R_1 - F_2)\right) + q(1-q)\left(\pi_1 + \pi_2 - F_2 + \delta V_1\right) + (1-q)^2\left(\pi_1 + \pi_2 + \delta V_{12}^{AP}(\delta, R_1, R_2)\right)$$

which we rewrite as

$$V_{12}^{AP}(\delta, R_1, R_2) = \frac{\pi_1 - qF_1}{1 - \delta(1 - q)} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)^2} + \frac{q(1 - q)(R_1 - F_2)}{2(1 - \delta(1 - q)^2)}$$
(4)

b-If cartel 2 is stable and robust and cartel 1 is stable but not robust, i.e. $\max(\tilde{\delta}_2, \hat{\delta}_2(R_1)) \leq \delta < \max(\tilde{\delta}_1, \hat{\delta}_1(R_2))$, the value of joint collusion is

$$V_{12}^{AP}(\delta, R_1, R_2) = \frac{\pi_1 - qF_1}{1 - \delta(1 - q)^2} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)} + \frac{q(1 - q)(R_2 - F_1)}{2(1 - \delta(1 - q)^2)}$$
(5)

c-If both cartels are either individually unstable or individually stable but not robust, i.e. $\delta < (\tilde{\delta}_k, \hat{\delta}_k(R_{-k}))$ for both k = 1, 2, Amnesty Plus induces the firms to report. The value of joint collusion is

$$V_{12}^{AP}(\delta, R_1, R_2) = \frac{\pi_1 - qF_1}{1 - \delta(1 - q)^2} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)^2} + \frac{q(1 - q)(R_1 + R_2 - F_1 - F_2)}{2(1 - \delta(1 - q)^2)}$$
(6)

d-If both cartels are individually stable and robust, i.e. $\delta \geq (\tilde{\delta}_k, \hat{\delta}_k(R_{-k}))$ for both k = 1, 2, the firms do not report these cartels, and the value of joint collusion is

$$V_{12}^{AP}(\delta, R_1, R_2) = \frac{\pi_1 - qF_1}{1 - \delta(1 - q)} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)}$$
(7)

4.3 Best Collusive Equilibrium

Amnesty Plus may enhance desistance through reporting and is therefore beneficial for competition after a first cartel conviction. It may however generate potentially conflicting effects at the stage of cartel formation: First, the *desistance effect* which occurs if Amnesty Plus induces the firms to report, and thus terminate, an individually stable collusive agreement after a first detection. This effect is either negative, i.e. it reduces the value of joint collusion, or zero. Second, the *reporting effect* which captures the expected *equilibrium* benefits from reporting under Amnesty Plus. This effect is either negative or zero or positive. We explore the net effect of Amnesty Plus in the subsequent analysis.

4.3.1 Neutrality of Amnesty Plus on Global Competition

Consider the interval $[0, \tilde{\delta}_1]$. Amnesty Plus is neutral, and the only equilibrium is (S_0, S_0) . To see this, note that if Amnesty Plus were to have an effect, it would have to make either individual or joint collusion sustainable, i.e. make either (S_1, S_1) or $(S_{12}^{AP}, S_{12}^{AP})$ an equilibrium. The former is clearly impossible because Amnesty Plus is irrelevant when firms collude in one market only. The latter cannot occur as well because, from the expressions in (2) and (6), we see that Amnesty Plus weakly decreases the value of jointly colluding over this interval:

$$V_{12}^{AP}(\delta, R_1, R_2) = V_{12}(\delta) + \frac{q(1-q)(R_1 + R_2 - F_1 - F_2)}{2(1 - \delta(1-q)^2)} \le V_{12}(\delta)$$

for all $\delta \in [0, \widetilde{\delta}_1[$.

4.3.2 The Anticompetitive Effect of Amnesty Plus

Consider the interval $[\tilde{\delta}_1, \tilde{\delta}_{12}]$ where (S_1, S_1) is the best collusive equilibrium in the EU. Amnesty Plus is anticompetitive if it induces the formation of cartel 2, i.e. it makes $(S_{12}^{AP}, S_{12}^{AP})$ the best collusive equilibrium, for discount factor values in this interval. This can happen only if Amnesty Plus increases the value of joint collusion.

Lemma 1 Amnesty Plus increases the value of joint collusion for δ in the interval $[\tilde{\delta}_1, \tilde{\delta}_{12}]$ if and only if cartel 1 is robust and the fine discount granted in market 1 in return for the disclosure of cartel 2 exceeds the fine that would have otherwise been imposed for the reported cartel 2:

$$V_{12}^{AP}(\delta, R_1, R_2) > V_{12}(\delta) \iff \delta \ge \widehat{\delta}_1(R_2) \text{ and } R_1 > F_2$$

Proof. See appendix C.

The net effect of Amnesty Plus is equal to $V_{12}^{AP}(\delta, R_1, R_2) - V_{12}(\delta)$. If this difference is positive, Amnesty Plus is potentially anticompetitive. If cartel 1 is robust, the value of joint

collusion is given by equation (4) for the US and by equation (1) for the EU. We can separate the difference of these two expressions into the desistance (Δ_D) and the reporting (Δ_R) effects of Amnesty Plus:

$$\Delta_D = \frac{\pi_1 - qF_1}{1 - \delta(1 - q)} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)^2} - \frac{\pi_1 - qF_1}{1 - \delta(1 - q)} - \frac{\pi_2 - qF_2}{1 - \delta(1 - q)^2} = 0$$
(8)
$$\Delta_R = \frac{q(1 - q)(R_1 - F_2)}{2(1 - \delta(1 - q)^2)} <> = 0$$

The desistance effect is zero because Amnesty Plus does not induce the reporting of the individually stable cartel 1 in this case. However, if $R_1 > F_2$, the reporting effect is strictly positive and Amnesty Plus increases the value of joint collusion.

Proposition 2 $(S_{12}^{AP}, S_{12}^{AP})$ is the best collusive equilibrium for a non-empty range of values of δ in the interval $[\tilde{\delta}_1, \tilde{\delta}_{12}]$ if and only if

$$R_2 < \bar{R}_2 \equiv \frac{\tilde{\delta}_{12}(\pi_1 - qF_1)}{1 - \tilde{\delta}_{12}(1 - q)} \tag{9}$$

$$R_1 > \underline{R}_1 \equiv F_2 + \frac{2\left(1 - \widetilde{\delta}_{12}(1-q)^2\right)}{q(1-q)} \left[2\pi_1 + 2\pi_2 - V_{12}(\widetilde{\delta}_{12})\right]$$
(10)

Proof. See appendix C.

Proposition 2 is central to our paper. It suggests that situations occur in which Amnesty Plus stabilizes the previously unstable cartel 2 and thereby increases the extent of collusion. If the fine in market 2 is small such that $F_2 < \bar{R}_2$, condition (9) always holds because of our assumption that $R_k \leq F_k$.¹⁶ Amnesty Plus is then anticompetitive if the AA overrewards applicants by granting a reduction R_1 in return for the reporting of cartel 2 that is too high. Condition (10) boils down to $R_1 > F_2$ if multimarket contact is procollusive, i.e. $\tilde{\delta}_{12} < \tilde{\delta}_2$.¹⁷ An agency that acts optimally would not agree to such a large discount. However, an agency that maximizes the number of convicted cartels rather than minimizing the number of cartels formed definitely has incentives to over-reward. As the number of cartels deterred is unobservable, an antitrust authority can only be assessed based on observable measures of performance such as the number of successfully prosecuted cartels. Maximal deterrence, though socially desirable, may therefore not be the primary objective of an antitrust authority (Harrington, 2010).

¹⁶If we suppose that there exists an increasing and continuous function g(.), verifying g(0)=0, such that $F_k \leq g(\pi_k)$, condition (9) always holds for a sufficiently small market 2.

¹⁷If $\tilde{\delta}_{12} < \tilde{\delta}_2$ then $V_{12}(\tilde{\delta}_{12}) = 2\pi_1 + 2\pi_2$ while, if $\tilde{\delta}_{12} = \tilde{\delta}_2$, this may not be true because it may happen that $V_{12}(\tilde{\delta}_{12}) < 2\pi_1 + 2\pi_2 \le V_{12}(\tilde{\delta}_{12})$.

Corollary 1 Amnesty Plus has no anticompetitive effect on cartel formation if the fine discount granted in market k in return for the disclosure of cartel -k does not exceed the fine that would have otherwise been imposed for the reported cartel -k, i.e. $R_k \leq F_{-k}$.

Proof. The second term of the right hand side in condition (10) of Proposition 2 is weakly positive. A fine reduction $R_k \leq F_{-k}$ violates this condition.

Corollary 1 suggests that the AA can avoid a procollusive effect of Amnesty Plus by fixing fine discounts such that $R_k \leq F_{-k}$. This result is crucial because it gives us a clear-cut policy rule which relies only on parameters set by the authority itself and is therefore easy to implement.

4.3.3 The Procompetitive Effect of Amnesty Plus

Consider now the interval $[\tilde{\delta}_{12}, 1]$ where (S_{12}, S_{12}) is the best collusive equilibrium in the EU. Amnesty Plus is procompetitive if it either prevents or defers the formation of cartel 2, i.e. if it makes either (S_1, S_1) or $(S_{1\to 2}, S_{1\to 2})$ the best collusive equilibrium. We divide this interval into two sub-intervals. We first examine $[\tilde{\delta}_{12}, \tilde{\delta}_2]$ where only cartel 1 is individually stable and Amnesty Plus can completely deter the formation of cartel 2, and second, we look at $[\tilde{\delta}_2, 1]$ where both cartels are individually stable and Amnesty Plus can only defer the formation of cartel 2. We focus on a situation where multimarket contact is procollusive such that $\tilde{\delta}_{12} < \tilde{\delta}_2$.

Amnesty Plus prevents the formation of cartel 2, i.e. it makes (S_1, S_1) the best collusive equilibrium, for at least some values of δ in the interval $[\tilde{\delta}_{12}, \tilde{\delta}_2]$ if and only if it lowers the value of joint collusion such that forming both cartels is no longer incentive compatible. Note that Amnesty Plus neutralizes the procollusive effect of multimarket contact in this case.

Proposition 3 Amnesty Plus prevents the formation of cartel 2 for a non-empty range of values of δ in the interval $[\tilde{\delta}_{12}, \tilde{\delta}_2]$ if and only if $R_2 > \bar{R}_2$ or $R_1 < F_2$:

- If $R_2 > \bar{R}_2$, (S_1, S_1) is the best collusive equilibrium for a non-empty range of values of δ in the interval $[\tilde{\delta}_{12}, \tilde{\delta}_2]$ for any $R_1 > 0$.

- If $R_2 \leq \bar{R}_2$, (S_1, S_1) is the best collusive equilibrium for a non-empty range of values of δ in the interval $[\tilde{\delta}_{12}, \tilde{\delta}_2]$ if and only if $R_1 < F_2$.

Proof. See appendix C.

Proposition 3 suggests that, for a high enough fine discount R_2 , Amnesty Plus causes desistance and thereby lowers the value of joint collusion such that forming both cartels is no longer incentive compatible for some values of δ . If, however, R_2 is too low to induce desistance, the reporting effect in market 2 must be strictly negative to break joint collusion. In the first case, cartel 1 is not robust for values of δ close enough to $\tilde{\delta}_{12}$. The value of joint collusion is given in (6) for the US and in (1) for the EU. Separating the difference of these expressions into the desistance and reporting effects, we get

$$\Delta_D = \frac{\pi_1 - qF_1}{1 - \delta(1 - q)^2} - \frac{\pi_1 - qF_1}{1 - \delta(1 - q)} < 0$$

$$\Delta_R = \frac{q(1 - q)(R_1 + R_2 - F_1 - F_2)}{2(1 - \delta(1 - q)^2)} \le 0$$
(11)

Amnesty Plus induces reporting in stage 4. Each firm's expected reporting benefits are $(R_1 + R_2 - F_1 - F_2)/2$ which must be weakly negative because $R_k \leq F_k$. Desistance from the stable cartel 1 after the detection of cartel 2 strictly lowers the value of joint collusion. Amnesty Plus therefore prevents the formation of cartel 2 for any fine discount R_1 if δ is close enough to $\tilde{\delta}_{12}$. In the second case, cartel 1 is robust for all δ in this interval. The desistance and reporting effects are given by the expressions in (8). Amnesty Plus can induce the reporting of only the unstable cartel 2 and therefore has no effect on desistance. However, if $R_1 < F_2$, Amnesty Plus lowers the value of joint collusion and may prevent the formation of cartel 2.

Amnesty Plus defers the formation of cartel 2, i.e. it makes $(S_{1\to 2}, S_{1\to 2})$ the best collusive equilibrium, for at least some values of δ in the interval $[\tilde{\delta}_2, 1]$ if and only if it lowers the value of jointly colluding such that either joint collusion is no more incentive compatible *or* is Pareto dominated by sequential collusion. We give here the intuitive arguments and provide the detailed formal analysis in appendix D.

Let us first sketch under what conditions joint collusion is no more incentive compatible. Loosely speaking, $(S_{12}^{AP}, S_{12}^{AP})$ is not an equilibrium for a non-empty range of values of δ in the interval $[\tilde{\delta}_2, 1]$ if R_1 and R_2 take intermediate values. On the one hand, at least one of the fine reductions must be high enough such that both firms report the remaining stable cartel in the reporting subgame of stage 4. On the other hand, the same fine reduction that induces the reporting must be sufficiently low such that the decrease in the expected fine does not compensate the firms for the enhanced desistance. If $(S_{12}^{AP}, S_{12}^{AP})$ is not an equilibrium, $(S_{1\to 2}, S_{1\to 2})$ is the best collusive equilibrium in this interval.

Let us now intuitively explain why the sequential equilibrium may Pareto dominate the joint equilibrium if the firms' discount factor is sufficiently close to 1. Amnesty Plus, if it induces both firms to report, erases future collusive profits in the remaining market. This is however not the case when firms collude sequentially. If the firms highly value *current* collusive profits and care less about the future, i.e. their δ is relatively low, they prefer to collude in both markets today and to incur the risk of being forced to globally compete in the future. If, however, the firms highly value *future* profits, they may be willing to sacrifice cartel profits today in return for a longer expected duration of collusion. $(S_{1\to 2}, S_{1\to 2})$ is then the best collusive equilibrium in this interval.

5 Extensions

5.1 Heterogenous Detection Probabilities

Amnesty Plus strategically links two markets. The direct consequence of this linkage is that Amnesty Plus may deploy its effects for parameter values where a standard leniency program cannot influence collusion at all. To see this, suppose that $q_1 > 0$ and $q_2 = 0$. Possible reasons for this difference may be that the AA concentrates on the discovery of big cartels or that consumers are more sensible to prices of a product with an important sales' volume and thus are more likely to complain to the authority about the prices in market 1. With $q_2 = 0$ a standard leniency program has no effect in market 2. This is however not true for Amnesty Plus. Amnesty Plus induces the reporting of the stable cartel 2 after the detection of cartel 1 if the size of the discount granted in market 1 is greater than the continuation value from colluding in market 2, which may happen even for $q_2 = 0$. Hence, provided that detection in market 1 occurs with a sufficiently high probability, Amnesty Plus may deter the formation of cartel 2 even for $q_2 = 0$.

5.2 More than two Firms

Consider $n \ge 2$ firms active on markets 1 and 2. Assume that if all the firms report the remaining cartel simultaneously in stage 4, each firm is first with probability $\frac{1}{n}$. As only the first informant is eligible for the fine discount under Amnesty Plus, a firm's expected payoff from reporting cartel -k, when everyone else does, is $\frac{1}{n}R_k - \frac{n-1}{n}F_{-k}$. We have $\frac{1}{n}[R_k - (n-1)F_{-k}] \le 0$ if and only if $R_k \le (n-1)F_{-k}$. Hence, to avoid a potential anticompetitive effect of Amnesty Plus, the AA would have to set the fine reductions such that $R_k \le (n-1)F_{-k}$.¹⁸ This constraint becomes slacker as the number of firms increases. Collusion however tends to be more important in highly concentrated markets, due to eased coordination and monitoring, than in markets where many small firms operate (Tirole, 1988). Moreover, if the AA wants to include a discount-setting rule in its amnesty plus policy that depends only on variables set, or directly observable, by itself then it should set up this rule to avoid the anticompetitive effect for any possible number of colluding firms. As the worst case scenario occurs for n = 2, the authority should adopt the rule $R_k \le F_{-k}$.

¹⁸Consider the case of collusive agreements which do not involve the same set of firms in both markets. Denote n_k the number of firms in cartel k and s the number of firms that participate in both cartels. If s = 0, Amnesty Plus has no effect. If $s \ge 1$, Amnesty Plus can increase the value of collusion for the firms involved in both cartels. But whenever this happens, Amnesty Plus also decreases the expected cartel profits for the firms colluding in one market only. To avoid an increase in the expected profits for any firm, it must hold that $R_k \le (s-1)F_{-k}$, which can be satisfied for strictly positive discounts only if s > 1. However, if we consider the weaker (and more relevant) requirement that Amnesty Plus should not increase the *total* value of each cartel then the discount-setting rule $R_k \le (n_{-k} - 1)F_{-k}$ is sufficient.

5.3 More than two Markets

Consider a set M of markets in which two identical firms interact. Denote $|M| = m \ge 2$ the number of markets. For a subset of markets $K \subseteq M$ denote Π_K the total profit each firm earns from collusion and F_K the total fine each firm pays if the AA detects the cartels in the subset K.¹⁹ For a subset of markets $L \subseteq M \setminus K$ let R_K^L be the fine discount the first firm gets under Amnesty Plus in return for reporting the cartels in subset L. Assume that $R_K^L \leq R_K^{L'}$ if $L \subseteq L'$.

Let us first define the strategies we consider under a leniency policy without Amnesty Plus. For any subset of markets $I \subseteq M$ denote s_I the following strategy over one period: collude in the subset I, neither deviate from the collusive agreements nor report. In particular, s_{\emptyset} consists of competing in all markets. We recursively define the strategies $S_{I,t}$ over the subgame starting from period t and denote $V_I(\delta)$ as each firm's expected payoff discounted to period twhen both firms play $S_{I,t}$.²⁰

 $S_{\emptyset,t}$: play s_{\emptyset} in period t and all subsequent periods.

If |I| = 1 then $S_{I,t}$ is the following strategy: play s_I in period t and any subsequent period as long as there is neither deviation from the collusive agreement nor reporting nor detection; if either deviation or reporting or detection occurs in period $t' \ge t$, play $S_{\emptyset,t'+1}$.

If $|I| \ge 2$ then $S_{I,t}$ is the following strategy: play s_I in period t and any subsequent period as long as there is neither deviation from the collusive agreements nor reporting nor detection; if detection of a subset of markets $J \subsetneq I$ occurs in some period $t' \ge t$ but neither deviation from any collusive agreement nor reporting, play $S_{L(I,J),t'+1}$ where $L(I,J) \subseteq I \smallsetminus J$ is such that $V_{L(I,J)}(\delta) \ge V_L(\delta)$ for any $L \subseteq I \searrow J$ if the set $R(I,J) = \{L \subseteq I \searrow J \mid V_L(\delta) \ge 2\Pi_L\}$ is not empty and $L(I,J) = \emptyset$ otherwise; if in some period $t'' \ge t$ reporting or deviation occurs, or all the cartels are detected, play $S_{\emptyset,t''+1}$.

Let us now define the strategies under a leniency policy with Amnesty Plus. For any subset of markets $I \subseteq M$, we recursively define the strategy s_I^{AP} over one period and the strategies $S_{I,t}^{AP}$ over the subgame starting from period t and note $V_I^{AP}(\delta)$ as each firm's expected payoff discounted to period t when both firms play $S_{I,t}^{AP}$. For any subset I such that $|I| \leq 1$, we define s_I^{AP} and $S_{I,t}^{AP}$ exactly as s_I and $S_{I,t}$. For any subset I such that $|I| \geq 2$, we define the strategies s_I^{AP} and $S_{I,t}^{AP}$ as follows:

 s_I^{AP} : collude in the subset I, neither deviate from the collusive agreements nor report; if detection of a subset of markets $J \subsetneq I$ occurs but neither deviation nor reporting, then report all the remaining cartels under Amnesty Plus if the set $R^{AP}(I, J) = \{L \subseteq I \setminus J / V_L^{AP}(\delta) \geq I \}$

¹⁹In this extension, we allow for substitutability and complementarity between markets, and thus, Π_K need not be equal to the sum of the profits in each of the markets in subset K. For an analysis of multimarket collusion with demand linkages see Choi and Gerlach (2009b).

²⁰We use |I| as a recursive variable: the definition of the collusive strategies over $I \neq \emptyset$ builds on the definitions of the collusive strategies over the sets whose cardinality is strictly less than |I|.

 $\max(2\Pi_L, \frac{R_J^{\Gamma_N J}}{\delta})$ is empty; otherwise, do not report any of the remaining cartels under Amnesty Plus.

 $S_{I,t}^{AP}$: play s_{I}^{AP} in period t and in any subsequent period as long as there is neither deviation from the collusive agreements nor detection; if in some period $t' \geq t$ detection of a subset $J \subsetneq I$ occurs but neither deviation nor reporting, then play $S_{L(I,J),t'+1}^{AP}$ where $L^{AP}(I,J) \subseteq I \smallsetminus J$ is such that $V_{L^{AP}(I,J)}^{AP}(\delta) \geq V_{L}^{AP}(\delta)$ for any $L \subseteq I \smallsetminus J$ if the set $R^{AP}(I,J)$ is not empty and $L^{AP}(I,J) = \emptyset$ otherwise; if in some period $t'' \geq t$ reporting or deviation occurs, or all the carteles are detected, play $S_{\emptyset,t''+1}^{AP}$.

The following proposition gives the natural extension of the discount-setting rule we suggest in Corollary 1 for m = 2 to the general case with $m \ge 2$ markets.

Proposition 4 If for all $K \subsetneq M$ and $L \subseteq M \setminus K$ it holds that:

$$R_K^L \le F_L$$

then for any $I \subseteq M$

$$V_I^{AP}(\delta) \le V_I(\delta)$$

which rules out any anticompetitive effect of Amnesty Plus on cartel formation.

Proof. See appendix C.

5.4 Partial Collusion

We have assumed that the firms collude at the monopoly price. Indeed, in our model, if collusion is incentive compatible in both markets, the firms have no incentives to collude at a price lower than the monopoly price because both cartel stability and expected collusive profits increase in industry profits. Partial collusion may however be optimal if $\delta \in [\tilde{\delta}_1, \tilde{\delta}_2[$. To see this, suppose that, if the firms collude, they can fix a price $p_k \in]c_k, p_k^m]$ where c_k is the marginal cost of production and p_k^m the monopoly price. We assume that a firm's profit function $\pi_k(p_k)$, when both firms choose p_k , is continuous, quasi-concave and reaches its maximum at $p_k^m < +\infty$. We denote $\bar{p}_k \in]c_k, p_k^m[$ the unique solution to the equation $\pi_k(p_k) - qF_k = 0.$

Consider first the situation for $\delta \in [\tilde{\delta}_1, \tilde{\delta}_2]$ under the EU Leniency Program. If both firms collude and fix a price $p_1 \in]c_1, p_1^m]$ in market 1 and a price $p_2 \in]c_2, p_2^m]$ in market 2, each firm's discounted expected total profit is

$$V_{12}(p_1, p_2, \delta) = \frac{\pi_1(p_1) - qF_1}{1 - \delta(1 - q)} + \frac{\pi_2(p_2) - qF_2}{1 - \delta(1 - q)^2}$$

Joint collusion at prices (p_1, p_2) is sustainable if and only if the participation constraint $p_k \ge \bar{p}_k$ holds for k = 1, 2 and $V_{12}(p_1, p_2, \delta) \ge 2\pi_1 + 2\pi_2$ which is equivalent to

$$\pi_1(p_1)\left(\frac{1}{1-\delta(1-q)}-2\right) - \frac{qF_1}{1-\delta(1-q)} + (12) + \pi_2(p_2)\left(\frac{1}{1-\delta(1-q)^2}-2\right) - \frac{qF_2}{1-\delta(1-q)^2} \ge 0$$

Optimal joint collusion at prices $(p_1, p_2) \in [\bar{p}_1, p_1^m] \times [\bar{p}_2, p_2^m]$ maximizes $V_{12}(p_1, p_2, \delta)$ subject to the incentive compatibility constraint given by (12). For all $\delta \in [\tilde{\delta}_1, \tilde{\delta}_2]$ the expression $(\frac{1}{1-\delta(1-q)}-2)$ is positive because $\tilde{\delta}_1 > \frac{1}{2(1-q)}$. The left hand side (LHS) of (12) therefore increases in p_1 which implies that full collusion in market 1, i.e. $p_1 = p_1^m$, is always optimal when jointly colluding. This, however, need not be true for market 2 because the LHS of the inequality in (12) decreases in p_2 if $\delta < \bar{\delta} = \frac{1}{2(1-q)^2}$. We distinguish three cases:

a- If $\overline{\delta} \leq \widetilde{\delta}_1$, equivalently $\frac{F_1}{\pi_1^m} \geq \frac{1}{1-q}$, the joint stability of the cartels increases with p_2 . Full collusion in market 2 is thus optimal when jointly colluding, and Proposition 1 remains valid. b- If $\widetilde{\delta}_1 < \overline{\delta} < \widetilde{\delta}_2$, equivalently $\frac{F_1}{\pi_1^m} < \frac{1}{1-q} < \frac{F_2}{\pi_2^m}$, the joint stability of the cartels strictly decreases in p_2 for $\delta \in]\widetilde{\delta}_1, \overline{\delta}[$, is independent of p_2 for $\delta = \overline{\delta}$, and strictly increases in p_2 for $\delta \in]\overline{\delta}, \widetilde{\delta}_2[$. Partial collusion arises (for some values of δ) in optimal joint collusion if the condition in (12) holds for $(p_1, p_2, \delta) = (p_1^m, \overline{p}_2, \overline{\delta})$ or, equivalently, if $F_2 \leq \frac{\pi_1^m - (1-q)F_1}{1-2q}$. If this inequality does not hold, full collusion is optimal, and Proposition 1 remains valid. If it holds, we can show that two thresholds $\delta_{12,p}$ and $\delta_{12,f}$ exist which satisfy $\widetilde{\delta}_1 < \delta_{12,p} < \delta_{12,f} < \widetilde{\delta}_2$ such that the price pair $(p_1(\delta), p_2(\delta))$ corresponding to optimal collusion contains $p_1(\delta) = p_1^m$ for all $\delta \in]\widetilde{\delta}_1, \widetilde{\delta}_2[$ and $p_2(\delta) \in]\overline{p}_2, p_2^m[$ strictly increasing over $]\delta_{12,p}, \delta_{12,p,f}[$ (partial collusion in market 2) and $p_2(\delta) = p_2^m$ for all $\delta \in [\delta_{12,f}, \widetilde{\delta}_2[$ (full collusion in market 2). Note that multimarket contact makes collusion easier by inducing either partial *or* full collusion in market 2.

c-If $\bar{\delta} \geq \tilde{\delta}_2$, equivalently $\frac{F_2}{\pi_2^m} \geq \frac{1}{1-q}$, the joint stability of the cartels strictly decreases in p_2 for all $\delta \in [\tilde{\delta}_1, \tilde{\delta}_2]$. We have two different situations: either joint collusion is not incentive compatible, and the firms compete in market 2, or collusion in market 2, partial at worst and full at best, is incentive compatible together with full collusion in market 1.

Consider now the situation under the US Leniency Program. We illustrate how the possibility for partial collusion affects our results by examining the case where $\tilde{\delta}_1 < \bar{\delta} < \tilde{\delta}_2$ and $F_2 \leq \frac{\pi_1^m - (1-q)F_1}{1-2q}$ such that partial collusion may arise in equilibrium. Suppose that R_2 is small enough such that cartel 1 is robust to a detection of cartel 2 over the entire interval. We can show that two thresholds $\delta_{12,p}^{AP}$ and $\delta_{12,f}^{AP}$ exist which satisfy $\tilde{\delta}_1 \leq \delta_{12,p}^{AP} \leq \delta_{12,f} \leq \tilde{\delta}_2$ such that the price pair $(p_1^{AP}(\delta), p_2^{AP}(\delta))$ corresponding to optimal collusion contains $p_1^{AP}(\delta) = p_1^m$ for all $\delta \in]\tilde{\delta}_1, \tilde{\delta}_2[$ and $p_2^{AP}(\delta)$ as a function over $]\tilde{\delta}_1, \tilde{\delta}_2[$ such that: $p_2^{AP}(\delta) = c_2$ for all $\delta \in]\tilde{\delta}_1, \delta_{12,p}^{AP}]$ (no

collusion in market 2), $p_2^{AP}(\delta) \in]\bar{p}_2, p_2^m[$ strictly increasing over $]\delta_{12,p}^{AP}, \delta_{12,f}^{AP}[$ (partial collusion in market 2) and $p_2^{AP}(\delta) = p_2^m$ for all $\delta \in [\delta_{12,f}^{AP}, \tilde{\delta}_2[$ (full collusion in market 2).²¹ Furthermore, we can establish that for all $p_2 \in]\bar{p}_2, p_2^m]$, we have $V_{12}(p_1, p_2, \delta) < V_{12}^{AP}(p_1, p_2, \delta, R_1, R_2)$ if and only if $R_1 > F_2$. Using the former result, it can be shown that, if $R_1 > F_2$, Amnesty Plus is anticompetitive in the sense that $\delta_{12,p}^{AP} < \delta_{12,p}, \ \delta_{12,f}^{AP} < \delta_{12,f}$ and for all $\delta \in]\tilde{\delta}_1, \tilde{\delta}_2[$ $p_2^{AP}(\delta) \ge p_2(\delta)$. If $R_1 < F_2$, the reverse is true, and Amnesty Plus is procompetitive.

6 Conclusion

This paper examines the effect of Amnesty Plus on the firms' incentives to form cartels. The firms repeatedly interact in two markets of different size and can use their multimarket contact to sustain collusion. While US success stories suggest that Amnesty Plus weakens cartel stability, our analysis shows that this is not correct in general.

We find that Amnesty Plus may increase cartel deterrence provided that the procollusive effect is avoided. The central implication of our analysis is that an antitrust authority can easily prevent this effect by adhering to the following rule: Set the absolute size of the fine discount granted in one market equal or below the fine the successful Amnesty Plus applicant would have incurred in the other market. We argue that this rule must be explicitly incorporated in the Amnesty Plus policy. One important reason is that, on top of pursuing a social welfare objective, an antitrust authority cares about performance. If performance is measured by the number of cartels dismantled, the antitrust authority may want to offer high discounts ex post, which may come with undesirable effects on deterrence ex ante.

Appendix

A The Sequential Equilibrium

We show that $(S_{2\to 1}, S_{2\to 1})$ can never be the best collusive equilibrium if q is sufficiently small. We proceed in 2 steps. In step 1, we show under which conditions $(S_{2\to 1}, S_{2\to 1})$ is an equilibrium. In step 2, we demonstrate that, when $(S_{2\to 1}, S_{2\to 1})$ is an equilibrium, there is always another equilibrium that Pareto dominates the latter if q is sufficiently small.

Step 1. The expected payoff associated with $(S_{2\to 1}, S_{2\to 1})$ is

$$V_{2\to 1}(\delta) = V_2(\delta) + q \frac{\delta}{1 - \delta(1 - q)} V_1(\delta)$$

 $(S_{2\to 1}, S_{2\to 1})$ is an equilibrium if and only if cartel 1 is individually stable, i.e. $\delta \geq \tilde{\delta}_1$, and $V_{2\to 1}(\delta) \geq 2\pi_2$. These two conditions hold if and only if $\delta \geq \max(\tilde{\delta}_1, \tilde{\delta}_{2\to 1})$ where $\tilde{\delta}_{2\to 1}$ is such that $V_{2\to 1}(\tilde{\delta}_{2\to 1}) = 2\pi_2$.

²¹We cannot exclude that one or even two of these intervals are empty.

Step 2. Note first that for $\delta \geq \tilde{\delta}_2$, $V_{1\to 2}(\delta) > V_{2\to 1}(\delta)$ if and only if $V_1(\delta) > V_2(\delta)$ which always holds because of our assumptions $\pi_1 > \pi_2$ and $\frac{F_2}{\pi_2} > \frac{F_1}{\pi_1}$. Hence, $(S_{1\to 2}, S_{1\to 2})$ always strictly Pareto dominates $(S_{2\to 1}, S_{2\to 1})$ for any δ in this range. Consider now the interval $[\max(\tilde{\delta}_1, \tilde{\delta}_{2\to 1}), \tilde{\delta}_2]$. The equilibrium (S_1, S_1) strictly Pareto dominates $(S_{2\to 1}, S_{2\to 1})$ if and only if $V_1(\delta) > V_{2\to 1}(\delta)$. We can write this inequality as

$$\frac{\pi_2 - qF_2}{\pi_1 - qF_1} < \frac{1 - \delta}{1 - \delta(1 - q)}$$

As the right hand side (RHS) of the above inequality is decreasing in δ , this condition holds for all $\delta \in [\max(\tilde{\delta}_1, \tilde{\delta}_{2 \to 1}), \tilde{\delta}_2]$ if and only if it holds for $\delta = \tilde{\delta}_2$, i.e.

$$\frac{(1-q)(\pi_2 - qF_2)^2}{(\pi_1 - qF_1)(\pi_2(1-2q) - qF_2)} < 1$$

As the LHS of the above inequality is continuous in q and tends to $\frac{\pi_2}{\pi_1} < 1$ when $q \to 0$, there exists a threshold $\tilde{q} > 0$ such that $V_1(\delta) > V_{2\to 1}(\delta)$ for all $q \in [0, \tilde{q}]$ and all $\delta \in [\max(\tilde{\delta}_1, \tilde{\delta}_{2\to 1}), \tilde{\delta}_2]$. Hence, if $q < \tilde{q}$, (S_1, S_1) always strictly Pareto dominates $(S_{2\to 1}, S_{2\to 1})$ for any δ in this range.

B The Effect of Multimarket Contact

Multimarket contact is procollusive, i.e. $\tilde{\delta}_{12} < \tilde{\delta}_2$, if and only if

$$V_{12}(\widetilde{\delta}_2^-) > 2\pi_1 + 2\pi_2$$

which we rewrite as

$$\pi_1 > \frac{F_1}{F_2} \pi_2 + \frac{\left(\frac{\pi_2}{F_2} - q\right) (\pi_2 + qF_2)}{1 + q - q(1 - q)\frac{F_2}{\pi_2}} \tag{B-1}$$

This condition holds only if the markets are sufficiently different in terms of profitability. To see this, we use our assumptions on the relative size of the fines and of the fine-profit ratios and write

$$\frac{\pi_2}{\pi_1}F_1 < F_2 \le F_1$$

If $\pi_2 \to \pi_1$, the above inequality implies that $F_2 \to F_1$, and the RHS of the inequality in (B-1) converges to

$$\pi_1 > \pi_1 + \frac{\left(\frac{\pi_1}{F_1} - q\right)(\pi_1 + qF_1)}{1 + q - q(1 - q)\frac{F_1}{\pi_1}} \tag{B-2}$$

Since $(1+q) - q(1-q)\frac{F_1}{\pi_1} > (1+q) - q(1+q)\frac{F_1}{\pi_1} = (1+q)\frac{\pi_1 - qF_1}{\pi_1}$ and $\pi_1 - qF_1 > 0$, the second expression in the RHS of the inequality in (B-2) is strictly positive. Hence, the condition in (B-1) is not satisfied, and multimarket contact cannot help to stabilize an individually unstable cartel if markets 1 and 2 are too close in terms of profitability. In this case, multimarket contact is neutral, i.e. $\tilde{\delta}_{12} = \tilde{\delta}_2$. However, if market 1 is sufficiently more profitable than market 2, in the sense that the condition in (B-1) holds, then multimarket contact is procollusive, i.e. $\tilde{\delta}_{12} < \tilde{\delta}_2$.

This finding contrasts with the irrelevance result in Bernheim and Whinston (1990). In our model, the latter takes the form of the special case q = 0 in which multimarket contact cannot affect the firms' ability to collude as the individual stability constraints are identical for both markets. If the presence of an antitrust authority creates an asymmetry between the markets in terms of collusion sustainability, due to e.g. heterogenous detection probabilities or fine-profit ratios, then multimarket contact may ease collusion.

C Proofs

Proof of Proposition 1. We proceed in three steps. In step 1, we determine the range of discount factors for which (S_{12}, S_{12}) is an equilibrium. In step 2, we show that the sequential collusion equilibrium can never be the best collusive equilibrium of the game. In step 3, we conclude.

Step 1. The value of joint collusion $V_{12}(\delta)$ is given by:

$$V_{12}(\delta) = \begin{cases} \frac{\pi_1 - qF_1}{1 - \delta(1 - q)^2} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)^2} & \text{if} & \delta < \widetilde{\delta}_1 \\ \frac{\pi_1 - qF_1}{1 - \delta(1 - q)} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)^2} & \text{if} & \widetilde{\delta}_1 \le \delta < \widetilde{\delta}_2 \\ \frac{\pi_1 - qF_1}{1 - \delta(1 - q)} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)} & \text{if} & \widetilde{\delta}_2 \le \delta \end{cases}$$
(C-1)

If $\delta < \widetilde{\delta}_1$, (S_{12}, S_{12}) is not an equilibrium because $V_{12}(\delta) \le V_1(\delta) + V_2(\delta) < 2\pi_1 + 2\pi_2$. If $\delta \ge \widetilde{\delta}_2$, (S_{12}, S_{12}) is an equilibrium because $V_{12}(\delta) = V_1(\delta) + V_2(\delta) \ge 2\pi_1 + 2\pi_2$. Consider now $\delta \in [\widetilde{\delta}_1, \widetilde{\delta}_2]$. Note first that

$$V_{12}(\widetilde{\delta}_1) = 2\pi_1 + \frac{\pi_2 - qF_2}{1 - \widetilde{\delta}_1(1 - q)^2} < 2\pi_1 + V_2(\widetilde{\delta}_1) < 2\pi_1 + 2\pi_2$$

It follows from the continuity and strict monotonicity of $V_{12}(\delta)$ that $V_{12}(\delta) < 2\pi_1 + 2\pi_2$ for any $\delta \in]\tilde{\delta}_1, \tilde{\delta}_2[$ if $V_{12}(\tilde{\delta}_2^-) \leq 2\pi_1 + 2\pi_1$. However, if $V_{12}(\tilde{\delta}_2^-) > 2\pi_1 + 2\pi_1$ then a threshold exists in the interval $]\tilde{\delta}_1, \tilde{\delta}_2[$ such that $V_{12}(\delta) \geq 2\pi_1 + 2\pi_2$ for the discount factor values above this threshold and $V_{12}(\delta) < 2\pi_1 + 2\pi_2$ for values below. Thus, a (unique) threshold $\tilde{\delta}_{12} \in]\tilde{\delta}_1, \tilde{\delta}_2]$ always exists such that (S_{12}, S_{12}) is an equilibrium for $\delta \geq \tilde{\delta}_{12}$ and (S_{12}, S_{12}) is not an equilibrium for $\delta < \tilde{\delta}_{12}$. If $V_{12}(\tilde{\delta}_2^-) > 2\pi_1 + 2\pi_1$ then $\tilde{\delta}_{12} < \tilde{\delta}_2$. Otherwise $\tilde{\delta}_{12} = \tilde{\delta}_2$. Step 2. We show that whenever $(S_{1\to 2}, S_{1\to 2})$ is an equilibrium, it is strictly dominated by the equilibrium (S_{12}, S_{12}) and thus cannot be the best collusive equilibrium. We know that $(S_{1\to 2}, S_{1\to 2})$ is an equilibrium if and only if $\delta \geq \tilde{\delta}_2$. However, for $\delta \geq \tilde{\delta}_2$, the strategy pair (S_{12}, S_{12}) constitutes an equilibrium as well and yields a collusive payoff of $V_{12}(\delta) =$ $V_1(\delta) + V_2(\delta)$ (Step 1). Since $V_1(\delta) + V_2(\delta) > V_1(\delta) + q \frac{\delta}{1-\delta(1-q)}V_2(\delta)$, $(S_{1\to 2}, S_{1\to 2})$ can never be the best collusive equilibrium. Notice that we exclude $\delta = 1$ because (S_{12}, S_{12}) and $(S_{1\to 2}, S_{1\to 2})$ yield the same payoff in that case, and both are best collusive equilibria.

Step 3. We conclude that:

- If $\delta < \widetilde{\delta}_1$, neither (S_1, S_1) nor $(S_{1 \to 2}, S_{1 \to 2})$ nor (S_{12}, S_{12}) is an equilibrium. - If $\widetilde{\delta}_1 \le \delta < \widetilde{\delta}_{12}$, the only collusive equilibrium is (S_1, S_1) . - If $\delta \ge \widetilde{\delta}_{12}$ then (S_{12}, S_{12}) is an equilibrium and yields a higher payoff than (S_1, S_1) and $(S_{1 \to 2}, S_{1 \to 2})$, whenever it is an equilibrium.

Proof of Lemma 1. Consider first $\delta < \hat{\delta}_1(R_2)$ where cartel 1 is not robust to a detection of cartel 2. The value of joint collusion is

$$V_{12}^{AP}(\delta, R_1, R_2) = \frac{\pi_1 - qF_1}{1 - \delta(1 - q)^2} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)^2} + \frac{q(1 - q)(R_1 + R_2 - F_1 - F_2)}{2(1 - \delta(1 - q)^2)}$$

Since $R_k \leq F_k$, we know from (C-1) in Proof of Proposition 1 that $V_{12}^{AP}(\delta, R_1, R_2) \leq V_{12}(\delta)$. Amnesty Plus cannot increase the value of joint collusion.

Consider now $\delta \geq \hat{\delta}_1(R_2)$. The value of joint collusion is

$$V_{12}^{AP}\left(\delta, R_1, R_2\right) = \underbrace{\frac{\pi_1 - qF_1}{1 - \delta\left(1 - q\right)} + \frac{\pi_2 - qF_2}{1 - \delta\left(1 - q\right)^2}}_{V_{12}(\delta)} + \frac{q\left(1 - q\right)\left(R_1 - F_2\right)}{2\left(1 - \delta\left(1 - q\right)^2\right)}$$

Amnesty Plus therefore increases the value of jointly colluding, i.e. $V_{12}^{AP}(\delta, R_1, R_2) > V_{12}(\delta)$, if and only if $R_1 > F_2$

Proof of Proposition 2. The value of joint collusion $V_{12}^{AP}(\delta, R_1, R_2)$ is strictly increasing and right-continuous in δ over $[\tilde{\delta}_1, \tilde{\delta}_{12}]$. Therefore, a necessary and sufficient condition for $(S_{12}^{AP}, S_{12}^{AP})$ to be the best collusive equilibrium over a non-empty sub-interval of $[\tilde{\delta}_1, \tilde{\delta}_{12}]$ is that $V_{12}^{AP}(\tilde{\delta}_{12}^-, R_1, R_2) > 2\pi_1 + 2\pi_2$. The Proof of Lemma 1 shows that if $\tilde{\delta}_{12} \leq \hat{\delta}_1(R_2)$ then $V_{12}^{AP}(\tilde{\delta}_{12}^-, R_1, R_2) < V_{12}(\tilde{\delta}_{12}^-) \leq V_{12}(\tilde{\delta}_{12}) = 2\pi_1 + 2\pi_2$. However, if $\hat{\delta}_1(R_2) < \tilde{\delta}_{12}$ (equivalent to the condition in (9)) then $V_{12}^{AP}(\tilde{\delta}_{12}^-, R_1, R_2) = V_{12}(\tilde{\delta}_{12}^-) + \frac{q(1-q)(R_1-F_2)}{2(1-\tilde{\delta}_{12}(1-q)^2)} > 2\pi_1 + 2\pi_2$ if and only if the condition in (10) holds.

Proof of Proposition 3. Assume first that $R_2 > \bar{R}_2$, which implies that $\hat{\delta}_1(R_2) > \tilde{\delta}_{12}$. For $\delta \in [\tilde{\delta}_{12}, \hat{\delta}_1(R_2)]$ cartel 1 is not robust and the Proof of Lemma 1 shows that $V_{12}^{AP}(\tilde{\delta}_{12}, R_1, R_2) < V_{12}(\tilde{\delta}_{12}) = 2\pi_1 + 2\pi_2$. Hence, for any $\delta \in [\tilde{\delta}_{12}, \hat{\delta}_1(R_2)]$ sufficiently close

to $\tilde{\delta}_{12}$, it must hold that $V_{12}^{AP}(\delta, R_1, R_2) < 2\pi_1 + 2\pi_2$ which implies that (S_{12}, S_{12}) is not an equilibrium and that (S_1, S_1) is then the best collusive equilibrium.

Assume now that $R_2 \leq \overline{R}_2$, which implies that $\widehat{\delta}_1(R_2) \leq \widetilde{\delta}_{12}$. For any $\delta \in [\widetilde{\delta}_{12}, \widetilde{\delta}_2[$, cartel 1 is then robust, and, consequently,

$$V_{12}^{AP}(\delta, R_1, R_2) = \frac{\pi_1 - qF_1}{1 - \delta(1 - q)} + \frac{\pi_2 - qF_2}{1 - \delta(1 - q)^2} + \frac{q(1 - q)(R_1 - F_2)}{2(1 - \delta(1 - q)^2)}$$

= $V_1(\delta) + \frac{2(\pi_2 - qF_2) + q(1 - q)(R_1 - F_2)}{2(1 - \delta(1 - q)^2)}$

 (S_1, S_1) is the best collusive equilibrium for a given δ if and only if either

$$V_{12}^{AP}(\delta, R_1, R_2) < 2\pi_1 + 2\pi_2 \tag{C-2}$$

or

$$V_{12}^{AP}(\delta, R_1, R_2) < V_1(\delta)$$
(C-3)

As we initially assumed that $q \leq \frac{\pi_2}{2\pi_2 + F_2}$ which implies that the numerator of $V_{12}^{AP}(\delta, R_1, R_2) - V_1(\delta) = \frac{2(\pi_2 - qF_2) + q(1-q)(R_1 - F_2)}{2(1-\delta(1-q)^2)}$ is strictly positive, and because $V_1(\delta)$ is increasing in δ , $V_{12}^{AP}(\delta, R_1, R_2)$ is also increasing in δ over $[\tilde{\delta}_{12}, \tilde{\delta}_2[$. Hence, (S_1, S_1) is the best collusive equilibrium for a non-empty range of values of δ in $[\tilde{\delta}_{12}, \tilde{\delta}_2[$ if and only if at least one of the conditions (C-2) and (C-3) holds for $\delta = \tilde{\delta}_{12}$, i.e.

$$V_{12}^{AP}(\widetilde{\delta}_{12}, R_1, R_2) < \max\left(2\pi_1 + 2\pi_2, V_1\left(\widetilde{\delta}_{12}\right)\right)$$

which amounts to

$$V_{12}(\widetilde{\delta}_{12}) + \frac{q(1-q)(R_1 - F_2)}{2(1 - \delta(1-q)^2)} < \max\left(2\pi_1 + 2\pi_2, V_1\left(\widetilde{\delta}_{12}\right)\right)$$

Since $V_1(\tilde{\delta}_{12}) < V_{12}(\tilde{\delta}_{12}) = 2\pi_1 + 2\pi_2$ the latter condition can be rewritten as $\frac{q(1-q)(R_1-F_2)}{2(1-\delta(1-q)^2)} < 0$ which is the same as $R_1 < F_2$

Proof of Proposition 4. Assume that fine discounts under Amnesty Plus are such that $R_K^L \leq F_L$ for all $K \subsetneq M$ and $L \subseteq M \setminus K$. For every $i \in \{2, 3, ..., m\}$, denote $M_i = \{I \subseteq M \text{ such that } |I| \leq i\}$. Let us prove by recursive induction on i that, for any $i \in \{2, 3, ..., m\}$, $V_I^{AP}(\delta) \leq V_I(\delta)$ for all $I \in M_i$.

For i = 2, the result is readily derived from our main analysis. Consider any $i \ge 3$ and assume that $V_I^{AP}(\delta) \le V_I(\delta)$ for all $I \in M_{i-1}$. To complete the proof, we need to show that the latter inequality also holds for any $I \in M_i$. To do so, it is sufficient to establish that the inequality is true for any subset I of i markets, i.e. such that |I| = i. Consider such a subset. $V_{I}(\delta)$ is recursively defined as:

$$V_{I}(\delta) = \Pi_{I} + (1-q)^{i} \, \delta V_{I}(\delta) + \sum_{\substack{J \subsetneq I \\ J \neq \varnothing}} q^{|J|} \, (1-q)^{i-|J|} \, [-F_{J} + Y(I,J)] - q^{i} F_{I}$$

where Y(I, J) = 0 if $R(I, J) = \emptyset$ and $Y(I, J) = \delta V_{L(I,J)}(\delta)$ otherwise, which yields:

$$V_{I}(\delta) = \frac{1}{1 - (1 - q)^{i} \delta} \left[\prod_{\substack{I \neq \emptyset \\ J \neq \emptyset}} q^{|J|} (1 - q)^{i - |J|} [-F_{J} + Y(I, J)] - q^{i} F_{I} \right]$$

 $V_I^{AP}(\delta)$ is recursively defined as:

$$V_{I}^{AP}(\delta) = \Pi_{I} + (1-q)^{i} \,\delta V_{I}^{AP}(\delta) + \sum_{\substack{J \subsetneq I \\ J \neq \varnothing}} q^{|J|} \,(1-q)^{i-|J|} \left[-F_{J} + Y^{AP}(I,J) \right] - q^{i} F_{I}$$

where $Y^{AP}(I,J) = \frac{1}{2}(R_J^{I\setminus J} - F_J^{I\setminus J})$ if $R^{AP}(I,J) = \emptyset$ and $Y^{AP}(I,J) = \delta V_{L^{AP}(I,J)}^{AP}(\delta)$ otherwise, which yields

$$V_{I}^{AP}(\delta) = \frac{1}{1 - (1 - q)^{i} \delta} \left[\Pi_{I} + \sum_{\substack{J \subsetneq I \\ J \neq \varnothing}} q^{|J|} (1 - q)^{i - |J|} \left[-F_{J} + Y^{AP}(I, J) \right] - q^{i} F_{I} \right]$$

Let us show that for any non-empty set $J \subsetneq I$, it holds that $Y^{AP}(I, J) \leq Y(I, J)$ which is a sufficient condition for the inequality $V_I^{AP}(\delta) \leq V_I(\delta)$ to hold.

Assume first that J is such that $R(I, J) = \emptyset$, i.e. for any $L \subseteq I \setminus J$, it holds that $V_L(\delta) < 2\Pi_L$. Since any $L \subseteq I \setminus J$ belongs to M_{i-1} , we have: $V_L^{AP}(\delta) \leq V_L(\delta) < 2\Pi_L \leq \max(2\Pi_L, \frac{R_J^{I \setminus J}}{\delta})$. Therefore, $R^{AP}(I, J) = \emptyset$. Thus, in this case, we get $Y(I, J) = Y^{AP}(I, J) = 0$.

Assume now that J is such that $R(I, J) \neq \emptyset$. If $R^{AP}(I, J) = \emptyset$ then $Y^{AP}(I, J) = \frac{1}{2}(R_J^{I\setminus J} - F_J^{I\setminus J}) \leq 0 \leq \delta V_{L(I,J)}(\delta) = Y(I,J)$. If $R^{AP}(I,J) \neq \emptyset$ then by definition of L(I,J), we have $V_{L^{AP}(I,J)}(\delta) \leq V_{L(I,J)}(\delta)$ and since $L^{AP}(I,J)$ belongs to M_{i-1} , we also have $V_{L^{AP}(I,J)}^{AP}(\delta) \leq V_{L(I,J)}(\delta)$. Combining the latter two inequalities we obtain $V_{L^{AP}(I,J)}^{AP}(\delta) \leq V_{L(I,J)}(\delta)$, which implies that $Y^{AP}(I,J) \leq Y(I,J)$.

We can conclude that $Y^{AP}(I,J) \leq Y(I,J)$ holds for any non-empty set $J \subsetneq I$, which implies that $V_I^{AP}(\delta) \leq V_I(\delta)$.

D The Procompetitive Effect of Amnesty Plus if $\delta \in [\widetilde{\delta}_2, 1[$

Amnesty Plus defers the formation of cartel 2, i.e. it makes $(S_{1\to 2}, S_{1\to 2})$ the best collusive equilibrium, for at least some values of δ in $[\tilde{\delta}_2, 1]$ if and only if it lowers the value of jointly colluding such that either joint collusion is no more incentive compatible or is Pareto dominated by sequential collusion. First, we show for which specific values of R_1 and R_2 , the strategy pair $(S_{12}^{AP}, S_{12}^{AP})$ cannot be an equilibrium for at least some values in the interval, and, second, we provide conditions under which $(S_{1\to 2}, S_{1\to 2})$ Pareto dominates $(S_{12}^{AP}, S_{12}^{AP})$.

The present discounted expected payoff $V_{12}^{AP}(\delta, R_1, R_2)$, each firm gets when they both play the strategy S_{12}^{AP} , is right-continuous and strictly increasing in δ over the interval $[\tilde{\delta}_2, 1[$. Hence, $(S_{12}^{AP}, S_{12}^{AP})$ is not an equilibrium for a non-empty range of values of δ in $[\tilde{\delta}_2, 1[$ if and only if

$$V_{12}^{AP}(\tilde{\delta}_2, R_1, R_2) < 2\pi_1 + 2\pi_2$$
 (D-4)

The value $V_{12}^{AP}(\tilde{\delta}_2, R_1, R_2)$ depends on the equilibrium payoff in the reporting subgame of stage 4. We therefore examine the condition in (D-4) for each of the four possible scenarios that arise from the comparison of the individual stability threshold $\tilde{\delta}_2$ and the robustness thresholds:

a- If $\tilde{\delta}_2 \geq \hat{\delta}_1(R_2)$ and $\tilde{\delta}_2 \geq \hat{\delta}_2(R_1)$, both cartels are individually stable and robust for $\delta = \tilde{\delta}_2$ and the value of joint collusion is equal to

$$V_{12}^{AP}(\widetilde{\delta}_2, R_1, R_2) = \frac{\pi_1 - qF_1}{1 - \widetilde{\delta}_2(1 - q)} + \frac{\pi_2 - qF_2}{1 - \widetilde{\delta}_2(1 - q)} > 2\pi_1 + 2\pi_2$$

It is straightforward that in this case the condition in (D-4) cannot hold. The fine reductions R_1 and R_2 are both too small to trigger reporting in the reporting subgame. Amnesty Plus has no effect and $V_{12}^{AP}(\tilde{\delta}_2, R_1, R_2) = V_{12}(\tilde{\delta}_2)$.

b- If $\tilde{\delta}_2 \geq \hat{\delta}_1(R_2)$ and $\tilde{\delta}_2 < \hat{\delta}_2(R_1)$, cartel 1 is individually stable and robust whereas cartel 2 is stable but not robust for $\delta = \tilde{\delta}_2$. The value of joint collusion is equal to

$$V_{12}^{AP}(\widetilde{\delta}_2, R_1, R_2) = \frac{\pi_1 - qF_1}{1 - \widetilde{\delta}_2(1 - q)} + \frac{\pi_2 - qF_2}{1 - \widetilde{\delta}_2(1 - q)^2} + \frac{q(1 - q)(R_1 - F_2)}{2\left(1 - \widetilde{\delta}_2(1 - q)^2\right)}$$

We can thus rewrite condition (D-4) as

$$R_1 < F_2 + \frac{2\left(1 - \widetilde{\delta}_2(1-q)^2\right)}{q(1-q)} \left(2\pi_1 + 2\pi_2 - V_1(\widetilde{\delta}_2) - \frac{\pi_2 - qF_2}{1 - \widetilde{\delta}_2(1-q)^2}\right)$$

which suggests that, if Amnesty Plus can induce the reporting of cartel 2 in the reporting

subgame, the fine reduction in market 1 must be sufficiently low. Otherwise, the decrease of the expected fine would compensate the firms for the enhanced desistance, and the procompetitive effect cannot occur.

c-If $\tilde{\delta}_2 < \hat{\delta}_1(R_2)$ and $\tilde{\delta}_2 \ge \hat{\delta}_2(R_1)$, cartel 2 is individually stable and robust whereas cartel 1 is individually stable but not robust for $\delta = \tilde{\delta}_2$. The value of joint collusion is equal to

$$V_{12}^{AP}(\widetilde{\delta}_2, R_1, R_2) = \frac{\pi_1 - qF_1}{1 - \widetilde{\delta}_2(1 - q)^2} + \frac{\pi_2 - qF_2}{1 - \widetilde{\delta}_2(1 - q)} + \frac{q(1 - q)(R_2 - F_1)}{2\left(1 - \widetilde{\delta}_2(1 - q)^2\right)}$$

Condition (D-4) becomes

$$R_2 < F_1 + \frac{2\left(1 - \tilde{\delta}_2(1-q)^2\right)}{q(1-q)} \left(2\pi_1 - \frac{\pi_1 - qF_1}{1 - \tilde{\delta}_2(1-q)^2}\right)$$

A similar argument as above applies, and the procompetitive effect cannot occur. d- If $\tilde{\delta}_2 < \hat{\delta}_1(R_2)$ and $\tilde{\delta}_2 < \hat{\delta}_2(R_1)$, both cartels are stable but not robust for $\delta = \tilde{\delta}_2$. The value of joint collusion is

$$V_{12}^{AP}(\widetilde{\delta}_2, R_1, R_2) = \frac{\pi_1 - qF_1}{1 - \widetilde{\delta}_2(1 - q)^2} + \frac{\pi_2 - qF_2}{1 - \widetilde{\delta}_2(1 - q)^2} + \frac{q(1 - q)(R_1 + R_2 - F_1 - F_2)}{2\left(1 - \widetilde{\delta}_2(1 - q)^2\right)}$$

We rewrite the condition in (D-4) as

$$R_1 + R_2 < (F_1 + F_2)\frac{2-q}{1-q} - \frac{4\tilde{\delta}_2(1-q)(\pi_1 + \pi_2)}{q}$$

In this case, Amnesty Plus triggers the reporting in each possible reporting subgame of stage 4. The fine reductions must be sufficiently low such that the expected fines do not decrease too much.

We now provide sufficient conditions under which $(S_{1\to 2}, S_{1\to 2})$ Pareto dominates $(S_{12}^{AP}, S_{12}^{AP})$. Since $V_{1\to 2}(1^-) = V_1(1^-) + V_2(1^-) > 2\pi_1 + 2\pi_2$ and $V_{1\to 2}(\delta)$ is continuous and increasing on $[\tilde{\delta}_2, 1[$, a threshold $\tilde{\delta}_{1\to 2} \in [\tilde{\delta}_2, 1[$ exists such that for δ values in this interval, we have $V_{1\to 2}(\delta) \ge 2\pi_1 + 2\pi_2$ if and only if $\delta \ge \tilde{\delta}_{1\to 2}$. This implies that the comparison of $V_{12}^{AP}(\delta, R_1, R_2)$ to $V_{1\to 2}(\delta)$ is mainly relevant over the interval $[\tilde{\delta}_{1\to 2}, 1[$. In what follows, we therefore concentrate on sufficiently high values of δ .

Consider the case where $\hat{\delta}_1(R_2) > 1$. Cartel 1 is then not robust for any value of δ in this interval. If, moreover, $\hat{\delta}_2(R_1) > 1$ the value of joint collusion for $\delta = 1^-$ is

$$V_{12}^{AP}(1^-, R_1, R_2) = \frac{\pi_1 - qF_1}{1 - (1 - q)^2} + \frac{\pi_2 - qF_2}{1 - (1 - q)^2} + \frac{q(1 - q)(R_1 + R_2 - F_1 - F_2)}{2(1 - (1 - q)^2)}$$

If, however, $\widehat{\delta}_2(R_1) \leq 1$ we have

$$V_{12}^{AP}(1^-, R_1, R_2) = \frac{\pi_1 - qF_1}{1 - (1 - q)^2} + \frac{\pi_2 - qF_2}{q} + \frac{q(1 - q)(R_2 - F_1)}{2(1 - (1 - q)^2)}$$

In both cases it is true that $V_{12}^{AP}(1^-, R_1, R_2) < V_1(1^-) + V_2(1^-) = V_{1\to 2}(1^-)$ which implies that $V_{12}^{AP}(\delta, R_1, R_2) < V_{1\to 2}(\delta)$ for a non-empty range of values of δ sufficiently close to 1. Hence, for this range of values, Amnesty Plus defers the formation of cartel 2.

Consider now the case where $\hat{\delta}_1(R_2) \leq 1$. Amnesty Plus defers the formation of cartel 2 for values of δ sufficiently close to 1 if

$$\frac{\pi_2 - qF_2}{q} < R_1 < F_2 + \underbrace{2\frac{\pi_2 - qF_2}{q(1-q)}\left(\frac{1 - (1-q)^2}{q} - 1\right)}_{>0}$$

The LHS ensures that cartel 2 is not robust, i.e. $\hat{\delta}_2(R_1) > 1$, and the RHS implies that $V_{12}^{AP}(1^-, R_1, R_2) < V_1(1^-) + V_2(1^-) = V_{1 \to 2}(1^-)$ given that $\hat{\delta}_1(R_2) \le 1$ and $\hat{\delta}_2(R_1) > 1$.

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