Information Aggregation and Investment Decisions

Elias Albagli
Harvard University

Christian Hellwig
UCLA

Aleh Tsyvinski
Yale University

December 2009
Preliminary

Abstract

This paper studies an environment in which information aggregation interacts with investment decisions. The first contribution of the paper is to develop a tractable model of such interactions. The second contribution is to solve the model in closed form and derive a series of implications that result from the interplay between information aggregation and the value of market information for the firms' decision problem. We show that the model generates an information aggregation wedge between price and expected dividend value. The information aggregation wedge is asymmetric: larger on the upside, when there is a lot of investment, and shares are over-valued, than on the downside, when there is little investment, and shares are under-valued. On average the share price exceeds the expected dividend value. We therefore arrive at a theory of share price over-valuation due to information feedback. Third, we discuss the role of managerial incentives which are tied to the firm's share price. We show that our model provides a theory linking price-based incentives to asset over-valuation and excess volatility of investment. When the managers seek to maximize share prices instead of expected dividends, their decisions will attribute too much weight to market information, relative to the optimum. The resulting prices are even more volatile and higher on average, expected dividends are lower, and the information aggregation wedge gets exacerbated.

1 Introduction

A key role played by asset prices in a market economy is the aggregation of information about the value of firms. By pooling together knowledge that is originally dispersed among individual investors, prices provide new sources of information that help shape investor expectations and portfolio decisions. To the extent that the information pooled through prices is not already known within the firm, prices will affect the firm's management's assessment of the value of its investment and profit opportunities and guide the firm in its investment decisions. When information aggregation is perfect, prices perfectly reflect current expectations of future dividends and provide an extremely parsimonious and effective way of conveying to firms the relevant information for guiding investment, or for providing incentives to managers to act in the firms' best interests. When information aggregation is imperfect however, prices can deviate from expected dividends in systematic ways.

In this paper, we examine the informational role of asset prices in partially aggregating information about fundamentals, and providing guidance for real investment decisions of firms. In particular, we ask how the noise in the information aggregation process affects the accuracy and effectiveness of prices for conveying information, valuing expected future dividends, and providing managerial incentives. To this end, we develop and characterize a model in which the value of a firm's dividends depends on fundamentals and the firm's choice of investment. Information about fundamentals is dispersed among traders in a financial market, and is imperfectly aggregated in the firm's stock price. This leads to a price-contingent investment decision that characterizes a feedback relation between the firm's decision problem and the valuation of the firm in the stock market. Our model features systematic departures of prices from fundamentals through noisy information aggregation, and we focus on how this feedback between information aggregation and firm decisions influences asset prices, investment, and expected dividends. We also use our model to reassess the effects of managerial incentives that are closely based on the firm's share price performance for investment, share prices and efficiency. Specifically, our paper makes three contributions.

First, we propose a tractable model to analyze information aggregation through prices and its effects on endogenous investment decisions by firms. The key difficulty is that the methods and assumptions used to characterize equilibria in the canonical models of information aggregation are not applicable in our context.[1] Such models are based on CARA preferences and tractability typically requires normally distributed dividends, conditional on information. This requirement

[1]See Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981), and Dow and Rahi (2003).

cannot be met if the market price is not a sufficient statistic for the firm's decision. These models are thus not suited to analyze a more realistic description of markets where both dispersed information and firm-level private information affect relevant payoffs, investment decisions, and stock prices.

Instead, by assuming that informed traders are risk-neutral and face constraints on their portfolio holdings, we gain substantial flexibility in specifying the firm's profit function, firm-specific private information, and decision problem. Moreover, with an appropriate assumption about noise trader's demand for the firm's shares, we maintain the tractability of normally distributed priors, private signals and market signals for updating purposes. The modeling setup we chose is useful more broadly to analyze questions that require less restrictive assumptions about payoff distributions than the canonical CARA-normal model of noisy information aggregation in asset markets.

Second, we solve our model in closed form and derive a series of implications that result from the interplay between information aggregation and the value of market information for the firms' decision problem. Most importantly, we show that the interaction between information aggregation through prices and the feedback from prices to firm investment causes shares to be over-valued relative to expected dividends on average. A natural consequence of information aggregation through prices is that market-generated information has a stronger impact on prices than on expected dividends, leading to an *information aggregation wedge* between prices and expected dividends–a form of predictability in stock returns, and excess sensitivity of prices to changes in expected expected fundamentals.[2] The excess sensitivity of prices results because the underlying shocks that shift the market information also shift the share owners' private information, which is reflected in prices but not in expected firm value from an outsider's point of view.[3] Although from the perspective of individual informed traders the noise in market information is uncorrelated with the noise in his private signals, the aggregation of individual trading decisions in the market causes the *average* private signal of share owners to positively co-move with with the market information. This positive co-movement causes prices to respond too strongly to the underlying shocks, relative to expected firm value.

With exogenous dividends or when investment decisions do not respond to share prices, this

[2]See Vives (2008) for a discussion of this excess sensitivity of prices, relative to fundamentals.

[3]Market-generated information reflects shocks to dividends and shocks to the supply of shares. The former directly affects the distribution of private signals about dividends, while the latter shifts the informed traders' private expectations. by changing the amount of shares that must be absorbed: since more optimistic traders are more likely to buy more shares, all else equal, the larger the supply, the more pessimistic the informed traders that buy the asset become on average.

information aggregation wedge is symmetric and under-valuations are as large and equally likely on average as overvaluations. This implies that the average price equals the aver firm value, even though the price tends to be too high in good states of the world, and too low in bad states. This no longer holds when the firm's investment responds to the market information: because the firm responds to the information conveyed by the price investing more in good states than in bad ones, the firm's investment increases the upside risk for shareholders and reduces the downside risk. But this also exacerbates the price over-reaction on the upside, and dampens it on the downside. As a result, the information aggregation wedge is asymmetric: larger in absolute terms in good states when there is a lot of investment and shares are over-valued, than in bad states when investment is low and shares are under-valued. On average, the share price will then exceed the expected dividend value. We thus arrive at a theory of share price over-valuation due to information aggregation and feedback from prices to investment decisions.

Third, we analyze the efficiency implications of managerial compensation schemes tied to the firm's share price. Because the share price can differ from expected dividends as a result of the information aggregation wedge, investment decisions that seek to maximize the former need not be in the best interest of shareholders. We find that a manager who wishes to maximize prices–a weighted sum of expected dividends and the information aggregation wedge–will attribute too much weight to market information and overinvest in good states when the wedge is positive, while underinvesting in bad states when it is negative. The resulting price is even more volatile and higher on average, expected dividends are lower, and the information aggregation wedge is exacerbated. Our model thus provides a theory linking price-based incentives to asset over-valuation and excess volatility of prices and investment.

We check the robustness and magnitude of these effects to parameter changes and alternative model assumptions. The interaction between the information aggregation wedge with the information value of the price for the firm is summarized by two key parameters. First, the over-reaction of market prices to new information, measured by the sensitivity of the marginal trader's beliefs to a change in the market signal relative to the sensitivity of the firm's posterior beliefs, determines the magnitude of the information aggregation wedge between the share price and the firm's expected dividends. Second, the prior variability of the firm's posterior expectation of fundamentals captures the firm's updating of information from market prices, and hence indexes the information value of the share price. Both of these parameters depend on three underlying structural parameters, namely the magnitude of shocks to firm fundamentals, the precision of shareholder's private information, and the magnitude of noise trader shocks, or equivalently, the quality of market information. More precisely, the variability of the firm's posterior expectation is increasing in the quality of market information and the magnitude of fundamental shocks, while the information aggregation wedge is decreasing in these same parameters, but increasing in the precision of the shareholder's private information. The comparative statics then trade off these two effects, reaching the biggest magnitudes when the shareholder's private information is very precise, and market information is sufficiently informative so that the firm responds to it, but not so informative to crowd out the shareholder's private information–which is the source of the discrepancy between firm and market expectations, or the information aggregation wedge.

We also discuss the role of payoff assumptions, showing that at a more general level share price over and under-valuation is linked to the convexity or concavity of the firm's expected dividends–a consequence of the firm's investment response to market-generated information. Furthermore, we discuss the role of our limits to arbitrage assumption that reduces the ability of traders to arbitrage away the information aggregation wedge. We propose a generalization of our noisy demand assumption by considering uninformed traders–agents which trade partly for exogenous motives and partly in response to expected returns, conditional on public information. The price elasticity of uninformed trader's demand is then inversely related to the price impact of private information, and the information aggregation wedge is largest in illiquid markets when informed traders have a large price impact. Thus, the better uninformed traders are able to arbitrage away the discrepancy between expected dividends and price, the smaller is the information aggregation wedge and the resulting over-valuation and investment distortions. This is not surprising, as the information aggregation wedge results from the discrepancy in the expectations between the marginal informed trader's and uninformed traders', becoming larger as the informed shareholder's impact on price increases and the uninformed trader's impact on price decreases.

Our paper is related to a large literature on REE models with endogenous investment. Leland (1992) addresses welfare considerations in a model with insider trading, but in a context where the firm perfectly observes fundamentals. Subrahmanyam and Titman (1999) constrain dividends to be independent of the investment decision by separating the payoffs into two components: a growth option that depends on the investment decision, and the actual dividend perceived by stockholders–an increasing function of fundamentals but independent of investment. As a result, prices convey dispersed information about fundamentals but do not internalize the effect of endogenous investment. Our model is not only able to handle this price-investment simultaneity in closed-form, but shows how it is actually the source of over-valuation and inefficiency. Dow and Gorton (1997) study a dynamic model of feedback effects from asset prices to managerial incentives in a Glosten-Milgrom (1985) environment where prices accurately reflect public information, and incentive distortions arise from differences in horizons between managers and shareholders.

Closest to our model, Dow and Rahi (2003) study risk-sharing, hedging and welfare in a CARA-gaussian setting with endogenous investment, but place less emphasis on the asset pricing implications of the feedback effect. Furthermore, CARA imposes substantive restrictions on the information structure allowed.[4] Dow, Goldstein and Guembel (2007) study how information is produced in the markets where prices affect investments, while Goldstein and Guembel (2008) focus on the strategic aspect of the feedback effect. They show that when traders exploit the impact of their demand on prices and investment, manipulative short-selling strategies that distort the firm's investment decision can be profitable. Our model considers a richer information structure where the firm observes private information not available to markets. Moreover, they consider a binary-state model which is somewhat less appealing for making asset pricing predictions. For the formulation of our asset market model, we draw on Hellwig, Mukherji, and Tsyvinski (2006).

Our paper also relates to the literature on stock-option compensation schemes, managerial incentives and corporate investment. Stein (1989) argues that managers might reduce investment in hard-to-measure assets to boost reported profits and share prices in the short-run. In contrast, Bebchuk and Stole (1993) suggest that the nature of the investment distortion depends on the particular information structure, and may lead to overinvesting as managers try to impress the stock market. Bebchuk and Fried (2003) provide a good survey on executive compensation and agency problems more generally.

In section 2, we describe our basic model. In section 3, we derive the equilibrium characterization. In section 4, we present our main result on information feedback and asset overvaluation. In section 5, we discuss the effects of tying managerial incentives to share prices. In section 6, we discuss various extensions and robustness exercises to our baseline model.

2 Model Description

In this section we describe the main model, asset markets, and the equilibrium concept.

A single firm is presented with an investment opportunity and has to decide whether to undertake it or not. There is a unit measure of traders, who each initially hold one share of the firm. Prior to the firm's investment decision, the shareholders observe private information on the return

[4] Indeed, normality of traders' final wealth is only obtained after imposing the restrictive assumption that the firm's decision can be directly inferred from the share price.

to the investment and decide whether or not to sell their share. The firm can use the observation of the price to update its belief about the profits from the investment.

2.1 Agents, Production, and Information Structure

We denote the firm's binary investment decision by $I \in [0, 1]$, where $I = 1$ denotes the decision to invest, and $I = 0$ stands for no investment. If the firm does not invest, $I = 0$, profits are equal to zero. The parameter θ determines the firm's dividend value without investment, as well as the additional return from investing. More specifically, the firms dividends, when it does not invest, are equal to $\rho \cdot \theta$, with $\rho > 0$. If the firm invests, its dividends are equal to $\rho \cdot \theta + \theta - F$. Here, θ denotes the additional revenue resulting from the investment, and F the cost of the investment. The firm's fundamental θ is distributed according to $\theta \sim \mathcal{N}(\mu, \lambda^{-1})$, where the mean is μ, and λ^{-1} is the unconditional variance. The random variable of the costs for the firm F is stochastic and independent of θ, and distributed according to cdf $G(\cdot)$ and density $g(\cdot)$. We let $\underline{F}$ denote the lower bound of the distribution of F, but allow for the case where F is unbounded below (i.e. $\underline{F} = -\infty$). For technical reasons, we assume that

$$\rho + G(F) + (F - \mu) g(F) > 0, \tag{1}$$

for all $F \geq \underline{F}$. This imposes a lower bound on the sensitivity of the outside dividend to the fundamental (the condition holds automatically with $\underline{F} \geq \mu$). Profits are disbursed fully as dividends to the final holders of the firm's shares.

We interpret the cost component F as the *firm-specific* information about the project at hand. For simplicity, we assume that the firm observes the realization of F without noise, and that no other agent observes any information about it. The firm-specific component F can be thought of as summarizing all the payoff-relevant characteristics of the project about which the firm holds very precise information. Examples might include proprietary technical specifications, quality of a new product to be introduced in the market, or the unit cost of production. Henceforth, we will refer to the private cost, or F, interchangeably, but its interpretation can be more general.

Revenue θ captures the *market-specific* component of profits. It stands for those characteristics affecting the project's outcome where knowledge is dispersed through the market. We can interpret this component as related to demand conditions (how successful/fashionable will the new product be) or to the firm's position in relation to its competitors.[5]

[5] See Miller and Rock (1985) and Rock (1986) for further discussions on market- vs. firm- specific sources of information.

This categorization of the payoff components is meant to capture the following intuition. The outcome of an investment decision depends partly on the realization of certain characteristics which the firm observes closely (the firm-specific component F), but it might be hard for the firm to get reliable information about other aspects directly, introducing uncertainty in the outcome. Part of this uncertainty can be reduced by learning information from the financial market.

Information about θ is revealed only to the traders, through noisy private signals. Each trader i observes a private signal $x_i \sim \mathcal{N}(\theta, \beta^{-1})$, which is i.i.d. across agents, and normally distributed with mean θ and variance β^{-1}.

2.2 The Asset Market

After observing their private signals x_i, traders decide whether to keep or sell their shares in a financial market. We formulate the trading environment as a Bayesian game between a unit measure of risk-neutral share-holders and a fictitious 'Walrasian auctioneer' who observes the shareholder's asset supply and the random demand of shares by noise traders, and then selects a market-clearing price.[6] The game is structured as follows. In a first stage, each shareholder submits a schedule of bids that represents his supply of shares at the prevailing market price. Shareholders are risk-neutral, and their supply is restricted to $a \in [0, 1]$. With risk-neutrality, their bids will be equal to either 0 or 1 almost everywhere. Such supplies can therefore be directly interpreted as an order to sell/not to sell a single share at given price P, or as a limit order, if the resulting schedule is monotone in P. Once all shareholders have submitted orders, nature draws the realization of demand (due to noise traders) in the second stage, and the auctioneer selects a price P at which aggregate asset supply meets stochastic demand and all orders can be executed. In a third stage, the firm observes the realized share price P and the realization of its the privately observed cost F and then decides whether or not to invest.

Trader i makes her supply decision contingent on observing her private signal x_i and on the market-clearing price P. It thus chooses a_i to maximize her expected net wealth

$$w_i = a_i \cdot [\rho\theta + (\theta - F)\, I - P]\,,$$

conditional on her signal x_i, and the market price P. We denote by $a_i = a(x_i, P)$ her resulting supply schedule. Aggregating individual shareholders' supply we find the aggregate market supply

[6] While the equilibrium has the same features as a noisy rational expectations equilibrium, this enables us to conduct the entire analysis using standard game-theoretic tools.

$A(\theta, P)$ for the firm's shares:[7]

$$A(\theta, P) = \int a(x, P) d\Phi(\sqrt{\beta}\,(x - \theta)), \tag{2}$$

where $\Phi(\cdot)$ denotes a cumulative standard normal distribution, and $\Phi(\sqrt{\beta}\,(x - \theta))$ therefore represents the cross-sectional distribution of private signals x_i across shareholders, conditional on the realization of θ.

The aggregate demand of shares is noisy, due to trading motives unrelated to information about dividends. Specifically, we assume that the total number of shares demanded is $D = \Phi\,(u)$, where $u$is normally distributed with mean zero and variance δ^{-1}. This demand assumption is adapted from Hellwig, Mukherji, and Tsyvinski (2006), and will enable us to characterize the informational content of prices in a particularly simple form. The Walrasian auctioneer thus selects the market-clearing price to set $A(\theta, P) = \Phi(u)$, for all θ and u.

The novelty of our model is to endogenize the firm's investment decision as a response to the information conveyed by prices in presence of both market-specific and firm-specific information. Since the firm is interested in learning information about output θ and (as we will show below) asset prices provide a noisy signal about it, the investment decision will be contingent on the market-clearing share price P. Rational traders understand this endogenous investment channel and adjust their expectations of dividend value accordingly, creating a two-way feedback effect: Share prices aggregate and transmit information, and thus impact investment decisions. Investment decisions affect traders' beliefs about dividends and their demand for shares, which in turn moves asset prices.

2.3 Equilibrium

A Perfect Bayesian Equilibrium consists of posterior beliefs $H\,(\theta|P)$ for the firm and $H\,(\theta|x, P)$ for traders, an investment function $I(F, P)$ for the firm and a demand function $d(x, P)$ for the traders, and an equilibrium price function $P\,(\theta, u)$, such that (i) the firm's investment decision $I(F, P)$ and the traders' demand function $d(x, P)$ are sequentially rational given their respective posteriors $H\,(\theta|P)$ and $H\,(\theta|x, P)$, (ii) for all (θ, u), the asset market clears, and (iii) the traders' and firm's posterior beliefs $H\,(\theta|x, \cdot)$ and $H\,(\theta|\cdot)$ satisfy Bayes' Rule whenever applicable, i.e. for all p such that $\{(\theta, u) : P\,(\theta, u) = p\}$ is non-empty.

[7] For this aggregation, we follow the common practice of assuming that the Law of Large Numbers applies to the continuum of traders, so that conditional on θ, the cross-sectional distribution of signal realizations ex post is the same as the ex ante distribution of any individual traders' signal.

In addition, we focus on equilibria in which the posterior beliefs for traders satisfy

$$H(\theta|x,P)=\frac{\int_{-\infty}^{\theta}\sqrt{\beta}\phi\left(\sqrt{\beta}\left(x-\theta'\right)\right)dH\left(\theta'|P\right)}{\int_{-\infty}^{+\infty}\sqrt{\beta}\phi\left(\sqrt{\beta}\left(x-\theta'\right)\right)dH\left(\theta'|P\right)},$$

so that traders' beliefs conditional on prices but not on their private signal are the same as the firm's posterior beliefs. In other words, the firm and the traders agree on the interpretation of the information contained in prices. On the equilibrium path, this condition is trivially satisfied as a consequence of Bayesian updating; the requirement thus only serves to impose structure on out-of-equilibrium beliefs for price realizations p such that $\{(\theta,u) : P(\theta,u)=p\}$ is empty. Since private signals are log-concave, it also follows that $H(\cdot|x,P)$ must be first-order stochastically increasing in x (Milgrom and Weber (1982)). Our additional requirement imposes the same type of monotonicity on out of equilibrium beliefs, i.e. for a given price, a higher signal makes the trader more optimistic about the project both on and off the equilibrium path.

3 Asset market equilibrium

In this section we characterize the equilibrium in the asset market assuming that the firm's investment strategy is characterized by a threshold rule $\tilde{F}(P)$, such that the firm invests if and only if $F \leq \tilde{F}(P)$:

$$I(F,P)=\begin{cases}1 \text{ if } F\leq\tilde{F}(P)\\ 0 \text{ otherwise}\end{cases}.$$

Notice that this allows for the possibility that $\tilde{F}(P) \leq \underline{F}$, in which case the firm never invests, no matter its cost realization. In our benchmark model where the firm maximizes expected dividends, the firms' optimal strategy is to invest whenever $F \leq \mathbb{E}(\theta|P) = \int \theta dH(\theta|P)$, i.e. whenever the expected return on the project exceeds the privately observed investment cost F. By setting $\tilde{F}(P) = \mathbb{E}(\theta|P)$, our benchmark model admits exactly such a threshold characterization for firm behavior. However, by allowing for arbitrary threshold rules at this stage, we are also laying the ground work for the latter parts of the analysis when we vary the managers' objective function.

Shareholder optimality condition: Given this threshold rule for the firm, it's optimal for a shareholder to sell their share if and only if the price exceeds her expectation of dividends. A shareholder with private signal x will want to sell at price P, whenever

$$\begin{aligned}P &\geq \rho\int\theta dH(\theta|x,P)+\int\left[I(f,P)\cdot\left(\int\theta dH(\theta|x,P)\ -f\right)\right]dG(f)\\ &= (\rho+G(F^*(P)))\int\theta dH(\theta|x,P)-\int_{\underline{F}}^{\tilde{F}(P)}fdG(f).\end{aligned}$$

The first integral in the equation above is shareholder's expectation of the payoff if the firm does not invest. The second integral is shareholder's expectation of the payoff if the firm takes investment decision $I(f,P)$. Note that the shareholder condition on two signals: his own private signal x and on the signal in price P.

Because $H(\theta|\cdot,P)$ is first-order stochastically increasing in x and dividends are monotone in θ, the shareholder's decisions are monotone in x and can be characterized by a threshold function $\hat{x}(P) \in \mathbb{R} \cup \{\pm\infty\}$, such that

$$a(x_i,P) = \begin{cases} 1 \text{ if } x_i \leq \hat{x}(P) \\ 0 \text{ otherwise} \end{cases},$$

i.e., a shareholder sells his share if and only if his signal is below a threshold $\hat{x}(P)$. By defining $\hat{x}(P)$ on the extended real line we also allow for the possibility that all or no shareholders sell the asset. These thresholds are uniquely defined by

$$\begin{aligned} \hat{x}(P) &= +\infty \text{ if } \left(\rho + G\left(F^*(P)\right)\right) \lim_{x\to\infty} \int \theta dH\left(\theta|x,P\right) - \int_{\underline{F}}^{\tilde{F}(P)} f dG(f) \leq P \\ \hat{x}(P) &= -\infty \text{ if } \left(\rho + G\left(F^*(P)\right)\right) \lim_{x\to-\infty} \int \theta dH\left(\theta|x,P\right) - \int_{\underline{F}}^{\tilde{F}(P)} f dG(f) \geq P \\ P &= \left(\rho + G\left(\tilde{F}(P)\right)\right) \int \theta dH\left(\theta|\hat{x}(P),P\right) - \int_{\underline{F}}^{\tilde{F}(P)} f dG(f) \text{ otherwise.} \end{aligned} \tag{3}$$

Thus, we allow for three cases: (i) if even the most optimistic traders do not believe that the expected dividend value exceeds the price, they all sell and the threshold above which a shareholder is willing to hold on to his share is $+\infty$. (ii) if even the most pessimistic trader believes that the expected dividend value is sufficiently high, then all traders will hold on and the threshold for selling is $-\infty$. (iii) in the intermediate case, some sell while others hold on to their shares. The threshold $\hat{x}(P)$ then takes on an interior real value defined by the marginal trader who is just indifferent between purchasing and not purchasing the asset.

Market-clearing and information aggregation: Market-clearing then requires that for all (θ,u), price P is selected such that the aggregate demand is equal to aggregate supply:

$$A(\theta,P) = \Phi(u). \tag{4}$$

Since $\Phi(u) \in (0,1)$, $\hat{x}(P) = +\infty$ and $\hat{x}(P) = -\infty$ imply that P cannot clear markets. For finite $\hat{x}(P)$, we can rewrite (4) as

$$\Phi(u) = \int_{-\infty}^{\hat{x}(P)} 1 \cdot d\Phi(\sqrt{\beta}\,(x-\theta)) = \Phi(\sqrt{\beta}\,(\hat{x}(P)-\theta)).$$

The price function $P(\theta,u)$ clears the market, if and only if $\hat{x}(P)=\theta+1/\sqrt{\beta}\cdot u$ for all (θ,u). This defines a correspondence $\hat{P}(\theta,u)$,

$$\hat{P}(\theta,u)=\left\{P\in\mathbb{R}:\hat{x}(P)=\theta+\frac{1}{\sqrt{\beta}}u\right\} \tag{5}$$

of market-clearing prices, i.e. a price P clears the market in state (θ,u) if and only if $P\in\hat{P}(\theta,u)$. Below, we focus on equilibria in which the price is conditioned on (θ,u) through $z\equiv\theta+1/\sqrt{\beta}\cdot u$. This additional requirement only imposes structure on the selection of prices, when there are multiple market-clearing prices for some z.

With this characterization of market-clearing, we can show that in any such equilibrium the resulting price function $P(z)$ must be invertible. This in turn implies that the information conveyed by P is isomorphic to directly observing the value of $z=\hat{x}(P)$, which leads to the following characterization of equilibrium beliefs:

Lemma 1 (Information Aggregation) *In any equilibrium with conditioning on z, the equilibrium price function $P(z)$ is invertible. Equilibrium beliefs for price realizations that are observed along the equilibrium path are determined by*

$$H(\theta|P) = \Phi\left(\sqrt{\lambda+\beta\delta}\left(\theta-\frac{\lambda\mu+\beta\delta\hat{x}(P)}{\lambda+\beta\delta}\right)\right) \tag{6}$$

$$H(\theta|x,P) = \Phi\left(\sqrt{\lambda+\beta+\beta\delta}\left(\theta-\frac{\lambda\mu+\beta x+\beta\delta\hat{x}(P)}{\lambda+\beta+\beta\delta}\right)\right) \tag{7}$$

Invertibility of the price function with respect to z is a fundamental equilibrium property, because the inference of z is necessary to clear the market in all states. If the price function was not invertible, then there would exist a price realization P for which multiple thresholds $\hat{x}(P)$ are consistent with best-response behavior. Moreover, each of these thresholds clears the market in some state z. But then, the observation of P alone does not inform traders which of the states consistent with P has been realized, and hence which best-response threshold is the one consistent with market-clearing. Conditioning the threshold $\hat{x}(P)$ on P alone implies that trading behavior is the same for all z that are consistent with P. But since trading must be different in different states, the market must fail to clear for some realizations of z. Thus, invertibility of the price function is fundamentally linked to the property that prices must be sufficiently informative to enable traders to follow strategies that allow the markets to clear at all times.

Solving the RHS in (3), and using this characterization of posterior beliefs, we find the shareholders' optimal demand threshold $\hat{x}(P)$:

$$P=\left(\rho+G\left(\tilde{F}(P)\right)\right)\frac{\lambda\mu+\beta(1+\delta)\hat{x}(P)}{\lambda+\beta+\beta\delta}-\int_{\underline{F}}^{\tilde{F}(P)}f\,dG(f). \tag{8}$$

Expected dividends of the marginal trader correspond to her posterior of θ, times ρ plus the probability of investment (first term of (8)), minus the expected cost of investment, conditional on the project being undertaken (second term of (8)). The price P enters this condition through several channels. First, it determines the cost of the asset and hence the opportunity cost of a purchase on the LHS of (8). Second, it feeds into the assets' expected dividend value conditional on investment, since it affects the posterior beliefs about θ (though its effect on $\hat{x}(P)$). Finally, it impacts the likelihood of investment by shifting the firm's investment threshold $\tilde{F}(P)$ and hence the probability of investment, as well as its expected cost.

Next, notice that with invertibility of the price function we can also redefine the firm's investment threshold as a function of z. With a slight abuse of notation, we write $\tilde{F}(z) = \tilde{F}(P(z))$. The equilibrium price function is then derived from the market-clearing requirement that $\hat{x}(P) = z$, for all $z = \theta + 1/\sqrt{\beta} \cdot u$, and the investment threshold threshold $\tilde{F}(z)$. Substituting these conditions into (8), our first proposition characterizes the equilibrium in the asset market, as a function of the firm's investment threshold $\tilde{F}(z)$:

Proposition 1 (Asset market equilibrium) *For a given investment threshold $\tilde{F}(z)$, define $P(z)$ by:*

$$P(z) = \left(\rho + G\left(\tilde{F}(z)\right)\right) \frac{\lambda\mu + \beta(1+\delta) z}{\lambda + \beta + \beta\delta} - \int_{\underline{F}}^{\tilde{F}(z)} f dG(f). \tag{9}$$

If $P(z)$ is strictly increasing, the unique asset market equilibrium is characterized by the price function $P(z)$ and the shareholders' threshold function $\hat{x}(p) = z = P^{-1}(p)$. The firm's expected dividends, conditional on z, are given by

$$V(z) = \left(\rho + G\left(\tilde{F}(z)\right)\right) \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta} - \int_{\underline{F}}^{\tilde{F}(z)} f dG(f). \tag{10}$$

Proposition 1 fully characterizes the asset market equilibrium for a given investment threshold $\tilde{F}(z)$. Assuming that the price function is invertible, the shareholders infer the state z from the price and condition their decisions accordingly. The above characterization shows that for each z there is a unique price function $P(z)$ that is consistent with the shareholder's optimality condition, as well as market clearing at every realization of z. This function characterizes the asset market equilibrium if (and only if) it is invertible. The function $V(z)$ then characterizes the firm's expected dividends from the perspective of its manager, which is equivalent to that of an uninformed outsider.

Both the equilibrium price function and the firm's expected dividends from the perspective of its management, can be decomposed into three terms. First, $\rho \cdot \mathbb{E}(\theta|x = z, z) = \rho \cdot \frac{\lambda\mu+\beta(1+\delta)z}{\lambda+\beta+\beta\delta}$ and $\rho \cdot \mathbb{E}(\theta|z) = \rho \cdot \frac{\lambda\mu+\beta\delta z}{\lambda+\beta\delta}$ denote the expected dividend if the investment is not undertaken, from the marginal trader's and the firm's perspective, respectively. Second, $G\left(\tilde{F}(z)\right)\cdot\mathbb{E}(\theta|x=z,z)$ and $G\left(\tilde{F}(z)\right)\cdot\mathbb{E}(\theta|z)$ denote the expected revenue resulting from the investment, from the marginal trader's and the firm's perspective. The third term, $\int_{\underline{F}}^{\tilde{F}(z)} f dG(f)$, denotes the expected cost of the investment.

The two expressions differ in how expectations over θ are formed: The price $P(z)$ reflects how dividends are valued by the shareholders, and more specifically, by the marginal trader who is just indifferent between holding and selling her share, and who therefore determines the price. This trader receives the market signal z in addition to his own private signal x, the realization of which must exactly equal z for the market to clear. The marginal trader's expectation of θ, $\mathbb{E}(\theta|x=z,z)$, thus behaves as if this trader had received one signal z with precision $\beta(1+\delta)$.[8] In contrast, the manager's expectation $\mathbb{E}(\theta|z)$ is based only on the information contained in the price, i.e., a signal equal to z with precision $\beta\delta$, which corresponds to the true distribution of z. Thus, $V(z)$ also corresponds to how a risk-neutral social planner would value the firm's expected dividends at an interim stage when the market information has become available.

4 Firm Value and Price Volatility

In this section, we characterize the equilibrium prices and expected dividends in the benchmark model when the firm maximizes expected dividends. The firm will find it optimal to invest whenever

$$F \leq \tilde{F}(P) = \int \theta dH(\theta|P) = \frac{\lambda\mu + \beta\delta\hat{x}(P)}{\lambda + \beta\delta}, \tag{11}$$

i.e. investment will take place only when the expectation of the investment return θ is larger than the investment cost F. The firm's investment threshold depends on the market-clearing price through the information P conveys about θ, which is captured by the traders' demand schedule $\hat{x}(P)$. If $\hat{x}(P)$ is higher, more traders are willing to hold the asset, which suggests a higher value of θ. This makes the firm more inclined to invest –the threshold for investment decisions goes up. The investment sensitivity to market-specific information is determined by $\beta\delta$: the precision of the market signal $\hat{x}(P)$. If market-specific information is more precise, the firm relies to a larger extent on market-generated information about profits. Substituting the investment threshold into the above pricing function and using the market-clearing condition $\hat{x}(P) = z$, we arrive at the following equilibrium characterization:

[8] It should be clear that these expectations are perfectly consistent with Bayesian rationality for any individual trader, as they each have access to an independent private signal, as well as the market signal. The perfect correlation between these two signals for the marginal trader instead results from the market-clearing condition.

Theorem 1 (Equilibrium Characterization) *There exists a unique equilibrium, in which the price function is given by*

$$P(z) = \left(\rho + G\left(\tilde{F}(z)\right)\right) \frac{\lambda\mu + \beta(1+\delta) z}{\lambda + \beta + \beta\delta} - \int_{\underline{F}}^{\tilde{F}(z)} f dG(f), \tag{12}$$

the firm's investment threshold is

$$\tilde{F}(z) = \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta},$$

and the firm's expected dividends are given by

$$V(z) = \left(\rho + G\left(\tilde{F}(z)\right)\right) \tilde{F}(z) - \int_{\underline{F}}^{\tilde{F}(z)} f dG(f). \tag{13}$$

The theorem not only provides the characterization, but it also verifies the existence of the equilibrium by checking that the candidate price function is indeed invertible. Because the marginal trader's expectation of θ, $\mathbb{E}(\theta|x = z, z)$, is not aligned with the firm's expectation $\mathbb{E}(\theta|z)$, there may be a range of realizations of z for which their assessment on the desirability of the investment differs, implying that the net return on the investment that is captured by the second and third terms together is locally decreasing in z. In fact, this is inevitably the case for sufficiently low z, whenever $G(F)/g(F)$ is non-decreasing and converges to 0 as $F \to -\infty$. Without the exogenous dividend term, this would lead to non-invertibility of the price function. Assuming that the firm's dividend without investment is sufficiently increasing in θ is a simple sufficient condition to overcome the non-invertibility. This assumption implies that a higher fundamental increases both the exogenous dividends and the returns to the investment. The linearity of dividends is assumed for simplicity, but this assumption can be replaced by arbitrary exogenous dividends (and the analysis goes through with only minor changes), as long as the resulting price function satisfies monotonicity. On the other hand, if the price function is not invertible, the analysis applies as long as one assumes that shareholders and the firms can infer z through other means than the price (perhaps some additional measure of order flow or market activity). This non-invertibility issue as well as other technical considerations that arise when the exogenous dividend is zero are discussed in full detail in section 6 and appendix B.

4.1 Implications for equilibrium prices and dividends

We now use the characterization in Theorem 1 to draw conclusions for expected dividends, prices, and their volatility. Our main result here is a theorem stating that the expectation of the price is higher than the average expected dividends. The key intuition for this result lies in understanding the value of information for the traders.

For this analysis, it is convenient to redefine the "state" z in terms of the firm's posterior expectation: $Z = \tilde{F}(z) = \frac{\lambda\mu+\beta\delta z}{\lambda+\beta\delta}$. Notice that from an ex ante perspective Z is distributed according to $Z \sim \mathcal{N}\left(\mu, \sigma_Z^2\right)$, where $\sigma_Z^2 = \beta\delta / (\lambda+\beta\delta) \cdot \lambda^{-1}$ measures the prior uncertainty about the firm's posterior expectation, summarizing the informational value of the price for the firm's decision. σ_Z^2 is increasing in the precision of the noise in the market signal $\beta\delta$, and the prior variance λ^{-1} of the fundamental shocks.

We can then rewrite the price and expected dividend as a function of Z:

$$\begin{aligned} V(Z) &= (\rho + G(Z)) Z - \int_{\underline{F}}^{Z} f dG(f) = \rho Z + \int_{\underline{F}}^{Z} G(f) df && (14) \\ P(Z) &= (\rho + G(Z)) (\mu + \gamma (Z-\mu)) - \int_{\underline{F}}^{Z} f dG(f) && (15) \\ &= V(z) + (\gamma - 1)(Z-\mu)(\rho + G(Z)), \\ \text{where } \gamma &= \frac{\beta+\beta\delta}{\lambda+\beta+\beta\delta}\frac{\lambda+\beta\delta}{\beta\delta}. \end{aligned}$$

The parameter $\gamma > 1$ captures the wedge between the marginal trader's and the firm's expectations. The marginal trader's expectation attributes a precision $\beta + \beta\delta$ to the market signal z, while the firm attributes a precision of $\beta\delta$. The marginal trader's posterior expectation is $\mu + \gamma(Z-\mu)$, while the outsider's or the firm's posterior is Z. The parameter γ thus measures how much more the marginal trader reacts to the information contained in market prices, relative to firm's managers.

The market equilibrium thus drives a wedge, $W(Z)$, between the market price of the firm's shares and its expected dividends.

$$W(Z) \equiv (\gamma - 1)(Z-\mu)(\rho + G(Z)). \tag{16}$$

We refer to $W(Z)$ as the information aggregation wedge, as it is the direct result of noisy information aggregation through prices. All our results below can be traced to specific properties of this wedge. The signal Z affects this wedge through two channels. First, holding fixed the investment probability $G(Z)$, the wedge is increasing in the gap between the firm's prior and posterior. When the signal and the posterior are above the prior mean, $Z > \mu$, the wedge $W(z)$ is positive and the price exceeds the expected dividend. When the signal is below the mean, $Z < \mu$, the wedge $W(Z)$ is negative and the price is smaller than the expected dividend. The fact that the price is more responsive to changes in z than the expected dividend reflects the fact that the marginal trader attributes more precision to the market signal than the firm does. In addition, Z affects the magnitude of the wedge through the firm's investment probability $G(Z)$.

The functions $V(Z)$ and $P(Z)$ are plotted in figure 1. Notice that the expected dividend value

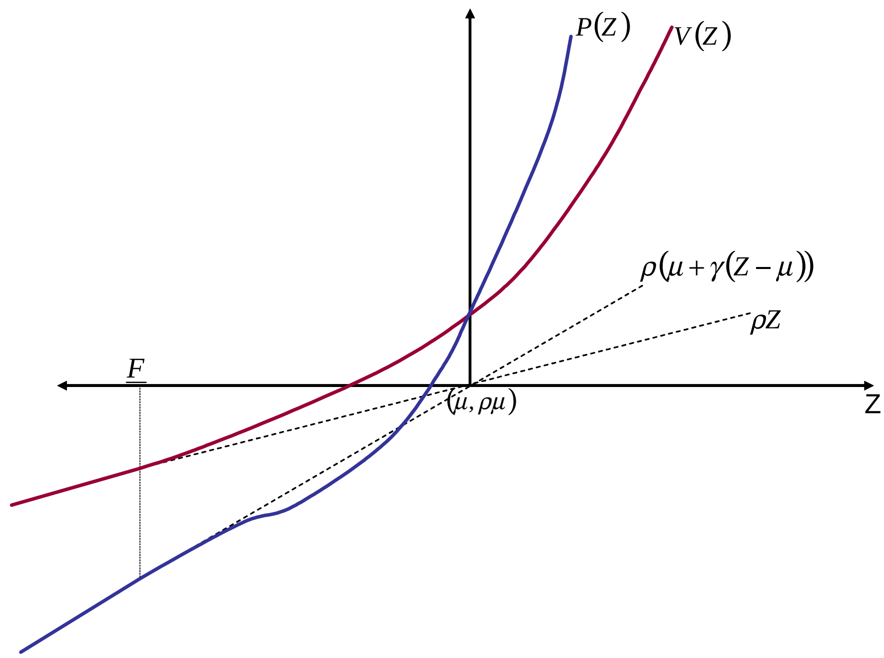

Figure 1: Equilibrium Prices and Expected Dividends

$V(Z)$ is increasing and convex in Z, and the price function is also increasing with a slope that strictly exceeds the slope V.

Our next theorem compares the unconditional mean and variance of the price with the unconditional expectation and variance of dividends:

Theorem 2 (Price and expected firm value) *bIn the equilibrium characterized in Theorem 1,*

(i) *The unconditional expectation of the price exceeds the unconditional expectation of dividends:* $\mathbb{E}(P(z)) > \mathbb{E}(V(z))$.

(ii) *The unconditional volatility of prices exceeds the unconditional volatility of dividends:* $Var(P(z)) > Var(V(z))$.

Both of these results follow from the information aggregation through prices, and the interaction of prices with investment decisions. The second statement– prices are more variable than expected dividends–is a direct consequence of the fact that the marginal trader reacts 'too much' to the information content of the market signal z: as long as $P'(z) > V'(z)$, for all z (subject to ρ satisfying the lower bound specified by equation (1)). This result is a natural consequence of information aggregation through prices, and does not rely specifically on endogenous investment by firms: in order to make the asset relatively less attractive at higher prices (and hence obtain downward-sloping demand for the asset and market-clearing prices), it is necessary that expected dividends increase less than one-for-one with an increase in prices.[9]

The first statement follows directly from taking the expectation of (16), and noting that

$$\mathbb{E}(P(z)) - \mathbb{E}(V(z)) = \mathbb{E}W(z) = (\gamma - 1)\, Cov\,(G(Z), Z) > 0, \tag{17}$$

since $G(\cdot)$ is an increasing function. Equation (17) shows that two conditions need to be met to obtain that the information aggregation wedge is positive in expectation. First we need $\gamma > 1$, so that the marginal trader's expectations respond more aggressively to market information than the firm's expectations. In addition, we need uncertainty about the firm's posterior Z ($\sigma_Z^2 > 0$), so that the firm learns something valuable from the market price and adjusts its expectation and its investment decision, captured by $G(Z)$, accordingly. To better understand the role of these two conditions, we begin by analyzing two benchmarks that violate each of these conditions.

Pricing and investment with common information: A simple way to eliminate the information aggregation wedge from a model like ours is to assume that the shareholders have homogeneous information (no private signals), and to replace the market signal with an exogenous public signal of the same precision, $z \sim \mathcal{N}\left(\theta, (\beta\delta)^{-1}\right)$. In this case, the firm's posterior Z, its investment threshold, and its interim and ex ante expected dividends $V(z)$ are the same as in our benchmark model, but there is a marked difference in market pricing. In particular, since all shareholders have the same information z, and the same posterior expectations about dividends $V(z)$, they all must be indifferent between selling and not selling their share (since some, but not all shareholders must sell in equilibrium to meet the stochastic demand). The price at which they are indifferent is $P(z) = V(z)$, so that there is no information aggregation wedge in equilibrium. Furthermore, since the function $V(z)$ is invertible, it is irrelevant whether the firm directly observes z, or whether it infers z from the observed price.

Moreover, from (14), we see that the function $V(Z)$ is strictly convex. This implies that $\mathbb{E}(V(Z)) > V(\mathbb{E}(Z)) = V(\mu)$. Thus, the firm's expected dividend is strictly increased by the fact that it learns from the price, and reacts to its information. In summary, the information contained in prices increases expected dividends, but when prices reveal common information–without aggregating private signals–prices are equal to expected dividend values in all states and there is no over-valuation effect: the firm's share price accurately reflects expected dividends at

[9] Strictly speaking, the lower bound assumption on ρ is not necessary for the result to obtain, but it facilitates the proofs. Without it, we always obtain over-reaction of prices on the upside, but due to possible non-monotonicity in the expected returns, prices may be less responsive to z than expected dividends on the downside.

the interim stage. Obviously, in order to obtain that prices exceed expected dividends on average, it is necessary that share prices differ from expected dividends for some realizations of Z, and information aggregation is the channel in our model that allows this to happen.

Information aggregation with exogenous investment: A simple way to eliminate the effect on firm decisions in our model is to assume that the firm's investment threshold is set exogenously, before the information in the price is realized. Specifically, using the characterization from Theorem 1 we find that for a fixed exogenous investment threshold $\tilde{F}$, the information aggregation wedge is given by

$$W(Z) = (\gamma - 1)\left(\rho + G\left(\tilde{F}\right)\right)(Z - \mu).$$

Therefore, the wedge is linear in Z with $P(\mu) = V(\mu)$. It follows immediately that $\mathbb{E}(W(Z)) = 0$; although information aggregation generates a wedge between price and expected dividends, this wedge is zero on average, which implies that although the shares may be mispriced for a particular realization of Z they are neither over- nor under-valued on average. The firm's response to the market information is therefore a key ingredient for our over-valuation result.

Now, how does the information feedback lead to an expected information aggregation wedge that is positive on average? Notice first that in the above expression $W'(Z)$ increases with the firm's threshold $\tilde{F}$, so that, in absolute value the wedge becomes larger, when there is more investment and more exposure of dividends to changes in θ. When the threshold varies with Z, i.e. when investment is determined by an exogenous, increasing threshold rule $\tilde{F}(Z)$, the wedge is given by

$$W(Z) = (\gamma - 1)\left(\rho + G\left(\tilde{F}(Z)\right)\right)(Z - \mu),$$

and its expected value is $\mathbb{E}(W(Z)) = (\gamma - 1)\, cov\left(G\left(\tilde{F}(Z)\right), Z\right) > 0$. Thus, by increasing investment in good states, the firm increases the size of the information aggregation wedge on the upside (when it is positive), while decreasing investment in bad states (when Z is below μ) reduces the wedge in absolute terms. Thus, by aligning its investment with the state the firm benefits even more from the gains of a positive shock, while the lower investment when Z is low reduces its exposure to negative shocks. The bigger upside gains and the smaller downside losses both increase the firm's expected dividends. This reflects the value of the information for the firm's decision.

Now, why does aligning the firm's investment decision with Z raise the expected price even more than the expected dividend? Because the marginal trader's expectation of θ is more sensitive to z than the firm's own expectation, the marginal trader will react more to both good and bad news than the firm. This means that when the firm's investment decision is an increasing function of Z, the market will value the returns to this investment even more highly than the firm does in good states ($Z > \mu$). On the other hand, in bad states, a smaller investment level reduces the responsiveness of dividends to θ, which reduces the size of the wedge precisely in those states where it is negative. Because of the excess sensitivity of market expectations to Z, the market thus values the reduced exposure in low states and the increased exposure in high states even more than the firm does, which explains why expected share prices increase by even more than expected dividends, and the wedge is now positive in expectation, $\mathbb{E}(W(Z)) > 0$.

The magnitude of the wedge is then tightly linked to the covariance between the firm's posterior Z, and its decision $G\left(\tilde{F}(Z)\right)$. As long as the firm is learning from prices, $\sigma_Z^2 > 0$, any increasing threshold function for investment will lead to positive correlation between investment and the state Z, and prices will be over-valued relative to expected dividends. The over-valuation result is thus not tightly linked to our specific model of firm decisions, but it arises naturally whenever investment is an increasing function of Z. The over-valuation results are therefore not specific to the firm's decision to maximize expected profits, but they hold generically whenever the firm responds to market information according to some increasing threshold rule $\tilde{F}(Z)$, but the size of the overvaluation will depend on the shape of the function $\tilde{F}(Z)$.

Finally, it is worth noting that although the overvaluation of shares results from the interaction between information provided by prices and the firm's investment decision, this over-valuation is not symptomatic of inefficient investment decisions. In our model, investment is efficient, when it maximizes the expected value of dividends conditional on z, which is the case with the investment decisions characterized by the threshold $\tilde{F}(Z) = Z$.

Comparative statics: We conclude by discussing how the model's parameters affect the magnitude of the information aggregation wedge, as well as expected prices and dividends. Above, we have summarized the model through the two parameters σ_Z^2 and γ that capture the information content of the price for the firm (in the form of uncertainty about the firm's posterior in σ_Z^2), and the wedge γ between the firm's and the market's expectation of θ. The next proposition discusses the comparative statics with respect to these two parameters. We then analyze how they are affected by the underlying parameters β, δ, and λ.

Proposition 2 (Comparative Statics) *Our model exhibits the following comparative statics for the expected prices, dividends and the information aggregation wedge:*

(i) For a given value of γ, $\mathbb{E}(P(z))$, $\mathbb{E}(V(z))$, and $\mathbb{E}(W(z))$ are all increasing in σ_Z^2.

(ii) For given value of σ_Z^2, $\mathbb{E}(V(z))$ does not depend on γ, while $\mathbb{E}(P(z))$ and $\mathbb{E}(W(z))$ are increasing in γ.

It follows from our discussion above that the expected dividend, the expected price, and the expected information aggregation wedge are all increasing in the prior uncertainty about the firm's posterior Z; σ_Z^2. Recall that σ_Z^2 captures the value of the market information to the firm; it is therefore not surprising that the firm's expected dividends are increasing in σ_Z^2. Moreover, since market expectations are more sensitive to Z than the firm's expectations, the impact of σ_Z^2 on the expected price is even stronger than in expected dividends, and therefore also impacts the wedge positively. Since the firm's investment threshold is given by Z, more uncertainty about Z automatically increases the covariance between $G(Z)$ and Z that determines the information aggregation wedge. On the other hand, expected dividends $\mathbb{E}(V(z))$ are not a function of γ, the parameter which measures the discrepancy between firm and market expectations, while the expected price level and hence the information aggregation wedge scale up with γ .

The primitive parameters β, δ, and λ affect expected dividends $\mathbb{E}(V(z))$ through σ_Z^2, which is increasing in both the precision $\beta\delta$ of the market signal, and in the prior uncertainty λ^{-1}. Both better market information, or a more variable prior, raise the expected dividends of the firm. This result can easily be explained in terms of the option value of the firm's investment. Because the firm will tend to invest in states with high realization of θ, but avoid investment when θ is low as it learns information from the price, the dividend becomes a convex function of its posterior expectation Z. It's expected value is therefore an increasing function of the prior uncertainty about the firm's posterior expectation. The firm's posterior becomes more uncertain, in turn, if either the underlying return becomes more variable (an increase in λ^{-1}), or if the firm receives a more precise signal of dividends (an increase in $\beta\delta$).

The same parameters affect the expected information aggregation wedge $\mathbb{E}(W(z))$ through both σ_Z^2 and γ. Notice that the comparative statics of these two parameters go in opposite directions– γ is decreasing in λ^{-1}, decreasing in β, and decreasing in δ. The overall effects of these primitive parameters on the information aggregation wedge and the expected prices are therefore ambiguous: more prior uncertainty, or more precise market information increase σ_Z^2, and hence the value of market information to the firm, but decrease the information aggregation wedge γ. In expectation, the information aggregation wedge is therefore largest at intermediate parameter values when the prior uncertainty λ^{-1} is sufficiently high to generate enough upside option value from the investment, the market information $\beta\delta$ is precise enough that the firm wants to respond to it, and the private information β is precise enough so that the market holds an informational advantage. Yet at the same time λ^{-1} must remains sufficiently large relative to $\beta\delta$ so that the market information doesn't completely crowd out the prior, which would eliminate the information wedge from our model.[10]

The comparative statics of the expected price $\mathbb{E}(P(z))$ add up the comparative statics of $\mathbb{E}(V(z))$ and $\mathbb{E}(W(z))$. For low levels of the parameters β, δ, and λ^{-1}, where these two go in the same direction, the expected price level is therefore increasing in the precision of market information and the prior uncertainty, and increasing in the precision of private signals.[11]

5 Tying managerial incentives to share prices

Up to this point we have assumed that the firm's managers act in the best interest of their shareholders ex post, after markets have closed. Formally, the firm decided to maximize the expected value of dividends $V(z)$. In this section we use our model to discuss the effects of managerial incentives that directly tie CEO compensation to the firm's stock market performance (through stocks or options). As well known result in economies with complete markets and common information about fundamentals is that $P(z) = V(z)$ in all states–maximizing ex ante or ex post shareholder value are one and the same. However, in our model information aggregation drives a wedge between ex ante and ex post share-holder values, or prices and expected dividends. Moreover, this wedge is directly affected by the firms' investment decision.

Thus, in our model the incentives of the initial shareholders who seek to maximize the price of the shares need not be aligned with the final shareholders who seek to maximize the expected dividend. Since this wedge is affected by the firm's investment decisions, our setup allows to discuss the effects of managerial incentives on investment, stock prices and efficiency.

To analyze this connection, we assume that the firm's managers decide to set $I(F, P)$ to maximize a linear combination of its dividend value $V(z)$ and its stock price $P(z)$. All other elements of the model are kept the same. Formally, we assume that at an initial stage, the managers form an investment plan that is made contingent on P, while the shareholders decide whether or not to sell their share based on their private signal and P. As before, an equilibrium is characterized by a pair of threshold functions $\hat{x}(P)$ and $\tilde{F}(P)$, one for the shareholders and one for the firm, and

[10] Alternatively, an increase in γ, holding σ_Z^2 fixed is equivalent to an increase in the precision of private information β, holding constant the precision of market information $\beta\delta$ and the prior uncertainty λ^{-1}. But then one easily checks that the biggest information aggregation wedge arises in a double limit where $\beta \to \infty$ and $\delta \to 0$, such that $\beta\delta$ converges to an interior constant, at which market information is neither too informative, nor too uninformative.

[11] Once the effects of these parameters on $\mathbb{E}(V(z))$ and $\mathbb{E}(W(z))$ go in opposite directions, we have not been able to ascertain whether $\mathbb{E}(P(z))$ simply inherits the c.s. properties of $\mathbb{E}(V(z))$, or whether there is the possibility that more precise market information increases expected dividends but lowers expected prices.

an invertible price function $P(z)$, such that shareholders and the firm behave optimally, and the market clears.

For a given threshold function $\tilde{F}(z) = \tilde{F}(P(z))$, by Proposition 1, the market-clearing price function is

$$P(z) = \left(\rho + G\left(\tilde{F}(z)\right)\right) \frac{\lambda\mu + \beta(1+\delta)z}{\lambda + \beta + \beta\delta} - \int_{\underline{F}}^{\tilde{F}(z)} f dG(f),$$

and the shareholder's threshold satisfies $\hat{x}(P(z)) = z$. What changes is how the investment threshold is determined. Specifically, we assume that the manager sets $\tilde{F}(z)$ to maximize $\alpha\mathbb{E}(P(z)) + (1-\alpha)\mathbb{E}(V(z))$, for some $\alpha \in [0,1]$. The parameter α thus measures how strongly incentives are based on prices, relative to expected dividends. At one extreme ($\alpha = 0$), it captures our initial model with expected dividend maximization, at the other ($\alpha = 1$), the manager's incentives are only based on the share price.

We solve this problem by maximizing the objective function pointwise for all z, and then checking that the resulting price function is invertible. Formally, we have

$$\begin{aligned}
\tilde{F}(z) &\in \arg\max_{\tilde{F}} \{\alpha P(z) + (1-\alpha) V(z)\} = \arg\max_{\tilde{F}} \{V(z) + \alpha W(z)\} \\
&= \arg\max_{\tilde{F}} \left\{ \left(\rho + G\left(\tilde{F}\right)\right) [\mu + k(Z-\mu)] - \int_{\underline{F}}^{\tilde{F}} f dG(f) \right\}, \qquad (18)
\end{aligned}$$

where, as before the firm's posterior expectation and the wedge between firm and market expectations are

$$\begin{aligned}
Z &= \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta} \text{ and } \gamma = \frac{\beta + \beta\delta}{\lambda + \beta + \beta\delta}\frac{\lambda + \beta\delta}{\beta\delta}, \text{ and} \\
k &= 1 + \alpha(\gamma - 1)
\end{aligned}$$

determines the strength of the distortion in incentives, which depends on the size of the information aggregation wedge γ and the weight given to prices α. Notice in particular that $k \in [\gamma, 1]$. Taking first-order conditions to determine the investment threshold $\tilde{F}(Z)$ and checking that the resulting price function is invertible, we have the following equilibrium characterization:

Proposition 3 (Equilibrium with price-based incentives) *In the unique PBE with price-based incentives, the investment threshold $\tilde{F}(Z)$, the price function $P(Z)$, and the expected dividends $V(Z)$ are given by*

$$\begin{aligned}
\tilde{F}(Z) &= \mu + k(Z-\mu), && (19) \\
P(Z) &= \rho(\mu + k(Z-\mu)) + \int_{\underline{F}}^{\mu+k(Z-\mu)} G(f)\,df + (\gamma - k)(Z-\mu)(\rho + G(\mu + k(Z-\mu))) && (20) \\
V(Z) &= \rho(\mu + k(Z-\mu)) + \int_{\underline{F}}^{\mu+k(Z-\mu)} G(f)\,df - (k-1)(Z-\mu)(\rho + G(\mu + k(Z-\mu))) && (21)
\end{aligned}$$

The effect of price-based incentives on prices and expected firm value is illustrated by figure 2. Our next theorem summarizes the effect of price-based managerial incentives on share prices, expected dividends and efficiency by considering the comparative statics with respect to k:

Theorem 3 (Tying managerial incentives to share-prices) *In the unique Perfect Bayesian Equilibrium:*

(i) *the volatility of investment is increasing in k: $\tilde{F}'(Z) = k > 1$.*

(ii) *expected dividends $V(Z)$ are decreasing in k, for all $Z \neq \mu$.*

(iii) *the share price $P(Z)$ is increasing in k, for all $Z \neq \mu$.*

Thus, the more managerial incentives are based on share prices the more volatile investment is. The managers find it optimal to react more strongly to the information generated by the market signal; this over-reaction (relative to our benchmark model) is captured by the parameter k. As a result, investment becomes more volatile than would be optimal from purely an efficiency point of view: when Z is higher than μ, the firm's investment threshold is too high and the firm invests too often, including states in which the cost F exceeds the expected gains from investment, Z. On the other hand, when Z is below μ, the firm's investment threshold is too low, and the firm foregoes investments in states where the expected gains Z exceed the investment cost F. As a result of this strategy, expected dividends are lower for all Z but prices are higher.

A simple intuition for this result can be gained from the tension between expected dividends and prices in the manager's objective function. Formally, the objective adds the wedge (weighted by the price weight α) to the expected dividends:

$$\mathbb{E}(V(z)) + \alpha\mathbb{E}(W(z)) = \mathbb{E}(V(z)) + (k-1)\, cov\left(G\left(\tilde{F}(Z)\right), Z\right).$$

We know from before that a threshold strategy $\tilde{F}(Z) = Z$ maximizes the first term $\mathbb{E}(V(z))$. Any small departure from this rule thus generates second-order losses in expected dividends. On the other hand, a more volatile investment strategy in the form $\tilde{F}(Z) = \mu + k(Z-\mu)$ (with $k > 1$) increases the covariance between the firm's investment and the state Z, generating first-order increases in the expected information aggregation wedge. By making the investment decision more volatile, the managers manipulate the share price to their advantage–to the detriment of shareholders who see the dividend value of the shares reduced.

The investment distortions that arise from stock price compensation schemes exacerbate the over-valuation result highlighted by Theorem 2. This is particularly clear when $\alpha = 1$, in which case the managers are purely interested in maximizing share prices. The parameter k is then equal to γ,

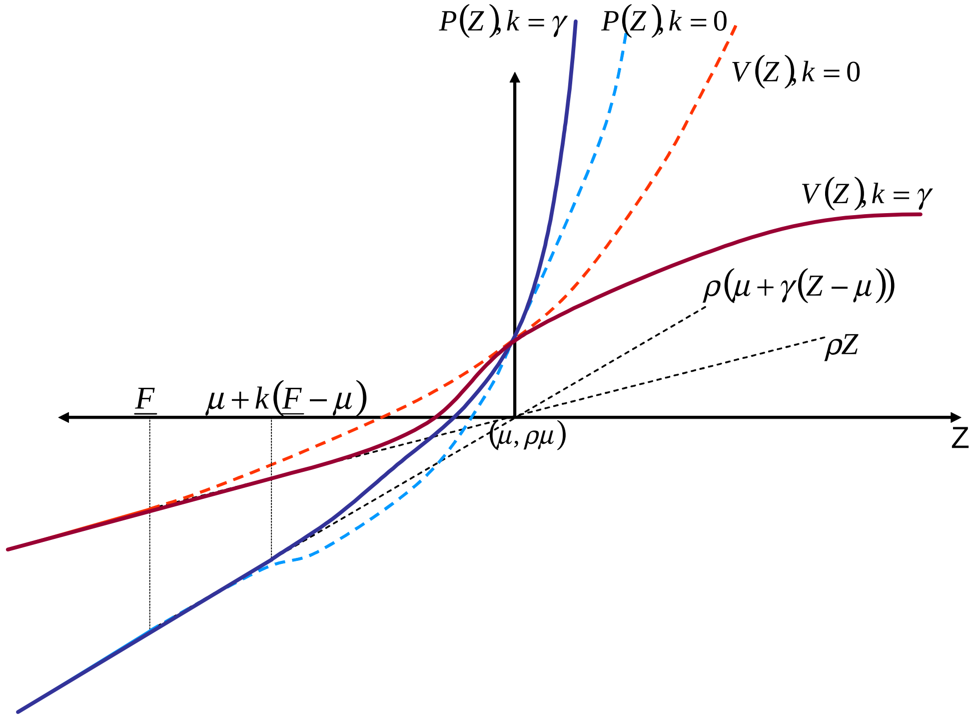

Figure 2: The effect of price-based incentives on share price and firm value

i.e. the managers act as if their expectations matched that of the marginal trader, who attributes a precision $\beta + \beta\delta$ instead of $\beta\delta$ to the market signal. Investment thus becomes excessively sensitive to the information content of share prices. Effectively, managers act if they are over-confident in the information content of market prices, and hence they rely too heavily on market information.[12]

Our model thus provides an argument against tying executive compensation too closely to short-term market valuations. When prices aren't aligned with expected dividends, and when, as in our model, the price-dividend gap responds to the firm's investment decision, price-based incentives lead to inefficient investments that lower the dividend value of firms, and artificially drive up prices. These results originate from the excess sensitivity to market information: when the market reveals good news, the firm over-reacts and increases investment too much, whereas when the market reveals bad news, there is too little investment and some projects are not under-taken for fear of lowering share prices even though they would benefit the firm by raising expected dividend.

Finally, notice that although the investment inefficiency is detrimental to the firm's ex post dividend value (and hence its subsequent shareholders), tying managerial incentives to share prices

[12]For intermediate values of α, the over-reliance in market information is still present, but smaller in magnitude.

may be in the interests of initial shareholders, especially if the latter do not expect to hold on to the stocks until the dividends are paid out.[13]

6 Discussion

In this section, we discuss how our main two results from the previous section, i.e. the over-valuation of shares and the excess sensitivity of investment to prices when managerial incentives are linked to share prices, depend on different features of the environment. We consider several departures from our benchmark model including different assumptions regarding the firm dividends and the firm's investment decision, the role of information aggregation and the price impact. We also discuss an extension with inside information by the managers into the project's expected revenues.

6.1 Generalizing the firm's dividends and investment decision

Here we discuss how the results of our model depend on the assumptions about the firm's dividends and decision problem. In principle, our analysis can be extended to a much richer set of assumptions about firm dividends and the firms' decision problem. Let $A \subseteq \mathbb{R}$ denote a compact set of actions from which the firm chooses, and $\pi : \mathbb{R} \times A \to \mathbb{R}$ the firm's payoffs. Keeping the market structure, traders' private information and preferences and the demand for shares the same as in our benchmark model, the market equilibrium is characterized by a threshold function $\hat{x}(P)$ for shareholders and a price function $P(z)$, where z is defined as before. The firm's decision a is then measurable with respect to z, and for a given decision function $a(z)$, the price and dividend values, conditional on z, are

$$\begin{aligned} P(z) &= \int \pi(\theta, a(z))\, d\Phi\left(\sqrt{\lambda+\beta+\beta\delta}\left(\theta - \frac{\lambda\mu + \beta(1+\delta)z}{\lambda+\beta+\beta\delta}\right)\right) \\ V(z) &= \int \pi(\theta, a(z))\, d\Phi\left(\sqrt{\lambda+\beta\delta}\left(\theta - \frac{\lambda\mu + \beta\delta z}{\lambda+\beta\delta}\right)\right). \end{aligned}$$

These functions characterize a market equilibrium, provided that $P(z)$ is strictly increasing. Under dividend value maximization, the firm's optimal decision satisfies

$$a(z) \in \arg\max_{a\in A} \int \pi(\theta, a)\, d\Phi\left(\sqrt{\lambda+\beta\delta}\left(\theta - \frac{\lambda\mu + \beta\delta z}{\lambda+\beta\delta}\right)\right),$$

[13] In practice, the optimal incentive scheme from the perspective of initial shareholders will involve a more complicated weighting rule between prices and expected dividends, with weights that are contingent on Z–higher when shareholders view selling as more likely.

while under share price maximization the firm's decision satisfies

$$a(z) \in \arg\max_{a \in A} \int \pi(\theta, a)\, d\Phi\left(\sqrt{\lambda + \beta + \beta\delta}\left(\theta - \frac{\lambda\mu + \beta(1+\delta)z}{\lambda + \beta + \beta\delta}\right)\right).$$

As before, we observe that linking managerial incentives to prices leads the firm to rely too heavily on market-generated information.[14] The key feature driving the inefficiency resulting from price-based incentives thus generalizes to a much wider class of models.

Exogenous dividend assumption: One assumption that we can discuss in detail is the role of the exogenous dividends in our model. In our baseline model, we have assumed that the firm generates a dividend of $\rho \cdot \theta$, if it doesn't invest, and an additional $\theta - F$ if it does. This assumption guaranteed that shareholders always have an incentive to trade on their private information (so that the price always reveals z), and ρ was taken to be sufficiently large to guarantee that the price function is invertible.

If we keep the return from investing $\theta - F$ the same, it is straight-forward to extend our analysis to nearly arbitrary exogenous dividend functions $\pi(\theta) \neq 0$. This doesn't change the firm's incentives to invest, and hence its response to market information, but it alters the exogenous component of the price function, the expected dividends and the information aggregation wedge so that

$$\begin{aligned} P(Z) &= \int \pi(\theta)\, d\Phi\left(\sqrt{\lambda + \beta + \beta\delta}\,(\theta - \mu - \gamma(Z - \mu))\right) + G(Z)(\mu + \gamma(Z - \mu)) - \int_{\underline{F}}^{Z} f dG(f) \\ V(Z) &= \int \pi(\theta)\, d\Phi\left(\sqrt{\lambda + \beta\delta}\,(\theta - Z)\right) + ZG(Z) - \int_{\underline{F}}^{Z} f dG(f), \end{aligned}$$

and these price and expected dividend functions characterize an equilibrium as long as $P'(\cdot) > 0$ everywhere. As before, the price and expected dividend value are decomposed into an exogenous term and a component that depends on the firms' investment choice. The price and expected dividends reflect expectations about both components of dividends, and without imposing additional structure on the dividend function, over- or under-valuation effects can result from both the exogenous and the endogenous part of the dividends.

More specifically, assume that $\pi(\cdot)$ is monotone increasing, and abstract for a moment from the possibility that the firm undertakes an investment (so that the dividend is exogenous). Price and dividend value are then given by

$$\begin{aligned} P(z) &= \int \pi(\theta)\, d\Phi\left(\sqrt{\lambda + \beta + \beta\delta}\,(\theta - \mu - \gamma(Z - \mu))\right) \\ V(Z) &= \int \pi(\theta)\, d\Phi\left(\sqrt{\lambda + \beta\delta}\,(\theta - Z)\right) \end{aligned}$$

[14] If the firm's managers maximize a weighted average of the dividend value and the price, then, as before, their decisions will reflect a weighted average of the firm's and the market's expectations about θ.

We have the following proposition:

Proposition 4 *The sign of the expected information aggregation wedge depends on the concavity of $\pi(\cdot)$:*

$$\begin{aligned}\pi''(\cdot) &< 0 \Longrightarrow \mathbb{E}(P(z)) < \mathbb{E}(V(z)) \\ \pi''(\cdot) &= 0 \Longrightarrow \mathbb{E}(P(z)) = \mathbb{E}(V(z)) \\ \pi''(\cdot) &> 0 \Longrightarrow \mathbb{E}(P(z)) > \mathbb{E}(V(z))\end{aligned}$$

This proposition allows us to re-interpret our original results on over-valuation of assets as a consequence of the convexity of dividends. In our benchmark model, the convexity resulted from the fact that the information contained in market prices allowed the firm to better align its investment with the state–matching high revenues with investment, while avoiding low ones with no investment. If the underlying optimal payoff function was instead concave, the firm would on average be under-valued, but the information contained in prices would still reduce the concavity and thereby increase the expected price by more than the expected dividend.

The model without exogenous dividends: In appendix B, we analyze our model and provide an equilibrium characterization when there are no exogenous dividends: $\pi(\theta) = 0$. In this case, market prices aggregate information about only the endogenous investment component, while other exogenous dividends are already known to the market, and thus fully reflected in prices. As a result, there is also a feedback in the informativeness of prices: when the firm doesn't invest with probability 1, private signals are uninformative of dividends, and hence so is the price. This feedback can lead to multiple equilibria and indeterminacy, as well as non-existence of equilibrium.

As a result of this feedback, we have to separately consider two possibilities: if the price $P \neq 0$, there is a positive probability of investment, traders respond to their private signals, and prices fully reveal Z through a market-clearing price function $P(Z)$ (which has the same features as above, with $\rho = 0$). Alternatively, if the price is $P = 0$, there is also the possibility that investment occurs with zero probability, the private signals are worthless, and traders are indifferent at this price between owning and not owning a share.

The model without exogenous dividend thus admits a rich set of possible equilibrium outcomes, including the potential for multiplicity, indeterminacy and non-existence. Multiplicity and indeterminacy are a consequence of the feedback on the signals' informativeness: when the firm is certain not to invest, private signals are worthless and traders are indifferent at a price of $P = 0$. Since $P = 0$ is then at most partially revealing of Z, it is possible to sustain (almost) arbitrary selections from the correspondence $\{0, P(Z)\}$ as equilibrium prices.

Non-existence on the other hand is linked to the non-invertibility of the price function, and hence the non-monotonicity of $P(\cdot)$. In our benchmark model, this non-invertibility occurs with unbounded support of F, and a thin-tail assumption on the distribution $G(\cdot)$. By removing the possibility of no information aggregation and no investment at a price of 0 from the set of possible market-clearing outcomes, the exogenous dividend $\pi(\theta)$ makes non-invertibility even more of an issue, unless $\pi'(\theta)$ is bounded sufficiently far away from zero so that the resulting price function is guaranteed to be monotone.

These multiplicity, indeterminacy and non-existence issues are interesting in their own right, but they are somewhat distracting from the main contribution of our paper. Our baseline model allowed us to focus on those cases where we have a unique equilibrium.

6.2 Price impact of information

In the baseline model, we have assumed that the demand for shares is exogenous and inelastic to the price and dividend expectations. While motivated by analytical tractability, this assumption nevertheless leads to the question of why the counter-party to the informed traders is not responding at all to prices. Perhaps this market structure might be interpreted as the interaction between active traders who respond to their own private information and that contained in the price, and passive traders whose demand for the assets is determined by institutional constraints, and thus unresponsive to any source of information.

We offer an extension of our benchmark model that highlights the robustness of our benchmark results to alternative market structures. The demand structure in our benchmark model maximized the price impact of private information. We propose a generalization of our original noisy demand assumption by considering uninformed traders: agents which trade partly from exogenous motives and partly in response to gaps between the price and their dividend expectation, conditional on the information contained in the price. We chose a demand formulation that maintains the benefits of tractability, yet allows considering an additional comparative static on the information aggregation wedge that relates naturally to the concept of market liquidity– the price impact of any given trade. The key parameter is the demand elasticity of uninformed traders: the higher the response to perceived mispricing, the lower the price impact of private information and hence the smaller the over-valuation and investment distortions. If the uninformed traders' demand is perfectly elastic, prices are equated to expected dividend values and the information aggregation wedge disappears.

Specifically, we consider the following formulation for asset demand:

$$D = \Phi\left(u + \eta\left(\mathbb{E}\left(V|P\right) - P\right)\right),$$

where as before $u \sim \mathcal{N}\left(0, \delta^{-1}\right)$. The expression captures the idea that the uninformed traders' demand is increasing in expected return $V - P$, conditional on observing the price. The parameter η captures the elasticity of demand w.r.t. expected returns.

We again proceed by characterizing the equilibrium threshold $\hat{x}(P)$ for the informed traders, and the market-clearing price function $P(z)$, for a given investment threshold function $\tilde{F}(P)$. With a threshold $\hat{x}(P)$ for informed traders, the market-clearing condition implies $\Phi\left(\sqrt{\beta}\left(\hat{x}(P) - \theta\right)\right) = \Phi\left(u + \eta\left(\mathbb{E}\left(V|P\right) - P\right)\right)$, or

$$z = \hat{x}(P) - \eta/\sqrt{\beta}\left(\mathbb{E}\left(V|P\right) - P\right),$$

and the observation of P is isomorphic to $z \sim \mathcal{N}\left(\theta, (\beta\delta)^{-1}\right)$. As before, we define $Z = \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta}$ as the firm's posterior expectation. For a firm's threshold function $\tilde{F}(Z)$, the expected dividend function is the same as before:

$$V(Z) = \left(\rho + G\left(\tilde{F}(Z)\right)\right) Z - \int_{\underline{F}}^{\tilde{F}(Z)} f dG(f).$$

After substituting $\hat{x}(P) = z + \eta/\sqrt{\beta}\left(\mathbb{E}\left(V|P\right) - P\right) = z + \eta/\sqrt{\beta}\left(V(Z) - P(Z)\right)$, the price function is

$$P(Z) = \left(\rho + G\left(\tilde{F}(Z)\right)\right)\left(\mu + \gamma\left(Z - \mu\right)\right) + \frac{\sqrt{\beta}\eta\left(\rho + G\left(F(z)\right)\right)}{\lambda + \beta + \beta\delta}\left(V(Z) - P(Z)\right) - \int_{\underline{F}}^{\tilde{F}(Z)} f dG(f)$$

and the information aggregation wedge can be solved for as:

$$P(Z) - V(Z) = \frac{\lambda + \beta + \beta\delta}{\lambda + \beta + \beta\delta + \sqrt{\beta}\eta\left(\rho + G\left(\tilde{F}(Z)\right)\right)}\left(\gamma - 1\right)\left(\rho + G\left(\tilde{F}(Z)\right)\right)\left(Z - \mu\right).$$

The information aggregation wedge is thus inversely related to the uninformed traders' demand elasticity parameter η. The higher is η, the less the private information of traders affects prices, and the smaller is the information aggregation wedge and the gap between realized and expected prices and dividend values.

Let us now assume that the firm maximizes $\alpha P(z) + (1 - \alpha) V(z)$. We again proceed by maximizing the objective pointwise w.r.t. $\tilde{F}$, and then check invertibility. Taking first-order conditions, we have:

$$\tilde{F}(Z) = Z + \alpha\left\{\frac{\lambda + \beta + \beta\delta}{\lambda + \beta + \beta\delta + \sqrt{\beta}\eta\left(\rho + G\left(\tilde{F}(Z)\right)\right)}\right\}^2\left(\gamma - 1\right)\left(Z - \mu\right)$$

Therefore, as long as $\alpha = 0$, the investment decision remains undistorted as before. When $\alpha > 0$, there is over-investment for $Z > \mu$, and underinvestment when $Z < \mu$, but this inefficiency is reduced by the elasticity of the uninformed traders' demand η.

For all η, we have $\tilde{F}(Z) \in [Z, \mu + k(Z-\mu)]$, with $\lim_{\eta\to\infty} \tilde{F}(Z) = Z$ and $\lim_{\eta\to 0} \tilde{F}(Z) = \mu + k(Z-\mu)$. We thus obtain efficient investment as the uninformed demand becomes infinitely elastic, or when uninformed traders arbitrage away the difference between price and expected dividends.

While linking managerial incentives to prices continues to generate excess sensitivity of investment to market-based signals, the elasticity of uninformed traders' demand reduces the extent of the inefficiency, which vanishes in the limit when $\eta \to \infty$. In this case, it is uninformed traders who are pricing the shares, completely arbitraging away any gap between the price and expected dividends.

7 Concluding remarks

This paper examines the role of asset prices in aggregating information about fundamentals and providing guidance for real investment decisions. We develop a model in which the outcome of a firm's project depends on fundamentals and the choice of investment. Information about the fundamentals is dispersed among agents in a financial market and imperfectly aggregated in the firm's stock price, upon which the firm conditions its investment decision. Market-generated information thus enhances firm value by encouraging investment when high prices communicate good fundamentals, but limiting the loses by discouraging investment when low prices signal poor realizations.

We also show that in the presence of information aggregation and noise trading prices can diverge from expected dividends due to an information aggregation wedge. The wedge originates from an overweighing of market-generated signals by the marginal informed trader–which must hold in order to make the asset relatively less attractive at higher prices, consistent with market-clearing. Moreover, because the firm responds to the information conveyed by the price investing more in good states than in bad ones, it exacerbates the price over-reaction on the upsides more than the price underreaction on the downsides. As a result, the information aggregation wedge is asymmetric: larger on the upside, when there is a lot of investment and shares are over-valued, than on the downside when investment is low and shares are under-valued. On average, the share price will exceed the expected dividend value, conditional on public information.

Finally, we discuss the role of managerial incentives that are tied to the firm's share price, showing how price-based compensation lead to asset over-valuation and excess volatility of investment in the presence of the information aggregation wedge. While our model has taken the manager's objective as given, the design of optimal incentive structures in the presence of a wedge between expected dividend and prices remains an important question for future research. Our model of financial markets with information aggregation appears to provide a promising building block for future work in this direction, as well as for other questions that require a flexible payoff structure for analyzing the interplay between managerial incentives, corporate decisions, and market prices.

References

[1] Bebchuk, Lucian A. and Jesse M. Fried (2003). "Executive Compensation as an Agency Problem" Journal of Economic Perspectives, 17. 71-92.

[2] Bebchuk, Lucian A. and Lars A. Stole (1993). "Do Short-term Objectives Lead to Under or Overinvestment in Long-term Projects?" Journal of Finance, 48. 719-729.

[3] Diamond, Douglas and Robert Verrecchia (1981). "Information Aggregation in a Noisy Rational Expectations Economy," Journal of Financial Economics, 9(3). 221-235.

[4] Dow, James and Gary Gorton (1997). "Stock Market Efficiency and Economic Efficiency: Is There a Connection?," The Journal of Finance, 52 (3). 1087-1129.

[5] Dow, James and Rohit Rahi (2003). "Informed Trading, Investment, and Welfare," The Journal of Business, 76(3).

[6] Dow, James, Itay Goldstein and Alexander Guembel (2008). "Incentives for Information Production in Markets Where Prices Affect Real Investment," 2008. Unpublished manuscript.

[7] Glosten, L. and Paul Milgrom (1985). "Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders," Journal of Financial Economics,14. 71-100.

[8] Goldstein, Itay and Alexander Guembel (2008). "Manipulation and the Allocational Role of Prices," Review of Economic Studies, 75(1), 133-164.

[9] Grossman, Sanford J. and Joseph E. Stiglitz (1980). "On the Impossibility of Informationally Efficient Markets," The American Economic Review, 70 (3), 393-408.

[10] Hellwig, Christian, Arijit Mukherji, and Aleh Tsyvinski (2006). "Self-Fulfilling Currency Crises: The Role of Interest Rates," American Economic Review, 96 (5). 1769-1787.

[11] Hellwig, Martin (1980). "On the aggregation of Information in Competitive Markets," Journal of Economic Theory, 22. 477-498.

[12] Leland, Hayne (1992). "Insider Trading: Should It Be Prohibited?," The Journal of Political Economy, 100 (4). 859-887.

[13] Milgrom, Paul R. and Robert J. Weber (1982). A Theory of Auctions and Competitive Bidding," Econometrica, 50 (5). 1089-1122.

[14] Miller, Merton H. and Kevin Rock (1985). "Dividend Policy under Asymmetric Information," The Journal of Finance, 40 (4). 1031-1051.

[15] Rock, Kevin (1986). "Why New Issues are Underpriced," Journal of Financial Economics, 15 (1-2). 187-212.

[16] Stein, Jeremy (1989). "Efficient Capital Markets, Inefficient Firms: a Model of Myopic Corporate Behavior," Quarterly Journal of Economics, 104. 655-669.

[17] Subrahmanyam, Avanidhar and Sheridan Titman (1999). "The Going-Public Decision and the Development of Financial Markets," The Journal of Finance, 54 (3). 1045-1082.

[18] Vives, Xavier (2008). *Information and Leaning in Markets.* Princeton University Press.

8 Appendix A: Proofs

Proof of Lemma 1. Common knowledge of the equilibrium thresholds $\hat{x}(P)$, along with the market-clearing condition implies that $z = \theta + 1/\sqrt{\beta} \cdot u = \hat{x}(P)$ is common knowledge among all agents, i.e. the common knowledge of P implies common knowledge of z. But since P is a function of z, $P(z)$ must be invertible, so that observing P is equivalent to observing z. The above formulae then follow immediately from Bayesian updating with normal priors and signals, using the fact that conditional on θ, $z = \hat{x}(P)$ is normally distributed with mean θ and variance $\beta\delta$. ■

Proof of Proposition 1. Substituting the market-clearing condition $\hat{x}(P) = z$ and the investment threshold $\tilde{F}(z) = \tilde{F}(P(z))$ into (8), a price function $P(z)$ is part of an equilibrium if and only if it satisfies (9) and is invertible. Given the investment threshold $\tilde{F}(z)$, the firm's expected dividend value is $V(z) = \left(\rho + G\left(\tilde{F}(z)\right)\right)\mathbb{E}(\theta|z) - \int_{\underline{F}}^{\tilde{F}(z)} f dG(f)$. ■

Proof of Theorem 1. Follows directly from Proposition 1 and equation (11). ■

Proof of Theorem 2. (i) $\mathbb{E}(P(z)) - \mathbb{E}(V(z)) = (\gamma - 1) Cov(G(Z), Z)$, which is strictly positive, since $G(\cdot)$ is strictly increasing in Z. (ii) Since $Var(P(z)) = Var(V(z)) + cov(V(z), W(z)) + Var(W(z))$, it suffices to show that $cov(V(z), W(z)) > 0$. But this follows immediately since $V'(z) > 0$ and $W'(z) = P'(z) - V'(z) > 0$. ■

Proof of Proposition 2. (i) Since $V'(Z) = G(Z) > 0$ and $V''(Z) = g(Z) > 0$, $V(Z)$ is increasing and convex, so an increase in σ_Z^2 increases $\mathbb{E}(V(z))$. Moreover,

$$\mathbb{E}(Z-\mu)(\rho + G(Z)) = (\rho + G(\mu))\mathbb{E}(Z-\mu) + \mathbb{E}(G(Z) - G(\mu))(Z-\mu) = \mathbb{E}(G(Z) - G(\mu))(Z-\mu).$$

Since $(G(Z) - G(\mu))(Z-\mu)$ is strictly positive for $Z \neq \mu$, and increasing in absolute value as $|Z-\mu|$ increases, it follows that an increase in σ_Z^2 shifts the variance of the firm's posterior to higher values of $|Z-\mu|$, and therefore increases $\mathbb{E}(G(Z) - G(\mu))(Z-\mu)$. The result for $\mathbb{E}(P(z))$ then follows from the statements about $\mathbb{E}(V(z))$ and $\mathbb{E}(W(z))$.

(ii) is immediate given that γ does not affect $V(Z)$, but linearly scales up $W(Z)$. ■

Proof of Proposition 3. Equation (19) follows immediately from (18). Substituting this into the pricing equation, we find:

$$\begin{aligned} P(Z) &= (\rho + G(\mu + k(Z-\mu)))(\mu + \gamma(Z-\mu)) - \int_{\underline{F}}^{\mu+k(Z-\mu)} f dG(f) \\ &= (\rho + G(\mu + k(Z-\mu)))(\mu + k(Z-\mu)) \\ &\quad - \int_{\underline{F}}^{\mu+k(Z-\mu)} f dG(f) + (\gamma - k)(Z-\mu)(\rho + G(\mu + k(Z-\mu))) \\ &= \rho(\mu + k(Z-\mu)) + \int_{\underline{F}}^{\mu+k(Z-\mu)} G(f)\, df + (\gamma - k)(Z-\mu)(\rho + G(\mu + k(Z-\mu))). \end{aligned}$$

To check that this is an equilibrium, we check that the price function is invertible:

$$\begin{aligned} P'(Z) &= \gamma(\rho + G(\mu + k(Z-\mu))) + (\gamma - k) k (Z-\mu) g(\mu + k(Z-\mu)) \\ &= k(\rho + G(\mu + k(Z-\mu))) \\ &\quad + (\gamma - k)(\rho + G(\mu + k(Z-\mu)) + k(Z-\mu) g(\mu + k(Z-\mu))), \end{aligned}$$

which is strictly positive, because of our initial assumption on ρ and G. The expected dividend function (21) then follows from rearranging $V(Z) = (\rho + G(\mu + k(Z-\mu))) Z - \int_{\underline{F}}^{\mu+k(Z-\mu)} f dG(f)$ along the same lines as the price function. ■

Proof of Theorem 3. Part (i) is immediate. For parts (ii) and (iii) notice that

$$\begin{aligned} \frac{\partial V}{\partial k} &= g(\mu + k(Z-\mu))(Z-\mu)(Z - (\mu + k(Z-\mu))) \\ &= g(\mu + k(Z-\mu))(Z-\mu)^2 (1-k) < 0, \text{ and} \\ \frac{\partial P}{\partial k} &= g(\mu + k(Z-\mu))(Z-\mu)(\mu + \gamma(Z-\mu) - (\mu + k(Z-\mu))) \\ &= g(\mu + k(Z-\mu))(Z-\mu)^2 (\gamma - k) > 0, \text{ for } Z \neq \mu. \end{aligned}$$

■

Proof of Proposition 4. Using the fact that $\pi(\theta) = \pi(\mu) + \pi'(\mu)(\theta - \mu) + \int_\mu^\theta (\pi'(\theta') - \pi'(\mu)) d\theta'$, we write $V(Z)$ and $P(Z)$ as follows:

$$\begin{aligned} V(Z) &= \pi(\mu) + \pi'(\mu)(Z-\mu) + \int_{-\infty}^{\infty} \int_\mu^\theta (\pi'(\theta') - \pi'(\mu)) d\theta' d\Phi\left(\sqrt{\lambda + \beta\delta}(\theta - Z)\right) \\ P(Z) &= \pi(\mu) + \pi'(\mu)\gamma(Z-\mu) + \int_{-\infty}^{\infty} \int_\mu^\theta (\pi'(\theta') - \pi'(\mu)) d\theta' d\Phi\left(\sqrt{\lambda + \beta + \beta\delta}(\theta - \mu - \gamma(Z-\mu))\right) \end{aligned}$$

Taking expectations and integrating out the variable Z, we have

$$\begin{aligned}
\mathbb{E}\left(V\left(z\right)\right) &= \pi\left(\mu\right)+\int_{-\infty}^{\infty}\int_{\mu}^{\theta}\left(\pi'\left(\theta'\right)-\pi'\left(\mu\right)\right)d\theta'\int_{-\infty}^{\infty}\sqrt{\lambda+\beta\delta}\phi\left(\sqrt{\lambda+\beta\delta}\left(\theta-Z\right)\right)\frac{1}{\sigma_Z}\phi\left(\frac{Z-\mu}{\sigma_Z}\right)dZd\theta \\
&= \pi\left(\mu\right)+\int_{-\infty}^{\infty}\int_{\mu}^{\theta}\left(\pi'\left(\theta'\right)-\pi'\left(\mu\right)\right)d\theta'\sqrt{\lambda}\phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)d\theta \\
\mathbb{E}\left(P\left(z\right)\right) &= \pi\left(\mu\right)+\int_{-\infty}^{\infty}\int_{\mu}^{\theta}\left(\pi'\left(\theta'\right)-\pi'\left(\mu\right)\right)d\theta'\int_{-\infty}^{\infty}\frac{1}{\sigma_P}\phi\left(\frac{\theta-\mu-\gamma\left(Z-\mu\right)}{\sigma_P}\right)\frac{1}{\sigma_Z}\phi\left(\frac{Z-\mu}{\sigma_Z}\right)dZd\theta \\
&= \pi\left(\mu\right)+\int_{-\infty}^{\infty}\int_{\mu}^{\theta}\left(\pi'\left(\theta'\right)-\pi'\left(\mu\right)\right)d\theta'\frac{1}{\tilde{\sigma}_P}\phi\left(\frac{\theta-\mu}{\tilde{\sigma}_P}\right)d\theta,
\end{aligned}$$

where $\sigma_P^2=\left(\lambda+\beta+\beta\delta\right)^{-1}$, and $\tilde{\sigma}_P^2=\sigma_P^2+\gamma^2\sigma_Z^2$. Therefore,

$$\begin{aligned}
\mathbb{E}\left(W\left(z\right)\right) &= \int_{-\infty}^{\infty}\int_{\mu}^{\theta}\left(\pi'\left(\theta'\right)-\pi'\left(\mu\right)\right)d\theta'\left\{\frac{1}{\tilde{\sigma}_P}\phi\left(\frac{\theta-\mu}{\tilde{\sigma}_P}\right)-\sqrt{\lambda}\phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)\right\}d\theta \\
&= \int_{\mu}^{\infty}\int_{\mu}^{\theta}\left(\pi'\left(\theta'\right)-\pi'\left(\mu\right)\right)d\theta'\left\{\frac{1}{\tilde{\sigma}_P}\phi\left(\frac{\theta-\mu}{\tilde{\sigma}_P}\right)-\sqrt{\lambda}\phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)\right\}d\theta \\
&\quad+\int_{-\infty}^{\mu}\int_{\mu}^{\theta}\left(\pi'\left(\theta'\right)-\pi'\left(\mu\right)\right)d\theta'\left\{\frac{1}{\tilde{\sigma}_P}\phi\left(\frac{\theta-\mu}{\tilde{\sigma}_P}\right)-\sqrt{\lambda}\phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)\right\}d\theta \\
&= \int_{\mu}^{\infty}\left(\pi'\left(\theta\right)-\pi'\left(\mu\right)\right)\left\{\Phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)-\Phi\left(\frac{\theta-\mu}{\tilde{\sigma}_P}\right)\right\}d\theta,
\end{aligned}$$

where we have proceeded by integration by parts. Since

$$\begin{aligned}
\tilde{\sigma}_P^2 &= \frac{1}{\lambda+\beta+\beta\delta}+\left(\frac{\beta+\beta\delta}{\lambda+\beta+\beta\delta}\frac{\lambda+\beta\delta}{\beta\delta}\right)^2\frac{\beta\delta}{\lambda+\beta\delta}\lambda^{-1} \\
&= \frac{1}{\lambda}\left(1+\frac{\beta+\beta\delta}{\lambda+\beta+\beta\delta}\left(\gamma-1\right)\right)>\frac{1}{\lambda},
\end{aligned}$$

it follows that $\Phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)-\Phi\left(\left(\theta-\mu\right)/\tilde{\sigma}_P\right)>0$, when $\theta<\mu$, and $\Phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)-\Phi\left(\left(\theta-\mu\right)/\tilde{\sigma}_P\right)>0$, when $\theta>\mu$.

Thus, clearly, when $\pi''\left(\cdot\right)=0$, i.e. when $\pi\left(\cdot\right)$ is linear, $\mathbb{E}\left(W\left(z\right)\right)=0$. When instead $\pi''\left(\cdot\right)>0$, $\pi'\left(\theta\right)-\pi'\left(\mu\right)$ is increasing in θ, and positive (negative), when $\theta>\mu$ ($\theta<\mu$), so $\left(\pi'\left(\theta\right)-\pi'\left(\mu\right)\right)\left\{\Phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)-\Phi\left(\frac{\theta-\mu}{\tilde{\sigma}_P}\right)\right\}>0$ for all $\theta\neq\mu$, and hence $\mathbb{E}\left(W\left(z\right)\right)>0$. When instead $\pi''\left(\cdot\right)<0$, $\pi'\left(\theta\right)-\pi'\left(\mu\right)$ is decreasing in θ, and positive (negative), when $\theta<\mu$ ($\theta>\mu$), so $\left(\pi'\left(\theta\right)-\pi'\left(\mu\right)\right)\left\{\Phi\left(\sqrt{\lambda}\left(\theta-\mu\right)\right)-\Phi\left(\frac{\theta-\mu}{\tilde{\sigma}_P}\right)\right\}<0$ for all $\theta\neq\mu$, and hence $\mathbb{E}\left(W\left(z\right)\right)<0$. ■

9 Appendix B: The model without exogenous dividends

In this appendix, we discuss the model without exogenous dividends: $\pi\left(\theta\right)=0$. In this case, market prices aggregate information about only the endogenous investment component, while other exogenous dividends are already known to the market, and thus fully reflected in prices. As a result, there is also a feedback in the informativeness of prices: when the firm doesn't invest with probability 1, private signals are uninformative of dividends, and hence so is the price. This feedback can lead to multiple equilibria.

More specifically, two different possibilities arise, depending on whether $\tilde{F}(P) \leq \underline{F}$ or $\tilde{F}(P) > \underline{F}$, i.e. whether there is a positive probability of investment, and fundamentals are partitioned into two sets, one for which the investment probability is zero, and one for which it is positive. If $\tilde{F}(P) \leq \underline{F}$, the firm does not invest, regardless of F, and the expected dividends are zero, regardless of private signals. Therefore, the trader's demand takes the form

$$d(x_i, P) = \begin{cases} 1 \text{ if } P < 0 \\ [0,1] \text{ if } P = 0 \\ 0 \; P > 0 \end{cases}.$$

That is, all shareholders are just indifferent between holding the asset or not, when $P = 0$. Since $\Phi(u) \in (0,1)$, we never observe a price on the equilibrium path for which either all shareholders sell or don't sell their share. This implies that $P = 0$ clears the market provided that $\tilde{F}(0) \leq \underline{F}$, whereas any price $P \neq 0$ is ruled out as a market-clearing price with $\tilde{F}(P) \leq \underline{F}$.

If instead $\tilde{F}(P) > \underline{F}$, there is a positive probability of investment, which makes the dividends uncertain, and the private signals informative. In this case, the same analysis as in Proposition 1 applies, with $\rho = 0$. A shareholder with private signal x will keep the stock, whenever $x \geq \hat{x}(P)$, where $\hat{x}(P)$ is implicitly defined by

$$P = G\left(\tilde{F}(P)\right) \int \theta d\Phi \left(\sqrt{\lambda + \beta + \beta\delta} \left(\theta - \frac{\lambda\mu + (\beta + \beta\delta)\,\hat{x}(P)}{\lambda + \beta + \beta\delta}\right)\right) - \int_{\underline{F}}^{\tilde{F}(P)} f dG(f),$$

and the market clears, if $\hat{x}(P) = z$. Now, given an investment threshold function $\tilde{F}(z) = \tilde{F}(P(z))$, we have the following equilibrium characterization:

Proposition 5 *Define* $\tilde{P}(Z)$

$$\begin{aligned} \tilde{P}(Z) &= \int_{\underline{F}}^{\tilde{F}(z)} G(f)\, df + (\gamma - 1)(Z - \mu)\, G\left(\tilde{F}(z)\right), \\ \textit{where } Z &= \frac{\lambda\mu + \beta\delta z}{\lambda + \beta\delta}. \end{aligned}$$

If $P(Z)$ characterizes an equilibrium, then: (i) $P(Z) \in \left\{0, \tilde{P}(Z)\right\}$ for all Z, and $P(Z) \neq 0$ only if $\tilde{F}(Z) > \underline{F}$, whereas $P(Z) = 0$ only if $\tilde{F}(Z) \leq \underline{F}$, and (ii) $P(Z)$ is invertible over $\mathcal{S}$, where $\mathcal{S} = \{Z \in \mathbb{R} : P(Z) \neq 0\}$.

The characterization of $\tilde{P}(Z)$ follows from the same arguments as above, and the supplementary conditions then take into account that P must be invertible, when it's different from 0, and $P(Z) = 0$ must be occurs only if the firm doesn't invest with probability 1.

Next, we characterize the firm's optimal behavior. As before, we define the firm's objective as a weighted average of dividend value and price (with weights $1-\alpha$ and α, respectively, and find that, as before, that conditional on Z, the firm's optimal investment threshold is

$$\tilde{F}(Z) = \mu + k(Z-\mu),$$

and the resulting maximized objective is

$$(1-\alpha)V(Z) + \alpha P(Z) = \int_{\underline{F}}^{\mu+k(Z-\mu)} G(f)\,df.$$

This function is strictly positive whenever $\mu + k(Z-\mu) > \underline{F}$, and 0 otherwise. The resulting price function $\tilde{P}(Z)$ on the other hand is

$$P(Z) = \int_{\underline{F}}^{\mu+k(Z-\mu)} G(f)\,df + (\gamma - k)(Z-\mu)(G(\mu + k(Z-\mu))).$$

In addition, when $P(Z) = 0$, it must be optimal for the firm not to invest, which requires that $\mathbb{E}(\mu + k(Z-\mu)\,|Z \in \mathbb{R}\setminus\mathcal{S}) \leq \underline{F}$. Summarizing, the equilibrium imposes the following requirements on the set $\mathcal{S}$, for which the firm invests with positive probability:

Proposition 6 *Any equilibrium is characterized by a price function $P(Z) \in \left\{0, \tilde{P}(Z)\right\}$, and a set $\mathcal{S} \subseteq \mathbb{R}$, such that*

$$P(Z) = \begin{cases} 0 \text{ if } Z \in \mathbb{R}\setminus\mathcal{S} \\ \tilde{P}(Z) \text{ if } Z \in \mathcal{S} \end{cases}$$

The firm's investment threshold satisfies

$$\tilde{F}(Z) = \begin{cases} \underline{F} \text{ if } Z \in \mathbb{R}\setminus\mathcal{S} \\ \mu + k(Z-\mu) \text{ if } Z \in \mathcal{S} \end{cases}$$

The set $\mathcal{S}$ satisfies the following necessary and sufficient conditions for equilibrium: (i) $\tilde{P}(Z)$ for $Z \in \mathcal{S}$, (ii) $\mathbb{E}(\mu + k(Z-\mu)\,|Z \in \mathbb{R}\setminus\mathcal{S}) \leq \underline{F}$, and (iii) $Z \in \mathbb{R}\setminus\mathcal{S}$ if $\mu + k(Z-\mu) \leq \underline{F}$.

Proof. If $P(Z)$ is an equilibrium price function, $P(Z) \in \left\{0, \tilde{P}(Z)\right\}$ for all Z, where $\tilde{P}(Z)$ is defined as above. $P(Z)$ must be invertible over the range in which $P(z) \neq 0$, i.e. the set $\mathcal{S}$ for which $P(z) = \tilde{P}(Z)$. When $P = 0$, the firm must find it optimal not to invest, requiring

$\mathbb{E}\left(\mu+k\left(Z-\mu\right)|Z\in\mathbb{R}\smallsetminus\mathcal{S}\right)\leq\underline{F}$. And finally, whenever $\mu+k\left(Z-\mu\right)\leq\underline{F}$, the firm would find it optimal not to invest, if it knows Z, requiring $\tilde{P}\left(Z\right)=0$, so that necessarily $z\in\mathbb{R}\smallsetminus\mathcal{S}$. Moreover, by construction, for any set $\mathcal{S}=\{Z\in\mathbb{R}:P\left(Z\right)b\neq0\}$ that satisfy these conditions, the corresponding price function and threshold functions for traders and the firm constitute an equilibrium. ■

Thus, without exogenous dividends, the realizations of Z are divided into two sets $\mathcal{S}$, and $\mathbb{R}\smallsetminus\mathcal{S}$, such that when $z\in\mathbb{R}\smallsetminus\mathcal{S}$, the price is zero and the firm does not invest, and when $Z\in\mathcal{S}$, the firm invests with positive probability, the price differs from zero, and aggregates private information. To satisfy the equilibrium conditions, $P\left(Z\right)$ must be invertible over the range with positive probability of investment $\mathcal{S}$, and the firm does not find it optimal to invest, when $Z\in\mathbb{R}\smallsetminus\mathcal{S}$. Moreover, when $\mu+k\left(Z-\mu\right)\leq\underline{F}$, the information the market price conveys is so negative that the firm would find it optimal not to invest. In these cases it is also necessary that $z\in\mathbb{R}\smallsetminus\mathcal{S}$.

We next examine what these restrictions imply for the equilibrium set. Taking derivatives, we have

$$\tilde{P}'\left(Z\right)=\gamma G\left(\mu+k\left(Z-\mu\right)\right)+\left(\gamma-k\right)k\left(Z-\mu\right)g\left(\mu+k\left(Z-\mu\right)\right).$$

Thus, when $\underline{F}\geq\mu$, $\tilde{P}'\left(Z\right)>0$ for all $\mu+k\left(Z-\mu\right)\geq\underline{F}$, so $\tilde{P}\left(Z\right)$ is invertible over its entire range, and therefore any subset $\mathcal{S}$ of $\{\mu+k\left(Z-\mu\right)\geq\underline{F}\}$. Moreover, for any $\mathcal{S}\subseteq\{Z:\mu+k\left(Z-\mu\right)\geq\underline{F}\}$, $\mathbb{E}\left(\mu+k\left(Z-\mu\right)|Z\in\mathcal{S}\right)\geq\underline{F}\geq\mu$, and therefore $\mathbb{E}\left(\mu+k\left(Z-\mu\right)|Z\in\mathbb{R}\smallsetminus\mathcal{S}\right)\leq\mu\leq\underline{F}$, implying that whenever $P=0$ is observed, it is indeed optimal not to invest, regardless of F. Thus, when $\underline{F}\geq\mu$, the additional requirements are automatically satisfied, and impose no additional restrictions: any set $\mathcal{S}$ that includes $\{Z:\mu+k\left(Z-\mu\right)\leq\underline{F}\}$ characterizes an equilibrium. Notice that this includes a 'least informative' equilibrium, in which $\mathcal{S}=\varnothing$, corresponding to an equilibrium in which the firm never invests, and a 'most informative' equilibrium, in which $\mathcal{S}=\{Z:\mu+k\left(Z-\mu\right)\leq\underline{F}\}$, and whenever $\mu+k\left(Z-\mu\right)\geq\underline{F}$, the equilibrium price is positive, and there is a positive chance of investment. The condition that $\underline{F}\geq\mu$ says that the prior mean is sufficiently pessimistic so that the firm would not want to invest if it didn't receive any further information about the profitability of the investment.

When instead $\underline{F}<\mu$, the additional restrictions have some bite: there now exists $\underline{Z}<\mu$ at which $\tilde{P}\left(Z\right)$ reaches a minimum; for Z, such that $\mu+k\left(Z-\mu\right)\in\left(\underline{F},\mu+k\left(\underline{Z}-\mu\right)\right)$, $\tilde{P}'\left(Z\right)<0$, while for $Z>\underline{Z}$, $\tilde{P}'\left(Z\right)>0$.[15] In addition, there exists $\bar{Z}<\infty$ such that $\mathbb{E}\left(\mu+k\left(Z-\mu\right)|Z\leq\bar{Z}\right)=\underline{F}$.

Thus both the equilibrium with $P\left(Z\right)=\tilde{P}\left(Z\right)$ whenever $\mu+k\left(Z-\mu\right)>\underline{F}$, and the equilibrium in which $P\left(Z\right)=0$ for all Z fail to exist; the former because the resulting price function is not

[15] To avoid some technicalities, we assume that $G\left(F\right)/g\left(F\right)$ is non-decreasing and that $\gamma>k$. When $\gamma=k$, $\mu+k\left(\underline{Z}-\mu\right)=\underline{F}$, and non-invertibility is not an issue.

invertible, and the latter because the firm would want to invest with positive probability if it obtained no additional information. The invertibility requirement then implies that $\mathcal{S}$ must include some of the domain of Z over which $\tilde{P}(Z)$ is not invertible, so as to restore invertibility over $\mathbb{R}\diagdown\mathcal{S}$. For example, this is satisfied by any set $\mathcal{S}=\{Z \leq Z'\}$, for any $Z' \geq \underline{Z}$. In addition, we need that conditional on $P(Z)=0$, it is optimal not to invest. For any set $\mathcal{S}=\{Z \leq Z'\}$, this optimality condition is satisfied whenever $Z' \leq \bar{Z}$, and if $\underline{Z} < \bar{Z}$, we have thus constructed a continuum of monotone equilibria for any set $\mathcal{S}=\{Z \leq Z'\}$, for $Z' \in \left[\underline{Z}, \bar{Z}\right]$. As $\underline{Z}$ becomes larger, or $\bar{Z}$ smaller, these additional requirements on $\mathcal{S}$ become more and more restrictive, until the point where $\underline{Z}=\bar{Z}$ (at which point there is a unique threshold equilibrium). When instead $\underline{Z} \geq \bar{Z}$, the requirements of invertibility at $P \neq 0$ and no investment at $P=0$ become so restrictive that they are mutually contradictory, and hence an equilibrium fails to exist.

Depending on parameters, any of these three scenarios is possible: Non-existence occurs whenever the distribution of F is unbounded below and has sufficiently thin tails, or $\underline{F}$ is finite, but very low. The intermediate case in which $\underline{F} < \underline{Z} < \bar{Z} < \infty$ occurs when $\underline{F}$ is sufficiently high, but lower than μ. These statements are summarized in the following proposition:

Proposition 7 (Equilibrium existence and multiplicity) *Define*

$$\begin{aligned} \underline{Z} &= \inf\left\{Z : \mu + k(Z-\mu) \geq \underline{F} : \tilde{P}'\left(Z'\right) > 0 \text{ for all } Z' > Z\right\} \\ \text{and } \bar{Z} &= \sup\left\{Z : \mu + k(Z-\mu) \geq \underline{F} : \mathbb{E}\left(\mu + k(Z-\mu)\,|Z \leq \bar{Z}\right) \leq \underline{F}\right\}. \end{aligned}$$

(i) if $\underline{F} \geq \mu$, then $\mu + k(\underline{Z}-\mu) = \underline{F}$, and $\bar{Z} = +\infty$, and any selection $P(Z) \in \left\{0, \tilde{P}(Z)\right\}$ characterizes an equilibrium.

(ii) if $\underline{F} < \mu$, and $\underline{Z} \leq \bar{Z}$, then for any $Z' \in \left[\underline{Z}, \bar{Z}\right]$, there exists an equilibrium, in which $P(Z)=0$ if and only if $Z \leq Z'$.

(iii) if $\underline{Z} > \bar{Z}$, an equilibrium does not exist.

Proof. (i) and (ii) are proved in the text.

(iii) If $\underline{Z} > \bar{Z}$, we show that the two equilibrium conditions on $\mathcal{S}$ are mutually contradictory, i.e. $\mathbb{E}\left(\mu + k(Z-\mu)\,|Z \in \mathbb{R}\diagdown\mathcal{S}\right) > \mathbb{E}\left(\mu + k(Z-\mu)\,|Z \leq \bar{Z}\right) = \underline{F}$ for any $\mathcal{S}$, such that $\tilde{P}(z)$ is invertible over $\mathcal{S}$. To show this consider an arbitrary $\mathcal{S}$, such that $\tilde{P}(z)$ is invertible over $\mathcal{S}$. Let

$\bar{\mathcal{S}} = (\mathbb{R} \setminus \mathcal{S}) \cap \{Z > \bar{Z}\}$ and $\underline{\mathcal{S}} = \mathcal{S} \cap \{Z \leq \bar{Z}\}$, and notice that

$$\begin{aligned}
\mathbb{E}(\mu + k(Z-\mu) | Z \in \mathbb{R} \setminus \mathcal{S}) &= \mathbb{E}\left(\mu + k(Z-\mu) | Z \leq \bar{Z}\right) \\
&+ \int_{\bar{\mathcal{S}}} \left(\mu + k(Z-\mu) - \mathbb{E}\left(\mu + k(Z-\mu) | Z \leq \bar{Z}\right)\right) \psi(Z)\, dZ \\
&- \int_{\underline{\mathcal{S}}} \left(\mu + k(Z-\mu) - \mathbb{E}\left(\mu + k(Z-\mu) | Z \leq \bar{Z}\right)\right) \psi(Z)\, dZ,
\end{aligned}$$

where we use $\Psi(\cdot)$ to denote the prior distribution of Z, which is normal with mean μ (and $\psi(\cdot)$ the corresponding density).

Now, for each $Z \in \underline{\mathcal{S}}$, there exists $Z'(Z) \in \bar{\mathcal{S}}$ such that $\tilde{P}(Z) = \tilde{P}(Z'(Z))$. Moreover, $\mu > Z'(Z) > Z$, so, $\psi(Z) < \psi(Z'(Z))$ and

$$\mu + k(Z-\mu) \leq \mathbb{E}\left(\mu + k(Z-\mu) | Z \leq \bar{Z}\right) < \mu + k\left(Z'(Z) - \mu\right),$$

where the first inequality uses the fact that $\mu + k(Z-\mu) \geq \underline{F}$ for $Z \in \underline{\mathcal{S}}$, and the second the fact that $Z'(Z) > \bar{Z}$. But then,

$$\begin{aligned}
& \left(\mu + k(Z-\mu) - \mathbb{E}\left(\mu + k(Z-\mu) | Z \leq \bar{Z}\right)\right) \psi(Z) \\
< \; & \left(\mu + k\left(Z'(Z) - \mu\right) - \mathbb{E}\left(\mu + k(Z-\mu) | Z \leq \bar{Z}\right)\right) \psi\left(Z'(Z)\right),
\end{aligned}$$

or

$$\begin{aligned}
& \int_{\underline{\mathcal{S}}} \left(\mu + k(Z-\mu) - \mathbb{E}\left(\mu + k(Z-\mu) | Z \leq \bar{Z}\right)\right) \psi(Z)\, dZ \\
< \; & \int_{\underline{\mathcal{S}}} \left(\mu + k\left(Z'(Z) - \mu\right) - \mathbb{E}\left(\mu + k(Z-\mu) | Z \leq \bar{Z}\right)\right) \psi\left(Z'(Z)\right) dZ \\
\leq \; & \int_{\bar{\mathcal{S}}} \left(\mu + k(Z-\mu) - \mathbb{E}\left(\mu + k(Z-\mu) | Z \leq \bar{Z}\right)\right) \psi(Z)\, dZ
\end{aligned}$$

which implies$\mathbb{E}(\mu + k(Z-\mu) | Z \in \mathbb{R} \setminus \mathcal{S}) > \mathbb{E}\left(\mu + k(Z-\mu) | Z \leq \bar{Z}\right) = \underline{F}$ ■

The model without exogenous dividend thus admits a rich set of possible equilibrium outcomes, including the potential for multiplicity, indeterminacy and non-existence. Multiplicity and indeterminacy are a consequence of the feedback on the signals' informativeness: when the firm is certain not to invest, private signals are worthless and traders are indifferent at a price of $P = 0$. Since $P = 0$ is then at most partially revealing of Z, it is possible to sustain (almost) arbitrary selections from the correspondence $\left\{0, \tilde{P}(Z)\right\}$ as equilibrium prices.

Assuming that the exogenous dividend $\pi(\theta)$, with $\pi'(\theta)$ strictly positive introduces an exogenous motive for information aggregation and hence trading based on private signals. This breaks the feedback cycle that is responsible for multiplicity and indeterminacy in our benchmark model.

Non-existence on the other hand is linked to the non-invertibility of the price function, and hence the non-monotonicity of $\tilde{P}(\cdot)$. In our benchmark model, this non-invertibility occurs with unbounded support of F, and a thin-tail assumption on the distribution $G(\cdot)$. By removing the possibility of no information aggregation and no investment at a price of 0 from the set of possible market-clearing outcomes, the exogenous dividend $\pi(\theta)$ makes non-invertibility even more of an issue, unless $\pi'(\theta)$ is bounded sufficiently far away from zero so that the resulting price function is guaranteed to be monotone.

These multiplicity, indeterminacy and non-existence issues are interesting in their own right, but they are somewhat distracting from the main contribution of our paper. The assumptions of our baseline model focus our analysis on those cases where we have a unique equilibrium.