Career Length: Effects of Curvature of Earnings Profiles, Earnings Shocks, Taxes, and Social Security*

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Abstract

The same high labor supply elasticity that characterizes a representative family model with indivisible labor and employment lotteries can also emerge without lotteries when self-insuring individuals choose career lengths. Off corners, the more elastic the earnings profile is to accumulated working time, the longer is a worker's career. Negative (positive) unanticipated earnings shocks reduce (increase) the career length of a worker holding positive assets at the time of the shock, while the effects are the opposite for a worker with negative assets. By inducing a worker to retire at an official retirement age, government provided social security can attenuate responses of career lengths to earnings profile slopes, earnings shocks, and taxes.

Key words: Career length, indivisible labor, earnings profile, earnings shocks, taxes, social security, labor supply elasticity.

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1 Introduction

A reformation incorporated between the first and second published versions of Edward Prescott's Nobel lecture testifies to a recent paradigm shift in macro-labor. In the first version, Prescott (2005) relied on an aggregation theory of Rogerson (1988) that features a labor supply indivisibility, employment lotteries, and complete insurance markets. In the second version, Prescott (2006) embraced a Ljungqvist and Sargent (2007) time-averaging setup with finitely lived individuals and incomplete markets that Prescott (2007) had discussed at an intervening 2006 NBER Macroeconomics Annual meeting. Instead of the older model's infinitely lived representative family that chooses a fraction of its members to send to work via employment lotteries, the newer time averaging model focuses attention on finitely lived individual workers' choices of career lengths. The newer model retains a labor supply indivisibility from the old model but combines it with incomplete markets and life cycles within an overlapping generations model.

As demonstrated by Ljungqvist and Sargent (2011), this paradigm shift represents meaningful progress because economists from a high labor supply elasticity camp and from a low labor supply elasticity camp can now agree about the key objects and forces in play. There remains ample room for disagreement about the balance among these forces. Advocates of high labor supply elasticities like Prescott stress the point that, despite the change in paradigm, a high elasticity continues to prevail at an interior solution for career length. Advocates of low labor supply elasticities study settings where career lengths are at a corner solution. Individuals can be put at a career-ending corner either by an official retirement age affiliated with government retirement programs or by the arrival of large negative and persistent shocks to their earnings capacities.

In a stylized time averaging model, this paper analytically establishes several findings that shed light on key forces also at work in more complicated settings that can be studied only with numerical simulations.[1] We obtain sharp outcomes by assuming preferences that are consistent with balanced growth. Such preferences are widely used in macroeconomic models because they are consistent with the fact that only modest changes in per capita hours of work have accompanied large increases in per capita incomes since World War II.[2] Principal outcomes of our analyses are these:

[1]For example, see French (2005), Low et al. (2010), and Kitao et al. (2008).

[2]An important exception has been the European employment experience, to which we will return in section 7.4.

1. Our first finding is that the more elastic are earnings to accumulated working time, the longer is a worker's career. This result suggests the possibility that it is a higher *slope* of the wage-experience profile of high wage workers, and not the *level* of the wage *per se*, that explains why people with higher wages and higher educations are more likely to retire later in life. Stark evidence for such a relationship is provided by Eckstein and Wolpin (1989) in their study of married women's labor force participation. They estimate a labor supply model in which wages depend positively on past work experience. Interpreting their empirical findings in light of our model requires properly adjusting for differences in model specifications because in contrast to our time averaging framework, they assume that households can neither save nor borrow, and they allow the disutility of work to vary with work experience. They estimate that the disutility of work actually increases with experience. Based on a misspecification analysis, we show that their estimate of an increasing disutility of work would reflect a falling marginal value of additional savings for retirement when viewed through the lens of a time averaging model.

   Remark 1.1. *Heckman (1993) argued that the relatively high labor supply elasticity of married women is mainly due to a higher elasticity of their labor force participation rate. Since married women have historically been second earners within household, it is likely that their participation decisions reflect interior solutions to career length choices. Another potential source of variations in career length, perhaps especially for primary workers, comes from shocks to workers' earnings capacities. We address this source in our second finding.*

2. In a time averaging model of lifetime labor supply, savings is an essential state variable. We find that the sign of a worker's savings balance determines how planned career length responds to an unanticipated multiplicative shock to earnings, a shock that leaves the elasticity of earnings to accumulated working time unchanged. Specifically, our second finding is that a negative (positive) earnings shock reduces (increases) the career length of a worker holding positive assets at the time of the shock, while the effects are the opposite for a worker with negative assets. In light of the increased variability observed to have confronted individual workers for both transitory and permanent components of labor earnings, our finding that negative permanent earnings shocks shorten careers for workers in mid- and late-age having positive life cycle savings identifies a force that can help to explain the increased incidence of early retirement

in recent decades.

Remark 1.2. *A multiplicative shift of the earnings profile could also result from a change in proportional taxation. Under the assumption that tax revenues are not returned to tax payers as transfers and also that they are not used to finance goods and services that are close substitutes to private consumption, our analysis of changes in career length in response to unanticipated earnings shocks would also apply to unanticipated changes in tax rates. Note that zero life cycle savings at the beginning of life implies that the level of a tax rate that is constant over an entire life cycle does not affect the optimal career length. This is another manifestation of preferences that are consistent with balanced growth, i.e., variations in the net-of-tax wage rate provoke completely offsetting income and substitution effects. This is not the outcome under Prescott's (2002) assumption that tax revenues are handed back lump sum to households, which is also the subject of our third finding.*

3. When tax revenues are returned as lump transfers to households, our third finding extends our earlier result (Ljungqvist and Sargent 2007) that the elasticity of aggregate labor supply with respect to the net-of-tax rate is high in the time averaging model, and is also of a magnitude similar to what it would be in an employment lottery model with its characteristic high labor supply elasticity. When the utility function is additively separable in consumption and leisure, that labor supply elasticity equals one, which is consistent with Prescott's (2002) assertion that a net-of-tax rate of 0.60 in the U.S. as compared to 0.40 in France can explain why French labor supply was depressed by 30 percent relative to that of the U.S. For preferences that are not additively separable, we show that the labor supply elasticity increases with the curvature of the utility function (the coefficient of relative risk aversion) as well as with the elasticity of earnings to accumulated working time. For example, at a coefficient of relative risk aversion of 2, the U.S. labor supply elasticity with respect to the net-of-tax rate is above 1.4 and increases strongly with increases in the elasticity of earnings to accumulated working time.

Remark 1.3. *One might be tempted to conclude that our third finding vindicates the influential framework for aggregate analysis advocated in Prescott's (2005) Nobel lecture that features a high labor supply elasticity founded on Rogerson's (1988) aggregation theory with employment lotteries and complete insurance markets. We recommend caution before jumping to that conclusion because, as shown in Ljungqvist and Sargent*

(2007), a model with such a high labor supply elasticity fails to explain employment outcomes once labor supply responses to both taxes and nonemployment benefits are calibrated to fit the welfare states of Europe. With Prescott's high labor supply elasticity, the puzzle becomes not why Europeans work so little *but rather why they work so* much *compared to Americans. Our fourth finding offers a possible explanation to this puzzle.*

4. Our fourth finding is that social security tax and benefit rules can put a kink into a worker's budget set that can lead to a corner solution for career length at an official retirement age. That effect extinguishes the high labor supply elasticity that would be associated with an interior solution.

   Remark 1.4. *In section 7, we suggest that this force attenuated labor supply elasticities in ways that can help explain how the European welfare states with generous benefits and high taxation could operate successfully in the post-World War II era until the late 1970s without causing any major differences in labor market outcomes vis a vis the U.S. Our analysis also contains clues about why outcomes in Europe deteriorated after the late 1970s.*

Section 2 describes a lifetime labor supply problem in which a finitely lived worker confronts a labor supply indivisibility, chooses when to work, and smooths consumption by trading a risk-free bond. How career lengths are affected by the shape of an experience-earnings profile, unanticipated earnings shocks, taxes, and social security are studied in sections 3, 4, 5 and 6, respectively. Implications for social security reform are discussed in section 7, where we also briefly indicate how the constellation of forces identified by our experiments may have balanced out in ways that can help explain variations in labor market outcomes across time and space. Appendix A compares our time averaging model to a corresponding employment lottery model with complete markets. Throughout, we focus exclusively on the extensive margin and exclude movements along the intensive margin.[3]

[3]Prescott et al. (2009) adopt and extend the Ljungqvist and Sargent (2007) time-averaging setup by adding an intensive margin to the individual's labor supply decision. They reaffirm Ljungqvist and Sargent's results about the elasticity of equilibrium employment to a labor tax rate under that extension. Rogerson and Wallenius (2009) also introduced human capital, but instead of making human capital endogenous as Ljungqvist and Sargent (2007) did, they assumed that workers face an exogenously given age-specific labor productivity that induces the young and the old to work less because their productivities are lower.

2 A lifetime labor supply problem

A worker's preferences are ordered by

$$\int_0^1 u(c_t, 1 - n_t)dt, \tag{1}$$

where $c_t \geq 0$ and $n_t \in \{0, 1\}$ are consumption and labor supply at time t, respectively. That $n_t \in \{0, 1\}$ asserts that labor supply is indivisible. A worker with past employment spells totaling $h_t = \int_0^t n_s ds$ has the opportunity to work at earnings

$$w_t = W h_t^{\phi}, \qquad W > 0, \;\; \phi \in [0, 1]. \tag{2}$$

Because the worker can borrow and lend at a zero interest rate, she faces the life-time budget constraint $\int_0^1 c_t dt \leq \int_0^1 w_t n_t dt$.[4] An optimal plan prescribes a fraction $T \in [0, 1]$ of a lifetime devoted to work. The worker is indifferent about the timing of her labor supply. Therefore, we are free to assume that the worker frontloads work at the beginning of life so that the present value of labor income for someone who works a fraction T of her lifetime is

$$\int_0^T W t^{\phi} \, d\, t = W \frac{T^{\phi+1}}{\phi + 1} \equiv W \, e(T; \phi). \tag{3}$$

Following King et al. (1988), we assume a utility function that is consistent with balanced growth and has a constant intertemporal elasticity of substitution in consumption equal to $1/\gamma$, namely,

$$u(c_t, 1 - n_t) = \frac{c_t^{1-\gamma}}{1 - \gamma} v(1 - n_t) \tag{4a}$$

for $0 < \gamma < 1$ and $\gamma > 1$, while for $\gamma = 1$,

$$u(c_t, 1 - n_t) = \log(c_t) + v(1 - n_t), \tag{4b}$$

where the total time endowment is normalized to one, so $1 - n_t$ is leisure at time t. In the

[4]We retain the assumption of Ljungqvist and Sargent (2007) that the worker's subjective discount rate and the market interest rate are equal. For simplicity, we assume that both rates are equal to zero. If instead they were both strictly positive, Ljungqvist and Sargent (2007) show that the worker would prefer to shift her labor supply to the end of life. Why? Because at a given lifetime disutility of work, working later in life would mean spending more total time working. That would push the worker further up the experience-earnings profile and thereby increase the present value of lifetime earnings.

multiplicatively separable case of (4a), the function $v(\cdot)$ is (i) increasing and concave if $\gamma < 1$ and (ii) decreasing and convex if $\gamma > 1$, provided that an additional condition on the second derivative of $v(\cdot)$ that assure overall concavity of $u(\cdot)$ is satisfied (see King et al. (1988, p. 202)). In the additively separable case (4b), all that we require is that $v(\cdot)$ is increasing and concave.

Under our assumption of indivisible labor, the precise curvature of $v(\cdot)$ is not an issue because we evaluate the function at only two points, $n_t \in \{0, 1\}$. Hence, in the multiplicatively separable case of (4a), we can normalize $v(1) = 1$ and let $v(0) = B$, so that the worker's lifetime utility in (1) can be written

$$\int_0^1 \left[\frac{c_t^{1-\gamma}}{1-\gamma} \max\{1 - n_t,\, B n_t\} \right] dt, \tag{5}$$

where for $0 < \gamma < 1$ ($\gamma > 1$), we require $0 < B < 1$ ($B > 1$) in order to satisfy the above conditions that make utility decrease in labor supply. For $\gamma = 1$, we normalize $v(1) = 0$, so that the worker's lifetime utility can be expressed as

$$\int_0^1 \Big[\log(c_t) - B n_t \Big] dt, \qquad B > 0. \tag{6}$$

Since the subjective discount rate equals the market interest rate, the optimal consumption plan prescribes constant consumption when working (i.e., $n_t = 1$), $c_t = \bar{c}$. Consumption is also constant when not working (i.e., $n_t = 0$), but possibly at a different level, $c_t = c$. Marginal utilities of consumption should be equated across spells of working and not working:

$$u_1(\bar{c}, 0) = u_1(c, 1) \quad \Longrightarrow \quad \begin{cases} \bar{c} = c\, B^{1/\gamma}, & \text{for } 0 < \gamma < 1 \text{ and } \gamma > 1; \\ \bar{c} = c, & \text{for } \gamma = 1. \end{cases} \tag{7}$$

The consumption plan must also satisfy the worker's present value budget constraint

$$T\bar{c} + (1 - T)c = W\, e(T; \phi). \tag{8}$$

After imposing (7), the present value budget constraint (8) implies

$$c = \begin{cases} \dfrac{W\, e(T;\phi)}{T\, B^{1/\gamma} + 1 - T}, & \text{for } 0 < \gamma < 1 \text{ and } \gamma > 1; \\[2ex] W\, e(T;\phi), & \text{for } \gamma = 1. \end{cases} \tag{9}$$

By using (7) and (9), we can eliminate consumption from the worker's lifetime utility (1), $T\,u(\bar{c},0) + (1-T)\,u(c,1)$, so that her optimization problem can be expressed in terms of a single choice variable $T \in [0,1]$. At an interior solution, the optimal career length is determined by the first-order condition at equality:[5]

$$\bar{T}(\phi) = \begin{cases} \dfrac{(1-\gamma)(\phi+1)}{\big[(1-\gamma)(\phi+1)+\gamma\big]\big(1-B^{1/\gamma}\big)}\,, & \text{for } 0<\gamma<1 \text{ and } \gamma>1; \\ \dfrac{\phi+1}{B}\,, & \text{for } \gamma = 1. \end{cases} \tag{10}$$

According to (10), the following restrictions on parameters are necessary for interior solutions:[6]

$$\big[(1-\gamma)(\phi+1)+\gamma\big]B^{1/\gamma} < (>)\gamma, \quad \text{for } 0<\gamma<1 \ (\gamma>1); \tag{11a}$$

$$\phi+1 \ < \ B, \quad \text{for } \gamma = 1. \tag{11b}$$

We will impose these throughout our analysis. Because preferences are consistent with balanced growth, the optimal career length $\bar{T}(\phi)$ in (10) does not depend on the earnings level parameter W. Therefore, there exists an expression $W\,a(t,\phi)$ for the worker's savings at time t of her lifetime, where the function $a(t,\phi)$ is common to all workers with the same curvature parameter ϕ for earnings.

2.1 Initial assets

Suppose that the worker starts with some initial assets A_0. At an interior solution, the optimal career length $\hat{T} \in (0,1)$ is determined implicitly by the first-order condition:

$$\hat{T} = \begin{cases} \bar{T}(\phi) - \dfrac{A_0\gamma(\phi+1)}{\big[(1-\gamma)(\phi+1)+\gamma\big]W\,\hat{T}^{\phi}}\,, & \text{for } 0<\gamma<1 \text{ and } \gamma>1; \\ \bar{T}(\phi) - \dfrac{A_0(\phi+1)}{W\,\hat{T}^{\phi}}\,, & \text{for } \gamma = 1. \end{cases} \tag{12}$$

Parameter restriction (11a) guarantees a positive denominator in (12). Thus, negative (positive) initial assets lengthen (shorten) the optimal career length relative to $\bar{T}(\phi)$, i.e., if

[5] For $0<\gamma<1$ and $\gamma>1$, the second order condition is calculated to be the negative of the inverse of $\bar{T}(\phi)$. Hence, if there exists an interior solution, $\bar{T}(\phi) \in (0,1)$, the second-order condition is trivially satisfied, $-1/\bar{T}(\phi) < 0$.

[6] Parameter restriction (11a) implies the earlier restrictions that if $0<\gamma<1$ ($\gamma>1$), then $0<B<1$ ($B>1$). Moreover, if $\gamma>1$, parameter restriction (11a) ensures that $[(1-\gamma)(\phi+1)+\gamma]>0$.

$A_0 < (>)\, 0$, then $\hat{T} > (<)\, \bar{T}(\phi)$. From hereon, we will assume that $A_0 = 0$, but the outcomes in (12) will be useful later when we study unanticipated earnings shocks in section 4.

3 Effect of earnings profile on career length

An elasticity parameter $\phi = 0$ means constant earnings, $w_t = W$, while $\phi > 0$ indicates an earnings profile that increases in cumulated time worked h_t, but at a decreasing rate (except for the linear specification, $\phi = 1$). A higher value of ϕ implies a slower relative decay in the slope of the earnings profile with respect to time worked.

Finding 1: $T'(\phi) > 0$, so that career length increases with increases in the elasticity of earnings to accumulated working time.

For $0 < \gamma < 1$ and $\gamma > 1$,

$$\bar{T}'(\phi) = \frac{(1-\gamma)\gamma}{\left[(1-\gamma)(\phi+1)+\gamma\right]^2 \left(1 - B^{1/\gamma}\right)} > 0\,, \tag{13}$$

where the strict inequality follows from the above parameter restrictions, i.e., if $0 < \gamma < 1$ ($\gamma > 1$), then $0 < B < 1$ ($B > 1$).

As an illustration, for $\gamma = 1$ and a disutility of work $B = 1.6$, figure 1 depicts two earnings profiles with elasticity parameters $\phi = 0.3$ and $\phi = 0.5$, respectively, with the optimal fraction of lifetime spent working, $\bar{T}(\phi)$, marked by a circle on each profile. As a normalization, we set level parameters $W = 1$ and $W = e(\bar{T}(0.3), 0.3)/e(\bar{T}(0.3), 0.5)$, respectively, so both earnings profiles yield the same present value of labor income when the same fraction $\bar{T}(0.3)$ is devoted to work. While that choice is optimal for a worker with profile $\phi = 0.3$, the agent with the higher $\phi = 0.5$ will choose to work a bigger fraction of her lifetime.

Viewed as a model of self-financed retirement, the streamlined model with the interior solutions presented here asserts that workers who retire later are those with earnings profiles that are more elastic to accumulated working time. In the remaining sections of this paper, we discuss how this outcome is modified when other features affect the worker's budget set, such as unanticipated earnings shocks, taxes, and government supplied social security. But first we discuss a piece of empirical evidence.

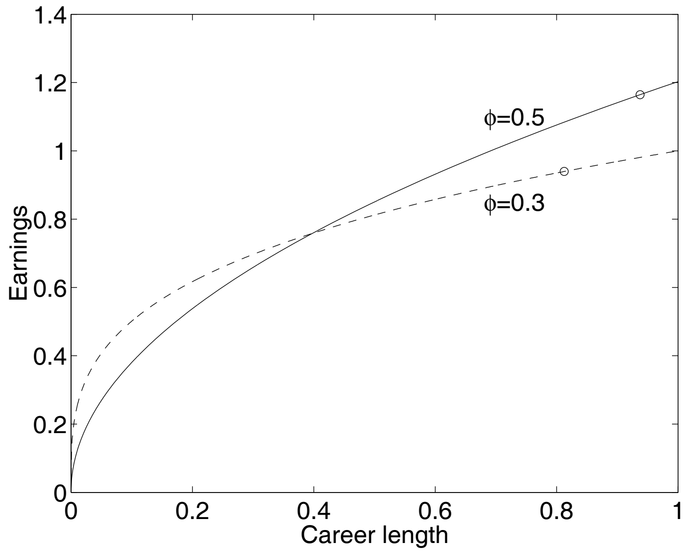

Figure 1: Two earnings profiles with $\phi = 0.3$ (dashed line) and $\phi = 0.5$ (solid line), respectively. For $\gamma = 1$ and a disutility of work $B = 1.6$, the circle on each profile denotes the optimal career length $\bar{T}(\phi)$.

3.1 A reinterpretation of empirical evidence

For a specification that posits that wages depend on past work experience, Eckstein and Wolpin (1989) estimated a dynamic model of married women's labor force participation. After estimating their model, they performed counterfactual experiments by perturbing the slope of the wage-experience profile away from their estimated value and found the following outcomes:

> Halving the slope of the log wage-experience profile implies that for a woman with ten years of experience at age 39, the expected additional number of years of work to age 60 will fall from 16.7 to 1.2. Doubling the coefficient implies that all women will work in every year subsequent to age 39 independent of work experience at age 39. (Eckstein and Wolpin 1989, p. 388)

We can reinterpret the simulation results of Eckstein and Wolpin in terms of responses of an interior solution for $\bar{T}(\phi)$ to the earnings-experience curvature parameter ϕ in (10) in our time-averaging setting. To do so, we have to resort to a misspecification analysis because the forces driving outcomes in our model differ substantially from those in Eckstein and Wolpin's. In contrast to us, Eckstein and Wolpin (i) assume that households can neither save nor borrow, and (ii) allow the disutility of work to vary with work experience and estimate that it actually increases with experience. Workers' inability to borrow or save in Eckstein and Wolpin's model completely disarms the mechanism at work in our time averaging model, whereby workers use the credit market to smooth consumption and to 'convexify' the indivisiblity in their instantaneous labor supply opportunities by choosing fractions of their lifetimes to work. This is not the force that drives career length outcomes in Eckstein and Wolpin (1989). Instead, their career length effect rests on an estimated schedule of disutilities of work that increases with past work experience. But we can reinterpret their result in terms of a specification analysis in which our model generates life-cycle employment and wage data that we mistakenly use to estimate Eckstein and Wolpin's model. We would estimate an increasing disutility of work, but that would be an artifact of misspecified preferences and mistaken exclusion of a credit market. Specifically, the estimate of an increasing disutility of work would truly reflect a falling marginal value of additional savings for retirement in the time averaging model.

For a formal exposition of our misspecification analysis, consider the Eckstein-Wolpin preference specification $\int_0^1 [c_t - \tilde{B}_t n_t] dt$ where $\tilde{B}_t = \tilde{b} h_t$, $\tilde{b} > 0$. A worker with these preferences would also be indifferent about the timing of her labor supply. Therefore, we continue to assume that the worker frontloads her work at the beginning of time so that the lifetime utility of consumption and lifetime disutility of labor for someone who works a fraction T of her lifetime are $W\, e(T; \phi)$ and $\int_0^T \tilde{b} t\, dt = \tilde{b} T^2/2$, respectively. Thus, the worker's optimal lifetime labor supply solves

$$\max_{T \in [0,1]} \left\{ W\, e(T; \phi) - \tilde{b} \frac{T^2}{2} \right\}, \tag{14}$$

with a first-order condition at an interior solution,

$$W T^{\phi} - \tilde{b} T = 0, \tag{15}$$

and a second-order condition,

$$W \phi T^{\phi - 1} - \tilde{b} < 0. \tag{16}$$

By substituting the interior solution $\bar{T}(\phi)$ from (10), for $\gamma = 1$, into (15), we can solve for the parameter value $\tilde{b}^\star = W((\phi + 1)/B)^{\phi - 1}$, at which (15) would result in the same choice of labor supply as for our time averaging model. Furthermore, by plugging the expressions for $\bar{T}(\phi)$ and $\tilde{b}^\star$ into (16), we find that the second-order condition reduces to $\phi - 1 < 0$, which holds for our assumptions (except for a borderline linear specification with $\phi = 1$). Hence, we have shown that the optimal labor supply of our time averaging model can be reproduced in an alternative model where utility is linear in consumption and the disutility of work increases with past work experience. In the alternative model with the market interest rate being equal to the worker's subjective discount rate, the worker would not regret that a credit market is absent.

4 Effect of earnings shocks on career length

We conduct a standard experiment of considering a multiplicative earnings shock, i.e., a shift in the parameter W in earnings expression (2). According to (10), the optimal career length does not respond to a shift in W at the very beginning of a lifetime, which reflects that preferences are consistent with balanced growth. Concerning an unanticipated permanent mid-career earnings shock, we have the following proposition.[7]

Finding 2: An unanticipated permanent negative (positive) earnings shock, in the form of a shift in W, reduces (increases) the career length of a worker holding positive assets at the time of the shock. Opposite effects prevail for a worker with negative assets.

Consider an unanticipated mid-career earnings shock at time $\hat{t} \in (0, \bar{T}]$. In particular, for $t < \hat{t}$, we assume that the worker had conformed to an optimal plan associated with earnings parameters W and ϕ. At time $\hat{t}$, the earnings profile unexpectedly jumps from Wt^{ϕ} to $\hat{W}t^{\phi}$ for $t \in [\hat{t}, 1]$, and the worker reoptimizes by choosing a new career length $\hat{T} \in [\hat{t}, 1]$. The worker's asset stock at time $\hat{t}$ is given by $Wa(\hat{t};\phi)$. Compare this asset stock to $\hat{W}a(\hat{t};\phi)$, i.e., the asset stock of a worker of the same age but who has always faced earnings profile $\hat{W}t^{\phi}$. There are three possibilities; $Wa(\hat{t};\phi)$ is either smaller than, bigger than, or equal to $\hat{W}a(\hat{t};\phi)$.

1. If $Wa(\hat{t},\phi) < \hat{W}a(\hat{t},\phi)$, then there exists a number $A_0 < 0$ such that the worker's actual assets are equal to the hypothetical assets of someone who has always faced earnings parameter $\hat{W}$ but whose initial assets were $A_0 < 0$. As of time $\hat{t}$, these two

[7] Alternatively, we could have modeled a stochastic earnings process which would have entailed the study of precautionary savings dynamics over the worker's life cycle or more precisely, over her working career. But such an added complication would have detracted from the transparency of our first-order findings that are driven by the assumptions of preferences that are consistent with balanced growth and incomplete markets. We study permanent shocks because the worker would seek to undo temporary negative shocks by postponing her labor supply. Recall that the worker is otherwise indifferent to the timing of her labor supply and in particular, a temporary interruption of a career does not in of itself degrade human capital.

workers face the same continuation problem and hence, the new optimal career length $\hat{T} > \bar{T}(\phi)$, as given by (12).

2. If $Wa(\hat{t},\phi) > \hat{W}a(\hat{t},\phi)$, an analogous argument with $A_0 > 0$ establishes that $\hat{T} < \bar{T}(\phi)$. The only caveat is that we have a corner solution with $\hat{T} = \hat{t}$, i.e., the initial assets $A_0 > 0$ needed to generate the same hypothetical asset holdings of someone who has always faced earnings parameter $\hat{W}$ would be inconsistent with this person still working at time $\hat{t}$.

3. If $Wa(\hat{t},\phi) = \hat{W}a(\hat{t},\phi)$, then $\hat{T} = \bar{T}(\phi)$. This can only happen if $a(\hat{t},\phi) = 0$, i.e., the two asset stocks are zero.

4.1 Explicit expressions for the case $\gamma = 1$

We let $\bar{T} = \bar{T}(\phi)$ denote the optimal fraction of her lifetime that the worker intends to devote to work before the realization of the unanticipated earnings shock, and, as above, $\hat{T}$ be the optimal fraction after the earnings shock. With frontloaded working time, before the earnings shock, the original optimal savings profile for $t \le \bar{T}$ is

$$A_t = \int_0^t \left[W s^\phi - We(\bar{T};\phi)\right] ds = \frac{W\,t}{\phi+1}\left[t^\phi - \left(\frac{\phi+1}{B}\right)^{\phi+1}\right] \equiv Wa(t;\phi), \qquad (17)$$

where we have used $c_t = We(\bar{T};\phi)$ and $\bar{T}(\phi) = (\phi+1)/B$, as given by (9) and (10), respectively. For $\phi > 0$, there exists a cutoff value $\bar{t}(\phi)$ such that accumulated assets are negative for $t \in (0, \bar{t}(\phi))$ and positive for $t > \bar{t}(\phi)$. (Workers who expect rising earnings borrow when young, repay when older, then lend when even older.) We can solve (17) for $\bar{t}(\phi)$ to get

$$\bar{t}(\phi) = \left(\frac{\phi+1}{B}\right)^{\frac{\phi+1}{\phi}} \in \left(0, \bar{T}(\phi)\right). \qquad (18)$$

The limit point of $\bar{t}(\phi)$ in (18) is zero as $\phi \to 0$, so we define $\bar{t}(0) = 0$. Thus, with a front loaded lifetime labor supply, asset holdings are always nonnegative for a worker with a flat $\phi = 0$ earnings profile.

An unanticipated mid-career earnings shock occurs at time $\hat{t} \in (0, \bar{T}]$, when the earnings profile unexpectedly jumps from Wt^ϕ to $\hat{W}t^\phi$ for $t \in [\hat{t}, 1]$. Subject to the asset stock $Wa(\hat{t};\phi)$ that had been accumulated under the old plan, the wage jump from W to $\hat{W}$ prompts the worker to maximize the remainder of her lifetime utility

$$\int_{\hat{t}}^{1} \Big[\log(\hat{c}_t) - B\hat{n}_t\Big] dt \tag{19}$$

by choosing new values $\hat{c}_t \geq 0$ and $\hat{n}_t \in \{0, 1\}$ of consumption and labor supply, respectively, for $t \in [\hat{t}, 1]$. The worker's revised optimal plan prescribes a constant consumption path over the interval $[\hat{t}, 1]$ and a fraction $\hat{T} \in [\hat{t}, 1]$ of her lifetime devoted to work.[8]

For the worker who after the unanticipated wage shock at $\hat{t}$ chooses to work a fraction $T \in [\hat{t}, 1]$ of her total lifetime, the sum of the financial assets already accumulated at time $\hat{t}$, $W a(\hat{t}; \phi)$, and the present value of future labor income becomes

$$\begin{aligned} W a(\hat{t}; \phi) + \int_{\hat{t}}^{T} \hat{W} s^{\phi}\, ds &= \hat{W} \left\{ \left(\frac{W}{\hat{W}} - 1 \right) a(\hat{t}; \phi) + \frac{1}{\phi + 1} \left[T^{\phi+1} - \hat{t} \left(\frac{\phi + 1}{B} \right)^{\phi+1} \right] \right\} \\ &\equiv \hat{W}\, \hat{e}(T; \hat{t}, W/\hat{W}, \phi), \end{aligned} \tag{20}$$

where the first equality is obtained by adding and subtracting $\hat{W} a(\hat{t}; \phi)$. This time $\hat{t}$ present value of financial plus non-financial wealth must equal the present value of consumption over the period $[\hat{t}, 1]$, so it follows that $\hat{W} \hat{e}(T; \hat{t}, W/\hat{W}, \phi)/(1 - \hat{t}\,)$ is the constant consumption rate over the remaining lifetime $1 - \hat{t}$.

The worker's optimal lifetime labor supply thus solves

$$\max_{T \in [\hat{t}, 1]} \left\{ (1 - \hat{t}\,) \log \left[\frac{\hat{W} \hat{e}(T; \hat{t}, W/\hat{W}, \phi)}{1 - \hat{t}} \right] - B(T - \hat{t}\,) \right\}. \tag{21}$$

The first-order condition for T is

$$\frac{(1 - \hat{t}\,) T^{\phi}}{\left(\frac{W}{\hat{W}} - 1 \right) a(\hat{t}; \phi) + \frac{1}{\phi+1} \left[T^{\phi+1} - \hat{t} \left(\frac{\phi+1}{B} \right)^{\phi+1} \right]} - B \begin{cases} < 0, & \text{corner soln } \hat{T} = \hat{t}; \\ = 0, & \text{interior soln } \hat{T} \in [\hat{t}, 1]; \\ > 0, & \text{corner soln } \hat{T} = 1; \end{cases} \tag{22}$$

where $\hat{T}$ is the optimal lifetime labor supply after the earnings shock at time $\hat{t}$. We let $\hat{T}(\hat{t}, W/\hat{W}, \phi)$ denote an interior solution that is determined implicitly by (22) at equality,

[8] We implicitly impose the restriction that parameters are such that any negative asset holdings at time $\hat{t}$ are strictly less than the present value of future labor income if the worker works for the rest of her lifetime.

i.e.,

$$(1-\hat{t}\,)\hat{T}^{\phi} = B\left\{\left(\frac{W}{\hat{W}}-1\right)a(\hat{t};\phi) + \frac{1}{\phi+1}\left[\hat{T}^{\phi+1} - \hat{t}\,\bar{T}^{\phi+1}\right]\right\}, \tag{23}$$

where we have invoked $(\phi+1)/B = \bar{T}(\phi)$. An interior solution for the post-shock career length $\hat{T}$ relates to the original career length $\bar{T}$ in the following way:[9]

$$\hat{T}(\hat{t}, W/\hat{W}, \phi)\left\{\begin{array}{l} < \bar{T}(\phi) \\ = \bar{T}(\phi) \\ > \bar{T}(\phi) \end{array}\right\} \text{ if } \left(\frac{W}{\hat{W}}-1\right)a(\hat{t};\phi)\left\{\begin{array}{l} > 0; \\ = 0; \\ < 0; \end{array}\right. \tag{24}$$

Evidently, the sign of the revision $\hat{T}-\bar{T}$ to an unanticipated earnings shock depends (i) on whether $\hat{W} > W$ or $\hat{W} < W$, and (ii) on whether the worker's asset holdings at the time of the shock, $A_{\hat{t}}$, are positive or negative. In response to a *negative* earnings shock, $\hat{W} < W$, the worker reduces (increases) her lifetime labor supply if her time $\hat{t}$ asset holdings are positive (negative), i.e., if $a(\hat{t};\phi) > 0$ ($a(\hat{t};\phi) < 0$), which means that the shock occurs at a time $\hat{t} > \bar{t}(\phi)$ ($\hat{t} < \bar{t}(\phi)$), where $\bar{t}(\phi)$ is defined in (18). In contrast, in response to a *positive* earnings shock, $\hat{W} > W$, the worker increases (decreases) her lifetime labor supply if her current asset holdings are positive (negative).

In the case of a flat $\phi = 0$ earnings profile and a frontloaded lifetime labor supply, asset holdings are always nonnegative in the initial plan, and, hence, the worker's labor supply response depends only on the sign of the earnings shock. Specifically, when $\phi = 0$, we can rewrite first-order condition (23) at an interior solution as

$$\hat{T} = \bar{T} - (1-\bar{T})\left(\frac{W}{\hat{W}}-1\right)\hat{t}\;\left\{\begin{array}{lll} < \bar{T} & \text{if} & \hat{W} < W, \\ = \bar{T} & \text{if} & \hat{W} = W, \\ > \bar{T} & \text{if} & \hat{W} > W. \end{array}\right. \tag{25}$$

As could be anticipated from (24), a worker with a flat earnings profile will reduce (increase) her lifetime labor supply in response to a negative (positive) earnings shock.

[9]Suppose that $(W/\hat{W}-1)\,a(\hat{t};\phi) > (<) 0$ but that, contrary to (24), lifetime labor supply satisfies $\hat{T} \geq (\leq)\bar{T}(\phi)$. According to (23), this would imply

$$(1-\hat{t}\,)\hat{T}^{\phi} > (<) B\frac{1}{\phi+1}\left[\hat{T}^{\phi+1} - \hat{t}\,\hat{T}^{\phi+1}\right],$$

which leads to the contradiction that $\hat{T} < (>)(\phi+1)/B = \bar{T}(\phi)$. When $(W/\hat{W}-1)\,a(\hat{t};\phi) = 0$, the equality $\hat{T} = \bar{T}$ can be confirmed by plugging that solution into (23) to verify that $\hat{T} = (\phi+1)/B = \bar{T}(\phi)$.

4.2 Interpretation of wealth and substitution effects

For a worker with positive asset holdings at $\hat{t}$, a *negative* earnings shock means that returns to working fall relative to the marginal value of her wealth. That induces the worker to enjoy more leisure because doing that has now become relatively less expensive. But with negative asset holdings at $\hat{t}$, a negative earnings shock compels the worker to supply more labor both to pay off time $\hat{t}$ debt and to moderate the adverse effect of the shock on her future consumption.

With a *positive* earnings shock, leisure becomes more expensive, causing the worker to substitute away from leisure and toward consumption. This force makes lifetime labor supply increase for a worker with positive wealth. But why does a positive earnings shock lead to a *reduction* in life-time labor supply when time $\hat{t}$ assets are negative?

In the case of a positive earnings shock and negative time $\hat{t}$ assets, consider a hypothetical asset path that would have prevailed if the worker had enjoyed the higher earnings profile associated with $\hat{W}$ from the beginning starting at $t = 0$. Along that hypothetical path, the worker would have been even further in debt at $\hat{t}$ (since assets would be scaled by $\hat{W}$ rather than W in (17)). So at $\hat{t}$, the worker actually finds herself *richer* at $\hat{t}$ than she would have in our hypothetical scenario. Because there is less debt to be repaid at $\hat{t}$, the worker chooses to supply less labor than she would have in the hypothetical scenario.

To construct another revealing hypothetical path in the case of a positive earnings shock and negative time $\hat{t}$ assets, suppose instead that the worker had known her actual earnings profile *including* the positive earnings shock at $\hat{t}$ from time $t = 0$ on. That would have induced her to choose a higher consumption level prior to time $\hat{t}$. That would leave her *more* in debt at time $\hat{t}$. We conclude that in the actual situation with a positive earnings shock and negative asset holdings at $\hat{t}$, it is not optimal to make up for what would have been past underconsumption relative to our hypothetical path. Instead, the worker chooses to enjoy more leisure because she has relatively less debt at $\hat{t}$ than she would along the hypothetical path.

5 Effect of taxes on career length

A multiplicative shock to the earnings profile in section 4 could also come from a change in proportional taxation. Under the assumption that tax revenues are not returned to tax payers as transfers nor are they used to finance goods and services that are close substitutes to private consumption, the preceding analysis of changes in career length in response to unanticipated earnings shocks would also apply to unanticipated changes in tax rates. And for the same reason that a shift in W at the very beginning of a lifetime does not affect the optimal career length, the level of a constant tax rate does not affect the worker's labor supply, i.e., income and substitution effects cancel with variations in the net-of-tax wage rate under the assumption that preferences are consistent with balanced growth.[10] But if instead all tax receipts are rebated lump sum to workers, then labor supply responds in ways that we study next.

We introduce a government that taxes labor income at the rate τ and runs a balanced budget by returning the tax receipts lump sum to workers (or by using the revenues to finance a social security system in section 6). Newborn workers enter the economy at a rate that keeps the population and age structure constant over time. Our focus is not on the determination of intertemporal prices in this overlapping generations environment with its possible dynamic inefficiencies,[11] so we retain our small open economy assumption of an exogenously given interest rate. All workers have the same preference specification and the same earnings profile parameters W and ϕ. As we will show, in a time averaging model, career length is highly responsive to the labor tax.

Finding 3: Given that tax revenues are handed back lump sum to households, the elasticity of aggregate labor supply with respect to the net-of-tax rate, $[(1-\tau)/T]\,\partial T/\partial(1-\tau)$, is high, as it also is in a corresponding employment-lottery model.

Let x be the present value of lump-sum transfers that each worker receives over her lifetime, as determined by the government budget constraint

$$\tau W\, e(T^\star;\phi) = x, \tag{26}$$

where $T^\star$ is the equilibrium career length. Note that given a zero interest rate and a lifetime of unit length, x is the instant-by-instant per capita lump-sum transfer that satisfies the government's static budget constraint (26) as well as the present value of total lump-sum transfers paid to a worker over her lifetime.

A worker again chooses constant consumption paths $\bar{c}$ and c while working and not

[10] Prescott (2002, p. 7) noted that "If [labor tax] revenues are used for some public good or are squandered, private consumption will fall, and the tax wedge will have little consequence for labor supply."

[11] For a treatment of overlapping generations models, see e.g. Ljungqvist and Sargent (2004).

working, respectively, but now budget constraint (8) is replaced by

$$T\bar{c} + (1-T)c = (1-\tau)W\, e(T;\phi) + x. \tag{27}$$

Substituting (26) into the first-order condition for the worker's optimization problem shows that the equilibrium career length is

$$T^{\star}(\tau) = \begin{cases} \dfrac{(1-\tau)(1-\gamma)(\phi+1)}{\big[(1-\tau)(1-\gamma)(\phi+1)+\gamma\big]\big(1-B^{1/\gamma}\big)}\,, & \text{for } 0<\gamma<1 \text{ and } \gamma>1; \\ \dfrac{(1-\tau)(\phi+1)}{B}\,, & \text{for } \gamma=1, \end{cases} \tag{28}$$

where an interior solution is guaranteed by parameter restrictions (11). The economy's elasticity of aggregate labor supply with respect to the net-of-tax rate becomes[12]

$$\frac{\partial T^{\star}(\tau)}{\partial(1-\tau)}\,\frac{1-\tau}{T^{\star}(\tau)} = \frac{\gamma}{(1-\tau)(1-\gamma)(\phi+1)+\gamma} > 0\,, \tag{29}$$

where parameter restriction (11a) guarantees a positive denominator. As shown in appendix A, this high elasticity is similar to that of a corresponding employment lotteries model, and the two elasticities are in fact the same for $\phi = 0$ (when aggregate labor supplies are also identical), and for $\gamma = 1$, regardless of the value of ϕ. An important ingredient of this high elasticity is that the government rebates tax revenues lump sum to workers.

To illustrate the sensitivity of career length to labor taxation when tax revenues are rebated lump sum, figure 2 depicts a version of Prescott's (2002) assertion that differences in tax rates explain differences in employment outcomes between the U.S. and Western Europe. According to Prescott, the net-of-tax rate, $1-\tau$, is 0.60 in the U.S. as compared to 0.40 in France, which can explain why French labor supply is depressed by 30 percent relative to that of the U.S. In particular, for $\gamma = 1$, the elasticity (29) is equal to one and hence a one-third lower net-of-tax rate should indeed result in one-third shorter career length. Figure 2 shows also how the answer to this net-of-tax differential changes with γ, the curvature of the utility function (the coefficient of relative risk aversion), as well as with ϕ, the elasticity of earnings to accumulated working time. In the figure, we have calibrated the disutility of

[12]Note that we have computed a labor supply elasticity with respect to the net-of-tax rate $(1-\tau)$ rather than to disposable wage income *per se*. As pointed out above and emphasized in footnote 10, what matters for the effect of taxes on labor supply is how wage income is split into two parts: one that goes directly to the worker as disposable wage income, another that is first paid to the government as taxes, but then returned to the worker in the form of lump-sum transfers.

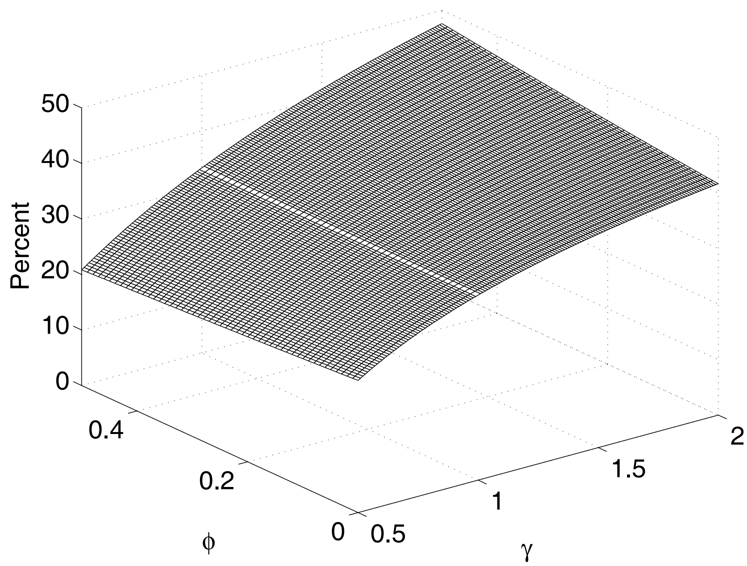

Figure 2: How much labor supply is depressed in Europe with tax rate $\tau_{EU} = 0.6$ as compared to the U.S. with tax rate $\tau_{US} = 0.4$, as a function of the coefficient of relative risk aversion (γ) and the elasticity of earnings to accumulated working time (ϕ). The disutility of work B is calibrated so that the U.S. career length is 2/3 (of adult life).

work B so that the U.S. career length is 2/3 (of adult life).

6 Effect of social security on career length

Our fourth and last finding is straightforward and possibly very pertinent. Many government provided social security programs are associated with implicit tax wedges that can cause workers to retire at particular ages. At such corner solutions, variations in taxes and benefits within some range do not alter a worker's choice of career length.

Finding 4: Social security tax and benefit rules that put a kink in a worker's budget set can cause a corner solution to career lengths at an official retirement age and thereby extinguish the high labor supply elasticity that would prevail at an interior solution.

To illustrate such outcomes, we derive equilibrium outcomes for the case with $\gamma = 1$ and the following particular social security arrangement.

Instead of returning all tax receipts lump sum to workers as in section 5, we now assume that all revenues are used to finance a social security system in which workers are eligible to retire and collect benefits after an official retirement age R. Only those labor earnings accruing *before* R are subject to a flat rate social security tax $\tau \in (0, 1)$. Benefits *after* the worker's chosen retirement date T, which may or may not equal R, are computed as a replacement rate ρ times a worker's average earnings prior to R. Thus, labor earnings after R are not taxed; neither do they affect the base for calculating benefits. Workers who choose to retire after R collect no benefits until they actually retire.[13]

To construct an equilibrium, we set the two parameters R and τ of the social security system, and then solve residually for a replacement rate ρ that is consistent with a balanced government budget. To simplify the task of characterizing equilibria, we restrict attention to policies with $R \in (0.5, 1)$, and we bound the disutility of work from above:

$$B \leq \frac{\phi + 1 - \tau}{1 - R}. \tag{30}$$

We shall show that these parameter restrictions deliver two equilibrium outcomes. First, the equilibrium career length, denoted $\tilde{T}$, is *longer* than the official retirement period, i.e., $\tilde{T} > 1 - R$. Second, workers strictly prefer to supply their labor *before* rather than after the official retirement age, i.e., $\int_0^R n_t dt = \min\{\tilde{T}, R\}$.[14] These outcomes simplify the task of characterizing an equilibrium while also being consistent with empirical facts about how primary workers distribute work over their lives, since the unit length of lifetime refers to a worker's adulthood.

[13]While our specification of social security taxes and benefits is overly simple, it captures key features of some real-world programs. The assumption that the replacement rate is a function of average earnings but not career length, is a good approximation to programs that compute benefits on the basis of fewer years than a primary worker's normal choice of career length, a feature that makes the first-order condition with respect to career length reflect a worker's marginal rather than inframarginal lifetime labor supply. (We elaborate on this point in section 7.2.) As an example, U.S. social security benefits are computed based on the average of a worker's highest 35 years of earnings. As for our assumption that someone who works beyond the official retirement age R receives no social security benefits until she actually retires, Schulz (2001, pp. 141-2) describes how this was the situation in the U.S. social security system between 1950 and 1972, after the repeal in 1950 of an earlier provision of a 1 percent increase in benefits for each year of delay. After 1972, a delayed retirement credit was reintroduced, but it is only with rules that recently became effective that the compensation is high enough for there to be no loss in the actuarial value of a worker's lifetime benefits. We consider implications of those recent major policy changes in the U.S. from the perspective of our framework in section 7.3.

[14]As shown in appendix B, the key to having workers prefer to supply their labor before rather than after the official retirement age is that the part of the equilibrium career length during which social security taxes are paid be longer than the part of the equilibrium retirement period during which benefits are collected, an outcome ensured by parameter restrictions (30) and $R \in (0.5, 1)$. This outcome makes the social security tax τ needed to balance the government's budget be lower than the social security replacement rate ρ. When $\rho > \tau$, a worker would not want to try to avoid the social security tax by postponing labor supply until after the official retirement age: lost social security benefits would outweigh tax savings.

With our parameter restrictions and these conjectured equilibrium outcomes, the government budget constraint is[15]

$$\tau W \min\{e(\tilde{T};\phi),\, e(R;\phi)\} = \Big(1 - \max\{R, \tilde{T}\}\Big) \frac{\rho}{\min\{\tilde{T}, R\}} W \min\{e(\tilde{T};\phi),\, e(R;\phi)\}, \quad (31)$$

where the left side is tax revenues and the right side is social security benefits. The first (second) argument of the max and min operators in (31) presumes an equilibrium outcome in which workers retire before (after) the official retirement age. That is, if the equilibrium career length $\tilde{T}$ is shorter (longer) than the official retirement age R, tax revenues are $\tau W e(\tilde{T};\phi)$ ($\tau W e(R;\phi)$) and social security pays a benefit of $\rho W e(\tilde{T};\phi)/\tilde{T}$ ($\rho W e(R;\phi)/R$) over the eligible nonworking period that lasts $1 - R$ ($1 - \tilde{T}$). Note that the unit length of a lifetime implies that an age interval corresponds both to a fraction of a worker's lifetime and also to a fraction of the population within that age interval at any point in time. From (31) we can solve for the replacement rate,

$$\rho = \frac{\min\{R, \tilde{T}\}}{1 - \max\{R, \tilde{T}\}}\, \tau. \quad (32)$$

Again, with our parameter restrictions and conjectured equilibrium outcomes, a worker's optimal career length solves[16]

$$\max_{T \in (0,1]} \Big\{ \log\Big[(1-\tau) W \, \min\{e(T;\phi),\, e(R;\phi)\} + W \, \max\{0,\, e(T;\phi) - e(R;\phi)\} \\ + \rho W \, \min\{(1-R)e(T;\phi)/T,\, (1-T)e(R;\phi)/R\} \Big] - BT \Big\}, \quad (33)$$

where the arguments of the max and min operators inside the log function appear in the same order as in (31) and (32), i.e., the first (second) argument refers to the case when the worker chooses to work shorter (longer) than the official retirement age.

Case with $\tilde{T} \le R$

In the case of an optimal career length $T \le R$, the first-order condition of (33) at an interior

[15] Division by $\min\{\tilde{T}, R\}$ in (31), as well as division by $(1 - \max\{R, \tilde{T}\})$ in (32), is permissible since $R \in (0.5, 1)$ and equilibrium career length can be neither $\tilde{T} = 0$, because a worker with preferences described by (4) would never choose zero consumption if that can be avoided, nor $\tilde{T} = 1$, as discussed below.

[16] Regarding our exclusion of $T = 0$ from the choice set, see footnote 15.

solution (with respect to $T \leq R$) becomes

$$\frac{\phi+1}{T} - \frac{\rho(1-R)/T}{(1-\tau)T + \rho(1-R)} - B = 0. \tag{34}$$

By government budget balance in (32), $\rho = \tau\tilde{T}/(1-R)$, which can be substituted into (34) to yield an expression for equilibrium career length,

$$\tilde{T} = \frac{\phi+1-\tau}{B} \equiv R^{+}(\tau). \tag{35}$$

Given an equilibrium with $\tilde{T} \leq R$, equilibrium expression (35) implies $R \geq R^{+}(\tau)$. If $R^{+}(\tau) \in (0.5, 1)$, it can be verified that $R^{+}(\tau)$ is the lowest possible official retirement age $R \in (0.5, 1)$ for which equilibrium expression (35) holds, namely, $\tilde{T} = R$ for $R = R^{+}(\tau)$.

Case with $\tilde{T} \geq R$

In the case of an optimal career length $T \geq R$, the first-order condition of (33) at an interior solution (with respect to $T \geq R$) becomes

$$\frac{-\frac{\rho}{R}\frac{R^{\phi+1}}{\phi+1} + T^{\phi}}{\left[\rho\frac{1-T}{R} - \tau\right]\frac{R^{\phi+1}}{\phi+1} + \frac{T^{\phi+1}}{\phi+1}} - B \geq 0, \tag{36}$$

which holds with equality except under a binding corner solution with $T = 1$. However, such a corner solution can be ruled out as an equilibrium because government budget balance in (32) would imply that the replacement rate goes to infinity; hence, it must be optimal for a worker to retire prior to the end of her lifetime. After substituting $\rho = \tau R/(1-\tilde{T})$ into (36) at equality, we obtain an expression for equilibrium career length,

$$\tilde{T} = \frac{\phi + 1 - \tau\frac{R}{1-\tilde{T}}\left(\frac{R}{\tilde{T}}\right)^{\phi}}{B}. \tag{37}$$

Given an equilibrium with $\tilde{T} \geq R$, equilibrium expression (37) implies

$$R \leq \frac{\phi + 1 - \tau\frac{R}{1-R}}{B}, \tag{38}$$

where the right side is an upper bound for the right side of (37), attained at $\tilde{T} = R$ because the right side of (37) is a decreasing function in $\tilde{T}$.[17] Next, we implicitly define $R^{-}(\tau)$ as a fixed point of (38) at equality,

$$R^{-}(\tau) = \frac{\phi + 1 - \tau \dfrac{R^{-}(\tau)}{1 - R^{-}(\tau)}}{B}. \tag{39}$$

Over the interval $[0, 1]$, there exists a unique fixed point $R^{-}(\tau) \in (0, 1)$, since the left side of (39) is a straight line with intercept zero and a positive slope, while the right side is a strictly decreasing function that starts at $(\phi + 1)/B > 0$ and has minus infinity as the limit when $R^{-}(\tau) \to 1$. If $R^{-}(\tau) \in (0.5, 1)$, it can be verified that $R^{-}(\tau)$ is the highest possible official retirement age $R \in (0.5, 1)$ for which equilibrium expression (37) holds, namely, $\tilde{T} = R$ for $R = R^{-}(\tau)$. Moreover, if $R^{-}(\tau) \in (0.5, 1)$, it follows from (35) and (39) that $R^{-}(\tau) < R^{+}(\tau)$.[18]

We can now state a proposition that describes how the retirement age $\tilde{T}$ chosen in equilibrium depends on the official social security retirement age. The proof appears in appendix B.

Proposition 1: Given an official retirement age $R \in (0.5, 1)$ and a tax rate $\tau \in (0, 1)$ that satisfy (30), the equilibrium career length $\tilde{T}(R, \tau)$ is unique and can be characterized in terms of $R^{+}(\tau)$ and $R^{-}(\tau)$, as defined in (35) and (39):

i) If $R \geq R^{+}(\tau)$, then $\tilde{T}(R, \tau) = R^{+}(\tau)$ (retirement *before* the official retirement age).

ii) If $R \leq R^{-}(\tau)$, then $\tilde{T}(R, \tau) \in [R^{-}(\tau), R^{+}(\tau))$, $\tilde{T}(R^{-}(\tau), \tau) = R^{-}(\tau)$ and $\partial \tilde{T}(R, \tau)/\partial R < 0$ (retirement *after* the official retirement age).

[17] The derivative of the right side of (37) with respect to $\tilde{T}$ is

$$-\frac{\tau R^{\phi+1}}{\tilde{T}^{\phi}(1 - \tilde{T})B} \left[\frac{1}{1 - \tilde{T}} - \frac{\phi}{\tilde{T}} \right] < 0,$$

where the strict inequality follows from $\phi \in [0, 1]$ and $\tilde{T} \in [R, 1)$, where $R \in (0.5, 1)$.

[18] Given that $R^{-}(\tau) \in (0.5, 1)$, the following strict inequality holds

$$\Big(R^{-}(\tau) = \Big) \qquad \frac{\phi + 1 - \tau \dfrac{R^{-}(\tau)}{1 - R^{-}(\tau)}}{B} < \frac{\phi + 1 - \tau}{B} \qquad \Big(= R^{+}(\tau) \Big).$$

iii) Otherwise, $\tilde{T}(R, \tau) = R$ (retirement *at* the official retirement age).

Figure 3 displays the equilibrium career length as a function of R and τ, and figure 4 compares equilibrium outcomes in two economies with different values of ϕ. We proceed to explain the shapes of these functions when $\tilde{T} < R$ and when $\tilde{T} > R$.

6.1 Possible corner solution at the official retirement age

According to Proposition 1, there exist a range of official retirement ages sufficiently *high* that they induce equilibrium retirements *before* the official retirement age and a range of official retirement ages sufficiently *low* that they induce equilibrium retirements *after* the official retirement age. Between these two intervals there exists an intermediate interval of official retirement ages that induce equilibrium retirement *at* the official retirement age. In this middle range, the coincidence of official and actual retirement ages indicates a kink in implicit taxation that occurs at the official retirement age – a situation commonly said to describe actual social security arrangements.[19]

For an R in our high range in which $\tilde{T} = R^{+}(\tau) < R$ so that workers retire *before* the official retirement age, the effect of the social security tax on career length in (35) is quantitatively similar to the effect of a labor tax in (28) in the style of Prescott's (2002) analysis in which all tax receipts are handed back lump sum to workers. Indeed, as can be seen by comparing formula (28) for T^* with formula (35) for $\tilde{T}$, when $\phi = 0$ the lifetime labor supply effects are actually identical to the labor supply effects obtained by Prescott (2002). The reasons are that (a) under our assumption that average lifetime earnings alone determine the replacement rate without regard to career length, when $\phi = 0$ workers regard the social security contribution purely as a tax and perceive no extra benefits accruing to them from paying it, while (b) the present value of future social security payments operates like a lump sum transfer when optimal career length falls short of the official retirement age. When $\phi > 0$, the social security tax in (35) is less distorting than Prescott's (2002) labor tax, since longer careers now have the advantageous effect of increasing social security benefits due to the higher average lifetime earnings when a worker moves up along the

[19] In the empirical analysis of Rust and Phelan (1997), the peaks in the distribution of retirement in the U.S. at age 62 and 65 (the ages of early and normal eligibility for social security benefits, respectively) are rationalized as artifacts of particular details of the rules for social security and for public health insurance for the elderly (Medicare). Hairault et al. (2010) analyze how social security rules in France in conjunction with specific income support programs for workers between age 55 and 59, shape implicit taxation and cause French nonemployment to rise sharply even prior to age 60 when workers become eligible for social security.

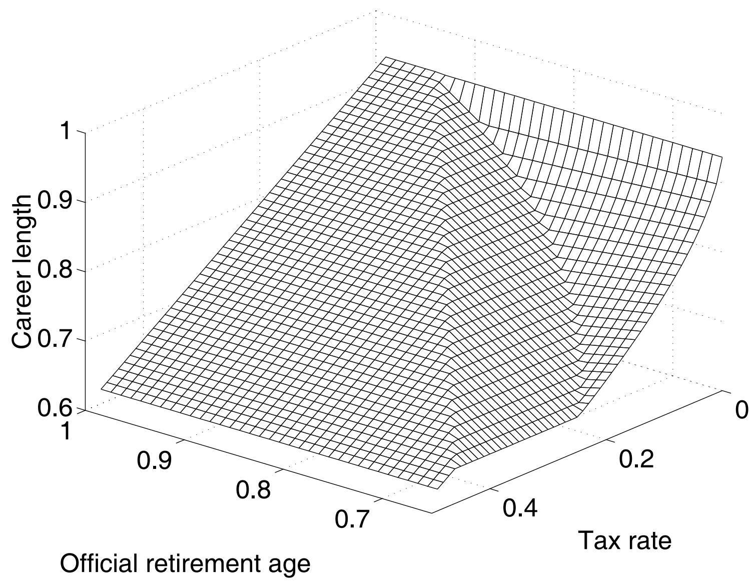

Figure 3: Equilibrium career length $\tilde{T}(R, \tau)$ as a function of the official retirement age R and the tax rate τ when the earnings profile parameter is $\phi = 0.5$, and the preference parameters are $\gamma = 1$ and $B = 1.6$.

earnings profile. But besides the longer career length $\tilde{T}$ in (35) as compared to T^* in (28) when $\phi > 0$, lifetime labor supply in the region with $\tilde{T} < R$, remains highly responsive to tax changes, as illustrated by the downward-sloping plane to the left in figure 3 (where, as explained above, the value of R has no effect).

For an R sufficiently low that $\tilde{T} = R^{-}(\tau) > R$ so that workers retire *after* the official retirement age, the marginal decision on career length is distorted by the loss of benefits incurred from working beyond the official retirement age. In this region with $\tilde{T} > R$, the tax rate and the official retirement age both affect $\tilde{T}$ through their effects on the equilibrium replacement rate as determined by $\rho = \tau R/(1 - \tilde{T})$ in (32). Specifically, for an unchanged career length $\tilde{T}$, the replacement rate rises in response to an increase in either τ or R. A worker who faces the resulting higher opportunity cost of retirement benefits foregone while working beyond the official retirement age would choose to reduce her career length. Thus, in the region with $\tilde{T} > R$, lifetime labor supply falls in response to an increase in either τ or R. This effect is depicted by the bowl-shaped surface at the far right in figure 3 where career length $\tilde{T}$ decreases with increases in both the tax rate and the official retirement age. The latter effect is easier to discern in figure 4 where the downward-sloping portions of both of the equilibrium career length functions refer to the region with $\tilde{T} > R$. These equilibrium

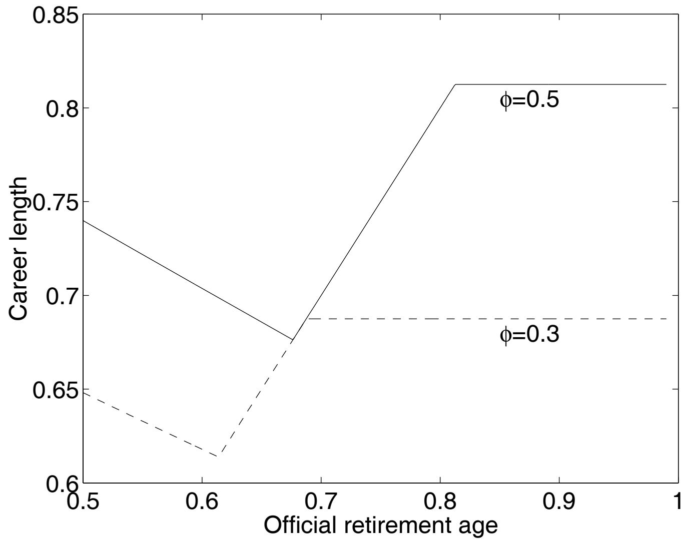

Figure 4: Equilibrium career length $\tilde{T}(R,\tau)$ as a function of the official retirement age R, in two economies with earnings profile parameter $\phi = 0.3$ (dashed line) and $\phi = 0.5$ (solid line), respectively. Both economies have the same tax rate $\tau = 0.2$, and the preference parameters are $\gamma = 1$ and $B = 1.6$.

forces underpin the analytical derivative shown above in case ii) of Proposition 1.

For an R within our intermediate range, there are no effects of the tax on lifetime labor supply so long as workers choose to remain at the corner solution highlighted in case iii) of Proposition 1. For example, at $R = 0.65$ in figure 3, any tax rate between 0.25 and 0.45 would induce equilibrium retirement *at* the official retirement age. For the parameterization in figure 3, such a corner solution prevails for a tax range that is wider than 15 percentage points up until an official retirement age of 0.80. Thereafter, the tax range associated with a corner solution narrows and eventually, at a high enough official retirement age, there exists only equilibria with equilibrium retirement before the official retirement age.

To provide another perspective, figure 4 depicts equilibrium career lengths $\tilde{T}(R,\tau)$ as functions of the official retirement age R in two economies with distinct earnings profile parameters $\phi = 0.3$ and $\phi = 0.5$, respectively. The two economies share the same tax rate $\tau = 0.2$ and the same disutility of work $B = 1.6$.[20] (We use the same preference and earnings profile parameters as in figure 1.) The figure illustrates how the presence of social security modifies but does not remove the tendency for workers with a higher earnings-curve elasticity parameter ϕ to retire at a later age, as studied in section 3. However, the existence

[20]The solid line in figure 4 is a slice of figure 3 at $\tau = 0.20$ (but for a somewhat wider range of R).

of our intermediate range of official retirement ages in which equilibrium career length equals the official retirement age opens up the possibility that the equilibrium career lengths are identical across two economies with different ϕ's but identical τ's and R's. This is evidently the case in figure 4 when R is approximately two thirds of a worker's (adult) lifetime.

7 Concluding remarks

We close with some thoughts about how observed career lengths might be interpreted through the lens of our model, as well as what our model says about effects of policy changes on prospective career lengths.

7.1 Diverse workers retiring at the same official retirement age

The two equilibrium mappings in figure 4 refer to two distinct economies with the only difference in primitives being the earnings-profile parameter ϕ. But we can also imagine the outcomes depicted there to refer to two groups of workers who live in the *same* economy, in particular, an economy in which the government runs a balanced social security budget for each group of workers, there being identical policy parameters τ and R across the two groups but different replacement rates ρ determined by (32)). This interpretation reminds us of a feature of real-world social security programs that tends to increase the range of official retirement ages for which an equilibrium would imply that heterogeneously situated workers all end up choosing identical career lengths by retiring at the official retirement age. Thus, real-world social security programs often redistribute from high to low income earners, and the former workers usually have more elastic (higher ϕ) earnings profiles than the latter workers. It follows that if the redistribution associated with social security payout rules ends up lowering and raising the implicit returns to work for high and low income workers, respectively, it tends to lower and raise the corresponding equilibrium mappings in figure 4 for high and low income workers, respectively. That would seem to widen the range of official retirement ages for which both groups of workers find it optimal to choose to retire at the official retirement age. To execute a precise analysis, we would need to specify the details of such a social security program and derive an equilibrium.

7.2 Retirement as a marginal decision off a corner solution

An important message of a life cycle model like ours with indivisible labor (and no intensive margin) is that a marginal labor supply decision is about the choice of retirement age. Hence, at an interior solution for career length, how taxes and social security affect actual retirement age are important determinants of the aggregate labor supply. To emphasize this point, consider an implicit tax and benefit system with the following characteristics. The system is such that a worker with earnings profile (2) chooses to supply labor at least during all of some initial phase of life $P \in (0,1)$ that we can call the 'prime of life'. After paying taxes and receiving government transfers including the discounted value of future social security benefits, this yields a present value of disposable lifetime income equal to m. When contemplating any additional old-age labor supply, $T > P$, we assume that the worker faces an 'effective' tax rate $\breve{\tau} \in [0,1)$ that incorporates both positive and negative effects that extra earnings might have on future social security benefits. Given a policy configuration that makes a prime-age labor supply of P optimal, the remainder problem that pins down the optimal career length is

$$\max_{T\in[P,1]} \Big\{ \log\left[(1-\breve{\tau})W\left(e(T;\phi)-e(P;\phi)\right)+m\right] - BT \Big\}. \tag{40}$$

The first-order condition is

$$\frac{(1-\breve{\tau})W\dfrac{\partial e(T;\phi)}{\partial T}}{(1-\breve{\tau})W\left(e(T;\phi)-e(P;\phi)\right)+m} \geq B. \tag{41}$$

The equilibrium career length $\breve{T}$ that satisfies this first-order condition depends on a worker's equilibrium consumption level. In our stationary economy with identical agents, we represent equilibrium consumption as a fraction of a worker's lifetime labor earnings

$$(1-\breve{\tau})W\left(e(\breve{T};\phi)-e(P;\phi)\right)+m = (1-\nu)e(\breve{T};\phi), \tag{42}$$

where the economy's resource constraint implies $\nu \in [0,1)$.[21] For example, ν would be strictly positive if the government uses some lifetime tax receipts to finance a public good that is not a perfect substitute with private consumption. In sections 5 and 6, we imposed $\nu = 0$ by assuming that all tax receipts were either handed back lump sum to workers or fully used to

[21] We exclude $\nu = 1$ because we want a worker's consumption to be positive in an equilibrium so that her lifetime utility remains well-defined.

finance a social security system. More generally, let $\nu \in [0, 1)$ and substitute (42) into (41) to arrive at the following expression for equilibrium career length:

$$\breve{T}(\breve{\tau}, \nu) = \min\left\{\frac{(1-\breve{\tau})(\phi+1)}{(1-\nu)B},\ 1\right\}. \tag{43}$$

The equilibrium career length in (43) depends only on the tax rate $\breve{\tau}$ in old age and the equilibrium fraction ν of lifetime earnings of which the government deprives workers. Remarkably, the exact details of the tax and social security system during the prime-age period do not enter here at all.

Before concluding that an optimal tax policy would set the tax distortion for older workers to zero, recall our presumption about the implicit tax and social security system, namely, that the system is such that workers choose to supply labor P early, i.e., while they are prime aged. Purely age-related tax relief proposals targeted to older workers are subject to the objection that they would motivate workers to postpone labor market participation in order to enjoy more favorable tax treatment over their lives. That would surely happen in the formal framework of this paper with its ample room for workers to engage in labor supply arbitrage over their life cycle.

However, features omitted from our model could limit the extensive intertemporal substitution underlying the caveat made in the previous paragraph. Factors that should make workers reluctant strategically to postpone their lifetime labor supplies are incomplete markets and uncertainties about future health status and how various aspects of individual labor careers will play out. Hence, young workers enter the labor market not only because of impediments to borrowing against future labor earnings, but also because of an interest in resolving uncertainties about their destinies in the labor market. Similarly, established workers are unlikely to put careers on hold and to engage in spells of temporary early retirement. Intermittent interruptions are not good for careers. For these reasons, we still suspect that if the goal is to increase total labor supplied over the life cycle, well designed policies will feature tax and benefit reforms targeted at older workers

7.3 Implications of recent changes in U.S. social security rules

Recently, there have been major changes in the U.S. social security rules. The Full Retirement Age (FRA) is gradually being increased from 65 to 67. In 2000, the earnings test through age 69 for persons who choose to work beyond the FRA was removed. In addition, for us an important change is the gradual increase in the Delayed Retirement Credit (DRC) for someone who reaches the FRA in 2009 and who delays claiming benefits, which tops out at an annualized credit of 8 percent per each year of delay until age 70. Thus, the kink in a worker's budget set associated with the FRA has been smoothed out because, as pointed out by Schulz (2001, p. 142), for there to be no loss in the actuarial value of a worker's lifetime benefits, the benefit level needs to be increased by about 8 percent for each year of delay.

In terms of our analysis, these changes have almost removed any *effective* official retirement age R. Thus, without the constraining influence from an official retirement age, the current U.S. social security rules are best approximated with case i) of Proposition 1, where workers can be thought of as retiring *before* the official retirement age.

In terms of the implied aggregate labor supply elasticities, any reforms that move people from the corner case iii) to the interior case i) of Proposition 1 are very important. For as we pointed out in subsection 6.1, the lifetime labor supply elasticity at a case i) interior solution becomes almost as large as found with Prescott's (2002) labor tax with tax receipts handed back lump sum. Reforms that move significant measures of people from the corner to the interior would substantially raise aggregate labor supply elasticities.

7.4 Confronting observations about career lengths

When combined with our section 4 analysis of unanticipated earnings shocks, events that move many workers between the interior solution of section 3 and the corner solution of section 6 manifest themselves as variations in career length around the official retirement age. In particular, the disutility of work B can take such a value that while workers had originally planned to retire at an official retirement age, large unforeseen earnings shocks can impel workers to alter planned career lengths, given their life cycle savings accumulated up to that point. It is natural to ask whether, by pushing workers on and off the corner solution associated with the official social security retirement age, an interplay among these forces can help explain the increased incidence of early retirement observed in the last few decades (see e.g. the country studies compiled by Gruber and Wise (2004)). More generally, nonemployment has risen especially among older workers in Europe – a key feature of the trans-Atlantic employment puzzle posed by Krugman (1987, p. 68): "no strong case exists that Europe's welfare states were much more extensive or intrusive in the 1970s than in the 1960s, and no case at all exists that there was more interference in markets in the 1980s than in the 1970s. Why did a social system that seemed to work extremely well in the 1960s work increasingly badly thereafter?" To address these observations, we suspect that it will be useful to combine the forces isolated in this paper with insights from empirical studies that have documented increased variability of both transitory and permanent components of individual workers' earnings (see e.g. the literature review of Katz and Autor (1999)), and thereby enrich our earlier efforts to solve the problem posed by Krugman (Ljungqvist and Sargent (1998, 2008)) by incorporating a more serious model of career length.[22]

A Equivalence between employment lotteries and time averaging?

Ljungqvist and Sargent (2007) found that in models with indivisible labor, a high disutility of labor that results in an interior solution is the source of a high aggregate labor supply elasticity, not the Rogerson aggregation theory based on employment lotteries and complete markets. The time-averaging model with indivisible labor and a high disutility of labor yields a high aggregate labor supply elasticity for a variety of specifications, including ones in which experience affects earnings.

But an *exact* equivalence of aggregate outcomes under individual time-averaging, on the one hand, and employment lotteries with complete markets, on the other hand, hinges on work experience not affecting earnings. Ljungqvist and Sargent (2007, sections 3.5, 3.6) analyze an increasing experience-earnings profile that is a step function with two flat spots and show that the equivalence between the lotteries and time-averaging models breaks down. It also break down for the specification that we have adopted in this paper. An increasing earnings-experience profile creates a nonconvexity over careers and allows a representative family to achieve aggregate allocations with employment lotteries that individuals cannot attain by time averaging.

Thus, consider a representative family consisting of a continuum $j \in [0, 1]$ of ex ante identical workers like those in section 2. The family chooses a consumption and employment allocation $c_t^j \geq 0$, $n_t^j \in \{0, 1\}$ to maximize

$$\int_0^1 \int_0^1 u\left(c_t^j, 1 - n_t^j\right) dt\, dj \tag{44}$$

[22] Kitao et al. (2008) pursue an analysis along those lines.

subject to

$$\int_0^1 \int_0^1 \Big[w_t^j n_t^j - c_t^j \Big] dt\, dj \geq 0\,, \tag{45}$$

where w_t^j is the potential earnings of worker j at time t which depends on her past work experience, as described in (2).

As in Ljungqvist and Sargent (2007, section 3.6), the family solves this problem by administering a lifetime employment lottery once and for all before time 0 that assigns a fraction $N \in [0,1]$ of people to work always ($n_t^j = 1$ for all $t \in [0,1]$ for these unlucky people) and a fraction $1 - N$ always to enjoy leisure ($n_t^j = 0$ for all $t \in [0,1]$ for these lucky ones). An individual who works throughout her lifetime generates present-value labor income equal to $We(1;\phi)$, as defined in (3). Since the subjective discount rate equals the market interest rate, the optimal plan prescribes constant consumption to those who work, $c_t^j = \bar{c}$ when $n_t^j = 1$, and also to those who do not work, but possibly at a different level, $c_t^j = c$ when $n_t^j = 0$. Thus, the family's optimal labor supply solves

$$\max_{N \in [0,1]} \Big\{ N\, u(\bar{c}, 0) + (1 - N) u(c, 1) \Big\}, \tag{46}$$

subject to $N\bar{c} + (1 - N)c = NWe(1;\phi)$. Following the steps in section 2, at an interior solution, the optimal fraction of individuals sent to work is determined by the first-order condition,

$$\bar{T}(\phi) = \begin{cases} \dfrac{1-\gamma}{1 - B^{1/\gamma}}\,, & \text{for } 0 < \gamma < 1 \text{ and } \gamma > 1; \\ \dfrac{1}{B}\,, & \text{for } \gamma = 1. \end{cases} \tag{47}$$

Hence, members of the representative family on average work less than individuals who are left to 'time average', as characterized by (10).[23] The latter individuals confront a difficult choice between enjoying leisure and earning additional labor income at the peak of their lifetime earnings potential. This tension is not experienced by the individuals who

[23] Ljungqvist and Sargent (2007, sections 3.5, 3.6) obtain a similar outcome in their model with an experience-earnings profile that has two flat spots. With time-averaging, those individuals who work enough to lift themselves beyond the lower flat part of the experience-earnings profile devote a fraction of their lifetimes to work that is higher than is the fraction of people working in the employment-lottery model. But for someone in the time-averaging model who chooses to work sufficiently little that she stays on the first flat segment of the experience-earnings profile, the optimal fraction of her lifetime devoted to work equals the fraction of people who work in the employment-lottery model. The latter outcome is consistent with the analysis here in the following sense. Under a flat experience-earnings profile, workers who 'time average' choose the same life-time labor supply as the average work in an employment-lottery model. For the employment-lottery model, equation (47) shows that the representative family chooses a fraction of family members who work that does not depend on whether the experience-earnings profile slopes upward.

follow the instructions of the family planner who uses lotteries to convexify the indivisibility brought by careers. Of course, in the special ($\phi = 0$) case when work experience does not affect earnings, the aggregate labor supplies are exactly the same across a Rogerson (1988) employment-lottery model and a Ljungqvist and Sargent (2007) time-averaging model, and people enjoy the same expected lifetime utilities.

We can concisely summarize the message of this appendix by comparing the responses of aggregate time spent employed to labor tax rate τ for the employment-lottery model,[24]

$$N^\star(\tau) = \begin{cases} \dfrac{(1-\tau)(1-\gamma)}{\big[(1-\tau)(1-\gamma)+\gamma\big]\big(1-B^{1/\gamma}\big)}\,, & \text{for } 0<\gamma<1 \text{ and } \gamma>1; \\ \dfrac{(1-\tau)}{B}\,, & \text{for } \gamma = 1, \end{cases} \tag{48}$$

and for the time-averaging model in (28). As noted above, individuals in the time-averaging model choose a longer career length than the average lifetime labor supply in the employment lottery model, at an interior solution. Therefore, if the equilibria without taxation are characterized by a corner solution, e.g. due to a binding official retirement age, successive increases in taxation will first reduce employment in the economy with employment lotteries while the labor supply in the economy with time averaging is more robust. Though at interior solutions, the elasticity of aggregate labor supply with respect to the net-of-tax rate in the employment lotteries model is equal to

$$\frac{\partial N^\star(\tau)}{\partial(1-\tau)}\,\frac{1-\tau}{N^\star(\tau)} = \frac{\gamma}{(1-\tau)(1-\gamma)+\gamma} > 0\,, \tag{49}$$

which can be compared to elasticity (29) for the time averaging model. The two elasticities are naturally the same when $\phi = 0$ since the aggregate labor supplies are then identical. The elasticities are also the same and equal to one for $\gamma = 1$, regardless of the value of ϕ. For other values of γ, the elasticity in the time averaging model is smaller (larger) than that of the employment lotteries models for $0 < \gamma < 1$ ($\gamma > 1$). So, yes, the exact equivalence between the models breaks down, but nevertheless with a high disutility of labor like those calibrated in the real business cycle literature, a high labor supply elasticity can still come through in both frameworks.

[24]See Ljungqvist and Sargent (2007, sections 4.2, 4.3) for the same exercise in their model with an experience-earnings profile that has two flat spots.

B Proof of Proposition 1

The formulation of optimization problem (33) is predicated on workers preferring to supply their labor before rather than after the official retirement age, i.e., $\int_0^R n_t dt = \min\{\tilde{T},\, R\}$. We proceed as if this is true in the following equilibrium characterization, and then afterwards verify its correctness under parameter restriction (30) and $R \in (0.5, 1)$.

i) If $R \geq R^{+}(\tau)$, then $\tilde{T}(R,\tau) = R^{+}(\tau)$.

   For any $R \in (0.5, 1)$ that satisfies $R \geq R^{+}(\tau)$, the constant career length $\tilde{T} = R^{+}(\tau)$ is an equilibrium since it satisfies both the government budget constraint (31) and a worker's first-order condition (34) for the case with $T \leq R$, as summarized in equilibrium expression (35). It remains just to show that there cannot exist another equilibrium in which workers choose a career length longer than R, i.e., we will show that equilibrium expression (37) cannot hold when $R \geq R^{+}(\tau)$. For any $R \in (0.5, 1)$ that satisfies $R \geq R^{+}(\tau)$, it follows from the fact that (35) holds with equality that the right-hand side of (37), evaluated at $\tilde{T} = R$, must fall below the left-hand side of (37). Next, since the left-hand side of (37) is strictly increasing in $\tilde{T}$, while the right-hand side is decreasing in $\tilde{T}$ (see footnote 17), we can rule out the existence of any $\tilde{T} > R$ at which equilibrium expression (37) would hold.

ii) If $R \leq R^{-}(\tau)$, then $\tilde{T}(R,\tau) \in [R^{-}(\tau), R^{+}(\tau))$, $\tilde{T}(R^{-}(\tau),\tau) = R^{-}(\tau)$ and $\partial\tilde{T}(R,\tau)/\partial R < 0$.

   Since $R^{-}(\tau) \in (0, 1)$ as established in section 6, it follows that if there is any $R \in (0.5, 1)$ that satisfies $R \leq R^{-}(\tau)$, it must be that $R^{-}(\tau) \in (0.5, 1)$. Moreover, since $R^{-}(\tau)$ is the fixed point of (39), it follows that equilibrium expression (37) for an interior solution with $\tilde{T} \geq R$ holds for $\tilde{T} = R = R^{-}(\tau)$, i.e., $\tilde{T}\left(R^{-}(\tau),\tau\right) = R^{-}(\tau)$. Next, since the right-hand side of (37) is strictly decreasing in R, it follows that for $R < R^{-}(\tau)$, the right-hand side of (37) lies strictly above the left-hand side of (37) when evaluated at $\tilde{T} = R^{-}(\tau)$. Together with the fact that the right-hand side of (37) is strictly decreasing in $\tilde{T}$ (see footnote 17) while the left-hand side of (37) is strictly increasing, it follows that, for $R \in (0.5,\, R^{-}(\tau))$, the solution to (37) is unique and has $\tilde{T} > R^{-}(\tau)$. Note that the existence of an interior solution $\tilde{T} < 1$ is ensured since the right-hand side of (37) goes to minus infinity when $\tilde{T} \to 1$.

   To establish the upper bound $\tilde{T} < R^{+}(\tau)$, we show that $R^{+}(\tau)$ is strictly greater than the right-hand side of (37) for all $\tilde{T} \geq R \in (0.5, 1)$, i.e.,

$$\frac{\phi + 1 - \tau}{B} > \frac{\phi + 1 - \tau \dfrac{R}{1-\tilde{T}} \left(\dfrac{R}{\tilde{T}}\right)^{\phi}}{B}, \tag{50}$$

which can be simplified to

$$R^{\phi+1} > (1 - \tilde{T})\tilde{T}^{\phi}. \tag{51}$$

Note that for the inadmissible values $R = 0.5$ and $\tilde{T} = 0.5$, the left- and right-hand side of (51) are equal. Next, since the left-hand side is strictly increasing in R while the right-hand side is strictly decreasing in $\tilde{T}$,[25] it follows that inequality (51) holds for all $\tilde{T} \geq R \in (0.5, 1)$.

Given the upper bound $R^{+}(\tau) > \tilde{T}$, it also follows that there cannot exist another equilibrium in which workers choose a career length shorter than R, i.e., equilibrium expression (35) cannot hold when $R \leq R^{-}(\tau)$. Specifically, for any $R \in (0.5, 1)$ that satisfies $R \leq R^{-}(\tau)$, we have shown the existence of an equilibrium with $\tilde{T} \geq R$ with an upper bound $R^{+}(\tau) > \tilde{T}$, and therefore, $R^{+}(\tau) > R$. The latter inequality rules out the existence of another equilibrium with career length shorter than R, because as shown in case i) above, the equilibrium career length in such an equilibrium would be $R^{+}(\tau)$ which now lies *above* rather than below R, i.e., a contradiction.

To establish that $\partial\tilde{T}(R, \tau)/\partial R < 0$, we form an implicit function for (37),

$$F(\tilde{T}, R) \equiv \frac{\phi + 1 - \tau \dfrac{R}{1-\tilde{T}} \left(\dfrac{R}{\tilde{T}}\right)^{\phi}}{B} - \tilde{T} = 0, \tag{53}$$

and use the implicit function theorem,

$$\frac{\partial\tilde{T}}{\partial R} = -\frac{\partial F(\tilde{T}, R)/\partial R}{\partial F(\tilde{T}, R)/\partial\tilde{T}} = -\frac{\tau(\phi+1)R^{\phi}}{\tau R^{\phi+1}\left\{\dfrac{1}{1-\tilde{T}} - \dfrac{\phi}{\tilde{T}}\right\} + B(1-\tilde{T})\tilde{T}^{\phi}} < 0, \tag{54}$$

where the strict inequality is assured by the nonnegativity of the expression in braces

[25] The derivative of the right-hand side of (51) with respect to $\tilde{T}$ is

$$-\tilde{T}^{\phi-1}[\tilde{T} - (1-\tilde{T})\phi] < 0, \tag{52}$$

where the strict inequality follows from $\phi \in [0, 1]$ and $\tilde{T} \in [R, 1)$, where $R \in (0.5, 1)$.

because $\phi \in [0,1]$ and $\tilde{T} \in [R, 1)$, where $R \in (0.5, 1)$.

iii) Otherwise, $\tilde{T}(R, \tau) = R$.

For any $R \in (0.5, 1)$ that satisfies neither $R \geq R^{+}(\tau)$ nor $R \leq R^{-}(\tau)$, the equilibrium career length $\tilde{T}$ is characterized neither by expression (35) for an an interior solution with respect to $\tilde{T} \leq R$, nor by expression (37) for an interior solution with respect to $\tilde{T} \geq R$. Thus, the equilibrium career length is at a corner solution with $\tilde{T} = R$.

The range of official retirement ages for which the equilibrium career length is at a corner solution, is given by $R \in (\max\{0.5,\, R^{-}(\tau)\},\, \min\{1,\, \max\{0.5,\, R^{+}(\tau)\}\})$. This range reflects the fact, as shown above, that the equilibrium sets for case i) and ii) are disjoint in the policy space ($R \in (0.5, 1)$, $\tau \in (0, 1)$). In particular, if $R^{-}(\tau) \in (0.5, 1)$, it follows from (35) and (39) that $R^{-}(\tau) < R^{+}(\tau)$ (see footnote 18).

Returning to the assertion underlying the formulation of optimization problem (33), namely, that workers prefer to supply their labor *before* rather than *after* the official retirement age, we now verify its correctness for the three cases above. In particular, we show that an infinitesimal shift of labor supply from before to after the official retirement age R reduces the present value of a worker's disposable income. (Note that we hold total labor supply constant in these perturbations so the disutility of work remains unchanged.)

i) Suppose that $\tilde{T}(R, \tau) < R$, when the worker under the solution above pays total taxes equal to $\tau W e(\tilde{T}; \phi)$ and collect total social security benefits equal to $(1-R)\rho W e(\tilde{T}; \phi)/\tilde{T}$. After an infinitesimal shift of labor supply from before to after the official retirement age, the worker saves on taxes at the rate $\tau W[\partial e(T; \phi)/\partial T]$ for $T = \tilde{T}$, but loses both on a shorter time of collecting social security, at the rate $-[\partial(1-T)/\partial T]\rho W e(\tilde{T}; \phi)/\tilde{T}$ for $T = R$, and on the lower benefit level caused by lower average labor earnings prior to the official retirement age, at the rate $(1-R)\rho W[\partial e(T; \phi)T^{-1}/\partial T]$ for $T = \tilde{T}$. The worker loses from such a shift in labor supply if the implied savings on taxes fall short of the implied losses on social security collection,

$$\tau W \tilde{T}^{\phi} < \rho W \frac{\tilde{T}^{\phi}}{\phi + 1} + (1-R)\rho W \frac{\phi \tilde{T}^{\phi-1}}{\phi + 1}. \tag{55}$$

After invoking (32), i.e., $\rho = \tau \tilde{T}/(1-R)$, this condition simplifies to $(1-R) < \tilde{T}$ which is indeed true for equilibrium career length (35) under parameter restriction (30).

ii) Suppose that $\tilde{T}(R,\tau) > R$, when the worker under the solution above pays total taxes equal to $\tau We(R;\phi)$ and collect total social security benefits equal to $(1-\tilde{T})\rho We(R;\phi)/R$. The condition corresponding to (55) becomes

$$\tau WR^{\phi} < \rho W \frac{R^{\phi}}{\phi+1} + (1-\tilde{T})\rho W \frac{\phi R^{\phi-1}}{\phi+1}. \tag{56}$$

After invoking (32), i.e., $\rho = \tau R/(1-\tilde{T})$, this condition simplifies to $(1-\tilde{T}) < R$ which is indeed true for equilibrium career length $\tilde{T}(R,\tau) > R$ and $R \in (0.5, 1)$.

iii) Suppose that $\tilde{T}(R,\tau) = R$ and hence, the calculation in (55) still applies. But now it follows immediately that condition $(1-R) < \tilde{T}$ is true for equilibrium career length $\tilde{T}(R,\tau) = R$ and $R \in (0.5, 1)$.

References

Eckstein, Zvi and Kenneth I. Wolpin. 1989. Dynamic Labour Force Participation of Married Women and Endogenous Work Experience. *Review of Economic Studies* 56:375–390.

French, Eric. 2005. The Effects of Health, Wealth, and Wages on Labour Supply and Retirement Behaviour. *Review of Economic Studies* 72:395–427.

Gruber, Jonathan and David A. Wise. 2004. *Social Security Programs and Retirement around the World*. Chicago: University of Chicago Press.

Hairault, Jean-Olivier, Francois Langot, and Thepthida Sopraseuth. 2010. Distance to Retirement and Older Workers' Employment: The Case for Delaying the Retirement Age. *Journal of the European Economic Association* 8 (5):1034–1076.

Heckman, James J. 1993. What Has Been Learned About Labor Supply in the Past Twenty Years? *American Economic Review* 83:116–121.

Katz, Lawrence F. and David H. Autor. 1999. Changes in the Wage Structure and Earnings Inequality. In *Handbook of Labor Economics*, vol. 3A, edited by Orley Ashenfelter and David Card. Amsterdam: Elsevier.

King, Robert G., Charles I. Plosser, and Sergio T. Rebelo. 1988. Production, Growth and Business Cycles. *Journal of Monetary Economics* 21:195–232.

Kitao, Sagiri, Lars Ljungqvist, and Thomas J. Sargent. 2008. A Life Cycle Model of Trans-Atlantic Employment Experiences. Working paper, USC and NYU.

Krugman, Paul R. 1987. Slow Growth in Europe: Conceptual Issues. In *Barriers to European Growth: A Transatlantic View*, edited by Robert Z. Lawrence and Charles L. Schultze. Washington, D.C.: Brookings Institution.

Ljungqvist, Lars and Thomas J. Sargent. 1998. The European Unemployment Dilemma. *Journal of Political Economy* 106:514–550.

———. 2004. *Recursive Macroeconomic Theory, Second Edition.* Cambridge, Mass.: MIT Press.

———. 2007. Do Taxes Explain European Employment? Indivisible Labor, Human Capital, Lotteries, and Savings. In *NBER Macroeconomics Annual 2006*, edited by Daron Acemoglu, Kenneth Rogoff, and Michael Woodford. Cambridge, Mass.: MIT Press.

———. 2008. Two Questions about European Unemployment. *Econometrica* 76:1–29.

———. 2011. A Labor Supply Elasticity Accord? Forthcoming in the *American Economic Review, papers and proceedings.*

Low, Hamish, Costas Meghir, and Luigi Pistaferri. 2010. Wage Risk and Employment Risk over the Life Cycle. *American Economic Review* 100 (4):1432–1467.

Prescott, Edward C. 2002. Prosperity and Depression. *American Economic Review* 92:1–15.

———. 2005. The Transformation of Macroeconomic Policy and Research. In *Les Prix Nobel 2004*. Stockholm: Almqvist & Wiksell International [http://nobelprize.org /economics/laureates/2004/prescott-lecture.pdf].

———. 2006. Nobel Lecture: The Transformation of Macroeconomic Policy and Research. *Journal of Political Economy* 114 (2):203–235.

———. 2007. Comment. In *NBER Macroeconomics Annual 2006*, edited by Daron Acemoglu, Kenneth Rogoff, and Michael Woodford. Cambridge, Mass.: MIT Press.

Prescott, Edward C., Richard Rogerson, and Johanna Wallenius. 2009. Lifetime Aggregate Labor Supply with Endogenous Workweek Length. *Review of Economic Dynamics* 12:23–36.

Rogerson, Richard. 1988. Indivisible Labor, Lotteries, and Equilibrium. *Journal of Monetary Economics* 21:3–16.

Rogerson, Richard and Johanna Wallenius. 2009. Micro and macro elasticities in a life cycle model with taxes. *Journal of Economic Theory* 144 (6):2277–2292.

Rust, John and Christopher Phelan. 1997. How Social Security and Medicare Affect Retirement Behavior In a World of Incomplete Markets. *Econometrica* 65 (4):781–831.

Schulz, James H. 2001. *The Economics of Aging, Seventh Edition*. Westport, Conn.: Auburn House.