

# Market penetration and Receiving Party Pays regime\*

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## Abstract

In this paper we study how the access price affects the choice of the tariff regime taken by the network operators. We show that for high values of the access price, that is taken as a parameter by the firms, networks decide to charge only the callers. Otherwise, for low values of the access charge, networks charge also the receivers. Moreover, we compare market penetration and total welfare between the two price regimes. Our model suggests that, for high values of call externality, market penetration and total welfare are larger in Receiving Party Pays regime when access charge is close to zero.

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# 1 Introduction

In order to provide interconnection among all users, telecom networks need access to rival's consumers. Access is provided after the payment of a termination charge (or access price). This charge is a part of the cost of off-net calls and consequently affects the price of calls.

Last years in Europe have seen a growing discussion among regulatory authorities about regulation of the termination charges or the access prices. On the one hand, the European Commission (2008) recommended to lower termination charge in order to lower average price per minute. On the other hand, some of them (e.g. Ofcom, the UK telecommunications regulator) is worried that this could bring network operators to charge consumers for receiving a call or, in other words, to switch from a Calling Party Pays (CPP) regime to a Receiving Party Pays (RPP) regime.

Ofcom has expressed several concerns about the introduction of a RPP tariff regime.<sup>1</sup> The main objections of the UK telecommunications regulatory authority are that it would be disruptive to customers, it would meet with consumer resistance and it might lead to customers turning off their mobile phones. There exist many works about this last concern (e.g. Bomsel et al. (2003), Cadman (2007) and Samarajiva and Melody (2000)) but they are not providing a theoretical background to their analysis. Our intention is to model the two tariff regimes and provide a theoretical framework to compare them.

Littlechild (2006) reports some difference between the two regimes. Some data are summarized in Table 1.

	Min. of use (per months)	Penetration (% , 2005)
<i>RPP countries</i>		
USA	630	61
Canada	359	47
Hong Kong	387	106
Singapore	282	90
<i>CPP countries</i>		
UK	151	104
Germany	76	87
France	225	74
Italy	120	110
Spain	135	99

Table 1: Mobile market structure in selected countries (2005).

In RPP countries minutes of usage are grater than in CPP countries. To understand the reason of that, it is important to notice that RPP countries usually adopt Bill & Keep (BaK) as

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<sup>1</sup>See Oftel (2002) and Ofcom (2005).

interconnection arrangement. This means that network operators pay a price equal to zero (or close to) in order to terminate phone calls. Hence, a BaK policy reduces the marginal costs of traffic and therefore usage prices, driving higher usage.

Data about penetration are ambiguous. In 2005 penetration in US and Canada is far below penetration in EU but in other BaK countries as Hong Kong and Singapore penetration is above EU average. Therefore, these data do not clearly show that penetration is lower in BaK countries. Moreover, in CPP countries penetration might be overstated because of the traditionally greater number of prepaid schemes and multiple SIM cards.<sup>2</sup>

In this paper we provide a theoretical framework to explain these differences and give an interpretation. In particular, we show how the choice of the access price determines the retail pricing regime: our model confirms that CPP is a choice of the telecommunications industry in response to access prices above termination cost. Otherwise for access prices below cost networks prefer to charge receivers as well. Moreover, we compare our predictions for two regimes with actual data in order to give some indications to the regulator. It turns out that with high call externality, BaK arrangement (which is associated to RPP regimes) implies higher usage and higher market penetration. Moreover, BaK maximizes social welfare with respect to any other policy.

From our model it turns out that is very important the assessment of the call externality. Indeed, as the European Commission (2008) says, the welfare-maximizer policy about the access price may be very different for different values of the call externality. For low values our model suggests that the optimal policy is to set the access price close to the termination cost and, consequently, to induce the industry to adopt a CPP price regime. Otherwise, for high values of call externality, the optimal policy should be BaK (in this case the industry would adopt a RPP regime). The reason is that RPP regimes internalize the call externality by making the receiver paying for receiving the call. Hence, when this externality is relevant, RPP regimes are more efficient.

**Related literature.** The literature on telecommunications emerged from the seminal papers by Armstrong (1998), Laffont et al. (1998a) and Laffont et al. (1998b). In their papers they model telecommunications competition between two network operators that compete for consumers that obtain utility only by making calls. Laffont et al. (1998a) analyze network interconnection in an unregulated environment where price discrimination is excluded. They show that for non-linear retail prices, high interconnection tariffs raise final retail prices and reduce social welfare. Gans and King (2001) corrected the above analyses and found that, under price discrimination and non-linear pricing, networks prefer an access price below cost.<sup>3</sup>

These papers inspired many works. Jeon et al. (2004) extend these models and allow consumers to obtain utility by receiving calls. In this framework, networks find optimal to charge customers also for receiving a call. On the one hand, without network based discrimination, networks set

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<sup>2</sup>A report of Analysis Mason (2008, pag. 8) for Ofcom says:

While looking at the comparative statics, it is important to note that the standard penetration data [...] measures the number of subscriptions in circulation, and not the number of users who hold mobile subscriptions, which in the case of Hong Kong, Singapore and the UK is much lower [...].

<sup>3</sup>For a good survey of the literature, see Armstrong (2002).

prices equal to the perceived marginal cost. On the other hand, in presence of network based discrimination, networks set high off-net prices (for high values of the externality, interconnection breaks down) and on-net prices lower than the marginal cost. To avoid multiplicity of equilibria, they introduce a noise in the utility of the receiver.

Cambini and Valletti (2008) use a model where the demand of phone calls between each pair of customers is jointly determined. They show that under certain conditions the connectivity breakdown is eliminated. Moreover, they explain the relationship between the access charge and the structure of the retail prices chosen by the network operators (networks choose to charge the receiver only if the access charge is sufficiently low). Lopez (2008) extends Jeon et al. (2004) in another direction. He introduces a noise also in the utility of the caller. Also in this framework, networks set prices equal to the perceived marginal cost. Moreover, he shows that firm's profits do not depend on the access charge.

Also Hermalin and Katz (2009) allow consumers to obtain utility by receiving calls but, differently from the previous papers, they assume that networks compete on quantities. It turns out that a regulator can not induce efficient off-net prices through the access charge.

In this paper we modify the framework described in Jeon et al. (2004) and we incorporate market expansion in the benchmark model. Under no network-based discrimination, we consider the case where networks charge a strictly positive charge to the receivers (Receiver Party Pays regime) and the case where networks do not charge consumers for receiving a call (Caller Party Pays regime). Therefore, we are able to compare equilibrium prices, market penetration and profits in the two different tariff regimes.

In Section 2 we explain the model. In Section 3 we solve the model and we find the equilibrium prices and quantities in the two tariff regime. In Section 4 we simulate the equilibrium results and we compare the solutions in the two cases. Section 5 concludes.

## 2 The Model

We generalize the model introduced by Jeon et al. (2004) allowing for market expansion.

**Networks.** We consider two mobile networks  $i = 1, 2$  located at two points of an infinite Hotelling line. We normalize to one the distance between the two networks. Mobile networks incur a fixed cost per consumer  $f$  and have on-net call cost of  $c = 2c_0 + c_1$ , where  $c_0$  is the marginal cost of originating or terminating a call and  $c_1$  is the marginal cost of transmitting a call. Let  $a$  denote the access charge or termination charge. The marginal cost of an off-net call is therefore  $c + (a - c_0)$  for the caller's network and  $c_0 - a$  for the receiver's network.

**Tariffs.** Mobile network  $i$  offers a multi-part tariff  $(p_i, r_i, F_i)$  where  $p_i$  is the caller's usage price (notice that we only consider the case of non network-based discrimination),  $r_i$  is the receiver's usage price and  $F_i$  is the fixed part.

**Consumers.** Consumers are differentiated along the Hotelling line. This line represents the preferences of the consumers over one characteristic of the networks. For instance, consumers may prefer the well know phone operator instead of a new one.

A consumer located at  $x$  and selecting network  $i$  incurs a transportation cost equal to  $t|x - x_i|$ .

The utility of placing a call is  $u(q)$ , where  $q$  denotes the length of the call. As in Jeon et al. (2004), we assume that the marginal utility that a receiver derives from receiving a call is subject to a noise  $\varepsilon$  which introduces uncertainty about the willingness to pay for receiving a call independently on the price she is paying.<sup>4</sup> For instance, it could be the case that the receiver is unwilling to talk on the phone when she is driving or working and by considering the noise we have a more realistic model. Hence the receiver's utility is  $\tilde{u}(q) + \varepsilon q$  and we assume that  $\varepsilon$  follows the distribution function  $F$  with support  $[\underline{\varepsilon}, \bar{\varepsilon}]$ , zero mean and density  $f$ . For simplicity we consider  $\tilde{u}(q) = \beta u(q)$ , with  $\beta > 0$ . The length of calls is determined by the first one who interrupts the conversation. The caller equates her marginal utility to the usage price  $p_i$  and she would hang up when her marginal utility is  $p_i$ .

The receiver equates her marginal utility  $\tilde{u}' + \varepsilon$  to the receiving price  $r_i$ . Therefore the receiver will solve  $\beta u' + \varepsilon = r_j$  for  $u'$  and therefore he would hang up when her marginal utility is  $\frac{r_j - \varepsilon}{\beta}$ . Hence, the volume of calls is  $q(\max\{p_i, (r_j - \varepsilon)/\beta\})$ .

The volume of calls is determined by the pair  $(p_i, r_j)$  and a realized value  $\varepsilon$  of the random variable:

$$D(p_i, r_j) = \left[1 - F(r_j - \beta p_i)\right] q(p_i) + \int_{\underline{\varepsilon}}^{r_j - \beta p_i} q\left(\frac{r_j - \varepsilon}{\beta}\right) f(\varepsilon) d\varepsilon. \quad (1)$$

This means that with probability  $[1 - F(r_j - \beta p_i)]$  the caller hang up first and the call lasts  $q(p_i)$  minutes. With probability  $F(r_j - \beta p_i)$  the receiver hang up first and the call lasts  $q(\frac{r_j - \varepsilon}{\beta})$  minutes. Therefore network  $i$  does not know a priori who will be the first one to hang up and consequently, who will determine the length of the call.

The utility of a consumer that joined network  $i$  derives by making calls to a consumer that joined network  $j$ :

$$U(p_i, r_j) = \left[1 - F(r_j - \beta p_i)\right] u(q(p_i)) + \int_{\underline{\varepsilon}}^{r_j - \beta p_i} u\left(q\left(\frac{r_j - \varepsilon}{\beta}\right)\right) f(\varepsilon) d\varepsilon. \quad (2)$$

The utility of a consumer that joined network  $i$  derives by receiving calls from a consumer that joined network  $j$ :

$$\begin{aligned} \tilde{U}(p_i, r_j) = & \int_{r_j - \beta p_i}^{\bar{\varepsilon}} \left[ \tilde{u}(q(p_i)) + q(p_i) \varepsilon \right] f(\varepsilon) d\varepsilon + \\ & + \int_{\underline{\varepsilon}}^{r_j - \beta p_i} \left[ \tilde{u}\left(q\left(\frac{r_j - \varepsilon}{\beta}\right)\right) + q\left(\frac{r_j - \varepsilon}{\beta}\right) \varepsilon \right] f(\varepsilon) d\varepsilon + \end{aligned} \quad (3)$$

Therefore, we can write the net surplus of a consumer that joined network  $i$  as follows:

$$\begin{aligned} w_i = & v_0 + n_i U(p_i, r_i) + n_j U(p_i, r_j) + n_i \tilde{U}(p_i, r_i) + n_j \tilde{U}(p_j, r_i) \\ & - p_i \left[ n_i D(p_i, r_i) + n_j D(p_i, r_j) \right] - r_i \left[ n_i D(p_i, r_i) + n_j D(p_j, r_i) \right] - F_i \end{aligned} \quad (4)$$

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<sup>4</sup>This noise results in a positive probability for both the caller and the receiver of hanging up first.

where  $v_0$  is a subscriber's utility from other mobile services.

The profits of network  $i$  is given by:

$$\begin{aligned} \pi_i = & n_i \left[ n_i(p_i - c)D(p_i, r_i) + n_j[p_i - c - (a - c_0)]D(p_i, r_j) + n_j(a - c_0)D(p_j, r_i) \right. \\ & \left. + r_i(n_i D(p_i, r_i) + n_j D(p_j, r_i)) + F_i - f \right] \end{aligned} \quad (5)$$

where  $n_i(p_i - c)D(p_i, r_i)$  are the profits per user for making on-net calls,  $n_j[p_i - c - (a - c_0)]D(p_i, r_j)$  are the profits per user for making off-net calls,  $n_j(a - c_0)D(p_j, r_i)$  are the profits for terminating off-net calls,  $r_i[n_i D(p_i, r_i) + n_j D(p_j, r_i)]$  are the profits for receiving calls and  $F_i$  is the fixed part of the multi part tariff.

Notice that the expression of the profits takes different forms when the caller or the receiver determines the length of the call. On the one hand, when  $\beta p_i < r_j$  the receiver will hang up first and then the length of the call depends only on the price  $r$ . On the other hand, when  $\beta p_i > r_j$  the caller will hang up first and the length of the call depends only on the price  $p$ . But the network do not know who will be the first one to hang up. To model that, Jeon et al. (2004) introduce a noise in the volume of calls. Therefore, we can maximize the expression of the profits that depends on the noise. In this case the profits are differentiable for all positive prices  $(p_i, r_i)$ .

### 3 The equilibrium

In order to analyze market penetration we consider elastic subscriber participation. Explicitly, we model consumer demand as the Hotelling model with hinterlands.<sup>5</sup> If the two networks offer utilities  $w_1$  and  $w_2$ , then network  $i$  attracts:

$$n_i = \frac{1}{2} + \frac{w_i - w_j}{2t} + \lambda w_i \quad (6)$$

where  $\lambda \geq 0$  represents the magnitude of market expansion possibilities. This is one of the novelties we introduce in our model with respect to Jeon et al. (2004) because it allows us to analyze how different values of the access price affect the equilibrium market penetration and the effects of the latter on welfare. In order to have non explosive market share  $\lambda$  must be small enough.<sup>6</sup> We impose:

$$\lambda < \min \left\{ \frac{1}{2(U^{rpp} + \tilde{U}^{rpp} - cD^{rpp})}, \frac{1}{2(U^{cpp} + \tilde{U}^{cpp} - p^{cpp}D^{cpp})} \right\}. \quad (7)$$

The equilibrium is given by the vector of prices  $(p_i, r_i, F_i)$  that maximize operator  $i$ 's profits as defined by equation (5). The only restrictions we impose are the non-negativity of the prices. When the operator is charging strictly positive prices to its users we have a RPP regime. Otherwise, if

<sup>5</sup>For more details see Armstrong and Wright (2009).

<sup>6</sup>From equations (15) and (20) the derivatives of the market size with respect to the fixed part are

$$\frac{\partial N^{rpp}}{\partial F_i^{rpp}} = -\frac{\lambda}{\gamma} \quad \text{and} \quad \frac{\partial N^{cpp}}{\partial F_i^{cpp}} = -\frac{\lambda}{\delta}.$$

We must have  $\gamma > 0$  and  $\delta > 0$ .

the receiving price is zero we have a CPP regime<sup>7</sup>. Under the assumption of a balanced calling pattern<sup>8</sup>, we characterize the equilibrium prices given the access charge.

### 3.1 The case $a < c_0$ : the Receiving Party Pays regime

Network operators are free to charge costumers for making and receiving a call. Therefore, network  $i$  sets a caller's usage price  $p_i$  and a receiver's usage price  $r_i$  that maximize consumers surplus that will be extracted through the fixed part  $F_i$ . Using the usual maximization procedure, the equilibrium retail prices are:

**Proposition 3.1** (Equilibrium retail prices). *The symmetric equilibrium retail prices  $(p, r, F)$  are:*

$$\begin{aligned} p^{rpp} &= c + (a - c_0) \\ r^{rpp} &= c_0 - a \\ F^{rpp} &= f + \frac{t\phi}{\gamma(a) + [3 + \gamma(a)]\lambda t} \end{aligned}$$

where  $\gamma(a) \equiv 1 - 2\lambda[U^{rpp}(a) + \tilde{U}^{rpp}(a) - cD^{rpp}(a)]$  and  $\phi \equiv 1 + 2\lambda v_0 - 2\lambda f$ .

*Proof.* See appendix. □

Notice that, as in the case of inelastic demand described by Jeon et al. (2004), the usage prices are equal to the perceived marginal cost. Moreover, the fixed part is higher than the fixed fee per user as long as  $\phi > 0$ . Finally, the sum of the usage price is constant and equal to the marginal cost  $c$ . The access charge determines the distribution of cost between caller and receiver. Furthermore, notice that  $\gamma(a)$  represents an inverse measure of the net surplus consumers derive when they join a call in a RPP regime ( $U^{rpp} + \tilde{U}^{rpp} - cD^{rpp}$ ): the bigger is  $\gamma(a)$ , the lower is this surplus.

It is easy to compute the total size of the market and the profits in the symmetric equilibrium:

$$N^{rpp} = \frac{[\gamma + (1 + \gamma)\lambda t]}{\gamma[\gamma + (3 + \gamma)\lambda t]} \phi \quad (8)$$

and

$$\pi_i^{rpp} = \frac{N}{2} [F - f] = \frac{\gamma + (1 + \gamma)\lambda t}{2\gamma[\gamma + (3 + \gamma)\lambda t]^2} t\phi^2. \quad (9)$$

Notice that, in order to have positive equilibrium quantities, we have to impose:

$$\phi > 0 \iff \lambda > -\frac{1}{2(v_0 - f)} \quad \text{if } v_0 > f,$$

that is always verified.<sup>9</sup>

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<sup>7</sup>Other cases where prices other than the receiving one are zero can not be an equilibrium.

<sup>8</sup>This assumption says that the percentage of calls originated and terminated on a given network reflects the market share of this network.

<sup>9</sup>If  $v_0 < f$ , we would have an upper bound for  $\lambda$ :

$$\lambda < \frac{1}{2(f - v_0)}.$$

**Vanishing noise.** As the noise  $\varepsilon$  tends to zero, it can be shown that the caller and the receiver demand the same length of communication when:

$$a = c_0 - \frac{\beta c}{1 + \beta} \equiv a_I.$$

If  $a > a_I$  then the caller is determining the length of the call with probability converging to one (caller sovereignty). Given that the equilibrium calling price is increasing in  $a$ , by reducing the access price the length of the call increases. Otherwise, if  $a < a_I$ , the receiver is determining the length of the call (receiver sovereignty). Since the equilibrium receiving price is decreasing in  $a$ , by increasing the access price the length of the call increases. Hence, at  $a = a_I$ , the call is the longest possible.

Since the access price  $a$  is nonnegative, an equilibrium where both caller and receiver can determine the length of a call exists only if

$$\beta \leq \frac{c_0}{c - c_0} < 1,$$

otherwise in equilibrium we can have only caller sovereignty and the longest call happens at  $a = 0$ .

### 3.1.1 Comparative statics

The regulator of a RPP country sets the access price belows the termination cost of a call. The choice of  $a$  should take into consideration the following results.

Remember that we define  $\gamma(a)$  as an inverse measure of the surplus that users derive when they join a call.

**Lemma 3.1** (Call surplus).  *$\gamma(a)$  is an decreasing function of  $a$  when  $a < a_I$  and an increasing function when  $a > a_I$ .*

*Proof.* See appendix. □

This result implies that the surplus the users derive by making and receiving a call is maximized in  $a = a_I$ .

**Regulatory effects on the fixed fee.** The fixed fee depends on the access price only through the impact of the latter on the net surplus the consumers derive by joining a call. Hence in RPP regimes we do not have a “waterbed effect”: operators do not gain profits by terminating calls and therefore they can not use them to reduce the fixed fee below the fixed cost in order to expand the market. The effect of a variation of the access price on the equilibrium fixed fee is:

$$\frac{\partial F}{\partial a} = \frac{\partial F}{\partial \gamma} \cdot \frac{\partial \gamma}{\partial a}.$$

**Proposition 3.2** (Fixed fee). *A variation of the access price affects the fixed fee through its effect on the “call surplus” ( $\partial F / \partial \gamma$ ). When the access price is set belows the level that makes caller and receiver to hang up at the same time, the fixed fee is increasing in  $a$ . When it is set above such level, the fixed fee is decreasing in  $a$ .*



*Proof.* See appendix. □

This proposition establishes a direct relationship between the surplus of the consumers and the fixed fee. The access price determines the length of a call (and, consequently, the utilities users derive) affecting the net surplus the consumers obtain when they join a call (“call surplus effect”): the higher is this surplus, the higher is the fixed fee operators choose in order to extract it.

**Regulatory effects on the market size.** The size of the market is affected by the choice of the access price through its effect on the call surplus:

$$\frac{\partial N^{rpp}}{\partial a} = \frac{\partial N^{rpp}}{\partial \gamma} \cdot \frac{\partial \gamma}{\partial a}.$$

**Proposition 3.3** (Market penetration). *The size of the market is maximized when the access price is set at the value that makes caller and receiver eager to hang up at the same time.*

*Proof.* See appendix. □

This result indicates that, when it wants to set an access price belows the termination cost, the regulator obtains the largest market by choosing  $a = a_I$ . Notice that at  $a = a_I$  operators are charging the highest fixed fee but the market size is still the largest: the higher surplus of a call more than compensates the consumers for the increased fixed fee.

**Regulatory effects on the industry profits.** The industry profits depend on the access price through its effects on the market size and the fixed fee:

$$\frac{\partial \pi}{\partial a} = \left( \frac{\partial \pi}{\partial N} \frac{\partial N}{\partial \gamma} + \frac{\partial \pi}{\partial F} \frac{\partial F}{\partial \gamma} \right) \frac{\partial \gamma}{\partial a}.$$

**Proposition 3.4** (Profits). *Networks’ profits are maximized when the access price is set at the value that makes caller and receiver eager to hang up at the same time.*

*Proof.* See appendix. □

Profits are an increasing function of the market size and of the fixed fee. According to the previous propositions these variables are maximized at  $a = a_I$ . Therefore operators gain the maximum profits when the regulator chooses an access price is  $a_I$ .

### 3.2 The case $a \geq c_0$ : the Calling Party Pays regime

We consider now the corner solution in which the networks do not charge consumers for incoming calls, therefore  $r$  will be equal to 0. As we saw in the previous section, the price for receiving a call decreases in the access price. When the access charge is equal to the cost of terminating a call ( $a = c_0$ ), the price for receiving a call is equal to zero. For higher values of  $a$ , the equilibrium prices are the corner solution of our maximization problem.

**Proposition 3.5** (Equilibrium retail prices). *When the access price is above termination cost, the equilibrium retail prices are:*

$$\begin{aligned} p^{cpp} &= c + \frac{a - c_0}{2} \\ r^{cpp} &= 0 \\ F^{cpp} &= f + \frac{\delta t - \frac{a - c_0}{2} D[\delta + (2 + \delta)\lambda t]}{\delta[\delta + (3 + \delta)\lambda t] - \lambda(a - c_0)D[\delta + (2 + \delta)\lambda t]} \phi \end{aligned}$$

where  $\delta(a) \equiv 1 - 2\lambda[U^{cpp}(a) + \tilde{U}^{cpp}(a) - p^{cpp}(a)D^{cpp}(a)]$ .

*Proof.* See appendix. □

These equilibrium prices hold when noise vanishes and caller and receiver want hang up at the same time. In this case the marginal utility of making and receiving a call are equal to the calling and receiving prices respectively. Since the noise does not depend on the level of the prices, there is always a strictly positive probability that one of the two hang up first.<sup>10</sup> Moreover notice that, similarly to  $\gamma(a)$  in RPP,  $\delta(a)$  is also an inverse measure of the net surplus of joining a call in CPP ( $U^{cpp} + \tilde{U}^{cpp} - p^{cpp}D^{cpp}$ ).

The total size of the market is:

$$N^{cpp} = \frac{\delta + (1 + \delta)\lambda t}{\delta[\delta + (3 + \delta)\lambda t] - \lambda(a - c_0)D[\delta + (2 + \delta)\lambda t]} \phi.$$

The profits in equilibrium are:

$$\begin{aligned} \pi^{cpp} &= \frac{N}{2} [N(p - c)D + F - f] \\ &= \frac{[\delta + (1 + \delta)\lambda t][\delta - \frac{a - c_0}{2} D\lambda]}{2[\delta[\delta + (3 + \delta)\lambda t] - \lambda(a - c_0)D[\delta + (2 + \delta)\lambda t]]^2} t\phi^2. \end{aligned}$$

In order to have a positive market size the access price has to satisfy:

$$a \leq c_0 + \frac{[\delta + (3 + \delta)\lambda t]\delta}{[\delta + (2 + \delta)\lambda t]\lambda D}. \quad (10)$$

Notice that this condition also implies positive profits in equilibrium.

### 3.2.1 Comparative statics

The regulator in CPP country is setting the access price above the termination cost. It should consider the following results.

**Lemma 3.2** (Call surplus).  *$\delta(a)$  is an increasing function of  $a$ .*

*Proof.* See appendix. □

This result implies that the surplus the users derive by making and receiving a call decreases when the regulator increases the access price.

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<sup>10</sup>In the appendix we consider also the case when the caller determines unilaterally the length of the call. In that case we have a discontinuity in the calling price.

**Regulatory effects on the fixed fee.** The equilibrium fixed fee depends on the access price through two different terms:  $\delta(a)$ , which reflects the surplus a consumer receives by joining a call, and  $T(a) \equiv (a - c_0)D^{cpp}$ , which are the profits an operator gains when it terminates a call. The total effect of a variation of the access price on the equilibrium fixed fee is given by:

$$\frac{\partial F^{cpp}}{\partial a} = \frac{\partial F^{cpp}}{\partial T} \cdot \frac{\partial T}{\partial a} + \frac{\partial F^{cpp}}{\partial \delta} \cdot \frac{\partial \delta}{\partial a}.$$

**Proposition 3.6** (Regulatory effects on the CPP fixed fee). *A variation of the access price affects the fixed fee through the sum of two effects: the “waterbed effect” ( $\partial F/\partial T$ ) and the “call surplus effect” ( $\partial F/\partial \delta$ ). Both effects are negative and their magnitude is increasing and, therefore, in CPP the fixed fee is decreasing in the access price.*

*Proof.* See appendix. □

Here we have two effects. As in the RPP case, the fixed fee increases in the direction the surplus that can be extracted from the consumers increases (call surplus effect). In the CPP case, when the access price diminishes this surplus (and the fixed fee) increases. Moreover here we have also a waterbed effect: the operators are gaining a positive profit by terminating calls. This profit is used to subsidy the fixed fee and to reduce it below the fixed cost in order to expand the market share.

### 3.3 Comments

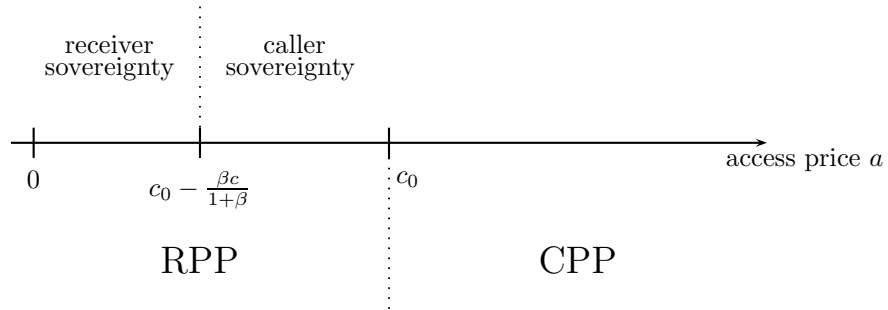


Figure 1: Access price

In our model the telecom industry chooses the retail pricing regime according to the level of the access price. In Fig. 1 we summarize some result. When the access price is below the termination cost  $c_0$  networks want to charge customers for receiving a call. The equilibrium prices are equal to the perceived marginal cost. Moreover, as the noise vanishes, caller and receiver decide to hang up at the same time only when the access price is  $c_0 - \frac{\beta}{1+\beta}c$ . For higher values, the price for calling is higher than the price for receiving and therefore the caller will hang up first (caller sovereignty). For lower values, the receiver will hang up first (receiver sovereignty). Moreover, for higher values of the access price networks charge only the caller.

Finally, we have seen that in RPP the fixed fee is always above the per user fixed cost  $f$  and that there is only a consumer surplus effect: the fixed fee is used to extract the surplus. Conversely, in CPP there is also a waterbed effect: the profits to terminate the call are used to subsidize the fixed cost and the fixed fee is decreasing in the access price.

## 4 Comparison

In this section we compare the two regimes. Table 2 resumes the equilibrium values we found above.

	Receiving Party Pays ( $a < c_0$ )	Calling Party Pays ( $a > c_0$ )
$p$	$c + (a - c_0)$	$c + \frac{a - c_0}{2}$
$r$	$c_0 - a$	0
Fixed part $F$	$f + \frac{t}{\gamma + (3 + \gamma)\lambda t} \phi$	$f + \frac{\delta t - \frac{a - c_0}{2} D[\delta + (2 + \delta)\lambda t]}{\delta[\delta + (3 + \delta)\lambda t] - \lambda(a - c_0)D[\delta + (2 + \delta)\lambda t]} \phi$
Market size $N$	$\frac{\gamma + (1 + \gamma)\lambda t}{\gamma[\gamma + (3 + \gamma)\lambda t]} \phi$	$\frac{\delta + (1 + \delta)\lambda t}{\delta[\delta + (3 + \delta)\lambda t] - \lambda(a - c_0)D[\delta + (2 + \delta)\lambda t]} \phi$
Profits $\pi$	$\frac{\gamma + (1 + \gamma)\lambda t}{2\gamma[\gamma + (3 + \gamma)\lambda t]^2} t \phi^2$	$\frac{[\delta + (1 + \delta)\lambda t][\delta - \frac{a - c_0}{2} D\lambda]}{2[\delta[\delta + (3 + \delta)\lambda t] - \lambda(a - c_0)D[\delta + (2 + \delta)\lambda t]]^2} t \phi^2$

where  $\phi \equiv 1 + 2\lambda v_0 - 2\lambda f$ .

Table 2: Comparison

**Usage price.** The calling price in CPP is always higher than the calling price in RPP. Fig. 2 illustrates the calling price (the thin line) and the receiver price (the thick line). We represent the optimal prices  $p$  and  $r$  according to the possible levels of the access price  $a$ . We chose the cost parameters following Hoernig and Harbord (2010). The dashed line  $c_0$  marks the threshold between the RPP regime (to the left) and the CPP regime (to the right). The dotted line  $a_I$  divides the regions where, as the noise vanishes, we have receiver sovereignty (to the left) and caller sovereignty (to the right).

Remember that the cost of a call is  $c$ . Part of this cost is borne by caller's network and part by the receiver's network. In RPP regime ( $a < c_0$ ) it is efficient to set the total price of a call equal to the marginal cost  $c$ . The access price  $a$  determines how caller and receiver share the cost of the call. In this case the slope of the calling price with respect to the access charge is one.

In CPP the slope of the calling price is smaller. If we allow networks to charge negative prices, therefore, when the access charge is above cost, they would subsidize receiver and they would charge callers the same RPP price. Since in our model we impose strictly positive prices, networks can not subsidize receivers, therefore they use the termination profits ( $\frac{a - c_0}{2}$ ) to reduce calling price. Hence the calling price in CPP will be  $p^{cpp} = c + \frac{a - c_0}{2}$ .

**Length of a call.** As usual, the demand function in terms of length of a call,  $q(\cdot)$ , is a decreasing function of the retail prices ( $q' < 0$ ). Moreover we saw that, as the noise vanishes, the length of the call is determined by the caller with probability converging to one when  $a > a_I$ .

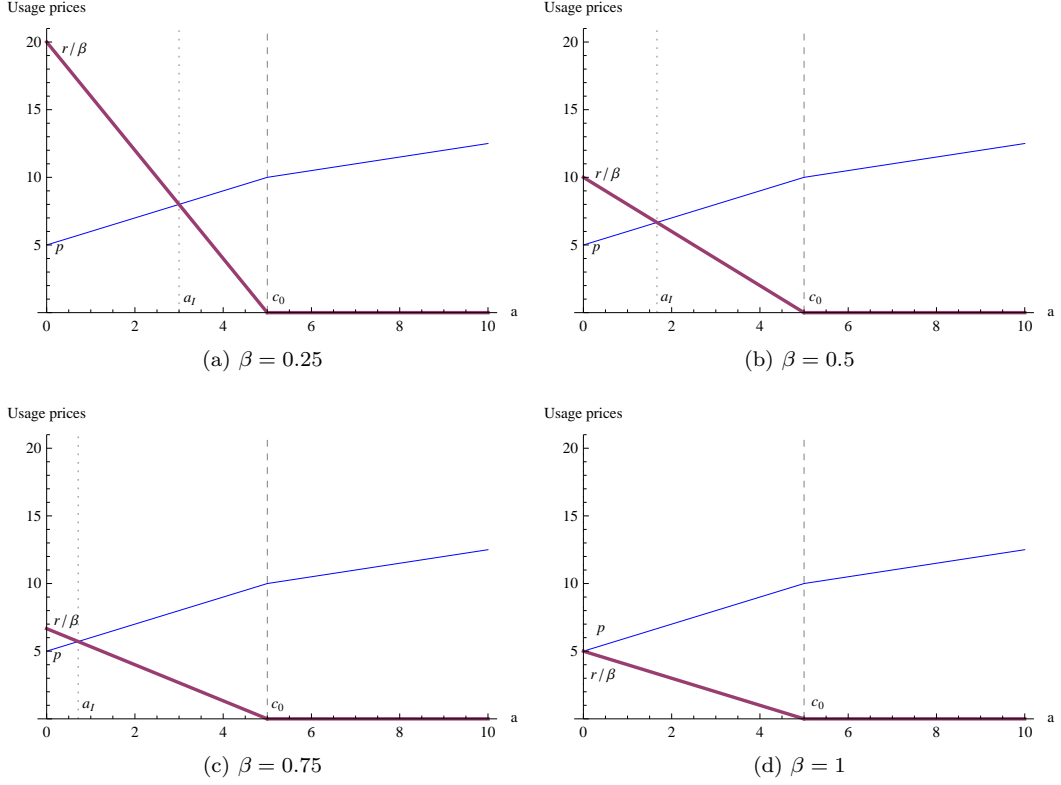


Figure 2: Usage prices  $p$  and  $r$ . Parameter values:  $c_0 = 1$ ,  $c = 2$ .

**Proposition 4.1** (Length of a call.). *Calls last more under RPP regime when calls externality is high enough, i.e.:*

$$\beta \geq \frac{c_0}{c}.$$

*If  $\beta < c_0/c$  calls last more under RPP only if the access price is high enough, i.e.:*

$$c_0 - \beta c < a < c_0.$$

*Proof.* See Appendix. □

This proposition tries to explain some empirical evidence: under RPP regimes the length of calls is significantly longer than under CPP. According with proposition 4.1 calls are longer under RPP if the externality the receivers perceived is high enough.

We represented the length of a call for different values of  $\beta$  in Fig. 3. From now on, we use a constant elasticity demand function  $q(p) = p^{-\eta}$  (as in Hoernig (2007)) where  $\eta > 1$  and  $u(q) = \frac{\eta}{\eta-1} q^{\frac{\eta-1}{\eta}}$ . The dotted line is still representing the threshold between receiver sovereignty and caller sovereignty while the dashed line separates RPP regime from CPP regime.

Notice that the longest length is attained for  $a_I$ , when caller and receiver want to hang up at the same time. The reason is that in RPP the price of a call is shared between caller and receiver, taking into account the positive externality on the receiver and, therefore, the calls tend to be

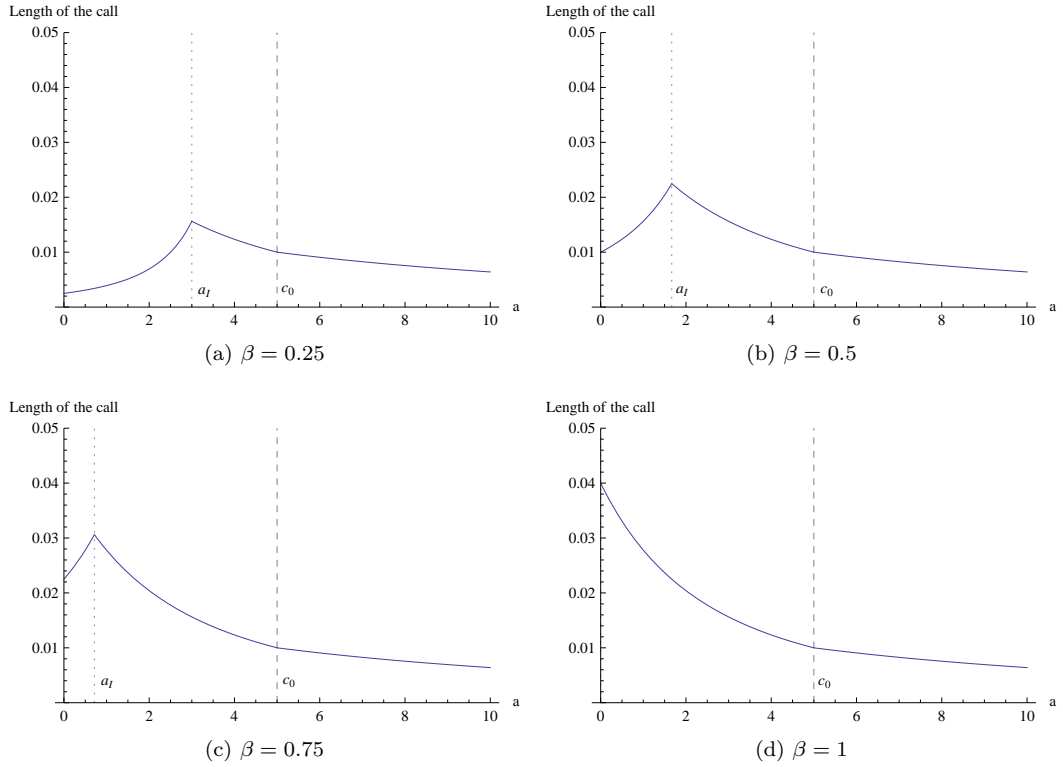


Figure 3: Length of a call  $q$ . Parameter values:  $c_0 = 1$ ,  $c = 2$ ,  $t = 1500$  and  $\eta = 2$ .

longer. But this is true only when caller and receiver are eager to hang up more or less at the same time. Otherwise who is bearing the higher price prefers to end the call earlier and, given that in RPP the variation of the retail prices is steeper, the length of the call drops quicker than in CPP.

For low values of call externality  $\beta$ , BaK produces shorter calls than values of access charge just above termination cost.

For high values of  $\beta$  (Fig. 3c and 3d) access prices close (or equal) to zero imply longer calls in RPP than CPP. The value of  $a$  that makes caller and receiver to hang up at the same time shifts towards zero because the receiver is eager to pay for receiving a call. This allows the regulator to set zero access price and keeping calls longer than CPP regimes. This confirms the expectation of Ofcom (2009, pag. 37):

[...] international comparisons provide evidence that this relationship between termination rates, and take-up and usage, exists. A simple analysis of cross-country data [...] suggests that countries that have, broadly speaking, systems that adopt reciprocity or “bill and keep”-like arrangements – US, Hong Kong and Singapore (and to a lesser degree Canada) have higher usage than countries with “Calling Party’s Network Pays” regimes.

**Fixed part.** Fig. 4 illustrates the comparison between the fixed part in the two price regimes. First, notice that the value of  $\lambda$  is chosen according to equation (7). It is worthwhile to notice

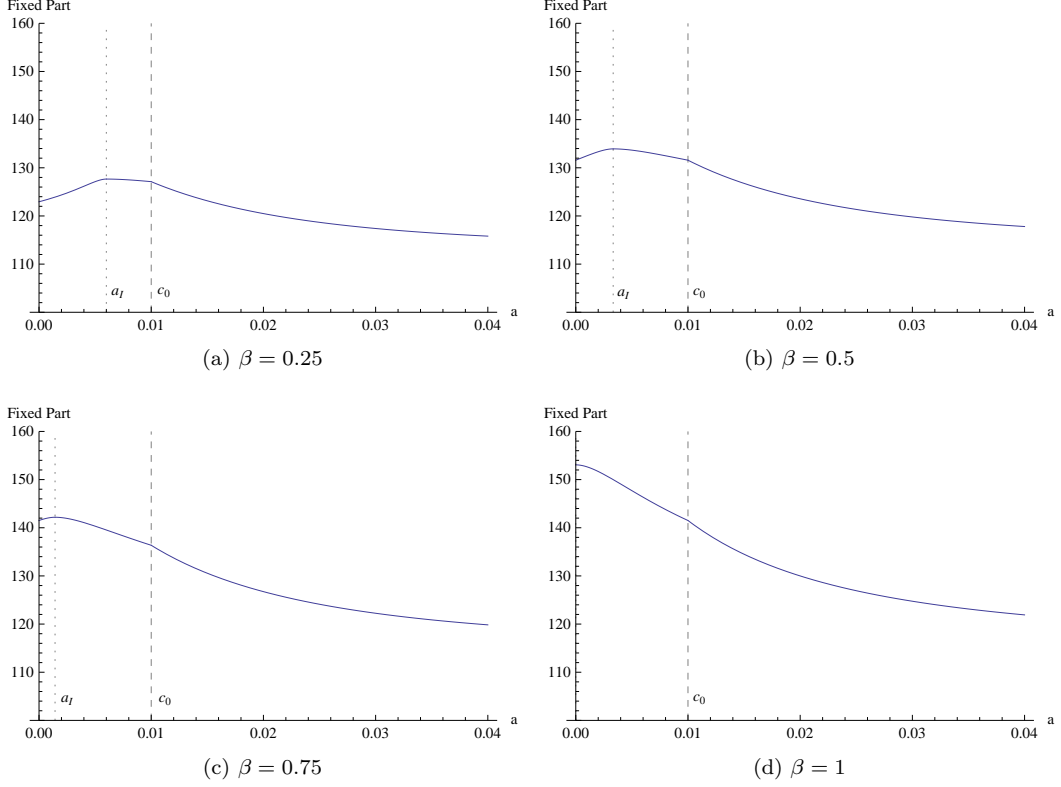


Figure 4: Fixed part  $F$ . Parameter values:  $c_0 = 1$ ,  $c = 2$ ,  $t = 1500$ ,  $\lambda = 0.002$ ,  $\eta = 2$ ,  $f = 0$  and  $v_0 = 250$ .

that at  $a = a_I$  we have the highest fixed part in RPP. The reason is straightforward:  $a_I$  maximizes consumer surplus of joining a call and therefore networks can extract a higher surplus through the fixed part. Indeed this is also the reason why in these graphs higher fixed tariffs are associated to longer calls.

For low values of calls externality, the relationship between equilibrium values in RPP and in CPP is not univocally determined.

For high values of  $\beta$ , fixed part in RPP is higher than fixed part in CPP. In particular, BaK determines higher fixed fee than any other value of the access charge. The reason is that, when access charge is below cost, calls last more, therefore the consumers' surplus that networks can extract is higher. This coincides with many empirical observations. For instance, Ofcom (2009, pag. 37) expects:

High termination rates tend to lead to a retail price structure with relatively high off-net call charges (since operators 'cover' their wholesale cost of each minute of a call with a corresponding retail charge) and lower subscription charges (since subscribers



generate incoming calls that provide call termination revenue). [...] Equally, if termination rates are low, consumers will tend to face higher subscription fees but lower or no charges to make (or receive) calls.

**Market penetration.** Fig. 5 illustrates that there is not a clear relationship between market penetration in the two regimes. For low values of receiver externality, there are values of access charge such that penetration is higher in CPP regimes. Conversely, for high receiver externality, RPP regimes present a high number of subscribers. This indeterminacy is also present in empirical evidence: on the one hand Littlechild (2006) shows how CPP are denoted by higher market penetration, on the other hand Analysis Mason (2008) states that actual data misrepresent true values of penetration by overestimating penetration in CPP countries. Moreover, high penetration is explained through the higher surplus the consumers receive. Once again in RPP we have the highest penetration at  $a = a_I$ .

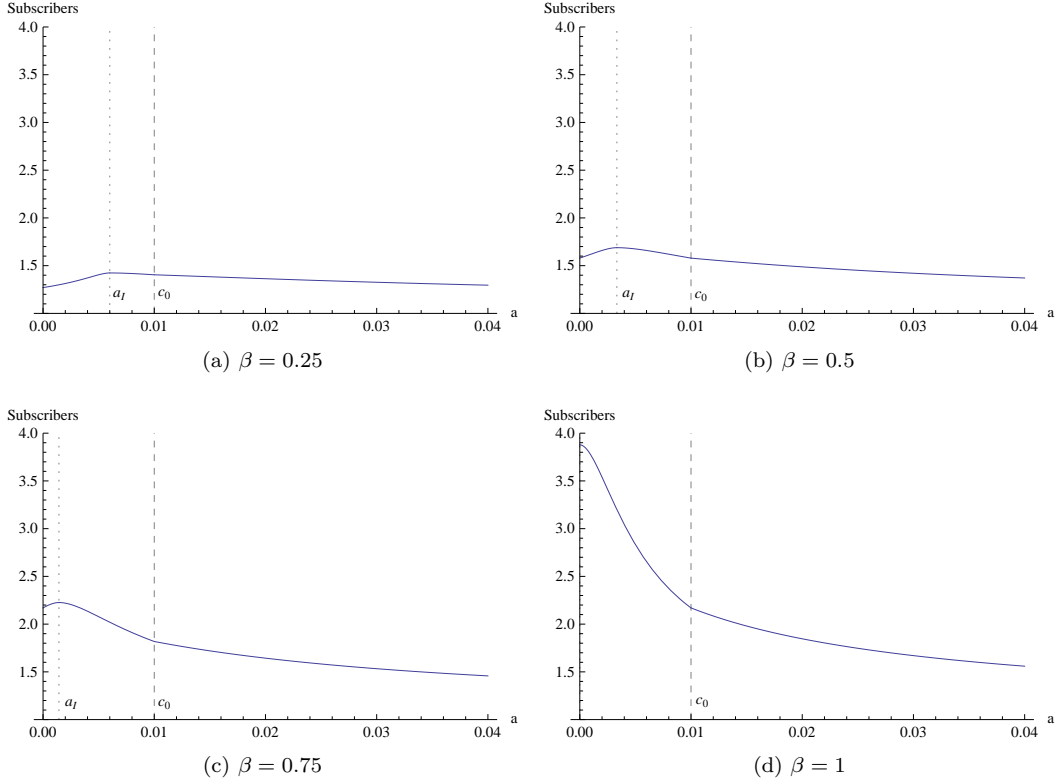


Figure 5: Market penetration  $N$ . Parameter values:  $c_0 = 1$ ,  $c = 2$ ,  $t = 1500$ ,  $\lambda = 0.002$ ,  $\eta = 2$ ,  $f = 0$  and  $v_0 = 250$ .

Graphics show, once again, that RPP regimes are more sensible to variations of the perceived externality: penetration is increasing in  $\beta$ .

## 4.1 Welfare analysis

We compare the welfare in the two regimes in Fig. 6. Total welfare is given by a weighted sum of consumers surplus and industry profits.<sup>11</sup> As it is clear, the highest welfare in RPP is attained at  $a = a_I$ . At this value of the access price, consumer surplus of a call is maximized and the network can obtain the highest profits by extracting it. In CPP the highest welfare is associated to values of the access price close to the termination cost.

Notice that as the receiver externality increases, the welfare is getting higher in RPP regimes. This fact remarks once again the importance for the regulator of having a very precise knowledge of the values of  $\beta$  when choosing the access price: very low values of  $a$  (accompanied by a RPP regime) are socially optimal only if the receiving externality is high. European Commission (2008) ends up to the same conclusion:

RPP might not be efficient if the calling party values the call highly but the called party does not and, as a result, an efficient call might not be completed. The reverse issue may arise in the CPP system, where an efficient call may not be initiated even if the called party values it highly but the calling party does not.

Indeed, if a regulator considers that in its country the externality is very low BaK is not the welfare maximizing policy.

Finally, assigning different weights to consumers surplus and industry profits results do not change qualitatively.

## 5 Conclusions

Regulatory authorities are concerned about reduction of mobile termination rates but there is a lack of theoretical analysis that could give them hints about the consequences of such a policy.

The European Commission (2009) proposed a robust reduction of the mobile termination rates during the next years. In another document, the European Commission (2008, pag. 26) noticed that “RPP may evolve after a reduction of the regulated termination charge or as a response to a Bill and Keep system”. Ofcom (2005) warned that RPP regimes could find the opposition of consumers who do not want to be charged for incoming calls.

In our paper we provide a theoretical framework that is useful to compare the two tariff regimes. We confirm the relationship between interconnection arrangements and retail price structure. It turns out that does not exist one tariff regime superior to the other other in terms of retail prices, usage, market penetration and overall welfare for all values of the access price.

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<sup>11</sup>The consumer surplus is given by the net surplus consumers perceive in equilibrium according with equation (4) minus the total amount of the transportation costs:

$$CS^{rpp} = \left( v_0 + N^{rpp}(U^{rpp} + \tilde{U}^{rpp} - cD^{rpp}) - F^{rpp} \right) N^{rpp} - \left[ \left( \frac{N^{rpp} - 1}{2} \right)^2 + \frac{1}{4} \right] t;$$

$$CS^{cpp} = \left( v_0 + N^{cpp}(U^{cpp} + \tilde{U}^{cpp} - p^{cpp}D^{cpp}) - F^{cpp} \right) N^{cpp} - \left[ \left( \frac{N^{cpp} - 1}{2} \right)^2 + \frac{1}{4} \right] t.$$

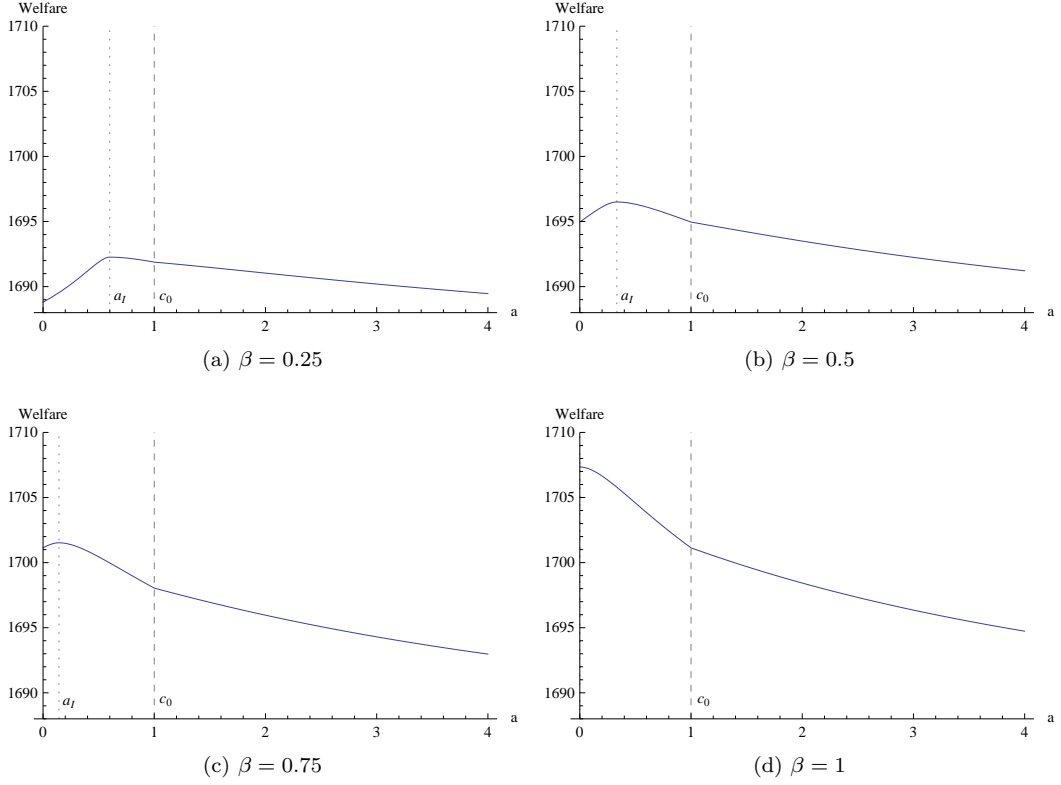


Figure 6: Total welfare  $W$ . Parameter values:  $c_0 = 1$ ,  $c = 2$ ,  $t = 1500$ ,  $\lambda = 0.002$ ,  $\eta = 2$ ,  $f = 0$  and  $v_0 = 1000$ .

Using realistic values of the industry parameters, we find out that the level of the call externality is crucial. When it takes high values, market penetration and total welfare are higher in a RPP regime with access charges close to zero.

## A Proofs

**Proof of Proposition 3.1** To find the usage prices we maximize profits with respect to  $p_i$  and  $r_i$  keeping market share  $n_i$  constant:

$$\begin{aligned} \max_{p_i, r_i} \quad & \pi_i \\ \text{s.t.} \quad & p_i, r_i \geq 0 \end{aligned}$$

We look for the interior solutions where  $p_i, r_i > 0$ . For a given  $n_i$ , the first order derivative of  $\pi_i$  with respect to  $p_i$  when  $r = r_i = r_j$  is:

$$\begin{aligned} & q'[1 - F(r - \beta p_i)]\{(n_i + n_j)(u' - c) - n_j(a - c_0) + n_i(\tilde{u}' + E[\varepsilon|\varepsilon \geq r - \beta p_i]) \\ & - n_i \frac{1}{1 + 2t\lambda}(\tilde{u}' + E[\varepsilon|\varepsilon \geq r - \beta p_i] - r)\} = 0. \end{aligned} \quad (11)$$

Similarly, for a given  $n_i$ , the first order derivative with respect to  $r_i$  when  $p = p_i = p_j$  is:

$$n_i(u' - c) + n_j(a - c_0) + (n_i + n_j)\tilde{u}' + n_i \frac{1}{1 + 2t\lambda}(u' - p) + \frac{E[\varepsilon q' | \varepsilon \leq r_i - \beta p]}{E[q' | \varepsilon \leq r_i - \beta p]} = 0. \quad (12)$$

As the noise vanishes, when the caller and the receiver want to hang up at the same time we have that  $u' = p$  and  $\tilde{u}' = r$ . In a symmetric equilibrium the first order conditions turn out to be:

$$\begin{aligned} p &= (c - r) + \frac{1}{2}(c + a - c_0 - c + r) \\ r &= (c_0 - a) + \frac{1}{2}(c - p - c_0 + a). \end{aligned}$$

Notice that both conditions hold for  $p = c + a - c_0$  and  $r = c_0 - a$ . To find the fixed part of the two-part tariff, we derive profits with respect to  $F_i$ :

$$\begin{aligned} \frac{\partial \pi_i}{\partial F_i} &= \frac{\partial n_i}{\partial F_i} \left[ n_i(p_i - c)D(p_i, r_i) + n_j(p_i - c - (a - c_0))D(p_i, r_j) + n_j(a - c_0)D(p_j, r_i) \right. \\ &\quad \left. + r_i \left( n_i D(p_i, r_i) + n_j D(p_j, r_i) \right) + F_i - f \right] \\ &\quad + n_i \left[ \frac{\partial n_i}{\partial F_i} (p_i - c)D(p_i, r_i) + \frac{\partial n_j}{\partial F_i} (p_i - c - (a - c_0))D(p_i, r_j) + \frac{\partial n_j}{\partial F_i} (a - c_0)D(p_j, r_i) \right. \\ &\quad \left. + r_i \left( \frac{\partial n_i}{\partial F_i} D(p_i, r_i) + \frac{\partial n_j}{\partial F_i} D(p_j, r_i) \right) + 1 \right] \end{aligned}$$

Using equilibrium prices:

$$\begin{aligned} \frac{\partial \pi_i}{\partial F_i} &= \frac{\partial n_i}{\partial F_i} \left[ n_i(p_i - c)D(p_i, r_i) + n_j(a - c_0)D(p_j, r_i) \right. \\ &\quad \left. + r_i \left( n_i D(p_i, r_i) + n_j D(p_j, r_i) \right) + F_i - f \right] \\ &\quad + n_i \left[ \frac{\partial n_i}{\partial F_i} (p_i - c)D(p_i, r_i) + \frac{\partial n_j}{\partial F_i} (a - c_0)D(p_j, r_i) \right. \\ &\quad \left. + r_i \left( \frac{\partial n_i}{\partial F_i} D(p_i, r_i) + \frac{\partial n_j}{\partial F_i} D(p_j, r_i) \right) + 1 \right] \\ &= \frac{\partial n_i}{\partial F_i} [F_i - f] + n_i = 0 \end{aligned}$$

Therefore the fixed part is:

$$F_i = f - \frac{n_i}{\frac{\partial n_i}{\partial F_i}} \quad (13)$$

Combining (4) with (6) we find the total size of the market  $N$  and the number of consumers  $n_i$ : We obtain:

$$\begin{aligned} N &= \frac{1 - \lambda(F_i + F_j - 2v_0)}{1 - 2\lambda(U + \tilde{U} - cD)} \quad \text{and} \\ n_i &= \frac{N}{2} + \frac{(F_j - F_i)(1 + \lambda t)}{2t} \end{aligned} \quad (14)$$

Let us write the market size as follows:

$$N = \frac{1 - \lambda(F_i + F_j - 2v_0)}{\gamma} \quad (15)$$

where  $\gamma(a) \equiv 1 - 2\lambda(U^{rpp} + \tilde{U}^{rpp} - cD^{rpp})$ . The derivative of the market share of network  $i$  with respect to  $F_i$  is:

$$\begin{aligned}\frac{\partial n_i}{\partial F_i} &= \frac{1}{2} \left[ \frac{\partial N}{\partial F_i} - \frac{1 + \lambda t}{t} \right] \\ &= -\frac{1}{2} \frac{\lambda t + \gamma(1 + \lambda t)}{\gamma t} \quad (< 0)\end{aligned}\tag{16}$$

Substituting (14) and (15) into (13) and looking for the symmetric equilibrium, we have:

$$F_i = f + \frac{N}{2} \frac{2\gamma t}{\lambda t + \gamma(1 + \lambda t)}.$$

Solving for  $F$  we obtain:

$$F = f + \frac{t\phi}{\gamma + (3 + \gamma)\lambda t}.$$

**Proof of Lemma 3.1** Remember that  $\gamma(a) \equiv 1 - 2\lambda(U^{rpp} + \tilde{U}^{rpp} - cD^{rpp})$ . Its derivative is:

$$\frac{\partial \gamma}{\partial a} = -2\lambda \left[ \frac{\partial U}{\partial a} + \frac{\partial \tilde{U}}{\partial a} - c \frac{\partial D}{\partial a} \right].$$

Let us first compute the derivative of the volume of calls with respect to the access price.

$$\frac{\partial D}{\partial a} = \frac{\partial D}{\partial p} \frac{\partial p}{\partial a} + \frac{\partial D}{\partial r} \frac{\partial r}{\partial a}$$

where<sup>12</sup>

$$\begin{aligned}\frac{\partial D}{\partial p} &= \frac{\partial F(r - \beta p)}{\partial p} \beta q(p) + [1 - F(r - \beta p)] q' + q(p) f(r - \beta p) (-\beta) = [1 - F(r - \beta p)] q' \\ \frac{\partial D}{\partial r} &= -f(r - \beta p) q(p) + q(p) f(r - \beta p) + \frac{1}{\beta} \int_{\varepsilon}^{r - \beta p} q' f(\varepsilon) d\varepsilon = \frac{1}{\beta} E[q' | \varepsilon \leq r - \beta p] F(r - \beta p).\end{aligned}$$

Hence, we have

$$\frac{\partial D}{\partial a} = [1 - F(r - \beta p)] q' - \frac{1}{\beta} E[q' | \varepsilon \leq r - \beta p] F(r - \beta p).$$

The derivative of the utility derived by making calls with respect to the access price is:

$$\frac{\partial U}{\partial a} = \frac{\partial U}{\partial p} \frac{\partial p}{\partial a} + \frac{\partial U}{\partial r} \frac{\partial r}{\partial a}$$

where

$$\begin{aligned}\frac{\partial U}{\partial p} &= \frac{\partial F(r - \beta p)}{\partial p} \beta u(q) + [1 - F(r - \beta p)] u'(q) q' + u(q) f(r - \beta p) (-\beta) \\ &= [1 - F(r - \beta p)] u'(q) q' \\ \frac{\partial U}{\partial r} &= -\frac{F(r - \beta p)}{\partial r} u(q) + u(q) f(r - \beta p) + \frac{1}{\beta} \int_{\varepsilon}^{r - \beta p} u'(q) q' f(\varepsilon) d\varepsilon \\ &= \frac{1}{\beta} E[u' q' | \varepsilon \leq r - \beta p] F(r - \beta p).\end{aligned}$$

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<sup>12</sup>Hereinafter  $q' < 0$  denotes the derivative of the lenght of a call with respect to the usage price.

Hence, we have

$$\frac{\partial U}{\partial a} = \left[1 - F(r - \beta p)\right] u' q' - \frac{1}{\beta} E \left[ u' q' \middle| \varepsilon \leq r - \beta p \right] F(r - \beta p).$$

The derivative of the utility derived by receiving calls with respect to the access price is:

$$\frac{\partial \tilde{U}}{\partial a} = \frac{\partial \tilde{U}}{\partial p} \frac{\partial p}{\partial a} + \frac{\partial \tilde{U}}{\partial r} \frac{\partial r}{\partial a}$$

where

$$\begin{aligned} \frac{\partial \tilde{U}}{\partial p} &= \tilde{u}' q' [1 - F(r_j - \beta p_i)] + \beta \tilde{u}(q(p_i)) f(r_j - \beta p_i) + q' [1 - F(r_j - \beta p_j)] E[\varepsilon | \varepsilon \geq r_j - \beta p_i] \\ &\quad + \beta q(p_i)(r_j - \beta p_i) f(r_j - \beta p_i) - \beta \tilde{u}(q(p_i)) f(r_j - \beta p_i) - \beta q(p_i)(r_j - \beta p_i) f(r_j - \beta p_i) \\ &= \left( \tilde{u}' + E[\varepsilon | \varepsilon \geq r - \beta p] \right) \left[ 1 - F(r - \beta p) \right] q' \\ \frac{\partial \tilde{U}}{\partial r} &= - \frac{F(r - \beta p)}{\partial r} \tilde{u}(q) + q f(r - \beta p)(r - \beta p) + \tilde{u}(q) f(r - \beta p) + \frac{1}{\beta} \int_{\varepsilon}^{r - \beta p} \tilde{u}'(q) q' f(\varepsilon) d\varepsilon \\ &\quad + (r - \beta p) q(p) f(r - \beta p) + \frac{1}{\beta} \int_{\varepsilon}^{r - \beta p} q' \varepsilon f(\varepsilon) d\varepsilon \\ &= \frac{1}{\beta} E \left[ q' (\tilde{u}' + \varepsilon) \middle| \varepsilon \leq r - \beta p \right] F(r - \beta p). \end{aligned}$$

Hence, we have

$$\frac{\partial \tilde{U}}{\partial a} = \left( \tilde{u} + E[\varepsilon | \varepsilon \geq r - \beta p] \right) \left[ 1 - F(r - \beta p) \right] q' - \frac{1}{\beta} E \left[ q' (\tilde{u}' + \varepsilon) \middle| \varepsilon \leq r - \beta p \right] F(r - \beta p).$$

Hence, the derivative of  $\gamma(a)$  is:

$$\begin{aligned} \frac{\partial \gamma(a)}{\partial a} &= 2\lambda \left\{ \frac{1}{\beta} F(r - \beta p) E[(u'(q) + \tilde{u}'(q) + \varepsilon - c) q' | \varepsilon \leq r - \beta p] \right. \\ &\quad \left. + \left[ 1 - F(r - \beta p) \right] \left[ c - u'(q) - \tilde{u}'(q) - E[\varepsilon | \varepsilon \geq r - \beta p] \right] q' \right\}. \end{aligned}$$

As the noise vanishes we get:

$$\frac{\partial \gamma(a)}{\partial a} = 2\lambda \left\{ \frac{1}{\beta} F(r - \beta p) \left[ u'(q) + \tilde{u}'(q) - c \right] q' - \left[ 1 - F(r - \beta p) \right] \left[ u'(q) - \tilde{u}'(q) - c \right] q' \right\}.$$

Notice that when  $a > a_I$  ( $a < a_I$ ) the caller (the receiver) wants to hang up first and therefore we have  $\tilde{u}'(q) > r$  ( $u'(q) > p$ ). This implies that  $u'(q) + \tilde{u}'(q) - c > 0$ . Moreover remember that  $F(r - \beta p)$  denotes the probability that the receiver hang up first: as the noise tends to zero this probability is equal to 1 in receiver sovereignty and equal to 0 in consumer sovereignty. Finally we have:

$$\frac{\partial \gamma}{\partial a} = \begin{cases} \frac{2\lambda}{\beta} \left[ u'(q) + \tilde{u}'(q) - c \right] q' < 0 & \text{if } a < a_I; \\ -2\lambda \left[ u'(q) + \tilde{u}'(q) - c \right] q' > 0 & \text{if } a > a_I. \end{cases}$$

**Proof of Proposition 3.2** We have:

$$\frac{\partial F}{\partial \gamma} = -\frac{(1 + \lambda t)\phi t}{[\gamma + (3 + \gamma)\lambda t]^2} < 0.$$

In Proposition 3.1 we proved the sign of  $\partial\gamma/\partial a$ . Hence we have:

$$\frac{\partial F}{\partial a} > 0 \quad \text{if } a < a_I \quad \text{and} \quad \frac{\partial F}{\partial a} < 0 \quad \text{if } a > a_I.$$

**Proof of Proposition 3.3** Let us take the derivative with respect to the access price of the market size in (8):

$$\frac{\partial N}{\partial a} = -\frac{\phi[\gamma(1 + \lambda t) + \lambda t]^2 + 2\lambda^2 t^2}{\gamma^2[\gamma + (3 + \gamma)\lambda t]^2} \frac{\partial \gamma}{\partial a}.$$

Notice that the fraction is strictly positive. Therefore the sign of the derivative is the opposite of the sign of  $\partial\gamma/\partial a$  as given in Lemma 3.1.

**Proof of Proposition 3.4** In propositions 3.2 and 3.3 we found that:

$$\frac{\partial F}{\partial \gamma} < 0 \quad \text{and} \quad \frac{\partial N}{\partial \gamma} < 0.$$

Moreover notice that, according to equation (9), we have:

$$\frac{\partial \pi}{\partial N} = \frac{F - f}{2} > 0 \quad \text{and} \quad \frac{\partial \pi}{\partial F} = \frac{N}{2} > 0.$$

Therefore the sign of the derivative of the profits with respect to the access price is the opposite of the sign of  $\partial\gamma/\partial a$  given in Lemma 3.1.

**Proof of Proposition 3.5**

$$\begin{aligned} \max_{p_i, r_i} \quad & \pi_i \\ \text{s.t.} \quad & p_i, r_i \geq 0 \end{aligned}$$

We look for the corner solution of this problem when  $r_i = 0$ . Given market share constant, the first order derivative of the profits in (5) with respect to  $p_i$  is:

We derive the indifference condition  $n_i = 1/2 + \frac{1}{2t}(w_i - w_j) + \lambda w_i$  with respect to  $p_i$  and we obtain:

$$\frac{\partial n_i}{\partial p_i} = \frac{1}{2t} \left[ \frac{\partial w_i}{\partial p_i} - \frac{\partial w_j}{\partial p_i} \right] + \lambda \frac{\partial w_i}{\partial p_i} = \left( \frac{1}{2t} + \lambda \right) \frac{\partial w_i}{\partial p_i} - \frac{1}{2t} \frac{\partial w_j}{\partial p_i} \quad (17)$$

where the derivative of the surplus with respect to  $p_i$  are:

$$\begin{aligned} \frac{\partial w_i}{\partial p_i} = & n_i \frac{\partial U_{ii}}{\partial p_i} + n_j \frac{\partial U_{ij}}{\partial p_i} + n_i \underbrace{\frac{\partial \tilde{U}_{ii}}{\partial p_i}}_{=0} + n_j \underbrace{\frac{\partial U_{ji}}{\partial p_i}}_{=0} - n_i D_{ii} - n_j D_{ij} \\ & - p_i \left[ n_i \frac{\partial D_{ii}}{\partial p_i} + n_j \frac{\partial U_{ij}}{\partial p_i} \right] - r_i \left[ n_i \frac{\partial D_{ii}}{\partial p_i} + n_j \underbrace{\frac{\partial U_{ji}}{\partial p_i}}_{=0} \right] - \frac{\partial F_i}{\partial p_i} \\ \frac{\partial w_j}{\partial p_i} = & n_i \frac{\partial \tilde{U}_{ij}}{\partial p_i} - r_j n_i \frac{\partial D_{ij}}{\partial p_i}. \end{aligned}$$

From (17) we have:

$$\begin{aligned} \frac{\partial F_i}{\partial p_i} = & -\frac{1}{1+2\lambda t} n_i \left[ \frac{\partial \tilde{U}_{ij}}{\partial p_i} - r_j \frac{\partial D_{ij}}{\partial p_i} \right] + n_i \frac{\partial U_{ii}}{\partial p_i} + n_j \frac{\partial U_{ij}}{\partial p_i} + n_i \frac{\partial \tilde{U}_{ii}}{\partial p_i} - n_i D_{ii} - n_j D_{ij} \\ & - p_i \left[ n_i \frac{\partial D_{ii}}{\partial p_i} + n_j \frac{\partial U_{ij}}{\partial p_i} \right] - r_i \left[ n_i \frac{\partial D_{ii}}{\partial p_i} \right] \end{aligned} \quad (18)$$

The derivative of the profits with respect to  $p_i$  is:

$$\frac{\partial \pi_i}{\partial p_i} = n_i \left[ D_{ii} + n_i(p_i - c) \frac{\partial D_{ii}}{\partial p_i} + n_j D_{ij} + n_j[p_i - c - (a - c_0)] \frac{\partial D_{ij}}{\partial p_i} + n_i r_i \frac{\partial D_{ii}}{\partial p_i} + \frac{\partial F_i}{\partial p_i} \right] \quad (19)$$

We substitute (18) into (19). Setting (19) equal to zero we obtain:

$$-n_i c \frac{\partial D_{ii}}{\partial p_i} - n_j(c + a - c_0) \frac{\partial D_{ij}}{\partial p_i} + n_i \frac{\partial U_{ii}}{\partial p_i} + n_j \frac{\partial U_{ij}}{\partial p_i} + n_i \frac{\partial \tilde{U}_{ii}}{\partial p_i} - n_i \frac{1}{1+2\lambda t} \left[ \frac{\partial \tilde{U}_{ij}}{\partial p_i} - r_i \frac{\partial D_{ij}}{\partial p_i} \right] = 0$$

We have:

$$\begin{aligned} & -cn_i(1 - F[r_i - \beta p_i])q'(p_i) - n_j[c + (a - c_0)](1 - F[r_j - \beta p_i])q'(p_i) + n_i(1 - F[r_i - \beta p_i])u'(q)q'(p_i) \\ & + n_j(1 - F[r_j - \beta p_i])u'(q)q'(p_i) + n_i[\tilde{u}'(q) + E(\varepsilon|\varepsilon \geq r_i - \beta p_i)](1 - F[r_i - \beta p_i])q'(p_i) \\ & - n_i \frac{1}{1+2\lambda t} \left[ [\tilde{u}'(q) + E(\varepsilon|\varepsilon \geq r_i - \beta p_i)](1 - F[r_i - \beta p_i])q'(p_i) - r_i(1 - F[r_j - \beta p_i])q'(p_i) \right] = 0 \end{aligned}$$

We look for the symmetric equilibrium where  $r_i = r_j = r$ . We have:

$$(u' - c)(n_i + n_j) - n_j(a - c_0) + \left[ 1 - \frac{1}{1+2\lambda t} \right] n_i [\tilde{u}' + E(\varepsilon|\varepsilon \geq r - \beta p)] + n_i \frac{1}{1+2\lambda t} r = 0$$

We look for a symmetric equilibrium when the caller and the receiver want to hang up at the same time (i.e.  $u' = p$  and  $\tilde{u}' = r$  as the noise vanishes).<sup>13</sup> Solving for  $p$  we finally have:

$$p = c + \frac{a - c_0}{2} - \frac{\lambda t}{1+2\lambda t} \left[ r + E(\varepsilon|\varepsilon \geq r - \beta p) \right] + \frac{1}{1+2\lambda t} \frac{r}{2}$$

Assuming that  $\varepsilon \geq -\beta p$  holds for every value of  $\varepsilon$ , we have that  $E(\varepsilon|\varepsilon \geq -\beta p) = 0$ , so, when  $r = 0$ , the equilibrium calling price becomes:

$$p = c + \frac{a - c_0}{2}$$

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<sup>13</sup>Since the random noise does not depend on the prices, the receiver may want to hang up first even though she does not pay for receiving a call. Otherwise, if we consider the case where the caller decides to hang up with probability one, the marginal utility of the receiver is  $\tilde{u}' = \beta u' = \beta p$ . In this case the equilibrium retail prices become:

$$\begin{aligned} p^{cpp} &= \left( c + \frac{a - c_0}{2} \right) \mu \\ r^{cpp} &= 0 \\ F^{cpp} &= f + \frac{\delta t - (p^{ccc} - c)D[\delta + (2 + \delta)\lambda t]}{\delta[\delta + (3 + \delta)\lambda t] - 2\lambda(p^{cpp} - c)D[\delta + (2 + \delta)\lambda t]} \phi \end{aligned}$$

where  $\delta(a) \equiv 1 - 2\lambda(U^{cpp} + \tilde{U}^{cpp} - p^{cpp}D^{cpp})$  and  $\mu \equiv \frac{1+2\lambda t}{1+2\lambda t + \beta\lambda t}$ . Notice that in this case there is a discontinuity in the calling price at  $a = c_0$ .



Let us now derive the fixed part  $F_i$  and the size of the market. The surplus of the consumer joining network  $i$ , given the equilibrium retail prices, is:

$$w_i = v_0 + N(U + \tilde{U} - pD) - F_i$$

therefore the quantity of consumers that join network  $i$  is:

$$n_i = \frac{1}{2} + \frac{w_i - w_j}{2t} + \lambda w_i$$

$$\frac{1}{2} + \frac{F_j - F_i}{2t} + \lambda[v_0 + N(U + \tilde{U} - pD) - F_i]$$

Adding up the market shares:

$$n_i + n_j = 1 + \lambda[2v_0 + 2N(U + \tilde{U} - pD) - F_i - F_j] \quad \text{solving for } N$$

$$N = \frac{1 - \lambda[F_i + F_j - 2v_0]}{1 - 2\lambda(U + \tilde{U} - pD)}$$

$$N = \frac{1 - \lambda[F_i + F_j - 2v_0]}{\delta} \quad (20)$$

where  $\delta(a) \equiv 1 - 2\lambda(U^{cpp} + \tilde{U}^{cpp} - pD^{cpp})$ .

$$n_i = \frac{N}{2} + \frac{(F_j - F_i)(1 + \lambda t)}{2t}$$

To find the fixed part  $F_i$ , we derive the profits with respect to  $F_i$  and we substitute the equilibrium retail price. We obtain:

$$\frac{\partial \pi_i}{\partial F_i} = \frac{\partial n_i}{\partial F_i} [N(p - c)D + F_i - f] + n_i \left[ \left( \frac{\partial n_i}{\partial F_i} + \frac{\partial n_j}{\partial F_i} \right) (p - c)D + 1 \right] = 0$$

$$\frac{\partial n_i}{\partial F_i} F_i = f - \left[ N(p - c)D + n_i \frac{\partial N}{\partial F_i} (p - c)D + n_i \right]$$

$$F_i^{cpp} = f + \frac{\delta t - (p - c)D[\delta + (2 + \delta)\lambda t]}{\delta[\delta + (3 + \delta)\lambda t] - 2\lambda(p - c)D[\delta + (2 + \delta)\lambda t]} \phi$$

**Proof of Lemma 3.2** This proof is similar to the proof of Lemma 3.1. Here we get:

$$\frac{\partial \delta}{\partial a} = -2\lambda \left\{ [1 - F(r - \beta p)] (u' + \tilde{u}' - p)q' - \frac{D^{cpp}}{2} \right\} > 0.$$

**Proof of Proposition 3.6** We analyze the four derivates separately. The sign of  $\partial F / \partial T$  is given by:

$$\text{sign} \left( \frac{\partial F}{\partial T} \right) = \text{sign} (-(\lambda t + \delta + \delta \lambda t)(2\lambda t + \delta + \delta \lambda t)) < 0,$$

that is, there is a waterbed effect.

Moreover, we have:

$$\frac{\partial T}{\partial a} = D + (a - c_0)D' > 0 \iff a < c_0 - \frac{D}{D'}.$$

Notice that  $D' < 0$  and, therefore, for all values of the access price bigger than  $c_0$ , i.e., the values of  $a$  that determines a CPP regime, the profits for terminating a call are increasing in  $a$  until

$a < c_0 - D/D'$ . Without restricting too much our analysis we can assume a regulator is interested in the values of  $a$  close to  $c_0$  and that satisfy this condition.

The sign of  $\partial F/\partial \delta$  is:

$$\text{sign}\left(\frac{\partial F}{\partial \delta}\right) = \text{sign}\left(-2t(1 + \lambda t)\delta^2 + [2\lambda^2 t^2 + (1 + \lambda t)(4\lambda t + \delta + \delta\lambda t)\delta]\right) < 0$$

when condition (10) holds, that is, for all the values of  $a$  that makes it profitable for an operator to stay in the market. This implies a positive consumer surplus effect.<sup>14</sup>

In the proof of Proposition 3.2 we proved that  $\partial \delta/\partial a > 0$ .

**Proof of Proposition 4.1** The condition on  $\beta$  comes comparing the higher possible receiving price<sup>15</sup> ( $r = c_0$ ) with the lower possible price in CPP ( $p = c$ ):

$$\frac{c_0}{\beta} < c \iff \beta > \frac{c_0}{c}.$$

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<sup>14</sup>Remember that  $\delta(a)$  represents an inverse measure of the surplus derived by the consumers when they join a call.

<sup>15</sup>Remember that the marginal utility of the receiver has to be equal to  $r/\beta$  in order to maximize her utility.

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