Amplification of Uncertainty in Illiquid Markets *

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Abstract

This paper argues that the capacity of financial markets to aggregate information is diminished in times of distress, resulting in countercyclical economic uncertainty. I build a rational expectations equilibrium model in which informed financial intermediaries face countercyclical fund outflows, which expose them to non-fundamental price fluctuations during contractions. This reduces information-based trading and price informativeness. Uncertainty spikes as conditions deteriorate due to amplification mechanisms arising from the dispersed nature of information and externalities in a dynamic environment. Heightened uncertainty increases risk premia, Sharpe ratios and price volatility even when attitude towards risk and the unconditional variance of fundamentals are constant. I trace the implications for real allocations in an economy with partial investment irreversibilities in which firms learn about demand condiitons from stock prices. Uncertainty reduces the level of real investment as a precautionary response of firms.

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1 Introduction

Asset prices play a key role in the allocation of resources. By aggregating information about economic fundamentals, they reduce the uncertainty about the future prospects of firms and help guide both portfolio allocations of investors, as well as real allocations through firm's production decisions. The idea that asset prices have an informational role beyond clearing current output markets can be traced to Hayek (1945), and was formalized by the *rational expectations equilibrium* (REE) models of the 1980's.[1]

Economic recessions –including the latest subprime episode– offer some stylized facts that beg the question of whether the information aggregation process depends itself upon the business cycle. These facts include i) a tightening of funding constraints of financial intermediaries in the form of capital outflows and margin calls;[2] ii) steep declines in the prices of risky assets and spikes in volatility; and iii) sharp and persistent contraction in aggregate real investment.

This paper investigates the relation between the tightening of funding constraints through fund outflows, and the fluctuations of asset prices and real investment. I argue that the endogenous variation of the informational content of asset prices provides a key linking mechanism.

More specifically, the paper makes three contributions. First, I offer a tractable model where countercyclical uncertainty arises endogenously from the diminished capacity of financial markets to aggregate information about economic fundamentals in times of distress.[3] The model combines two literatures that are central in macroeconomics and finance: the REE analysis of information aggregation, and the *limits of arbitrage* (De Long, Shleifer, Summers and Waldmann (1990); Shleifer and Vishny (1997)). I consider agents that react to private, heterogeneous information trading with agents that have non-informational motives (noise trading), resulting in the partial aggregation of information about fundamentals into asset prices. The limits of arbitrage qualify the extent of such aggregation through the trading restrictions that arise from funding constraints in actual markets. I argue that because funding constraints are particularly binding in times of distress, the informativeness of asset prices is diminished during contractions.[4]

The second contribution of this paper is to trace the asset pricing implications of countercyclical uncertainty. I argue that the model can shed light into the observed time-variation of expected returns and Sharpe ratios.[5] As economic conditions and the informativeness of prices deteriorate, traders demand a larger compensation for holding risk. Higher expected returns translate into

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[1]Grossman and Stiglitz (1980); Hellwig (1980); Diamond and Verrecchia (1981), and Kyle (1985).

[2]The tightening of funding constraints during contractions dates at least to the great depression, as documented by Galbraith (1954).

[3]I will refer to uncertainty as measurable risk: the variance of fundamentals, conditional on information.

[4]Countercyclical uncertainty is documented in Veronesi (1999), Rich and Tracy (2003) and Bollerslev and Zhou (2007).

[5]Hansen and Singleton (1983); Campbell and Shiller (1988); Fama and French (1989); Ferson and Harvey (1991).

heightened price variability in the presence of noise trading, consistent with countercyclical stock market volatility.[6] Importantly, the model delivers these results in an environment when both the attitude towards risk and the unconditional volatility of fundamentals remain constant.

As a third contribution, I study the implications for real allocations in an production economy featuring partial irreversibilities in investment.[7] I show that when firms learn information about productivity conditions from stock prices,[8] heightened noise reduces the accuracy of investment decisions. Moreover, I provide a closed-form solution for the investment problem in which the *level* of investment is negatively affected by the conditional variance of fundamentals. Time variability in information aggregation can thus provides a theoretical foundation for the negative impact of uncertainty shocks on investment documented in recent empirical work (Bloom (2009)).

The central feature of the model is that traders with private information are financial intermediaries who invest on behalf of households. Intermediaries choose asset positions in a first stage, trading against noise traders. In a second stage, a fraction of households face liquidity shocks that trigger the redemption of funding from the intermediaries, forcing them to liquidate positions. Assets liquidated in the second stage are absorbed by uninformed traders –rational investors who learn from prices– at a price that can diverge from fundamentals due noise trading in this stage as well. Liquidations therefore expose informed traders to a second source of risk beyond uncertainty about fundamentals: *noise trading risk* –the price impact of exogenous trading motives.

When economic conditions deteriorate, liquidity needs of households cause more funding withdrawals. This lowers expected returns and increasing trading risks for intermediaries in the second stage. In anticipation, intermediaries trade less aggressively in response to private information in the first stage, which reduces the informativeness of the asset price. A key element of the equilibrium is that uninformed traders, who act as liquidity providers when intermediaries are forced to reverse positions, have an endogenous demand elasticity for the asset. Indeed, their willingness to absorb net supply depends on the informativeness of the price. When the price is noisy, uninformed traders require larger compensation for bearing risk, lowering the average price and increasing its volatility in the second stage. This reduces expected returns and increases the risk for intermediaries in the first stage, who therefore trade even less aggressively and reduce the informativeness of the price even further. Endogenous supply of liquidity by uninformed traders amplifies the effect of funding withdrawals on uncertainty, and can result in multiple trading equilibria.

This paper relates to three other broad strands of literature that cover countercyclical uncertainty, asset pricing models of asymmetric information and financial constraints, and time-varying risk premium.

First, several papers have studied countercyclical uncertainty and business cycle learning dy-

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[6]Schwert (1989); Bollerslev, Chou and Kroner (1992).

[7]Bernanke (1993); Dixit and Pindyck (1994); Bertola and Caballero (1994)

[8]Dow and Gorton (1997); Dow and Rahi (2003); Goldstein and Guembel (2008).

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namics. While Van Niewerburgh and Veldkamp (2006) and Angeletos and Lao (2008) focus on neoclassical production economies with no role for financial frictions, Veronesi (1999) discusses learning in financial markets in a model of regime shifts, but in a representative agent framework without financing constraints.

A second line of literature features models of asymmetric information and financial constraints. Xiong (2001) and Kyle and Xiong (2001) build equilibrium models in which arbitrageurs' wealth losses destabilize asset prices by increasing risk aversion. He and Krishnamurthy (2008) study the impact of wealth shocks on the contracting problem between households and intermediaries. Brunnermeier and Pedersen (2009) focus on the feedback between funding liquidity and non-fundamental volatility.[9] While accurately predicting several moments of pricing data, these models do not offer enough tractability to focus on the informational role of equilibrium prices, assuming instead exogenous information asymmetries across agents. By focusing on dynamic risk considerations triggered by withdrawals, I am able to provide closed-form solutions for the informativeness of asset prices as a function of underlying fundamentals, allowing an explicit analysis of time-varying information aggregation. Closest to this paper, Barlevy and Veronesi (2003) and Yuan (2005) simultaneously deal with funding constraints and learning to explain stock market crashes. My paper differs from these last two contributions in the specification of information asymmetries, limits of arbitrage, and the dynamic nature of risk.

A third line of research that relates to my results studies the origins of time variation in the risk premium. Campbell and Cochrane (1999) build a representative agent model with external habit formation, arguing that the effective risk aversion of households spikes as consumption falls towards habit levels in recessions. The alternative explanation stresses exogenous time variation in the volatility of the dividend generating process of risky assets (Barsky and DeLong (1993); Bansal and Yaron (2004)). While the attitude towards risk and the exogenous amount of it are likely to be higher in recessions,[10] I argue that uncertainty arising from the *endogenous* fluctuation of asset price informativeness can play a role by itself. Alternatively, my explanation can serve as a microfoundation for the empirical work that models time-variation in the volatility process through reduced-form equations. To my knowledge, no related paper has formally studied the connection between endogenous information aggregation and time-variation in the price of risk.[11]

The remainder of the paper is structured as follows. The next section describes a simple REE model in which funding constraints exogenously limit market participation to a subset of informed traders, with the aim of elucidating the broad link between real market constraints and the cyclical variation of price informativeness. Section 3 introduces dynamic risk in a three stage model. In

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[9]Gromb and Vayanos (2002) analyze arbitrage and welfare in a multi-period model with collateral constraints.

[10]Bekaert, Engstrom and Xing (2009) find that both varying risk aversion and conditional variance of fundamentals have explanatory power for the variance decomposition of the term structure, equity prices and risk premia.

[11]I am indebted to Andy Atkeson for pointing this out.

section 4, I discuss the asset pricing implications of the model. Section 5 discusses the impact on real allocations when asset prices provide information for real investment decisions. Section 6 offers testable predictions of the model and brief remarks for policy. Section 7 concludes.

2 A noisy REE model with financial constraints

2.1 Setup

This section illustrates how financial constraints can hinder information aggregation using a simple REE model of an endowment economy with asymmetric information, which will be extended in subsequent sections. There are two stages: 1, and 2. A consumption good is produced in the random amount θ at stage 2. I refer to θ as the dividend, or economic fundamentals interchangeably, which follows a normal distribution with zero mean and variance λ_θ^{-1}.

The economy is populated by traders that exchange claims on the risky endowment through shares in a financial market that opens in stage 1. In stage 2, θ is revealed and traders are paid according to their net positions, and consume.

Traders can be either informed traders or noise traders. There are a continuum of informed traders indexed by $i \in [0, 1]$, each endowed with one share. At stage 1, they observe a private signal s_i about the value of the dividend:

$$s_i = \theta + \epsilon_i; \ \epsilon_i \sim \mathcal{N}\left(0, \lambda_\epsilon^{-1}\right), \tag{1}$$

The signal consists of the true realization of θ plus idiosyncratic noise ϵ_i, which is identically and independently distributed across traders conditional on θ. Informed traders have CARA preferences with risk aversion γ over the consumption of terminal wealth at stage 2: $U\left(W_{i,2}\right) = -\exp\left(-\gamma \cdot W_{i,2}\right)$.

Noise traders have other (unmodeled) trading motives and supply the random amount of n shares, with $n \sim \mathcal{N}(0, \lambda_n^{-1})$.

To illustrate the impact of funding constraints on information aggregation, suppose informed traders have a funding status $f_i \in \{0, 1\}$, so that only traders with $f_i = 1$ can exchange claims on the financial market. $F \equiv \int f_i di$ measures the fraction of informed traders that are allowed to trade, which I label *aggregate funding liquidity*. Limits to trading can arise from losses on prior positions. If the wealth (capital) of traders is low enough, they are likely to face constraints in raising funds for further trading. Alternatively, agents that use informed traders as intermediaries might face liquidity needs that force them to make withdrawals. I explore the latter channel in section 3, but for now I take F as exogenous, focusing on the informational properties of the share price that result from its variation. I assume that liquid traders ($f_i = 1$) can borrow at a riskless

rate normalized at zero.

2.2 Equilibrium

A competitive equilibrium is 1) a share price function $P_1(\theta, n; F)$, 2) demand schedules by informed traders $t_i = t(s_i, P_1; f_i, F)$, and 3) prior beliefs $H(\theta)$, and posterior beliefs $H(\theta \mid s_i, P_1)$ such that $\forall i \in [0, 1]$: (i) If $f_i = 0$, $t_i = 0$ and if $f_i = 1$ asset demands are optimal given posterior beliefs and aggregate funding liquidity F; (ii) The share price clears the market; and (iii) Posterior beliefs are updated through Bayes law.

The price function $P_1(\cdot)$ depends on the realization of the fundamental (θ), noise trading (n), and aggregate funding liquidity F. Condition (i) states that informed traders who can trade $(f_i = 1)$ maximize expected utility given posterior beliefs and aggregate funding conditions, since F will affect the informativeness of the share price. Condition (ii) imposes market clearing for any realization of the noisy asset supply, while (iii) restricts beliefs to follow Bayes rule: the conditional distribution of D is updated from the observation of signals s_i and the share price P_1.

The solution method follows three steps (Grossman (1976)). First, I conjecture that the price function is linear in the shocks;

$$P_1(\theta, n, \cdot) = A_1 + A_2 \cdot \theta + A_3 \cdot n \tag{2}$$

so that informed traders back out a noisy signal about θ from the observation of price labeled $\hat{p}_1$: the *informational content* of the price,

$$\hat{p}_1 \equiv \frac{P_1 - A_1}{A_2} = \theta - \mathbf{\Delta} \cdot n \tag{3}$$

where $\mathbf{\Delta} = -A_3/A_2$. $\hat{p}_1$ given θ is distributed normally with mean θ and variance $\lambda_1^{-1} = \lambda_n^{-1} \cdot \mathbf{\Delta}^2$. The variance of the noise in the price signal is the product of two terms: the variance of noise trading shocks (λ_n^{-1}) and the *noise amplifier* $\mathbf{\Delta}$. The latter captures the response of the price to innovations in noise trading relative to fundamentals (the ratio $-A_3/A_2$). When high, noise trading has a large impact on the price, which becomes a poor aggregator of dispersed information about the fundamental θ.

Informed traders' posterior beliefs of θ depend on private signals and the market-clearing price. Applying the projection theorem (Appendix A), the first two moments of informed traders' posterior beliefs are given by

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\begin{aligned}
\mathbb{E}[\theta \mid s_i, \hat{p}_1] &= a_0 \cdot s_i + a_1 \cdot \hat{p}_1 \\
\mathbb{V}[\theta \mid s_i, \hat{p}_1] &= \left[\lambda_\theta + \lambda_\epsilon + \lambda_n/\mathbf{\Delta}^2\right]^{-1}
\end{aligned}
\tag{4}
$$

where a_o, and a_1 are the Bayesian weights assigned to the private and public signals that depend on their relative precision.

The second step is to compute the optimal demands that follow from the posterior beliefs characterized by (4). Informed traders' terminal wealth is given by $W_{i,2} = t_i \cdot (\theta - P_1) + \theta$, which conditional on demand t_i and the information set $\{s_i, \hat{p}_1\}$ is normally distributed. Maximizing the expectation of exponential utility is then equivalent to maximizing a mean-variance utility augmented by risk-aversion (γ), leading to demands

$$t_i = \frac{\mathbb{E}\left[\theta \mid s_i, \hat{p}_1\right] - P_1}{\Sigma} - 1 \tag{5}$$

where $\Sigma = \gamma \cdot \mathbb{V}\left[\theta \mid s_i, \hat{p}_1\right]$ is the risk-aversion adjusted variance conditional on information, which is common across traders. Demand schedules in (5) have two components. The first term is the ratio between expected returns and risk-aversion adjusted variance, and captures information-based motives for trade. The -1 term is the hedging motive arising from the initial risky endowment of one share.

To solve the linear equilibrium the third step imposes market-clearing:

$$F \cdot \int t_i di = n \tag{6}$$

where the left hand side is the aggregate demands of informed traders, which equals the noisy asset supply. Solving for P_1 yields the coefficients in (2) through the method of undetermined coefficients. These are functions of the primitive parameters and aggregate funding liquidity; F. The following proposition summarize the main results.

Proposition 1 (Equilibrium): *There exists a unique linear equilibrium price function $P_1(\theta, n, \cdot)$:*[12]

$$P_1 = \tilde{A}_1 + \tilde{A}_2 \cdot \theta + \tilde{A}_3 \cdot n$$

with coefficients

$$\tilde{A}_1 = -\Sigma; \quad \tilde{A}_2 = a_0 + a_1; \quad \tilde{A}_3 = -\frac{\Sigma}{F} \cdot \frac{a_0 + a_1}{a_0}$$

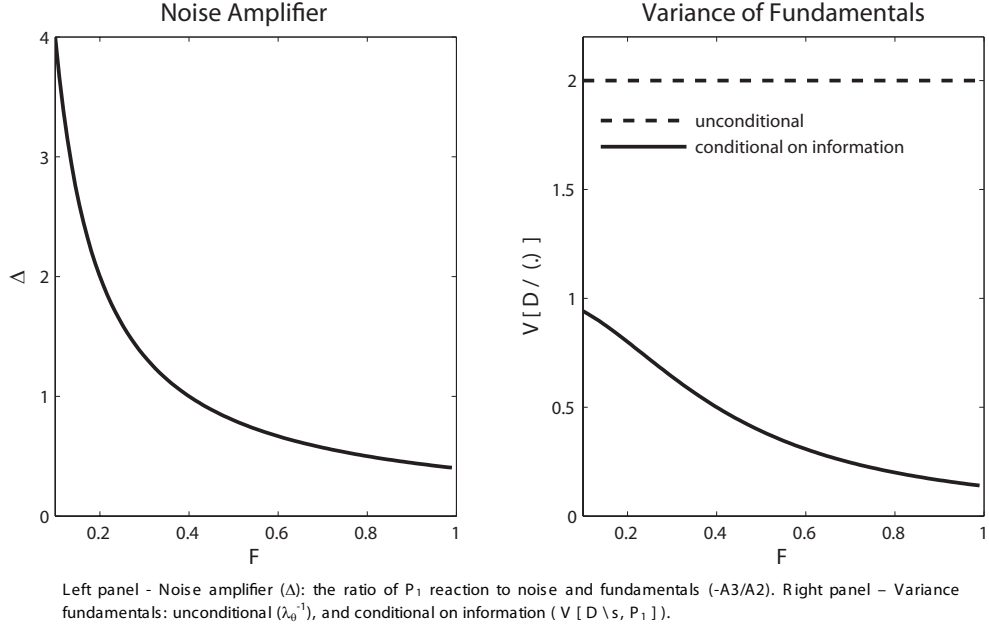*where Σ, a_0 and a_1 are given by (4) as a function of the noise amplifier $\mathbf{\Delta}$:*

$$\mathbf{\Delta}(F) = \frac{\gamma}{\lambda_\epsilon} \cdot \frac{1}{F} \tag{7}$$

Proof. *In appendix A.* ∎

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[12]I ignore whether there exist non-linear equilibria.

Figure 1: Noise and Uncertainty

Left panel - Noise amplifier (Δ): the ratio of P_1 reaction to noise and fundamentals (-A3/A2). Right panel – Variance fundamentals: unconditional (λ_θ^{-1}), and conditional on information (V [D \s, P_1]).

Expression (7) gives the noise amplifier $\boldsymbol{\Delta}(\cdot)$ as a function of funding liquidity F. Figure 1 plots the impact of funding on $\boldsymbol{\Delta}(\cdot)$, and the conditional variance of fundamentals $\mathbb{V}[\theta \mid s, \tilde{p}_1]$. As $F \to 1$, all informed traders impound their private information into the price. The noise amplifier reaches its lower bound $\boldsymbol{\Delta}(1) = \gamma \cdot \lambda_\epsilon^{-1}$, an increasing function of the variance of idiosyncratic noise in private signals (λ_ϵ^{-1}) and risk aversion (γ).[13]

Conversely, as $F \to 0$ the asset price loses informational value completely ($\boldsymbol{\Delta} \to \infty$). Since information is dispersed, traders learn additional information from the price in this economy. As a result, a higher value of $\boldsymbol{\Delta}$ increases the conditional variance of fundamentals. Economic uncertainty, defined as the variance of the dividend conditional on information, is therefore decreasing in funding liquidity: $\partial \mathbb{V}[\theta \mid \cdot] / \partial F < 0$ (expression (4)).

3 A noisy REE model with dynamic risk

This section provides a model of dynamic risk arising from funding constraints *going forward*: even if current financing is available, traders might fear tightening of constraints in the near future. The following quote from Shleifer and Vshny (1997) captures the essence of the argument,

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[13]The noise amplifier in this limit is equivalent to the one that obtains in the endogenous information acquisition model of Grossman and Stiglitz (1980), when the unit measure of traders choose to become informed.

"Arbitrageurs can become most constrained .. when the mispricing they have bet against gets even worse... the fear of this scenario would make them more cautious when they put on their initial trades, and hence less effective in bringing about market efficiency".

Mounting evidence suggests that this "fear of illiquidity" mechanism is important empirically, particularly when asset prices fall at the outset of contractions. Several studies document predictability of mutual fund net inflows from lagged fund performance.[14] In the time series, Warther (1995) documents a positive correlation between mutual fund performance and *aggregate* inflows into the industry. Reliance on debt raises similar concerns. Brunnermeier and Pedersen (2009) argue that margin requirements tighten when counter-party risk increases during crises. Moreover, rolling-over short-term debt becomes increasingly difficult as liquidity dries up[15] and can escalate to virtual market freezes, as illustrated by the subprime episode.[16] In short, all sources of financing become increasingly fragile as crises unfold.

In practice, funding constraints arise both from the behavior of financial intermediaries' creditors (debt) and its clients (capital). I focus on the latter. Modeling risky debt is burdensome because it usually requires imposing limited liability constraints (Bernanke and Gertler (1989); Holmstrom and Tirole (1997)), a property at odds with CARA preferences which allow negative consumption. However, CARA permits tractable information aggregation in a risk-averse setting. Focusing exclusively on capital constraints enables me to use a more parsimonious model to analyze the interaction between limited arbitrage and price informativeness.[17]

The model presented below studies dynamic risk in a setting with two trading stages. At stage 1, risk-averse informed traders raise funds from a continuum of clients. They benefit from their private, heterogeneous information trading against noise traders, sharing profits with clients. Funding is fragile however, as some clients may withdraw funds at stage 2 depending on their idiosyncratic *liquidity shocks*. This forces informed traders to liquidate the corresponding fraction of positions at a price that can differ from fundamentals due to noise trading at this stage as well.

The unwinding of informed traders' positions and noise trading at stage 2 is absorbed by uninformed traders: rational investors who learn information about the fundamental θ from prices. A key observation is that although the variance of noise trading is an exogenous parameter, its impact on stage 2 prices –noise trading risk– is endogenous to informed traders' decisions in stage 1. The less informed traders react to their private information in stage 1, the less revealing equilibrium prices are. This increases uncertainty for uninformed traders and reduces their willingness to provide liquidity at stage 2. The price impact from asset liquidations and noise in stage 2 is

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[14]Chevalier and Ellison (1997); Wermers (1999); Coval and Stafford (2007).

[15]See Acharya, Gale and and Yorulmazer (2009).

[16]See Gorton and Metrick (2009).

[17]Xiong (2001) and Kyle and Xiong (2001) derive limited arbitrage from decreased risk-tolerance of CRRA traders following wealth losses. Power utility is much less tractable for modeling information aggregation, however. See Mertens (2009) for a discussion on non-linear methods to overcome this problem.

thus enhanced, lowering expected returns and increasing risk from early liquidations. Both effects induce more cautious trades by informed traders in response to private information, creating a reinforcement between noise trading risk and price informativeness. The impact of funding fragility is thus amplified through the endogeneity of noise trading risk, and can even lead to multiple trading equilibria in the asset market.

3.1 Setup

There are three stages: 1, 2 and 3. The single risky asset in the economy pays a liquidation value of D in stage 3 according to

$$D = D_{-1} + \theta + \mu$$

where $D_{-1} = \bar{D} + \theta_{-1}$ is common knowledge at stage 1, given by a mean $\bar{D}$ plus the realization of the lagged dividend innovation θ_{-1}. Both θ_{-1} and θ are drawn independently from a normal distribution with zero mean and variance λ_θ^{-1}. Henceforth, I will refer to θ_{-1} as *lagged economic conditions*. The innovation $\theta + \mu$ becomes common knowledge at stage 3, but part of the uncertainty about θ will reduced by trading in prior stages. The term μ is a normally distributed white noise with variance λ_μ^{-1}.

3.1.1 Agents

There are two kinds of agents: traders and clients. Traders can be of three types; informed, uninformed, or noise traders. Informed traders are a continuum of mass 1 indexed by $i \in [0, 1]$. They are born in stage 1 with CARA preferences (risk-aversion γ) over the consumption of terminal wealth $W_{i,3}$. At stage 1, each observes a private signal about θ given by (1). Uninformed traders are born in stage 2 in unit mass, and have CARA preferences (with risk-aversion γ_u) over the consumption of terminal wealth $W_{u,3}$. They have no private information about θ but make rational inferences from the asset price. Noise traders are born in stages 1 and 2 in masses n_1 and n_2, which are drawn independently from a zero-mean normal distribution with variance λ_n^{-1}.

Financial intermediation arises from the need to finance purchases or sales of the risky asset. I treat long and short positions symmetrically by assuming both require funding the entire price.[18] Informed traders have no endowments and cannot borrow at the riskless rate, so they must raise funding from clients to take asset positions at stage 1. I assume linear contracts: to raise an amount X, an informed trader receives 1 unit of the consumption good from a continuum (mass X) of clients, who can choose whether they want to settle their investment at stage 2 through

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[18]Short positions also require capital, since borrowing shares require withholding the proceeds of the sale plus the margin requirement in deposit accounts. This provides a safeguard for the lender against counterparty risk.

early withdrawals, or maintain it until stage 3.[19] I assume clients are in relatively large mass so that informed traders can always raise funding from a continuum of them. The informed trader retains a fraction $1 - c$ of trading profits.

Clients have risk-neutral preferences and can only participate in the asset market through intermediation by the informed traders. At stage 2, clients receive an endowment which is an increasing function of lagged economic conditions $g(\theta_{-1})$ plus a client-specific *liquidity shock* $\ell_j \sim \mathcal{N}\left(L, \lambda_\ell^{-1}\right)$. Liquidity shocks are unknown to all agents in stage 1. In the spirit of Diamond and Dybvig (1983) and the ensuing the bank-runs literature, clients with low values of the liquidity shock will withdraw funding from informed traders. In particular, client j withdraws at stage 2 whenever $g(\theta_{-1}) + \ell_j < 0$. I assume (with no loss of generality) a simple function $g(\theta_{-1}) = \theta_{-1}$. The fraction of positions that informed traders will keep until stage 3, which I label funding liquidity F, is then given by

$$F(\theta_{-1}) = 1 - \Pr(\theta_{-1} + \ell_j < 0) = \Phi\left(\sqrt{\lambda_\ell} \cdot (L + \theta_{-1})\right) \tag{8}$$

while the complement $1 - F(\cdot)$ must be liquidated at stage 2 to meet withdrawals. For tractability, I assume withdrawal decisions are independent of the price at stage 2, and that informed traders do not voluntarily change net positions at this stage. Without these assumptions, normality of conditional wealth breaks down, and the CARA framework is no longer tractable. [20]

3.1.2 Asset market

The timing of trades in the financial market are as follows (figure 2). At stage 1, informed traders submit price-contingent demand schedules. Although individual clients' liquidity shocks are unknown at stage 1, with the law of large numbers informed traders can perfectly forecast the fraction of early withdrawals $(1 - F(\cdot))$ from the knowledge of θ_{-1}. Demand schedules therefore depend on private signals s_i and anticipated funding liquidity: $t_i = t(s_i, P_1; F(\cdot))$.
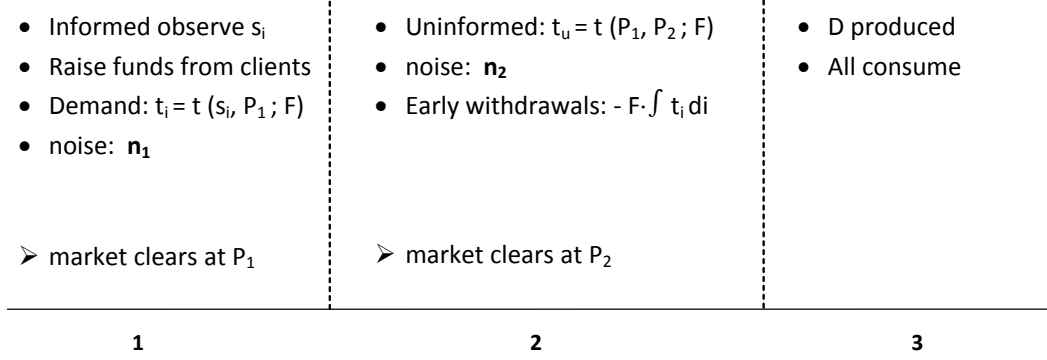
I assume the average supply of shares in stage 1 is unity, so the economy bears aggregate risk. In addition, noise traders supply the random amount of n_1 shares. A market auctioneer then selects price P_1 at which all price-contingent demands can be executed.

At stage 2 informed traders are forced to unwind a fraction $1 - F(\cdot)$ of positions to meet withdrawals. Uninformed traders also bid for the shares at stage 2 submitting demand schedules

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[19]Holmstrom and Milgrom (1987) show that linear contracts are optimal in principal-agent problem with CARA preferences. Kyle, Ou-Yang and Wei (2010) solve the optimal linear contract in a CARA setup with endogenous effort choice –the precision of information– by intermediaries.

[20]Voluntary liquidations at stage 2 would skew stage 2 prices conditional on information at stage 1, breaking the normality of traders wealth. These assumptions do not compromise the main qualitative results however, since what matters is that funding constraints place *some* limits in the positions that traders can take in the future, affecting their incentives to trade in the present.

Figure 2: Asset Market Summary

• Informed observe s_i	• Uninformed: $t_u = t\,(P_1, P_2\,;\,F)$	• D produced
• Raise funds from clients	• noise: $\mathbf{n_2}$	• All consume
• Demand: $t_i = t\,(s_i, P_1\,;\,F)$	• Early withdrawals: - F·∫ $t_i\,di$	
• noise: $\mathbf{n_1}$		
➤ market clears at P_1	➤ market clears at P_2	
1	**2**	**3**

to the market auctioneer. For simplicity, I assume they are unconstrained to finance trading positions at stage 2 (i.e., they can borrow at the risk-free rate). Uninformed traders condition both on the price at stage 2; P_2, as well as on P_1 and funding liquidity: $t_u = t\,(P_1, P_2; F)$. The second draw of noise traders supply n_2 shares and the auctioneer selects the price P_2 at which uninformed demands can be executed given n_2 and informed traders' early liquidations. At stage 3, $\theta + \mu$ is revealed and agents consume the dividend according to their net positions.

3.2 Equilibrium

A competitive sequential equilibrium is defined by 1) a sequence of share price functions $P_1\,(\theta, n_1; F(\theta_{-1}))$, $P_2\,(\theta, n_1, n_2; F(\theta_{-1}))$; 2) demands by informed, $t_i = t\,(s_i, P_1; F(\theta_{-1}))$, and uninformed traders, $t_u = t\,(P_1, P_2; F(\theta_{-1}))$; and 3) a set of prior beliefs $H\,(D \mid D_{-1})$ for all agents, posterior beliefs $H\,(D \mid D_{-1}; s_i, P_1, F(\theta_{-1}))$ and $H\,(P_2 \mid D_{-1}; s_i, P_1, F(\theta_{-1}))$ for informed traders, and $H\,(D \mid D_{-1}; P_1, P_2, F(\theta_{-1}))$ for uninformed traders such that, $\forall\ i \in [0, 1]$ and u: (i) Asset demands are optimal given funding liquidity $F(\theta_{-1})$, and posterior beliefs $H\,(D \mid \cdot)$ and $H\,(P_2 \mid \cdot)$, (ii) The asset price clears the market at each stage, and (iii) Posterior beliefs are updated using Bayes law.

The main object of the equilibrium are the price functions $\{P_1\,(\cdot)\,, P_2\,(\cdot)\}$, which depend on the realization of the shocks up to each stage and funding liquidity $F(\theta_{-1})$ –which is known at stage 1. Under condition (i), informed traders maximize expected utility given funding liquidity and posterior beliefs about the dividend and price P_2, since a fraction $1 - F(\cdot)$ of the positions taken in stage 1 will pay according to the latter. Condition (ii) imposes market clearing at each trading round for all realization of the noisy asset supplies. Condition (iii) imposes Bayesian updating of the conditional distributions of D and P_2 on all available information, including prices. I now briefly sketch the main steps to solve the equilibrium, with details relegated to Appendix B.

Step 1: Price conjectures and beliefs

Conjecture 1 *(Affine prices)*: *(i) There exists an equilibrium where the price in stage 1, P_1, is a linear combination of the dividend's prior expectation (D_{-1}), the dividend innovation (θ), and the noise trading shock (n_1);*

$$P_1 = A_o + A_1 \cdot D_{-1} + A_2 \cdot \theta + A_3 \cdot n_1, \tag{9}$$

(ii) In this equilibrium, the price in stage 2 is a linear combination of the dividend's prior expectation (D_{-1}), the shocks $\{\theta, n_1, n_2\}$, the share price P_1, and its informational content $\tilde{p}_1$:

$$P_2 = B_0 + B_1 \cdot D_{-1} + B_2 \cdot \theta + B_3 \cdot n_1 + B_4 \cdot n_2 + B_5 \cdot \tilde{p}_1 + B_6 \cdot P_1 \tag{10}$$

According to part (i) of the conjecture, $P_1(\cdot)$ is informationally equivalent to a noisy public signal about θ; $\tilde{p}_1 \equiv \theta - \Delta \cdot n_1$, where $\Delta = -A_3/A_2$. The noise in the signal depends on the variance of noise trading and the noise amplifier Δ. Since informed traders must liquidate some positions at stage 2, they also form beliefs about P_2. Part (ii) of the conjecture states that in the equilibrium the price function $P_2(\cdot)$ is also an affine combination of the underlying shocks up to stage 2, in addition to the price in stage 1, P_1, and its informational content $\tilde{p}_1$.

Informed traders' posterior beliefs about the dividend D and the price P_2 depend on private information and the endogenous public signal provided by the price in stage 1, $\tilde{p}_1$. The projection theorem gives the first two moments of informed traders' posterior beliefs about D. Beliefs about price P_2 can be similarly computed from (10) using expression (4).

Uninformed traders also use share prices to form beliefs. Although the equilibrium also implies an informational role for P_2, I assume for simplicity that uninformed traders only make inferences from P_1 through the endogenous signal $\tilde{p}_1$ –i.e., uninformed traders process information in prices with a lag. I relax this assumption in Appendix B, and argue that while adding significant complexity to the solution, an additional public signal in P_2 does not change the qualitative results of the model. Given information $\Omega_u : \{D_{-1}; P_1\} = \{D_{-1}; \tilde{p}_1\}$, the first two moments of uninformed traders' beliefs correspond to

$$\mathbb{E}[D \mid \Omega_u] = D_{-1} + \mathbb{E}[\theta \mid \Omega_u] = D_{-1} + b_1 \cdot \tilde{p}_1 \tag{11}$$
$$\mathbb{V}[D \mid \Omega_u] = \mathbb{V}[\theta \mid \Omega_u] + \mathbb{V}[\mu \mid \Omega_u] = \left[\lambda_\theta + \lambda_n/\Delta^2\right]^{-1} + \lambda_\mu^{-1};$$

Step 2: Optimal demands Since informed traders' face withdrawals at stage 2, terminal wealth is given by
$$W_{i,3} = (1-c) \, t_i \left[F \cdot (D - P_1) + (1-F) \cdot (P_2 - P_1)\right]$$

From (10), wealth follows a normal distribution conditional on information at stage 1. Maximizing CARA expected utility then leads to linear demands:

$$t_i = \frac{F \cdot \mathbb{E}\left[D \mid \Omega_i\right] + (1 - F) \cdot \mathbb{E}\left[P_2 \mid \Omega_i\right] - P_1}{\Sigma} \tag{12}$$

where $\Sigma = (1 - c)\gamma\left[F^2 \mathbb{V}\left[D \mid \Omega_i\right] + (1 - F)^2 \mathbb{V}\left[P_2 \mid \Omega_i\right] + 2F(1 - F) Cov\left[D, P_2 \mid \Omega_i\right]\right]$ is the risk aversion-adjusted variance. Asset demands of informed traders depend on expectations about the dividend and price P_2. Importantly, response to private signals is tempered by P_2 volatility.[21]

Uninformed traders participate at stage 2 and choose demands to maximize the expected utility over the terminal wealth $W_u = t_u \cdot (D - P_2)$. Normality of wealth conditional on information and CARA preferences also yields linear demands:

$$t_u = \frac{\mathbb{E}\left[D \mid \Omega_u\right] - P_2}{\Sigma_u} \tag{13}$$

with risk-aversion adjusted variance $\Sigma_u = \gamma_u \cdot \mathbb{V}\left[D \mid \Omega_u\right]$. Uninformed traders' demands are proportional to the expected profit per share ($\mathbb{E}\left[D \mid \Omega_u\right] - P_2$), tempered by the risk-aversion adjusted variance; Σ_u.

Step 3: Market clearing I now solve the coefficients in the price conjectures. Working backwards, I impose the market-clearing condition at stage 2:

$$t_u - (1 - F) \cdot \int t_i di = n_2 \tag{14}$$

where the left hand side is given by uninformed demands plus the unwinding of informed traders' positions from stage 1, which must equal the second draw of noise trading n_2. Solving for P_2 gives the coefficients in part (ii) of Conjecture 1.

Now I impose the market clearing condition at stage 1:

$$\int t_i di = 1 + n_1 \tag{15}$$

where the left hand side contains aggregate demands by informed traders, which must equal the unit supply of shares plus noise trading. Solving for P_1 yields the coefficients in part (i) of Conjecture 1, as a function of the coefficients in part (ii) of the conjecture. I summarize the main properties of the equilibrium in the following proposition, referring proofs to Appendix B.

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[21]The profit splitting rule acts as a risk-aversion moderator: the lower is the fraction of profits that corresponds to informed traders $(1 - c)$, the more aggressive they trade (see Kyle, Ou-Yang and Wei (2010)

Proposition 2 *(Existence and uniqueness):* (*i*) *All linear price equilibria satisfy the system of equations:*[22]

$$\boldsymbol{\Delta} = \frac{\Sigma}{a_0 \left(F + (1-F) B_2\right)}, \tag{16a}$$

$$\Sigma = (1-c)\gamma \left\{ \mathbb{V}\left[\theta \mid s, \tilde{p}_1\right] \left(F + (1-F) B_2\right)^2 + (1-F)^2 \Sigma_u^2 \lambda_n^{-1} + F^2 \lambda_\mu^{-1} \right\}, \tag{16b}$$

$$\Sigma_u = \gamma_u \left\{ \mathbb{V}\left[\theta \mid \tilde{p}_1\right] + \lambda_\mu^{-1} \right\}, \tag{16c}$$

$$B_2 = \frac{-\Sigma_u (1-F) F \cdot a_0}{\Sigma + \Sigma_u (1-F)^2 \cdot a_0}, \tag{16d}$$

where $F(\theta_{-1})$ *is given by equation (8),* $\mathbb{V}\left[\theta \mid s, \tilde{p}_1\right]$ *and* $\mathbb{V}\left[\theta \mid \tilde{p}_1\right]$ *are functions of* $\boldsymbol{\Delta}$ *given by (4) and (11), and* $a_0 = \lambda_\epsilon \cdot \mathbb{V}\left[\theta \mid s, \tilde{p}_1\right]$. (*ii*) *A linear equilibrium always exists; and* (*iii*) *There exists* $\delta_0 > 0$, *s.t. whenever the sufficient condition* $\lambda_\mu < \delta_0$ *holds, the equilibrium is unique. If* $\lambda_\mu > \delta_0$, *additional equilibria might exist.*

Proposition 2 validates expressions (9) and (10) in Conjecture 1 as equilibrium outcomes: beliefs based on the proposed price functions lead to asset demands that sustain such beliefs in equilibrium. Equilibrium multiplicity is an interesting property of the two-stage trading environment that arises from higher-order beliefs of informed traders. In order to predict price P_2, they form expectations about uninformed traders' demands, and thus about uninformed trader's beliefs about the dividend.

In one of these equilibria, informed traders respond aggressively to private signals in stage 1, and the price P_1 is nearly revealing. Facing low risk, uninformed traders absorb the supply of the asset with little effect on price P_2. Low P_2 volatility, in turn, sustains the optimality of informed traders' aggressive response to information in the first stage.

In the other equilbria, informed traders react mildly to private information in stage 1 and P_1 reveals little information about the forthcoming dividend. This reduces uninformed traders' willingness to absorb supply, increasing P_2 volatility. High payoff uncertainty of the positions liquidated in stage 2 thus sustains the mild response of informed traders to information at stage 1. This leads to an equilibrium with higher noise in prices –the *noisy* equilibrium.

The inclusion of μ in the dividend equation adds uncertainty which cannot be mitigated through information aggregation in prices. No matter how precise the price signal is, uninformed traders conditional variance of D lies strictly above the variance of μ; λ_μ^{-1}. This bounds the volatility of P_2 from below so that aggressive trading of informed traders is not sustained as an equilibrium. Consequently, the near-revealing equilibrium vanishes for high enough λ_μ. The noisy equilibrium, on the other hand, exists for all parameter values. In what follows, I restrict attention to the noisy

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[22]I ignore whether there exist other non-linear equilibria.

equilibrium.[23]

I now state the relation between funding liquidity and information aggregation in the asset price (proven in Appendix B):

Proposition 3 *(**Liquidity and noise**): There exists $\delta_1 > 0$, s.t. if the sufficient condition $\lambda_\mu < \delta_1$ holds, there exists a threshold $\bar{\theta}$ s. t. the noise amplifier $\mathbf{\Delta}(\cdot)$ is strictly decreasing in θ_{-1} for all values of lagged economic conditions $\theta_{-1} < \bar{\theta}$, where $\partial\bar{\theta}/\partial\gamma_u > 0$ and $\partial\bar{\theta}/\partial\lambda_n^{-1} > 0$.*

Proposition 3 is the main result of the model. I discuss in detail the forces at work in generating this result in the next subsection.

3.3 Illiquidity and the amplification of uncertainty

Figure 3 plots the impact of lagged economic conditions θ_{-1} on funding liquidity $F(\cdot)$ and the noise amplifier $\mathbf{\Delta}(\cdot)$ (Table 1 specifies the benchmark parameters used in all the figures below). As economic conditions deteriorate and future withdrawals become more pervasive, informed traders become more cautious when trading at stage 1 in response to private information. In equilibrium, this reduces the aggregation of private information into the signal provided by the asset price.

To understand the mechanisms involved in the amplification of uncertainty, consider an informed trader who expects a high dividend at stage 3 and must choose her initial trade in stage 1. If she expects a relatively large fraction of withdrawals $1 - F$ in stage 2, she will respond moderately to private information for two separate reasons. First, liquidation of positions at stage 2 is risky in the presence of noise trading, so the risk-aversion adjusted variance of returns (Σ) increases as funding F drops. I call this effect *noise trading risk*. But a second effect regarding the conditional *mean* of returns is also at work: the higher the fraction of withdrawals, the larger the amount of trades that the trader expects to reverse at stage 2 together with all other informed traders. This reversal of positions causes an opposite price pressure in P_2 which reduces expected profits. I call this second effect the *expected trade reversal*.

Equation (16a) can be decomposed to illustrate the impact of each of these effects in the noise amplifier; $\mathbf{\Delta}$. The noise trading risk effect is captured by the term $(1 - F)^2 \Sigma_u^2 \lambda_n^{-1}$ in the expression of Σ (16b). As F drops, informed traders become more exposed to the risk of price fluctuations in stage 2. Moreover, note that the risk aversion-adjusted variance of uninformed traders Σ_u also increases as F falls and the price becomes more noisy. Uninformed traders will only supply liquidity when uncertainty is higher if expected returns increase as well. Noise trades therefore

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[23]It is worthwhile noting that the impact of funding liquidity F on the noise amplifier $\mathbf{\Delta}$ is qualitatively similar in both equilibria.

Table 1

Baseline parameters	
Unconditional variance of θ	Average dividend
$\lambda_\theta^{-1} = 2$	$\check{D} = 5$
Unconditional variance of μ	Liquidity shock
$\lambda_\mu^{-1} = 0.4$	$L = 6; \ \lambda_l^{-1} = 50$
Variance of private signals	Risk-aversion
$\lambda_\epsilon^{-1} = 2$	$\gamma = 1; \gamma u = 2$
Noise trading shock variance	Profit sharing rule
$\lambda_n^{-1} = 2.5$	$c = 0.9$

Figure 3: Liquidity and Noise
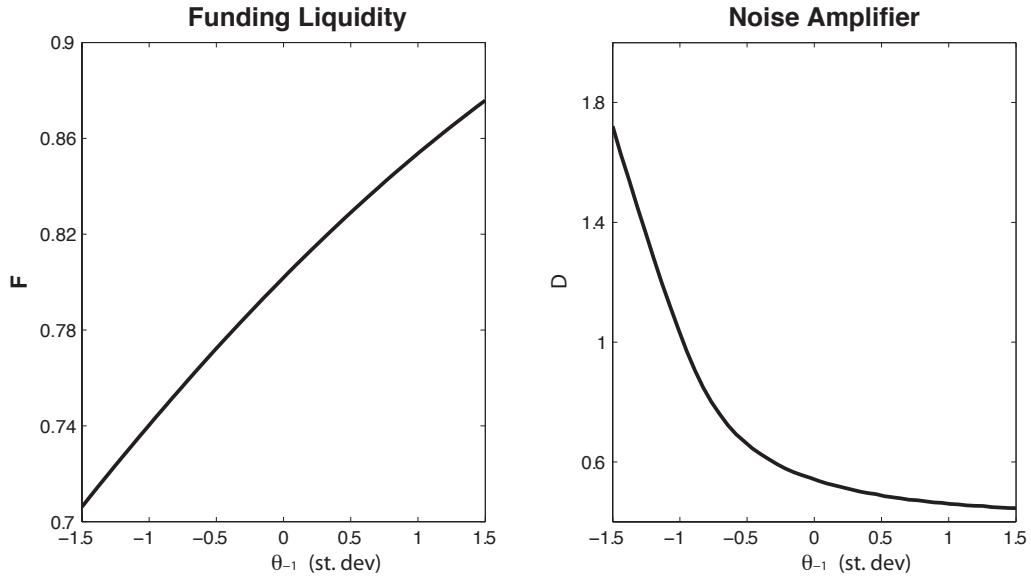
Left panel - Aggregate funding liquidity (F): fraction of positions held by informed traders (intermediaries) until stage 3.
Right panel - Noise amplifier (Δ): ratio of P1 reaction to noise and fundamentals (-A3/A2 in Conjecture 1).

have a larger price impact at stage 2, increasing the risk for intermediaries.[24] As an attenuation of the effect however, note that more noise in the price increases the reliance of informed traders on their private signals, captured by the coefficient a_0. Under the conditions in Proposition 3, the increase in Σ will dominate the increase in a_0. Since the ratio Σ/a_0 is proportional to the noise amplifier, prices become more noisy as liquidity tightens through the noise trading risk effect (left panel of Figure 4).
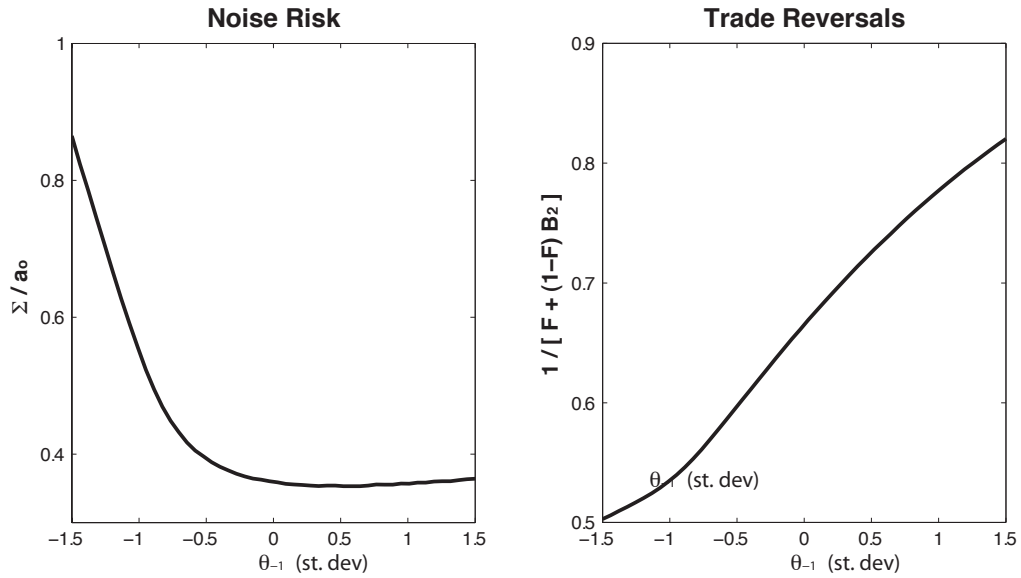
The expected trade reversal effect is captured by the term $F + (1 - F) B_2$ in (16a) (Right panel of Figure 4), which corresponds to the increase in expected profits per share for a given increase in the expected fundamental. Informed traders anticipate that only a fraction F of positions pay the dividend D, while $(1 - F)$ pay P_2. D also affects P_2 through B_2 (equation (10)). But from expression (16d), note that B_2 is actually negative: $-1 < B_2 < 0$. This reflects the price discount required by uninformed traders to absorb the positions unwind by informed traders at stage 2. Moreover, since uninformed traders' willingness to provide liquidity depends on price informativeness, B_2 also becomes more negative as F falls. The term $F + (1 - F) B_2$ is therefore lower than one and decreases as F drops, further increasing the noise amplifier $\boldsymbol{\Delta}$.

Figure 5 shows the path of prices after a one-standard deviation positive innovation in θ. The left panel corresponds to a case where the lagged fundamental θ_{-1} is relatively low (-1 st. dev.), while the right panel considers a high value (+1 st. dev.). The solid line in both panels is the simulated trajectory of prices when both realizations of the noise trading shock are zero. The solid line thus illustrates the expected trade reversal: if informed traders bought shares in stage 1, they must sell a fraction of them in stage 2, which lowers P_2. The dashed lines plot the price impact of a positive innovation in noise trading n_2. This stochastic fluctuation in the price corresponds to the noise trading risk. The figure makes clear that both effects become more pervasive when funding liquidity is low. Moreover, since both effects multiply each other in the denominator of $\boldsymbol{\Delta}(\theta_{-1})$ in expression (16a), the spike in the noise amplifier can be quite marked, as is apparent from Figure 3.

Both effects can be related to a "fire sales" argument. Early withdrawals force informed traders to unwind initial positions at stage 2 prices. But this happens precisely when other informed traders (on average) are also reversing their trades, which causes a reversal in the price in expected terms. Moreover, the presence of noise makes the fire sale risky as well. The key element added by the REE framework is that informed traders consider the future effects of the fire sale when choosing their initial trades. In response to larger future withdrawals, informed traders' demands become less sensitive to private information, which reduces the informativeness of the price P_1.

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[24]Note that it is not immediate that lower F increases total risk since intermediaries become less exposed to risk in fundamentals. As the condition in Proposition 3 states, noise trading risk will always dominate for sufficiently low $\overline{\theta}$, such that $F(\theta_{-1}) < F(\overline{\theta})$. The threshold value $\overline{\theta}$ depends positively on the variance of noise trading and the risk aversion of uninformed traders.

Figure 4: Amplification Effects–Noise Trading Risk and Expected Trade Reversals

Noise Risk

Trade Reversals

Left panel -- Noise trading risk effect: the ratio between the risk aversion-adjusted variance of informed traders (Σ) and the Bayesian weight of private information in the expectation of θ (a_0). Right panel -- Expected trade reversal effect: the anticipated discount in P_2 relative to the expected dividend ($F + (1-F) B_2 < 1$).

Figure 5: Illiquidity and Dynamic Risk

Prices
($\theta_{-1} = -1$ st. dev.)
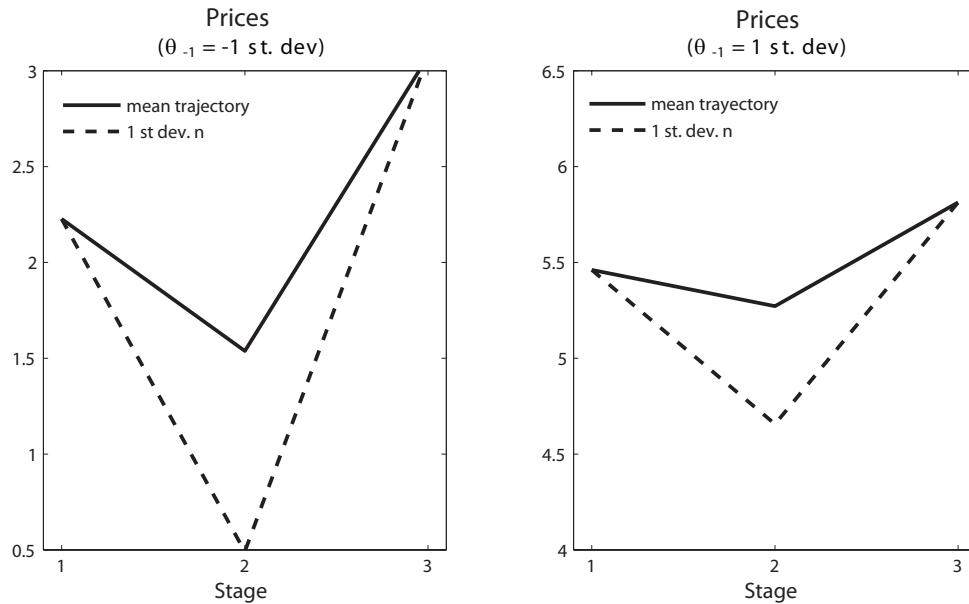
Prices
($\theta_{-1} = 1$ st. dev.)

Left panel – Price trajectory for low lagged fundamentals ($\theta_{-1} = -1$ st. dev.). Solid line plots the mean path of prices ($n_1 = n_2 = 0$). The dashed line plots the reaction in P_2 from a 1 st. dev. innovation in n_2. Right panel – Price trajectory for high lagged fundamentals ($\theta_{-1} = 1$ st. dev.). Solid line plots the mean path of prices ($n_1 = n_2 = 0$). The dashed line plots the reaction in P_2 from a 1 st. dev. innovation in n_2. Both panels assume a positive 1 st. dev. innovation in θ.

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Figure 6: Countercyclical Uncertainty

Informed Traders   **Uninformed Traders**

Left panel - Variance of fundamentals: unconditional (λ_d^i) and conditional on informed traders' information (V[D | s, P$_1$]).
Right panel - Variance of fundamentals: unconditional (λ_d^i) and conditional on uninformed traders' information (V[D | P$_1$]).
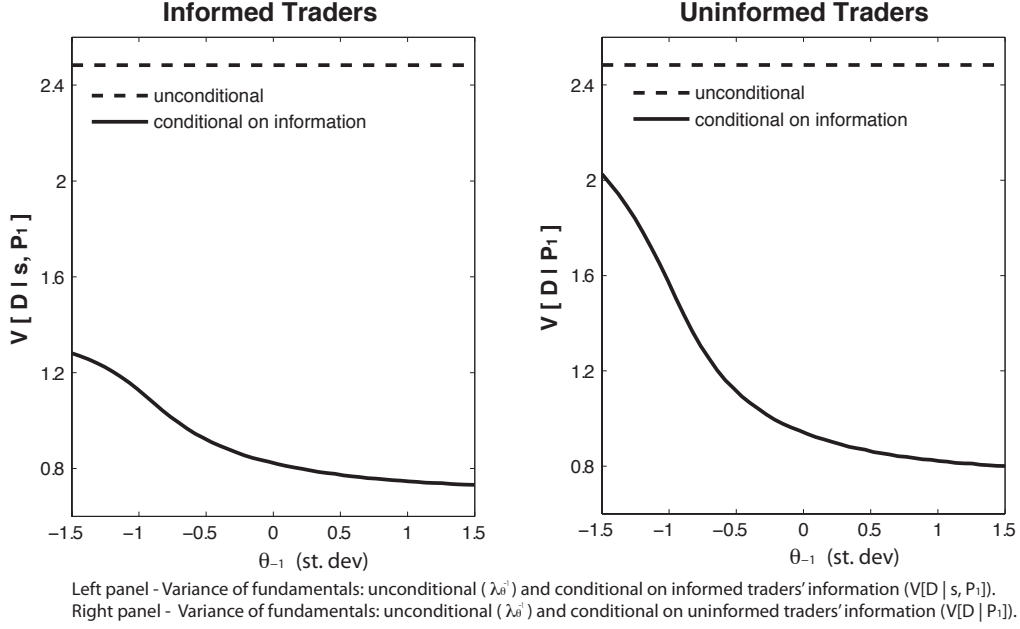
The anticipation of the aggregate effect on prices of trading decisions in stage 1 does not mean that informed traders fully internalize the consequences of their actions, however. On the contrary, the competitive nature of the equilibrium makes each individual informed trader ignore the impact of their trading choices on the informativeness of the price. This imposes an information externality to other informed traders as well as to uninformed traders, since all traders learn from prices in a dispersed information environment. Interestingly, the social inefficiency of private decisions hits back on informed traders, as heightened uncertainty of uninformed traders increase the returns they require to absorb supply of the asset at stage 2.

Figure 6 plots the effect of lagged economic conditions on the dividend's conditional variance, for both informed and uninformed traders. Better economic conditions improve funding liquidity, which increases the informativeness of the asset price. The residual variance of fundamentals conditional on information is then decreasing on θ_{-1} for both informed and uninformed traders, since all agents learn from the asset price in an environment with dispersed private information. Of course, the effect is particularly pronounced for uninformed traders who lack private information.

4 Asset pricing implications

Two salient features of asset prices during economic slumps are the increase in expected excess returns of risky assets–the risk premium– and spikes in the volatility of stock markets. I argue in this section that countercyclical price informativeness is consistent with both observations. Moreover, I stress the amplification effects of modeling endogenous informativeness of prices in a context where both (i) information is heterogeneous across informed agents, and (ii) liquidity provision by uninformed traders depends endogenously on price informativeness.

4.1 Risk premium

Excess returns of risky assets are predictable at low frequencies. Fama and French (1989) document that variables related to business cycle conditions,such as default and term spreads, track excess returns of corporate bonds and stocks in a similar way as the dividend yield. This suggest that return predictability reflects cyclical variation in the price of risk. This interpretation is supported by the literature on volatility tests that cannot otherwise explain the "excess volatility" of price/dividend ratios.[25]

Countercyclical price informativeness suggests one possible explanation for time variation in the risk premium. I view this mechanism as complementary to the existing theories that focus on cyclical variation in the attitude towards risk (Campbell and Cochrane (1999)), or an exogenous conditional heteroskedastic dividend process (Barsky and Delong (1993); Bansal and Yaron (2004)). The informational channel stressed in section 3 suggests that financial markets are poor aggregators of information about the value of future dividends when traders are funding constrained. As the resulting risk goes up, the corresponding reward for bearing should move in the same direction. Importantly, this is true in the model even when the risk aversion of agents as well as the unconditional variance of dividends are held constant.

I compute the risk premium as the expected return on holding a position on the risky asset between stages 1 and 3, conditioning on prior information D_{-1}. Conditional on the realization of the shocks $\{\theta, n_1\}$, the holding period return is given by [26]

$$\text{Ret}_{1,3} \equiv \frac{D - P_1}{\bar{D}} = \frac{\theta - A_2 \cdot \tilde{p}_1 - A_0}{\bar{D}}$$

---

[25]See Campbell and Shiller (1988), Campbell (1991), and Cochrane (1992).

[26]Since prices can be negative with positive probability, I compute expected returns using the unconditional mean $\bar{D}$ on the denominator. This understates the cyclical variation in risk premium that would result from using P_1 instead.

The risk premium is just the average of $\text{Ret}_{1,3}$ over the joint normal distribution of $\{\theta, n_1\}$:

$$
\begin{aligned}
RP &\equiv \mathbb{E}\left[\text{Ret}_{1,3} \mid D_{-1}\right] = -\frac{A_0}{\bar{\bar{D}}} \\
&= \left[\Sigma + \Sigma_u \left(1 - F\right)^2\right] / \bar{D}
\end{aligned}
\tag{17}
$$

where A_0 is the intercept of the price function in Conjecture 1.

Note that the risk aversion-adjusted variance of uninformed traders; Σ_u, also appears in expression (17) although I defined the risk premium as the discount required by informed traders at stage 1. The intuition is that informed traders will require a high premium when they expect price P_2 to be more volatile. The volatility of P_2 will be higher when noise trading in stage 2 has a large price impact, which is the case when uninformed traders risk aversion-adjusted variance Σ_u is large. The next proposition states the conditions under which the risk premium is strictly decreasing in lagged economic conditions θ_{-1}. I provide all proofs of this section in Appendix C.

Proposition 4 *(Risk-premium):* *Under the parameter restrictions of propositions 2 and 3, the risk premium is strictly decreasing in lagged economic conditions for all $\theta_{-1} < \bar{\theta}$.*

The left panel of Figure 7 plots the negative relation between the risk premia and θ_{-1}. As θ_{-1} falls and funding tightens, the rise in the noise amplifier $\mathbf{\Delta}$ increases the conditional variance of fundamentals. Correspondingly, traders will require a higher return for holding the asset. Note that tighter funding not only increases the risk premium by directly rasing the share of asset liquidated early on but it also increases the conditional variance of the remaining positions that pay according to D. This is a direct implication of a dispersed information environment where all traders learn from prices.

The right panel of Figure 6 plots the Sharpe ratio: the quotient between the risk-premia and the conditional standard deviation of the dividend return,

$$
SR \equiv \frac{RP}{\sqrt{\mathbb{V}\left[D - P_1 \mid \Omega_i\right] / \bar{D}^2}} = \frac{-A_0}{\sqrt{\mathbb{V}\left[D \mid \Omega_i\right]}}
\tag{18}
$$

which is decreasing in θ_{-1} as well, as made precise by the next proposition:

Proposition 5 *(Sharpe ratio):* *Under the parameter restrictions of propositions 2 and 3, the Sharpe ratio is strictly decreasing in lagged economic conditions for all $\theta_{-1} < \bar{\theta}'$.*

Propositions 4 and 5 follows directly from CARA preferences in the presence of aggregate risk (positive average asset supply). Market clearing implies that in expected terms, the profit per share is proportional to the risk aversion-adjusted variance of informed traders Σ. Dividing by

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Figure 7: The Price of Risk

Left panel- Risk premium: expected return of holding the asset between stage 1 and 3, conditional on prior information (D₋₁).
Right panel -Sharpe ratio: ratio between the mean and st. dev. of the return, conditional on prior information (D₋₁).
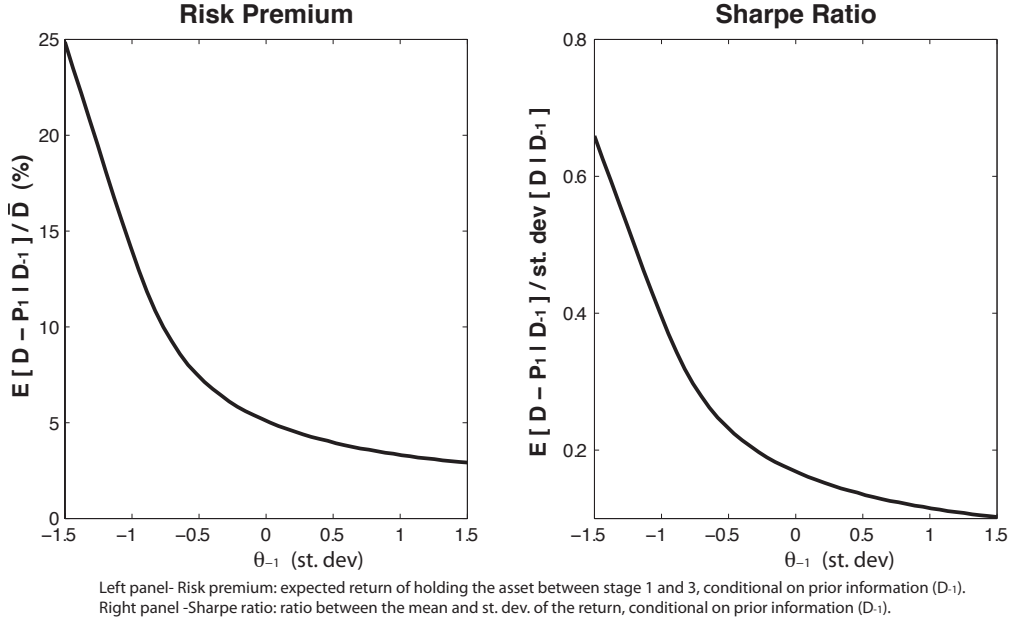
the standard deviation of D conditional on information at stage 1 gives a Sharpe ratio that is proportional to the conditional standard deviation of the dividend, and thus decreasing in θ_{-1}.

The benchmark parameters in Table 1 imply a mean expected excess return on equity of 5.2%. Of course, this figure depends on an arbitrary choice of parameters, mainly the value of the average dividend $\bar{D}$. What is less arbitrary from the figure is the considerable variation of the equity premium, which oscillates by a factor of 8 within 3 standard deviations around the mean of lagged economic conditions. Importantly, the Sharpe ratio follows a similar countercyclical behavior, so that the slope of the mean-variance frontier is higher in contractions as documented in the data.[27]

These results are only meant to be interpreted qualitatively, since a three-period CARA preferences framework is hardly suited for a quantitative asset pricing discussion. Within these limitations however, it is worth noting that the model has the potential to generate an interesting time-variation in the forecastability of returns and the price of risk.

4.2 Price volatility

I now discuss the impact of funding liquidity on the volatility of price P_1 conditional on prior information; $\mathbb{V}[P_1 \mid D_{-1}]$. I focus on P_1 because P_2 behaves much like P_1 except for scale effects.

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[27]See Harvey (1989), and Ferson and Harvey (1991).

Figure 8: Price Volatility

Left panel - Price volatility: variance of P_1 conditional on prior information (D_{-1}). Right panel - Volatility decomposition: fraction of P_1 variance explained by noise trading shocks (n) and fundamental innovations (θ).
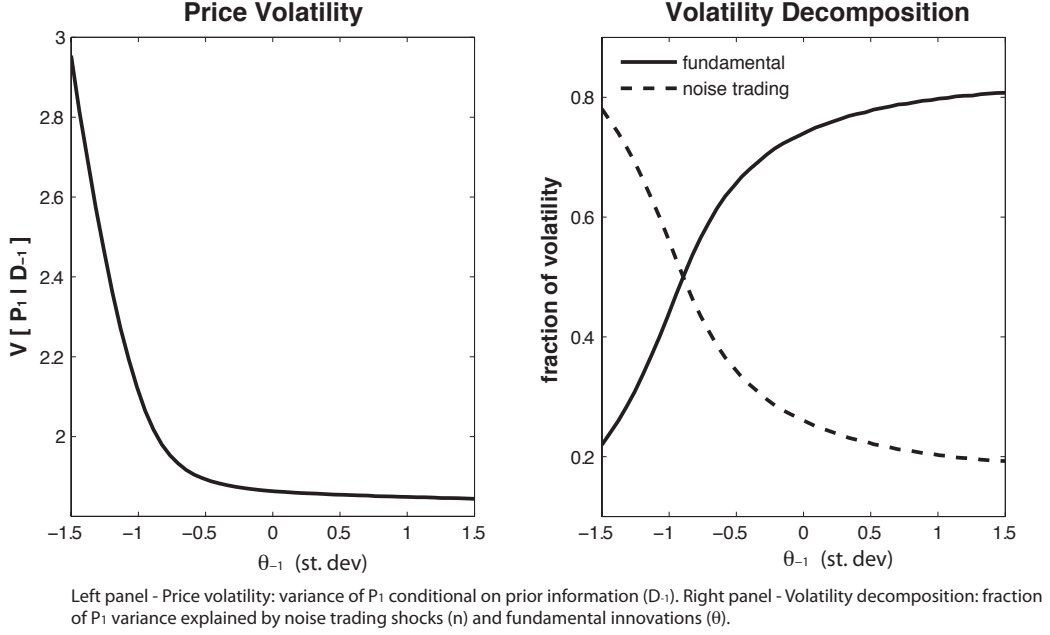
The left panel in Figure 8 shows the relation between funding liquidity, and price volatility. As funding liquidity tightens, volatility spikes considerably. The key driver of the increase in the conditional variance of both informed and uninformed traders. Since in equilibrium the supply of shares unloaded by noise traders must always be absorbed, higher risk premium translates into a larger price response to noise n_1.

The right panel of figure 8 decomposes price variability into its two sources. In the absence of noise, price volatility is driven exclusively by dividend innovations and is therefore given by the unconditional variance of θ; λ_θ^{-1}. The interaction between noise and risk-aversion prevents full revelation from the observation of the price, introducing non-fundamental volatility coming from noisy supply innovations. For relatively low values of θ_{-1}, this second source of volatility becomes dominant and price volatility drops as economic conditions improve. But as the price becomes increasingly more informative for higher θ_{-1}, traders also weight less the prior belief $\mathbb{E}[\theta] = 0$ and the price becomes more sensitive to innovations in fundamentals. The latter effect can dominate for large enough θ_{-1} depending on parameter choices.[28]

As a digression, statements about price volatility must be weighted by the limitations inherent in a three-period model, as in a fully dynamic context the mapping between endogenous uncertainty and volatility is more involved. Since prices must eventually reflect dividend innovations at *some*

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[28]This depends mainly on the variance of noise trading λ_n^{-1} and risk aversion (γ and γ_U). An increase in either parameter will expand to the region in which volatility is strictly decreasing in θ_{-1}.

frequency in the data, varying price informativeness is likely to affect the timing of price movements in anticipation of dividends, but not price volatility defined over low enough frequencies.[29] The analysis above is probably best suited to make predictions about relatively high frequency stock market fluctuations closely tied to the presence of noise. Interestingly, this suggests that spikes in volatility during recessions should coincide with increased forecastability at high frequencies due to return reversals, a testable prediction I expand further on below.

Wang (1993) studies an intertemporal equilibrium model of asymmetric information where price fluctuations are driven by dividend innovations and noise. He finds that increasing information asymmetry (or the uncertainty about fundamentals averaged across agents) can indeed generate spikes in price volatility when noise trading is important. [30] His conclusions are confirmed in the three-period analysis provided here as volatility is indeed decreasing in price informativeness.

4.3   Amplification effects

Two important elements contributing to the amplification of uncertainty in financial markets under funding constraints are the heterogeneity (dispersion) of private information, and the endogenous nature of noise trading risk in a dynamic setup. To illustrate the importance of each element in generating the results, I consider two modifications to the benchmark model of section 3. First, I fix the uncertainty of informed traders by assuming they receive common private signals (so they don't learn from the price). As a second modification, I fix the uncertainty of uninformed traders by assuming they behave like naive investors who don't learn from the price. This amounts to fixing the elasticity of their demand for shares to an exogenous constant independent of economic conditions.

4.3.1   A common information setup

I now consider an economy identical to that in section 3, except informed traders observe the same private signal s about the dividend innovation θ:

$$s_i = s_j = s = \theta + \epsilon, \quad \forall i \in [0,1]; \quad \text{with} \quad \epsilon \sim N\left(0, \lambda_\epsilon^{-1}\right)$$

I solve for the equilibrium of this economy in Appendix C, limiting the discussion here to the main contrasts with the dispersed information benchmark.

With common private signals, informed traders learn nothing from the asset price. As economic conditions worsen and funding tightens, informed traders are subject to higher risk from early

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[29]I thank Jeremy Stein pointing this out.

[30]Campbell and Kyle (1993) also provide an intertemporal model of asymmetric information in which uncertainty about asset returns and price volatility are countercyclical, but they consider an exogenous information structure.

Figure 9: Dispersed vs. Common Private Information

Left panel- Risk premium: endogenous uncertainty (solid line) vs. exogenous uncertainty of informed traders (dashed).
Right panel -Sharpe ratio: endogenous uncertainty (solid line) vs. exogenous uncertainty of informed traders (dashed).
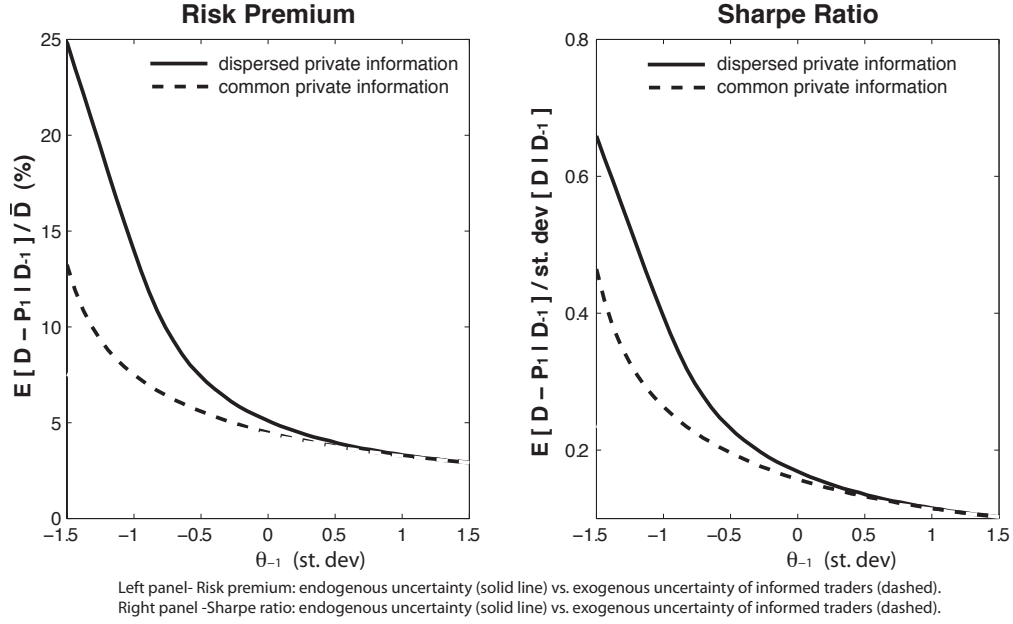
liquidations, but not from the portion of their portfolios that pay according to the dividend. Figure 9 compares the resulting risk premium and Sharpe ratio between the benchmark model of dispersed information and the common private signal case. It is apparent that the effect of funding liquidity in the price of risk is less pronounced when informed traders do not learn from prices. Since price volatility is intimately tied to required returns, the results also extends to the variance of P_1 (not shown).

Naturally, this contrast depends on the assumed parameters. If the dispersion of private signals (λ_ϵ^{-1}) is very low, or the variance of noise trading shocks (λ_n^{-1}) is very large, both cases will exhibit roughly similar asset pricing sensitivity to changes in economic conditions. Intuitively, when private information is very precise informed traders have little to learn from prices. When noise trading shocks are too volatile on the other hand, the market mechanism is unable to provide valuable information about fundamentals to begin with.

4.3.2 Exogenous liquidity provision

As explained in section 3, dynamic risk considerations for informed traders are especially important in the presence of uninformed traders which learn from the share price. The endogenous uncertainty of uninformed traders affects the terms at which they are willing to supply liquidity at stage 2, absorbing the net supply from intermediaries' liquidations and noise trading. When the price at stage 1 is noisy, required returns by uninformed traders increase making the noise trading risk and

Figure 10: Endogenous vs. Exogenous Uninformed Trader's Uncertainty

Left panel- Risk premium: endogenous uncertainty (solid line) vs. exogenous uncertainty of uninformed traders (dashed).
Right panel -Sharpe ratio: endogenous uncertainty (solid line) vs. exogenous uncertainty of uninformed traders (dashed).
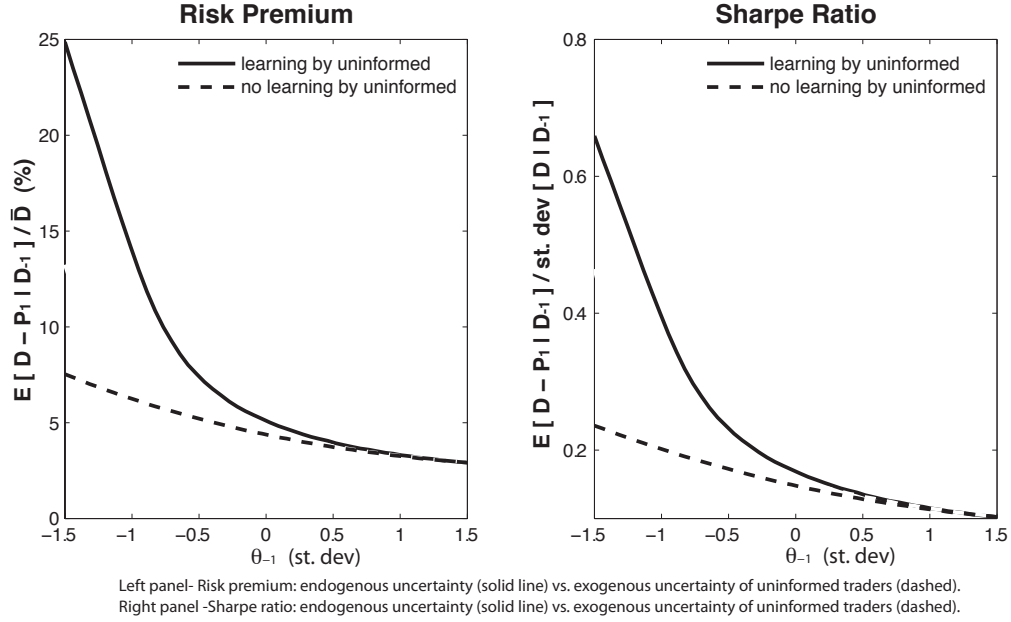
trade reversal effect more pronounced. This further reduces informed trading in stage 1 creating an amplification effect between a given decrease in funding liquidity F and the resulting noise in asset prices.

Figure 10 compares the risk premium and Sharpe ratio between the benchmark model and a fictitious economy where uninformed traders do not learn from the price. The figure fixes the uncertainty of uninformed traders at the level attained in the benchmark model at $+1.5$ standard deviations of lagged fundamentals θ_{-1}. As funding F drops, intermediaries are still more exposed to the price fluctuations and lower expected returns described by the noise trading risk and trade reversal effects. However, since the required return per share by uninformed traders is insensitive to F, the volatility of P_2 and the expected return on shares liquidated at stage 2 remain constant. The increase in risk for intermediaries is therefore more modest in this alternative setup.

5   Real investment implications

Countercyclical uncertainty not only matters for understanding the behavior of asset prices, but it is likely to play an important role in real allocations decisions through its impact on aggregate investment. Several authors have pointed out that higher risk premia directly reduces investment by raising firms' cost of capital.[31]   In this section I explore a different channel by considering

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[31]See Lettau and Ludvigson (2002), and more recently Hassan and Mertens (2009).

the investment problem of a firm that learns information about its productivity –more broadly interpreted as future demand conditions for its products, or the likely success of an investment project– from the price of its shares in the stock market. This learning channel is usually referred to as the *feedback*[32] literature: firms learn information about the likely success of projects by observing their share prices (or an industrial average, or even aggregate indexes) to the extent that prices aggregate information which can be of value to the firm, but is originally dispersed across agents in the economy. Although recent empirical work documents a link between asset prices and investment working through information flows (Chen, Goldstein, and Jiang (2007)), the relevance of this channel is far from being settled (Morck, Shleifer, and Vishny (1990)).

Another view which fits more naturally with the single asset framework in this paper is to interpret "fundamentals" as aggregate demand in the economy, and aggregate investment as the representative firm's investment decision. The asset market then plays a coordination role between households and firms, conveying information about consumption plans of the former into production decisions of the latter.

I capture the impact of uncertainty on investment decisions following the literature on investment irreversibilities.[33] This literature highlights how partial irreversibilities generate a "real option value" on investment decisions that create a wedge between the marginal product of capital which justifies investment and disinvestment. This wedge generates a region of inaction in which investment is fixed for some range of the underlying state variables. When uncertainty increases, the option value of delaying decisions is raised, expanding the inaction region. In simple terms, firms prefer to wait until the dust settles before undertaking investment decisions that will later be regretted.

Below I present a tractable model which exhibits the feature that a spike in uncertainty coming from a noisier asset price increases the expected losses from investment, which leads to a reduction in the scale of investment with respect to the level attained in a deterministic environment. This reduction is increasing in the variance of fundamentals, conditional on information.

5.1 Setup

Technology I consider a single period. A firm produces a consumption good using capital $k \geq 0$. Investment translates into units of the consumption good given the realization of a random

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[32]See Dow and Gorton (1997), Subrahmanayam and Titman (1999), Dow and Rahi (2003), and Goldstein and Guembel (2008).

[33]See Bernanke (1993), Dixit and Pindyck (1994), Bertola and Caballero (1994) and the recent evidence in Bloom, Bond, and Van Reenen (2007) and Bloom (2009).

productivity parameter θ through the profit function $\Pi(\theta, k)$:

$$\Pi(\theta; k) = \theta \cdot k - \frac{1}{2}k^2 - L(\theta; k), \quad \text{with} \tag{19}$$
$$L(\theta; k) = \left[\frac{a}{2}(\theta - k)^2\right]k$$

where θ is normally distributed with mean of zero and variance λ_θ^{-1}.

Profits are adapted from a standard quadratic costs of investment function modified to include the loss function $L(\theta; k)$ which is quadratic in the difference $(\theta - k)$. This term captures the cost of investment being "out of line" with respect to the ex-post optimal investment which, as I will show in a moment, is given by θ. The term is multiplied by k, making losses proportional to investment –a given deviation from the ex-post optimum should be more costly for a larger investment scale.

The asset market and investment Modeling feedback effects between asset prices and investment is a complex task. A rigorous approach needs to deal with the fact that prices simultaneously reflect dispersed information about fundamentals and the reaction of investment to prices, since investment affects expected dividends. Albagli, Hellwig and Tsyvinski (2010) explicitly consider such interaction in an environment that allows for differential information observed by the firm and traders, but where the firm still learns valuable information from its share price. In this paper I follow the simpler route of assuming a separation between the profits of the firm in (19), and the dividend paid out to shareholders (see for instance Subrahmanayam and Titman (1999)). In particular, shares pay the terminal dividend θ so that the asset price will aggregate information about θ and affect the investment decision, but the latter will not be incorporated in traders beliefs about dividend value.

I consider the simple trading environment of section 2 where only a fraction F of traders participate in the asset market. I focus on characterizing the firm's investment decision given the information about θ contained in the share price of expression (2), with the noise amplifier given by expression (7).

5.2   The investment problem

The firm maximizes expected profits in (19), given its posterior beliefs $H(\theta \mid \Omega_f)$. To keep the analysis simple, I assume the firm only learns information about θ from its share price, so that $\Omega_f : \{\tilde{p}_1\}$, where $\tilde{p}_1$ is given by equation (3). It is straightforward to include private information in the firm's beliefs, but this will not change the qualitative results discussed here. The firm's

problem then reduces to

$$\max_{k} \ \mathbb{E}\left[\Pi \mid \Omega_f\right] = k \cdot \mathbb{E}\left[\theta \mid \tilde{p}_1\right] - \frac{1}{2}k^2 - \frac{a}{2}k \cdot E\left[(\theta - k)^2 \mid \tilde{p}_1\right]$$

The f.o.c. of this problem is derived in Appendix D, and leads to an optimal investment of

$$k^* = \frac{2}{3}\mathbb{E}\left[\theta \mid \tilde{p}_1\right] - \frac{1}{3a} + \sqrt{\left(\frac{1}{3a} + \frac{\mathbb{E}\left[\theta \mid \tilde{p}_1\right]}{3}\right)^2 - \frac{1}{3}\mathbb{V}\left[\theta \mid \tilde{p}_1\right]} \qquad (20)$$

The original problem is a polynomial of order three in k, with a negative coefficient next to the cubic term. The global solution of this problem is therefore always at $k = -\infty$, which is ruled out by the requirement $k \geq 0$. The positive root in the f.o.c., when real, is the local maximum of the problem. But note that for some values of $E\left[\theta \mid \tilde{p}_1\right]$, the squared term inside the square root can fall below $1/3 \cdot V\left[\theta \mid \tilde{p}_1\right]$. The complex root then indicates the inexistence of a local maximum for $k \geq 0$, leading to the corner solution $k = 0$.
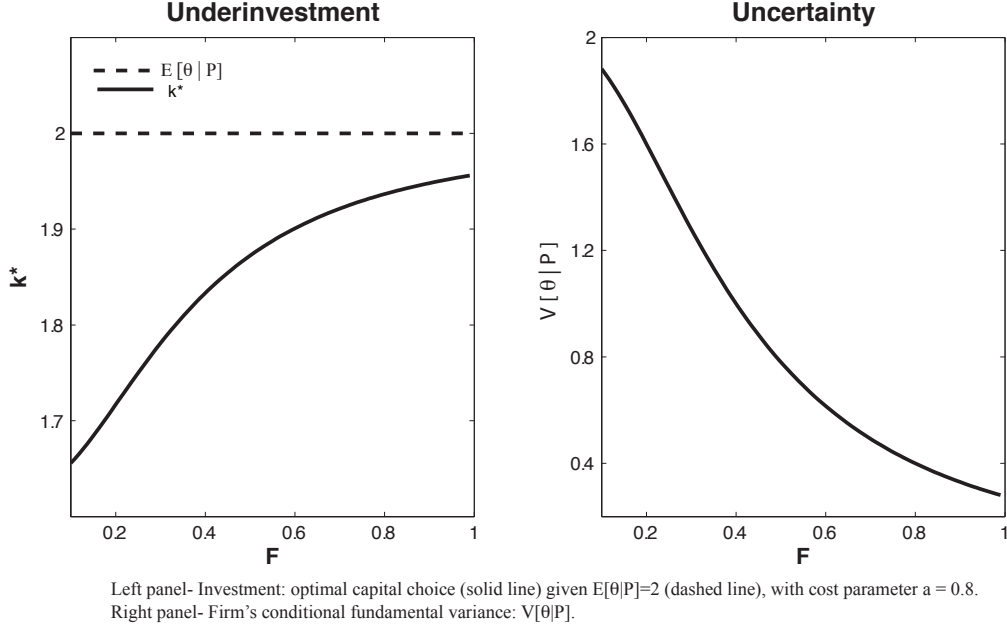
5.3   Market illiquidity and real investment

To understand the impact of varying price informativeness on investment decisions, consider for a moment a perfect information benchmark in which $V\left[\theta \mid \tilde{p}_1\right] \to 0$. From expression (20), this yields the simple investment decision $k^* = \theta$. An analogous result obtains in the standard quadratic problem with uncertainty, but no loss term: $k^* = \mathbb{E}\left[\theta \mid \tilde{p}_1\right]$. In contrast, the loss function $L\left(\theta; k\right)$ introduces an additional cost of investment "mistakes" –my proxy for partial irreversibilities in a static model–which holds back investment in a more uncertain environment (higher $\mathbb{V}\left[\theta \mid \tilde{p}_1\right]$). Figure 10 plots the implications of uncertainty in the underinvestment of the firm relative to its expectations about fundamentals. For each value of the expected productivity θ (dotted line), the firm chooses a lower investment level (solid line), a gap that widens as funding in the asset market tightens through its impact on price informativeness.

Modeling the reaction of investment to changes in both the first and second moments of productivity is important for explaining the *growth rate* or *steepness* asymmetry– the empirical regularity that investment tends to contract sharply at the outset of a slump but builds up only gradually when growth resumes. In the model above, investment asymmetry results from endogenous information aggregation in asset prices: when a boom turns into a bust, investment will be lower not only because the conditional first moment $\mathbb{E}\left[\theta \mid \tilde{p}_1\right]$ is likely to be lower but also because uncertainty about it is larger when traders in the asset market face tighter constraints. Investment thus falls sharply, contributing to the steep fall on GDP that characterize economic contractions.

Two theoretical papers bear a similar prediction about investment dynamics. Van Niewer-

Figure 11: Uncertainty and Underinvestment

Left panel- Investment: optimal capital choice (solid line) given E[θ|P]=2 (dashed line), with cost parameter a = 0.8.
Right panel- Firm's conditional fundamental variance: V[θ|P].

burgh and Veldkamp (2006) use a DSGE real business cycle model where capital and labor amplify the effect of unobserved productivity in total output, which is also affected by an unobservable disturbance. This creates asymmetric learning dynamics as larger hiring of inputs during expansions raises the signal-to-noise ratio of observable output, improving inferences about productivity. Chamley and Gale (1994) build a model where investment generates positive information externalities between players introducing a motive for strategic delay that is consistent with investment data. However, neither paper discusses the role of asset markets in generating endogenous uncertainty about fundamentals.

The above analysis suggests endogenous uncertainty about fundamentals can have welfare effects that go beyond redistribution of wealth between asset market participants. To the extent that real resources are guided by asset prices, funding liquidity affects economic efficiency by reducing the precision of investment. Moreover, as the model shows in convenient closed form, the first moment of investment falls below the expected value of fundamentals as a precautionary measure.

The next proposition states these welfare results formally. The appropriate concept of firm value is the ex-ante expectation of firm's profits, since the interest here is how production outcomes will be affected by investment decisions over all possible realizations of $\tilde{p}_1$.

Proposition 6 *(firm value):* *The unconditional expected profit of the firm is strictly increasing*

in funding liquidity F;

$$\frac{\partial \mathbb{E}\left[\Pi\left(\theta, k\right)\right]}{\partial F} > 0$$

Proof. *In Appendix D.* ∎

The intuition of the result follows directly from decomposing the impact of F into its effect on average profits per unit of capital (the intensive margin), and its effect on total investment (the extensive margin). The proof shows that the reduction in uncertainty brought along by a higher level of F increases expected average profits as the firm's decision becomes more accurate -the intensive margin of profits increases. Moreover, as the level of investment increases with F the extensive margin also contributes in raising the value of the firm, since average expected profits are positive, conditional on $k^* > 0$.

6 Testable predictions and policy implications

6.1 Testable predictions

The model offered advances some new asset pricing predictions. First, it predicts that the autocorrelation of stock returns in high-frequency data associated with high trading volume should decline (or become more negative) when funding conditions are tight. As Campbell, Grossman and Wang (1993) and Wang (1994) argue, returns explained by noise trading tend to reverse at high frequencies, which is supported empirically using high trading volume days as a proxy for noise trading. The analysis above implies that such reversals should be stronger during contractions, since noisy trading has a larger impact on asset prices when uncertainty is higher.

Pastor and Stambaugh (2003) find an analogous pattern for the cross-section of stocks returns. Stocks with higher "liquidity betas" exhibit larger return reversal associated with trading volume. An interesting extension would be to test for differential impacts across the business cycle, or conditional on institutional investors' funding restrictions.

The analysis also suggests models of information aggregation may have some bite in explaining certain asset pricing anomalies, such as the post-earnings announcements drift (PEAD). A learning approach to this anomaly (Hong, Lim and Stein (2000)) suggests drifts could reflect the slow diffusion of information. It follows that PEAD should be more pronounced when funding constraints limit information aggregation. A finding in this direction is provided by Chordia and Shivakumar (2006), who document that the profitability of PEAD strategies are significant *negative* predictors of future economic activity. While this result challenges the interpretation of PEAD excess returns as compensation for risk–since they actually provide hedges according to this evidence–it is consistent with this alternative interpretation in which high PEAD returns reflect slow incorporation

of information into prices during periods of financial turmoil.

Finally, the model has direct implications on the precision and relative dispersion of professional forecasts. If forecasters follow Bayesian updating rules when making predictions (about either macroeconomic aggregates, or individual firms' profits), they will tend to base those predictions less in information inferred from asset markets during contractions, when prices become less reliable. One should expect a shift towards private sources of information, and therefore an increase in the cross-sectional dispersion of forecasts. Since overall uncertainty is higher, one should also expect higher mean prediction errors. Both predictions seem to hold in the data (See Veronesi (1999) and Van Niewerburgh and Veldkamp (2006)).

6.2 Policy implications

The funding problems of banks and financial institutions in the midst of the sub-prime crisis underscores the importance of prompt public liquidity provision. The model suggest that the success of interventions, however, will depends on how liquidity is distributed across players.

The argument developed above explains uncertainty and non-fundamental volatility as the joint product of funding constraints and dispersed information. In models that considers only the former, the size of the liquidity pool is a sufficient statistic for the success of the policy, since knowledge about the environment is not the element driving the gap between prices and fundamentals.

Things change dramatically if information is dispersed. Indeed, risk-averse traders who happen to find themselves sitting on a pile of cash may have little use for it when uncertainty is an endogenous state-variable. If the problem is a group problem –the failure of the market in aggregating disseminated pieces of information though trading– it is intuitively very appealing that the solution should be a group solution.

A second issue relates to the optimal taxation of dividends vs. capital gains from trading. In the model, prices become less informative because funding constraints shorten the effective trading horizon of informed traders, exposing them to additional risk. A marginal decrease in tax rates applied to dividends–with the corresponding increase in capital gains if one wishes to maintain fiscal neutrality–would tend to increase the incentives to react to private information about fundamentals, and could potentially reduce equilibrium uncertainty. This insight applies more generally to models that stress other reasons for the short trading horizons, such as **?**.

7   Conclusion

I develop a tractable model in which the diminished capacity of distressed financial markets to aggregate information explains countercyclical uncertainty. Building on a standard noisy REE

model with dispersed information, I incorporate funding constraints on informed traders that are more likely to bind in periods of financial distress. Countercyclical risk premia, Sharpe ratios, and stock price volatility follow directly from this mechanism even when preferences towards risk and the unconditional volatility of fundamentals remain constant.

I argue that adding dispersed information and dynamic risk considerations into the analysis delivers strong internal amplification mechanisms. Moreover, dispersed information and the endogenous aggregation capacity of financial markets is the appropriate conceptual benchmark for understanding the impact of uncertainty in real investment decisions and for guiding policy actions.

Future work may proceed in several directions. First, a quantitative assessment about the contribution of the informational mechanism for asset pricing and real investment phenomena seems in order. This is likely to be a complicated task, since the appropriate benchmark of CRRA utility (or even Epstein-Zin preferences) calls for the use of nonlinear methods to tract information aggregation—but one that should be tackled nonetheless to gauge the relevance of endogenous price informativeness. Second, extending the framework to multiple assets and allowing traders to learn from a richer set of signals can provide answers regarding the comovement of individual stocks and aggregate market indexes. As Morck, Yeung and Yu (2000) argue, comovement seems to increase during volatile markets in the time-series, and it is also higher in countries with less developed financial systems. Finally, the model can easily fit additional information about economic conditions –such as public news– whose impact on stock prices might endogenously depend on the business cycle.

8 Appendix

8.1 Appendix A

Signal extraction problem: projection theorem The inference problem analyzed in sections 2 and 3, generally speaking, amounts to forecasting a $Nx1$ vector X (with unconditional mean $\mathbb{E}[X]$) from the observation of a $Mx1$ vector of correlated signals Ω (with unconditional mean $\mathbb{E}[\Omega]$). If X and Ω are jointly normally distributed, and Σ_{XX}, $\Sigma_{\Omega\Omega}$ and $\Sigma_{X\Omega}$ are the variance-covariance matrices of X, Ω and between X and Ω respectively, then the projection theorem gives the following results for the conditional moments of $X \mid \Omega$:

$$
\begin{aligned}
\mathbb{E}[X \mid \Omega] &= \mathbb{E}[X] + \Sigma_{X\Omega}[\Sigma_{\Omega\Omega}]^{-1}(\Omega - \mathbb{E}[\Omega]), \\
\mathbb{V}[X \mid \Omega] &= \Sigma_{X\Omega}[\Sigma_{\Omega\Omega}]^{-1}(\Sigma_{X\Omega})'
\end{aligned}
$$

Applied to the specific problem analyzed in the text, X corresponds to θ (with $\mathbb{E}[\theta] = 0$), and the vector of signals becomes $\Omega_i = \{s_i, \tilde{p}_1\} = \{\theta + s_i, \theta - \Delta \cdot n_1\}$ for informed traders, and $\Omega_u = \{\tilde{p}_1\} = \{\theta + s_i\}$ for the uninformed, where $\tilde{p}_1$ is an object of equivalent informational content as P_1. The Bayesian weights of informed traders $\{a_0, a_1\}$ and uninformed traders $\{b_1\}$ are given by

$$
\begin{aligned}
a_0 &= \lambda_\epsilon \cdot \mathbb{V}[\theta \mid \Omega_i]; \; a_1 = \lambda_n \Delta^{-2} \cdot \mathbb{V}[\theta \mid \Omega_i], \\
b_1 &= \lambda_n \Delta^{-2} \cdot \mathbb{V}[\theta \mid \Omega_u]
\end{aligned}
$$

where
$$
\mathbb{V}[\theta \mid \Omega_i] = \left[\lambda_\theta + \lambda_\epsilon + \lambda_n/\Delta^2\right]^{-1}, \quad \mathbb{V}[\theta \mid \Omega_u] = \left[\lambda_\theta + \lambda_n/\Delta^2\right]^{-1}
$$

Note that the conditional second moments of θ do not depend on the trader-specific private signal s_i, and are therefore common across traders of the same type.

For proving Proposition 2, it will be of use to state the conditional expectation of the noise trading shock, n_1. Applying the theorem once again, we find

$$
\mathbb{E}[n_1 \mid s_i, \tilde{p}_1] = (a_0/\Delta) \cdot s_i - ((1 - a_1)/\Delta) \cdot \tilde{p}_1 \tag{21}
$$

Proof of Proposition 1: To solve the proposed linear equilibrium of section 2, I replace informed traders beliefs computed in (4) in demands from (5). This allows to write the market-

clearing condition (6) as

$$n = F \cdot \left(\int \frac{a_0 \cdot s_i + a_1 \cdot \tilde{p}_1 - P_1}{\Sigma} di - 1 \right), \text{ or}$$
$$F + n = F \cdot \frac{a_0 \cdot \theta + a_1 \cdot \tilde{p}_1 - P_1}{\Sigma}$$

where the second line follows from the zero mean of idiosyncratic noise ϵ_i. This gives price P_1 as

$$P_1 = -\Sigma + a_0 \cdot \theta + a_1 \cdot \tilde{p}_1 - \Sigma/F \cdot n$$
$$= -\Sigma + (a_0 + a_1) \cdot \theta + (A_3/A_2 \cdot a_1 - \Sigma/F) \cdot n$$

Comparing with the price conjecture in 2 (method of undetermined coefficients) yields the results in the proposition. `QED`.

8.2   Appendix B

Proof of Proposition 2:   Part (i)

To solve the proposed linear equilibrium of section 3, I replace informed traders beliefs about θ and P_2 using (4) and (10), and uninformed beliefs about D using (11), in the market-clearing condition at stage 2:

$$n_2 = \frac{D_{-1} + b_1 \cdot \tilde{p}_1 - P_2}{\Sigma_u} - \frac{(1 - F)}{\Sigma} \cdot$$
$$[\int \left\{ F(D_{-1} + \mathbb{E}[\theta \mid s, \tilde{p}_1]) + (1 - F) \left(\begin{array}{c} B_0 + B_1 D_{-1} + B_2 \mathbb{E}[\theta \mid s, \tilde{p}_1] \\ +B_3 \mathbb{E}[n_1 \mid s, \tilde{p}_1] + B_5 \tilde{p}_1 + B_6 P_1 \end{array} \right) \right\} di - P_1]$$

where $\mathbb{E}[\theta \mid s, \tilde{p}_1] = a_0 \cdot s_i + a_1 \cdot \tilde{p}_1$, and $\mathbb{E}[n_1 \mid s, \tilde{p}_1]$ is given by 21. This given price P_2 as a function of the variables $\{\theta, n_1, n_2, \tilde{p}_1, P_1\}$, which can be compared to the price conjecture in (10) to obtain (through the method of undetermined coefficients)

$$B_0 = B_3 = 0; \tag{22}$$
$$B_1 = \frac{\Sigma - \Sigma_u (1 - F) F}{\Sigma + \Sigma_u (1 - F)^2}; \quad B_2 = \frac{-\Sigma_u \cdot a_0 (1 - F) \cdot F}{\Sigma + \Sigma_u a_0 (1 - F)^2}; \quad B_4 = -\Sigma_u;$$
$$B_5 = \frac{\Sigma b_1 - \Sigma_u a_1 (1 - F) (F + (1 - F) B_2)}{\Sigma + \Sigma_u (1 - F)^2}; \quad B_6 = \frac{(1 - F) \Sigma_u}{\Sigma + \Sigma_u (1 - F)^2}$$

Similarly, the market-clearing conditions at stage 1 implies a price P_1 as a function of $\{D_{-1}, \theta, n_1, \tilde{p}_1\}$.

Regrouping the observed terms in P_1 into the left hand side allows to solve for $\tilde{p}_1$;

$$\frac{P_1 + \left[\Sigma + \Sigma_u\,(1-F)^2\right] - D_{-1} - \tilde{p}_1\,(a_1(F + (1-F)B_2) + (1-F)b_1)}{\left[\frac{\Sigma + \Sigma_u(1-F)^2}{\Sigma}\right] a_0 \cdot (F + (1-F)B_2)} = \theta - n_1\frac{\Sigma}{a_0 \cdot (F + (1-F)B_2)}$$

$$\equiv \tilde{p}_1$$

which implies that the noise amplifier $\boldsymbol{\Delta}$ is given by equation (16a), as stated in the proposition.

To compute the risk-aversion adjusted variance of informed traders; Σ, note that the conditional covariance between P_2 and D is $B_2\mathbb{V}\left[\theta \mid s, \tilde{p}_1\right]$. From the definition of Σ in the text, this gives equation (16b). The expression for Σ_u comes from (11). This completes the four equations in the proposition that any linear equilibrium must satisfy, in the unknowns $\{\boldsymbol{\Delta}, \Sigma, \Sigma_u, B_2\}$.

Part (ii)

The system of equations in Proposition 2 can be reduced to two equations in the unknowns $\{\boldsymbol{\Delta}, \Sigma\}$:

$$\Sigma\left(\Sigma, \boldsymbol{\Delta}\right) = (1-c)\,\gamma\,(1-F)^2\,\gamma_u^2\left[\left[\lambda_\theta + \lambda_n/\boldsymbol{\Delta}^2\right]^{-1} + \lambda_\mu^{-1}\right]^2 \lambda_n^{-1} \tag{23}$$

$$+ (1-c)\,\gamma F^2\left\{\frac{\Sigma^2 \cdot \left[\lambda_\theta + \lambda_\epsilon + \lambda_n/\boldsymbol{\Delta}^2\right]^{-1}}{\left[\Sigma + \gamma_u\left(\left[\lambda_\theta + \lambda_n/\boldsymbol{\Delta}^2\right]^{-1} + \lambda_\mu^{-1}\right)(1-F)^2\,\lambda_\epsilon\left[\lambda_\theta + \lambda_\epsilon + \lambda_n/\boldsymbol{\Delta}^2\right]^{-1}\right]^2} + \lambda_\mu^{-1}\right\}$$

$$\boldsymbol{\Delta}\left(\Sigma, \boldsymbol{\Delta}\right) = \frac{\Sigma + \gamma_u\left(\left[\lambda_\theta + \lambda_n/\boldsymbol{\Delta}^2\right]^{-1} + \lambda_\mu^{-1}\right)(1-F)^2\,\lambda_\epsilon\left[\lambda_\theta + \lambda_\epsilon + \lambda_n/\boldsymbol{\Delta}^2\right]^{-1}}{\left[\lambda_\theta + \lambda_\epsilon + \lambda_n/\boldsymbol{\Delta}^2\right]^{-1}} \tag{24}$$

Proving existence of equilibria amounts to showing that the loci of combinations $(\Sigma, \boldsymbol{\Delta})$ that satisfy each equation intersects at least once, for all parameter values. I provide figure A1 as a graphical intuition of the proof below. I begin with the loci $\Sigma = \hat{\Sigma}\left(\Sigma, \boldsymbol{\Delta}\right)$ that satisfies equation (23), for a given value of $\boldsymbol{\Delta}$. First, note that the derivative of the RHS of (23) w.r.t. Σ is less than unity, and that $\hat{\Sigma}\left(0, \boldsymbol{\Delta}\right) > 0$. Since the derivative of the left hand side (23) is one, there is a unique value $\Sigma^*\left(\boldsymbol{\Delta}\right)$ that satisfies (23), for each $\boldsymbol{\Delta}$. To characterize the loci $\Sigma = \hat{\Sigma}\left(\Sigma, \boldsymbol{\Delta}\right)$ in the $(\boldsymbol{\Delta}, \Sigma)$ space, note that its intercept is given by

$$\hat{\Sigma}\left(\Sigma, 0\right) \equiv \underline{\Sigma} = (1-c)\,\gamma\lambda_\mu^{-1}\left[(1-F)^2\,\gamma_u^2\lambda_\mu^{-1}\lambda_n^{-1} + F^2\right] \tag{25}$$

and the limit $\hat{\Sigma}\left(\Sigma, \boldsymbol{\Delta} \longrightarrow \infty\right) \equiv \overline{\Sigma}$ solves

$$\overline{\Sigma} = (1-c)\,\gamma \cdot \tag{26}$$

$$\left\{(1-F)^2\,\gamma_u^2\left[\lambda_\theta^{-1} + \lambda_\mu^{-1}\right]^2 \lambda_n^{-1} + F^2\left[\lambda_\mu^{-1} + \frac{\overline{\Sigma}^2}{\left[\overline{\Sigma} + \gamma_u\left(\lambda_\theta^{-1} + \lambda_\mu^{-1}\right)(1-F)^2\,\lambda_\epsilon\left[\lambda_\theta + \lambda_\epsilon\right]^{-1}\right]^2}\right]\right\}$$

Figure 12: Uniqueness vs. Multiplicity
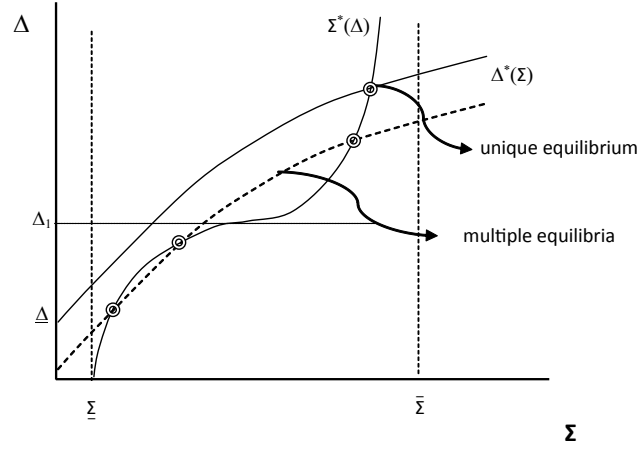
with $\overline{\Sigma} > \underline{\Sigma}$. This upper limit $\overline{\Sigma}$ ensures that the implicit function $\Sigma^* (\Delta)$ becomes concave in Δ, for $\Delta > \Delta_1$. Without imposing any parameter restrictions, these results imply the function $\Sigma^* (\Delta)$ always has a solution $\forall \Delta \in \mathbb{R}^+$, and that $\Sigma^* (\Delta) < \overline{\Sigma}$.

Similarly, the loci $\Delta = \hat{\Delta} (\Sigma, \Delta)$ that characterizes the implicit solution $\Delta^* (\Sigma)$ in equation (24) is continuous in $\Sigma \in \mathbb{R}^+$, and always has a solution $\Delta^* (\Sigma) > 0$. It follows that represented in the (Σ, Δ) space, the loci $\Sigma^* (\Delta)$ from equation one will intersect the loci $\Delta^* (\Sigma)$ at least once, since the former is a locally convex (for $\Delta > \Delta_1$), continuous correspondence with image $\Delta \in [0, \infty)$ on the domain $(0, \overline{\Sigma})$, and the latter is a continuous, positive-valued function $\forall \Sigma \in \mathbb{R}^+$. This implies that, over the interval $\Sigma \in [\underline{R}, \overline{\Sigma})$, equations (23) and (24) will always be satisfied simultaneously for at least one pair (Σ^*, Δ^*). QED.

Part (iii)

Finding conditions for equilibrium uniqueness amounts to finding the parameter subspace for which the loci $\Sigma^* (\Delta)$ and $\Delta^* (\Sigma)$ from equations (23) and (24) intersect only once. For this I must first restrict $\Delta^* (\Sigma)$ to be single-valued (since $\Sigma^* (\Delta)$ is single-valued without restrictions, from the analysis in part (ii)). $\Delta^* (\Sigma)$ is single-valued whenever the derivative of the right hand side of (24) w.r.t. Δ is less than one. It is simple to show that this derivative is actually negative whenever $\gamma_u \lambda_\mu^{-1} \left(2 + \lambda_\theta^{-1} \lambda_\mu^{-1} \right) > 1$, or $\lambda_\mu < \kappa_0 (\gamma_u, \lambda_\theta)$.

To see under what conditions these single-valued functions (assuming $\lambda_\mu < \kappa_0 (\cdot)$) cross only once, its necessary to characterize the intercepts and slopes of the implicit functions $\Sigma^* (\Delta)$ and $\Delta^* (\Sigma)$, which I do in the (Σ, Δ) space. Since both functions are continuous, I can use the implicit

function theorem for the latter task. Starting with $\Delta^*(\Sigma)$, its intercept $\underline{\Delta}$ solves

$$\underline{\Delta} = \gamma_u (1 - F)^2 \left[\lambda_\mu^{-1} + \left(\lambda_\theta + \lambda_n/\underline{\Delta}^2 \right)^{-1} \right] \tag{27}$$

which is increasing λ_μ^{-1}. To find the slope, I totally differentiate the function

$$\begin{aligned} \mathsf{f} \quad &: \quad \Delta - \hat{\Delta}(\Sigma, \Delta) = 0, \\ \frac{d\Delta}{d\Sigma} \quad &= \quad -\frac{\partial \mathsf{f}/\partial \Sigma}{\partial \mathsf{f}/\partial \Delta} = \frac{\partial \hat{\Delta}(\cdot)/\partial \Sigma}{1 - \partial \hat{\Delta}(\cdot)/\partial \Delta} \end{aligned}$$

to find $d\Delta/d\Sigma > 0$ under the condition already stated $(\lambda_\mu < \kappa_0(\cdot))$. Moreover, the function is concave, with a slope that approaches $(\lambda_\epsilon + \lambda_\theta)/\lambda_\epsilon$ as $\Sigma \to \infty$.

Regarding $\Sigma^*(\Delta)$, I already established that $\Sigma^*(0) = \underline{R}$, so that $\Sigma^*(\Delta)$ does not intercept the Δ axis (since $\Sigma \geq 0$). With respect to its slope, I totally differentiate the function

$$\begin{aligned} \mathsf{f} \quad &: \quad \Sigma - \hat{\Sigma}(\Sigma, \Delta) = 0, \\ \frac{d\Sigma}{d\Delta} \quad &= \quad -\frac{\partial \mathsf{f}/\partial \Delta}{\partial \mathsf{f}/\partial \Sigma} = \frac{\partial \Sigma(\cdot)/\partial \Delta}{1 - \partial \Sigma(\cdot)/\partial \Sigma} \end{aligned}$$

to find with some algebra that a sufficient condition for a positive slope is

$$2\gamma (1 - c) \left[F^2 \lambda_\mu^{-1} + (1 - F)^2 \gamma_u^2 \lambda_\mu^{-1} \lambda_n^{-1} \right] > F^2 \lambda_n,$$

or $\lambda_\mu < \kappa_1(\cdot)$. Recall as well that this function is concave for $\Delta > \Delta_1$, which implies a convex loci in the (Σ, Δ) space, with $\Sigma < \bar{\Sigma}$. These results together imply one can always find a finite value $\lambda_\mu = \kappa_2$, s.t. $\forall \lambda_\mu < \kappa_2$, the concave function $\Delta^*(\Sigma)$ intersects only once with the (locally) convex function $\Sigma^*(\Delta)$. This case corresponds to the solid line $\Delta^*(\Sigma)$ in figure A1. In contrast, if $\lambda_\mu \to 0$, then it cannot be granted that the two loci will not intersect more than once (dotted line). Therefore, whenever $\lambda_\mu \leq \delta_0$ with $\delta_0 \equiv \min(\kappa_0(\cdot), \kappa_1(\cdot), \kappa_2)$, the equilibrium in Proposition 2 is unique. `QED`.

Proof of Proposition 3: I will follow a similar reasoning as in the previous proof, finding conditions under which an increase in F moves the loci $\Sigma^*(\Delta)$ and $\Delta^*(\Sigma)$ in a direction that implies an unambiguous decrease in $\boldsymbol{\Delta}(F)$. First, note that the intercept of the implicit function $\Delta^*(\Sigma)$ given by (27) is decreasing in F, which can be shown by totally differentiating

$$\begin{aligned} \mathsf{f} \quad &: \quad \underline{\Delta} - \hat{\underline{\Delta}}(F, \Delta) = 0, \\ \frac{d\underline{\Delta}}{dF} \quad &= \quad -\frac{\partial \mathsf{f}/\partial F}{\partial \mathsf{f}/\partial \Delta} = \frac{\partial \hat{\underline{\Delta}}(\cdot)/\partial F}{1 - \partial \hat{\underline{\Delta}}(\cdot)/\partial \Delta} \end{aligned}$$

which is negative whenever $\partial \hat{\underline{\Delta}}\left(\cdot\right)/\partial \Delta < 1$. With some algebra, this can be translated into the sufficient condition $2\gamma_u \left(1-F\right)^2 \left(\lambda_\theta + \lambda_n/\underline{\Delta}^2\right)^{-1} < \underline{\Delta}$, which is always satisfied by some $\lambda_\mu \leq \kappa_3$, with $\kappa_3 > 0$, since $\underline{\Delta} > \lambda_\mu^{-1}$. Regarding the slope of $\Delta^*\left(\Sigma\right)$, I totally differentiate the function

$$
\begin{aligned}
\mathsf{f} \quad &: \quad \Delta - \hat{\Delta}\left(F, \Delta\right) = 0, \\
\frac{d\Delta}{dF} \quad &= \quad -\frac{\partial \mathsf{f}/\partial F}{\partial \mathsf{f}/\partial \Delta} = \frac{\partial \hat{\Delta}\left(\cdot\right)/\partial F}{1 - \partial \hat{\Delta}\left(\cdot\right)/\partial \Delta}
\end{aligned}
$$

to find $d\Delta/dF < 0$ whenever $1 - \partial \hat{\Delta}\left(\cdot\right)/\partial \Delta > 0$, or when condition $\lambda_\mu \leq \kappa_0\left(\cdot\right)$ in the previous proof holds.

It remains to find the conditions under which the intercept and the slope of the implicit function $\Sigma^*\left(\Delta\right)$ have a negative w.r.t. F. The derivative of the intercept $\underline{R}$ in equation (25) is proportional to $F - \gamma_u^2 \left(1-F\right)\lambda_\mu^{-1}\lambda_n^{-1}$, which is always negative for values of F below $\bar{F}_1 \equiv \gamma_u^2 \lambda_\mu^{-1}\lambda_n^{-1}/\left(1+_u^2 \lambda_\mu^{-1}\lambda_n^{-1}\right)$. With respect to the slope of $\Sigma^*\left(\Delta\right)$, I totally differentiate the function

$$
\begin{aligned}
\mathsf{f} \quad &: \quad \Sigma - \hat{\Sigma}\left(\Sigma, F\right) = 0, \\
\frac{d\Sigma}{dF} \quad &= \quad -\frac{\partial \mathsf{f}/\partial F}{\partial \mathsf{f}/\partial \Sigma} = \frac{\partial \Sigma\left(\cdot\right)/\partial F}{1 - \partial \Sigma\left(\cdot\right)/\partial \Sigma}
\end{aligned}
$$

which amounts to finding restrictions under which $\partial \Sigma\left(\cdot\right)/\partial F < 0$. But note that there always exists a value $F > 0$, s.t. the derivative of the right hand side of equation (23) is negative w.r.t. F, which can be easily shown by evaluating $\partial \Sigma\left(\cdot\right)/\partial F$ at $F = 0$. Moreover, we can find a lower bound $\bar{F}_2$ s.t. the derivative is negative at $F \leq \bar{F}_2$. Taking the partial of equation (23) w.r.t F gives

$$
\begin{aligned}
\partial \Sigma\left(\cdot\right)/\partial F \quad &\propto \quad -\lambda_n^{-1} + \frac{F}{1-F}\frac{\left[\mathbb{V}\left[\theta \mid \Omega_i\right]\cdot\Gamma\left(F\right) + \lambda_\mu^{-1}\right]}{\gamma_u\left[\mathbb{V}\left[\theta \mid \Omega_u\right] + \lambda_\mu^{-1}\right]} + 2\frac{F^2}{1-F}\Gamma\left(F\right)a_0\mathbb{V}\left[\theta \mid \Omega_i\right] \\
&< \quad -\lambda_n^{-1} + \frac{F}{1-F}\frac{1}{\gamma_u} + 2\frac{F^2}{1-F}\left[\lambda_\epsilon + \lambda_\theta\right]^{-1},
\end{aligned}
$$

where $\Gamma\left(F\right) \equiv \Sigma^2/\left[\Sigma + \gamma_u\left[\mathbb{V}\left[\theta \mid \Omega_u\right] + \lambda_\mu^{-1}\right]\left(1-F\right)^2 a_0\right]^2 < 1$, so that $\bar{F}_2 > \breve{F}$, where $\breve{F}$ makes the last term in the inequality equal to zero. It follows that both the derivative of the intercept, and the slope of $\Sigma^*\left(\Delta, \cdot\right)$ is negative for all $F \leq \bar{F} \equiv \min\left(\bar{F}_1, \bar{F}_2\right)$. Note that both thresholds $\bar{F}_1$ and $\bar{F}_2$ are increasing functions of the variance of noise trading shocks λ_n^{-1}, and the risk aversion of uninformed traders increase.

Finally, since there is a one-to-one, monotonically increasing relation between θ_{-1} and F (equation (8)), it follows that the threshold $\bar{F}$ can be mapped into a threshold $\bar{\theta}$, s.t., $\Delta'\left(\theta_{-1}\right) < 0$, $\forall \theta_{-1} \leq \bar{\theta} \equiv \lambda_\ell^{-1/2}\cdot\Phi^{-1}\left(\bar{F}\right)$, with $\partial\bar{\theta}/\partial\lambda_n^{-1} > 0$, and $\partial\bar{\theta}/\partial\gamma_u$. In summary, whenever the sufficient

conditions $\lambda_\mu < \delta_1 \equiv \min\left(\kappa_0\left(\cdot\right), \kappa_3\right)$, and $\theta_{-1} < \bar{\theta}\left(\lambda_n^{-1}, \gamma_u\right)$ hold, $\mathbf{\Delta}'\left(\theta_{-1}\right) < 0$. `QED`.

Equilibrium when uninformed traders learn from P_2: From Conjecture **??**, uninformed traders can back out an additional noisy signal about θ;

$$\tilde{p}_2 \equiv \frac{P_2 - B_0 - B_1 \cdot D_{-1} - B_5 \cdot \tilde{p}_1 - B_6 \cdot P_1}{B_2} = \theta + \frac{B_3}{B_2}n_1 + \frac{B_4}{B_2}n_2 \qquad (28)$$

The equilibrium solution proceeds in the same steps as before, with the difference that uninformed traders beliefs' now correspond to

$$
\begin{aligned}
\mathbb{E}\left[\theta \mid \tilde{p}_1, \tilde{p}_2\right] &= b_1\tilde{p}_1 + b_2\tilde{p}_2, &&(29)\\
\mathbb{V}\left[\theta \mid \tilde{p}_1, \tilde{p}_2\right] &= \left[\lambda_\theta + \lambda_n\left(\Delta^{-2} + \left(\left(\frac{B_3}{B_2}\right)^2 + \left(\frac{B_4}{B_2}\right)^2\right)^{-1}\right)\right]
\end{aligned}
$$

which will imply a different solution for the linear coefficients of Conjecture **??**. These are now given by

$$
\begin{aligned}
B_0 &= B_3 = 0; & B_1 + B_5 = 1, &&(30)\\
B_2 &= \frac{b_2 - \Sigma_u \cdot a_0\left(1 - F\right) \cdot F}{\Sigma + \Sigma_u a_0\left(1 - F\right)^2}; & B_4 = \Sigma_u\left[\frac{b_2\Sigma - \Sigma_u a_0\left(1 - F\right) \cdot F}{\Sigma_u a_0\left(1 - F\right)\left(F + \left(1 - F\right)b_2\right)}\right]; \\
B_6 &= \frac{b_1\Sigma - \Sigma_u a_1\left(1 - F\right)\left(F + \left(1 - F\right)B_2\right)}{\Sigma + \Sigma_u\left(1 - F\right)^2}
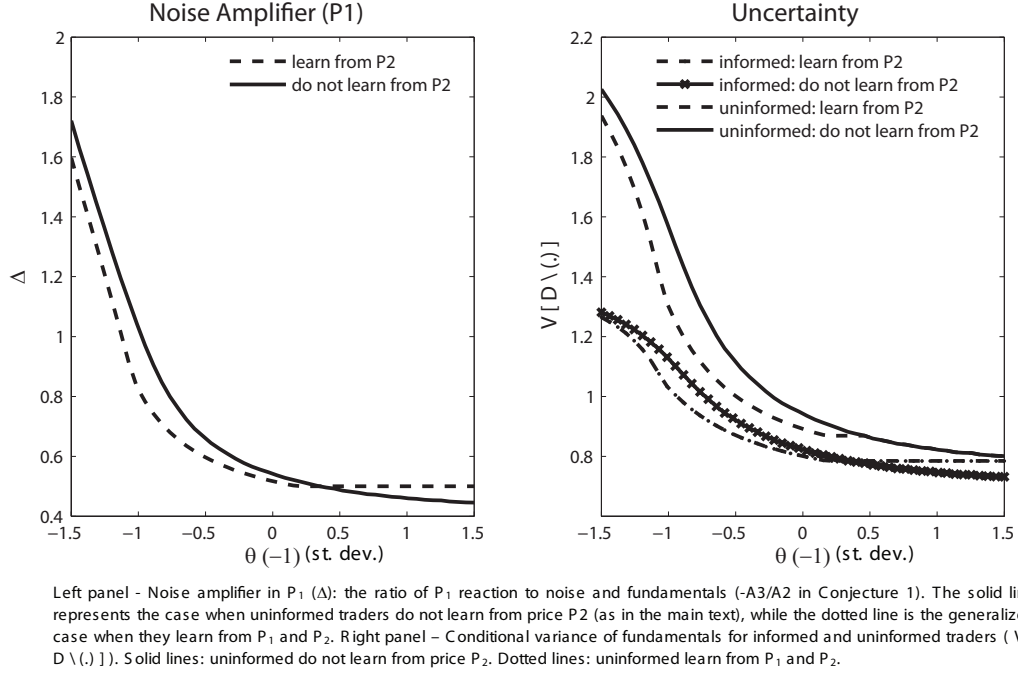\end{aligned}
$$

The equilibrium is now characterized by a system of seven equations: Δ from 16a; B_2 and B_4 from (30); b_2 given by

$$b_2 = \frac{\lambda_n\left(B_2/B_4\right)^2}{\lambda_\theta + \lambda_n\left(\Delta^{-2} + \left(B_2/B_4\right)^2\right)}$$

from the projection theorem; Σ and Σ_u from (16b) and (29), and $a_0 = \lambda_\epsilon\mathbb{V}\left[\theta \mid s, \tilde{p}_1\right]$ as before.

I have not been able to provide an analytically find specific parameter conditions that can ensure uniqueness. Figure A2 plots one equilibrium simulated for starting values of the iteration in the neighborhood of the equilibrium in which uninformed traders do not learn from P_2. The results are fairly intuitive: first, since P_2 provides additional information, the conditional variance of uninformed traders is lower in this case (panel B), which increases their willingness to absorb informed traders' liquidations and noise. This alleviates to some extent the noise risk and trade reversal effects discussed in section 3, increasing the incentives for informed traders to react to information at stage 1. Consequently, the noise amplifier $\mathbf{\Delta}$ at stage 1 is lower as well. The new equilibrium behaves qualitatively very similar to the previous one, and I suspect that similar

Figure 13: Countercyclical Uncertainty

Left panel - Noise amplifier in P_1 (Δ): the ratio of P_1 reaction to noise and fundamentals (-A3/A2 in Conjecture 1). The solid line represents the case when uninformed traders do not learn from price P2 (as in the main text), while the dotted line is the generalized case when they learn from P_1 and P_2. Right panel – Conditional variance of fundamentals for informed and uninformed traders (V [D \ (.)]). Solid lines: uninformed do not learn from price P_2. Dotted lines: uninformed learn from P_1 and P_2.

restrictions on the variance of μ can lead to equilibrium uniqueness, but the precise characterization is out of the scope of the present paper.

8.3 Appendix C

Proof of Proposition 4: This proof follows directly from the proof of Proposition 3. Taking the partial derivative of (17) w.r.t. θ_{-1} gives

$$\partial RP/\partial \theta_{-1} = \left[\partial \Sigma/\partial F + \partial \Sigma_u/\partial F \cdot (1 - F)^2 - 2\Sigma_u (1 - F)\right] \partial F/\partial \theta_{-1}$$

Recall that under the condition $F \leq \bar{F}$, both the intercept and slope of the implicit function $\Sigma^* (\Delta, \cdot)$ are decreasing in F. I also showed that the derivative and intercept of the implicit function $\Delta^* (\Sigma, \cdot)$ are decreasing in F under the restriction $\lambda_\mu < \delta_1$. It follows that the solution $\Sigma^* (\Delta, \cdot)$ has a negative derivative w.r.t. F under these restrictions, which takes care of the term $\partial \Sigma/\partial F$. Moreover, since Σ_u is increasing in the conditional variance $\mathbb{V} [\theta \mid \Omega_u] = [\lambda_\theta + \lambda_n/\mathbf{\Delta}^2]^{-1}$, it follows that under the same restrictions $\partial \Sigma_u/\partial F < 0$. Finally, $\partial F/\partial \theta_{-1} > 0$ from equation (8). QED.

Proof of Proposition 5: The derivative of the Sharpe ratio w.r.t. θ_{-1} is proportional to

$$\frac{-2\left(1-F\right)\gamma_u^2\left(\mathbb{V}\left[\theta\mid\Omega_u\right]+\lambda_\mu^{-1}\right)^2\lambda_n^{-1}}{\left(\mathbb{V}\left[\theta\mid\Omega_i\right]+\lambda_\mu^{-1}\right)^{1/2}}$$

$$-\left(1-F\right)^2\gamma_u^2\lambda_n^{-1}\frac{\mathbb{V}\left[\theta\mid\Omega_u\right]+\lambda_\mu^{-1}}{\Delta^3\left(\mathbb{V}\left[\theta\mid\Omega_i\right]+\lambda_\mu^{-1}\right)^{1/2}}\left[4\mathbb{V}\left[\theta\mid\Omega_u\right]-\mathbb{V}\left[\theta\mid\Omega_i\right]^2\frac{\mathbb{V}\left[\theta\mid\Omega_u\right]+\lambda_\mu^{-1}}{\mathbb{V}\left[\theta\mid\Omega_i\right]+\lambda_\mu^{-1}}\right]$$

$$+2F\frac{\Gamma\left(F\right)\mathbb{V}\left[\theta\mid\Omega_i\right]+\lambda_\mu^{-1}}{\mathbb{V}\left[\theta\mid\Omega_i\right]+\lambda_\mu^{-1}}+\frac{F^2}{\mathbb{V}\left[\theta\mid\Omega_i\right]+\lambda_\mu^{-1}}\cdot\left\{\Gamma\left(F\right)\partial\mathbb{V}\left[\theta\mid\Omega_i\right]/\partial\Delta\cdot\Delta'\left(\cdot\right)+\Gamma'\left(F\right)\partial\mathbb{V}\left[\theta\mid\Omega_i\right]/\partial\Delta\right.$$

$$\left.-\frac{1}{2}\left(\mathbb{V}\left[\theta\mid\Omega_i\right]+\lambda_\mu^{-1}\right)^{-1/2}\left(\Gamma\left(F\right)\mathbb{V}\left[\theta\mid\Omega_i\right]+\lambda_\mu^{-1}\right)\partial\mathbb{V}\left[\theta\mid\Omega_i\right]/\partial\Delta\cdot\Delta'\left(\cdot\right)\right\}$$

where the first term is negative, and the second can be shown to be negative under the restriction $\lambda_\mu < \delta$. The third term is positive, while the fourth is of ambiguous sign, depending on the value of F. It follows that it is possible to find a value $\bar{F}_3$ s.t. $\forall F < \bar{F}_3$, the whole expression is negative. This can be translated into a corresponding threshold for θ_{-1}, such that $\partial S\Sigma/\partial\theta_{-1} < 0\ \forall\theta_{-1} \leq \bar{\theta}' \equiv \lambda_\ell^{-1/2}\cdot\Phi^{-1}\left(\bar{F}_3\right)$, as stated in the proposition. QED.

Equilibrium characterization in the common information economy   I proceed in similar steps as in section 3. First, I conjecture the linear price functions

$$
\begin{aligned}
P_1 &= A_o + A_1\cdot D_{-1} + A_2\cdot\left(\theta+\epsilon\right) + A_3\cdot n_1\\
P_2 &= B_0 + B_1\cdot D_{-1} + B_2\cdot\left(\theta+\epsilon\right) +\\
&\quad +B_4\cdot n_2 + B_5\cdot P_1 + B_6\cdot\tilde{p}_1
\end{aligned}
$$

where I have already used the fact that the partial effect of n_1 on P_2 is zero ($B_3 = 0$, as in previous section). The key difference in this setting is that aggregating across informed traders will not wash out the common noise ϵ, so that prices will inherit $s = \theta + \epsilon$. Another distinction is that informed traders can perfectly infer the noise trading shock n_1, since they observe the price as well as the common signal ϵ. I use these facts to replace the conditional moments of θ and P_2 in informed traders' demands.

Using the market-clearing condition at stage 2 allows to solve for the $B's$ coefficients of the conjecture about P_2, which I then replace in the market-clearing condition at stage 1. From the resulting P_1, I can recover the public signal

$$\tilde{p}_1 = \left(\theta+\epsilon\right) - \boldsymbol{\Delta}\cdot n_1$$

where $\mathbf{\Delta}$ is now given by

$$\mathbf{\Delta} = \frac{\Sigma}{F \cdot a_0 + (1 - F) \cdot B_2}$$

Note that with common information however, uninformed traders are the only agents making inferences from the asset price P_1, which now captures both noise trading shocks and the common noise ϵ.

The solution for the coefficients in price P_2 are given by

$$B_0 = 0; \; B_1 = \frac{\Sigma - \Sigma_u \cdot (1 - F) F}{\Sigma + \Sigma_u (1 - F)^2}; \;\; B_2 = \frac{-\Sigma_u (1 - F) F \cdot a_0}{\Sigma + \Sigma_u (1 - F)^2} \tag{31}$$

$$B_4 = -\Sigma_u; \;\; B_5 = \frac{\Sigma_u \cdot (1 - F)}{\Sigma + \Sigma_u (1 - F)^2}; B_6 = \frac{b_1 \Sigma}{\Sigma + \Sigma_u (1 - F)^2};$$

$$\text{with} \quad b_1 = \lambda_\epsilon \lambda_n / [\lambda_\theta (\lambda_n + \lambda_\epsilon \Delta^2) + \lambda_\epsilon \lambda_n]$$

What remains is finding the expressions for the total risk (weighted by risk-aversion) faced by informed traders; Σ. Note that, conditional on the signal ϵ, the conditional variance of P_2 is just $(\Sigma_u)^2 \cdot \lambda_n^{-1}$, which gives

$$\Sigma = \gamma \left[F^2 \left(\frac{1}{\lambda_\theta + \lambda_\epsilon} + \frac{1}{\lambda_\mu} \right) + (1 - F)^2 (\Sigma_u)^2 \cdot \lambda_n^{-1} \right]$$

where $\Sigma_u = \gamma_u \left[(\lambda_\epsilon^{-1} + \lambda_n^{-1} \Delta^2)^{-1} + \lambda_\mu^{-1} \right]$. This last expression, together with (31), allows to solve for Δ.

8.4 Appendix D

First order condition of the investment problem: I can rewrite the investment problem in the maximand as

$$\mathbb{E}[\theta \mid \tilde{p}_1] - \frac{1}{2} k^2 - \frac{a}{2} k \cdot E[\theta^2 - 2\theta k + k^2 \mid \tilde{p}_1]$$

$$= \mathbb{E}[\theta \mid \tilde{p}_1] - \frac{1}{2} k^2 - \frac{a}{2} k \cdot \left[\mathbb{V}[\theta \mid \tilde{p}_1] + (\mathbb{E}[\theta \mid \tilde{p}_1])^2 - 2\mathbb{E}[\theta \mid \tilde{p}_1] k + k^2 \right]$$

taking the derivative w.r.t. k leads to a quadratic equation in k, which can be solved to yield the result in expression (20).

Proof of Proposition 6: I first show that conditional on $\tilde{p}_1$, the firm does weakly better on average when the precision of information is larger - i.e., the expected profit is increasing in F. I

can write the firm's profit expectation as

$$\mathbb{E}\left[\Pi\left(\theta,k\right)\mid\tilde{p}_1\right]=k^*\cdot\left\{\frac{\mathbb{E}\left[\Pi\left(\theta,k\right)\mid\tilde{p}_1\right]}{k^*}\right\}$$

and its derivative with respect to F as

$$\frac{\partial\mathbb{E}\left[\Pi\left(\theta,k\right)\mid\tilde{p}_1\right]}{\partial F}=\frac{\partial k^*}{\partial F}\cdot\frac{\mathbb{E}\left[\Pi\left(\theta,k\right)\mid\tilde{p}_1\right]}{k^*}+k^*\cdot\partial\left\{\frac{\mathbb{E}\left[\Pi\left(\theta,k\right)\mid\tilde{p}_1\right]}{k^*}\right\}/\partial k^*$$

I distinguish two possible cases. First, if either expectations on fundamentals are too low, and/or the conditional variance of fundamentals is too large, the firm does not invest, and expected profits do not change with marginal increases in F. In the second situation, investment k^* is positive, which is the only case when the firm expects the average profit per unit of k^* to be positive as well. This makes the first term of the last expression strictly positive, for $k^*>0$.

The second term requires more analysis. Its partial derivative w.r.t. F can be written as

$$=-\frac{k^*}{2}\frac{\partial\mathbb{V}\left[\theta\mid\tilde{p}_1\right]}{\partial F}\left\{\frac{\partial k^*}{\partial\mathbb{V}\left[\theta\mid\tilde{p}_1\right]}+a\left(1+2\left(\mathbb{E}\left[\theta\mid\tilde{p}_1\right]-k^*\right)\frac{\partial\left(\mathbb{E}\left[\theta\mid\tilde{p}_1\right]-k^*\right)}{\partial\mathbb{V}\left[\theta\mid\tilde{p}_1\right]}\right)\right\}$$

Since $\partial\mathbb{V}\left[\theta\mid p_1\right]/\partial F<0$, it suffices to show that the term in brackets is strictly positive, for $k^*>0$. We can write

$$\frac{\partial k^*}{\partial\mathbb{V}\left[\theta\mid\tilde{p}_1\right]}=-\frac{1}{6}\left(SQR\right)^{-1/2},\quad\text{and}$$

$$\frac{\partial\left(\mathbb{E}\left[\theta\mid\tilde{p}_1\right]-k^*\right)}{\partial\mathbb{V}\left[\theta\mid\tilde{p}_1\right]}=\frac{1}{6}\left(SQR\right)^{-1/2}$$

where

$$SQR=\left(\frac{1}{3a}+\frac{\mathbb{E}\left[\theta\mid\tilde{p}_1\right]}{3}\right)^2-\frac{1}{3}\mathbb{V}\left[\theta\mid\tilde{p}_1\right]$$

which is strictly positive under the requirement $k^*>0$. This allows to rewrite the bracket term as

$$\frac{1}{6}\left(SQR\right)^{-1/2}\left[\frac{2\mathbb{E}\left[\theta\mid\tilde{p}_1\right]}{3}+\frac{2}{3a}-2\left(SQR\right)^{1/2}\right]+a$$

which can be further manipulated by replacing the solution k^* from (20), to yield

$$k^*+3\left(\frac{1}{3a}-\left(SQR\right)^{1/2}\right)+a=k^*+3\left(\frac{2\mathbb{E}\left[\theta\mid\tilde{p}_1\right]}{3}-k^*\right)+a$$
$$=2\left(\mathbb{E}\left[\theta\mid\tilde{p}_1\right]-k^*\right)+a$$

which is always strictly positive, for any arbitrarily small variance $\mathbb{V}\left[\theta\mid\tilde{p}_1\right]$, since $\mathbb{E}\left[\theta\mid\tilde{p}_1\right]-k^*>0$.

It remains to show that F raises the unconditional profit expectation. This can be restated as the integral of conditional profits, over the distribution of $\tilde{p}_1$. Labeling the cdf of $\tilde{p}_1$ by $G(\tilde{p}_1)$, one can write

$$\frac{\partial \mathbb{E}\left[\mathbb{E}\left[\Pi(\theta, k) \mid \tilde{p}_1\right]\right]}{\partial F} = \int \left(\frac{\partial \mathbb{E}\left[\Pi(\theta, k) \mid \tilde{p}_1\right]}{\partial F}\right) dG(\tilde{p}_1) > 0$$

which follows from the fact that the conditional result $\partial \mathbb{E}\left[\Pi(\theta, k) \mid \tilde{p}_1\right]/\partial F \geq 0$ holds for any realization of the price P_1. Since at least for some realizations the firm will choose a positive level of investment, it follows that the impact of F on firm value is strictly positive. QED.

References

[1] Acharya, Viral, Douglas Gale, and Tanju Yorulmazer (2009). Rollover Risk and market Freezes, Working paper.

[2] Albagli, Elias, Christian Hellwig, and Aleh Tsyvinski (2010). Information Aggregattion and Investment Decisions, Working paper.

[3] Angeletos, George-Marios and Jennifer Lao (2008). Dispersed Information over the Business Cycle: Optimal Fiscal and Monetary Policy, Working Paper.

[4] Bansal, Ravi and Amir Yaron (2004). Risks for the Long run: A Potential Resolution of Asset Pricing Puzzles, Journal of Finance, 59, 14811509.

[5] Barlevy, Gadi and Pietro Veronesi (2003). Rational panics and stock market crashes, Journal of Economic Theory, 110(2).

[6] Barsky, Robert and Bradford DeLong (1993). Why Does the Stock Market Fluctuate, Quarterly Journal of Economics, 108, 291312.

[7] Bernanke, Ben (1993). Irreversibility, Uncertainty, and Cyclical Investment, Quarterly Journal of Economics, 98(1), 85106.

[8] —— and Mark Gertler (1989). Agency Costs, Net Worth, and Business Fluctuations, American Economic Review, 79(1), 1431.

[9] Bertola, Giussepe and Ricardo Caballero (1994). Irreversibility and Aggregate Investment, Review of Economic Studies, 61, 223246.

[10] Bloom, Nick (2009). The Impact of Uncertainty Shocks, Econometrica, 77, 623685.

[11] ——, Stephen Bond, and John Van Reenen (2007). Uncertainty and Investment Dynamics, Review of Economic Studies, 74, 391415.

[12] Bollerslev, T. and H. Zhou (2007). Expected Stock Returns and Variance Risk Premia, Technical Report, Federal Reserve Board Finance and Discussion Series 2007-11.

[13] Bollerslev, Tim, Ray Y. Chou, and Kenneth F. Kroner (1992). ARCH modeling in finance : A review of the theory and empirical evidence, Journal of Econometrics, 52 (1-2), 5 - 59.

[14] Brunnermeier, Markus and Lasse Heje Pedersen (2009). Market Liquidity and Funding Liquidity, Review of Financial Studies, 22 (6), 22012238.

[15] Campbell, John (1991). A Variance Decomposition for Stock Returns, The Economic Journal, 101 (405), 157179.

[16] —— and Albert S. Kyle (1993). Smart Money, Noise Trading and Stock Price Behavior, The Review of Economic Studies, 60 (1), 134.

[17] —— and John Cochrane (1999). By Force of Habit: a Consumption-based Explanation of Aggregate Stock Market Behavior, Journal of Political Economy, 107(2), 205251.

[18] —— and Robert J. Shiller (1988). The Dividend-price Ratio and Expectations of Future Dividends and Discount Factors, The Review of Financial Studies, 1 (3), 195228.

[19] ——, Sanford Grossman, and Jiang Wang (1993). Trading Volume and serial correlation in stock returns, The Quarterly Journal of Economics, 108 (1), 905939.

[20] Chamley, Christophe and Douglas Gale (1994). Information Revelation and Strategic Delay in a Model of Investment, Econometrica, 62 (5), 10651085.

[21] Chen, Qi, Itay Goldstein, and Wei Jiang (2007). Price Informativeness and Investment Sensitivity to Stock Price, Review of Financial Studies, 20 (3), 619650.

[22] Chevalier, Judith A. and Glenn Ellison (1997). Risk-taking by Mutual Funds as a Response to Incentives, Journal of Political Economy, 1997, 55, 11671200.

[23] Chordia, Tarun and Lakashmanan Shivakumar (2006). Earnings and Price Momentum, Journal of Financial Economics, 80, 627656.

[24] Cochrane, John(1992). Explaining the Variance of Price-Dividend Ratios, Review of Financial Studies, 2, 46385.

[25] Coval, Joshua and Erik Stafford (2007). Asset Fire Sales (and Purchases) in Equity Markets, Journal of Financial Economics, 2007, 86, 479512.

[26] Diamond, Douglas and Philip Dybvig (1983). Bank Runs, Deposit Insurance, and Liquidity, Journal of Political Economy, 91(3), 401419.

[27] —— and Robert Verrecchia (1981). Information Aggregation in a Noisy Rational Expectations Economy, Journal of Financial Economics, 9(3), 221235.

[28] Dixit, A. and R. Pindyck. *Investment Under Uncertainty*, Princeton University Press, 1994.

[29] Dow, James and Gary Gorton (1997). Stock Market Efficiency and Economic Efficiency: Is There a Connection?, The Journal of Finance, 52 (3), 10871129.

[30] Dow, James and Rohit Rahi (2003). Informed Trading, Investment, and Welfare, The Journal of Business, 76(3).

[31] Fama, Eugene F. and Kenneth R. French (1989). Business Conditions and Expected Returns on Stocks and Bonds, Journal of Financial Economics, 25 (1), 2349.

[32] Ferson, Wayne and Campbell Harvey (1991). The Variation of Economic Risk Premiums, Journal of Political Economy, 99 (2), 385415.

[33] Froot, Kenneth, David Scharfstein, and Jeremy Stein (1992). Herd on the Street: Informational Inefficiencies in a Market with Short-Term Speculation, Journal of Finance, 47 (4).

[34] Galbraith, John Kenneth (1954). *The Great Crash.* Houghton Mifflin, Boston, New York.

[35] Goldstein, Itay and Alexander Guembel (2008). Manipulation and the Allocational Role of Prices, Review of Economic Studies, 75(1), 133164.

[36] Gorton, Gary and Andrew Metrick (2009). The Run on the Repo and the Panic of 2007-2008, Working paper.

[37] Gromb, Denis and Dimitri Vayanos (2002). Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs, Journal of Financial Economics, 66 (2-3), 361407.

[38] Grossman, Sanford (1976). On the Efficiency of Competitive Stock Markets Where Trades Have Diverse Information, The Journal of Finance, 31 (2), 573585.

[39] —— and Joseph E. Stiglitz (1980). On the Impossibility of Informationally Efficient Markets, The American Economic Review, 70 (3), 393408.

[40] Hansen, Lars and Kenneth Singleton (1983). Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns, The Journal of Political Economy, 91 (2), 249 265.

[41] Hassan, Tarek and Thomas Mertens (2009). The Social Cost of Near-Rational Investment, Working paper.

[42] Hayek, Friedrich (1945). "The use of knowledge in society". American Economic Review 35 (4), 519-30.

[43] Hellwig, Martin (1980). On the aggregation of information in competitive markets, Journal of Economic Theory, 22, 477498.

[44] Holmstrom, B., and P. Milgrom (1987). Aggregation and Linearity in the Provision of Intertemporal Incentives," Econometrica, 55, 303328.

[45] Holmstrom, Bengt and Jean Tirole (1997). Financial Intermediation, Loanable Funds, and the Real Sector, Quarterly Journal of Economics, 112 (3), 663-691.

[46] Hong, Harrison, Terence Lim, and Jeremy C. Stein (2000). Bad News Travels Slowly: Size, Analyst Coverage, and the Profitability of Momentum Strategies, Journal of Finance, 55 (1), 265295.

[47] He, Zhiguo and Arvind Krishnamurthy (2008). A Model of Capital and Crises, Working Paper.

[48] Kyle, Albert (1985). "Continuous Auctions and Insider Trading," Econometrica, 53(6), 1315-35.

[49] ——, Hui Ou-Yang and Bin Wei (2010). A model of Portfolio Delegation and Strategic Trading," Working paper.

[50] —— and Wei Xiong (2010). Contagion as a Wealth Effect, The Journal of Finance, 56 (4), 14011440.

[51] Lettau, Martin and Sydney Ludvigson (2002). Time-varying Risk Premia and the Cost of Capital: An Alternative Implication of the Q theory of Investment, Journal of Monetary Economics, 40 (1), 3166.

[52] Long, J. Bradford De, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann (1990). Noise Trader Risk in Financial Markets, Journal of Political Economy, 98 (4), 703738.

[53] Mertens, Thomas (2009). Excessively Volatile Stock Markets: Equilibrium Computation and Policy Analysis, Working paper.

[54] Morck, Randall, Andrei Shleifer, and Robert Vishny (1990). The Stock Market and Investment: is the Market a Sideshow?, Brookings Papers on Economic Activity, 2, 157215.

[55] ——, Bernard Yeung, and Wayne Yu (2000). The Information Content of Stock Markets: why do Emerging Markets have Synchronous Stock Price Movements?, Journal of Financial Economics, 58 (1-2), 215 260.

[56] Pastor, Lubos and Robert F. Stambaugh (2003). Liquidity Risk and Expected Stock Returns, Journal of Political Economy, 111 (3), 642685.

[57] Rich, Robert and Joseph Tracy (2003). Modeling Uncertainty: Predictive Accuracy as a Proxy for Predictive Confidence, Technical Report, Federal Reserve Bank of New York Staff Reports, no. 161.

[58] Schwert, G. William (1989). Why Does Stock Market Volatility Change Over Time?, The Journal of Finance, 44 (5), 11151153.

[59] Shleifer, Andrei and Robert W. Vishny (1997). The Limits of Arbitrage, Journal of Finance, 52, 3555.

[60] Subrahmanyam, Avanidhar and Sheridan Titman (1999). The Going-Public Decision and the Development of Financial Markets, The Journal of Finance, 54 (3), 10451082.

[61] Van Niewerburgh, Stijn and Laura Veldkamp (2006). Learning Asymmetries in Real Business Cycles, Journal of Monetary Economics, 53, 753772.

[62] Veronesi, Pietro (1999). Stock Market Overreaction to Bad News in Good Times: a Rational Expectations Equilibrium Model, Review of Financial Studies, 12(5), 9751007.

[63] Wang, Jiang (1993). A Model of Intertemporal Asset Prices Under Asymmetric Information, The Review of Economic Studies, 60 (2), 249282.

[64] Warther, Vincent (1995). Aggregate Mutual Fund Flows and Security Returns, Journal of Financial Economics, 39, 209235.

[65] Wermers, Russ (1999). Mutual Fund Herding and the Impact on Stock Prices, Journal of Finance, 54 (2), 581622.

[66] Xiong, Wei (2010). Convergence Trading with Wealth Effects: an Amplification Mechanism in Financial Markets, Journal of Financial Economics, 62 (2), 247 292.

[67] Yuan, Kathy (2005). Asymmetric Price Movements and Borrowing Constraints: a Rational Expectations Equilibrium Model of Crises, Contagion and Confusion, Journal of Finance, 60(1), 379 411.