Participation Quorums in Costly Meetings

Sabine Flamand Orestis Troumpounis

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Abstract

In most meetings of shareholders, members of societies, and clubs, the number of attendees must exceed an exogenously given participation requirement in order for a decision to be taken. Otherwise, the meeting has to be postponed. In order to understand the effect of such a participation requirement on individuals' behavior and the decision outcome, we model a setup of repeated meetings based on Osborne et al. (2000). We show that the decision is delayed when the quorum requirement is high and members are not harmed by postponing the decision. If this is the case, the number of attendees taking the decision may be smaller than in the no-quorum case. Finally, we show that in order to avoid policy distortions, the required number of participants should be even.

1 Introduction

In any kind of decision making processes where a participation quorum requirement applies, the decision is valid only if participation is higher than a given level. Examples are various and include large elections such as referenda and presidential elections, but even more frequently smaller scale decision making meetings such as meetings of faculty members, shareholders, neighbors, or members of many societies, and clubs[1].

Although economists have extensively studied alternative voting rules, the use of a participation quorum has received very little attention. Recently, different authors (Maniquet and Morelli, 2010; Herrera and Mattozzi, 2009; Aguiar-Conraria and Magalhães, 2010,; Houy, 2009) have studied the effect of a participation quorum in referenda. In this case, when not enough voters participate,

[1]The voting procedure in shareholders' meetings varies among most countries and is determined by each county's corporate law (Dornseifer, 2005).

the referendum is considered to be invalid and the outcome is a reversal to the status quo. On the contrary, in decision making meetings, in case the quorum is not fulfilled, the meeting has to be repeated since a decision must be taken (i.e. in contrast to the case of referenda there is no reversal to the status quo). The importance of analyzing such a situation lies in the fact that, although such a participation requirement is widely applied, as we show, it often has negative welfare implications, including policy distortions.

We build on the model of costly meetings by Osborne et al. (2000)[2]. In the general setup, which may apply to any meeting and which we refer to as the No-Quorum Game, a group of individuals has to decide on a policy through a decision making meeting. Each member of the group independently decides whether to participate, at a cost, in the meeting, whose outcome is assumed to be a compromise between the participants' favorite policies. In deciding whether to attend or not, each member compares the cost of attending with the impact of her presence on the policy decision.

Similar to Bulkley et al. (2001), we characterize the Nash equilibrium of the one-shot *attendance* game by using a constructive approach as follows: We assume a sequential exit process that starts with the full set of attendees, and at each step of which the individual with the largest benefit from not attending the meeting is the one who exits. The process finishes when no further attendee is willing to exit. We fully characterize this exit process, and we argue that it ends at a unique Nash equilibrium belonging to the set of equilibria of the original Osborne et al. (2000) one-shot attendance game. Through this exit process, we are able to characterize the unique Nash equilibrium of any such attendance game, which consists in a set of attendees and the corresponding policy decision.

More importantly, in order to extend the attendance game to decision making meetings for which a participation quorum applies (which we refer to as the *quorum* game), we introduce two additional notions: the participation quorum itself, and how much members are harmed by the fact that a decision is delayed in case the meeting has to be postponed. Our ultimate goal is to analyze the effect of such a participation quorum on the equilibrium set of attendees, the policy decision and on total welfare.

As in Osborne et al. (2000), the equilibrium of the No-Quorum game that is reached through the exit process is such that moderates tend to abstain, while

[2]For other applications of Osborne et al. (2000), see, for example, Aragones and Sánchez-Pagés (2009)

extremists tend to attend. Introducing a participation quorum in this setup leads to several insights. First, there is no situation in which some individuals show up in the first meeting and the quorum is not met. That is, either the participation quorum is fulfilled and the decision is taken in the first meeting, or everyone abstains in the first meeting and the decision is taken in the second one.

Second, the introduction of a quorum might either increase or decrease the number of attendees in equilibrium, depending on whether the second meeting is ever reached. More specifically, if the quorum requirement is higher than the number of members who would attend if there was no quorum, the decision is taken in the first meeting when individuals are harmed “enough” by delaying the decision. In that case, the quorum is binding, and its introduction yields to an increase in the equilibrium number of attendees with respect to the no-quorum case. Conversely, if the second meeting is ever reached, and provided that individuals value strictly less a decision that has been postponed, the introduction of a quorum has the opposite effect of lowering the number of attendees in equilibrium.

Third, given that we focus on symmetric individuals’ favorite policies, introducing a quorum has in most cases no effect on the chosen policy, no matter whether the second meeting is reached or not. In terms of welfare, it turns out that introducing a quorum, while having no direct policy effects, always yields to a loss of aggregate welfare, even in the cases for which less individuals attend the meeting as compared to the no-quorum equilibrium.

However, there does exist a situation in which the introduction of a quorum might yield to a policy distortion, this situation being that the quorum is odd and strictly higher than the number of attendees would there be no quorum. Intuitively, such a situation might, in some cases, allow an individual to free ride on the attendance of another committee member. As a result, the equilibrium policy ends up being biased. This result clearly has negative implications from a welfare perspective. Indeed, not only does the introduction of a quorum force more individuals to attend the meeting, which, given that the policy remains unaffected, is always a pure welfare loss, but it might also yield to policy distortions when it is odd, which is even worse in terms of aggregate welfare.

The paper is structured as follows: In Section 2, we present the model and we describe formally the attendance and the quorum game. In Section 3 we present the results of our analysis and in Section 4 we conclude. All proofs can be found in the Appendix.

2 The Model

2.1 The Setup

The policy space is continuous, one-dimensional, and represented by the interval $[0, 1]$. There is a finite group of $N \geqslant 2$ individuals who must collectively choose a policy, that is, a point $x \in [0, 1]$. We denote individual i's favorite policy/position by $x_i \in [0, 1]$. We assume that individuals' preferences are single-peaked and uniformly distributed in the $[0, 1]$ interval, so that the distance between the ideal policy of any two individuals is $d = 1/(N-1)$.

Each individual cares about the remoteness (but not the direction) of the collectively chosen policy x from his favorite policy x_i. Specifically, let individual i's valuation of the distance between policy x and his ideal point x_i be $V_i(|x_i - x|)$, where $V : [0, 1] \to [0, 1]$ is a continuous function. We assume that with respect to the distance between x and x_i, V is strictly increasing, and strictly convex. Furthermore, notice that it is symmetric with respect to x_i, that is, the valuation of a policy x depends only on the distance between x and x_i and not on the direction in which x differs from x_i.

Each individual chooses whether or not to attend a meeting, at which a policy is to be selected. Hence, the available actions of individual i are either to attend the meeting or to abstain. Every individual who attends a meeting bears a cost $c > 0$. The final utility of individual i is then given by

$$U_i = 1 - V_i(|x_i - x|) - c\alpha_i$$

where $\alpha_i = 1$ if i attends the meeting (and so pays a cost c) and $\alpha_i = 0$ if i abstains. Therefore, $U(.)$ is strictly decreasing and strictly concave with respect to the distance between x and x_i.

The incentive of every individual to attend a meeting lies in the fact that he can affect the chosen policy to some extent. For simplicity, we assume that the chosen policy x is the median of the favorite policies of the members who decide to attend[3]. Furthermore, given the symmetric distribution of individuals' preferences in $[0, 1]$, we assume that the default policy (i.e. the one chosen when everyone abstains) is given by $x = \frac{1}{2}$. This is the most natural assumption, since

[3]In case the number of attendees is even the median is defined by $\frac{x_{\frac{N}{2}} + x_{\frac{N}{2}+1}}{2}$

any other choice for the default would mean introducing an arbitrary bias in favor of some individuals.

2.2 The Exit Process

The attendance game of Osborne et al. (2000) is a one-shot game in which all individuals simultaneously decide whether to abstain or to attend a (costly) meeting. In this setup, there may exist multiple pure strategy Nash equilibria, where in general, moderates tend to abstain, while extremists tend to pay the cost and attend[4].

Given that that the ultimate goal of our paper is to study the effect of an exogenously given (quorum) voting rule on the decision process, the multiplicity of equilibria may be problematic, as it may not allow us to draw sharp comparisons among different quorum rules regarding the decision outcome. In order for the model of Osborne et al. (2000) to serve our purposes, we are interested in an equilibrium refinement such that the Nash equilibrium of the attendance game is unique.

Similar to Bulkley et al. (2001), we assume that the attendance decision of each member is determined by a sequential exit process. At each step of the process, given any set of attendees, we assume that the attendee to exit is the individual with the highest potential benefit from doing so. Our exit process begins with the full set of individuals attending, and continues successively until no attendee has an incentive to exit.

Formally, let A be any set of attendees with decision outcome M. For some $i \in A$, let $A' = A \setminus \{i\}$ with decision outcome M'. Let the benefit of exit of attendee i be given by the following function:

$$b_i(A) = U_i^{Exit} - U_i^{Attend} = V_i(|x_i - M|) - V_i(|x_i - M'|) + c$$

Let E be the set of attendees with a positive benefit of exiting (i.e. $E = \{i : i \in A \text{ and } b_i(A) > 0\}$). The exit process is defined as follows:

The process starts with the full set of individuals attending (i.e. $A = \{1, 2, ..., N\}$) and proceeds in the following way:

For any committee A:

1. If $E = \emptyset$ then the process terminates;

[4]For the issue of multiplicity of equilibria, see Dhillon and Lockwood (2002)

2. If $E \neq \emptyset$ then attendee j exits, where j is defined by: $b_j(A)$ be the maximal element of $\{b_i(A)\}, i \in E$;

3. If there are more than one maximal elements in E, then the attendee who exits is drawn randomly among the ones having the maximal $b_i(A)$.

Intuitively, as in Osborne et al. (2000), when an attendee decides to exit, he does not have to pay the cost of attending (i.e. c), while he suffers some disutility in terms of the decision outcome since as a result of exiting, the decision moves further from his ideal policy (i.e. $V_i(|x_i - M|) - V_i(|x_i - M'|)) < 0$). Notice that at each step of the exit process, several attendees may have a positive benefit from exiting. Having assumed that at each step the attendee to exit is the one who has the highest benefit from doing so, and given that all individuals have to pay the same cost in case of attending, the individual that actually exits is the attendee with the smallest disutility from doing so in terms of the policy outcome (i.e. the attendee with the lowest $V_i(|x_i - M'|) - V_i(|x_i - M|))$).

One can think of the above situation as following: Suppose that the committee members meet in a room where the meeting has to take place. Once in the room, the individuals have to decide whether to leave the room or to stay and pay the (opportunity) cost of attending the meeting, and thus influence to some extent the decision taken. Given such a situation, and for a given cost, we assume that the first individual to leave the room is the one for whom the benefit of leaving is the highest. After the first individual has left, the next member to exit (if any) is the one with the highest benefit from leaving given the remaining set of people in the room, and so on.

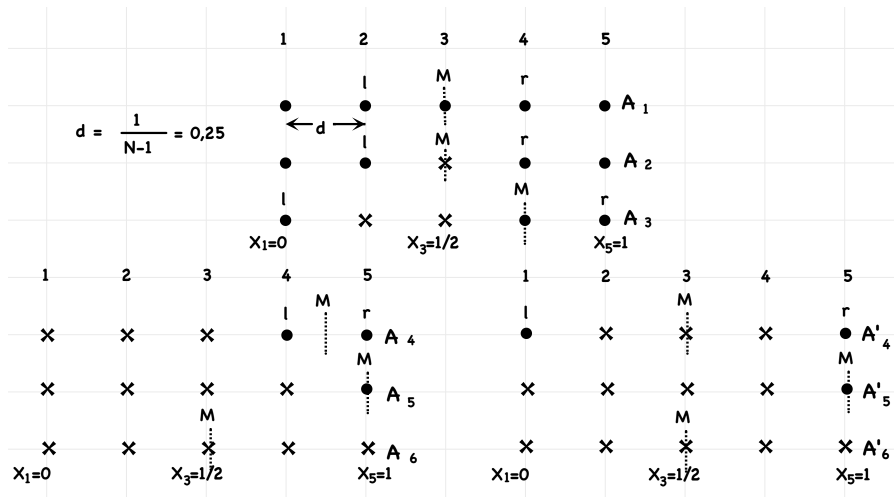

Figure 1: An Example of a 5 members committee. We denote by x a member who abstains, and with a circle a member who attends.

Example 1. *In order to illustrate how a decision is reached given the exit process defined above, suppose* $N = 5$*, so that we start with a (full) set of 5 attendees as in situation* A_1*. Let* M *denote the chosen policy for any given set of attendees, and let* M' *denote the new chosen policy resulting from the exit of some attendee from the committee. Then, for any given set of attendees, let* m *denote the attendee located at* M *(when the number of attendees is odd), and let* l *and* r *denote the first attendees on the left and on the right of* M *respectively. Consider situation* A_1 *in the figure. In that situation, we have that* $M = \frac{1}{2}$*, which coincides with the location of* $m = 3$*, while* $l = 2$ *and* $r = 4$*. If* m *exits the committee, the policy remains at* $\frac{1}{2}$*, that is,* $M = M'$*, so that* m *is willing to exit whenever* $V_m(0) > V_m(0) - c$*, or, equivalently,* $c > 0$*. If any* $i \neq m$ *exits, he would suffer a strictly positive loss in terms of distance (i.e.* $V_i(|x_i - M|) - V_i(|x_i - M'|) < 0$ *for all* $i \neq m$*). Hence,* m *is the first attendee to leave (i.e.* $b_3(A)$ *is the max element of* E*), so that we go to situation* A_2*.*

At A_2*, observe that both* l *and* r *are willing to leave if and only if* $V(\frac{1}{4}) > V(\frac{1}{2}) - c$*, or, equivalently,* $c > V(\frac{1}{2}) - V(\frac{1}{4})$*. If this inequality is not satisfied, the exit process stops here, and we are at an equilibrium (with the corresponding policy being* $M = \frac{1}{2}$*). Suppose now, on the contrary, that the inequality is satisfied so that* l *and* r *want to leave. Notice then that at this stage, both attendees 1 and 5 might also want to exit. However, their disutility in terms of distance* *from doing so is given by* $V(\frac{3}{4}) - V(\frac{1}{2}) > V(\frac{1}{2}) - V(\frac{1}{4})$ *by the strict convexity of* $V(.)$*. Therefore, the disutility from leaving is strictly higher for* l *and* r *than it is for* 1 *and* 5*, so that either one of the formers leaves first. Suppose, WLOG, that* l *exits, so that we go to* A_3*.*

At A_3*, it is clear that* $m = 4$ *would exit before* $r = 5$*. Indeed,* m *wants to exit if and only if* $c > V(\frac{1}{4})$*, which is true since we have assumed at stage* A_2 *that* $c > V(\frac{1}{2}) - V(\frac{1}{4})$ *and V(.) is strictly convex. Hence,* m *wants to leave. Then, observe that the potential disutility of* r *from exiting is given by* $V(\frac{5}{8}) - V(\frac{1}{4}) > V(\frac{1}{4})$*, so that* m *exits before* r*. Now, attendee* l *wants to exit if and only if* $c > V(\frac{7}{8}) - V(\frac{3}{4})$*. As we cannot compare the potential disutility from exiting between* m *and* l*, either of the two might leave first.*

Suppose that $V(.)$ *is such that* m *exits before* l*, so that we are at stage* A'_4*. Again, observe that both* $l = 1$ *and* $r = 5$ *have the same incentives to leave the committee, that is, they want to exit if and only if* $c > V(1) - V(\frac{1}{2})$*. If this inequality is not satisfied, the exit process stops here, and we are at an equilibrium. Indeed, neither* l *nor* r *want to exit, and none of the abstainers want to attend: it is direct that neither* 4 *nor* 2 *(by symmetry) want to attend, as they just left. Then, it is also direct that abstainer* 3 *has no incentive to attend, as there would be no effect on the policy (which is already located at his ideal point) while he would have to bear the cost of attending. Hence, as no individual wants to deviate, we are at an equilibrium (with the corresponding policy being* $M = \frac{1}{2}$*). Suppose now, on the contrary, that* $c > V(1) - V(\frac{1}{2})$*, and suppose, WLOG, that* l *leaves, so that we reach* A'_5*.*

At A'_5*, only attendee* m *is left, and the chosen policy* M *is at his ideal point. As* $c > V(\frac{1}{2})$*,* m *leaves, and we reach* A'_6*, at which no attendee is left. As no abstainer wants to attend, we are at an equilibrium. Indeed, it is direct that neither* 5 *nor* 1 *(by symmetry) want to attend, as they just left. Then, it follows directly that neither* 2 *nor* 4 *want to attend either, as their potential benefit from doing so is strictly lower than it is for* 1 *and* 5*. Finally, as* M *is already located at* 3*'s ideal point, he has no incentive to attend either. Therefore, we are at an equilibrium, and the chosen policy is the default (i.e.* $M = \frac{1}{2}$*).*

Suppose now that at A_3*,* $V(.)$ *is such that* l *exits before* m*, so that we go to* A_4*. The policy* M *is now located at* $\frac{7}{8}$*, and both* l *and* r *want to exit if and only* *if $c > V(\frac{1}{4}) - V(\frac{1}{8})$. As we have assumed that $c > V(\frac{7}{8}) - V(\frac{3}{4})$ at the previous stage, it follows directly that $c > V(\frac{1}{4}) - V(\frac{1}{8})$, so that both l and r want to leave. Suppose, WLOG*[5] *that l leaves, so that we go to stage A_5, which is identical to stage A'_5. From there on, we just saw that we end up at an equilibrium at which no attendee is left, and at which the chosen policy is the default (i.e. $M = \frac{1}{2}$).*

2.3 The Quorum Game

In order to study the effect of a participation quorum, we introduce two additional ingredients in the attendance game: the quorum itself, and how much individuals discount the fact that the decision may be delayed in case the meeting has to be postponed.

Regarding the participation quorum, we start the analysis with the simplest case. In particular, we assume that in the first meeting, the number of attendees must be at least $Q \in (0, N]$ in order for a policy to be chosen. In case the attendees of the first meeting do not fulfill the quorum, the meeting has to be repeated. We assume that in the second meeting no participation quorum applies.

Individual i's valuation of the policy chosen (in terms of final utility) in the second meeting is given by $\beta(1 - V_i(|x_i - x|))$, where $\beta \in [0, 1]$ denotes the loss in terms of utility that individuals suffer as a result of the decision being delayed. In situations where the final decision is the same no matter if delayed or not, a low β captures the idea that individuals are in a hurry to take the decision the soonest possible. On the contrary, individuals with a high β do not really care about postponing the decision to the following meeting, while individuals with $\beta = 1$ are indifferent whether the decision is taken in the first or in the second meeting.

- **Stage 1** *(1st Meeting):* Following the exit process defined above, individuals decide whether to attend the meeting or not. If the number of attendees is larger than Q, then a policy x is decided according to the compromise function (here the median). Each individual gets final utility $U_{1i} = 1 - V_i(|x_i - x|) - c\alpha_i$. If the quorum is not met, the game goes to stage two.

[5] This is WLOG because we are at the 5 individuals' example. Otherwise, it wouldn't be general to assume that l leaves first at this stage. In any case, this is of little importance, as it turns out that no matter what is the number of individuals, the structure of the equilibrium that is reached by applying the exit process is always the same (see section 3).

- **Stage 2** *(2nd Meeting):* Following the exit process defined above, individuals decide whether to attend the meeting or not. No matter what is the number of participants this time, a policy x is to be chosen according to the same compromise function. Each individual gets final utility $U_{2i} = \beta(1 - V_i(|x_i - x|)) - c\alpha_i$.

3 Results

3.1 The Benchmark: The No-Quorum Game

In this section, we present the results for a one-shot attendance game for which no quorum applies. Naturally, these results are similar to the ones described in Osborne et al. (2000). To be more precise, our results characterize a unique equilibrium that belongs to the set of equilibria of the original work by Osborne et al. (2000). The novelty of our results lies in the fact that the equilibrium we obtain is unique, thanks to the equilibrium refinement through the defined exit process we based on Bulkley et al. (2001).

Given the exit process we have defined previously, the following lemmas apply for the equilibrium:

Lemma 1. *The equilibrium number of attendees is even.*

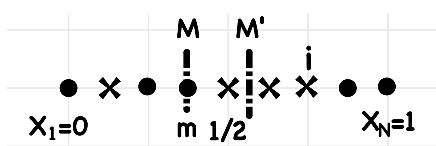

Figure 2: The equilibrium number of attendees cannot be odd.

Lemma 1 is intuitive. In any situation where the number of attendees is odd (see Figure 2), the policy is located at attendee m's ideal point. Such a situation cannot be an equilibrium, since the non-attendee i who mirrors m with respect to M' is willing to attend if m does so (M' being the new policy would m leave). Indeed, observe that the policy moves by the same distance Δ would m leave or i attend (i.e. it goes to M'). However, given that Δ occurs at a further distance from i than it does from m, and given that $V(.)$ is strictly convex, i always wants to attend provided that m attends.

Lemma 2. *The equilibrium is such that there are no gaps between two given attendees on each side of* $\frac{1}{2}$.

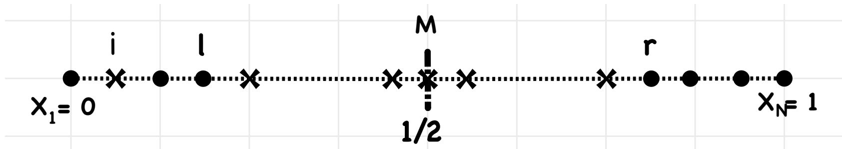

Figure 3: No gaps in equilibrium.

Lemma 2 is a consequence of the exit process. Starting from the full set of attendees N, and as $c > 0$, the first attendee to leave is m. Then, as the process keeps going, whenever the number of attendees is even, it is either l or r who leaves first. In order to see this, observe that if any attendee on the left of l leaves, the effect on the policy is exactly the same as if l leaves (i.e. it will coincide with x_r), while, being further from M, the disutility of doing so is strictly higher than the one of l by the strict convexity of $V(.)$. As the same holds for any attendee on the right of r, the first one to leave will be either l or r (and thus, individual i in the figure cannot have left before l).

By the same reasoning, whenever the number of attendees is odd, m always leaves before all the attendees between himself and the extreme of the policy line (i.e. 0 or 1), while the same holds for the first attendee on the other side on 1/2 (i.e. l or r). Therefore, it is always either m, or r, or l leaving first, and there can be no gaps between any two attendees on each side of 1/2 in equilibrium.

Corollary 1. *If there are attendees on both sides of* 1/2*, the individuals located at* 0 *and* 1 *attend.*

At any stage of the exit process such that there are attendees on both sides of 1/2, the attendee located at 0 (respectively 1) has strictly higher disutility from leaving than l (respectively r). Hence, it follows directly that any such stage, both the individuals at 0 and 1 attend.

Lemma 3. *The equilibrium is balanced. That is, the number of attendees on each side of* $\frac{1}{2}$ *is the same.*

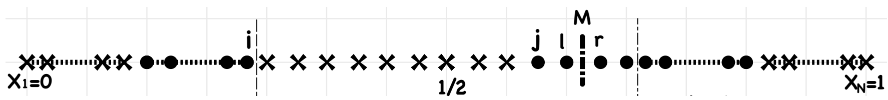

Figure 4: The equilibrium number of attendees can not be unbalanced.

Given that at an equilibrium, the number of attendees is even (Lemma 1), and that there can be no gaps between any two attendees on each side of $\frac{1}{2}$ (Lemma 2), then a situation in which the number of attendees is unbalanced is such that the number of attendees between 0 and 1/2 is strictly smaller (or higher) than the number of attendees between 1/2 and 1. Such a situation, as depicted in Figure 4, cannot be an equilibrium since if l and r are attending, any given abstainer is also willing to attend provided that $V(.)$ is strictly convex.

We are now ready to characterize the (unique) equilibrium of the attendance game given the exit process:

Proposition 1. *For any $c > 0$, let t be the unique solution of $c = V(2t) - V(t)$. If $t \geqslant \frac{1}{2}$, then for all $N \geqslant 2$, $A^* = 0$. If $t < \frac{1}{2}$, then, for all $N \geqslant 2$, there exists a unique equilibrium at which any individual i with $x_i \in \left[0, \frac{1}{2} - t\right) \cup \left(\frac{1}{2} + t, 1\right]$ attends. The equilibrium number of attendees is given by $A^* = 2(k+1)$, where $k \in \mathbb{N}$ is the maximum natural number such that $k < (\frac{1}{2} - t)(N-1)$.*

The characterization, and, more importantly, the uniqueness of the equilibrium rely on the above lemmas and the associated corollaries.

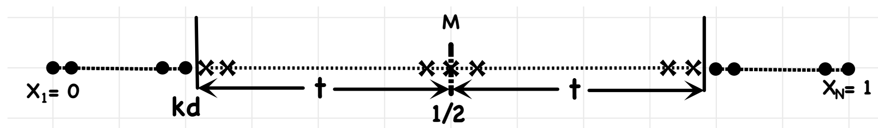

Figure 5: The Unique No-Quorum Equilibrium.

The threshold t represents the distance from $\frac{1}{2}$ such that an individual is indifferent between abstaining and attending. Observe that t does not depend on N. That is, no matter what is the number of individuals, the attendance condition is always the same, as for any given c, what matters for individual decisions is the *absolute* distance from the policy, which does not depend on N. On the contrary, t is increasing in c, as a higher participation cost obviously pushes the attendance threshold towards the extremes, meaning that the exit process continues further[6].

If $t \geqslant \frac{1}{2}$, it means that c is so high that the abstention interval includes the whole policy line, so that no one attends (i.e. $A^* = 0$)[7]. If, on the contrary, $t < \frac{1}{2}$, it means that the policy line will contain both an abstention interval and two attendance regions on each side of $\frac{1}{2}$. Given the exit process, the equilibrium is characterized by an equal number of consecutive attendees on each side of $\frac{1}{2}$, from the extremists (i.e. the attendees located at 0 and 1) to the last individuals for whom it is worth attending given c (i.e. attendees l and r). Consider the individuals located on the left of $\frac{1}{2}$. The exit process continues until reaching the attendee located at kd, who is the first attendee for whom the benefit of leaving is negative (since $kd < \frac{1}{2} - t$). Therefore, the equilibrium number of attendees on the left of $\frac{1}{2}$ is given by $k+1$, and thus $A^* = 2(k+1)$.

Corollary 2. *The equilibrium number of attendees is zero if and only if* $c \geqslant V(1) - V(\frac{1}{2})$.

From the above corollary, there exists a threshold value of c (which corresponds to $t = 1/2$) such that no individual wants to attend the meeting. As it was the case for t, observe that the threshold value of c is constant. In particular, it does not depend on the number of individuals N.

Corollary 3. *The equilibrium policy* x^* *is always* $\frac{1}{2}$.

Given the symmetric structure of the equilibrium, it turns out that the equilibrium policy is always located at the middle of the policy line (i.e. at $\frac{1}{2}$) no matter what is the number of attendees.

Finally, Proposition 2 below gives some comparative statics results regarding the participation cost c and the number of individuals N on the equilibrium number of attendees.

Proposition 2. *The equilibrium number of attendees is decreasing in the attendance cost. Furthermore, both the number of attendees and abstainers is nondecreasing in the number of individuals.*

The comparative statics results in the above proposition are intuitive. For given N, an increase in the attendance cost c implies that the exit process

[6]see Proposition 2 and its proof.

[7]we assume that whenever an individual is indifferent between attending and abstaining, he abstains.

continues further. That is, the most moderate attendees from the original set of attendees might not find it worthwhile any longer to attend, given that the cost of participation has increased. Therefore, the equilibrium number of attendees decreases. Moreover, given that an increase in the number of individuals N does not alter the indifference threshold on the policy line between attending and abstaining, it follows that such an increase directly translates into a higher (or at least equal) equilibrium number of both attendees and abstainers.

3.2 The Quorum Game

Having characterized the unique equilibrium of the No-Quorum Game, we can apply the results to the analysis of the Quorum Game. Remember that the Quorum Game consists of two meetings. In the second meeting there is no quorum requirement, while in the first one, a participation quorum $Q \in (0, N]$ has to be fulfilled in order for a decision to be taken.

Notation 1. *We use subindexes 1 and 2 to refer to the first and second round of the quorum game, while no subindex refers to the one-shot no-quorum game.*

We begin the analysis of the quorum game with the second meeting, if this were to be required.

Lemma 4 (Second Meeting)**.** *For any $c > 0$, let t_2 be the unique solution of $c = \beta[V(2t_2) - V(t_2)]$. If $t_2 \geqslant \frac{1}{2}$, then for all $N \geqslant 2$, $A_2^* = 0$. If $t_2 < \frac{1}{2}$, then, for all $N \geqslant 2$, there exists a unique equilibrium at which any individual i with $x_i \in \left[0, \frac{1}{2} - t_2\right) \cup \left(\frac{1}{2} + t_2, 1\right]$ attends. The equilibrium number of attendees is given by $A_2^* = 2(k_2 + 1)$, where $k_2 \in \mathbb{N}$ is the maximum natural number such that $k_2 < (\frac{1}{2} - t_2)(N - 1)$.*

Lemma 4 has exactly the same flavor as Proposition 1. The equilibrium has a very similar structure, the chosen policy at the second stage is $x_2^* = 1/2$, and the only (but important) difference is that the number of attendees may decrease.

Proposition 3. $A_2^* \leqslant A^*$

The number of attendees in the second meeting, if affected, is lower than that of the no-quorum game. Remember that in the second meeting, each individual discounts the decision taken (i.e. $\beta \in [0, 1]$). More specifically, the effect of one's attendance in terms of final utility is discounted, and hence some moderates that would have incentives to attend the one-shot no-quorum meeting may not participate in the second one. This is the case if their incentives to attend in terms of their (small) influence on the postponed decision do not compensate the cost of attending.

In order to characterize the equilibrium of the quorum game, the following definitions will be helpful. Thereafter we present the main contribution of our analysis in Proposition 4.

Definition 1. *Let the threshold values of the discount factor be defined as follows:*

$$\beta_1 = 1 - \frac{c}{1-V(|kd-\frac{1}{2}|)}, \beta_2 = \frac{c}{V(|1-2kd|)-V(|kd-\frac{1}{2}|)}, \beta_3 = 1 - \frac{c}{1-V(|(\frac{Q}{2}-1)d-\frac{1}{2}|)},$$

$$\beta_4 = \frac{1-c}{1-V(|(\frac{Q-1}{2})d-\frac{1}{2}|)}, \beta_5 = \frac{1-V(|1-(\frac{Q-1}{2})d|)}{1-V(\frac{1}{2})}$$

Proposition 4. *For any $c > 0$, $N \geqslant 2$ and $Q \in (0, N]$ there exists a unique equilibrium that is characterized as follows:*

Table 1: Equilibrium of the Quorum Game

Q	$Q < A^*$	$Q = A^*$		$Q > A^*$			
				Q even		Q odd	
β	$\beta \in [0,1]$	$\beta \in (\beta_1, \beta_2]$	otherwise	$\beta < \beta_3$	otherwise	$\beta < min\{\beta_4, \beta_5\}$	otherwise
A_1^*	A^*	0	Q	Q	0	Q	0
A_2^*	–	$2(k_2+1)$	–	–	$2(k_2+1)$	–	$2(k_2+1)$
x^*	1/2	1/2	1/2	1/2	1/2	$(\frac{Q-1}{2})d$	1/2

According to the above proposition, the effect of the quorum on the equilibrium (i.e. A_1^*, A_2^* and x^*) depends on the level of the quorum itself (i.e. Q), and on how much individuals discount a delayed decision (i.e. β). To be more precise, the only case for which the equilibrium outcome is independent of β (and the decision always taken in the first meeting), is when the quorum is lower than the no-quorum attendance rate (i.e. $Q < A^*$). On the contrary, when $Q \geqslant A^*$, how much individuals discount a delayed decision is crucial.

1. $Q < A^*$: If the quorum is smaller than the no-quorum attendance rate, then it has no effect on the equilibrium outcome. This is a consequence of the fact that the quorum does not alter the exit decision of any of the individuals during the first meeting and hence the policy decision remains unaffected.

2. $Q = A^*$: When the quorum is equal to the number of attendees in the no-quorum game, we know that A^* and hence the quorum is even. Moreover, we know that individual l located at kd is the first attendee who decided not to exit the no-quorum game (see Figure 6)[8].

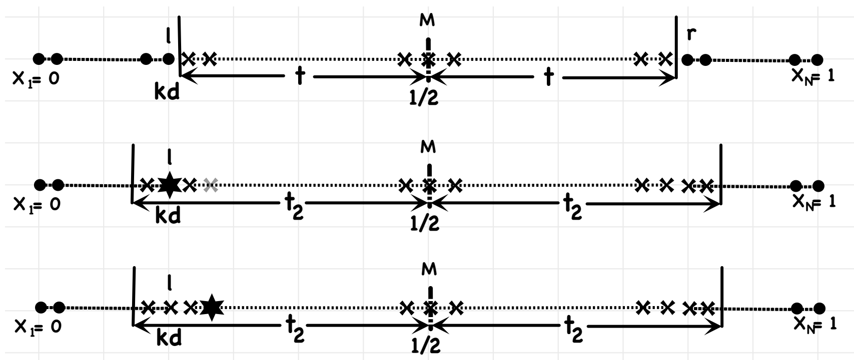

Figure 6: The meeting at the top is the one of the No-Quorum Game. At the middle the case of $Q = A^*$, and at the bottom the case of $Q > A^*$. The individual with a "star" is the pivotal individual (located at kd if $Q = A^*$, at $(\frac{Q}{2} - 1)d$ if $Q > A^*$ and even, and at $(\frac{Q-1}{2})d$ if $Q > A^*$ and odd).

Starting analyzing the exit process of the quorum game with the full set of attendees, the decision of all attendees between l and r is the same as in the no-quorum game, meaning that they exit. Notice though that because of the presence of the quorum, attendee l is now pivotal on whether the quorum is going to be met or not. Hence, his cost-benefit exit calculation is altered compared to the no-quorum case.

By exiting, he actually postpones the decision to the next meeting, while he knows that the policy will remain the same (that is, $x_2^* = 1/2$). The cost of such a decision stems from the fact that the decision is delayed, while its benefit lies in the fact that under certain conditions, he will not have to pay the cost of attending the second meeting.

When will the exit decision be worthwhile for the pivotal individual? Or, said in other words, when is the cost of exiting lower than the benefit? On the one hand, he must not discount the future "too much" (i.e. $\beta > \beta_1$), so that

[8]WLOG we analyze the decision of individual l, while the same holds for individual r.

he does not care to delay the decision (i.e. low cost of exiting). On the other hand, however, he has to discount the future "enough" (i.e. $\beta < \beta_2$), so that he has no incentives to attend the second meeting (i.e. high benefit of exiting). Notice that if the pivotal individual exits (who is also the individual with the highest incentives to attend), then all the remaining attendees exit as well so as to avoid paying the cost of attending a meeting that is not fulfilling the quorum. Therefore, if $\beta \in (\beta_1, \beta_2)$, then $A_1^* = 0, A_2^* = 2(k_2 + 1)$ and $x^* = 1/2$.

If the above restriction on the discount factor does not hold (i.e. $\beta \notin (\beta_1, \beta_2)$), for example because the pivotal individual discounts the future a lot, then the quorum equilibrium is identical to the no-quorum one and the quorum is binding in the first round.

3. $Q > A^*$: If the quorum is larger than the number of attendees in the no-quorum game, its effect on the equilibrium varies depending on whether the quorum is even or odd.

(a) Q Even: The intuition in this case is similar to the one where $Q = A^*$. However, an important difference is that when $Q > A^*$, the pivotal individual is now located at $(\frac{Q}{2} - 1)d$, and hence is more moderate than the pivotal individual at $Q = A^*$[9].

During the exit process of the quorum game, the pivotal attendee is the first individual whose exit decision may be altered because of the presence of the quorum. By exiting, he actually postpones the decision to the next meeting, being sure that the policy will remain the same (that is $x_2^* = 1/2$). Moreover, and contrary to the $Q = A^*$ case, he definitely has no incentives to attend the second meeting, given that he is an abstainer in the no-quorum game.

The benefit of attending stems from the fact that the decision is not delayed, while it implies an extra attendance cost. Hence, the pivotal individual is willing to pay the cost, attend and make the quorum binding in the first meeting if he discounts a lot a delayed decision (i.e. $\beta < \beta_3$). On the contrary, if $\beta \geqslant \beta_3$, the pivotal individual has no incentives to pay the cost, the quorum is not met and as before no one attends the first meeting (i.e. $A_1^* = 0, A_2^* = 2(k_2 + 1)$ and $x^* = 1/2$).

(b) Q Odd: The intuition in this case is similar but with a very important difference with respect to the case of an even quorum. The pivotal individual is located at $(\frac{Q-1}{2})d$, who is again more moderate than the pivotal individual at

[9]WLOG we analyze the individual located at $(\frac{Q}{2} - 1)d$. Because of symmetry, the same analysis holds for his mirror with respect to 1/2.

$Q = A^*$[10].

Through the exit process of the quorum game, the pivotal attendee is the first individual whose exit decision may be altered because of the presence of the quorum. By exiting, he actually postpones the decision to the next meeting, being sure that the policy will remain the same (that is $x_2^* = 1/2$). As before, he definitely has no incentives to attend the second meeting, given that he is an abstainer in the no-quorum game.

The benefit of attending has now an additional component compared to the case of an even quorum. In addition to not being delayed, the elected policy moves to his ideal point (since the pivotal attendee is the new median). As before, the cost is what the individual has to pay in order to attend the meeting. Hence, the pivotal individual is willing to pay the cost, attend and make the quorum binding in the first meeting if he discounts a lot a delayed decision (i.e. $\beta < \beta_4$). On the contrary, if $\beta > \beta_4$, the pivotal individual has no incentives to pay the cost, the quorum is not met and as before no one attends the first meeting (i.e. $A_1^* = 0, A_2^* = 2(k_2 + 1)$ and $x^* = 1/2$).

Notice though that the attendance of the pivotal member implies a policy distortion. Indeed, the policy now moves to $x^* = \frac{Q-1}{2}d \leqslant \frac{1}{2}$, and given that the exit process is altered, it deserves a closer look. In order to guarantee that the exit process terminates at an equilibrium, we have to make sure that no further attendees have incentives to exit, while no further abstainers want to attend.

The process actually terminates if the attendee with the highest benefit of exiting is not willing to do so. Notice that the policy is biased towards the left (i.e. $x^* = \frac{Q-1}{2}d \leqslant \frac{1}{2}$). Moreover, any attendee that exits guarantees a delayed policy outcome of 1/2. The individual with the highest incentives to do so is then the extremist located the furthest from $x^* = \frac{Q-1}{2}d$, that is, the individual located at 1. In order for this extremist not to exit, it has to be the case that he is harmed a lot by postponing the decision to the second meeting (i.e. $\beta < \beta_5$).

The individual with the highest benefit of entering is the mirror (with respect to 1/2) of the individual located at x^*. Observe that this individual is never attending a no-quorum meeting, precisely because it is not worthwhile for him to pay the cost so as to push the policy from x^* to 1/2. Therefore, he has no incentives to attend here either. Notice that in fact, this individual free rides on the attendance of his 'mirror" who participates so as to avoid postponing the decision.

[10] For the same reasoning as Footnote 7, WLOG we analyze the individual located at $(\frac{Q-1}{2})d$.

To sum up, our results regarding the Quorum Game can be summarized as following:

- The quorum is met in the first round when:
  1. The quorum is lower than the no-quorum attendance rate.
  2. The quorum is higher than the no-quorum attendance rate and members discount a lot a delayed decision.
- If the second meeting takes place, the number of attendees is smaller or equal than in the no-quorum case.
- When the quorum is larger or equal than the no-quorum attendance rate, then either no member attends the first meeting or the quorum is binding. Notice that there exists no equilibrium in which a subset of members decide to attend the first meeting and the quorum is not met[11].
- The quorum has a different effect when it is even or odd. An odd quorum may create policy distortions.

Finally, Corollary 4 below describes the effect of the quorum regarding total welfare:

Corollary 4. *Total welfare is higher or equal under no quorum than under a quorum requirement.*

Whenever the quorum has no effect on the equilibrium, that is, when the decision is taken in the first meeting and both the number of attendees and the equilibrium policy remain the same, total welfare is clearly not affected.

Whenever the quorum is binding, but the equilibrium policy unchanged, total welfare is strictly lower than under the no-quorum equilibrium. Indeed, as the presence of the quorum "forces" some moderates to attend (and thus bear the corresponding cost) in order for the meeting not to be postponed, total welfare is strictly lower following the introduction of the quorum. If, in addition to that, the equilibrium policy gets distorted (i.e. $x^* \neq \frac{1}{2}$), the effect of the quorum is even worse from a welfare perspective.

Finally, when the decision is postponed to the second meeting, and even though less individuals attend, total welfare is lowered as a result of the quorum being present. That is, the fact that less individuals pay the cost of attending does not compensate the aggregate loss of welfare in terms of policy valuation.

[11] A possible way to obtain such a result would be to introduce an exogenous probability that each individual may be prevented from attending the meeting.

4 Conclusion

We believe that this piece of research is a first step towards the understanding of a widely used but not formally analyzed voting rule. Clearly, our results have important policy implications, and we think that policy makers deciding on the use of a participation quorum in small decision making meetings should take our results into consideration.

In particular, in meetings where the preferences of individuals are symmetrically distributed, introducing a participation quorum is a bad idea since it creates welfare losses. As explained, these losses may be the result of the decision being delayed, or the consequence of the fact that individuals who are better off abstaining are now "forced" to attend in order to fulfill the quorum.

One standard argument in favor of the use of a participation quorum is that it allows to protect individuals from a policy decision being taken by a small minority of the members. However, it turns out that the policy is in general not affected as it coincides with the one that would have been taken had there been no quorum. This is so because in a world where the the costly committee meetings are ruled by the "citizen-candidate" spirit of Osborne et al. (2000), the individuals who are really afraid of a unwanted decision have incentives to attend anyway. In order to protect those individuals, there is then no need for a participation quorum since their participation is in line with their own interest, and hence this argument does not apply.

A final argument in favor of a participation quorum is that it guarantees the legitimacy of the decision taken. Think for instance of a meeting of the shareholders of an important enterprise deciding on the enterprise's strategy. Suppose that all shareholders have a very high opportunity cost of attending the meeting, so that the equilibrium number of attendees under no quorum is very small. In such cases, one can think of a participation quorum as a tool to protect the legitimacy of the decision, since a decision taken by very few shareholders might have a rather negative impact on the enterprise's image. According to our results, if the shareholders are in a hurry, introducing a quorum then has the effect of increasing the number of shareholders who take part in the decision. In such situations, our analysis has policy recommendations as well. We showed that the use of a participation quorum always lowers welfare. In any case, however, if there is a quorum to be introduced, it should require an even number of individuals so as to avoid policy distortions (and thus minimize welfare losses).

As the present paper constitutes a first attempt to analyze the use of a participation quorum in meetings, many paths for future research remain open. Among those, a very important feature which we wish to capture is the case in which a positive number of members show up at the first meeting, but the quorum is not fulfilled. A way to allow for such a possibility could be the introduction of an exogenous probability that each member is prevented from attending the meeting. Intuitively, this would affect the number of attendees in each meeting and, more importantly, it would give rise to situations in which some individuals end up paying the cost of attending (even though the meeting has to be postponed), because they believe the quorum will be met.

So far our analysis has been focused on the case of uniformly distributed individuals' favorite positions. Although our results would still be valid for other symmetric distributions, we believe it is important to extend our analysis to asymmetric setups. Indeed, one of the main arguments favoring the use of a participation quorum is the fact that one wants to prevent the decision from being taken by a (non-representative) minority. However, in symmetric situations such as the one we have been analyzing here, there is no way a minority could possibly exploit a majority.

5 Appendix

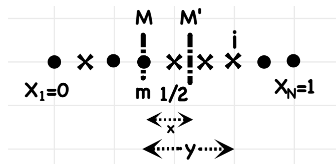

Figure 7: Proof of Lemma 1.

Proof of Lemma 1. Suppose the number of attendees is odd and individual m, by leaving, does not change the location of the policy M. Then he's willing to leave if and only if $V(0) > V(0) - c$ or, equivalently, $c > 0$. Therefore, m is always leaving.

Suppose the number of attendees is odd and individual m, by leaving, changes the location of the policy. Suppose, furthermore, that c is such that m is not willing to leave, and let the corresponding policy given this set of attendees be M. If this is the case, there is always a non-attendee i who is willing to attend, and whose location is such that by doing so, the policy moves to the same location as it does if m leaves. Let the distance between individual m and the new policy M' in case he leaves be x. Furthermore, let the distance between the non-attendee i mentioned above and the policy M be y, where $y = 2x$. If m is not willing to leave, it means that $-V(x) < -V(0) - c$, or, equivalently, $c < V(x)$. Now, notice that i is willing to attend if and only if $-V(x) - c > -V(y)$, or, equivalently, $c < V(y) - V(x)$. As $y = 2x$ and $V(.)$ is strictly convex, it follows that $V(x) < -V(x) + V(y)$, or, equivalently, $V(2x) > 2V(x)$. Therefore, i wants to attend.

□

Proof of Lemma 2. Given the exit process, we know that at any stage, the first attendee to leave (if any) is the one with the lowest potential disutility from leaving. Starting from the full set of attendees N, where N is odd, we know that as $c > 0$, the first attendee to leave is m. Let l and r be the first attendees on the left and on the right of M respectively[12]. The potential disutility from leaving is the same for both of them, and is given by $V(2d) - V(d)$. Consider now the next attendees on the left of l and on the right of r respectively. Their

[12]Observe that assuming that N is odd is without loss of generality, since assuming instead that N is even simply means that we start from here on.

disutility from leaving is identical, and is given by $V(3d) - V(2d)$, which is strictly higher than $V(2d) - V(d)$ given the strict convexity of $V(.)$. Obviously, this will also be true for any pair of attendees who are located even further from M. Therefore, the first one to leave is either l or r. From there on, if c is high enough so that the exit process keeps going,

1. At any stage for which the number of attendees is odd (case A_3 in Example 1), either m, or one (and only one) of the attendees l or r is the first one to leave. Given the exit process, a situation in which the number of attendees is odd is such that m is next to r (l), while, as some attendees have already left, there are gaps between m and l (r). Suppose, WLOG, that m is next to r (as in case A_3) and let kd be the distance between m and l (so that $k = 3$ in the example). We aim at showing that the first one to leave is either m or l (that is, either the median attendee or the furthest one next to him). If m leaves, his disutility from doing so is given by $V(\frac{(k-1)d}{2})$. If r leaves, he will suffer a disutility of $V(\frac{kd}{2}+d)-V(d)$. As $V(\frac{(k-1)d}{2}) = V(\frac{(k+1)d}{2} - d) < V(\frac{(k+1)d}{2}) - V(d)$, it follows that m always leaves before r. Obviously, this is also true for any attendee on the right of r. Indeed, if any such attendee leaves, the effect on the policy will be exactly the same as if r leaves, while, being strictly further, the disutility from leaving in terms of distance will be strictly higher than the one of r by the strict convexity of $V(.)$.
   Now, if l leaves, his disutility from doing so is given by $V((k+\frac{1}{2})d)-V(kd)$. We do not know whether it is m or l who has the lowest disutility from leaving. However, by the same reasoning as the one just described above, we know that any attendee on the left of l will suffer a strictly higher disutility from leaving than l, so that l always leaves first. Therefore, either m or l leaves first.

2. At any stage for which the number of attendees is even (cases A_4 and A_4' in Example 1), so that the policy is $M = \frac{(x_l+x_r)}{2}$, either l or r is the first one to leave. Let kd be the distance between i and M, $i = l, r$ (so that $k = \frac{1}{2}$ in situation A_4 and $k = 2$ in situation A_4'). The disutility from leaving for individual $i = l, r$ is given by $V(2kd) - V(kd)$. Consider now the next attendees on the left of l and on the right of r respectively. Their disutility from leaving is strictly higher than $V(2kd) - V(kd)$ given the strict convexity of $V(.)$. Furthermore, this is also true for any pair of attendees who are located even further from M.

□

Proof of Corollary 1. Consider any stage of the exit process such that there are still attendees on both sides of 1/2 (i.e. we rule out situations such as A_4 in Example 1). From Lemma 3, we know that, no matter whether the number of attendees is even or odd, the first one to leave is always either m, or l, or r. Therefore, it follows directly that at any such stage (i.e. such that there are attendees on both sides of 1/2), the attendee located at 0 (respectively 1) has strictly higher disutility from leaving than l (respectively r). Hence, at any such stage, both the individuals at 0 and 1 attend. □

Figure 8: Proof of Lemma 3.

Proof of Lemma 3. Let $L \geqslant 0$ be the number of attendees on the left of 1/2. Similarly, let R be the number of attendees on the right of 1/2, and assume, WLOG, that $R \geqslant L$, and so $R = L + K$, where $K > 0$ and is even by Lemma 1. From Lemma 2, it follows that the L and R attendees respectively on the left and on the right of 1/2 are consecutive (i.e. there are no gaps between them). Let M be the chosen policy given this set of attendees, which is located at the median of the K attendees. Observe that if the attendees l and r are not willing to leave, it means that c is small enough so that it is worth attending to prevent M from moving by $\frac{1}{2}d$, that is, it means that $c < V(d) - V(\frac{1}{2}d)$. Let i and j be the first attendees on the left and on the right of 1/2 respectively, and consider any non-attendee between i and j. If any such non-attendee were to come back, the policy would move to x_l, and he would do so if and only if $c < V((k+\frac{1}{2})d) - V(kd)$, where $kd > d$ is the distance between any non-attendee and l. It turns out that if r attends, any abstainer between i and j wants to attend as well, that is, $V((k+\frac{1}{2})d) - V(kd) > V(d) - V(\frac{1}{2}d)$ for all $k > 1$ by the strict convexity of $V(.)$. Therefore, the equilibrium is such that $K = 0$ (i.e. balanced). □

Proof of Proposition 1. By lemmas 1 to 3 and the associated corollaries, we know that the equilibrium reached by means of the exit process is such that there is an equal number of attendees on each side of $\frac{1}{2}$ without gaps between any two of them, and such that the individuals located at the extremes of the policy line attend. Let t be the unique solution of $c = V(2t) - V(t)$ and consider a situation such as A_1 in Figure 9. By definition of t (and by the lemmas and associated corollaries), any such situation, that is, any situation with an equal number of consecutive attendees starting from the extremes on both sides of $\frac{1}{2}$, and such that only individuals with $x_i \in \left[0, \frac{1}{2} - t\right) \cup \left(\frac{1}{2} + t, 1\right]$ attend, is an equilibrium. Indeed, notice that the individuals with the highest potential benefit from leaving are l and r. However, as by definition of t, $b_l(A) = b_r(A) < 0$, they both attend. Furthermore, as by definition of t, $b_i(A) > 0$ for any i with $x_i \in \left[\frac{1}{2} - t, \frac{1}{2} + t\right]$, none of such individuals is willing to attend. Therefore, the situation depicted in the figure is an equilibrium.

In order to derive the equilibrium number of attendees A^*, observe that the exit process stops at the individual located at kd (i.e. attendee l), who is the first attendee (on the left side of $\frac{1}{2}$) for whom $b_i(A) < 0$. Therefore, the equilibrium number of attendees on the left of $\frac{1}{2}$ is given by $k + 1$, and thus $A^* = 2(k + 1)$.

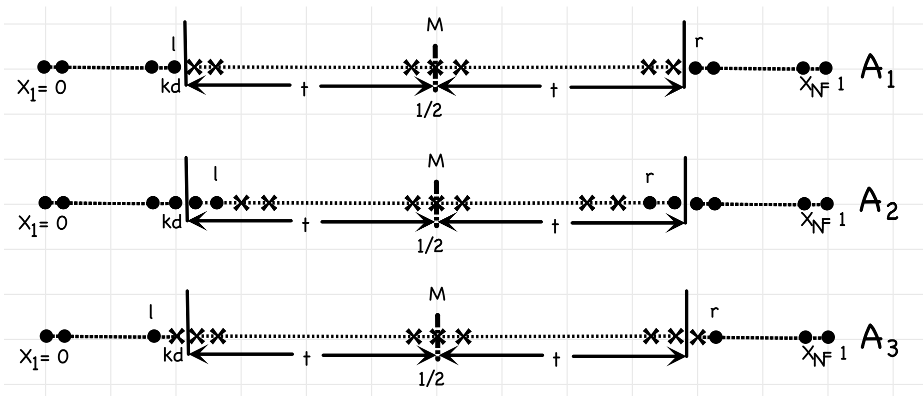

Figure 9: Proof of Proposition 1: Uniqueness

It now remains to be shown that the equilibrium is unique. Consider a situation such as A_2 in the figure. That is, a situation that satisfies the equilibrium requirements as described in the lemmas, but such that there is one (or more) pair(s) of consecutive attendees in the interval $\left[\frac{1}{2} - t, \frac{1}{2} + t\right]$ (or, equivalently, such that $x_l, x_r \in \left[\frac{1}{2} - t, \frac{1}{2} + t\right]$). As, by definition of t, $b_i(A) \geqslant 0$ for any i with

$x_i \in \left[\frac{1}{2} - t, \frac{1}{2} + t\right]$, it follows that, for any t, any such situation cannot be an equilibrium.

Finally, consider a situation such as A_3 in the figure. That is, a situation that satisfies the equilibrium requirements as described in the lemmas, but such that there is one (or more) pair(s) of consecutive abstainers in the interval $\left[0, \frac{1}{2} - t) \cup (\frac{1}{2} + t, 1\right]$. As, by definition of t, $b_i(A) < 0$ for any i with $x_i \in \left[0, \frac{1}{2} - t) \cup (\frac{1}{2} + t, 1\right]$, it follows that, for any t, any such situation cannot be an equilibrium.

□

Proof of Corollary 2. According to the exit process, if everybody leaves, the last attendee to exit is either the one located at 0 or the one located at 1. This individual is indifferent to leave if and only if $c = V(1) - V(\frac{1}{2})$, or, equivalently, $t = \frac{1}{2}$. Obviously, if $c > V(1) - V(\frac{1}{2})$, or, equivalently, $t > \frac{1}{2}$, this last attendee wants to exit, which means that no one attends.

□

Proof of Proposition 2. The equilibrium value of t is implicitly defined by $\phi(t) = c - V(2t) + V(t) = 0$. We have that

$$\frac{\partial \phi}{\partial t} = -2V'(2t) + V'(t) < 0$$

$$\frac{\partial \phi}{\partial c} = 1 > 0$$

and thus, by the implicit function theorem: $\frac{\partial t}{\partial c} = -\frac{\frac{\partial \phi}{\partial c}}{\frac{\partial \phi}{\partial t}} > 0$. Therefore, as the threshold for attendance is moving towards the extremes of the policy line as c increases, it follows directly that the equilibrium number of attendees decreases.

Given that t does not depend on the number of individuals N, it is also direct that the *absolute* number of both attendees and abstainers is increasing in N. Indeed, for a given $c > 0$ and given the (fixed) distance t, increasing the (uniformly distributed) number of individuals obviously means that there will be more (or at least an equal number of) individuals both between 0 and t (respectively t and 1) and in the abstention interval. □

Proof of Proposition 3. As t is the unique solution of $c = V(2t) - V(t)$ and t_2

is the unique solution of $c = \beta\left[V(2t_2) - V(t_2)\right]$, we have that

$$\beta = \frac{V(2t) - V(t)}{V(2t_2) - V(t_2)}$$

As $\beta \leqslant 1$, it follows that $t_2 \geqslant t$, and thus $k_2 \leqslant k$ and $A_2^* \leqslant A^*$. $\square$

Proof of Proposition 4. 1. Suppose $Q < A^*$ and consider attendees l and r of the no-quorum game for given N and c. Thus, $b_l(A^*) = b_r(A^*) < 0$, the set E is empty and there are A^* attendees in equilibrium. Let now introduce a quorum $Q < A^*$. All moderates located between l and r of the no-quorum game exit, as before. Then, $b_l(A_1^*) = b_r(A_1^*) < 0$ and the set E is empty, so that the exit process stops at l and r. Therefore, $A_1^* = A^*$ and $x_1^* = x^* = \frac{1}{2}$. Said in words, the attendance decision of any individual is the same as in the no-quorum case (in particular, l is not pivotal regarding the quorum requirement), and hence the equilibrium in unaffected.

2. Suppose $Q = A^*$. All moderates located between l and r of the no-quorum game exit, as before. Then, WLOG, given that l is now pivotal regarding Q, he exits if and only if

$$1 - V(|kd - \frac{1}{2}|) - c < \beta\left[1 - V(|kd - \frac{1}{2}|)\right] - \alpha_l c$$

where $\alpha_l = 1$ if and only if

$$\beta > \frac{c}{V(|1 - 2kd|) - V(|kd - \frac{1}{2}|)} = \beta_2$$

and $\alpha_l = 0$ otherwise. Suppose that $\beta < \beta_2$, so that l would not attend in the second round (otherwise, the second round is never reached as l has no incentive to exit in the first round). Then, he exits in the first round if and only if

$$\beta > 1 - \frac{c}{1 - V(|kd - \frac{1}{2}|)} = \beta_1$$

Therefore, l exits in the first round if and only if $\beta_1 < \beta < \beta_2$. Suppose this is the case. Notice that l is the abstainer with the highest potential benefit from attending so as to fulfill Q, and thus no other abstainer has an incentive to attend either. Given that l exits, all other attendees exit as well since the quorum is not met and $c > 0$. Hence, $A_1^* = 0$ and we go to stage 2. From there on, the characterization of the equilibrium and its uniqueness follow the same reasoning as in Proposition 1 (with t_2 being the unique solution of $c = \beta\,[V(2t_2) - V(t_2)]$ as the new attendance threshold and $x_2^* = \frac{1}{2}$).

If $\beta < \beta_1$, l has no incentive to exit in the first round. The exit process stops here, and as no individual has an incentive to deviate, we are at an equilibrium, and it follows that $A_1^* = A^*$ and $x_1^* = x^* = \frac{1}{2}$. Similarly, if $\beta > \beta_2$, l would attend in the second round, so that he has no incentive to exit in the first round, and the same conclusion applies.

3. Suppose $Q > A^*$ is **even**. Then, during the exit process, WLOG, the attendee i located at $(\frac{Q}{2} - 1)d > kd$ is pivotal regarding Q, so that he exits if and only if

$$\beta > 1 - \frac{c}{1 - V(|(\frac{Q}{2} - 1)d - \frac{1}{2}|)} = \beta_3$$

Suppose this is the case, so that i exits. Notice that i is the abstainer with the highest potential benefit from attending so as to fulfill Q, and thus no other abstainer has an incentive to attend either. Given that i exits, all other attendees exit as well since the quorum is not met and $c > 0$. Hence, $A_1^* = 0$ and we go to stage 2. From there on, the characterization of the equilibrium and its uniqueness follow the same reasoning as in Proposition 1.

Suppose now that $\beta < \beta_3$, so that i attends. Given that i is the attendee with the highest potential benefit from exiting, and $b_i(A) < 0$, the exit process stops here, and since no individual has an incentive to deviate, we are at an equilibrium. Thus, the equilibrium number of attendees is given by $A_1^* = Q$ and the equilibrium policy is $x_1^* = \frac{1}{2}$.

4. Suppose $Q > A^*$ is **odd**. Then, during the exit process, WLOG, the attendee i located at $(\frac{Q-1}{2})d > kd$ is pivotal regarding Q, so that he exits if and only if

$$\beta > \frac{1 - c}{1 - V(|(\frac{Q-1}{2})d - \frac{1}{2}|)} = \beta_4$$

Suppose this is the case, so that i exits. Notice that i is the abstainer with the highest potential benefit from attending so as to fulfill Q, and thus no other abstainer has an incentive to attend either. Given that i exits, all other attendees exit as well since the quorum is not met and $c > 0$. Hence, $A_1^* = 0$ and we go to stage 2. From there on, the characterization of the equilibrium and its uniqueness follow the same reasoning as in Proposition 1.

Suppose now that $\beta < \beta_4$, so that i attends. Notice that at this stage, the attendee located at 1 has the highest potential from exiting. Indeed, he's the furthest from the policy $M = (\frac{Q-1}{2})d$, would it be implemented, and he's pivotal with respect to Q, so that it could be beneficial for him to exit in order for the decision to be postponed and the policy to be closer to his ideal point. Formally, 1 exits if and only if

$$\beta > \frac{1 - V(|1 - (\frac{Q-1}{2})d|)}{1 - V(\frac{1}{2})} = \beta_5$$

Suppose that $\beta < min\{\beta_4, \beta_5\}$, so that both i and 1 attend. Given that 1 is the attendee with the highest potential benefit from exiting, and $b_1(A) < 0$, the exit process stops here, and since no individual has an incentive to deviate, we are at an equilibrium. Thus, the equilibrium number of attendees is given by $A_1^* = Q$ and the equilibrium policy is $x_1^* = (\frac{Q-1}{2})d$.

□

Proof of Corollary 4. Total welfare under the No-Quorum equilibrium is given by

$$W = N - \sum_{i=1}^{N} V(|x_i - \frac{1}{2}|) - A^* c$$

Then, total welfare in the first meeting under the Quorum equilibrium is given by

$$W_1 = N - \textstyle\sum_{i=1}^{N} V(|x_i - \frac{(Q-1)}{2}d|) - Qc \text{ if } Q > A^* \text{ odd and } \beta < min\{\beta_4, \beta_5\}$$

$$W_1 = N - \textstyle\sum_{i=1}^{N} V(|x_i - \frac{1}{2}|) - Qc \text{ otherwise}$$

Finally, total welfare in the second meeting under the Quorum equilibrium is given by

$$W_2 = \beta \left[N - \sum_{i=1}^{N} V(|x_i - \frac{1}{2}|) \right] - A_2^* c$$

1. If $Q < A^*$, we know from Proposition 3 that $A_1^* = A^*$ and $x^* = x_1^* = \frac{1}{2}$, and thus $W = W_1$.

   Then, if $Q = A^*$ and $\beta \notin (\beta_1, \beta_2)$, we know from Proposition 3 that $A_1^* = Q = A^*$ and $x^* = x_1^* = \frac{1}{2}$, and thus $W = W_1$.

2. If $Q = A^*$ and $\beta \in (\beta_1, \beta_2)$, $Q > A^*$ even and $\beta > \beta_3$, or $Q > A^*$ odd and $\beta > min\{\beta_4, \beta_5\}$, we know from Proposition 3 that the policy x is decided at the second meeting. Notice that... TO BE COMPLETED

3. If $Q > A^*$ even and $\beta < \beta_3$, we know from Proposition 3 that Q is binding in the first meeting and $x_1^* = x^* = \frac{1}{2}$. As $Q > A^*$, it follows directly that $W > W_1$.

   Then, if $Q > A^*$ odd and $\beta < min\{\beta_4, \beta_5\}$, we know from Proposition 3 that Q is binding in the first meeting and $x_1^* = \frac{(Q-1)}{2}d$. As $Q > A^*$ and $\sum_{i=1}^{N} V(|x_i - x|)$ is minimized at $x = \frac{1}{2}$, it follows directly that $W > W_1$.

$\square$

References

Aguiar-Conraria, L. and P. Magalhães (2010a): "How quorum rules distort referendum outcomes: evidence from a pivotal voter model," *European Journal of Political Economy.*

——— (2010b): "Referendum design, quorum rules and turnout," *Public Choice*, 144, 63–81.

Aragones, E. and S. Sánchez-Pagés (2009): "A theory of participatory democracy based on the real case of Porto Alegre," *European Economic Review*, 53, 56–72.

Bulkley, G., G. Myles, and B. Pearson (2001): "On the membership of decision-making committees," *Public Choice*, 106, 1–22.

Dhillon, A. and B. Lockwood (2002): "Multiple equilibria in the citizen-candidate model of representative democracy: a comment," *Journal of Public Economic Theory*, 4, 171–184.

Dornseifer, F. (2005): *Corporate business forms in Europe: a compendium of public and private limited companies in Europe*, Sellier European Law Pub.

Herrera, H. and A. Mattozzi (2009): "Quorum and turnout in referenda," *Journal of the European Economic Association.*

Houy, N. (2009): "A characterization of majority voting rules with quorums," *Theory and decision*, 67, 295–301.

Maniquet, F. and M. Morelli (2010): "Approval Quorums Dominate Participation Quorums," *Working Paper, Series EUI ECO, 2010/13.*

Osborne, M., J. Rosenthal, and M. Turner (2000): "Meetings with costly participation," *American Economic Review*, 90, 927–943.