Sovereign Default Risk and Uncertainty Premia [*]

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Abstract

This paper develops a general equilibrium model of sovereign debt with endogenous default. Foreign lenders fear that the probability model which dictates the evolution of the endowment of the borrower is misspecified. To compensate for the risk and uncertainty-adjusted probability of default, they demand higher returns on their bond holdings. In contrast with the existing literature on sovereign default, we are able to match the average bond spreads observed in the data together with the standard empirical regularities of emerging economies.

The technical contribution of the paper lies in extending the methodology of McFadden (1981) to compute equilibrium allocations and prices using the discrete state space (DSS) technique in the context of risk and uncertainty aversion on the lenders' side.

1   Introduction

Business cycles in emerging economies differ from those in the developed ones. Among the most striking differences, we observe that output tends to exhibit large swings, consumption is more volatile relative to output, and net exports are countercyclical. Real interest rates are not only more volatile but also countercyclical, as documented by Neumeyer and Perri (2005) and Uribe and Yue (2006). Also, for emerging economies sovereign default is a recurrent event. The presence of default risk, thereby, implies positive and time-varying spreads for sovereign debt[1].

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[1]By spreads we mean the differential between the sovereign bond yield and the risk-free interest rate.

In this paper we develop an infinite horizon model using Eaton and Gersovitz (1981) general equilibrium framework to explain these business cycle features with a special focus on the high average level on bond spreads. In our model, an emerging economy can trade one-period discount bonds with international creditors in financial markets. Debt repayments cannot be enforced and the emerging economy may decide to default. These default contingencies emerge from the fact that bond repayments cannot be conditioned on each particular state of the economy. Lenders in equilibrium, anticipate the default strategies of the emerging economies and demand higher returns on their sovereign bond holdings to compensate for the default risk. In case of default, the economy is temporarily excluded from financial markets and suffers a direct output cost.

The vast majority of sovereign default models are built under the assumption that international creditors are risk neutral, in which case the equilibrium bond prices are simply given by the discounted conditional probability of not defaulting next period[2]. Consequently, the pricing rule in these environments prescribes a strong connection between equilibrium prices and default probability. When calibrated to the data, matching the default frequency to historical levels (the consensus number for Argentina is around 3 percent annually), delivers spreads that are too low relative to those observed in the data. Also, the introduction of plausible degrees of risk aversion on the lenders' side with time-separable preferences has shown not to be sufficient for the recovering of those high spreads.

The novelty in our paper comes from the fact that lenders are assumed to fear that the probability model governing the evolution of the stochastic endowment of the borrower may be subject to specification errors. In turn, they contemplate a set of alternative models. This assumption intends to capture the fact that lenders are aware of the limited availability of reliable official data, measurement errors, and lags in the release of the official statistics with subsequent revisions. To express fears about model misspecification, following Hansen and Sargent (2005) we assume that lenders exhibit multiplier preferences. In our context, the lender wants to guard himself against a worst-case conditional density for the endowment of the borrower by slanting probabilities towards the states associated with low continuation values. In our model, these low-utility states for the lender coincide with those in which the borrower defaults on its debt.

As this paper demonstrates, from an asset pricing perspective, the key element in generating high spreads while matching the default frequency is a sufficiently negative correlation of the market stochastic discount factor with the country's default decisions. With fears about model misspecification, the stochastic discount factor has an additional component given by

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[2]See Arellano (2008) and Aguiar and Gopinath (2006), for example.

the probability distortion inherited in the worst-case density for the endowment of the borrower. This probability distortion, which is low when the borrower repays and high when the borrower defaults, induces in general a negative comovement between the stochastic discount factor and the default decisions of the borrower, which, as mentioned before, is necessary to explain high bond spreads.

The main result of our paper is that by introducing fears about model misspecification for the lenders our calibrated model matches the high bond spreads observed in the data, for the Argentinean economy, together with standard features of its business cycle while keeping the default frequency at historical levels. In the simulations we also find that, under plausible values of the parameters, risk aversion alone on the lender's side with time-separable preferences is not sufficient to generate the observed risk premia.

The technical contribution of this paper relates to the way we solve the model numerically using the discrete state space (DSS) technique, in the context of risk aversion and model uncertainty. Since default is a discrete choice, it can occur that -under DSS technique- debt policy rule is not continuous in the current state variables and prices. In turn, the discontinuity in the debt policy function with respect to bond prices translates into discontinuity in the lenders' Euler equation, which may lead to convergence problems. We handle this technical complication with the introduction of an i.i.d. preference shock. This preference shock enters additively in the borrower's utility when he decides to repay its debt and it is drawn from a logistic distribution, following McFadden (1981) and Rust (1994). As a result, the default decision, that originally was a discrete variable taking values of 0 or 1, becomes a continuous variable, that now summarizes a probability which depends on the spread of continuation values of repaying and defaulting on the outstanding debt.

Several papers on sovereign default are related to ours. Arellano (2008) and Aguiar and Gopinath (2006) were the first to extend Eaton and Gersovitz (1981) general equilibrium framework with endogenous default and risk neutral lenders to study the business cycles of emerging economies. Lizarazo (2010) endows the lenders with constant relative risk aversion (CRRA) preferences. Borri and Verdelhan (2010) have studied the setup with positive comovement between lender's consumption and output in the emerging economy in addition to time-varying risk aversion on the lenders' side.

From a technical perspective, Chatterjee and Eyingungor (2010) proposes an alternative approach to handle convergence issues. The authors consider an i.i.d. output shock drawn from a continuous distribution with a very small variance. Once this i.i.d. shock is incorporated, they are able to show the existence of a unique equilibrium price function for long-term debt with the property that the return on debt is increasing in the amount borrowed.

To our knowledge, the paper that is the closest to ours is Costa (2009). That paper also assumes that foreign lenders want to guard themselves against specification errors in the endowment of the borrower, but this is achieved in a different form. In our model, lenders are endowed with Hansen and Sargent (2005) multiplier preferences. With these preferences, lenders contemplate a set of alternative models and want to guard themselves against the model that minimizes their lifetime utility. In contrast, in Costa (2009) the worst-case density minimizes the expected value of the bond. Moreover, in Costa (2009) lenders are assumed to live for one period only.

The paper is organized as follows. Section 2 describes the benchmark model. In section 3, we calibrate our model to Argentinean data. In section 4, we describe the numerical algorithm with preference shock. Section 5 explains how to calibrate the parameter that measures the lender's concerns about model misspecification. Section 6 concludes.

2   Benchmark Model

In our model an emerging economy interacts with a continuum of foreign lenders of measure 1. The emerging economy is populated by a representative risk-averse household and a government, with its evil alter ego, which represents his doubts about model misspecification. The emerging economy can trade a one-period discount bond with identical atomistic lenders to smooth consumption over time. Throughout the paper we will refer to the emerging economy as the *borrower*. Debt contracts cannot be enforced and the borrower may decide to default at any point of time.

The lender distrusts the probability model governing the stochastic process of the endowment of the borrower, which we will refer to as the approximating model. For this reason, she contemplates a set of alternative models that are statistical perturbations of the approximating model, and wishes to design a decision rule that performs well across this set of densities[3].

Time is discrete $t = 0, 1, \ldots$ Let $(\Omega, \mathcal{F}_\Omega, P_\Omega)$ be the underlying probability space. We assume that each period t the borrower and lenders receive an exogenous stochastic endowment, y_t^B and y_t^L, respectively, where for each t, $y_t^i : \Omega \to \mathbb{Y} \subseteq \mathbb{R}_+$ for $i = B, L$. We assume that $(y_t^B)_t$ and $(y_t^L)_t$ are independent Markov processes, with transition densities for y_t^B and y_t^L given by $f^B(y_{t+1}^B | y_t^B)$ and $f^L(y_{t+1}^L | y_t^L)$, respectively. We denote the endowment vector as $y_t \equiv (y_t^B, y_t^L)$

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[3]In order to depart as little as possible from Eaton and Gersovitz (1981) framework, throughout the paper we assume that the lender only distrusts the probability model dictating the evolution of the endowment of the borrower, not the distribution of any other source of uncertainty, such as the stochastic endowment of the lender or the random variable that indicates whether the borrower re-enters financial markets or not.

and the history as $y^t = (y_0, ..., y_t)$ (y^{it} are defined by analogy), for all $t \geq 0$. The joint density for the endowment vector y_t is thus defined as $f_{Y'|Y}(y_{t+1}|y_t) = f^L(y^L_{t+1}|y^L_t)f^B(y^B_{t+1}|y^B_t)$. Finally, we denote $\mathcal{F}^t_Y$ as the sigma algebra generated by y^t.

We follow Arellano (2008) and adopt a recursive formulation for both the borrower and lender's problem. We still use t and $t+1$ to denote current and next period's variables, respectively.

2.1   Timing Protocol

We assume that all economic agents, lenders (and their evil alter ego) and the government (that cares about the consumption of the representative household), act sequentially, choosing their allocations period by period.

At the beginning of every period that the government enters with access to financial markets, it has to decide whether to repay its debt or not. In the former case, it has to choose next period's bond holdings. In case the government defaults on its debt, it incurs two types of costs. First, it is temporary excluded from financial markets. Second, it suffers a direct output loss.

The timing protocol within the period is as follows. First, the endowments are realized. The government observes the endowments and decides whether to repay its debt or not. If it decides to repay, it chooses new bond holdings and how much to consume. Then atomistic lenders—taking prices as given—choose how much to save and how much to consume. The evil agent, who is a metaphor for the lenders' fears about model misspecification, chooses the probability distortions to minimize the lenders' expected utility. Due to the zero-sumness of the game between the lender and its evil agent, different timing protocols of their moves yield the same solution. If the government decides to default, both the government and the lenders switch to autarky.

There are two stages the economy can be in at the beginning of each period t: financial autarky and a *normal* stage, in which the economy has access to financial markets and the government has to make a decision whether to repay its debt or not. In case the government defaults, it switches to financial autarky for a random number of periods. In case the government decides to repay, it enters a *continuation* stage within the period in which it repays its outstanding debt and has to choose new bond holdings and consumption. Next period, it will be entering a new normal stage in which it has to decide whether to repay its new debt contract, etc.

2.2 Sovereign Debt Markets

Financial markets are incomplete. Only a non-contingent one-period discount bond can be traded between the borrower and the lenders; however, the borrower can default on this bond at any time, thereby adding some degree of state contingency.

Bond holdings of the government and the individual lenders, which are $\mathcal{F}_Y^{t-1}$ measurable, are denoted by $B_t \in \mathbb{B} \subseteq \mathbb{R}$ and $b_t \in \mathbb{B} \subseteq \mathbb{R}$, respectively. In case it purchases bonds, the government is saving and bond holdings B_t is positive; otherwise, if it sell bonds, the government borrows from the lenders and B_t is negative.

The borrower can buy a quantity B_{t+1} of bonds at a price q_t. A debt contract is given by a vector (B_{t+1}, q_t) of quantities of bonds and corresponding bond prices. The price q_t depends on the borrower's demand for debt at time t, B_{t+1}, and his endowment y_t^B, since these variables affect his incentives to default. The higher the level of indebtness and/or the lower the (persistent) borrower's endowment, the higher the possibilities the borrower will default next period and, hence, the lower the bond prices in the current period. Also, prices are function of the representative lender's current debt level (which in equilibrium is given by $-B_t$), as well as B_{t+1} and the endowment y_t^L, through the stochastic discount factor; we formalize this below.

We refer to $q(y_t, B_t, \cdot) : \mathbb{B} \longrightarrow \mathbb{R}_+$ as the bond price function. Thus, we can define the set of debt contracts available to the borrower at a given state (y_t, B_t) as $\mathbb{B} \times q(y_t, B_t, \mathbb{B})$.

2.3 Preferences

A representative household in the emerging economy derives utility from consumption of a single good in the economy. His preferences over consumption plans can be described by the expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} (\beta^B)^t U(c_t) \tag{1}$$

where E_0 denotes the mathematical expectation operator conditional on time zero information, and $\beta^B \in (0, 1)$ denotes the time discount factor and the period utility function U is assumed to have the CRRA form,

$$U(c) = \begin{cases} \frac{c^{1-\sigma^B}}{1-\sigma^B} & \text{if } \sigma^B \neq 1 \\ \log(c) & \text{if } \sigma^B = 1 \end{cases}$$

where σ^B is the coefficient of relative risk aversion of the household.

The government in this economy, who is benevolent and maximizes the household's utility (1), may have access to international financial markets, where it can trade a one-period

discount bond with the foreign lenders. While with access to the financial markets, it can sell or purchase bonds from the lenders and make a lump-sum transfer across households to help them smooth consumption over time. Debt is also used to front load consumption as the borrower is relatively more impatient than the international creditors, i.e. $\beta^B < \beta^L$.

2.4   Borrowers' Problem

Let $V^B(y_t, B_t)$ denote the value for the government in the normal stage, at state (y_t, B_t). Formally, $V^B(y_t, B_t)$ is given by

$$V^B(y_t, B_t) = \max\left\{V_d^B(y_t), V_c^B(y_t, B_t)\right\},$$

where $V_d^B(y_t)$ is the value of defaulting, and $V_c^B(y_t, B_t)$ is the value of repayment at (y_t, B_t). Throughout the paper we will use subscripts d and c to denote the values for *autarky* (or *default*) and *continuation* stage, respectively.

When in the normal stage, the government evaluates the present lifetime utility of households if debt contracts are honored against the present lifetime utility of households if they are repudiated. If the former outweighs the latter, the government decides to comply with the contracts, pays back the debt carried from the last period B_t and chooses next period's bond holdings B_{t+1}. Otherwise, if the utility of defaulting on the debt and switching to financial autarky is higher, the government decides to default on the sovereign debt.

Consequently, the value $V_c^B(y_t)$ of repaying is given by

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\begin{aligned}
V_c^B(y_t, B_t) &= \max_{c_t^B, B_{t+1}} U(c_t^B) + \beta^B \int_{\mathbb{Y}\times\mathbb{Y}} V^B(y_{t+1}, B_{t+1}) f_{Y'|Y}(y_{t+1}|y_t) dy_{t+1} \\
\text{s.t. } c_t^B &= y_t^B - q_t B_{t+1} + B_t
\end{aligned}
$$

where $q_t \equiv q(y_t, B_t, B_{t+1})$. We also assume throughout the paper that the borrower faces an exogenous borrowing constraint, $B_{t+1} \geq \underline{B}$ which rules out Ponzi schemes and is assumed not to be ever binding.

Finally, the autarky value $V_d^B(y_t)$ is given by

$$V_d^B(y_t) = U(h^d(y_t^B)) + \beta^B \int_{\mathbb{Y}\times\mathbb{Y}} \left((1-\pi)V_d^B(y_{t+1}) + \pi V^B(y_{t+1}, 0)\right) f_{Y'|Y}(y_{t+1}|y_t) dy_{t+1}$$

where π is the probability of re-entering financial markets next period. In that event, the borrower enters next period in the normal stage starting over with no debt, $B_{t+1} = 0$.[4] The

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[4]Notice that we assume there is no debt renegotiation nor any form of debt restructuring mechanism. Yue (2010) models a debt renegotiation process as a Nash bargaining game played by the borrower and lenders. For more examples of debt renegotiation, see Benjamin and Wright (2009) and Pitchford and Wright (2010).

function $h^d(\cdot)$ represents an ad-hoc direct output cost in terms of consumption units that the borrower incurs when being excluded from financial markets. This output loss function is consistent with evidence that shows that countries experience a fall in output in default times due to the lack of short-term trade credit. Mendoza and Yue (2010) endogenize this output loss as an outcome that results from the substitution of imported inputs by less-efficient domestic ones as credit lines are cut when the country declares a default. Notice that in autarky the borrower has no decision to make and simply consumes $h^d(y_t^B)$.

The default decisions can be characterized in terms of default sets and a default indicator. Let $D : \mathbb{B} \to \mathcal{Y}$ where $\mathcal{Y}$ is the space of subsets in $\mathbb{Y}^2$. Then we define the default set for a given debt level B

$$D(B) \equiv \{y : V_c^B(y, B) < V_d^B(y)\}$$

to be the set of endowment realizations in which the government finds it optimal to default.

In a similar fashion, let $\delta : \mathbb{Y}^2 \times \mathbb{B} \to \{0, 1\}$ denote the default indicator, that takes value 0 in case of default; and 1, otherwise; i.e.,

$$\delta(y, B) = \mathcal{I}\left\{V_c^B(y, B) \geq V_d^B(y)\right\}$$

We now characterize the borrower's problem

LEMMA 2.1. *Suppose $\mathbb{Y}$ and $\mathbb{B}$ are convex, compact subsets of $\mathbb{R}$, y_t is an i.i.d. vector, the price function $q(y_t, B_t, B_{t+1})$ is continuous in B_{t+1}, and assumption B.1 in the appendix holds. Then the following hold has to be true:*

1. *(V_c^B, V_d^B) are continuous functions.*

2. *(V_c^B, V_d^B) are increasing in B_t and y_t^B.*

3. *The correspondence $(y, B) \mapsto B^*(y, B)$, where*

$$B^*(y_t, B_t) \equiv \arg \max_{B_{t+1} \in \mathbb{B}} \left\{U(y_t^B - q_t B_{t+1} + B_t) + \beta^B E\left[V^B(y_{t+1}, B_{t+1})|y_t\right]\right\}$$

   is non-empty, compact-valued and upper hemi-continuous.

Remark 2.1. *(1) The i.i.d. assumption is used in results (1) and (2). It is easy to relax it to allow for time-dependence. For result (1) we need that the operator $w \mapsto E[w|y]$ preserves continuity; see Stokey and Lucas (1989) for sufficient conditions for this. For result (2) we need that the operator $w \mapsto E[w|y]$ preserves monotonicity; by assuming first order stochastic dominance of $F_{Y'|Y}$, the desired property could be obtained.*

Pouzo (2010) assumes a debt restructuring mechanism in which the borrower receives random exogenous offers to repay a fraction of the defaulted debt. A positive rate of debt recovery gives rise to positive prices for defaulted debt which can be traded amongst lenders in secondary markets.

2.5 Lenders and their Fears about Model Misspecification

Lender's period payoff is also given by a CRRA utility function, with coefficient of relative risk aversion of the lender σ^L.

We assume that the lender is concerned about model misspecification. While the lender fully trusts the transition density $f^L(y^L_{t+1}|y^L_t)$ for her own endowment, she distrusts the probability model governing the stochastic process of the endowment of the borrower, given by $f^B(y^B_{t+1}|y^B_t)$, which we will refer to as the approximating model. For this reason, she contemplates a set of alternative densities that are statistical perturbations of the approximating model, and wishes to design a decision rule that performs well across this set of densities. These alternative transition densities, denoted by $\widetilde{f}^B(y^B_{t+1}|y^B_t)$, are assumed to be absolutely continuous with respect to f^B, i.e. if $f^B(\mathcal{A}_1|\mathcal{A}_2) = 0$ for some given Borel sets $\mathcal{A}_1, \mathcal{A}_2 \subset \mathbb{Y}$, then it has to be true that $\widetilde{f}^B(\mathcal{A}_1|\mathcal{A}_2) = 0$. To construct any of these distorted probabilities $\widetilde{f}^B_{Y'|Y}(y^B_{t+1}|y^B_t)$ we can use a nonnegative $\mathcal{F}^t_Y-$measurable function m_{t+1} that satisfies $E(m_{t+1}|\mathcal{Y}^t) = 1$. The function m_{t+1} takes the form of a conditional likelihood ratio, i.e.

$$m_{t+1}(y^B_{t+1}|y^t) = \begin{cases} \frac{\widetilde{f}^B(y^B_{t+1}|y^B_t)}{f^B(y^B_{t+1}|y^B_t)} & \text{if } f^B(y^B_{t+1}|y^B_t) > 0 \\ 1 & \text{if } f^B(y^B_{t+1}|y^B_t) = 0 \end{cases}$$

The discrepancy between the distorted and approximating probability distribution, $\widetilde{f}^B(y^B_{t+1}|y^B_t)$ and $f^B(y^B_{t+1}|y^B_t)$ respectively, is measured by the relative entropy, which, for given history y^t, takes the form

$$\int (m_{t+1} \log m_{t+1}|y^t) f^B(y^B_{t+1}|y^B_t) dy^B_{t+1}$$

Following Hansen and Sargent (2007b) and references therein, to express fears about model misspecification we endow lenders with multiplier preferences. We are thinking of the lenders as playing a zero-sum game against their evil alter ego who represents their doubts about model misspecification. While the lender chooses bond holdings to maximize his utility, the evil alter ego chooses conditional likelihood ratios $m_{t+1}(y^B_{t+1}|y^t)$, to minimize it. Thus, preferences over consumption plans for lenders are then described by

$$\min_{\{m_{t+1}\}} U(c_t) + \beta^L \int_{\mathbb{Y} \times \mathbb{Y}} \left[m_{t+1}(y^B_{t+1}|y^t)\theta \log m_{t+1}(y^B_{t+1}|y^t) + m_{t+1}(y^B_{t+1}|y^t)W^L_{t+1}(y^{t+1}) \right] f_{Y'|Y}(y_{t+1}|y_t)dy_{t+1}$$

where $\theta \in (\underline{\theta}, +\infty]$ is a penalty parameter that measures the degree of concern about model misspecification and W^L_{t+1} is the continuation value for the lender at time $t+1$ (the index $t+1$ denotes other state variables which enter the function, the precise definition of W^L is in section 2.6). The minimization problem conveys the ambiguity aversion. Through the entropy term, the evil alter ego is being penalized whenever he chooses distorted probabilities that

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differ from the approximating model. The higher the value of θ, the more the evil alter ego is penalized. In the extreme case of $\theta = +\infty$, there is no concerns about model misspecification and we are back to the standard environment where both borrower and lender share the same model, given by f.

The minimization problem of the evil alter ego yields the following specification for m_{t+1}

$$m_{t+1}(y_{t+1}^B|y^t) = \frac{\exp\left\{-\frac{\mathcal{W}_{t+1}^{L*}(y^t, y_{t+1}^B)}{\theta}\right\}}{\int_{\mathbb{Y}} \exp\left\{-\frac{\mathcal{W}_{t+1}^{L*}(y^t, y^{B\prime})}{\theta}\right\} f^B(y^{B\prime}|y_t^B)dy^{B\prime}} \tag{2}$$

where $\mathcal{W}_{t+1}^{L*}(y^t, y_{t+1}^B)$ is the $t+1$-equilibrium value for the lender *before* the realization of his own endowment, y_{t+1}^L, i.e. $\mathcal{W}_{t+1}^L(y^t, y_{t+1}^B) = \int_{\mathbb{Y}} W_{t+1}^L(y^t, y_{t+1}^B, y_{t+1}^L)f^L(y_{t+1}^L|y_t^L)dy_{t+1}^L$, Through its choice of m_{t+1}, the evil alter ego pessimistically twists the conditional distribution f^B by putting more weight in continuation outcomes associated to lower utility for the lender.

Plugging this expression back into the lifetime utility yields the risk-sensitivity recursion developed by Hansen and Sargent (2007a)

$$W_t^L(y^t) = U(c_t^L) - \theta\beta^L \log\left(\int_{\mathbb{Y}} \exp\left\{-\frac{\mathcal{W}_{t+1}^L(y^t, y_{t+1}^B)}{\theta}\right\} f^B(y_{t+1}^B|y_t^B)dy_{t+1}^B\right) \tag{3}$$

where the second term is the risk-sensitivity operator T^θ, defined as in Hansen and Sargent (2007a)

$$T^\theta[W_{t+1}^L] = -\theta \log\left(\int_{\mathbb{Y}} \exp\left\{-\frac{\mathcal{W}_{t+1}^L(y^t, y_{t+1}^B)}{\theta}\right\} f^B(y_{t+1}^B|y_t^B)dy_{t+1}^B\right).$$

If we let $\theta \longrightarrow +\infty$, the recursion (3) converges to the value function with standard expected utility. At the same time, the probability distortion given by expression (2) converges to 1, and the worst-case distorted density induced by the evil alter ego converges to the approximating density.

2.6 Lender's Problem

Since lender are atomistic, each individual lender take as given the aggregate debt. The lender has a "perceived" law of motion for this variable, which only in equilibrium will be required to coincide with the actual one. We denote b_t as the individual lender's debt.

In a normal stage, that is when lender and borrower can engage in a financial relationship, the lender's value function is $W_c^L(y_t, B_t, b_t)$ is given by:

$$W_c^L(y_t, B_t, b_t) = \max_{c_t^L, b_{t+1}} \left\{U(c_t^L) + \beta^L T^\theta \left[\mathcal{W}^L(y_t, y_{t+1}^B, B_{t+1}, b_{t+1})\right](y_t, B_{t+1}, b_{t+1})\right\}$$

$$s.t. \ c_t^L = y_t^L + q_t b_{t+1} - b_t$$

$$B_{t+1} = \Gamma^L(y_t, B_t).$$

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where $\mathcal{W}^L(y_t, y_{t+1}^B, B_{t+1}, b_{t+1}) \equiv \int_{\mathbb{Y}} \delta(y_{t+1}^B, y_{t+1}^L, B_{t+1}) W_c^L(y_{t+1}^B, y_{t+1}^L, B_{t+1}, b_{t+1}) +$
$(1 - \delta(y_{t+1}^B, y_{t+1}^L, B_{t+1})) W_d^L(y_{t+1}^B, y_{t+1}^L) f^L(y_{t+1}^L | y_t^L) dy_{t+1}^L$, and $\Gamma^L : \mathbb{Y}^2 \times \mathbb{B} \to \mathbb{B}$ is the perceived law of motion of the individual lender for the debt holdings of the borrower, B_{t+1}.

Let $W_d^L(y_t)$ denote the value of a lender in the financial autarky at state (y_t) given by

$$W_d^L(y_t) = U(y_t^L) + \beta^L T^\theta [(1 - \varrho) \mathcal{W}_d^L(y_t, y_{t+1}^B) + \varrho \mathcal{W}^L(y_t, y_{t+1}^B, 0, 0)](y_t)$$

where $\mathcal{W}_d^L(\cdot, \cdot)$ and $\mathcal{W}^L(\cdot, \cdot, \cdot, \cdot)$ are the $t + 1$-equilibrium values for the lender *before* the realization of y_{t+1}^L in financial autarky and in the normal stage, respectively, and ϱ is a random variable that is equal to 0 with probability $(1 - \pi)$ and 1 with probability π.

In contrast with the borrower's case, no output loss is assumed for the lender during financial autarky.

LEMMA 2.2. *Suppose $\mathbb{Y}$ and $\mathbb{B}$ are convex, compact subsets of $\mathbb{R}$, y_t is an i.i.d. random vector, and assumption B.1 holds. Then the following holds:*

1. *(W_c^L, W_d^L) are continuous functions.*

2. *(W_c^L, W_d^L) are decreasing in b_t and increasing in y_t^L.*

3. *(W_c^L, W_d^L) are strictly concave in y_t^L and b_t.*

4. *The correspondence $(y, B, b) \mapsto b^*(y, B, b)$, where*

$$b^*(y_t, B_t, b_t) \equiv \arg \max_{b_{t+1} \in \mathbb{B}} \left\{ U(y_t^L - b_t + q_t b_{t+1}) + \beta^L T^\theta \left[\mathcal{W}^L(y_{t+1}^B, B_{t+1}, b_{t+1}) \right] (y_t, B_{t+1}, b_{t+1}) \right\}$$

is single-valued (i.e., it is a function) and continuous.

Remark 2.2. *(1) The i.i.d. assumption is used in results (1) and (2). It is easy to relax it to allow for time-dependence. For result (1) we need that the operator $w \mapsto E[w|y]$ preserves continuity; see Stokey and Lucas (1989) for sufficient conditions for this. For result (2) we need that the operator $w \mapsto E[w|y]$ preserves monotonicity; by assuming first order stochastic dominance of $F_{Y'|Y}$, the desired property could be obtained.*

2.7 Recursive Equilibrium

We are interested in a recursive equilibrium in which all agents choose sequentially.

Definition 2.1. *A collection of policy functions $\{c^B, c^L, B, b, m, \delta\}$ is given by mappings for consumption $c^B : \mathbb{Y}^2 \times \mathbb{B} \to \mathbb{R}_+$ and $c^L : \mathbb{Y}^2 \times \mathbb{B}^2 \to \mathbb{R}_+$, bond holdings $B : \mathbb{Y}^2 \times \mathbb{B} \to \mathbb{B}$ and $b : \mathbb{Y}^2 \times \mathbb{B}^2 \to \mathbb{B}$ for borrower and individual lender, respectively, probability distortions $m : \mathcal{V} \to L^1(\mathbb{Y}^2 \times \mathbb{B}^2)$ and default decisions, $\delta : \mathbb{Y}^2 \times \mathbb{B} \to \{0, 1\}$.*

Definition 2.2. *A collection of value functions $\{V_c^B, V_d^B, W_c^L, W_d^L\}$ is given by mappings $V_c^B : \mathbb{Y}^2 \times \mathbb{B} \to \mathbb{R}$, $V_d^B : \mathbb{Y} \to \mathbb{R}$, $W_c^L : \mathbb{Y}^2 \times \mathbb{B}^2 \to \mathbb{R}$, $W_d^L : \mathbb{Y} \to \mathbb{R}$.*

Definition 2.3. *A price schedule is given by $q : \mathbb{Y}^2 \times \mathbb{B}^3 \to \mathbb{R}_+$.*

Definition 2.4. *A recursive equilibrium is a collection of policy functions $\{c^{*,B}, c^{*,L}, B^*, b^*, m^*, \delta^*\}$, a collection of value functions $\{V_c^{*,B}, V_d^{*,B}, W_c^{*,L}, W_d^{*,L}\}$, a perceived law of motion for the borrower's bond holdings, and price schedule such that:*

1. *Given perceived laws of motion for debt and price schedule, policy functions and value functions solve the borrower and individual lender's problems.*

2. *Given the policy functions and value functions for the lender, m^* solves the evil agent's minimization problem.*

3. *Bond prices $q(y, B, B^*(y, B))$ clear the financial markets, i.e.,*

$$B^*(y, B) + b^*(y, B, B) = 0, \ \forall y \in \mathbb{Y}^2, B \in \mathbb{B}$$

4. *The actual and perceived law of motions for debt holdings coincide, i.e.,*

$$B^*(y, B) = \Gamma(y, B).$$

After imposing the market clearing condition given by point 3 above, vector (y_t, B_t) is sufficient to describe the state variables for any agent in this economy. Hence, from here on, we consider (y_t, B_t) as the state vector, common to the borrower and the individual lenders.

By taking FOCs with respect to b_{t+1} in the lender's optimization problem[5], and imposing the equilibrium conditions, we derive the lenders' intertemporal Euler equation

$$q_t \nabla_c U(c^{*,L}(y_t, B_t)) =$$

$$\beta^L \int_{D(B^*(y_t, B_t))^C} \nabla_c U(c^{*,L}(y_{t+1}, B^*(y_t, B_t))) m[\mathcal{W}^{*,L}](y_t, y_{t+1}^B, B_{t+1}^*) f_{Y'|Y}(y_{t+1}|y_t) dy_{t+1}$$

where $\nabla_x f(x)$ denotes the partial derivative of f with respect to x and $m[\mathcal{W}^{*,L}](y_t, y_{t+1}^B, B_t^*)$ is the multiplicative distortion to the conditional density f^B given by expression (2)

The left hand side of this expression represents the marginal cost in terms of utility of forgoing consumption in the current period to purchase an additional bond from the borrower. The right hand side reflects the discounted expected (under the distorted model) marginal

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[5]We assume differentiability of $W^L(y_t, B_t, b_t)$ with respect to b_t, which, given Lemma 2.2 and following Stokey and Lucas (1989) results, it can be proved.

benefit. A larger default risk next period reduces the expected (under the distorted model) marginal benefit of trading the bond.

Then, in equilibrium, we have the following bond pricing function for the borrower evaluated at B_{t+1} is given by

$$q(y_t, B_t, B_{t+1}) = \beta^L \int_{D(B_{t+1})^c} \frac{\nabla_c U(c_{t+1}^{*,L})}{\nabla_c U(c_t^L)} m^*[\mathcal{W}^{*,L}](y_t, y_{t+1}^B, B_{t+1}) f_{Y'|Y}(y_{t+1}|y_t) dy_{t+1}. \quad (4)$$

where subscripts t and $t+1$ summarize the state variables, and where consumption for the lender is given by

$$
\begin{aligned}
c_t^L &= y_t^L + q(y_t, B_t, B_{t+1})B_{t+1} - B_t \\
c_{t+1}^{*,L} &= c^L(y_{t+1}, B_{t+1}) = y_{t+1}^L + q(y_{t+1}, B_{t+1}, B^*(y_{t+1}, B_{t+1}))B^*(y_{t+1}, B_{t+1}) - B_{t+1}
\end{aligned}
$$

In equilibrium, for each state of the economy (y_t, B_t) only one debt contract is traded between the borrower and the lenders and, hence, we observe a particular quantity of new bond holdings, $B^*(y_t, B_t)$, with an associated price q_t.
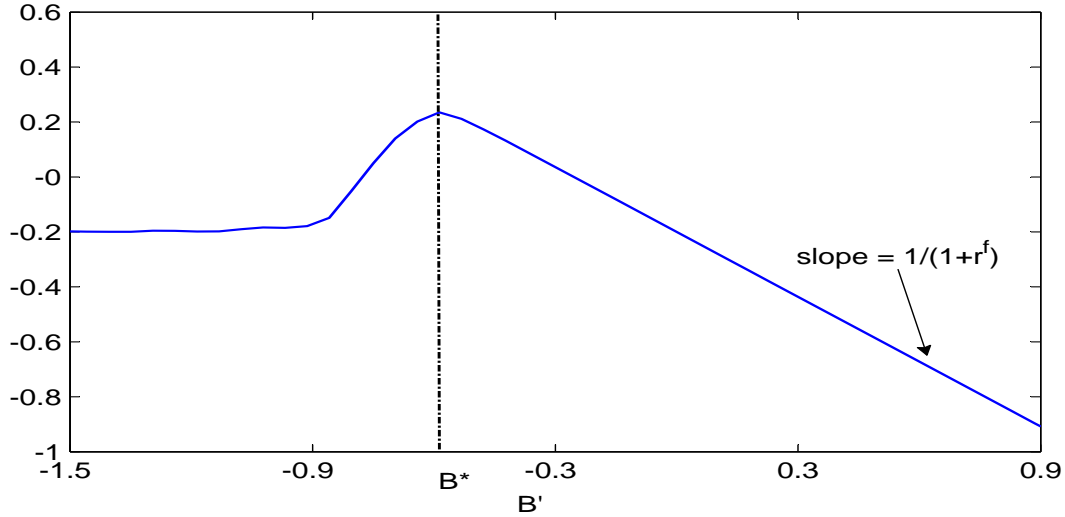

Figure 1: Borrower's debt revenue.

In addition, a risk-free rate $r^f(y_t, B_t)$ in the normal stage is determined by[6]

$$\frac{1}{1+r^f(y_t, B_t)} = \beta^L \left[\int_{D(B^*_{t+1})^c} \frac{\nabla_c U(c^{*,L}_{t+1})}{\nabla_c U(c^{*,L}_t)} m^*[\mathcal{W}^{*,L}](y_t, y^B_{t+1}, B^*_{t+1}) f_{Y'|Y}(y_{t+1}|y_t) dy_{t+1} \right.$$
$$\left. + \int_{D(B^*_{t+1})} \frac{\nabla_c U(y^L_{t+1})}{\nabla_c U(c^{*,L}_t)} m^*[\mathcal{W}^{*,L}](y_t, y^B_{t+1}, B^*_{t+1}) f_{Y'|Y}(y_{t+1}|y_t) dy_{t+1} \right]$$

The first term in the square brackets refers to the value of the repayment of the claim in states where the borrower repays. The second term relates to the value of the repayment when the borrower defaults on the risky bond, and the lender consumes his endowment y^L_{t+1}. Since all lenders are identical, there is no actual trading of the risk-free asset. In the absence of fears about model uncertainty, i.e. $\theta = +\infty$, there is no probability distortion, equilibrium bond prices are given by

$$q_t = q(y_t, B_t, B^*_{t+1}) = \beta^L \int_{D(B^*_{t+1})^c} \frac{\nabla_c U(c^{*,L}(y_{t+1}, B^*(y_t, B_t)))}{\nabla_c U(c^{*,L}(y_t, B_t))} f_{Y'|Y}(y_{t+1}|y_t) dy_{t+1} \quad (5)$$

In this environment the stochastic discount factor is given by the lender's time discount factor β^L times the ratio of marginal utilities for the lender. In Arellano (2008), lenders are risk neutral and, hence, the stochastic discount factor is equal to β^L. Equilibrium bond prices then turn to be the discounted probability of not defaulting. Lizarazo (2010) relaxes the assumption of risk neutrality by exploring the implications of assuming that lenders are risk averse with decreasing absolute risk aversion (DARA). When lenders are risk averse, they demand an excess risk premium in order to be willing to take the default risk inherited in the sovereign bonds and therefore the endogenous credit limits faced by the borrower are more stringent. For any given degree of risk aversion for the lender, the larger the correlation between next period's consumption of the lender and bond repayments is, the less valuable the bond is to the lender as it does not serve as an insurance mechanism. Hence, lenders will trade the bond only at prices sufficiently low. Conversely, if the bond repays on average in states in which lender's consumption is low, the lenders value it more and hence demand lower returns or high prices for it. Borri and Verdelhan (2010) exploit this insight by endowing the lenders with external habit formation and assuming some positive correlation between lender's consumption and borrower's endowment.

In contrast with the case with risk-neutral lenders, equilibrium bond prices and capital outflows depend not only on the economic fundamentals of the emerging economy but also on

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[6]We can view the lenders trading among themselves a claim that pays off one unit of consumption in every state next period.

characteristics of the international lenders, more specifically, on their degree of risk aversion and income process[7].

Under model uncertainty, the modified stochastic discount factor is then given by three multiplicative components,

$$\beta^L \frac{\nabla_c U(c_{t+1}^{*,L})}{\nabla_c U(c_t^{*,L})} m^*[\mathcal{W}^{*,L}](y_t, y_{t+1}^B, B_{t+1}^*)$$

In addition to the discount factor times the standard ratio of marginal utilities for the lender, we have the probability distortion $m^*[\cdot]$ induced by the evil alter ego. The lender in this economy distrusts the conditional density f^B and wants to guard himself against a worst-case distorted density for y_{t+1}^B given by $m^*[\mathcal{W}^{*,L}](y_t, y_{t+1}^B, B_{t+1}^*) f^B(y_{t+1}^B | y_t^B)$. The evil alter ego, who represents his doubts about model misspecification, will be selecting this worst-case distorted density by slanting probabilities towards the states associated with the lowest continuation utility for the lender. In the presence of default risk and given a deterministic endowment for the lender, these states associated to low utility coincide with the states in which the borrower defaults and thereby the lender receives no repayment.

For the particular case with deterministic endowment for the lender, we plot in figure 2 the conditional distorted density and approximating density for y_{t+1}^B given current state y_t^B and choice of B_{t+1}. The dotted line corresponds to the default probability conditional on the realization of y_{t+1}^B, which equals $1 - \delta(y_{t+1}^B, B_{t+1})$[8]. The evil alter ego takes away probability mass from those states in which the borrower does not default, and puts it in turn on those low realizations of y_{t+1}^B in which default is optimal for the borrower [9]. This tilting of the probabilities by the evil alter ego generates the endogenous hump of the distorted density over the interval of y_{t+1}^B associated with default risk, as observed in the figure.

This discrepancy between probability models is key to generate high bonds spreads in the model while, at the same time, keeping the actual default frequency at historical levels.

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[7]This is consistent with the findings by Longstaff et al. (2009) that attribute a large portion of the sovereign credit risk to global macroeconomic factors. Introducing fears about model misspecification could potentially amplify the magnitude of these effects.

[8]In our model, the default decision is dictated not by an indicator, that takes only values of 0 and 1, but by a mapping into a probability. This non-standard specification for the default decision results from the introduction of an i.i.d. preference shock to handle some computational issues. For details, see section 5.

The distorted probability exhibits a smooth downturn over the support associated to default risk due to a gradual decline in the default probability.

[9]In the case of two or more borrowing economies, we conjecture that the probability distortions would induce some correlation in the distorted stochastic processes for the endowments of the borrowers, even if they are independent according to the approximating distributions. We find this an interesting issue for future research.
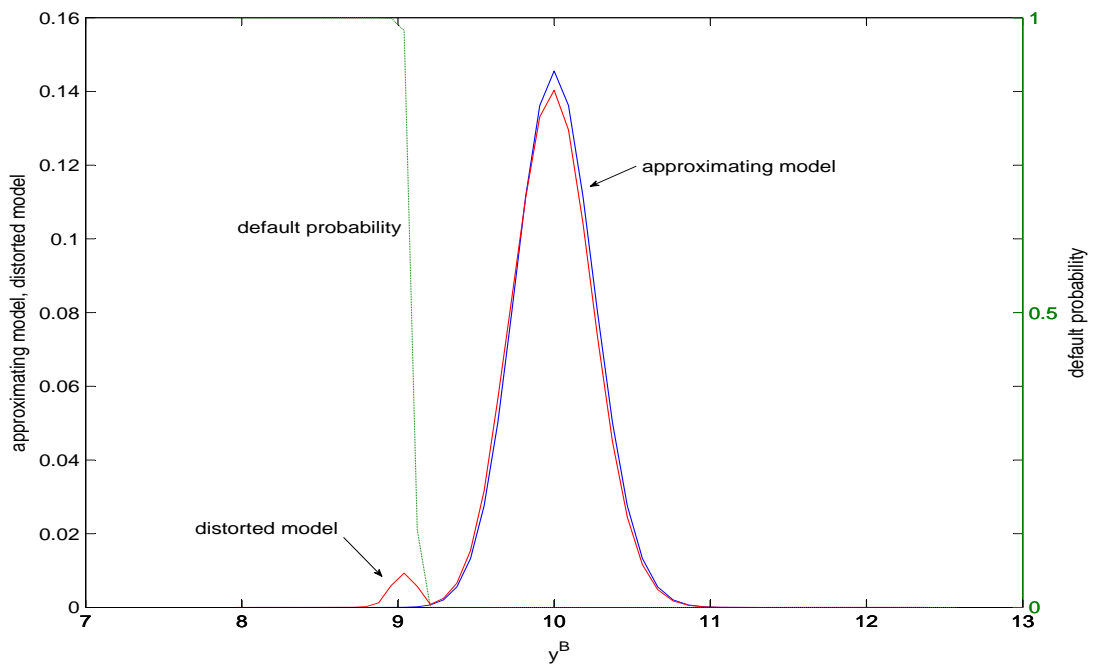
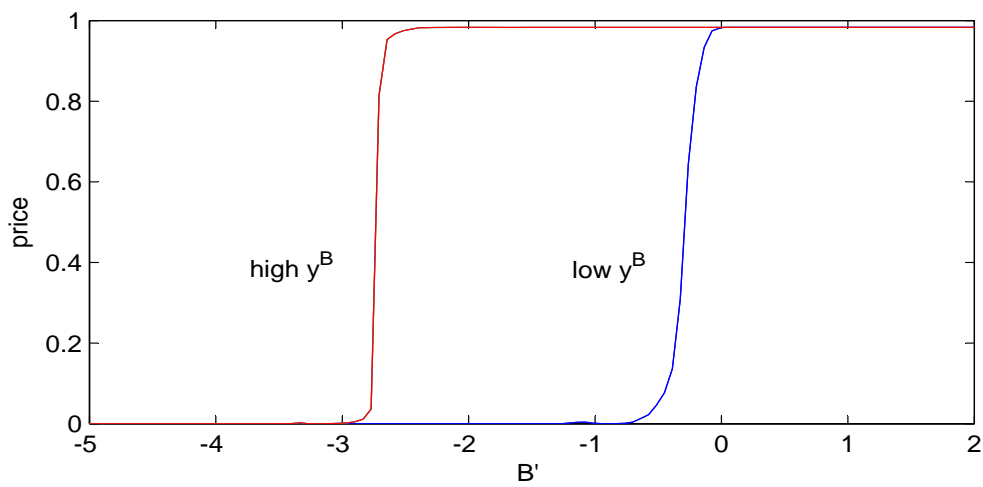Figure 2: Approximating and distorted densities.

Figure 3: Equilibrium prices for different endowment realizations of the borrower.

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Introducing a stochastic endowment for the lender, may alter the dynamics as the evil alter ego may decide to combine time-varying distortions to both the conditional distribution of the borrower and lender's endowment. In periods with high level of indebtness and low endowment for the borrower, in which the borrower is more prone to default, the evil alter ego may find optimal to distort relatively more the transition density of the borrower's endowment. In contrast, in periods with low default risk, the lender may prefer to distort mostly the transition density of the lender's endowment. The extent to which the evil alter ego will be distorting one or another will depend on the amount borrowed as well as on the endowments.

2.8   Benchmark Case: i.i.d. endowment shock

In this subsection, we impose the assumption of i.i.d. endowment shocks, permanent exclusion from financial markets following a default event, i.e. $\pi = 0$, and no output loss in financial autarky, i.e. $h^d(y^B) = y^B$.

LEMMA 2.3. *Suppose $\mathbb{Y}$ and $\mathbb{B}$ are convex, compact subsets of $\mathbb{R}$ hold. Let $R_t^{\sigma,\theta} \equiv 1/q_t^{\sigma,\theta} = q^{\sigma,\theta}$ be the return of the risky debt ($R_t^{\sigma,\infty}$ is the return without the fears about model misspecification (i.e., $m[\cdot] \equiv 1$) and $R_t^{0,\infty}$ is the return without fears about model misspecification and risk-neutral lenders). Suppose the endowment is an i.i.d. random variable. Then it follows:*

$$R_t^{\sigma,\theta} = R_{f,t}^{\sigma,\theta} \left\{ \frac{1}{1 - \mathrm{Pr}_t^{\sigma,\theta}(D(B_{t+1}))} \right\}$$

where $R_{f,t}^{\sigma,\theta} = (1 + r_t^{\sigma,\theta})$ is the gross risk-free rate return. The function $\mathrm{Pr}_t^{\sigma,\theta}$ is a probability measure given by

$$Pr_t^{\sigma,\theta}(A) \equiv \int_{\mathbb{Y}^2 \cap A} \frac{\nabla_c U(c_{t+1}^L) m[\mathcal{W}^L](y_t, y_{t+1}^B, B_{t+1}) f_{Y'|Y}(y_{t+1}|y_t)}{\int_{\mathbb{Y}} \nabla_c U(c_{t+1}^L) m[\mathcal{W}^L](y_t, y_{t+1}^B, B_{t+1}) f_{Y'|Y}(y_{t+1}|y_t) dy_{t+1}} dy_{t+1}$$

for all $A \in \mathcal{F}_Y$.

The lemma sharply characterizes the mark up over the risk-free rate as the inverse of the probability of no default. The main point of the theorem is that this probability is adjusted by (a) risk aversion of the lenders and (b) the degree of model uncertainty. In a way, the lemma extends results found for risk-neutral agents and without model uncertainty in previous studies (see Arellano (2008)).

The next corollary shows how the premium is decomposed as a function of the probability of no default—this is the term that appear in the standard models of default—,and an additional term which takes into account the risk aversion and the fear of misspecification.

COROLLARY 2.1. *Under the same assumptions of lemma 2.3, it follows:*

$$\frac{R_t^{\sigma,\theta}}{R_{f,t}^{\sigma,\theta}} = \left(Pr_t^{0,\infty}(D(B_{t+1})^C) + \int_{y_{t+1} \in D(B_{t+1})^c} \left\{ \frac{\nabla_c U(c_{t+1}^L) \exp\left\{-\frac{\mathcal{W}_{t+1}^L}{\theta}\right\}}{E_t\left[\nabla_c U(c_{t+1}^L) \exp\left\{-\frac{\mathcal{W}_{t+1}^L}{\theta}\right\}\right]} - 1 \right\} f_{Y'|Y}(y_{t+1}|y_t) dy_{t+1} \right)^{-1}$$

where $Cov_t^{0,\infty}$ is the covariance with respect to $Pr_t^{0,\infty}(\cdot)$.

We manage to decompose the markup over the risk-free rate into two parts, first the *objective* probability of no default and a second term that accounts for (a) risk aversion and (b) model uncertainty. Under risk neutrality and no model uncertainty the whole second term vanishes, but if we increase either the risk aversion or degree of model uncertainty, $\nabla_c U(c_{t+1}^L) \exp\left\{-\frac{\mathcal{W}_{t+1}^L}{\theta}\right\}$ becomes a non-degenerate random variable and its deviations from the mean increase, thereby implying a larger correction. The exact magnitude will be determined in the simulations.

3 Numerical Results

This section analyzes the quantitative implications of the model. The benchmark model is calibrated for Argentina for the period spanned from the second quarter of 1983 to the last quarter of the year 2001, when Argentina defaulted on its foreign debt. In table 1 we present the parameter values for the benchmark model.

The model is solved numerically using value function iteration. To that end, we apply the discrete state space (DSS) technique.

Table 1: Parameter Values

		Parameter	Value
Borrower			
	Risk aversion	σ^B	2
	Time discount factor	β^B	0.912
	Tuning parameter	h	0.0003
	Probability of reentry	π	0.282
	Output cost parameter	κ	0.90
	AR(1) coefficient for y_t^B	ρ^B	0.945
	Std. deviation of ε_t^B	σ_ε^B	0.025
Lender			
	Risk aversion	σ^L	0
	Robustness parameter	θ	0.1
	Time discount factor	β^L	0.983
	Constant for y^L	α^L	3.83

We will refer to h as our tuning parameter. When solving the model using the DSS technique, the discontinuity of the optimal debt policy implies discontinuity in the price function. Due to the presence of lender's risk aversion, the potentially discontinuous optimal debt policy appears in the Euler equation. This may translate into lack of convergence for the iterations. To handle this computational difficulty, we introduce an i.i.d. preference shock in the spirit of McFadden (1981) and Rust (1994). This preference shock, denoted by v, is drawn from a logistic(h) distribution $F_\nu(v) = \frac{1}{1+\exp\{-h^{-1}v\}}$, where the parameter h controls for the variance of the distribution. In the computation we set it to very low values with the purpose of guaranteeing convergence while having marginal effects on the equilibrium dynamics. More specifically, we select its value to obtain in the simulations that at most 10 percent of the times the borrower defaults the preference shock prompts the declaration of default[10]. For details on the computational algorithm see section 4.

The coefficient of relative risk aversion for the borrower is set to 2, which is standard in the sovereign default literature. The re-entry probability π is set to 0.282, as in Arellano (2008), which is consistent with Gelos et al. (2004) estimates of average time period of less than one year a country is excluded from financial markets following a default. The lender's discount factor β^L is set equal to the reciprocal of a risk-free rate of 1.7 percent, which is the average

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[10]For the calibrated model, we have that only 8 percent of the defaults are caused by a low enough realization of the preference shock.

quarterly interest rate of a five-year US treasury bond for the period in consideration.

The interest rate series for Argentina are constructed by adding the quarterly EMBI+ spreads from JP Morgan's EMBI+ database on Argentinean foreign currency denominated bonds to the the average interest rate of five-year US treasury bonds. Interest rates are reported as percentages in annual terms.

Time series at a quarterly frequency for output, consumption and net exports for Argentina are taken from the Ministry of Finance (MECON). All these series are seasonally adjusted, in logs and filtered using a linear trend. Net exports are computed as a percentage of output.

We assume that the endowment of Argentina follows a log-normal AR(1) process,

$$\log y_{t+1}^B = \rho^B \log y_t^B + \sigma_\varepsilon^B \varepsilon_{t+1}^B$$

where the shock $\varepsilon_{t+1}^B \sim i.i.d.\mathcal{N}(0,1)$. We estimate this stochastic process using the output time series. The endowment space is discretized into 21 points and the stochastic process is approximated to a Markov chain using Tauchen and Hussey (1993) quadrature-based method. Finally, we set the lender's log endowment to be constant and equal to 3.83 to replicate the size of the Argentinean economy relative to the U.S.

Following Arellano (2008), we consider the following functional form for the output costs of default

$$h^d(y^B) = \begin{cases} \widehat{y}^B & \text{if } y^B > \widehat{y}^B \\ y^B & \text{if } y^B \leq \widehat{y}^B \end{cases}$$

with $\widehat{y}^B = \kappa E(y^B)$. For output levels below the threshold $\widehat{y}^B$, there is no output loss in autarky. For output levels over $\widehat{y}^B$, this direct cost as a percentage of output is increasing in output. This particular specification for the output costs of default has significant implications for the dynamics of debt and default events in the model. For high levels of output, a borrower is more severely punished and, hence, has less incentives to default. As the likelihood of default is lower, lenders demand low returns on their bond holdings. Faced with low returns, or equivalently higher bond prices, the borrower responds by borrowing large amounts of debt when output is high. For low levels of output, the costs of defaulting are lower, hence, the default risk is higher, and so are the bond returns. Repaying the debt is in turn more costly, driving consumption even lower. If the borrower is hit by a sequence long enough of bad output realizations, it eventually finds it optimal to declare default.

To compute the business cycle statistics, we run $1,000$ Monte Carlo simulations of the model with $1,000$ periods each. As in Arellano (2008), to replicate the period for Argentina between default events, from 1983:Q3 to 2001:Q4, we consider sub-samples of 74 periods with access to financial markets followed by a default event. We then compute the mean statistics for these sub-samples.

We calibrate the model by setting the borrower's discount factor β^B to match an annual frequency of default of 3 percent, and selecting penalty parameter θ that measures the concern about model misspecification to match a detection error probability of 0.2. For details about the calibration of θ see section 5. The calibrated value for β^B is 0.912, which is an intermediate value between the 0.953 set by Arellano (2008), and 0.80 as used in other studies, such as Aguiar and Gopinath (2007).

Table 2: Business Cycle Statistics for Model, data and Arellano

Statistic	Data	Arellano	Model
$\text{mean}(r - r^f)$	10.25	3.58	9.71
$\text{std.dev.}(r - r^f)$	5.58	6.36	12.28
$\text{mean}(-b/y^B)$	53.3	5.95	5.96
$\text{std.dev.}(c^B)/\text{std.dev. }(y^B)$	1.10	1.10	1.08
$\text{std.dev.}(tb/y^B)$	1.75	1.50	1.63
$\text{corr}(y^B, c^B)$	0.98	0.97	0.97
$\text{corr}(y^B, r - r^f)$	-0.88	-0.29	-0.71
$\text{corr}(y^B, tb/y^B)$	-0.64	-0.25	-0.18
Default frequency	3.00	3.00	3.00

The model matches the standard empirical regularities of the Argentinean economy. First, we can account for almost all the average bond spreads observed in the data matching at the same time the historical frequency of default of 3 percent. While for the period in consideration annual bond spreads for Argentina were on average 10.25 percent in the data, our model can explain up to 9.71 percent which is almost three times higher than the 3.58 percent obtained by Arellano (2008). In this particular case, since these statistics for our model correspond to the case of risk-neutral lenders, i.e. $\sigma^L = 1$, the differential in spreads relative to Arellano (2008) is entirely due to these fears about model misspecification. In our model, lenders charge an additional uncertainty premium on bond holdings to get compensates for bearing the default risk under the worst-case density.

We consider this is an important contribution of the paper. To our knowledge, there is only one other paper that has been able to match the average bond spreads observed in the data, Chatterjee and Eyingungor (2010). In their model, the borrower issues long-term debt. In contrast with models with one-period bonds, they are able to generate positive spreads even when there is no default risk in the near future. The authors reproduce the average high spreads for Argentina, but at the cost of doubling the default frequency to 6 percent annually.

Furthermore, we want to stress that we replicate this feature of the bond spreads in a general equilibrium framework. Arellano (2008) and some recent studies on long-term debt, such as Arellano and Ramanarayanan (2010) and Hatchondo et al. (2010), have been able to account for it but by assuming an ad-hoc functional form for the stochastic discount factor, which depends on the output shock to the borrowing economy. Our paper can then be seen as providing microfoundations for such a functional form.

Also, the introduction of plausible degrees of risk aversion on the lenders' side with time-separable preferences has shown not to be sufficient to recover the high spreads observed in the data. With constant relative risk aversion, as in Lizarazo (2010), matching high spreads calls for very large risk aversion coefficient and implausible risk-free rates, as Mehra and Prescott (1985) and Weil (1989). in the context of studies on equity premium. Borri and Verdelhan (2010) have studied the setup with positive comovement between lender's consumption and output in the emerging economy in addition to time-varying risk aversion on the lenders' side. To generate endogenous time-varying risk aversion for lenders, they endow them with Campbell and Cochrane (1999) preferences with external habit formation. However, they find that even with these additional components average bond spreads generated by the model are far below from those in the data[11]

Second, the model can deliver strongly countercyclical and very volatile bond spreads. When output goes up, the default risk decreases, and, hence, lenders demand lower returns on their bond holdings. The correlation between borrower's output and bond spreads is slightly below the one observed in the data, but still higher than in Arellano (2008). The model, however, delivers twice as much volatility of the bond spreads as in the data. Along these two dimensions, it is worth noting that Hatchondo et al. (2010) showed that the computational method used to solve numerically the model has important implications for the results. In particular, when solving the model of Arellano (2008), they found that if finer grids for the endowment and assets are considered using the DSS technique, or value functions are approximated with splines or Chebyshev polynomials, standard deviation of bond spreads falls by more than fifty percent, and the negative correlation between output and bond spreads is higher. It should not be surprising anyhow that even in these cases interest rates are not as countercyclical as observed in the data since we consider an endowment economy and there is no feedback back from interest rates into output.

Third, the model reproduces higher volatility of consumption relative to output, and volatility and countercyclicality of net exports. In our model, these features result from the

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[11]Borri and Verdelhan (2010) report average bond spreads of 4.27 percent for an annual default frequency of 3.11 percent.

short-term maturity of debt and the particular specification for the output costs of default. Notice that to maintain the same level of indebtness b, the borrower needs to refinance it at the new price $q(y, b, b)$ which, given our specification for the output loss function, is very sensitive to fluctuations in output. As output increases, bond prices are higher reflecting the smaller default likelihood. With cheaper debt, borrowing increases substantially, and consumption rises even more than output. Conversely, in downturns, default risk is higher; in turn, servicing the debt is more costly, and consumption exhibits a relatively larger decline. Consequently, consumption and net exports are more volatile, and the latter are countercyclical.

Our model falls short in explaining quantitatively the negative comovement between output and net exports, which is a common feature sovereign models fail to account.

Table 3: Business Cycle Statistics for Different Degrees of Robustness

Statistic	$\theta = +\infty$	$\theta = 2$	$\theta = 1$	$\theta = 0.5$	$\theta = 0.25$	$\theta = 0.1$
mean$(r - r^f)$	3.41	3.81	4.75	5.77	7.11	9.72
std.dev.$(r - r^f)$	6.15	6.54	7.49	7.22	9.21	12.28
mean$(-b/y^B)$	12.81	13.96	12.82	11.21	9.15	5.96
std.dev.(c^B)/std.dev.(y^B)	1.15	1.19	1.18	1.15	1.13	1.08
std.dev.(tb/y^B)	1.93	2.54	2.45	2.35	2.21	1.63
corr(y^B, c^B)	0.96	0.94	0.94	0.94	0.95	0.97
corr$(y^B, r - r^f)$	-0.56	-0.59	-0.60	-0.69	-0.66	-0.71
corr$(y^B, tb/y^B)$	-0.33	-0.27	-0.26	-0.23	-0.21	-0.18
Default frequency	3.00	3.00	3.00	3.00	3.00	3.00

In table 3 we report the business cycle statistics from the simulations of our model for different degrees of model uncertainty and no risk aversion on the lender's side, i.e. $\sigma^L = 0$. We start with no fears about model misspecification, i.e. $\theta = +\infty$, and lower the penalty parameter up to 0.1, for which we obtain a detection error probability of 20 percent. As we reduce the value of θ, we observe that the frequency of default goes down. To keep it at the historical level of 3 percent, we adjust β^B downwards.

The first finding is that both the mean and the standard deviation of bond spreads increase with the lender's concerns about model misspecification. A lower θ implies a higher distorted probability associated to low utility states for the lender, which in turn implies lower and more volatile bond prices.

Second, the level of indebtness falls significantly. As the demand for bonds decreases, price and quantity demanded go down. Average debt-to-out ratios for the borrower move from around 12 percent to slightly below 6 percent.

Finally, our models explain a higher countercyclicality of interest rates which is consistent with what we observe in the data. As output declines, the default probability under the approximating model increases. The evil alter ego in turn induces a higher probability distortion in the conditional density of next period's output of the borrower in the interval of states associated with default risk. The higher distorted probability of default then implies higher bond spreads in equilibrium. When output increases, the opposite occurs: the distorted probability of default goes down and so do the bond spreads.

Table 4: Business Cycle Statistics for Different Degrees of Risk Aversion

Statistic	$\sigma^L = 0$	$\sigma^L = 1$	$\sigma^L = 2$
mean($r - r^f$)	3.41	3.91	3.59
std.dev.($r - r^f$)	6.15	7.97	8.36
mean($-b/y^B$)	12.81	12.73	11.12
std.dev.(c^B)/std.dev.(y^B)	1.15	1.15	1.08
std.dev.(tb/y^B)	1.93	1.94	1.64
corr(y^B, c^B)	0.96	0.96	0.97
corr($y^B, r - r^f$)	-0.56	-0.55	-0.48
corr($y^B, tb/y^B$)	-0.33	-0.33	-0.22
Default frequency	3.00	3.00	3.00

Finally, as display in table 4, plausible degrees of risk aversion with no fears about model misspecification, i.e. $\theta = +\infty$, are not enough to generate sufficiently high bond spreads while keeping the default frequency as observed in the data[12].To show that, we considered a stochastic process for the lender's endowment given by

$$\ln y_{t+1}^L = \alpha^L + \sigma_\varepsilon^L \varepsilon_{t+1}^L$$

where $\varepsilon_{t+1}^L \sim i.i.d.\mathcal{N}(0,1)$. Shocks ε_{t+1}^B and ε_{t+1}^L are assumed to be independent. The parameter σ_ε^L is set to 0.008, which corresponds to the estimated conditional standard deviation of the U.S. output.

4 Numerical Algorithm

Solving sovereign default models using the discrete state space (DSS) technique may encounter convergence problems for some particular environments. Those environments include models

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[12]For each value of σ^L, the discount factor for the borrower, β^B, is calibrated to replicate a default frequency of 3 percent annually.
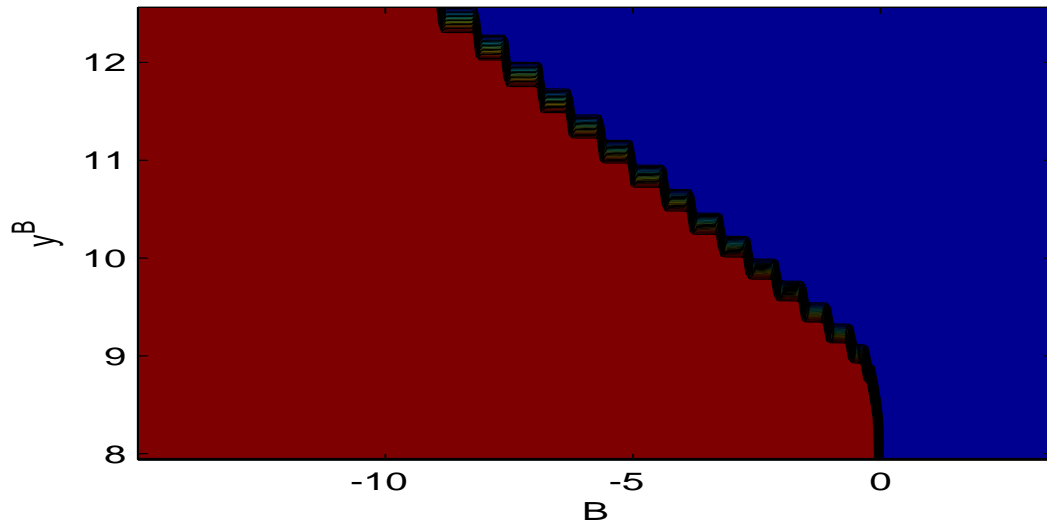
Figure 4: Borrower default policy function. Blue is the non-default region and Red is the default region.
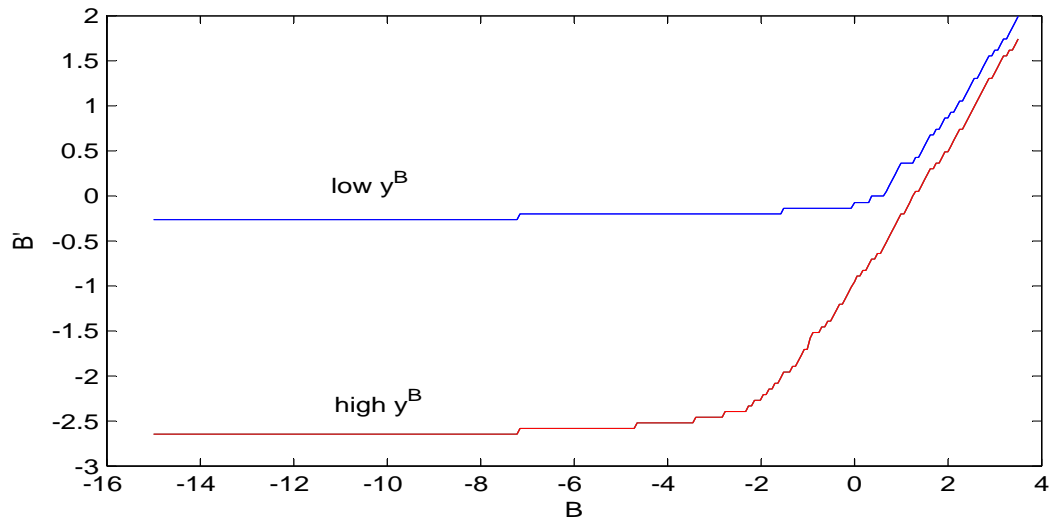

Figure 5: Borrower debt policy function.

with larger maturity structure of debt as Chatterjee and Eyingungor (2010), as well as environments with risk averse lenders, as first analyzed by Lizarazo (2010). The borrower's budget set is in general nonconvex and his value function is not strictly concave in the bond holdings as a consequence of the option to default. Due to this lack of convexity of the borrower's problem the debt policy function is not continuous in the current endowment and debt level, and prices. This discontinuity of the debt policy function in prices may induce discontinuity in the lender's Euler equation (5), which may translate into lack of convergence.

More specifically, in our environment the expectations term on the right hand side of the price equation (4) depend on the optimal debt policy through the lender's consumption next period. The discontinuity of the debt policy in prices, when using grids for debt and endowment, may generate discrete changes on it for small variation in prices. When solving the model numerically, the code may fluctuate between two different debt plans and never converge.

To handle this technical complication we propose an approach based on the introduction of an i.i.d. preference shock based on McFadden (1981) and Rust (1994). This preference shock[13], denoted by v, is drawn from a logistic(h) distribution $F_\nu(v) = \frac{1}{1+\exp\{-h^{-1}v\}}$, where the parameter h controls for the variance of the distribution. To our knowledge, in the sovereign default literature, there is only one more paper, Chatterjee and Eyingungor (2010), that provides a similar approach to deal with a problem of the same nature. In a model with long-term debt, Chatterjee and Eyingungor (2010) proposes an accurate method based on the introduction of a continuous i.i.d. shock to output. Once this i.i.d. shock is incorporated, they are able to show the existence of a unique equilibrium price function for long-term debt with the property that the return on debt is increasing in the amount borrowed.

Formally, let $(\nu_t)_t$ be a sequence of ii-logistic(h) random variables; i.e., $F_\nu(v) = \frac{1}{1+\exp\{-h^{-1}v\}}$. In our numerical simulations we set $h \approx 0$ so $F_\nu(v) \approx 1\{\nu \geq 0\}$

Following McFadden (1981) and Rust (1994), we assume that ν_t enters the utility in an additive separable fashion when the economy is not in default, i.e.,

$$U(c_t) + \nu_t \delta_t(y_t, B_t).$$

We can cast the new Bellman equation problem as

$$V(y_t, B_t, v_t) = \max\{V_c^B(y_t, B_t) + \nu_t, V_d^B(y_t)\}$$

---

[13]The shock ν could be interpreted as an error of an agents that intends to behave according to a certain payoff, but she incorrectly calculates the payoff by adding a noise.

where

$$V_c^B(y_t, B_t) = \max_{c_t, B_{t+1}^B} \left\{ U(c_t^B) + \beta^B \int_{\mathbb{Y} \times \mathbb{Y}} \int_{\mathbb{R}} V(y_{t+1}, B_{t+1}, v_{t+1}) F_\nu(dv_{t+1}) f_{Y'|Y}(y_{t+1}|y_t) dy_{t+1} \right. .$$

Note that, since

$$V(y_{t+1}, B_{t+1}, v_{t+1}) = \max\{ V_c^B(y_{t+1}, B_{t+1}) - V_d^B(y_{t+1}) + v_{t+1}, 0 \} + V_d^B(y_{t+1})$$

it follows, by our choice of F_ν, that

$$\int_{\mathbb{R}} V(y_{t+1}, B_{t+1}, v_{t+1}) F_\nu(dv_{t+1}) = \int_{-\Delta V^B(t+1)}^{\infty} \{\Delta V^B(t+1) + v_{t+1}\} F_\nu(dv_{t+1}) + V_d^B(y_{t+1}) \quad (6)$$

where $\Delta V^B(t+1) \equiv V_c^B(y_{t+1}, B_{t+1}) - V_d^B(y_{t+1})$. Moreover

$$\int_{-\Delta V^B(t+1)}^{\infty} \{\Delta V^B(t+1) + v_{t+1}\} F_\nu(dv_{t+1}) = \int_0^{\infty} v_{t+1} f_\nu(v_{t+1} - \Delta V^B(t+1)) dv_{t+1}$$

where f_v is the logistic(h) density function. Doing integration by parts and some algebra, it follows

$$\int_0^{\infty} v_{t+1} f_\nu(v_{t+1} - \Delta V^B(t+1)) dv_{t+1} = -v_{t+1}(1 - F_\nu(v_{t+1} - \Delta V^B(t+1))) \mid_0^{\infty}$$

$$+ \int_0^{\infty} 1 - F_\nu(v_{t+1} - \Delta V^B(t+1)) dv_{t+1}$$

$$= \int_0^{\infty} 1 - F_\nu(v_{t+1} - \Delta V^B(t+1)) dv_{t+1}$$

Finally,

$$\int_0^{\infty} 1 - F_\nu(v_{t+1} - \Delta V^B(t+1)) dv_{t+1} = -h \log\left(1 + \exp\{-h^{-1}(v_{t+1} - \Delta V^B(t+1))\}\right) \mid_0^{\infty}$$

$$= h \log\left(1 + \exp\{h^{-1} \Delta V^B(t+1)\}\right).$$

Therefore,

$$\int_{-\Delta V^B(t+1)}^{\infty} \{\Delta V^B(t+1) + v_{t+1}\} F_\nu(dv_{t+1}) = h \log\left(1 + \exp\{h^{-1} \Delta V^B(t+1)\}\right),$$

and the Bellman equation is given by

$$V_c^B(y_t, B_t) = \max_{c_t, B_{t+1}} \left\{ U(c_t^B) + \beta^B \int_{\mathbb{Y} \times \mathbb{Y}} \left(h \log\left(1 + \exp\{h^{-1} \Delta V^B(t+1)\}\right) + V_d^B(y_{t+1}) \right) f_{Y'|Y}(y_{t+1}|y_t) dy_{t+1} \right. .$$

This is the new Bellman operator. The default policy function now is a mapping $(y_t, B_t) \mapsto \int_{\mathbb{R}} 1\{V_d^B(y_t) \leq V_c^B(y_t, B_t) + v_t\} F_\nu(dv_t)$. In particular, the default function turns now to be a probability,

$$(y_t, B_t) \mapsto \frac{1}{1 + \exp\{-h^{-1} \Delta V^B(t+1)\}}. \quad (7)$$

The fact that we obtain closed forms for continuation values (in the normal stage) and for the default indicator, given by expressions (6) and (7), respectively, has important implications in terms of computation. In contrast with Chatterjee and Eyingungor (2010), our approach has the advantage that it does not rely on constructing an additional grid for the i.i.d. shock.

5   Detection Error Probabilities

This section describes how to calibrate the penalty parameter θ using Bayesian detection error probabilities. We proceed by assuming that the lender in this economy is concerned about probability models that are difficult to distinguish from each other given the dataset available. To that extent, we consider likelihood ratio tests for distinguishing the approximating model from some worst-case model associated to some penalty parameter θ. Let model A be the approximating model and model W be the worst-case model for some θ. We are interested in computing after observing some number of realizations how often we commit errors type I through the likelihood ratio test, that is, rejecting a null hypothesis that is true. In particular, how often we cannot reject that model W generated the data when model A actually did. Similarly, how many times the likelihood ratio test says model A generated the data when in fact model W did.

We start by setting an initial debt level and endowment vector. We first simulate time series for 74 quarters for output of the lender using the conditional density f^L. We then generate time series of similar length for output of the borrower under the approximating model, given by the transition density f^B, and compute bond holdings and default decisions using optimal policies. We evaluate the likelihood functions L_A and L_W of the approximating model and the worst-case model, respectively. The process is repeated 1,000 times. We then compute the percentage of the times the likelihood ratio test falsely indicates that the worst-case model generated the data,

$$p_A = \Pr \left(\log \frac{L_W}{L_A} > 0 \mid A \right)$$

The percentage p_A approximates the probability that the likelihood ratio indicates that the worst-case model generated the data when the approximating model actually did.

We repeat the same exercise by simulating the time series for the output of the borrower now under the worst-case model. To do so, we need to generate the time series for output of the lender using f^L, and apply again the optimal debt and default policy functions. We then calculate the percentage of the times the likelihood ratio test falsely says that the

approximating model generated the data,

$$p_W = \Pr\left(\log \frac{L_A}{L_W} > 0 \mid W\right)$$

The percentage p_W approximates the probability that the likelihood ratio indicates that the approximating model generated the data when the it was actually generated by the worst-case model. The detection error probability is obtained then by averaging p_A and p_W

$$p(\theta^{-1}) = \frac{1}{2}(p_A + p_W)$$

where θ is the penalty parameter associated to the worst-case model. If the approximating model coincides with the worst-case model, they are indistinguishable from each other and the detection error probability is 0.5. If instead they are distant from each other, the detection error probability is below 0.5, getting closer to 0 as the discrepancy between models is larger.

We consider a reasonable value for the detection error probability is 0.2. We calibrate then θ to match this detection error probability. Since the function $p(\theta^{-1})$ differs for different approximating models, we will need to calibrate it for each stochastic process for the endowment we consider. In particular, when we switch from a deterministic endowment for the lender into a stochastic one. Same argument applies for different degrees of risk aversion on the lender's side.

6   Conclusion

The paper accounts for the high bond spreads observed for emerging economies, in particular for Argentina, while, first, keeping the default frequency at historical levels, and, at the same time, matching the standard empirical regularities of these economies. We achieve this by introducing fears about model misspecification on the lenders. Lenders in this economy fear that the probability model governing the evolution of the endowment of the borrower is misspecified. They contemplate a set of alternative probability models and seek decisions rule that perform well across them. To compensate for a risk and uncertainty adjusted probability of default they demand higher returns on their bond holdings.

Also, we propose an approach to tackle convergence issues when solving the model numerically using the DSS technique. This approach is based on the introduction of an i.i.d. preference shock for the borrower drawn from a logistic distribution.

In future research we plan to extend the framework to account for a process for the borrower's endowment with stochastic trend. As pointed out by Aguiar and Gopinath (2007), shocks to trend growth -rather than transitory fluctuations- are the primary source of fluctuations in emerging markets. We would like to explore the implications that substantial

volatility in trend growth jointly with the inability of clearly identify transitory from trend shocks have on the dynamics of prices and allocations in emerging economies.

A   Fears about Model Misspecification of Two Stochastic Processes

In this section we allow the lender to distrust both the stochastic process for the endowment of the borrower, y^B, as well as of his own, y^L, but to a different extent. We derive the exponential twisting formulas for the probability distortions and the associated risk-sensitivity operators for a general environment. We assume the endowment vector $y_t = (y_t^B, y_t^L)$ follows a Markov process with joint conditional density $f(y_{t+1}^B, y_{t+1}^L | y_t^B, y_t^L)$.

Let a nonnegative $m(y_{t+1}^B, y_{t+1}^L | y^t)$ denote the probability distortion to the joint conditional density $f(y_{t+1}^B, y_{t+1}^L | y_t^B, y_t^L)$.

We first apply Bayes' rule to decompose the probability distortion $m(y_{t+1}^B, y_{t+1}^L | y^t) = m(y_{t+1}^B | y^t) m(y_{t+1}^L | y^t, y_{t+1}^B)$. Let the penalty parameter θ_1 govern the degree of model misspecification concerning the probability distribution of the next period endowment of the borrower y_{t+1}^B, conditional on current y_t^B and y_t^L. Let the penalty parameter θ_2 measure the concerns about model misspecification regarding the distribution of the next period endowment of the lender y_{t+1}^L, conditional on current y_t^B and y_t^L, and next period y_{t+1}^B.

To simplify the notation, let m_1 denote $m(y_{t+1}^B | y^t)$ and m_2 denote $m(y_{t+1}^L | y^t, y_{t+1}^B)$. We also omit other state variables.

The utility of the lender is then given by

$$W(y_t^B, y_t^L) = \min_{m_1, m_2 \geq 0} u(c_t) + \beta^L \int f(y_{t+1}^B | y_t^B, y_t^L) m_1 \left[\theta_1 \log m_1 \right.$$

$$\left. + \int f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) m_2 \left\{ \theta_2 \log m_2 + W(y_{t+1}^B, y_{t+1}^L) \right\} dy_{t+1}^L \right] dy_{t+1}^B$$

$$\text{s.t.} \int m_2 f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) dy_{t+1}^L = 1 \tag{8}$$

$$\int m_1 f(y_{t+1}^B | y_t^B, y_t^L) dy_{t+1}^B = 1 \tag{9}$$

Taking FOC with respect to m_2 yields

$$\theta_2 \log m_2 + \theta_2 + W(y_{t+1}^B, y_{t+1}^L) = 0$$

$$\log m_2 = -\frac{W(y_{t+1}^B, y_{t+1}^L)}{\theta_2} - 1$$

$$m_2 = \exp(-1) \exp\left(-\frac{W(y_{t+1}^B, y_{t+1}^L)}{\theta_2} \right)$$

Dividing both sides by $\int m_2 f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) dy_{t+1}^L$, and combining with constraint (9),

delivers

$$m_2 = \frac{\exp\left(-\frac{W(y_{t+1}^B, y_{t+1}^L)}{\theta_2}\right)}{\int \exp\left(-\frac{W(y_{t+1}^B, y_{t+1}^L)}{\theta_2}\right) f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) dy_{t+1}^L}$$

Replacing m_2 by this expression in $\int f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) m_2 \left\{ \theta_2 \log m_2 + W(y_{t+1}^B, y_{t+1}^L) \right\} dy_{t+1}^L$
yields

$$\int f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) m_2 \left\{ \theta_2 \log m_2 + W(y_{t+1}^B, y_{t+1}^L) \right\} dy_{t+1}^L$$

$$= \int f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) m_2 \left\{ -\theta_2 \log \int \exp\left(-\frac{W(y_{t+1}^B, y_{t+1}^L)}{\theta_2}\right) f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) dy_{t+1}^L \right\} dy_{t+1}^L$$

$$= \left\{ -\theta_2 \log \int \exp\left(-\frac{W(y_{t+1}^B, y_{t+1}^L)}{\theta_2}\right) f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) dy_{t+1}^L \right\} \int f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) m_2 dy_{t+1}^L$$

$$= -\theta_2 \log \int \exp\left(-\frac{W(y_{t+1}^B, y_{t+1}^L)}{\theta_2}\right) f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) dy_{t+1}^L \tag{10}$$

where the third equality uses constraint (9).

Replacing $\int f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) m_2 \left\{ \theta_2 \log m_2 + W(y_{t+1}^B, y_{t+1}^L) \right\} dy_{t+1}^L$ by expression (10), and
differentiating the utility of the lender with respect to m_1 we obtain

$$\theta_1 \log m_1 + \theta_1 = \theta_2 \log \int \exp\left(-\frac{W(y_{t+1}^B, y_{t+1}^L)}{\theta_2}\right) f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) dy_{t+1}^L$$

$$\log m_1 = \frac{\theta_2}{\theta_1} \log \int \exp\left(-\frac{W(y_{t+1}^B, y_{t+1}^L)}{\theta_2}\right) f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) dy_{t+1}^L - 1$$

$$m_1 = \exp(-1) \exp\left(\frac{\theta_2}{\theta_1} \log \int \exp\left(-\frac{W(y_{t+1}^B, y_{t+1}^L)}{\theta_2}\right) f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) dy_{t+1}^L\right)$$

Using constraint (8) yields

$$m_1 = \frac{\exp\left(\frac{\theta_2}{\theta_1} \log \int \exp\left(-\frac{W(y_{t+1}^B, y_{t+1}^L)}{\theta_2}\right) f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) dy_{t+1}^L\right)}{\int \exp\left(\frac{\theta_2}{\theta_1} \log \int \exp\left(-\frac{W(y_{t+1}^B, y_{t+1}^L)}{\theta_2}\right) f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) dy_{t+1}^L\right) f(y_{t+1}^B | y_t^B, y_t^L) dy_{t+1}^B}$$

Throughout the paper we assume no concerns about model misspecification regarding the
distribution of the next period endowment of the lender y^L, conditional on current y_t^B and y_t^L,
and next period y_{t+1}^B, i.e. $\theta_2 = +\infty$.
In this case, $m_2 = 1$ and $-\theta_2 \log \int \exp\left(-\frac{W(y_{t+1}^B, y_{t+1}^L)}{\theta_2}\right) f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) dy_{t+1}^L$ collapses to
$\int W(y_{t+1}^B, y_{t+1}^L) f(y_{t+1}^L | y_t^B, y_t^L, y_{t+1}^B) dy_{t+1}^L$, which we refer to as $\mathcal{W}^L(y_t, y_{t+1}^B)$.

B Proofs

The following is a technical high-level assumption, that ensures that positive consumption is always available.

ASSUMPTION B.1. *Given the price function q_t, for any $(y_t, B_t, b_t) \in \mathbb{Y}^2 \times \mathbb{B}^2$ and t, the following holds*

$$\{c : c = y_t^B + q_t B_{t+1} - B_t\} \cap \mathbb{R}_+ \neq \{\emptyset\}$$
$$\{c : c = y_t^L + q_t b_{t+1} - b_t\} \cap \mathbb{R}_+ \neq \{\emptyset\}.$$

This assumptions ensure that the budget constraint correspondence always yields positive consumption. For instance, let B_{max}, B_{min} and y_{min}^B be the maximum and minimum elements of $\mathbb{B}$ and $\mathbb{Y}$, it suffice to assume that $\beta B_{min} + y_{min}^B \geq B_{max}$ (where we are implicitly assuming that there is no default under B_{min}).

Proof of Lemma 2.1. In this proof we actually show a stronger result than lemma 2.1, by introducing the operator T^θ instead of the usual conditional expectation operator.

The definitions of $L^\infty(\mathbb{Y}^2 \times \mathbb{B} \times \{0,1\})$, $||\cdot||$ and $C(\mathbb{Y}^2 \times \mathbb{B} \times \{0,1\})$ are analogous to those in the proof of lemma 2.2.

Let $V(\cdot, \cdot, 0) = V_c(\cdot, \cdot)$ and $V(\cdot, \cdot, 1) = V_d(\cdot, \cdot)$. Let $L_0 : L^\infty(\mathbb{Y}^2 \times \mathbb{B} \times \{0,1\}) \to L^\infty(\mathbb{Y}^2 \times \mathbb{B})$ (by our choice of U and the assumption that $\mathbb{Y}^2 \times \mathbb{B}$ is compact in the product topology, it is easy to see that L_0 in fact maps L^∞ into itself) with

$$L_0[V](y_t, B_t) = \max_{B_{t+1}} \left\{ U(y_t^B + q_t B_{t+1} - B_t) + \beta^B E[V(y_{t+1}, B_{t+1})](y_t, B_{t+1}) \right\}.$$

Let $L_1 : L^\infty(\mathbb{Y}^2 \times \mathbb{B} \times \{0,1\}) \to L^\infty(\mathbb{Y}^2 \times \mathbb{B})$ (by our choice of U and the assumption that $\mathbb{Y}^2 \times \mathbb{B}$ is compact in the product topology, it is easy to see that L_1 in fact maps L^∞ into itself) with

$$L_1[V](y_t, B_t) = U(h^d(y_t^B)) + \beta^B E[(1 - \pi)V_d(y_{t+1}) + \pi V(y_{t+1}, 0)](y_t)$$

and $L(\cdot, \cdot, l) \equiv L_l$ for $l = \{0,1\}$. Note that, under assumption B.1, $L : L^\infty(\mathbb{Y}^2 \times \mathbb{B} \times \{0,1\}) \to L^\infty(\mathbb{Y}^2 \times \mathbb{B} \times \{0,1\})$.

STEP 1. Note that $L^\infty(\mathbb{Y}^2 \times \mathbb{B} \times \{0,1\})$ is a Banach space (endowed with the aforementioned norm, $||\cdot||$). Moreover, let $C(\mathbb{Y}^2 \times \mathbb{B} \times \{0,1\}) \equiv \{f \in L^\infty(\mathbb{Y}^2 \times \mathbb{B} \times \{0,1\}) : f \text{ is continuous in } (y, B)\}$. The space $(C(\mathbb{Y}^2 \times \mathbb{B} \times \{0,1\}), ||\cdot||)$ is a Banach space.

If V_c and V_d are continuous in (y, B), so is the envelope of this too. The mapping $v \mapsto E[v]$ also preserves continuity (note that $E[v]$ is constant as a function of y by the i.i.d. assumption, thus continuous). By our choice of the utility, U is continuous as a function of both y_t and B_t, under assumption B.1, so $L : C(\mathbb{Y}^2 \times \mathbb{B} \times \{0, 1\}) \to C(\mathbb{Y}^2 \times \mathbb{B} \times \{0, 1\})$.

Hence, in order to show existence, it is sufficient to check Blackwell's sufficient conditions. (1) Let V_1 and V_2 both in $C(\mathbb{Y}^2 \times \mathbb{B} \times \{0, 1\})$ such that $V_1(\cdot) \leq V_2(\cdot)$. Therefore, say $V_1(\cdot, \cdot, 1) = \max\{V_1(\cdot, \cdot, 0), V_1(\cdot, \cdot, 1)\}$ then

$$\max\{V_1(\cdot, \cdot, 0), V_1(\cdot, \cdot, 1)\} = V_1(\cdot, \cdot, 1) \leq V_2(\cdot, \cdot, 1) \leq \max\{V_2(\cdot, \cdot, 0), V_2(\cdot, \cdot, 1)\}$$

Applying conditional expectations does not modify it; thus, $L_0[V_1](\cdot) \leq L_0[V_2](\cdot)$. Similarly, we can show that $L_1[V_1](\cdot) \leq L_1[V_2](\cdot)$. Hence $L[V_1](\cdot) \leq L[V_2](\cdot)$. (2) Let $V + c$ with $c \in \mathbb{R}$; note that

$$\max\{V(\cdot, \cdot, 0) + c, V(\cdot, \cdot, 1) + c\} = \max\{V(\cdot, \cdot, 0), V(\cdot, \cdot, 1)\} + c.$$

It follows that discounting holds for L_0 and L_1. Therefore, it also holds for L.

By theorem 4.12 in Stokey and Lucas (1989), L is a contraction, and thus there exists a $V^* \in C(\mathbb{Y} \times \mathbb{B} \times \{0, 1\})$ such that $V^* = L[V^*]$. This proof result (1).

STEP 2. We now show result (2). Let $\mathcal{C} \equiv \{f \in L^\infty(\mathbb{Y}^2 \times \mathbb{B} \times \{0, 1\}) : f \text{ is nondecreasing} in y^B \text{ and } f \text{ is nondecreasing in } B\}$. Under $||\cdot||$, this set is a closed subset of $L^\infty(\mathbb{Y}^2 \times \mathbb{B} \times \{0, 1\})$. By our choice of U, $U(\cdot)$ is (strictly) increasing in B_t, thus

$$\left\{U(y_t^B + q_t B_{t+1} - B_{1,t}) + \beta^B E[V(y_{t+1}, B_{t+1})](y_t, B_{t+1})\right\}$$
$$< \left\{U(y_t^B + q_t B_{t+1} - B_{2,t}) + \beta^B E[V(y_{t+1}, B_{t+1})](y_t, B_{t+1})\right\}$$

for $B_{1,t} \geq B_{2,t}$. Applying "max" at both sides does not change the inequality, thus $L_0[V](y, B)$ is also nondecreasing in B.

Regarding y^B, note that by the i.i.d. assumption $E[V](y, B)$ is actually constant with respect to y^B, and thus trivially nondecreasing. By our choice of $U(\cdot)$, $U(\cdot)$ is also (strictly) increasing in y^B, so $L_0[V](y, B)$ is also nondecreasing in y^B.

Starting from an initial guess $V^0 \in \mathcal{C}$, it follows $(L[V^n])_n \subseteq \mathcal{C}$ and since we know it converges, it must be true that $\lim_{n \to \infty} L[V^n] = L[V^*] = W^* \in \mathcal{C}$ by closure of $\mathcal{C}$. We thus have that L_l maps nondecreasing (increasing) functions into decreasing (increasing) functions, so W^* is, in fact, increasing in y^B and B.

STEP 3. Result (3) follows from the theorem of the maximum, see Berge (1997). $\quad \square$

Proof of Lemma 2.2. Define the space of functions $L^\infty(\mathbb{Y}^2 \times \mathbb{B}^2 \times \{0,1\})$, and endow this space with the following norm $\max_{l=0,1} ||\cdot||_{L^\infty(\mathbb{Y}^2 \times \mathbb{B}^2)}$. Let $W(\cdot,\cdot,\cdot,0) = W_c(\cdot,\cdot,\cdot)$ and $W(\cdot,\cdot,\cdot,1) = W_d(\cdot,\cdot,\cdot)$. Let $L_0 : L^\infty(\mathbb{Y}^2 \times \mathbb{B}^2 \times \{0,1\}) \to L^\infty(\mathbb{Y}^2 \times \mathbb{B}^2)$, such that

$$L_0[W](y_t, B_t, b_t) = \max_{b_{t+1}} \left\{ U(y_t^L + q_t b_{t+1} - b_t) + \beta^L T^\theta[\mathcal{W}^L(y_t, y_{t+1}^B, \Gamma^L(y_t, B_t), b_{t+1})](y_t, B_{t+1}, b_{t+1}) \right\}$$

where $\mathcal{W}^L(y_t, y_{t+1}^B, B_{t+1}, b_{t+1}) \equiv \int_\mathbb{Y} \delta(y_{t+1}^B, y_{t+1}^L, B_{t+1}) W_c^L(y_{t+1}^B, y_{t+1}^L, B_{t+1}, b_{t+1}) + (1 - \delta(y_{t+1}^B, y_{t+1}^L, B_{t+1})) W_d^L(y_{t+1}^B, y_{t+1}^L) f^L(y_{t+1}^L | y_t^L) dy_{t+1}^L$. By our choice of U and assumption B.1, and the assumption that $\mathbb{Y}^2 \times \mathbb{B}^2$ is compact in the product topology, it is easy to see that L_0 in fact maps L^∞ into itself.

Let $L_1 : L^\infty(\mathbb{Y}^2 \times \mathbb{B}^2 \times \{0,1\}) \to L^\infty(\mathbb{Y}^2 \times \mathbb{B}^2)$, such that

$$L_1[W](y_t, B_t, b_t) = U(y_t^L) + \beta^L T^\theta[(1 - \varrho)\mathcal{W}_d^L(y_t, y_{t+1}^B) + \varrho\mathcal{W}^L(y_t, y_{t+1}^B, 0, 0)](y_t)$$

where $\mathcal{W}_d^L(\cdot,\cdot)$ and $\mathcal{W}^L(\cdot,\cdot,\cdot,\cdot)$ are the $t+1$-equilibrium values for the lender *before* the realization of y_{t+1}^L in financial autarky and in the normal stage, respectively. By our choice of U, assumption B.1, and the assumption that $\mathbb{Y}^2 \times \mathbb{B}^2$ is compact in the product topology, it is easy to see that L_1 in fact maps L^∞ into itself. Let $L(\cdot,\cdot,l) \equiv L_l$ for $l = \{0,1\}$. Note that $L : L^\infty(\mathbb{Y}^2 \times \mathbb{B}^2 \times \{0,1\}) \to L^\infty(\mathbb{Y}^2 \times \mathbb{B}^2 \times \{0,1\})$.

STEP 1. Note that $L^\infty(\mathbb{Y}^2 \times \mathbb{B}^2 \times \{0,1\})$ is a Banach space (endowed with the aforementioned norm). Moreover, let $C(\mathbb{Y}^2 \times \mathbb{B}^2 \times \{0,1\}) \equiv \{f \in L^\infty(\mathbb{Y}^2 \times \mathbb{B}^2 \times \{0,1\}) : f$ *is continuous in* $(y, B, b)\}$. The space $(C(\mathbb{Y}^2 \times \mathbb{B}^2 \times \{0,1\}), ||\cdot||)$ is a Banach space.

The mapping $w \mapsto T^\theta[w]$ preserves continuity (note that $T^\theta[w]$ is constant as a function of y by the i.i.d. assumption, thus continuous). By our choice of the utility, U is continuous as a function of y_t, B_t and b_t (under assumption B.1, so $L : C(\mathbb{Y}^2 \times \mathbb{B}^2 \times \{0,1\}) \to C(\mathbb{Y}^2 \times \mathbb{B}^2 \times \{0,1\})$.

Hence, in order to show existence, it is sufficient to check Blackwell's sufficient conditions. (1) Let W_1 and W_2 both in $C(\mathbb{Y}^2 \times \mathbb{B}^2 \times \{0,1\})$ such that $W_1(\cdot) \leq W_2(\cdot)$. Let $W_l^N(y, B, b) = (1 - \delta(y, B, b))W_l(y, B, b, 1) + \delta(y, B, b)W_l(y, B, b, 0)$, for $l = 1, 2$.

Therefore

$$(1 - \varrho)W_1(y, 0, 0, 1) + \varrho W_1^N(y, 0, 0) \leq (1 - \varrho)W_2(y, 0, 0, 1) + \varrho W_2^N(y, 0, 0)$$

taking conditional expectations with respect to y^L does not alter the inequality, applying $\exp\{-\cdot/\theta\}$ reverses it; applying conditional expectations with respect to y^B does not modify it; finally applying $-\theta \log(\cdot)$ reverses the inequality again to bring it to the original "position".

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Thus, $L_1[W_1](\cdot) \leq L_1[W_2](\cdot)$. Similarly, we can show that $L_0[W_1](\cdot) \leq L_0[W_2](\cdot)$. Hence $L[W_1](\cdot) \leq L[W_2](\cdot)$. (2) Let $W + c$ with $c \in \mathbb{R}$; note that

$$(1 - \varrho)(W(y, 0, 0, 1) + c) + \varrho(W^N(y, 0, 0) + c) = (1 - \varrho)W(y, 0, 0, 1) + \varrho W^N(y, 0, 0) + c.$$

Since c is constant, it is easy to see that $\beta^L T^\theta[\mathcal{W} + c] = \beta^L T^\theta[\mathcal{W}] + \beta^L c$. So, discounting holds for L_1. It is easy to see that it also holds for L_0. Therefore, it also holds for L.

By theorem 4.12 in Stokey and Lucas (1989), L is a contraction, and thus there exists a $W^* \in C(\mathbb{Y} \times \mathbb{B}^2 \times \{0, 1\})$ such that $W^* = L[W^*]$. This proves result (1).

STEP 2. We now show result (2). Let $\mathcal{C} \equiv \{f \in L^\infty(\mathbb{Y}^2 \times \mathbb{B}^2 \times \{0, 1\}) : f \text{ is nondecreasing}$ in y^B and f is nonincreasing in $b\}$. Under $|| \cdot ||$, this set is a closed subset of $L^\infty(\mathbb{Y}^2 \times \mathbb{B}^2 \times \{0, 1\})$. By our choice of U, $U(\cdot)$ is (strictly) decreasing in b, thus

$$\left\{ U(y_t^L + q_t b_{t+1} - b_{1,t}) + \beta^L T^\theta[\mathcal{W}(y_t, y_{t+1}^B, \Gamma^L(y_t, B_t), b_{t+1})](y_t, B_{t+1}, b_{t+1}) \right\}$$
$$< \left\{ U(y_t^L + q_t b_{t+1} - b_{2,t}) + \beta^L T^\theta[\mathcal{W}(y_t, y_{t+1}^B, \Gamma^L(y_t, B_t), b_{t+1})](y_t, B_{t+1}, b_{t+1}) \right\}$$

for $b_{1,t} \geq b_{2,t}$. Applying "max" at both sides does not change the inequality, thus $L_l[W](y, B, b)$ is also nonincreasing in b.

Regarding y^L, note that by the i.i.d. assumption $T^\theta[\mathcal{W}](y, B)$ is actually constant with respect to y^L, and thus trivially nondecreasing. By our choice of $U(\cdot)$, $U(\cdot)$ is also (strictly) increasing in y^L, so $L_l[W](y, B)$ is also nondecreasing in y^L.

Starting from an initial guess $W^0 \in \mathcal{C}$, it follows $(L[W^n])_n \subseteq \mathcal{C}$ and since we know it converges, it must be true that $\lim_{n \to \infty} L[W^n] = L[W^*] = W^* \in \mathcal{C}$ by closure of $\mathcal{C}$. We thus have that L_l maps nondecreasing (increasing) functions into decreasing (increasing) functions, so W^* is, in fact, increasing in y^L and decreasing in b.

STEP 3. We want to compute the second Gateaux of T^θ at $\mathcal{W}_0$ with direction $[\mathcal{W}, \mathcal{W}]$, i.e.,

$$\nabla^2 T^\theta[\mathcal{W}_0; \mathcal{W}, \mathcal{W}](y, B, b) \equiv \lim_{\lambda \to 0} \frac{T^\theta[\mathcal{W}_0 + \lambda\mathcal{W}](y, B, b) + T^\theta[\mathcal{W}_0 - \lambda\mathcal{W}](y, B, b) - T^\theta[\mathcal{W}_0](y, B, b)}{\lambda^2}$$

where $\mathcal{W}_0 \pm \lambda\mathcal{W} \in C(\mathbb{Y}^2 \times \mathbb{B}^2 \times \{0, 1\})$. It follows

$$\nabla^2 T^\theta[\mathcal{W}_0; \mathcal{W}, \mathcal{W}](y, B, b) = -\theta^{-1}\frac{E[\exp\{-\mathcal{W}_0/\theta\}\mathcal{W}_0^2]}{E[\exp\{-\mathcal{W}_0/\theta\}]} + \theta^{-1}\frac{(E[\exp\{-\mathcal{W}_0/\theta\}\mathcal{W}_0])^2}{(E[\exp\{-\mathcal{W}_0/\theta\}])^2},$$

which is clearly well-defined and negative. The technical lemma B.1, implies that $T^\theta[\mathcal{W}^L]$ maps concave functions onto concave functions.

Since U is concave too, the results in step 1, we can follow the same steps as theorem 4.8 in Stokey and Lucas (1989) to show that L_l maps concave functions onto (strictly) concave functions. The fact that W is strictly concave in y^L and b follows from similar arguments to those in step 2 and in Stokey and Lucas (1989).

STEP 4. From the theorem of the Maximum, see Berge (1997), we know that $(y, B, b) \mapsto b^*(y, B, b)$ is a non-empty, compact valued and upper hemi-continuous correspondence. Thus, we only need to show that $b^*(y, B, b)$ is single-valued. Suppose not, then there exist a (y, B, b) such that b_1' and b_2' are in $b^*(y, B, b)$ and $b_1' \neq b_2'$. This implies that

$$
U(y - b + qb_1') + \beta^L T^\theta [\mathcal{W}^*(y, y^{B'}, \Gamma^B(y, B), b_1')](y, B', b_1') = U(y - b + qb_2')
$$
$$
+ \beta^L T^\theta [\mathcal{W}^*(y, y^{B'}, \Gamma^B(y, B), b_2')](y, B', b_2').
$$

From result (3) we know that W^* is strictly concave and so is U, thus $b_\lambda' \equiv \lambda b_1' + (1 - \lambda) b_2' \in \mathbb{B}$ is such that

$$
U(y - b + qb_\lambda') + \beta^L T^\theta [\mathcal{W}^*(y', \Gamma^B(y, B), b_\lambda')](y, y^{B'}, B', b_\lambda') > U(y - b + qb_l')
$$
$$
+ \beta^L T^\theta [\mathcal{W}^*(y, y^{B'}, \Gamma^B(y, B), b_l')](y, B', b_1')
$$

for all $l = 0, 1$. But this is a contradiction to the fact that b_1' and b_2' are in $b^*(y, B, b)$. $\square$

LEMMA B.1. *Let $T^\theta : C(\mathbb{R}^d) \to C(\mathbb{R}^d)$ with $d < \infty$ and assume T^θ is twice Gateaux differentiable and its Gateaux derivative is negative at $\mathcal{W}_0 \in C(\mathbb{R}^d)$ in the following sense, $\nabla^2 T[\mathcal{W}_0; \mathcal{W}, \mathcal{W}] \leq 0$ for all $\mathcal{W} \in C(\mathbb{R}^d)$. Then, T^θ maps concave functions onto concave functions.*

Proof of lemma 2.3. To minimize the notational burden, we are going to omit the superscript $^{\sigma,\theta}$ in returns and prices. It follows from our equilibrium expression for the prices that

$$
\frac{1}{R_t} \equiv q_t = \beta^L E_t \left[(1 - \delta(y_{t+1}, B_{t+1})) \frac{\nabla_c U(c_{t+1}^L)}{\nabla_c U(c_t^L)} m[\mathcal{W}^L] \right] \tag{11}
$$

and

$$
\frac{1}{R_{f,t}} = \beta^L E_t \left[\frac{\nabla_c U(c_{t+1}^L)}{\nabla_c U(c_t^L)} m[\mathcal{W}^L] \right]. \tag{12}
$$

Thus, dividing the former by the latter equations it follows

$$
\frac{R_{f,t}}{R_t} = E_t \left[(1 - \delta(y_{t+1}, B_{t+1})) \left\{ \frac{\frac{\nabla_c U(c_{t+1}^L)}{\nabla_c U(c_t^L)} m[\mathcal{W}^L]}{E_t \left[\frac{\nabla_c U(c_{t+1}^L)}{\nabla_c U(c_t^L)} m[\mathcal{W}^L] \right]} \right\} \right]. \tag{13}
$$

The expression inside the curly brackets is defined as $\Pr_t^{\sigma,\theta}$ and the measure of $\delta(y_{t+1}, B_{t+1})$, since B_{t+1} is fixed and we only integrate across y_{t+1} is equivalent to the measure of $1 - D(B_{t+1})$, hence

$$\frac{R_{f,t}}{R_t} = 1 - Pr_t^{\sigma,\theta}(D(B_{t+1})). \tag{14}$$

$\square$

Proof of Corollary 2.1. It follows,

$$q_t = \beta^L E_t \left[\delta(y_{t+1}, B_{t+1}) \frac{\nabla_c U(c_{t+1}^L)}{\nabla_c U(c_t^L)} m[\mathcal{W}^L] \right]$$

$$= \beta^L E_t[\delta(y_{t+1}, B_{t+1})] E_t \left[\frac{\nabla_c U(c_{t+1}^L)}{\nabla_c U(c_t^L)} m[\mathcal{W}^L] \right] + \beta^L Cov_t \left(\delta(y_{t+1}, B_{t+1}), \frac{\nabla_c U(c_{t+1}^L)}{\nabla_c U(c_t^L)} m[\mathcal{W}^L] \right)$$

$$= \frac{1}{1+r_t} 1 - Pr_t(D(B_{t+1})) + \beta^L Cov_t \left(\delta(y_{t+1}, B_{t+1}), \frac{\nabla_c U(c_{t+1}^L)}{\nabla_c U(c_t^L)} m[\mathcal{W}^L] \right)$$

$$= \frac{1}{1+r_t} \left\{ 1 - Pr_t(D(B_{t+1})) + \beta^L(1+r_t) Cov_t \left(\delta(y_{t+1}, B_{t+1}), \frac{\nabla_c U(c_{t+1}^L)}{\nabla_c U(c_t^L)} m[\mathcal{W}^L] \right) \right\}$$

Since $\frac{1}{1+r_t} = \beta^L E_t \left[\frac{\nabla_c U(c_{t+1}^L)}{\nabla_c U(c_t^L)} m[\mathcal{W}^L] \right]$ it follows that

$$\beta^L(1+r_t) Cov_t \left(\delta(y_{t+1}, B_{t+1}), \frac{\nabla_c U(c_{t+1}^L)}{\nabla_c U(c_t^L)} m[\mathcal{W}^L] \right) \tag{15}$$

$$= \frac{Cov_t \left(\delta(y_{t+1}, B_{t+1}), \nabla_c U(c_{t+1}^L) \exp\left\{ -\frac{\mathcal{W}_{t+1}^L}{\theta} \right\} \right)}{E_t \left[\nabla_c U(c_{t+1}^L) \exp\left\{ -\frac{\mathcal{W}_{t+1}^L}{\theta} \right\} \right]}. \tag{16}$$

Since $Cov_t(X, Z) = E_t[(X - E[X])Z]$ it follows that

$$Cov_t \left(\delta(y_{t+1}, B_{t+1}), \nabla_c U(c_{t+1}^L) \exp\left\{ -\frac{\mathcal{W}_{t+1}^L}{\theta} \right\} \right) \tag{17}$$

$$= \int_{y_{t+1} \in D(B_{t+1})^c} \left\{ \nabla_c U(c_{t+1}^L) \exp\left\{ -\frac{\mathcal{W}_{t+1}^L}{\theta} \right\} - E_t \left[\nabla_c U(c_{t+1}^L) \exp\left\{ -\frac{\mathcal{W}_{t+1}^L}{\theta} \right\} \right] \right\} f_{Y'|Y}(y_{t+1}|y_t) dy_{t+1} \tag{18}$$

$\square$

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